1. (40 points)

i) Find the truth table of the proposition
$(p \vee \neg r) \rightarrow (\neg q \vee p)$

ii) Find the value of each of the following proposition given:
$P(x, y) : (x < 0) \wedge (y < 0) \rightarrow (xy > 1),\ \text{x, y are real numbers}$

a) $\forall x \forall y P(x, y)$
b) $\forall x \exists y P(x, y)$

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2. (40 points)

i) Prove the following set identity:
$(A - B) - (B - C) = A - B$

ii) Is the set identity $(A \cup B) \cap C = A \cup (B \cap C)$ true? If it is true give a proof, otherwise give a counterexample to show that it is false.

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3. (40 points)

i) For each of the following predicates, find all the values in the domain $D = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ such that the predicate is true:

a) $Q(n) : (n^2 > 15) \wedge (n \text{ is an odd number})$
b) $S(n) : (2n > 7) \rightarrow (n \equiv 1 \bmod 3)$

ii) Prove that $\min\{x, y\} + \max\{x, y\} = x + y$ for any real numbers x, y.

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4. (40 points)

Which of the following functions are injective? surjective? bijective? Explain your answers.

- $f : \mathbb{N} \rightarrow \mathbb{N}, f(n) = n - (-1)^n \qquad \text{where } \mathbb{N} = \{0, 1, 2, 3, \ldots\}$
- $g : \mathbb{Z} \rightarrow \mathbb{Z}, g(x) = \lceil \frac{x+1}{2} \rceil$
- $h : \mathbb{R}^2 \rightarrow \mathbb{R}^2, h(x, y) = (x + 1, y - 2)$

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5. (40 points)

What is the least integer n such that $f(x)$ is $O(x^n)$ for each of the following functions? Explain in detail.

a) $f(x) = 3\sqrt{x}x^3 - x^2 + 5$
b) $f(x) = 3x^5 \ln x + x \ln x$

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6. (30 points)

Prove that for every real numbers x, y the following inequalities holds:
$\lceil x \rceil + \lceil y \rceil - 1 \leq \lceil x + y \rceil \leq \lceil x \rceil + \lceil y \rceil$

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7. (40 points)

i) Show the steps of using Euclidean algorithm to find $\gcd(1104, 5037)$

ii) Find the binary representation of $(7\mathrm{AC})_{16}$

iii) Find the octal representation of $(1011101010)_2$

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8. (30 points)

Show the steps of using modular exponentiation algorithm to find $11^{53} \bmod 59$