

MTH 201 – Business Calculus: Final Exam 2024

1.

a)

Given

$$f(x) = e^x(x^2 + x + 1),$$

find

$$f'(x), \quad f''(x).$$

b)

Find local maximum and local minimum of the function

$$f(x) = (x^2 - 3)e^x.$$

2.

A rectangular box with a square base and a cover is to have a volume of

$$1000 \text{ ft}^3.$$

If the cost per square foot for the bottom is \$3, for the top is \$1, and for the sides is \$2, what should the dimensions be in order to minimize the cost?

3.

A company is designing an open-top, rectangular container that will be twice as long as it is wide and must have a volume of

$$10 \text{ m}^3.$$

The cost of making the bottom of container is \$5 per square meter, and the cost of the sides is \$4 per square meter. Find the dimensions of the container that will minimize total cost.

4.

If the total revenue

$$R(x) = 300x - x^2$$

and the total cost

$$C(x) = 40x + 5,$$

find the maximum profit and the number of units that must be produced and sold in order to yield this maximum profit.

5.

A manufacturer determines that in order to sell x units of a good, the price per unit, in dollars, must be

$$p(x) = 100 - x.$$

The manufacturer also determines that the total cost of producing x units is given by

$$C(x) = \frac{1}{3}x^3 - 6x^2 + 89x + 100.$$

a)

Find the total revenue $R(x)$.

b)

Find the total profit $P(x)$.

c)

How many units must the company produce and sell in order to maximize profit?

d)

What is the maximum profit?

e)

What price per unit must be charged in order to make this maximum profit?

6–8.

The photos do not show these questions clearly enough to transcribe completely.

9.

Evaluate the integral

$$\int (x + 2)e^{3x} dx.$$

10.

Shylls, Inc. determines that its marginal revenue per day is given by

$$R'(t) = 100e^t, \quad R(0) = 0,$$

where $R(t)$ is the total accumulated revenue, in dollars, on the t th day.

The company's marginal cost per day is given by

$$C'(t) = 100 - 0.2t, \quad C(0) = 0,$$

where $C(t)$ is the total accumulated cost, in dollars, on the t th day.

a)

Find the total profit from $t = 0$ to $t = 10$ (the first 10 days).

b)

Find the average daily profit for the first 10 days (from $t = 0$ to $t = 10$).

11.

Great Green, Inc. determines that its marginal revenue per day is given by

$$R'(t) = 75e^t - 2t, \quad R(0) = 0,$$

where $R(t)$ is the total accumulated revenue, in dollars, on the t th day.

The company's marginal cost per day is given by

$$C'(t) = 75 - 3t, \quad C(0) = 0,$$

where $C(t)$ is the total accumulated cost, in dollars, on the t th day.

a)

Find the total profit from $t = 0$ to $t = 10$ (the first 10 days).

b)

Find the average daily profit for the first 10 days. See textbook, page 429.

12.

A firm's marginal revenue and marginal cost functions are given by

$$MR = R'(x) = 240 - 0.6x^2$$

and

$$MC = C'(x) = 150 + 0.3x^2.$$

Fixed costs are 50.

a)

Write down an expression for total revenue and deduce the corresponding demand function.

b)

Write down an expression for the total cost function.

c)

Determine the maximum profit.

13.

Given the demand equation

$$D(x) = 13 - x, \quad 0 \leq x \leq 13$$

and supply equation

$$S(x) = \sqrt{x + 17},$$

calculate:

a)

The equilibrium point.

b)

The consumer surplus at the equilibrium point.

c)

The producer surplus at the equilibrium point.

14.

Find the consumer's surplus for the demand function

$$D(x) = 500 - 3\sqrt{x} - 0.3x^2$$

when

$$x = 9.$$

15.

Find the producer's surplus for the supply function

$$S(x) = 20\sqrt{x} - 15$$

when

$$x = 9.$$

16.

A computer company has the following Cobb-Douglas production function for a certain product:

$$p(x, y) = 200x^{3/4}y^{1/4},$$

where x is the labor, measured in dollars, and y is the capital, measured in dollars.

Suppose that the company can make a total investment in labor and capital of \$20000. What amount of labor and capital would maximize production?

17.

Find the relative maximum and minimum values of

$$f(x, y) = 3x^2 - 6xy + y^3 - 9y + 1.$$

18.

Consider the following data showing the average life expectancy of men in various years.

Note that x represents the actual year.

Year, x	1980	1990	2000	2010
Life Expectancy of Men, y (years)	70.0	71.8	74.1	75.4

a)

Find the regression line,

$$y = mx + b.$$

b)

Use the regression line to predict the life expectancy of men in 2020 and 2030.