Chapter 6.
(Linear) Momentum and Collisions

6.1. (Linear) Momentum

- This is a new fundamental quantity, like force, energy. It is a vector quantity (points in same direction as velocity).
- The linear momentum $\vec{p}$ of an object of mass *m* moving with a velocity $\vec{v}$ is defined to be the product of the mass and velocity:

$$\boxed{\vec{p} = m\vec{v}}$$

- The terms momentum and linear momentum will be used interchangeably in the text.
- Momentum depend on an object's mass and velocity

??? Momentum and Energy

MCQ: Two objects with masses m_1 and m_2 have equal kinetic energy. How do the magnitudes of their momenta compare?

(A) $p_1 < p_2$ (B) $p_1 = p_2$

(C) $p_1 > p_2$ (D) Not enough information is given

MCQ

??? Momentum and Energy

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(C) $p_1 > p_2$ (D) Not enough information is given

6.1. Linear Momentum, (cont'd)

- Linear momentum is a vector quantity: $\boxed{\vec{p} = m\vec{v}}$
  - Its direction is the same as the direction of the velocity
- The dimensions of momentum are **ML / T**
- The SI units of momentum are: **kg.m/s**
- Momentum can be expressed in component form:

$$p_x = mv_x \qquad p_y = mv_y \qquad p_z = mv_z$$

Newton's Law and Momentum

- Newton's Second Law can be used to relate the momentum of an object to the resultant force acting on it

$$\boxed{\vec{F}_{net} = m\vec{a} = m\frac{d\vec{v}}{dt} = \frac{d(m\vec{v})}{dt}}$$

(because m = const)

- The change in an object's momentum divided by the elapsed time equals the constant net force acting on the object

$$\boxed{\frac{d\vec{p}}{dt} = \vec{F}_{net}}$$

the 1st law of (linear) momentum

Incase, F is changing: $\frac{\Delta\vec{p}}{\Delta t} = \frac{\text{change in momentum}}{\text{time interval}} = \vec{F}_{average}$

<u>6.2. Impulse $\vec{I}$</u>

- There is an **impulse** *delivered to the object:*

$$\vec{I} = \int_{t_1}^{t_2} \vec{F} dt$$

✓is defined as the *impulse*

✓vector quantity, the direction depends on the direction of the force.

✓ Incase, $\vec{F} = \overrightarrow{const}$ (a single, constant force acts on the object) then $\vec{I} = \vec{F}\Delta t$. With direction is the same the direction of the force acting on the object.

Impulse-Momentum Theorem

- From relation (1st law): $\vec{\boldsymbol{F}}_{\boldsymbol{net}} = \dfrac{\boldsymbol{d\vec{p}}}{\boldsymbol{dt}}$

$$\boldsymbol{d\vec{p}} = \vec{\boldsymbol{F}}_{\boldsymbol{net}}\boldsymbol{dt}$$

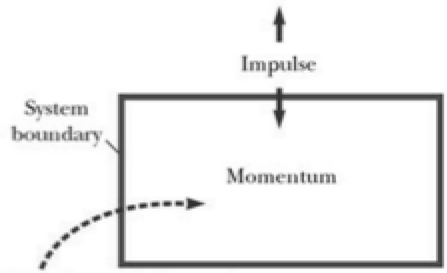
- The theorem states that the impulse acting on a system is equal to the change in momentum of the system

$$\Delta\vec{p} = \int_{\vec{p}_1}^{\vec{p}_2} \vec{p}dt = \int_{t_1}^{t_2} \vec{F}_{net}dt = \vec{I}$$

- Or: $\boldsymbol{\vec{I} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i}$

- Incase, F is changing

$$\Delta\vec{p} = \vec{F}_{net/average}\Delta t = \vec{I}$$



The change in the total momentum of the system is equal to the total impulse on the system.

Example 1: Calculating the Change of Momentum

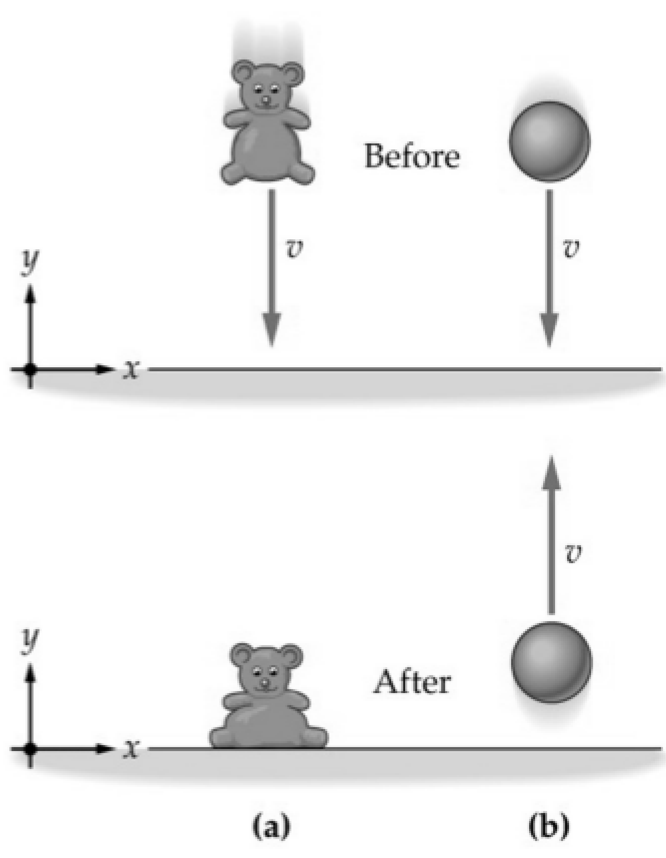
$$\begin{aligned}\Delta\vec{p} &= \vec{p}_{after} - \vec{p}_{before} \\ &= mv_{after} - mv_{before} \\ &= m(v_{after} - v_{before})\end{aligned}$$

For the teddy bear

$$\Delta p = m[0 - (-v)] = mv$$

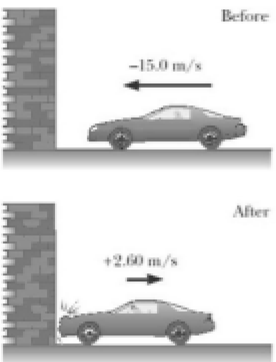
For the bouncing ball

$$\Delta p = m[v - (-v)] = 2mv$$



Example 2: How Good Are the Bumpers?

In a crash test, a car of mass 1.5×10^3 kg collides with a wall and rebounds as in figure. The initial and final velocities of the car are v_i=-15 m/s and v_f = 2.6 m/s, respectively. If the collision lasts for 0.15 s, find
(a) the impulse delivered to the car due to the collision
(b) the size and direction of the average force exerted on the car



$$p_i = mv_i = (1.5\times10^3kg)(-15m/s) = -2.25\times10^4 kg\cdot m/s$$

$$p_f = mv_f = (1.5\times10^3kg)(+2.6m/s) = +0.39\times10^4 kg\cdot m/s$$

$$\begin{aligned} I &= p_f - p_i = mv_f - mv_i \\ &= (0.39\times10^4 kg\cdot m/s) - (-2.25\times10^4 kg\cdot m/s) \\ &= 2.64\times10^4 kg\cdot m/s \end{aligned}$$

$$F_{av} = \frac{\Delta p}{\Delta t} = \frac{I}{\Delta t} = \frac{2.64\times10^4 kg\cdot m/s}{0.15s} = 1.76\times10^5 N$$

Example 3: Impulse-Momentum Theorem

- A child bounces a 100 g superball on the sidewalk. The velocity of the superball changes from 10 m/s downward to 10 m/s upward. If the contact time with the sidewalk is 0.1s, what is the magnitude of the impulse imparted to the superball?

  (A) *0*

  (B) *2* kg.m/s

  (C) *20* kg.m/s

  (D) *200* kg.m/s

  (E) *2000* kg.m/s

$$\boxed{\vec{I} = \Delta\vec{p} = m\vec{v}_f - m\vec{v}_i}$$

- what is the magnitude of the force between the sidewalk and the superball?

  (A) *0*

  (B) *2* N

  (C) *20* N

  (D) *200* N

  (E) *2000* N

$$\boxed{\vec{F} = \frac{\vec{I}}{\Delta t} = \frac{\Delta\vec{p}}{\Delta t} = \frac{m\vec{v}_f - m\vec{v}_i}{\Delta t}}$$

6.3. Conservation of Momentum

- In a system: Massese $m_1, m_2, \dots m_N$ Velocities: $\vec{v}_1$, $\vec{v}_2$

$$\frac{d\vec{p}_i}{dt} = \vec{F}_{i-external} + \vec{F}_{i-interna}$$

- Using 3rd Newton's Law => $\sum_{i=1}^{N} \vec{F}_{i-internal} = 0$, so

$$\sum_{i=1}^{N} \frac{d\vec{p}_i}{dt} = \sum_{i=1}^{N} \vec{F}_{i-external}$$

- Incase of isolated system: $\sum_{i=1}^{N} \vec{F}_{i-external} = 0$

$$=> \sum_{i=1}^{N} \vec{p}_i = const$$

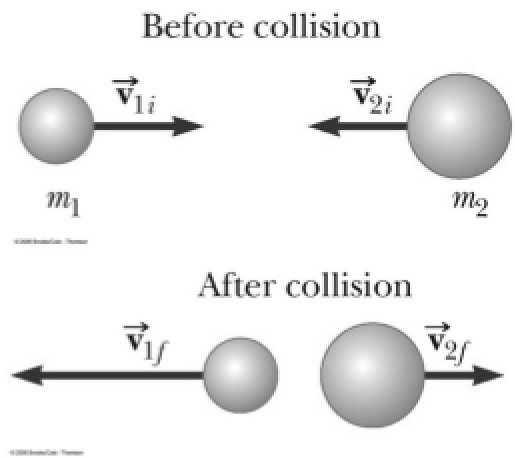
- the total momentum of the isolated system remains constant in time.
  - Isolated system: no external forces.
  - Closed system: no mass enters or leaves.
  - The linear momentum of each colliding body may change.
  - The total momentum $\vec{p}$ of the system cannot change.

6.4. Collision & Conservation of Momentum

- Start from impulse-momentum theorem

$$\vec{F}_{21}\Delta t = m_1\vec{v}_{1f} - m_1\vec{v}_{1i}$$

$$\vec{F}_{12}\Delta t = m_2\vec{v}_{2f} - m_2\vec{v}_{2i}$$

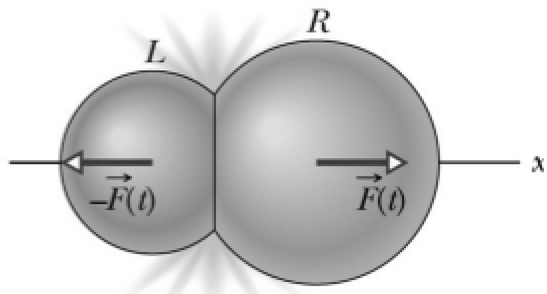


- Since: $\vec{F}_{21}\Delta t = -\vec{F}_{12}\Delta t$

- Then: $m_1\vec{v}_{1f} - m_1\vec{v}_{1i} = -(m_2\vec{v}_{2f} - m_2\vec{v}_{2i})$

- So:

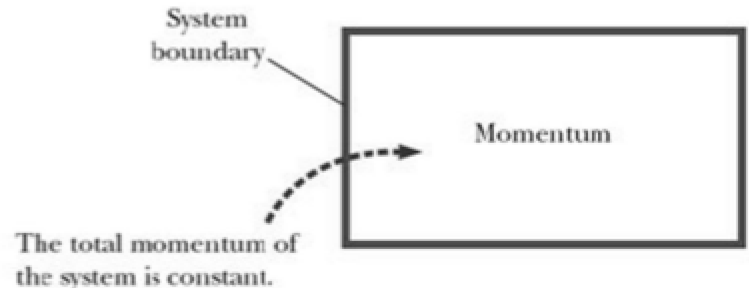
$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$



Conservation of Momentum: two objects

- When no external forces act on a system consisting of two objects that collide with each other, the total momentum of the system remains constant in time

$$\vec{F}_{net}\Delta t = \Delta\vec{p} = \vec{p}_f - \vec{p}_i$$



- When $\vec{F}_{net} = 0$

  then $\Delta\vec{p} = 0$

- For an isolated system

$$\vec{p}_f = \vec{p}_i$$

- Specifically, the total momentum before the collision will equal the total momentum after the collision:

$$m_1\vec{v}_{1i} + m_2\vec{v}_{2i} = m_1\vec{v}_{1f} + m_2\vec{v}_{2f}$$

Example 3: The Archer

An archer stands at rest on frictionless ice and fires a 0.5-kg arrow horizontally at 50.0 m/s. The combined mass of the archer and bow is 60.0 kg. With what velocity does the archer move across the ice after firing the arrow?

$m_1 = 60.0kg, m_2 = 0.5kg,$

$v_{1i} = v_{2i} = 0, v_{2f} = \dfrac{50m}{s},$

$v_{1f} = ?$

$$p_i = p_f$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$0 = m_1 v_{1f} + m_2 v_{2f}$$

$$v_{1f} = -\frac{m_2}{m_1} v_{2f} = -\frac{0.5kg}{60.0kg}(50.0m/s) = -0.417m/s$$

MCQ: Conservation of Momentum

A 100 kg man and 50 kg woman on ice skates stand facing each other. If the woman pushes the man backwards so that his final speed is 1 m/s, at what speed does she recoil?

(A) 0

(B) 0.5 m/s

(C) 1 m/s

(D) 1.414 m/s

(E) 2 m/s

6.4. Collision (cont's)
Types of Collisions

- Momentum is conserved in any collision
- **Inelastic collisions:** *rubber ball and hard ball*
  - Kinetic energy is not conserved
  - **Perfectly inelastic** collisions occur when the objects stick together
- **Elastic collisions:** *billiard ball*
  - both momentum and kinetic energy are conserved
- ***Actual collisions***
  - *Most collisions fall between elastic and perfectly inelastic collisions*

Collisions Summary

- In an elastic collision, both momentum and kinetic energy are conserved.
- In a non-perfect inelastic collision, momentum is conserved but kinetic energy is not. Moreover, the objects do not stick together.
- In a perfectly inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same.
- Elastic and perfectly inelastic collisions are limiting cases, most actual collisions fall in between these two types .
- Momentum is conserved in almost all collisions.

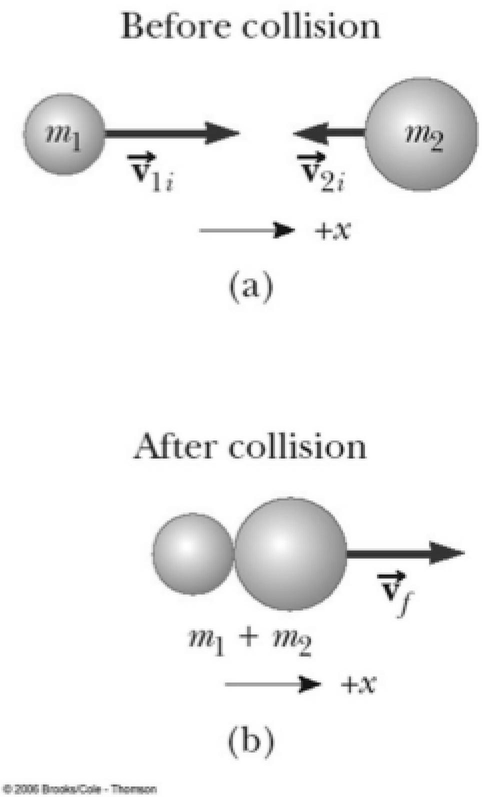
6.4.1. Perfectly Inelastic Collisions

- When two objects stick together after the collision, they have undergone a perfectly inelastic collision
- Conservation of momentum

$$\boxed{m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f}$$

- Kinetic energy is NOT conserved

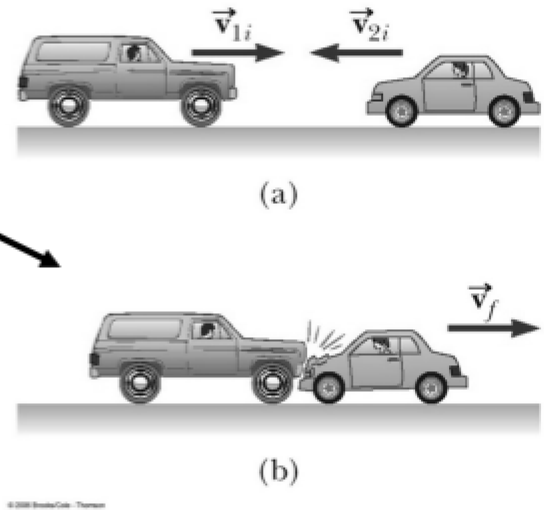
$$\boxed{v_f = \frac{m_1 v_{1i} + m_2 v_{2i}}{m_1 + m_2}}$$



An SUV Versus a Compact

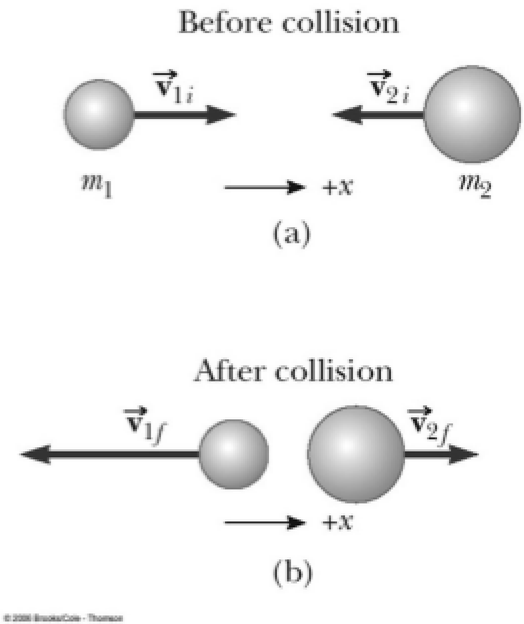
- An SUV with mass 1.80×10^3 kg is travelling eastbound at +15.0 m/s, while a compact car with mass 9.0×10^2 kg is travelling westbound at -15.0 m/s. The cars collide head-on, becoming entangled.

a) Find the speed of the entangled cars after the collision.
b) Find the change in the velocity of each car.
c) Find the change in the kinetic energy of the system consisting of both cars.



6.4.2. Elastic Collisions

- Both momentum and kinetic energy are conserved



$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

$$\frac{1}{2} m_1 v_{1i}^2 + \frac{1}{2} m_2 v_{2i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$

- Typically have two unknowns
- Momentum is a vector quantity
  - Direction is important
  - Be sure to have the correct signs
- Solve the equations simultaneously

Summary of Types of Collisions

- In an **elastic collision**, both momentum and kinetic energy are conserved

$$v_{1i} + v_{1f} = v_{2f} + v_{2i}$$

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

- In an inelastic collision, momentum is conserved but kinetic energy is not

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$

- In a *perfectly* inelastic collision, momentum is conserved, kinetic energy is not, and the two objects stick together after the collision, so their final velocities are the same

$$m_1 v_{1i} + m_2 v_{2i} = (m_1 + m_2) v_f$$

Example 5. Conservation of Momentum

- An object of mass m moves to the right with a speed v. It collides head-on with an object of mass $3m$ moving with speed $v/3$ in the opposite direction. If the two objects stick together, what is the speed of the combined object, of mass $4m$, after the collision?

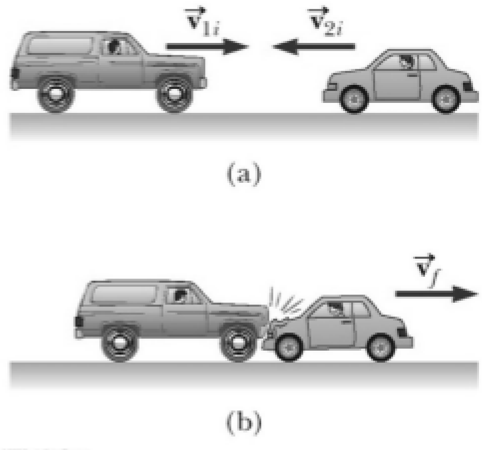
  (A) 0

  (B) v/2

  (C) v

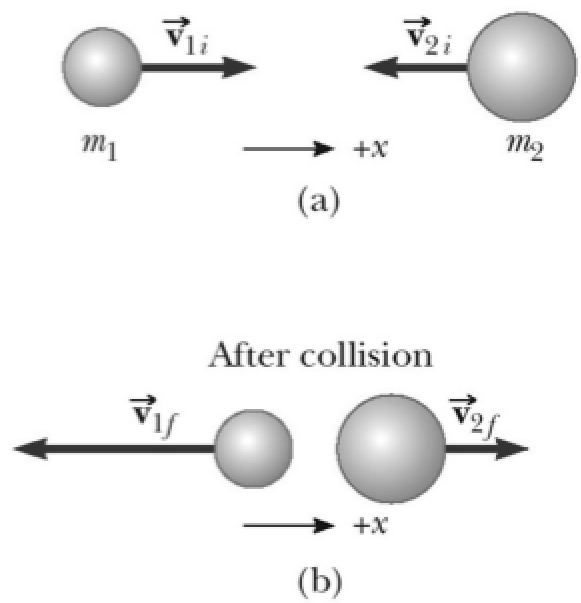
  (D) 2v

  (E) 4v



Problem Solving for 1D Collisions, 1

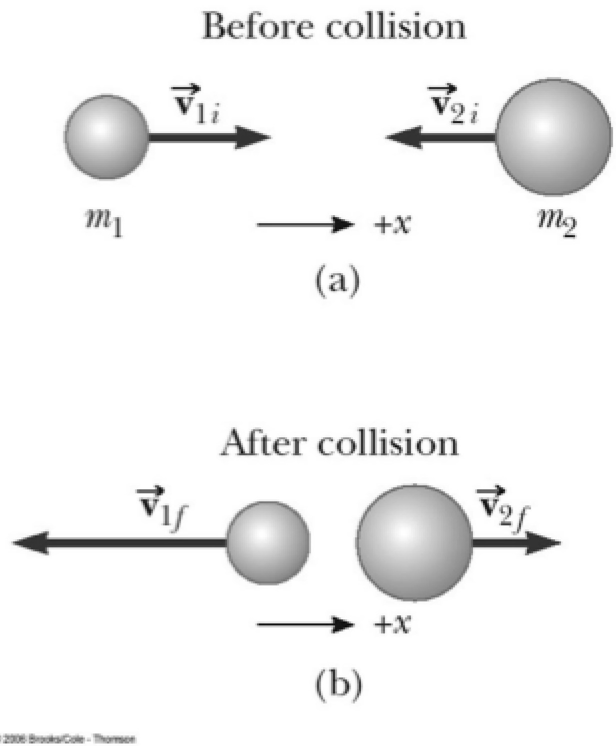
- **Coordinates:** Set up a coordinate axis and define the velocities with respect to this axis
  - It is convenient to make your axis coincide with one of the initial velocities
- **Diagram:** In your sketch, draw all the velocity vectors and label the velocities and the masses



Problem Solving for 1D Collisions, 2

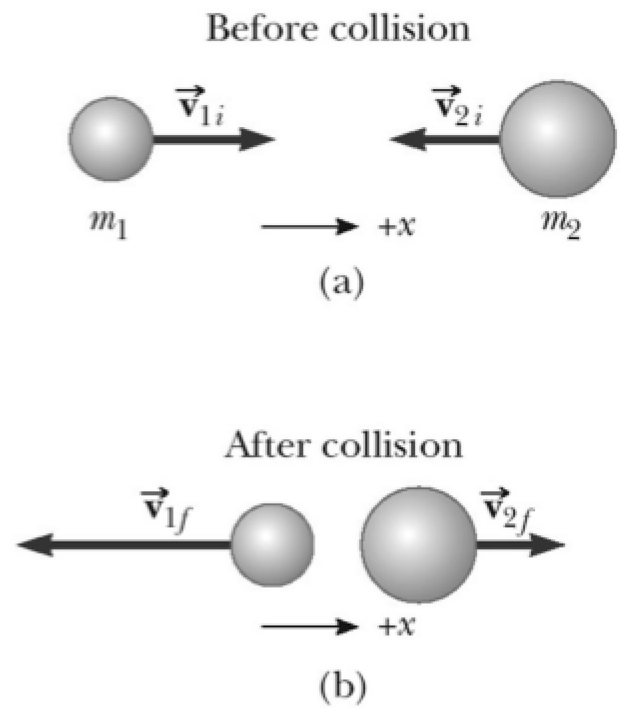
- **Conservation of Momentum:** Write a general expression for the total momentum of the system *before* and *after* the collision
  - Equate the two total momentum expressions
  - Fill in the known values

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f}$$



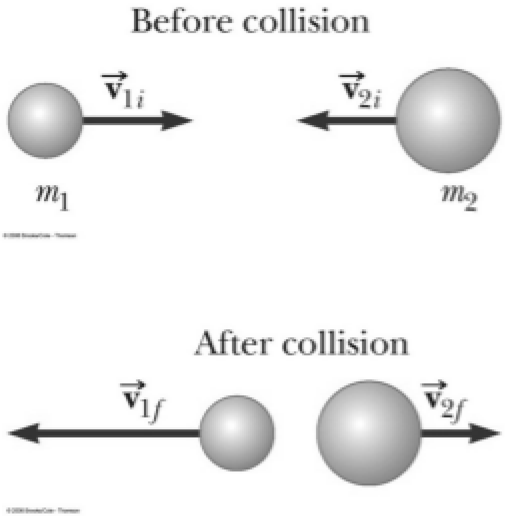
Problem Solving for 1D Collisions, 3

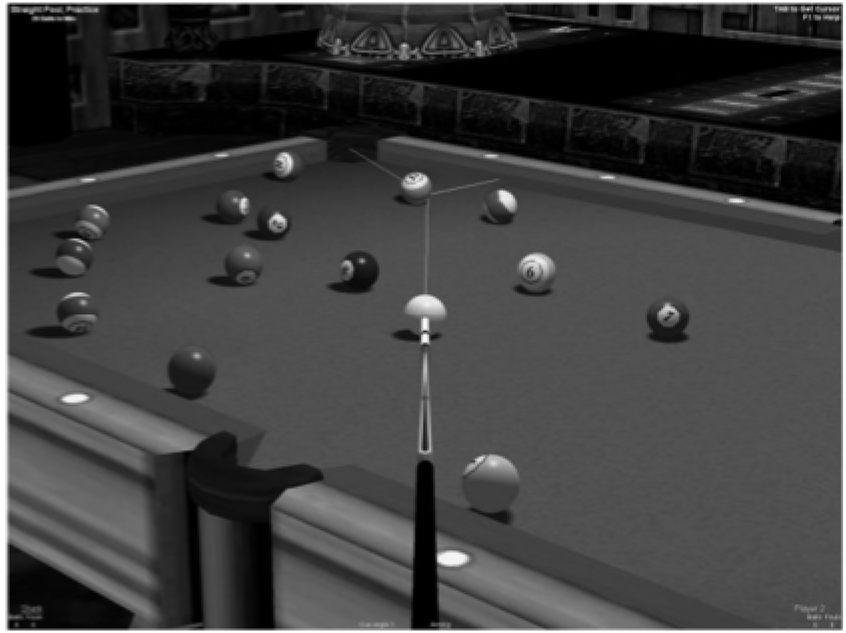
- **Conservation of Energy:** If the collision is elastic, write a second equation for conservation of KE, or the alternative equation $v_{1i} + v_{1f} = v_{2f} + v_{2i}$
    - This only applies to perfectly elastic collisions



- **Solve:** the resulting equations

One-Dimension vs Two-Dimension

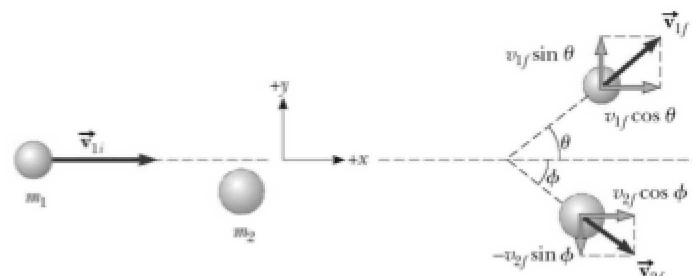




6.4.4. Two-Dimensional Collisions

- For a general collision of two objects in two-dimensional space, the conservation of momentum principle implies that the ***total momentum of the system in each direction is conserved***

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$
$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$



(a) Before the collision (b) After the collision

6.4.4. Two-Dimensional Collisions

- The momentum is conserved in all directions

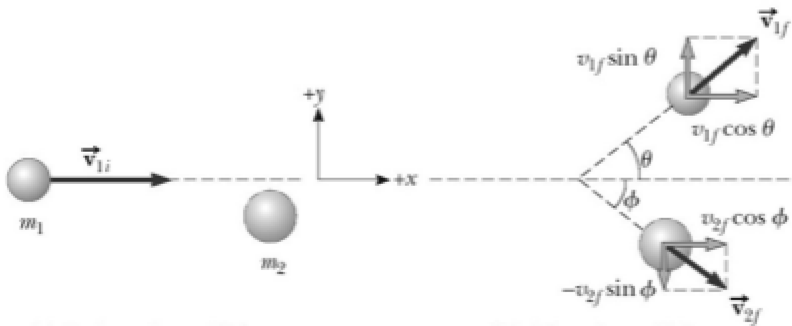
$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$
$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

- Use subscripts for
  - Identifying the object
  - Indicating initial or final values
  - The velocity components
- If the collision is elastic, use conservation of kinetic energy as a second equation
  - Remember, the simpler equation can only be used for one-dimensional situations

$$\cancel{v_{1i} + v_{1f} = v_{2f} + v_{2i}}$$

Glancing Collisions

- The "after" velocities have x and y components
- Momentum is conserved in the x direction and in the y direction
- Apply conservation of momentum separately to each direction



(a) Before the collision (b) After the collision

$$m_1 v_{1ix} + m_2 v_{2ix} = m_1 v_{1fx} + m_2 v_{2fx}$$

$$m_1 v_{1iy} + m_2 v_{2iy} = m_1 v_{1fy} + m_2 v_{2fy}$$

Example 6: 2-D Collision,

- Particle 1 is moving at velocity $\vec{\mathbf{v}}_{1i}$ and particle 2 is at rest
- In the *x*-direction, the initial momentum is $m_1 v_{1i}$
- In the *y*-direction, the initial momentum is 0.
- After the collision, the momentum in the *x*-direction is $m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\phi$
- After the collision, the momentum in the *y*-direction is $m_1 v_{1f} \sin\theta + m_2 v_{2f} \sin\phi$

$$m_1 v_{1i} + 0 = m_1 v_{1f} \cos\theta + m_2 v_{2f} \cos\varphi$$
$$0 + 0 = m_1 v_{1f} \sin\theta - m_2 v_{2f} \sin\varphi$$

- If the collision is elastic, apply the kinetic energy equation

$$\frac{1}{2} m_1 v_{1i}^2 = \frac{1}{2} m_1 v_{1f}^2 + \frac{1}{2} m_2 v_{2f}^2$$