CALCULUS II - MIDTERM EXAM - SUMMER 2026

Course Code: MAT1126
Exam Time: 90 minutes
FULL NAME:
STUDENT ID:

GENERAL INSTRUCTIONS:

1. No discussion is allowed in the exam room.
2. No electronic devices are allowed except for basic scientific calculators.
3. Show all your work for Part B to receive full credit.

SCORE SUMMARY TABLE:

PROBLEM	POINTS	SCORE	PROBLEM	POINTS	SCORE
A1	5		B1		
A2	5		B2		
A3	5		B3		
A4	5		B4		
A5	5		B5		
A6	5				
A7	5				
A8	5				
			TOTAL		

PART A: Multiple Choice or Sort Answers (40 POINTS)
Circle or Write only the final result for each question.

Problem A1. If $\int \sin kx \sin 2x dx = \frac{1}{2}\left[\frac{\sin 3x}{3} - \frac{\sin 7x}{7}\right] + C$ then $k =$?
Write (exact) value here: $k =$

Problem A2. Find $P(10)$, where $P(t)$ is the solution of the initial-value problem:
$P' = 0.2P - \frac{0.2P^2}{2000}, \quad P(0) = 500$
Circle the best answer.
A. 1000
B. 1422
C. 1650
D. 2000

Problem A3. Evaluate $\int_0^1 2x^3 \sin(x^2) dx$

Write (exact) value here:

Problem A4. Set up an integral for the length of the curve
$y = \int_0^x \sqrt{\sec^4 t - 1} dt, \quad 0 \le x \le \frac{\pi}{4}$
Circle the best answer:
A. $\int_0^{\pi/4} \sec^2 x dx$
B. $\int_0^{\pi/4} \sqrt{\tan^4 x} dx$
C. $\int_0^{\pi/4} \sec x dx$
D. A, B, C are all incorrect.

Problem A5. Evaluate $\int_0^{\pi/2} e^{-x} \cos x dx$
Circle the best answer.
A. $\frac{1}{2}(e^{-\pi/2} + 1)$
B. $\frac{1}{2}(1 - e^{-\pi/2})$
C. A, B, D are all incorrect.
D. $e^{-\pi/2} - 1$

Problem A6. Evaluate if it converges. $\int_0^\infty x^2 e^{-x^3} dx$
Write (exact) value here:

Problem A7. Evaluate $\int_0^1 \frac{x-2}{\sqrt{x^2+1}} dx$
Write (exact) value here:

Problem A8. Use Euler's method with step size 0.1 to estimate $y(0.2)$, where $y(x)$ is the solution of the initial-value problem: $y' = 2x - y, \quad y(0) = 1$
Write (exact) value here:

PART B: WRITE YOUR FULL ANSWERS (60 POINTS)

Problem B1 (15 pts).
a. Use trigonometric substitution to find:
$\int \frac{1}{x^2\sqrt{4x^2+25}} dx$

b. Evaluate the integral:
$\int \frac{\sqrt{7+x}}{2x} dx$

Problem B2. (15 points) Given the curve C defined by:
$y = \frac{x^3}{6} + \frac{1}{2x}, \quad 1 \le x \le 2$
a. Find the area of the resulting surface obtained by rotating the curve C about the x-axis.
b. Set up an integral for the area of the resulting surface obtained by rotating the curve C about the y-axis. Either evaluate it or use the Midpoint Rule to approximate it with $n = 4$. Estimate the error.

Problem B2. (15 points) Given the curve C defined by:
$y = \frac{x^4}{8} + \frac{1}{4x^2}, \quad 1 \le x \le 2$
a. Find the area of the resulting surface obtained by rotating the curve C about the y-axis.
b. Set up an integral for the area of the resulting surface obtained by rotating the curve C about the x-axis. Either evaluate it or use the Midpoint Rule to approximate it with $n = 4$. Estimate the error.

Problem B3. (10 points) Determine whether the improper integral:
$\int_1^\infty \frac{\arctan x}{x^2+2x+2} dx$
converges or diverges. Justify your answer carefully.

Problem B4. (10 points)
a. Solve the differential equation:
$\sin x \cdot y' = y \ln y \quad (y > 0)$
b. Find the orthogonal trajectories of the family of curves given by $y = kx^3$ (where $k \in \mathbb{R}$).

Problem B5. (10 points) If m and n are positive integers, prove that:
$\int_0^1 x^m (\ln x)^n dx = \frac{(-1)^n n!}{(m+1)^{n+1}}$

— END OF EXAM —