Applications of differentiation
MTH 201 - Business Calculus

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April 2026

Outline

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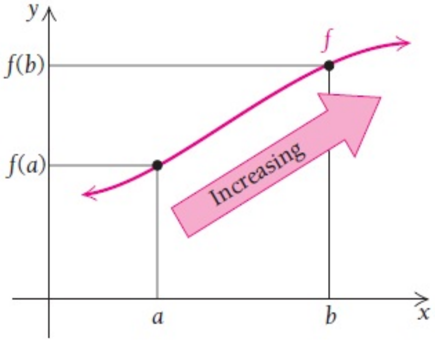

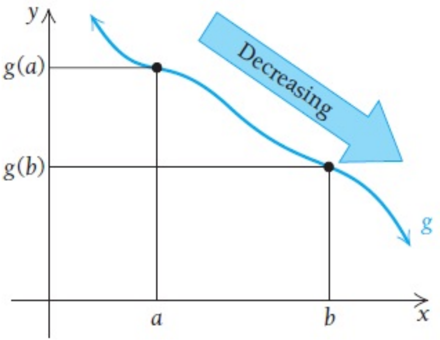

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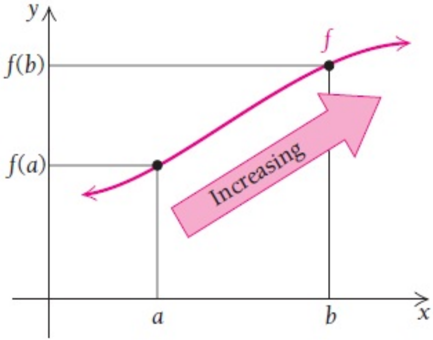

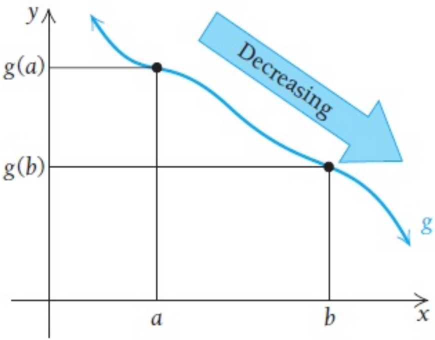

A function g is decreasing over I if, for every $a, b \in I$, and $a < b$, then $g(a) > g(b)$.

Theorem (Increasing/Decreasing Test)

Let f be differentiable over an open interval I.
If $f'(x) > 0$ on I, then f is increasing on that interval.
If $f'(x) < 0$ on I, then f is decreasing on that interval.

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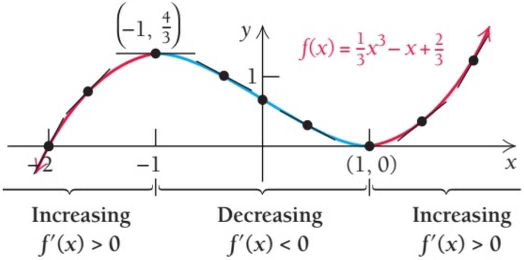

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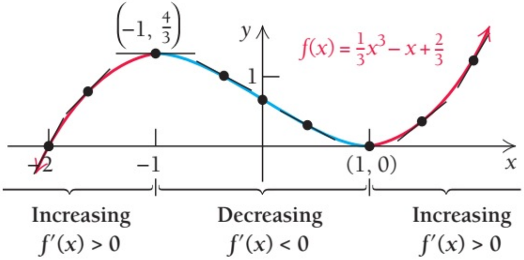

Exercise. For what values of c is the function $f(x) = \ln(x^2+1) - cx$ increasing on $(-\infty, \infty)$?

Relative maximum and minimum values

Definition

Let D be the domain of f. $f(c)$ is a relative minimum if there exists within D an open interval I containing c such that $f(c) \leq f(x)$, for all x in I;
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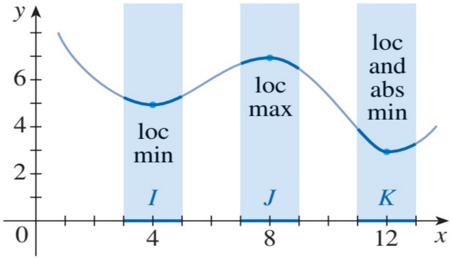

Critical values

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A *critical value* *of a function* f *is any number* c *in the domain of* f *for which the tangent line at* $(c, f(c))$ *is horizontal or for which the derivative does not exist.*
That is, c *is a critical value if* $f'(c)$ *exists and* $f'(c) = 0$ *or* $f'(c)$ *does not exist.*

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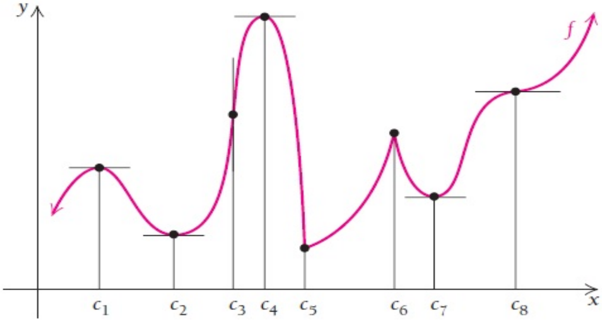
Consider the graph of a continuous function f in the following figure



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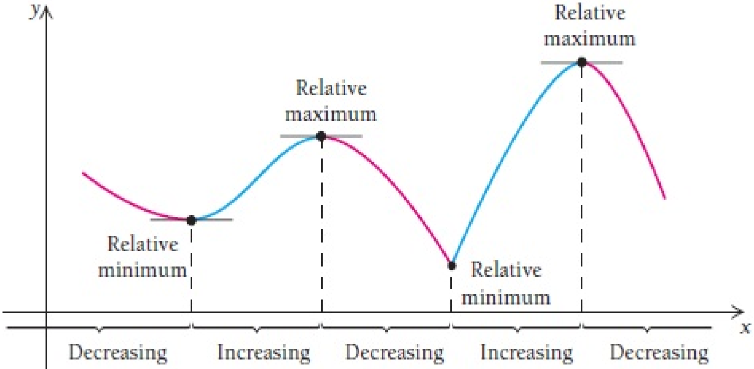

Relative maximum and minimum values

Theorem (First derivative test for relative extrema)

Suppose that c is a critical value of a continuous function f, and that f is differentiable at every point in some interval containing c except possibly at c itself. Moving across c from left to right,

1. *if f' changes from negative to positive at c, then f has a local minimum at c. That is, f is decreasing to the left of c and increasing to the right of c;*
2. *if f' changes from positive to negative at c, then f has a local maximum at c. That is, f is increasing to the left of c and decreasing to the right of c;*
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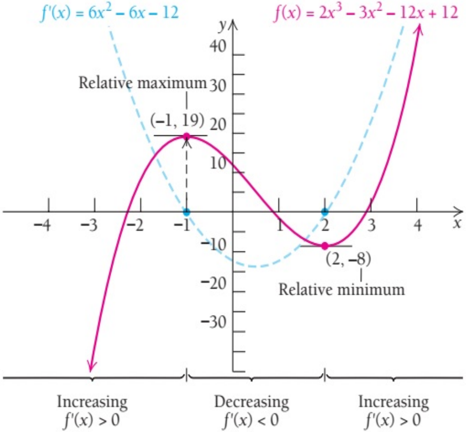

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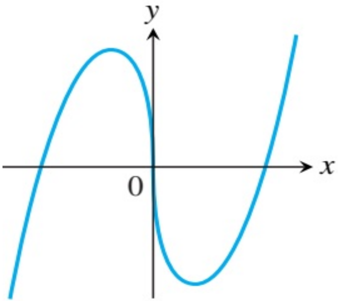
f does not have an extreme value at $x = 0$; f has a local minimum at $x = 1$, the value of the local minimum is $f(1) = -6$; f has a local maximum at $x = -1$, $f(-1) = 6$.



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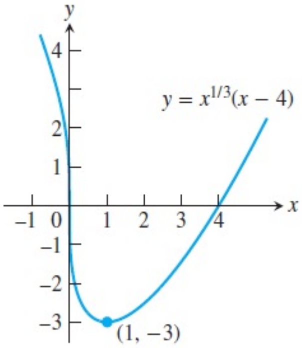

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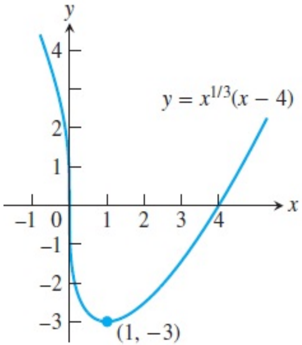

Exercise 3. Find any relative extrema of each function

a) $f(x) = 3x^4 - 15x^2 + 2$ b) $g(x) = \dfrac{3x}{x^2+4}$ c) $h(x) = x\sqrt{8-x^2}$ d) $p(x) = 4\ln x - x^3$.

Outline

Second derivative test for relative extrema

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Theorem

Suppose f'' is continuous on an open interval that contains $x = c$ and $f'(c) = 0$.

1. *If $f''(c) < 0$, then f has a relative maximum at $x = c$.*
2. *If $f''(c) > 0$, then f has a relative minimum at $x = c$.*
3. *If $f''(c) = 0$, then the test fails. The first derivative test can be used to determine whether $f(c)$ is a relative extremum.*

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Example 1. Find any relative extrema of each function. State where the function is increasing or decreasing.

a) $f(x) = 20x^3 - 3x^5$ b) $g(x) = 4x - 6x^{2/3}$ c) $h(x) = (x-1)^{2/3} - (x+1)^{2/3}$.

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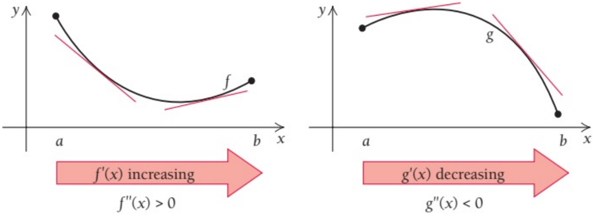
Example 2. Find the relative extrema of the function f given by $f(x) = \dfrac{\ln x}{x}$, then show that $\pi^e < e^\pi$.

Concavity

Increasing and decreasing derivatives



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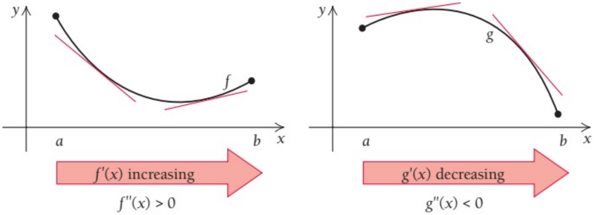

Definition

The graph of a differentiable function $y = f(x)$ *is*

i) *concave up on an open interval* I *if* f' *is increasing on* I.

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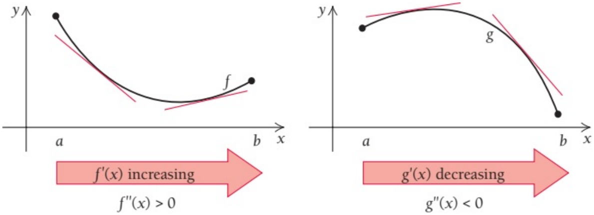

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i) *concave up on an open interval I if f' is increasing on I.*

ii) *concave down on an open interval I if f' is decreasing on I.*

If the graph of $y = f(x)$ is concave up on I then all the tangent lines for f are below the graph.

Concavity

Theorem (Test for concavity)

1. *If $f''(x) > 0$ on an interval I, then the graph of f is concave up. (f' is increasing, so f is turning up on I.)*
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Example. Determine the concavity of

a) $f(x) = 80 - 9x^2 - x^3$ b) $g(x) = \dfrac{x}{x^2 + 1}$ c) $h(x) = x^4 - 4x^3 + 10$.

Definition (Point of inflection)

A point where the graph of a function has a tangent line and where the concavity changes is a point of inflection.

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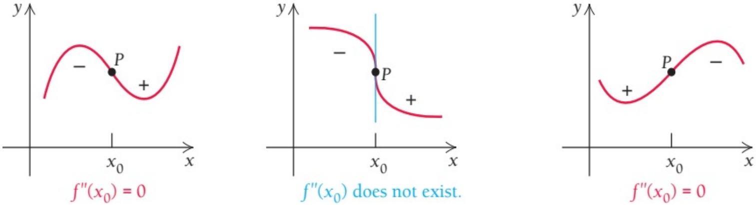

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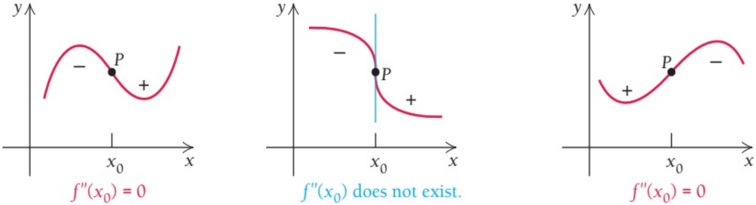
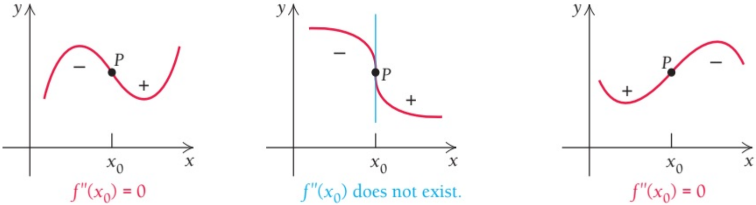


Theorem (Finding points of inflection)

If a function f has a point of inflection, it must occur at a point x_0, in the domain of f, where

$$f''(x_0) = 0 \quad \text{or} \quad f''(x_0) \text{ does not exist.}$$

Points of inflection



To find candidates for points of inflection, we look for numbers x_0 for which $f''(x_0) = 0$ or for which $f''(x_0)$ does not exist. Then, if $f''(x)$ changes sign as x moves through x_0, we have a point of inflection at x_0.

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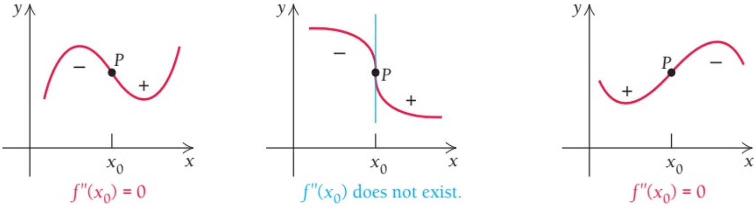

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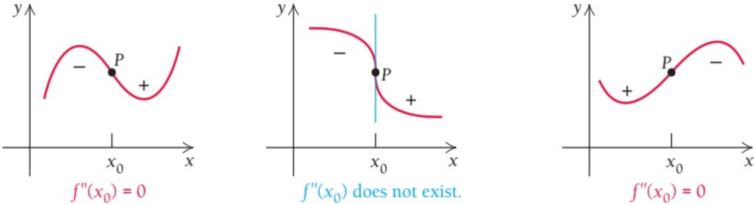

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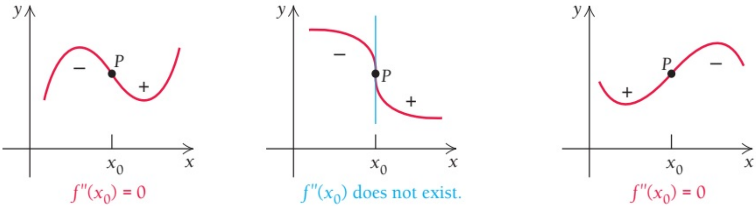
The curve $y = x^4$ has no inflection point at $x = 0$.



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The curve $y = x^4$ has no inflection point at $x = 0$.
The curve $y = x^{1/3}$ has a point of inflection at $x = 0$, but y'' does not exist there.

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Answer. We have $f'(x) = 15x^4 - 60x^2$ and $f''(x) = 60x^3 - 120x$

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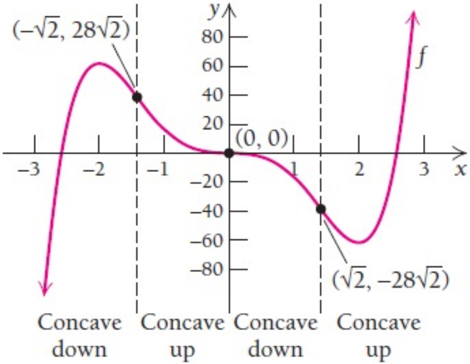

Exercises

Exercise. For each function, list the coordinates of any extrema or points of inflection. State where the function is increasing or decreasing, where its graph is concave up or concave down

a) $f(x) = 8x^3 - 6x + 1$ b) $f(x) = x^4 - 6x^2$

c) $f(x) = x\sqrt{4 - x^2}$ d) $f(x) = (x+1)e^{-x}$ e) $f(x) = x^3 + 3x^2 - 9x - 13.$

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a) $f(x) = 8x^3 - 6x + 1$ b) $f(x) = x^4 - 6x^2$

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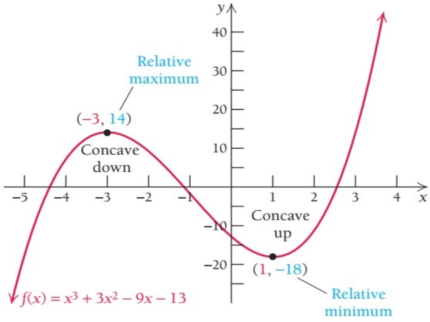

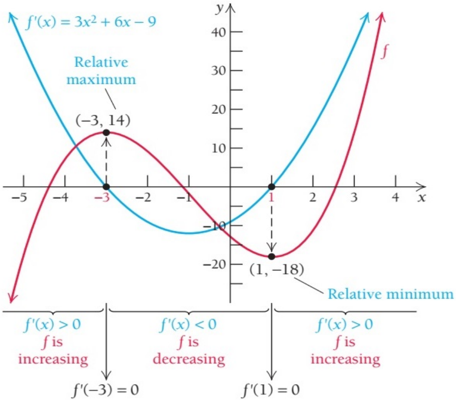

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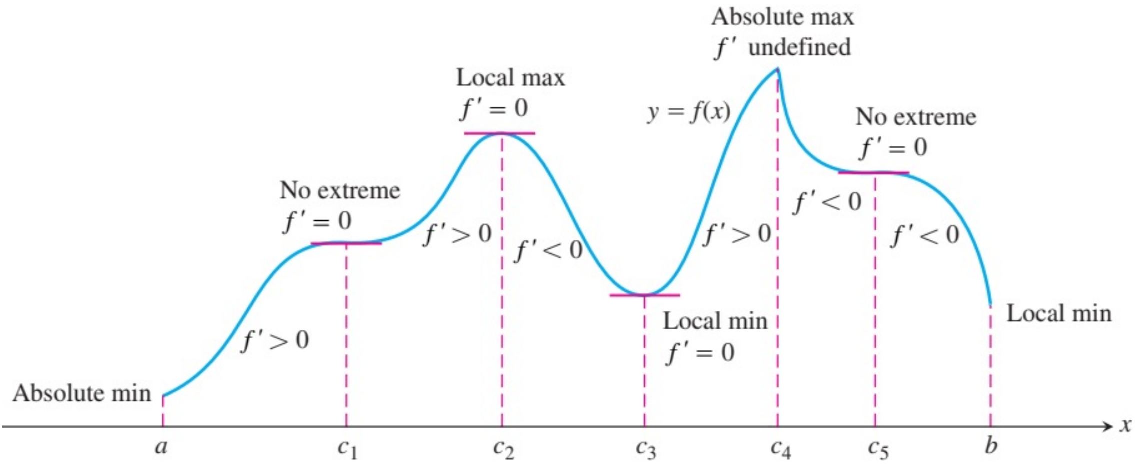

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Definition

Suppose that f is a function with domain I.
$f(c)$ is an absolute minimum if $f(c) \leq f(x)$ for all x in I.
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Problem. *How to find absolute maximum and minimum values over closed intervals?*

Theorem

If f is continuous on a closed interval $[a, b]$, then f attains both an absolute maximum value M and an absolute minimum value m in $[a, b]$. That is, there are numbers x_1 and x_2 in $[a, b]$ with $f(x_1) = m$, $f(x_2) = M$ and $m \leq f(x) \leq M$ for every other x in $[a, b]$.

Finding absolute maximum and minimum values

Theorem

Suppose f is a continuous function defined over a closed interval $[a, b]$. To find the absolute maximum and minimum values over $[a, b]$:
a) First find $f'(x)$.
b) Then determine all critical values in $[a, b]$. That is, find all c in $[a, b]$ for which $f'(c) = 0$ or $f'(c)$ does not exist.
c) List the values from step (b) and the endpoints of the interval: $a, c_1, c_2, ..., c_n, b$.
d) Evaluate $f(x)$ for each value in step (c): $f(a), f(c_1), f(c_2), ..., f(c_n), f(b)$.
The largest of these is the absolute maximum of f over $[a, b]$. The smallest of these is the absolute minimum of f over $[a, b]$.

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Example. Find the absolute maximum and minimum values of $f(x) = x^4 - 2x^3$ over the interval $[-2, 2]$.

Theorem

Suppose f is a function such that $f'(x)$ exists for every x in an interval I and there is exactly one (critical) value c in I, for which $f'(c) = 0$. Then

$$f(c) \text{ is the absolute maximum value over } I \text{ if } f''(c) < 0$$

or

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Example. Find the absolute maximum and minimum values of each function over the indicated interval

a) $f(x) = x^2 + \dfrac{432}{x}$ over $(0, \infty)$

b) $g(x) = -x^3 + x^2 + 5x - 1$ over $(0, \infty)$

c) $h(x) = \frac{1}{3}x^3 + 2x^2 + x$ over $[-4, 0]$.

Finding absolute maximum and minimum values

Exercise 1. For a dosage of x cubic centimeters (cc) of a certain drug, the resulting blood pressure B is approximated by $B(x) = 305x^2 - 1830x^3$, $0 \leq x \leq 0.16$. Find the maximum blood pressure and the dosage at which it occurs.

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Exercise 2. The percentage of unemployed workers in service occupations can be modeled by $p(x) = -0.039x^3 + 0.594x^2 - 1.967x + 7.555$, where x is the number of years since 2003. (*Source*: Based on data from www.bls.gov.) According to this model, in what year during the period 2003–2013 was this percentage a maximum?

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Exercise 3. The total-cost and total-revenue functions for producing x items are $C(x) = 5000 + 600x$ and $R(x) = -\frac{1}{2}x^2 + 1000x$, where $0 \leq x \leq 600$.
a) Find the total-profit function $P(x)$.
b) Find the number of items, x, for which total profit is a maximum.
c) Average profit is given by $A(x) = P(x)/x$. Find $A(x)$.
d) Find the number of items, x, for which average profit is a maximum.