

Exam Questions Transcript

Question 1

1. (30 points) Find the following integrals:

a) $\int \frac{x}{\sqrt{x^4+3}} dx$

b) $\int \frac{\cos x dx}{\sqrt{3-\sin x}}$

c) $\int \frac{dx}{x(1+\ln^2 x)}$

Question 2

2. (30 points) Find the following integrals

a) $\int \frac{x^3+x^2}{x^2+x-2} dx$

b) $\int \sin 5x \cos 3x dx$

c) $\int x \tan^{-1} x dx$

Question 3

3. (30 points) Find the value of n so that the magnitude of the error will be less than 5×10^{-4} when n subdivisions are used to approximate the integral $\int_1^4 (x + \sqrt{x}) dx$ by using

a) The Trapezoidal rule;

b) The Midpoint rule.

Question 4

4. (30 points) Find the following improper integrals if they converge, if not explain why they diverge.

a) $\int_0^{+\infty} \frac{x dx}{(1+x^2)^4}$

b) $\int_{-1}^4 \frac{dx}{\sqrt{|x|}}$

Question 5

5. (30 points) Determine whether the integral is convergent or divergent (No need to find the exact values of the integral)

a) $\int_1^{+\infty} \frac{1}{\sqrt{x^5+2}} dx$

b) $\int_1^{\infty} \frac{\cos^2 x}{1+x^2} dx$

Question 6

6. (30 points) Use the integrating factor to solve the initial value problem:

$$\sin(x)y' = \cos(x)y + 1, \quad y\left(\frac{\pi}{4}\right) = 0.$$

Question 7

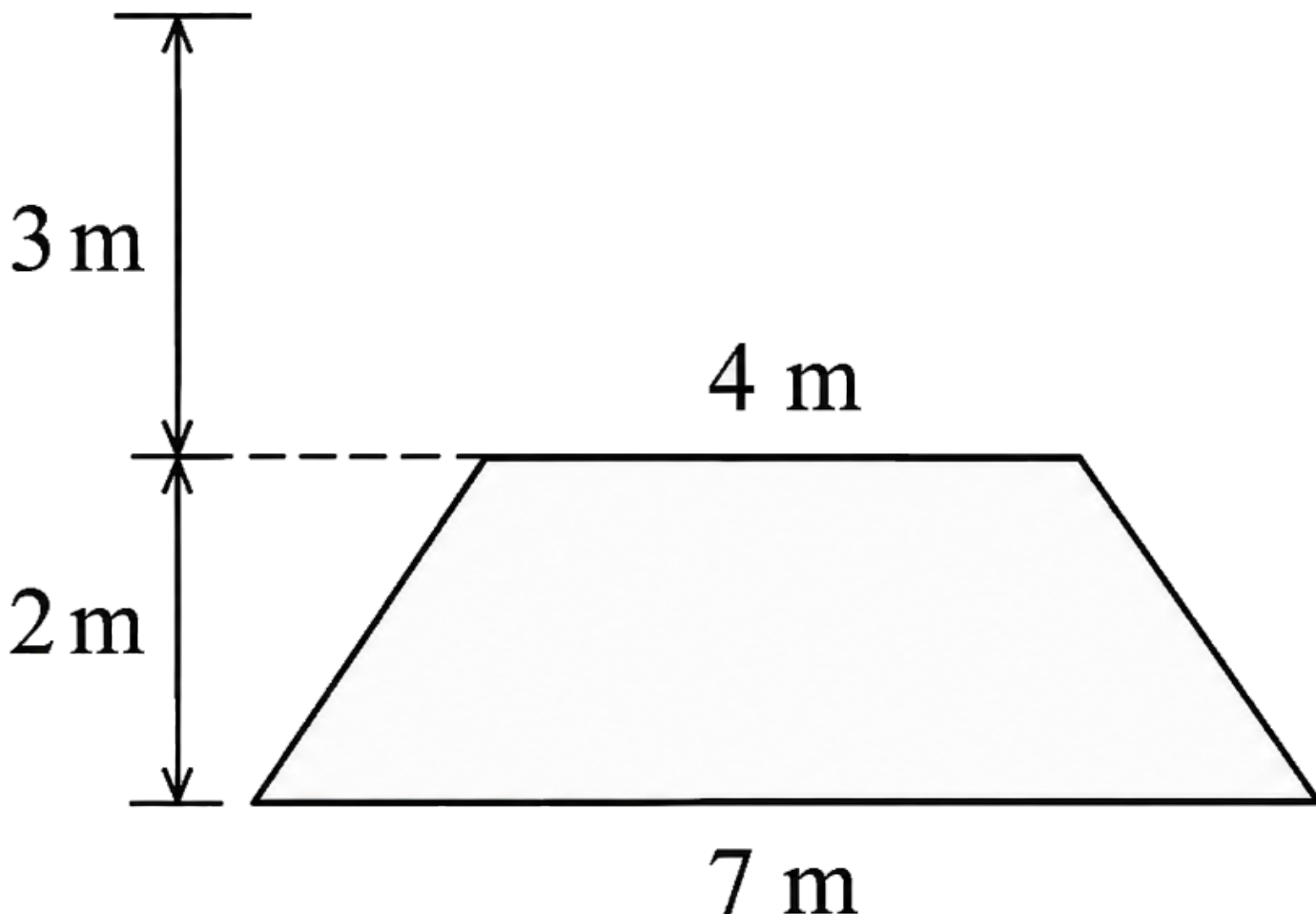
7. (30 points) a) Write the integral expressing the arc length of the curve $y = \tan^{-1} x, 0 \leq x \leq 2$. (do not compute the value of the integral)

b) Sketch the picture and write the integral expressing the surface area of the solid generated by rotating the following curve about the y-axis: $y = 1 - \frac{x^2}{4}, 0 \leq x \leq 2$.

(do not compute the value of the integral)

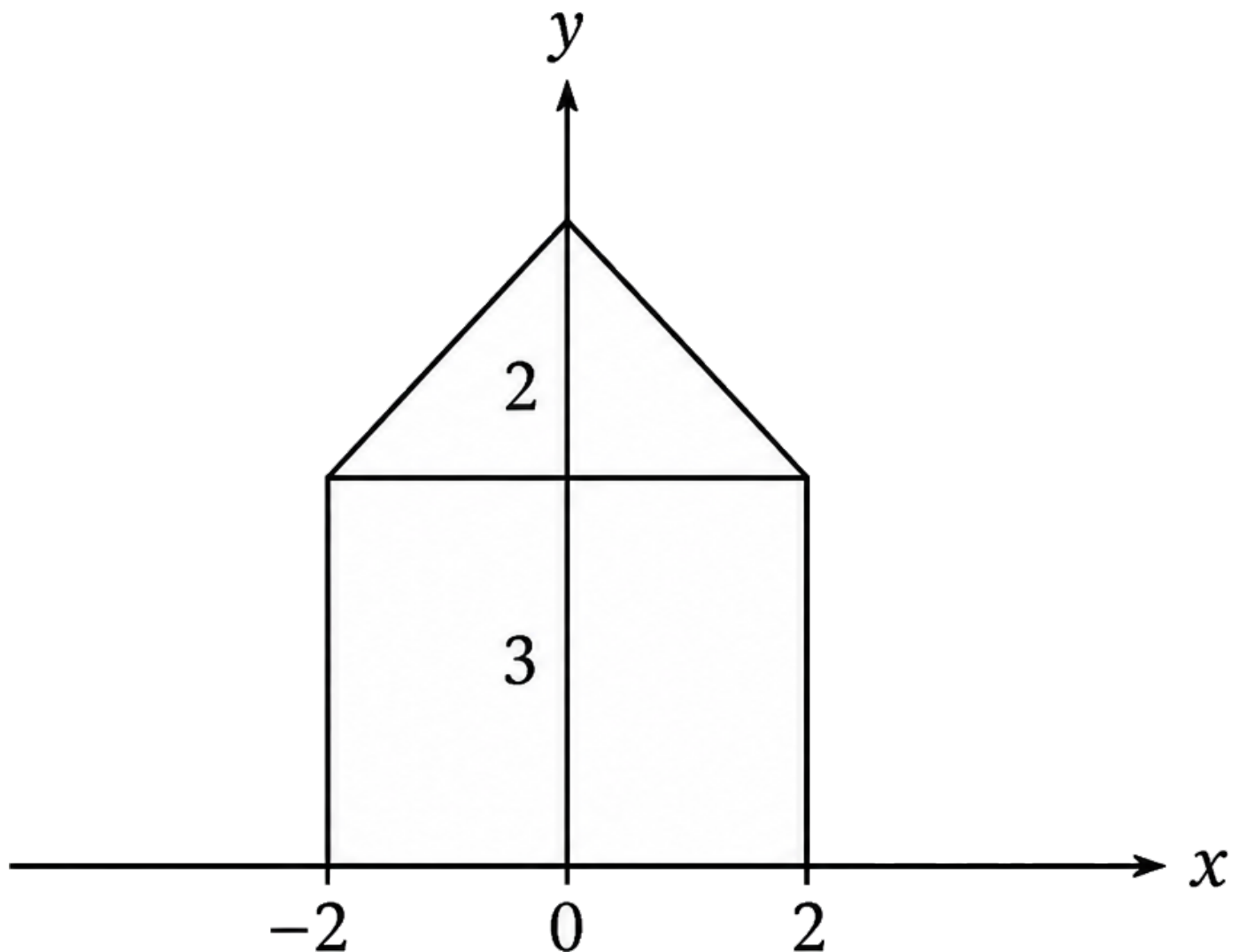
Question 8

8. (30 points) A vertical gate of a dam has the shape of an isosceles trapezoid with the 4m top, the 7m bottom and 2m height. Given that the top of the gate is 3m below the water surface. Explain how to approximate the hydrostatic force against one side of the gate by a Riemann sum. Then express the force as an integral and evaluate it.



Question 9

9. (30 points) Find the moment about the x-axis and the y-coordinate of the centroid of the region which is an isosceles triangle of height 2 on top of a rectangle of base 4 and height 3 as in the following picture.

**Question 10**

10. (30 points) a) Find the general solution by using separable of variables

$$\sqrt{1-x^2}y' = xy.$$

b) Use Euler's method to approximate the solution of the following differential equation with n steps of size h

$$y' = e^{xy}, \quad y(0) = 1, \quad n = 3, \quad h = 0.1.$$