Applications of differentiation (cont.)
MTH 201 - Business Calculus

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Outline

Maximum-Minimum problems

A strategy for solving maximum–minimum problems

1. Read the problem carefully. If relevant, make a drawing.
2. Make a list of appropriate variables and constants, noting what varies, what stays fixed, and what units are used. Label the measurements on your drawing, if one exists.
3. Translate the problem to an equation involving a quantity Q to be maximized or minimized. Try to represent Q in terms of the variables of step 2.
4. Try to express Q as a function of one variable. Use the procedures developed in the previous sections to determine the maximum or minimum values and the points at which they occur.

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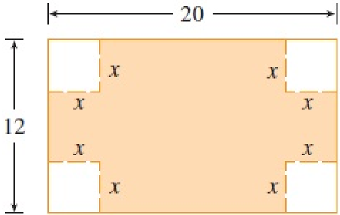

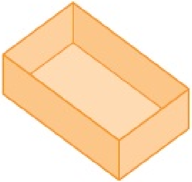

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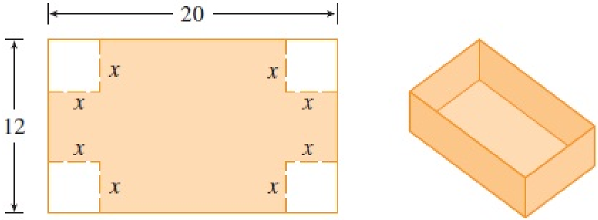

Step 2. Write an equation for the volume of the box...

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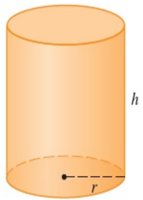

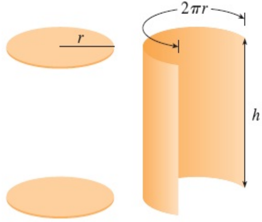

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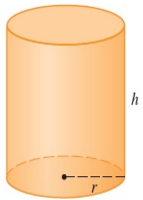

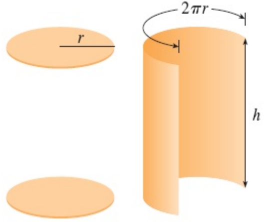

The surface area $A = 2\pi r^2 + \dfrac{2000}{r}$.

Example 3. (Minimizing cost of material) A rectangular box with a square base and a cover is to have a volume of 1000 ft^3. If the cost per square foot for the bottom is \$3, for the top is \$1, and for the sides is \$2, what should the dimensions be in order to minimize the cost?

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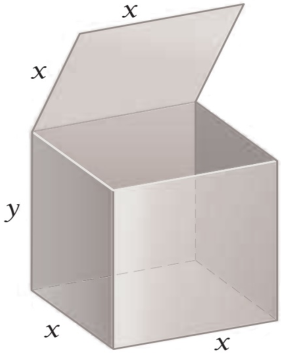

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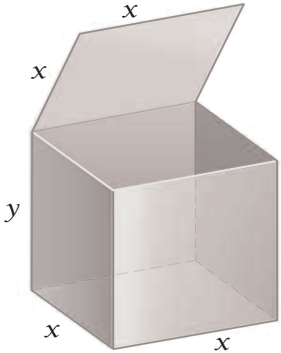

The cost $C = 3x^2 + x^2 + 8xy$ where $V = x^2y = 1000.$

Example 4. (Business: Maximizing Profit) A stereo manufacturer determines that in order to sell x units of a new stereo, the price per unit, in dollars, must be

$$p(x) = 100 - x.$$

The manufacturer also determines that the total cost of producing x units is given by

$$C(x) = \frac{1}{3}x^3 - 6x^2 + 89x + 100.$$

a) Find the total revenue $R(x)$.

b) Find the total profit $P(x)$.

c) How many units must the company produce and sell in order to maximize profit?

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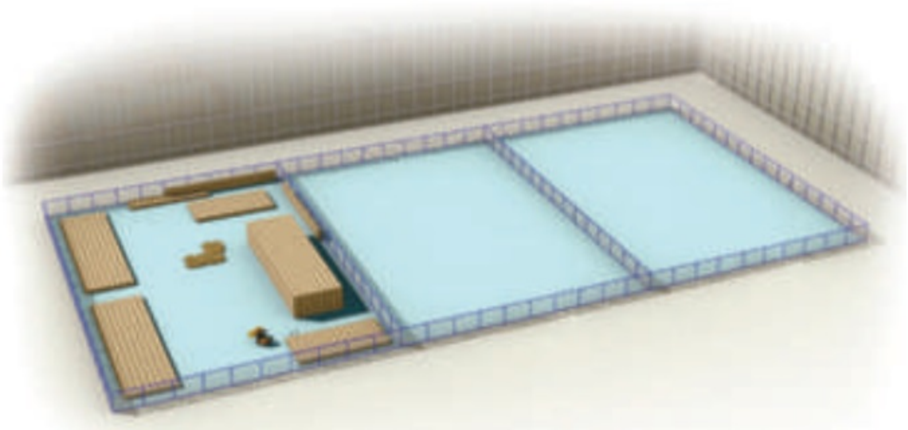
Note that maximum profit occurs at those x-values for which

$$R'(x) = C'(x), \quad \text{and} \quad R''(x) < C''(x).$$

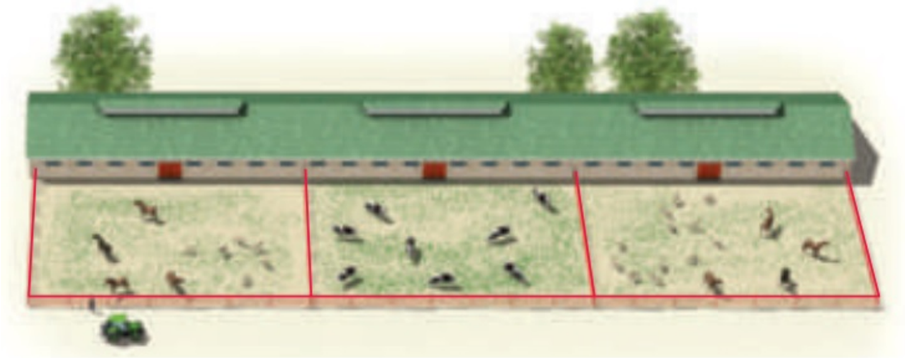
Exercise 1. (Maximizing area) A rancher wants to enclose two rectangular areas near a river, one for sheep and one for cattle. There are 240 m of fencing available. What is the largest total area that can be enclosed?

Exercise 2. (Maximizing area) Hentz Industries plans to enclose three parallel rectangular areas for sorting returned goods. The three areas are within one large rectangular area and 1200 m of fencing is available. What is the largest total area that can be enclosed?

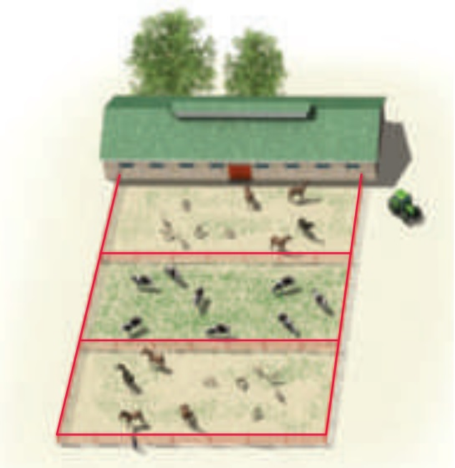


Exercise 3. (Maximizing area) Grayson Farms plans to enclose three parallel rectangular livestock pens within one large rectangular area using 600 m of fencing. One side of the enclosure is a pre-existing stone wall.

a) If the three rectangular pens have their shorter sides perpendicular to the stone wall, find the largest possible total area that can be enclosed.



b) If the three rectangular pens have their longer sides parallel to the stone wall, find the largest possible total area that can be enclosed.



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Exercise 5. (Minimizing surface area) Open Air Waste Management is designing a rectangular construction dumpster that will be twice as long as it is wide and must hold 12 m^3 of debris. Find the dimensions of the dumpster that will minimize its surface area.

Exercise 6. (Maximizing profit) Find the maximum profit and the number of units that must be produced and sold in order to yield the maximum profit. Assume that revenue $R(x)$ and cost $C(x)$ are in thousands of dollars, and x is in thousands of units.

a) $R(x) = 9x - 2x^2$, $C(x) = x^3 - 3x^2 + 4x + 1$

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Exercise 7. (Minimizing cost) A rectangular parking area measuring 5000 m^2 is to be enclosed on three sides using chain-link fencing that costs \$4.50 per meter. The fourth side will be a wooden fence that costs \$7 per meter. What dimensions will minimize the total cost to enclose this area, and what is the minimum cost (rounded to the nearest dollar)?

Outline

Marginal cost, revenue and profit

Definition

Let $R(x), C(x)$ and $P(x)$ represent, respectively, the total revenue, cost and profit from the production and sale of x items (demand).

- *The marginal revenue at x, denoted by $MR(x)$, is the derivative of total revenue with respect to demand x: $MR(x) = R'(x)$.*
- *The marginal cost at x, denoted by $MC(x)$, is the derivative of total cost with respect to demand x: $MC(x) = C'(x)$.*
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We have $MR(x) = R'(x) = \lim_{\Delta x \to 0} \dfrac{R(x+\Delta x) - R(x)}{\Delta x} \approx \dfrac{R(x+\Delta x) - R(x)}{\Delta x}$ for Δx is small.

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In particular, if $\Delta x = 1$ then $R'(x) \approx R(x+1) - R(x)$.

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Example 1. Suppose the monthly cost, in dollars, of producing x daypacks is $C(x) = 0.001x^3 + 0.07x^2 + 19x + 700$, and currently 25 daypacks are produced monthly.

a) What is the current monthly cost?

b) What would be the additional cost of increasing production to 26 daypacks monthly?

c) What is the marginal cost when $x = 25$?

d) Use marginal cost to estimate the difference in cost between producing 25 and 27 daypacks per month.

e) Use the answer from part (d) to predict $C(27)$.

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Example 2. The U.S. gross domestic product, in billions of current dollars, may be modeled by the function $P(x) = 567 + x(36x^{0.6} - 104)$, where x is the number of years since 1960. (*Source*: U.S.Bureau for Economic Analysis.) Use $P'(x)$ to estimate how much the gross domestic product increased from 2014 to 2015.

Outline

1. Maximum-Minimum problems: Business and economics applications
2. Marginals and differentials
3. **Elasticity of demand**

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- Total revenue is decreasing at those x-values for which $E(x) > 1$ (*elastic*).

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a) 10, b) 50, c) 90.

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Solution a) The demand is inelastic. b) unit elastic. c) elastic.

Elasticity

The price elasticity at a point (**point elasticity**) $E = -\dfrac{P}{Q} \times \dfrac{dQ}{dP}$.

Example 1 Given the demand function $P = 100 - Q$, calculate the price elasticity of demand when the price is

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Example 2 Given the demand equation

$$P = -Q^2 - 10Q + 150.$$

Find the price elasticity of demand when $Q = 4$. Estimate the percentage change in price needed to increase demand by 10%.

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Example 3 Given the demand function $P = 1000 - 2Q$, calculate the **arc elasticity** as P falls from 210 to 200.