ĐẠI HỌC
BÁCH KHOA HÀ NỘI
HANOI UNIVERSITY
OF SCIENCE AND TECHNOLOGY

TROY PHYSICS at HUST
General Physics 2252

Lecturer: Asso. Prof. Thoan Nguyen-Hoang
Office: 107B – C9
Office hours: 10:00-12:00 am Mon., Tues.
Telephone: 0914566144
Email: *thoan.nguyenhoang@hust.edu.vn*

TEAMS CODES:

For Wednessday Class: *mvtk5yp*

For Thursday class:

ONE LOVE. ONE FUTURE.

Course Information: Materials

- Primary Textbook: "**College Physics**" *Volume 1, 9th edition by Raymond A. Serway, and Chris Vuille.*

Lecture Note will be provided after each lesson.

- Books for additional reading:

1. D. Halliday, R. Resnick, J. Walker, "Fundamentals of Physics", 9th ed., John Wiley & Sons, 2011.
2. Lương Duyên Bình (Chủ biên), "Vật lý dại cương" tập 1: Cơ- Nhiệt, NXB Giáo dục Viêt Nam, 2010.

- Lab Material: "*Physics Laboratory Manual* ", handout

Course Information: Evaluation and Grading

- Attendance, Assignments and homeworks: 25%
- Midterm exam: 25% (week 8: chapters 1–6)
- Lab. Reports : 10 %
- Final exam : 40% (week 15)

- Grade A: 90% – 100%
- Grade B: 80% – 89%
- Grade C: 70% – 79%
- Grade D: 60% – 69%
- Grade F: 59% or lower

Course Information: Homework

- Homework problem assignment:

List of Homeworks

Chapter 1: Units, Estimation, Dimensional analysis, Coordinate system, Uncertainty in Measurement, Trigonometry: (1) -39, 43, 55

Chapter 2: Motion in one dimension: (2) - 6, 11, 18, 20, 21, 33, 37, 47, 59

Chapter 3: Vectors and Two-dimensional motion: (3) - 14, 23 , 41, 58

Chapter 4: The laws of Motion (4) -7,15,21, 38,41

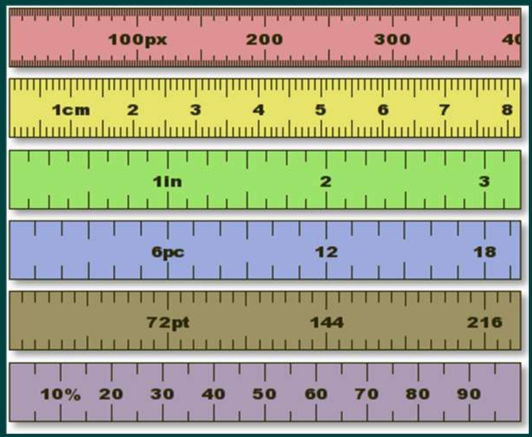
Chapter 7: Rotation Motion and the Law of Gravity: (7) – 3, 4, 14, 15, 25, 27, 34, 63,70

…

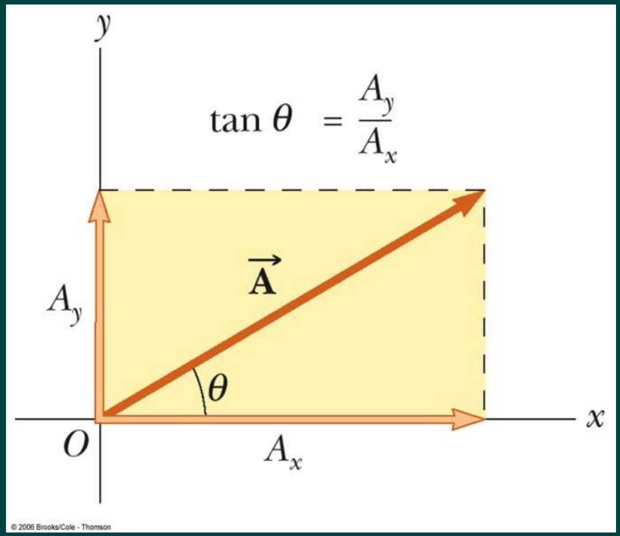
- Due date: The week after finishing each chapter.

Lecture 1

- Brief Introduction to Physics
- Chapter 1 – Measurements
  - 1.1. Quantities
  - 1.2. SI units
  - 1.3. Significant figures
  - 1.4. Measurements
  - 1.5. Micrometer
  - 1.6. Vernier caliper



- Chapter 2 – Motion in one dimension
  - 2.1. Motion
  - 2.2. Position and displacement
  - 2.3. Average velocity and average speed
  - 2.4. Instantaneous velocity and speed
  - 2.5. Acceleration
  - 2.6. Constant acceleration: A special case
  - 2.7. Free fall acceleration



Introduction: Physics and Mechanics

- **Physics** deals with the nature and properties of **matter** and **energy**. Common language is mathematics. Physics is based on experimental **observations** and quantitative **measurements**.
- The study of physics can be divided into six main areas:
  - **Classical mechanics** => Basic Physics (this course)
  - Electromagnetism
  - Optics
  - Relativity
  - **Thermodynamics** => Basic Physics (this course)
  - Quantum mechanics
- **Classical mechanics** deals with the motion and equilibrium of material bodies and the action of forces.

Introduction: Classical Mechanics

- Classical mechanics deals with the motion of objects
- Classical Mechanics: Theory that predicts qualitatively & quantitatively the results of experiments for objects that are NOT…

too small: atoms and subatomic particles – Quantum Mechanics

too fast: objects close to the speed of light – Special Relativity

too dense: black holes, the early Universe – General Relativity

- Classical mechanics concerns the motion of objects that are large relative to atoms and move at speeds much slower than the speed of light (i.e. nearly everything!)




Chapter 1. Physical quantities & Measurement technique

1.1. Quantities

1.2. SI units

1.3. Significant figures

1.4. Measurements

1.5. Micrometer

1.6. Vernier caliper

1.1. Quantities

To be quantitative in Physics requires measurements

How tall is Lionel Messi? How about his weight?

- Height: 1.70 m (5 ft 7 in)
- Weight: 72 kg (159 lbs)

- Number + Unit

Lionel Messi - Wikipedia

"thickness is 10." has no physical meaning.

Both **numbers and units** necessary for any meaningful physical quantities.

1.1. Quantities

A physical quantity is a feature of something that can be measured.

- Examples: length, mass, weight, . . .
- Every quantity has a **numerical value** and a **unit**.
- Large and small quantities are usually expressed in scientific notation – a simple number multiplied by a power of ten.

$0.00000012 = 0.12\times10^{-6} = 1.2\times10^{-7} = 12\times10^{-8}$

$34000000000000000000000000000000000000=34\times10^{36}$

Type Quantities

- Many things can be measured: distance, speed, energy, time, force ......
- These are related to one another: speed = distance / time
- Choose basic quantities (DIMENSIONS):
  - **LENGTH**
  - **MASS**
  - **TIME, ...**
- Define other units in terms of these.

1.1. Quantities: ... scalar and vectors

- Physical quantities are classified into **scalars** and **vectors.**

  A scalar has magnitude only.

  A vector has both magnitude and direction.

♣ Underline the following quantities which are vectors.

speed, velocity, force, pressure, acceleration, mass

speed, <u>velocity</u>, <u>force</u>, pressure, <u>acceleration</u>, mass

- The resultant of two scalars is the sum of their magnitudes.
- To find the resultant of two vectors, we use the parallelogram law or the triangle law.

1.1. Quantities

1.2. SI units

1.3. Significant figures

1.4. Measurements

1.5. Micrometer

1.6. Vernier caliper

1.2. SI Unit

- **There are many unit systems:**

FPS (foot-pound-second), CGS (centimeter, gram, second)

MKS (meter-kilogram-second). The MKS is also called the **SI (international system)**.

- In SI, there are 7 base **quantities** and seven base **units**.

Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric Current	ampere	A
Thermodynamic Temperature	kelvin	K
Luminous Intensity	candela	cd
Amount of Substance	mole	mol

1.2. SI Unit

Prefixes for SI Units (Multiples)

- 3,000 m = 3×1,000 m
  = 3×10^3 m = 3 km
- 1,000,000,000 = 10^9 = 1G
- 1,000,000 = 10^6 = 1M
- 1,000 = 10^3 = 1k

- 141 kg = ? g
- 1 GB = ? Byte = ? MB

10^x	Prefix	Symbol
x=18	exa	E
15	peta	P
12	tera	T
9	giga	G
6	mega	M
3	kilo	k
2	hecto	h
1	deca	da

2. SI Unit

Prefixes for SI Units (submultiples)

10^x	Prefix	Symbol
x=-1	deci	d
-2	centi	c
-3	milli	m
-6	micro	µ
-9	nano	n
-12	pico	p
-15	femto	f
-18	atto	a

- 0.003 s = 3 × 0.001 s
  = 3×10^{-3} s = 3 ms
- 0.01 = 10^{-2} = centi
- 0.001 = 10^{-3} = milli
- 0.000 001 = 10^{-6} = micro
- 0.000 000 001 = 10^{-9} = nano
- 0.000 000 000 001 = 10^{-12}
  = pico = p
- 1 nm = ? m = ? cm
- 3 cm = ? m = ? mm

1.2. SI Unit

Derived Quantities and Units

- For a quantity apart from the base quantities, its unit is a combination of the base units, called a **derived unit**.
- Some derived units have their own names.

For example, the unit of force is newton (**N = kg.m.s⁻²**).

the unit of energy is Joule (**J =kg.m².s⁻²**)

- Quantities which don't have a named unit are expressed in terms of other units.

For example, the unit of **speed** is meter-per-second (**m.s⁻¹**). m/s

the unit of **acceleration** is meter-per-second square (**m.s⁻²**). m/s²

- Different units can be multiplied together or divided by one another. But they can never be added or subtracted.

1.2. SI Unit

Derived Quantities and Units

- Multiply and divide units just like numbers
- Derived quantities: area, speed, volume, density ……
  - Area = Length × Length — SI unit for area = m^2
  - Volume = Length × Length × Length — SI unit for volume = m^3
  - Speed = Length / time — SI unit for speed = m/s
  - Density = Mass / Volume — SI unit for density = kg/m^3 $= kg.m^{-3}$
- In 2008 Olympic Game, Usain Bolt sets world record at 9.69 s in Men's 100 m Final. What is his average speed ?

$$\text{speed} = \frac{100\ \text{m}}{9.69\ \text{s}} = \frac{100}{9.69} \cdot \frac{\text{m}}{\text{s}} = 10.32\ \text{m/s}$$

Other Unit System

- U.S. customary system: foot, slug, second
- CGS system: cm, gram, second
- We will use SI units in this course, but it is useful to know conversions between systems.
  - 1 mile = 1609 m = 1.609 km 1 ft = 0.3048 m = 30.48 cm
  - 1 m = 39.37 in. = 3.281 ft 1 in. = 0.0254 m = 2.54 cm
  - 1 lb = 0.465 kg 1 oz = 28.35 g 1 slug = 14.59 kg
  - 1 day = 24 hours = 24 * 60 minutes = 24 * 60 * 60 seconds

  - More can be found in Appendices A & D in your textbook.

Dimensions, Units and Equations

- Quantities have dimensions:
  - Length – L, Mass – M, and Time - T
- Quantities have units: Length – m, Mass – kg, Time – s
- To refer to the dimension of a quantity, use square brackets, e.g. $[F]$ means dimensions of force.

Quantity	Area	Volume	Speed	Acceleration
Dimension	$[A]$ = L^2	$[V]$ = L^3	$[v]$ = L/T	$[a]$ = L/T^2
SI Units	m^2	m^3	m/s	m/s^2

Dimensional Analysis

- Necessary either to derive a math expression, or equation or to check its correctness.
- Quantities can be added/subtracted only if they have the same dimensions.
- The terms of both sides of an equation must have the same dimensions.

  - a, b, and c have units of meters, $s = a$, what is [s] ?
  - a, b, and c have units of meters, $s = a + b$, what is [s] ?
  - a, b, and c have units of meters, $s = (2a + b)b$, what is [s] ?
  - a, b, and c have units of meters, $s = (a + b)^3/c$, what is [s] ?
  - a, b, and c have units of meters, $s = (3a + 4b)^{1/2}/9c^2$, what is [s] ?

1.3. Significant figures

Examples: 0012304.005

12.300

120300

♣ **The five rules**

1. All non-zero digits are significant.

2. Zeros between two non-zero digits are significant.

3. Leading zeros are not significant.

4. Trailing zeros in a number containing a decimal point are significant.

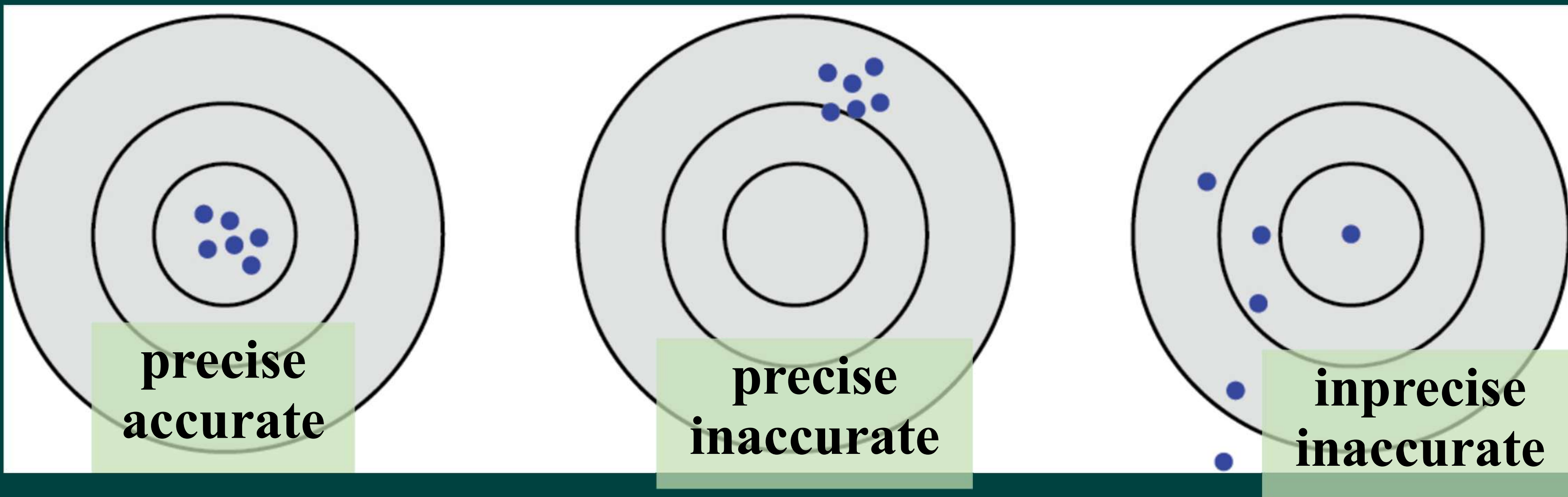
5. The significance of trailing zeros in a number not containing a decimal point needs to be indicated when it is necessary.

For examples: 120300, $120\bar{3}00$, 120300 (4sf).

1.4. Measurements

1.4.1. Precision and accuracy

A measurement which is precise may be not accurate.



Accuracy refers to the closeness of the measured value to the "true value".

Precision refers to the closeness of different measurements to each other.

1.4. Measurements

- The **result** of a physical measurement has two components:

  the **average value** (the best estimate possible value),

  the **uncertainty** (a range of values within which you expect the true value to lie).

- For example, when you measure the width L of a table:

i	L_i/cm	$\Delta L_i/\text{cm}$
1	95.8	0.42
2	95.5	0.12
3	95.6	0.22
4	94.8	0.58
5	95.2	0.18

The average:

$$\langle L \rangle = 95.38\ cm$$

$$\Delta L_i = |\langle L \rangle - Li\,|$$

$$\langle \Delta L \rangle = 0.304\ \text{cm}$$

$$L = (95.38 \pm 0.30)\ \text{cm}$$

1.4. Measurements

1.4.2. Two rules in writing the results

- Take **no more than 2** significant figures for the uncertainty.
- Take the same position for the least significant figure in the average value and the uncertainty

1.4.3. Percentage uncertainty

$$\textbf{\textit{Percentage uncertainty}} = \frac{\text{actual uncertainty}}{\text{average value}}$$

- For example:

$$\delta_L = \frac{\Delta L}{\langle L \rangle} = \frac{0.30\cancel{4}}{95.38} = 0.0032 = 0.32\%$$

1.4.4. Combining uncertainties in indirect measurements

- For quantities which are added or subtracted, we add the actual uncertainties.

$$A = B + 2C - 3D$$

$$\rightarrow dA = dB + 2dC - 3dD$$

$$\rightarrow \Delta A = \Delta B + 2\Delta C + 3\Delta D$$

- For quantities which are multiplied together or divided by one another, we add the percentage uncertainties.

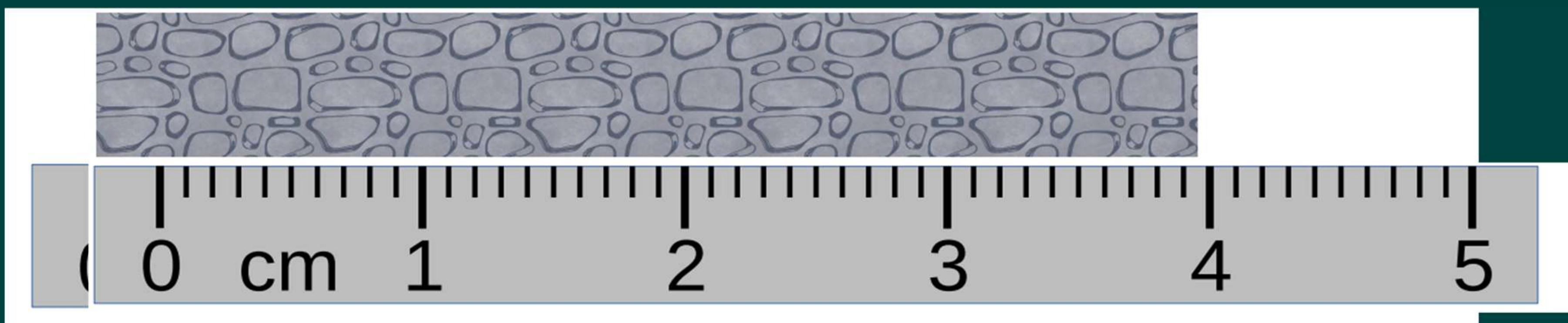
$$A = \frac{BC^2}{D^3} \rightarrow lnA = lnB + 2lnC - 3lnD$$

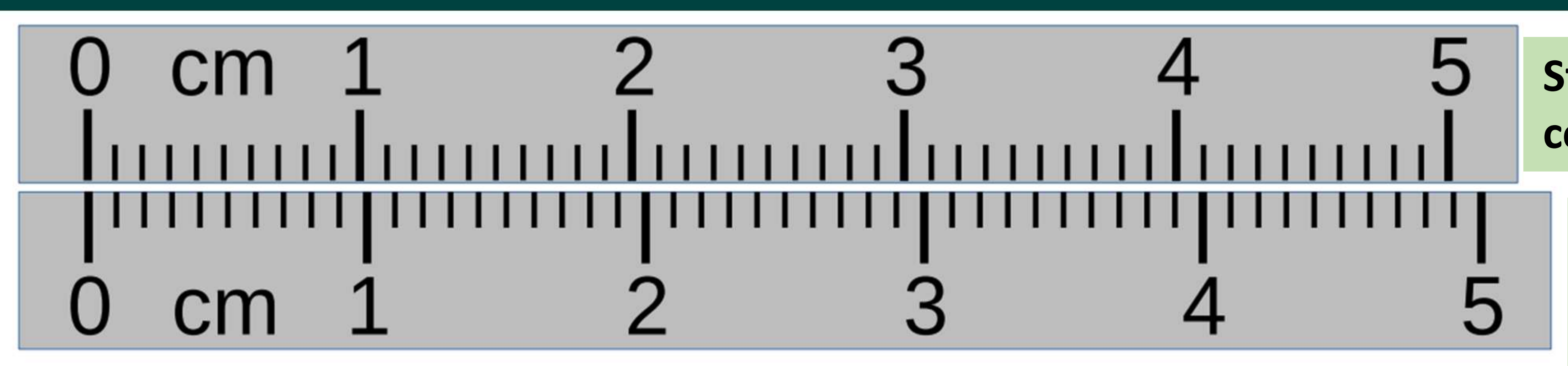
$$\frac{\mathrm{d}A}{A} = \frac{\mathrm{d}B}{B} + 2\frac{\mathrm{d}C}{C} - 3\frac{\mathrm{d}D}{D} \quad \rightarrow \quad \boxed{\frac{\Delta A}{A} = \frac{\Delta B}{B} + 2\frac{\Delta C}{C} + 3\frac{\Delta D}{D}}$$

1.4.5. Two types of error

- **A systematic error** results in readings which are ***always above*** or ***always below*** the true value. For examples, **zero error** and **incorrectly calibrated scale**.

zero error



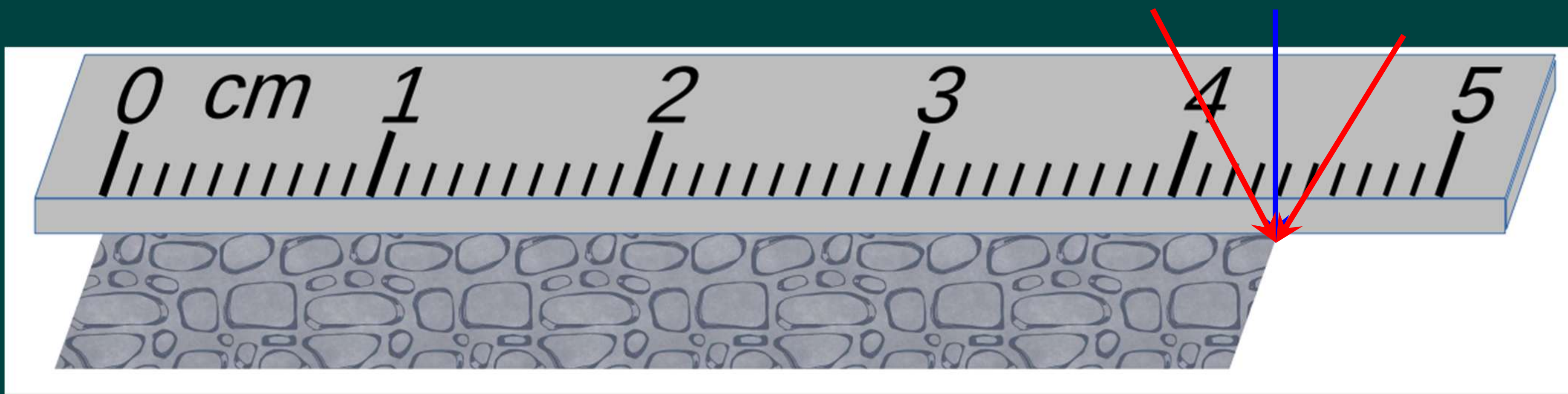


Standard/ corrected scale

Incorrectly calibrated scale

1.4.5. Two types of error

- **A systematic error** results in readings which are **always above** or **always below** the true value. For examples, **zero error** and **incorrectly calibrated scale**.
- **A random error** results in readings which are scattered about the true values. An example is parallax error, which is caused by incorrect viewing angle.



1.1. Quantities

1.2. SI units

1.3. Significant figures

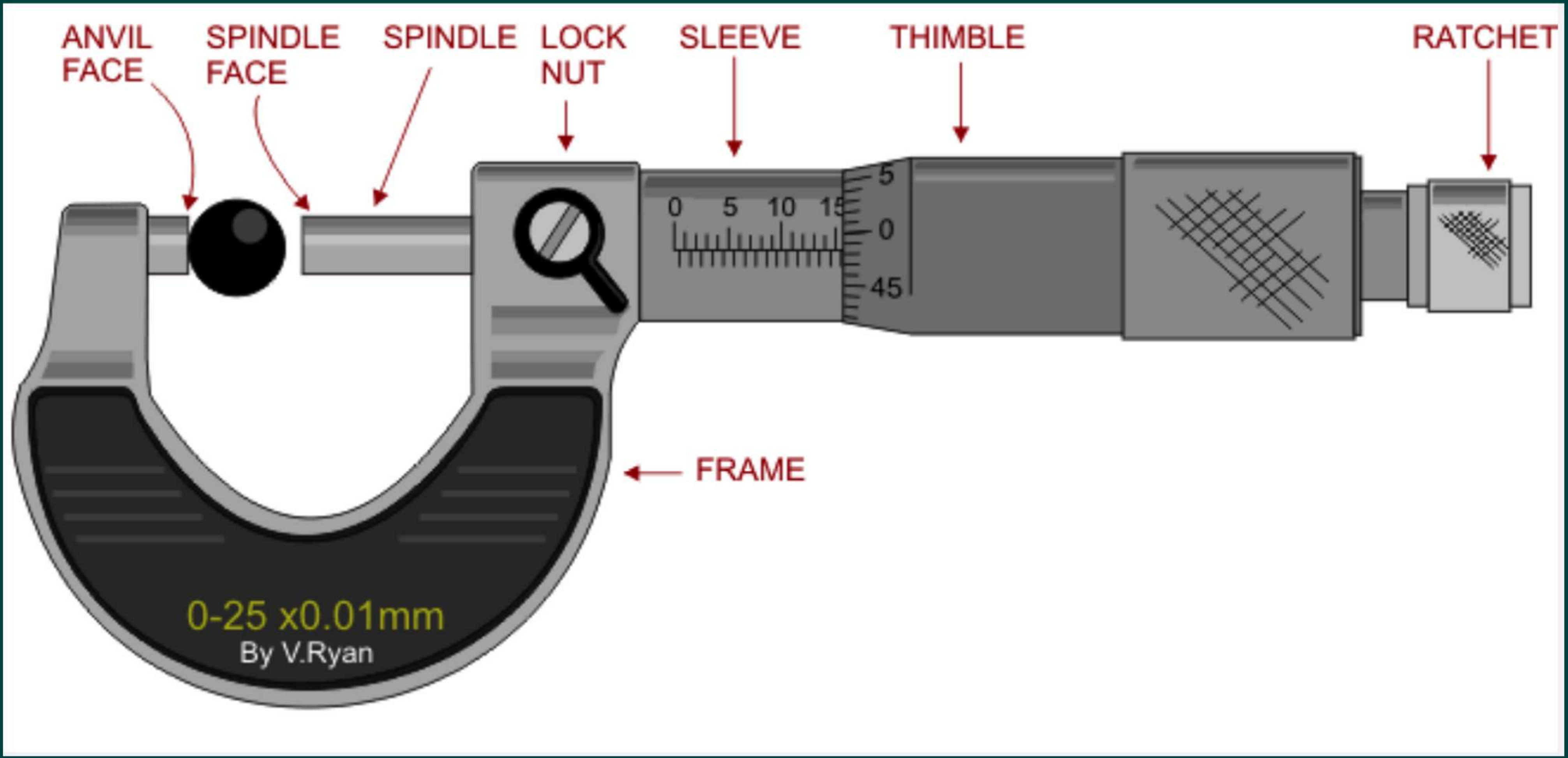
1.4. Measurements

1.5. Micrometer

1.6. Vernier caliper

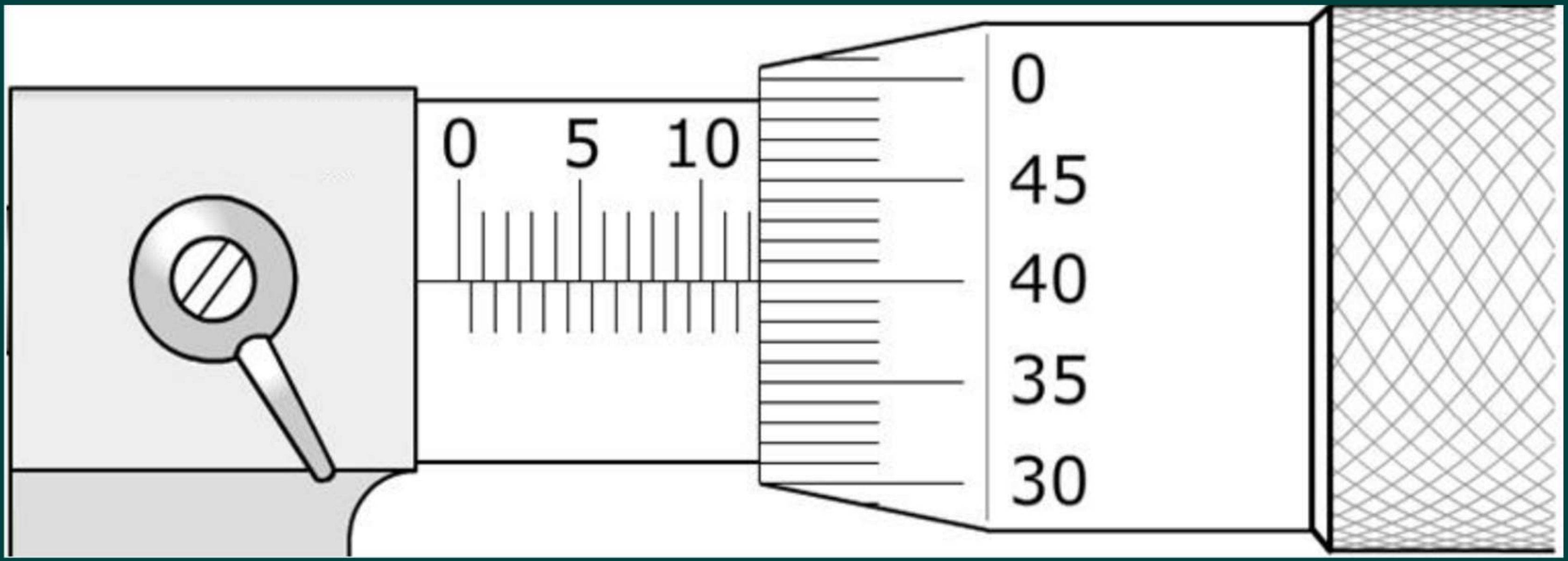
1.5. Micrometer

- **Full name: micrometer screw gauge**



1.5. Micrometer

- **Reading**

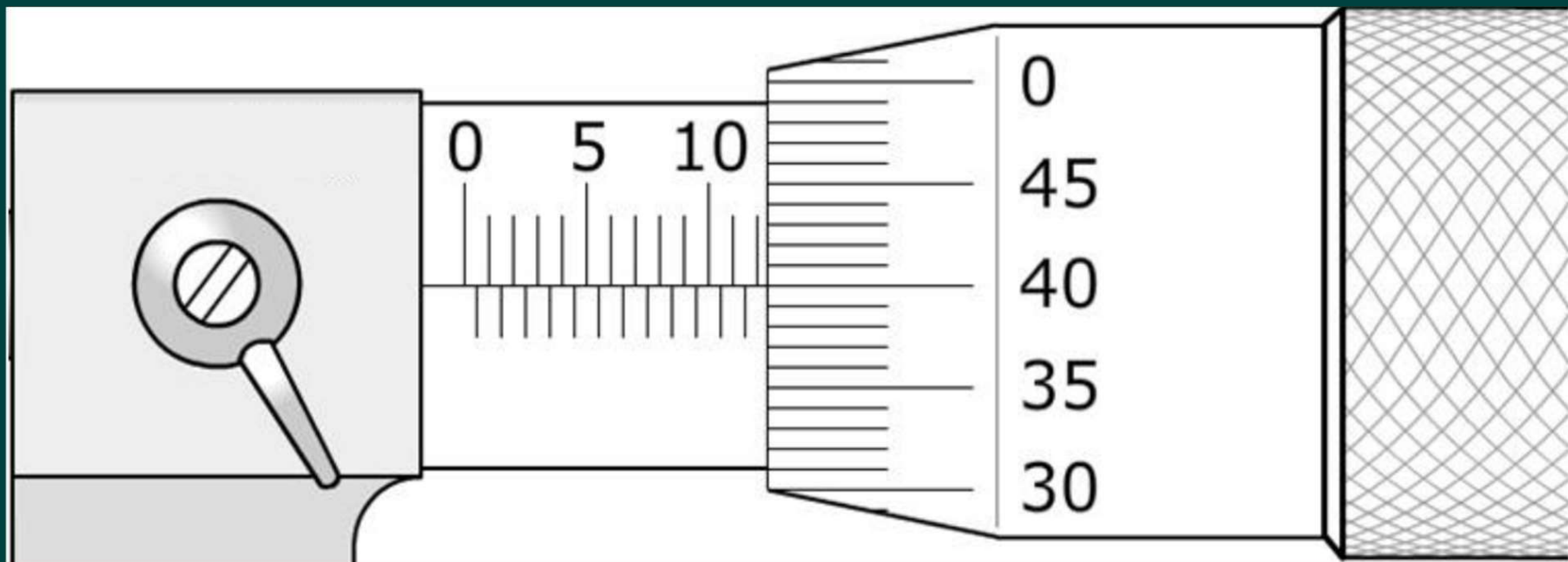


- The smallest division on the barrel scale is 0.50 mm, and on the thimble scale is 0.01 mm.
- The reading on the micrometer in the figure is

12.40 mm

1.5. Micrometer

- **Reading**



- The reading on the micrometer in the figure is 12.40 mm.

1.5. Micrometer

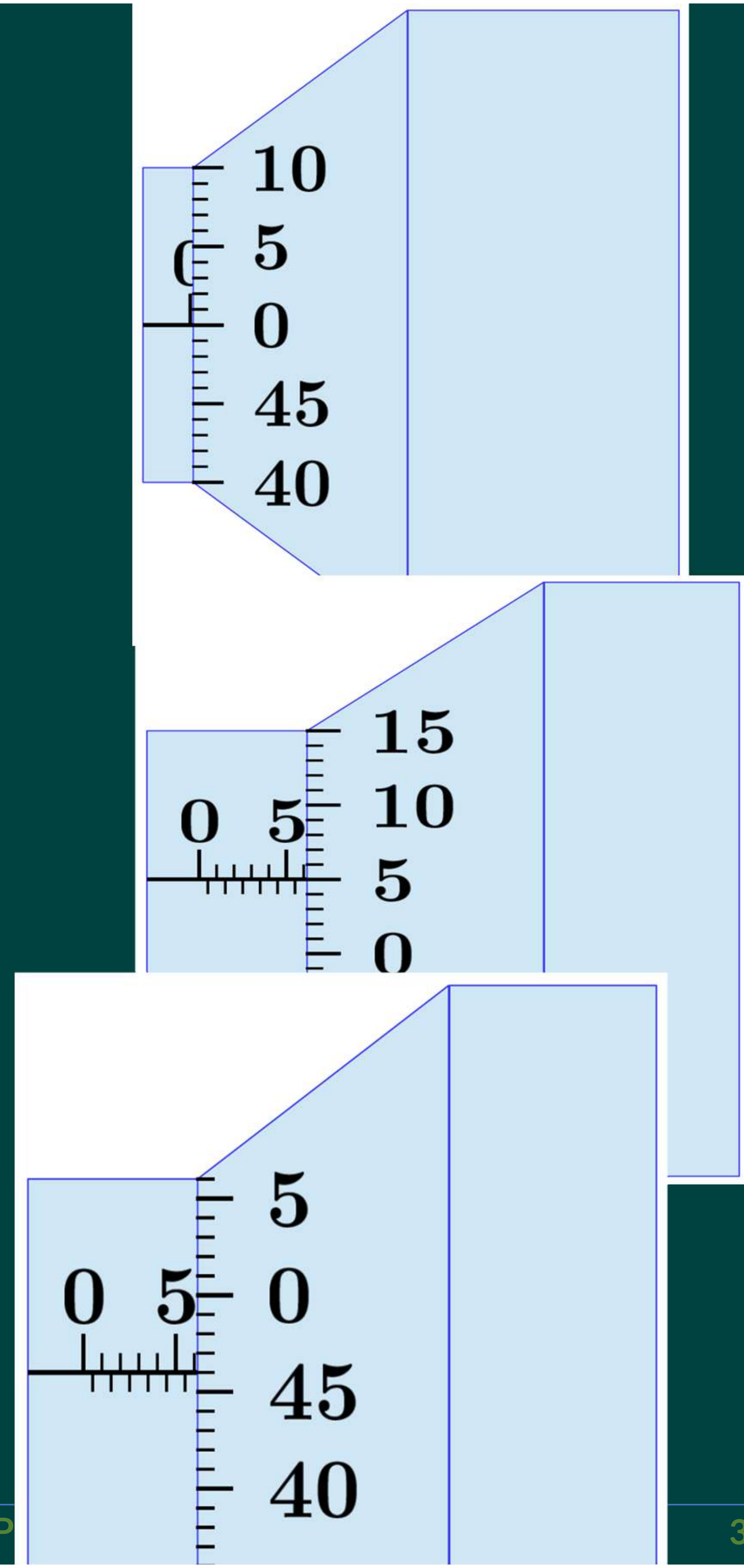
- **Reading**

- Even when the reading is zero, we somehow see the zero mark on the barrel.

- State the reading:

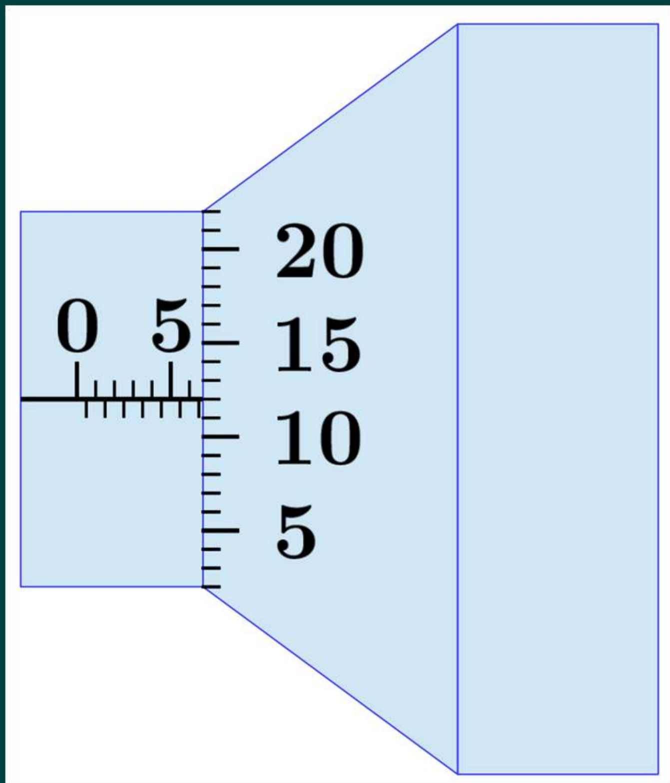
  6.05 mm.

  5.96 mm.

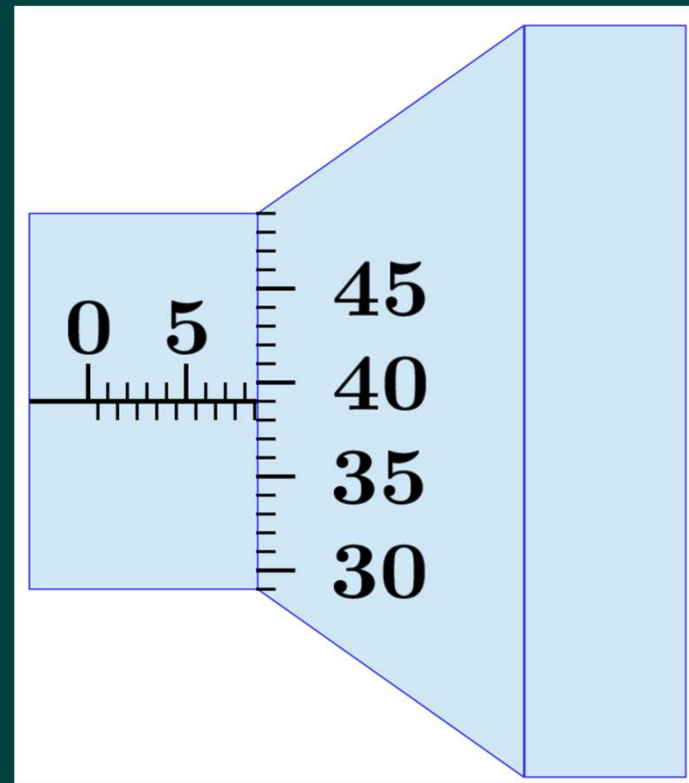


1.5. Micrometer

- Reading



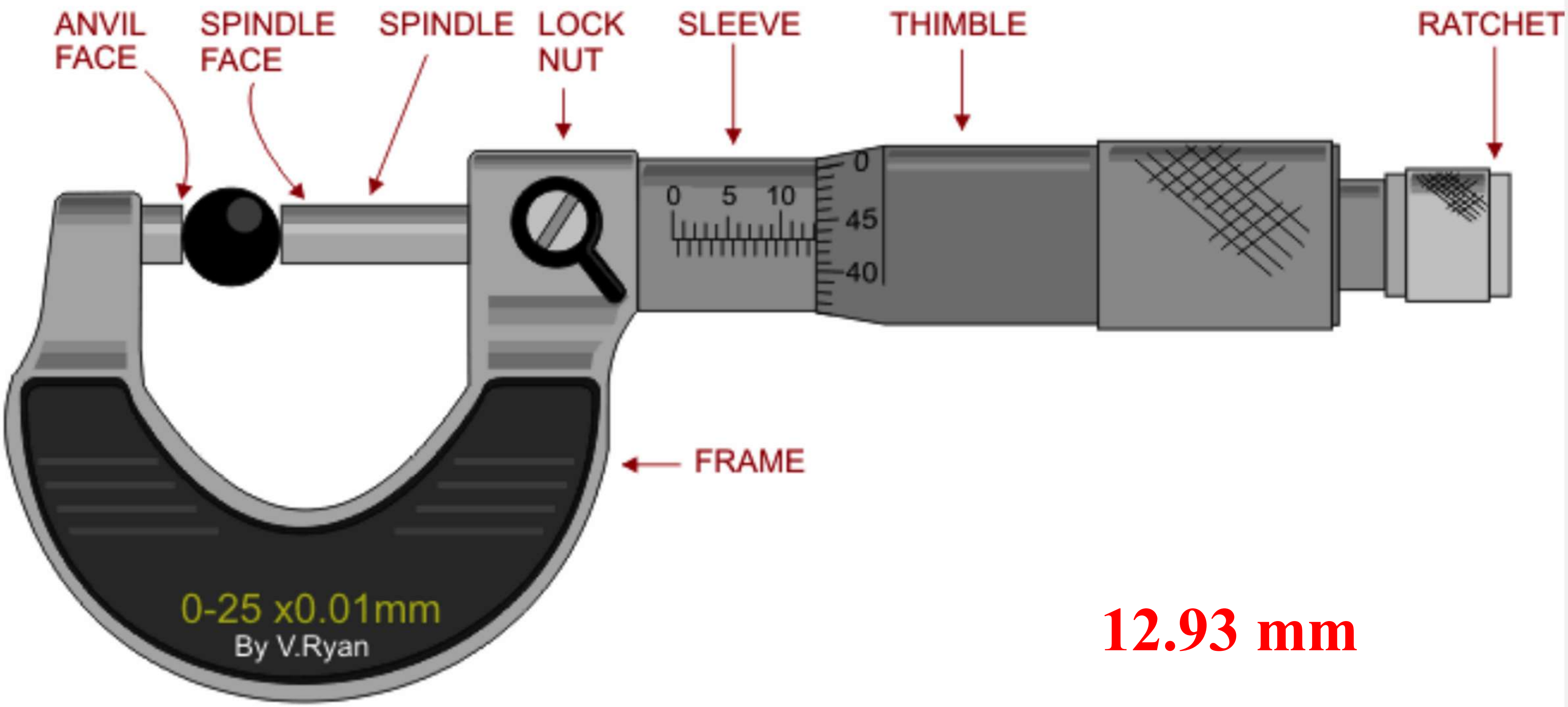
6.62 mm



8.39 mm

1.5. Micrometer

- **What is the reading in the following micrometer?**



12.93 mm

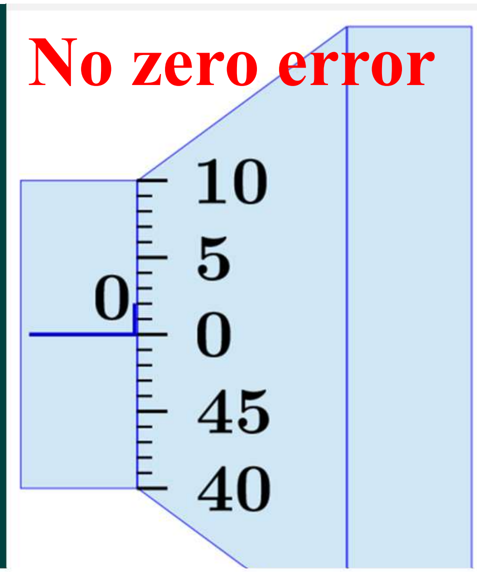
1.5. Micrometer

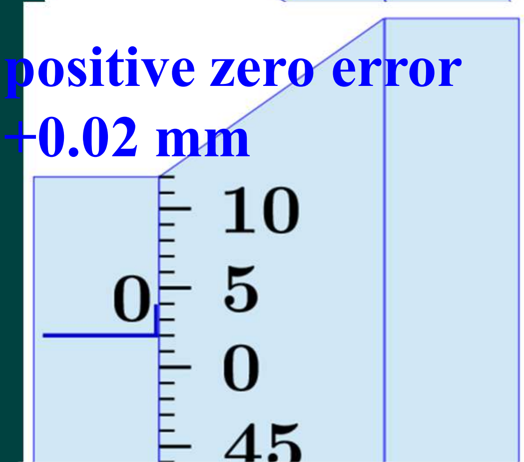
1.5. 4. Determine zero error

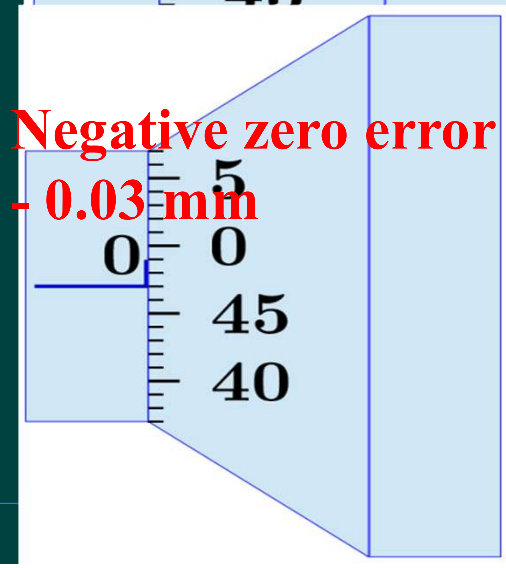
when the two measuring faces are in contact.

- If the zero mark on the thimble lines up with the index line on the barrel, there is **no zero error**.
- The zero mark before the index line means a **positive zero error**.
- The zero mark after the index line means a **negative zero error**.

actual size = reading - zero error



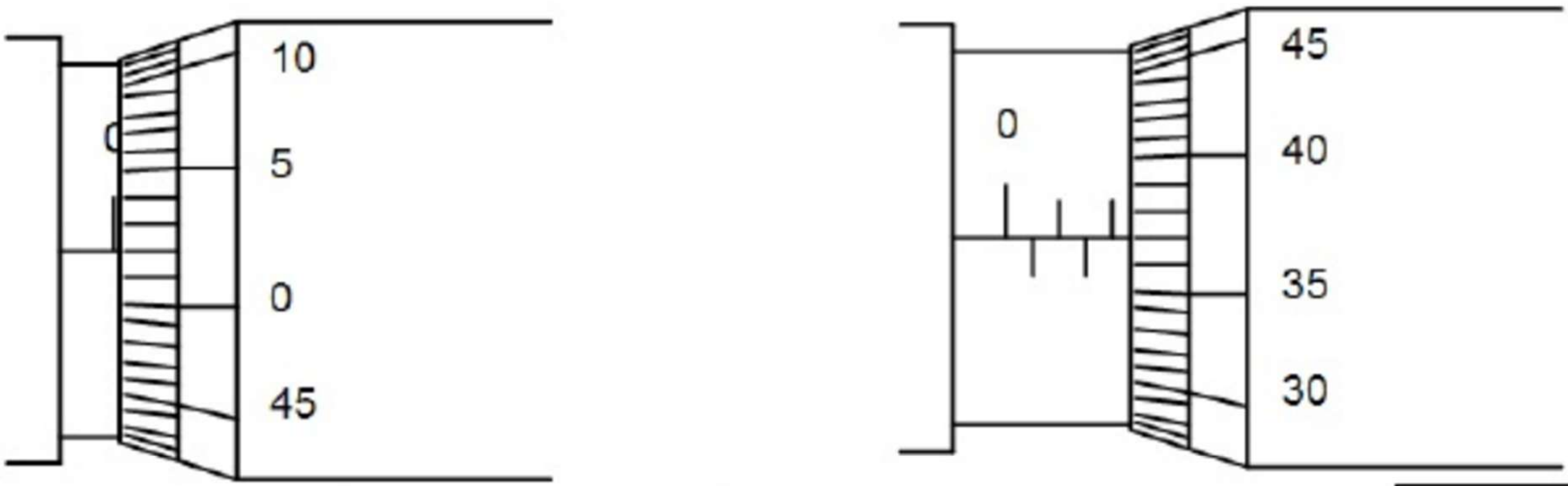




1.5. Micrometer

An example

State the reading of the micrometer screw gauge in the figure below.



Zero error = **+0.02 mm**

Observed reading = **2.37 mm**

Therefore, corrected reading = **2.37 – 0.02 = 2.35 mm**

1.1. Quantities

1.2. SI units

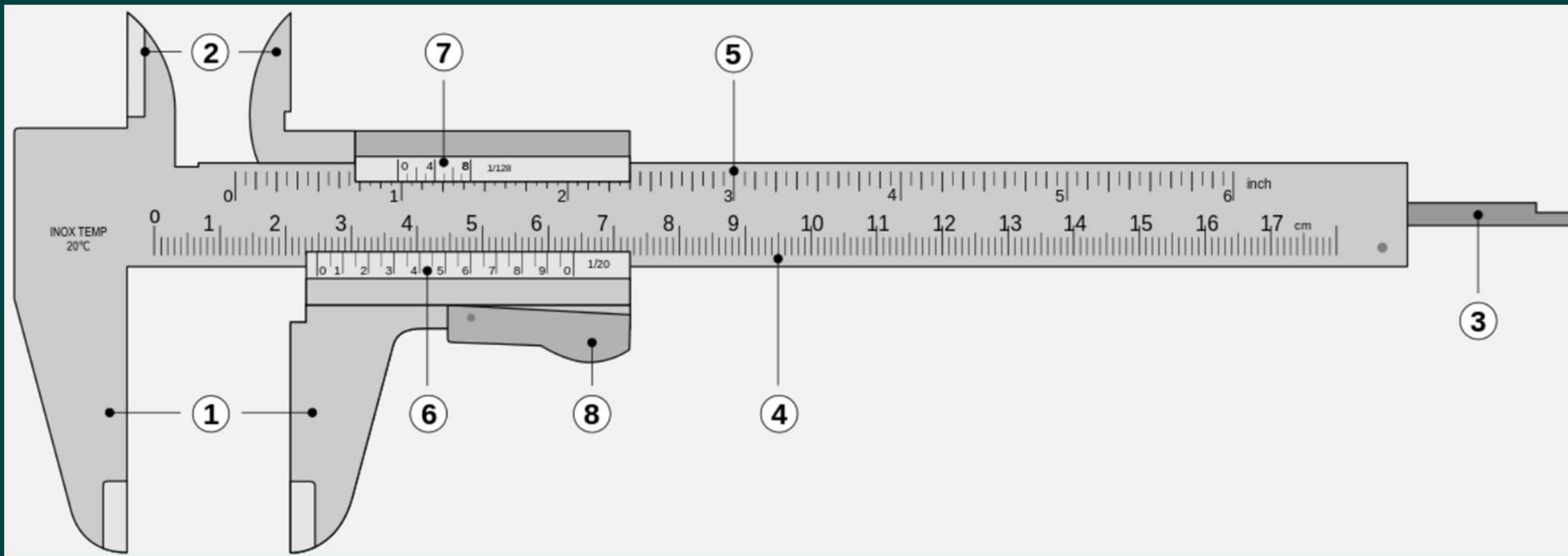
1.3. Significant figures

1.4. Measurements

1.5. Micrometer

1.6. Vernier caliper

1.6. Vernier caliper



1 Outside large jaws (prongs)

2 Inside small jaws

3 Depth rod (probe)

4 Metric main scale

5 Imperial main scale

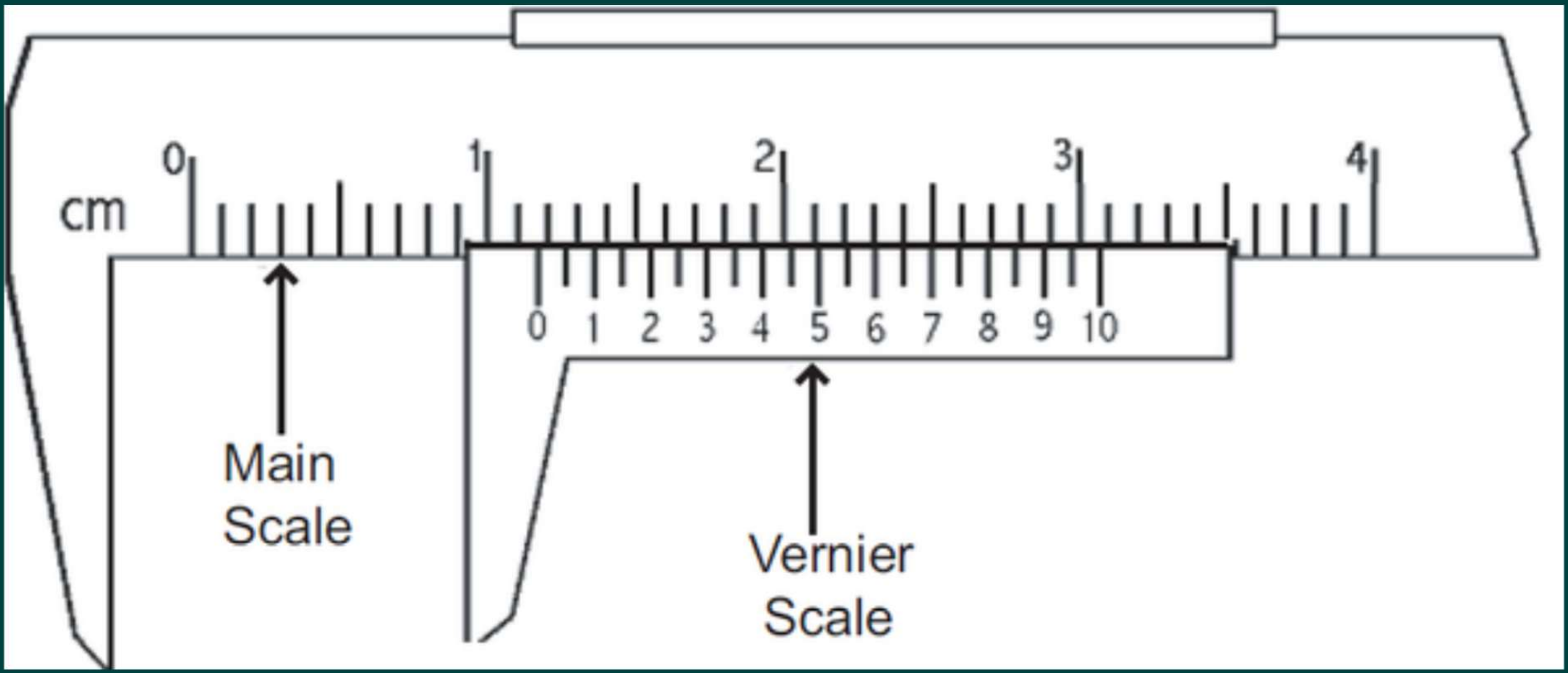
6 Metric vernier scale

7 Imperial vernier scale

8 Retainer

1.6. Vernier caliper

1.6.1. Reading



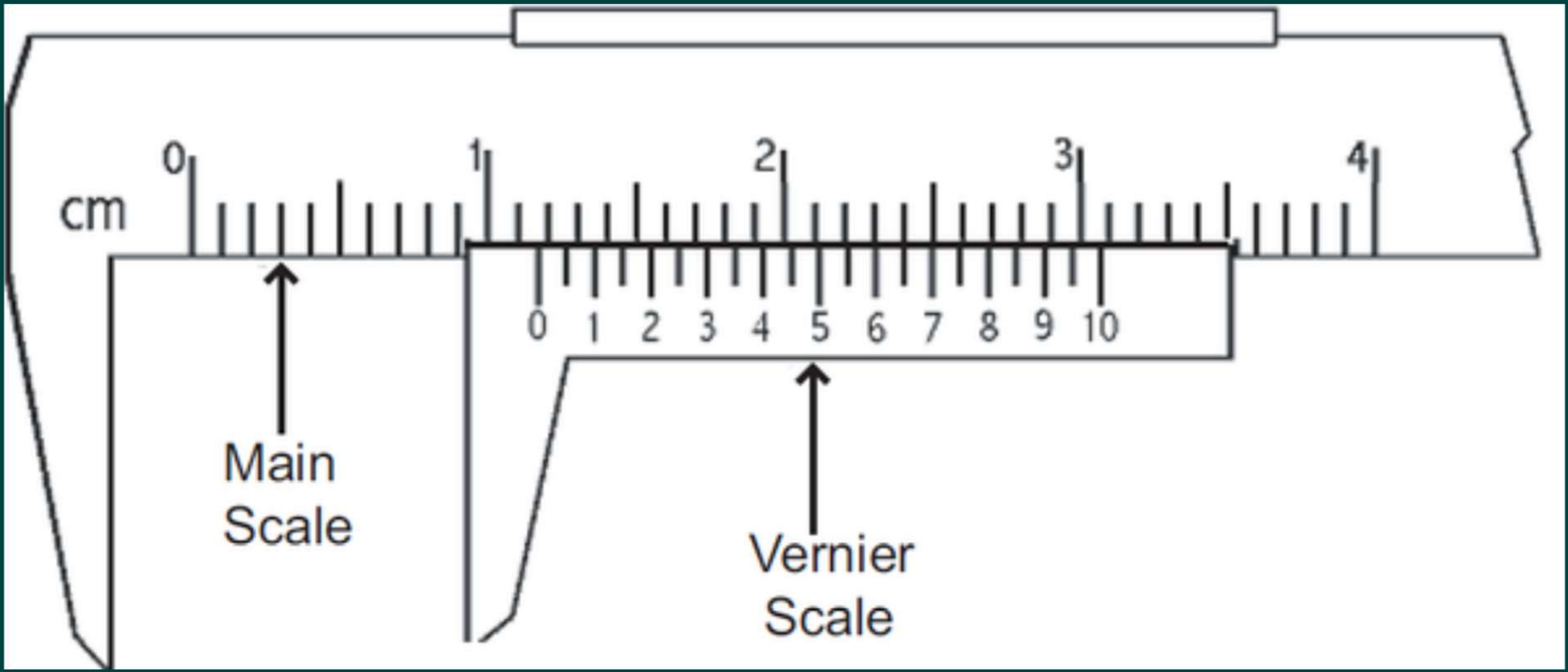
reading = on main scale + on vernier scale

= 11 mm + 0.65 mm

= 11.65 mm

1.6. Vernier caliper

Reading



reading = on main scale + on vernier scale

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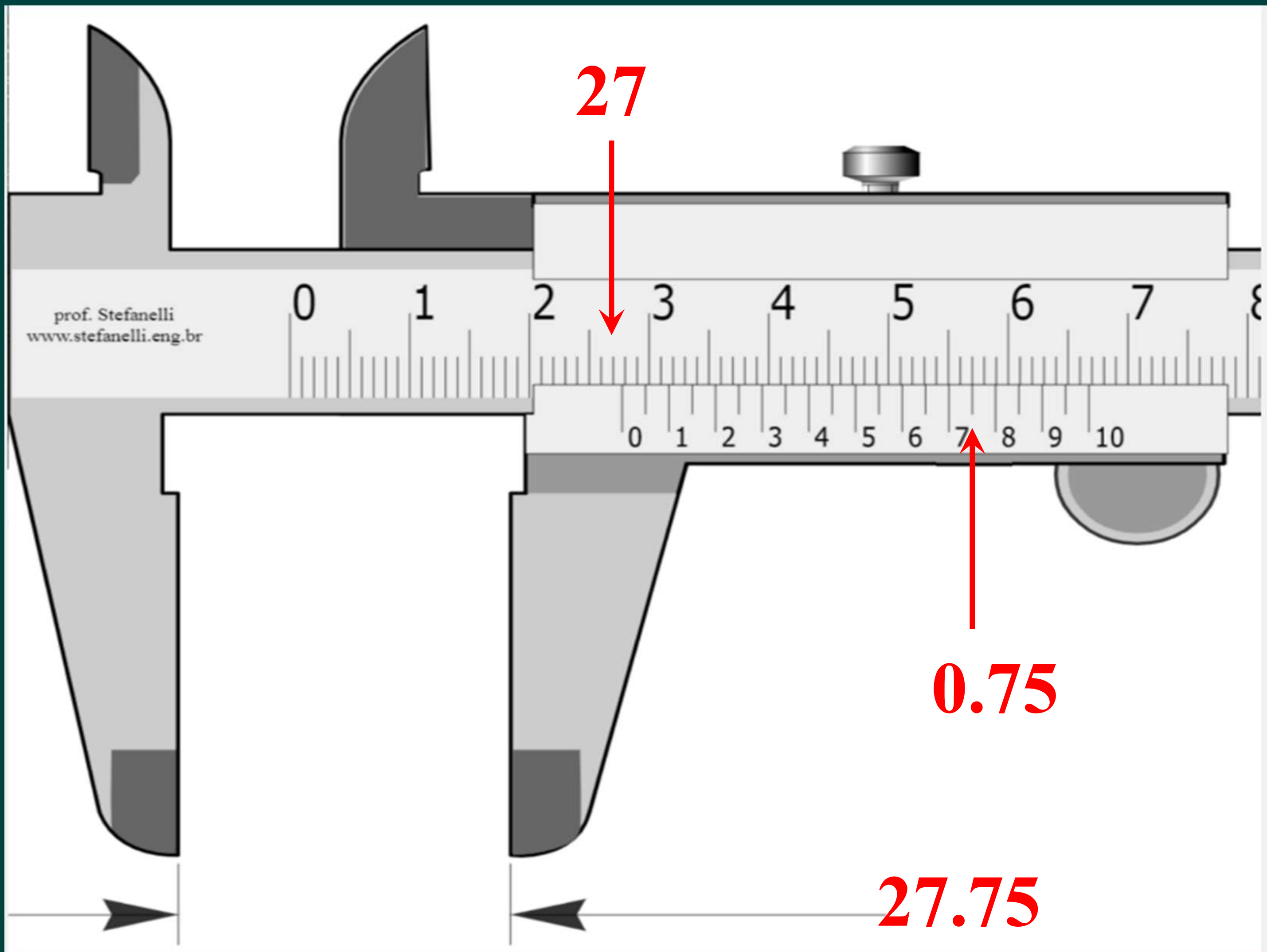
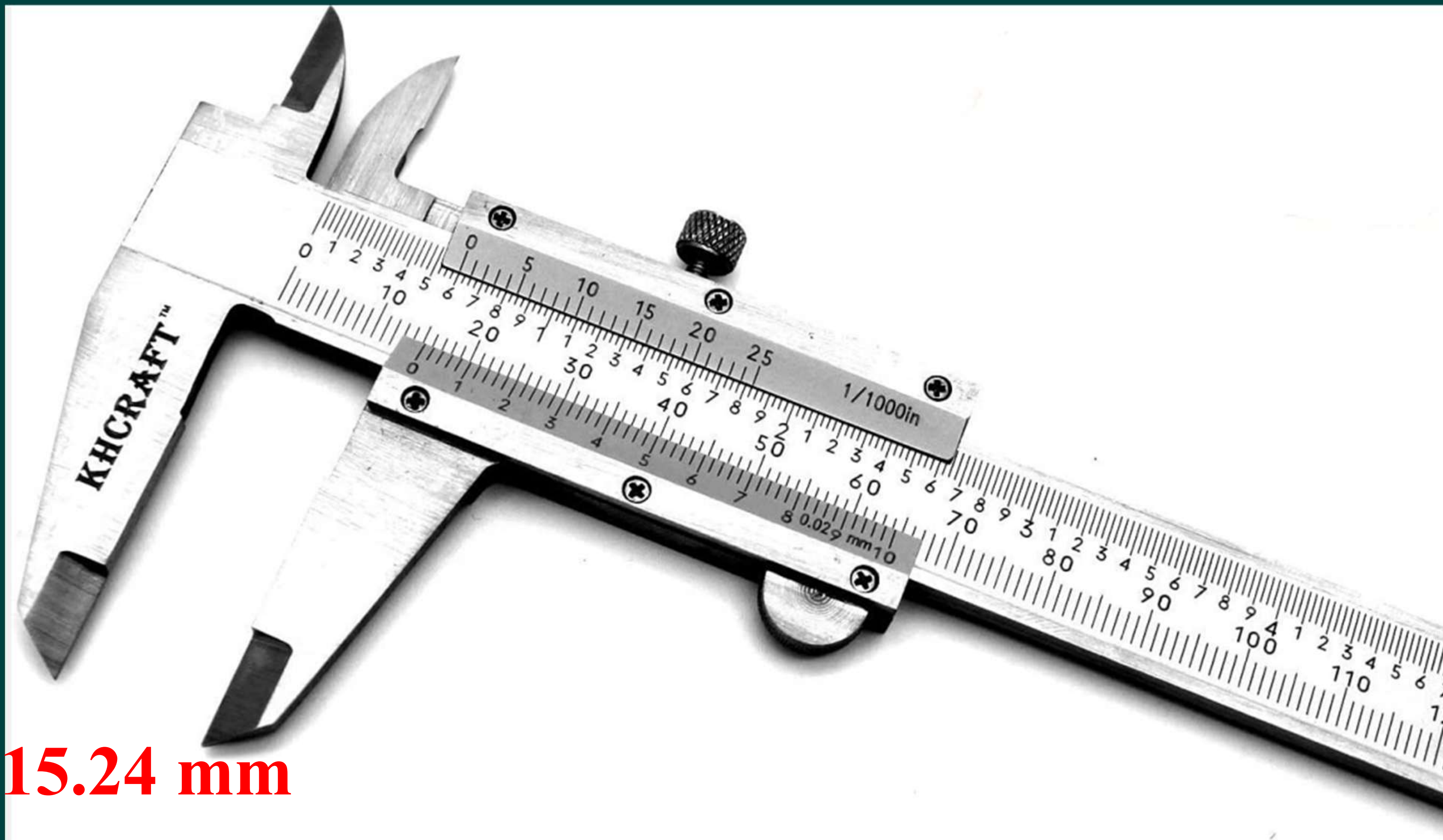
1.6. Vernier caliper

Reading



1.6. Vernier caliper

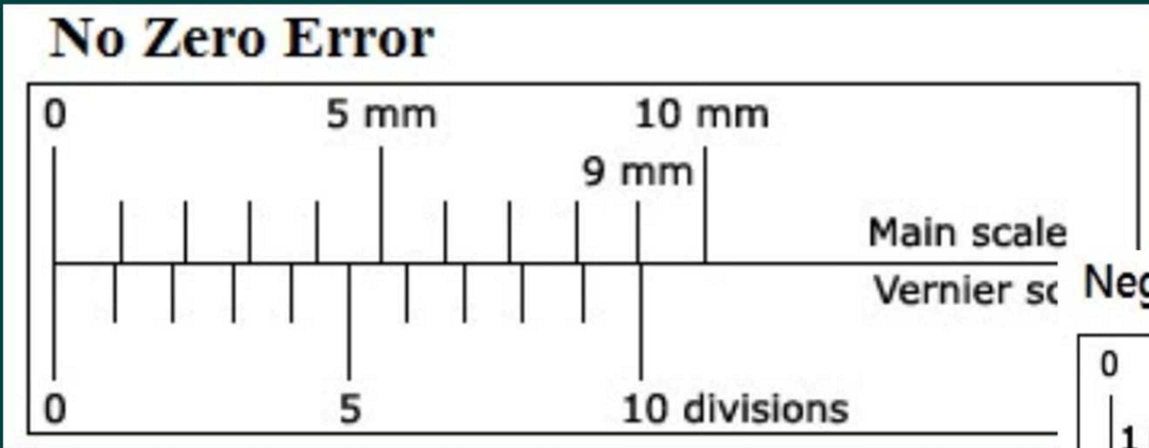
Reading

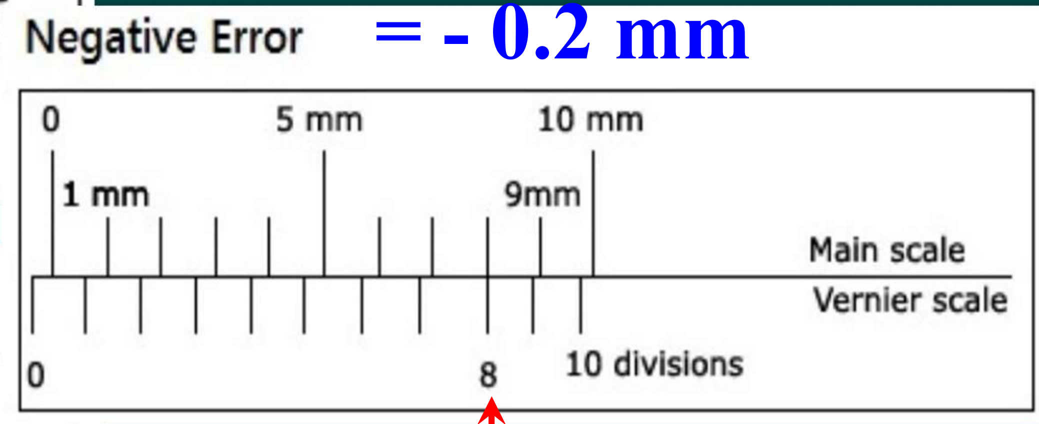


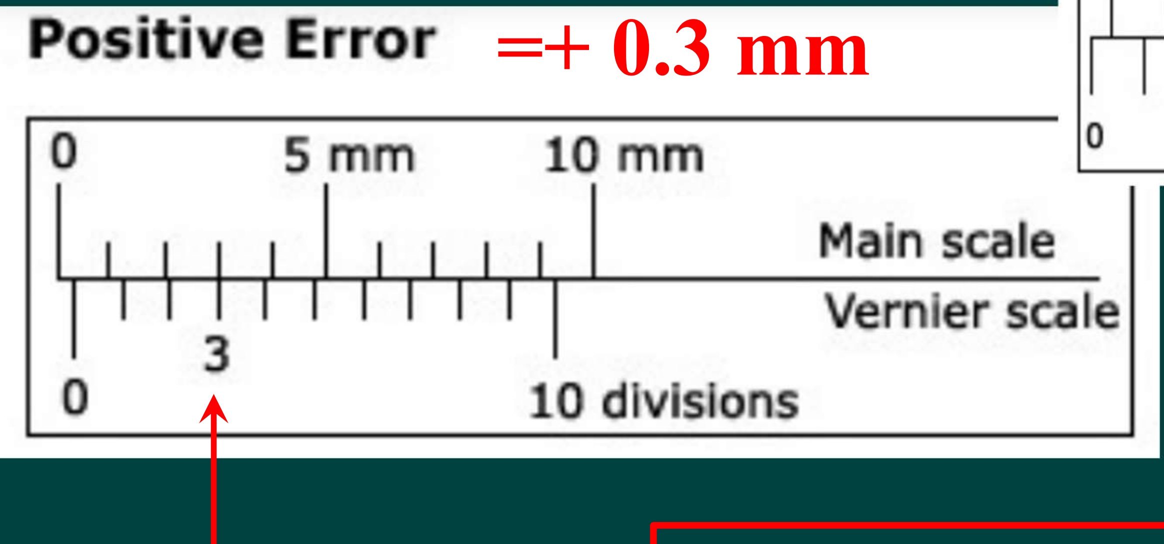
1.6. Vernier caliper

1.6.2. Zero error

- Consider when the two outside jaws are in contact







actual size = reading - zero error

Chapter 1. => Summary

- The three fundamental physical dimensions of mechanics are length, mass and time, which in the SI system have the units meter (m), kilogram (kg), and second (s), respectively
- The method of dimensional analysis is very powerful in solving physics problems.
- Units in physics equations must always be consistent. Converting units is a matter of multiplying the given quantity by a fraction, with one unit in the numerator and its equivalent in the other units in the denominator, arranged so the unwanted units in the given quantity are cancelled out in favor of the desired units.

Chapter 2.
Motion in one dimension

(Motion along a straight line)

Linear motion (displacement, velocity, and acceleration) in one dimension

ONE LOVE. ONE FUTURE.

Nguyễn Hoàng Thoan - FEP

ONE LOVE. ONE FUTURE.

Chapter 2. Motion along a straight line

2.1. Motion

2.2. Position and displacement

2.3. Velocity and speed

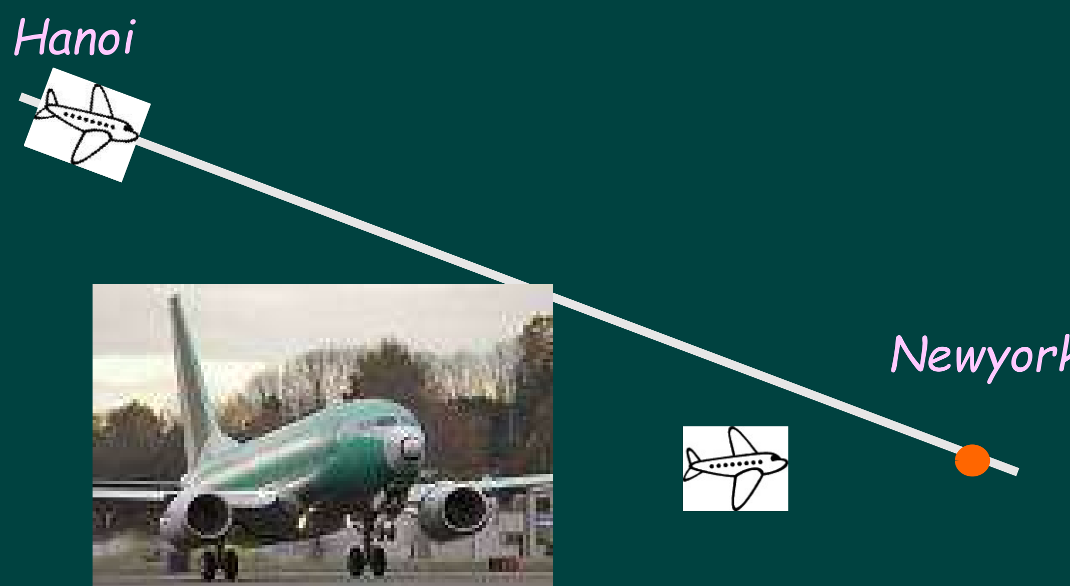
2.4. Acceleration

2.5. Constant acceleration: A special case

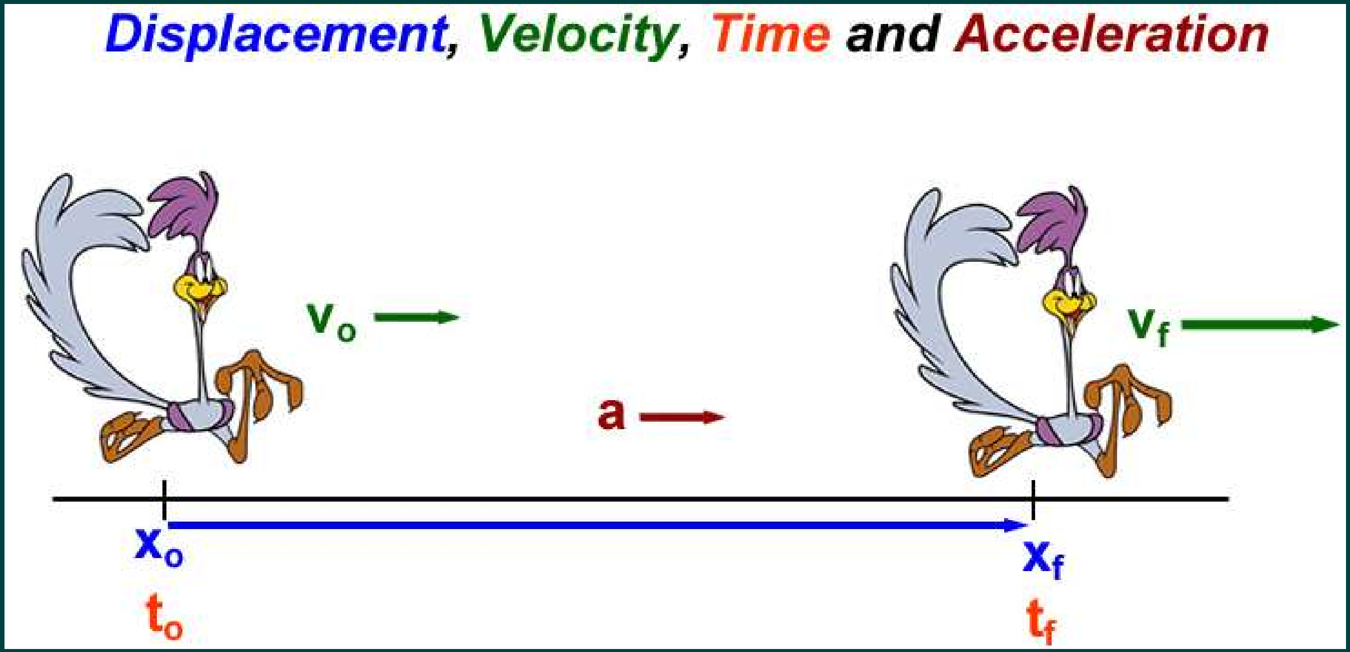
2.6. Free fall acceleration

2.1. Motion

- Everything moves! Motion is one of the main topics in Physics I
- In the spirit of taking things apart for study, then putting them back together, we will first consider only **motion along a straight line**.
- Simplification: Consider a moving object as **a particle**, i.e. it moves like a particle —a "**point object**"



Four Basic Quantities in Kinematics



- Any motion involves three concepts

    Displacement

    Velocity

    Acceleration

- These concepts can be used to study objects in motion.

Chapter 2. Motion along a straight line

2.2. One Dimensional Position x, …..

❖ Position

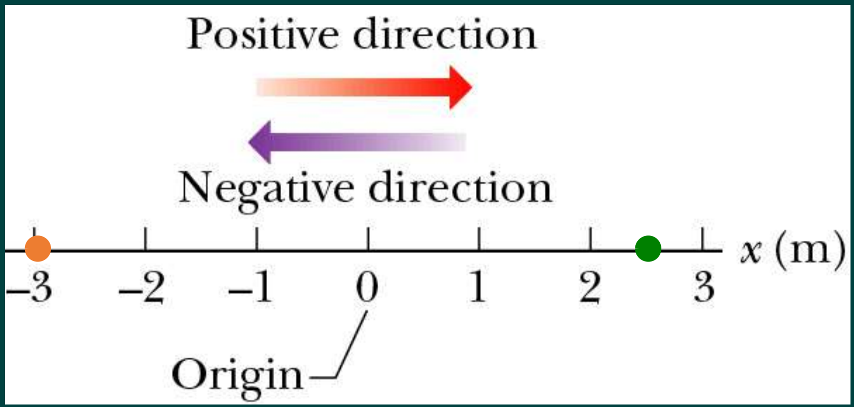
- **Motion can be defined as the change of position over time.**
- How can we represent position along a straight line?
- Position definition:

Defines a starting point: origin (x = 0), x relative to origin

Direction: positive (normally: right or up), negative (normally: left or down)

It depends on time: t = 0 (start clock), x(t=0) does not have to be zero.

- Position has units of [Length]: meters.

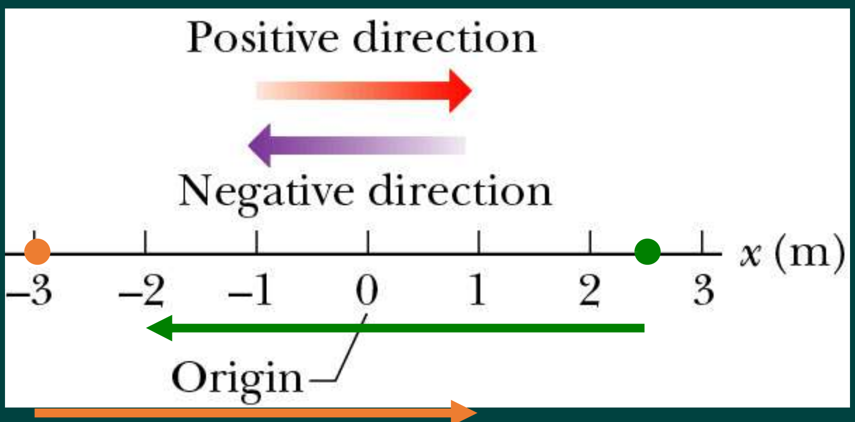


x = + 2.5 m

x = - 3 m

2.2. ….. and Displacement

- Displacement is a change of position in time.
- Displacement: $\Delta x = x_f(t_f) - x_i(t_i)$
  - f stands for final and i stands for initial.
- It is a vector quantity.
- It has both magnitude and direction: + or – sign
- It has unit of [length]: meters.



$x_1(t_1) = +2.5$ m
$x_2(t_2) = -2.0$ m
$\Delta x = -2.0$ m $- 2.5$ m $= -4.5$ m

$x_1(t_1) = -3.0$ m
$x_2(t_2) = +1.0$ m
$\Delta x = +1.0$ m $+ 3.0$ m $= +4.0$ m

Distance and Position-time graph

- Displacement in space

  From A to B: $\Delta x = x_B - x_A = 52\ m - 30\ m = 22\ m$

  From A to C: $\Delta x = x_c - x_A = 38\ m - 30\ m = 8\ m$

- **Distance** is the length of a path followed by a particle

  from A to B: $d = |x_B - x_A| = |52\ m - 30\ m| = 22\ m$

  from A to C: $d = |x_B - x_A| + |x_C - x_B| = 22\ m + |38\ m - 52\ m| = 36\ m$

- **Displacement is not Distance.**

B
C
- A
D
E
F

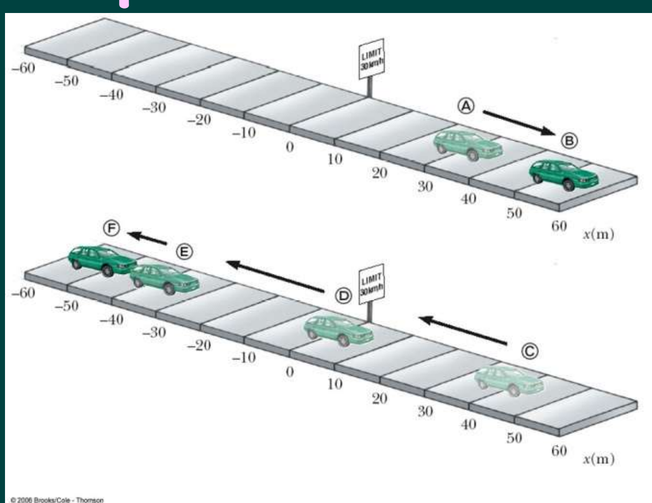
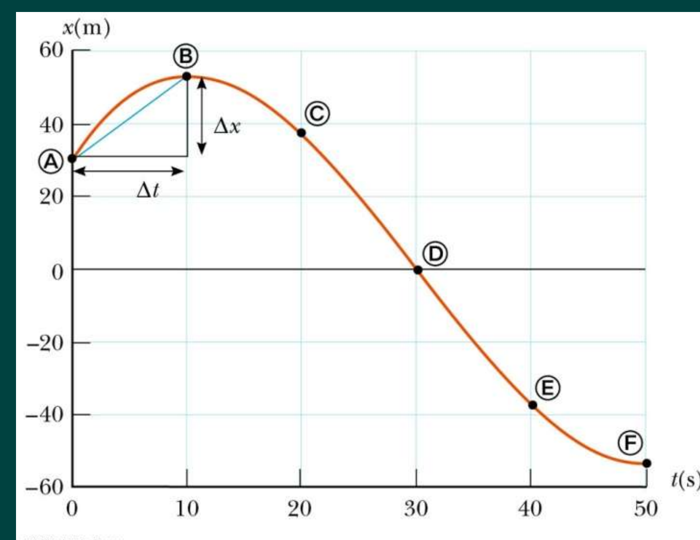


TABLE 2.1

Position of the Car at Various Times

Position	t (s)	x (m)
Ⓐ	0	30
Ⓑ	10	52
Ⓒ	20	38
Ⓓ	30	0
Ⓔ	40	−37
Ⓕ	50	−53



2.3. Velocity and speed

- Velocity is the rate of change of position.
- Velocity is a vector quantity.
- Velocity has both magnitude and direction.
- Velocity has a unit of [length/time]: meter/second.
- We will be concerned with four quantities, defined as:

displacement

Average velocity

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

distance

Average speed

$$s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}$$

Instantaneous velocity

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

displacement

Instantaneous speed

$$s = \lim_{\Delta t \to 0} \frac{\text{distance}}{\Delta t}$$

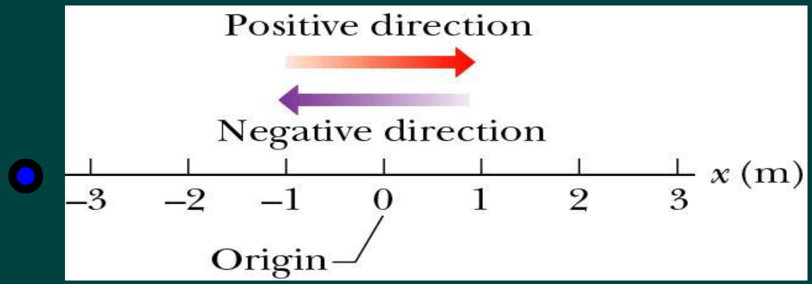
2.3.1. Average Velocity

- Average velocity

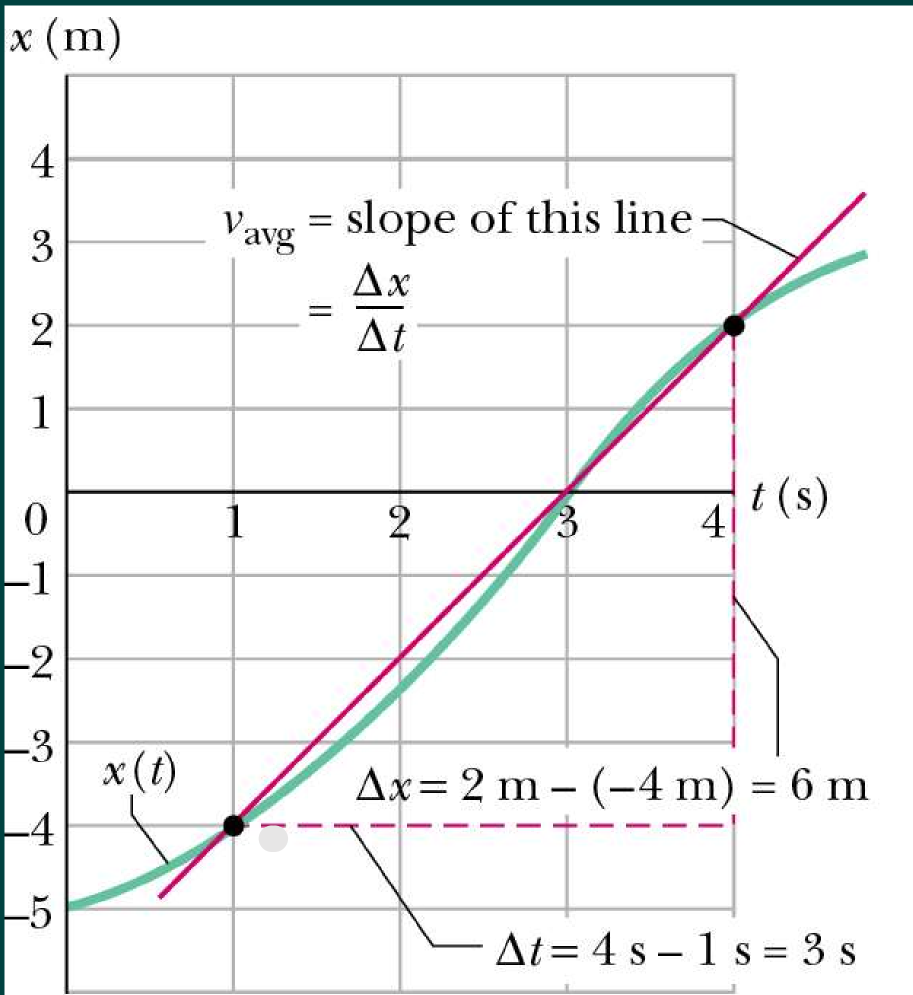
$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

is the slope of the line segment between end points on a graph.

- Dimensions: length/time (L/T) [m/s].
- SI unit: m/s.
- It is a vector (i.e. is signed), and displacement direction sets its sign.



2.3.2. Average Speed



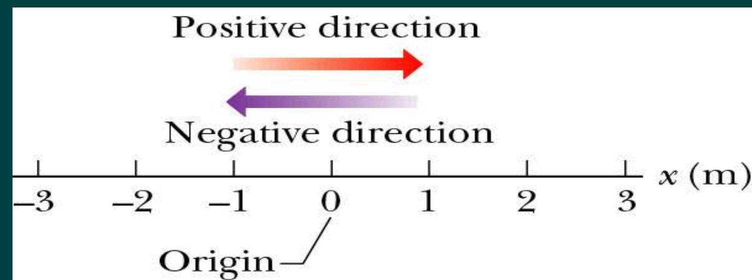
- **Average speed:**

$$s_{avg} = \frac{\text{total distance}}{\Delta t}$$

- Dimension: length/time, [m/s].
- Scalar: No direction involved.
- Not necessarily close to V_{avg}:

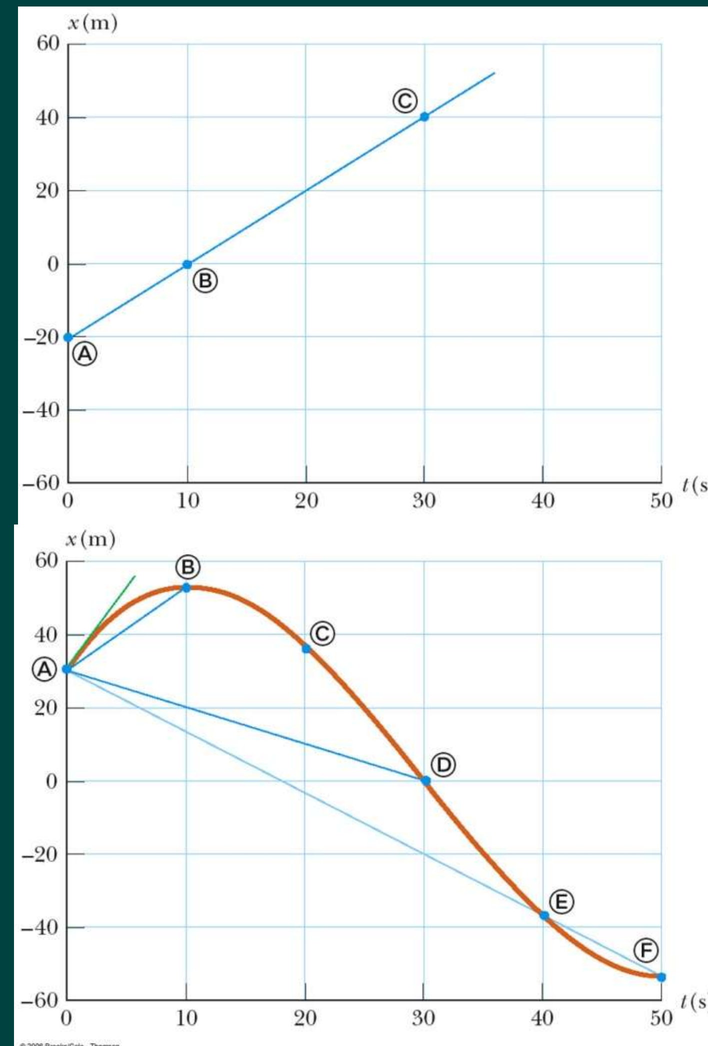
For example:

- S_{avg} = (6m + 6m)/(3s+3s) = 2 m/s
- V_{avg} = (0 m)/(3s+3s) = 0 m/s



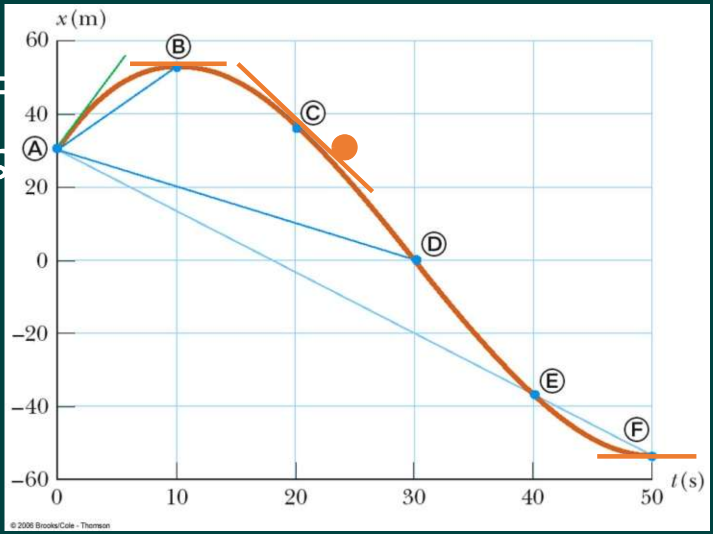
Graphical Interpretation of Velocity

- Velocity can be determined from a position-time graph
- Average velocity equals the slope of the line joining the initial and final positions. It is a vector quantity.
- An object moving with a constant velocity will have a graph that is a straight line.



2.3.2 Instantaneous Velocity

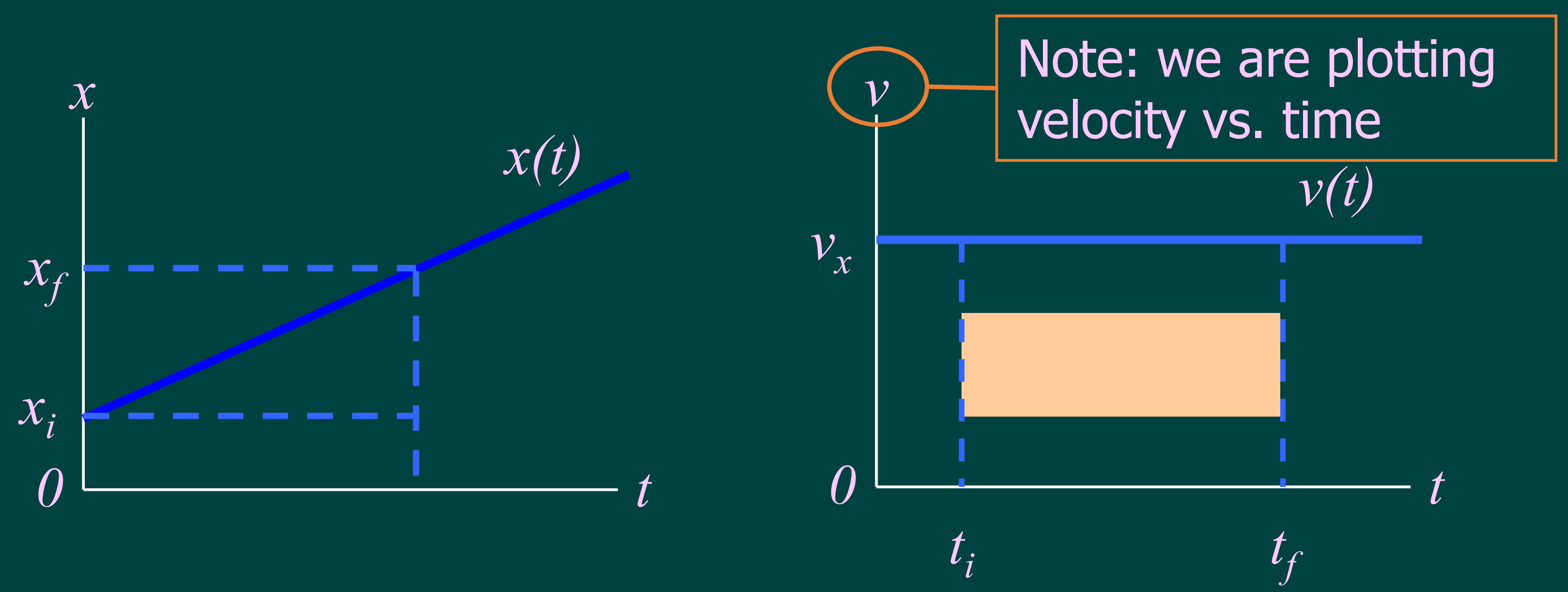
- Instantaneous means "at some given instant". The instantaneous velocity indicates what is happening at every point of time.
- Limiting process:
  - Chords approach the tangent as Δt ⇒
  - Slope measure rate of change of pos



- Instantaneous velocity:
- $$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$
- It is a vector quantity.
- Dimension: length/time (L/T), [m/s].
- It is the slope of the tangent line to x(t).
- Instantaneous velocity v(t) is a function of time.

Uniform Velocity

- Uniform velocity is the special case of constant velocity
- In this case, instantaneous velocities are always the same, all the instantaneous velocities will also equal the average velocity
- Begin with $x_f = x_i + v_x \Delta t$ then $v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$



2.4. Acceleration

- Changing velocity (non-uniform) means an acceleration is present.
- Acceleration is the rate of change of velocity.
- Acceleration is a **vector** quantity.
- Acceleration has both magnitude and direction.
- Unit (a dimensions): length/time^2: [m/s^2].
- Definition:
  - Average acceleration: $$\boldsymbol{a_{avg}} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$
  - Instantaneous acceleration: $$\boldsymbol{a} = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}$$

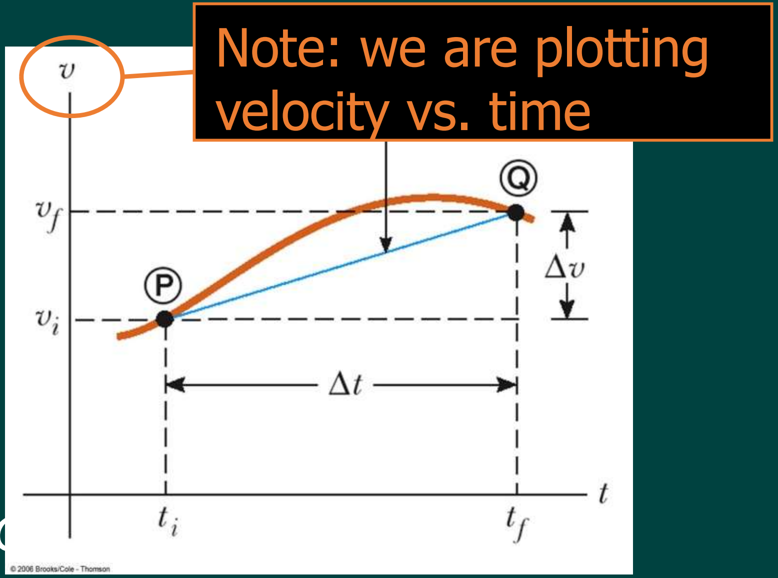
Average Acceleration

- Average acceleration

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}$$

- Velocity as a function of time

$$v_f(t) = v_i + a_{avg}\Delta t$$



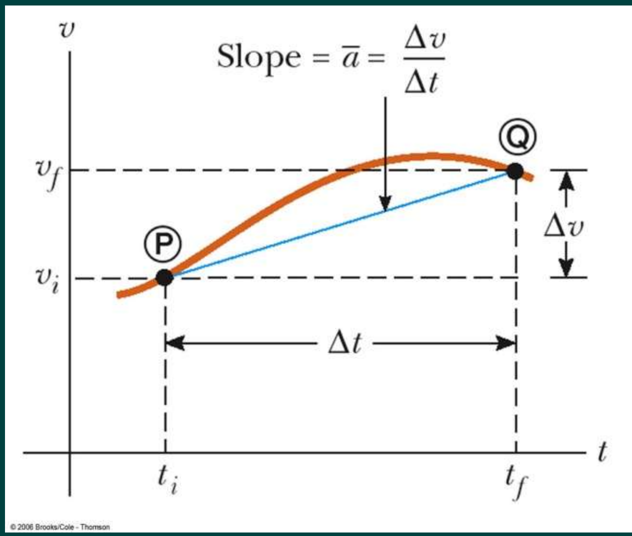
- Positive or negative acceleration
  - When the sign of the velocity and the acceleration are the same (either positive or negative), then the speed is increasing
  - When the sign of the velocity and the acceleration are in the opposite directions, the speed is decreasing (**de**acceleration)
- Average acceleration is the slope of the line connecting the initial and final velocities on a velocity-time graph.

Instantaneous and Uniform Acceleration

- The limit of the average acceleration as the time interval goes to zero

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

- When the instantaneous accelerations are always the same, the acceleration will be uniform. The instantaneous acceleration will be equal to the average acceleration
- Instantaneous acceleration is the slope of the tangent to the curve of the velocity-time graph



Kinematic Variables: x, v, a

- Position is a function of time: $x = x(t)$
- Velocity is the rate of change of position.

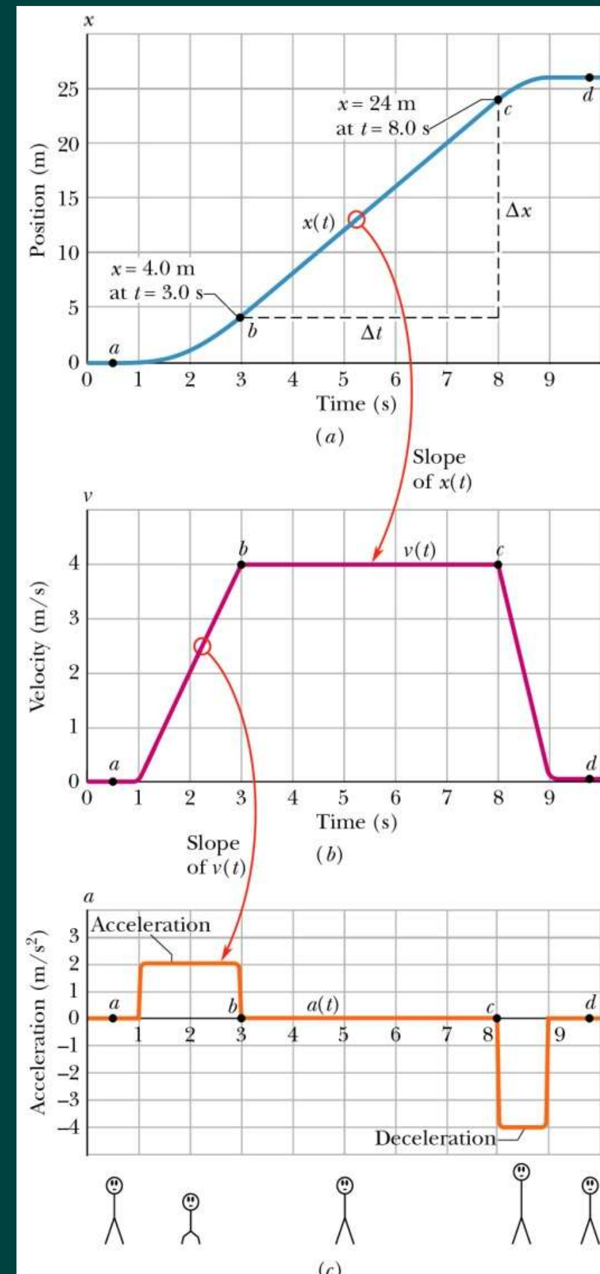
$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

- Acceleration is the rate of change of velocity.

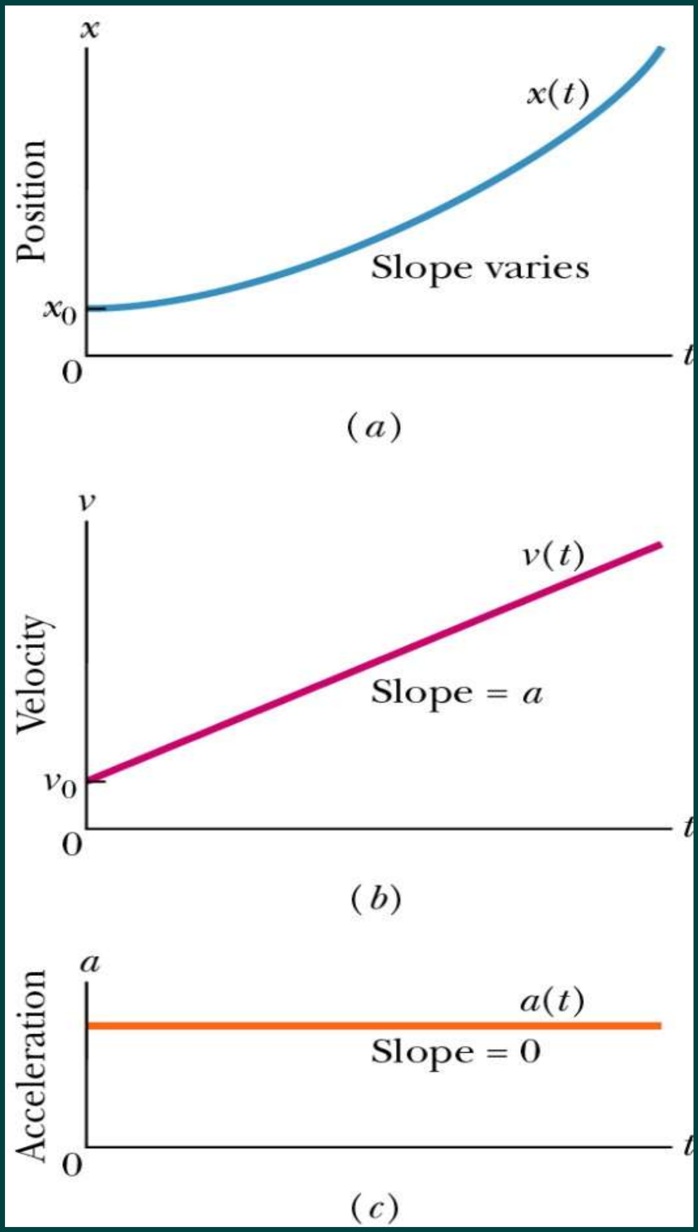
$$a = \lim_{\Delta t \to 0} \frac{\Delta xv}{\Delta t} = \frac{dv}{dt}$$

- $Position \xrightarrow{\frac{d}{dt}} Velocity \xrightarrow{\frac{d}{dt}} Acceleration$
- Graphical relationship between x, v, and a

This same plot can apply to an elevator that is initially stationary, then moves upward, and then stops. Plot v and a as a function of time.



2.5. Special Case: Motion with Uniform Acceleration (our typical case)



- Acceleration is a constant: $a = const$
- Kinematic Equations (which we will derive in a moment)

$$v = v_0 + at \qquad (1)$$

$$\Delta x = \bar{v}t = \frac{1}{2}(v_0 + v)t$$

Or:

$$\Delta x = v_0 t + \frac{1}{2}at^2 \qquad (2)$$

and

$$v^2 = {v_0}^2 + 2a\Delta x \qquad (3)$$

2.5. Motion with Uniform Acceleration

Derivation of the Equation (1)

- Given initial conditions:

$$a(t) = \text{constant} = a,\ v(t=0) = v_0, x(t=0) = x_0$$

- Start with definition of average acceleration:

$$\boldsymbol{a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0} = \frac{v - v_0}{t - 0} = \frac{v - v_0}{t} = a}$$

- We immediately get the first equation: $\boldsymbol{v = v_0 + at}$
- *Shows velocity as a function of acceleration and time*
- *Use when you don't know and aren't asked to find the displacement*

2.5. Motion with Uniform Acceleration

Derivation of the Equation (2)

- Given initial conditions:
  - $a(t) = \text{constant} = a,\ v(t=0) = v_0, x(t=0) = x_0$
- Start with definition of average velocity:

$$\boldsymbol{v_{avg} = \frac{x - x_0}{t} = \frac{\Delta x}{t}}$$

- Since velocity changes at a constant rate, we have

$$\boldsymbol{\Delta x = v_{avg} t = \frac{1}{2}(v_0 + v)t}$$

- Gives displacement as a function of velocity and time
- Use when you don't know and aren't asked for the acceleration

2.5. Motion with Uniform Acceleration

Derivation of the Equation (3)

- Given initial conditions:
  - $a(t) = \text{constant} = a,\ v(t=0) = v_0, x(t=0) = x_0$
- Start with the two just-derived equations:

$$v = v_0 + at \quad \text{and} \quad \Delta x = v_{avg}t = \frac{1}{2}(v_0 + v)t$$

- We have

$$\Delta x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(v_0 + v_0 + at)t$$

Then, $$\Delta x = x - x_0 = v_0 t + \frac{1}{2}at^2$$

- Gives displacement as a function of all three quantities: time, initial velocity and acceleration
- Use when you don't know and aren't asked to find the final velocity

2.5. Motion with Uniform Acceleration

Derivation of the Equation (4)

- Given initial conditions:
  - $a(t) = \text{constant} = a,\ v(t=0) = v_0, x(t=0) = x_0$
- Rearrange the definition of average acceleration

$$\boldsymbol{a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t} = a}$$

to find the time $\boldsymbol{t = \frac{v - v_0}{a}}$

- Use it to eliminate t in the second equation:

$$\Delta x = \frac{1}{2}(v_0 + v)t = \frac{1}{2a}(v + v_0)(v - v_0) = \frac{v^2 - {v_0}^2}{2a}$$

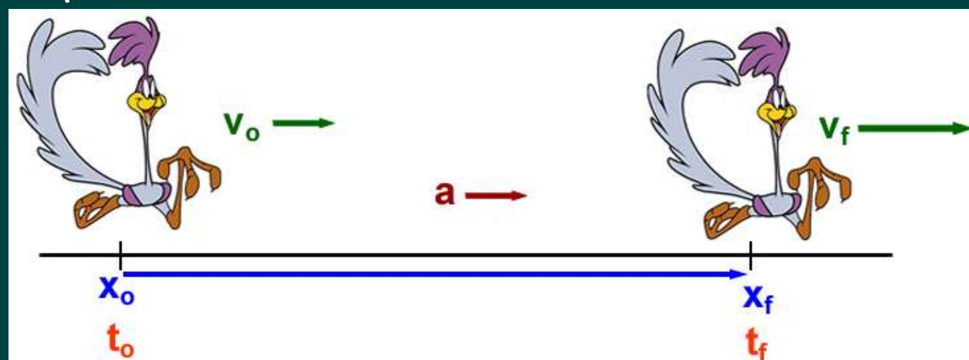
rearrange to get

$$\boldsymbol{v^2 = {v_0}^2 + 2a\Delta x = {v_0}^2 + 2a(x - x_0)}$$

2.5. Motion with Uniform Acceleration

Problem-Solving Hints

- Read the problem
- Draw a diagram
  - Choose a coordinate system, label initial and final points, indicate a positive direction for velocities and accelerations



- Label all quantities, be sure all the units are consistent
  - Convert if necessary
- Choose the appropriate kinematic equation
- Solve for the unknowns
  - You may have to solve two equations for two unk
- Check your results

$$v = v_0 + at$$

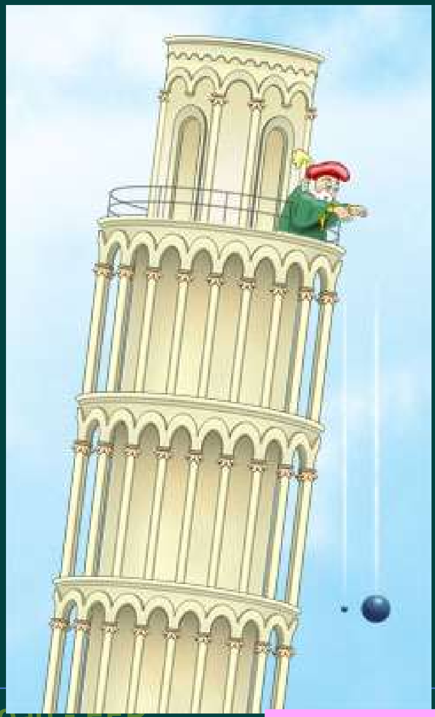
$$\Delta x = v_0 t + \tfrac{1}{2} a t^2$$

$$v^2 = v_0^{\,2} + 2a\Delta x$$

2.6. Free Fall Acceleration

- Earth gravity provides a constant acceleration. Most important case of constant acceleration.
- Free-fall acceleration is independent of mass.
- Magnitude: $|a| = g = 9.8$ m/s^2
- Direction: g always downward, so a_g is negative if we define "up" as positive,

  $a = -g = -9.8$ m/s^2
- Try to pick origin so that $x_i = 0$



An example: Free Fall for Rookie

A stone is thrown from the top of a building with an initial velocity of 20.0 m/s straight upward, at an initial height of 50.0 m above the ground. The stone just misses the edge of the roof on the its way down. Determine

(a) the time needed for the stone to reach its maximum height.

(b) the maximum height.

(c) the time needed for the stone to return to the height from which it was thrown and the velocity of the stone at that instant.

(d) the time needed for the stone to reach the ground

(e) the velocity and position of the stone at $t = 5.0$ s

Summary of chapter 2.

- Displacement: $\Delta x = x_f(t_f) - x_i(t_i)$ $\neq$ **distance (path length)**
- Velocity

  Average velocity: $v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$

  Average speed: $s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}$

  Instantaneous velocity: $v = \frac{dx}{dt}$

  Instantaneous speed: $s = \frac{ds}{dt}$ $= \left|\frac{dx}{dt}\right| = |v|$
- Acceleration

  Average acceleration: **:** $\boldsymbol{a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{t_f - t_i}}$

  Instantaneous acceleration: $\boldsymbol{a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2 x}{dt^2}}$
- 1-D motion with a=const
- Freely falling object:

Summary: motion in a straight line

- This is the simplest type of motion
- It lays the groundwork for more complex motion
- Kinematic variables in one dimension
  - Position $x(t)$ m L
  - Velocity $v(t)$ m/s L/T
  - Acceleration $a(t)$ m/s^2 L/T^2
  - All depend on time
  - All are vectors: magnitude and direction vector:
- **Equations for motion with constant acceleration**: missing quantities

	missing quantities
$v = v_0 + at$	$x - x_0$
$x - x_0 = v_0 t + \frac{1}{2} a t^2$	v
$v^2 = v_0^2 + 2a(x - x_0)$	t
$x - x_0 = \frac{1}{2}(v + v_0)t$	a
$x - x_0 = vt - \frac{1}{2} a t^2$	v_0