1. (50 points)
   a) The numbers 1 through 10 are arranged in any order around a circle. Prove that there will always be 3 consecutive numbers whose sum is at least 17.
   b) Put 20 balls into 10 box at random. Assume that the 10 boxes are distinguishable, and each of the 20 balls independently chooses one of the 10 boxes with probability 1/10. What is the probability that there exists at least one box containing exactly 10 balls. (Write your answer using binomial coefficients is fine)

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2. (40 points) Find the number of (unordered) five-card poker hands, selected from an ordinary 52-card deck, in each of the following cases.
   a) Containing four of a kind, that is, four cards of the same denomination
   b) Containing cards of exactly two suits
   c) Containing cards of all suits
   d) Containing two of one denomination, two of another denomination, and one of a third denomination

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3. (50 points) A laboratory stores five types of chemical vials: acid vials, base vials, salt vials, buffer vials, indicator vials. Assume each type is available in unlimited quantity, and vials of the same type are identical.
   How many ways are there to choose:
   a) 25 vials?
   b) 16 vials with at least 2 of each type?
   c) 18 vials with no more than 3 indicator vials?
   d) 20 vials with at least 4 acid vials and at least 5 salt vials?
   e) 22 vials with at least 1 acid vial, at least 2 base vials, at least 4 buffer vials, and no more than 5 indicator vials?

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4. (50 points)
   a) Give a formula for the coefficient of x^k in the expansion of $(x^2 - \frac{1}{x})^{100}$
   b) Let $a_1 = 0$ and $a_n = 4a_{\lfloor n/2 \rfloor} + n, n \geq 2$. Prove that $a_n \leq 4(n-1)^2$ for all $n \geq 1$.

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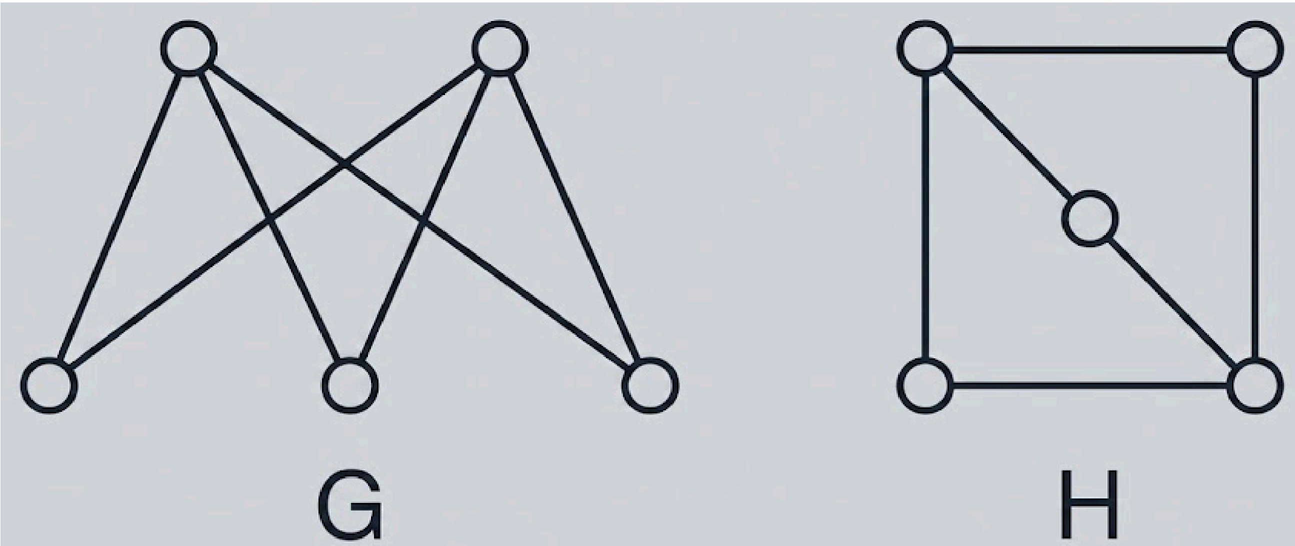
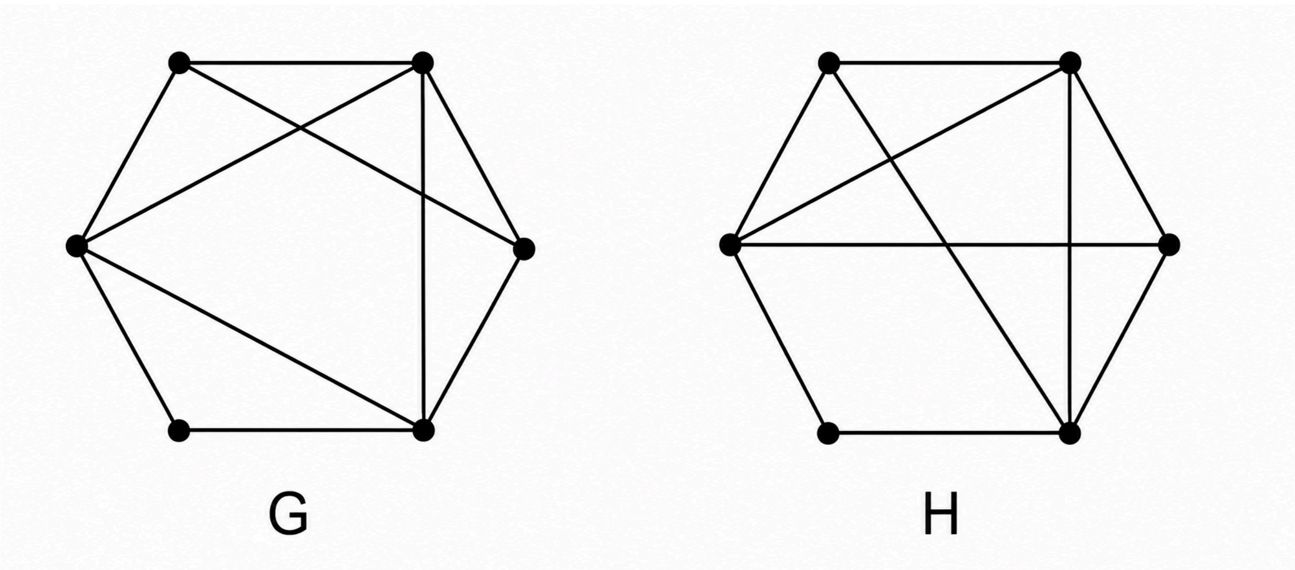
5. (40 points) Which of the following is a degree sequence of a simple graph? If it is, draw the graph. If it is not, explain why not.
   a) 5, 4, 2, 2, 1, 1
   b) 5, 4, 3, 2, 2, 0
   c) 5, 4, 3, 3, 2, 1
   d) 5, 4, 2, 2, 1, 0

6. (50 points)
   a) For which values of c, d the sequence $a_n = c2^n + d, n = 0, 1, 2, \ldots$ is a solution of the following recurrence relation:
   $a_n = 2a_{n-1} - a_{n-2} + 2^n, n \geq 2$.
   b) Find the solution of the following recurrence relation:
   $a_n = 2a_{n-1} - a_{n-2} + 2^n, n \geq 2$, and $a_0 = 1, a_1 = 2$.

7. (40 points) Are the following pairs of graphs isomorphic? If they are, state the correspondence of the vertices. If they are not, explain why not.
   a)



   b)



8. (40 points) For which values of $m, n > 1$, does the complete bipartite graph $K_{m,n}$ have a Hamilton circuit? Hamilton path? Write down the Hamilton circuit, path for such values of m, n.

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9. (50 points)
   i) Which of the following graphs have a Hamilton circuit? If they do, find it. If they do not, explain why not.
   ii) Which of the following graphs have a Hamilton path? If they do, find it. If they do not, explain why not.