Chapter 4: Integration
MTH 201 - Business Calculus

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Outline

1. **Antidifferentiation**
2. Area and definite integrals
3. Techniques of integration
4. Applications of definite integrals

Indefinite integration

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Initial conditions.

Simple power function and exponential functions

$$\int x^n dx = \frac{1}{n+1}x^{n+1} + C, \qquad \int \frac{1}{x}dx = \ln|x| + C, \qquad \int e^{mx}dx = \frac{1}{m}e^{mx} + C.$$

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a) $\displaystyle\int \frac{1}{x^3}dx,$ b) $\displaystyle\int \sqrt[3]{x}dx,$ c) $\displaystyle\int e^{-x}dx.$

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Rules of integration. Linearity

$$\boxed{\int [af(x) + bg(x)]\, dx = a \int f(x)\, dx + b \int g(x)\, dx}$$

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Example 2. Evaluate

$$\text{a) } \int \left(\frac{5}{x^2} - 2e^{3x}\right) dx, \qquad \text{b) } \int (x+1)\sqrt{x\sqrt{x}} dx.$$

Applications of indefinite integration in economics

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Example.

a) A firm's marginal cost function is $MC = 2$. Find an expression for the total cost function if the fixed costs are 500. Hence find the total cost of producing 40 goods.

b) The marginal revenue function of a monopolistic producer is

$$MR = 100 - 6Q.$$

Find the total revenue function and deduce the corresponding demand equation.

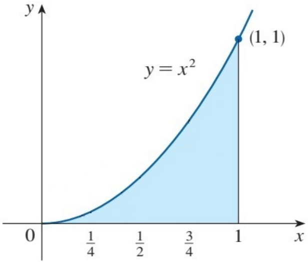
c) Find an expression for the savings function if the marginal propensity to save is given by

$$MPS = 0.4 - 0.1Y^{-1/2}$$

and savings are zero when income is 100.

Outline

Area under a curve



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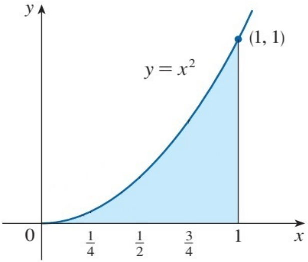
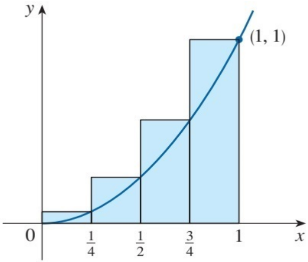
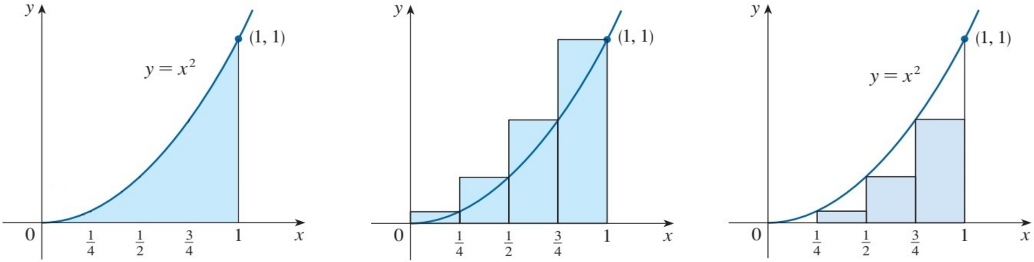
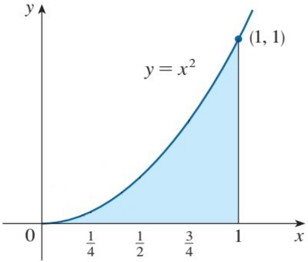


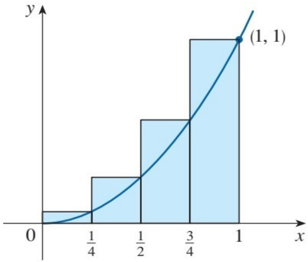


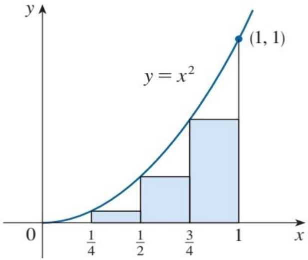


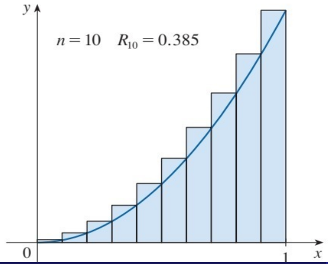
y
x
0
1/4
1/2
3/4
1
(1, 1)
y = x^2
(1, 1)
(1, 1)
y = x^2

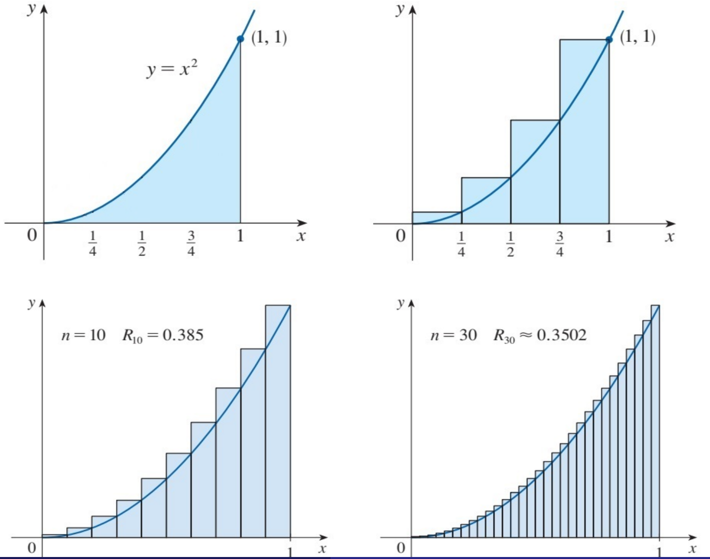
Area under a curve











y
x
0
$\frac{1}{4}$
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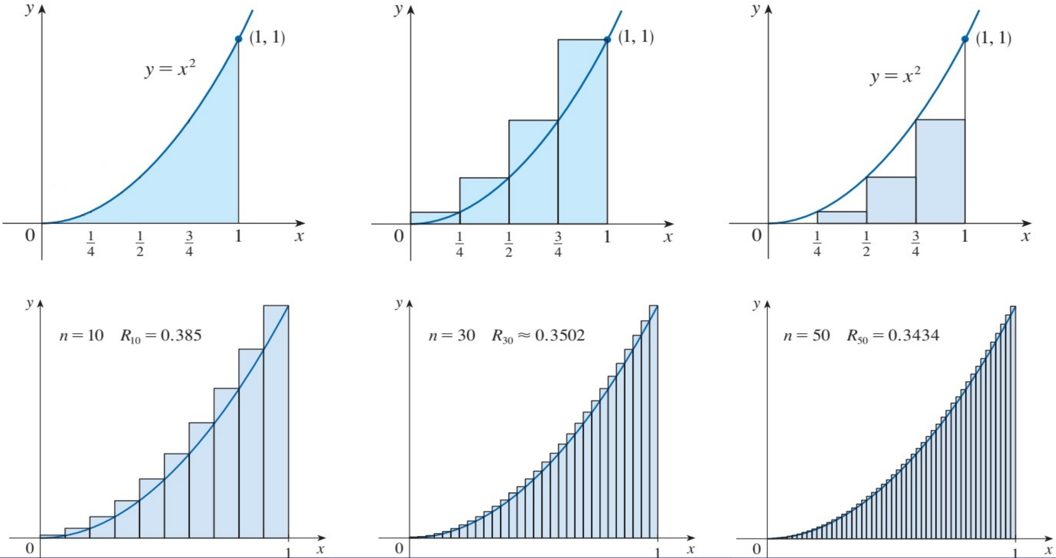
1
$(1, 1)$
$y = x^2$
$n = 10 \quad R_{10} = 0.385$
$n = 30 \quad R_{30} \approx 0.3502$

Area under a curve



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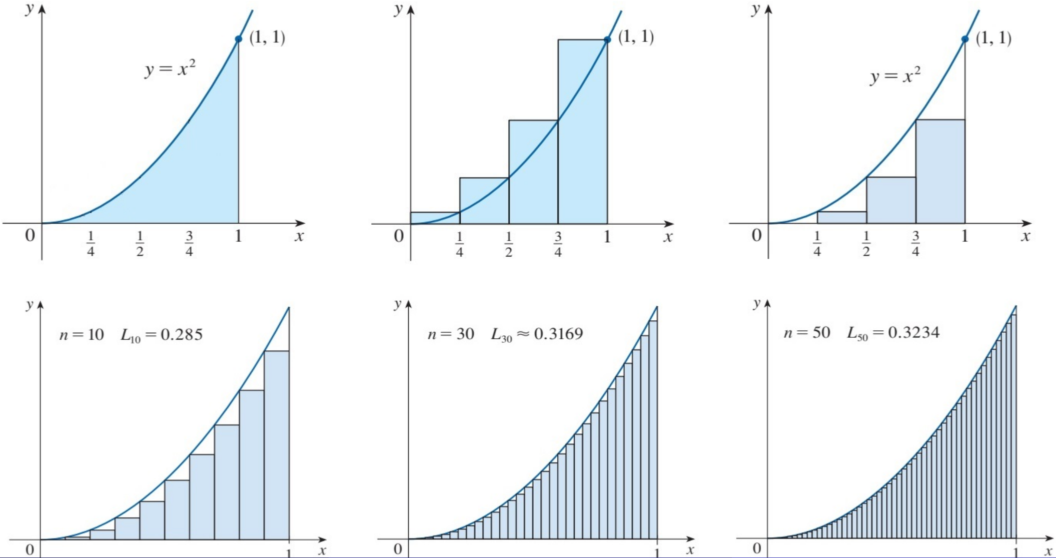

Area under a curve and Riemann sums

Let $f(x)$ be a function which is defined on a closed interval $[a, b]$

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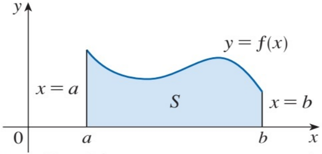

$S = \{(x, y) \mid a \leqslant x \leqslant b, 0 \leqslant y \leqslant f(x)\}$

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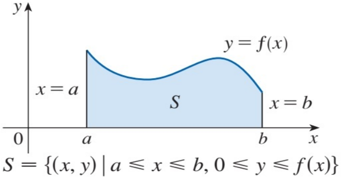

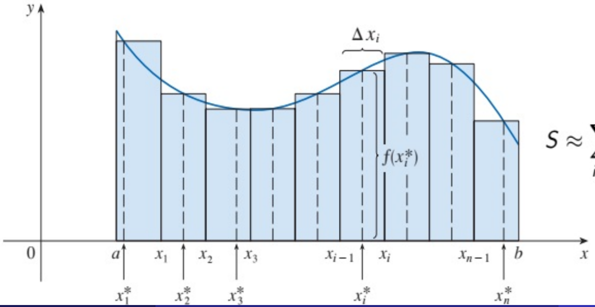

$$S \approx \sum_{i=1}^{n} f(x_i^*)\Delta x_i$$

Riemann sums

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- Choose $x_i^* \in [x_{i-1}, x_i]$, then define a sum (Riemann sum)

$$S_n = \sum_{i=1}^{n} f(x_i^*)\Delta x_i \text{ với } \Delta x_i = x_i - x_{i-1}.$$

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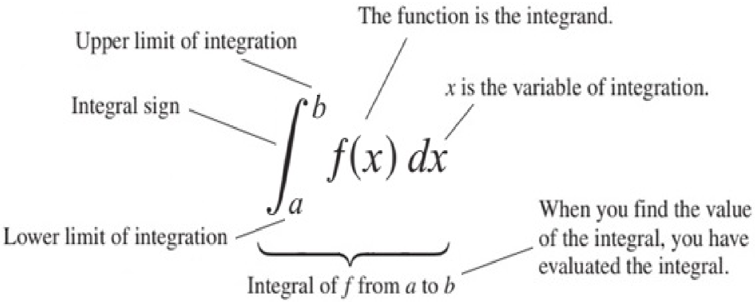
$$S_n = \sum_{i=1}^{n} f(x_i^*)\Delta x_i \text{ where } \Delta x_i = x_i - x_{i-1}.$$

Definition

Let $f(x)$ be a bounded function defined on a closed interval $[a, b]$. The definite integral of f over $[a, b]$ is the limit of the Riemann sums $\sum\limits_{i=1}^{n} f(x_i^*)\Delta x_i$ as $n \to +\infty$, $\lambda \to 0$ and the limit exists for every partition of $[a, b]$ and for any choice of x_i^* in $[x_{i-1}, x_i]$.

Definite integrals



When the definition is satisfied, we say the Riemann sums of f on $[a, b]$ converge to the definite integral $\int_a^b f(x)dx$ and that f is integrable over $[a, b]$.

Definite integrals

Remark

- *If* $b < a$ *then we define* $\int\limits_a^b f(x)dx := -\int\limits_b^a f(x)dx.$
- *If* $b = a$ *then we define* $\int\limits_a^b f(x)dx = 0.$

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Theorem

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Theorem

If $f(x)$ *is bounded on* $[a, b]$ *and is increasing/decreasing on* $[a, b]$*, then* f *is integrable on* $[a, b]$*.*

Properties of definite integrals

- *Linearity*

$$\int_a^b [\alpha f(x) + \beta g(x)]dx = \alpha \int_a^b f(x)dx + \beta \int_a^b g(x)dx$$

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  Suppose that all of the following definite integrals exist

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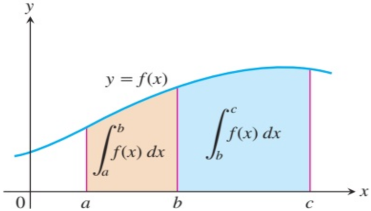

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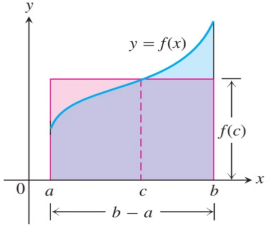

Definition

If f is a continuous function over $[a, b]$, then its average value on $[a, b]$, also called its mean value, is

$$av(f) = \frac{1}{b-a}\int_a^b f(x)dx.$$

The Fundamental Theorem of Calculus

Theorem (The Fundamental Theorem of Calculus part 1)

If $f(x)$ *is continuous on [a,b] then* $F(x) = \int\limits_a^x f(t)dt$ *is continuous on [a,b] and differentiable on* (a, b) *and its derivative is* $f(x)$*:* $F'(x) = f(x)$.

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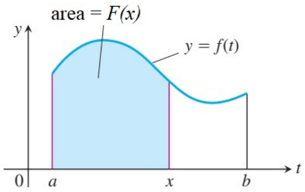

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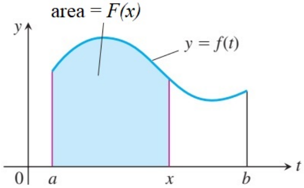

If f is continuous on $[a,b]$ then the function $F(x) = \int\limits_a^x f(t)dt$ is an anti-derivative of f.

The Fundamental Theorem of Calculus

Theorem (The Fundamental Theorem of Calculus part 2)

If f is continuous at every point of $[a, b]$ and F is any anti-derivative of f on $[a, b]$, then

$$\int_a^b f(x)dx = F(b) - F(a) \quad \textit{(Newton-Leibniz formula).}$$

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Example 1. Evaluate $\int_0^{\pi/2} \cos^2 x dx$, $\int_1^e \left(2 - 4x - \frac{1}{x}\right) dx$.

Example 2. (Physical Science: Distance). Suppose a particle's acceleration, in m/sec^2, is $a(t) = 12t^2 - 6$, with initial velocity $v(0) = 5m/sec$, and initial position (or distance) $s(0) = 10m$.

a) Find $s(t)$.

b) Find the particle's position (distance from the initial position) after 2 sec.

Area under a curve as a definite integral

Definition

If $y = f(x)$ *is nonnegative and integrable over a closed integral* $[a, b]$, *then the area under the curve* $y = f(x)$ *over* $[a, b]$ *is the integral of* f *from* a *to* b,

$$A = \int_a^b f(x)dx.$$

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Example. Compute the integral $\int_0^2 \sqrt{4 - x^2}dx$.

Areas between curves.

Example. The temperature, in degrees Fahrenheit, in Minneapolis on a winter's day can be modeled by the function $T(x) = -0.012x^3 + 0.38x^2 - 1.99x - 10.1$, where x is the number of hours from midnight $(0 \leq x \leq 24)$. Find the average temperature in Minneapolis during this 24-hour period.

Exercise 1. (Total and average daily profit) Shylls, Inc., determines that its marginal revenue per day is given by $R'(t) = 100e^t$, $R(0) = 0$, where $R(t)$ is the total accumulated revenue, in dollars, on the tth day. The company's marginal cost per day is given by $C'(t) = 100 - 0.2t$, $C(0) = 0$, where $C(t)$ is the total accumulated cost, in dollars, on the tth day.
a) Find the total profit from $t = 0$ to $t = 10$ (the first 10 days). Note:

$$P(T) = R(T) - C(T) = \int_0^T [R'(t) - C'(t)]dt.$$

b) Find the average daily profit for the first 10 days (from $t = 0$ to $t = 10$).

Exercise 2. (Total and average daily profit) Great Green, Inc., determines that its marginal revenue per day is given by $R'(t) = 75e^t - 2t$, $R(0) = 0$, where $R(t)$ is the total accumulated revenue, in dollars, on the tth day. The company's marginal cost per day is given by $C'(t) = 75 - 3t$, $C(0) = 0$, where $C(t)$ is the total accumulated cost, in dollars, on the tth day.
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a) Find the total profit from $t = 0$ to $t = 10$ (the first 10 days).

b) Find the average daily profit for the first 10 days (from $t = 0$ to $t = 10$).

Exercises 53, 54, 56, page 430; 66 page 431.

Outline

The Substitition Rule

The power rule in integral form. If u is any differentiable function, then

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Theorem (The Substitution Rule)

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$$\int f(g(x))g'(x)dx = \int f(u)du.$$

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$$\int f(g(x))g'(x)dx = \int f(u)du.$$

Example. Find $\int x(x-1)^9 dx$, $\int t\sqrt{2t-1}dt$, $\int \frac{\sqrt{x^2-4}}{x}dx$.

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Exercise. Evaluate $\int 3x^5\sqrt{x^3+1}dx$, $\int \dfrac{x}{\sqrt[3]{x^2+2}}dx$, $\int \dfrac{1}{e^x+1}dx$, $\int \sqrt{\dfrac{2-x}{x}}dx$.

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Exercises. 87, 88, 89, 90 page 438, 439.

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Exercises. 39, 40, 42 page 445-446.

Tables of integration formulas (page 447-448)

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Exercises. 39, 40, 42 page 445-446.

Tables of integration formulas (page 447-448)

Exercises. Evaluate $\int \dfrac{1}{x\sqrt{x^2+4}}dx$, $\int \dfrac{3\ln x}{x^2}dx$, $\int x\sqrt{2+7x}dx$, $\int e^x\sqrt{1+e^x}dx$, $\int x^3\sqrt{4+x^2}dx$.

Outline

An Economics Application: Consumer Surplus

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Suppose that $p = D(x)$ describes the demand function for a commodity. Then the consumer surplus is defined for the point (Q, P) as

$$CS = \int_0^Q D(x)dx - PQ.$$

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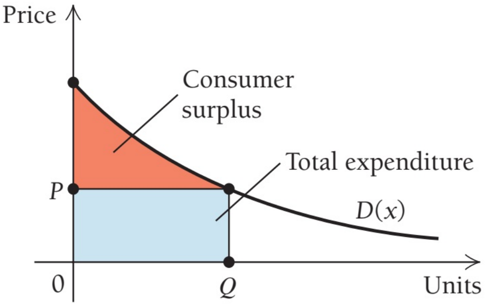
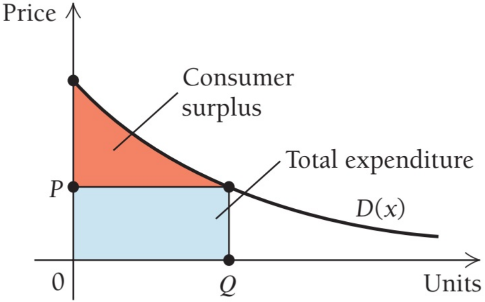


An Economics Application: Consumer Surplus

The total money spent on Q goods (at the price P) is PQ. The shaded area represents the benefit to the consumer of paying the fixed price of P.



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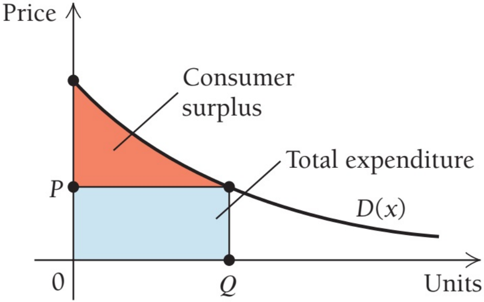

Example. Find the consumer surplus for the demand function given by $D(x) = x^2 - 6x + 16$ when $x = 1$.

Producer Surplus

Definition

Suppose that $p = S(x)$ *is the supply function for a commodity. Then the* *producer surplus* *is defined for the point* (Q, P) *as*

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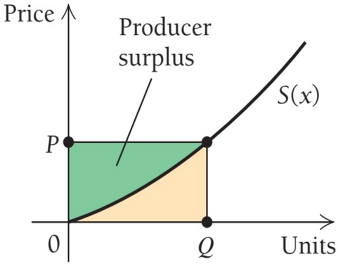
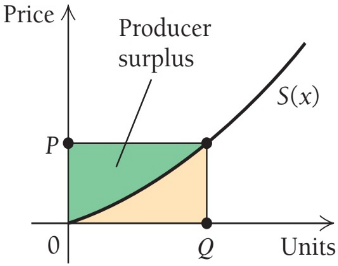


Producer Surplus

Total money received on selling Q goods (at the price P) is PQ. The shaded area therefore represents the benefit to the producer of selling at the fixed price of P.



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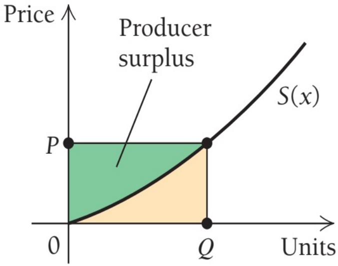

Example. Find the producer surplus for the supply function given by $S(x) = \frac{1}{3}x^2 + \frac{4}{3}x + 4$ when $x = 1$.

Consumer Surplus and Producer Surplus

The equilibrium point (x_E, p_E) in the following figure is the point at which the supply and demand curves intersect. It is the point at which sellers and buyers come together and purchases and sales actually occur.

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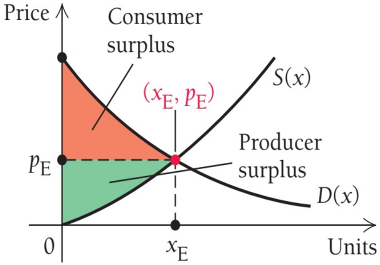

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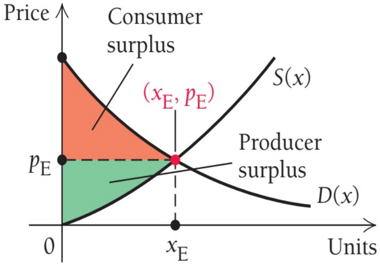

The consumer surplus and the producer surplus at the equilibrium point are

$$CS = \int_0^{x_E} D(x)dx - x_E p_E \qquad PS = x_E p_E - \int_0^{x_E} S(x)dx.$$

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Example. Given the demand equation $D(x) = \dfrac{1800}{\sqrt{x+1}}$ and supply equation $S(x) = 2\sqrt{x+1}$.

Calculate

a) The equilibrium point.

b) The consumer surplus at the equilibrium point.

c) The producer surplus at the equilibrium point.