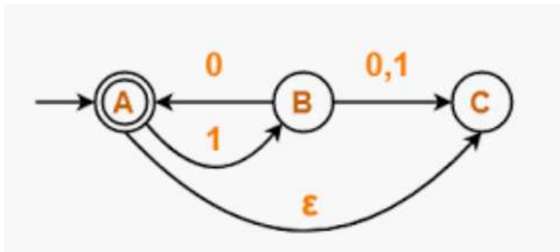
SOLUTION OF ASSIGNMENT 1: REGULAR LANGUAGES

1/ Let M be the following finite automaton with ε transition. Construct the equivalent DFA



Formal Description of M

Q = {A, B, C}

Σ = {0,1}

q_0 = A

F = {A}

δ is described in the following table

	0	1	ε
A	∅	{B}	{C}
B	{A, C}	{C}	∅
C	∅	∅	∅

D is the equivalent deterministic finite automaton,

D = (Q', Σ, δ', q_0', F')

Q'= {{A}, ∅, {B}, {C}, {A, C}}

q_0' = {A}

F'= {{A}, {A, C}}

δ' is described in the following table

	0	1
{A}	∅	{B}
∅	∅	∅
{B}	{A, C}	{C}
{C}	∅	∅
{A,C}	∅	{B}

Detail of computation (not necessary to present in the exam paper)

$\delta'(\{A\},1) = \{B\}$ because

ε-CLOSURE(A) $=\{A, C\}$

$\delta(A,1) = \{B\}$

$\delta(C,1) = \emptyset$

$\delta(A,1) \cup \delta(C,1) = \{B\}$

ε-CLOSURE(B)$= \{B\}$

2/ Use proper English to describe the language defined by the regular expression b*ab*ab*

Set of all strings over {a, b} that have exactly 2 a's.

3/ Convert the regular expression 1(10+2)* into an equivalent ε-NFA

4/ Prove that the following language is not regular using the pumping lemma.

$$L = \{a^n b^m \mid n \leq m\}$$

- Suppose L is regular. Since L is regular, we can apply the pumping lemma to L.
- Let p be the pumping length for L.
- Choose w = $\mathbf{a^p b^p}$. Note that $w \in L$ and $|w| = 2p \geq p$.
- From the pumping lemma, there exists some x, y, z where xyz = w and $|xy| \leq p$, $|y| > 0$, and $\forall\ i \geq 0$, $xy^iz \in L$.
- Because $|xy| \leq p$, $y = a^k$,(y is all a's).
- We choose **i = 2** and $xy^iz = \mathbf{a^{p+k}b^p}$. Because $|y| > 0$,

$xy^2z = xz \notin L$ (the string has **more** a's than b's). Thus for all possible x, y, z: xyz = w, $\exists i$, $xy^iz \notin L$. **Contradiction**

- L is not regular.