PART II – ALGEBRA MATRICES

Exercise 1.

In Problems 9–24, use the following matrices to evaluate the given expression.

$$A = \begin{bmatrix} 0 & 3 & -5 \\ 1 & 2 & 6 \end{bmatrix} \quad B = \begin{bmatrix} 4 & 1 & 0 \\ -2 & 3 & -2 \end{bmatrix} \quad C = \begin{bmatrix} 4 & 1 \\ 6 & 2 \\ -2 & 3 \end{bmatrix}$$

9. $A + B$ 10. $A - B$ 11. $4A$ 12. $-3B$

13. $3A - 2B$ 14. $2A + 4B$ 15. AC 16. BC

17. CA 18. CB 19. $C(A + B)$ 20. $(A + B)C$

21. $AC - 3I_2$ 22. $CA + 5I_3$ 23. $CA - CB$ 24. $AC + BC$

Exercise 2.

In Problems 25-30, find the product

25. $\begin{bmatrix} 2 & -2 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 2 & 1 & 4 & 6 \\ 3 & -1 & 3 & 2 \end{bmatrix}$ 26. $\begin{bmatrix} 4 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} -6 & 6 & 1 & 0 \\ 2 & 5 & 4 & -1 \end{bmatrix}$ 27. $\begin{bmatrix} 1 & 2 & 3 \\ 0 & -1 & 4 \end{bmatrix}\begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 2 & 4 \end{bmatrix}$

28. $\begin{bmatrix} 1 & -1 \\ -3 & 2 \\ 0 & 5 \end{bmatrix}\begin{bmatrix} 2 & 8 & -1 \\ 3 & 6 & 0 \end{bmatrix}$ 29. $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 4 & 1 \\ 3 & 6 & 1 \end{bmatrix}\begin{bmatrix} 1 & 3 \\ 6 & 2 \\ 8 & -1 \end{bmatrix}$ 30. $\begin{bmatrix} 4 & -2 & 3 \\ 0 & 1 & 2 \\ -1 & 0 & 1 \end{bmatrix}\begin{bmatrix} 2 & 6 \\ 1 & -1 \\ 0 & 2 \end{bmatrix}$

Exercise 3.

In Problems 31-40, each matrix is nonsingular. Find the inverse of each matrix

31. $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$ 32. $\begin{bmatrix} 3 & -1 \\ -2 & 1 \end{bmatrix}$ 33. $\begin{bmatrix} 6 & 5 \\ 2 & 2 \end{bmatrix}$ 34. $\begin{bmatrix} -4 & 1 \\ 6 & -2 \end{bmatrix}$ 35. $\begin{bmatrix} 2 & 1 \\ a & a \end{bmatrix}$ $a \neq 0$

36. $\begin{bmatrix} b & 3 \\ b & 2 \end{bmatrix}$ $b \neq 0$ 37. $\begin{bmatrix} 1 & -1 & 1 \\ 0 & -2 & 1 \\ -2 & -3 & 0 \end{bmatrix}$ 38. $\begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 3 \\ 1 & -1 & 0 \end{bmatrix}$ 39. $\begin{bmatrix} 1 & 1 & 1 \\ 3 & 2 & -1 \\ 3 & 1 & 2 \end{bmatrix}$ 40. $\begin{bmatrix} 3 & 3 & 1 \\ 1 & 2 & 1 \\ 2 & -1 & 1 \end{bmatrix}$

Exercise 4.

In Problems 61–66, show that each matrix has no inverse.

61. $\begin{bmatrix} 4 & 2 \\ 2 & 1 \end{bmatrix}$ 62. $\begin{bmatrix} -3 & \frac{1}{2} \\ 6 & -1 \end{bmatrix}$ 63. $\begin{bmatrix} 15 & 3 \\ 10 & 2 \end{bmatrix}$

64. $\begin{bmatrix} -3 & 0 \\ 4 & 0 \end{bmatrix}$ 65. $\begin{bmatrix} -3 & 1 & -1 \\ 1 & -4 & -7 \\ 1 & 2 & 5 \end{bmatrix}$ 66. $\begin{bmatrix} 1 & 1 & -3 \\ 2 & -4 & 1 \\ -5 & 7 & 1 \end{bmatrix}$

Exercise 5.

College Tuition: Nikki and Joe take classes at a community college, LCCC, and a local university, SIUE. The number of credit hours taken and the cost per credit hour (2009–2010 academic year, tuition only) are as follows:

	LCCC	SIUE
Nikki	6	9
Joe	3	12

Cost per Credit Hour	
LCCC	\$80.00
SIUE	\$277.80

(a) Write a matrix A for the credit hours taken by each student and a matrix B for the cost per credit hour.
(b) Compute AB and interpret the results.

Exercise 6.

School Loan: Interest Jamal and Stephanie each have school loans issued from the same two banks. The amounts borrowed and the monthly interest rates are given next (interest is compounded monthly):

	Lender 1	Lender 2
Jamal	\$4000	\$3000
Stephanie	\$2500	\$3800

	Monthly Interest Rate
Lender 1	0.011 (1.1%)
Lender 2	0.006 (0.6%)

(a) Write a matrix A for the amounts borrowed by each student and a matrix B for the monthly interest rates.
(b) Compute AB and interpret the results.
(c) Let $C = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. Compute $C(A + B)$ and interpret the result.

Exercise 7.

Cryptography: One method of encryption is to use a matrix to encrypt the message and then use the corresponding inverse matrix to decode the message. The encrypted matrix, E, is obtained by multiplying the message matrix, M, by a key matrix, K. The original message can be retrieved by multiplying the encrypted matrix by the inverse of the key matrix. That is $E = M.K$ and $M = E.K^{-1}$

(a) Given the key matrix $K = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, find its inverse, K^{-1},

(b) Use your result from part (a) to decode the encrypted matrix $E = \begin{bmatrix} 47 & 34 & 33 \\ 44 & 36 & 27 \\ 47 & 41 & 20 \end{bmatrix}$

(c) Each entry in your result for part (b) represents the position of a letter in the English alphabet ($A = 1, B = 2, C = 3$ and so on) What is the original message?

HOMEWORKS:
Exercise 1: 18, 20, 22
Exercise 2: 26, 28, 30
Exercise 3: 32, 34, 36, 38
Exercise 4: 62, 64, 66
Exercise 5:
Exercise 6