ASSIGNMENT 2: CONTEXT FREE LANGUAGES

1/ Convert the following CFG into an equivalent CFG in Chomsky normal form

$A \rightarrow BAB \mid B \mid \varepsilon$

$B \rightarrow 00 \mid \varepsilon$

Remove ε-rules

Nullable variables: A, B

Rules of A :

$A \rightarrow A \mid B \mid BA \mid AB \mid BB \mid BAB$

$B \rightarrow 00$

Remove $A \rightarrow A$. It has no meaning

Because ε is in the language generated by the given grammar, we need to add a new start variable

Add new start variable: S

$S \rightarrow A \mid \varepsilon$

$A \rightarrow B \mid BA \mid AB \mid BB \mid BAB$

$B \rightarrow 00$

$S \Rightarrow^* \{A, B\}$ only use unit rules

Add rules of S with non unit RHS of A and B

$S \rightarrow BA \mid AB \mid BB \mid BAB \mid 00$

$A \Rightarrow^* \{B\}$

Add rules of A with non unit RHS of B

$A \rightarrow 00$

We get the equivalent grammar without unit rule:

$S \rightarrow \varepsilon \mid BA \mid AB \mid BB \mid BAB \mid 00$

$A \rightarrow BA \mid AB \mid BB \mid BAB \mid 00$

$B \rightarrow 00$

Replace long rules BAB

Add a new variables C

$S \rightarrow BAB$ is replaced by $S \rightarrow BC$ and $C \rightarrow AB$

$A \rightarrow BAB$ is replace by $A \rightarrow BC$ (and $C \rightarrow AB$ generated by previous rule)

Replace terminals : Add a new variable D and new rule $D \rightarrow 0$

Replace $B \rightarrow 00$ by $B \rightarrow DD$ and $D \rightarrow 0$

The equivalent grammar in CNF is

$S \rightarrow \varepsilon \mid BA \mid AB \mid BB \mid BC \mid DD$

$A \rightarrow BA \mid AB \mid BB \mid BC \mid DD$

$B \rightarrow DD$

$C \rightarrow AB$

$D \rightarrow 0$

2/ Consider the following CFG G:

$S \rightarrow SS \mid T$

$T \rightarrow aTb \mid ab$

Describe L(G) and show that G is ambiguous

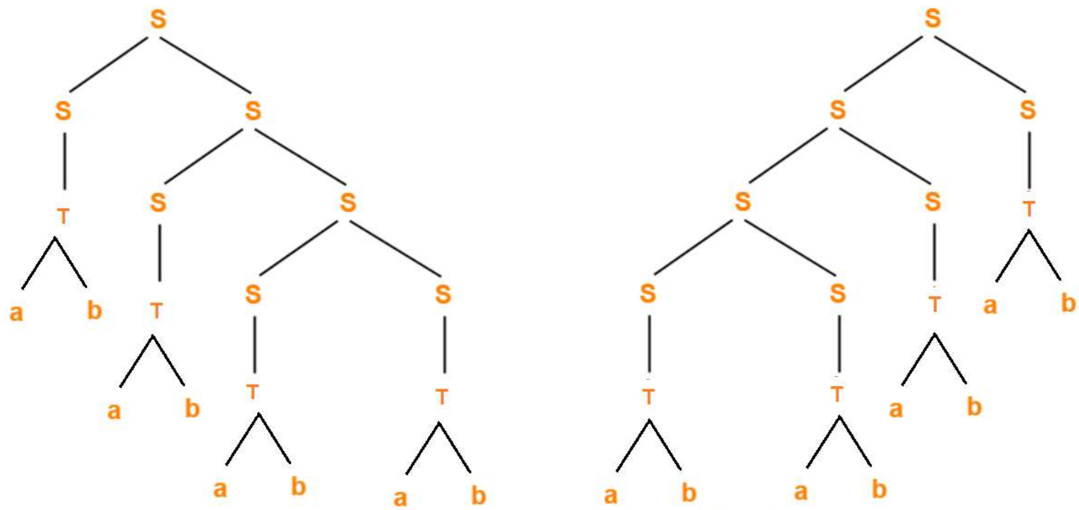
Describe L(G)

$L(G) = \{w_1w_2...w_n \mid w_i \in \{a^nb^n \mid n \geq 1\}\}$

It is because $T \rightarrow aTb \mid ab$ generate $\{a^nb^n \mid n \geq 1\}$ and $S \rightarrow SS$ generates SS...S and SS...S derives TT...T

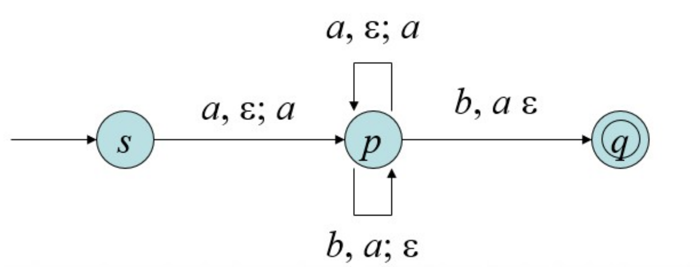
G is ambiguous

Because string abababab has at least 2 different derivation trees



Tree 1 Tree 2

3/ Write formal description of the push down automaton in the following state diagram:



Let M be the given automaton, so M = $(Q, \Sigma, \Gamma, \delta, q_1, F)$, where

$Q = \{s, p, q\}$,

$\Sigma = \{a,b\}$,

$\Gamma = \{0, \$\}$,

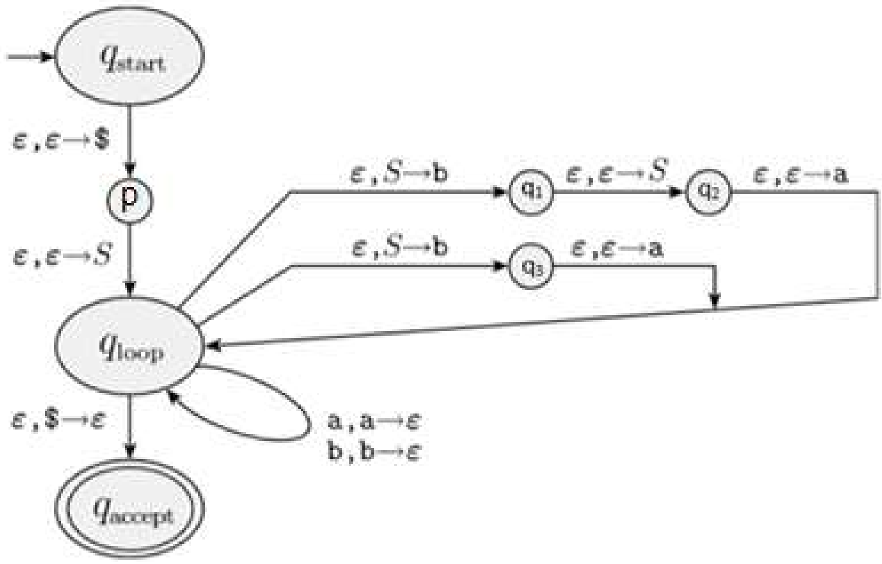
$F = \{q\}$, and δ is

$\delta(s, \varepsilon, a) = \{(p,a)\}$

$\delta(p, \varepsilon, a) = \{(p,a)\}$

$\delta(p, a, b) = \{(p, \varepsilon),(q, \varepsilon)\}$

4/ Write the computation of the automaton below with string aaabbb



(q_{start}, aaabbb, ε)

⇒ (p, aaabbb, \$)

⇒ (q_{loop}, aaabbb, S\$)

⇒ (q_1, aaabbb, b\$)

⇒ (q_2, aaabbb, Sb\$)

⇒ (q_{loop}, aaabbb, aSb\$)

⇒ (q_{loop}, aabbb, Sb\$)

⇒ (q_1, aabbb, bb\$)

⇒ (q_2, aabbb, Sbb\$)

⇒ (q_{loop}, aabbb, aSbb\$)

⇒ (q_{loop}, abbb, Sbb\$)

⇒ (q_3, abbb, bbb\$)

⇒ (q_{loop}, abbb, abbb\$)

⇒ (q_{loop}, bbb,bbb\$)

⇒ (q_{loop}, bb,bb\$)

⇒ (q_{loop}, b,b\$)

⇒ (q_{loop}, ε, \$)

$\Rightarrow (q_{accept}, \varepsilon, \varepsilon)$

5/ Prove that the following language is not regular using the pumping lemma for context free languages.

$$L = \{a^n \mid n \text{ is perfect square}\}$$

Suppose that L is context free

Since L is context free, we apply the Pumping Lemma for context free languages and assert the existence of a number $p > 0$ (pumping length) that satisfies the property

if s is any string in L of length at least p, then s can be divided into 5 pieces s = uvxyz satisfying the conditions:

$|vy| > 0$

$|vxy| \leq p$

for each $i \geq 0$, $uv^ixy^iz \in L$ (*)

We pick string $s = a^n$ with $n = p^2$.

Clearly $|s| = p^2 \geq p$

s = uvxyz like in (*)

Because $|vxy| \leq p$ so $|vy| \leq p$

$|vy| > 0$, so $|vy| \geq 1$. That means $|vy| = k$ where $1 \leq k \leq p$

Choose $i = 2$

$uv^ixy^iz = a^{n+k}$, $n=p^2$. $p^2 < \mathbf{p^2+k} \leq p^2 + p < p^2+2p+1=(k+1)^2$. That means p^2+k is not a perfect square.

The contradiction proved that L is not context free.