1. (40 points)

i) Find the truth table of the proposition: $\neg(p \wedge q) \vee (\neg q \vee r)$

ii) Determine the value of each of the following proposition given:
$P(x, y) : (x < y) \rightarrow (x^2 < y^2),\ \text{x, y are real numbers}$

a) $\forall x \forall y P(x, y)$
b) $\exists y \forall x P(x, y)$

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2. (40 points)

i) Prove the following set identity:
$(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

ii) Is the set identity $(A - B) \cup (B - C) = (A - C)$ true? If it is true give a proof, otherwise give a counterexample to show that it is false.

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3. (40 points)

i) For each of the following predicates, find all the values in the domain $D = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ such that the predicate is true:

a) $Q(n) : (n^2 < 15) \rightarrow (n \text{ is an even number})$
b) $S(n) : (2n > 10) \vee (n \mid 8)$

ii) Prove that $n = \lfloor \frac{n}{2} \rfloor + \lceil \frac{n}{2} \rceil$ for any positive integer n.

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4. (40 points)

Which of the following functions are injective? surjective? bijective? Explain your answers.

- $f : \mathbb{N} \rightarrow \mathbb{N}, f(n) = \begin{cases} \frac{n+1}{2}, & n \equiv 1 \bmod 2 \\ \frac{n}{2}, & n \equiv 0 \bmod 2 \end{cases} \qquad \text{where } \mathbb{N} = \{0, 1, 2, 3, \ldots\}$
- $g : \mathbb{R} \rightarrow \mathbb{Z}, g(x) = \lfloor x + 1 \rfloor$
- $h : \mathbb{Z}^+ \rightarrow \mathbb{Z}, h(x) = x^2 - 1 \qquad \text{where } \mathbb{Z}^+ = \{1, 2, 3, \ldots\}$

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5. (40 points)

What is the least integer n such that $f(x)$ is $O(x^n)$ for each of the following functions? Explain in detail.

a) $f(x) = x^{3/2} + x^2 + 5$
b) $f(x) = 2x^3 + x^4 \ln x - 2x^2$

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6. (30 points)

Let x, y be positive real numbers such that $x^2 - y^2 = 1$. Prove that x or y (or both) is not an integer.

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7. (40 points)

i) Show the steps of using Euclidean algorithm to find $\gcd(1411, 2703)$

ii) Find the binary representation of $(8\mathrm{FB})_{16}$

iii) Find the octal representation of $(10010110101)_2$

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8. (30 points)

Show the steps of using modular exponentiation algorithm to find $13^{61} \bmod 65$