TROY UNIVERSITY PROGRAM AT HUST

REVIEW

MTH112, PRE-CALCULUS ALGEBRA

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OUTLINE

- Real numbers
- Polynomials
- Factoring Polynomials
- Synthetic Division
- Rational Expressions
- nth Roots, Rational Exponents

Sets and Real numbers

Sets

- A **set** is a well-defined collection of distinct objects. The objects of a set are called its **elements.**
- If a set has no elements, it is called the **empty set**, or **null set,** and is denoted by the symbol ∅
- Examples:
  - The set of digits of the collection numbers 0;1;2;3;4;5;6;7;8 and 9. We denote $D = \{0; 1; 2; 3; 4; 5; 6; 7; 8; 9\}$
  - The set of all students at HUST
  -

Notation of Sets

- Roster method: the braces { }
- Set-builder notation:

$$D = \{x \mid x \text{ is a digit}\}$$

Read as "D is the set of all x such that x is a digit."

- Examples:
  - $E = \{x | x \ is \ an \ even \ digit\} = \{0; 2; 4; 6; 8\}$
  - $E = \{x | x \ is \ an \ odd \ digit\} = \{1; 3; 5; 7; 9\}$

Intersection and union

- If A and B are sets, the **intersection** of A with B, denoted $A \cap B$ is the set consisting of elements that belong to both A and B.
- The **union** of A with B, denoted $A \cup B$ is the set consisting of elements that belong to either A or B, or both.
- Example:
  - Let $A = \{1; 3; 5; 8\}$, $B = \{3; 5; 7\}$ and $C = \{2; 4; 6; 8\}$. Find $A \cap B$, $A \cup B$, $B \cap (A \cup C)$

Complement

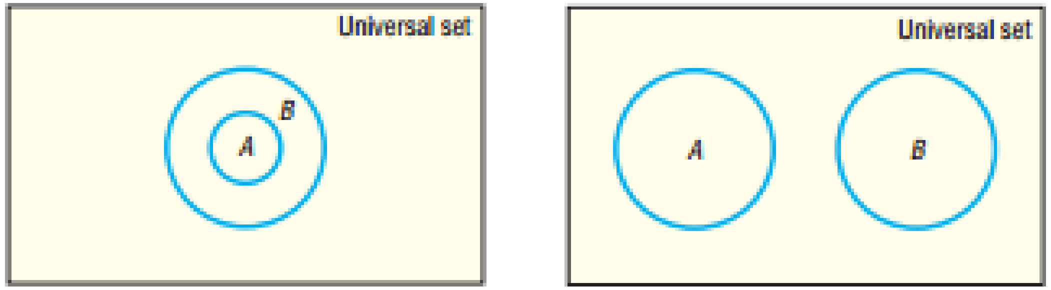
- A **universal set** *U*, the set consisting of all the elements that we wish to consider. Once a universal set has been designated, we can consider elements of the universal set not found in a given set.

- If *A* is a set, the **complement** of *A*, denoted $\bar{A}$ is the set consisting of all the elements in the universal set that are not in *A*

- It follows from the definition of complement that $A \cup \bar{A} = U$ and $A \cap \bar{A} = \emptyset$

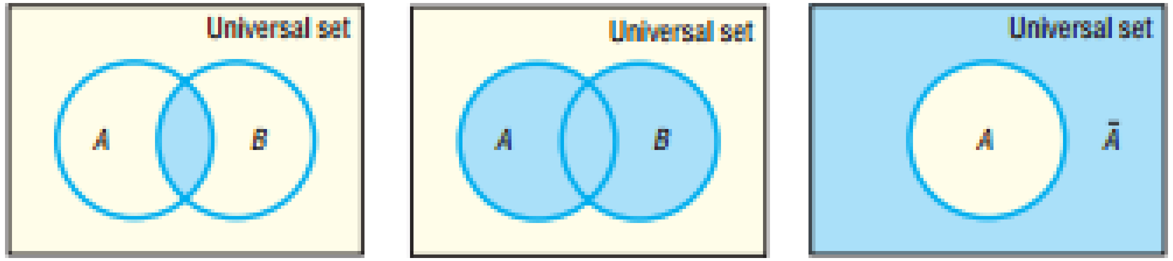
- Example: If the universal set is $U = \{1,2,3,4,5,6,7,8,9\}$ and if $A = \{1,3,5,7,9\}$, then $\bar{A} = \{2,4,6,8\}$

Venn diagram



(a) $A \subseteq B$
subset

(b) $A \cap B = \varnothing$
disjoint sets



(a) $A \cap B$
intersection

(b) $A \cup B$
union

(c) $\bar{A}$
complement

Classify Numbers

- The **counting numbers**, or **natural numbers,** are the numbers in the set $\{1,2,3,4,\ldots\}$.
- The **whole numbers** are the numbers in the set $\{0,1,2,3,4,\ldots\}$, that is, the counting numbers together with 0.
- The **integers** are the set of numbers $\{\ldots.,-3,-2,-1,0,1,2,3,\ldots.\}$.
-

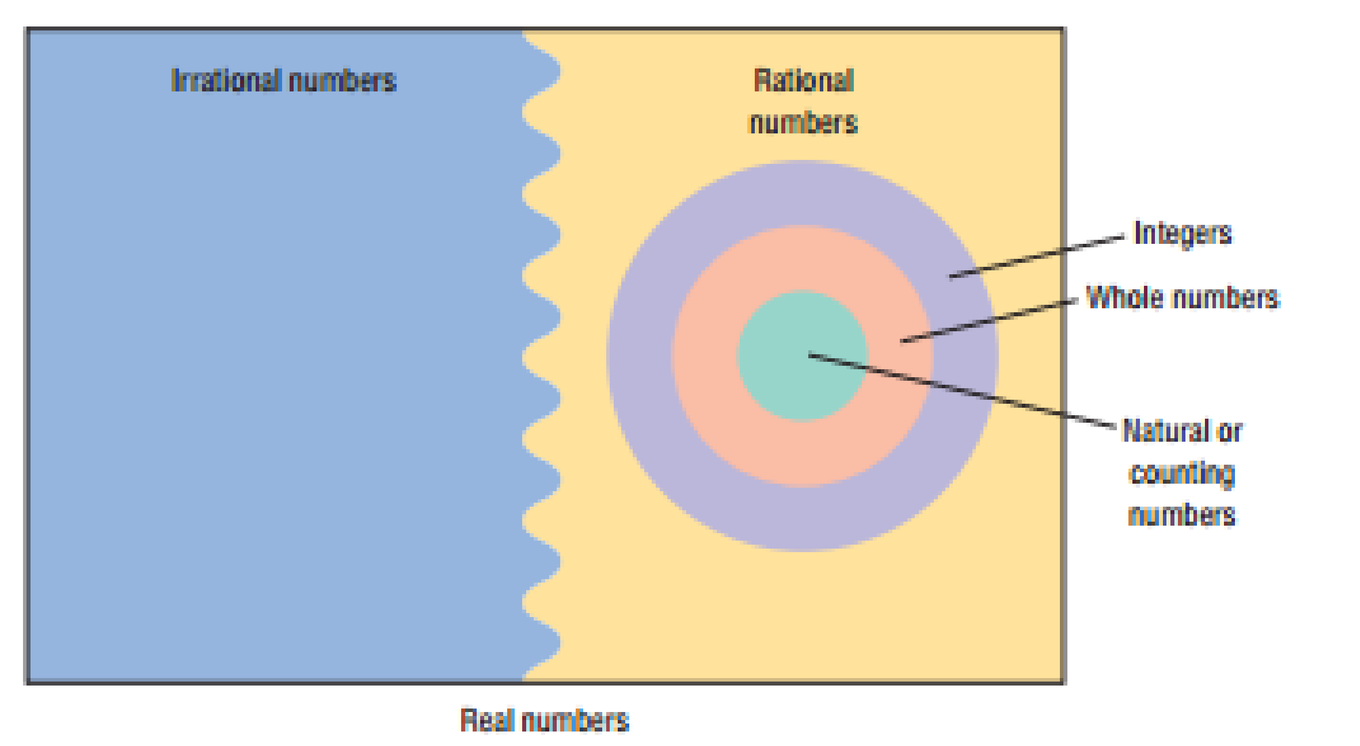
Classify Numbers

- A **rational number** is a number that can be expressed as a quotient $\frac{a}{b}$ of two integers. The integer *a* is called the **numerator,** and the integer *b*, which cannot be 0, is called the **denominator.** The rational numbers are the numbers in the set $\{x | x = \frac{a}{b}, \text{where a, b are integers and } b \neq 0\}$
- Rational numbers may be represented as **decimals**
- Examples: $\frac{3}{4} = 0.75\ \frac{5}{2} = 2.5, -\frac{2}{3} = -0.666 \ldots = -0.\overline{6}$

Classify numbers

- Some decimals do not fit into either of these categories. Such decimals represent **irrational numbers.**
- irrational numbers cannot be written in the form $\frac{a}{b}$ where a, b are integers and $b \neq 0$
- The set of **real numbers** is the union of the set of rational numbers with the set of irrational numbers.

Classify numbers



Approximations

- Every decimal may be represented by a real number (either rational or irrational), and every real number may be represented by a decimal. Using symbol $\approx$
- Example: $\sqrt{2} \approx 1.4142$, $\pi \approx 3.1416$.
- **Truncation:** Drop all the digits that follow the specified final digit in the decimal.
- **Rounding:** Identify the specified final digit in the decimal. If the next digit is 5 or more, add 1 to the final digit; if the next digit is 4 or less, leave the final digit as it is. Then truncate following the final digit.

Approximations

- Example: Approximating 20.98752 to Two decimal places by
  - Truncating: 20.98
  - Rounding 20.99

Operations

Operation	Symbol	Words
Addition	$a + b$	Sum: a plus b
Subtraction	$a - b$	Difference: a minus b
Multiplication	$a \cdot b$, $(a) \cdot b$, $a \cdot (b)$, $(a) \cdot (b)$, ab, $(a)b$, $a(b)$, $(a)(b)$	Product: a times b
Division	a/b or $\frac{a}{b}$	Quotient: a divided by b

Work with Properties of Real Numbers

- The **reflexive property** states that a number always equals itself; that is, $a = a$
- The **symmetric property** states that if $a = b$ then $b = a$
- The **transitive property** states that if $a = b$ and $b = c$ then $a = c$
- The **principle of substitution** states that if $a = b$ then we may substitute *b* for *a* in any expression containing *a*.

Some properties

- Commutative Properties

$$a + b = b + a \tag{1a}$$

$$a \cdot b = b \cdot a \tag{1b}$$

Some properties

- Associative Properties

$$a + (b + c) = (a + b) + c = a + b + c \quad (2a)$$

$$a \cdot (b \cdot c) = (a \cdot b) \cdot c = a \cdot b \cdot c \quad (2b)$$

Some properties

- Distributive Property

$$a \cdot (b + c) = a \cdot b + a \cdot c \tag{3a}$$

$$(a + b) \cdot c = a \cdot c + b \cdot c \tag{3b}$$

Some properties

- For each *nonzero* real number *a*, there is a real number $\frac{1}{a}$ called the **multiplicative inverse** of *a*, having the following property:
- Multiplicative Inverse Property

$$a \cdot \frac{1}{a} = \frac{1}{a} \cdot a = 1 \qquad \text{if } a \neq 0 \qquad (5b)$$

Difference & Quotient

- The **difference** also read "*a* less *b*" or "*a* minus *b*," is defined as $a - b = a + (-b)$
- If *b* is a nonzero real number, the **quotient** $\boldsymbol{\frac{a}{b}}$ also read as "*a* divided by *b*" or "the ratio of *a* to *b*," is defined as $\frac{a}{b} = a.\frac{1}{b}$ if $b \neq 0$

Some properties

- Multiplication by Zero: $a.0 = 0$
- Division Properties: $\frac{0}{a} = 0, \frac{a}{a} = 1$ if $a \neq 0$
- Rules signs

$$a(-b) = -(ab) \qquad (-a)b = -(ab) \qquad (-a)(-b) = ab$$

$$-(-a) = a \qquad \frac{a}{-b} = \frac{-a}{b} = -\frac{a}{b} \qquad \frac{-a}{-b} = \frac{a}{b} \qquad (10)$$

Some properties

- Zero-product Property: If $ab = 0$, then $a = 0$ or $b = 0$, or both
- Arithmetic of Quotients

$$\frac{a}{b} + \frac{c}{d} = \frac{ad}{bd} + \frac{bc}{bd} = \frac{ad + bc}{bd} \qquad \text{if } b \neq 0, d \neq 0 \qquad (13)$$

$$\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd} \qquad \text{if } b \neq 0, d \neq 0 \qquad (14)$$

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc} \qquad \text{if } b \neq 0, c \neq 0, d \neq 0 \qquad (15)$$

Least common multiples (LCM)

- The LCM of two numbers is the smallest number that each has as a common multiple.
- Example: LCM(15,12)=60

15, 30, 45, 60, 75, 90, 105, 120, . . .

12, 24, 36, 48, 60, 72, 84, 96, 108, 120, . . .

Polynomial

- Recognize monomials
- Know Formulas for Special Products
- Divide Polynomials Using Division

Recognize Monomials

- A **monomial** in one variable is the product of a constant and a variable raised to a nonnegative integer power. A monomial is of the form ax^k, where *a* is a constant, x is a variable, and is an integer. The constant *a* is called the **coefficient** of the monomial. If $a \neq 0$, then *k* is called the **degree** of the monomial.
- Example: $9x^6$, Coefficient: 9, Degree 6

Recognize Polynomials

- The sum or difference of two monomials having different degrees is called a **binomial**. Example: $x^2 - 2$
- The sum or difference of three monomials with three different degrees is called a **trinomial.** Example: $x^3 - 3x + 5$
- A **polynomial** in one variable is an algebraic expression of the form $a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$,

where $a_n, a_{n-1}, \dots, a_1, a_0$ are constants called the coefficient of the polynomial, n is an integer and x is a variable. If $a_n \neq 0$, it is called the leading coefficient, $a_n x^n$ is called the leading term.

Know Formulas for Special Products

Difference of Two Squares

$$(x - a)(x + a) = x^2 - a^2 \tag{2}$$

Squares of Binomials, or Perfect Squares

$$(x + a)^2 = x^2 + 2ax + a^2 \tag{3a}$$

$$(x - a)^2 = x^2 - 2ax + a^2 \tag{3b}$$

Know Formulas for Special Products

Cubes of Binomials, or Perfect Cubes

$$(x + a)^3 = x^3 + 3ax^2 + 3a^2x + a^3 \qquad \textbf{(4a)}$$

$$(x - a)^3 = x^3 - 3ax^2 + 3a^2x - a^3 \qquad \textbf{(4b)}$$

Difference of Two Cubes

$$(x - a)(x^2 + ax + a^2) = x^3 - a^3 \qquad \textbf{(5)}$$

Sum of Two Cubes

$$(x + a)(x^2 - ax + a^2) = x^3 + a^3 \qquad \textbf{(6)}$$

Divide Polynomials Using Division

- Example: Find the quotient and the remainder when $3x^3 + 4x^2 + x + 7$ is divided by $x^2 + 1$

Factoring Polynomials

- Factor the Difference of Two Squares and the Sum and Difference of Two Cubes
- Factor Perfect Squares
- Factor a Second-Degree Polynomial
- Factor by Grouping
- Factor a Second-Degree Polynomial
- Complete the Square

Factor the Difference of Two Squares and the Sum and Difference of Two Cubes

- Difference of Two Squares: $x^2 - a^2 = (x - a)(x + a)$
- Perfect Squares: $x^2 + 2ax + a^2 = (x + a)^2$
- $x^2 - 2ax + a^2 = (x - a)^2$
- Sum of two cubes: $x^3 + a^3 = (x + a)(x^2 - ax + a^2)$
- Difference of Two Cubes: $x^3 - a^3 = (x - a)(x^2 + ax + a^2)$

Factor Perfect Squares

- Factor completely:

  $x^2 + 6x + 9$

  $25x^2 + 30x + 9$

Factor a Second-Degree Polynomial

To factor a second-degree polynomial $x^2 + Bx + C$, find integers whose product is C and whose sum is B. That is, if there are numbers a, b, where $ab = C$ and $a + b = B$, then

$$\boxed{x^2 + Bx + C = (x + a)(x + b)}$$

- Factor completely: $x^2 + 7x + 10$

Any polynomial of the form $x^2 + a^2$, a real, is prime.

Factor by Grouping

- Sometimes a common factor does not occur in every term of the polynomial, but in each of several groups of terms that together make up the polynomial. When this happens, the common factor can be factored out of each group by means of the Distributive Property. This technique is called **factoring by grouping**
- Factor completely by grouping: $(x^2 + 2)x + (x^2 + 2).3$

Factor a Second-Degree Polynomial

Steps for Factoring $Ax^2 + Bx + C$,
when $A \neq 1$ and A, B, and C Have No Common Factors

STEP 1: Find the value of AC.
STEP 2: Find a pair of integers whose product is AC that add up to B. That is, find a and b so that $ab = AC$ and $a + b = B$.
STEP 3: Write $Ax^2 + Bx + C = Ax^2 + ax + bx + C$.
STEP 4: Factor this last expression by grouping.

- Factor completely: $2x^2 + 5x + 3$

nth Roots; Rational Exponents

- Work with nth Roots
- Simplify Radicals
- Rationalize Denominators
- Simplify Expressions with Rational Exponents

Work with nth Roots

- The **principal *n*th root of a real number *a*, $\boldsymbol{n \geq 2}$** an integer, symbolized by $\sqrt[n]{a}$ is defined as follows:

$$\sqrt[n]{a} = b \text{ means } a = b^n$$

- In general, if $n \geq 2$ is an integer and a is a real number, we have

$$\sqrt[n]{a^n} = a \text{ if } n \geq 3 \text{ is odd}$$

$$\sqrt[n]{a^n} = |a| \text{ if } n \geq 2 \text{ is even}$$

Simplify Radicals

Properties of Radicals

$\sqrt[n]{ab} = \sqrt[n]{a}\,\sqrt[n]{b}$		**(2a)**
$\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$	$b \neq 0$	**(2b)**
$\sqrt[n]{a^m} = (\sqrt[n]{a})^m$		**(2c)**

Rationalize Denominators

- When radicals occur in quotients, it is customary to rewrite the quotient so that the new denominator contains no radicals. This process is referred to as **rationalizing the denominator.**

If a Denominator Contains the Factor	Multiply by	To Obtain a Denominator Free of Radicals
$\sqrt{3}$	$\sqrt{3}$	$(\sqrt{3})^2 = 3$
$\sqrt{3} + 1$	$\sqrt{3} - 1$	$(\sqrt{3})^2 - 1^2 = 3 - 1 = 2$
$\sqrt{2} - 3$	$\sqrt{2} + 3$	$(\sqrt{2})^2 - 3^2 = 2 - 9 = -7$
$\sqrt{5} - \sqrt{3}$	$\sqrt{5} + \sqrt{3}$	$(\sqrt{5})^2 - (\sqrt{3})^2 = 5 - 3 = 2$
$\sqrt[3]{4}$	$\sqrt[3]{2}$	$\sqrt[3]{4} \cdot \sqrt[3]{2} = \sqrt[3]{8} = 2$

- Rationalize the denominator of each expression:

(a) $\dfrac{1}{\sqrt{3}}$ (b) $\dfrac{5}{4\sqrt{2}}$ (c) $\dfrac{\sqrt{2}}{\sqrt{3} - 3\sqrt{2}}$

Simplify Expressions with Rational Exponents

- If a is a real number and $n > 2$ is an integer, then $a^{1/n} = \sqrt[n]{a}$
- If a is a real number and m and n are integers containing no common factors with $n \geq 2$, then $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$ (provided that $\sqrt[n]{a}$ exists.