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TROY PHYSICS at HUST
General Physics 2252

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For Wednessday Class: *mvtk5yp*

For Thursday class: 2xez3yp

ONE LOVE. ONE FUTURE.

Chapter 3. Motion in Two Dimensions

3.1. Reminder of vectors and vector algebra

3.2. Displacement and position in 2-D

3.3. Average and instantaneous velocity in 2-D

3.4. Average and instantaneous acceleration in 2-D

3.5. Motion in two dimensions

3.6. Projectile motion

3.7. Uniform circular motion

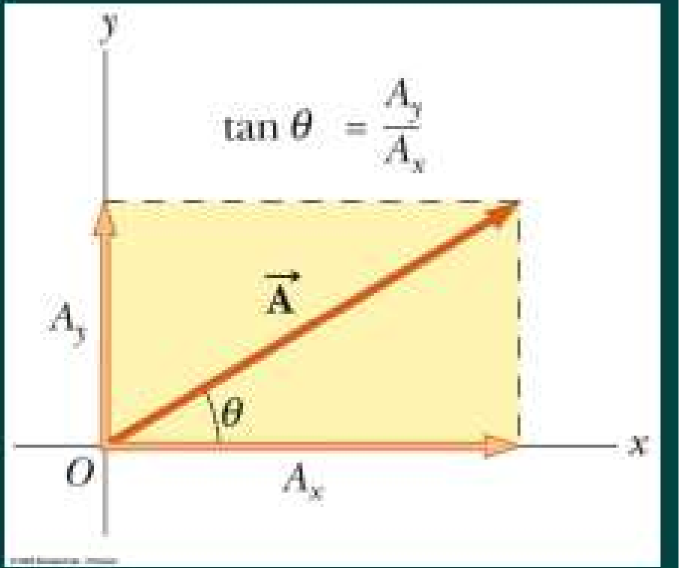
3.8. Relative velocity*

3.1. Reminder of Vector and vector algebra

- The components are the legs of the right triangle whose hypotenuse is *A*:

$$\vec{A} = \vec{A}_x + \vec{A}_y, \text{where: } \begin{cases} A_x = A\cos(\theta) \\ A_y = A\sin(\theta) \end{cases}$$

$$\begin{cases} |\vec{A}| = \sqrt{(A_x)^2 + (A_y)^2} \\ \tan(\theta) = \frac{A_y}{A_x} \text{ or } \theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \end{cases}$$



In case of 3D

$$\vec{A} = \vec{A}_x + \vec{A}_y + \vec{A}_z$$

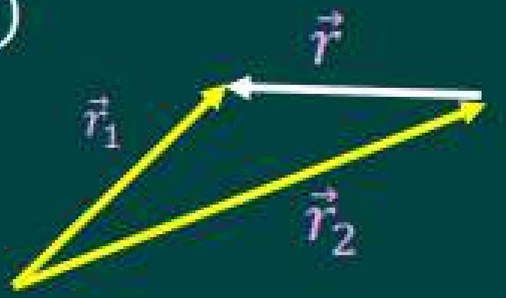
$$\begin{cases} A_x = A\sin(\varphi)\cos(\theta) \\ A_y = A\sin(\varphi)\sin(\theta) \\ A_z = A\cos(\varphi) \end{cases}$$

Vector Algebra

- Which diagram can represent

  $\vec{r} = \vec{r}_2 - \vec{r}_1$?

A)



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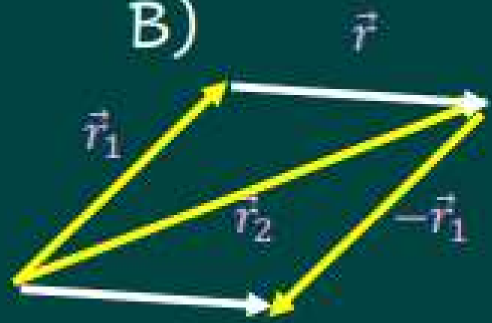

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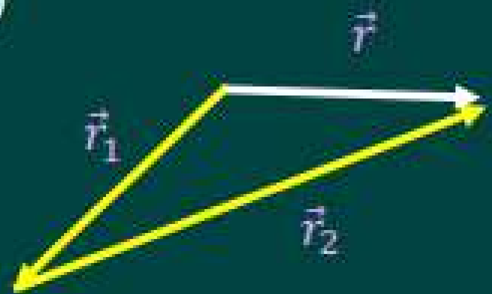

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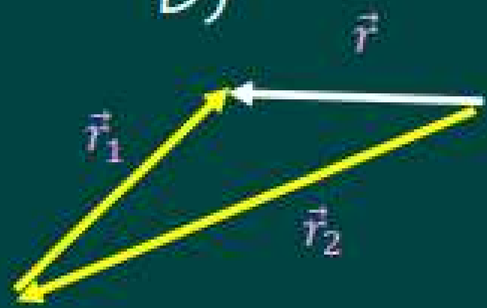
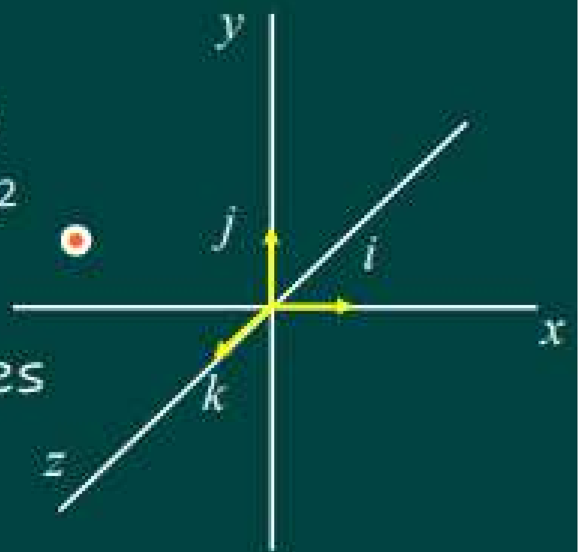


Motion in 2 and 3 dimensions

- Kinematic variables in one dimension
  - Position: $x(t)$ m
  - Velocity: $v(t)$ m/s
  - Acceleration: $a(t)$ m/s^2

- Kinematic variables in three dimensions
  - Position: $\vec{r}(t) = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ m
  - Velocity: $\vec{v}(t) = v_x\hat{\imath} + v_y\hat{\jmath} + v_z\hat{k}$ m/s
  - Acceleration: $\vec{a}(t) = a_x\hat{\imath} + a_y\hat{\jmath} + a_z\hat{k}$ m/s^2
- All are vectors: have direction and magnitudes

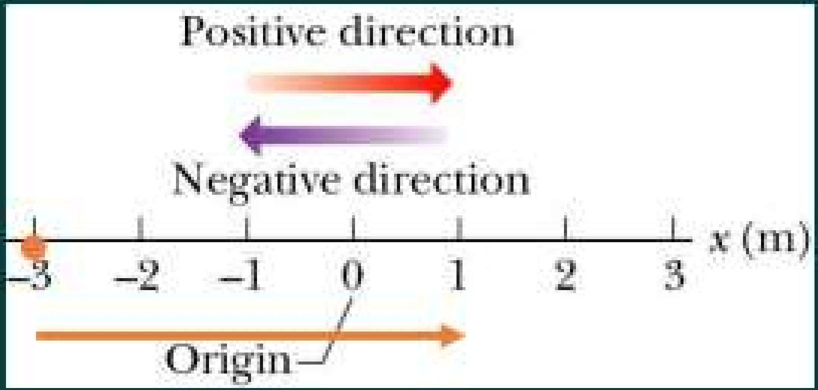


3.2. Position and Displacement

- **In one dimension**, displacement $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

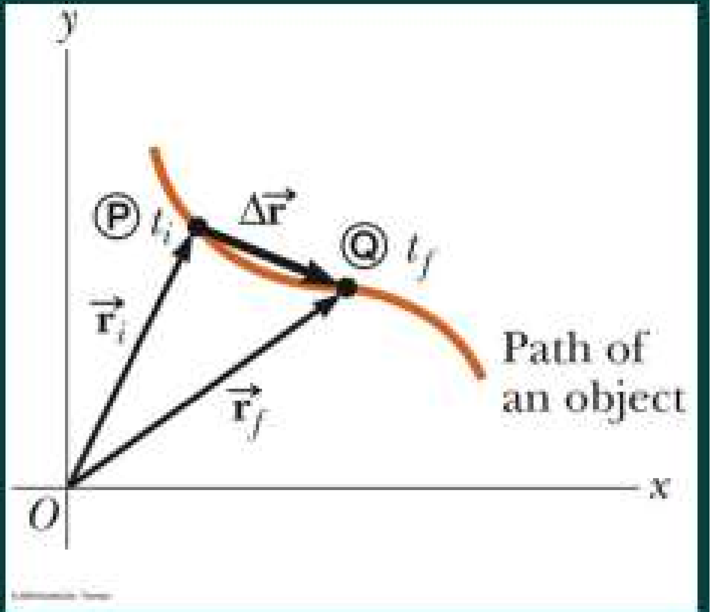
$$\Leftrightarrow \Delta x = x_2(t_2) - x_1(t_1)$$

$x_1(t_1) = -3.0$ m, $x_2(t_2) = +1.0$ m
$\Delta x = +1.0$ m $+ 3.0$ m $= +4.0$ m



- **In two dimensions**
  - **Position:** the position of an object is described by its position vector $\vec{r}(t)$ always points to particle from origin.
  - **Displacement:** $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$

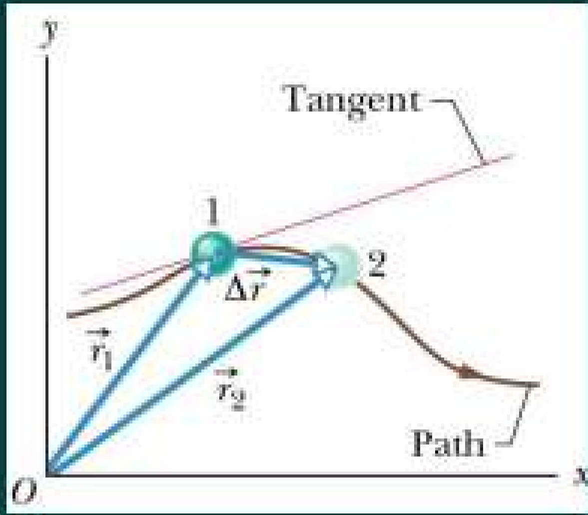
$$\Delta\vec{r} = (x_2\hat{\imath} + y_2\hat{\jmath}) - (x_1\hat{\imath} + y_1\hat{\jmath})$$
$$= (x_2 - x_1)\hat{\imath} + (y_2 - y_1)\hat{\jmath}$$
$$= \Delta x\hat{\imath} + \Delta y\hat{\jmath}$$



3.3. Average & Instantaneous Velocity

- Average velocity: $\vec{v}_{avg} \equiv \frac{\Delta\vec{r}}{\Delta t}$

$$\vec{v}_{avg} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath} = v_{avg,x}\hat{\imath} + v_{avg,y}\hat{\jmath}$$

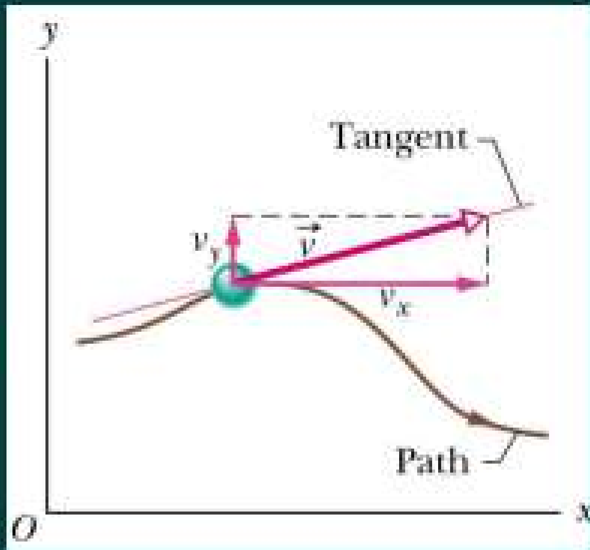


- Instantaneous velocity

$$\vec{v} \equiv \lim_{\Delta t \to 0} \vec{v}_{avg} = \lim_{\Delta t \to 0} \frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} = v_x\hat{\imath} + v_y\hat{\jmath}$$

- $\vec{v}$ is tangent to the path in x-y graph;



3.4. Average & Instantaneous Acceleration

- Average acceleration $\vec{a}_{avg} \equiv \frac{\Delta\vec{v}}{\Delta t}$

$$\vec{a}_{avg} = \frac{\Delta v_x}{\Delta t}\hat{\imath} + \frac{\Delta v_y}{\Delta t}\hat{\jmath} = a_{avg,x}\hat{\imath} + a_{avg,y}\hat{\jmath}$$

- Instantaneous acceleration

$$\vec{a} \equiv \lim_{\Delta t \to 0} \vec{a}_{avg} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} = a_x\hat{\imath} + a_y\hat{\jmath}$$

The average acceleration is defined as the rate at which the velocity changes. The instantaneous acceleration is the limit of the average acceleration as Δt approaches zero

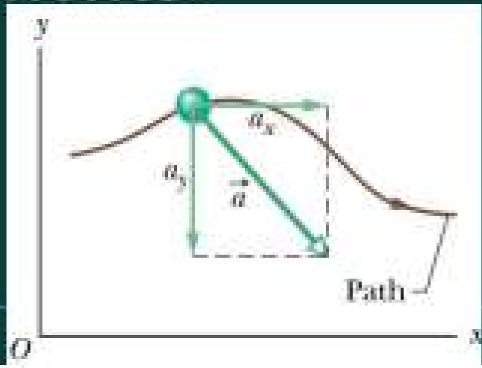
Notice that:

- In general: $d\vec{v} \neq dv$, so a $\neq \frac{dv}{dt}$
- $\vec{v}$ is always tangential to the orbit
- $\vec{a}$ is not necessary to be tangential to the orbit;

Tangential and **normal acceleration components:**

$$\vec{a} = \vec{a}_t + \vec{a}_n$$

- $\vec{a}_t$: tangential acceleration, and $a_t = \frac{dv}{dt}$
- $\vec{a}_n$: normal acceleration, $a_n = \frac{v^2}{R}$



3.5. Motion in 2 dimensions

- Motions in each dimension are independent components
- **Consider: Constant acceleration equations**

$$\vec{v} = \vec{v}_0 + \vec{a}t$$
$$\vec{r}_f - \vec{r}_i = \vec{v}_0 t + \frac{1}{2}\vec{a}t^2$$

- Constant acceleration equations hold in each dimension

$$v_x = v_{0x} + a_x t$$
$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$
$${v_x}^2 = {v_{0x}}^2 + 2a_x(x - x_0)$$

$$v_y = v_{0y} + a_y t$$
$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$$
$${v_y}^2 = {v_{0y}}^2 + 2a_y(y - y_0)$$

$At : t = 0$ beginning of the process
- $\vec{a} = a_x\hat{\imath} + a_y\hat{\jmath}$ where a_x and a_y are constants
- Initial velocity $\vec{\boldsymbol{v}}_0 = \boldsymbol{v}_{0x}\hat{\boldsymbol{\imath}} + \boldsymbol{v}_{0y}\hat{\boldsymbol{\jmath}}$
- initial displacement $\vec{\boldsymbol{r}}_0 = \boldsymbol{x}_0\hat{\boldsymbol{\imath}} + \boldsymbol{y}_0\hat{\boldsymbol{\jmath}}$;

Hints for solving problems

- Define coordinate system. Make sketch showing axes, origin.
- List known quantities. Find v_{0x}, v_{0y}, a_x, a_y, etc. Show initial conditions on sketch.
- List equations of motion to see which ones to use.
- Time t is the same for x and y directions.

$$x_0 = x(t = 0),\ y_0 = y(t = 0),$$

$$v_{0x} = v_x(t = 0),\ v_{0y} = v_y(t = 0).$$

- Have an axis point along the direction of **a** if it is constant.

$$v_x = v_{0x} + a_x t$$

$$x - x_0 = v_{0x}t + \frac{1}{2}a_x t^2$$

$${v_x}^2 = {v_{0x}}^2 + 2a_x(x - x_0)$$

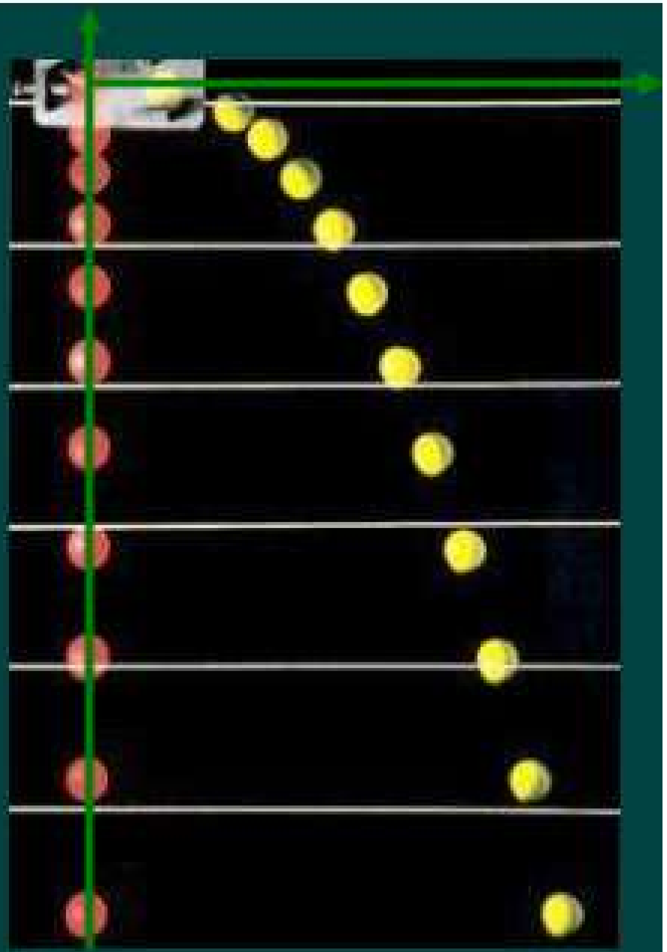
$$v_y = v_{0y} + a_y t$$

$$y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$$

$${v_y}^2 = {v_{0y}}^2 + 2a_y(y - y_0)$$

3.6. Projectile Motion

Horizontal projectile motion



- 2-D problem and define a coordinate system: x- horizontal, y- vertical (up +)
- Try to pick $x_0 = 0$, $y_0 = 0$ at $t = 0$
- Horizontal motion + Vertical motion
- Horizontal: $a_x = 0$, constant velocity motion
- Vertical: $a_y = -g = -9.8\ m/s^2$, $v_{0y} = 0$
- Equations:

Horizontal

$$v_x = v_{0x}$$

$$x - x_0 = v_{0x}t$$

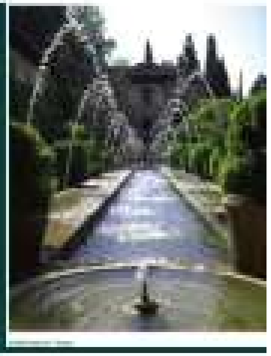
Vertical

$$v_y = -gt$$

$$y - y_0 = -\frac{1}{2}gt^2$$

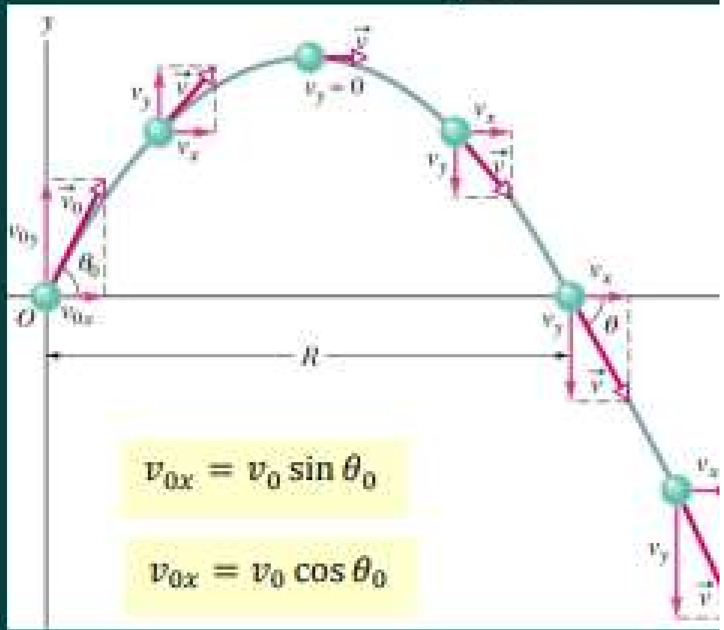
$$v_y^{\,2} = v_{0y}^{\,2} - 2g\,(y - y_0)$$

3.6. Projectile Motion (Cont.)

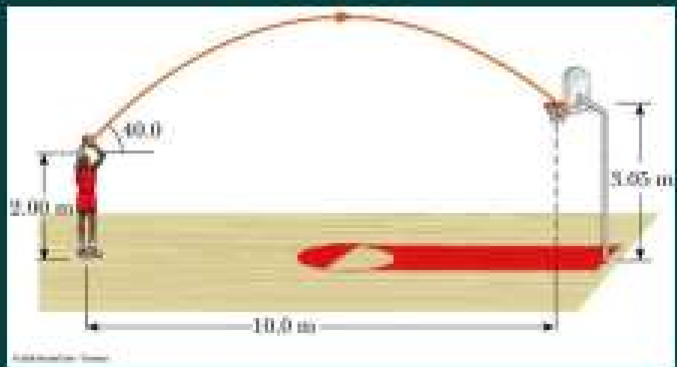


In general case:

- Horizontal: $a_x = 0$ and vertical: $a_y = -g$.
- Try to pick $x_0 = 0$, $y_0 = 0$ at $t = 0$.
- Velocity initial conditions:
  - v_0 can have x, y components.
  - v_{0x} is constant usually.
  - v_{0y} changes continuously.
- Equations:



Horizontal	Vertical
$v_x = v_{0x}$	$v_y = v_{0y} - gt$
$x = x_0 + v_{0x}t$	$y = y_0 + v_{0y}t - \frac{1}{2}gt^2$



Trajectory of Projectile Motion

- If: Initial conditions ($t = 0$): $x_0 = 0$, $y_0 = 0$
  $v_{0x} = v_0 \cos\theta_0$ and $v_{0y} = v_0 \sin\theta_0$
- Horizontal motion:

$$x = 0 + v_{0x}t \quad \Rightarrow \quad t = \frac{x}{v_{0x}}$$

- Vertical motion:

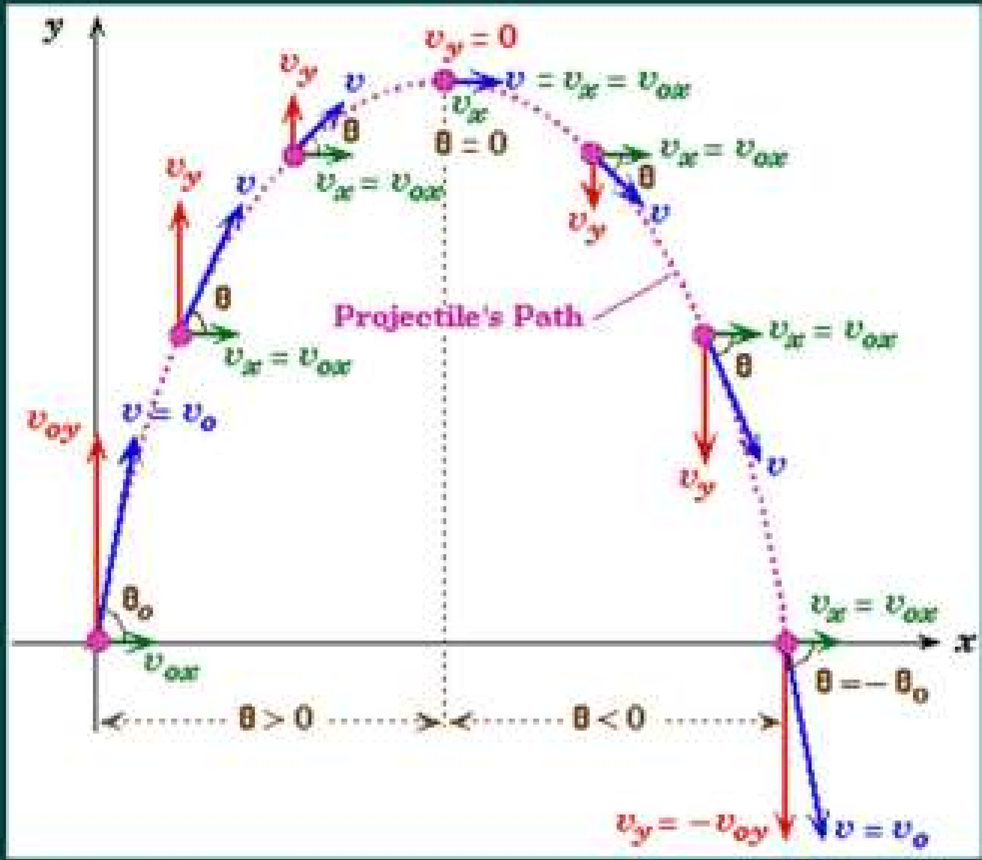
$$y = 0 + v_{0y}t - \frac{1}{2}gt^2$$

$$y = v_{0y}\left(\frac{x}{v_{0x}}\right) - \frac{g}{2}\left(\frac{x}{v_{0x}}\right)^2$$

$$y = x\tan\theta_0 - \frac{g}{2{v_0}^2\cos^2\theta_0}x^2$$

$\Rightarrow$ **Parabola;**

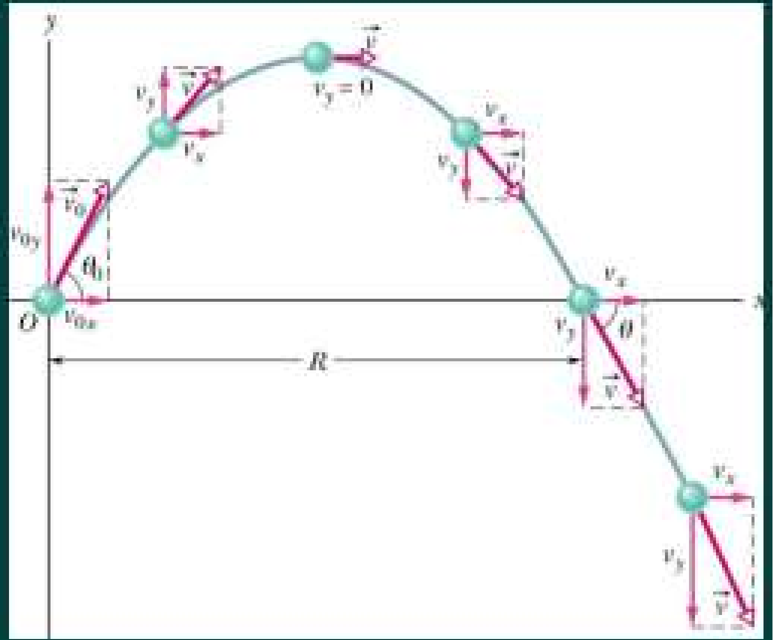
If: $\theta_0 = 0$ and $\theta_0 = 90$?



What is R and h ?

- Time and position of landing:

$$x = 0 + v_{0x}t \qquad 0 = 0 + v_{0y}t - \frac{1}{2}gt^2$$



$$t = \frac{2v_{0y}}{g} = \frac{2v_0 \sin\theta_0}{g}$$

$$R = x - x_0 = v_{0x}t = \frac{2v_0 \cos\theta_0 \, v_0 \sin\theta_0}{g} = \frac{{v_0}^2 \sin 2\theta_0}{g}$$

$$h = y - y_0 = v_{0y}t_h - \frac{1}{2}g{t_h}^2 = v_{0y}\frac{t}{2} - \frac{g}{2}\left(\frac{t}{2}\right)^2$$

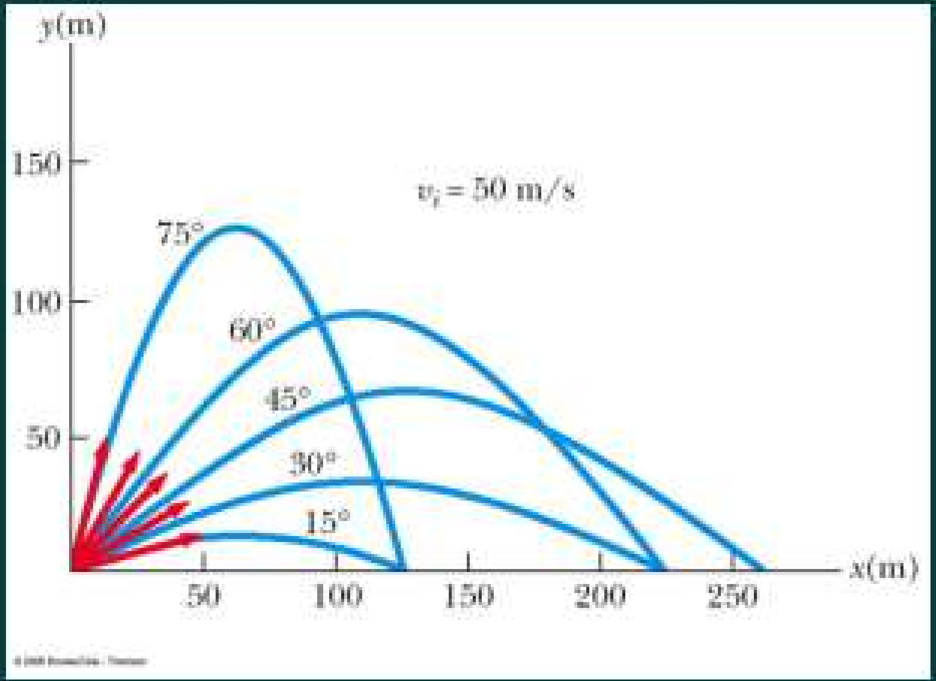
$$h = \frac{{v_0}^2 \sin^2\theta_0}{2g}$$

$$v_y = v_{0y} - gt = v_{0y} - g\frac{2v_{0y}}{g} = -v_{0y}$$

Projectile Motion at Various Initial Angles

- Complementary values of the initial angle result in the same range
  - The heights will be different
- The maximum range occurs at a projection angle of 45°

$$R = \frac{{v_0}^2 \sin 2\varphi}{g}$$



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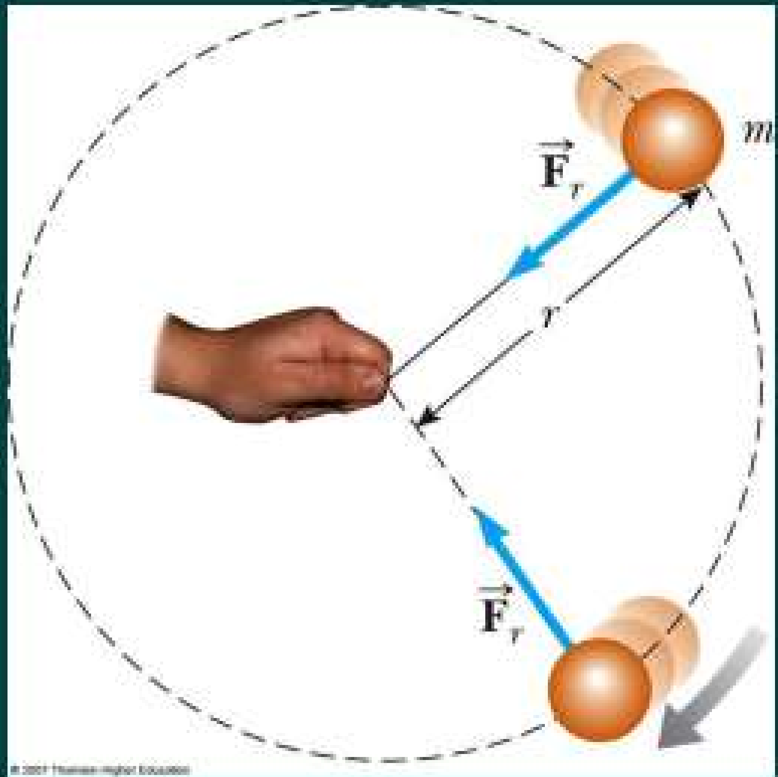
3.7. Uniform circular motion
3.8. Relative velocity*

3.7. Uniform circular motion



An object traveling in a circle, even though it moves with a constant speed, will have an acceleration

The centripetal acceleration is due to the change in the direction of the velocity

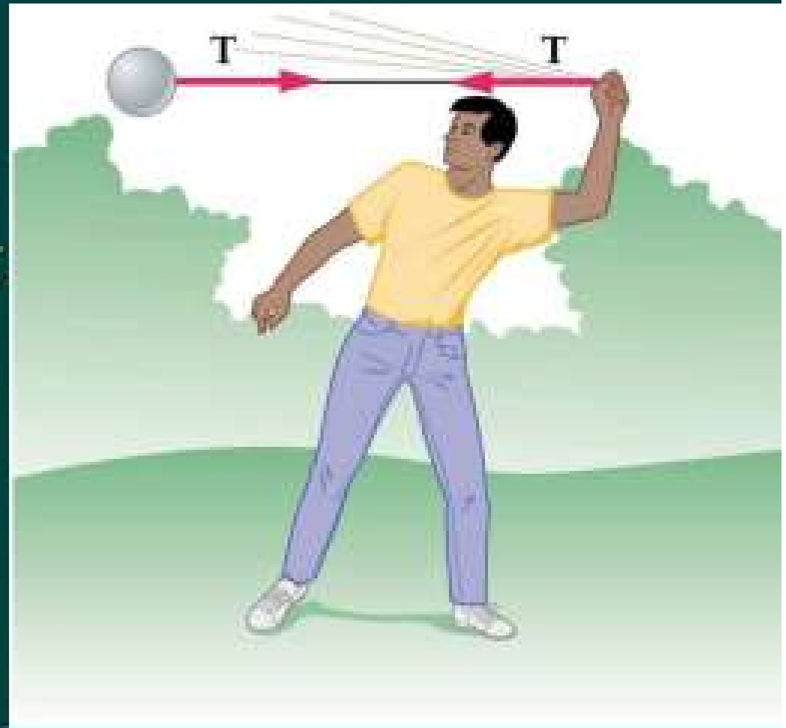
Uniform circular motion

Constant speed, or, constant magnitude of velocity	Motion along a circle: Changing direction of velocity

Circular Motion: Observations

Object moving along a curved path with **constant speed**

- Magnitude of velocity: same
- Direction of velocity: changing
- Velocity: changing
- Acceleration is NOT zero!
- **Net force acting on the object is NOT zero**
- "**Centripetal force**"



$$\vec{F}_{net} = m\vec{a}$$

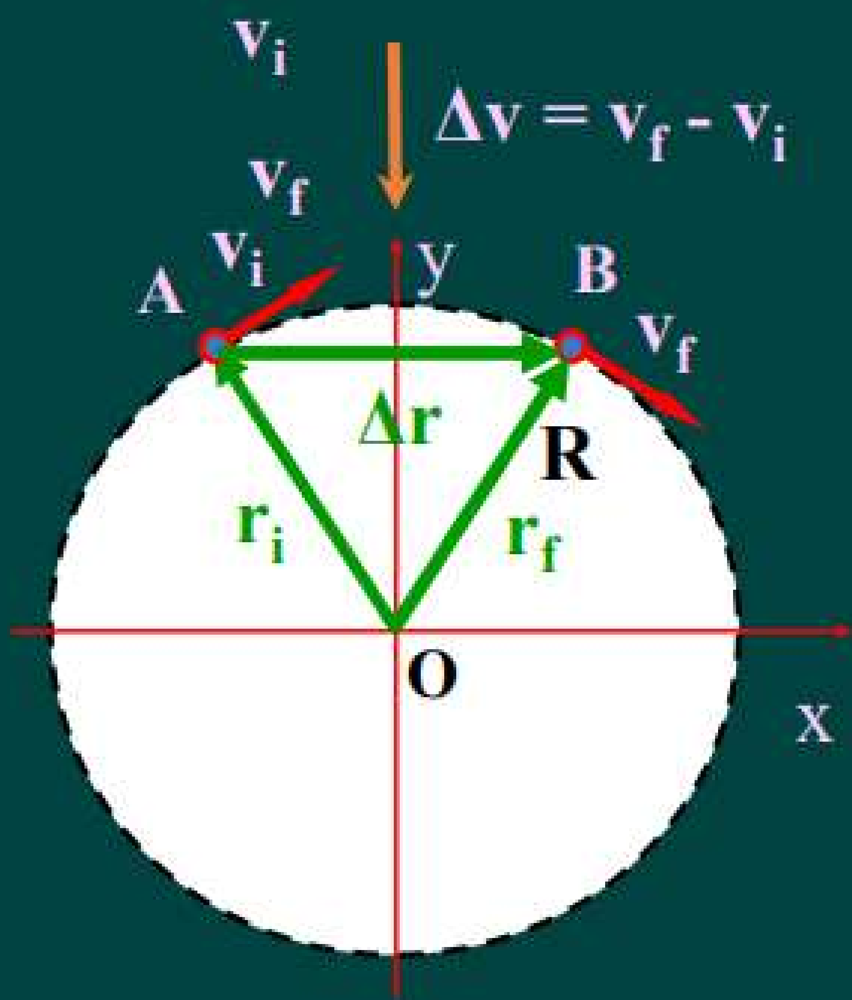
3.7. Uniform Circular Motion (cont.)

- Centripetal acceleration

$$\frac{\Delta v}{v} = \frac{\Delta r}{r} \quad \text{so,} \ \Delta v = \frac{v\Delta r}{r}$$

$$\frac{\Delta v}{\Delta t} = \frac{\Delta r}{\Delta t}\frac{v}{r} = \frac{v^2}{r}$$

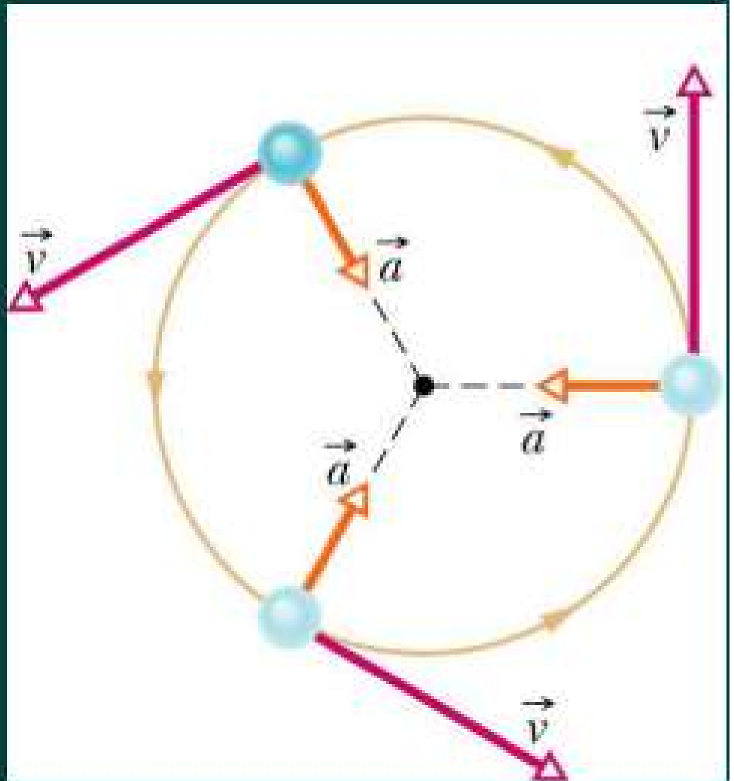
$$a_r = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$



- Direction: Centripetal

3.7. Uniform Circular Motion (cont.)

- **Velocity:**
  - Magnitude: constant v
  - The direction of the velocity is tangent to the circle
- **Angular velocity:**
  - $\omega = \frac{v}{r}$
- **Acceleration:** $\vec{a}_c \perp \vec{v}$
  - Magnitude: $a_c = \frac{v^2}{r}$
  - directed toward the center of the circle of motion
- **Period:**
  - time interval required for one complete revolution of the particle $T = \frac{2\pi r}{v} = \frac{2\pi}{\omega}$
- **Frequency:** $f = \frac{1}{T}$



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3.8. Relative velocity* (reading)

Summary of chapter 3.

- Position: $\vec{r}(t) = x\hat{\imath} + y\hat{\jmath}$
- Average velocity: $\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{\imath} + \frac{\Delta y}{\Delta t}\hat{\jmath} = v_{avg,x}\hat{\imath} + v_{avg,y}\hat{\jmath}$
- Instantaneous velocity: $\vec{v}(t) = \lim_{t\to 0}\frac{\Delta\vec{r}}{\Delta t} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{\imath} + \frac{dy}{dt}\hat{\jmath} = v_x\hat{\imath} + v_y\hat{\jmath}$

  $v_x \equiv \frac{dx}{dt}$ and $v_y \equiv \frac{dy}{dt}$
- Acceleration

  $$\vec{a}(t) = \lim_{t\to 0}\frac{\Delta\vec{v}}{\Delta t} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{\imath} + \frac{dv_y}{dt}\hat{\jmath} = a_x\hat{\imath} + a_y\hat{\jmath}$$

  $$a_x \equiv \frac{dv_x}{dt} = \frac{d^2x}{dt^2} \qquad a_y \equiv \frac{dv_y}{dt} = \frac{d^2y}{dt^2}$$
- $\vec{r}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ are not necessarily in the same direction.

Summary of chapter 3.

- If a particle moves with constant acceleration *a*, motion equations are

$$\vec{r}_f = \vec{r}_i + \vec{v}_i t + \frac{1}{2}\vec{a}t^2$$

$$\vec{r}_f = x_f\hat{\imath} + y_f\hat{\jmath} = (x_i + v_{xi}t + \frac{1}{2}a_{xi}t^2)\hat{\imath} + (y_i + v_{yi}t + \frac{1}{2}a_{yi}t^2)\hat{\jmath}$$

$$\vec{v} = \vec{v}_i + \vec{a}t$$

$$\vec{v}_f(t) = v_{fx}\hat{\imath} + v_{fy}\hat{\jmath} = (v_{ix} + a_x t)\hat{\imath} + (v_{iy} + a_y t)\hat{\jmath}$$

- Projectile motion is one type of 2-D motion under constant acceleration, where $a_x = 0$, $a_y = -g$.