

1. (50 points) a) Find the minimum number n of integers to be selected from $S = \{1, 2, \dots, 9\}$ to make sure that the difference of two of the n integers is 5.

b) Find the smallest number of people you need to choose at random so that the probability that at least two of them were both born on April 1 exceeds $1/2$.

2. (40 points) One hundred tickets, numbered 1, 2, 3, ... , 100, are sold to 100 different people for a drawing. Four different prizes are awarded, including a grand prize (a trip to Tahiti). How many ways are there to award the prizes if
- a) there are no restrictions?
 - b) the person holding ticket 47 wins one of the prizes?
 - c) the people holding tickets 19 and 47 both win prizes?
 - d) the people holding tickets 19, 47, and 73 all win prizes?

3. (40 points) a) Draw the undirected graph with the adjacency matrix:

$$\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

b) Give an example of two non-isomorphic graphs with the same degree sequence.

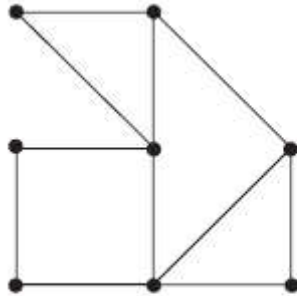
4. (40 points) Show that any positive integer can be written as sum of **distinct** powers of two.

5. (50 points) Find the solution of the recurrence relation

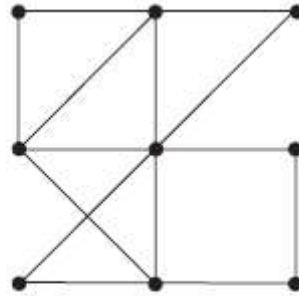
$$a_n = 4a_{n-1} - 3a_{n-2} + 2^n + n + 3, \quad a_0 = 1, a_1 = 4$$

6. (40 points)

- i) Which of the following graphs have an Euler circuit? If they do, find it. If they do not, explain why not.
- ii) Which of the following graphs have an Euler path? If they do, find it. If they do not, explain why not.



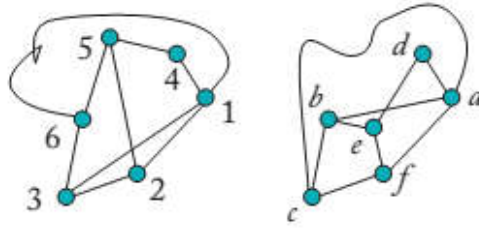
(a)



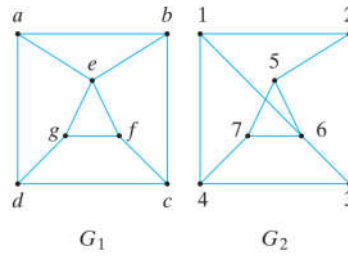
(b)

7. (40 points) Are the following pairs of graphs isomorphic? why and why not?
(state the correspondence of the vertices in each isomorphic pair)

a)



b)



8. (50 points) In three-dimensional space, 8 points are located such that no three of them lie on a straight line. 17 line segments are drawn, each with both endpoints at these points. Prove that three line segments form a triangle.

9. (50 points)

- i) Which of the following graphs have a Hamilton circuit? If they do, find it. If they do not, explain why not.
- ii) Which of the following graphs have a Hamilton path? If they do, find it. If they do not, explain why not.

