CHAPTER 1 – EQUATIONS AND INEQUALITIES

Exercise 1.
In problems 22-34, solve each equation.

22. $3x^4 + 4x^2 + 1 = 0$

23. $\sqrt{2x-3} + x = 3$

24. $\sqrt{2x-1} = x - 2$

25. $\sqrt[4]{2x+3} = 2$

26. $\sqrt[5]{3x+1} = -1$

27. $\sqrt{x+1} + \sqrt{x-1} = \sqrt{2x+1}$

28. $\sqrt{2x-1} - \sqrt{x-5} = 3$

29. $2x^{1/2} - 3 = 0$

30. $3x^{1/4} - 2 = 0$

31. $x^{-6} - 7x^{-3} - 8 = 0$

32. $6x^{-1} - 5x^{-1/2} + 1 = 0$

33. $x^2 + m^2 = 2mx + (nx)^2 \quad n \neq 1$

34. $b^2x^2 + 2ax = x^2 + a^2 \quad b \neq 1$

Exercise 2.
In Problems 35-46, solve each equation

35. $10a^2x^2 - 2abx - 36b^2 = 0$

36. $\dfrac{1}{x-m} + \dfrac{1}{x-n} = \dfrac{2}{x} \quad x \neq 0, x \neq m, x \neq n$

37. $\sqrt{x^2+3x+7} - \sqrt{x^2-3x+9} + 2 = 0$

38. $\sqrt{x^2+3x+7} - \sqrt{x^2+3x+9} = 2$

39. $|2x+3| = 7$

40. $|3x-1| = 5$

41. $|2-3x| + 2 = 9$

42. $|1-2x| + 1 = 4$

43. $2x^3 = 3x^2$

44. $5x^4 = 9x^3$

45. $2x^3 + 5x^2 - 8x - 20 = 0$

46. $3x^3 + 5x^2 - 3x - 5 = 0$

Exercise 3.

75. Find the number a for which $x = 4$ is a solution of the equation

$$x + 2a = 16 + ax - 6a$$

76. Find the number b for which $x = 2$ is a solution of the equation

$$x + 2b = x - 4 + 2bx$$

Exercise 4
a) A total of \$20,000 is to be s to be invested, some in bonds and some in certificates of deposit (CDs). If the amount invested in bonds is to exceed that in CDs by \$3000, how much will be invested in each type of investment?
b) Sandra, who is paid time-and-a half for hours worked in excess of 40 hours, had gross weekly wages of \$442 for 48 hours worked. What is her regular hourly rate?
c) Leigh is paid time-and-a-half for hours worked in excess of 40 hours and double-time for hours worked on Sunday. If Leigh had gross weekly wages of \$456 for working 50 hours, 4 of which were on Sunday, what is her regular hourly rate?
d) Going into the final exam, which will count as two tests, Brooke has test scores of 80, 83, 71, 61, and 95. What score does Brooke need on the final in order to have an average score of 80?
e) Going into the final exam, which will count as two-thirds of the final grade, Mike has test scores of 86, 80, 84, and 90. What score does Mike need on the final in order to earn a B, which requires an average score of 80? What does he need to earn an A, which requires an average of 90?
f) A builder of tract homes reduced the price of a model by 15%. If the new price is \$425,000, what was its original price? How much can be saved by purchasing the model?

Exercise 5.
In problems 25-28, solve each equation by factoring

25. $6x - 5 = \dfrac{6}{x}$

26. $x + \dfrac{12}{x} = 7$

27. $\dfrac{4(x-2)}{x-3} + \dfrac{3}{x} = \dfrac{-3}{x(x-3)}$

28. $\dfrac{5}{x+4} = 4 + \dfrac{3}{x-2}$

Exercise 6.

In problems 89-90, find the real solutions, if any, of each equation

89. $\frac{x}{x-2}+\frac{2}{x+1}=\frac{7x+1}{x^2-x-2}$

90. $\frac{3x}{x+2}+\frac{1}{x-1}=\frac{4-7x}{x^2+x-2}$

Exercise 7. Applications and Extensions

a) Pythagorean Theorem: How many right triangles have a hypotenuse that measures $2x+3$ meters and legs that measure $2x-5$ meters and $x+7$ meters? What are the dimensions of the triangle(s)?

b) Pythagorean Theorem: How many right triangles have a hypotenuse that measures $4x+5$ inches and legs that measure $3x+13$ inches and x inches? What are the dimensions of the triangle(s)?

c) Dimensions of a Window: The area of the opening of a rectangular window is to be 143 square feet. If the length is to be 2 feet more than the width, what are the dimensions?

d) Geometry: Find the dimensions of a rectangle whose perimeter is 26 meters and whose area is 40 square meters

e) Physics: A ball is thrown vertically upward from the top of a building 96 feet tall with an initial velocity of 80 feet per second. The distance s (in feet) of the ball from the ground after t seconds is $s=96+80t-16t^2$

(i) After how many seconds does the ball strike the ground?

(b) After how many seconds will the ball pass the top of the building on its way down?

Exercise 8.

In the following problems, write each expression in the standard form $a+bi$

29. $\left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)^2$

30. $\left(\frac{\sqrt{3}}{2}-\frac{1}{2}i\right)^2$

31. $(1+i)^2$

32. $(1-i)^2$

43. $i^7(1+i^2)$

44. $2i^4(1+i^2)$

45. $i^6+i^4+i^2+1$

46. $i^7+i^5+i^3+i$

Exercise 9.

In Problems 65 - 72, solve each equation in the complex number system

65. $x^2+x+1=0$

66. $x^2-x+1=0$

67. $x^3-8=0$

68. $x^3+27=0$

69. $x^4=16$

70. $x^4=1$

71. $x^4+13x^2+36=0$

72. $x^4+3x^2-4=0$

Exercise 10.

79. $2+3i$ is a solution of a quadratic equation with real coefficients. Find the other solution.

80. $4-i$ is a solution of a quadratic equation with real coefficients. Find the other solution.

Exercise 11. Applications and Extensions

a) Electrical Circuits The impedance Z, in ohms, of a circuit element is defined as the ratio of the phasor voltage V, in volts, across the element to the phasor current I, in amperes, through the elements. That is, $Z=\frac{V}{I}$. If the voltage across a circuit element is $18+i$ volts and the current through the element is $3-4i$ amperes, determine the impedance.

b) In an ac circuit with two parallel pathways, the total impedance Z, in ohms, satisfies the formula $\frac{1}{Z}=\frac{1}{Z_1}+\frac{1}{Z_2}$, where Z_1 is the impedance of the first pathway and Z_2 is the impedance of the second pathway. Determine the total impedance if the impedances of the two pathways are $Z_1=2+i$ ohms and $Z_2=4-3i$ ohms.

Exercise 11.

In Problems 65-72, find the real solutions of each equation

65. $\dfrac{1}{(x+1)^2} = \dfrac{1}{x+1} + 2$

66. $\dfrac{1}{(x-1)^2} + \dfrac{1}{x-1} = 12$

67. $3x^{-2} - 7x^{-1} - 6 = 0$

68. $2x^{-2} - 3x^{-1} - 4 = 0$

69. $2x^{2/3} - 5x^{1/3} - 3 = 0$

70. $3x^{4/3} + 5x^{2/3} - 2 = 0$

71. $\left(\dfrac{v}{v+1}\right)^2 + \dfrac{2v}{v+1} = 8$

72. $\left(\dfrac{y}{y-1}\right)^2 = 6\left(\dfrac{y}{y-1}\right) + 7$

Exercise 12.

In Problems 83-88, find the real solutions of each equation

83. $2x^3 + 4 = x^2 + 8x$

84. $3x^3 + 4x^2 = 27x + 36$

85. $5x^3 + 45x = 2x^2 + 18$

86. $3x^3 + 12x = 5x^2 + 20$

87. $x(x^2 - 3x)^{1/3} + 2(x^2 - 3x)^{4/3} = 0$

88. $3x(x^2 + 2x)^{1/2} - 2(x^2 + 2x)^{3/2} = 0$

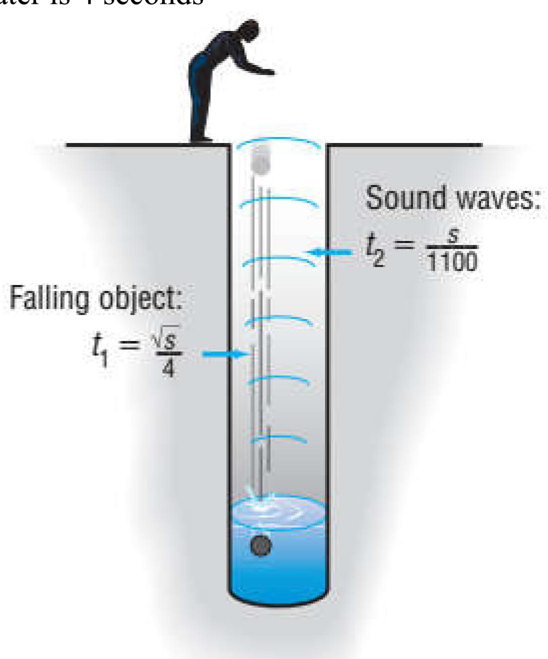
Exercise 13.

Physics: Using Sound to Measure Distance The distance to the surface of the water in a well can sometimes be found by dropping an object into the well and measuring the time elapsed until a sound is heard. If t_1 is the time (measured in seconds) that it takes for the object to strike the water, then t_1 will obey the equation $s = 16t_1^2$, where s is the distance (measured in feet). It follows that $t_1 = \frac{\sqrt{s}}{4}$. Suppose that t_2 is the time that it takes for the sound of the impact to reach your ears. Because sound waves are known to travel at a speed of approximately 1100 feet per second, the time t_2 to travel the distance s will be $t_2 = \frac{s}{1100}$

Now $t_1 + t_2$ is the total time that elapses from the moment that the object is dropped to the moment that a sound is heard. We have the equation

$$\text{Total time elapsed} = \frac{\sqrt{s}}{4} + \frac{s}{1100}$$

Find the distance to the water's surface if the total time elapsed from dropping a rock to hearing it hit water is 4 seconds



Exercise 14.

Foucault's Pendulum The period of a pendulum is the time it takes the pendulum to make one full swing back and forth. The period T, in seconds, is given by the formula $T = 2\pi\sqrt{\frac{l}{32}}$, where l is the length, in feet, of the pendulum. In 1851, Jean Bernard Leon Foucault demonstrated the axial rotation of Earth using a large pendulum that he hung in the Panthéon in Paris. The period of Foucault's pendulum was approximately 16.5 seconds. What was its length?

Exercise 15.

In Problems 95-97, find a and b

95. If $-3 < x < 0$, then $a < \dfrac{1}{x+4} < b$.

96. If $2 < x < 4$, then $a < \dfrac{1}{x-6} < b$.

97. If $6 < 3x < 12$, then $a < x^2 < b$.

Exercise 16.

Harmonic Mean. For $0 < a < b$, let h be defined by

$$\frac{1}{h} = \frac{1}{2}\left(\frac{1}{a} + \frac{1}{b}\right)$$

Show that $a < h < b$. The number h is called the harmonic mean of a and b

Exercise 17.

Computing Grades In your Economics 101 class, you have scores of 68, 82, 87, and 89 on the first four of five tests. To get a grade of B, the average of the first five test scores must be greater than or equal to 80 and less than 90.

(a) Solve an inequality to find the range of the score that you need on the last test to get a B.

(b) What score do you need if the fifth test counts double?

Exercise 18.

Sue wants to lose weight. For healthy weight loss, the American College of Sports Medicine (ACSM) recommends 200 to 300 minutes of exercise per week. For the first six days of the week, Sue exercised 40, 45, 0, 50, 25, and 35 minutes. How long should Sue exercise on the seventh day in order to stay within the ACSM guidelines?

Exercise 19.

Commonwealth Edison Company's charge for electricity in January 2010 was 9.44¢ per kilowatt-hour. In addition, each monthly bill contains a customer charge of \$12.55. If last year's bills ranged from a low of \$76.27 to a high of \$248.55, over what range did usage vary (in kilowatt-hours)?

Exercise 20.

The markup over dealer's cost of a new car ranges from 12% to 18%. If the sticker price is \$18,000, over what range will the dealer's cost vary?

Exercise 21.

In Problems 53-60, solve each inequality. Express your answer using set notation or interval notation. Graph the solution set.

53. $|3x + 4| < \frac{1}{2}$

54. $|1 - 2x| < \frac{1}{3}$

55. $|2x - 5| \geq 9$

56. $|3x + 1| \geq 10$

57. $2 + |2 - 3x| \leq 4$

58. $\frac{1}{2} + \left|\frac{2x - 1}{3}\right| \leq 1$

59. $1 - |2 - 3x| < -4$

60. $1 - \left|\frac{2x - 1}{3}\right| < -2$

Exercise 22.
"Normal" human body temperature is 98.6°F. If a temperature x that differs from normal by at least 1.5° is considered unhealthy, write the condition for an unhealthy temperature x as an inequality involving an absolute value, and solve for x

Exercise 23.
Household Voltage. In the United States, normal household voltage is 110 volts. However, it is not uncommon for actual voltage to differ from normal voltage by at most 5 volts. Express this situation as an inequality involving an absolute value. Use x as the actual voltage and solve for x.

Exercise 24.
a) Express the fact that x differs from 3 by less than $1/2$ as aninequality involving an absolute value. Solve for x.
b) Express the fact that x differs from -4 by less than 1 as an inequality involving an absolute value. Solve for x

Exercise 25.
Solve each inequality

91. Solve $|3x - |2x + 1|| = 4$.

92. Solve $|x + |3x - 2|| = 2$.

Exercise 26.
Betsy, a recent retiree, requires \$6000 per year in extra income. She has \$50,000 to invest and can invest in B-rated bonds paying 15% per year or in a certificate of deposit (CD) paying 7% per year. How much money should be invested in each to realize exactly \$6000 in interest per year?

Exercise 27.
Banking. A bank loaned out \$12,000, part of it at the rate of 8% per year and the rest at the rate of 18% per year. If the interest received totaled \$1000, how much was loaned at 8%?

Exercise 28.
Business: Blending Coffee A coffee manufacturer wants to market a new blend of coffee that sells for \$3.90 per pound by mixing two coffees that sell for \$2.75 and \$5 per pound, respectively. What amounts of each coffee should be blended to obtain the desired mixture? [Hint: Assume that the total weight of the desired blend is 100 pounds.]

Exercise 29.

Business: Mixing Nuts A nut store normally sells cashews for \$9.00 per pound and almonds for \$3.50 per pound. But at the end of the month the almonds had not sold well, so, in order to sell 60 pounds of almonds, the manager decided to mix the 60 pounds of almonds with some cashews and sell the mixture for \$7.50 per pound. How many pounds of cashews should be mixed with the almonds to ensure no change in the profit?

Exercise 30.

Business: Mixing Candy A candy store sells boxes of candy containing caramels and cremes. Each box sells for \$12.50 and holds 30 pieces of candy (all pieces are the same size). If the caramels cost \$0.25 to produce and the cremes cost \$0.45 to produce, how many of each should be in a box to make a profit of \$3?

Exercise 31.

In Problems 37-46, find the real solutions, if any, of each equation

37. $\sqrt{x^2+3x+7}-\sqrt{x^2-3x+9}+2=0$

38. $\sqrt{x^2+3x+7}-\sqrt{x^2+3x+9}=2$

39. $|2x+3|=7$

40. $|3x-1|=5$

41. $|2-3x|+2=9$

42. $|1-2x|+1=4$

43. $2x^3=3x^2$

44. $5x^4=9x^3$

45. $2x^3+5x^2-8x-20=0$

46. $3x^3+5x^2-3x-5=0$

HOMEWORKS

Exercise 7: b,c

Exercise 10: 79

Exercise 15: 95, 96

Exercise 16

Exercise 17

Exercise 25: 91

Exercise 26

Exercise 30.