PART A: Multiple choice or Short answer

(5 *mark for each correct answer/choice*)

Problem A1. Let p and q be the propositions:

p: 113 is a prime number.

q: $x^2 - 3x + m = 0$ always has a real solution for any real number m.

Circle the best answer.

A. $\neg p \rightarrow \neg q$ is true and $p \rightarrow q$ is false.

B. $\neg p \rightarrow \neg q$ is true and $p \rightarrow q$ is true.

C. $\neg p \rightarrow \neg q$ is false and $p \rightarrow q$ is false.

D. $\neg p \rightarrow \neg q$ is false and $p \rightarrow q$ is true.

Problem A2. Find the cardinality of 2^A, where A is the set of all even integers between 1 and 9.

Circle the best answer.

A. 32 B. 16 C. 4 D. 8

Problem A3. Express the negations of the following statement so that all negation symbols immediately precede predicates

$$\exists x \forall y((xy = 0) \rightarrow (x + y = 2))$$

Write your answer here:

Problem A4. Let

$X = \{x \mid 0 \leq x \leq 2,\ \ x \text{ are intergers}\}$, and

$Y = \{y \mid 0 \leq y^2 \leq 3,\ \ y \text{ are integers}\}$.

Find the product $X \times Y$ (*list all its elements*).

Write your answer here: $X \times Y =$

Problem A5. Let $f : \mathbf{R} \rightarrow \mathbf{R}$ be a real valued function, $f(x) = \ln x$,and g : $\mathbf{R} \rightarrow \mathbf{R}$ the ceiling function of x. Find $g(f(1/2))$.

Write your answer here: $g(f(1/2)) =$

Problem A6. Find the proposition(s) which is logically equivalent to the following proposition:

`*Every CSE student takes a calculus course.*'

p(x): `*x is a CSE student*'

q(x): `*x takes a calculus course.*'

Circle the best answer.

A. $\forall x\ (p\ (x) \wedge q(x))$ B. $\nexists x\ (p\ (x) \rightarrow q(x))$

C. $\forall x\ (p\ (x) \rightarrow q(x))$ D. The correct answer is different from A,B,C.

Problem A7. Find the possible cardinality of a domain **U** for the quantifiers in

$\exists x \exists y(x \neq y \land \forall z(x = z) \lor (y = z))$
such that this statement is true.
Write your answer here: $|U| =$

Problem A8. Let $f, g : \mathbf{X} \to \mathbf{X}$ be two functions.
*Circle the **incorrect** stamen(s)*

A. f is NOT onto if and only if

$$\exists y \forall x(f(x) \neq y),$$

B. f is injective if and only if

$$\forall y \forall x((x \neq y) \to (f(x) \neq f(y)),.$$

C. If f and $g^{\circ}f$ are both bijective then g is one to one.

D. $f^{\circ}g$ is onto if and only if both f and g are onto.

PART B: WRITE YOUR FULL ANSWERS.

Problem B1. Let A, B, C be subsets of U (Universal). Prove

a. Prove $\overline{A \cup B \cup C} = \bar{A} \cap \bar{B} \cap \bar{C}$

b. Prove $B - A = B \cap \bar{A}$

c. If $|A| = 5, |B| = 7, |C| = 8, |A \cap B| = 3, |B \cap C| = 3$ and $A \cap C$ is empty. Find $|A \cup B \cup C|$?

Problem B2. Use rules of inference to show that if $\forall x(P((x) \to Q(x))$ and $\forall x(P((x) \land Q(x) \to R(x)))$ are true, then $\forall x(\neg R((x) \to \neg P(x))$ is also true, where the domains of all quantifiers are the same.

Problem B3. Prove that if x^5 is irrational then x is irrational using

a. a proofs by contraposition (indirect proof).

b. a proofs by contradiction.

Problem B4. Let p, q and r be propositions. Prove or disprove that the following propositions are logical equivalent (Justify your answer):

$$(\neg p \land \neg r) \lor q \text{ and } (p \to q) \lor (r \to q)?$$

Problem B5. Let $f, g : \mathbf{R} \longrightarrow \mathbf{R}$ be functions defined by

$$f(x) = e^x, \quad g(x) = 3x^3 - x.$$

a. [*5 marks*] Find the formula of $g^{\circ}f$ and $f^{\circ}g$.

b. [*10 marks*] Find the inverse of $f^{\circ}g$ if exists. Justify your answer.

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