

PART A: Multiple choice or Short answer

(5 mark for each correct answer/choice)

Problem A1. Find the limit if it exists.

$$\lim_{x \rightarrow -\infty} (\sqrt{x^2 - 2x + 3} + x).$$

Write your answer here:

Problem A2. Let $f(x) = e^x + 2 \sin x$, and $g(x) = x + 1$ if $x > 0$ and $g(x) = x^2 + 3x + 1$ otherwise. If $h = g \circ f$, then $h'(0) = ?$

Write your answer: $h'(0) =$

Problem A3. Evaluate

$$\lim_{x \rightarrow -1} \frac{x^{2026} - 1}{x + 1}.$$

Write your answer here:

Problem A4. Find $f'(0)$ if it exists:

$$f(x) = \begin{cases} x^4 \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

Circle the best answer.

- A. 1
- B. 0
- C. 2
- D. It does not exist

Problem A5. Find the limit if it exists

$$\lim_{x \rightarrow -1} \frac{|x-2| - |x+3| + x}{x+1}.$$

Circle the best answer.

- A. -1
- B. $-\infty$
- C. 1
- D. A,B,C are incorrect

Problem A6. Find all vertical lines of the curve

$$y = \frac{2x^2 - 8x - 10}{(4x - 1)(x + 1)}$$

Write your answer:

Problem A7. If $f'(x) \leq 6$ for all x and $f(2) = -1$ then the largest possible value of $f(8)$ is ..

Circle the best answer.

- A. 35
- B. 31

C. 50

D. A,B,C are incorrect

Problem A8. Find the 2026th derivative of $f(x) = \sin(3x)$.

Write your answer here:

PART B: WRITE YOUR FULL ANSWERS. (If the space is not enough, please write on the back and indicate with a phrase like 'Problem ...continue on...')

Problem B1. Find all values of a and b such that f is differential everywhere:

$$f(x) = \begin{cases} a \cos x + b & \text{if } x < 0 \\ x^2 - 2x - 3 & \text{if } x \geq 0 \end{cases}$$

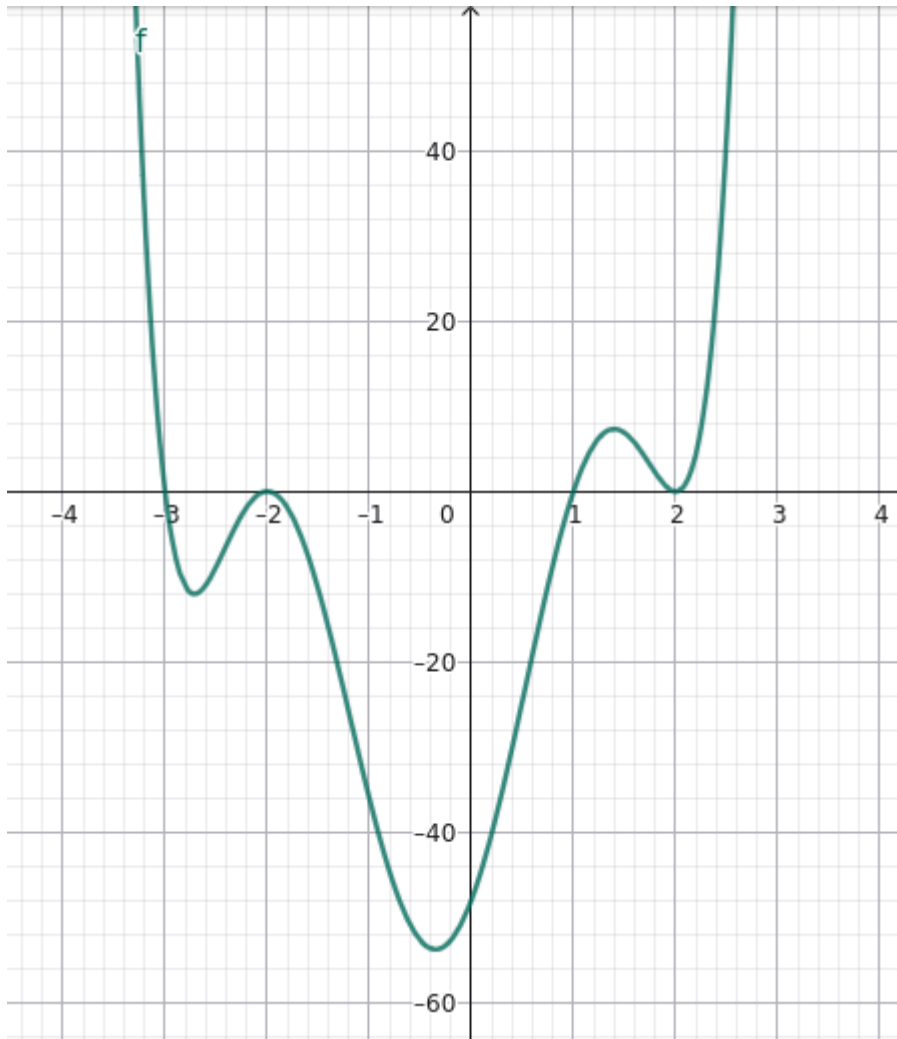
Problem B2. Find equations of the tangent line at the point $(1, 1)$ to the curve $y = (\sin x)^{\ln x}$

Problem B3. Let $f(x) = x^3 - 3x^2 + mx - 1$, where m is a parameter (a real number).

- Use the definition of derivative (Section 2.7, Definition 4 - Textbook) to find the derivative of f at the number a .
- Show that the equation $f(x) = 0$ has at least one real root.
- Find all m such that $f(x) = 0$ has exactly one real root.

Problem B4. Given the polynomial $f(x)$ with graph below (Assume all zeroes are real).

- State $\lim_{x \rightarrow \infty} f(x)$ and $\lim_{x \rightarrow -\infty} f(x)$?
- How many points where the tangent lines are horizontal? Estimate local minimal/maximal values of f ?
- Use the graph to name the zeroes of f , then state whether their multiplicity is even or odd
- state the minimum possible degree of f and write one possible function for f in factored form;



Problem B5. Suppose f is a function with the property that $|f(x)| \leq x^4$. Find $f'(0)$?

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