1. (50 points) a) Find the value of following integral if it converges $\int_{-\infty}^{0} \frac{dx}{(2x-1)^3}$

b) Draw the set of points whose polar coordinates satisfy $|r| = 1, 0 \leq \theta \leq \frac{\pi}{2}$

2. (50 points) a) Use the Taylor series to find the limits $\lim_{x \to 0} \frac{(1+x)^{\frac{1}{3}} - 1}{x}$

b) Find the first four non-zero terms of the Taylor series of $f(x) = x\sin(1+x)$ at $x = -1$.

3. (50 points) a) Find the Maclaurin series and its interval of convergence of the function $y = \ln(x^2+1)$.

b) Use the Maclaurin series to find the value $\int_0^{0.2} \ln(x^2+1)dx$ with an error of magnitude less than 0.0001.

4. (50 points) a) Find the slope of the polar curve at the given point:

$$r = \tan\theta, \theta = \frac{3\pi}{4}$$

b) Sketch the region and find the area of the interior of the curve $r = 4 - 4\cos(\theta)$ in polar coordinates

5. (50 points) a) Find the limit of the sequence if it converges or explain why it diverges

$$a_n = (-1)^n \frac{2\sqrt{n}+1}{\sqrt{n}+1}$$

b) Determine if the sequence is non-decreasing and if it is bounded from above $a_n = \dfrac{(n+2)}{\ln(n+2)}$

6. (50 points) Determine whether the series converge by using appropriate tests

a) $\sum_{n=2}^{\infty}\left(\frac{1}{1+n}-\frac{1}{1-n}\right)^{n}$

b) $\sum_{n=2}^{\infty}\frac{1}{n(\ln n)^{2}}$

7. (50 points) Determine whether the series converge absolutely, converge conditionally or diverge:

a) $\sum_{n=1}^{\infty} \frac{(-1)^n n \ln n}{2n^2+1}$

b) $\sum_{n=1}^{\infty} \frac{\sin(\frac{n\pi}{4})}{2n^2+1}$

8. (50 points) Find the radius of convergence and the interval of convergence of the series

$$f(x)=\sum_{n=1}^{\infty}\frac{(\pi)^{n}(2x-1)^{2n}}{(2n+1)!}$$

(be sure to check for convergence at the end points of the interval)