Artificial Intelligence

For HEDSPI Project

Lecture 8 – Constraint Satisfaction Problems

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Constraints Satisfaction Problems (CSPs)

- CSPs example
- Backtracking search
- Problem structure
- Local search for CSPs

CSP

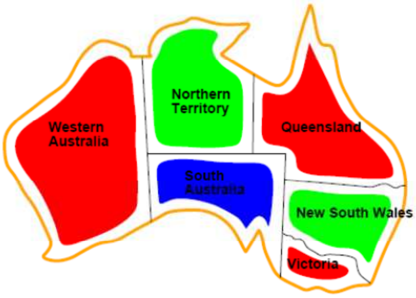
- Standard search problems
  - State is a “black-box”
    - Any data structure that implements initial states, goal states, successor function
- CSPs
  - State is composed of variables X_i with value in domain D_i
  - Goal test is a set of constraints over variables

Example: Map Coloring

- Variables
  - WA, NT, Q, NSW, V , SA
- Domain
  - D_i = {red, green, blue}
- Constraint
  - Neighbor regions must have different colors
    - WA /= NT
    - WA /= SA
    - NT /= SA
    - …

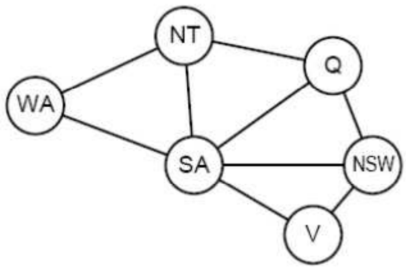
Example: Map Coloring

- Solution is an assignment of variables satisfying all constraints
  - WA=red, and
  - NT=green, and
  - Q=red, and
  - NSW=green, and
  - V=red, and
  - SA=blue



Constraint Graph

- Binary CSPs
  - Each constraint relates at most two variables
- Constraint graph
  - Node is variable
  - Edge is constraint



Varieties of CSPs

- Discrete variables
  - Finite domain, e.g, SAT Solving
  - Infinite domain, e.g., work scheduling
    - Variables is start/end of working day
    - Constraint laguage, e.g., $StartJob_1 + 5 <= StartJob_3$
    - Linear constraints are decidable, non-linear constraints are undecidable
- Continuous variables
  - e.g., start/end time of observing the universe using Hubble telescope
  - Linear constraints are solvable using Linear Programming

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Varieties of Constraints

- Single-variable constraints
  - e.g., SA /= green
- Binary constraints
  - e.g., SA /= WA
- Multi-variable constraints
  - Relate at least 3 variables
- Soft constraints
  - Priority, e.g., red better than green
  - Cost function over variables

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Example: Cryptarimetic

- Variables
  - F,T,O,U,R,W, X_1,X_2,X_3
- Domain
  - {0,1,2,3,4,5,6,7, 8,9}
- Constraints
  - Alldiff(F,T,O,U,R,W)
  - O+O = R+10*X_1
  - X_1+W+W= U+10*X_2
  - X_2+T+T= O+10*X_3
  - X_3=F

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   TWO
+  TWO
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  FOUR
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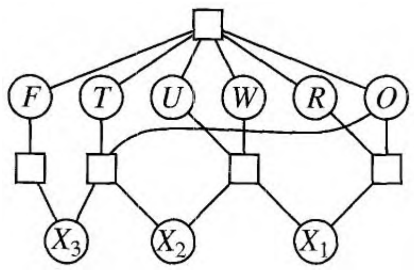

Real World CSP

- Assignment
  - E.g., who teach which class
- Scheduling
  - E.g., when and where the class takes place
- Hardware design
- Spreadsheets
- Transport scheduling
- Manufacture scheduling

CSPs by Standard Search

- State
  - Defined by the values assigned so far
- Initial state
  - The empty assignment
- Successor function
  - Assign a value to a unassigned variable that does not conflict with current assignment
    - Fail if no legal assignment
- Goal test
  - All variables are assigned and no conflict

CSP by Standard Search

- Every solution appears at depth d with n variables
  - Use depth-first search
- Path is irrelevant
- Number of leaves
  - $n!d^n$
    - Two many

Backtracking Search

- Variable assignments are commutative, e.g.,
  - {WA=red, NT =green}
  - {NT =green, WA=red}
- Single-variable assignment
  - Only consider one variable at each node
  - d^n leaves
- Backtracking search
  - Depth-first search+ Single-variable assignment
- Backtracking search is the basic uninformed algorithm for CSPs
  - Can solve n-Queen with n = 25

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Backtracking Search Algorithm

```
function BACKTRACKING-SEARCH(csp) returns solution/failure
   return RECURSIVE-BACKTRACKING({ }, csp)
function RECURSIVE-BACKTRACKING(assignment, csp) returns soln/failure
   if assignment is complete then return assignment
   var ← SELECT-UNASSIGNED-VARIABLE(VARIABLES[csp], assignment, csp)
   for each value in ORDER-DOMAIN-VALUES(var, assignment, csp) do
       if value is consistent with assignment given CONSTRAINTS[csp] then
           add {var = value} to assignment
           result ← RECURSIVE-BACKTRACKING(assignment, csp)
           if result ≠ failure then return result
           remove {var = value} from assignment
   return failure
```

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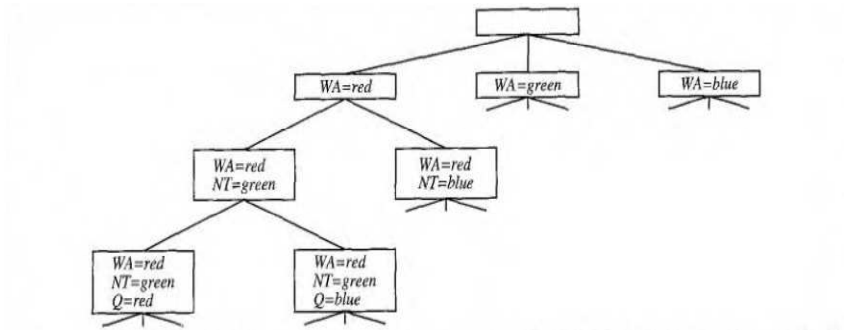
Backtracking Search Algorithm



Improving Backtracking Search

- Which variable should be assigned next?
- In what order should its values be tried?
- Can we detect inevitable failure early?
- Can we take advantage of problem structure?

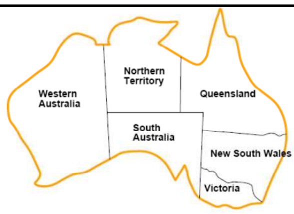
Choosing Variables

- Minimum remaining values (MRV)
  - Choose the variable with the fewest legal values
- Degree heuristic
  - Choose the variable with the most constraints on remaining variables

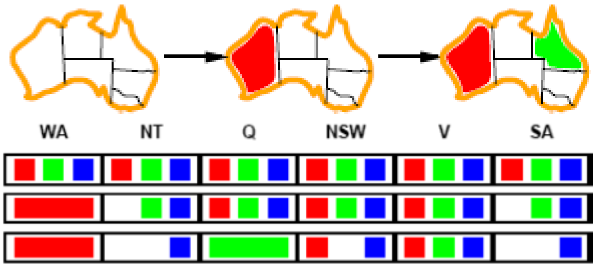
Choosing Values

- Least constraining value (LCV)
  - Choose the least constraining value
    - the one that rules out the fewest values in the remaining variables
- Keep track of remaining legal values for unassigned variables
  - Terminate search when any variable has no legal values

Forward Checking



- Constraint propagation



  - NT and SA cannot both be blue
- Simplest form of propagation makes each arc consistent
  - X -> Y is consistent iff for each value x of X there is some allowed value y for Y

Backtracking Example

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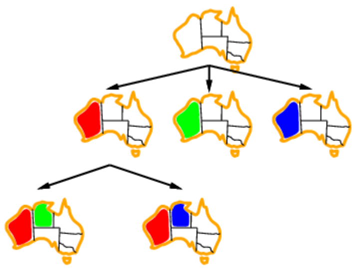
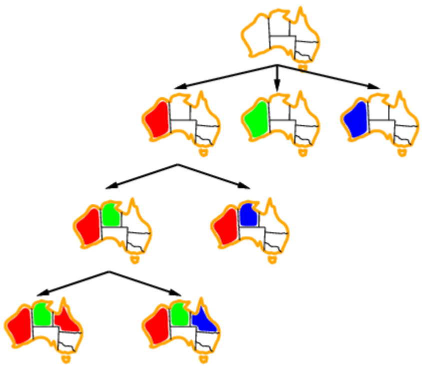


Backtracking Example

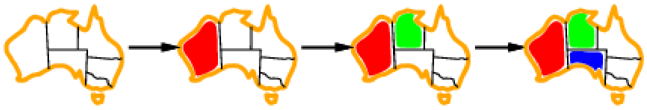


Improving Backtracking Efficiency

- General-purpose methods can give huge gains in speed:
  - Which variable should be assigned next?
  - In what order should its values be tried?
  - Can we detect inevitable failure early?

Most Constrained Variable

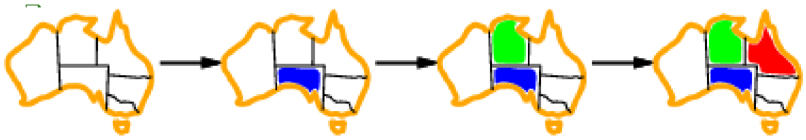
- Most constrained variable:
  choose the variable with the fewest legal values



- a.k.a. minimum remaining values (MRV) heuristic

Most Constraining Variable

- Tie-breaker among most constrained variables
- Most constraining variable:
  - choose the variable with the most constraints on remaining variables



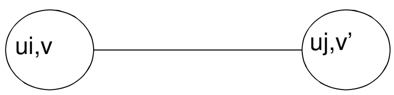
Least Constraining Value

- Given a variable, choose the least constraining value:
  - the one that rules out the fewest values in the remaining variables
  -

- Combining these heuristics makes 1000 queens feasible

Forward Checking (Haralick and Elliott, 1980)

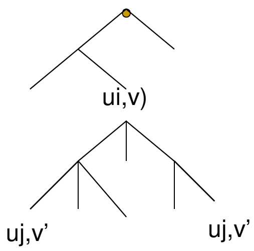
Variables: U = {u1, u2, … , un}
Values: V = {v1, v2, … , vm}
Constraint Relation: R = {(ui,v,uj,v') | ui having value v is compatible with uj having label v'}



If (ui,v,uj,v') is not in R, they are incompatible, meaning if ui has value v, uj cannot have value v'.

Forward Checking

Forward checking is based on the idea that
once variable ui is assigned a value v,
then certain future variable-value pairs (uj,v')
become impossible.



Instead of finding this out at many places on the tree,
we can rule it out in advance.

Data Structure for Forward Checking

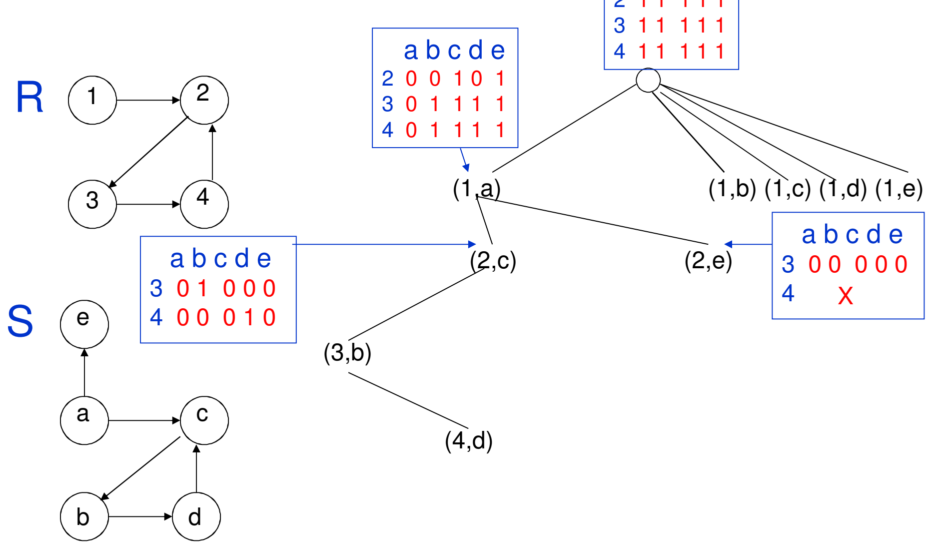
Future error table (FTAB)
One per level of the tree (ie. a stack of tables)

	v1	v2	. . .	vm	
u1					
u2					
:					
un					

What does it mean if a whole row becomes 0?

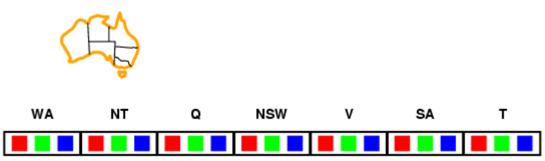
At some level in the tree,
for future (unassigned) variables u
FTAB(u,v) = 1 if it is still possible to assign v to u
0 otherwise

Graph Matching Example



Book's Forward Checking Example

- Idea:
  - Keep track of remaining legal values for unassigned variables
  - Terminate search when any variable has no legal values



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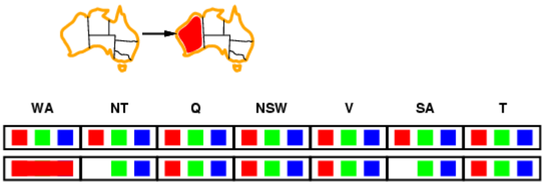

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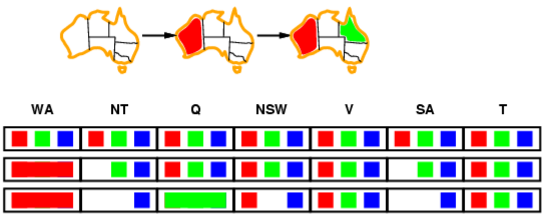

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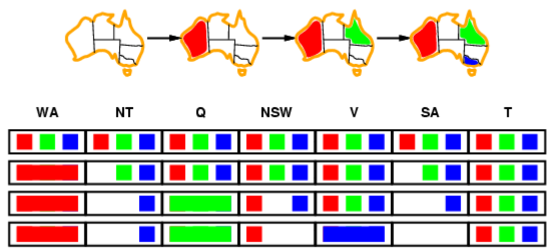
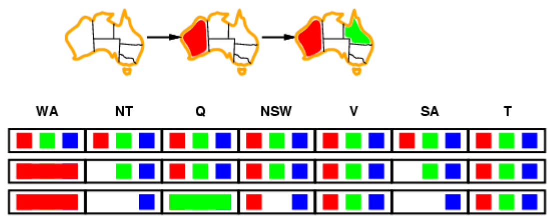


Constraint Propagation

- Forward checking propagates information from assigned to unassigned variables, but doesn't provide early detection for all failures:

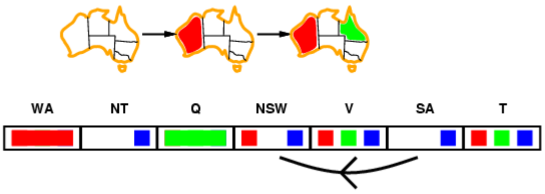


- NT and SA cannot both be blue!
- Constraint propagation repeatedly enforces constraints locally

Arc Consistency

- Simplest form of propagation makes each arc consistent
- *X* → *Y* is consistent iff

  for every value *x* of *X* there is some allowed value y of *Y*



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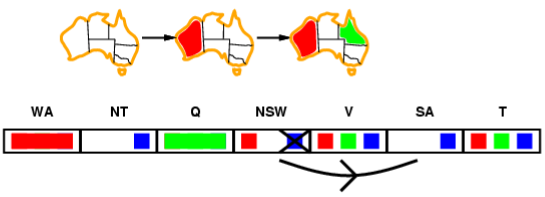

Iterative Algorithms for CSPs

- Hill-climbing, Simulated Annealing can be used for CSPs
  - Complete state, e.g., all variables are assigned at each node
- Allow states with unsatisfiable constraints
- Operators reassign variables
- Variable selection
    - Random
- Value selection by min-conflicts heuristic
  - Choose value that violates the fewest constraints
    - i.e., hill climbing with h(n) = total number of violated constraints

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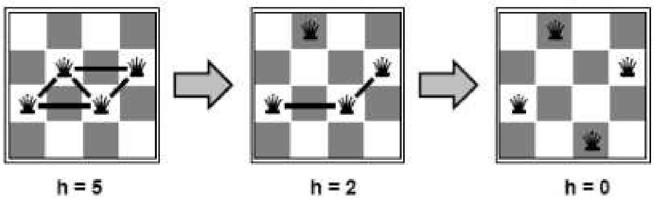
Example: 4-Queens

- State: 4 queens in four columns (4*4 = 256 states)
- Operators: move queen in column
- Goal test: no attacks
- Evaluation: h(n) = number of attacks



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Summary

- CSPs are a special kind of problem:
  - states defined by values of a fixed set of variables
  - goal test defined by constraints on variable values
- Backtracking = depth-first search with one variable assigned per node
- Variable ordering and value selection heuristics help significantly
- Forward checking prevents assignments that guarantee later failure
- Constraint propagation (e.g., arc consistency) does additional work to constrain values and detect inconsistencies
- The CSPs representation allows analysis of problem structure
- Tree-structured CSPs can be solved in linear time
- Iterative min-conflicts is usually effective in practice

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Exercice

- Solve the following cryptarithmetic problem by combining the heuristics
  - Constraint Propagation
  - Minimum Remaining Values
  - Least Constraining Values

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+  TWO
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  FOUR
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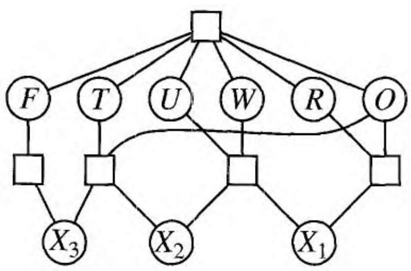

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Exercice

- $O+O = R+10*X_1$
- $X_1+W+W= U+10*X_2$
- $X_2+T+T= O+10*X_3$
- $X_3=F$

1. Choose X_3: domain {0,1}
2. Choose $X_3=1$: use constraint propagation F/=0
3. F = 1
4. Choose X_2: X_1 and X_2 have the same remaing values
5. Choose $X_2=0$
6. Choose X_1: X_1 has Minimum remaining values (MRV)
7. Choose $X_1=0$
8. Choose O: O must be even, less than 5 and therefore has MRV (T+T=O dư 1 và O+O=R+10*0)
9. Choose O=4
10. R=8
11. T=7
12. Choose U: U must be even, less than 9
13. U=6: constraint propagation
14. W=3

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