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Exercise 1. Find the inverse matrix of the following matrices

a) $A = \begin{bmatrix} 3 & 4 \\ 5 & 7 \end{bmatrix}$ b) $B = \begin{bmatrix} 3 & -4 & 5 \\ 2 & -3 & 1 \\ 3 & -5 & 1 \end{bmatrix}$ c) $C = \begin{bmatrix} 1 & -a & 0 & 0 \\ 0 & 1 & -a & 0 \\ 0 & 0 & 1 & -a \\ 0 & 0 & 0 & 1 \end{bmatrix}$.

Exercise 2. Find a such that matrix $A = \begin{bmatrix} a+1 & -1 & a \\ 3 & a+1 & 3 \\ a-1 & 0 & a-1 \end{bmatrix}$ is an inverse matrix

Exercise 3. Solve the following systems of linear equations

a) $\begin{cases} 3x_1 - 5x_2 + 2x_3 + 4x_4 = 2 \\ 7x_1 - 4x_2 + x_3 + 3x_4 = 5 \\ 5x_1 + 7x_2 - 4x_3 - 6x_4 = 3 \end{cases}$ b) $\begin{cases} 3x_1 - x_2 + 3x_3 = 1 \\ -4x_1 + 2x_2 + x_3 = 3 \\ -2x_1 + x_2 + 4x_3 = 4 \\ 10x_1 - 5x_2 - 6x_3 = -10 \end{cases}$

Exercise 4. Solve the following systems of linear equations

a) $\begin{cases} x + 2y - z + 3t = 12 \\ 2x + 5y - z + 11t = 49 \\ 3x + 6y - 4z + 13t = 49 \\ x + 2y - 2z + 9t = 33 \end{cases}$ b) $\begin{cases} x+2y+3z+4t=-4 \\ 3x+7y+10z+11t=-11 \\ x+2y+4z+2t=-3 \\ x+2y+2z+7=-6 \end{cases}$

Exercise 5. Find m such that the system of linear equations $\begin{cases} mx_1 + 2x_2 - x_3 = 3 \\ x_1 + mx_2 + 2x_3 = 4 \\ 2x_1 + 3x_2 + x_3 = -m \end{cases}$ has a unique solution

Exercise 6. Given a system of linear equations $\begin{cases} x_1 + 2x_2 - x_3 + mx_4 = 4 \\ -x_1 - x_2 + 3x_3 + 2x_4 = k \\ 2x_1 - x_2 - 3x_3 + (m-1)x_4 = 3 \\ x_1 + x_2 + x_3 + 2mx_4 = 5 \end{cases}$

a) Solve the system of linear equations when $m = 2, k = 5$

b) Find m, k that the system has a unique solution

c) Find m, k that the system has many infinitely many solutions

Exercise 7.

Given a matrix $A = \begin{bmatrix} m & 1 & 1 & 1 \\ 1 & m & 1 & m \\ 1 & 1 & 1 & m^2 \end{bmatrix}$ .Find m such that r(A) < 3

Exercise 8.. Determine the rank of the following exericses

a. $\begin{bmatrix} 1 & 3 & 5 & -1 \\ 2 & -1 & -3 & 4 \\ 5 & 1 & -1 & 7 \\ 7 & 7 & 9 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 2 & -1 & 3 & -2 & 4 \\ 4 & -2 & 5 & 1 & 7 \\ 2 & -1 & 1 & 8 & 2 \end{bmatrix}$ c. $\begin{bmatrix} 2 & -4 & 3 & 1 & 0 \\ 1 & -2 & 1 & -4 & 2 \\ 0 & 1 & -1 & 3 & 1 \\ 1 & -7 & 4 & -4 & 5 \end{bmatrix}$

d. $\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \\ 10 & m & 12 \end{bmatrix}$ e. $\begin{bmatrix} 3 & 1 & 1 & 4 \\ 2 & 2 & 4 & 3 \\ m & 4 & 10 & 1 \\ 1 & 7 & 17 & 3 \end{bmatrix}$ f. $\begin{bmatrix} m & 1 & 1 & 1 \\ 1 & m & 1 & 1 \\ 1 & 1 & m & 1 \\ 1 & 1 & 1 & m \end{bmatrix}$

Exercise 9. Solve the following systems

a. $\begin{cases} 2x_1 + x_2 - 2x_3 = 10 \\ 3x_1 + 2x_2 + 2x_3 = 1 \\ 5x_1 + 4x_2 + 3x_3 = 4 \end{cases}$ b. $\begin{cases} x_1 + 2x_2 + x_3 = 7 \\ 2x_1 - x_2 + 4x_3 = 17 \\ 3x_1 - 2x_2 + 2x_3 = 14 \end{cases}$ c. $\begin{cases} 2x_1 + x_2 - 3x_3 = 1 \\ 5x_1 + 2x_2 - 6x_3 = 5 \\ 3x_1 - x_2 - 4x_3 = 7 \end{cases}$

d. $\begin{cases} x_1 + 2x_2 - x_3 = 3 \\ 2x_1 + 5x_2 - 4x_3 = 5 \\ 3x_1 + 4x_2 + 2x_3 = 12 \end{cases}$ e. $\begin{cases} x_1 + x_2 = 7 \\ x_2 - x_3 + x_4 = 5 \\ x_1 - x_2 + x_3 + x_4 = 6 \\ x_2 - x_4 = 10 \end{cases}$

f. $\begin{cases} x_1 + 2x_2 + 3x_3 = 14 \\ 3x_1 - 2x_2 + x_3 = 10 \\ x_1 + x_2 + x_3 = 6 \\ 2x_1 + 3x_2 - x_3 = 5 \\ x_1 + x_2 = 3 \end{cases}$ g. $\begin{cases} x_1 + 2x_2 + 3x_3 + 4x_4 = 5 \\ 2x_1 + x_2 + 2x_3 + 3x_4 = 1 \\ 3x_1 + 2x_2 + x_3 + 2x_4 = 1 \\ 4x_1 + 3x_2 + 2x_3 + x_4 = -5 \end{cases}$

Exercise 10. Solve the following systems

a. $\begin{cases} mx + y + z = 1 \\ x + my + z = m \\ x + y + mz = m^2 \end{cases}$ b. $\begin{cases} x + y + (1-m)z = m+2 \\ (1+m)x - y + 2z = 0 \\ 2x - my + 3z = m+2 \end{cases}$

c. $\begin{cases} x_1 - 2x_2 + x_3 + 2x_4 = 1 \\ x_1 + x_2 - x_3 + x_4 = m \\ x_1 + 7x_2 - 5x_3 - x_4 = 4m \end{cases}$ d. $\begin{cases} 2x_1 - x_2 + x_3 + x_4 = 1 \\ x_1 + 2x_2 - x_3 + 4x_4 = 2 \\ x_1 + 7x_2 - 4x_3 + 11x_4 = m \end{cases}$

Exercise 11 $\begin{cases} mx + y + z = m \\ 2x + (1+m)y + (1+m)z = m-1 \\ x + y + mz = 1 \end{cases}$. Find m such that the system has solution