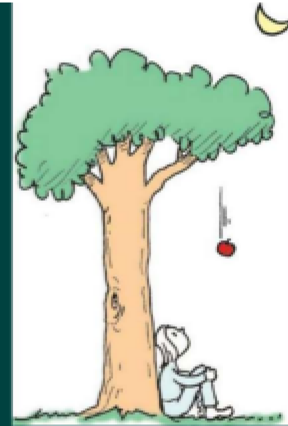
# Chapter 4. The Laws of Motion



1. Dynamics: Force and Mass
2. Newton's first law of motion
3. Newton's second law of motion
4. Types of forces
5. Newton's third law of motion
6. Newton's law of gravity
7. Solving a problem: Examples
8. Application of Newton's Laws

1642-1726

*Isaac Newton's work represents one of the greatest contributions to science ever made by an individual.*

GENERAL PHYSICS
PHYS252

## 4.1. Dynamics: force and mass

- Describes the relationship between the motion of objects in our everyday world and the forces acting on them.
- Language of Dynamics

**Force:** The measure of interaction between **two objects** (pull or push). It is a vector quantity – it has a **magnitude and direction.**

+ May be a **contact force** or a **field force**

**Contact forces** result from physical contact between two objects

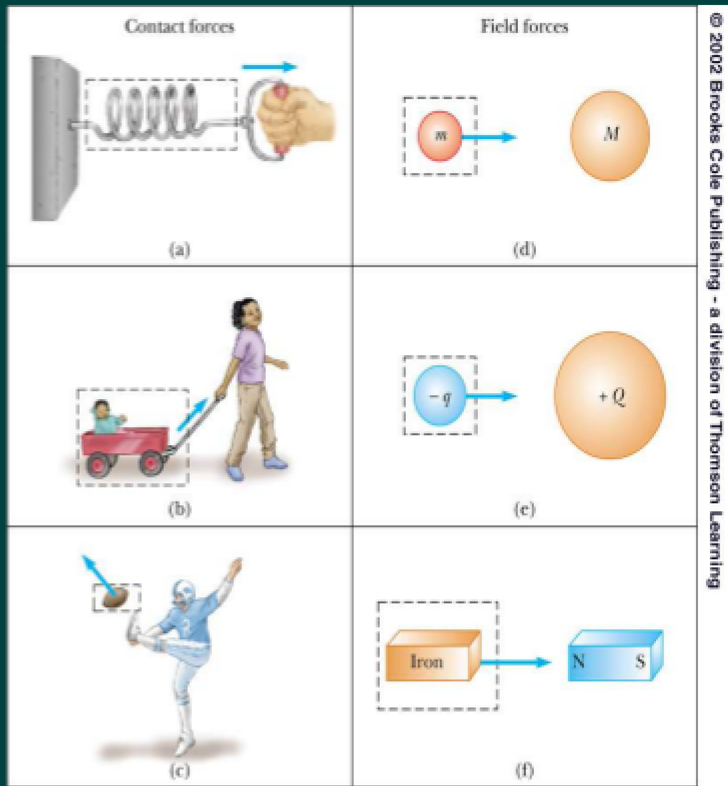
**Field forces** act between disconnected objects => Also called "action at a distance".

**Mass:** The measure of how difficult it is to change object's velocity (sluggishness or inertia of the object).

# Examples of contact forces and field forces

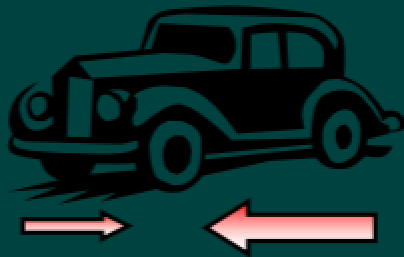
**Contact forces** result from physical contact between two objects

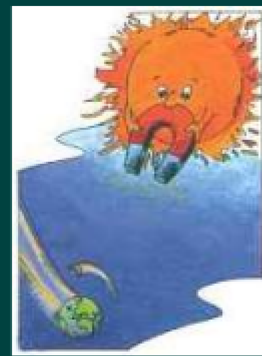
**Field forces** act between disconnected objects => Also called "action at a distance"

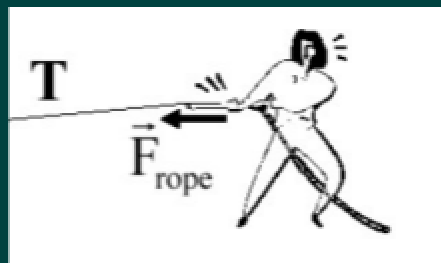


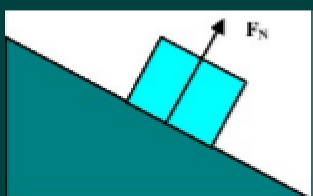
# Forces

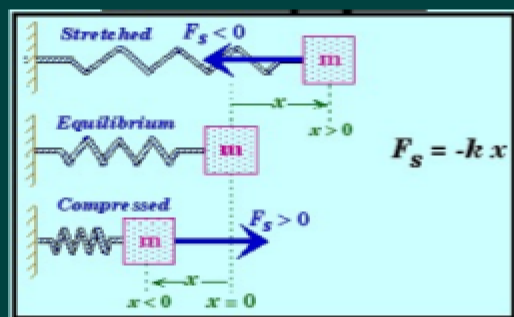
- Gravitational Force
- Archimedes Force
- Friction Force
- Tension Force
- Spring Force
- Normal Force







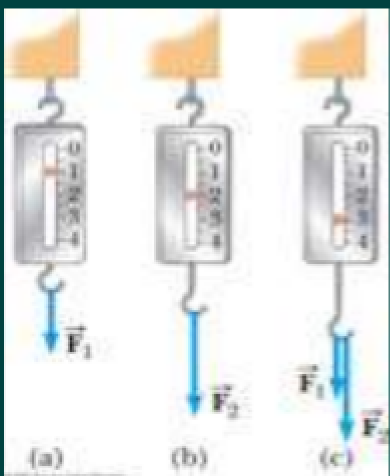



## Vector Nature of Force

- Vector force: has ***magnitude*** ***and*** ***direction***
- Net Force: a resultant force acting on object

$$\vec{F}_{net} = \sum \overrightarrow{F_i} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \ldots..$$

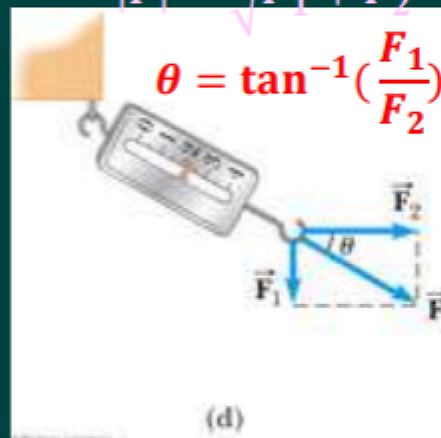
- You must use the rules of vector addition to obtain the net force on an object.




$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2$$
$$|\vec{F}| = F_1 + F_2$$

$$|\vec{F}| = \sqrt{F_1^2 + F_2^2} = 2.24\text{N}$$

$$\boldsymbol{\theta = \tan^{-1}\left(\frac{F_1}{F_2}\right)} = -26.6^{\circ}$$



# 4.2. Newton's First Law

## Aristotle and Galileo

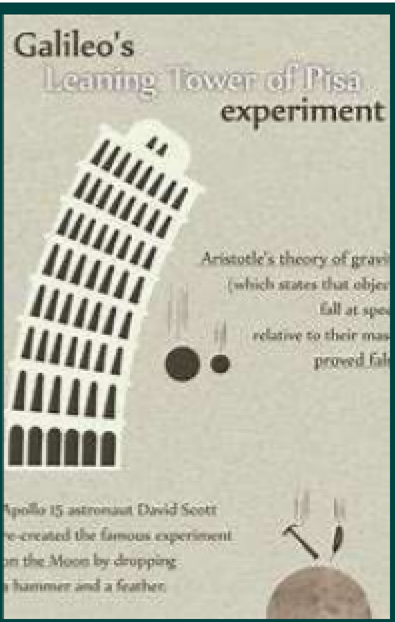
♣ **Aristotle** ($4^{th}$ century BC) was a Greek polymath.

- He was very influent in philosophy, science, and education.
- He classified motion into natural motion (motion of a body towards its natural place) and violent motion (motion of a body away from its natural place).
- He had two assertions about motion and force.

A heavier object falls faster than a lighter object.

The natural state of an object was at rest. To keep the object in motion, a force was necessary.

# Aristotle and Galileo




♠ **Galileo Galilei** (16th century) was an Italian polymath.

• His famous **Leaning Tower of Pisa experiment** showed that objects of different masses fall with the same acceleration.

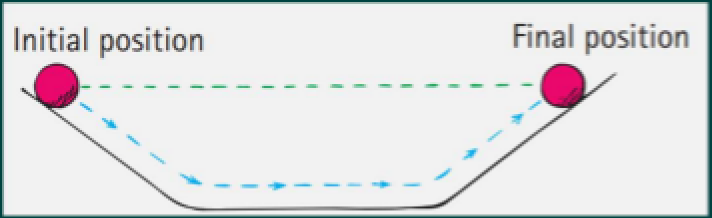
• In case the objects don't fall the same, he thought it is because of the air resistance. He postulated that in the absence of air, all objects would fall the same.

• The Moon has **no atmosphere**. If you can get there, like David Scott, you can check Galileo's theory.

• You can also check this theory on the Earth by using a vacuum tube.

# Aristotle and Galileo (cont.)

- Aristotle's first assertion is incorrect because it does not agree with experiment. How about his second assertion?
- Galileo did another experiment with a ball rolling on planes inclined at various angles with negligibly small friction





- He concluded that although a force is needed to start an object moving, no force is needed to keep it moving – except for the force needed to overcome friction.

# Aristotle and Galileo (cont.)

- Galileo made a revolution. He was at odds with others around him, who held to Aristotelian ideas.
- His assertions about motion and force are against Aristotle but are consistent with experiments.

**All objects would fall with the same constant acceleration in the absence of air resistance.**

**It is just as natural for an object to be in motion with a constant velocity as it is for it to be at rest**

## 4.2. Newton's First Law

- Isaac Newton based upon and continued with Galileo's ideas.
- His book "**The Principia**", one of the most important books in human history, includes the three Newton's laws of motion.

♣ Statement of the law

**Every object continues in a state of rest or of uniform velocity unless acted on by a nonzero net force.**

- This law can be regarded as a restatement of Galileo's idea.

It means that every object has **an inertia**

That is why it is also called **the law of inertia**

## 4.2. Newton's First Law

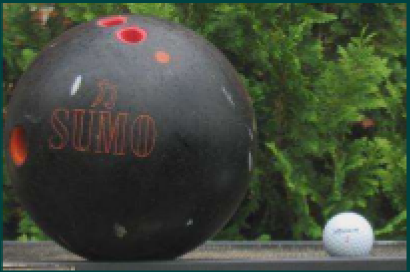
**# Statement:**

**"An object at rest tends to stay at rest and an object in motion tends to stay in motion with the same speed and in the same direction unless acted upon by an unbalanced force"**

- An object at rest remains at rest as long as no **net force** acts on it
- An object moving with **constant velocity** continues to move with the same speed and in the same direction (the same velocity) as long as no net force acts on it
- *"Keep on doing what it is doing".*
- When forces are balanced, the acceleration of the object is zero
  - Object at rest: $v = 0$ and $a = 0$
  - Object in motion: $v \neq 0$ and $a = 0$
- The net force is defined as the vector sum of all the external forces exerted on the object. If the net force is zero, forces are balanced. When forces are balances, the object can be stationary, or move with constant velocity.

# Mass and Inertia

- **Inertia** is a property of objects to resist changes of motion!
- **Mass** is a measure of the amount of **inertia**.
- **Mass** is a measure of the resistance of an object to changes in its velocity
- Mass is an inherent property of an object
- Scalar quantity and SI unit of mass: **kg**



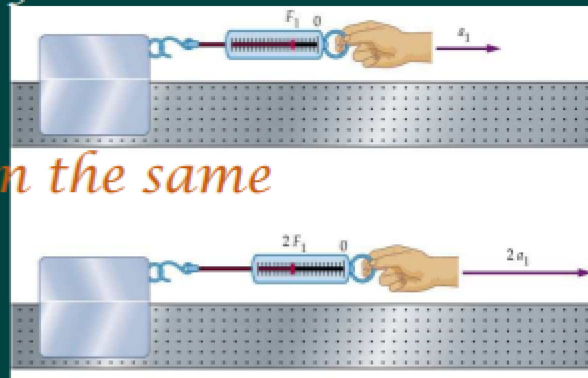
**Newton's first law** explains what happens to an object when no forces act on it: it either remains at rest or moves in a straight line with constant speed.

So, what will happen to an object if the net force is not zero?

Newton's 2nd law answers this question.

## 4.3. Newton's Second Law

*When the net force acting on an object is nonzero, the object is accelerated.*



*Its acceleration depends on:*

*+ the net force ($a \propto F$ and $\vec{a}$ is in the same direction as $\vec{F}$ ),*

*+ its mass ($a \propto 1/m$ )*

*Statement of the law:*

*"The acceleration of an object is*

*- directly proportional to the net force acting on it*

*- is in the same direction as the net force*

*- and inversely proportional to its mass"*

$$\boxed{\vec{a} = \frac{\sum \vec{F}}{m} = \frac{\vec{F}_{net}}{m}}$$

$$\boxed{\vec{F}_{net} = \sum \vec{F} = m\vec{a}}$$

**Note:** *the equations are in SI unit system*

# 4.3. Newton's Second Law

- For the same mass, by doubling the net force, the acceleration will be doubled.
- For the same net force, by doubling the mass, the acceleration will be halved.

$$a \propto F \text{ and } a \propto 1/m \rightarrow a \propto F/m \quad \text{or} \qquad a = k.\frac{F}{m}$$

- The constant *k* depends on the system of units in used.

We use the SI units and in this system k = 1.

$$\boxed{\vec{a} = \frac{\vec{F}_{net}}{m}}$$

## 4.3. Newton's Second Law

### Free fall

- **Free fall** is the fall of an object under the action of the force of gravity only (no air resistance).

$F = ma \rightarrow W = mg$ ( $g$ is the **free fall acceleration**)

- For an object of a double mass, the weight is double. Thus, $g$ has the same value for all objects (consistent with Galileo).

The approximate value: $\mathbf{g = 9.81\ ms^{-2}}$

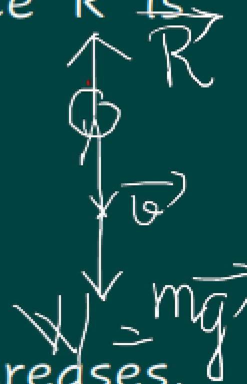
- Free fall can be observed in a vacuum tube or on the Moon.
- For a stone falling on Earth in a short distance, air resistance is negligible, and the fall is approximately a free fall.

## 4.3. Newton's Second Law

## Non-free fall

- **Non-free fall** is the fall when air resistance R is significant.

$$F_{net} = mg - R, \quad a = \frac{F_{net}}{m} \rightarrow \quad \boldsymbol{a = g - \frac{R}{m}}$$




- There is a mutual relation between speed and acceleration.
- As the speed increases, the air resistance increases, causing a decrease in acceleration.
- The speed keeps increasing (because of the acceleration) and the acceleration keeps decreasing (because of the increase in speed). This happens until the acceleration becomes zero.
- That is when the speed reaches a maximum constant value, called terminal speed.

## Units of Force

- From Newton's second law:

$$\vec{F}_{net} = \sum \vec{F} = m\vec{a}$$

- SI unit of force is a Newton (N)

$$1\ \mathrm{N} \equiv 1\ \frac{\mathrm{kg.m}}{\mathrm{s}^2}$$

- *US Customary unit of force is a pound (lb)*
  - *1 N = 0.225 lb*
  - *Weight, also measured in lbs. is a force (mass × acceleration). What is the acceleration in that case?*

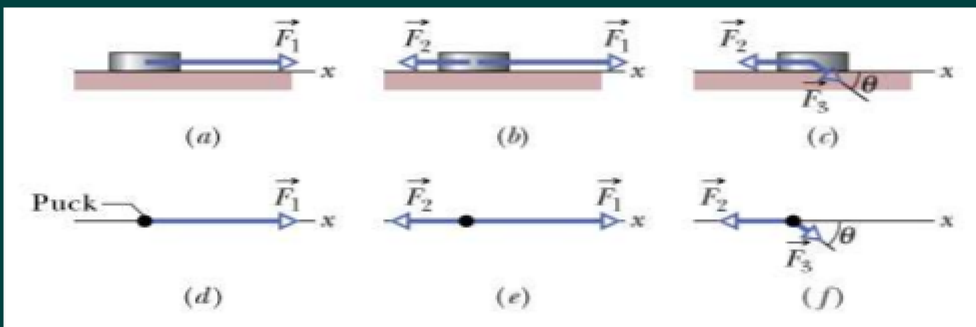
# More about Newton's 2nd Law

- You must be certain about which body we are applying it to
- $\vec{F}_{\text{net}}$ must be the vector sum of all the forces that act on that body
- Only forces that act on that body are to be included in the vector sum
- Net force component along an axis gives rise to the acceleration along that same axis

$$\boxed{F_{net,x} = ma_x} \qquad \boxed{F_{net,y} = ma_y}$$

# Sample Problem

- One or two forces act on a puck that moves over frictionless ice along an $x$ axis, in one-dimensional motion. The puck's mass is $m = 0.20$ kg. Forces $\boldsymbol{F_1}$ and $\boldsymbol{F_2}$ and are directed along the x axis and have magnitudes $F_1 = 4.0$ N and $F_2 = 2.0$ N. Force $\boldsymbol{F_3}$ is directed at angle $\theta = 30°$ and has magnitude $F_3 = 1.0$ N. In each situation, what is the acceleration of the puck?




$$a) F_1 = ma_x$$

$$a_x = \frac{F_1}{m} = \frac{4.0\text{N}}{0.2\text{kg}} = 20\text{m/s}^2$$

$$b) F_1 - F_2 = ma_x$$

$$a_x = \frac{F_1 - F_2}{m} = \frac{4.0\text{N} - 2.0\text{N}}{0.2\text{kg}} = 10\text{m/s}^2$$

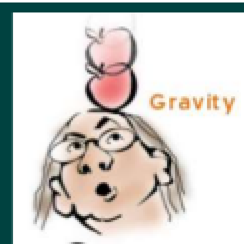
$$F_{net,x} = ma_x$$

$$c) F_{3,x} - F_2 = ma_x \qquad F_{3,x} = F_3 \cos\theta$$

$$a_x = \frac{F_3 \cos\theta - F_2}{m} = \frac{1.0\text{N} \cos 30° - 2.0\text{N}}{0.2\text{kg}}$$

$$= -5.7\text{m/s}^2$$

## 4.4. Types of Forces




### a) Gravitational Force

- Gravitational force is a vector
- Expressed by Newton's Law of Universal Gravitation:

$$F_g = G\frac{Mm}{R^2}$$

G – gravitational constant, $6.67\times10^{-11}$ N.m$^2$/kg$^2$.

M – mass of the Earth, $5.97\times10^{24}$ kg

m – mass of an object

R – radius of the Earth, 64000 km

- Direction: pointing downward

# Weight

- The magnitude of the gravitational force acting on an object of mass $m$ near the Earth's surface is called the weight $W$ of the object:

$$W = F_g = mg$$

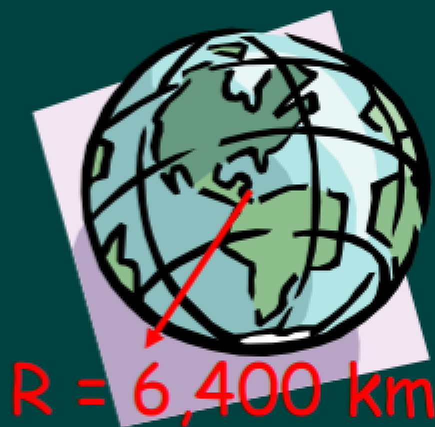
- $g$ can also be found from the **Law of Universal Gravitation**

$$F_g = G\frac{mM}{R^2}$$

$$\Rightarrow g = G\frac{M}{R^2} = 9.8\ \mathrm{m/s^2}$$

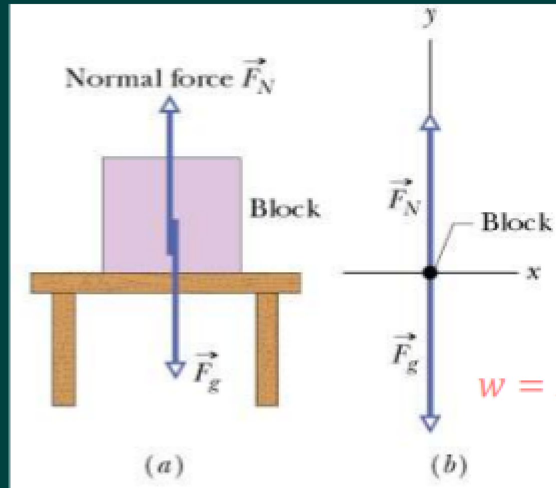
- Weight has a unit of: N

- Weight depends upon location



# Normal Force

- Force from a solid surface which keeps object from falling through
- Direction: always **perpendicular to the surface**
- Magnitude: depends on situation




$$w = F_g = mg$$

$$N - F_g = ma_y$$

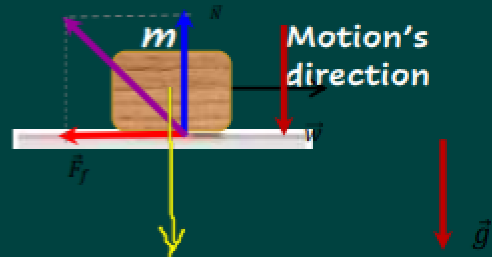
$$N - mg = ma_y$$

$$N = mg$$

## When an object is moving in contact with a surface, it is experienced by

(1) **Gravitational force** or weight the earth exerts on a body that is proportional to gravitational acceleration $g$ ⇒ according to Newton's second law : $\vec{w} = m\vec{g}$

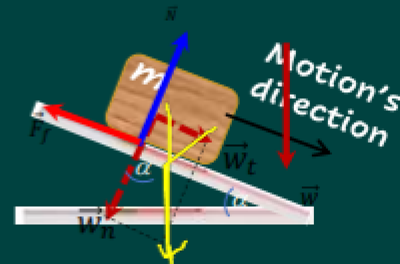
- Direction: vertical to the earth's surface.
- Heading: toward to the earth's surface.




(2) **Contact forces:** the surface exerts on object

- One component that acts perpendicular to the surface of contact, is so-called *Normal* ($\vec{N}$)

- Other one that acts parallel to the surface and in opposite to motion's direction, is so-called *Frictional force*.




$\vec{F_f} = \mu\vec{N}$ ($\mu$: coefficient of kinetic friction)

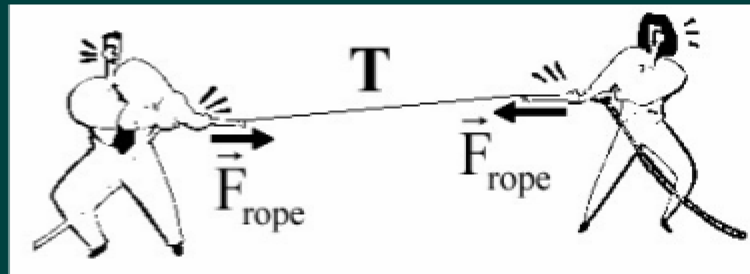
$w_t = w.\sin\alpha$

$w_n = w.\cos\alpha$

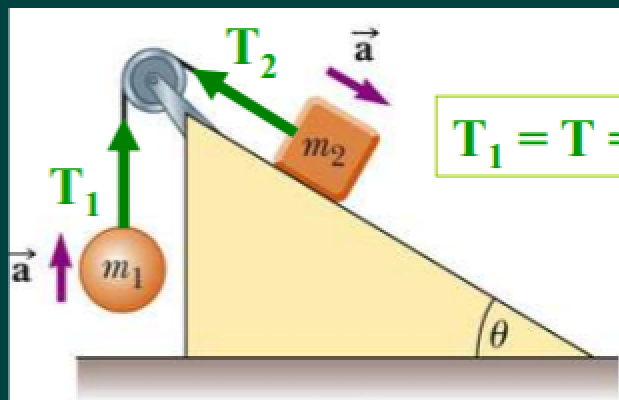
- The vector addition of normal and friction force gives the vector sum which is just *contact force*.

# Tension Force: $T$

- A taut rope exerts forces on whatever holds its ends
- Direction: always along the cord (rope, cable, string ......) and away from the object
- Magnitude: depend on situation




$$\left|\vec{F}_{on\ A}\right| = T = \left|\vec{F}_{on\ B}\right|$$



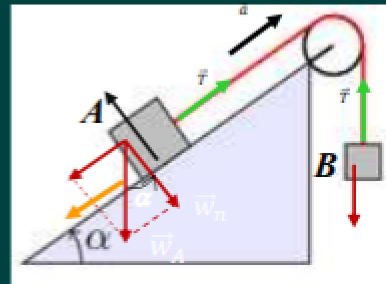

$$T_1 = T = T_2$$

# Example

**☛ Given**

♦ $m_A$ and $m_B$ are connected by a rope that both objects acted by the same tension force.

♦ $m_A$ moves on a ramp at an inclined angle of $\alpha$, motion's direction of $m_B$ is vertical, both have the same accelerations.




☞ Free body diagram and vector dynamic equations

♦ For A: $m_A\vec{a} = \vec{N} + \vec{w}_A + \vec{T} + \vec{F}_f$

(**noted**: $w_A$ has $w_n$- and $w_t$-components which are ⊥ and // to the ramp)

♦ For B: $m_B\vec{a} = \vec{w}_B + \vec{T}$

☞ Solution

♦ Take the projection of force vectors along the direction of acceleration vectors of each object resulting in

♦ For A: $\boxed{m_A a = T - \mu N - m_A g\sin\alpha \quad (1)}$

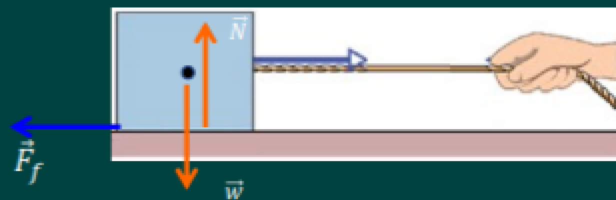
♦ For B: $\boxed{m_B a = m_B g - T \quad (2)}$

# Free body diagram

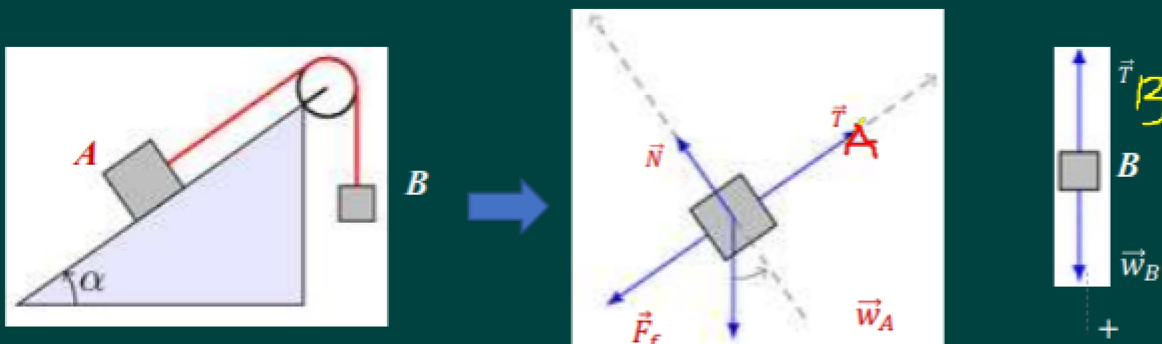
☞ For making the analysis of problem of particle's dynamic it is necessary to identify all the forces acting on the body, not forces body exert to the others.

☞ Free-body diagrams are essential to help identify the relevant forces,

♦ showing the chosen body by itself, "free" of its surroundings, with vectors drawn to demonstrate the magnitudes and directions of all the forces applied to the body that interact with it.




♦ When a problem involves more than one body, it must be considered apart, then the separate free-body diagram is drawn for each body.



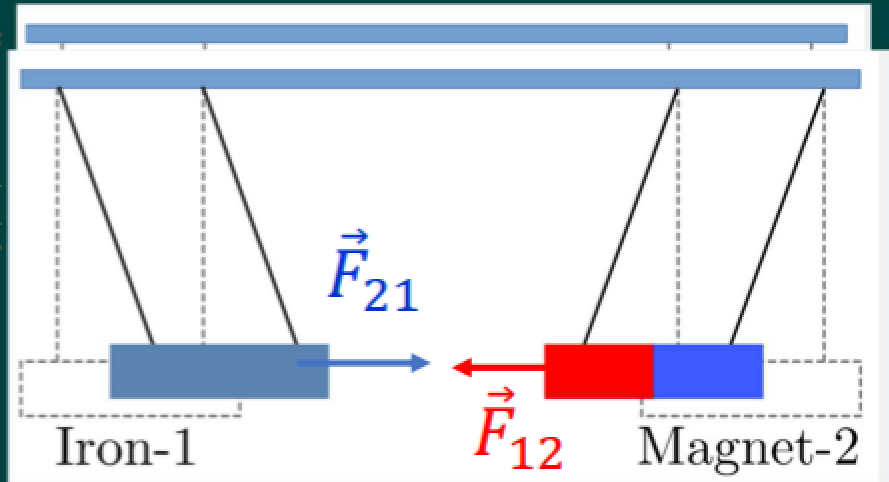
## 4.5. Newton's Third Law of motion

- A force acting on an object must be acted by another object

- The hammer acts an action force on the nail, pushing the nail into wood. What makes the hammer slows down and stops?
- The nail must act a reaction force back on the hammer.
- We say that the hammer and the nail interact each other.

# 4.5. Newton's Third Law of motion (cont.)

- Hang two bars which have identical shape and mass.
- The magnet attracts the iron bar, makes the cables hang the iron bar inclined.



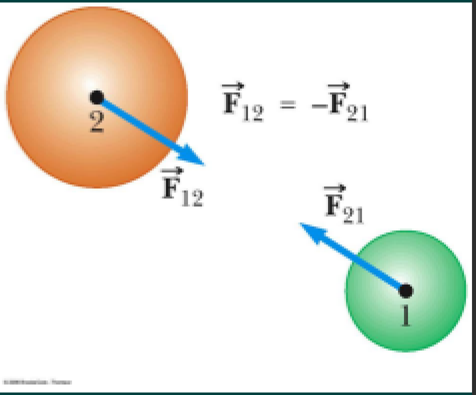

- However, the cables hang the magnet are also inclined. This means that the iron bar attracts the magnet too.
- Also, the angles of inclination are the same, meaning that the interacting forces have the same magnitude:

$$\vec{F}_{21} = -\vec{F}_{12}$$

## 4.5. Newton's Third Law

- When an object exerts a force on another, the second exerts an equal and opposite force on the first.

$$\vec{F}_{12} = -\vec{F}_{21}$$



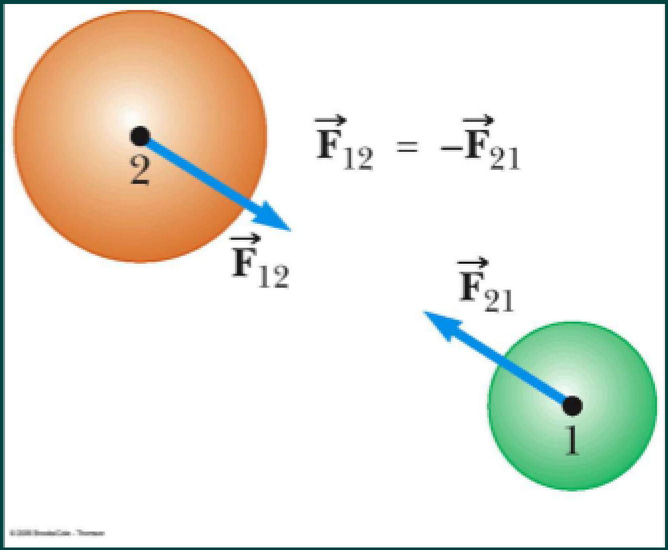

- Notice that $\vec{F}_{21}$ and $\vec{F}_{12}$ are interacting forces, one is called action and the other is called reaction. Neither of them exists without the other.
- However, the two forces do not balance each other since they act on different objects.

**If object 1 and object 2 interact, the force exerted by object 1 on object 2 is equal in magnitude but opposite in direction to the force exerted by object 2 on object 1:**

$$\vec{F}_{12} = -\vec{F}_{21}$$

## 4.5. Newton's Third Law (cont.)

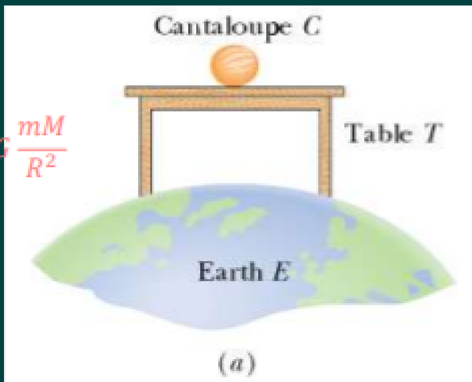
- $F_{12}$ may be called the *action* force and $F_{21}$ the *reaction* force
  - Actually, either force can be the action or the reaction force
- The action and reaction forces act on **different** objects

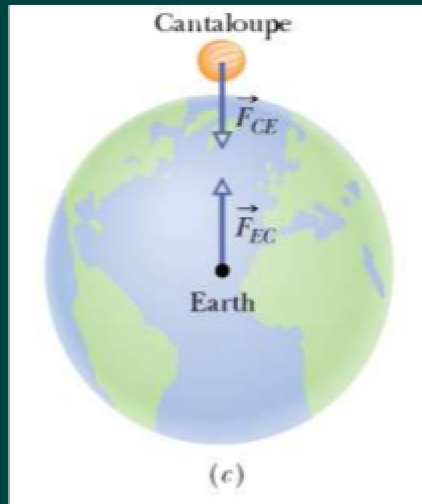



*Equivalent to saying a single isolated force cannot exist*

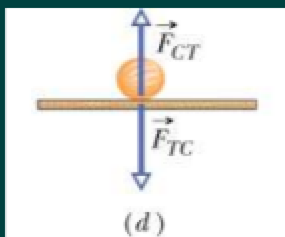
# Some Action-Reaction Pairs

$F_g = G\frac{mM}{R^2}$








$$F_g = G\frac{mM}{R^2}$$

$$F_g = mg = m\frac{GM}{R^2}$$

$$F_g = Ma = M\frac{Gm}{R^2}$$

# 4.6. Newton's law of Gravitation

**Universal gravitation:**

- My grandma told me that the Earth was flat, above the sky was the heaven and under the ground was the hell.
  - If the Earth is a sphere, how can people on the other side stay on the ground? Would they fall out of the Earth?
- Physics helps me know that people all around the spherical Earth are attracted by a force of gravity (gravitational force).

We are acted by the gravity because we have a mass.

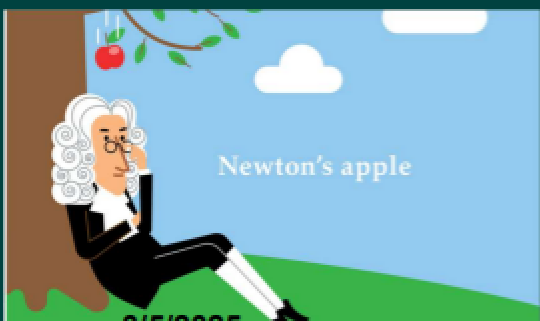
The Earth gives the force because it has a mass

**The origin of gravitational force is the mass.**

# 4.6. Newton's law of Gravitation

**Universal gravitation**:

- **Newton** was inspired by the **apple incident**.
  - If gravity acts at the top of a tree then perhaps it acts all the way to the Moon. This produces a force that keeps the Moon in its orbit around the Earth.
  - In the next step, he believed that what keeps the planets in their orbits around the Sun is also the gravitational force.
  - Furthermore, if gravity acts between these objects, why not between all objects on the universe?

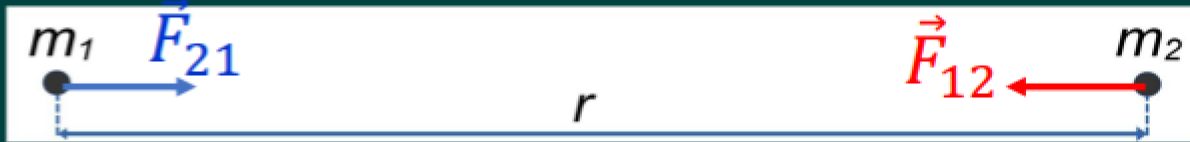



**Gravitational force is universal.**
(It appears for any objects which have masses.)

# 4.6. Newton's law of Gravitation

**Statement:**

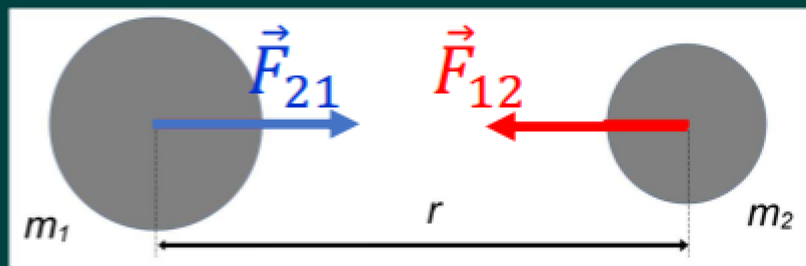
- Two point masses attract each other with a force which is proportional to the product of their masses and inversely proportional to the square of their separation.




$F \propto m_1 m_2$ $\quad F \propto \frac{1}{r^2}$ $\longrightarrow$ $\boxed{F = G\frac{m_1 m_2}{r^2}}$

- The **coefficient G** is called the **gravitational constant**.

$$\boxed{G = 6.67 \times 10^{-11}\,\mathrm{Nm^2\,kg^{-2}}}$$



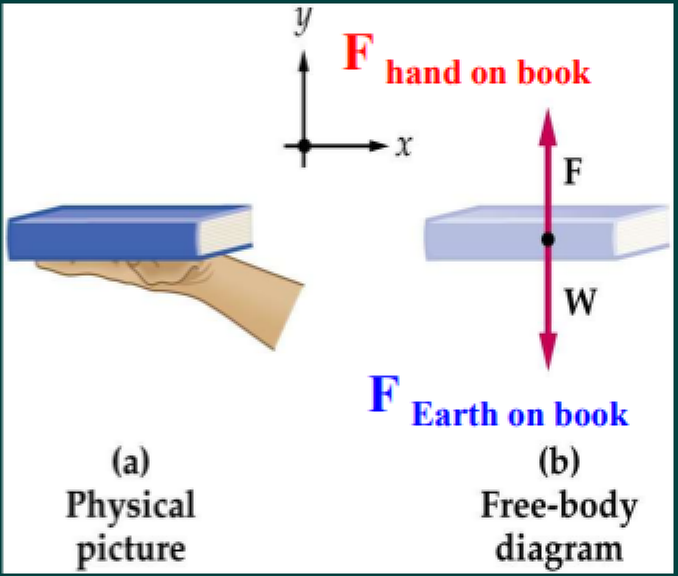

**Note that:**

- The law is stated for two point masses.
- However, it can be applied for 2 uniform spheres with $r$ is the distance between **their centers**.

## 4.7. Solving a problem

## Free Body Diagram

- The most important step in solving problems involving Newton's Laws is to draw the free body diagram
- Be sure to include only the forces acting on the object of interest
- Include any field forces acting on the object
- Do not assume the normal force equals the weight



## Hints for Problem-Solving

- **Read** the problem carefully at least once
- **Draw** a picture of the system, identify the object of primary interest, and indicate forces with arrows
- **Label** each force in the picture in a way that will bring to mind what physical quantity the label stands for (e.g., T for tension)
- **Draw** a free-body diagram of the object of interest, based on the labeled picture. If additional objects are involved, draw separate free-body diagram for them
- **Choose a convenient coordinate system** for each object
- **Apply Newton's second law.** The x- and y-components of Newton second law should be taken from the vector equation and written individually. This often results in two equations and two unknowns
- **Solve** for the desired unknown quantity, and substitute the numbers

$$F_{net,x} = ma_x \qquad F_{net,y} = ma_y$$

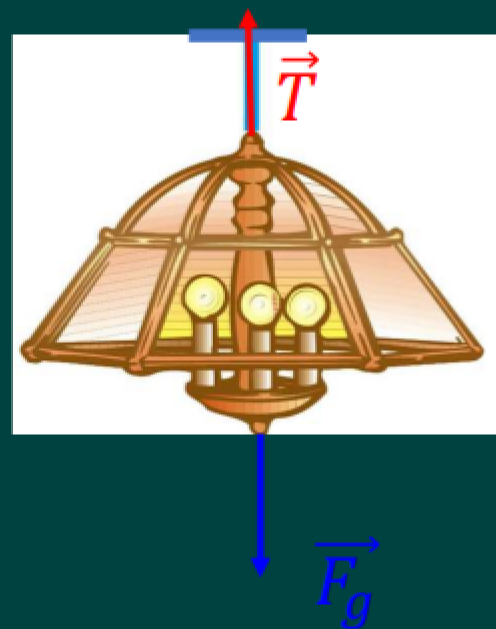
## 4.6.1. Objects in Equilibrium

- Objects that are either at rest or moving with constant velocity are said to be in **equilibrium**
- Acceleration of an object can be modeled as zero: $\vec{a} = 0$
- Mathematically, the net force acting on the object is zero.
- Equivalent to the set of component equations given by

$$\sum \vec{F} = 0 \qquad \begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum F_z = 0 \end{cases}$$

# Equilibrium: Example 1

- A lamp is suspended from a chain of negligible mass
- The forces acting on the lamp are
  - the downward force of gravity $F_g$
  - the upward tension in the chain: T
- Applying equilibrium gives

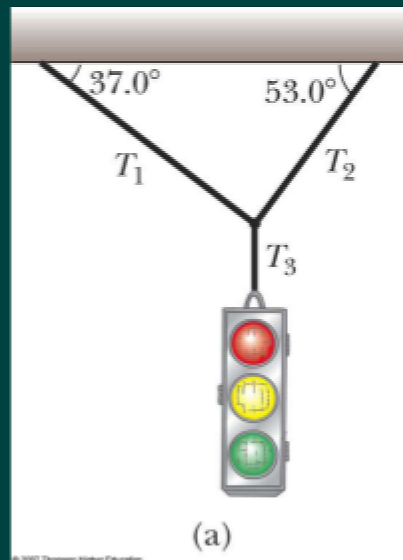



$$\vec{T} + \vec{F_g} = 0$$

$$\sum F_y = 0 \rightarrow T - F_g = 0 \rightarrow T = F_g$$

# Equilibrium, Example 2

- A traffic light weighing 100 N hangs from a vertical cable tied to two other cables that are fastened to a support. The upper cables make angles of 37° and 53° with the horizontal. Find the tension in each of the three cables.
  - Conceptualize the traffic light
    - Assume cables don't break
    - Nothing is moving
  - Categorize as an equilibrium problem
    - No movement, so acceleration is zero
    - Model as an object in equilibrium




$$\sum F_x = 0$$

$$\sum F_y = 0$$

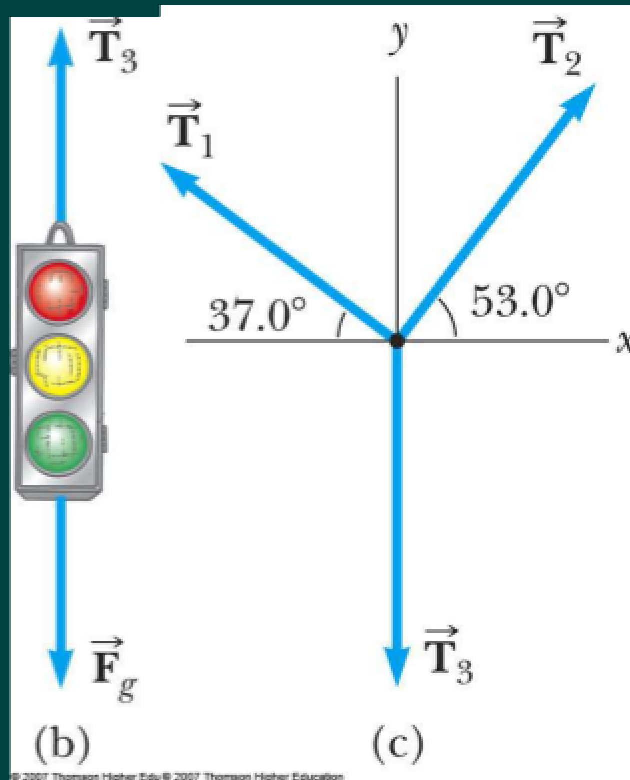
# Equilibrium, Example 2

- Need 2 free-body diagrams
  - Apply equilibrium equation to light

$$\sum F_y = 0 \rightarrow T_3 - F_g = 0$$
$$T_3 = F_g = 100N$$

  - Apply equilibrium equations to knot

$$\sum F_x = T_{1x} + T_{2x} = -T_1 \cos 37^\circ + T_2 \cos 53^\circ = 0$$
$$\sum F_y = T_{1y} + T_{2y} + T'_{3y} = T_1 \sin 37^\circ + T_2 \sin 53^\circ - 100N = 0$$
$$T_2 = T_1 \left( \frac{\cos 37^\circ}{\cos 53^\circ} \right) = 1.33T_1$$
$$T_1 = 60N \qquad T_2 = 1.33T_1 = 80N$$



## 4.6.2. Accelerating Objects

- If an object that can be modeled as a particle experiences an acceleration, there must be a nonzero net force acting on it
- Draw a free-body diagram
- Apply Newton's Second Law in component form

$$\sum \vec{F} = m\vec{a}$$

$$\sum F_x = ma_x \qquad \sum F_y = ma_y$$

# Accelerating Objects, Example

- A man weighs himself with a scale in an elevator. While the elevator is at rest, he measures a weight of 800 N.
  - What weight does the scale read if the elevator accelerates **upward** at 2.0 m/s²? a = 2.0 m/s²
  - What weight does the scale read if the elevator accelerates **downward** at 2.0 m/s²? a = - 2.0 m/s²

❑ **Upward:** $\sum F_y = N - mg = ma$

$$N = mg + ma = m(g + a)$$

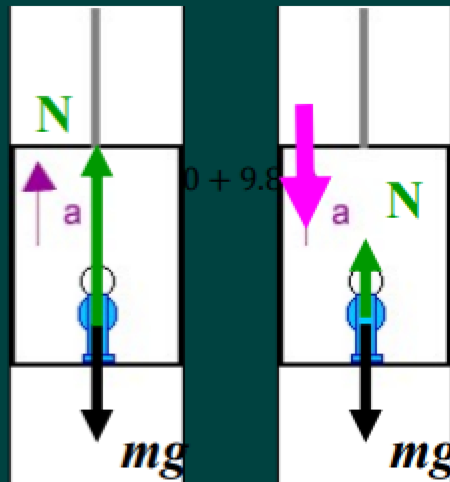
$$m = \frac{w}{g} = \frac{800\text{N}}{9.8\text{m/s}^2} = 80\text{N}$$

$$N = 80(2.0 + 9.8) = 944N$$

$$\boxed{N > mg}$$

❑ **Downward:** $N = 80(-2.0 + 9.8) = 624\text{N}$

$$\boxed{N < mg}$$



# Chapter 4. The Laws of Motion

1. Dynamics: Force and Mass
2. Newton's first law
3. Newton's second law
4. Types of forces
5. Newton's third law
6. Newton's law of gravity
7. Solving a problem: Examples
8. **Application of Newton's Laws**

1642-1726

*Isaac Newton's work represents one of the greatest contributions to science ever made by an individual.*

# Sumary: Newton's Laws

I. If no net **force** acts on a body, then the body's velocity cannot change. If $\vec{F}_{net} = 0$ then $\vec{v} = \overrightarrow{const}$ (or $\vec{a} = 0$)

II. The net **force** on a body is equal to the product of the body's mass and acceleration. $\vec{F}_{net} = \sum \vec{F} = m\vec{a}$

III. When two bodies interact, the force on the bodies from each other are always equal in magnitude and opposite in direction. $\vec{F}_{21} = -\vec{F}_{12}$

**Force is a vector**

**Unit of force in S.I.: N = kg.m/s$^2$**

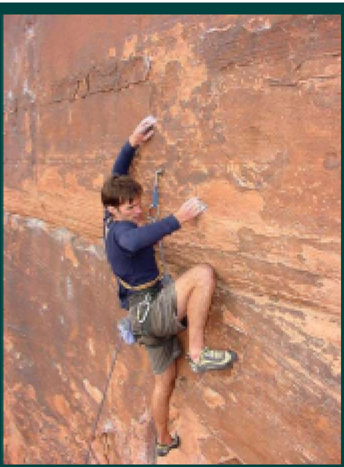
# Chapter 4. The Laws of Motion (cont.)
## 4.8. Applications of Newton's Laws

4.8.1. Frictional forces

4.8.2. Circular Motion

*Isaac Newton's work represents one of the greatest contributions to science ever made by an individual.*

## 4.8.1. Forces of Friction: $f$



- When an object is in motion on a surface or through a viscous medium, there will be a resistance to the motion. This resistance is called the **force of friction**
- This is due to the interactions between the object and its environment
- We will be concerned with two types of frictional force
  - Force of static friction: $f_s$
  - Force of kinetic friction: $f_k$

- Direction: opposite to the direction of the intended motion
  - **If moving: $f_k$ in direction opposite to the velocity**
  - **If stationary, $f_s$ in direction opposite to the vector sum of other forces**

## 4.8.1. Forces of Friction: Magnitude

- Magnitude: Friction is proportional to the normal force
  - Static friction: $F_f = F \leq \mu_s N$
  - Kinetic friction: $F_k = \mu_k N$
  - $\mu$ is the **coefficient of friction**
- The coefficients of friction are nearly independent of the area of contact (why?)

**TABLE 5.1**

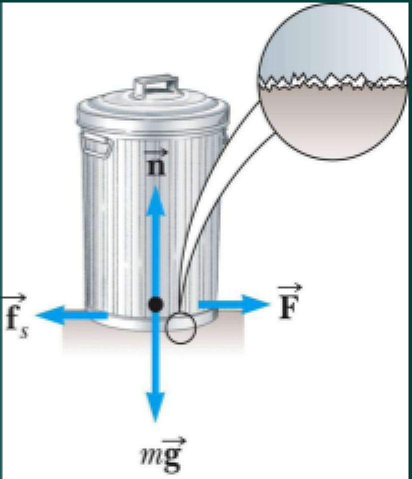
**Coefficients of Friction**

| | $\mu_s$ | $\mu_k$ |
|---|---|---|
| Rubber on concrete | 1.0 | 0.8 |
| Steel on steel | 0.74 | 0.57 |
| Aluminum on steel | 0.61 | 0.47 |
| Glass on glass | 0.94 | 0.4 |
| Copper on steel | 0.53 | 0.36 |
| Wood on wood | 0.25–0.5 | 0.2 |
| Waxed wood on wet snow | 0.14 | 0.1 |
| Waxed wood on dry snow | — | 0.04 |
| Metal on metal (lubricated) | 0.15 | 0.06 |
| Teflon on Teflon | 0.04 | 0.04 |
| Ice on ice | 0.1 | 0.03 |
| Synovial joints in humans | 0.01 | 0.003 |

*Note:* All values are approximate. In some cases, the coefficient of friction can exceed 1.0.

# Static Friction

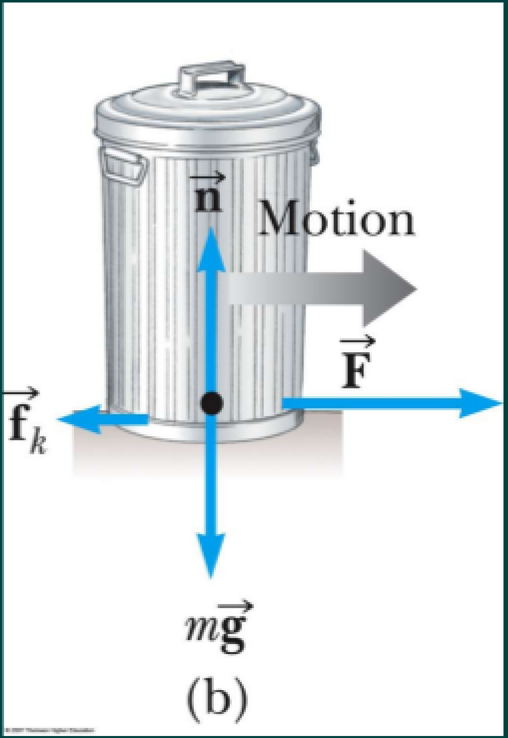
- Static friction acts to keep the object from moving
- If $\vec{F}$ increases, so does $\vec{f}_s$
- If $\vec{F}$ decreases, so does $\vec{f}_s$
- $f_s \le \mu_s N$
  - Remember, the equality holds when the surfaces are on the verge of slipping



(a)

# Kinetic Friction

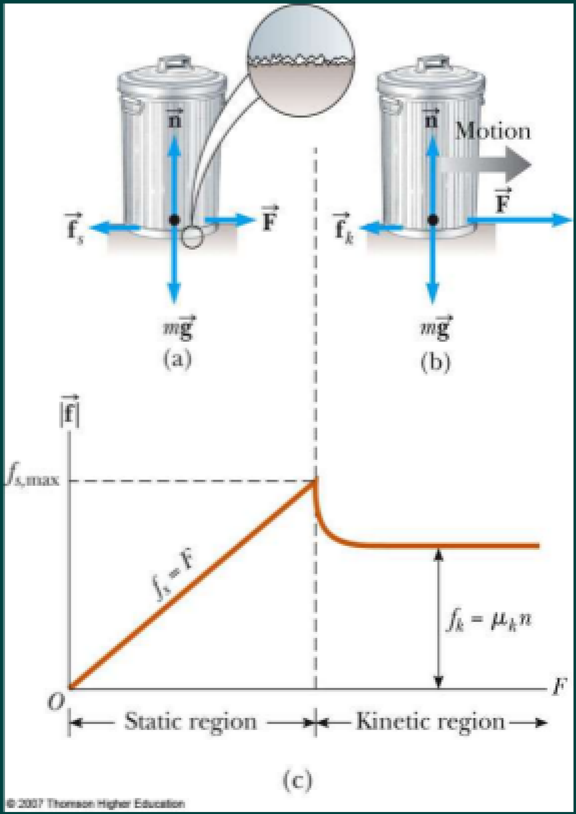
- The force of kinetic friction acts when the object is in motion
- Although $\mu_k$ can vary with speed, we shall neglect any such variations
- $f_k = \mu_k N$



(b)

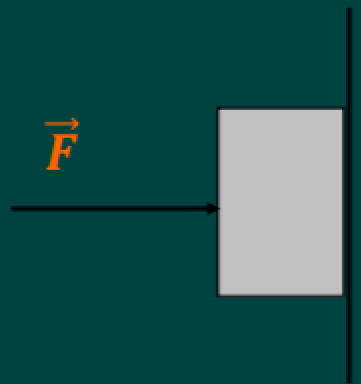
# Explore Forces of Friction

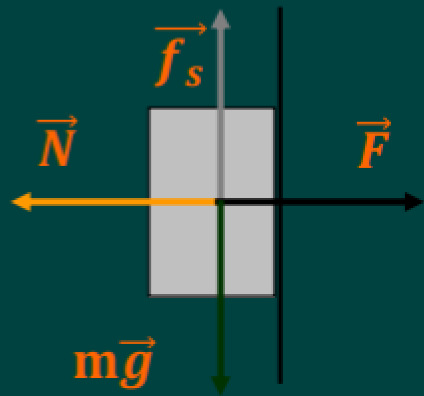
- Vary the applied force
- Note the value of the frictional force
    - Compare the values
- Note what happens when the can starts to move



# Example 1: Equilibrium

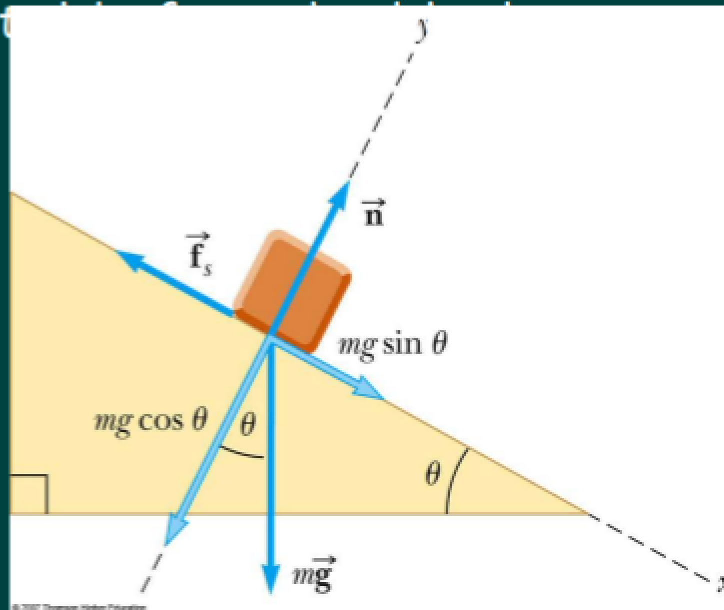
- What is the smallest value of the force F such that the 2.0-kg block will not slide down the wall? The coefficient of static friction between the block and the wall is 0.2. ?



## Example 2: Inclined Plane

- Suppose a block with a mass of 2.50 kg is resting on a ramp. If the coefficient of static friction between the block and ramp is 0.350, what maximum angle can the ramp make with the horizont
to slip down?




- Newton 2nd law:

$$\sum F_x = mg\sin\theta - \mu_s N = 0$$
$$\sum F_y = N - mg\cos\theta = 0$$

- Then

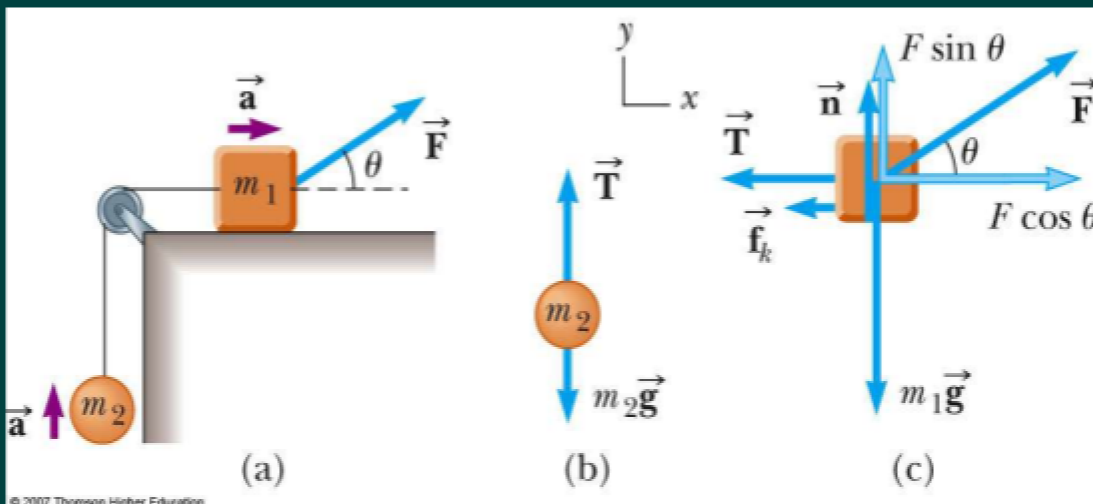
$N = mg\cos\theta$

$\sum F_y = mg\sin\theta - \mu_s mg\cos\theta = 0$

- So: $\tan\theta = \mu_s = 0.350$
- Then: $\theta = \tan^{-1}(0.350) = 19.3°$

# Example 3: Multiple Objects

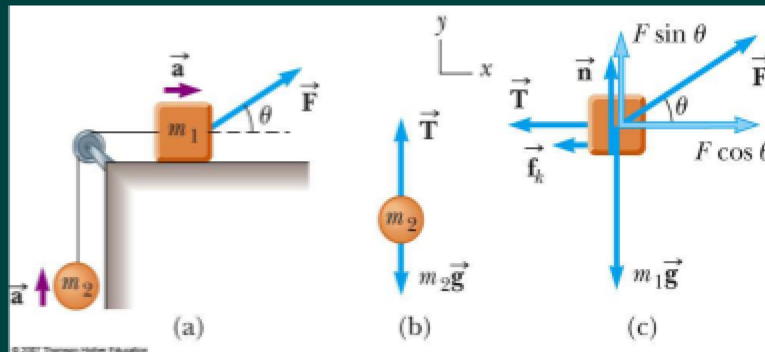
- A block of mass $m_1$ on a rough, horizontal surface is connected to a ball of mass $m_2$ by a lightweight cord over a lightweight, frictionless pulley as shown in figure. A force of magnitude F at an angle $\theta$ with the horizontal is applied to the block as shown and the block slides to the right. The coefficient of kinetic friction between the block and surface is $\mu_k$. Find the magnitude of acceleration of the two objects.



# Example 3: Multiple Objects (cont.)

- $\boldsymbol{m_1}$: $\quad \boldsymbol{m_1\vec{g} + \vec{N} + \vec{F} + \vec{T}_1 = m_1\vec{a}_1}$

$$\sum F_x = F\cos\theta - f_k - T = m_1 a_x = m_1 a$$
$$\sum F_y = N + F\sin\theta - m_1 g = 0$$




- $\boldsymbol{m_2}$: $\quad \boldsymbol{m_2\overrightarrow{a_2} + \vec{T}_2 = m_1\vec{a}_2}$

$$\sum F_y = T - m_2 g = m_2 a_y = m_2 a$$

$$T = m_2(a+g)$$

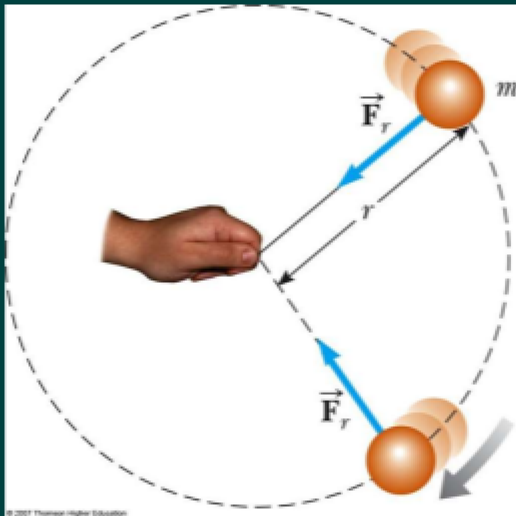
$$N = m_1 g - F\sin\theta$$

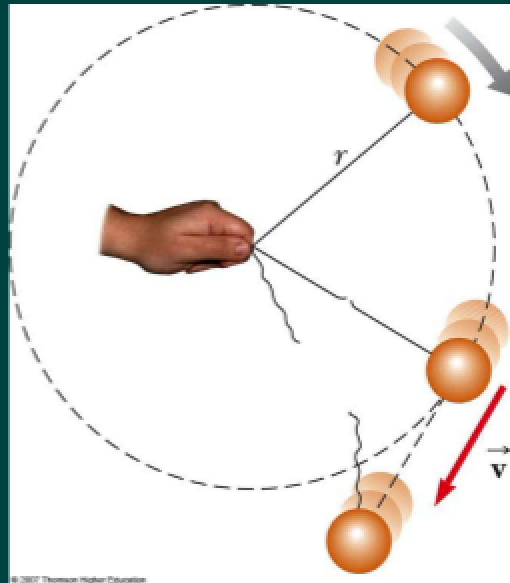
$$f_k = \mu_k N = \mu_k(m_1 g - F\sin\theta)$$

$$F\cos\theta - \mu_k(m_1 g - F\sin\theta) - m_2(a+g) = m_1 a$$

$$\boxed{a = \frac{F(\cos\theta + \mu_k\sin\theta) - (m_2 + \mu_k m_1)g}{m_1 + m_2}}$$

# 4.7.2. Uniform Circular Motion: Definition

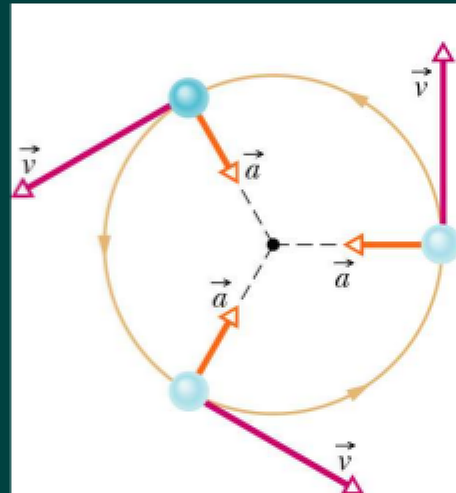



Uniform **circular motion**

| Uniform | circular motion |
|---|---|
| Constant speed, or, constant magnitude of velocity | Motion along a circle: Changing direction of velocity |

# 4.8.2 Uniform Circular Motion (review)




- Velocity:
  - *Magnitude: constant $v$*
  - The direction of the velocity is tangent to the circle
- Acceleration: $\vec{a}_c \perp \vec{v}$
  - Magnitude: $a_c = \frac{v^2}{r}$
  - directed toward the center of the circle of motion
- Period:
  - time interval required for one complete revolution of the particle $T = \frac{2\pi r}{v}$

✓ Acceleration is NOT zero!

✓ **Net force acting on the object is NOT zero**

✓ **"Centripetal force"**

# What provides Centripetal Force ?

- Centripetal force is not a new kind of force
- Centripetal force refers to any force that keeps an object following a circular path:

$$F_c = ma_c = \frac{mv^2}{r}$$

- Centripetal force maybe a combination of

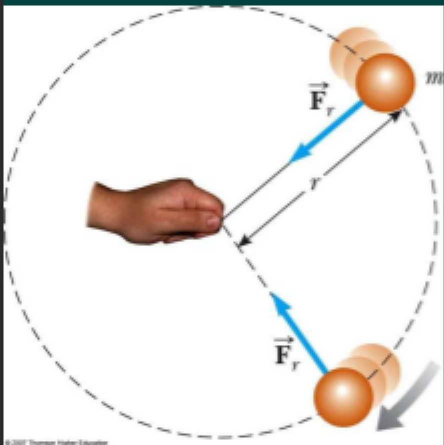
Gravitational force $m\mathbf{g}$: downward to the ground

Normal force $\mathbf{N}$: perpendicular to the surface

Tension force $\mathbf{T}$: along the cord and away from object

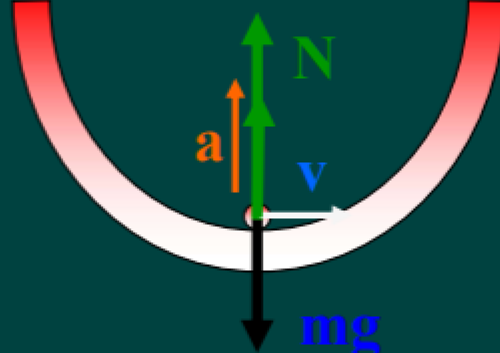
Static friction force: $f_s^{max} = \mu_s N$

# What provides Centripetal Force ?




$$F_{net} = N - mg = ma$$
$$N = mg + m\frac{v^2}{r}$$

$$F_{net} = T$$
$$= ma$$
$$T = \frac{mv^2}{r}$$

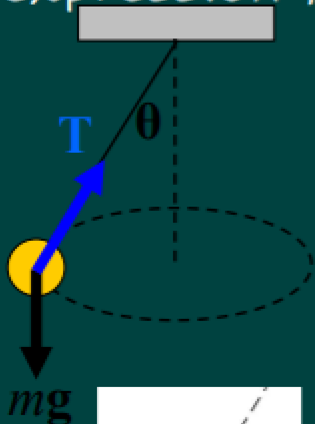


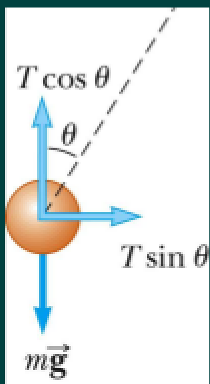
# Problem Solving Strategy

- **Draw a free body diagram**, showing and labeling all the forces acting on the object(s)
- **Choose a coordinate system** that has one axis perpendicular to the circular path and the other axis tangent to the circular path
- **Find the net force toward the center** of the circular path (this is the force that causes the centripetal acceleration, $F_c$)
- **Use Newton's second law**
  - The directions will be radial, normal, and tangential
  - The acceleration in the radial direction will be the centripetal acceleration
- **Solve for the unknown(s)**

## Example 1: The Conical Pendulum

- A small ball of mass $m$ = 5 kg is suspended from a string of length $L$ = 5 m. The ball revolves with constant speed $v$ in a horizontal circle of radius $r$ = 2.0 m. Find an expression for $v$ and $a$.




$$m = 5\,kg \quad L = 5\,m \quad r = 2\,m$$

$$\sum F_y = T\cos\theta - mg = 0$$

$$T\cos\theta = mg$$

$$\sum F_x = T\sin\theta = \frac{mv^2}{r}$$

$$\sin\theta = \frac{r}{L} = 0.4$$

$$\tan\theta = \frac{r}{\sqrt{L^2 - r^2}} = 0.44$$

$$T\sin\theta = \frac{mv^2}{r}$$

$$T\cos\theta = mg$$

$$\tan\theta = \frac{v^2}{gr}$$

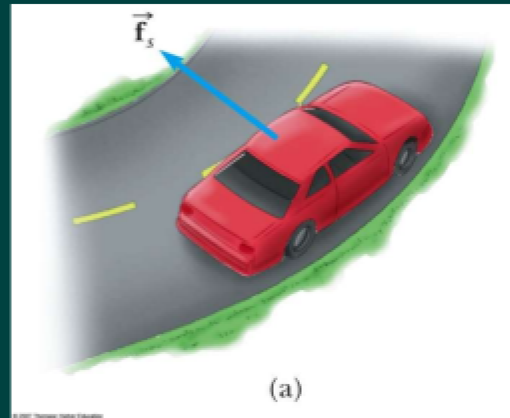
$$v = \sqrt{rg\tan\theta}$$

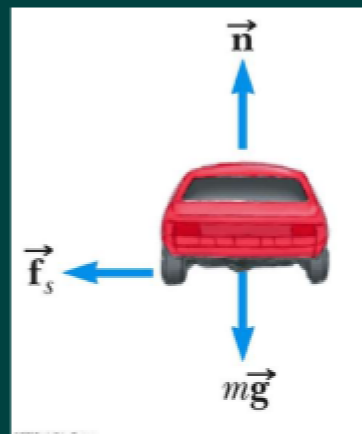
$$v = \sqrt{Lg\sin\theta\tan\theta} = 2.9\ \mathrm{m/s}$$

$$a = \frac{v^2}{r} = g\tan\theta = 4.3\ \mathrm{m/s^2}$$

# Example 2: Level Curves

- A 1500 kg car moving on a flat, horizontal road negotiates a curve as shown. If the radius of the curve is 35.0 m and the coefficient of static friction between the tires and dry pavement is 0.523, find the maximum speed the car can have and still make the turn successfully.

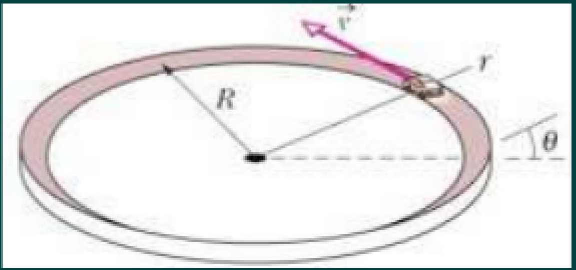


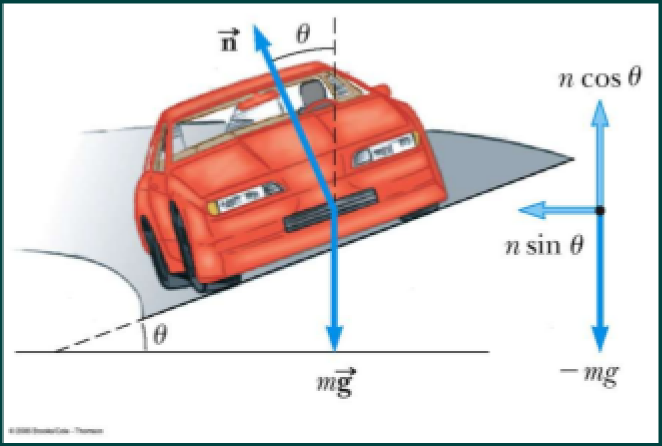




$$v = \sqrt{\mu r g}$$

# Example 3: Banked Curves

- A car moving at the designated speed can negotiate the curve. Such a ramp is usually banked, which means that the roadway is tilted toward the inside of the curve. Suppose the designated speed for the ramp is to be 13.4 m/s and the radius of the curve is 35.0 m. At what angle should the curve be banked?



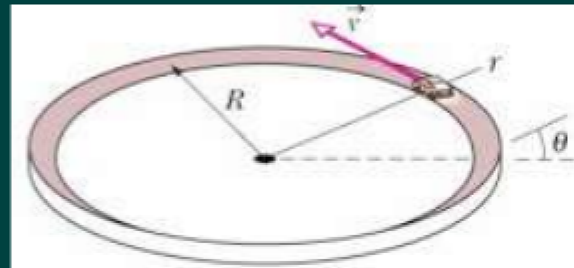



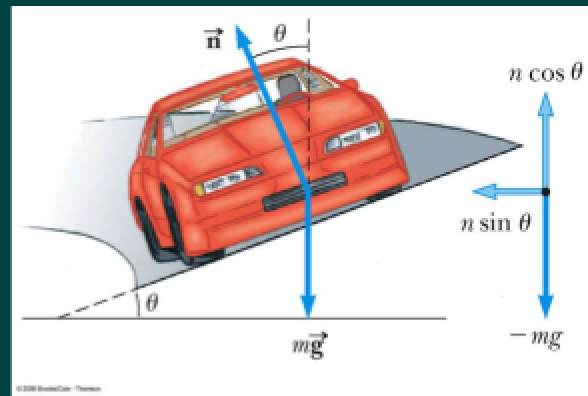
# Example 3: Banked Curves

$v = 13.4 \text{ m/s} \quad r = 35.0 \text{ m}$

$$\sum F_r = n\sin\theta = ma_c = \frac{mv^2}{r}$$

$$\sum F_y = n\cos\theta - mg = 0$$




$$n\cos\theta = mg$$

$$\tan\theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1}\left(\frac{13.4 \text{ m/s}}{(35.0 \text{ m})(9.8 \text{ m/s}^2)}\right) = 27.6^\circ$$