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The impact of distributional effects on secular stagnation

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Declaration of Authorship

I, Amir Hossein FOROOZANDE NEJAD, declare that this thesis titled, "The impact of distributional effects on secular stagnation" and the work presented in it are my own. I confirm that:

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- Where I have consulted the published work of others, this is always clearly attributed.
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UNIVERSITY OF SASKATCHEWAN

Abstract

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Master of Arts

The impact of distributional effects on secular stagnation

by Amir Hossein FOROOZANDE NEJAD

The experience of Japan in the 1990s and now of most industrial economies portray an environment where inflation is below target, and the growth rate is subpar. First, this paper, following the seminal work of Eggertsson and Mehrotra (2014), constructs an economy with a permanent negative natural interest rate, which crystallizes the notion of secular stagnation. I focus on demand-side secular forces that exacerbate the chronic excess saving and inadequate economic growth. I further explore the mechanism in which distributional effects and the risk premium between safe and other assets can put downward pressure on the natural interest rate by using a heterogenous overlapping generation model.

Acknowledgements

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Contents

To those who believed in me when I did not.

Chapter 1

Introduction to Secular Stagnation

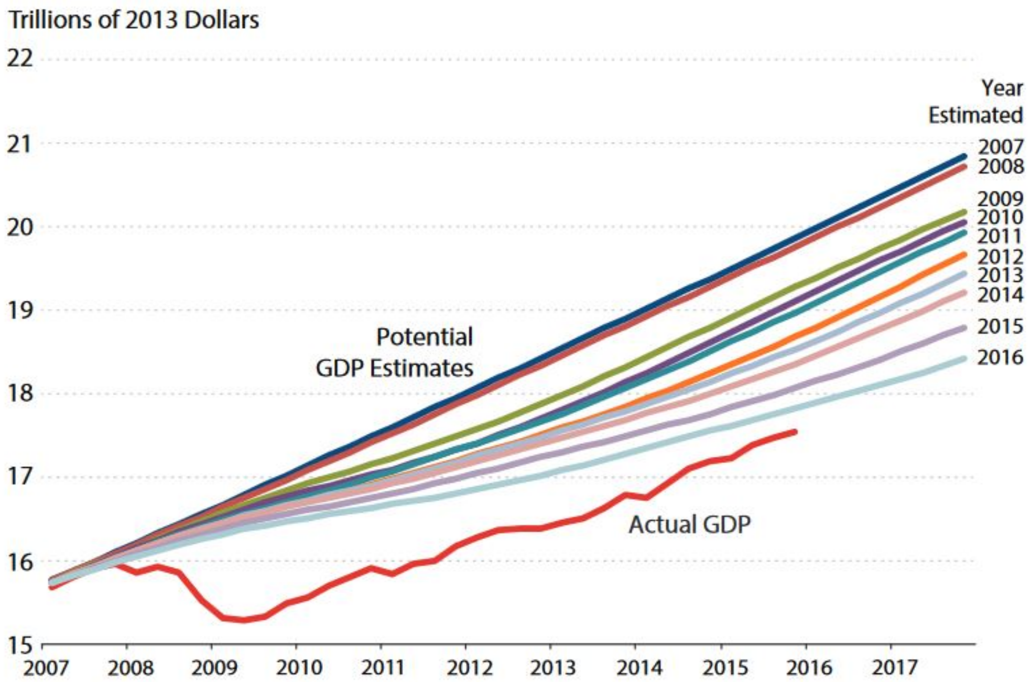


FIGURE 1.1: Downward Revision in Potential GDP, United States (CBO Budget and Economic Outlooks 2007-15; Bureau of Economic Analysis - Summers, L. H. (2014))

1.1 Introduction

One defining characteristic of the US economy is sluggish inflation which has been partially irresponsive to unemployment or monetary policy in the past two decades. After the 2008 financial crisis, unemployment rose sharply to levels unseen in the 50 years, while inflation jumped by a margin. In fact, since the mid-1990s, the core inflation has been hovering around 2% in the United States. This troubling phenomenon is coupled with subpar economic growth and recovery. Similarly, Europe experienced a prolonged discrepancy between estimates of potential and GDP growth, indicating a permanently depressed output slump.

During an IMF speech, Summers (2013b) highlighted the idea of secular stagnation as one potential explanation for the current economic climate. The concept of secular stagnation was first coined by Hansen (1939) in his presidential address to the American Economics Association. He proposed United States economy is experiencing a permanent shortfall in aggregate demand after a decade-long zero lower bound episode during the Great Depression. The issue of depressed aggregate demand and sluggish growth of the 1930s subsided after the US government significantly boosted its spending during world war two, followed by a baby boom. Summers (2013a) observed similar patterns in the developed economies when both the US and Europe were facing unprecedented sluggish recoveries with nominal interest rates that were at the zero lower bound, and potential GDP estimates revealing a permanently depressed output-slump comparable to that of Japan's lost decade[1].

Summers framed the secular stagnation hypothesis around the idea of the permanently negative natural rate of interest where full-employment is only consistent with rates below zero (Summers (2013a) and Summers (2014)). In his later works, Summers (2018) points out that a common perception of secular stagnation is that developed economies are doomed to remain stagnant at high levels of unemployment, while the hypothesis aims to explain how structural characteristics of an economy, such as population growth rate, income inequality, and financial frictions can elicit enough output gap to result in a permanent negative natural rate of interest. It is critical to not reduce this phenomenon to issues around the zero lower bound; nevertheless, it can manifest itself in the form of binding lower bound. A plethora of works around this matter shows the declining trend of natural rate in the developed countries over the past three decades and with being predominantly binding after the financial crash (see Krugman (2014), Pichelmann et al. (2015), Laubach and Williams (2016), and Kiley and Roberts (2017)).

The natural rate trends of Europe and the United States after the crisis of 2008 bear a striking resemblance to Japan in the 1990s. Japan was also facing a remarkably low population growth rate when the Japanese asset-bubble

[1]The reports of pre-covid years indicate that Europe's recovery is underwhelming, and the European Central bank predicts nominal interest rates on the main refinancing operations remain negative (Hartmann and Smets (2018)). Similarly, the United States economy inflation and unemployment were hovering around 2% and 4% respectively in pre-covid years. The projected 2% by Labor Statistics (2018) growth rate in 2018 was after the Federal Reserve (2014) extensive policies to boost the US economy.

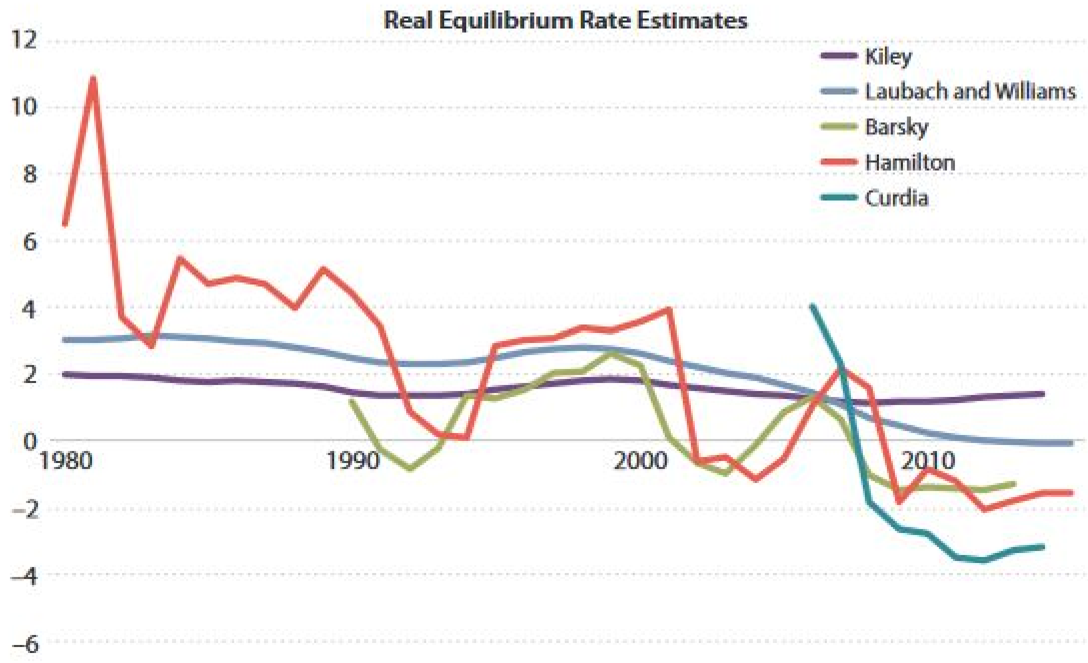


FIGURE 1.2: SOURCE: Michael Kiley "What Can the Data Tell Us About The Equilibrium Real Interest Rate," Laubach & Williams "Measuring The Natural Rate Of Interest Redux," Hamilton et al. "The Equilibrium Real Funds Rate," Vasco Curdia "Why So Slow? A Gradual Return For Interest Rates," Barsky et al. "The Natural Rate & Its Usefulness for Monetary Policy Making" - Summers, L. H. (2014)..

burst[2] plunged the inflated aggregate demand (see Illing, Ono, and Schlegl (2018) and Sudo and Takizuka (2020)). Further, Okazaki and Sudo (2018) demonstrate how tighter collateral constraints in the lost decades of Japan drove down the natural rate. I demonstrate the relation between tighter collateral constraints and the natural rate of interest in the second chapter of this project.

This work is an extension to Eggertsson and Mehrotra (2014), which formalizes the hypothesis of secular stagnation. Similarly, I use the overlapping generations models of varying degrees of complexity as the framework for analyzing how demand-side issues can perpetuate secular stagnation. Based on the existing literature on the topic, I choose slow-moving forces linked to structural characteristics that can endogenously drive the natural rate of

[2]A low real rate of return fosters rational asset bubbles as an alternative form to hold savings, and the asset bubble produces windfalls or rents that boost consumption and alleviate the interest rate market till the future expectation is above the current rate (see e.g. Baldwin and Teulings (2014) and Richard (2014)).

interest downwards for developed economies. A part of these experiments results in characteristics associated with the secular stagnation hypothesis, such as subpar growth, inflation below target, and a binding zero lower bound.

The base overlapping generations model utilizes a similar framework to Samuelson (1958), and I extend that to a 60-period OLG model for the quantitative section to incorporate richer transition dynamics. In this setting, the relative aggregate demand and supply determine the real interest rate and not merely the representative agent discount factor. This feature of overlapping generations models allows for the possibility of arbitrarily long periods of negative interest rates. Since the discount factor of a representative saver is no longer the only determinant of the natural rate of interest, the result for optimal policy differs from the standard New Keynesian models. For instance, Krugman, Dominquez, and Rogoff (1998) argue that if an exogenous shock gave rise to the Japanese zero lower bound (ZLB) episode in the 1990s, then the economy will revert itself to the pre-shock state. Conversely, the baseline model predicts that ZLB episodes can be permanent in the absence of other exogenous shocks and interferences.

The other distinction to the standard New Keynesian models of the zero lower bound is the role of fiscal policy. In comparison to the monetary policy, fiscal policy is a more potent tool for the treatment of the issues around secular stagnation as it can eliminate[3] the steady-state with the negative natural rate. Successful fiscal policy in both the baseline and quantitative model increases the natural rate of interest by reducing the excess saving. Households and, by proxy, the aggregate demand is not as forward-looking in the baseline model compared to the quantitative and New Keynesian models. This distinction implies that any governmental policy that relies on forward-looking agents has attenuated effects in the base model[4].

The project includes three chapters divided into fourteen sections as follows. I conduct a literature review and provide reasoning for my choice of model and its implications. I follow the sections in Eggertsson and Mehrotra (2014) to create the groundwork for later extensions in chapter two. Lastly, the third chapter is my original contribution. I study a range of scenarios that

[3] A well-designed fiscal policy shifts the aggregate demand curve to the right, which may eradicate the multiplicity of steady states.

[4] For instance, forward guidance policy is not as effective as in ZLB literature due to the same reason.

can affect the natural rate of interest and conclude it by including a quantitative model. In section 1.2, I review the existing literature on secular stagnation and the liquidity trap, then focus on trends that can result in such phenomena. To lay down the groundwork for further extensions, I put forward an endowment economy in section 2.1 and add Keneysian features to the baseline model in the subsequent section 2.2. Sections 2.3 and 2.4 incorporate nominal rigidities and competitive firms and close the base model by introducing a monetary policy rule. Next, section 2.5 focuses on aggregate supply and demand after the economy reaches steady states. The result in section 2.6 shows, a permanent deflationary steady-state with an output gap is attainable under certain conditions.

The results from section 3.1 become the foundation for the following extensions to the base model. In section 3.2, I review the literature relating inequality to secular stagnation and formalize a model with income inequality, bequest motive, and heterogeneity in marginal propensity to save. Section 3.2 reviews the issue relating to capital and secular stagnation and introduces two new models for capital and inelastic capital demand. Section 3.3 studies the policy implications of fiscal and monetary policy on secular stagnation. Section 3.4 complements the two models on intergenerational and aggregate risk with the existing literature on them. The quantitative model of the United States economy in 3.5 shows that the relationship between income inequality, and the relative price of capital and the real interest rate in the economy. Lastly, section 3.6 concludes the paper, highlighting the key insights and conclusions.

1.2 Literature Review

After the influential work of Eggertsson and Mehrotra (2014) around formalizing the secular stagnation hypothesis in pursuit of Summers (2013b), there has been a surge of studies around the topic. The hypothesis focuses on the downward trend of the real interest rate in the advanced economies, reflecting an excess of desired saving over investment, resulting in a persistent output gap and slow economic growth rate (Krugman (2014) and Eichengreen (2015)). The possibility of a drop in the natural rate of interest causing a temporary zero lower bound episode is already studied in the liquidity trap literature. However, Eggertsson and Mehrotra (2014) suspect that secular stagnation did not arise from the literature as New Keynesian models do not allow for such mechanism. Zero lower bound episodes within the New Keynesian models are the results of shocks to representative agents' discount factor, which has an inverse relationship to the long-run real interest rate. As the long-run real interest rate is set to be positive for the maximization problem, the ZLB episodes caused by shocks are not permanent (Krugman, Dominquez, and Rogoff (1998), Werning (2011), and Christiano, Eichenbaum, and Rebelo (2011)). Furthermore, Cochrane (2016) points out that New Keynesian models predict a sharp collapse in output and deflationary spirals for long ZLB episodes, which produce conflicting results compared to the contemporary economic performance[5].

Hence, the overlapping generations models become a desirable choice to address the shortcomings of ZLB New Keynesian models. Eggertsson and Mehrotra (2014) study the impacts of long-lasting zero lower bound episodes by adding nominal frictions seen in recessions (similar to Eggertsson and Krugman (2012) and Schmitt-Grohé and Uribe (2016)) and an overlapping generations model of Samuelson (1958). Work of J Caballero and Farhi (2018) also utilizes stylized stochastic overlapping generations model to study deflationary safety trap equilibrium with endogenous risk premia. They conclude that safety traps can be arbitrarily persistent despite infinitely lived assets reinforcing the supply side of the secular stagnation hypothesis. Similarly, Caballero, Farhi, and Gourinchas (2015) study the consequences of low equilibrium real interest rates with integrated but heterogeneous capital markets within the framework of an overlapping generations model with nominal rigidities designed to accentuate the heterogeneous relative demand for and supply of financial assets across markets. By including agents with a locally infinite risk aversion, they show a rise to an endogenous risk premium in the Uncovered Interest Parity condition, which creates a possibility of an asymmetric safety trap equilibrium, concluding portion of countries may experience a secular stagnation [6].

While the aforementioned papers provide a nuanced conceptual environment to study the natural rate of interest, it is equally noteworthy to highlight earlier models' contributions. There is a significant link between the theoretical side of the paper with the liquidity trap literature (see, e.g., Eggertsson

[5]Galí (2018) work encompasses a comprehensive review of the state of New Keynesian models in the recent year, concluding an evolving trend of the literature to cover the previously mentioned criticisms. For example, the work of Gabaix (2020) on bounded rationality affects within a New Keynesian framework, which results in a less costly ZLB episode compare to traditional models.

[6]They show home bias can lead to permanent real interest rate differentiation.

and Woodford (2004), Gauti and Woodford (2003), and Eggertsson and Krugman (2012)). The baseline model introduces nominal friction to the endowment economy, following Eggertsson and Krugman (2012) in the inclusion of financial frictions to the household problem and a similar inflation-targeting monetary regime. Schmitt-Grohé and Uribe (2016) provide the empirical evidence in favor of the existence of such a relationship by focusing on cross-country data. Another resemblance to the New Keynesian models is utilizing a hybrid Philips curve on the supply-side of the economy. This mechanism is inspired from the accelerationist Friedman-Phelps Phillips curve (see Phelps (1967) and Friedman (1968)) and downward nominal rigidity first discussed by Tobin (1972).

Other related work to the body of secular stagnation literature includes Eichengreen (2015), which classifies the tendencies behind an excess of desired saving over desired investment into four general categories: a rise in savings rates due to the rise of emerging markets; the scarcity of attractive investment opportunities, a reduction of the relative price of investment goods, and a decline in the rate of population growth. Hansen (1939) considered the decline in population growth rate as one of the primary forces behind the hypothesis of secular stagnation. More recent works of Carvalho, Ferrero, and Nechio (2016), Gagnon, Johannsen, and Lopez-Salido (2016), and Ikeda and Saito (2014) also study the impact of demographics on the decreasing trends of real interest rates in advanced economies. The empirics in all of those studies suggest a strong connection between the aging population trends of developed societies and the decline of real interest rates. Eggertsson, Mehrotra, and Summers (2016) study the secular stagnation within the framework of an open economy, finding that capital flows[7] transmit recession in a low-interest-rate environment is beggar-thy-neighbor.

Work of Caballero, Farhi, and Gourinchas (2015) in the context of the global economy also argue when real interest rates cannot play their equilibrium role, global output becomes the active margin. In this scenario, liquidity traps emerge and one country can drag the other one into it. For instance, during 2010, the developed countries were experiencing a zero lower bound episode while emerging markets were not. Devaluations in developed economies trigger capital inflows to emerging markets, lowering natural interest rates in emerging market economies. Lastly, IMF (2014) investigates changes in the relative price of investment in the advanced economies.

[7]Capital flows can propagate a recession when the zero lower bound becomes binding, dragging the domestic economy into secular stagnation.

They argue that a reduction in the relative price of investment will shift the demand for funds. While the effect of this shift on the real interest rate is ambiguous in their analysis, it can lower the rate according to conventional models used in the secular stagnation literature.

Chapter 2

The baseline Model

This section follows the base model in Eggertsson and Mehrotra (2014) paper. I start by introducing an endowment model, and the following sections are a modification to the existing model until reaching a comprehensive framework at the end of chapter two. The objective is to study the possibility of secular stagnation and create a foundation for further extensions in chapter three.

2.1 Endowment Economy

Eggertsson and Mehrotra (2014) use an overlapping generations model similar to Samuelson (1958) for analyzing an economy with a possible permanent negative real interest rate. In this section, I replicate a three-generation endowment economy. The goal of this section is to identify the determinants of the real interest rate and to study the effect of each determinant. In this three-period overlapping generations model, households can borrow or consume. In the first period, the young households are born who then enter the second period as middle-age and retire in the third one as old. The young have no initial wealth, do not receive any income, and only borrow from the middle-aged at an interest rate r_t. Next, middle-age households receive an endowment of Y_t^m for selling their labor to firms. Households maximize their lifetime utility using the following equation:

$$\max \mathbb{E} \left\{ \log(C_t^y) + \beta \log(C_{t+1}^m) + \beta^2 \log(C_{t+2}^o) \right\} \tag{2.1}$$

A representative utility-maximizing household born at time t consumes C^j where $j = \{y, m, o\}$ and faces following constraints in each period:

$$C_t^y = B_t^y \tag{2.2}$$
$$C_{t+1}^m = Y_{t+1}^m - (1+r_t)\, B_t^y + B_{t+1}^m \tag{2.3}$$
$$C_{t+2}^o = -\,(1+r_{t+1})\, B_{t+1}^m \tag{2.4}$$
$$(1+r_t)\, B_t^i \leq D_t \tag{2.5}$$

Equation (2.2) represents the budget constraint of the young households, where they borrow B_y^t from other generations and consume it. However, a limit, D_t, is imposed on the amount of borrowing or lending via one-period risk-free bonds. D_t [1] is simply an exogenous time-varying constant. Equation (2.3) denotes the middle-aged household budget constraint. They receive an endowment payment of Y_t^m, pay what they borrowed in the previous period, and save B_m^{t+1} for their retirement. Finally, in the last period of the household's life, they are hand-to-mouth[2] agents who use all of the savings from the last period.

The inequality (2.5) is a collateral constraint on borrowing, which is assumed to be permanently binding. This assumption further simplifies the future calculation of the steady state. This assumption implies that the limitation is tight enough to bind in every period.

$$C_t^y = B_t^y = \frac{D_t}{(1+r_t)} \tag{2.6}$$

Considering that the young generation is limited by (2.6) and consumption in the older generation is determined by saving in middle age, then the only control variable in the economy is the saving decision of the middle-aged.

$$\varphi = \log(B_t^y) + \beta \log(Y_{t+1}^m - (1+r_t)\, B_t^y + B_{t+1}^m) + \beta^2 \log(-\,(1+r_{t+1})\, B_{t+1}^m)$$

Now, the first order condition of the household's lifetime optimal utility (φ) can be written as:

[1] For (2.5) to hold, $D_t < \dfrac{Y_{t+1}^m}{(1+r_{t+1})(1+\beta)^2}$.

[2] In this context, hand-to-mouth are assigned to household who do not hold any savings and only consume their bringing from the previous period.

$$\frac{\partial \varphi}{\partial B^m_{t+1}} = \frac{\beta}{Y^m_{t+1} - (1+r_t)\, B^y_t + B^m_{t+1}} + \beta^2 \mathbb{E} \frac{-(1+r_{t+1})}{Y^o_{t+2} - (1+r_{t+1})\, B^m_{t+1}} = 0$$
$$\implies \frac{1}{Y^m_{t+1} - (1+r_t)\, B^y_t + B^m_{t+1}} = \beta \mathbb{E} \frac{1+r_{t+1}}{Y^o_{t+2} - (1+r_{t+1})\, B^m_{t+1}}$$

For middle age households at period t, the Euler equation can be written as:

$$\frac{1}{C^m_t} = \beta \mathbb{E} \frac{1+r_t}{C^o_{t+1}} \tag{2.7}$$

In the model, the demand arises from young borrowing and middle age supplying loans. In the equilibrium, the aggregate demand for one-period risk-free bonds is equal to the population of young households multiplied by individual borrowing, and middle age household saving multiplied by their population determines aggregate supply. Let us denote each generation population by N_t and population growth by $1 + g_t$, then the following equation holds:

$$\begin{aligned} N_t B^y_t &= -\, N_{t-1} B^m_t \\ (1+g_t) B^y_t &= -\, B^m_t \end{aligned} \tag{2.8}$$

ν^d_t denotes the aggregate demand for bonds. By using equation (2.6), the bond demand is equal to:

$$\begin{aligned} \nu^d_t &= (1+g_t) B^y_t \\ &= (1+g_t) \frac{D_t}{(1+r_t)} \end{aligned} \tag{2.9}$$

As households want to smooth their consumption over different periods to maximize their lifetime utility, loan supply is determined by relative income in middle age versus old age [3]. Assuming perfect foresight, the aggregate bond demand is derived by using the Euler equation (2.7) and household budget constraints.

$$\nu^s_t = \frac{\beta}{1+\beta}(Y^m_t - D_{t-1}) - \frac{1}{1+\beta} \frac{Y^o_{t+1}}{1+r_t} \tag{2.10}$$

[3] The income for old household is represented by Y^o_t, which is zero throughout the model.

A market clearing in the bonds market implies equilibrium in this economy. Thus, the intersection of aggregate bond supply (ν_t^s) and demand (ν_t^d) determines the equilibrium in the bond market and the economy. The real interest rate is obtained by rearranging the intersection point of equation (2.9) and equation (2.10).

$$\begin{aligned}
(1+g_t)\frac{D_t}{(1+r_t)} &= \frac{\beta}{1+\beta}(Y_t^m - D_{t-1}) - \frac{1}{1+\beta}\frac{Y_{t+1}^o}{1+r_t} \\
\frac{1}{1+r_t}((1+g_t)D_t + \frac{Y_{t+1}^o}{1+\beta}) &= \frac{\beta}{1+\beta}(Y_t^m - D_{t-1}) \qquad (2.11)\\
\Longrightarrow \; 1+r_t &= \frac{1+\beta}{\beta}\frac{(1+g_t)\,D_t}{Y_t^m - D_{t-1}} + \frac{1}{\beta}\frac{Y_{t+1}^o}{Y_t^m - D_{t-1}}
\end{aligned}$$

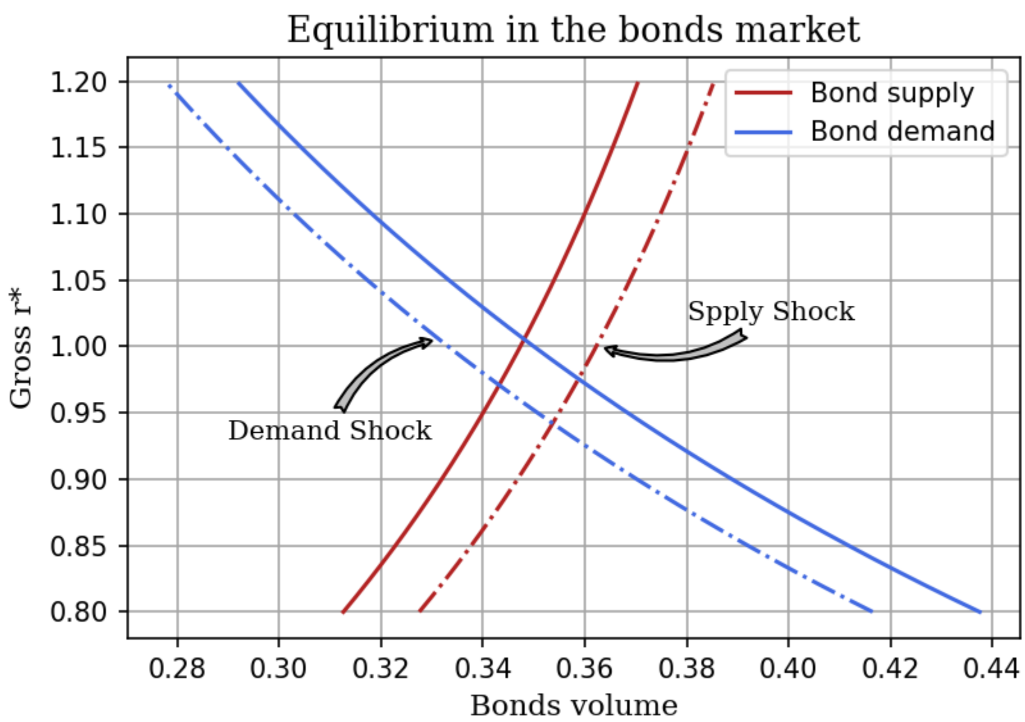


FIGURE 2.1: Equilibrium in the bonds market (own calculations - source is available in appendix B.1).

As it is evident, the determination of the real interest rate in this model is by the middle-aged income, debt limit, population growth, and the discount factor. This identification is clearly different from standard representative household models, where real interest is equal to the inverse of the discount factor. Equation (2.11) implies any distributional effects, which influence disposable income $(Y_t^m - D_t)$ can potentially impact the real interest rate. This approach will be the foundation of my analysis to understand how inequality, risk premium, and other forms of heterogeneity can put downward pressure on the real interest rate. Additionally, the slowdown of population growth can further reduce the interest rate in the economy. As the proportion of young households decreases, loan demand falls, shifting the loan demand curve in (2.1) to the left.

An equilibrium is now defined as a set of stochastic processes $\{C_t^y, C_{t,}^o, C_t^m, r_t, B_t^y, B_t^m\}$ that solve (2.2), (2.3), (2.4), (2.6), (2.7), and (2.8) given an exogenous process for $\{D_t, g_t\}$. The figure (2.1) is the graphical output of the original code and calibration cited in appendix B.1.

2.2 Price level Determination

Section one of chapter two is a basic framework for analyzing the real interest rate in the economy. To examine if a permanent negative real interest rate is possible within this framework, it is required to add a range of New Keynesian features to the model. Thus, in the following sections of chapter 2, I replicate features into the endowment economy in order to reach a comprehensive base model.

To investigate the behavior of price level and inflation, first, I include perfectly flexible nominal prices and then consider a case with nominal rigidities. The first extension adds a kink to the aggregates of the model, which suggests there will be no equilibrium if the real interest rate is negative and inflation fails to reach a certain threshold. It follows that, in the case of more realistic nominal frictions, there will be no equilibrium if the central banks fail to achieve the minimum level inflation.

In the first step of introducing perfectly flexible nominal price levels, I now assume that households have access to one-period nominal risk-free bonds controlled by the government[4]. At this stage, there are two options available for between generation saving. The nominal bonds with interest rate determined by the government and the one-period risk-free real debt. This assumption changes the budget constraints of the representative household of a cohort born at time t to:

[4]Eggertsson and Mehrotra (2014) extend the model to include micro-founded demand for money. However, the end results are similar to the base model.

$$C_t^y = B_t^y \tag{2.12}$$

$$C_{t+1}^m = Y_{t+1}^m - \frac{P_t\,(1+i_t)}{P_{t+1}} B_t^y + B_{t+1}^m \tag{2.13}$$

$$C_{t+2}^o = -\frac{P_{t+1}\,(1+i_{t+1})}{P_{t+2}} B_{t+1}^m \tag{2.14}$$

In this setting, P_t denotes the price level and i_t is the nominal interest rate. Assuming perfect foresight, the middle-aged households now save and consume according to the following Euler equation:

$$\frac{1}{C_t^m} = \beta \mathbb{E} \frac{1}{C_{t+1}^o} \frac{P_t\,(1+i_t)}{P_{t+1}} \tag{2.15}$$

Here, $\Pi_t = \frac{P_{t+1}}{P_t}$ is equal to the inflation rate. In addition, the nominal interest rate is limited by a non-negativity constraint. This introduces a zero lower bound for the nominal interest rate into the model, which becomes binding if its equilibrium value becomes negative.

$$i_t \geq 0 \tag{2.16}$$

Combining the two Euler equations (2.7), (2.15) and assuming perfect foresight, yields a standard Fisher relation:

$$(1+r_t) = \frac{(1+i_t)}{\Pi_t} \tag{2.17}$$

This equation states the relationship between nominal interest rate, inflation, and the real interest rate. An interesting thought experiment here is to imagine an economy where the real interest is negative (determined by equation (2.11)). As the zero lower bound on the nominal rate becomes binding, the equilibrium can only exists when the inflation is positive and satisfies the Fisher equation.

2.3 Aggregate Supply

To build a foundation for incorporating the supply side of the economy into the existing model, I assume perfectly competitive firms exist that are price takers in the market and maximize their profit each period. Firms produce Y_t in the form of endowment, and pay nominal profit Z_t and nominal wage W_t to their stakeholders. Considering there is no capital in the economy at this point, total output only depends on the exogenous level of labor supply L_t and labor share α.

$$Y_t = L_t^{\alpha}$$

The equation shows that firms face diminishing marginal returns to scale production. Further, firms maximize their profit through:

$$Z_t = \max_{L_t} P_t Y_t - W_t L_t \tag{2.18}$$

$$\text{s.t. } Y_t = L_t^{\alpha} \tag{2.19}$$

As there is no friction in the economy, the total labor demand is equal to:

$$w_t = \alpha L_t^{\alpha - 1} \tag{2.20}$$

Where $w_t = \frac{W_t}{P_t}$ is the real wage. This implies that similar to the previous section, output level is determined by the real interest rate. Labor L_t is equal to a constant term, which arbitrarily is equal to $L_t = \bar{L}$. For simplicity, assume that only middle-aged generation supply labor to firms, which changes their budget constraint.

$$C_{t+1}^{m} = \frac{W_{t+1}}{P_{t+1}} L_{t+1} + \frac{Z_{t+1}}{P_{t+1}} - (1 + r_t)\, B_t^{y} + B_{t+1}^{m} \tag{2.21}$$

$$C_{t+2}^{o} = -\,(1 + r_{t+1})\, B_{t+1}^{m} \tag{2.22}$$

I assume that the middle-aged generation will supply a constant level of labor $\bar{L}$ inelastically. Note that if the firms do not hire all available labor supplied, then labor demand L_t may be lower than labor supply $\bar{L}$ due to rationing. Under these assumptions, each of the generations' consumption-saving decision remains the same as before. The firms' labor demand condition is then given by equation (2.20).

Next, I move away from a frictionless economy by incorporating nominal rigidities into the model. This addition introduces a trade-off between inflation and employment level, which has a profound impact on the output level if the real interest rate falls below zero. The conventional choice is to include short-term rigidities [5]. However, following the Eggertsson and Mehrotra (2014), I choose the long-run nominal rigidities in the form of a permanent Philips curve.

This section provides a grounding for the unconventional choice of a permanent Philips curve. The model creates a permanent trade-off between inflation and the employment level in deflationary environments, and it is neutral in inflationary environments. The second part is in line with the prevalent view in the literature that there is no link between the two in the long run as the expectation for inflation adjusts (see Friedman (1977)). However, the ubiquitous acceptance of this view does not extend to the low inflation environments. In line with a strand of literature, this paper adopts a downward nominal wage rigidity similar to Schmitt-Grohé and Uribe (2016), where they show the existence of such a relationship by focusing on cross-country data. This idea was first discussed by Tobin (1972) after analyzing the wage policies of firms during the great depression[6]. Tobin observed that firms were averse to reducing wages of current or new employees during the economic downturn when there was a demand gap. This empirically significant phenomenon gave rise to the idea of wage rigidity.

Following Tobin's work, papers such as Akerlof et al. (1996), Kim and Ruge-Murcia (2009),Babecký et al. (2010), Benigno and Ricci (2011), Coibion, Gorodnichenko, and Wieland (2012), and more recently Daly and Hobijn (2014), Fallick, Lettau, and Wascher (2016) and, Hazell et al. (2020) contributed theoritically to the litrature of an upward sloping long-run Phillips curve. Empirically, amongst similar works[7] shows the existence of resistance toward wage cuts both in the normal and high unemployment periods.

I discussed a frictionless economy in which firms using the Cobb-Douglas production function generates output $Y_t = L_t^{\alpha}$ in each period. Previously,

[5] For example, see Greenwald and Stiglitz (1989).

[6] Tobin also analyzed the 1970's oil crisis and reached the same conclusion about downward nominal wage rigidity.

[7] For example, Bewley (1999) interviews the executives about cutting nominal wages, and Barattieri, Basu, and Gottschalk (2014) provide evidence for wage rigidities by analyzing US administrative data.

households would accept any optimal wage determined by firms. However, by adding downward nominal wage rigidities to the economy, they only take wages at period t equal or higher than the wage norm $\tilde{W}_t$. Thus, $\tilde{W}_t = \gamma W_{t-1} + (1-\gamma)W_t^{flex}$ is the lower bound of nominal accepted wage by households, where:

$$W_t^{flex} = P_t \alpha \bar{L}^{\alpha-1} \tag{2.23}$$

γ [8] intensifies the rigidity of wages in the economy. For instance, by increasing γ from zero to one, the minimum accepted nominal wage is going to be equal to the last period's nominal wages instead of a flexible wage. Wages in the economy are determined by

$$W_t = \max\left\{\tilde{W}_t, W_t^{\text{flex}}\right\} \text{ where } \tilde{W}_t = \gamma W_{t-1} + (1-\gamma)W_t^{\text{flex}} \tag{2.24}$$

The variable that determines the nominal wage norm in the economy is inflation. Intuitively, if the gross inflation is not less than one, then the nominal wage is going to be greater than $\tilde{W}$ and W_{t-1}.

$$W_t = \begin{cases} W_t^{flex} = P_t \alpha \bar{L}^{\alpha-1} & \text{if} \quad \Pi \geq 1 \\ \tilde{W}_t = \gamma W_{t-1} + (1-\gamma)W_t^{flex} & \text{if} \quad \Pi < 1 \end{cases} \tag{2.25}$$

System (2.25) implies that the wage norm is greater than the flexible wage if the gross inflation is less than one. In this case, the labor demand falls below the market-clearing value as households do not supply labor for a wage below the wage norm. Otherwise, households accept the flexible wage, and the labor market clears. Further, this dual mechanism simultaneously envelopes two views on the Philips curve. First, the accelerationist Friedman-Phelps Phillips curve (see Phelps (1967) and Friedman (1968)) due to the neutrality of inflation[9] when the net inflation is positive. Second, as the net inflation drops below zero, the model enforces a permanent trade-off between labor demand and inflation by introducing a lower bound for wages[10].

[8] It is possible to have a heterogeneous γ based on income distribution of households. However, Schmitt-Grohé and Uribe (2016) shows that γ is close for various income groups. Thus, this paper uses a homogeneous γ.

[9] Since wages are flexible in the inflationary environments, it implies that there is no long-run trade-offs between higher inflation and unemployment.

[10] Similar to the model in Schmitt-Grohé and Uribe (2016).

2.4 Monetary Policy Rule

The last modification to reach a comprehensive model in chapter two is including the monetary policy rule. To close out the model, assume that the central bank sets the nominal interest rate in the economy based on a Taylor rule given by

$$1 + i_t = \max\left(1, (1 + i^*)\left(\frac{\Pi_t}{\Pi^*}\right)^{\phi_\pi}\right) \tag{2.26}$$

In this equation, $\phi_\pi > 1; \Pi^*$ and i^* are constant parameters set by the central bank as part of their policy to maintain target inflation Π^* and the nominal interest at i^*. Intuitively, $\phi_\pi > 1$ is the response rate of the central bank to deviations from the target parameters. Higher Taylor coefficient ϕ_π implies a bigger overshoot[11] to coerce the inflation toward its target. Also, note that a precondition for equation (2.26) is that the nominal interest rate are not constrained by the zero lower bound.

Eggertsson and Mehrotra (2014) assert that variants of the Taylor rule do not have a significant effect on the model. This rule aims to introduce a mechanism comparable to an inflation-targeting regime, a common practice by various central banks in industrial economies. In this model, the target for the nominal interest rate will be determined by either the Fisher equation or the zero lower bound constraint. This relationship can be shown as following

$$(1 + i^*) = \max\left(1, \left(1 + r^f\right)\Pi^*\right) \text{ [12]} \tag{2.27}$$

The primary assumption here is that, at the ZLB, there is no possibility that the inflation rate is above its target. Essentially, the central bank does not allow such an outcome as it can raise the nominal interest rate. The sole equilibria then according to the policy regime would be ones in which inflation is below target. Note, there are equilibria with higher inflation targets that result in a negative natural rate of interest.

2.5 Steady State

Lastly, I defined the household side, the production side, rigidities, and monetary policy in the economy, I now analyze the equilibrium.

[11] It triggers a harsher nominal interest rate adjustments by the central bank in the economy.
[12] r^f is the full-employment real interest rate in the economy.

Definition 1. A competitive equilibrium is attainable if there exists a sequence of quantities $\{C^y_t, C^o_t, C^m_t, B^y_t, B^m_t, L_t, Y_t, Z_t\}$ and prices $\{P_t, W_t, W^{\text{flex}}_t, r_t, i_t\}$ that satisfy (2.2), (2.4),(2.6),(2.7),(2.8), (2.15),(2.17),(2.18), (2.19),(2.20),(2.21), (2.23), (2.24), and (2.26) given an exogenous process for $\{D_t, g_t\}$ and initial values for W_{-1} and B^m_{-1}.

I discussed how the downward nominal rigidity setup divides the economy into an environment with a permanent trade-off between employment and inflation level and an inflation-neutral one. Thus, to analyze the aggregates in the steady-states in labor and bonds markets, I will take both regimes into account. The two wage regimes create a kink at $\Pi = 1$[13]. At $\Pi \geq 1$, wages are above the wage norm ($W = W^{flex} = P\alpha\bar{L}^{\alpha-1}$), implying that the labor market clears at flexible wage. In this scenario, the economy is at the full-employment level as firms demand labor equal to the exogenous level of labor supply $\bar{L}$. Thus, the total output can be derived using equation (2.19).

$$Y = \bar{L}^{\alpha} = Y^f \text{ for } \Pi \geq 1 \tag{2.28}$$

The aggregate supply is independent of the inflation rate, and it is only determined by the amount of labor supplied (Friedman-Phelps Phillips curve), which manifests itself as a vertical line in graph (2.2).

If there is a steady-state in the deflationary environment $\Pi < 1$, then the wage norm is binding at all points. The households do not accept the flexible wage as it is below the wage norm, which forces firms to pay above the marginal productivity of labor at full-employment[14]. Let us denote the real wage by $w = \frac{W}{P}$ then rearranging (2.24) results in the following equation[15]

$$\begin{aligned} \tilde{w}_t &= \gamma\tilde{w}_{t-1}\Pi^{-1} + (1-\gamma)\alpha\bar{L}^{\alpha-1} \text{ for } \Pi < 1 \\ w &= \frac{(1-\gamma)\alpha\bar{L}^{\alpha-1}}{1-\gamma\Pi^{-1}} \end{aligned} \tag{2.29}$$

Labor demand diminishes as wages are now above the marginal productivity of labor at full-employment. Let us then derive a relationship between

[13] This is the point of intersection of two wage regimes.

[14] As firms are perfectly competitive, they always pay based on the marginal productivity of labor. In this scenario, $\tilde{w}_t > w^{flex}$ leads to lower demand for labor.

[15] This raises an issue as (2.29) is not continuous for all reasonable values of Π^{-1}, which needs further attention.

labor supply and inflation tracing the lower segment of the AS curve by substituting (2.29) into equation (2.20) and express it as

$$L_t = (\frac{(1-\gamma)\alpha\bar{L}^{\alpha-1}}{\alpha(1-\gamma\Pi^{-1})})^{\frac{1}{\alpha-1}} \tag{2.30}$$

As I have not introduced capital to the model, the level of labor supply determines the aggregate supply in deflationary environments. The results can be expressed in terms of output by using labor supply (2.30) and the production function (2.19).

$$\frac{\gamma}{\Pi} = 1 - (1-\gamma)\left(\frac{Y}{Y^f}\right)^{\frac{1-\alpha}{\alpha}} \quad \text{for } \Pi < 1 \tag{2.31}$$

As the gap widens between the wage and marginal productivity of labor at full-employment, labor demand declines. Intuitively, as prices are only sticky in nominal terms, raising inflation will effectively depreciate real wages, boosting labor demand and total output in turn. Thus, there is a positive relationship[16] between the output level and inflation.

Aggregate demand stems from young households borrowing bonds from the middle-aged and saving and consumption of middle-age households in period t. Both the level of borrowing and saving depend on the nominal interest rate and inflation at period t. Thus, similar to the previous part, I define two regimes for determining the aggregate demand in the economy. In scenarios where the nominal interest rate is above zero and the zero lower bound is not binding, the aggregate demand equation is defined by combining the real interest rate equation (2.11), Fisher relation (2.17), and monetary policy rule (2.26).

$$Y = D + \frac{(1+\beta)(1+g)D\Gamma^*}{\beta}\frac{1}{\Pi^{\phi_\pi - 1}} \text{ for } i > 0 \tag{2.32}$$

This equation is a standard aggregate demand with a negative slope where $\Gamma^* \equiv (1+i^*)^{-1}(\Pi^*)^{\phi_\pi}$ is the composite policy parameter in the monetary policy reaction function. ϕ_π represents the intensity of the central bank overshooting nominal interest rates based on the Taylor rule. In this model, $\phi_\pi > 1$ implies that the nominal interest rate changes more than one for one compared to the inflation rate. As a result, an increase in inflation encourages

[16] Equation (2.31) is a non-linear Phillips curve.

households to increase their savings, further reducing the total demand.

Next, consider a situation where the zero lower bound is binding ($i = 0$). The real interest rate is given by $(1 + r_t) = \frac{1}{\pi_t}$ and the aggregate demand by the following expression[17]

$$Y = D + \frac{(1+\beta)(1+g)D}{\beta}\Pi \text{ for } i = 0 \tag{2.33}$$

In contrast with equation (2.33), this equation is an upward sloping aggregate demand curve, where rising inflation increases the total demand. Similar expressions are common in the liquidity trap literature (see Eggertsson and Krugman (2012)). As the zero lower bound is binding and the central bank is not further adjusting the nominal interest rate, an increase in inflation results in a lower real interest rate (higher real interest rate boosts the consumption).

There exists a kink in the aggregate demand curve dividing the upper part (2.32) from the lower part (2.33) similar to the aggregate supply curve. Contrary to the aggregate supply curve, the kink is not independent of variables in the economy. The central bank's monetary policies are the determining factor. Let us demonstrate that by finding the intersection point of equation (2.32) and (2.33).

$$\Pi_{kink} = \left(\frac{1}{1+i^*}\right)^{\frac{1}{\phi\pi}} \Pi^* \tag{2.34}$$

Equating the aggregate demand equations for two regimes shows that kink occurs at the highest gross inflation resulting in a negative nominal interest rate, i.e., ZLB is binding, based on the Taylor rule. Moving below that point is a deflationary environment, in which the central bank is unable to attain the full-employment real interest rate through manipulating the nominal interest rate. Similar to the endowment economy, r^f[18] is calculated by combining equations (2.11) and (2.28).

$$1 + r_t^f = \frac{1+\beta}{\beta}\frac{(1+g_t)\,D_t}{Y^f - D_{t-1}}$$

[17] For the AS and AD diagrams and numerical examples I use parameter values and codes cited in appendix B.2.

[18] r^f matches the real interest rate in the endowment economy.

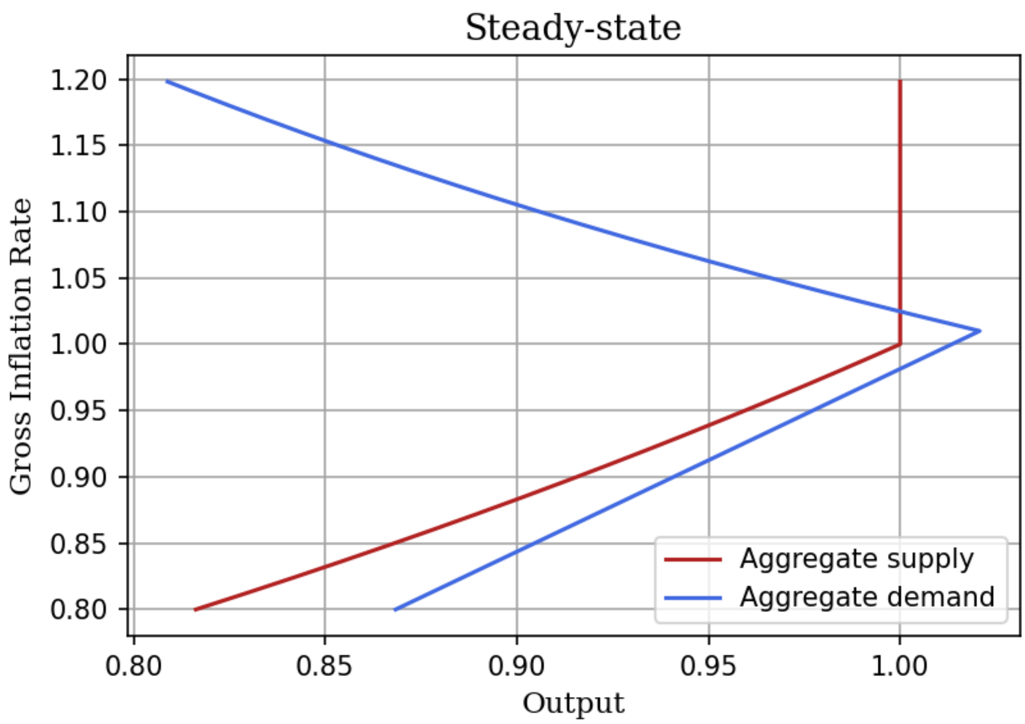


FIGURE 2.2: Steady-state aggregate demand and supply curves (own calculations - source is available in appendix B.2).

2.6 Equilibrium and Secular Stagnation

In previous sections, I defined aggregate supply and demand for the model. The intersection of the aggregate demand and aggregate supply curves determines the equilibrium of inflation and output in the economy. There are two possible scenarios for intersection points.

First, consider a case where the real interest rate is above zero, and both the nominal rate and inflation match their policy targets [19]. This implies that the aggregate demand curve intersects the vertical part of the aggregate supply curve. The intersection happens at a point where the labor market clears, and the economy is at the full-employment state with no rigidities. Thus, as the equilibrium occurs above r^f, only policy variables determine the exact intersection point[20].

In this setting, a shock that tightens the collateral constraint D_t can move

[19] In this scenario $1 + i^* = (1 + r^f)\Pi^*$, as the central bank regulates the economy using the monetary policy rule.

[20] Equation (2.32) shows how the central bank can alter the slope of the demand curve by choosing policy parameters. Also, there exist a unique equilibrium for small inflation values and high enough γ. As γ declines toward zero, it is possible to have more equilibria.

the economy into a steady-state with a permanent negative real interest rate. Tightening the collateral constraint implies that the younger households have to consume less as they cannot finance it through the bond market. In the next period, the newly turned middle-age households have less debt to pay, which in turn increases the aggregate saving. Thus, in the long run, a decline in the level of D_t will reduce the real interest rate by both increasing the aggregate saving and decreasing the demand. A large enough shock tightening the collateral constraint can shift the aggregate demand to the point, at which the zero lower bounds become binding. The zero lower bound and the permanent Philips curve do not allow for restoring the total consumption via the real interest rate adjustment. Similarly, this shift moves the economy out of the full-employment state[21]. Hence, forming a deflationary steady-state.

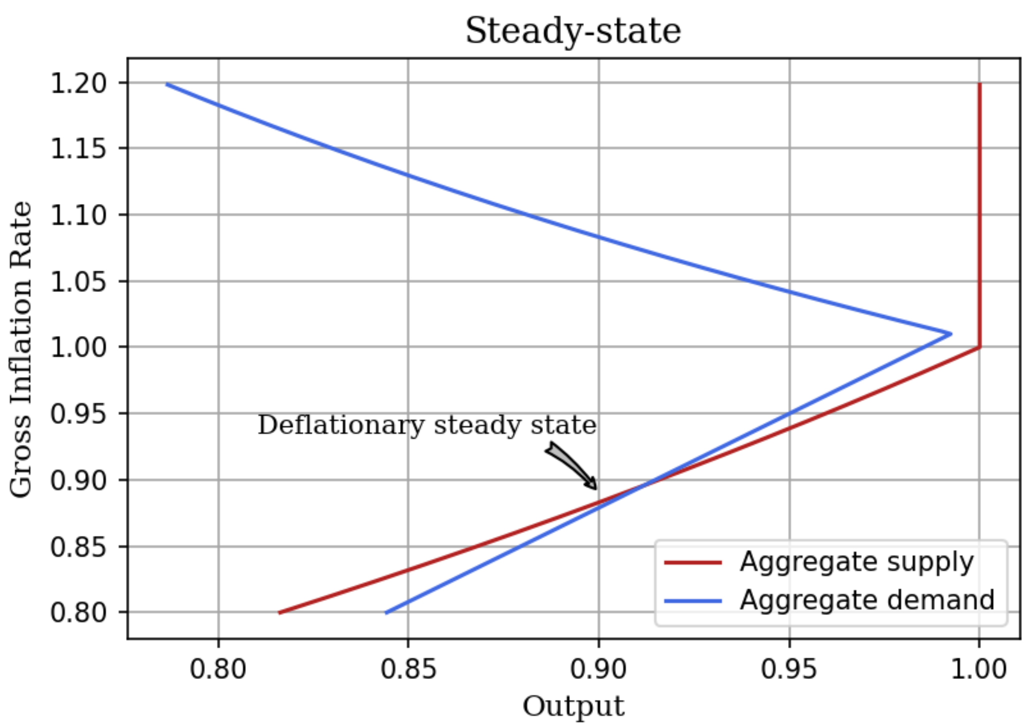


FIGURE 2.3: Steady state aggregate demand and supply curves (own calculations - source is available in appendix B.2)

Proposition. A unique determinate secular stagnation equilibrium exists if $\gamma > 0$ and $i^* = r^f < 0$.

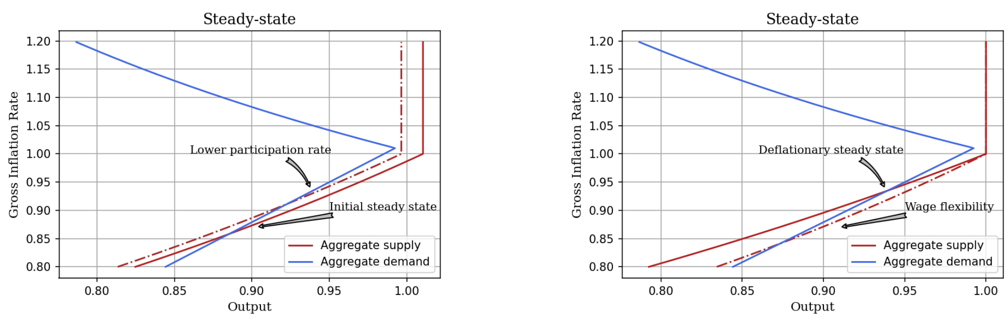
For proof check the appendix A $\square$

[21]labor demand will fall as a result of higher real wages.

Intuitively, this proposition mathematically displays that if the aggregate demand curve is steeper in the steady-state, then the equilibrium is locally determinate. The determinacy implies that the economy will not automatically recover from the long slump and may continue to have uniquely bounded business cycles in that state.

As discussed earlier, there is no reason to believe that the economy will pull towards full employment without government intervention. However, there are mechanisms, which can potentially return the economy to the post-shock state. First, assume there exists a temporary shock driving down the natural rate in the economy. Clearly, after the shock subsides, the economy adjusts itself. This mechanism is less relevant to the model as this work is mainly focused on secular forces[22].

The second mechanism is wage flexibility. Wage rigidity plays a crucial role in creating the slump in employment and output. Contrary to what one may expect, an increase in the flexibility of wages will not result in a push out of secular stagnation equilibrium. A decrease in γ will lower the expected inflation, further intensifying the output gap[23].



(A) hysteresis responses (B) wage flexibility responses

FIGURE 2.4: Adjustment mechanisms (own calculations - source is available in appendix B.2).

The last mechanism is the decline in the labor force participation rate. Workers are discouraged from participating in the labor market after prolonged periods of unemployment. A decreasing labor force participation rate puts upward pressure on wages. A large enough reduction of labor force can push the economy to achieve the full-employment output. In this scenario, even

[22]For instance, the slow population growth and inequality are both secular forces.

[23]A more flexible wage rigidity simply increases the slope of the aggregate supply curve, pushing down the point of intersection through increasing the real wage. This paradox between wage flexibility and shortfall in output was further discussed in Bhattarai, Eggertsson, and Schoenle (2014).

after the recovery, the output remains at a lower level compared to the pre-shock state.

The main conclusion from chapter two is that an economy can stay in a deflationary state indefinitely. I show an economy with such characteristics theoretically by following the baseline model in Eggertsson and Mehrotra (2014) and quantitatively by developing my original computational model in the Python environment (see appendix B.2). I encourage readers to test out my computational model of section two for various scenarios. However, I provide a qualitative summary of how a decrease in values of each parameter affects the steady state real interest rate in the final form of the model.

TABLE 2.1: Parameter effects

Description	**Symbol**	**Deflationary env**	**Inflationary env**
Rate of time preference	β	↑	↑
Wage adjustment	γ	↓	–
Gross inflation target	Π^*	–	↓
Taylor coefficient	ϕ^π	–	↑
Labor supply	$\bar{L}$	↑	↑
Labor share parameter	α	↓	–
Collateral constraint	D_t	↓	↓
Population growth rate	g_t	↓	↓

Note: Each value change represents a distinct shock impulse response.

Three parameters of interest in the baseline model are the population growth rate, collateral constraint, and rate of time preference. To examine how profound is the impact of a change in the mentioned variables on the real interest rate, I conduct a sensitivity analysis. The below table represents the percentage change in the real interest rate, as a result of 1%, 5%, and 10% increase in each variable. The source code and initial calibration are available in appendix B.2[24].

TABLE 2.2: Sensitivity Analysis

Description	**Symbol**	**1% Δ**	**5% Δ**	**10% Δ**
Rate of time preference	β	+1.605%	+6.42%	+10.514%
Collateral constraint	D_t	−4.806%	−11.68%	N/A
Population growth rate	g_t	−0.774%	−4.261%	−10.68%

Note: Each data point represents % of $\Delta(1+r)$.

[24] A 10% increase violates the following constraint: $D_t < \frac{Y^m_{t+1}}{(1+r_{t+1})(1+\beta)^2}$. Thus, it is recorded as N/A.

Chapter 3

The Extensions

In chapter three, I expand on the findings of the previous chapter by investigating structural and distributional characteristics of an economy, which introduces a range of new mechanisms for provoking a permanent negative natural rate. The following sections are standalone additions to the base model.

3.1 Income Inequality

In the previous sections, I formalized the baseline model, in which any force that affects the relative aggregate supply and demand becomes a determinant for the real interest rate. In the following sections, I will discuss economic trends that can put downward pressure on the natural rate in the economy. A closely related topic to the issue of excess savings is the connection between marginal propensity to save and consume of households and inequality. Furman, Stiglitz, et al. (1998) demonstrate a robust inverse relationship between the lifetime income and marginal propensity to consume. Dynan, Skinner, and Zeldes (2004) finds marginal propensity to save increase sharply with the level of permanent income, and reaches values of 0.5 or even higher for the highest income groups[1]. Bunting (1991) find substantial evidence that American households' marginal propensities to save uniformly increase with their quintile share of income rises, using consumer expenditure survey data for the United States. Lim (1980) also uncovers that inequality tends to boost aggregate saving rates. More recent empirical studies such as Carroll et al. (2017) and Alvarez-Cuadrado and El-Attar Vilalta (2018) find similar results to the earlier works in the literature. While there is

[1] Menchik and David (1983) utilizes the United States disaggregated income data to find if the elasticity of bequests to lifetime income is more considerable for the higher income groups. The result shows that the marginal propensity to bequeath is unambiguously higher for the wealthy.

a body of works on the effect of inequality on economic growth, this analysis mainly utilizes the difference in marginal propensity to save to understand the impact of inequality on the natural rate of interest.

As it is possible to introduce inequality into the model in many forms, and there is no general form to examine how real interest rates change with an increase in inequality, I confine my analysis to one proposed model. There exist a level of income inequality between generations in the baseline model, considering only middle-aged households receive income. To introduce within-generation income inequality, assume that there are two types of households based on their skill level. Let us denote high skill ones with s^h and low skill ones with s^l. The introduction of skills implies that wages are now proportional to the skill and supplied labor of households, which creates income heterogeneity. Further, following the literature on marginal propensity to save and consume, the high-income households have a higher marginal propensity to save. The distinction in saving rates comes from different discount factors[2]. Lastly, the high-income old group leaves bequests for their children, which changes the (2.1) for the high-income households to:

$$\max \mathbb{E} \left\{ \log(C_t^y) + \beta_h \log(C_{t+1}^{m,h}) + \beta_h^2 \left(\log(C_{t+2}^{o,h}) + \frac{\log(Q_{t+2})}{1+\beta} \right) \right\} \tag{3.1}$$

Bequest motive is denoted by Q_t[3] in the equation (3.1). When households die at period t, they leave Q_t amount of endowment to their children. It is noteworthy that there is no mobility between high-income and low-income groups, but it is possible to implement that in more sophisticated models[4]. A representative utility-maximizing household with high-income born at time t consumes $C^{j,h}$ where $j = \{y, m, o\}$ and faces following constraints in each

[2]Higher discount factor implies high-income households value utility derived from the saving in future more than their low-income counterparts, leading to a higher saving rate for the former group.

[3]The denominator of Q_t in equation (3.1) represent the value of bequest compare to consumption for the parents. The generalized form of this can account for how benevolent they are.

[4]For instance, one can use a finite Markov chain to create a stochastic overlapping generations model, where agents either stay or change their income group with a certain probability.

period:

$$C_t^y = B_t^y = \frac{D_t}{1+r_t} \tag{3.2}$$

$$C_{t+1}^{m,h} = Y_{t+1}^{m,h} - (1+r_t)\, B_t^y + B_{t+1}^{m,h} + \frac{Q_{t+1}}{1+g_t} \tag{3.3}$$

$$C_{t+2}^{o,h} = -\,(1+r_{t+1})\, B_{t+1}^{m,h} - Q_{t+2} \tag{3.4}$$

Similarly, for low-income households, I write down the utility function and budget constraints accordingly.

$$\max \mathbb{E} \left\{ \log(C_t^y) + \beta_l \log(C_{t+1}^{m,l}) + \beta_l^2 \log(C_{t+2}^{o,l}) \right\} \tag{3.5}$$

$$C_t^y = \frac{D_t}{1+r_t} \tag{3.6}$$

$$C_{t+1}^{m,l} = Y_{t+1}^{m,l} + \frac{D_{t+1}}{1+r_{t+1}} - D_{t+1} \tag{3.7}$$

$$C_{t+2}^{o,l} = Y_{t+2}^{o,l} - D_{t+2} \tag{3.8}$$

These households will work for the last two periods of their lives and rollover their debt to the next period. The collateral constraint is binding in both periods, which is only possible for low enough income levels and tight credit constraints. Thus, only high-income households supply loans in this environment to the young and low-income middle-age households. Further, the decomposition of population is $N_t = \eta N_t + (1-\eta)N_t$. Where ηN_t and $(1-\eta)N_t$ are the populations of low-income and high-income households respectively[5], and $\eta \in [0,1]$. First, I consider a case for the endowment economy, in which households receive an endowment proportional to their respective skills. Loan supply is determined by the relative income of high-income households in middle age versus old age, assuming perfect foresight.

$$\nu_t^s = \frac{(1-\eta)\beta_h}{1+\beta_h} \left(s_t^h Y_t + \frac{Q_t}{1+g_t} - D_{t-1} \right) + \frac{1-\eta}{1+\beta_h} \frac{Q_{t+1}}{1+r_t} \tag{3.9}$$

Y_t represent a constant level of endowment, and I derive an expression for bequest in the next period Q_{t+1}, assuming perfect foresight, by combining budget constraints and the Euler equation.

[5] I assume that the population growth rate for both groups is equal.

$$Q_{t+1} = \frac{(1+r_t)\,\beta_h}{\beta_h + (1+\beta_h)^2}\left(Y_t^m - D_{t-1} + \frac{Q_t}{1+g_{t-1}}\right) \tag{3.10}$$

In contrast with the baseline model, the loan demand is not solely from the young generation as low-income groups roll over their debt.

$$v_t^d = (1+g_t)\frac{D_t}{(1+r_t)} + \eta\frac{D_t}{(1+r_t)} \tag{3.11}$$

The intersection of loan demand (3.11) and loan supply (3.9) determines the real interest rate in the economy.

$$(1+r_t) = \frac{(1+\eta)}{(1-\eta)}\frac{(1+g_t)(1+\beta_h)D_t}{\beta_h\left(s_t^h Y_t + \frac{Q_t}{1+g_t} - D_{t-1}\right)} + \frac{Q_{t+1}}{\beta_h\left(s_t^h Y_t + \frac{Q_t}{1+g_t} - D_{t-1}\right)} \tag{3.12}$$

The introduction of inequality to the model polarized the households into the high-income group who only participate in the debt market by supplying loans and the low-income group who is credit constrained. As the level of collateral constraint determines the demand side of the bond market, high-income earners' excess savings in their middle period of life becomes the sole determinant of the real interest rate in the economy.

The first observation from loan supply (3.9) is that the bequest motive increases loan supply through two mechanisms. The high-income middle-age households have to save more[6] in order to deed a portion of their lifetime income to their children and have more income at their disposal after receiving the bequest. Consider a case where the values of parameters s^h and s^l are proportional to the share of total endowment households receive that corresponds to their skills, then it is evident increase in inequality will lead to a lower real interest rate. Mathematically, $\frac{\partial r_t}{\partial s_t^h} < 0$, implying that the value of real interest rate (3.12) has an inverse relationship with the value of s^h.

Second, by comparing the loan demand equation (3.11) to its baseline counterpart, there is a larger fraction of the population who are credit constrained. A shock to the collateral constraint has a more significant effect in decreasing the real interest rate under the existence of inequality due to

[6] If the denominator of bequest motive gets smaller in the high-income household utility function then the value of Q_{t+1} increases, further lowering the real interest rate.

affecting the low-income households as well. Furthermore, the real interest rate equation shows that an increase in η can push the economy out of a zero lower bound episode. Perhaps, it is an expected result based on the structure of the model. A surge in η increases the real interest rate by raising the population of credit constraint households and reducing the loan supply. However, it makes the economy more vulnerable to shock, which influences the loan demand. The results of this section further support the proposition that not all forms of income inequality have an adverse effect on the real interest rate. I show how a decrease in each parameter of interest affects the real interest rate in this section in the table below.

TABLE 3.1: Parameter effects

Description	**Symbol**	**Deflationary env**	**Inflationary env**
Rate of time preference (h)	β_h	$\uparrow$	$\uparrow$
Rate of time preference (l)	β_l	–	–
Collateral constraint (h)	D_t^h	$\downarrow$	$\downarrow$
Collateral constraint (l)	D_t^l	$\downarrow$	$\downarrow$
Bequest	Q_t	$\uparrow$	$\uparrow$
Gross inflation target	Π^*	–	$\downarrow$
Taylor coefficient	ϕ^π	–	$\uparrow$
Labor supply	$\bar{L}$	$\uparrow$	$\uparrow$
Population growth rate	g_t	$\downarrow$	$\downarrow$
Low-income Population share	η	$\downarrow$	$\downarrow$
High-income skill pay	s^h	$\uparrow$	$\uparrow$
Low-income skill pay	s^l	$\downarrow$	$\downarrow$

Note: (h) and (l) denote the high income and low income households, respectively.

Moreover, I perform several Sensitivity analyses on newly introduced variables to study their impact on the real interest rate. I increase the value of each variable by 1%, 5%, and 10% and record their effect on $\Delta(1+r)$ in the below table. The source code and initial calibration are available in appendix B.3.

TABLE 3.2: Sensitivity Analysis

Description	**Symbol**	**1% Δ**	**5% Δ**	**10% Δ**
Rate of time preference (h)	β_h	-0.641%	-2.6%	-5.843%
High-income skill pay	s^h	-1.516%	-5.194%	-9.848%
Low-income Population share	η	$+1.19\%$	$+6.601\%$	$+13.64\%$

Note: the following results are for an endowment economy.

3.2 Incorporating Capital

The core of demand-side secular stagnation is the IS-LM framework, stating the Wicksellian natural interest rate determined at the intersection of the investment demand and the supply of savings curves (see, Summers (2013b), Summers (2013a), and Summers (2014)). The baseline model abstracted from the inclusion of capital to focus issues around labor demand. In this section, I introduce this addition to the model and touch upon new mechanisms that can generate secular stagnation. Prevalent reasoning for the declining real interest rate is the observed trend of the relative price of capital goods in advanced economies. A lower relative price of capital implies that the economy can maintain the same level of investment projects by committing a smaller share of GDP. Eichengreen (2015) asserts with less investment spending and the same savings, the outcome can be a lower real interest rate and, potentially, a chronic excess saving.

Summers (2013a) also shares a similar view due to the evolving capital composition of major firms, which do not rely on immense amounts of physical capital to operate, and the falling relative price of investment goods compare to consumer goods, weakening the demand for investment. IMF (2014) investigates changes in the relative price of investment in he advanced economies, arguing a downward trend in the relative price of investment will shift the demand for funds. Furthermore, an argument of Hansen (1939) concentrates on how the low rate of population growth is pulling down the rate of investment. Slower population growth implied that capital had less additional labor to work on the margin, resulting in lower returns and investment[7].

Lastly, Caggese and Perez-Orive (2017) highlights another mechanism, which capital can affect the natural rate in the economy. The growing prevalence of intangible capital means the rising significance of reallocation of intangible assets, which have a low collateral value and have to be financed using retained earnings. The results show a novel misallocation effect of endogenously low interest rates and the existence of a significant impact lag[8].

[7]Goodhart and Erfurth (2014) questions the relationship between lower labor force growth and downward rate of investment, predicting a contradictory result to Hansen's argument.

[8]Their model shows that even though the rise of intangible technologies was already happening in the 1970s, while its net negative effects on output growth only started to become evident from the mid-1980s onward.

3.2.1 Introducing Capital

In this section, consider an economy similar to the baseline model with two new additions. I introduce capital by allowing middle-aged households to invest in capital through renting capital to perfectly competitive firms and supplying loans to young households. The firms now maximize period-by-period profits following a Cobb-Douglas production function using both labor and capital to produce output Y_t at period t.

$$Y_t = K_t^{1-\alpha} L_t^{\alpha}$$

Here, K_t denotes the capital stock at period t. The firms' labor is equal to marginal product of labor, which remains unchanged, but now firms also rent capital so that the marginal product of capital is equated to the rental rate of capital r_t^k as firms operate in a perfect competition setting :

$$r_t^k = (1-\alpha)\frac{Y_t}{K_t}$$

While I will introduce nominal rigidities into the model, for now, assume wages are flexible, implying that $L_t = \bar{L}$. Thus, I only focus on how the introduction of capital affects the natural rate of interest.

Analogous to the base model, households maximize their lifetime utility using an objective function identical to section 2.1. However, the modified budget constraints account for the middle generation investing in firms by renting capital and receiving the sold net value of capital after depreciation in the next period.

$$\begin{aligned}
&\max_{C_t, C_{t+1}, C_{t+2}} \mathbb{E}_t \left\{ \log\left(C_t\right) + \beta \log\left(C_{t+1}\right) + \beta^2 \log\left(C_{t+2}\right) \right\} \\
&\text{s.t.} \quad C_t = B_t \\
&\qquad C_{t+1} = w_{t+1} L_{t+1} + r_{t+1}^k K_{t+1} + B_{t+1} - p_{t+1}^k K_{t+1} - \frac{(1+i_t)}{\Pi_{t+1}} B_t \\
&\qquad C_{t+2} = p_{t+2}^k K_{t+1}(1-\delta) - \frac{(1+i_{t+1})}{\Pi_{t+2}} B_{t+1} \\
&\qquad B_{t+j} \leq \mathbb{E}_{t+j}\left(1 + r_{t+j+1}\right) D_{t+j} \text{ for } j = 0, 1
\end{aligned}$$

Here, p_t^k[9] is an exogenous relative price of capital goods, δ is the capital depreciation rate and K_t are households' purchases of the physical capital good. The middle-age households purchase K_t at period t and rent it out to the firms.

The households choose an optimal level of consumption using an Euler equation similar to equation (2.7). However, they have to currently choose an optimal level of capital investment in addition to supplying bonds to young households, using the following equation:

$$\frac{1}{C_t^m} = \beta \mathbb{E}_t \frac{1+i_t}{\Pi_{t+1} C_{t+1}^o} \frac{p_t^k - r_t^k}{C_t^m} = \beta \mathbb{E}_t \frac{p_{t+1}^k (1-\delta)}{C_{t+1}^o} \tag{3.13}$$

Assuming perfect foresight, I combine the two Euler equations for optimal choice of capital investment and consumption to derive the relationship between the relative price of capital and real interest rate[10].

$$\frac{p_t^k - r_t^k}{p_{t+1}^k (1-\delta)} = \frac{\Pi_{t+1}}{1+i_t} \tag{3.14}$$

Using the relationship between the relative price of capital and real interest rate, the adjusted household's budget constraints for each type at any point in time is given below:

$$C_t^y = \mathbb{E}_t \Pi_{t+1} \frac{D_t}{1+i_t} \tag{3.15}$$

$$C_t^m = Y_t - D_{t-1} - p_t^k K_t - B_t^m \tag{3.16}$$

$$C_t^o = p_t^k K_{t-1}(1-\delta) + B_{t-1}^m \frac{1+i_{t-1}}{\Pi} \tag{3.17}$$

On the supply side of the economy, firms choose the optimal level of capital and labor according to the profit maximization problem given below:

$$Z_t = \max_{L_t, K_t} P_t Y_t - W_t L_t - P_t r_t^k K_t \tag{3.18}$$

$$\text{s.t. } Y_t = A_t K_t^{1-\alpha} L_t^{\alpha} \tag{3.19}$$

[9] If the conversion of consumption good to investment good is without any cost, then $p_t^k = 1$. The more elaborate form is to have a two-sector economy for the production of "investment good" and a "consumption good" similar to Greenwood, Hercowitz, and Huffman (1988).

[10] As $r^k > 0$, this is a no arbitrage condition linking the capital rental rate to the real interest rate. Assuming that the relative price of capital is constant in a steady-state then there exist a lower bound on the steady-state real interest equal to $r \geq -\delta$.

The real flexible wage when the wage norm is not binding is equal to:

$$w_t = \alpha \frac{Y_t}{L_t} \tag{3.20}$$

In deflationary environments, the wage norm becomes binding, implying that households do not accept a wage equal to the marginal productivity of labor. Using a similar framework to the base model, the lower bound for wages at each period is equal to the weighted average of the wage in the previous period and the flexible wage.

$$\begin{aligned} \tilde{W}_t &= \gamma \tilde{W}_{t-1} + (1-\gamma) W_t^{\text{flex}} \ \text{ for } \Pi < 1 \\ \tilde{w}_t &= \gamma \tilde{w}_{t-1} \Pi^{-1} + (1-\gamma) \alpha A_t K_t^{1-\alpha} L_t^{\alpha-1} \\ w &= \frac{(1-\gamma)\alpha A_t K_t^{1-\alpha} \bar{L}_t^{\alpha-1}}{1-\gamma\Pi^{-1}} \end{aligned} \tag{3.21}$$

Steady-State:

Comparing the baseline model to the capital extension of it, there are four primary differences. First, there is a new lower bound set on the real interest rate as it can not fall below the depreciation rate of capital.

$$\begin{aligned} \frac{p_t^k - r_t^k}{p_{t+1}^k (1-\delta)} &= \frac{\Pi_{t+1}}{1+i_t} \\ \frac{p_t^k - r_t^k}{p_{t+1}^k (1-\delta)} &= \frac{1}{1+r_t} \\ r_t^k = p_t^k - \frac{p_{t+1}^k (1-\delta)}{1+r_t} &\geq 0 \end{aligned} \tag{3.22}$$

Assuming the relative price of capital is constant in the steady-state, equation (3.22) imposes a new constraint on the model. Next, the economy consists of two markets, bond and capital market, and both have to clear for the economy to reach equilibrium. This dual market framework can cause multiple kink points in the aggregate demand and supply curve. Lastly, the introduction of capital introduces the depreciation rate and the relative price of capital into the model. These two parameters can affect the real interest rate. To study their impact on the economy, I derive the equations that define the equilibrium real interest rate. To avoid multiple kinks and to simplify the final results, I assume an inflation target $\Pi^* = 1$ and an initial natural rate of interest of zero so that $i^* = 1$. The policy assumption divides the economy into two different environments as there is a point of unity for aggregate demand and supply kink points.

According to Taylor rule (2.27) and the policy assumption, if inflation is above zero then the zero lower bound is not binding, which result in a flexible wages ($w_t = \alpha \frac{Y_t}{L_t}$). This implies that the economy is at the full-employment level ($L_t = \bar{L}$). Thus, firms use their profit function (3.18) to determine optimal level of capital demand and then wage [11]:

$$
\begin{aligned}
r_t^k &= (1-\alpha)\frac{A_t K_t^{1-\alpha} L_t^{\alpha}}{K_t} \\
K_t &= ((1-\alpha)\frac{A_t}{r_t^k})^{-\alpha}
\end{aligned}
\tag{3.23}
$$

$$
\begin{aligned}
w_t &= \alpha \frac{A_t K_t^{1-\alpha} L_t^{\alpha}}{L_t} \\
w_t &= \alpha A_t \left(((1-\alpha)\frac{A_t}{r_t^k})^{-\alpha} \right)^{1-\alpha}
\end{aligned}
\tag{3.24}
$$

As this work mainly focuses on the real interest rate at the steady-state, I assume that both bond and capital markets are clear. The market-clearing condition for the goods market implies that loans demand is determined by (2.9). However, the loan supply (B_t^m) is derived by the equation below:

$$
v_t^s = \frac{\beta}{1+\beta}\left(Y_t - D_{t-1} - T_t^m\right) - \frac{\beta}{1+\beta}\left(p_t^k + \frac{p_{t+1}^k(1-\delta)}{\beta\,(1+r_t)}\right) K_t + \frac{T^o}{(1+r)(1+\beta)}
\tag{3.25}
$$

The aggregate demand now has two components, capital demand and bond demand. Equation (3.25) determines the bond demand in the economy, and

[11] Wage rigidity also exist in this model, but the wage norm is not binding in the inflationary environment.

equation (3.23) is the firm's capital demand. Thus, the aggregate demand and supply can be written as:

$$Y_t^s = A_t K_t^{1-\alpha} L_t^{\alpha} \tag{3.26}$$

$$Y_t^d = D + \frac{1+\beta}{\beta} B_g + \frac{1+\beta}{\beta} \frac{(1+g)}{1+r} D + p^k K \left(1 + \frac{1}{\beta} \frac{(1-\delta)}{1+r}\right) \tag{3.27}$$

When inflation is below unity, under the assumptions made, the zero lower bound and the bound on nominal wages is binding so that labor is rationed. Similar to the previous part, bellow equations determine the steady state.

$$L = \left(\frac{1 - \frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{1}{1-\alpha}} \bar{L} \tag{3.28}$$

$$K_t^{\alpha} = (1-\alpha) \frac{A_t L_t^{\alpha}}{r^k} \tag{3.29}$$

Similar to the full-employment environment, the aggregate supply is determined by equation (3.26). However, the zero lower bound alters the aggregate demand to the equation below:

$$Y_t^d = D + \frac{1+\beta}{\beta} B_g + \frac{1+\beta}{\beta} \Pi(1+g) D + p^k K \left(1 + \frac{\Pi}{\beta}(1-\delta)\right) \tag{3.30}$$

Similar to the baseline model, a change in the value of a parameter that increases the relative aggregate supply will decrease the real interest rate in the economy. The computational model in appendix B.3 uses this method to determine the real interest rate value at the steady-state.

The two parameters of interest in this section are the capital depreciation rate and the relative price of capital. Based on equations (3.25), (3.30) and (3.27), a decrease in the relative price of capital boosts the bond supply while it decreases the aggregate demand. This shock shifts the aggregate demand curve to the left, resulting in a lower real interest rate.

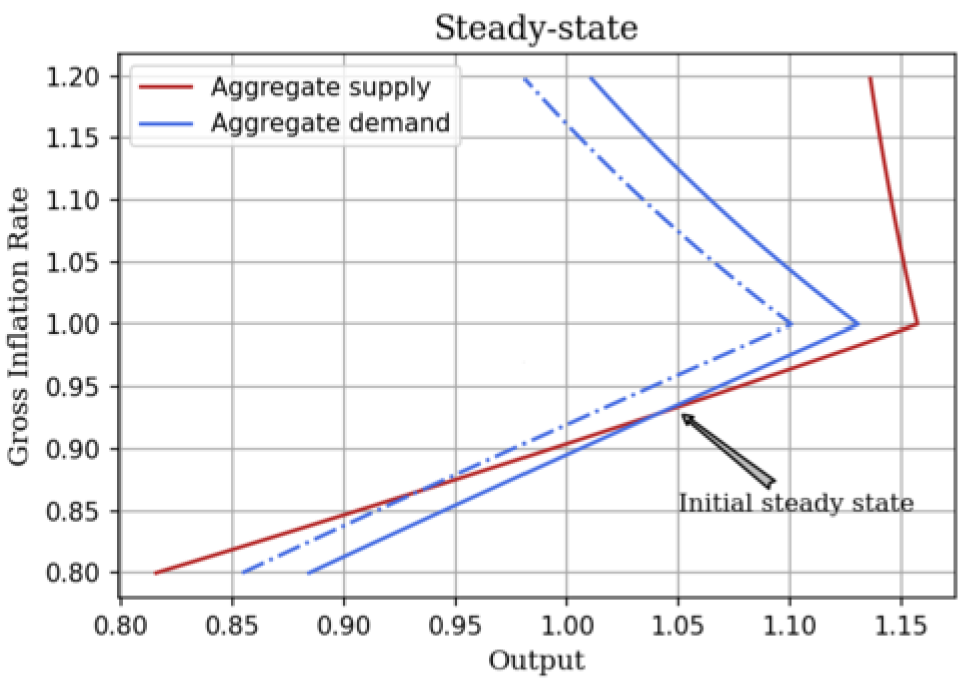


FIGURE 3.1: The relative price of capital shock (own calculations - source is available in appendix B.3)

Another experiment is a preference shock to less durable goods. This preference shock has a similar impact on the real interest rate and aggregate demand curve as a decrease in the relative price of capital. Quantitatively, I can show that by changing the value of δ and P^k in the computational model cited in appendix B.3. Two numerical examples are given below

TABLE 3.3: Parameter values

Description	**Symbol**	**Pre-shock value**	**Post-shock value**
Price of capital	P^k	0.35	0.3
Real interest rate	r_t	-0.016	-0.07
Depreciation rate	δ	0.79	0.7
Real interest rate	r_t	-0.016	0.04

Note: Each value change represents a distinct shock impulse response.

3.2.2 Inelastic capital demand

One observation from the Japanese experience is private capital being inelastic and unresponsive to policies. Eggertsson, Robbins, and Wold (2021) show that Tobin's Q has increased from 1 to 1.75 since 1970 in the United

States. Since Tobin's Q is the ratio of the asset's market value and its replacement cost, a Q value larger than one indicates the capital value exceeds the cost of obtaining it. However, The investment level is below the pre-crisis forecasts level in most developed countries (International Monetary Fund (2015)). These two facts create a puzzling case.

In this section, I study the convex cost function as one potential reason for the low investment rate in advanced economies. I capture this property in the model by adding convex adjustment cost. Consider the model in section 3.2.1 with new additions, first, firms rent capital to return the depreciated capital to old households, and they face a quadratic investment cost function, $\Lambda(K_{t-1}, I_t)$, similar to Caballero (1999).
The household budget constraints remain similar[12] to the previous section. However, the firm allocates the labor and capital following a modified profit function as it is facing new cost function.

$$Z_t = \max_{L_t} P_t Y_t - W_t L_t - P_t r_t^k K_t - \Lambda(K_{t-1}, I_t) \tag{3.31}$$

$$\text{s.t. } Y_t = A_t K_t^{1-\alpha} L_t^{\alpha} \tag{3.32}$$

$$K_{t+1} = (1-\delta)K_t + I_{t+1} \tag{3.33}$$

Assume $\Lambda(K_{t-1}, I_t) = P_t r_t^k (\frac{\alpha}{2})(\frac{I_t^2}{K_{t-1}})$, which implies that firms costs will increase disproportionately compare to their investment level. The following calculations represent the level of each variable in the steady state:

$$\begin{aligned} K_{t+1} &= (1-\delta)K_t + I_{t+1} \\ I_{t+1} &= \delta K_{t+1} \end{aligned} \tag{3.34}$$

$$Z_t = \max_{L_t} P_t Y_t - W_t L_t - P_t r_t^k K_t - P_t r_t^k (\frac{\alpha}{2})(\delta^2 K_t) \tag{3.35}$$

$$r_k = (1-\alpha)\frac{Y_t}{K_t + (\frac{\alpha}{2})\delta^2 K_t} \tag{3.36}$$

Adding adjustment cost to the model has two primary implications. First, equation (3.36) shows that the rental rate of capital is lower compared to equation (3.23). The investment to output ratio rises after the rental rate on

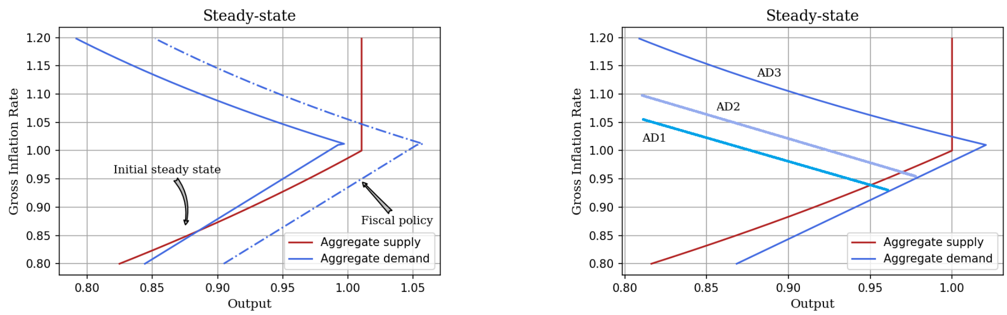
[12] In the base model, the firms rent the capital and give back the depreciated capital to the old household in the next period. However, it is possible that firms buy their capital and sell it back in the next period (or even borrow the not needed capital).

capital fall as shown in the equation below. Thus, the adjustment cost has no impact on the real interest rate at the steady state.

$$\frac{I}{Y} = \delta \frac{K}{Y} = \frac{\delta}{A} \left(\frac{\alpha A}{r_k} \right)^{1-\alpha} \tag{3.37}$$

However, in the transition path, adjustment cost can prolong the number required to reach the new steady-state as the cost function penalizes a large increase in investment. Hence, I conclude that the sluggish capital adjustment is not at the heart of the subpar investment level.

3.3 Fiscal and Monetary Policy



(A) Fiscal policy responses (B) Monetary policy responses

FIGURE 3.2: The policy implications (own calculations - source is available in appendix)

One takeaway from the results of the baseline model is that secular stagnation can be permanent in the absence of other exogenous shocks and interferences. In this scenario, the government has two tools, fiscal and monetary policy, to potentially push the economy into a more favorable state. The result of this section shows the different implications of the monetary and fiscal policy compared to the New Keynesian models of the liquidity trap.

According to the liquidity trap literature, the central banks can credibly commit[13] (check Krugman, Dominquez, and Rogoff (1998) for more on the topic of credibility) to an expansionary policy to depreciate the real interest rate and boost consumption in the current period. To test the effectiveness of this idea, I increase the inflation target Π^* to 1% and 5%, respectively. In the framework of this model, the higher inflation target allows for a higher

[13]The assumption here is that all the monetary policy attempts in this model are credible as I abstract from it.

policy response rate[14], which shifts the kink outward. The shift is evident on AD1, AD2, and AD3 curves in figure (2.4).

The success of such a policy, however, is dependant on the extent to which the central bank is willing to raise the inflation target. There exists a unique secular stagnation steady state in AD1, and the 1% boost in inflation target proves to be inadequate for creating the full-employment steady-state. After raising the target inflation by 6%, the AD3 curve intersects with the aggregate supply curve on three points, two of which are full-employment steady-states. In the first attempt, the conservative choice of a one percent inflation target increase fails to shift the kink far enough to intersect with the upper section of the aggregate supply curve. Krugman, Dominquez, and Rogoff (1998) coined the term ”law of the excluded middle” for this phenomenon.

The second observation from the experiment is the non-unique steady-states in the AD3 environment. Out of three intersections, the second one is considered locally indeterminate[15]. The two remaining intersections are locally determinate. Thus, the monetary policy did not eliminate the secular stagnation steady state.

Subsequently, to study the policy implications of fiscal policy in the form of lump-sum taxes, let us denote taxation for each generation by T_t^i so the household budget constraints can be written as:

$$C_t^y = B_t^y$$

$$C_{t+1}^m = Y_{t+1}^m - (1+r_t)\, B_t^y + B_{t+1}^m - T_{t+1}^m \tag{3.38}$$

$$C_{t+2}^o = Y_{t+2}^o - (1+r_{t+1})\, B_{t+1}^m - T_{t+2}^o \tag{3.39}$$

In addition to taxation, which could change the supply side, the government can also borrow from the middle-age household. This changes the asset market clearing (2.8) to:

$$\begin{aligned} N_t B_t^y + N_{t-1} B_t^g &= -\, N_{t-1} B_t^m \\ (1+g_t) B_t^y + B_t^g &= -\, B_t^m \end{aligned} \tag{3.40}$$

Where B^g denote the government debt. The government ability to borrow could be a new leverage to raise the real interest rate as it increases loan demand given below equation

[14]According to the monetary policy rule, changes in interest rate correspond more than one to one to changes in the real interest rate. An increase in Π^* raises that ratio.

[15]There exists an infinite number of initial price levels $P_0 > 0$ consistent with a perfect-foresight equilibrium.

$$\begin{aligned} L_t^d &= (1+g_t)B_t^y + B_t^g \\ &= (1+g_t)\frac{D_t}{(1+r_t)} + B_t^g \end{aligned} \tag{3.41}$$

$$v_t^s = \frac{\beta}{1+\beta}(Y^m - D - T^m) - \frac{1}{1+\beta}\frac{Y^o - T^o}{1+r} \tag{3.42}$$

In this setting, the government finances its spending through two distinct mechanisms, which are taxation and borrowing from households. Finally, the government budget constraint closes the systems for determining the asset market equilibrium.

$$T^m + B^g + \frac{1}{1+g}T^o + (1+g)T^y = G + (1+r)\frac{1}{1+g}B^g \tag{3.43}$$

Here, G denotes government spending normalized to the size of middle-age generation. The government has the ability to alter both loan supply and demand through new mechanisms. The government can increase the loan demand by borrowing from households and control the supply through taxation and redistribution. A fiscal policy regime is consistent with the government choice for the distribution of taxation and government spending for each period. Consider now a debt-preserving fiscal regime where tax on the young and government spending is equal to zero, and government debt remains at a certain level. Further, the relation between taxes on the middle-aged and the old, which insures the debt-preserving regime, is:

$$T^m = \frac{1}{\beta}\frac{1}{1+r}T^o \tag{3.44}$$

The value of either parameters can be exogenously determined as long as it satisfies the previous equations. To determine the full employment real interest rate, I define a fiscal policy and substitute it in (3.43) and then equate loan supply and demand equations[16].

$$Y = D + T^m + \frac{1+\beta}{\beta}B^g + \left(\frac{(1+g)(1+\beta)}{\beta}D - \frac{1}{\beta}T^o\right)\Pi \tag{3.45}$$

In the current specification of the fiscal policy, equation (3.44) eliminates

[16]Assume, $i = 0$ and $T_m = B_g$ to satisfy the debt-preserving policy.

the effect of policy on loan supply, implying the government can shift the aggregate demand curve by permanently increasing its debt level. The aggregate demand equation confirms the fact that the curve only shifts to the right as the government increases its debt level. Another important observation is the debt level being permanent. A temporary increase in the government debt level can not increase the steady-state interest rate as it does not change the expectation of the middle age households.

Previously, I considered the case where a successful fiscal policy can lift the economy out of secular stagnation. I now extend the fiscal policy to a scenario with income inequality similar to section 3.1. A generalized fiscal policy changes the budget constraint of high-income households born at time t to:

$$C^{m,h}_{t+1} = Y^{m,h}_{t+1} - (1+r_t)\,B^{y}_{t} + B^{m,h}_{t+1} + \frac{Q_{t+1}}{1+g_t} - T^{m,h}_{t+1} \tag{3.46}$$

$$C^{o,h}_{t+2} = -\,(1+r_{t+1})\,B^{m,h}_{t+1} - Q_{t+2} - T^{o,h}_{t+2} \tag{3.47}$$

Similarly, for low-income households, their budget constraints can be written as:

$$C^{m,l}_{t+1} = Y^{m,l}_{t+1} + \frac{D_{t+1}}{1+r_{t+1}} - D_{t+1} - T^{m,l}_{t+1} \tag{3.48}$$

$$C^{o,l}_{t+2} = Y^{o,l}_{t+2} - D_{t+2} - T^{o,l}_{t+2} \tag{3.49}$$

It is evident that the government can alter the supply and demand of bonds by taxing different income groups. Taxing high-income middle age will decrease the real interest rate by reducing bonds supply, while any transfer to low-income counterparts boosts the demand. The government's budget constraint follows the bellow equation in the equilibrium.

$$\eta T^{m,l} + (1-\eta)T^{m,h} + B^{g} + \frac{(\eta T^{o,l} + (1-\eta)T^{o,h})}{1+g} + (1+g)T^{y} = G + \frac{(1+r)B^{g}}{1+g} \tag{3.50}$$

Consider now a similar debt-preserving fiscal regime where tax on the young, old and government spending is equal to zero, and government debt remains at a certain level. Further, the relation between taxes on the middle-aged low-income and high-income households, which insures the debt-preserving regime, is:

$$T^{m,l} = \frac{(1-\eta)}{\eta} T^{m,h} \tag{3.51}$$

By redistributing income, this fiscal policy is lowering the loan supply and boosting the aggregate demand, which results in a higher real interest rate. Thus, a successful redistributive tax regime can provide a sufficient stimulus to pull the economy out of a secular stagnation equilibrium. In table 3.4, I compared the government purchases multiplier of two policies (with and without inequality) at the zero lower bound.

TABLE 3.4: Government purchases multiplier at zero lower bound

Financing	Multiplier	Value	Multiplier	Value
Increase in public debt	$\frac{1+\beta}{\beta}\frac{1}{1-\zeta\psi}$	>2	$\frac{1+\beta}{\beta}\frac{(1-\eta)}{s^h\eta-(1+\eta)\zeta\psi}$	>2
Tax on young generation	0	0	0	0
Tax on middle generation	$\frac{1}{1-\zeta\psi}$	>1	$\frac{(1-\eta)}{s^h\eta-(1+\eta)\zeta\psi}^{(**)}$	>1
Tax on old generation	$-\frac{1+g}{\beta}\frac{1}{1-\zeta\psi}$	<0	$-\frac{1+g}{\beta}\frac{(1-\eta)}{s^h\eta-(1+\eta)\zeta\psi}^{(**)}$	<0

Note: (**) indicates a policy that target high-income households.

I linearize the aggregate supply and demand curve using an assumption that an increase is not significant enough to boost the economy out of the ZLB episode to derive the above results. Further, $\zeta = \frac{1-\alpha}{\alpha}\frac{1-\gamma}{\gamma}$, $\psi = \frac{1+\beta}{\beta}(1+g)D$.

3.4 Risk

The previous sections abstract from the inclusion of risk and risk preferences in the models. Similarly, in the liquidity traps literature, the asset shortage that leads to zero lower bound episodes stems from an exogenous increase in the propensity to save (see, e.g., Krugman, Dominquez, and Rogoff (1998), Eggertsson and Woodford (2003), Christiano, Eichenbaum, and Rebelo (2011), and Werning (2011)). More sophisticated models (such as Eggertsson and Krugman (2012)) focus on the tightening collateral constraint and its effect on the natural rate.

Caballero and Krishnamurthy (2009) , Bernanke et al. (2011) identify that the economy suffers from a safe-asset shortage. This key insight becomes crucial as Barro et al. (2014) shows the relationship between safe assets and GDP within a heterogenous risk aversion environment. In this body of literature, a shortage of safe assets can lead to similar results as in the liquidity traps literature. Abel et al. (1989) demonstrates how accounting for aggregate risk can potentially lead to a negative risk-free interest rate, while the average and marginal return from capital (net of depreciation) remain positive.

Work of J Caballero and Farhi (2018) also utilizes stylized stochastic overlapping generations model to study the relationship between safe asset shortage and endogenous risk premia. They infer that safety traps can be arbitrarily persistent in the presence of a zero lower bound reinforcing the supply side of the secular stagnation hypothesis. Caballero, Farhi, and Gourinchas (2015) build on their previous work by extending the model to include open economy elements. Specifically, they question how heterogeneous capital markets within the framework of an overlapping generations model with nominal rigidities affect demand for and supply of financial assets across markets. A primary assumption of their work is agents with heterogeneous risk aversion. Locally infinite risk aversion agents who always demand safe assets deepens an endogenous risk premium in the Uncovered Interest Parity condition, which creates a possibility of an asymmetric safety trap equilibrium, concluding portion of countries may experience a secular stagnation.

3.4.1 Intergenerational Risk

A straightforward method to add risk into the model is to consider an event that results in consumption loss for old households. Let us denote the event of income loss, which is an exogenous time-varying constant, by H, which is the level of consumption loss for the affected group. The event H has the occurrence probability of ρ_H, where $0 < \rho_H < 1$. This modification implies that a portion of old households suffers from an income loss. Assuming the households can foresee this income loss, then the two groups budget constraints are:

$$C_t^{o,l} = B_{t-1}^m \frac{1+i_{t-1}}{\Pi_t} - H \tag{3.52}$$

$$C_t^{o,h} = B_{t-1}^m \frac{1+i_{t-1}}{\Pi_t} \tag{3.53}$$

Based on Euler equation, middle age households have to save more to account for generational risk. This change increases the loan supply and decreases the real interest rate.

$$\frac{1}{C_t^m} = \beta \frac{1}{(1+r_t)B_{t-1}^m - (\rho_H * H)} \frac{P_t\,(1+i_t)}{P_{t+1}} \tag{3.54}$$

The middle-aged decide their optimal level of saving based on the weighted average of the possible events. The $(\rho_H * H)$ represents the weighted future income loss that households compensate by boosting their savings. In this scenario, the aggregate demand remains similar to the base model while the aggregate supply increases. Thus, an increase in $(\rho_H * H)$ will put downward pressure on the real interest rate in the economy. Further, the households that do experience event H have made a sub-optimal saving decision. One solution is intergenerational risk-sharing, where the government tax the middle-aged and redistributes it to affected old households. This fiscal policy ensures that aggregate loan supply does not increase and households make optimal decisions.

The results[17] of this section further support the proposition that intergenerational risk can intensify the issues around excess saving, leading to a lower real interest rate. I show how qualitatively a decrease in each parameter of interest affects the real interest rate in this section in the table below. Furthermore, I demonstrate how increasing the initial parameter values by 5% quantitatively impact the gross real interest rate.

TABLE 3.5: Parameter effects

Description	**Symbol**	**Qualitative effect**	**Quantitative effect**
Future income loss	H	$\uparrow$	+0.294%
Occurrence probability	ρ_H	$\uparrow$	+0.294%

Source: initial calibration and computation codes are available in appendix B.2.1

3.4.2 Aggregate Risk

This section focuses on how the shortage of safe assets can put downward pressure on the real interest rate in the framework of an endowment economy with aggregate risk and heterogeneity in the level of risk aversion. The model is inspired from J Caballero and Farhi (2018) but within a three-period overlapping generations model of the base model.

The base model for this section is a three-period overlapping generations model, where agents work and consume in the subsequent period, and the

[17] A more sophisticated model is to depart from the full-commitment and no default case. In this scenario, a group of middle-aged households with a probability decide not to pay back their debt from the previous period. As both models highlight a mechanism of future income loss, the results are exact.

endowment lasts only for one period. These assumptions imply that utility-maximizing households have to use all the endowments from the previous period. Furthermore, there exists an exogenous time-varying constant Y, which denotes the total output in the economy similar to the baseline model. However, the output is subject to aggregate shocks represented by μ, which is the realization of a Poisson process with the Dirichlet process modeling the intensity. The fraction of the population who consumes at the end of a period is equal to:

$$\eta = \frac{1 + (1 + g)}{1 + (1 + g) + (1 + g)^2}$$

Here, g is the population growth rate. Thus, at the end of the period, middle age and old households consume η fraction of total wealth in the economy. As this analysis is focused on the steady-state, the market-clearing condition for the goods market implies that:

$$\kappa_t = \frac{Y_t}{\eta} \tag{3.55}$$

In this scenario, old and middle age consume part of the total wealth, and young and middle-aged households create their portfolio at the end of a period. There are two types of households, risk-neutral and Knightian[18] (infinitely risk-averse over short time intervals), during the formation of their portfolio. The fraction of Knightians in each generation is α. The wealth of Knightians indicated by κ^K and neutrals by κ^N.

Next, assume that ρ is the securitization capacity of the economy. Neutrals act as financial intermediaries who manage Lucas trees issue safe assets backed by risky assets. However, financial friction is present in the economy as there is a limit to securitization. The supply of safe assets before the shock at each period is given below:

$$V^s = \rho\mu\frac{Y_t}{\eta} \tag{3.56}$$

This equation links the supply of safe assets to the securitization capacity and the shock level. I also assume that only δ fraction of output is pledgable, and the rest belong to the young. Subsequently, based on the clearing condition

[18] Knightians are the group who always demand safe assets.

of the financial market, the supply of risky assets[19] before the shock is equal to:

$$V^r = (1-\rho)\mu\frac{Y_t}{\eta} \tag{3.57}$$

The supply of safe and risky assets before the realization of the Poisson event is characterized by:

$$V^S = \frac{\delta^S Y}{r^S} \tag{3.58}$$

$$V^r = \frac{(\delta - \delta^S)Y}{r} \tag{3.59}$$

Where r^S, δ^S and r are rate of return on safe assets, the dividend paid by safe assets and risky assets, respectively. Safe assets can be regarded as safe short-term debt that households roll over each period with rate of return r^S. The change of wealth for each group at an infinitely short moment after the creation of portfolios are:

$$\kappa^K_{t+dt} - \kappa^K_t = \dot{\kappa}^K_t = -\eta\kappa^K_t + \alpha(1-\delta)X + r^S\kappa^K_t \tag{3.60}$$

$$\kappa^N_{t+dt} - \kappa^N_t = \dot{\kappa}^N_t = -\eta\kappa^N_t + (1-\alpha)(1-\delta)X + r\kappa^N_t \tag{3.61}$$

κ^j_t, $j \in \{K, N\}$ is the level of wealth consumed by each group. Knightians also increase their wealth by $\alpha(1-\delta)X$ and collect interest rates $r^S\kappa^K_t$. Thus, the equilibrium in the economy can be shown as:

$$\kappa^K_t + \kappa^N_t = V^S + V^r \tag{3.62}$$

Now, there are two distinct equilibria based on the value of safe assets in the market. If the supply of safe assets is larger than the wealth of Knightians then the rates in the economy are as below:

$$r = r^S = \delta\eta \tag{3.63}$$

Notice that the rate of return is equal on all assets regardless of the type of marginal holder. However, if the supply of safe assets falls below the threshold, then the entirety of safe assets will be held by Knightians regardless of

[19]Assume $\rho > \alpha$, which ensures that the supply of safe assets is not less than the wealth held by Knightians.

their rate of return [20].

$$r^S = \delta\eta - (1-\delta)\eta\frac{\alpha - \rho\mu}{\rho\mu} < \delta\eta < \delta\eta + (1-\delta)\eta\frac{\alpha - \rho\mu}{1-\rho\mu} = r \tag{3.64}$$

It is evident that a safe asset shortage can put downward pressure on the safe assets rate of return. Based on equation (3.64), the decrease in rates of returns is related to multiple factors. A powerful shock can lower the supply of safe assets to the point where $\alpha > \rho\mu$. In this scenario, as marginal holders of safe assets are Knightians, the lower V^S commands a lower return on such assets. An event similar to the 2008 financial crisis, where a group of assets was deemed to be safe before the crisis and not after it, can lower the securitization capacity and the supply of safe assets. Lastly, unlike the base model, an increase in g_t can reduce the rate of return in the economy as the suppliers are middle age and old households.

I am not moving away from the endowment economy in this section. However, an interesting thought experiment is to consider an economy with a zero lower bound and safe asset shortage. Looking at equation (3.64), it is possible that the return on safe assets falls below zero. If there is no zero lower bound, the economy moves along the demand curve to find the equilibrium similar to the endowment economy. However, the introduction of the zero lower bounds prevents the interest rate from falling below zero like the base model. In this hypothetical situation, since the zero lower bound is binding, the equilibrium in the asset market can only be attained by a reduction in output. The economy remains in this deflationary environment permanently until a shock or policies boost V^S. The conclusion of this thought experiment is in line with the secular stagnation literature and the models in the previous sections.

[20] In a similar fashion, the dividend paid by safe assets also decreases to $\delta^\mu = \delta\rho\mu - (\alpha - \rho\mu)(1-\delta)$.

3.5 A Quantitative Life Cycle Model

In the following, I study the real interest rate of United States economy in 2009 using a medium-scale overlapping generations model. Households enter the market at age $y = 18$ and die at $y = 78$[21]. At the age of 18, households are randomly assigned to a skill group S_i, where θ_i, $i \in \{1, 2, 3, 4, 5\}$, represent the population in each group subject to $\sum \theta_i = 1$. Furthermore, I discretize the state space of heterogeneous deterministic lifetime skills to $S_i = (s_{18,i}, \ldots, s_{78,i})$. All households maximize their lifetime utility using a similar function to:

$$\max \sum_{s=1}^{J} \beta^{y-1} \left(\Pi_{j=1}^{y} \phi_{t+j-1}^{j-1} \right) \mathbb{E}_t \left[u(c_{t+y-1}^{y}, l_{t+y-1}^{y}) \right], \tag{3.65}$$

Where $\phi_t = \phi = 1$ as I abstract from population growth in this section. Instantaneous utility $u(c, 1-l)$ is specified as a function of consumption c and and leisure $1 - l$:

$$u\left(c_{s,y,t}, 1 - l_{s,y,t}\right) = \frac{c_{s,y,t}^{1-\sigma} - 1}{1-\sigma} + \chi_s^n b \left[1 - \left(\frac{1 - l_{s,y,t}}{\tilde{l}} \right)^{v} \right]^{\frac{1}{v}} \tag{3.66}$$

Unlike the base model, the labor in the quantitative section is endogenous. I chose the endogenous setup after realizing that the endogeneity of labor supply can put upward pressure on the real interest rate during my early experiment with the quantitative frameworks. Hence, the households derive utility both from leisure and labor[22]. Household work until they retire at the age of 65. After retirement, they only rely on the savings from previous periods. The household budget constraint is equal to:

$$c_{s,y,t} + \xi_t b_{s,y+1,t+1} = \left(r_t^k + \xi_t (1 - \delta) \right) b_{s,y,t} + w_t S_{y,i} n_{s,y,t} \tag{3.67}$$

I also assume that households enter the market with no initial wealth and have no wealth at the moment of death, implying $b_{t,17} = b_{t,79} = 0$. Furthermore, households can invest a portion of their income by purchasing

[21] Based on CDC report, Murphy, Xu, and Kochanek (2013), on the life expectancy in the US.

[22] The reasoning behind choosing the standard constant Frisch elasticity disutility of labor supply function model are the computional ease and significant lower computation time compare to similar labor-leisure models.

capital goods with the exogenous relative price of ξ_t. Firms will pay a return of r^k_{t+1}, which is the rental rate of capital, and capital has a resell value (net of depreciation) of $\xi_{t+1}(1-\delta)$. The firms maximize profits following a Cobb-Douglas[23] production function using both labor and capital to produce output Y_t at period t.

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}. \tag{3.68}$$

If the economy is in the steady-state, based on the market-clearing conditions, the labor and capital supply are equal to:

$$\begin{aligned} L_t &= \sum_{s=1}^{S} \sum_{y=18}^{Y} \theta_y S_{y,i} n_{j,s,t} \\ K_t &= \sum_{s=2}^{S} \sum_{y=18}^{Y} \theta_y b_{y,s,t} \end{aligned} \tag{3.69}$$

The first equation implies that the aggregate labor supply is proportional to the productivity and population of each skill. Similarly, the capital supply is a function of the saving decisions of each skill group and their respective population.

To solve the model computationally, I utilize the Auerbach and Kotlikoff method (Auerbach and Kotlikoff (1987)) to determine the transition path by recursively solving the policy functions through iterating on the entire transition path of the endogenous variables.

The focus of calibration is the United States economy in 2009. The set value for the parameters comes from two references. The statistical data about US demographics and productivity growth comes from government agency sources. Next, a portion of parameters is taken from the related literature.

[23]The result between CES and Cobb-Douglas functions were similar in my early experiments. Hence, I prioritized the function with less computation time.

Panel A: Data	Symbol	Value	Source
Mortality profile	Υ		US mortality tables, CDC
Income profile	$S_{y,i}$		Continuous Work History Sample
Population distribution	λ		Schwabish and Topoleski (2013)
Total fertility rate	n	1.88	UN fertility data
Productivity growth	g	0.65%	Fernald (2012)
Elliptical disutility of labor scale	b_s	0.501	Evans and Phillips (2017)
Elliptical disutility of labor shape	υ	1.554	Evans and Phillips (2017)
Panel B: Related literature			
Elasticity of intertemporal substitution	ρ	0.75	Gourinchas and Parker (2002)
Depreciation rate	δ	12%	Jorgenson (1996)
Parameters chosen to match targets	Symbol	Value	
Rate of time preference	β	0.98	
Borrowing limit (% of annual income)	D	23.4%	
Capital share parameter	α	0.24	

The result of the calibration shows that a permanent negative interest rate with parameters in the above table is possible. The real interest rate of steady-state is approximately -1.14%, which is 22%[24] higher than a similar model with an exogenous labor supply. This discrepancy proves my earlier point that abstracting from endogenous labor supply may result in exaggerated outcomes.

Lastly, the model approximation of the steady-state real interest rate is strikingly close to the average of the real equilibrium rate estimates in figure 1.2. As many in the literature, such as Eichengreen (2015), pointed out the negative connection between the relative price of capital and the natural rate of interest. Furthermore, the model confirms the inverse relationship between the relative price of capital, income inequality, and the real interest rate.

[24]In the model with exogenous labor supply, bequest motive and population dynamic the real interest rate is around 1.47%, while following a quite similar quantitative calibration of values. Another factor that could possible put downward pressure on the real interest rate in the Eggertsson, Mehrotra, and Summers (2016) is including stochastic elements (as evident by the result of section 3.4).

3.6 Conclusion

In this paper, I identified the various types of distributional effects altering the real interest rate in the economy and studied their impact. A consistent result throughout the project is the persistent deflationary steady state is a possibility in the absence of effective policy or shocks. Nevertheless, as Summers (2018) points out secular stagnation hypothesis is not an apocalyptic perception that developed economies are doomed to remain stagnant at high levels of unemployment; as this work, in line with the current literature, put forward effective policies that can push the economy into a desirable state.

Another takeaway is the ambiguous effect of formalizing issues related to capital, inequality, and risk into the overlapping generations model. There is no general result that defines how each of the additions affects the model. One reason is the vast scope of methods, which one can incorporate capital, inequality, and risk into the model. For example, a form of inequality that raises the loan demand can increase the real interest rate in the economy. However, it is possible to consider forms of inequality that intensify the issues of excess saving, leading to a lower natural rate. The more striking result is that an increase in population growth can put downward pressure on the real interest rate based on the aggregate risk model. These thought-provoking outcomes are areas for further quantitative studies.

My work in this paper also demonstrates the great challenge that policymakers face in addressing secular stagnation. The result of incorporating a range of features and structural issues into the base model is a testament to how distinct policy recommendations should be. There is no "silver bullet" policy to boost to economy into a more favorable state without identifying the case-specific causes.

Lastly, Eichengreen (2015) connects the fall of the relative price of capital to the secular stagnation hypothesis. He argues that the economy commits less of its GDP to maintain the same level of investment, which intensifies the issue of excess savings. The quantitative model of the United States economy shows that an increase in income inequality and a fall in the relative price of capital result in a decline in the real interest rate in the economy. The quantitative results further confirm the qualitative outcomes derived in this project and the existing literature.

Appendix A

Proof

Proposition 1. If $\gamma > 0, \Pi^* = 1$, and $i^* = r^J < 0$, then there exists a unique determinate secular stagnation equilibrium.

We can write down the aggregate supply and demand as:

$$\begin{aligned} Y_{AD} &= D + \psi\Pi \\ Y_{AS} &= \left(\frac{1-\frac{\gamma}{\Pi}}{1-\gamma}\right)^{\frac{\alpha}{1-\alpha}} Y^f \end{aligned} \tag{A.1}$$

The second derivative of inflation with respect to total output will result the following:

$$\begin{aligned} \frac{d^2\Pi}{dY^2} &= G(Y)\left((1+\phi)(1-\gamma)\left(\frac{Y}{Y^f}\right)^{\phi} + (\phi - 1)\right) \\ G(Y) &= \frac{\phi\gamma(1-\gamma)\left(\frac{Y}{Y^f}\right)^{\phi}}{Y^2\left(1-(1-\gamma)\left(\frac{Y}{Y^f}\right)^{\phi}\right)} \\ \phi &= \frac{1-\alpha}{\alpha}. \end{aligned} \tag{A.2}$$

Thus, the slope of the aggregate supply curve at equilibrium is equal to:

$$\frac{d\Pi}{dY} = \frac{1-\alpha}{\alpha}\frac{1}{\gamma}\frac{\Pi}{Y}(\Pi - \gamma) \tag{A.3}$$

If the slope of the aggregate supply curve is less than the slope of the aggregate demand curve at the intersection point, then it must be the case that

$$\begin{aligned} \frac{1-\alpha}{\alpha}\frac{\Pi}{Y}\left(\frac{\Pi}{\gamma} - 1\right) &< \psi^{-1} \\ \frac{1-\alpha}{\alpha}\frac{\psi\Pi}{Y}\left(\frac{\Pi}{\gamma} - 1\right) &< 1 \\ \frac{1-\alpha}{\alpha}\frac{Y-D}{Y}\left(\frac{\Pi}{\gamma} - 1\right) &< 1 \end{aligned} \tag{A.4}$$

$$s_y \frac{\alpha}{1-\alpha} + 1 > \frac{\Pi}{\gamma}$$
$$\frac{\gamma}{\Pi}\left(s_y \frac{\alpha}{1-\alpha} + 1\right) > 1 \qquad \blacksquare$$

Thus, the determinacy is a required condition for a unique secular stagnation steady state.

Appendix B

Computation and Calibration

In the following section, I present my original codes in python and the calibrations for each numerical simulation. The Python files are also available on my GitHub directory of the project.

B.1 Endowment Economy

I consider a shock to the bond supply and another shock to the bond demand in the endowment economy (section 2.1). The parameter values and the python codes are given below.

TABLE B.1: Parameter values

Description	**Symbol**	**Pre-shock value**	**Post-shock value**
Rate of time preference	β	0.98	0.98
Collateral constraint	D_t	0.28	0.28
Population growth rate	g_t	0.25	0.19
Total endowment	Y_t	1.01	0.98

B.2 Equilibrium in the baseline model

This section presents the numerical computation for sections 2.2 to 2.6. There are four python files for each aspect of the economy. The parameter values and the python codes are given below.

TABLE B.2: Parameter values

Description	**Symbol**	**Pre-shock value**	**Post-shock value**
Rate of time preference	β	0.985	0.985
Wage adjustment	γ	0.3	0.21
Gross inflation target	Π^*	1.01	1.01
Taylor coefficient	ϕ^π	2	2
Labor supply	$\bar{L}$	1	0.95
Labor share parameter	α	0.7	0.7
Collateral constraint	D_t	0.28	0.265
Population growth rate	g_t	0.25	0.19

Note: Each value change represents a distinct shock impulse response.

B.2.1 Intergenerational Risk

This section presents the numerical computation for sections 3.4.1. This is an additional python file[1] to the already existing ones. The new parameter values and the python codes are given below.

TABLE B.3: Parameter values

Description	**Symbol**	**Value**
Rate of time preference	H	0.1
Wage adjustment	ρ_H	15%

B.3 Incorporating Capital

This section presents the numerical computation for section 3.2. Similar to the previous baseline model, there are four python files. The codes for calculating the steady-state are omitted as they are identical to the previous section. The parameter values and the python codes are given below. The Python files are also available on my GitHub directory of the project.

[1]You can add risk to the main file by substituting the below code with its counterpart in the previous section.

TABLE B.4: Parameter values

Description	Symbol	Value
Rate of time preference	β	0.9
Wage adjustment	γ	0.25
Gross inflation target	Π^*	1
interest rate target	i^*	0
Taylor coefficient	ϕ^π	2
Labor supply	$\bar{L}$	1
Price of capital	P_k	0.35
Capital depreciation rate	δ	0.79
Labor share parameter	α	0.7
Collateral constraint	D_t	0.21
Population growth rate	g_t	0.2

B.4 The Quantitative Model

The numerical simulation for computing the steady-state and transition path of the quantitative section consists of over a thousand lines of code. Thus, I only highlight features and elements of the computational model here. However, the complete version is available on my GitHub.

B.4.1 Steady-State high-level algorithm

As part of the high-level algorithm, I am using to solve the quantitative model utilizes a bisection method in the outer loop guesses for steady-state aggregate capital and labor. In the inner loop, I use a series of root finders to find the remaining endogenous variables for each household. Below, I highlight this algorithm:

- Make an initial guess for aggregate labor and capital. With the initial guesses, the algorithm uniquely determines the real interest rate and wage in the economy.
- The inner loop algorithm uses a series of root finders and household saving and labor supply Euler equations to derive consumption, labor supply, and savings for each type of household by using the variables from the outer loop.
- The algorithm uses the bisection method to update the aggregate capital and labor with a convex combination of the initial guess and values from the inner loop. The updating continues until the difference

between the initial and updated guesses is less than a predetermined small value.

B.4.2 Transition path high-level algorithm

The high-level algorithm for solving the transition path is to an extent similar to the previous section with the addition that it assumes that the economy will eventually reach a steady state. Below, I highlight this algorithm:

- Given initial aggregate labor and capital, the algorithm guesses two-transition paths (one for each variable).
- Using the aggregate values, it calculates the remaining optimized endogenous variables for each period.
- The algorithm uses convex combinations of the initial paths and values from the inner loop to update the outer layer. The updating continues until the difference between the initial and updated transition-paths are less than a predetermined small value.

B.4.3 Additional information:

TABLE B.5: Quantitative model code info

Description	**info**
Lines of code	$\approx$ 1200
Steady-state computation time	4:30 min
Transition path computation time	28 min
Method of interpolation	Quadratic

System spec for computation time: 4GB of ram and AMD Phenom X3 CPU

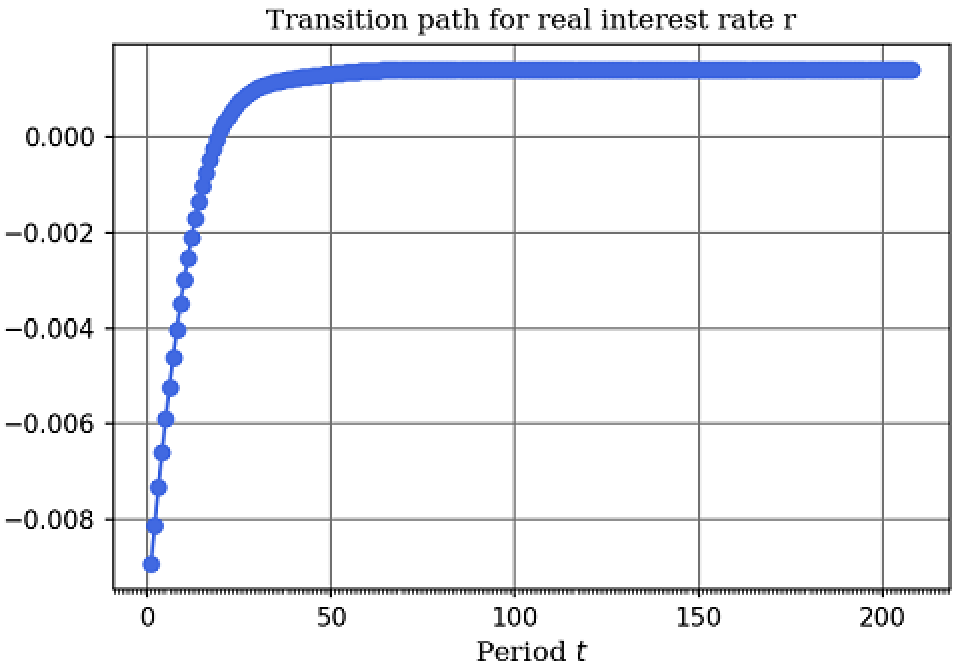


FIGURE B.1: The real interest rate transition path after a preference shock.

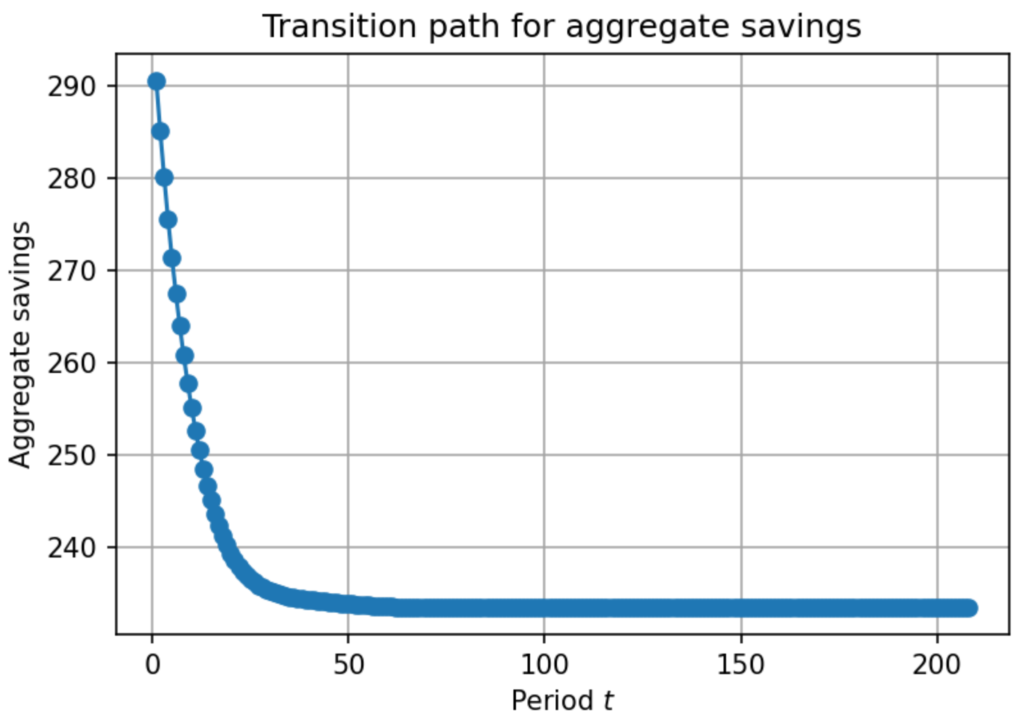


FIGURE B.2: The aggregate saving transition path after a preference shock.

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