

Required Exercises

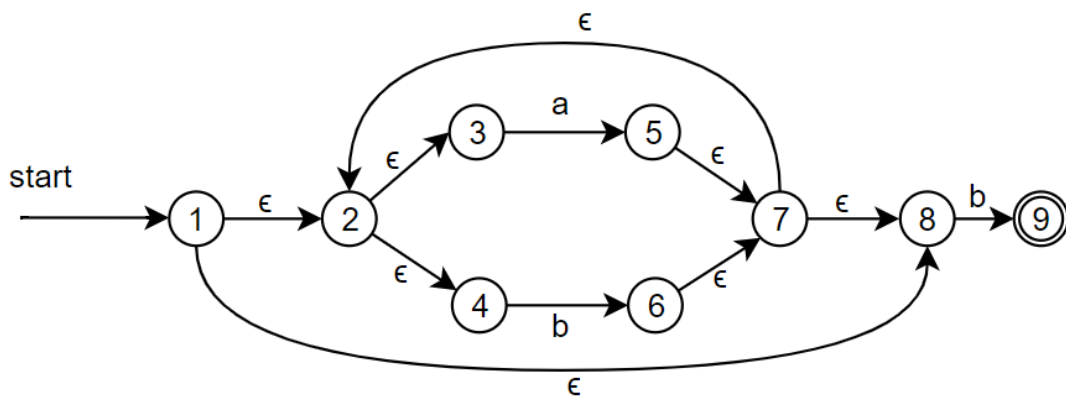
Exercise 1

Design NFAs and DFAs to recognize each of the following regular languages:

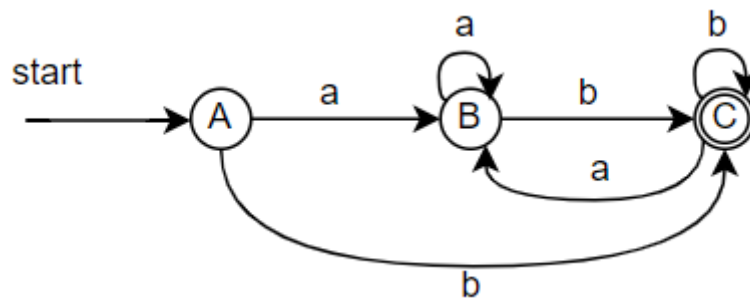
1. $L((a|b)^*b)$ [10 points]
2. $L(((\epsilon|a)^*b)^*)$ [10 points]
3. $L((a|b)^*a(a|b)(a|b))$ [10 points]
4. $L(a^*ba^*ba^*ba^*)$ [10 points]

1. $L((a|b)^*b)$

NFA:

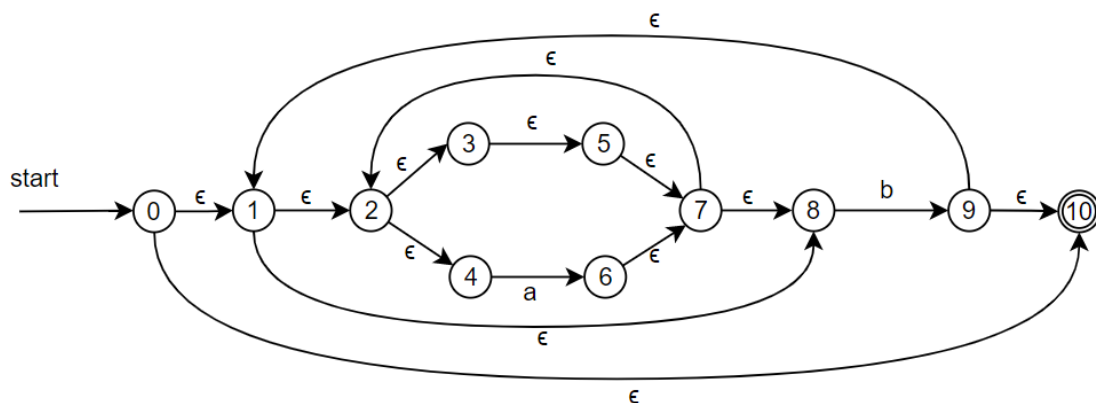


DFA:

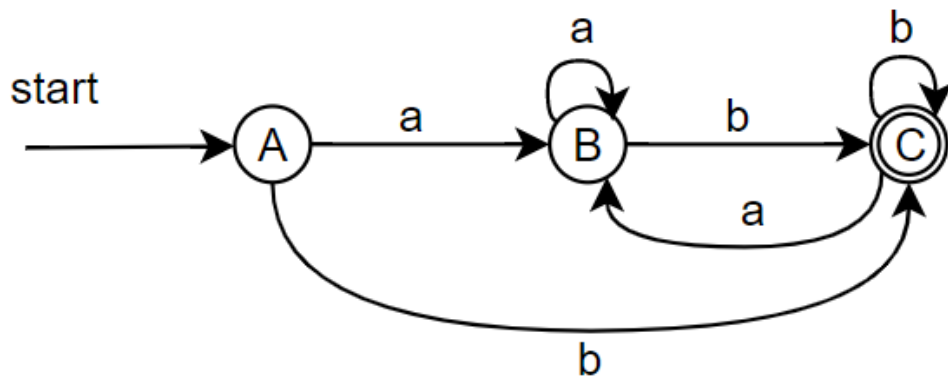


2. $L(((\epsilon|a)^*b)^*)$

NFA:

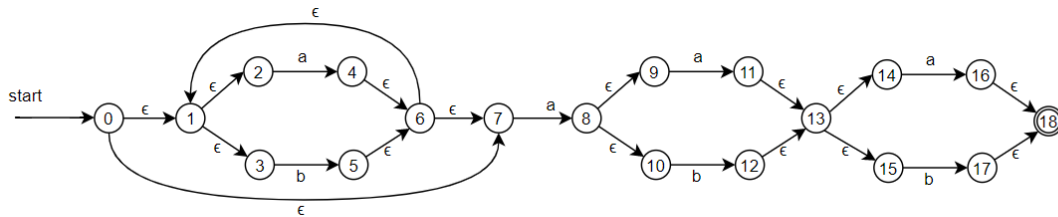


DFA:

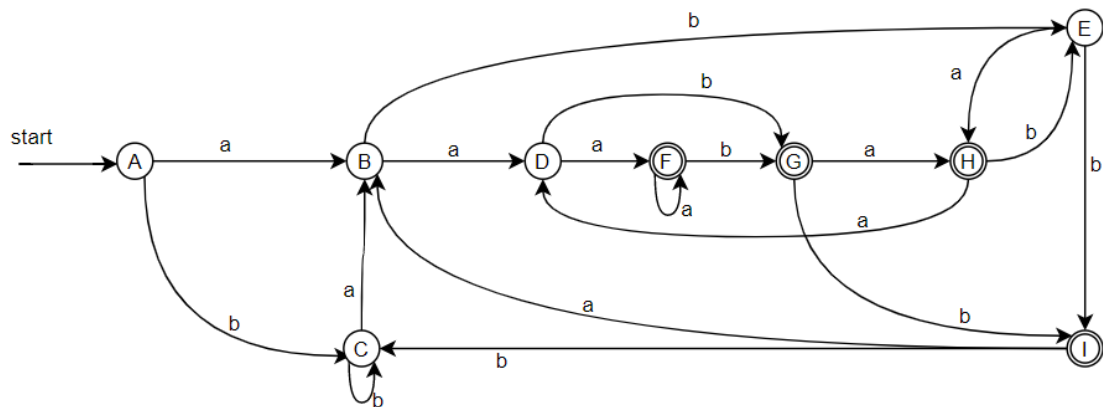


3. $L((a|b)^*a(a|b)(a|b))$

NFA:

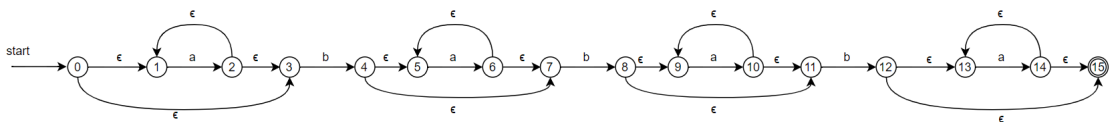


DFA:

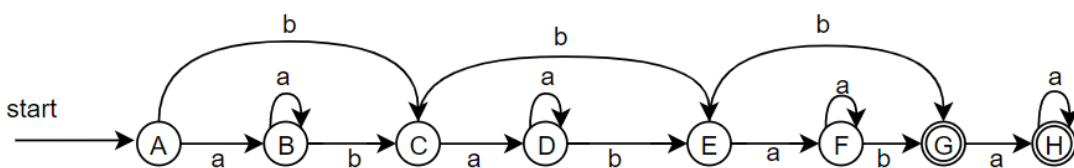


4. $L(a^*ba^*ba^*ba^*)$

NFA:



DFA:



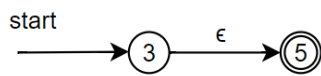
Exercise 2

Convert the following regular expressions to NFAs using the Thompson's Construction Algorithm (Algorithm 3.23 in the dragon book). Please put down the detailed steps.

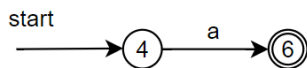
1. $((\epsilon|a)^*b)^*$ [10 points]
2. $(a|b)^*a(a|b)(a|b)$ [10 points]
3. $a^*ba^*ba^*ba^*$ [10 points]

1. $((\epsilon|a)^*b)^*$

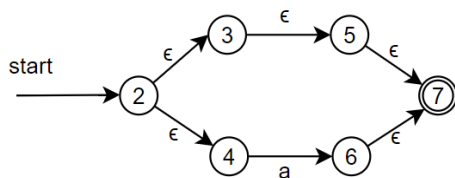
NFA for the first ϵ :



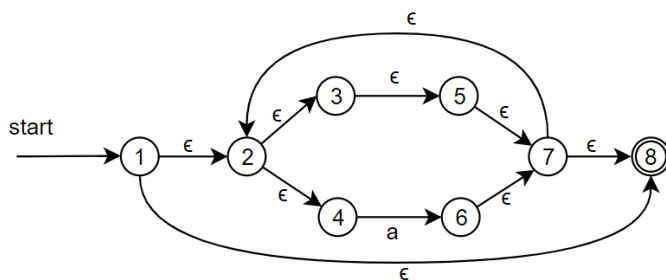
NFA for the first a :



NFA for $(\epsilon|a)$:



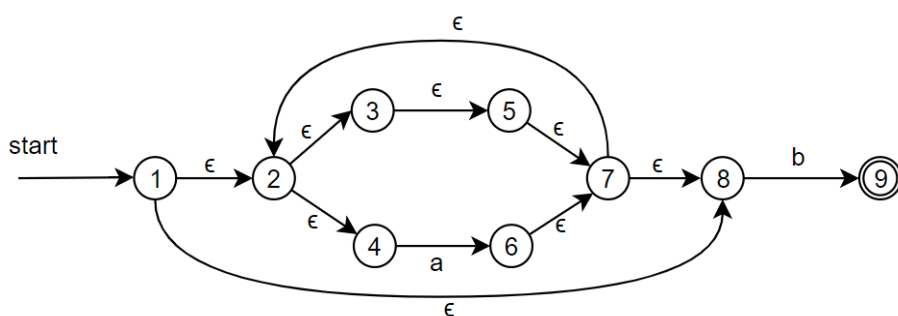
NFA for $(\epsilon|a)^*$:



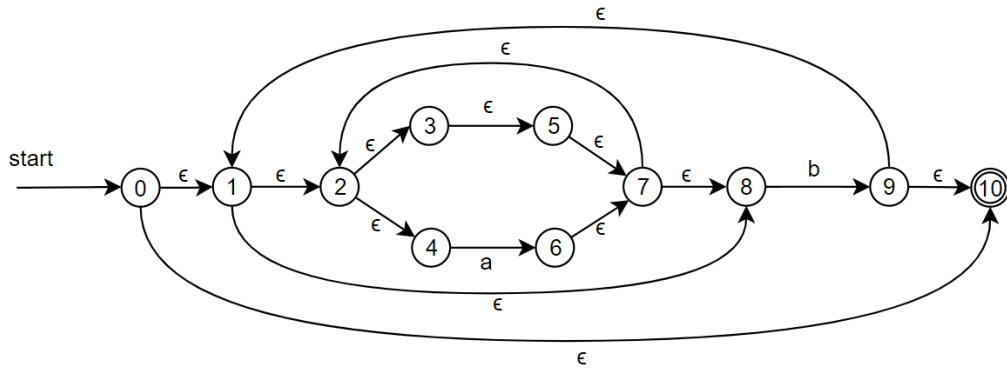
NFA for b :



NFA for $((\epsilon|a)^*b)$:



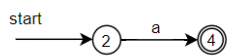
NFA for $((\epsilon|a)^*b)^*$:



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2. $(a|b)^*a(a|b)(a|b)$

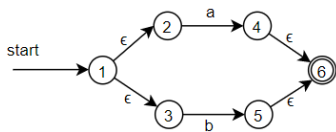
NFA for a:



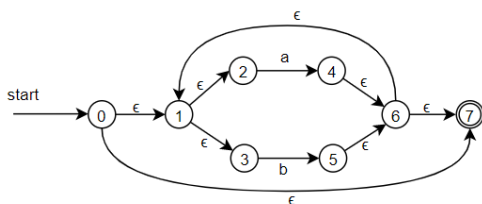
NFA for b:



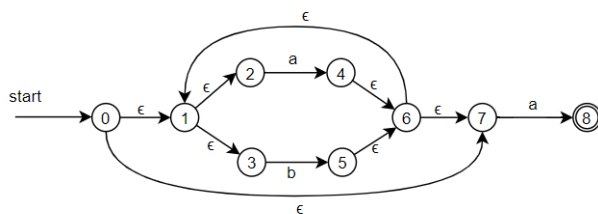
NFA for $(a|b)$:



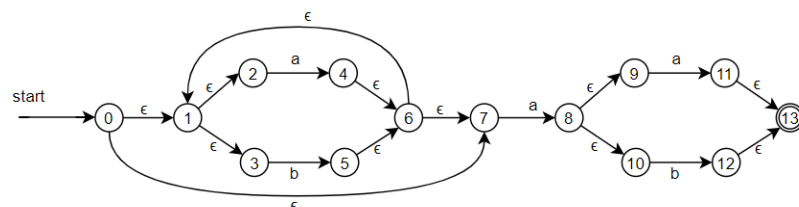
NFA for $(a|b)^*$:



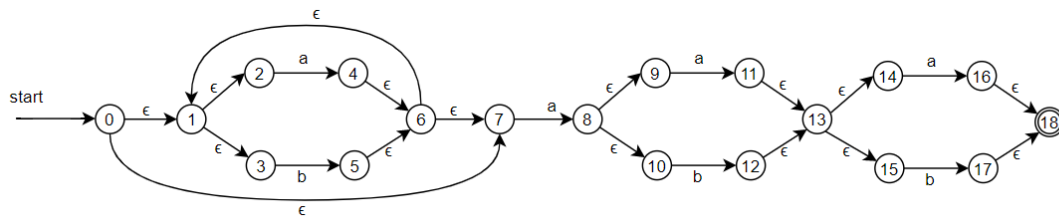
NFA for $(a|b)^*a$:



NFA for $(a|b)^*a(a|b)$:



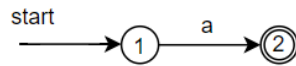
NFA for $(a|b)^*a(a|b)(a|b)$:



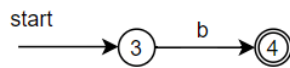
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3. $a^*ba^*ba^*ba^*$

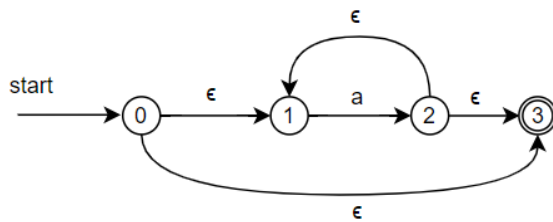
NFA for a:



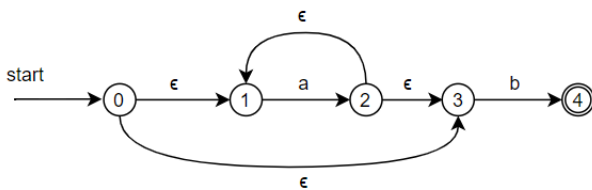
NFA for b:



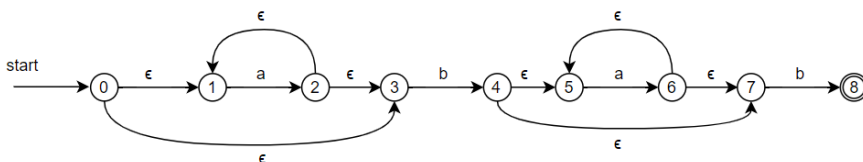
NFA for a^* :



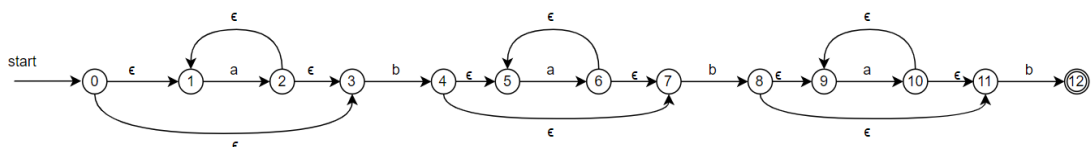
NFA for a^*b :



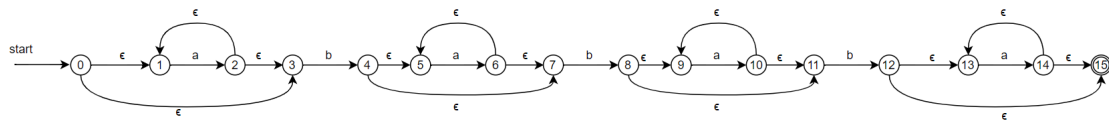
NFA for a^*ba^*b :



NFA for $a^*ba^*ba^*b$:



NFA for $a^*ba^*ba^*ba^*$:



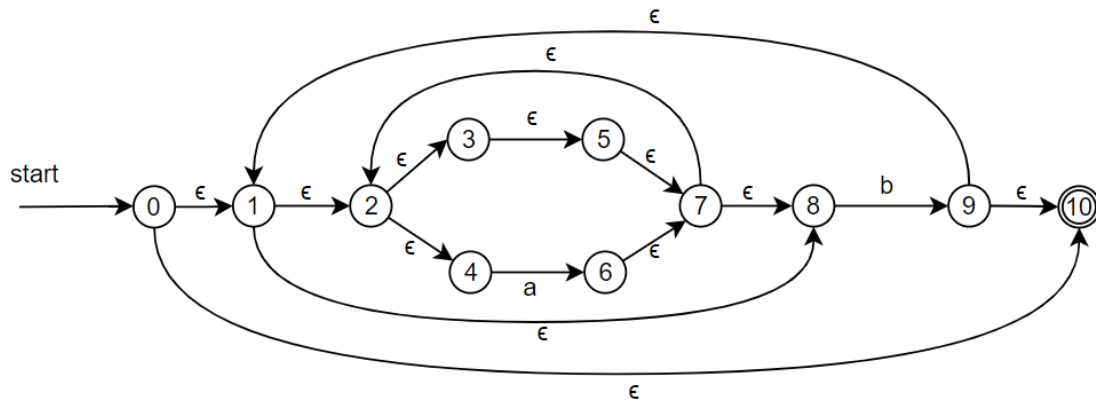
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Exercise 3

Convert the NFAs in Exercise 2 to DFAs using the Subset Construction Algorithm (Algorithm 3.20 in the dragon book). Please put down the detailed steps. [30 points in total; 10 points for each correct conversion]

1. $((\epsilon|a)^*b)^*$

NFA:



ϵ -closure($\{0\}$) = $\{0, 1, 10, 2, 3, 4, 5, 7, 8\}$

$A = \{0, 1, 10, 2, 3, 4, 5, 7, 8\}$

$Dtran[A, a] = \epsilon$ -closure(move(A, a)) = ϵ -closure($\{6\}$) = $\{6, 7, 2, 8, 3, 4, 5\}$

$B = \{6, 7, 2, 8, 3, 4, 5\}$

$Dtran[A, b] = \epsilon$ -closure(move(A, b)) = ϵ -closure($\{9\}$) = $\{9, 10, 1, 2, 3, 4, 5, 7, 8\}$

$C = \{9, 10, 1, 2, 3, 4, 5, 7, 8\}$

$Dtran[B, a] = \epsilon$ -closure(move(B, a)) = ϵ -closure($\{6\}$) = $\{6, 7, 2, 8, 3, 4, 5\} = B$

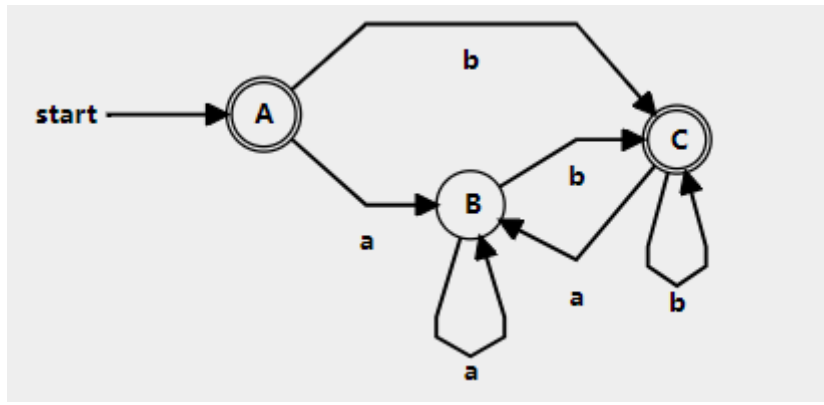
$Dtran[B, b] = \epsilon$ -closure(move(B, b)) = ϵ -closure($\{9\}$) = $\{9, 10, 1, 2, 3, 4, 5, 7, 8\} = C$

$Dtran[C, a] = \epsilon$ -closure(move(C, a)) = ϵ -closure($\{6\}$) = $\{6, 7, 2, 8, 3, 4, 5\} = B$

$Dtran[C, b] = \epsilon$ -closure(move(C, b)) = ϵ -closure($\{9\}$) = $\{9, 10, 1, 2, 3, 4, 5, 7, 8\} = C$

NFA State	DFA State	a	b
$\{0, 1, 10, 2, 3, 4, 5, 7, 8\}$	A	B	C
$\{6, 7, 2, 8, 3, 4, 5\}$	B	B	C
$\{9, 10, 1, 2, 3, 4, 5, 7, 8\}$	C	B	C

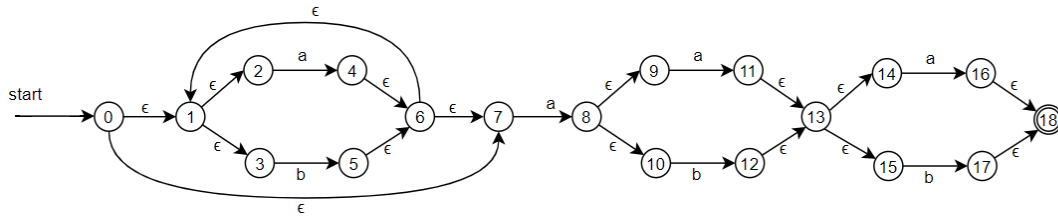
DFA:



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2. $(a|b)^*a(a|b)(a|b)$

NFA:



$\epsilon\text{-closure}(\{0\}) = \{0, 1, 2, 3, 7\}$

$A = \{0, 1, 2, 3, 7\}$

$D\text{tran}[A, a] = \epsilon\text{-closure}(\text{move}(A, a)) = \epsilon\text{-closure}(\{4, 8\}) = \{4, 8, 6, 1, 7, 2, 3, 9, 10\}$

$B = \{4, 8, 6, 1, 7, 2, 3, 9, 10\}$

$D\text{tran}[A, b] = \epsilon\text{-closure}(\text{move}(A, b)) = \epsilon\text{-closure}(\{5\}) = \{5, 6, 7, 1, 2, 3\}$

$C = \{5, 6, 7, 1, 2, 3\}$

$D\text{tran}[B, a] = \epsilon\text{-closure}(\text{move}(B, a)) = \epsilon\text{-closure}(\{4, 8, 11\}) = \{4, 8, 11, 6, 1, 7, 2, 3, 9, 10, 13, 14, 15\} = D$

$D\text{tran}[B, b] = \epsilon\text{-closure}(\text{move}(B, b)) = \epsilon\text{-closure}(\{5, 12\}) = \{5, 12, 6, 7, 1, 2, 3, 13, 14, 15\} = E$

$D\text{tran}[C, a] = \epsilon\text{-closure}(\text{move}(C, a)) = \epsilon\text{-closure}(\{4, 8\}) = \{4, 8, 6, 1, 7, 2, 3, 9, 10\} = B$

$D\text{tran}[C, b] = \epsilon\text{-closure}(\text{move}(C, b)) = \epsilon\text{-closure}(\{5\}) = \{5, 6, 7, 1, 2, 3\} = C$

$D\text{tran}[D, a] = \epsilon\text{-closure}(\text{move}(D, a)) = \epsilon\text{-closure}(\{4, 8, 11, 16\}) = \{4, 8, 11, 16, 6, 1, 7, 2, 3, 9, 10, 13, 14, 15, 18\} = F$

$D\text{tran}[D, b] = \epsilon\text{-closure}(\text{move}(D, b)) = \epsilon\text{-closure}(\{5, 12, 17\}) = \{5, 12, 17, 6, 7, 1, 2, 3, 13, 14, 15, 18\} = G$

$D\text{tran}[E, a] = \epsilon\text{-closure}(\text{move}(E, a)) = \epsilon\text{-closure}(\{4, 8, 16\}) = \{4, 8, 16, 6, 1, 7, 2, 3, 9, 10, 18\} = H$

$D\text{tran}[E, b] = \epsilon\text{-closure}(\text{move}(E, b)) = \epsilon\text{-closure}(\{5, 17\}) = \{5, 17, 6, 7, 1, 2, 3, 18\} = I$

$D\text{tran}[F, a] = \epsilon\text{-closure}(\text{move}(F, a)) = \epsilon\text{-closure}(\{4, 8, 11, 16\}) = \{4, 8, 11, 16, 6, 1, 7, 2, 3, 9, 10, 13, 14, 15, 18\} = F$

$D\text{tran}[F, b] = \epsilon\text{-closure}(\text{move}(F, b)) = \epsilon\text{-closure}(\{5, 12, 17\}) = \{5, 12, 17, 6, 7, 1, 2, 3, 13, 14, 15, 18\} = G$

$D\text{tran}[G, a] = \epsilon\text{-closure}(\text{move}(G, a)) = \epsilon\text{-closure}(\{4, 8, 16\}) = \{4, 8, 16, 6, 1, 7, 2, 3, 9, 10, 18\} = H$

$D\text{tran}[G, b] = \epsilon\text{-closure}(\text{move}(G, b)) = \epsilon\text{-closure}(\{5, 17\}) = \{5, 17, 6, 7, 1, 2, 3, 18\} = I$

$Dtran[H, a] = \epsilon\text{-closure}(\text{move}(H, a)) = \epsilon\text{-closure}(\{4, 8, 11\}) = \{4, 8, 11, 6, 1, 7, 2, 3, 9, 10, 13, 14, 15\} = D$

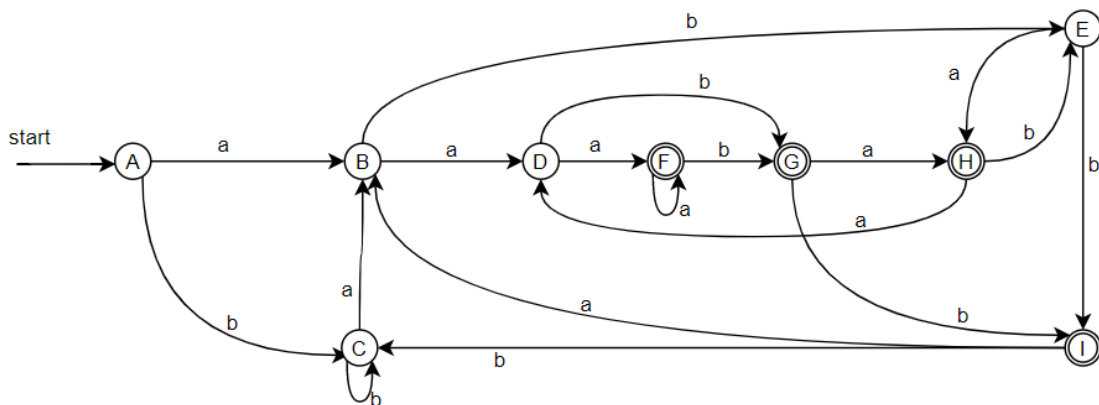
$Dtran[H, b] = \epsilon\text{-closure}(\text{move}(H, b)) = \epsilon\text{-closure}(\{5, 12\}) = \{5, 12, 6, 7, 1, 2, 3, 13, 14, 15\} = E$

$Dtran[I, a] = \epsilon\text{-closure}(\text{move}(I, a)) = \epsilon\text{-closure}(\{4, 8\}) = \{4, 8, 6, 1, 7, 2, 3, 9, 10\} = B$

$Dtran[I, b] = \epsilon\text{-closure}(\text{move}(I, b)) = \epsilon\text{-closure}(\{5\}) = \{5, 6, 7, 1, 2, 3\} = C$

NFA State	DFA State	a	b
$\{0, 1, 2, 3, 7\}$	A	B	C
$\{1, 2, 3, 4, 6, 7, 8, 9, 10\}$	B	D	E
$\{1, 2, 3, 5, 6, 7\}$	C	B	C
$\{1, 2, 3, 4, 6, 7, 8, 9, 10, 13, 14, 15\}$	D	F	G
$\{1, 2, 3, 5, 6, 7, 12, 13, 14, 15\}$	E	H	I
$\{1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 18\}$	F	F	G
$\{1, 2, 3, 5, 6, 7, 12, 13, 14, 15, 17, 18\}$	G	H	I
$\{1, 2, 3, 4, 6, 7, 8, 9, 10, 16, 18\}$	H	D	E
$\{1, 2, 3, 5, 6, 7, 17, 18\}$	I	B	C

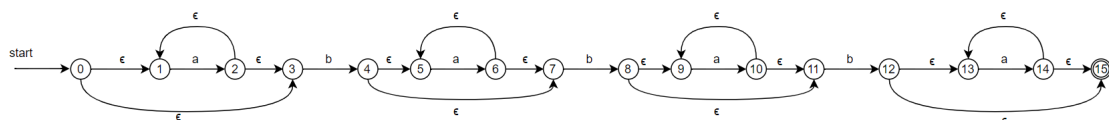
DFA:



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3. $a^*ba^*ba^*ba^*$

NFA:



$\epsilon\text{-closure}(\{0\}) = \{0, 1, 3\}$

$A = \{0, 1, 3\}$

$Dtran[A, a] = \epsilon\text{-closure}(\text{move}(A, a)) = \epsilon\text{-closure}(\{2\}) = \{1, 2, 3\}$

$B = \{1, 2, 3\}$

$Dtran[A, b] = \epsilon\text{-closure}(\text{move}(A, b)) = \epsilon\text{-closure}(\{4\}) = \{4, 5, 7\}$

$C = \{4, 5, 7\}$

$Dtran[B, a] = \epsilon\text{-closure}(\text{move}(B, a)) = \epsilon\text{-closure}(\{2\}) = \{1, 2, 3\} = B$

$Dtran[B, b] = \epsilon\text{-closure}(\text{move}(B, b)) = \epsilon\text{-closure}(\{4\}) = \{4, 5, 7\} = C$

$Dtran[C, a] = \epsilon\text{-closure}(\text{move}(C, a)) = \epsilon\text{-closure}(\{6\}) = \{6, 5, 7\} = D$

$Dtran[C, b] = \epsilon\text{-closure}(\text{move}(C, b)) = \epsilon\text{-closure}(\{8\}) = \{8, 9, 11\} = E$

$Dtran[D, a] = \epsilon\text{-closure}(\text{move}(D, a)) = \epsilon\text{-closure}(\{6\}) = \{6, 5, 7\} = D$

$Dtran[D, b] = \epsilon\text{-closure}(\text{move}(D, b)) = \epsilon\text{-closure}(\{8\}) = \{8, 9, 11\} = E$

$Dtran[E, a] = \epsilon\text{-closure}(\text{move}(E, a)) = \epsilon\text{-closure}(\{10\}) = \{10, 9, 11\} = F$

$Dtran[E, b] = \epsilon\text{-closure}(\text{move}(E, b)) = \epsilon\text{-closure}(\{12\}) = \{12, 13, 15\} = G$

$Dtran[F, a] = \epsilon\text{-closure}(\text{move}(F, a)) = \epsilon\text{-closure}(\{10\}) = \{10, 9, 11\} = F$

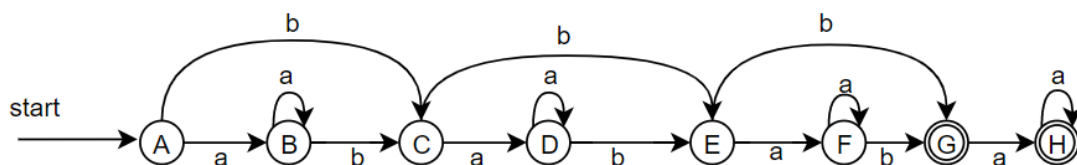
$Dtran[F, b] = \epsilon\text{-closure}(\text{move}(F, b)) = \epsilon\text{-closure}(\{12\}) = \{12, 13, 15\} = G$

$Dtran[G, a] = \epsilon\text{-closure}(\text{move}(G, a)) = \epsilon\text{-closure}(\{14\}) = \{13, 14, 15\} = H$

$Dtran[H, a] = \epsilon\text{-closure}(\text{move}(H, a)) = \epsilon\text{-closure}(\{14\}) = \{13, 14, 15\} = H$

NFA State	DFA State	a	b
$\{0, 1, 3\}$	A	B	C
$\{1, 2, 3\}$	B	B	C
$\{4, 5, 7\}$	C	D	E
$\{6, 5, 7\}$	D	D	E
$\{8, 9, 11\}$	E	F	G
$\{10, 9, 11\}$	F	F	G
$\{12, 13, 15\}$	G	H	
$\{13, 14, 15\}$	H	H	

DFA:



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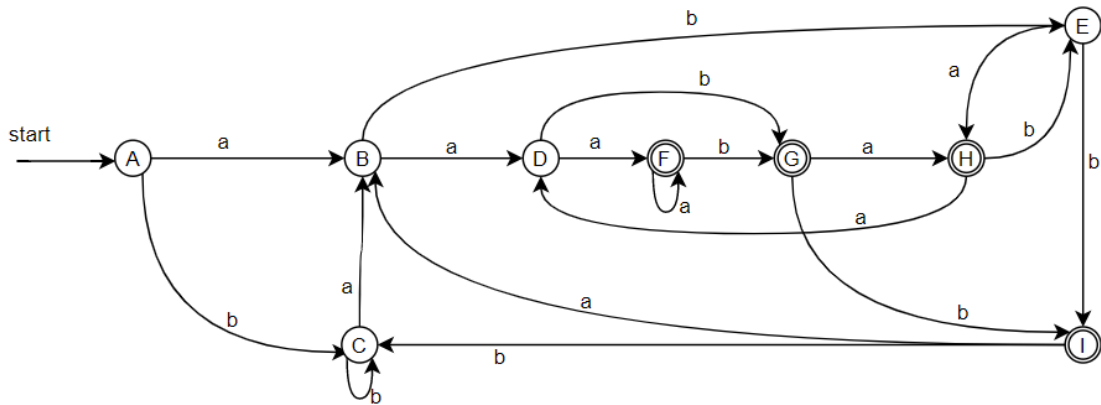
Optional Exercises

Exercise 1

Exercise 1: Minimize the number of states of the DFAs you have built for regular expressions 2 and 3 in Exercise 2 using the State-Minimization Algorithm (Algorithm 3.39 in the dragon book). Please put down the detailed steps. [10 points for each correct minimization process]

1. $(a|b)^*a(a|b)(a|b)$

初始DFA:



对于每个状态，输入a, b后转移到的状态如下表所示

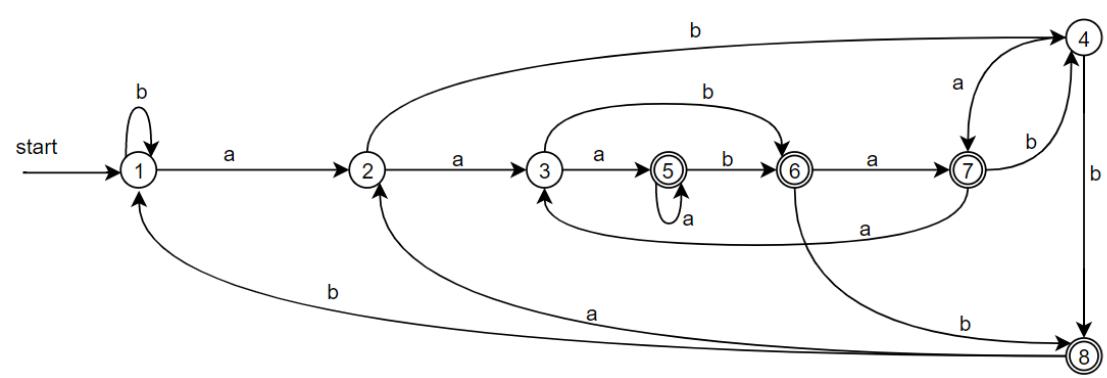
State	a	b
A	B	C
B	D	E
C	B	C
D	F	G
E	H	I
F	F	G
G	H	I
H	D	E
I	B	C

1. 将{A, B, C, D, E, F, G, H, I}按非接收态和接收态分为{A, B, C, D, E}和{F, G, H, I}, $\Pi_{new} = \{A, B, C, D, E\} \{F, G, H, I\}$
2. 对于{A, B, C, D, E}, 在输入b上, 状态A, B, C都转移到{A, B, C, D, E}的某个成员上, 状态D, E都转移到{F, G, H, I}的某个成员上, 故这一轮 $\Pi_{new} = \{A, B, C\} \{D, E\} \{F, G, H, I\}$
3. 对于{F, G, H, I}, 在输入b上, 状态F, G都转移到{F, G, H, I}的某个成员上, 状态H转移到{D, E}的成员E上, I转移到{A, B, C}的成员C上, 故这一轮 $\Pi_{new} = \{A, B, C\} \{D, E\} \{F, G\} \{H\} \{I\}$
4. 对于{A, B, C}, 在输入b上, 状态A, C都转移到{A, B, C}的成员C上, 状态B转移到{D, E}的成员E上, 故这一轮 $\Pi_{new} = \{A, C\} \{B\} \{D, E\} \{F, G\} \{H\} \{I\}$
5. 对于{D, E}, 在输入b上, 状态D转移到{F, G}中的成员G上, 状态E转移到{I}的成员I上, 故这一轮 $\Pi_{new} = \{A, C\} \{B\} \{D\} \{E\} \{F, G\} \{H\} \{I\}$
6. 对于{F, G}, 在输入a上, 状态F转移到{F, G}中的成员F上, 状态G转移到{H}的成员H上, 故 $\Pi_{final} = \{A, C\} \{B\} \{D\} \{E\} \{F\} \{G\} \{H\} \{I\}$

根据 $\Pi_{final} = \{A, C\} \{B\} \{D\} \{E\} \{F\} \{G\} \{H\} \{I\}$ 将状态A, C合并, 得到状态最小化的DFA

DFA State	Min-DFA State	a	b
A, C	1	2	1
B	2	3	4
D	3	5	6
E	4	7	8
F	5	5	6
G	6	7	8
H	7	3	4
I	8	2	1

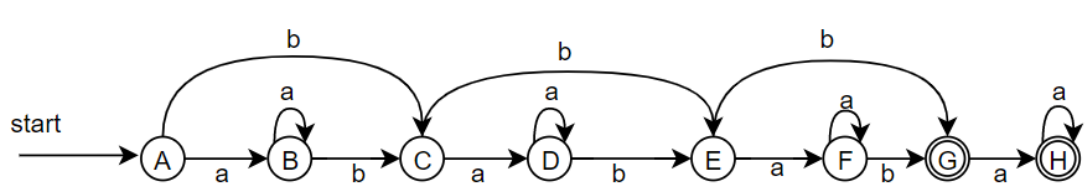
状态最小化DFA:



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2. $a^*ba^*ba^*ba^*$

初始DFA:



对于每个状态，输入a, b后转移到的状态如下表所示

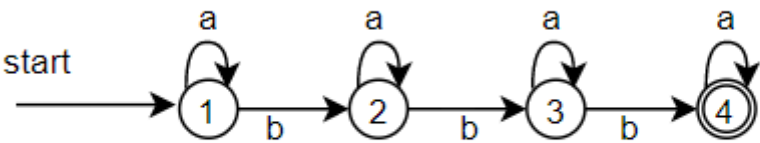
State	a	b
A	B	C
B	B	C
C	D	E
D	D	E
E	F	G
F	F	G

State	a	b
G	H	
H	H	

1. 将{A, B, C, D, E, F, G, H}按非接收态和接收态分为{A, B, C, D, E, F} 和{G, H}, $\Pi_{new} = \{A, B, C, D, E, F\} \{G, H\}$
2. 对于{A, B, C, D, E, F}, 在输入b上, 状态A, B, C, D都转移到{A, B, C, D, E, F}的某个成员上, 状态E, F都转移到{G, H}的成员G上, 故这一轮 $\Pi_{new} = \{A, B, C, D\} \{E, F\} \{G, H\}$
3. 对于{A, B, C, D}, 在输入b上, 状态A, B都转移到{A, B, C, D}的成员C上, 状态C, D都转移到{E, F}的成员E上, 故 $\Pi_{final} = \{A, B\} \{C, D\} \{E, F\} \{G, H\}$

根据 $\Pi_{final} = \{A, B\} \{C, D\} \{E, F\} \{G, H\}$ 将状态合并, 得到状态最小化的DFA

DFA State	Min-DFA State	a	b
A, B	1	1	2
C, D	2	2	3
E, F	3	3	4
G, H	4	4	



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