TROY UNIVERSITY PROGRAM AT HUST

Chapter 4 – Linear and Quadratic Functions

MTH112, PRE-CALCULUS ALGEBRA

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Outline

- Linear Functions and Their Properties
- Linear Models: Building Linear Functions from Data
- Quadratic Functions and Their Properties
- Build Quadratic Models from Verbal Descriptions and from Data
- Inequalities Involving Quadratic Function

Linear Functions and Their Properties

- Graph Linear Functions
- Use Average Rate of Change to Identify Linear Functions
- Determine Whether a Linear Function Is Increasing, Decreasing, or Constant
- Build Linear Models from Verbal Descriptions

Graph Linear Functions

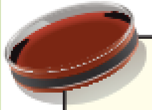
- A **linear function** is a function of the form $f(x) = mx + b$. The graph of a linear function is a line with slope *m* and *y*-intercept *b*. Its domain is the set of all real numbers.

Use Average Rate of Change to Identify Linear Functions

- Linear functions have a constant average rate of change. That is, the average rate of change of a linear function $f(x) = mx + b$ is $\frac{\Delta y}{\Delta x} = m$

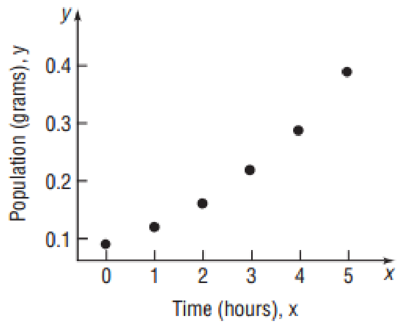
Use Average Rate of Change to Identify Linear Functions

- A strain of E-coli Beu 397-recA441 is placed into a Petri dish at 30° Celsius and allowed to grow. The data shown in Table are collected. The population is measured in grams and the time in hours. Plot the ordered pairs *(x, y)* in the Cartesian plane and use the average rate of change to determine whether the function is linear.



Time (hours), *x*	Population (grams), *y*	*(x, y)*
0	0.09	(0, 0.09)
1	0.12	(1, 0.12)
2	0.16	(2, 0.16)
3	0.22	(3, 0.22)
4	0.29	(4, 0.29)
5	0.39	(5, 0.39)

Figure



Table

Time (hours), *x*	Population (grams), *y*	Average Rate of Change $= \frac{\Delta y}{\Delta x}$
0	0.09	
		$\frac{0.12 - 0.09}{1 - 0} = 0.03$
1	0.12	
		0.04
2	0.16	
		0.06
3	0.22	
		0.07
4	0.29	
		0.10
5	0.39	

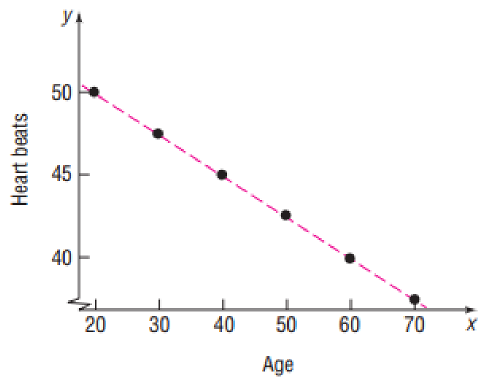
Use Average Rate of Change to Identify Linear Functions

- The data in Table represent the maximum number of heartbeats that a healthy individual should have during a 15-second interval of time while exercising for different ages. Plot the ordered pairs *(x, y)* in the Cartesian plane and use the average rate of change to determine whether the function is linear.

Age, x	Maximum Number of Heartbeats, y	(x, y)
20	50	(20, 50)
30	47.5	(30, 47.5)
40	45	(40, 45)
50	42.5	(50, 42.5)
60	40	(60, 40)
70	37.5	(70, 37.5)

Source: American Heart Association

Figure



Table

Age, x	Maximum Number of Heartbeats, y	Average Rate of Change $= \frac{\Delta y}{\Delta x}$
20	50	
		$\frac{47.5 - 50}{30 - 20} = -0.25$
30	47.5	
		-0.25
40	45	
		-0.25
50	42.5	
		-0.25
60	40	
		-0.25
70	37.5	

Determine Whether a Linear Function Is Increasing, Decreasing, or Constant

- A linear function $f(x) = mx + b$ is increasing over its domain if its slope, *m*, is positive. It is decreasing over its domain if its slope, *m*, is negative. It is constant over its domain if its slope, *m*, is zero.
- Example

Determine whether the following linear functions are increasing, decreasing, or constant.

(a) $f(x) = 5x - 2$ (b) $g(x) = -2x + 8$

(c) $s(t) = \frac{3}{4}t - 4$ (d) $h(z) = 7$

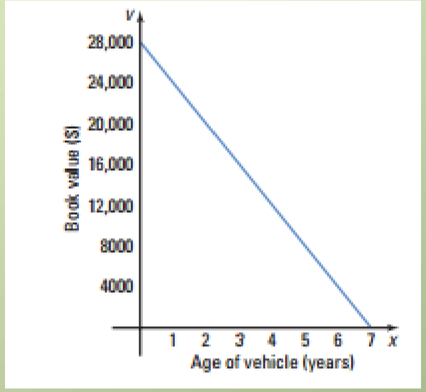
Build Linear Models from Verbal Descriptions

- If the average rate of change of a function is a constant *m*, a linear function *f* can be used to model the relation between the two variables as follows: $f(x) = mx + b$, where b is the value of f at 0, that is, $b = f(0)$
- Example

Book value is the value of an asset that a company uses to create its balance sheet. Some companies depreciate their assets using straight-line depreciation so that the value of the asset declines by a fixed amount each year. The amount of the decline depends on the useful life that the company places on the asset. Suppose that a company just purchased a fleet of new cars for its sales force at a cost of \$28,000 per car. The company chooses to depreciate each vehicle using the straight-line method over 7 years. This means that each car will depreciate by $\frac{\$28{,}000}{7} = \$4000$ per year.

(a) Write a linear function that expresses the book value *V* of each car as a function of its age, *x*.

(b) Graph the linear function.



Supply and Demand

The **quantity supplied** of a good is the amount of a product that a company is willing to make available for sale at a given price. The **quantity demanded** of a good is the amount of a product that consumers are willing to purchase at a given price. Suppose that the quantity supplied, S, and quantity demanded, D, of cellular telephones each month are given by the following functions:

$$S(p) = 60p - 900$$

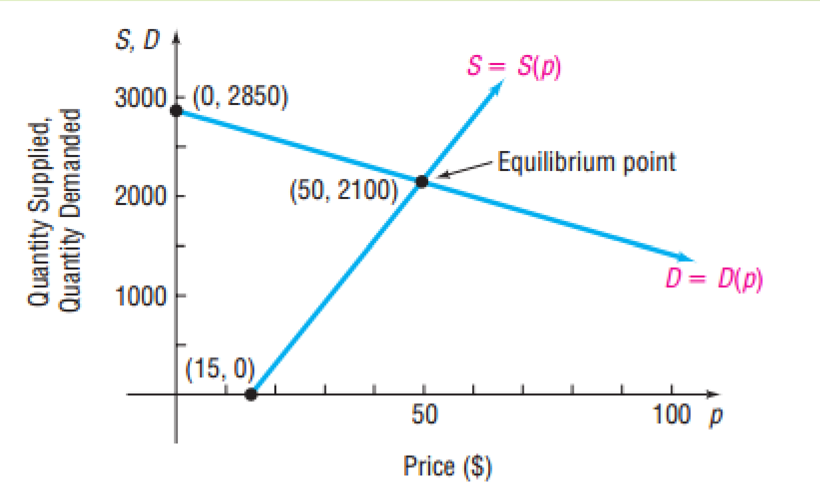
$$D(p) = -15p + 2850$$

where p is the price (in dollars) of the telephone.

(a) The **equilibrium price** of a product is defined as the price at which quantity supplied equals quantity demanded. That is, the equilibrium price is the price at which $S(p) = D(p)$. Find the equilibrium price of cellular telephones. What is the **equilibrium quantity,** the amount demanded (or supplied) at the equilibrium price?

(b) Determine the prices for which quantity supplied is greater than quantity demanded. That is, solve the inequality $S(p) > D(p)$.

(c) Graph $S = S(p)$, $D = D(p)$ and label the equilibrium price.



Linear Models: Building Linear Functions from Data

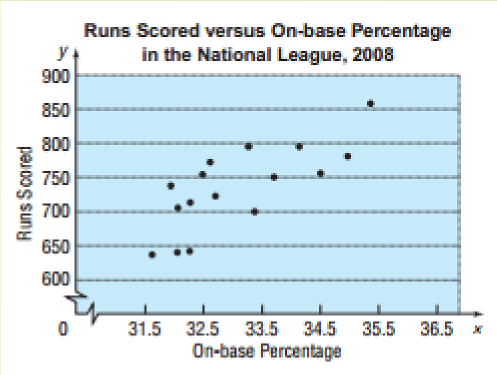
- Draw and Interpret Scatter Diagrams
- Distinguish between Linear and Nonlinear Relations
- Use a Graphing Utility to Find the Line of Best Fit

Draw and Interpret Scatter Diagrams

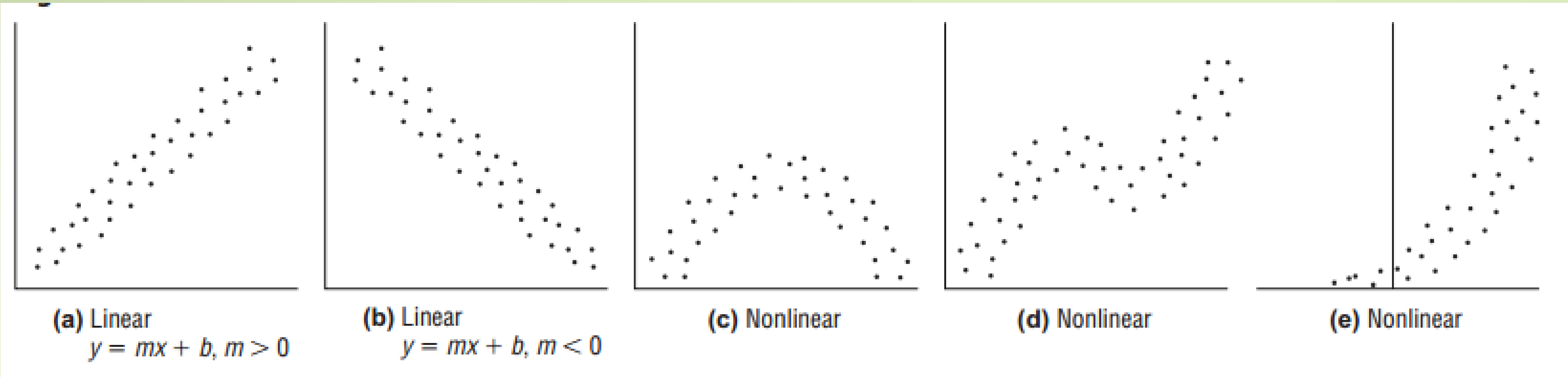
- Example: In baseball, the on-base percentage for a team represents the percentage of time that the players safely reach base. The data given in Table represent the number of runs scored y and the on-base percentage x for teams in the National League during the 2008 baseball season.

Team	On-Base Percentage, x	Runs Scored, y	(x, y)
Atlanta	34.5	753	(34.5, 753)
St. Louis	35.0	779	(35.0, 779)
Colorado	33.6	747	(33.6, 747)
Houston	32.3	712	(32.3, 712)
Philadelphia	33.2	799	(33.2, 799)
San Francisco	32.1	640	(32.1, 640)
Pittsburgh	32.0	735	(32.0, 735)
Florida	32.6	770	(32.6, 770)
Chicago Cubs	35.4	855	(35.4, 855)
Arizona	32.7	720	(32.7, 720)
Milwaukee	32.5	750	(32.5, 750)
Washington	32.3	641	(32.3, 641)
Cincinnati	32.1	704	(32.1, 704)
San Diego	31.7	637	(31.7, 637)
NY Mets	34.0	799	(34.0, 799)
Los Angeles	33.3	700	(33.3, 700)

Source: Based on data from http://www.baseball-reference.com. A Sports Reference, LLC, web site.



Distinguish between Linear and Nonlinear Relations



Finding a Model for Linearly Related Data

- Example: Use the data in Table to:
  - Select two points ((32.7,720) and (35.4,855)) and find an equation of the line containing the points
  - Graph the line on the scatter diagram obtained in Example.

Team	On-Base Percentage, x	Runs Scored, y	(x, y)
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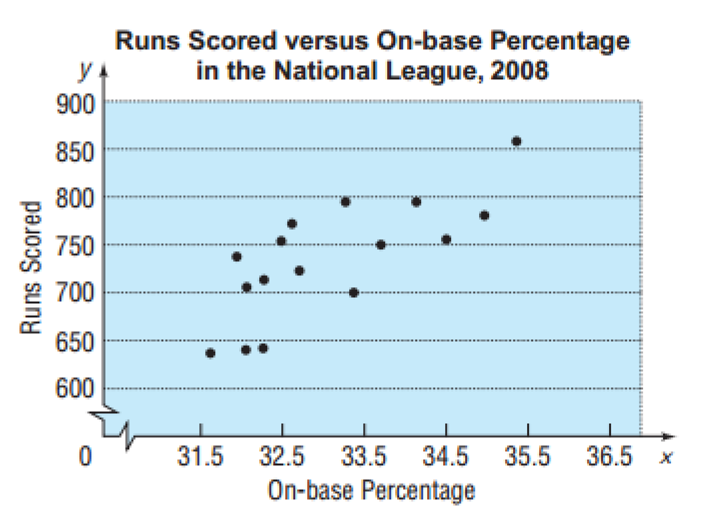


Quadratic Functions and Their Properties

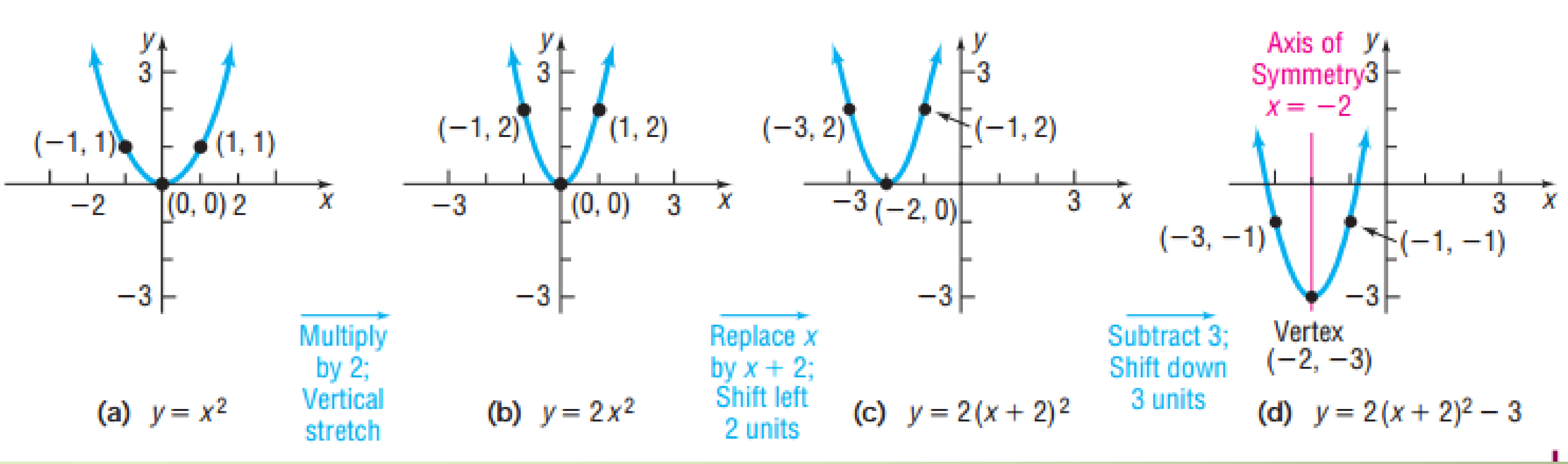
- Graph a Quadratic Function Using Transformations
- Identify the Vertex and Axis of Symmetry of a Quadratic Function
- Graph a Quadratic Function Using Its Vertex, Axis, and Intercepts
- Find a Quadratic Function Given Its Vertex and One Other Point
- Find the Maximum or Minimum Value of a Quadratic Function

Graph a Quadratic Functions

- A **quadratic function** is a function of the form $f(x) = ax^2 + bx + c$, where a,b and c are real numbers and $a \neq 0$. The domain of a quadratic function is the set of all real numbers.
- If $h = -\frac{b}{2a}$ and $k = \frac{4ac-b^2}{4a}$, then $f(x) = ax^2 + bx + c = a(x-h)^2 + k$
- The graph of $f(x) = a(x-h)^2 + k$ is the parabola $y = ax^2$ shifted horizontally h units (replace x by x-h) and vertically k units (add k)/.

Graphing a Quadratic Function Using Transformations

- Graph the function $f(x) = 2x^2 + 8x + 5$. Find the vertex and axis of symmetry



Identify the Vertex and Axis of Symmetry of a Quadratic Function

- Properties of the Graph of a Quadratic Function $f(x) = ax^2 + bx + c$, where $a \neq 0$.
- Vertex = $\left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$.
- Axis of symmetry: the line $x = -\frac{b}{2a}$
- Parabola opens up if $a > 0$,; the vertex is a minimum point
- Parabola opens downs if $a < 0$; the vertex is a maximum point

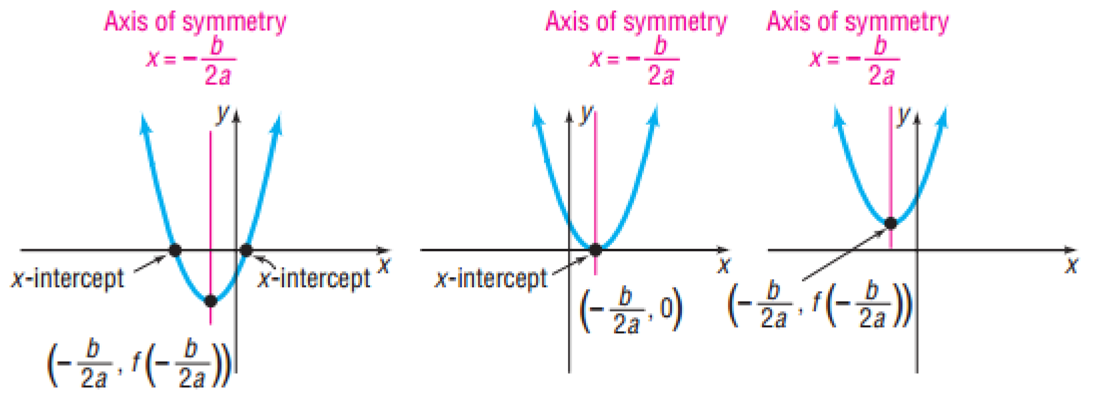
Identify the Vertex and Axis of Symmetry of a Quadratic Function

- Example: Without graphing, locate the vertex and axis of symmetry of the parabola defined by $f(x) = -3x^2 + 6x + 1$. Does it open up or down?

Graph a Quadratic Function Using Its Vertex, Axis, and Intercepts

The x-Intercepts of a Quadratic Function

1. If the discriminant $b^2 - 4ac > 0$, the graph of $f(x) = ax^2 + bx + c$ has two distinct x-intercepts so it crosses the x-axis in two places.
2. If the discriminant $b^2 - 4ac = 0$, the graph of $f(x) = ax^2 + bx + c$ has one x-intercept so it touches the x-axis at its vertex.
3. If the discriminant $b^2 - 4ac < 0$, the graph of $f(x) = ax^2 + bx + c$ has no x-intercepts so it does not cross or touch the x-axis.



(a) $b^2 - 4ac > 0$
Two x-intercepts

(b) $b^2 - 4ac = 0$
One x-intercept

(c) $b^2 - 4ac < 0$
No x-intercepts

Graphing a Quadratic Function Using Its Vertex, Axis, and Intercepts

- Let $f(x) = -3x^2 + 6x + 1$ be a Quadratic Function. Determine the domain and the range of f? Determine where f is increasing and where it is decreasing.

Find a Quadratic Function Given Its Vertex and One Other Point

Given the vertex (h, k) and one additional point on the graph of a quadratic function $f(x) = ax^2 + bx + c, a \neq 0$, we can use

$$\boxed{f(x) = a(x - h)^2 + k \qquad \textbf{(3)}}$$

to obtain the quadratic function.

Find a Quadratic Function Given Its Vertex and One Other Point

- Determine the quadratic function whose vertex is $(1, -5)$ and whose y-intercept is -3.

Build Quadratic Models from Verbal Descriptions and from Data

- Build Quadratic Models from Verbal Descriptions
- Build Quadratic Models from Data (p. 304)

Build Quadratic Models from Verbal Descriptions

In economics, revenue *R*, in dollars, is defined as the amount of money received from the sale of an item and is equal to the unit selling price *p*, in dollars, of the item times the number x of units actually sold. That is

$$R = xp$$

The Law of Demand states that *p* and x are related: As one increases, the other decreases.The equation that relates *p* and *x* is called the **demand equation.** When the demand equation is linear, the revenue model is a quadratic function.

Build Quadratic Models from Verbal Descriptions

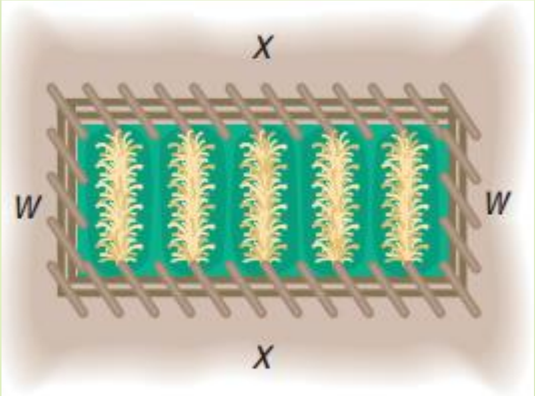
Example on Maximizing Revenue: The marketing department at Texas Instruments has found that, when certain calculators are sold at a price of *p* dollars per unit, the number x of calculators sold is given by the demand equation

$$x = 21{,}000 - 150p$$

a) Find a model that expresses the revenue *R* as a function of the price *p*.
b) What is the domain of *R*?
c) What unit price should be used to maximize revenue?
d) If this price is charged, what is the maximum revenue?
e) How many units are sold at this price?
f) Graph *R*.
g) What price should Texas Instruments charge to collect at least $675,000 in revenue?

Build Quadratic Models from Verbal Descriptions

Example on Maximizing the Area Enclosed by a Fence: A farmer has 2000 yards of fence to enclose a rectangular field. What are the dimensions of the rectangle that encloses the most area?



Build Quadratic Models from Verbal Descriptions

Example on Analyzing the Motion of a Projectile: A projectile is fired from a cliff 500 feet above the water at an inclination of 45° to the horizontal, with a muzzle velocity of 400 feet per second. In physics, it is established that the height *h* of the projectile above the water can be modeled by

$$h(x) = -\frac{32x^2}{(400)^2} + x + 500$$

Where x is the horizontal distance of the projectile from the base of the cliff

(a) Find the maximum height of the projectile.
(b) How far from the base of the cliff will the projectile strike the water?

Build Quadratic Models from Data

The data in Table represent the percentage D of the population that is divorced for various ages x in 2007.

a) Draw a scatter diagram of the data treating age as the independent variable.
Comment on the type of relation that may exist between age and percentage of the population divorced.

b) Use a graphing utility to find the quadratic function of best fit that models the
relation between age and percentage of the population divorced.

c) Use the model found in part (b) to approximate the age at which the percentage of the population divorced is greatest.

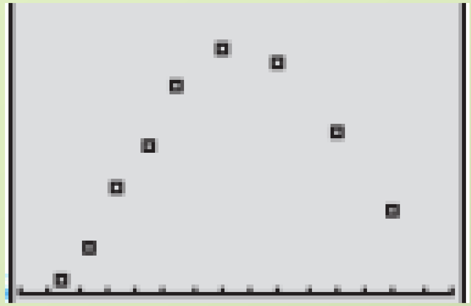
d) Use the model found in part (b) to approximate the highest percentage of the
population that is divorced.

e) Use a graphing utility to draw the quadratic function of best fit on the scatter
diagram

Age, x	Percentage Divorced, D
22	0.8
27	2.8
32	6.4
37	8.7
42	12.3
50	14.5
60	13.8
70	9.6
80	4.9

Source: United States Statistical Abstract, 2009

Build Quadratic Models from Data



- The scatter diagram, $a < 0$
- Upon executing the QUADratic REGression program: $D(x) = -0.0136x^2 + 1.4794x - 26.3412$
- The age with the greatest percent:

$$-\frac{b}{2a} = -\frac{1.4794}{2(-0.0136)} \sim 54 \text{ years}$$