MTH1126 — FINAL EXAM

Date: September 8, 2026
Exam time: 120 minutes

NAME: _____

STUDENT ID NUMBER: _____

General Instructions

- No discussion is allowed in the exam room.
- No electronic devices (except calculator) are allowed.

Score Table

Problem	Points	Score
A1	4	
A2	4	
A3	4	
A4	4	
A5	4	
A6	4	
A7	4	
A8	4	
A9	4	
A10	4	
B1	20	
B2	15	

B3	10	
B4	10	
B5	5	
TOTAL	**100**	

PART A: Multiple Choice or Short Answer

4 marks for each correct answer/choice.

Problem A1

Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}.$$

Write your exact answer here:

Problem A2

Convert the point

$$\left(4, \frac{5\pi}{6}\right)$$

from polar to Cartesian coordinates.

Write your exact answer here:

Problem A3

Let

$$r = 2 + 2\sin\theta$$

be a cardioid. Find the slope of the tangent line when

$$\theta = \frac{\pi}{6}.$$

Circle the best answer.

- **A.** 0
- **B.** 1
- **C.** -1
- **D.** A, B, C are incorrect

Problem A4

Find an equation for the hyperbola with vertices

$$(\pm 4, 0)$$

and foci

$$(\pm 6, 0).$$

Write your exact answer here:

Problem A5

Set up an integral that represents the length of the curve

$$x = t + \sin t, \qquad y = 1 - \cos t, \qquad 0 \le t \le \pi.$$

Circle the best answer.

A.

$$\int_0^{\pi} \sqrt{(1+\cos t)^2 + \sin^2 t}\, dt$$

B.

$$\int_0^{\pi} \sqrt{(1-\cos t)^2 + \sin^2 t}\, dt$$

C.

$$\int_0^{\pi} \left((1+\cos t)^2 + \sin^2 t\right)\, dt$$

D. A, B, C are incorrect

Problem A6

Find the vertices and foci of the ellipse

$$16x^2 + 25y^2 = 400.$$

Vertices: _____

Foci: _____

Problem A7

Find the number r such that

$$\sum_{n=2}^{\infty} 5r^n = 5.$$

Write your exact answer in rational form:

Problem A8

Find the tangent line at $t = 0$ to the curve

$$x = t - \cos t, \qquad y = 1 + \sin t.$$

Write your exact answer here:

Problem A9

Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n2^n}.$$

Circle the best answer.

- **A.** $\ln \frac{3}{2}$
- **B.** $\ln 2$
- **C.** $\ln \frac{4}{3}$
- **D.** Divergence

Problem A10

Find the area enclosed by

$$x = \cos t, \qquad y = \sin t, \qquad 0 \le t \le \frac{\pi}{3},$$

the x-axis, and the vertical line $x = 1$.

Circle the best answer.

- **A.** $\frac{\pi}{6} - \frac{\sqrt{3}}{8}$
- **B.** $\frac{\pi}{6} + \frac{\sqrt{3}}{8}$
- **C.** $\frac{\pi}{3}$
- **D.** A, B, C are incorrect

PART B: WRITE YOUR FULL ANSWERS

Problem B1 — 20 points

A curve C is defined by

$$x = 2 + 4\sin t, \qquad y = 3\cos t.$$

(a)

Find the points on C where the x-coordinate is 4. Find the slope of the tangent line at the point in the first quadrant.

(b)

Find the points on C where the tangent is horizontal.

(c)

Let R be the part of the interior of C lying in the first quadrant.

Set up an integral representing the volume obtained by rotating R about the x-axis, and another integral for rotation about the y-axis. Do not evaluate.

Problem B2 — 15 points

Find the radius of convergence and interval of convergence of

$$\sum_{n=1}^{\infty} \frac{3^{n+1}(x-1)^n}{2n+1}.$$

Problem B3 — 10 points

Solve the differential equation

$$y' = 8xe^{-\sin x} - y\cos x.$$

Problem B4 — 10 points

(a)

Prove that the series

$$\sum_{n=1}^{\infty} \frac{1}{(n+2)\left(\ln(n+2)\right)^2}$$

is convergent.

(b)

Approximate its sum using the first six terms and estimate the error involved in this approximation.

Problem B5 — 5 points

Prove that $\sin x$ is equal to the sum of its Maclaurin series.