TROY UNIVERSITY PROGRAM AT HUST

Chapter 6 – Exponential and Logarithmic Function

MTH112, PRE-CALCULUS ALGEBRA

DR. DOAN DUY TRUNG

Outline

- Composite Functions
- One-to-One Functions; Inverse Functions
- Exponential Functions
- Logarithmic Functions
- Properties of Logarithms
- Logarithmic and Exponential Equations
- Financial Models
- Exponential Growth and Decay Models;
- Building Exponential, Logarithmic, and Logistic Models from Data

Form a Composite Function

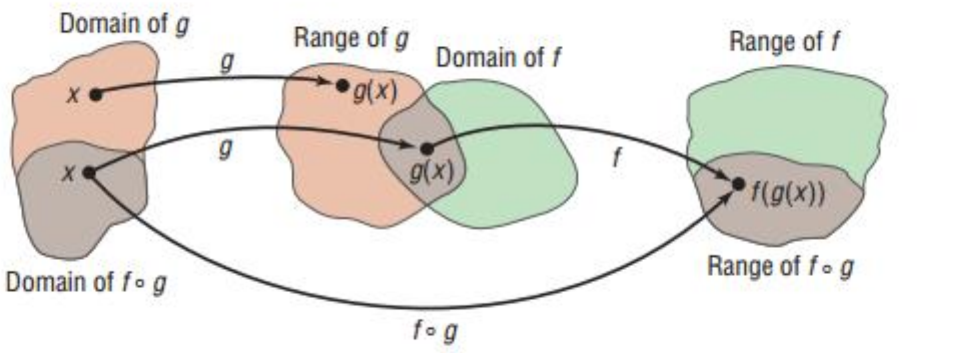
DEFINITION

Given two functions f and g, the **composite function,** denoted by $f \circ g$ (read as "f composed with g"), is defined by

$$\boxed{(f \circ g)(x) = f(g(x))}$$

The domain of $f \circ g$ is the set of all numbers x in the domain of g such that $g(x)$ is in the domain of f.

Figure 2



Form a Composite Function

Suppose that $f(x) = 2x^2 - 3$ and $g(x) = 4x$. Find:

(a) $(f \circ g)(1)$ (b) $(g \circ f)(1)$ (c) $(f \circ f)(-2)$ (d) $(g \circ g)(-1)$

Find the Domain of a Composite Function

- Example 1

Suppose that $f(x) = x^2 + 3x - 1$ and $g(x) = 2x + 3$.

Find: (a) $f \circ g$ (b) $g \circ f$

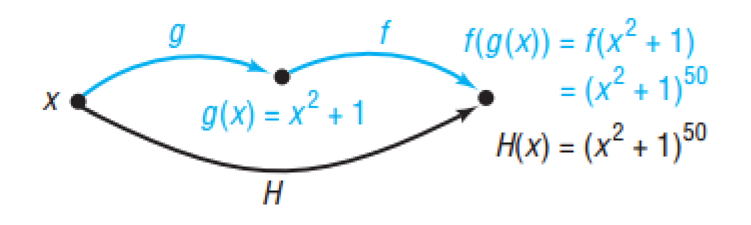
Then find the domain of each composite function.

- Example 2

Find the domain of $f \circ g$ if $f(x) = \dfrac{1}{x + 2}$ and $g(x) = \dfrac{4}{x - 1}$.

Finding the Components of a Composite Function

Find functions f and g such that $f \circ g = H$ if $H(x) = (x^2 + 1)^{50}$.

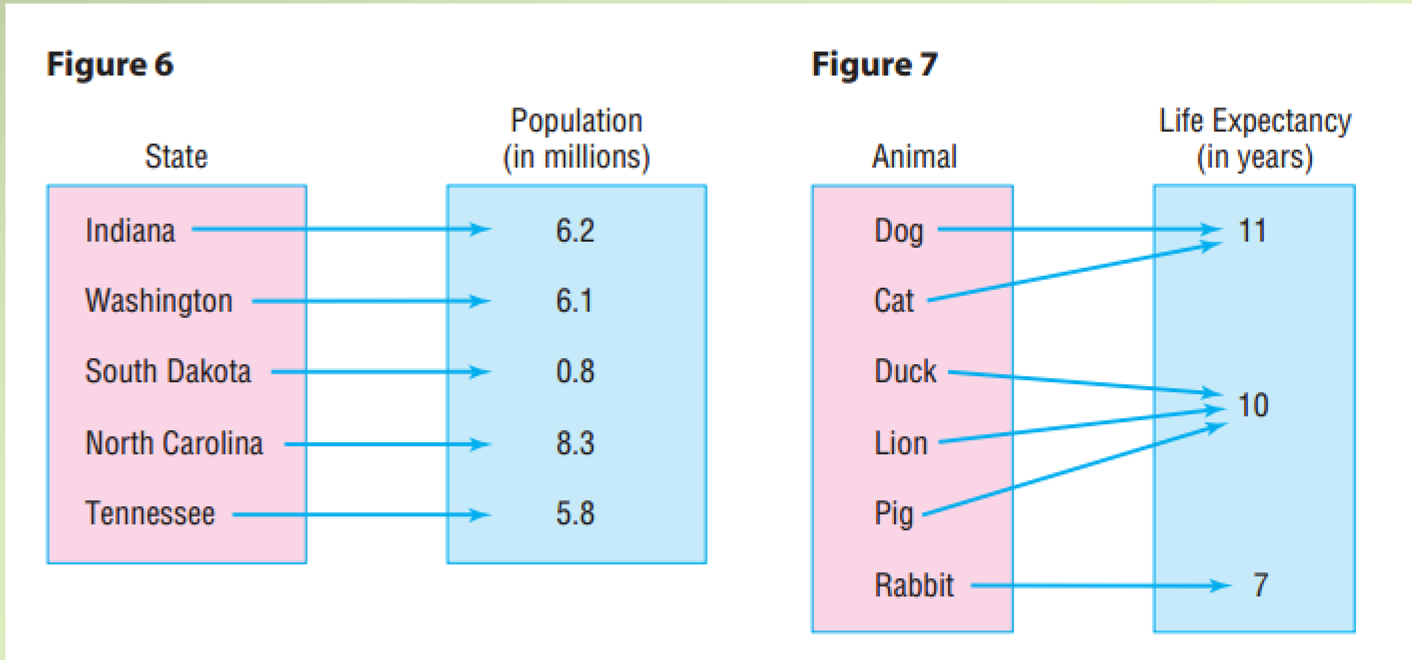


Find functions f and g such that $f \circ g = H$ if $H(x) = \dfrac{1}{x + 1}$.

One-to-One Functions, Inverse Functions

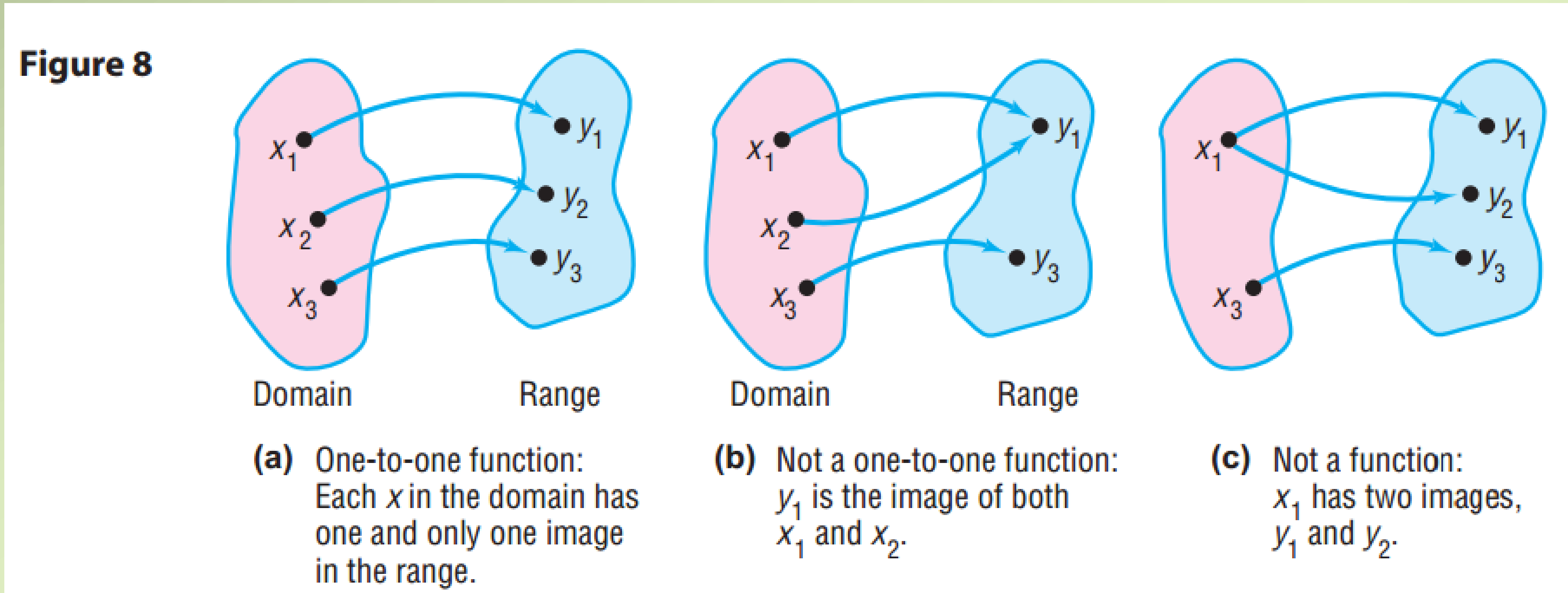
- Determine Whether a Function Is One-to-One
- Determine the Inverse of a Function Defined by a Map or a Set of Ordered Pairs
- Obtain the Graph of the Inverse Function from the Graph of the Function
- Find the Inverse of a Function Defined by an Equation

Determine Whether a Function Is One-to-One



DEFINITION A function is **one-to-one** if any two different inputs in the domain correspond to two different outputs in the range. That is, if x_1 and x_2 are two different inputs of a function f, then f is one-to-one if $f(x_1) \neq f(x_2)$.

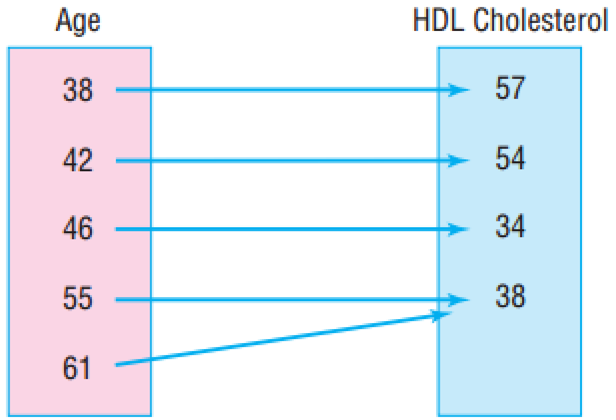
Determine Whether a Function Is One-to-One



Determine Whether a Function Is One-to-One

Determine whether the following functions are one-to-one.

(a) For the following function, the domain represents the age of five males and the range represents their HDL (good) cholesterol (mg/dL).



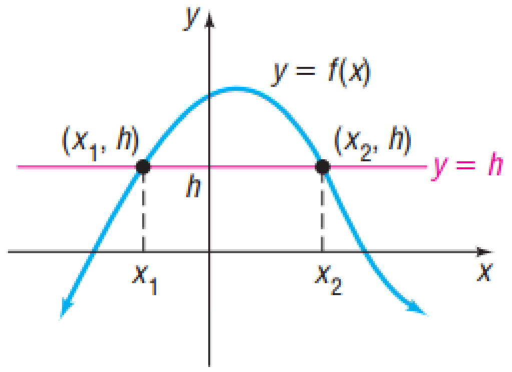
(b) $\{(-2, 6), (-1, 3), (0, 2), (1, 5), (2, 8)\}$

Horizontal-line Test

Horizontal-line Test

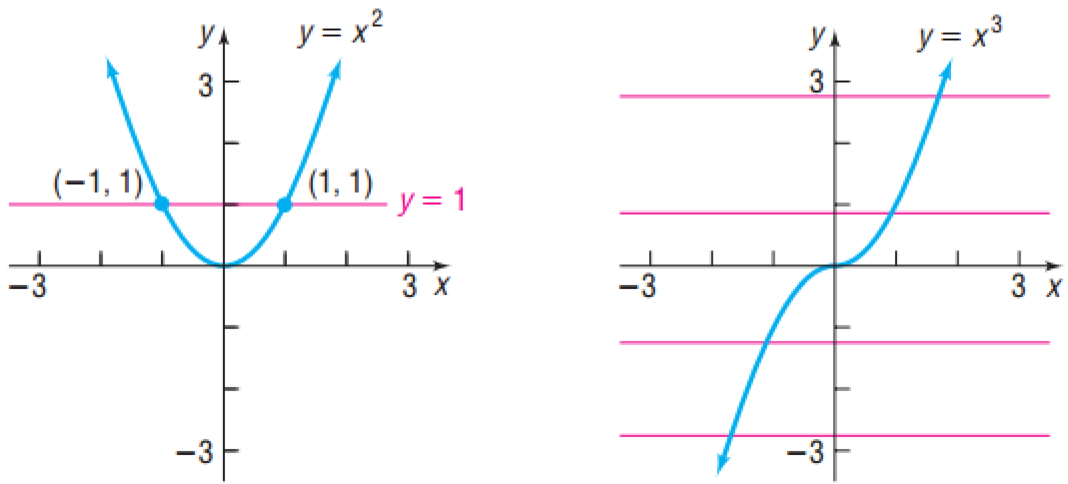
If every horizontal line intersects the graph of a function f in at most one point, then f is one-to-one.

Figure 9
$f(x_1) = f(x_2) = h$ and $x_1 \neq x_2$; f is not a one-to-one function.



For each function, use its graph to determine whether the function is one-to-one.

(a) $f(x) = x^2$ (b) $g(x) = x^3$



(a) A horizontal line intersects the graph twice; f is not one-to-one

(b) Every horizontal line intersects the graph exactly once; g is one-to-one

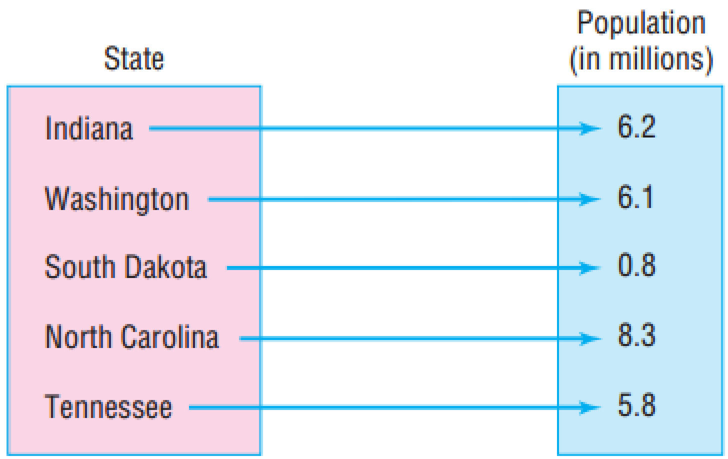
Determine the Inverse of a Function Defined by a Map or a Set of Ordered Pairs

Suppose that f is a one-to-one function. Then, to each x in the domain of f, there is exactly one y in the range (because f is a function); and to each y in the range of f, there is exactly one x in the domain (because f is one-to-one). The correspondence from the range of f back to the domain of f is called the **inverse function of f.** The symbol f^{-1} is used to denote the inverse of f.

Determine the Inverse of a Function Defined by a Map or a Set of Ordered Pairs

- Example 1

Find the inverse of the following function. Let the domain of the function represent certain states, and let the range represent the state's population (in millions). State the domain and the range of the inverse function.



- Example 2

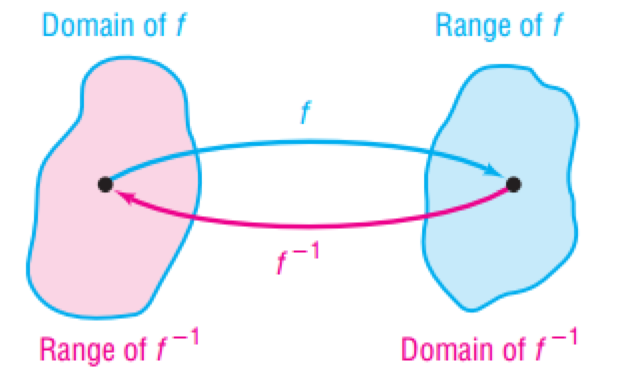
Find the inverse of the following one-to-one function:

$$\{(-3, -27), (-2, -8), (-1, -1), (0, 0), (1, 1), (2, 8), (3, 27)\}$$

State the domain and the range of the function and its inverse.

Determine the Inverse of a Function Defined by a Map or a Set of Ordered Pairs

$$\text{Domain of } f = \text{Range of } f^{-1} \qquad \text{Range of } f = \text{Domain of } f^{-1}$$



$$f^{-1}(f(x)) = x \quad \text{where } x \text{ is in the domain of } f$$
$$f(f^{-1}(x)) = x \quad \text{where } x \text{ is in the domain of } f^{-1}$$

Verifying Inverse Functions

- Examples

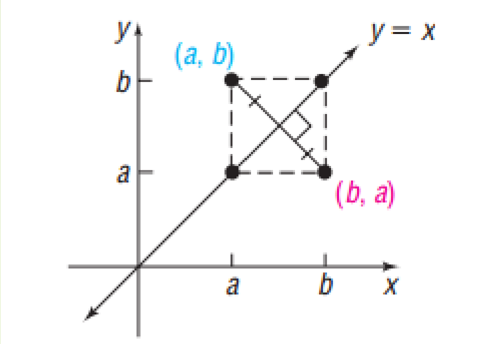
(a) Verify that the inverse of $g(x) = x^3$ is $g^{-1}(x) = \sqrt[3]{x}$

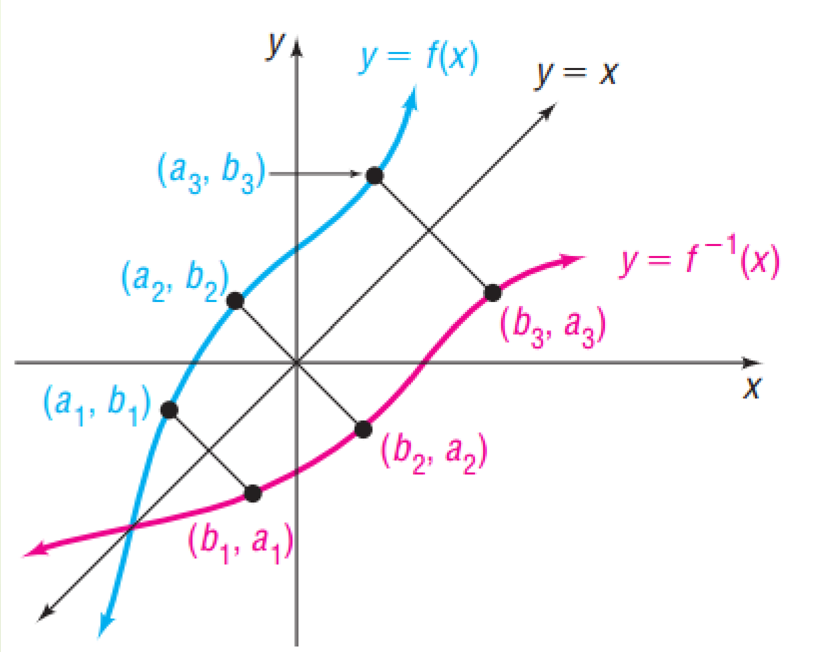
(b) Verify that the inverse of $f(x) = 2x + 3$ is $f^{-1}(x) = \frac{1}{2}(x - 3)$

Verify that the inverse of $f(x) = \dfrac{1}{x-1}$ is $f^{-1}(x) = \dfrac{1}{x} + 1$. For what values of x is $f^{-1}(f(x)) = x$? For what values of x is $f(f^{-1}(x)) = x$?

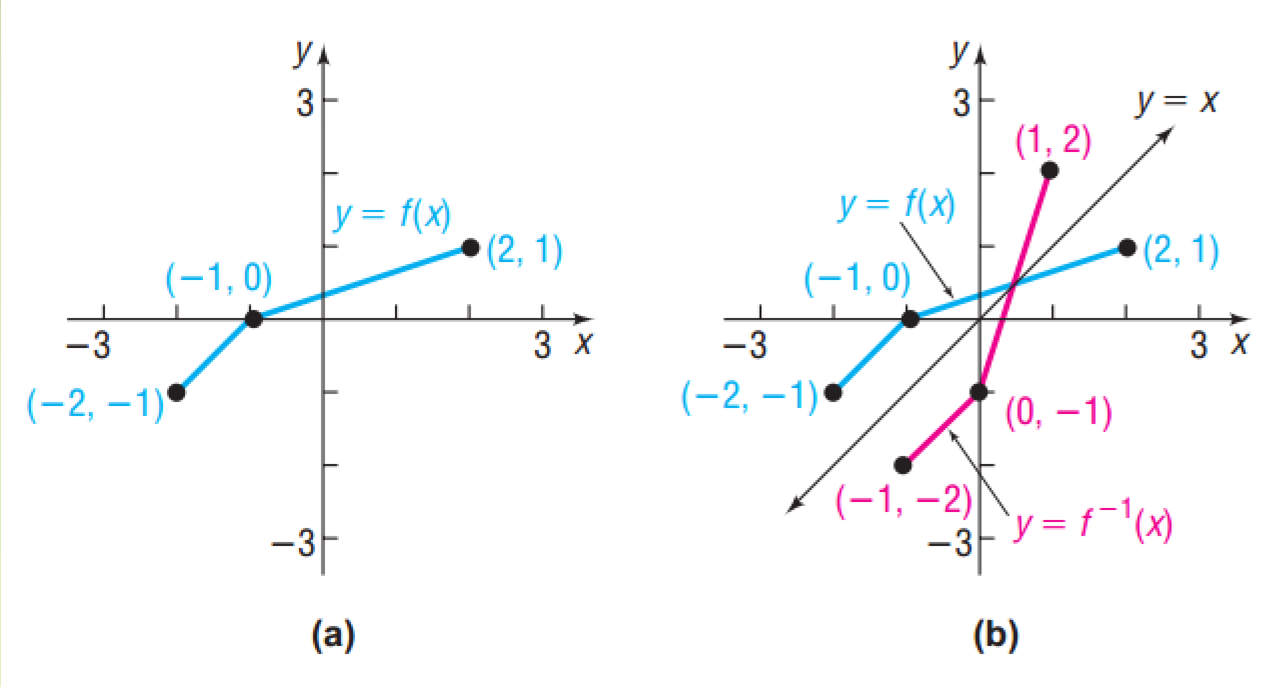
Obtain the Graph of the Inverse Function from the Graph of the Function

THEOREM The graph of a one-to-one function f and the graph of its inverse f^{-1} are symmetric with respect to the line $y = x$.





Obtain the Graph of the Inverse Function from the Graph of the Function



Find the Inverse of a Function Defined by an Equation

Procedure for Finding the Inverse of a One-to-One Function

STEP 1: In $y = f(x)$, interchange the variables x and y to obtain

$$x = f(y)$$

This equation defines the inverse function f^{-1} implicitly.

STEP 2: If possible, solve the implicit equation for y in terms of x to obtain the explicit form of f^{-1}:

$$y = f^{-1}(x)$$

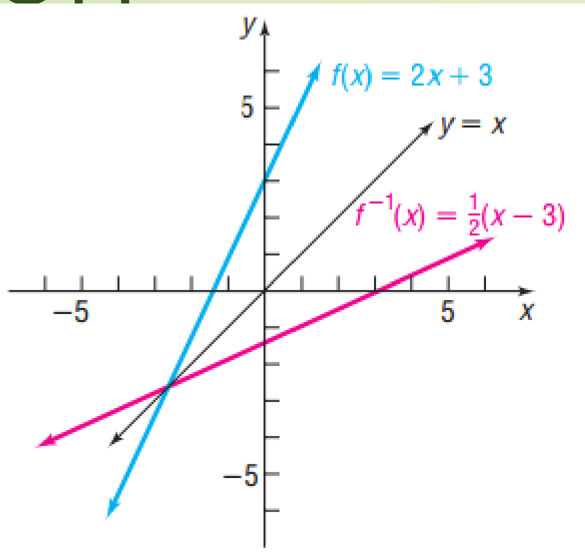
STEP 3: Check the result by showing that

$$f^{-1}(f(x)) = x \quad \text{and} \quad f(f^{-1}(x)) = x$$

Find the Inverse of a Function Defined by an Equation

- Example 1

Find the inverse of $f(x) = 2x + 3$.



- Example 2

The function

$$f(x) = \frac{2x + 1}{x - 1} \qquad x \neq 1$$

is one-to-one. Find its inverse and check the result.

Exponential Functions

- Evaluate Exponential Functions
- Graph Exponential Functions
- Define the Number *e*
- Solve Exponential Equations

Evaluate Exponential Functions

THEOREM

Laws of Exponents

If s, t, a, and b are real numbers with $a > 0$ and $b > 0$, then

$$a^s \cdot a^t = a^{s+t} \qquad (a^s)^t = a^{st} \qquad (ab)^s = a^s \cdot b^s$$

$$1^s = 1 \qquad a^{-s} = \frac{1}{a^s} = \left(\frac{1}{a}\right)^s \qquad a^0 = 1 \tag{1}$$

DEFINITION

An **exponential function** is a function of the form

$$\boxed{f(x) = Ca^x}$$

where a is a positive real number $(a > 0)$, $a \neq 1$, and $C \neq 0$ is a real number. The domain of f is the set of all real numbers. The base a is the **growth factor,** and because $f(0) = Ca^0 = C$, we call C the **initial value.**

Evaluate Exponential Functions

THEOREM For an exponential function $f(x) = Ca^x$, where $a > 0$ and $a \neq 1$, if x is any real number, then

$$\boxed{\frac{f(x+1)}{f(x)} = a \quad \text{or} \quad f(x+1) = af(x)}$$

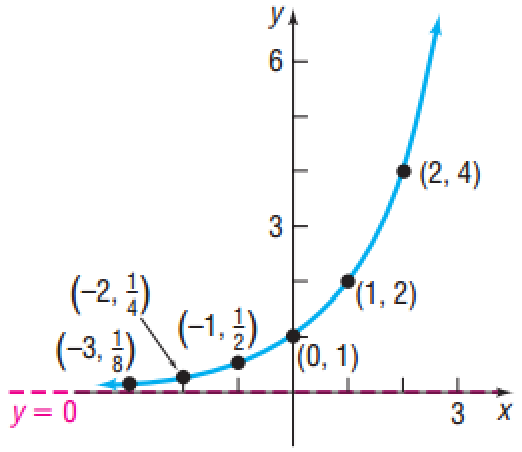
Graph Exponential Functions

Graph the exponential function: $f(x) = 2^x$

Table 3

x	$f(x) = 2^x$
-10	$2^{-10} \approx 0.00098$
-3	$2^{-3} = \frac{1}{8}$
-2	$2^{-2} = \frac{1}{4}$
-1	$2^{-1} = \frac{1}{2}$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$
10	$2^{10} = 1024$

Figure 18

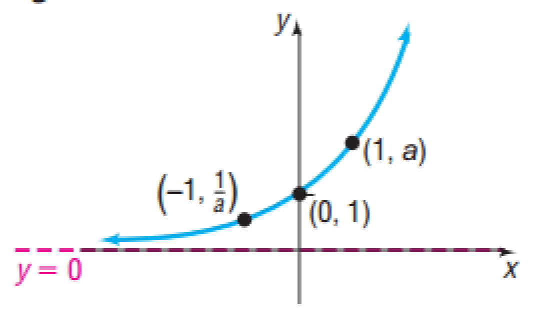


Properties of the Exponential Function $f(x) = a^x, a > 1$

Properties of the Exponential Function $f(x) = a^x$, $a > 1$

1. The domain is the set of all real numbers or $(-\infty, \infty)$ using interval notation; the range is the set of positive real numbers or $(0, \infty)$ using interval notation.
2. There are no x-intercepts; the y-intercept is 1.
3. The x-axis $(y = 0)$ is a horizontal asymptote as $x \to -\infty$ $\left[\lim_{x \to -\infty} a^x = 0\right]$.
4. $f(x) = a^x$, where $a > 1$, is an increasing function and is one-to-one.
5. The graph of f contains the points $(0, 1)$, $(1, a)$, and $\left(-1, \frac{1}{a}\right)$.
6. The graph of f is smooth and continuous, with no corners or gaps. See Figure 21.

Figure 21

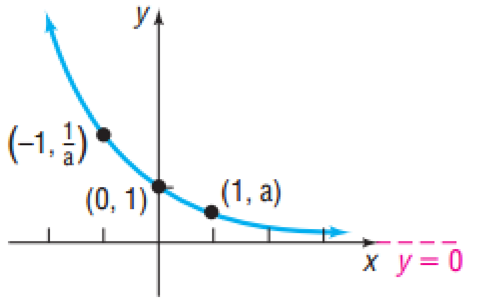


Properties of the Exponential Function $f(x) = a^x, 0 < a < 1$

Properties of the Exponential Function $f(x) = a^x$, $0 < a < 1$

1. The domain is the set of all real numbers or $(-\infty, \infty)$ using interval notation; the range is the set of positive real numbers or $(0, \infty)$ using interval notation.
2. There are no x-intercepts; the y-intercept is 1.
3. The x-axis ($y = 0$) is a horizontal asymptote as $x \to \infty$ $\left[\lim_{x\to\infty} a^x = 0\right]$.
4. $f(x) = a^x, 0 < a < 1$, is a decreasing function and is one-to-one.
5. The graph of f contains the points $\left(-1, \frac{1}{a}\right)$, $(0, 1)$, and $(1, a)$.
6. The graph of f is smooth and continuous, with no corners or gaps. See Figure 25.

Figure 25



Define the Number e

DEFINITION

The **number *e*** is defined as the number that the expression

$$\left(1 + \frac{1}{n}\right)^n \qquad \textbf{(2)}$$

approaches as $n \to \infty$. In calculus, this is expressed using limit notation as

$$e = \lim_{n\to\infty}\left(1 + \frac{1}{n}\right)^n$$

Solve Exponential Equations

$$\text{If} \quad a^u = a^v, \quad \text{then} \quad u = v. \qquad \textbf{(3)}$$

Solve: $e^{-x^2} = (e^x)^2 \cdot \dfrac{1}{e^3}$

Logarithmic Functions

- Change Exponential Statements to Logarithmic Statements and Logarithmic Statements to Exponential Statements
- Evaluate Logarithmic Expressions
- Determine the Domain of a Logarithmic Function
- Graph Logarithmic Functions
- Solve Logarithmic Equations

Logarithmic Functions

DEFINITION The **logarithmic function to the base a,** where $a > 0$ and $a \neq 1$, is denoted by $y = \log_a x$ (read as "y is the logarithm to the base a of x") and is defined by

$$\boxed{y = \log_a x \quad \text{if and only if} \quad x = a^y}$$

The domain of the logarithmic function $y = \log_a x$ is $x > 0$.

Change Exponential Statements to Logarithmic Statements and Logarithmic Statements to Exponential Statements

Change each exponential statement to an equivalent statement involving a logarithm.

(a) $1.2^3 = m$ (b) $e^b = 9$ (c) $a^4 = 24$

Change each logarithmic statement to an equivalent statement involving an exponent.

(a) $\log_a 4 = 5$ (b) $\log_e b = -3$ (c) $\log_3 5 = c$

Determine the Domain of a Logarithmic Function

Domain of the logarithmic function = Range of the exponential function = $(0, \infty)$

Range of the logarithmic function = Domain of the exponential function = $(-\infty, \infty)$

$y = \log_a x$ (defining equation: $x = a^y$)

Domain: $0 < x < \infty$ Range: $-\infty < y < \infty$

- Example

Find the domain of each logarithmic function.

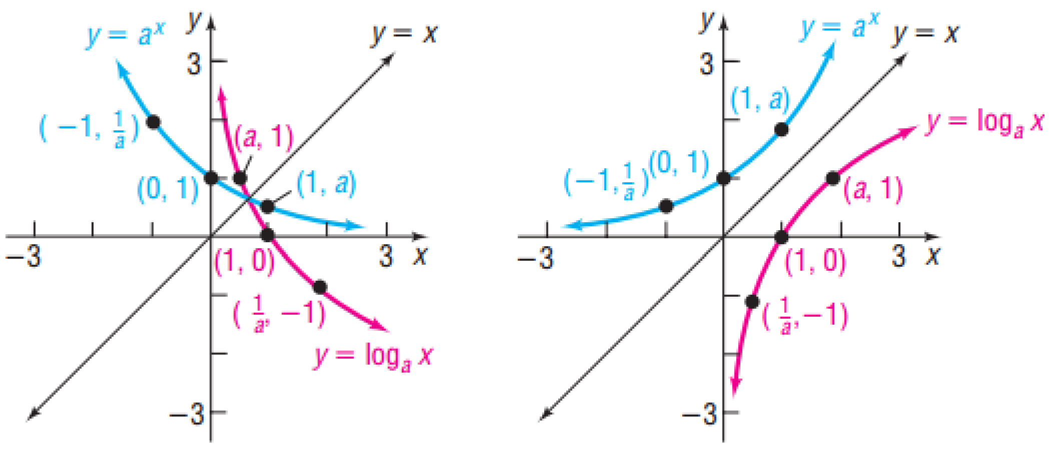
(a) $F(x) = \log_2(x + 3)$ (b) $g(x) = \log_5\left(\dfrac{1 + x}{1 - x}\right)$ (c) $h(x) = \log_{1/2}|x|$

Graph Logarithmic Functions

Properties of the Logarithmic Function $f(x) = \log_a x$

1. The domain is the set of positive real numbers or $(0, \infty)$ using interval notation; the range is the set of all real numbers or $(-\infty, \infty)$ using interval notation.
2. The x-intercept of the graph is 1. There is no y-intercept.
3. The y-axis $(x = 0)$ is a vertical asymptote of the graph.
4. A logarithmic function is decreasing if $0 < a < 1$ and increasing if $a > 1$.
5. The graph of f contains the points $(1, 0)$, $(a, 1)$, and $\left(\frac{1}{a}, -1\right)$.
6. The graph is smooth and continuous, with no corners or gaps.

Graph Logarithmic Functions



(a) $0 < a < 1$ **(b)** $a > 1$

Natural logarithm function

- If the base of a logarithmic function is the number *e*, then we have the **natural logarithm function.** This function occurs so frequently in applications that it is given a special symbol, **ln** (from the Latin, *logarithmus naturalis*). That is,

$$y = \ln x \quad \text{if and only if} \quad x = e^y \qquad \textbf{(1)}$$

- If the base of a logarithmic function is the number 10, then we have the **common logarithm function.** If the base *a* of the logarithmic function is not indicated, it is understood to be 10. That is,

$$y = \log x \quad \text{if and only if} \quad x = 10^y$$

Solve Logarithmic Equations

- Equations that contain logarithms are called **logarithmic equations.**
- **Example 1**

Solve:

(a) $\log_3(4x - 7) = 2$ (b) $\log_x 64 = 2$

- **Example 2**

Solve: $e^{2x} = 5$

Solve Logarithmic Equations

- Example

Blood alcohol concentration (BAC) is a measure of the amount of alcohol in a person's bloodstream. A BAC of 0.04% means that a person has 4 parts alcohol per 10,000 parts blood in the body. Relative risk is defined as the likelihood of one event occurring divided by the likelihood of a second event occurring. For example, if an individual with a BAC of 0.02% is 1.4 times as likely to have a car accident as an individual that has not been drinking, the relative risk of an accident with a BAC of 0.02% is 1.4. Recent medical research suggests that the relative risk R of having an accident while driving a car can be modeled by an equation of the form

$$R = e^{kx}$$

where x is the percent of concentration of alcohol in the bloodstream and k is a constant.

(a) Research indicates that the relative risk of a person having an accident with a BAC of 0.02% is 1.4. Find the constant k in the equation.

(b) Using this value of k, what is the relative risk if the concentration is 0.17%?

(c) Using this same value of k, what BAC corresponds to a relative risk of 100?

(d) If the law asserts that anyone with a relative risk of 5 or more should not have driving privileges, at what concentration of alcohol in the bloodstream should a driver be arrested and charged with a DUI (driving under the influence)?

Properties of Logarithms

- Work with the Properties of Logarithms
- Write a Logarithmic Expression as a Sum or Difference of Logarithms
- Write a Logarithmic Expression as a Single Logarithm
- Evaluate Logarithms Whose Base Is Neither 10 Nor e

Work with the Properties of Logarithms

Properties of Logarithms

In the properties given next, M and a are positive real numbers, $a \neq 1$, and r is any real number.

The number $\log_a M$ is the exponent to which a must be raised to obtain M. That is,

$$a^{\log_a M} = M \qquad \textbf{(1)}$$

The logarithm to the base a of a raised to a power equals that power. That is,

$$\log_a a^r = r \qquad \textbf{(2)}$$

$$\log_a 1 = 0 \qquad \log_a a = 1$$

Properties of Logarithms

THEOREM

Properties of Logarithms

In the following properties, M, N, and a are positive real numbers, $a \neq 1$, and r is any real number.

The Log of a Product Equals the Sum of the Logs

$$\log_a(MN) = \log_a M + \log_a N \quad (3)$$

The Log of a Quotient Equals the Difference of the Logs

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N \quad (4)$$

The Log of a Power Equals the Product of the Power and the Log

$$\log_a M^r = r \log_a M \quad (5)$$

$$a^x = e^{x \ln a} \quad (6)$$

Properties of Logarithm

THEOREM

Properties of Logarithms

In the following properties, M, N, and a are positive real numbers, $a \neq 1$.

$$\text{If } M = N, \text{ then } \log_a M = \log_a N. \quad \textbf{(7)}$$

$$\text{If } \log_a M = \log_a N, \text{ then } M = N. \quad \textbf{(8)}$$

Change of Base Formula

THEOREM

Change-of-Base Formula

If $a \neq 1$, $b \neq 1$, and M are positive real numbers, then

$$\log_a M = \frac{\log_b M}{\log_b a} \qquad \textbf{(9)}$$

Since calculators have keys only for $\boxed{\text{log}}$ and $\boxed{\text{ln}}$, in practice, the Change-of-Base Formula uses either $b = 10$ or $b = e$. That is,

$$\log_a M = \frac{\log M}{\log a} \quad \text{and} \quad \log_a M = \frac{\ln M}{\ln a} \qquad \textbf{(10)}$$

Logarithmic and Exponential Equations

- Solve Logarithmic Equations
- Solve Exponential Equations
- Solve Logarithmic and Exponential Equations Using a Graphing Utility

Solve Logarithmic Equations

$$y = \log_a x \quad \text{is equivalent to} \quad x = a^y \qquad a > 0, a \neq 1$$

If $\log_a M = \log_a N$, then $M = N$ $\qquad$ M, N, and a are positive and $a \neq 1$.

Solve Logarithmic Equations

- Example 1

Solve: $2\log_5 x = \log_5 9$

- Example 2

Solve: $\log_5(x + 6) + \log_5(x + 2) = 1$

- Example 3

$\ln x = \ln(x + 6) - \ln(x - 4)$

Solve Exponential Equations

If $a^u = a^v$, then $u = v$ $\quad a > 0, a \neq 1$

- Example 1

Solve: (a) $2^x = 5$ (b) $8 \cdot 3^x = 5$

- Example 2

Solve: $5^{x-2} = 3^{3x+2}$

- Example 3

Solve: $4^x - 2^x - 12 = 0$

Financial Models

- Determine the Future Value of a Lump Sum of Money
- Calculate Effective Rates of Return
- Determine the Present Value of a Lump Sum of Money
- Determine the Rate of Interest or Time Required to Double a Lump Sum of Money

Determine the Future Value of a Lump Sum of Money

THEOREM

Simple Interest Formula

If a principal of P dollars is borrowed for a period of t years at a per annum interest rate r, expressed as a decimal, the interest I charged is

$$I = Prt \qquad \textbf{(1)}$$

Interest charged according to formula (1) is called **simple interest.**

In working with problems involving interest, we define the term **payment period** as follows:

Annually:	Once per year	**Monthly:**	12 times per year
Semiannually:	Twice per year	**Daily:**	365 times per year*
Quarterly:	Four times per year		

Determine the Future Value of a Lump Sum of Money

- When the interest due at the end of a payment period is added to the principal so that the interest computed at the end of the next payment period is based on this new principal amount the interest is said to have been **compounded.**
- **Compound interest** is interest paid on the principal and previously earned interest.

Computing Compound Interest

- Example 1

A credit union pays interest of 8% per annum compounded quarterly on a certain savings plan. If \$1000 is deposited in such a plan and the interest is left to accumulate, how much is in the account after 1 year?

Compound Interest Formula

THEOREM

Compound Interest Formula

The amount A after t years due to a principal P invested at an annual interest rate r compounded n times per year is

$$\boxed{A = P \cdot \left(1 + \frac{r}{n}\right)^{nt} \qquad \textbf{(2)}}$$

Comparing Investments Using Different Compounding Periods

THEOREM **Continuous Compounding**

The amount A after t years due to a principal P invested at an annual interest rate r compounded continuously is

$$A = Pe^{rt} \tag{4}$$

Example

The amount A that results from investing a principal P of \$1000 at an annual rate r of 10% compounded continuously for a time t of 1 year is

$$A = \$1000e^{0.10} = (\$1000)(1.10517) = \$1105.17$$

Calculate Effective Rates of Return

The **effective rate of interest** is the equivalent annual simple interest rate that would yield the same amount as compounding *n* times per year, or continuously, after 1 year

THEOREM

Effective Rate of Interest

The effective rate of interest r_e of an investment earning an annual interest rate r is given by

Compounding n times per year: $r_e = \left(1 + \frac{r}{n}\right)^n - 1$

Continuous compounding: $r_e = e^r - 1$

Calculate Effective Rates of Return

Example

Suppose you want to open a money market account. You visit three banks to determine their money market rates. Bank A offers you 6% annual interest compounded daily and Bank B offers you 6.02% compounded quarterly. Bank C offers 5.98% compounded continuously. Determine which bank is offering the best deal.

Bank A

$$\begin{aligned} r_e &= \left(1 + \frac{0.06}{365}\right)^{365} - 1 \\ &\approx 1.06183 - 1 \\ &= 0.06183 \\ &= 6.183\% \end{aligned}$$

Bank B

$$\begin{aligned} r_e &= \left(1 + \frac{0.0602}{4}\right)^{4} - 1 \\ &\approx 1.06157 - 1 \\ &= 0.06157 \\ &= 6.157\% \end{aligned}$$

Bank C

$$\begin{aligned} r_e &= e^{0.0598} - 1 \\ &\approx 1.06162 - 1 \\ &= 0.06162 \\ &= 6.162\% \end{aligned}$$

Determine the Present Value of a Lump Sum of Money

The **present value** of *A* dollars to be received at a future date is the principal that you would need to invest now so that it will grow to *A* dollars in the specified time period.

THEOREM

Present Value Formulas

The present value P of A dollars to be received after t years, assuming a per annum interest rate r compounded n times per year, is

$$P = A \cdot \left(1 + \frac{r}{n}\right)^{-nt} \qquad (5)$$

If the interest is compounded continuously,

$$P = Ae^{-rt} \qquad (6)$$

Computing the Value of a Zero-coupon Bond

Example

A zero-coupon (noninterest-bearing) bond can be redeemed in 10 years for $1000. How much should you be willing to pay for it now if you want a return of

(a) 8% compounded monthly? (b) 7% compounded continuously?

For a return ò 8

Determine the Rate of Interest or Time Required to Double a Lump Sum of Money

Example: What annual rate of interest compounded annually should you seek if you want to double your investment in 5 years.

Exponential Growth and Decay Models;

- Find Equations of Populations That Obey the Law of Uninhibited Growth
- Find Equations of Populations That Obey the Law of Decay

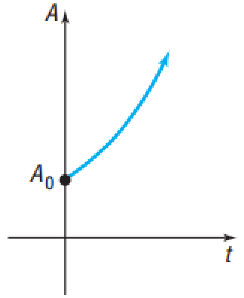
Find Equations of Populations That Obey the Law of Uninhibited Growth

Many natural phenomena have been found to follow the law that an amount A varies with time t according to the function

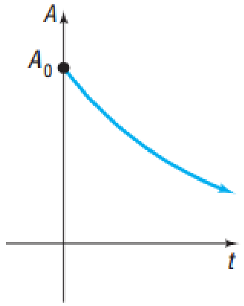
$$A(t) = A_0 e^{kt}$$

Where A_0 is the original amount $(t = 0)$ and $k \neq 0$ is a constant.

An amount *A* varies over time according to equation (1), it is said to follow the **exponential law** or the **law of uninhibited growth ($k > 0$)** **or decay ($k < 0$)**.



(a) $A(t) = A_0 e^{kt}, k > 0$
Exponential growth



(b) $A(t) = A_0 e^{kt}, k < 0$
Exponential decay

Find Equations of Populations That Obey the Law of Uninhibited Growth

Uninhibited Growth of Cells

A model that gives the number N of cells in a culture after a time t has passed (in the early stages of growth) is

$$N(t) = N_0 e^{kt} \qquad k > 0 \qquad \textbf{(2)}$$

where N_0 is the initial number of cells and k is a positive constant that represents the growth rate of the cells.

Find Equations of Populations That Obey the Law of Uninhibited Growth

Bacterial Growth

A colony of bacteria that grows according to the law of uninhibited growth is modeled by the function $N(t) = 100e^{0.045t}$, where N is measured in grams and t is measured in days.

(a) Determine the initial amount of bacteria.

(b) What is the growth rate of the bacteria?

(c) What is the population after 5 days?

(d) How long will it take for the population to reach 140 grams?

(e) What is the doubling time for the population?

Find Equations of Populations That Obey the Law of Decay

Uninhibited Radioactive Decay

The amount A of a radioactive material present at time t is given by

$$\boxed{A(t) = A_0 e^{kt} \qquad k < 0 \qquad \textbf{(3)}}$$

where A_0 is the original amount of radioactive material and k is a negative number that represents the rate of decay.

All radioactive substances have a specific **half-life**, which is the time required for half of the radioactive substance to decay. In **carbon dating**, we use the fact that all living organisms contain two kinds of carbon, carbon 12 (a stable carbon) and carbon 14 (a radioactive carbon with a half-life of 5600 years).

Example

Estimating the Age of Ancient Tools

Traces of burned wood along with ancient stone tools in an archeological dig in Chile were found to contain approximately 1.67% of the original amount of carbon 14. If the half-life of carbon 14 is 5600 years, approximately when was the tree cut and burned?