TROY UNIVERSITY PROGRAM AT HUST

Chapter 2 - Graphs

MTH112, PRE-CALCULUS ALGEBRA

DR. DOAN DUY TRUNG

Outline

- The Distance and Midpoint Formulas
- Graphs of Equations in Two Variables; Intercepts; Symmetry
- Lines
- Circles
- Variation

The Distance and Midpoint Formulas

- Use the Distance Formula
- Use the Midpoint Formula

Use the Distance Formula

- Distance Formula

Distance Formula

The distance between two points $P_1 = (x_1, y_1)$ and $P_2 = (x_2, y_2)$, denoted by $d(P_1, P_2)$, is

$$d(P_1, P_2) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \qquad \textbf{(1)}$$

- Find the distance d between the points

(a) $(1; 3)$ and $(5; 6)$ (b) $(-4,5)$ and $(3,2)$

Use Midpoint Formula

- Midpoint Formula

Midpoint Formula

The midpoint $M = (x, y)$ of the line segment from $P_1 = (x_1, y_1)$ to $P_2 = (x_2, y_2)$ is

$$M = (x, y) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \quad \textbf{(2)}$$

Graphs of Equations in Two Variables: Intercepts, Symmetry

- Graph Equations by Plotting Points
- Find Intercepts from a Graph
- Find Intercepts from an Equation
- Test an Equation for Symmetry with Respect to the x-Axis, the y-Axis, and the Origin
- Know How to Graph Key Equations

Graph Equations by Plotting Points

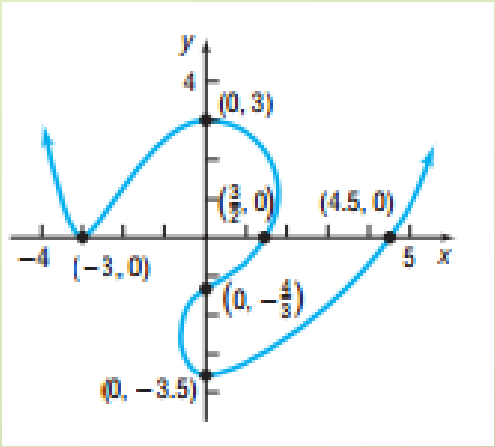
- Example on Determining if the following points are on the graph of the equation $2x - y = 6$

(a) (2,3) (b) (2,-2)

Find Intercepts from a Graph

- The points, if any, at which a graph crosses or touches the coordinate axes are called the **intercepts.**
- The intercepts of the graph are the points

$(-3,0)$, $(0,3)$, $\left(\frac{3}{2},0\right)$, $\left(0,-\frac{4}{3}\right)$, $(0,-3.5)$, $(4.5,0)$



- Find the x-intercept(s) and the y-intercept(s) of the graph of $y = x^2 - 4$. Then graph $y = x^2 - 4$ by plotting points

Symmetry

- A graph is said to be **symmetric with respect to the x-axis** if, for every point (x, y) on the graph, the point (x,-y) is also on the graph.
- A graph is said to be **symmetric with respect to the y-axis** if, for every point (x,y) on the graph, the point (-x,y) is also on the graph
- A graph is said to be **symmetric with respect to the origin** if, for every point (x,y) on the graph, the point (-x,-y) is also on the graph

Lines

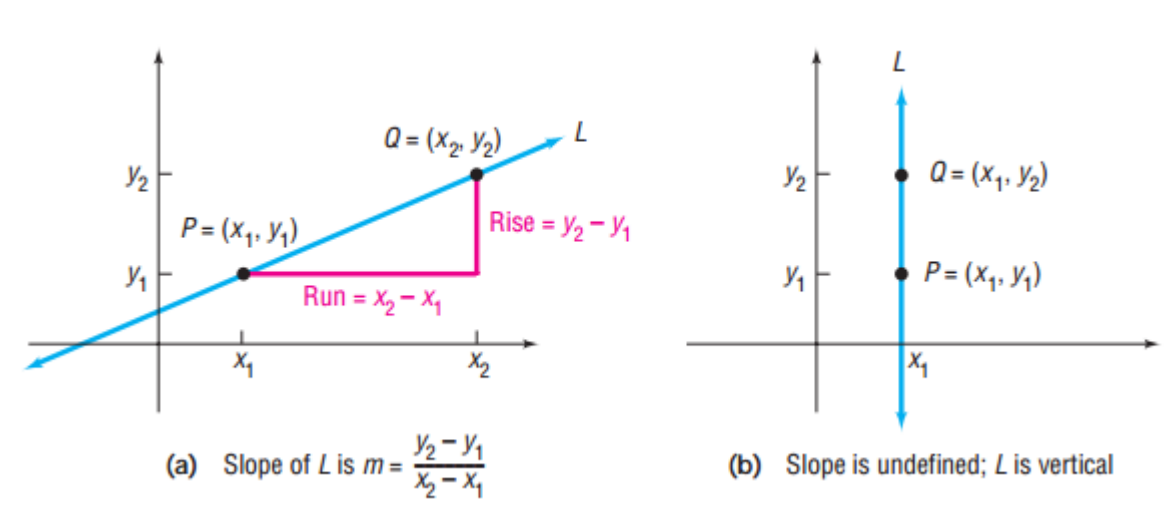
- Calculate and Interpret the Slope of a Line
- Find the Equation of a Vertical Line
- Use the Point–Slope Form of a Line; Identify Horizontal Lines
- Find the Equation of a Line Given Two Points
- Write the Equation of a Line in Slope–Intercept Form
- Identify the Slope and *y*-Intercept of a Line from Its Equation
- Graph Lines Written in General Form Using Intercepts
- Find Equations of Parallel Lines
- Find Equations of Perpendicular Lines

Calculate and Interpret the Slope of a Line

Let $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ be two distinct points. If $x_1 \neq x_2$, the **slope** m of the nonvertical line L containing P and Q is defined by the formula

$$m = \frac{y_2 - y_1}{x_2 - x_1} \qquad x_1 \neq x_2 \qquad \textbf{(1)}$$

If $x_1 = x_2$, L is a **vertical line** and the slope m of L is **undefined** (since this results in division by 0).



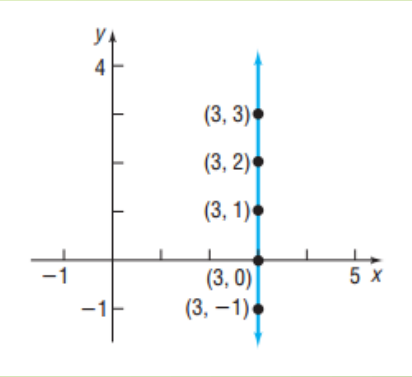
Vertical line and Horizontal line

Equation of a Vertical Line

A vertical line is given by an equation of the form

$$x = a$$

where a is the x-intercept.



Equation of a Horizontal Line

A horizontal line is given by an equation of the form

$$y = b$$

where b is the y-intercept.

Use the Point–Slope Form of a Line; Identify Horizontal Lines

- Point-Slope Form of an Equation of a Line

Point–Slope Form of an Equation of a Line

An equation of a nonvertical line with slope m that contains the point (x_1, y_1) is

$$y - y_1 = m(x - x_1) \qquad (2)$$

- Find the equation of the line with slope 4 and containing the point (1,2).

Write the Equation of a Line in Slope–Intercept Form

- An equation of a line with slope m and y-intercept b is $y = mx + b$
- Example: Find the slope m and y-intercept b of the equation $2x + 4y = 8$. Graph the equation.

Parallel Lines

- Two nonvertical lines are parallel if and only if their slopes are equal and they have different *y*-intercepts .
- Example: Show that the lines given by the following equations are parallel:

$L_1: 2x + 3y = 6$ $\qquad\qquad$ $L_2: 4x + 6y = 0$

- Example: Find an equation for the line that contains the point $(2, -3)$ and is parallel to the line $2x + y = 6$
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Perpendicular Lines

- Two nonvertical lines are perpendicular if and only if the product of their slopes is . -1.
- Example: Find an equation of the line that contains the point $(1, -2)$ and is perpendicular to the line $x + 3y = 6$. Graph the two lines.

Circles

- Write the Standard Form of the Equation of a Circle
- Graph a Circle
- Work with the General Form of the Equation of a Circle

Write the Standard Form of the Equation of a Circle

- A **circle** is a set of points in the *xy*-plane that are a fixed distance *r* from a fixed point . The fixed distance *r* is called the **radius,** and the fixed point (h, k) is called the **center** of the circle,
- The standard form of an equation of a circle with radius r and center (h,k) is $(x-h)^2+(y-k)^2=r^2$
- Example: For the circle $(x+3)^2+(y-2)^2=16$, find the intercepts, if any of its graph.

Work with the General Form of the Equation of a Circle

- When its graph is a circle, the equation $x^2 + y^2 + ax + by + c = 0$ is referred to as the general form of the equation of a circle.

Variation

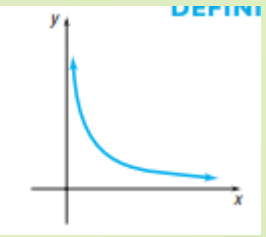
- Construct a Model Using Direct Variation
- Construct a Model Using Inverse Variation
- Construct a Model Using Joint Variation or Combined Variation

Construct a Model Using Direct Variation

- Let x and *y* denote two quantities. Then *y* **varies directly** with x, or *y* is **directly proportional to** x, if there is a nonzero number *k* such that $y = kx$. The number *k* is called the **constant of proportionality**

- Example on Mortgage Payments: The monthly payment *p* on mortgage varies directly with the amount borrowed *B*. If the monthly payment on a 30-year mortgage is $6.65 for every $1000 borrowed, find a formula that relates the monthly payment *p* to the amount borrowed *B* for a mortgage with these terms. Then find the monthly payment *p* when the amount borrowed *B* is $120,000.

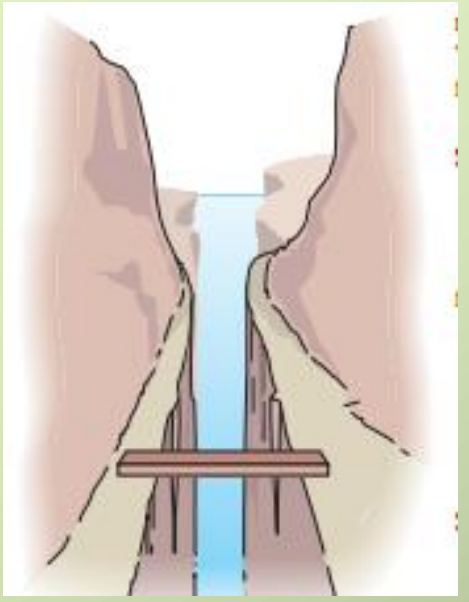
Construct a Model Using Inverse Variation



- Let x and *y* denote two quantities. Then *y* **varies inversely** with x, or *y* is **inversely proportional to** x, if there is a nonzero constant *k* such that $y = \frac{k}{x}$

Construct a Model Using Inverse Variation

- The maximum weight W that can be safely supported by a 2-inch by 4-inch piece of pine varies inversely with its length l. Experiments indicate that the maximum weight that a 10-foot-long 2-by-4 piece of pine can support is 500 pounds. Write a general formula relating the maximum weight W (in pounds) to length l (in feet). Find the maximum weight W that can be safely supported by a length of 25 feet.



Construct a Model Using Joint Variation or Combined Variation

- When a variable quantity Q is proportional to the product of two or more other variables, we say that Q **varies jointly** with these quantities. Finally, combinations of direct and/or inverse variation may occur. This is usually referred to as **combined variation.**
- Example on Force of the Wind on a Window: The force *F* of the wind on a flat surface positioned at a right angle to the direction of the wind varies jointly with the area *A* of the surface and the square of the speed of the wind. A wind of 30 miles per hour blowing on a window measuring 4 feet by 5 feet has a force of 150 pounds. What is the force on a window measuring 3 feet by 4 feet caused by a wind of 50 miles per hour?

$F = kAv^2$, where k is the constant of proportionality.