SOME BASIC TRIG - KNOW THIS!

FUNDAMENTAL TRIG IDENTITIES (IDs)

Memorize these in both "directions" (i.e., left-to-right and right-to-left).

Reciprocal Identities

$$\csc x = \frac{1}{\sin x} \qquad \sin x = \frac{1}{\csc x}$$

$$\sec x = \frac{1}{\cos x} \qquad \cos x = \frac{1}{\sec x}$$

$$\cot x = \frac{1}{\tan x} \qquad \tan x = \frac{1}{\cot x}$$

Warning: Remember that the reciprocal of $\sin x$ is $\csc x$, not $\sec x$.

Note: We typically treat "0" and "undefined" as reciprocals when we are dealing with trig functions. Your algebra teacher will not want to hear this, though!

Quotient Identities

$$\tan x = \frac{\sin x}{\cos x} \qquad \text{and} \qquad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \cot^2 x = \csc^2 x$$

$$\tan^2 x + 1 = \sec^2 x$$

Tip: The 2nd and 3rd IDs can be obtained by dividing both sides of the 1st ID by $\sin^2 x$ and $\cos^2 x$, respectively.

Tip: The squares of $\csc x$ and $\sec x$, which have the "Up-U, Down-U" graphs, are all alone on the right sides of the last two IDs. They can never be 0 in value. (Why is that? Look at the left sides.)

Cofunction Identities

If x is measured in radians, then:

$$\sin x = \cos\left(\frac{\pi}{2} - x\right)$$
$$\cos x = \sin\left(\frac{\pi}{2} - x\right)$$

We have analogous relationships for tan and cot, and for sec and csc; remember that they are sometimes undefined.

Think: Cofunctions of complementary angles are equal.

Even / Odd (or Negative Angle) Identities

Among the six basic trig functions, cos (and its reciprocal, sec) are even:

$$\cos(-x) = \cos x$$
$$\sec(-x) = \sec x, \text{ when both sides are defined}$$

However, the other four (sin and csc, tan and cot) are odd:

$$\sin(-x) = -\sin x$$
$$\csc(-x) = -\csc x, \text{ when both sides are defined}$$

$$\tan(-x) = -\tan x, \text{ when both sides are defined}$$
$$\cot(-x) = -\cot x, \text{ when both sides are defined}$$

Note: If f is an even function (such as cos), then the graph of $y = f(x)$ is symmetric about the y-axis.

Note: If f is an odd function (such as sin), then the graph of $y = f(x)$ is symmetric about the origin.

MORE TRIG IDENTITIES – MEMORIZE!

SUM IDENTITIES

Memorize:

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

Think: "Sum of the mixed-up products"
(Multiplication and addition are commutative, but start with the $\sin u \cos v$ term in anticipation of the Difference Identities.)

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

Think: "Cosines [product] – Sines [product]"

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

Think: $"\dfrac{\text{Sum}}{1-\text{Product}}"$

DIFFERENCE IDENTITIES

Memorize:

Simply take the Sum Identities above and change every sign in sight!

$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

(Make sure that the right side of your identity for $\sin(u+v)$ started with the $\sin u \cos v$ term!)

$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Obtaining the Difference Identities from the Sum Identities:

Replace v with $(-v)$ and use the fact that sin and tan are odd, while cos is even.

For example,

$$\begin{aligned}\sin(u-v) &= \sin\left[u+(-v)\right]\\ &= \sin u \cos(-v) + \cos u \sin(-v)\\ &= \sin u \cos v - \cos u \sin v\end{aligned}$$

<u>DOUBLE-ANGLE (Think: Angle-Reducing, if $u > 0$) IDENTITIES</u>

Memorize:

(Also be prepared to recognize and know these "right-to-left")

$$\sin(2u) = 2\sin u \cos u$$

<u>Think</u>: "Twice the product"

Reading "right-to-left," we have:

$$2\sin u \cos u = \sin(2u)$$

(This is helpful when simplifying.)

$$\cos(2u) = \cos^2 u - \sin^2 u$$

<u>Think</u>: "Cosines – Sines" (again)

Reading "right-to-left," we have:

$$\cos^2 u - \sin^2 u = \cos(2u)$$

Contrast this with the Pythagorean Identity:

$$\cos^2 u + \sin^2 u = 1$$

$$\tan(2u) = \frac{2\tan u}{1 - \tan^2 u}$$

(Hard to memorize; we'll show how to obtain it.)

Notice that these identities are "angle-reducing" (if $u > 0$) in that they allow you to go from trig functions of $(2u)$ to trig functions of simply u.

Obtaining the Double-Angle Identities from the Sum Identities:

Take the Sum Identities, replace v with u, and simplify.

$$\begin{aligned}\sin(2u) &= \sin(u+u)\\ &= \sin u \cos u + \cos u \sin u \quad \text{(From Sum Identity)}\\ &= \sin u \cos u + \sin u \cos u \quad \text{(Like terms!!)}\\ &= 2 \sin u \cos u\end{aligned}$$

$$\begin{aligned}\cos(2u) &= \cos(u+u)\\ &= \cos u \cos u - \sin u \sin u \quad \text{(From Sum Identity)}\\ &= \cos^2 u - \sin^2 u\end{aligned}$$

$$\begin{aligned}\tan(2u) &= \tan(u+u)\\ &= \frac{\tan u + \tan u}{1 - \tan u \tan u} \quad \text{(From Sum Identity)}\\ &= \frac{2 \tan u}{1 - \tan^2 u}\end{aligned}$$

This is a "last resort" if you forget the Double-Angle Identities, but you will need to recall the Double-Angle Identities quickly!

One possible exception: Since the $\tan(2u)$ identity is harder to remember, you may prefer to remember the Sum Identity for $\tan(u+v)$ and then derive the $\tan(2u)$ identity this way.

If you're quick with algebra, you may prefer to go in reverse: memorize the Double-Angle Identities, and then guess the Sum Identities.

Memorize These Three Versions of the Double-Angle Identity for $\cos(2u)$:

Let's begin with the version we've already seen:

Version 1: $\cos(2u) = \cos^2 u - \sin^2 u$

Also know these two, from "left-to-right," and from "right-to-left":

Version 2: $\cos(2u) = 1 - 2\sin^2 u$

Version 3: $\cos(2u) = 2\cos^2 u - 1$

Obtaining Versions 2 and 3 from Version 1

It's tricky to remember Versions 2 and 3, but you can obtain them from Version 1 by using the Pythagorean Identity $\sin^2 u + \cos^2 u = 1$ written in different ways.

To obtain Version 2, which contains $\sin^2 u$, we replace $\cos^2 u$ with $\left(1 - \sin^2 u\right)$.

$$\begin{aligned}
\cos(2u) &= \cos^2 u - \sin^2 u && \text{(Version 1)} \\
&= \underbrace{\left(1 - \sin^2 u\right)}_{\text{from Pythagorean Identity}} - \sin^2 u \\
&= 1 - \sin^2 u - \sin^2 u \\
&= 1 - 2\sin^2 u && (\Rightarrow \text{Version 2})
\end{aligned}$$

To obtain Version 3, which contains $\cos^2 u$, we replace $\sin^2 u$ with $\left(1 - \cos^2 u\right)$.

$$\begin{aligned}
\cos(2u) &= \cos^2 u - \sin^2 u && \text{(Version 1)} \\
&= \cos^2 u - \underbrace{\left(1 - \cos^2 u\right)}_{\text{from Pythagorean Identity}} \\
&= \cos^2 u - 1 + \cos^2 u \\
&= 2\cos^2 u - 1 && (\Rightarrow \text{Version 3})
\end{aligned}$$

POWER-REDUCING IDENTITIES ("PRIs")

(These are called the "Half-Angle Formulas" in some books.)

Memorize: **Then,**

$$\sin^2 u = \frac{1-\cos(2u)}{2} \quad \text{or} \quad \frac{1}{2} - \frac{1}{2}\cos(2u)$$

$$\cos^2 u = \frac{1+\cos(2u)}{2} \quad \text{or} \quad \frac{1}{2} + \frac{1}{2}\cos(2u)$$

$$\tan^2 u = \frac{\sin^2 u}{\cos^2 u} = \frac{1-\cos(2u)}{1+\cos(2u)}$$

Actually, you just need to memorize one of the $\sin^2 u$ or $\cos^2 u$ identities and then switch the visible sign to get the other. Think: "sin" is "bad" or "negative"; this is a reminder that the minus sign belongs in the $\sin^2 u$ formula.

Obtaining the Power-Reducing Identities from the Double-Angle Identities for $\cos(2u)$

To obtain the identity for $\sin^2 u$, start with Version 2 of the $\cos(2u)$ identity:

$$\cos(2u) = 1 - 2\sin^2 u$$

Now, solve for $\sin^2 u$.

$$2\sin^2 u = 1 - \cos(2u)$$

$$\sin^2 u = \frac{1-\cos(2u)}{2}$$

To obtain the identity for $\cos^2 u$, start with Version 3 of the $\cos(2u)$ identity:

$$\cos(2u) = 2\cos^2 u - 1$$

Now, switch sides and solve for $\cos^2 u$.

$$2\cos^2 u - 1 = \cos(2u)$$

$$2\cos^2 u = 1 + \cos(2u)$$

$$\cos^2 u = \frac{1+\cos(2u)}{2}$$

HALF-ANGLE IDENTITIES

Instead of memorizing these outright, it may be easier to derive them from the Power-Reducing Identities (PRIs). We use the substitution $\theta = 2u$. (See **Obtaining …** below.)

The Identities:

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1+\cos\theta}{2}}$$

$$\tan\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} = \frac{1-\cos\theta}{\sin\theta} = \frac{\sin\theta}{1+\cos\theta}$$

For a given θ, the choices among the $\pm$ signs depend on the Quadrant that $\frac{\theta}{2}$ lies in. Here, the $\pm$ symbols indicate incomplete knowledge; unlike when we deal with the Quadratic Formula, we do not take both signs for any of the above formulas for a given θ. There are no $\pm$ symbols in the last two $\tan\left(\frac{\theta}{2}\right)$ formulas; there is no problem there of incomplete knowledge regarding signs.

One way to remember the last two $\tan\left(\frac{\theta}{2}\right)$ formulas: Keep either the numerator or the denominator of the radicand of the first formula, stick $\sin\theta$ in the other part of the fraction, and remove the radical sign and the $\pm$ symbol.

Obtaining the Half-Angle Identities from the Power-Reducing Identities (PRIs):

For the $\sin\left(\frac{\theta}{2}\right)$ identity, we begin with the PRI:

$$\sin^2 u = \frac{1-\cos(2u)}{2}$$

Let $u = \frac{\theta}{2}$, or $\theta = 2u$.

$$\sin^2\left(\frac{\theta}{2}\right) = \frac{1-\cos\theta}{2}$$

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}} \quad \text{(by the Square Root Method)}$$

Again, the choice among the $\pm$ signs depends on the Quadrant that $\frac{\theta}{2}$ lies in.

The story is similar for the $\cos\left(\frac{\theta}{2}\right)$ and the $\tan\left(\frac{\theta}{2}\right)$ identities.

What about the last two formulas for $\tan\left(\frac{\theta}{2}\right)$? The key trick is multiplication by trig conjugates. For example:

$$\begin{aligned}
\tan\left(\frac{\theta}{2}\right) &= \pm\sqrt{\frac{1-\cos\theta}{1+\cos\theta}} \\
&= \pm\sqrt{\frac{(1-\cos\theta)}{(1+\cos\theta)}\cdot\frac{(1-\cos\theta)}{(1-\cos\theta)}} \\
&= \pm\sqrt{\frac{(1-\cos\theta)^2}{1-\cos^2\theta}} \\
&= \pm\sqrt{\frac{(1-\cos\theta)^2}{\sin^2\theta}} \\
&= \pm\sqrt{\left(\frac{1-\cos\theta}{\sin\theta}\right)^2} \\
&= \pm\left|\frac{1-\cos\theta}{\sin\theta}\right| \qquad \left(\text{because } \sqrt{blah^2} = |blah|\right)
\end{aligned}$$

Now, $1-\cos\theta \geq 0$ for all real θ, and $\tan\left(\frac{\theta}{2}\right)$ has the same sign as $\sin\theta$ (can you see why?), so …

$$=\frac{1-\cos\theta}{\sin\theta}$$

To get the third formula, use the numerator's (instead of the denominator's) trig conjugate, $1+\cos\theta$, when multiplying into the numerator and the denominator of the radicand in the first few steps.

PRODUCT-TO-SUM IDENTITIES (Given as necessary on exams)

These can be verified from right-to-left using the Sum and Difference Identities.

The Identities:

$$\sin u \sin v = \frac{1}{2}\left[\cos(u-v)-\cos(u+v)\right]$$

$$\cos u \cos v = \frac{1}{2}\left[\cos(u-v)+\cos(u+v)\right]$$

$$\sin u \cos v = \frac{1}{2}\left[\sin(u+v)+\sin(u-v)\right]$$

$$\cos u \sin v = \frac{1}{2}\left[\sin(u+v)-\sin(u-v)\right]$$

SUM-TO-PRODUCT IDENTITIES (Given as necessary on exams)

These can be verified from right-to-left using the Product-To-Sum Identities.

The Identities:

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

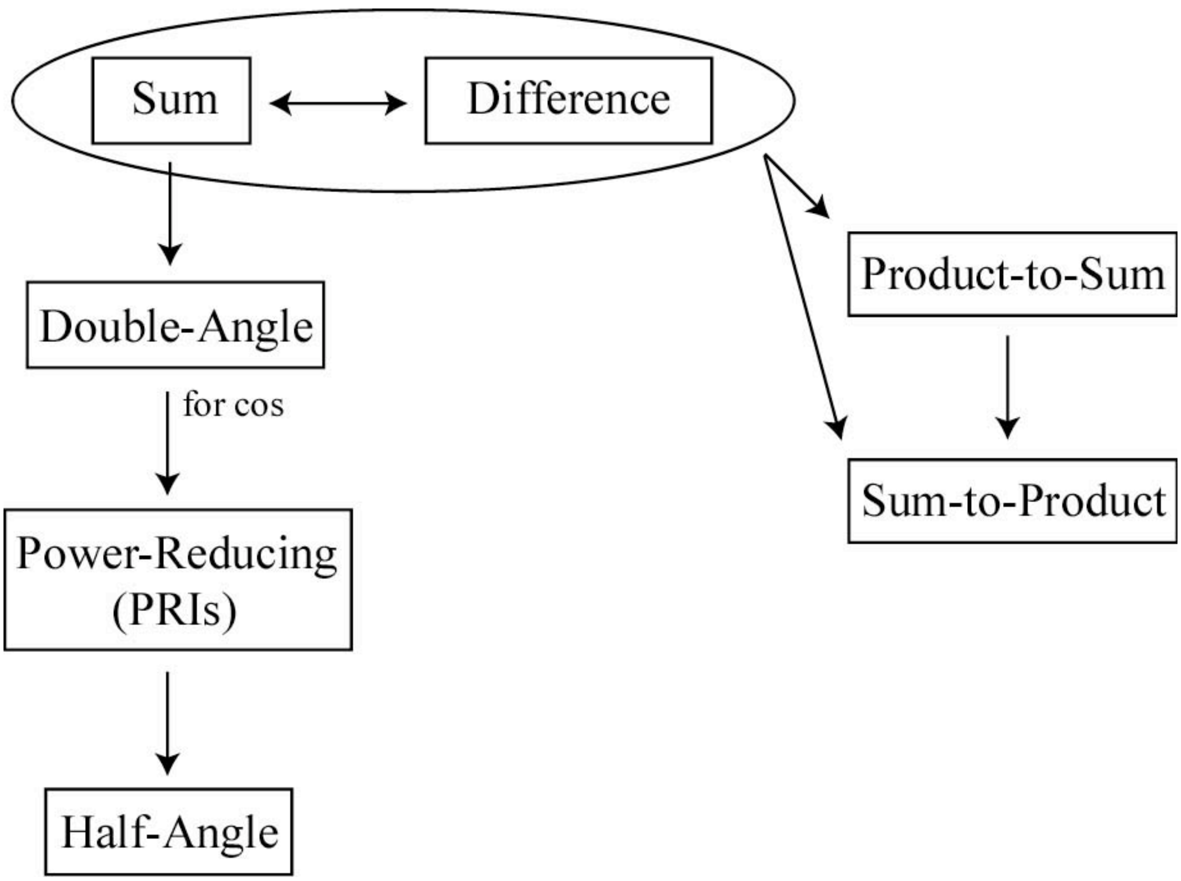
$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

SECTIONS 5.4 and 5.5: MORE TRIG IDENTITIES

PART A: A GUIDE TO THE HANDOUT

See the Handout on my website.

The identities (IDs) may be derived according to this flowchart:



In Calculus: The Double-Angle and Power-Reducing IDs are most commonly used among these, though we will discuss a critical application of the Sum IDs in Part C.

Some proofs are on pp.403-5. See p.381 for notes on Hipparchus, the "inventor" of trig, and the father of the Sum and Difference IDs.

PART B: EXAMPLES

Example: Finding Trig Values

Find the exact value of $\sin 15^\circ$.

Note: Larson uses radians to solve this in Example 2 on p.381, but degrees are usually easier to deal with when applying these identities, since we don't have to worry about common denominators.

Solution (Method 1: Difference ID)

We know trig values for 45° and 30°, so a Difference ID should work.

$$\sin 15^\circ = \sin\left(45^\circ - 30^\circ\right)$$

Use: $\sin\left(u - v\right) = \sin u \cos v - \cos u \sin v$

$$\begin{aligned}
&= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ \\
&= \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) \\
&= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \\
&= \frac{\sqrt{6} - \sqrt{2}}{4}
\end{aligned}$$

Warning: $\sqrt{6} - \sqrt{2} \neq \sqrt{4}$. We do **not** have sum and difference rules for radicals the same way we have product and quotient rules for them.

Solution (Method 2: Half-Angle ID)

We know trig values for 30°, so a Half-Angle ID should work.

$$\sin 15^\circ = \sin\left(\frac{30^\circ}{2}\right)$$

Use: $\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$

$$= \pm\sqrt{\frac{1-\cos 30^\circ}{2}}$$

$$= \pm\sqrt{\frac{\left(1-\frac{\sqrt{3}}{2}\right)}{2} \cdot \frac{2}{2}}$$

$$= \pm\sqrt{\frac{2-\sqrt{3}}{4}}$$

$$= \pm\frac{\sqrt{2-\sqrt{3}}}{2}$$

We know $\sin 15^\circ > 0$, since 15° is an acute Quadrant I angle. We take the "+" sign.

$$= \frac{\sqrt{2-\sqrt{3}}}{2}$$

In fact, $\frac{\sqrt{2-\sqrt{3}}}{2}$ is equivalent to $\frac{\sqrt{6}-\sqrt{2}}{4}$, our result from Method 1. They are both positive in value, and you can see (after some work) that their squares are equal.

Example: Simplifying and/or Evaluating

Find the exact value of: $\dfrac{\tan 25^\circ + \tan 20^\circ}{1 - \tan 25^\circ \tan 20^\circ}$

Solution

We do not know the exact tan values for 25° or 20°, but observe that the expression follows the template for the Sum Formula for tan:

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

We will use this ID "in reverse" (i.e., from right-to-left):

$$\frac{\tan u + \tan v}{1 - \tan u \tan v} = \tan(u+v)$$

$$\begin{aligned} \frac{\tan 25^\circ + \tan 20^\circ}{1 - \tan 25^\circ \tan 20^\circ} &= \tan\left(25^\circ + 20^\circ\right) \\ &= \tan 45^\circ \\ &= 1 \end{aligned}$$

Example: Simplifying Trig Expressions

Simplify: $\dfrac{1}{\sin(3\theta)\cos(3\theta)}$

Solution

We will take the Double-Angle ID: $\sin(2u) = 2\sin u \cos u$ and use it "in reverse": $2\sin u \cos u = \sin(2u)$.

Let $u = 3\theta$. Observe:

$$\begin{aligned} 2\sin(3\theta)\cos(3\theta) &= \sin\left[2(3\theta)\right] \\ 2\sin(3\theta)\cos(3\theta) &= \sin(6\theta) \\ \sin(3\theta)\cos(3\theta) &= \frac{1}{2}\sin(6\theta) \end{aligned}$$

Note: We also get this result from the Product-to-Sum Identities, but they are harder to remember!

Therefore,

$$\begin{aligned} \frac{1}{\sin(3\theta)\cos(3\theta)} &= \frac{1}{\frac{1}{2}\sin(6\theta)} \\ &= 2\csc(6\theta) \end{aligned}$$

Examples: Verifying Trig IDs

Examples 5 and 6 on p.382 of Larson show how these IDs can be used to verify Cofunction IDs and Reduction IDs.

Example: These IDs can be used to verify something like: $\sin(\theta + \pi) = -\sin\theta$. Can you see why this is true using the Unit Circle?

Examples: Solving Trig Equations

See Example 8 on p.383 of Larson.

Example

Solve: $\sin x - \cos(2x) = 0$

Solution

We will use the Double-Angle ID for $\cos(2x)$.

$$\begin{aligned} \sin x - \cos(2x) &= 0 \\ \sin x - \left(\cos^2 x - \sin^2 x\right) &= 0 \\ \sin x - \cos^2 x + \sin^2 x &= 0 \end{aligned}$$

Warning: Remember to use grouping symbols if you are subtracting a substitution result consisting of more than one term.

Use the basic Pythagorean Identity to express $\cos^2 x$ in terms of a power of $\sin x$.

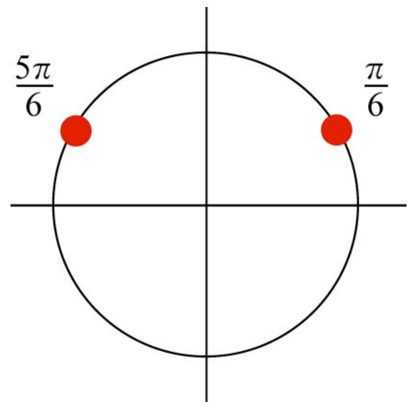
$$\begin{aligned} \sin x - \left(1 - \sin^2 x\right) + \sin^2 x &= 0 \\ \sin x - 1 + \sin^2 x + \sin^2 x &= 0 \\ 2\sin^2 x + \sin x - 1 &= 0 \end{aligned}$$

You can use the substitution $u = \sin x$, or you can factor directly.

$$(2\sin x - 1)(\sin x + 1) = 0$$

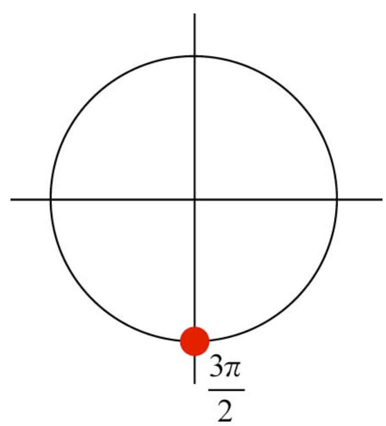
First factor:

$$2\sin x - 1 = 0$$
$$\sin x = \frac{1}{2}$$
$$x = \frac{\pi}{6} + 2\pi n, \text{ or } x = \frac{5\pi}{6} + 2\pi n \quad (n \text{ integer})$$



Second factor:

$$\sin x + 1 = 0$$
$$\sin x = -1$$
$$x = \frac{3\pi}{2} + 2\pi n \quad (n \text{ integer})$$

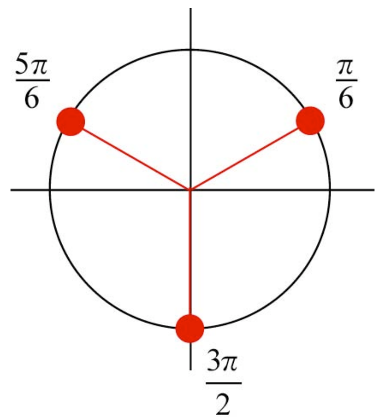


Solution set:

$$\left\{ x \,\middle|\, x = \frac{\pi}{6} + 2\pi n,\ x = \frac{5\pi}{6} + 2\pi n, \text{ or } x = \frac{3\pi}{2} + 2\pi n \quad (n \text{ integer}) \right\}$$

<u>A More Efficient Form!</u>

Look at the red points (corresponding to solutions) we've collected on the Unit Circle:



The solutions exhibit a "period" of $\frac{2\pi}{3}$, corresponding to "third-revolutions" about the Unit Circle.

Here is a much more efficient form for the solution set:

$$\left\{ x \,\middle|\, x = \frac{\pi}{6} + \frac{2\pi}{3} n \quad (n \text{ integer}) \right\}$$

Examples: Using Right Triangles

Example

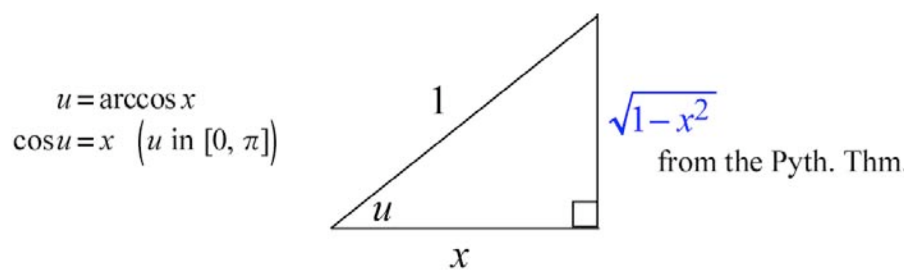
Express $\sin\left(2\arccos x\right)$ as an equivalent algebraic expression in x.
Assume x is in $\left[-1, 1\right]$, the domain of the arccos function.

Solution

We use the Double-Angle ID: $\sin\left(2u\right) = 2\sin u\cos u$, where $u = \arccos x$.

$$\sin\left(2\arccos x\right) = 2\sin\left(\arccos x\right)\cos\left(\arccos x\right)$$

Since x is assumed to be in $\left[-1, 1\right]$, we know that $\cos\left(\arccos x\right) = x$.
We will use a right triangle model to reexpress $\sin\left(\arccos x\right)$.



$$\sin\left(\arccos x\right) = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$

We then have ... $\sin\left(2\arccos x\right) = 2\left(\sqrt{1-x^2}\right)\left(x\right)$

$$= 2x\sqrt{1-x^2}$$

See Example 4 on p.381 of Larson. When rewriting $\cos\left(\arctan 1 + \arccos x\right)$, we let $u = \arctan 1$ and $v = \arccos x$, and we can apply the Sum Identity for $\cos\left(u+v\right)$.

It may help to recognize that $\arctan 1 = \dfrac{\pi}{4}$.

Examples: Using Power-Reducing IDs (PRIs)

See Example 5 on p.389. In Calculus: You will need to do this when you do advanced techniques of integration in Calculus II: Math 151 at Mesa. In the next Example, we will explain one of the more confusing steps in the solution:

Example

Express $\cos^2(2x)$ in terms of first powers of cosines.

Solution

We use the PRI: $\cos^2 u = \dfrac{1+\cos(2u)}{2}$, where $u = 2x$.

$$\begin{aligned}\cos^2(2x) &= \frac{1+\cos\left[2(2x)\right]}{2} \\ &= \frac{1+\cos(4x)}{2}\end{aligned}$$

Example: Product-to-Sum ID

Example

Apply a Product-to-Sum ID to reexpress $\sin(6\theta)\sin(4\theta)$ as an equivalent expression.

Solution

The relevant ID is: $\sin u \sin v = \frac{1}{2}\left[\cos(u-v) - \cos(u+v)\right]$

$$\begin{aligned}\sin(6\theta)\sin(4\theta) &= \frac{1}{2}\left[\cos(6\theta-4\theta) - \cos(6\theta+4\theta)\right] \\ &= \frac{1}{2}\left[\cos(2\theta) - \cos(10\theta)\right]\end{aligned}$$

Example: Sum-to-Product ID

Example

Apply a Sum-to-Product ID to reexpress $\sin(6\theta)+\sin(4\theta)$ as an equivalent expression.

Warning: This is **not** equivalent to $\sin(10\theta)$.

Solution

The relevant ID is: $\sin x+\sin y=2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$$\begin{aligned}\sin(6\theta)+\sin(4\theta)&=2\sin\left(\frac{6\theta+4\theta}{2}\right)\cos\left(\frac{6\theta-4\theta}{2}\right)\\&=2\sin\left(\frac{10\theta}{2}\right)\cos\left(\frac{2\theta}{2}\right)\\&=2\sin(5\theta)\cos\theta\end{aligned}$$

Example: Extending IDs

Example 4 on p.389 in Larson shows how a Triple-Angle ID can be derived from the Double-Angle IDs.

PART C: APPLICATIONS IN CALCULUS

Review difference quotients and derivatives ("slope functions") in Notes 1.57 and 1.58.

The Sum IDs help us show that:

If $f(x)=\sin x$, then the derivative $f'(x)=\cos x$.
If $f(x)=\cos x$, then the derivative $f'(x)=-\sin x$.

Let's consider $f(x)=\sin x$. We will use a limit definition for the derivative:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}=\lim_{h\to 0}\frac{\sin(x+h)-\sin x}{h}$$

We will use a Sum ID to expand $\sin(x+h)$.
Example 7 on p.383 works this out in a slightly different way.

$$=\lim_{h\to 0}\frac{\sin x\cos h+\cos x\sin h-\sin x}{h}$$

$$=\lim_{h\to 0}\frac{(\sin x\cos h-\sin x)+\cos x\sin h}{h} \qquad \text{(Group terms with } \sin x.\text{)}$$

$$=\lim_{h\to 0}\frac{(\sin x)(\cos h-1)+\cos x\sin h}{h} \qquad \text{(Factor } \sin x \text{ out of the group.)}$$

$$=\lim_{h\to 0}\left[(\sin x)\underbrace{\left(\frac{\cos h-1}{h}\right)}_{\to 0}+(\cos x)\underbrace{\left(\frac{\sin h}{h}\right)}_{\to 1}\right]$$

As $h\to 0$, $\dfrac{\cos h-1}{h}\to 0$, or, equivalently, $\dfrac{1-\cos h}{h}\to 0$, if you use the book's result.

Also, $\dfrac{\sin h}{h}\to 1$.

$$=\cos x$$