1. (50 points)

   a) Find the value of following integral if it converges or show that it diverges:
   $\int_1^5 \frac{dx}{\sqrt{x-1}}$

   b) Draw the set of points whose polar coordinates satisfy $-1 \leq r \leq 1, \frac{\pi}{4} \leq \theta \leq \frac{\pi}{2}$.

2. (50 points)

   a) Use the Taylor series to find the limits $\lim_{x\to 0} \frac{(1+x)^{\frac{1}{3}}-1}{x}$

   b) Find the first three non-zero terms of the Taylor series of $f(x) = \cos x$ at $x = \frac{\pi}{2}$

3. (50 points)

   a) Use the Maclaurin series to express the integral $\int_0^x \frac{\sin t}{t} dt$ as a power series in x

   b) Use part a) to find $\int_0^{0.5} \frac{\sin t}{t} dt$ with an error of magnitude less than 0.0001.

4. (50 points)

   a) Find the slope of the tangent line to the following curve at (-1,1)
   $x(t) = t^3, y(t) = \ln(-\cos(\pi t)) + t^2$.

   b) Sketch the region and find the area of the interior of the curve $r = 4 - 4\cos(\theta)$ in polar coordinates

5. (50 points)

   a) Find the limit of the sequence if it converges or explain why it diverges
   $a_n = \frac{n}{n+1} - \frac{n+1}{n}$

   b) Find the values of the following series if it converges or explain why it diverges
   $\sum_{n=1}^{\infty} \frac{1}{n^2+13n+42}$

6. (50 points) Determine whether the series converge by using appropriate tests

   a) $\sum_{n=1}^{\infty} \frac{n}{2^{-n}+3^{-n}}$

   b) $\sum_{n=2}^{\infty} \frac{1}{n(\ln n)^2}$

7. (50 points) Determine whether the series converge absolutely, converge conditionally or diverge:

   a) $\sum_{n=2}^{\infty} \frac{(-1)^n}{\sqrt{n(n-1)}}$

   b) $\sum_{n=1}^{\infty} \frac{\sin(\frac{n\pi}{4})}{2n^2+1}$

8. (50 points) Let $f(x) = \sum_{n=1}^{\infty} \frac{nx^{n-1}}{2^{n-1}}$.

   a) Find the radius of convergence of f.

   b) Show that $y = f(x)$ is a solution of the differential equation

$2y - 2y' + xy' = 0.$

c) Find $f(1)$.