CALCULUS II - MIDTERM EXAM - SUMMER 2026

Course Code: MAT1126
Exam Time: 90 minutes
FULL NAME: _____
STUDENT ID: _____

GENERAL INSTRUCTIONS:

1. No discussion is allowed in the exam room.
2. No electronic devices are allowed except for basic scientific calculators.
3. Show all your work for Part B to receive full credit.

SCORE SUMMARY TABLE:

PROBLEM	POINTS	SCORE	PROBLEM	POINTS	SCORE
A1	5		B1		
A2	5		B2		
A3	5		B3		
A4	5		B4		
A5	5		B5		
A6	5				
A7	5				
A8	5				
			TOTAL		

PART A: Multiple Choice or Sort Answers (40 POINTS)
Circle or Write only the final result for each question.

Problem A1. Evaluate if it converges. $\int_e^\infty \frac{1}{x(\ln x)^2}dx$
Write (exact) value here:

Problem A2. Find $\int_1^e x \ln x dx$
Write (exact) value here:

Problem A3. Evaluate $\int_0^1 3t^5 e^{t^3} dt$
Write (exact) value here:

Problem A4. Set up an integral for the length of the curve
$y = \int_0^x \sqrt{t^2+2t}dt, \quad 0 \le x \le 2$
Circle the best answer:
A. $\int_0^2 (t^2+2t)^{1/2}dt$
B. $\int_0^2 (1+x^2+2x)^{1/2}dx$
C. $\int_0^2 \sqrt{(t-1)^2}dt$
D. A, B, C are all incorrect.

Problem A5. Evaluate $\int_0^\pi e^x \sin x dx$
Circle the best answer.
A. $\frac{1}{2}(e^\pi + 1)$
B. $e^\pi - 1$
C. A, B, D are all incorrect.
D. $\frac{1}{2}(1 - e^\pi)$

Problem A6. If $\int \sin 5x \cos mx dx = -\frac{1}{2}\left[\frac{\cos 8x}{8} + \frac{\cos 2x}{2}\right] + C$ then $m =$?
Write (exact) value here: $m =$

Problem A7. Evaluate $\int_0^3 \frac{t+1}{\sqrt{t^2+9}}dt$
Circle the best answer.
A. 1
B. $\sqrt{18} + \ln(3)$
C. $3\sqrt{2} - 3 + \ln(1+\sqrt{2})$
D. A, B, C are all incorrect.

Problem A8. Use Euler's method with step size 0.1 to estimate $y(0.2)$, where $y(x)$ is the solution of the initial-value problem: $y' = x + y^2, y(0) = 1$
Write (exact) value here:

PART B: WRITE YOUR FULL ANSWERS (60 POINTS)

Problem B1 (15 pts). a. Use trigonometric substitution to prove that:

$\int \frac{1}{\sqrt{a^2-(x+b)^2}}dx = \arcsin\left(\frac{x+b}{a}\right) + C, \quad (a > 0)$

b. Evaluate the integral:

$\int \frac{x+3}{\sqrt{5-4x-x^2}}dx$

Problem B2 (15 pts). Given the region R bounded by the curve:

$y = \frac{x^3}{6} + \frac{1}{2x}, \quad 1 \le x \le 2$

a. Find the area of the resulting surface obtained by rotating the curve about the y-axis.

b. Set up an integral for the area of the resulting surface obtained by rotating the curve about the x-axis. Approximate the value of this integral using the Trapezoidal Rule with $n = 4$. Estimate the error.

Problem B3 (10 pts). Determine whether the following improper integral converges or diverges. Justify your answer using a comparison test:

$\int_1^\infty \frac{2+\cos x}{x^2+\sqrt{x}}dx$

Problem B4 (10 pts). A large mixing tank initially holds 600 liters of a liquid buffer solution containing a 3% concentration of an active pharmaceutical ingredient (by volume). A stronger solution containing 8% of the active ingredient is pumped into the tank at a rate of 12 liters per minute. The tank is kept perfectly mixed, and the resulting mixture is pumped out at the exact same rate. What is the percentage of the active ingredient in the tank after exactly one hour (60 minutes)?

Problem B5 (10 pts). Let n be a non-negative integer. Define $I_n = \int_0^\infty x^n e^{-x} dx$. Prove that $I_n = n!$ for all $n \ge 0$.

— END OF EXAM —