MTH1126 — FINAL EXAM

Date: September 8, 2026
Exam time: 120 minutes

NAME: _____

STUDENT ID NUMBER: _____

General Instructions

- No discussion is allowed in the exam room.
- No electronic devices (except calculator) are allowed.

Score Table

Problem	Points	Score
A1	4	
A2	4	
A3	4	
A4	4	
A5	4	
A6	4	
A7	4	
A8	4	
A9	4	
A10	4	
B1	15	
B2	10	

B3	20	
B4	10	
B5	5	
TOTAL	**100**	

PART A: MULTIPLE CHOICE OR SHORT ANSWER

4 marks for each correct answer/choice.

Problem A1

Convert the point

$$\left(4, \frac{5\pi}{6}\right)$$

from polar coordinates to Cartesian coordinates.

Write your exact answer here:

Problem A2

Set up an integral that represents the length of the curve

$$x = t - \sin t, \qquad y = 1 - \cos t, \qquad 0 \le t \le 2\pi.$$

Circle the best answer.

A.

$$\int_0^{2\pi} \sqrt{(1+\cos t)^2 + \sin^2 t}\, dt$$

B.

$$\int_0^{2\pi} \sqrt{(1-\cos t)^2 + \sin^2 t}\, dt$$

C.

$$2\int_0^{\pi} \left((1+\cos t)^2 + \sin^2 t\right)\, dt$$

D. A, B, C are incorrect

Problem A3

Find the area enclosed by the curve

$$x = \cos t, \qquad y = \sin t, \qquad 0 \le t \le \frac{\pi}{2},$$

and the coordinate axes.

Circle the best answer.

- **A.** $\frac{\pi}{4}$
- **B.** $\frac{\pi}{2}$
- **C.** 1
- **D.** A, B, C are incorrect

Problem A4

Find the equation of the tangent line at

$$t = \frac{\pi}{2}$$

to the parametric curve

$$x = t + \cos t, \qquad y = t - \sin t.$$

Write your exact answer here:

Problem A5

Let

$$r = 2 + 2\sin\theta$$

be a cardioid. Find the slope of the tangent line when

$$\theta = \frac{\pi}{6}.$$

Circle the best answer.

- **A.** $\frac{1}{\sqrt{3}}$
- **B.** $-\frac{1}{\sqrt{3}}$
- **C.** 1
- **D.** A, B, C are incorrect

Problem A6

Find the value of r such that

$$\sum_{n=1}^{\infty} \frac{r^n}{3^n} = 6.$$

Write your exact answer here:

$$r = \underline{\qquad\qquad\qquad\qquad}$$

Problem A7

Find the sum of the series

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}.$$

Write your exact answer here:

Problem A8

Find the vertices and foci of the ellipse

$$25x^2 + 16y^2 = 400.$$

Write your exact answer here:

Problem A9

Find an equation for the hyperbola whose vertices are

$$(\pm 4, 0)$$

and whose foci are

$$(\pm 6, 0).$$

Write your exact answer here:

Problem A10

Find the sum of the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n-1} \left(\frac{1}{2}\right)^{n-1}.$$

Circle the best answer.

- **A.** $\ln \frac{3}{2}$
- **B.** $\ln \frac{4}{3}$
- **C.** $\frac{1}{2} \ln 3$
- **D.** Divergence

PART B: WRITE YOUR FULL ANSWERS

Problem B1 — 15 points

Find the radius of convergence and the interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{4^n (x-2)^n}{n+1}.$$

Problem B2 — 10 points

Solve the differential equation

$$y' = 4xe^{-\sin x} - y \cos x.$$

Problem B3 — 20 points

A curve C is defined by the parametric equations

$$x = 2 + 4 \sin t, \qquad y = 3 \cos t.$$

(a)

Find the points on C where the x-coordinate is 4. Find the slope of the tangent line at the point in the first quadrant.

(b)

Find all points on C where the tangent is horizontal.

(c)

Let R be the region bounded by the curve C and the first quadrant.

Set up integrals that represent the volume obtained by rotating R.
You do not need to evaluate the integrals.

1. About the x-axis.
2. About the y-axis.

Problem B4 — 10 points

(a)

Prove that the series

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

is convergent.

(b)

Approximate the sum

$$\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$$

by using the first six terms. Estimate the error involved in this approximation.

Problem B5 — 5 points

Prove that

$$\sin x$$

is equal to the sum of its Maclaurin series.