

Review exercises for midterm - TROY 2026

1. Differentiate

a) $y = \frac{5x^2 + x - 1}{x^3 - 9x^2}$ b) $f(x) = x^2(4x + 2)^{3/4}$ c) $f(x) = 3e^{2x} \ln x$ d) $f(x) = x^4 e^{3x}$.

2. Find the second derivative of

a) $f(x) = x\sqrt{x^2 + 1}$ b) $g(x) = \frac{x^2}{x+1}$ c) $h(x) = (x^4 + x)^{2/3}$.

3. Find the points on the graph of $f(x) = \frac{1}{3}x^3 + 2x^2 + 12x$ at which the tangent line has slope 9. Find the equation of the tangent line at these points.

4. A spherical balloon is inflated with helium at the rate of 400π cm³/min. How fast is the balloon's radius increasing at the instant the radius is 20 cm? How fast is the surface area increasing?

5. Find any relative extrema of each function. State where the function is increasing or decreasing.

a) $f(x) = 20x^3 - 3x^5$ b) $g(x) = (x - 1)^{2/3} - (x + 1)^{2/3}$ c) $h(x) = 8x^3 - 6x + 1$

6. Indicate where the following function is increasing or decreasing, list any extrema, and determine points of inflection.

a) $f(x) = 3x^4 - 15x^2 + 2$ b) $g(x) = \frac{3x}{x^2 + 4}$ c) $h(x) = x\sqrt{8 - x^2}$.

7. The total revenue and total cost functions for producing x clocks are $R(x) = 520x - 0.01x^2$ and $C(x) = 120x + 100000$ where $0 \leq x \leq 25000$. What is the maximum annual profit?

8. Find the absolute maximum and minimum values of the following functions, if they exist

a) $f(x) = \frac{x^2}{1+x^3}$ over $[0; \infty)$ b) $g(x) = x^2 + \frac{128}{x}$ over $(0; \infty)$
c) $p(x) = \frac{1}{3}x^3 + 2x^2 + x$ over $[-4; 0]$ d) $h(x) = 5x^2 + \frac{10}{x}$ over the interval $(0; \infty)$.

9. The total cost and total revenue functions for a company are

$$C(x) = \frac{1}{3}x^3 - 15x^2 + 480x + 750, R(x) = x(536 - 2x).$$

a) Write down the equation for the profit function. Hence determine the level of output, x , for which profit is maximized and minimized.

b) Write down the equation of the marginal profit curve. Hence find the maximum value of marginal profit.

10. A rectangular box with a square base and a cover is to have a volume of 1000 ft³. If the cost per square foot for the bottom is \$3, for the top is \$1, and for the sides is \$2, what should the dimensions be in order to minimize the cost?

11. A firm's demand function is given by $D(x) = 100 - 4\sqrt{x} - 3x$

a) Write down an expression for total revenue, $R(x)$, in terms of x .

b) Find an expression for the marginal revenue, MR , and find the value of MR when $x = 9$.

c) Use the result of part b) to *estimate* the change in R when x increases by 0.25 units from its current level of 9 units and compare this with the exact change in R .