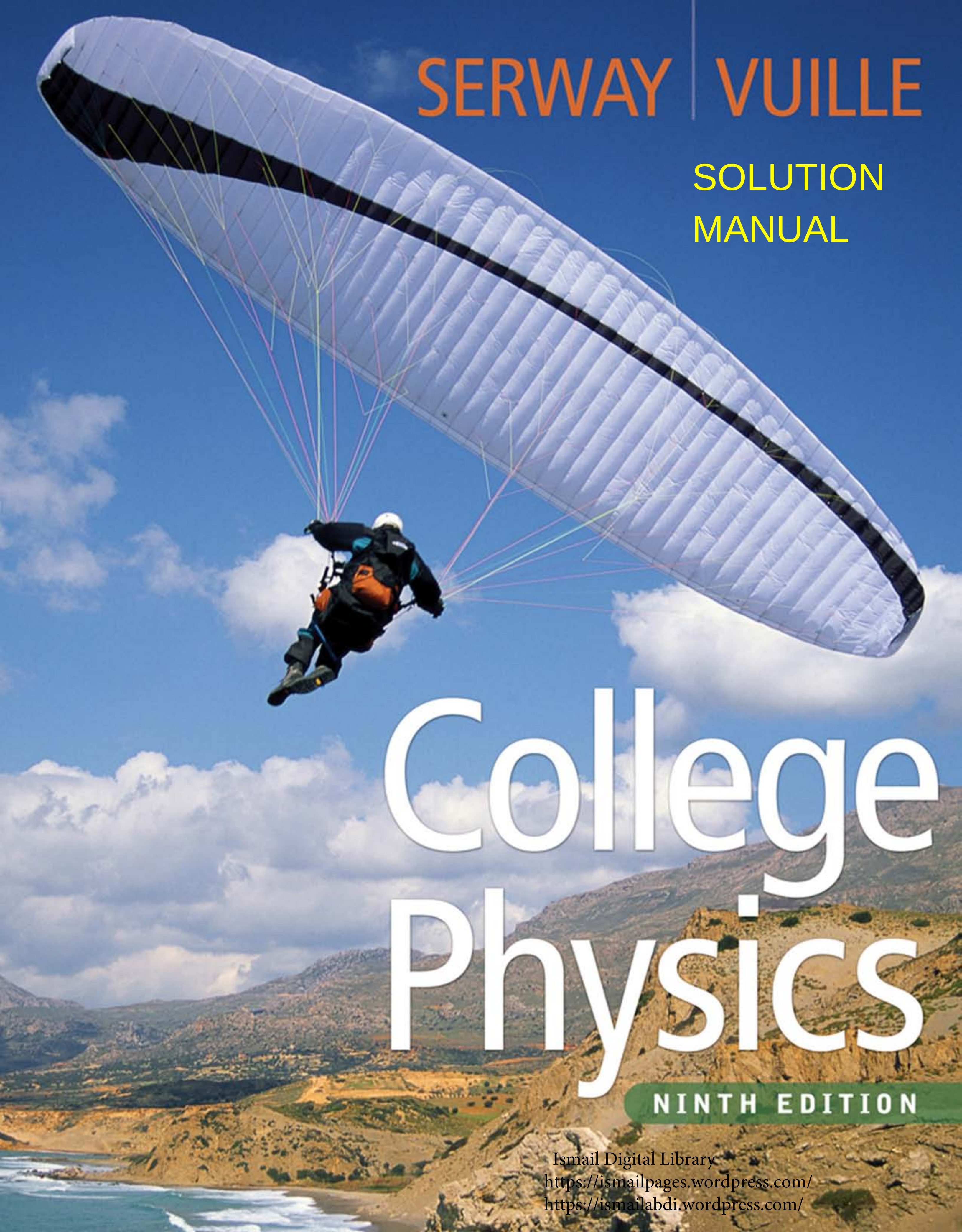

SERWAY | VUILLE

SOLUTION MANUAL

College Physics

NINTH EDITION

Instructor's Solutions Manual

for

Serway and Vuille's

College Physics

Ninth Edition

Charles Teague
Emeritus, Eastern Kentucky University

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1 2 3 4 5 6 7 14 13 12 11 10

TABLE OF CONTENTS

TABLE OF CONTENTS

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PREFACE

This manual is written to accompany *College Physics*, Ninth Edition, by Raymond A. Serway and Chris Vuille. For each chapter in that text, the manual includes solutions to all end-of-chapter problems, more detailed answers to Quick Quizzes and Multiple Choice Questions than available in the main text, and answers to the even-numbered Conceptual Questions.

Considerable effort has been made to insure that the solutions and answers given in this manual comply with the rules on significant figures and rounding given in the Chapter 1 of the textbook. This means that intermediate answers are rounded to the proper number of significant figures when written, and that rounded value is used in all subsequent calculations. Users should not be concerned if their answers differ slightly in the last digit from the answers given here. Most often, this will be caused by choosing to round intermediate answers at different stages of the solution.

You are encouraged to keep this manual out of the hands of students as instructors in many colleges throughout the country use this textbook, and many of them use graded problem assignments as part of the final course grade. Additionally, even when the problems are not used in such a direct fashion, it is advantageous for students to struggle with some problems in order to improve their problem-solving skills. Feel free to post answers and solutions to selected questions and problems, but please preserve the manual as a whole. You may also encourage students to purchase a copy of the Student Solutions Manual with Study Guide, which provides chapter summaries as well as detailed solutions to selected problems in the main text.

Attempting to keep the manual of manageable size, and recognizing that the primary users will be instructors well versed in the field, answers and solutions are kept fairly brief. Answers to conceptual questions have been shortened by not offering detailed arguments that lead to the answer. Problem solutions often omit commentary, intermediate steps, as well as initial steps that could be necessary for clear understanding by students. On occasions where selected problem solutions are to be shared with students, you may wish to supply intermediate steps and additional comments as needed. An electronic version of this manual can be obtained by requesting the Instructor's Power Lecture CD from your local Cengage Learning Sales Representative. Contact information for your sales representative is available under the "Find Your Rep" tab found at the bottom of the page at www.academic.cengage.com.

We welcome your comments on the accuracy of the solutions as presented here, as well as suggestions for alternative approaches.

Charles Teague

1

Introduction

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. Using a calculator to multiply the length by the width gives a raw answer of $6783\ \mathrm{m}^2$, but this answer must be rounded to contain the same number of significant figures as the least accurate factor in the product. The least accurate factor is the length, which contains either 2 or 3 significant figures, depending on whether the trailing zero is significant or is being used only to locate the decimal point. Assuming the length contains 3 significant figures, answer (c) correctly expresses the area as $6.78\times 10^3\ \mathrm{m}^2$. However, if the length contains only 2 significant figures, answer (d) gives the correct result as $6.8\times 10^3\ \mathrm{m}^2$.

2. Both answers (d) and (e) could be physically meaningful. Answers (a), (b), and (c) must be meaningless since quantities can be added or subtracted only if they have the same dimensions.

3. According to Newton's second law, $Force = mass \times acceleration$. Thus, the units of *Force* must be the product of the units of *mass* (kg) and the units of *acceleration* $\left(\mathrm{m/s^2}\right)$. This yields $\mathrm{kg\cdot m/s^2}$, which is answer (a).

4. The calculator gives an answer of 57.573 for the sum of the 4 given numbers. However, this sum must be rounded to 58 as given in answer (d) so the number of decimal places in the result is the same (zero) as the number of decimal places in the integer 15 (the term in the sum containing the smallest number of decimal places).

5. The required conversion is given by:

$$h = \left(2.00\ \cancel{\mathrm{m}}\right)\left(\frac{1\,000\ \cancel{\mathrm{mm}}}{1.00\ \cancel{\mathrm{m}}}\right)\left(\frac{1.00\ \text{cubitus}}{445\ \cancel{\mathrm{mm}}}\right) = 4.49\ \text{cubiti}$$

This result corresponds to answer (c).

6. The given area $\left(1\,420\ \mathrm{ft}^2\right)$ contains 3 significant figures, assuming that the trailing zero is used only to locate the decimal point. The conversion of this value to square meters is given by:

$$A = \left(1.42\times 10^3\ \mathrm{ft}^2\right)\left(\frac{1.00\ \mathrm{m}}{3.281\ \mathrm{ft}}\right)^2 = 1.32\times 10^2\ \mathrm{m}^2 = 132\ \mathrm{m}^2$$

Note that the result contains 3 significant figures, the same as the number of significant figures in the least accurate factor used in the calculation. This result matches answer (b).

7. You cannot add, subtract, or equate a number apples and a number of days. Thus, the answer is yes for (a), (c), and (e). However, you can multiply or divide a number of apples and a number of days. For example, you might divide the number of apples by a number of days to find the number of apples you could eat per day. In summary, the answers are (a) yes, (b) no, (c) yes, (d) no, and (e) yes.

8. The given Cartesian coordinates are $x = -5.00$, and $y = 12.00$, with the least accurate containing 3 significant figures. Note that the specified point (with $x < 0$ and $y > 0$) is in the second quadrant. The conversion to polar coordinates is then given by:

$$r = \sqrt{x^2 + y^2} = \sqrt{(-5.00)^2 + (12.00)^2} = 13.0$$

$$\tan\theta = \frac{y}{x} = \frac{12.00}{-5.00} = -2.40 \quad \text{and} \quad \theta = \tan^{-1}(-2.40) = -67.3° + 180° = 113°$$

Note that 180° was added in the last step to yield a second quadrant angle. The correct answer is therefore (b) (13.0, 113°).

9. Doing dimensional analysis on the first 4 given choices yields:

(a) $\dfrac{[v]}{[t^2]} = \dfrac{\mathrm{L/T}}{\mathrm{T}^2} = \dfrac{\mathrm{L}}{\mathrm{T}^3}$ (b) $\dfrac{[v]}{[x^2]} = \dfrac{\mathrm{L/T}}{\mathrm{L}^2} = \mathrm{L}^{-1}\mathrm{T}^{-1}$

(c) $\dfrac{[v^2]}{[t]} = \dfrac{(\mathrm{L/T})^2}{\mathrm{T}} = \dfrac{\mathrm{L}^2/\mathrm{T}^2}{\mathrm{T}} = \dfrac{\mathrm{L}^2}{\mathrm{T}^3}$ (d) $\dfrac{[v^2]}{[x]} = \dfrac{(\mathrm{L/T})^2}{\mathrm{L}} = \dfrac{\mathrm{L}^2/\mathrm{T}^2}{\mathrm{L}} = \dfrac{\mathrm{L}}{\mathrm{T}^2}$

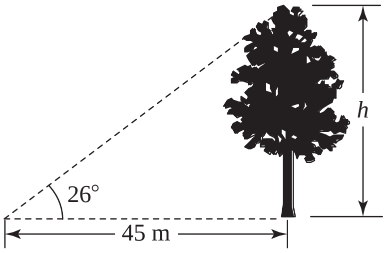
Since acceleration has units of length divided by time squared, it is seen that the relation given in answer (d) is consistent with an expression yielding a value for acceleration.

10. The number of gallons of gasoline she can purchase is

$$\#\text{ gallons} = \frac{\text{total expenditure}}{\text{cost per gallon}} \approx \frac{33\ \cancel{\text{Euros}}}{\left(1.5\ \dfrac{\cancel{\text{Euros}}}{\cancel{\text{L}}}\right)\left(\dfrac{1\ \cancel{\text{L}}}{1\ \cancel{\text{quart}}}\right)\left(\dfrac{4\ \cancel{\text{quarts}}}{1\ \text{gal}}\right)} \approx 5\text{ gal}$$

so the correct answer is (b).

11. The situation described is shown in the drawing at the right.



From this, observe that $\tan 26° = \dfrac{h}{45\text{ m}}$, or

$$h = (45\text{ m})\tan 26° = 22\text{ m}$$

Thus, the correct answer is (a).

12. Note that we may write $1.365\,248\,0 \times 10^7$ as $136.524\,80 \times 10^5$. Thus, the raw answer, including the uncertainty, is $x = (136.524\,80 \pm 2) \times 10^5$. Since the final answer should contain all the digits we are sure of and one estimated digit, this result should be rounded and displayed as $137 \times 10^5 = 1.37 \times 10^7$ (we are sure of the 1 and the 3, but have uncertainty about the 7). We see that this answer has three significant figures and choice (d) is correct.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. Atomic clocks are based on the electromagnetic waves that atoms emit. Also, pulsars are highly regular astronomical clocks.

4. **(a)** $\sim 0.5 \text{ lb} \approx 0.25 \text{ kg}$ or $\sim 10^{-1} \text{ kg}$

(b) $\sim 4 \text{ lb} \approx 2 \text{ kg}$ or $\sim 10^{0} \text{ kg}$

(c) $\sim 4000 \text{ lb} \approx 2000 \text{ kg}$ or $\sim 10^{3} \text{ kg}$

6. Let us assume the atoms are solid spheres of diameter 10^{-10} m. Then, the volume of each atom is of the order of 10^{-30} m^3. (More precisely, volume $= 4\pi r^3/3 = \pi d^3/6$.) Therefore, since $1 \text{ cm}^3 = 10^{-6} \text{ m}^3$, the number of atoms in the 1 cm^3 solid is on the order of $10^{-6}/10^{-30} = 10^{24}$ atoms. A more precise calculation would require knowledge of the density of the solid and the mass of each atom. However, our estimate agrees with the more precise calculation to within a factor of 10.

8. Realistically, the only lengths you might be able to verify are the length of a football field and the length of a housefly. The only time intervals subject to verification would be the length of a day and the time between normal heartbeats.

10. In the metric system, units differ by powers of ten, so it's very easy and accurate to convert from one unit to another.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** L/T^2 **(b)** L

4. All three equations are dimensionally incorrect.

6. **(a)** $\text{kg}\cdot\text{m}/\text{s}$ **(b)** $Ft = p$

8. **(a)** 22.6 **(b)** 22.7 **(c)** 22.6 is more reliable

10. **(a)** $3.00 \times 10^8 \text{ m/s}$ **(b)** $2.997\,9 \times 10^8 \text{ m/s}$ **(c)** $2.997\,925 \times 10^8 \text{ m/s}$

12. **(a)** $346 \text{ m}^2 \pm 13 \text{ m}^2$ **(b)** $66.0 \text{ m} \pm 1.3 \text{ m}$

14. **(a)** 797 **(b)** 1.1 **(c)** 17.66

16. 3.09 cm/s

18. **(a)** $5.60 \times 10^2 \text{ km} = 5.60 \times 10^5 \text{ m} = 5.60 \times 10^7 \text{ cm}$

(b) $0.491\,2 \text{ km} = 491.2 \text{ m} = 4.912 \times 10^4 \text{ cm}$

(c) $6.192 \text{ km} = 6.192 \times 10^3 \text{ m} = 6.192 \times 10^5 \text{ cm}$

(d) $2.499 \text{ km} = 2.499 \times 10^3 \text{ m} = 2.499 \times 10^5 \text{ cm}$

20. 10.6 km/L

22. 9.2 nm/s

24. $2.9 \times 10^2 \text{ m}^3 = 2.9 \times 10^8 \text{ cm}^3$

26. $2.57 \times 10^6 \text{ m}^3$

28. $\sim 10^8$ steps

30. $\sim 10^8$ people with colds on any given day

32. **(a)** $4.2 \times 10^{-18}\ \text{m}^3$ **(b)** $\sim 10^{-1}\ \text{m}^3$ **(c)** $\sim 10^{16}$ cells

34. **(a)** $\sim 10^{29}$ prokaryotes **(b)** $\sim 10^{14}$ kg

(c) The very large mass of prokaryotes implies they are important to the biosphere. They are responsible for fixing carbon, producing oxygen, and breaking up pollutants, among many other biological roles. Humans depend on them!

36. 2.2 m

38. 8.1 cm

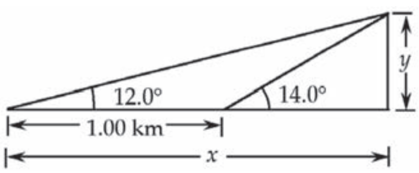
40. $\Delta s = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_1 - \theta_2)}$

42. 2.33 m

44. **(a)** 1.50 m **(b)** 2.60 m

46. 8.60 m

48. **(a)** and **(b)**



(c) $y/x = \tan 12.0°,\ \ y/(x - 1.00\ \text{km}) = \tan 14.0°$ **(d)** 1.44×10^3 m

50. $y = \dfrac{d \cdot \tan\theta \cdot \tan\phi}{\tan\phi - \tan\theta}$

52. **(a)** 1.609 km/h **(b)** 88 km/h **(c)** 16 km/h

54. Assumes population of 300 million, average of 1 can/week per person, and 0.5 oz per can.

(a) $\sim 10^{10}$ cans/yr **(b)** $\sim 10^5$ tons/yr

56. **(a)** 7.14×10^{-2} gal/s **(b)** $2.70 \times 10^{-4}\ \text{m}^3/\text{s}$ **(c)** 1.03 h

58. **(a)** $A_2/A_1 = 4$ **(b)** $V_2/V_1 = 8$

60. **(a)** $\sim 10^2$ yr **(b)** $\sim 10^4$ times

62. $\sim 10^4$ balls/yr. Assumes 1 lost ball per hitter, 10 hitters per inning, 9 innings per game, and 81 games per year.

PROBLEM SOLUTIONS

1.1 Substituting dimensions into the given equation $T = 2\pi\sqrt{\ell/g}$, and recognizing that 2π is a dimensionless constant, we have

$$[T] = \sqrt{\frac{[\ell]}{[g]}} \qquad \text{or} \qquad \mathrm{T} = \sqrt{\frac{\mathrm{L}}{\mathrm{L/T^2}}} = \sqrt{\mathrm{T}^2} = \mathrm{T}$$

Thus, the $\boxed{\text{dimensions are consistent}}$.

1.2 (a) From $x = Bt^2$, we find that $B = \dfrac{x}{t^2}$. Thus, B has units of

$$[B] = \frac{[x]}{[t^2]} = \boxed{\frac{\mathrm{L}}{\mathrm{T}^2}}$$

(b) If $x = A\sin(2\pi ft)$, then $[A] = [x]/[\sin(2\pi ft)]$

But the sine of an angle is a dimensionless ratio.

Therefore, $[A] = [x] = \boxed{\mathrm{L}}$

1.3 (a) The units of volume, area, and height are:

$$[V] = \mathrm{L}^3,\ [A] = \mathrm{L}^2,\ \text{and}\ [h] = \mathrm{L}$$

We then observe that $\mathrm{L}^3 = \mathrm{L}^2\mathrm{L}$ or $[V] = [A][h]$

Thus, the equation $V = Ah$ is $\boxed{\text{dimensionally correct}}$.

(b) $V_{\text{cylinder}} = \pi R^2 h = (\pi R^2)h = Ah$, where $\boxed{A = \pi R^2}$

$V_{\text{rectangular box}} = \ell wh = (\ell w)h = Ah$, where $\boxed{A = \ell w = length \times width}$

1.4 (a) In the equation $\frac{1}{2}mv^2 = \frac{1}{2}mv_0^2 + \sqrt{mgh}$, $[mv^2] = [mv_0^2] = \mathrm{M}\left(\dfrac{\mathrm{L}}{\mathrm{T}}\right)^2 = \dfrac{\mathrm{ML}^2}{\mathrm{T}^2}$

while $\left[\sqrt{mgh}\right] = \sqrt{\mathrm{M}\left(\dfrac{\mathrm{L}}{\mathrm{T}^2}\right)\mathrm{L}} = \dfrac{\mathrm{M}^{\frac{1}{2}}\mathrm{L}}{\mathrm{T}}$. Thus, the equation is $\boxed{\text{dimensionally incorrect}}$.

(b) In $v = v_0 + at^2$, $[v] = [v_0] = \dfrac{\mathrm{L}}{\mathrm{T}}$ but $[at^2] = [a][t^2] = \left(\dfrac{\mathrm{L}}{\mathrm{T}^2}\right)(\mathrm{T}^2) = \mathrm{L}$. Hence, this equation is $\boxed{\text{dimensionally incorrect}}$.

(c) In the equation $ma = v^2$, we see that $[ma] = [m][a] = \mathrm{M}\left(\dfrac{\mathrm{L}}{\mathrm{T}^2}\right) = \dfrac{\mathrm{ML}}{\mathrm{T}^2}$, while $[v^2] = \left(\dfrac{\mathrm{L}}{\mathrm{T}}\right)^2 = \dfrac{\mathrm{L}^2}{\mathrm{T}^2}$.

Therefore, this equation is also $\boxed{\text{dimensionally incorrect}}$.

1.5 From the universal gravitation law, the constant G is $G = Fr^2/Mm$. Its units are then

$$[G] = \frac{[F][r^2]}{[M][m]} = \frac{(\cancel{\mathrm{kg}}\cdot\mathrm{m/s^2})(\mathrm{m}^2)}{\cancel{\mathrm{kg}}\cdot\mathrm{kg}} = \boxed{\frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}}$$

1.6 (a) Solving $KE = \dfrac{p^2}{2m}$ for the momentum, p, gives $p = \sqrt{2m(KE)}$ where the numeral 2 is a dimensionless constant. Dimensional analysis gives the units of momentum as:

$$[p] = \sqrt{[m][KE]} = \sqrt{\mathrm{M}\left(\mathrm{M}\cdot\mathrm{L}^2/\mathrm{T}^2\right)} = \sqrt{\mathrm{M}^2\cdot\mathrm{L}^2/\mathrm{T}^2} = \mathrm{M}(\mathrm{L}/\mathrm{T})$$

Therefore, in the SI system, the units of momentum are $\boxed{\mathrm{kg}\cdot(\mathrm{m/s})}$.

(b) Note that the units of force are $\mathrm{kg}\cdot\mathrm{m/s^2}$ or $[F] = \mathrm{M}\cdot\mathrm{L/T^2}$. Then, observe that

$$[F][t] = \left(\mathrm{M}\cdot\mathrm{L/T^2}\right)\cdot\mathrm{T} = \mathrm{M}(\mathrm{L/T}) = [p]$$

From this, it follows that force multiplied by time is proportional to momentum: $\boxed{Ft = p}$. (See the impulse–momentum theorem in Chapter 6, $F\cdot\Delta t = \Delta p$, which says that a constant force F multiplied by a duration of time Δt equals the change in momentum, Δp.)

1.7 $Area = (length)\times(width) = (9.72\ \mathrm{m})(5.3\ \mathrm{m}) = \boxed{52\ \mathrm{m}^2}$

1.8 (a) Computing $\left(\sqrt{8}\right)^3$ without rounding the intermediate result yields

$\left(\sqrt{8}\right)^3 = \boxed{22.6}$ to three significant figures.

(b) Rounding the intermediate result to three significant figures yields

$$\sqrt{8} = 2.8284 \quad\rightarrow\quad 2.83$$

Then, we obtain $\left(\sqrt{8}\right)^3 = (2.83)^3 = \boxed{22.7}$ to three significant figures.

(c) $\boxed{\text{The answer 22.6 is more reliable}}$ because rounding in part (b) was carried out too soon.

1.9 (a) 78.9 ± 0.2 has $\boxed{\text{3 significant figures}}$ with the uncertainty in the tenths position.

(b) 3.788×10^9 has $\boxed{\text{4 significant figures}}$

(c) 2.46×10^{-6} has $\boxed{\text{3 significant figures}}$

(d) $0.003\,2 = 3.2\times10^{-3}$ has $\boxed{\text{2 significant figures}}$. The two zeros were originally included only to position the decimal.

1.10 $c = 2.997\,924\,58\times10^8\ \mathrm{m/s}$

(a) Rounded to 3 significant figures: $c = \boxed{3.00\times10^8\ \mathrm{m/s}}$

(b) Rounded to 5 significant figures: $c = \boxed{2.997\,9\times10^8\ \mathrm{m/s}}$

(c) Rounded to 7 significant figures: $c = \boxed{2.997\,925\times10^8\ \mathrm{m/s}}$

1.11 Observe that the length $\ell = 5.62$ cm, the width $w = 6.35$ cm, and the height $h = 2.78$ cm all contain 3 significant figures. Thus, any product of these quantities should contain 3 significant figures.

(a) $\ell w = (5.62\ \mathrm{cm})(6.35\ \mathrm{cm}) = \boxed{35.7\ \mathrm{cm}^2}$

(b) $V = (\ell w)h = \left(35.7\ \mathrm{cm}^2\right)(2.78\ \mathrm{cm}) = \boxed{99.2\ \mathrm{cm}^3}$

continued on next page

(c) $wh = (6.35\text{ cm})(2.78\text{ cm}) = \boxed{17.7\text{ cm}^2}$

$V = (wh)\ell = (17.7\text{ cm}^2)(5.62\text{ cm}) = \boxed{99.5\text{ cm}^3}$

(d) In the rounding process, small amounts are either added to or subtracted from an answer to satisfy the rules of significant figures. For a given rounding, different small adjustments are made, introducing a certain amount of randomness in the last significant digit of the final answer.

1.12 (a) $A = \pi r^2 = \pi(10.5\text{ m} \pm 0.2\text{ m})^2 = \pi\left[(10.5\text{ m})^2 \pm 2(10.5\text{ m})(0.2\text{ m}) + (0.2\text{ m})^2\right]$

Recognize that the last term in the brackets is insignificant in comparison to the other two. Thus, we have

$$A = \pi\left[110\text{ m}^2 \pm 4.2\text{ m}^2\right] = \boxed{346\text{ m}^2 \pm 13\text{ m}^2}$$

(b) $C = 2\pi r = 2\pi(10.5\text{ m} \pm 0.2\text{ m}) = \boxed{66.0\text{ m} \pm 1.3\text{ m}}$

1.13 The least accurate dimension of the box has two significant figures. Thus, the volume (product of the three dimensions) will contain only two significant figures.

$$V = \ell \cdot w \cdot h = (29\text{ cm})(17.8\text{ cm})(11.4\text{ cm}) = \boxed{5.9 \times 10^3\text{ cm}^3}$$

1.14 (a) The sum is rounded to $\boxed{797}$ because 756 in the terms to be added has no positions beyond the decimal.

(b) $0.003\,2 \times 356.3 = (3.2 \times 10^{-3}) \times 356.3 = 1.14016$ must be rounded to $\boxed{1.1}$ because 3.2×10^{-3} has only two significant figures.

(c) $5.620 \times \pi$ must be rounded to $\boxed{17.66}$ because 5.620 has only four significant figures.

1.15 $d = (250\,000\text{ mi})\left(\dfrac{5\,280\text{ ft}}{1.000\text{ mi}}\right)\left(\dfrac{1\text{ fathom}}{6\text{ ft}}\right) = \boxed{2 \times 10^8\text{ fathoms}}$

The answer is limited to one significant figure because of the accuracy to which the conversion from fathoms to feet is given.

1.16 $v = \dfrac{\ell}{t} = \dfrac{186\ \cancel{\text{furlongs}}}{1\ \cancel{\text{fortnight}}}\left(\dfrac{1\ \cancel{\text{fortnight}}}{14\ \cancel{\text{days}}}\right)\left(\dfrac{1\ \cancel{\text{day}}}{8.64 \times 10^4\text{ s}}\right)\left(\dfrac{220\ \cancel{\text{yds}}}{1\ \cancel{\text{furlong}}}\right)\left(\dfrac{3\ \cancel{\text{ft}}}{1\ \cancel{\text{yd}}}\right)\left(\dfrac{100\text{ cm}}{3.281\ \cancel{\text{ft}}}\right)$

giving $v = \boxed{3.09\text{ cm/s}}$

1.17 $6.00\text{ firkins} = 6.00\ \cancel{\text{firkins}}\left(\dfrac{9\ \cancel{\text{gal}}}{1\ \cancel{\text{firkin}}}\right)\left(\dfrac{3.786\ \cancel{\text{L}}}{1\ \cancel{\text{gal}}}\right)\left(\dfrac{10^3\ \cancel{\text{cm}^3}}{1\ \cancel{\text{L}}}\right)\left(\dfrac{1\text{ m}^3}{10^6\ \cancel{\text{cm}^3}}\right) = \boxed{0.204\text{ m}^3}$

1.18 (a) $\ell = (348\text{ mi})\left(\dfrac{1.609\text{ km}}{1.000\text{ mi}}\right) = \boxed{5.60 \times 10^2\text{ km}} = \boxed{5.60 \times 10^5\text{ m}} = \boxed{5.60 \times 10^7\text{ cm}}$

(b) $h = (1\,612\text{ ft})\left(\dfrac{1.609\text{ km}}{5\,280\text{ ft}}\right) = \boxed{0.491\,2\text{ km}} = \boxed{491.2\text{ m}} = \boxed{4.912 \times 10^4\text{ cm}}$

(c) $h = (20\,320\text{ ft})\left(\dfrac{1.609\text{ km}}{5\,280\text{ ft}}\right) = \boxed{6.192\text{ km}} = \boxed{6.192 \times 10^3\text{ m}} = \boxed{6.192 \times 10^5\text{ cm}}$

continued on next page

(d) $d = (8\,200 \text{ ft})\left(\dfrac{1.609 \text{ km}}{5\,280 \text{ ft}}\right) = \boxed{2.499 \text{ km}} = \boxed{2.499 \times 10^3 \text{ m}} = \boxed{2.499 \times 10^5 \text{ cm}}$

In (a), the answer is limited to three significant figures because of the accuracy of the original data value, 348 miles. In (b), (c), and (d), the answers are limited to four significant figures because of the accuracy to which the kilometers-to-feet conversion factor is given.

1.19 $v = 38.0 \dfrac{\cancel{\text{m}}}{\cancel{\text{s}}}\left(\dfrac{1 \cancel{\text{km}}}{10^3 \cancel{\text{m}}}\right)\left(\dfrac{1 \text{ mi}}{1.609 \cancel{\text{km}}}\right)\left(\dfrac{3\,600 \cancel{\text{s}}}{1 \text{ h}}\right) = 85.0 \text{ mi/h}$

$\boxed{\text{Yes, the driver is exceeding the speed limit by 10.0 mi/h}}$.

1.20 $\textit{efficiency} = 25.0 \dfrac{\cancel{\text{mi}}}{\cancel{\text{gal}}}\left(\dfrac{1 \text{ km}}{0.621 \cancel{\text{mi}}}\right)\left(\dfrac{1 \cancel{\text{gal}}}{3.786 \text{ L}}\right) = \boxed{10.6 \text{ km/L}}$

1.21 (a) $r = \dfrac{\textit{diameter}}{2} = \dfrac{5.36 \text{ in}}{2}\left(\dfrac{2.54 \text{ cm}}{1 \text{ in}}\right) = \boxed{6.81 \text{ cm}}$

(b) $A = 4\pi r^2 = 4\pi(6.81 \text{ cm})^2 = \boxed{5.83 \times 10^2 \text{ cm}^2}$

(c) $V = \dfrac{4}{3}\pi r^3 = \dfrac{4}{3}\pi(6.81 \text{ cm})^3 = \boxed{1.32 \times 10^3 \text{ cm}^3}$

1.22 $\textit{rate} = \left(\dfrac{1}{32}\ \dfrac{\cancel{\text{in}}}{\cancel{\text{day}}}\right)\left(\dfrac{1 \cancel{\text{day}}}{24 \cancel{\text{h}}}\right)\left(\dfrac{1 \cancel{\text{h}}}{3\,600 \text{ s}}\right)\left(\dfrac{2.54 \cancel{\text{cm}}}{1.00 \cancel{\text{in}}}\right)\left(\dfrac{10^9 \text{ nm}}{10^2 \cancel{\text{cm}}}\right) = \boxed{9.2 \text{ nm/s}}$

This means that the proteins are assembled at a rate of many layers of atoms each second!

1.23 $c = \left(3.00 \times 10^8 \dfrac{\cancel{\text{m}}}{\cancel{\text{s}}}\right)\left(\dfrac{3\,600 \cancel{\text{s}}}{1 \text{ h}}\right)\left(\dfrac{1 \cancel{\text{km}}}{10^3 \cancel{\text{m}}}\right)\left(\dfrac{1 \text{ mi}}{1.609 \cancel{\text{km}}}\right) = \boxed{6.71 \times 10^8 \text{ mi/h}}$

1.24 Volume of house $= (50.0 \text{ ft})(26 \text{ ft})(8.0 \text{ ft})\left(\dfrac{2.832 \times 10^{-2} \text{ m}^3}{1 \text{ ft}^3}\right)$

$$= \boxed{2.9 \times 10^2 \text{ m}^3} = \left(2.9 \times 10^2 \text{ m}^3\right)\left(\frac{100 \text{ cm}}{1 \text{ m}}\right)^3 = \boxed{2.9 \times 10^8 \text{ cm}^3}$$

1.25 Volume $= \left(25.0 \cancel{\text{acre}} \cancel{\text{ft}}\right)\left(\dfrac{1 \text{ m}}{3.281 \cancel{\text{ft}}}\right)\left[\left(\dfrac{43\,560 \cancel{\text{ft}^2}}{1 \cancel{\text{acre}}}\right)\left(\dfrac{1 \text{ m}}{3.281 \cancel{\text{ft}}}\right)^2\right] = \boxed{3.08 \times 10^4 \text{ m}^3}$

1.26 Volume of pyramid $= \dfrac{1}{3}(\text{area of base})(\text{height})$

$$= \frac{1}{3}\left[(13.0 \text{ acres})\left(43\,560 \text{ ft}^2/\text{acre}\right)\right](481 \text{ ft}) = 9.08 \times 10^7 \text{ f}$$

$$= \left(9.08 \times 10^7 \text{ ft}^3\right)\left(\frac{2.832 \times 10^{-2} \text{ m}^3}{1 \text{ ft}^3}\right) = \boxed{2.57 \times 10^6 \text{ m}^3}$$

1.27 Volume of cube $= L^3 = 1$ quart (Where L = length of one side of the cube.)

Thus, $L^3 = \left(1 \cancel{\text{quart}}\right)\left(\dfrac{1 \cancel{\text{gallon}}}{4 \cancel{\text{quarts}}}\right)\left(\dfrac{3.786 \cancel{\text{liter}}}{1 \cancel{\text{gallon}}}\right)\left(\dfrac{1000 \text{ cm}^3}{1 \cancel{\text{liter}}}\right) = 947 \text{ cm}^3$

and $L = \sqrt[3]{947 \text{ cm}^3} = \boxed{9.82 \text{ cm}}$

1.28 We estimate that the length of a step for an average person is about 18 inches, or roughly 0.5 m.

Then, an estimate for the number of steps required to travel a distance equal to the circumference of the Earth would be

$$N = \frac{\text{Circumference}}{\text{Step Length}} = \frac{2\pi R_E}{\text{Step Length}} \approx \frac{2\pi\left(6.38\times10^6\ \text{m}\right)}{0.5\ \text{m/step}} \approx 8\times10^7\ \text{steps}$$

or $\boxed{N \sim 10^8\ \text{steps}}$

1.29. We assume an average respiration rate of about 10 breaths/minute and a typical life span of 70 years. Then, an estimate of the number of breaths an average person would take in a lifetime is

$$n = \left(10\ \frac{\text{breaths}}{\cancel{\text{min}}}\right)\left(70\ \cancel{\text{yr}}\right)\left(\frac{3.156\times10^7\ \cancel{\text{s}}}{1\ \cancel{\text{yr}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \cancel{\text{s}}}\right) = 4\times10^8\ \text{breaths}$$

or $\boxed{n \sim 10^8\ \text{breaths}}$

1.30 We assume that the average person catches a cold twice a year and is sick an average of 7 days (or 1 week) each time. Thus, on average, each person is sick for 2 weeks out of each year (52 weeks). The probability that a particular person will be sick at any given time equals the percentage of time that person is sick, or

$$\textit{probability of sickness} = \frac{2\ \text{weeks}}{52\ \text{weeks}} = \frac{1}{26}$$

The population of the Earth is approximately 7 billion. The number of people expected to have a cold on any given day is then

$$\textit{Number sick} = (\textit{population})(\textit{probability of sickness}) = \left(7\times10^9\right)\left(\frac{1}{26}\right) = 3\times10^8\ \text{or}\ \boxed{\sim 10^8}$$

1.31 (a) Assume that a typical intestinal tract has a length of about 7 m and average diameter of 4 cm. The estimated total intestinal volume is then

$$V_{\text{total}} = A\ell = \left(\frac{\pi d^2}{4}\right)\ell = \frac{\pi(0.04\ \text{m})^2}{4}(7\ \text{m}) = 0.009\ \text{m}^3$$

The approximate volume occupied by a single bacterium is

$$V_{\text{bacteria}} \sim (\textit{typical length scale})^3 = \left(10^{-6}\ \text{m}\right)^3 = 10^{-18}\ \text{m}^3$$

If it is assumed that bacteria occupy one hundredth of the total intestinal volume, the estimate of the number of microorganisms in the human intestinal tract is

$$n = \frac{V_{\text{total}}/100}{V_{\text{bacteria}}} = \frac{\left(0.009\ \text{m}^3\right)/100}{10^{-18}\ \text{m}^3} = 9\times10^{13} \quad \text{or} \quad n \sim \boxed{10^{14}}$$

(b) The large value of the number of bacteria estimated to exist in the intestinal tract means that they are probably not dangerous. Intestinal bacteria help digest food and provide important nutrients. Humans and bacteria enjoy a mutually beneficial symbiotic relationship.

1.32 (a) $V_{\text{cell}} = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(1.0\times10^{-6}\ \text{m}\right)^3 = \boxed{4.2\times10^{-18}\ \text{m}^3}$

(b) Consider your body to be a cylinder having a radius of about 6 inches (or 0.15 m) and a height of about 1.5 meters. Then, its volume is

$$V_{\text{body}} = Ah = \left(\pi r^2\right)h = \pi(0.15\ \text{m})^2(1.5\ \text{m}) = 0.11\ \text{m}^3\ \text{or}\ \boxed{\sim 10^{-1}\ \text{m}^3}$$

continued on next page

(c) The estimate of the number of cells in the body is then

$$n = \frac{V_{\text{body}}}{V_{\text{cell}}} = \frac{0.11 \text{ m}^3}{4.2\times10^{-18} \text{ m}^3} = 2.6\times10^{16} \text{ or } \boxed{\sim 10^{16}}$$

1.33 A reasonable guess for the diameter of a tire might be 3 ft, with a circumference ($C = 2\pi r = \pi D$ = distance travels per revolution) of about 9 ft. Thus, the total number of revolutions the tire might make is

$$n = \frac{\textit{total distance traveled}}{\textit{distance per revolution}} = \frac{(50\,000 \text{ mi})(5\,280 \text{ ft/mi})}{9 \text{ ft/rev}} = 3\times10^7 \text{ rev, or } \boxed{\sim 10^7 \text{ rev}}$$

1.34 Answers to this problem will vary, dependent on the assumptions one makes. This solution assumes that bacteria and other prokaryotes occupy approximately one ten-millionth (10^{-7}) of the Earth's volume, and that the density of a prokaryote, like the density of the human body, is approximately equal to that of water (10^3 kg/m^3).

(a) $$\textit{estimated number} = n = \frac{V_{\text{total}}}{V_{\substack{\text{single}\\\text{prokaryote}}}} \sim \frac{(10^{-7})V_{\text{Earth}}}{V_{\substack{\text{single}\\\text{prokaryote}}}} \sim \frac{(10^{-7})R^3_{\text{Earth}}}{(\textit{length scale})^3} \sim \frac{(10^{-7})(10^6 \text{ m})^3}{(10^{-6} \text{ m})^3} \sim \boxed{10^{29}}$$

(b) $$m_{\text{total}} = (\textit{density})(\textit{total volume}) \sim \rho_{\text{water}}\left(nV_{\substack{\text{single}\\\text{prokaryote}}}\right) = \left(10^3 \frac{\text{kg}}{\text{m}^3}\right)(10^{29})(10^{-6} \text{ m})^3 \sim \boxed{10^{14} \text{ kg}}$$

(c) The very large mass of prokaryotes implies they are important to the biosphere. They are responsible for fixing carbon, producing oxygen, and breaking up pollutants, among many other biological roles. Humans depend on them!

1.35 The x coordinate is found as $x = r\cos\theta = (2.5 \text{ m})\cos 35° = \boxed{2.0 \text{ m}}$

and the y coordinate $y = r\sin\theta = (2.5 \text{ m})\sin 35° = \boxed{1.4 \text{ m}}$

1.36 The x distance out to the fly is 2.0 m and the y distance up to the fly is 1.0 m. Thus, we can use the Pythagorean theorem to find the distance from the origin to the fly as

$$d = \sqrt{x^2 + y^2} = \sqrt{(2.0 \text{ m})^2 + (1.0 \text{ m})^2} = \boxed{2.2 \text{ m}}$$

1.37 The distance from the origin to the fly is r in polar coordinates, and this was found to be 2.2 m in Problem 36. The angle θ is the angle between r and the horizontal reference line (the x axis in this case). Thus, the angle can be found as

$$\tan\theta = \frac{y}{x} = \frac{1.0 \text{ m}}{2.0 \text{ m}} = 0.50 \qquad \text{and} \qquad \theta = \tan^{-1}(0.50) = 27°$$

The polar coordinates are $\boxed{r = 2.2 \text{ m and } \theta = 27°}$

1.38 The x distance between the two points is $|\Delta x| = |x_2 - x_1| = |-3.0 \text{ cm} - 5.0 \text{ cm}| = 8.0 \text{ cm}$ and the y distance between them is $|\Delta y| = |y_2 - y_1| = |3.0 \text{ cm} - 4.0 \text{ cm}| = 1.0 \text{ cm}$. The distance between them is found from the Pythagorean theorem:

$$d = \sqrt{|\Delta x|^2 + |\Delta y|^2} = \sqrt{(8.0 \text{ cm})^2 + (1.0 \text{ cm})^2} = \sqrt{65 \text{ cm}^2} = \boxed{8.1 \text{ cm}}$$

1.39 Refer to the Figure given in Problem 1.40 below. The Cartesian coordinates for the two given points are:

$$x_1 = r_1\cos\theta_1 = (2.00 \text{ m})\cos 50.0° = 1.29 \text{ m} \qquad x_2 = r_2\cos\theta_2 = (5.00 \text{ m})\cos(-50.0°) = 3.21 \text{ m}$$

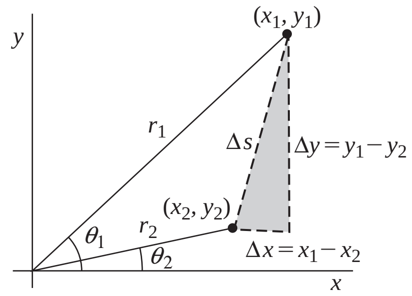
$$y_1 = r_1\sin\theta_1 = (2.00 \text{ m})\sin 50.0° = 1.53 \text{ m} \qquad y_2 = r_2\sin\theta_2 = (5.00 \text{ m})\sin(-50.0°) = -3.83 \text{ m}$$

continued on next page

The distance between the two points is then:

$$\Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(1.29\ \text{m} - 3.21\ \text{m})^2 + (1.53\ \text{m} + 3.83\ \text{m})^2} = \boxed{5.69\ \text{m}}$$

1.40 Consider the Figure shown at the right. The Cartesian coordinates for the two points are:



$$x_1 = r_1 \cos\theta_1 \qquad x_2 = r_2 \cos\theta_2$$
$$y_1 = r_1 \sin\theta_1 \qquad y_2 = r_2 \sin\theta_2$$

The distance between the two points is the length of the hypotenuse of the shaded triangle and is given by

$$\Delta s = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$$

or

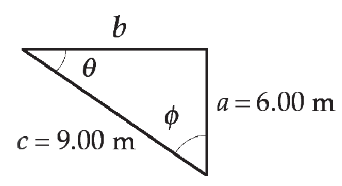
$$\Delta s = \sqrt{\left(r_1^2 \cos^2\theta_1 + r_2^2 \cos^2\theta_2 - 2r_1 r_2 \cos\theta_1 \cos\theta_2\right) + \left(r_1^2 \sin^2\theta_1 + r_2^2 \sin^2\theta_2 - 2r_1 r_2 \sin\theta_1 \sin\theta_2\right)}$$

$$= \sqrt{r_1^2\left(\cos^2\theta_1 + \sin^2\theta_1\right) + r_2^2\left(\cos^2\theta_2 + \sin^2\theta_2\right) - 2r_1 r_2\left(\cos\theta_1\cos\theta_2 + \sin\theta_1 \sin\theta_2\right)}$$

Applying the identities $\cos^2\theta + \sin^2\theta = 1$ and $\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2 = \cos(\theta_1 - \theta_2)$, this reduces to

$$\boxed{\Delta s = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2\left(\cos\theta_1\cos\theta_2 + \sin\theta_1\sin\theta_2\right)}} = \boxed{\sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_1 - \theta_2)}}$$

1.41 (a) With $a = 6.00$ m and b being two sides of this right triangle having hypotenuse $c = 9.00$ m, the Pythagorean theorem gives the unknown side as



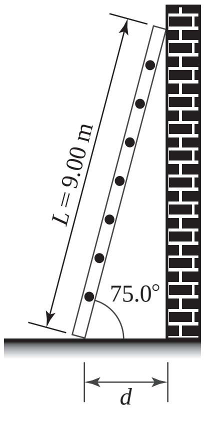
$$b = \sqrt{c^2 - a^2} = \sqrt{(9.00\ \text{m})^2 - (6.00\ \text{m})^2} = \boxed{6.71\ \text{m}}$$

(b) $\tan\theta = \dfrac{a}{b} = \dfrac{6.00\ \text{m}}{6.71\ \text{m}} = \boxed{0.894}$

(c) $\sin\phi = \dfrac{b}{c} = \dfrac{6.71\ \text{m}}{9.00\ \text{m}} = \boxed{0.746}$

1.42 From the diagram, $\cos(75.0°) = d/L$

Thus, $d = L\cos(75.0°) = (9.00\ \text{m})\cos(75.0°) = \boxed{2.33\ \text{m}}$

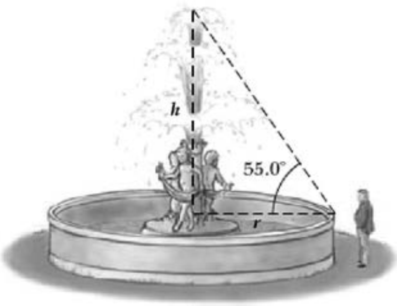


1.43 The circumference of the fountain is $C = 2\pi r$, so the radius is

$$r = \frac{C}{2\pi} = \frac{15.0 \text{ m}}{2\pi} = 2.39 \text{ m}$$

Thus, $\tan(55.0°) = \frac{h}{r} = \frac{h}{2.39 \text{ m}}$ which gives

$$h = (2.39 \text{ m})\tan(55.0°) = \boxed{3.41 \text{ m}}$$



1.44 (a) $\sin\theta = \frac{\text{opposite side}}{\text{hypotenuse}}$ so, opposite side $= (3.00 \text{ m})\sin(30.0°) = \boxed{1.50 \text{ m}}$

(b) $\cos\theta = \frac{\text{adjacent side}}{\text{hypotenuse}}$ so, adjacent side $= (3.00 \text{ m})\cos(30.0°) = \boxed{2.60 \text{ m}}$

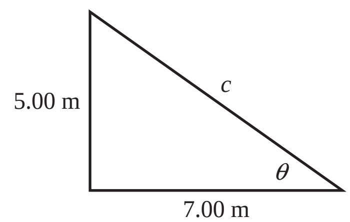
1.45 (a) The side opposite $\theta = \boxed{3.00}$ (b) The side adjacent to $\phi = \boxed{3.00}$

(c) $\cos\theta = \frac{4.00}{5.00} = \boxed{0.800}$ (d) $\sin\phi = \frac{4.00}{5.00} = \boxed{0.800}$

(e) $\tan\phi = \frac{4.00}{3.00} = \boxed{1.33}$

1.46 Using the diagram at the right, the Pythagorean theorem yields

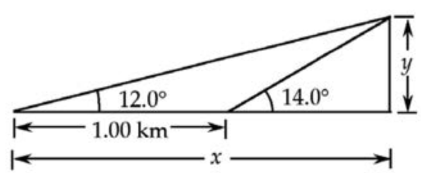
$$c = \sqrt{(5.00 \text{ m})^2 + (7.00 \text{ m})^2} = \boxed{8.60 \text{ m}}$$



1.47 From the diagram given in Problem 1.46 above, it is seen that

$$\tan\theta = \boxed{\frac{5.00}{7.00}} = 0.714 \quad \text{and} \quad \theta = \tan^{-1}(0.714) = \boxed{35.5°}$$

1.48 (a) and (b) See the Figure given at the right.



(c) Applying the definition of the tangent function to the large right triangle containing the 12.0° angle gives:

$$\boxed{y/x = \tan 12.0°} \qquad \textbf{[1]}$$

Also, applying the definition of the tangent function to the smaller right triangle containing the 14.0° angle gives:

$$\boxed{\frac{y}{x - 1.00 \text{ km}} = \tan 14.0°} \qquad \textbf{[2]}$$

(d) From Equation [1] above, observe that $x = y/\tan 12.0°$

Substituting this result into Equation [2] gives

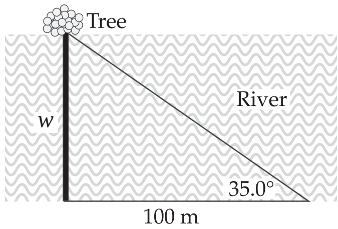
$$\frac{y \cdot \tan 12.0°}{y - (1.00 \text{ km})\tan 12.0°} = \tan 14.0°$$

continued on next page

Then, solving for the height of the mountain, y, yields

$$y = \frac{(1.00\ \text{km})\tan 12.0°\tan 14.0°}{\tan 14.0° - \tan 12.0°} = 1.44\ \text{km} = \boxed{1.44\times 10^3\ \text{m}}$$

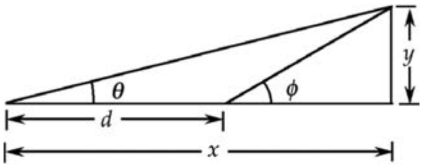
1.49 Using the sketch at the right:



$$\frac{w}{100\ \text{m}} = \tan 35.0°\ ,\ \text{ or}$$

$$w = (100\ \text{m})\tan 35.0° = \boxed{70.0\ \text{m}}$$

1.50 The figure at the right shows the situation described in the problem statement.



Applying the definition of the tangent function to the large right triangle containing the angle θ in the Figure, one obtains

$$y/x = \tan\theta \qquad \textbf{[1]}$$

Also, applying the definition of the tangent function to the small right triangle containing the angle ϕ gives

$$\frac{y}{x-d} = \tan\phi \qquad \textbf{[2]}$$

Solving Equation [1] for x and substituting the result into Equation [2] yields

$$\frac{y}{y/\tan\theta - d} = \tan\phi \qquad \text{or} \qquad \frac{y\cdot\tan\theta}{y - d\cdot\tan\theta} = \tan\phi$$

The last result simplifies to $\quad y\cdot\tan\theta = y\cdot\tan\phi - d\cdot\tan\theta\cdot\tan\phi$

Solving for y: $y(\tan\theta - \tan\phi) = -d\cdot\tan\theta\cdot\tan\phi \quad$ or

$$y = -\frac{d\cdot\tan\theta\cdot\tan\phi}{\tan\theta - \tan\phi} = \boxed{\frac{d\cdot\tan\theta\cdot\tan\phi}{\tan\phi - \tan\theta}}$$

1.51 (a) Given that $a \propto F/m$, we have $F \propto ma$. Therefore, the units of force are those of ma,

$$[F] = [ma] = [m][a] = \text{M}\left(\text{L}/\text{T}^2\right) = \boxed{\text{M L T}^{-2}}$$

(b) $[F] = \text{M}\left(\dfrac{\text{L}}{\text{T}^2}\right) = \dfrac{\text{M}\cdot\text{L}}{\text{T}^2}$ so newton $= \boxed{\dfrac{\text{kg}\cdot\text{m}}{\text{s}^2}}$

1.52 (a) $1\ \dfrac{\text{mi}}{\text{h}} = \left(1\ \dfrac{\text{mi}}{\text{h}}\right)\left(\dfrac{1.609\ \text{km}}{1\ \text{mi}}\right) = \boxed{1.609\ \dfrac{\text{km}}{\text{h}}}$

(b) $v_{\text{max}} = 55\ \dfrac{\text{mi}}{\text{h}} = \left(55\ \dfrac{\text{mi}}{\text{h}}\right)\left(\dfrac{1.609\ \text{km/h}}{1\ \text{mi/h}}\right) = \boxed{88\ \dfrac{\text{km}}{\text{h}}}$

(c) $\Delta v_{\text{max}} = 65\ \dfrac{\text{mi}}{\text{h}} - 55\ \dfrac{\text{mi}}{\text{h}} = \left(10\ \dfrac{\text{mi}}{\text{h}}\right)\left(\dfrac{1.609\ \text{km/h}}{1\ \text{mi/h}}\right) = \boxed{16\ \dfrac{\text{km}}{\text{h}}}$

1.53 (a) Since $1\text{ m} = 10^2\text{ cm}$, then $1\text{ m}^3 = (1\text{ m})^3 = (10^2\text{ cm})^3 = (10^2)^3\text{ cm}^3 = 10^6\text{ cm}^3$, giving

$$\text{mass} = (\text{density})(\text{volume}) = \left(\frac{1.0\times10^{-3}\text{ kg}}{1.0\text{ cm}^3}\right)(1.0\text{ m}^3)$$

$$= \left(1.0\times10^{-3}\,\frac{\text{kg}}{\text{cm}^3}\right)(1.0\text{ m}^3)\left(\frac{10^6\text{ cm}^3}{1\text{ m}^3}\right) = \boxed{1.0\times10^3\text{ kg}}$$

As a rough calculation, treat each of the following objects as if they were 100% water.

(b) cell: $\text{mass} = \text{density}\times\text{volume} = \left(10^3\,\frac{\text{kg}}{\text{m}^3}\right)\frac{4}{3}\pi\left(0.50\times10^{-6}\text{ m}\right)^3 = \boxed{5.2\times10^{-16}\text{ kg}}$

(c) kidney: $\text{mass} = \text{density}\times\text{volume} = \rho\left(\frac{4}{3}\pi r^3\right) = \left(10^3\,\frac{\text{kg}}{\text{m}^3}\right)\frac{4}{3}\pi\left(4.0\times10^{-2}\text{ m}\right)^3 = \boxed{0.27\text{ kg}}$

(d) fly: $\text{mass} = \text{density}\times\text{volume} = (\text{density})(\pi r^2 h)$

$$= \left(10^3\,\frac{\text{kg}}{\text{m}^3}\right)\pi\left(1.0\times10^{-3}\text{ m}\right)^2\left(4.0\times10^{-3}\text{ m}\right) = \boxed{1.3\times10^{-5}\text{ kg}}$$

1.54 Assume an average of 1 can per person each week and a population of 300 million.

(a) $$\text{number cans/year} = \left(\frac{\text{number cans/person}}{\text{week}}\right)(\text{population})(\text{weeks/year})$$

$$\approx \left(1\,\frac{\text{can/person}}{\text{week}}\right)\left(3\times10^8\text{ people}\right)(52\text{ weeks/yr})$$

$$\approx 2\times10^{10}\text{ cans/yr, or } \boxed{\sim10^{10}\text{ cans/yr}}$$

(b) $$\text{number of tons} = (\text{weight/can})(\text{number cans /year})$$

$$\approx \left[\left(0.5\,\frac{\text{oz}}{\text{can}}\right)\left(\frac{1\text{ lb}}{16\text{ oz}}\right)\left(\frac{1\text{ ton}}{2\,000\text{ lb}}\right)\right]\left(2\times10^{10}\,\frac{\text{can}}{\text{yr}}\right)$$

$$\approx 3\times10^5\text{ ton/yr, or } \boxed{\sim10^5\text{ ton/yr}}$$

Assumes an average weight of 0.5 oz of aluminum per can.

1.55 The term s has dimensions of L, a has dimensions of LT^{-2}, and t has dimensions of T. Therefore, the equation, $s = k\,a^m\,t^n$ with k being dimensionless, has dimensions of

$$\text{L} = \left(\text{LT}^{-2}\right)^m(\text{T})^n \qquad\text{or}\qquad \text{L}^1\text{T}^0 = \text{L}^m\,\text{T}^{n-2m}$$

The powers of L and T must be the same on each side of the equation. Therefore, $\text{L}^1 = \text{L}^m$ and $\boxed{m = 1}$

Likewise, equating powers of T, we see that $n - 2m = 0$, or $\boxed{n = 2m = 2}$

$\boxed{\text{Dimensional analysis cannot determine the value of } k}$, a dimensionless constant.

1.56 (a) The rate of filling in gallons per second is

$$rate = \frac{30.0\text{ gal}}{7.00\text{ min}}\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{7.14\times10^{-2}\text{ gal/s}}$$

continued on next page

(b) Note that $1\ \text{m}^3 = \left(10^2\ \text{cm}\right)^3 = \left(10^6\ \text{cm}^3\right)\left(\dfrac{1\ \text{L}}{10^3\ \text{cm}^3}\right) = 10^3\ \text{L}$. Thus,

$$rate = 7.14\times10^{-2}\ \frac{\text{gal}}{\text{s}}\left(\frac{3.786\ \text{L}}{1\ \text{gal}}\right)\left(\frac{1\ \text{m}^3}{10^3\ \text{L}}\right) = \boxed{2.70\times10^{-4}\ \text{m}^3/\text{s}}$$

(c) $$t = \frac{V_{\text{filled}}}{rate} = \frac{1.00\ \text{m}^3}{2.70\times10^{-4}\ \text{m}^3/\text{s}} = 3.70\times10^3\ \text{s}\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{1.03\ \text{h}}$$

1.57 The volume of paint used is given by $V = Ah$, where A is the area covered and h is the thickness of the layer. Thus,

$$h = \frac{V}{A} = \frac{3.79\times10^{-3}\ \text{m}^3}{25.0\ \text{m}^2} = 1.52\times10^{-4}\ \text{m} = 152\times10^{-6}\ \text{m} = \boxed{152\ \mu\text{m}}$$

1.58 (a) For a sphere, $A = 4\pi R^2$. In this case, the radius of the second sphere is twice that of the first, or $R_2 = 2R_1$.

Hence, $$\frac{A_2}{A_1} = \frac{4\pi R_2^2}{4\pi R_1^2} = \frac{R_2^2}{R_1^2} = \frac{(2R_1)^2}{R_1^2} = \boxed{4}$$

(b) For a sphere, the volume is $V = \dfrac{4}{3}\pi R^3$

Thus, $$\frac{V_2}{V_1} = \frac{(4/3)\pi R_2^3}{(4/3)\pi R_1^3} = \frac{R_2^3}{R_1^3} = \frac{(2R_1)^3}{R_1^3} = \boxed{8}$$

1.59 The estimate of the total distance cars are driven each year is

$$d = (\text{cars in use})(\text{distance traveled per car}) = \left(100\times10^6\ \text{cars}\right)\left(10^4\ \text{mi/car}\right) = 1\times10^{12}\ \text{mi}$$

At a rate of 20 mi/gal, the fuel used per year would be

$$V_1 = \frac{d}{rate_1} = \frac{1\times10^{12}\ \text{mi}}{20\ \text{mi/gal}} = 5\times10^{10}\ \text{gal}$$

If the rate increased to $25\ \text{mi/gal}$, the annual fuel consumption would be

$$V_2 = \frac{d}{rate_2} = \frac{1\times10^{12}\ \text{mi}}{25\ \text{mi/gal}} = 4\times10^{10}\ \text{gal}$$

and the fuel savings each year would be

$$savings = V_1 - V_2 = 5\times10^{10}\ \text{gal} - 4\times10^{10}\ \text{gal} = \boxed{1\times10^{10}\ \text{gal}}$$

1.60 (a) The amount paid per year would be

$$\text{annual amount} = \left(1\,000\ \frac{\text{dollars}}{\text{s}}\right)\left(\frac{8.64\times10^4\ \text{s}}{1.00\ \text{day}}\right)\left(\frac{365.25\ \text{days}}{\text{yr}}\right) = 3.16\times10^{10}\ \frac{\text{dollars}}{\text{yr}}$$

Therefore, it would take $\dfrac{10\times10^{12}\ \text{dollars}}{3.16\times10^{10}\ \text{dollars/yr}} = 3\times10^2\ \text{yr}$, or $\boxed{\sim10^2\ \text{yr}}$

(b) The circumference of the Earth at the equator is

$$C = 2\pi r = 2\pi\left(6.378\times10^6\ \text{m}\right) = 4.007\times10^7\ \text{m}$$

continued on next page

The length of one dollar bill is 0.155 m, so the length of ten trillion bills is $\ell = \left(0.155\ \dfrac{\text{m}}{\text{dollar}}\right)\left(10\times10^{12}\ \text{dollars}\right) = 1\times10^{12}\ \text{m}$. Thus, the ten trillion dollars would encircle the Earth

$$n = \frac{\ell}{C} = \frac{1\times10^{12}\ \text{m}}{4.007\times10^{7}\ \text{m}} = 2\times10^{4},\ \text{or}\ \boxed{\sim10^{4}\ \text{times}}$$

1.61 (a) $1\ \text{yr} = \left(1\ \cancel{\text{yr}}\right)\left(\dfrac{365.2\ \cancel{\text{days}}}{1\ \cancel{\text{yr}}}\right)\left(\dfrac{8.64\times10^{4}\ \text{s}}{1\ \cancel{\text{day}}}\right) = \boxed{3.16\times10^{7}\ \text{s}}$

(b) Consider a segment of the surface of the Moon which has an area of 1 m^2 and a depth of 1 m. When filled with meteorites, each having a diameter 10^{-6} m, the number of meteorites along each edge of this box is

$$n = \frac{\text{length of an edge}}{\text{meteorite diameter}} = \frac{1\ \text{m}}{10^{-6}\ \text{m}} = 10^{6}$$

The total number of meteorites in the filled box is then

$$N = n^{3} = \left(10^{6}\right)^{3} = 10^{18}$$

At the rate of 1 meteorite per second, the time to fill the box is

$$t = 10^{18}\ \text{s} = \left(10^{18}\ \text{s}\right)\left(\frac{1\ \text{y}}{3.16\times10^{7}\ \text{s}}\right) = 3\times10^{10}\ \text{yr},\ \text{or}\ \boxed{\sim10^{10}\ \text{yr}}$$

1.62 We will assume that, on average, 1 ball will be lost per hitter, that there will be about 10 hitters per inning, a game has 9 innings, and the team plays 81 home games per season. Our estimate of the number of game balls needed per season is then

$$\begin{aligned}\text{number of balls needed} &= (\text{number lost per hitter})(\text{number hitters/game})(\text{home games/year})\\ &= (1\ \text{ball per hitter})\left[\left(10\ \frac{\text{hitters}}{\text{inning}}\right)\left(9\ \frac{\text{innings}}{\text{game}}\right)\right]\left(81\ \frac{\text{games}}{\text{year}}\right)\\ &= 7300\ \frac{\text{balls}}{\text{year}} \quad \text{or} \quad \boxed{\sim10^{4}\ \frac{\text{balls}}{\text{year}}}\end{aligned}$$

1.63 The volume of the Milky Way galaxy is roughly

$$V_G = At = \left(\frac{\pi d^2}{4}\right)t \approx \frac{\pi}{4}\left(10^{21}\ \text{m}\right)^2\left(10^{19}\ \text{m}\right) \quad \text{or} \quad V_G \sim 10^{61}\ \text{m}^3$$

If, within the Milky Way galaxy, there is typically one neutron star in a spherical volume of radius $r = 3\times10^{18}$ m, then the galactic volume per neutron star is

$$V_1 = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(3\times10^{18}\ \text{m}\right)^3 = 1\times10^{56}\ \text{m}^3 \quad \text{or} \quad V_1 \sim 10^{56}\ \text{m}^3$$

The order of magnitude of the number of neutron stars in the Milky Way is then

$$n = \frac{V_G}{V_1} \sim \frac{10^{61}\ \text{m}^3}{10^{56}\ \text{m}^3} \quad \text{or} \quad \boxed{n \sim 10^{5}\ \text{neutron stars}}$$

2

Motion in One Dimension

QUICK QUIZZES

1. (a) 200 yd (b) 0 (c) 0

2. (a) False. The car may be slowing down, so that the direction of its acceleration is opposite the direction of its velocity.

(b) True. If the velocity is in the direction chosen as negative, a positive acceleration causes a decrease in speed.

(c) True. For an accelerating particle to stop at all, the velocity and acceleration must have opposite signs, so that the speed is decreasing. If this is the case, the particle will eventually come to rest. If the acceleration remains constant, however, the particle must begin to move again, opposite to the direction of its original velocity. If the particle comes to rest and then stays at rest, the acceleration has become zero at the moment the motion stops. This is the case for a braking car—the acceleration is negative and goes to zero as the car comes to rest.

3. The velocity-vs.-time graph (a) has a constant slope, indicating a constant acceleration, which is represented by the acceleration-vs.-time graph (e).

Graph (b) represents an object whose speed always increases, and does so at an ever increasing rate. Thus, the acceleration must be increasing, and the acceleration-vs.-time graph that best indicates this behavior is (d).

Graph (c) depicts an object which first has a velocity that increases at a constant rate, which means that the object's acceleration is constant. The motion then changes to one at constant speed, indicating that the acceleration of the object becomes zero. Thus, the best match to this situation is graph (f).

4. Choice (b). According to *graph b*, there are some instants in time when the object is simultaneously at two different x-coordinates. This is physically impossible.

5. (a) The *blue graph* of Figure 2.14b best shows the puck's position as a function of time. As seen in Figure 2.14a, the distance the puck has traveled grows at an increasing rate for approximately three time intervals, grows at a steady rate for about four time intervals, and then grows at a diminishing rate for the last two intervals.

(b) The *red graph* of Figure 2.14c best illustrates the speed (distance traveled per time interval) of the puck as a function of time. It shows the puck gaining speed for approximately three time intervals, moving at constant speed for about four time intervals, then slowing to rest during the last two intervals.

(c) The *green graph* of Figure 2.14d best shows the puck's acceleration as a function of time. The puck gains velocity (positive acceleration) for approximately three time intervals, moves at constant velocity (zero acceleration) for about four time intervals, and then loses velocity (negative acceleration) for roughly the last two time intervals.

6. Choice (e). The acceleration of the ball remains constant while it is in the air. The magnitude of its acceleration is the free-fall acceleration, $g = 9.80\text{ m/s}^2$.

7. Choice (c). As it travels upward, its speed decreases by 9.80 m/s during each second of its motion. When it reaches the peak of its motion, its speed becomes zero. As the ball moves downward, its speed increases by 9.80 m/s each second.

8. Choices (a) and (f). The first jumper will always be moving with a higher velocity than the second. Thus, in a given time interval, the first jumper covers more distance than the second, and the separation distance between them *increases*. At any given instant of time, the velocities of the jumpers are definitely different, because one had a head start. In a time interval after this instant, however, each jumper increases his or her velocity by the same amount, because they have the same acceleration. Thus, the difference in velocities *stays the same*.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. Once the arrow has left the bow, it has a constant downward acceleration equal to the free-fall acceleration, g. Taking upward as the positive direction, the elapsed time required for the velocity to change from an initial value of 15.0 m/s upward $\left(v_0 = +15.0\text{ m/s}\right)$ to a value of 8.00 m/s downward $\left(v_f = -8.00\text{ m/s}\right)$ is given by

$$\Delta t = \frac{\Delta v}{a} = \frac{v_f - v_0}{-g} = \frac{-8.00\text{ m/s} - (+15.0\text{ m/s})}{-9.80\text{ m/s}^2} = 2.35\text{ s}$$

Thus, the correct choice is (d).

2. In Figure MCQ2.2, there are five spaces separating adjacent oil drops, and these spaces span a distance of $\Delta x = 600$ meters. Since the drops occur every 5.0 s, the time span of each space is 5.0 s and the total time interval shown in the figure is $\Delta t = 5(5.0\text{ s}) = 25\text{ s}$. The average speed of the car is then

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{600\text{ m}}{25\text{ s}} = 24\text{ m/s}$$

making (b) the correct choice.

3. The derivation of the equations of kinematics for an object moving in one dimension (Equations 2.6 through 2.10 in the textbook) was based on the assumption that the object had a constant acceleration. Thus, (b) is the correct answer. An object having constant acceleration would have constant velocity only if that acceleration had a value of zero, so (a) is not a necessary condition. The speed (magnitude of the velocity) will increase in time only in cases when the velocity is in the same direction as the constant acceleration, so (c) is not a correct response. An object projected straight upward into the air has a constant acceleration. Yet its position (altitude) does not always increase in time (it eventually starts to fall back downward) nor is its velocity always directed downward (the direction of the constant acceleration). Thus, neither (d) nor (e) can be correct.

4. The bowling pin has a constant downward acceleration $\left(a=-g=-9.80\ \text{m/s}^2\right)$ while in flight. The velocity of the pin is directed upward on the upward part of its flight and is directed downward as it falls back toward the juggler's hand. Thus, only (d) is a true statement.

5. The initial velocity of the car is $v_0=0$ and the velocity at time t is v. The constant acceleration is therefore given by $a=\Delta v/\Delta t=\left(v-v_0\right)/t=(v-0)/t=v/t$ and the average velocity of the car is $\overline{v}=\left(v+v_0\right)/2=(v+0)/2=v/2$. The distance traveled in time t is $\Delta x=\overline{v}t=vt/2$. In the special case where $a=0$ $\left(\text{and hence } v=v_0=0\right)$, we see that statements (a), (b), (c), and (d) are all correct. However, in the general case $\left(a\neq 0, \text{ and hence } v\neq 0\right)$, only statements (b) and (c) are true. Statement (e) is not true in either case.

6. The motion of the boat is very similar to that of a object thrown straight upward into the air. In both cases, the object has a constant acceleration which is directed opposite to the direction of the initial velocity. Just as the object thrown upward slows down and stops momentarily before it starts speeding up as it falls back downward, the boat will continue to move northward for some time, slowing uniformly until it comes to a momentary stop. It will then start to move in the southward direction, gaining speed as it goes. The correct answer is (c).

7. In a position versus time graph, the velocity of the object at any point in time is the slope of the line tangent to the graph at that instant in time. The speed of the particle at this point in time is simply the magnitude (or absolute value) of the velocity at this instant in time. The displacement occurring during a time interval is equal to the difference in x-coordinates at the final and initial times of the interval $\left(\Delta x=x|_{t_f}-x|_{t_i}\right)$.

The average velocity during a time interval is the slope of the straight line connecting the points on the curve corresponding to the initial and final times of the interval $\left[\overline{v}=\Delta x/\Delta t=\left(x_f-x_i\right)/\left(t_f-t_i\right)\right]$. Thus, we see how the quantities in choices (a), (e), (c), and (d) can all be obtained from the graph. Only the acceleration, choice (b), *cannot be obtained* from the position versus time graph.

8. From $\Delta x=v_0t+\frac{1}{2}at^2$, the distance traveled in time t, starting from rest $\left(v_0=0\right)$ with constant acceleration a, is $\Delta x=\frac{1}{2}at^2$. Thus, the ratio of the distances traveled in two individual trials, one of duration $t_1=6$ s and the second of duration $t_2=2$ s, is

$$\frac{\Delta x_2}{\Delta x_1}=\frac{\frac{1}{2}at_2^2}{\frac{1}{2}at_1^2}=\left(\frac{t_2}{t_1}\right)^2=\left(\frac{2\text{ s}}{6\text{ s}}\right)^2=\frac{1}{9}$$

and the correct answer is (c).

9. The distance an object moving at a uniform speed of $v=8.5$ m/s will travel during a time interval of $\Delta t=1/1\,000\text{ s}=1.0\times 10^{-3}$ s is given by

$$\Delta x=v(\Delta t)=(8.5\text{ m/s})\left(1.0\times 10^{-3}\text{ s}\right)=8.5\times 10^{-3}\text{ m}=8.5\text{ mm}$$

so the only correct answer to this question is choice (d).

10. Once either ball has left the student's hand, it is a freely falling body with a constant acceleration $a=-g$ (taking upward as positive). Therefore, choice (e) cannot be true. The initial velocities of the red and blue balls are given by $v_{iR}=+v_0$ and $v_{iB}=-v_0$, respectively. The velocity of either ball when it has a displacement from the launch point of $\Delta y=-h$ (where h is the height of the building) is found from $v^2=v_i^2+2a(\Delta y)$ as follows:

$$v_R=-\sqrt{v_{iR}^2+2a(\Delta y)_R}=-\sqrt{\left(+v_0\right)^2+2(-g)(-h)}=-\sqrt{v_0^2+2gh}$$

and

$$v_B = -\sqrt{v_{iB}^2 + 2a(\Delta y)_B} = -\sqrt{(-v_0)^2 + 2(-g)(-h)} = -\sqrt{v_0^2 + 2gh}$$

Note that the negative sign was chosen for the radical in both cases since each ball is moving in the downward direction immediately before it reaches the ground. From this, we see that choice (c) is true. Also, the speeds of the two balls just before hitting the ground are

$$|v_R| = \left|-\sqrt{v_0^2 + 2gh}\right| = \sqrt{v_0^2 + 2gh} > v_0 \text{ and } |v_B| = \left|-\sqrt{v_0^2 + 2gh}\right| = \sqrt{v_0^2 + 2gh} > v_0$$

Therefore, $|v_R| = |v_B|$, so both choices (a) and (b) are false. However, we see that both final speeds exceed the initial speed and choice (d) is true. The correct answer to this question is then (c) and (d).

11. At ground level, the displacement of the rock from its launch point is $\Delta y = -h$, where h is the height of the tower and upward has been chosen as the positive direction. From $v^2 = v_o^2 + 2a(\Delta y)$, the speed of the rock just before hitting the ground is found to be

$$|v| = \left|\pm\sqrt{v_0^2 + 2a(\Delta y)}\right| = \sqrt{v_0^2 + 2(-g)(-h)} = \sqrt{(12 \text{ m/s})^2 + 2(9.8 \text{ m/s}^2)(40.0 \text{ m})} = 30 \text{ m/s}$$

Choice (b) is therefore the correct response to this question.

12. Once the ball has left the thrower's hand, it is a freely falling body with a constant, non-zero, acceleration of $a = -g$. Since the acceleration of the ball is not zero at any point on its trajectory, choices (a) through (d) are all false and the correct response is (e).

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. Yes. The particle may stop at some instant, but still have an acceleration, as when a ball thrown straight up reaches its maximum height.

4. (a) No. They can be used only when the acceleration is constant.

(b) Yes. Zero is a constant.

6. (a) In Figure (c), the images are farther apart for each successive time interval. The object is moving toward the right and speeding up. This means that the acceleration is positive in Figure (c).

(b) In Figure (a), the first four images show an increasing distance traveled each time interval and therefore a positive acceleration. However, after the fourth image, the spacing is decreasing, showing that the object is now slowing down (or has negative acceleration).

(c) In Figure (b), the images are equally spaced, showing that the object moved the same distance in each time interval. Hence, the velocity is constant in Figure (b).

8. (a) At the maximum height, the ball is momentarily at rest (i.e., has zero velocity). The acceleration remains constant, with magnitude equal to the free-fall acceleration g and directed downward. Thus, even though the velocity is momentarily zero, it continues to change, and the ball will begin to gain speed in the downward direction.

(b) The acceleration of the ball remains constant in magnitude and direction throughout the ball's free flight, from the instant it leaves the hand until the instant just before it strikes the ground. The acceleration is directed downward and has a magnitude equal to the freefall acceleration g.

10. (a) Successive images on the film will be separated by a constant distance if the ball has constant velocity.

(b) Starting at the right-most image, the images will be getting closer together as one moves toward the left.

(c) Starting at the right-most image, the images will be getting farther apart as one moves toward the left.

(d) As one moves from left to right, the balls will first get farther apart in each successive image, then closer together when the ball begins to slow down.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 2×10^4 mi **(b)** $\Delta x/2R_E = 2.4$

4. **(a)** 10.04 m/s **(b)** 7.042 m/s

6. **(a)** 5.00 m/s **(b)** 1.25 m/s **(c)** -2.50 m/s

(d) -3.33 m/s **(e)** 0

8. **(a)** $+4.0$ m/s **(b)** -0.50 m/s **(c)** -1.0 m/s

(d) 0

10. **(a)** 2.3 min **(b)** 64 mi

12. **(a)** L/t_1 **(b)** $-L/t_2$ **(c)** 0

(d) $2L/(t_1+t_2)$

14. **(a)** 1.3×10^2 s **(b)** 13 m

16. **(a)** The trailing runner's speed must be greater than that of the leader, and the leader's distance from the finish line must be great enough to give the trailing runner time to make up the deficient distance.

(b) $t = d/(v_1 - v_2)$ **(c)** $d_2 = v_2 d/(v_1 - v_2)$

18. **(a)** Some data points that can be used to plot the graph are as given below:

x(m)	5.75	16.0	35.3	68.0	119	192
t(s)	1.00	2.00	3.00	4.00	5.00	6.00

(b) 41.0 m/s, 41.0 m/s, 41.0 m/s

(c) 17.0 m/s, much smaller than the instantaneous velocity at $t = 4.00$ s

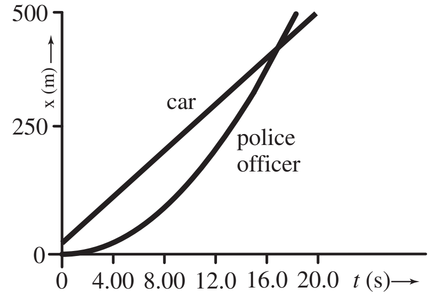
20. **(a)** $20.0 \text{ m/s}, 5.00 \text{ m/s}$ **(b)** 263 m

22. 0.391 s

24. **(i)** (a) 0 (b) 1.6 m/s^2 (c) 0.80 m/s^2

(ii) (a) 0 (b) 1.6 m/s^2 (c) 0

26. The curves intersect at $t = 16.9$ s.



28. $a = 2.74\times10^5 \text{ m/s}^2 = \left(2.79\times10^4\right)g$

30. **(a)**

(b) $v_f^2 = v_i^2 + 2a(\Delta x)$ **(c)** $a = \left(v_f^2 - v_i^2\right)/2(\Delta x)$ **(d)** 1.25 m/s^2

(e) 8.00 s

32. **(a)** 13.5 m **(b)** 13.5 m **(c)** 13.5 m

(d) 22.5 m

34. **(a)** 20.0 s **(b)** No, it cannot land safely on the 0.800 km runway.

36. **(a)** 5.51 km **(b)** $20.8 \text{ m/s}, 41.6 \text{ m/s}, 20.8 \text{ m/s}, 38.7 \text{ m/s}$

38. **(a)** 107 m **(b)** 1.49 m/s^2

40. **(a)** $v = a_1 t_1$ **(b)** $\Delta x = \frac{1}{2} a_1 t_1^2$

(c) $\Delta x_{\text{total}} = \frac{1}{2} a_1 t_1^2 + a_1 t_1 t_2 + \frac{1}{2} a_2 t_2^2$

42. 95 m

44. 29.1 s

46. 1.79 s

48. **(a)** Yes. **(b)** $v_{\text{top}} = 3.69 \text{ m/s}$ **(c)** $\left|\Delta\vec{\mathbf{v}}\right|_{\text{downward}} = 2.39 \text{ m/s}$

(d) No, $\left|\Delta\vec{\mathbf{v}}\right|_{\text{upward}} = 3.71 \text{ m/s}$. The two rocks have the same acceleration, but the rock thrown downward has a higher average speed between the two levels, and is accelerated over a smaller time interval.

50. **(a)** 21.1 m/s **(b)** 19.6 m **(c)** 18.1 m/s, 19.6 m

52. **(a)** $v = \left|-v_0 - gt\right| = \left|v_0 + gt\right|$ **(b)** $d = \frac{1}{2}gt^2$

(c) $v = \left|v_0 - gt\right|,\ d = \frac{1}{2}gt^2$

54. **(a)** 29.4 m/s **(b)** 44.1 m

56. **(a)** $-202\ \text{m/s}^2$ **(b)** 198 m

58. **(a)** 4.53 s **(b)** 14.1 m/s

60. **(a)** $v_i = h/t + gt/2$ **(b)** $v = h/t - gt/2$

62. See Solutions Section for Motion Diagrams.

64. Yes. The minimum acceleration needed to complete the 1 mile distance in the allotted time is $a_{\min} = 0.032\ \text{m/s}^2$, considerably less than what she is capable of producing.

66. **(a)** $y_1 = h - v_0 t - \frac{1}{2}gt^2,\ \ y_2 = h + v_0 t - \frac{1}{2}gt^2$ **(b)** $t_2 - t_1 = 2v_0/g$

(c) $v_{1f} = v_{2f} = -\sqrt{v_0^2 + 2gh}$ **(d)** $y_2 - y_1 = 2v_0 t$ as long as both balls are still in the air.

68. 3.10 m/s

70. **(a)** 3.00 s **(b)** $v_{0,2} = -15.2\ \text{m/s}$

(c) $v_1 = -31.4\ \text{m/s},\ v_2 = -34.8\ \text{m/s}$

72. **(a)** 2.2 s **(b)** $-21\ \text{m/s}$

74. **(a)** only if acceleration $= 0$ **(b)** Yes, for all initial velocities and accelerations.

PROBLEM SOLUTIONS

2.1 We assume that you are approximately 2 m tall and that the nerve impulse travels at uniform speed. The elapsed time is then

$$\Delta t = \frac{\Delta x}{v} = \frac{2\ \text{m}}{100\ \text{m/s}} = 2\times10^{-2}\ \text{s} = \boxed{0.02\ \text{s}}$$

2.2 (a) At constant speed, $c = 3\times10^8\ \text{m/s}$, the distance light travels in 0.1 s is

$$\Delta x = c(\Delta t) = \left(3\times10^8\ \text{m/s}\right)(0.1\ \text{s})$$

$$= \left(3\times10^7\ \cancel{\text{m}}\right)\left(\frac{1\ \text{mi}}{1.609\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{km}}}{10^3\ \cancel{\text{m}}}\right) = \boxed{2\times10^4\ \text{mi}}$$

(b) Comparing the result of part (a) to the diameter of the Earth, D_E, we find

$$\frac{\Delta x}{D_E} = \frac{\Delta x}{2R_E} = \frac{3.0\times10^7\ \cancel{\text{m}}}{2\left(6.38\times10^6\ \cancel{\text{m}}\right)} \approx \boxed{2.4} \qquad \left(\text{with } R_E = \text{Earth's radius}\right)$$

2.3 Distances traveled between pairs of cities are

$$\Delta x_1 = v_1(\Delta t_1) = (80.0 \text{ km/h})(0.500 \text{ h}) = 40.0 \text{ km}$$

$$\Delta x_2 = v_2(\Delta t_2) = (100 \text{ km/h})(0.200 \text{ h}) = 20.0 \text{ km}$$

$$\Delta x_3 = v_3(\Delta t_3) = (40.0 \text{ km/h})(0.750 \text{ h}) = 30.0 \text{ km}$$

Thus, the total distance traveled is $\Delta x = (40.0 + 20.0 + 30.0)$ km $= 90.0$ km, and the elapsed time is $\Delta t = 0.500 \text{ h} + 0.200 \text{ h} + 0.750 \text{ h} + 0.250 \text{ h} = 1.70 \text{ h}$.

(a) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{90.0 \text{ km}}{1.70 \text{ h}} = \boxed{52.9 \text{ km/h}}$

(b) $\Delta x = \boxed{90.0 \text{ km}}$ (see above)

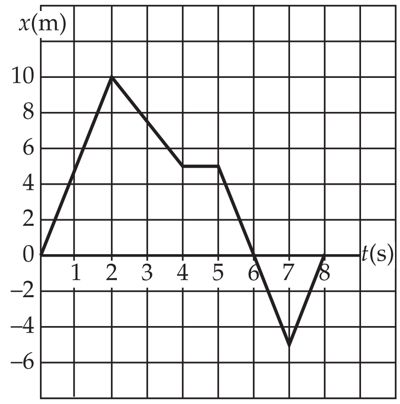
2.4 (a) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{2.000 \times 10^2 \text{ m}}{19.92 \text{ s}} = \boxed{10.04 \text{ m/s}}$

(b) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{1.000 \cancel{\text{ mi}}}{228.5 \text{ s}}\left(\dfrac{1.609 \cancel{\text{ km}}}{1 \cancel{\text{ mi}}}\right)\left(\dfrac{10^3 \text{ m}}{1 \cancel{\text{ km}}}\right) = \boxed{7.042 \text{ m/s}}$

2.5 (a) Boat A requires 1.0 h to cross the lake and 1.0 h to return, total time 2.0 h. Boat B requires 2.0 h to cross the lake at which time the race is over. $\boxed{\text{Boat A wins, being 60 km ahead of B}}$ when the race ends.

(b) Average velocity is the net displacement of the boat divided by the total elapsed time. The winning boat is back where it started, its displacement thus being zero, yielding an average velocity of $\boxed{\text{zero}}$.

2.6 The average velocity over any time interval is



$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

(a) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{10.0 \text{ m} - 0}{2.00 \text{ s} - 0} = \boxed{5.00 \text{ m/s}}$

(b) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{5.00 \text{ m} - 0}{4.00 \text{ s} - 0} = \boxed{1.25 \text{ m/s}}$

(c) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{5.00 \text{ m} - 10.0 \text{ m}}{4.00 \text{ s} - 2.00 \text{ s}} = \boxed{-2.50 \text{ m/s}}$

(d) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{-5.00 \text{ m} - 5.00 \text{ m}}{7.00 \text{ s} - 4.00 \text{ s}} = \boxed{-3.33 \text{ m/s}}$

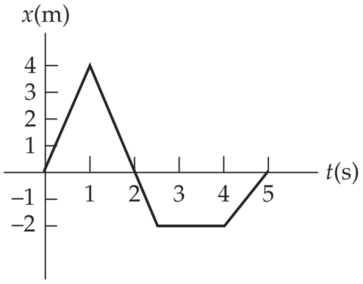
(e) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{x_2 - x_1}{t_2 - t_1} = \dfrac{0 - 0}{8.00 \text{ s} - 0} = \boxed{0}$

2.7 (a) Displacement $= \Delta x = (85.0 \text{ km/h})(35.0 \text{ min})\left(\dfrac{1 \text{ h}}{60.0 \text{ min}}\right) + 130 \text{ km} = \boxed{180 \text{ km}}$

(b) The total elapsed time is $\Delta t = (35.0 \text{ min} + 15.0 \text{ min})\left(\dfrac{1 \text{ h}}{60.0 \text{ min}}\right) + 2.00 \text{ h} = 2.83 \text{ h}$

so, $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{180 \text{ km}}{2.84 \text{ h}} = \boxed{63.6 \text{ km/h}}$

2.8 The average velocity over any time interval is



$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i}$$

(a) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{4.0\ \text{m} - 0}{1.0\ \text{s} - 0} = \boxed{+4.0\ \text{m/s}}$

(b) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{-2.0\ \text{m} - 0}{4.0\ \text{s} - 0} = \boxed{-0.50\ \text{m/s}}$

(c) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{0 - 4.0\ \text{m}}{5.0\ \text{s} - 1.0\ \text{s}} = \boxed{-1.0\ \text{m/s}}$

(d) $\bar{v} = \dfrac{\Delta x}{\Delta t} = \dfrac{0 - 0}{5.0\ \text{s} - 0} = \boxed{0}$

2.9 The plane starts from rest $(v_0 = 0)$ and maintains a constant acceleration of $a = +1.3\ \text{m/s}^2$. Thus, we find the distance it will travel before reaching the required takeoff speed $(v = 75\ \text{m/s})$, from $v^2 = v_0^2 + 2a(\Delta x)$, as

$$\Delta x = \frac{v^2 - v_0^2}{2a} = \frac{(75\ \text{m/s})^2 - 0}{2(1.3\ \text{m/s}^2)} = 2.2\times 10^3\ \text{m} = 2.2\ \text{km}$$

Since this distance is less than the length of the runway, $\boxed{\text{the plane takes off safely.}}$

2.10 (a) The time for a car to make the trip is $t = \dfrac{\Delta x}{v}$. Thus, the difference in the times for the two cars to complete the same 10 mile trip is

$$\Delta t = t_1 - t_2 = \frac{\Delta x}{v_1} - \frac{\Delta x}{v_2} = \left(\frac{10\ \text{mi}}{55\ \text{mi/h}} - \frac{10\ \text{mi}}{70\ \text{mi/h}}\right)\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = \boxed{2.3\ \text{min}}$$

(b) When the faster car has a 15.0 min lead, it is ahead by a distance equal to that traveled by the slower car in a time of 15.0 min. This distance is given by $\Delta x_1 = v_1(\Delta t) = (55\ \text{mi/h})(15\ \text{min})$.

The faster car pulls ahead of the slower car at a rate of

$$v_{\text{relative}} = 70\ \text{mi/h} - 55\ \text{mi/h} = 15\ \text{mi/h}$$

Thus, the time required for it to get distance Δx_1 ahead is

$$\Delta t = \frac{\Delta x_1}{v_{\text{relative}}} = \frac{(55\ \text{mi/h})(15\ \text{min})}{15.0\ \text{mi/h}} = 55\ \text{min}$$

Finally, the distance the faster car has traveled during this time is

$$\Delta x_2 = v_2(\Delta t) = (70\ \text{mi/h})(55\ \text{min})\left(\frac{1\ \text{h}}{60\ \text{min}}\right) = \boxed{64\ \text{mi}}$$

2.11 (a) From $v_f^2 = v_i^2 + 2a(\Delta x)$, with $v_i = 0$, $v_f = 72\ \text{km/h}$, and $\Delta x = 45$ m, the acceleration of the cheetah is found to be

$$a = \frac{v_f^2 - v_i^2}{2(\Delta x)} = \frac{\left[\left(72\ \frac{\cancel{\text{km}}}{\cancel{\text{h}}}\right)\left(\frac{10^3\ \text{m}}{1\ \cancel{\text{km}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3\,600\ \text{s}}\right)\right]^2 - 0}{2(45\ \text{m})} = \boxed{4.4\ \text{m/s}^2}$$

continued on next page

(b) The cheetah's displacement 3.5 s after starting from rest is

$$\Delta x = v_i t + \frac{1}{2}at^2 = 0 + \frac{1}{2}\left(4.4 \text{ m/s}^2\right)(3.5 \text{ s})^2 = \boxed{27 \text{ m}}$$

2.12 (a) $\bar{v}_1 = \dfrac{(\Delta x)_1}{(\Delta t)_1} = \dfrac{+L}{t_1} = \boxed{+L/t_1}$

(b) $\bar{v}_2 = \dfrac{(\Delta x)_2}{(\Delta t)_2} = \dfrac{-L}{t_2} = \boxed{-L/t_2}$

(c) $\bar{v}_{\text{total}} = \dfrac{(\Delta x)_{\text{total}}}{(\Delta t)_{\text{total}}} = \dfrac{(\Delta x)_1 + (\Delta x)_2}{t_1 + t_2} = \dfrac{+L - L}{t_1 + t_2} = \dfrac{0}{t_1 + t_2} = \boxed{0}$

(d) $(ave.\ speed)_{\text{trip}} = \dfrac{total\ distance\ traveled}{(\Delta t)_{\text{total}}} = \dfrac{|(\Delta x)_1| + |(\Delta x)_2|}{t_1 + t_2} = \dfrac{|+L| + |-L|}{t_1 + t_2} = \boxed{\dfrac{2L}{t_1 + t_2}}$

2.13 (a) The total time for the trip is $t_{\text{total}} = t_1 + 22.0 \text{ min} = t_1 + 0.367 \text{ h}$, where t_1 is the time spent traveling at $v_1 = 89.5 \text{ km/h}$. Thus, the distance traveled is $\Delta x = v_1 t_1 = \bar{v} t_{\text{total}}$, which gives

$$(89.5 \text{ km/h})t_1 = (77.8 \text{ km/h})(t_1 + 0.367 \text{ h}) = (77.8 \text{ km/h})t_1 + 28.5 \text{ km}$$

or, $(89.5 \text{ km/h} - 77.8 \text{ km/h})t_1 = 28.5 \text{ km}$

From which, $t_1 = 2.44 \text{ h}$ for a total time of $t_{\text{total}} = t_1 + 0.367 \text{ h} = \boxed{2.81 \text{ h}}$

(b) The distance traveled during the trip is $\Delta x = v_1 t_1 = \bar{v} t_{\text{total}}$, giving

$$\Delta x = \bar{v} t_{\text{total}} = (77.8 \text{ km/h})(2.81 \text{ h}) = \boxed{219 \text{ km}}$$

2.14 (a) At the end of the race, the tortoise has been moving for time t and the hare for a time $t - 2.0 \text{ min} = t - 120 \text{ s}$. The speed of the tortoise is $v_t = 0.100 \text{ m/s}$, and the speed of the hare is $v_h = 20 v_t = 2.0 \text{ m/s}$. The tortoise travels distance x_t, which is 0.20 m larger than the distance x_h traveled by the hare. Hence,

$$x_t = x_h + 0.20 \text{ m}$$

which becomes $v_t\, t = v_h (t - 120 \text{ s}) + 0.20 \text{ m}$

or $(0.100 \text{ m/s})t = (2.0 \text{ m/s})(t - 120 \text{ s}) + 0.20 \text{ m}$

This gives the time of the race as $t = \boxed{1.3 \times 10^2 \text{ s}}$

(b) $x_t = v_t t = (0.100 \text{ m/s})\left(1.3 \times 10^2 \text{ s}\right) = \boxed{13 \text{ m}}$

2.15 The maximum allowed time to complete the trip is

$$t_{\text{total}} = \frac{\text{total distance}}{\text{required average speed}} = \frac{1600 \text{ m}}{250 \text{ km/h}}\left(\frac{1 \text{ km/h}}{0.278 \text{ m/s}}\right) = 23.0 \text{ s}$$

The time spent in the first half of the trip is

$$t_1 = \frac{\text{half distance}}{\bar{v}_1} = \frac{800 \text{ m}}{230 \text{ km/h}}\left(\frac{1 \text{ km/h}}{0.278 \text{ m/s}}\right) = 12.5 \text{ s}$$

continued on next page

Thus, the maximum time that can be spent on the second half of the trip is

$$t_2 = t_{\text{total}} - t_1 = 23.0 \text{ s} - 12.5 \text{ s} = 10.5 \text{ s}$$

and the required average speed on the second half is

$$\bar{v}_2 = \frac{\text{half distance}}{t_2} = \frac{800 \text{ m}}{10.5 \text{ s}} = 76.2 \text{ m/s}\left(\frac{1 \text{ km/h}}{0.278 \text{ m/s}}\right) = \boxed{274 \text{ km/h}}$$

2.16 (a) In order for the trailing athlete to be able to catch the leader, his speed (v_1) must be greater than that of the leading athlete (v_2), and the distance between the leading athlete and the finish line must be great enough to give the trailing athlete sufficient time to make up the deficient distance, *d*.

(b) During a time *t* the leading athlete will travel a distance $d_2 = v_2 t$ and the trailing athlete will travel a distance $d_1 = v_1 t$. Only when $d_1 = d_2 + d$ (where *d* is the initial distance the trailing athlete was behind the leader) will the trailing athlete have caught the leader. Requiring that this condition be satisfied gives the elapsed time required for the second athlete to overtake the first:

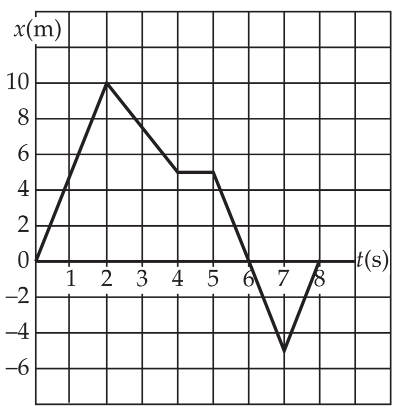
$$d_1 = d_2 + d \quad \text{or} \quad v_1 t = v_2 t + d$$

giving $\quad v_1 t - v_2 t = d \quad$ or $\quad t = \boxed{d/(v_1 - v_2)}$

(c) In order for the trailing athlete to be able to at least tie for first place, the initial distance *D* between the leader and the finish line must be greater than or equal to the distance the leader can travel in the time *t* calculated above (i.e., the time required to overtake the leader). That is, we must require that

$$D \geq d_2 = v_2 t = v_2\left[d/(v_1 - v_2)\right] \quad \text{or} \quad \boxed{D \geq \frac{v_2 d}{v_1 - v_2}}$$

2.17 The instantaneous velocity at any time is the slope of the *x* vs. *t* graph at that time. We compute this slope by using two points on a straight segment of the curve, one point on each side of the point of interest.



(a) $$v_{t=1.00 \text{ s}} = \frac{10.0 \text{ m} - 0}{2.00 \text{ s} - 0} = \boxed{5.00 \text{ m/s}}$$

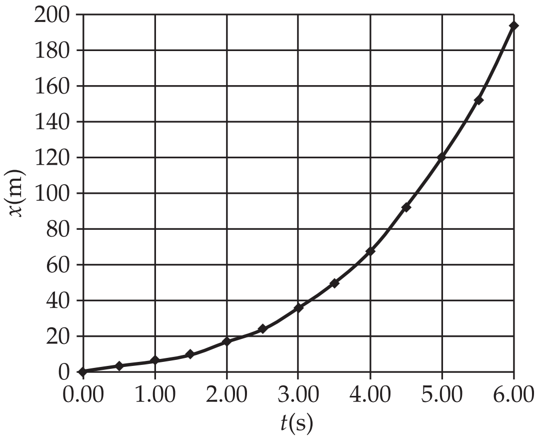
(b) $$v_{t=3.00 \text{ s}} = \frac{(5.00 - 10.0) \text{ m}}{(4.00 - 2.00) \text{ s}} = \boxed{-2.50 \text{ m/s}}$$

(c) $$v_{t=4.50 \text{ s}} = \frac{(5.00 - 5.00) \text{ m}}{(5.00 - 4.00) \text{ s}} = \boxed{0}$$

(d) $$v_{t=7.50 \text{ s}} = \frac{0 - (-5.00 \text{ m})}{(8.00 - 7.00) \text{ s}} = \boxed{5.00 \text{ m/s}}$$

2.18 (a) A few typical values are

t (s)	x (m)
1.00	5.75
2.00	16.0
3.00	35.3
4.00	68.0
5.00	119
6.00	192



(b) We will use a 0.400 s interval centered at t = 400 s. We find at t = 3.80 s, x = 60.2 m and at t = 4.20 s, x = 76.6 m. Therefore,

$$v = \frac{\Delta x}{\Delta t} = \frac{16.4 \text{ m}}{0.400 \text{ s}} = \boxed{41.0 \text{ m/s}}$$

Using a time interval of 0.200 s, we find the corresponding values to be: at t = 3.90 s, x = 64.0 m and at t = 4.10 s, x = 72.2 m. Thus,

$$v = \frac{\Delta x}{\Delta t} = \frac{8.20 \text{ m}}{0.200 \text{ s}} = \boxed{41.0 \text{ m/s}}$$

For a time interval of 0.100 s, the values are: at t = 3.95 s, x = 66.0 m, and at t = 4.05 s, x = 70.1 m. Therefore,

$$v = \frac{\Delta x}{\Delta t} = \frac{4.10 \text{ m}}{0.100 \text{ s}} = \boxed{41.0 \text{ m/s}}$$

(c) At t = 4.00 s, x = 68.0 m. Thus, for the first 4.00 s,

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{68.0 \text{ m} - 0}{4.00 \text{ s} - 0} = \boxed{17.0 \text{ m/s}}$$

This value is $\boxed{\text{much less}}$ than the instantaneous velocity at t = 4.00 s.

2.19 Choose a coordinate axis with the origin at the flagpole and east as the positive direction. Then, using $x = x_0 + v_0 t + \frac{1}{2}at^2$ with a = 0 for each runner, the x-coordinate of each runner at time t is

$$x_A = -4.0 \text{ mi} + (6.0 \text{ mi/h})t \quad \text{and} \quad x_B = 3.0 \text{ mi} + (-5.0 \text{ mi/h})t$$

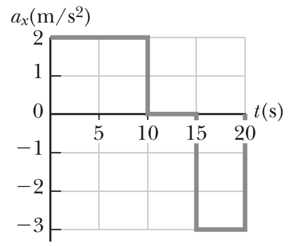
When the runners meet, $x_A = x_B$

giving $-4.0 \text{ mi} + (6.0 \text{ mi/h})t = 3.0 \text{ mi} + (-5.0 \text{ mi/h})t$

or $(6.0 \text{ mi/h} + 5.0 \text{ mi/h})t = 3.0 \text{ mi} + 4.0 \text{ mi}$

This gives the elapsed time when they meet as $t = (7.0 \text{ mi})/(11.0 \text{ mi/h}) = 0.64$ h. At this time, $x_A = x_B = -0.18$ mi. Thus, they meet $\boxed{\text{0.18 mi west of the flagpole}}$.

2.20 From the figure at the right, observe that the motion of this particle can be broken into three distinct time intervals, during each of which the particle has a constant acceleration. These intervals and the associated accelerations are



$$0 \le t < 10.0 \text{ s}, \quad a = a_1 = +2.00 \text{ m/s}^2$$

$$10 \le t < 15.0 \text{ s}, \quad a = a_2 = 0$$

and $$15.0 \le t < 20.0 \text{ s}, \quad a = a_3 = -3.00 \text{ m/s}^2$$

(a) Applying $v_f = v_i + a(\Delta t)$ to each of the three time intervals gives

for $0 \le t < 10.0$ s, $\quad v_{10} = v_0 + a_1(\Delta t_1) = 0 + (2.00 \text{ m/s}^2)(10.0 \text{ s}) = \boxed{20.0 \text{ m/s}}$

for $10.0 \text{ s} \le t < 15.0$ s, $\quad v_{15} = v_{10} + a_2(\Delta t_2) = 20.0 \text{ m/s} + 0 = 20.0 \text{ m/s}$

for $15.0 \text{ s} \le t < 20.0$ s, $\quad v_{20} = v_{15} + a_3(\Delta t_3) = 20.0 \text{ m/s} + (-3.00 \text{ m/s}^2)(5.00 \text{ s}) = \boxed{5.00 \text{ m/s}}$

(b) Applying $\Delta x = v_i(\Delta t) + \frac{1}{2}a(\Delta t)^2$ to each of the time intervals gives

for $0 \le t < 10.0$ s,

$$\Delta x_1 = v_0 \Delta t_1 + \frac{1}{2}a_1(\Delta t_1)^2 = 0 + \frac{1}{2}(2.00 \text{ m/s}^2)(10.0 \text{ s})^2 = 1.00 \times 10^2 \text{ m}$$

for $10.0 \text{ s} \le t < 15.0$ s,

$$\Delta x_2 = v_{10}\Delta t_2 + \frac{1}{2}a_2(\Delta t_2)^2 = (20.0 \text{ m/s})(5.00 \text{ s}) + 0 = 1.00 \times 10^2 \text{ m}$$

for $15.0 \text{ s} \le t < 20.0$ s,

$$\Delta x_3 = v_{15}\Delta t_3 + \frac{1}{2}a_3(\Delta t_3)^2$$

$$= (20.0 \text{ m/s})(5.00 \text{ s}) + \frac{1}{2}(-3.00 \text{ m/s}^2)(5.00 \text{ s})^2 = 62.5 \text{ m}$$

Thus, the total distance traveled in the first 20.0 s is

$$\Delta x_{\text{total}} = \Delta x_1 + \Delta x_2 + \Delta x_3 = 100 \text{ m} + 100 \text{ m} + 62.5 \text{ m} = \boxed{263 \text{ m}}$$

2.21 We choose the positive direction to point away from the wall. Then, the initial velocity of the ball is $v_i = -25.0$ m/s and the final velocity is $v_f = +22.0$ m/s. If this change in velocity occurs over a time interval of $\Delta t = 3.50$ ms (i.e., the interval during which the ball is in contact with the wall), the average acceleration is

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_i}{\Delta t} = \frac{+22.0 \text{ m/s} - (-25.0 \text{ m/s})}{3.50 \times 10^{-3} \text{ s}} = \boxed{1.34 \times 10^4 \text{ m/s}^2}$$

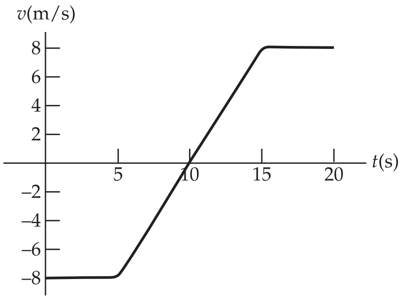
2.22 From $a = \Delta v / \Delta t$, the required time is

$$\Delta t = \frac{\Delta v}{a} = \left(\frac{60.0 \text{ mi/h} - 0}{7g}\right)\left(\frac{1g}{9.80 \text{ m/s}^2}\right)\left(\frac{0.447 \text{ m/s}}{1 \text{ mi/h}}\right) = \boxed{0.391 \text{ s}}$$

2.23 From $a = \dfrac{\Delta v}{\Delta t}$, we have $\Delta t = \dfrac{\Delta v}{a} = \dfrac{(60-55) \text{ mi/h}}{0.60 \text{ m/s}^2}\left(\dfrac{0.447 \text{ m/s}}{1 \text{ mi/h}}\right) = \boxed{3.7 \text{ s}}$

2.24 **(i)** (a) From $t = 0$ to $t = 5.0$ s,

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{-8.0 \text{ m/s} - (-8.0 \text{ m/s})}{5.0 \text{ s} - 0} = \boxed{0}$$



(b) From $t = 5.0$ s to $t = 15$ s,

$$\bar{a} = \frac{8.0 \text{ m/s} - (-8.0 \text{ m/s})}{15 \text{ s} - 5.0 \text{ s}} = \boxed{1.6 \text{ m/s}^2}$$

(c) From $t = 0$ to $t = 20$ s,

$$\bar{a} = \frac{8.0 \text{ m/s} - (-8.0 \text{ m/s})}{20 \text{ s} - 0} = \boxed{0.80 \text{ m/s}^2}$$

(ii) At any instant, the instantaneous acceleration equals the slope of the line tangent to the v vs. t graph at that point in time.

(a) At $t = 2.0$ s, the slope of the tangent line to the curve is $\boxed{0}$.

(b) At $t = 10$ s, the slope of the tangent line is $\boxed{1.6 \text{ m/s}^2}$.

(c) At $t = 18$ s, the slope of the tangent line is $\boxed{0}$.

2.25 (a) $\bar{a} = \dfrac{\Delta v}{\Delta t} = \dfrac{175 \text{ mi/h} - 0}{2.5 \text{ s}} = \boxed{70.0 \text{ mi/h} \cdot \text{s}}$

or $$\bar{a} = \left(70.0 \frac{\cancel{\text{mi}}}{\cancel{\text{h}} \cdot \text{s}}\right)\left(\frac{1609 \text{ m}}{1 \cancel{\text{mi}}}\right)\left(\frac{1 \cancel{\text{h}}}{3600 \text{ s}}\right) = \boxed{31.3 \text{ m/s}^2}$$

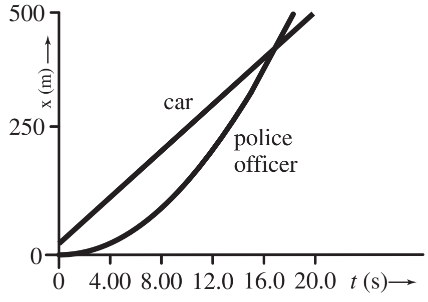
Alternatively, $$\bar{a} = \left(31.3 \frac{\text{m}}{\text{s}^2}\right)\left(\frac{1\,g}{9.80 \text{ m/s}^2}\right) = \boxed{3.19g}$$

(b) If the acceleration is constant, $\Delta x = v_0 t + \frac{1}{2}at^2$

$$\Delta x = 0 + \frac{1}{2}\left(31.3 \frac{\text{m}}{\text{s}^2}\right)(2.50 \text{ s})^2 = \boxed{97.8 \text{ m}}$$

or $$\Delta x = (97.8 \text{ m})\left(\frac{3.281 \text{ ft}}{1 \text{ m}}\right) = \boxed{321 \text{ ft}}$$

2.26 As in the algebraic solution to Example 2.5, we let t represent the time the trooper has been moving.



We graph

$$x_{\text{car}} = 24.0 \text{ m} + (24.0 \text{ m/s})t$$

and

$$x_{\text{trooper}} = (1.50 \text{ m/s}^2)t^2$$

The curves intersect at $t = \boxed{16.9 \text{ s}}$

2.27 Apply $\Delta x = v_0 + \frac{1}{2}at^2$ to the 2.00-second time interval during which the object moves from $x_i = 3.00$ cm to $x_f = -5.00$ cm. With $v_0 = 12.0$ cm/s, this yields an acceleration of

$$a = \frac{2\left[\left(x_f - x_i\right) - v_0 t\right]}{t^2} = \frac{2\left[(-5.00 - 3.00)\ \text{cm} - (12.0\ \text{cm/s})(2.00\ \text{s})\right]}{(2.00\ \text{s})^2}$$

or $\quad a = \boxed{-16.0\ \text{cm/s}^2}$

2.28 From $v^2 = v_0^2 + 2a(\Delta x)$, we have $\left(10.97\times10^3\ \text{m/s}\right)^2 = 0 + 2a(220\ \text{m})$ so that

$$a = \frac{v^2 - v_0^2}{2(\Delta x)} = \frac{\left(10.97\times10^3\ \text{m/s}\right)^2 - 0}{2(220\ \text{m})} = \boxed{2.74\times10^5\ \text{m/s}^2}$$

$$= \left(2.74\times10^5\ \text{m/s}^2\right)\left(\frac{1\ g}{9.80\ \text{m/s}^2}\right) = \boxed{2.79\times10^4\ \text{times}\ g!}$$

2.29 (a) $\Delta x = \bar{v}(\Delta t) = \left[\left(v + v_0\right)/2\right]\Delta t$ becomes

$$40.0\ \text{m} = \left(\frac{2.80\,\text{m/s} + v_0}{2}\right)(8.50\ \text{s})$$

which yields $\quad v_0 = \frac{2}{8.50\ \text{s}}(40.0\ \text{m}) - 2.80\,\text{m/s} = \boxed{6.61\ \text{m/s}}$

(b) $a = \frac{v - v_0}{\Delta t} = \frac{2.80\ \text{m/s} - 6.61\ \text{m/s}}{8.50\ \text{s}} = \boxed{-0.448\ \text{m/s}^2}$

2.30 (a)

(b) The known quantities are initial velocity, final velocity, and displacement. The kinematics equation that relates these quantities to acceleration is $\boxed{v_f^2 = v_i^2 + 2a(\Delta x)}$

(c) $a = \boxed{\dfrac{v_f^2 - v_i^2}{2(\Delta x)}}$

(d) $a = \dfrac{v_f^2 - v_i^2}{2(\Delta x)} = \dfrac{(30.0\ \text{m/s})^2 - (20.0\ \text{m/s})^2}{2\left(2.00\times10^2\ \text{m}\right)} = \boxed{1.25\ \text{m/s}^2}$

(e) Using $a = \Delta v/\Delta t$, we find that $\quad \Delta t = \dfrac{\Delta v}{a} = \dfrac{v_f - v_i}{a} = \dfrac{30.0\ \text{m/s} - 20.0\ \text{m/s}}{1.25\ \text{m/s}^2} = \boxed{8.00\ \text{s}}$

2.31 (a) With $v = 120$ km/h, $v^2 = v_0^2 + 2a(\Delta x)$ yields

$$a = \frac{v^2 - v_0^2}{2(\Delta x)} = \frac{\left[(120\ \text{km/h})^2 - 0\right]}{2(240\ \text{m})}\left(\frac{0.278\ \text{m/s}}{1\ \text{km/h}}\right)^2 = \boxed{2.32\ \text{m/s}^2}$$

(b) The required time is $\quad \Delta t = \dfrac{v - v_0}{a} = \dfrac{(120\ \text{km/h} - 0)}{2.32\ \text{m/s}^2}\left(\dfrac{0.278\ \text{m/s}}{1\ \text{km/h}}\right) = \boxed{14.4\ \text{s}}$

2.32 (a) From $v_f^2 = v_i^2 + 2a(\Delta x)$, with $v_i = 6.00\ \text{m/s}$ and $v_f = 12.0\ \text{m/s}$, we find

$$\Delta x = \frac{v_f^2 - v_i^2}{2a} = \frac{(12.0\ \text{m/s})^2 - (6.00\ \text{m/s})^2}{2\left(4.00\ \text{m/s}^2\right)} = \boxed{13.5\ \text{m}}$$

(b) In this case, the object moves in the same direction for the entire time interval and the total distance traveled is simply the magnitude or absolute value of the displacement. That is,

$$d = |\Delta x| = \boxed{13.5\ \text{m}}$$

(c) Here, $v_i = -6.00\ \text{m/s}$ and $v_f = 12.0\ \text{m/s}$, and we find

$$\Delta x = \frac{v_f^2 - v_i^2}{2a} = \boxed{13.5\ \text{m}} \qquad \text{[the same as in part (a)]}$$

(d) In this case, the object initially slows down as it travels in the negative *x*-direction, stops momentarily, and then gains speed as it begins traveling in the positive *x*-direction. We find the total distance traveled by first finding the displacement during each phase of this motion.

While coming to rest $\left(v_i = -6.00\ \text{m/s},\ v_f = 0\right)$,

$$\Delta x_1 = \frac{v_f^2 - v_i^2}{2a} = \frac{(0)^2 - (-6.00\ \text{m/s})^2}{2\left(4.00\ \text{m/s}^2\right)} = -4.50\ \text{m}$$

After reversing direction $\left(v_i = 0,\ v_f = 12.0\ \text{m/s}\right)$,

$$\Delta x_2 = \frac{v_f^2 - v_i^2}{2a} = \frac{(12.0\ \text{m/s})^2 - (0)^2}{2\left(4.00\ \text{m/s}^2\right)} = 18.0\ \text{m}$$

Note that the net displacement is $\Delta x = \Delta x_1 + \Delta x_2 = -4.50\ \text{m} + 18.0\ \text{m} = 13.5\ \text{m}$, as found in part (c) above. However, the total distance traveled in this case is

$$d = |\Delta x_1| + |\Delta x_2| = |-4.50\ \text{m}| + |18.0\ \text{m}| = \boxed{22.5\ \text{m}}$$

2.33 (a) $a = \dfrac{v - v_0}{\Delta t} = \dfrac{24.0\ \text{m/s}^2 - 0}{2.95\ \text{s}} = \boxed{8.14\ \text{m/s}^2}$

(b) From $a = \Delta v/\Delta t$, the required time is $\Delta t = \dfrac{v_f - v_i}{a} = \dfrac{20.0\ \text{m/s} - 10.0\ \text{m/s}}{8.14\ \text{m/s}^2} = \boxed{1.23\ \text{s}}$

(c) $\boxed{\text{Yes.}}$ For uniform acceleration, the change in velocity Δv generated in time Δt is given by $\Delta v = a(\Delta t)$. From this, it is seen that doubling the length of the time interval Δt will always double the change in velocity Δv. A more precise way of stating this is: “When acceleration is constant, velocity is a linear function of time.”

2.34 (a) The time required to stop the plane is $t = \dfrac{v - v_0}{a} = \dfrac{0 - 100\ \text{m/s}}{-5.00\ \text{m/s}^2} = \boxed{20.0\ \text{s}}$

(b) The minimum distance needed to stop is

$$\Delta x = \overline{v}t = \left(\frac{v + v_0}{2}\right)t = \left(\frac{0 + 100\ \text{m/s}}{2}\right)(20.0\ \text{s}) = 1000\ \text{m} = 1.00\ \text{km}$$

Thus, the plane requires a minimum runway length of 1.00 km.

$\boxed{\text{It cannot land safely on a 0.800 km runway.}}$

2.35 We choose $x = 0$ and $t = 0$ at the location of Sue's car when she first spots the van and applies the brakes. Then, the initial conditions for Sue's car are $x_{0S} = 0$ and $v_{0S} = 30.0\ \text{m/s}$. Her constant acceleration for $t \geq 0$ is $a_S = -2.00\ \text{m/s}^2$. The initial conditions for the van are $x_{0V} = 155\ \text{m}$, $v_{0V} = 5.00\ \text{m/s}$, and its constant acceleration is $a_V = 0$. We then use $\Delta x = x - x_0 = v_0 t + \frac{1}{2}at^2$ to write an equation for the x-coordinate of each vehicle for $t \geq 0$. This gives

Sue's Car: $x_S - 0 = (30.0\ \text{m/s})t + \dfrac{1}{2}(-2.00\ \text{m/s}^2)t^2$ or $x_S = (30.0\ \text{m/s})t - (1.00\ \text{m/s}^2)t^2$

Van: $x_V - 155\ \text{m} = (5.00\ \text{m/s})t + \dfrac{1}{2}(0)t^2$ or $x_V = 155\ \text{m} + (5.00\ \text{m/s})t$

In order for a collision to occur, the two vehicles must be at the same location (i.e., $x_S = x_V$). Thus, we test for a collision by equating the two equations for the x-coordinates and see if the resulting equation has any real solutions.

$$x_S = x_V \quad \Rightarrow \quad (30.0\ \text{m/s})t - (1.00\ \text{m/s}^2)t^2 = 155\ \text{m} + (5.00\ \text{m/s})t$$

$$\text{or} \quad (1.00\ \text{m/s}^2)t^2 - (25.00\ \text{m/s}) + 155\ \text{m} = 0$$

Using the quadratic formula yields

$$t = \frac{-(-25.00\ \text{m/s}) \pm \sqrt{(-25.00\ \text{m/s})^2 - 4(1.00\ \text{m/s}^2)(155\ \text{m})}}{2(1.00\ \text{m/s}^2)} = 13.6\ \text{s} \quad \text{or} \quad \boxed{11.4\ \text{s}}$$

The solutions are real, not imaginary, so $\boxed{\text{a collision will occur}}$. The smaller of the two solutions is the collision time. (The larger solution tells when the van would pull ahead of the car again if the vehicles could pass harmlessly through each other.) The x-coordinate where the collision occurs is given by

$$x_{\text{collision}} = x_S\big|_{t=11.4\ \text{s}} = x_V\big|_{t=11.4\ \text{s}} = 155\ \text{m} + (5.00\ \text{m/s})(11.4\ \text{s}) = \boxed{212\ \text{m}}$$

2.36 The velocity at the end of the first interval is

$$v = v_0 + at = 0 + (2.77\ \text{m/s})(15.0\ \text{s}) = 41.6\ \text{m/s}$$

This is also the constant velocity during the second interval and the initial velocity for the third interval. Also, note that the duration of the second interval is $t_2 = (2.05\ \text{min})(60.0\ \text{s/1 min}) = 123\ \text{s}$.

(a) From $\Delta x = v_0 t + \dfrac{1}{2}at^2$, the total displacement is

$$\begin{aligned}(\Delta x)_{\text{total}} &= (\Delta x)_1 + (\Delta x)_2 + (\Delta x)_3 \\ &= \left[0 + \frac{1}{2}(2.77\ \text{m/s}^2)(15.0\ \text{s})^2\right] + \left[(41.6\ \text{m/s})(123\ \text{s}) + 0\right] \\ &\quad + \left[(41.6\ \text{m/s})(4.39\ \text{s}) + \frac{1}{2}(-9.47\ \text{m/s}^2)(4.39\ \text{s})^2\right]\end{aligned}$$

or $(\Delta x)_{\text{total}} = 312\ \text{m} + 5.12 \times 10^3\ \text{m} + 91.4\ \text{m} = 5.52 \times 10^3\text{m} = \boxed{5.52\ \text{km}}$

continued on next page

(b) $\bar{v}_1 = \dfrac{(\Delta x)_1}{t_1} = \dfrac{312\ \text{m}}{15.0\ \text{s}} = \boxed{20.8\ \text{m/s}}$

$\bar{v}_2 = \dfrac{(\Delta x)_2}{t_2} = \dfrac{5.12\times 10^3\ \text{m}}{123\ \text{s}} = \boxed{41.6\ \text{m/s}}$

$\bar{v}_3 = \dfrac{(\Delta x)_3}{t_3} = \dfrac{91.4\ \text{m}}{4.39\ \text{s}} = \boxed{20.8\ \text{m/s}}$, and the average velocity for the

total trip is $\bar{v}_{\text{total}} = \dfrac{(\Delta x)_{\text{total}}}{t_{\text{total}}} = \dfrac{5.52\times 10^3\ \text{m}}{(15.0+123+4.39)\ \text{s}} = \boxed{38.8\ \text{m/s}}$

2.37 Using the uniformly accelerated motion equation $\Delta x = v_0 t + \frac{1}{2}at^2$ for the full 40 s interval yields $\Delta x = (20\ \text{m/s})(40\ \text{s}) + \frac{1}{2}(-1.0\ \text{m/s}^2)(40\ \text{s})^2 = 0$, which is obviously wrong. The source of the error is found by computing the time required for the train to come to rest. This time is

$$t = \frac{v - v_0}{a} = \frac{0 - 20\ \text{m/s}}{-1.0\ \text{m/s}^2} = 20\ \text{s}$$

Thus, the train is slowing down for the first 20 s and is at rest for the last 20 s of the 40 s interval.

The acceleration is not constant during the full 40 s. It is, however, constant during the first 20 s as the train slows to rest. Application of $\Delta x = v_0 t + \frac{1}{2}at^2$ to this interval gives the stopping distance as

$$\Delta x = (20\ \text{m/s})(20\ \text{s}) + \frac{1}{2}(-1.0\ \text{m/s}^2)(20\ \text{s})^2 = \boxed{200\ \text{m}}$$

2.38 $v_0 = 0$ and $v_f = \left(40.0\ \dfrac{\text{mi}}{\text{h}}\right)\left(\dfrac{0.447\ \text{m/s}}{1\ \text{mi/h}}\right) = 17.9\ \text{m/s}$

(a) To find the distance traveled, we use

$$\Delta x = \bar{v}t = \left(\frac{v_f + v_0}{2}\right)t = \left(\frac{17.9\ \text{m/s} + 0}{2}\right)(12.0\ \text{s}) = \boxed{107\ \text{m}}$$

(b) The constant acceleration is $a = \dfrac{v_f - v_0}{t} = \dfrac{17.9\ \text{m/s} - 0}{12.0\ \text{s}} = \boxed{1.49\ \text{m/s}^2}$

2.39 At the end of the acceleration period, the velocity is

$$v = v_0 + at_{\text{accel}} = 0 + (1.5\ \text{m/s}^2)(5.0\ \text{s}) = 7.5\ \text{m/s}$$

This is also the initial velocity for the braking period.

(a) After braking, $v_f = v + at_{\text{brake}} = 7.5\ \text{m/s} + (-2.0\ \text{m/s}^2)(3.0\ \text{s}) = \boxed{1.5\ \text{m/s}}$

(b) The total distance traveled is

$$\Delta x_{\text{total}} = (\Delta x)_{\text{accel}} + (\Delta x)_{\text{brake}} = (\bar{v}t)_{\text{accel}} + (\bar{v}t)_{\text{brake}} = \left(\frac{v + v_0}{2}\right)t_{\text{accel}} + \left(\frac{v_f + v}{2}\right)t_{\text{brake}}$$

$$\Delta x_{\text{total}} = \left(\frac{7.5\ \text{m/s} + 0}{2}\right)(5.0\ \text{s}) + \left(\frac{1.5\ \text{m/s} + 7.5\ \text{m/s}}{2}\right)(3.0\ \text{s}) = \boxed{32\ \text{m}}$$

2.40 For the acceleration period, the parameters for the car are: initial velocity $= v_{ia} = 0$, acceleration $= a_a = a_1$, elapsed time $= (\Delta t)_a = t_1$, and final velocity $= v_{fa}$. For the braking period, the parameters are: initial velocity $= v_{ib} =$ final velocity of acceleration period $= v_{fa}$, acceleration $= a_b = a_2$, and elapsed time $= (\Delta t)_b = t_2$.

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(a) To determine the velocity of the car just before the brakes are engaged, we apply $v_f = v_i + a(\Delta t)$ to the acceleration period and find

$$v_{ib} = v_{fa} = v_{ia} + a_a(\Delta t)_a = 0 + a_1 t_1 \qquad \text{or} \qquad v_{ib} = \boxed{a_1 t_1}$$

(b) We may use $\Delta x = v_i(\Delta t) + \frac{1}{2}a(\Delta t)^2$ to determine the distance traveled during the acceleration period (i.e., before the driver begins to brake). This gives

$$(\Delta x)_a = v_{ia}(\Delta t)_a + \tfrac{1}{2}a_a(\Delta t)_a^2 = 0 + \frac{1}{2}a_1 t_1^2 \qquad \text{or} \qquad (\Delta x)_a = \boxed{\frac{1}{2}a_1 t_1^2}$$

(c) The displacement occurring during the braking period is

$$(\Delta x)_b = v_{ib}(\Delta t)_b + \tfrac{1}{2}a_b(\Delta t)_b^2 = (a_1 t_1)t_2 + \frac{1}{2}a_2 t_2^2$$

Thus, the total displacement of the car during the two intervals combined is

$$(\Delta x)_{\text{total}} = (\Delta x)_a + (\Delta x)_b = \boxed{\frac{1}{2}a_1 t_1^2 + a_1 t_1 t_2 + \frac{1}{2}a_2 t_2^2}$$

2.41 The time the Thunderbird spends slowing down is

$$\Delta t_1 = \frac{\Delta x_1}{\bar{v}_1} = \frac{2(\Delta x_1)}{v + v_0} = \frac{2(250\text{ m})}{0 + 71.5\text{ m/s}} = 6.99\text{ s}$$

The time required to regain speed after the pit stop is

$$\Delta t_2 = \frac{\Delta x_2}{\bar{v}_2} = \frac{2(\Delta x_2)}{v + v_0} = \frac{2(350\text{ m})}{71.5\text{ m/s} + 0} = 9.79\text{ s}$$

Thus, the total elapsed time before the Thunderbird is back up to speed is

$$\Delta t = \Delta t_1 + 5.00\text{ s} + \Delta t_2 = 6.99\text{ s} + 5.00\text{ s} + 9.79\text{ s} = 21.8\text{ s}$$

During this time, the Mercedes has traveled (at constant speed) a distance

$$\Delta x_M = v_0(\Delta t) = (71.5\text{ m/s})(21.8\text{ s}) = 1\,559\text{ m}$$

and the Thunderbird has fallen behind a distance

$$d = \Delta x_M - \Delta x_1 - \Delta x_2 = 1\,559\text{ m} - 250\text{ m} - 350\text{ m} = \boxed{959\text{ m}}$$

2.42 The car is distance d from the dog and has initial velocity v_0 when the brakes are applied, giving it a constant acceleration a.

Apply $\bar{v} = \Delta x/\Delta t = (v_f + v_0)/2$ to the entire trip (for which $\Delta x = d + 4.0$ m, $\Delta t = 10$ s, and $v_f = 0$) to obtain

$$\frac{d + 4.0\text{ m}}{10\text{ s}} = \frac{0 + v_0}{2} \qquad \text{or} \qquad v_0 = \frac{d + 4.0\text{ m}}{5.0\text{ s}} \qquad \textbf{[1]}$$

Then, applying $v_f^2 = v_0^2 + 2a(\Delta x)$ to the entire trip yields $0 = v_0^2 + 2a(d + 4.0\text{ m})$.

Substitute for v_0 from Equation [1] to find that

$$0 = \frac{(d + 4.0\text{ m})^2}{25\text{ s}^2} + 2a(d + 4.0\text{ m}) \qquad \text{and} \qquad a = -\frac{d + 4.0\text{ m}}{50\text{ s}^2} \qquad \textbf{[2]}$$

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Finally, apply $\Delta x = v_0 t + \frac{1}{2}at^2$ to the first 8.0 s of the trip (for which $\Delta x = d$).

This gives $\quad d = v_0(8.0\text{ s}) + \frac{1}{2}a(64\text{ s}^2)$ **[3]**

Substitute Equations [1] and [2] into Equation [3] to obtain

$$d = \left(\frac{d+4.0\text{ m}}{5.0\text{ s}}\right)(8.0\text{ s}) + \frac{1}{2}\left(-\frac{d+4.0\text{ m}}{50\text{ s}^2}\right)(64\text{ s}^2) = 0.96d + 3.8\text{ m}$$

which yields $\quad d = 3.8\text{ m}/0.04 = \boxed{95\text{ m}}$

2.43 (a) Take $t = 0$ at the time when the player starts to chase his opponent. At this time, the opponent is distance $d = (12\text{ m/s})(3.0\text{ s}) = 36\text{ m}$ in front of the player. At time $t > 0$, the displacements of the players from their initial positions are

$$\Delta x_{\text{player}} = (v_0)_{\text{player}}\, t + \frac{1}{2}a_{\text{player}}t^2 = 0 + \frac{1}{2}(4.0\text{ m/s}^2)t^2 \qquad [1]$$

and $$\Delta x_{\text{opponent}} = (v_0)_{\text{opponent}}\, t + \frac{1}{2}a_{\text{opponent}}t^2 = (12\text{ m/s})t + 0 \qquad [2]$$

When the players are side-by-side, $\Delta x_{\text{player}} = \Delta x_{\text{opponent}} + 36\text{ m}$ **[3]**

Substituting Equations [1] and [2] into Equation [3] gives

$$\frac{1}{2}(4.0\text{ m/s}^2)t^2 = (12\text{ m/s})t + 36\text{ m} \qquad \text{or} \qquad t^2 + (-6.0\text{ s})t + (-18\text{ s}^2) = 0$$

Applying the quadratic formula to this result gives

$$t = \frac{-(-6.0\text{ s}) \pm \sqrt{(-6.0\text{ s})^2 - 4(1)(-18\text{ s}^2)}}{2(1)}$$

which has solutions of $t = -2.2$ s and $t = +8.2$ s. Since the time must be greater than zero, we must choose $t = \boxed{8.2\text{ s}}$ as the proper answer.

(b) $$\Delta x_{\text{player}} = (v_0)_{\text{player}}\, t + \frac{1}{2}a_{\text{player}}t^2 = 0 + \frac{1}{2}(4.0\text{ m/s}^2)(8.2\text{ s})^2 = \boxed{1.3\times10^2\text{ m}}$$

2.44 The initial velocity of the train is $v_0 = 82.4\text{ km/h}$ and the final velocity is $v = 16.4\text{ km/h}$. The time required for the 400 m train to pass the crossing is found from $\Delta x = \bar{v}t = [(v+v_0)/2]t$ as

$$t = \frac{2(\Delta x)}{v+v_0} = \frac{2(0.400\text{ km})}{(82.4+16.4)\text{ km/h}} = (8.10\times10^{-3}\text{ h})\left(\frac{3600\text{ s}}{1\text{ h}}\right) = \boxed{29.1\text{ s}}$$

2.45 (a) From $v^2 = v_0^2 + 2a(\Delta y)$ with $v = 0$, we have

$$(\Delta y)_{\text{max}} = \frac{v^2 - v_0^2}{2a} = \frac{0-(25.0\text{ m/s})^2}{2(-9.80\text{ m/s}^2)} = \boxed{31.9\text{ m}}$$

(b) The time to reach the highest point is

$$t_{\text{up}} = \frac{v-v_0}{a} = \frac{0-25.0\text{ m/s}}{-9.80\text{ m/s}^2} = \boxed{2.55\text{ s}}$$

(c) The time required for the ball to fall 31.9 m, starting from rest, is found from

$$\Delta y = (0)t + \frac{1}{2}at^2 \text{ as } t = \sqrt{\frac{2(\Delta y)}{a}} = \sqrt{\frac{2(-31.9\text{ m})}{-9.80\text{ m/s}^2}} = \boxed{2.55\text{ s}}$$

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(d) The velocity of the ball when it returns to the original level (2.55 s after it starts to fall from rest) is

$$v = v_0 + at = 0 + \left(-9.80\ \text{m/s}^2\right)(2.55\ \text{s}) = \boxed{-25.0\ \text{m/s}}$$

2.46 We take upward as the positive y-direction and $y = 0$ at the point where the ball is released. Then, $v_{0y} = -8.00\ \text{m/s}$, $a_y = -g = -9.80\ \text{m/s}^2$, and $\Delta y = -30.0\ \text{m}$ when the ball reaches the ground.

From $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$, the velocity of the ball just before it hits the ground is

$$v_y = -\sqrt{v_{0y}^2 + 2a_y(\Delta y)} = -\sqrt{(8.00\ \text{m/s})^2 + 2\left(-9.80\ \text{m/s}^2\right)(-30.0\ \text{m})} = -25.5\ \text{m/s}$$

Then, $v_y = v_{0y} + a_y t$ gives the elapsed time as

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-25.5\ \text{m/s} - (-8.00\ \text{m/s})}{-9.80\ \text{m/s}^2} = \boxed{1.79\ \text{s}}$$

2.47 (a) The velocity of the object when it was 30.0 m above the ground can be determined by applying $\Delta y = v_0 t + \frac{1}{2}at^2$ to the last 1.50 s of the fall. This gives

$$-30.0\ \text{m} = v_0(1.50\ \text{s}) + \frac{1}{2}\left(-9.80\ \frac{\text{m}}{\text{s}^2}\right)(1.50\ \text{s})^2 \qquad \text{or} \qquad v_0 = \boxed{-12.7\ \text{m/s}}$$

(b) The displacement the object must have undergone, starting from rest, to achieve this velocity at a point 30.0 m above the ground is given by $v^2 = v_0^2 + 2a(\Delta y)$ as

$$(\Delta y)_1 = \frac{v^2 - v_0^2}{2a} = \frac{(-12.7\ \text{m/s})^2 - 0}{2\left(-9.80\ \text{m/s}^2\right)} = -8.23\ \text{m}$$

The total distance the object drops during the fall is then

$$\left|(\Delta y)_{\text{total}}\right| = \left|(-8.23\ \text{m}) + (-30.0\ \text{m})\right| = \boxed{38.2\ \text{m}}$$

2.48 (a) Consider the rock's entire upward flight, for which $v_0 = +7.40\ \text{m/s}$, $v_f = 0$, $a = -g = -9.80\ \text{m/s}^2$, $y_i = 1.55$ m (taking $y = 0$ at ground level), and $y_f = h_{\max}$ = maximum altitude reached by rock. Then applying $v_f^2 = v_i^2 + 2a(\Delta y)$ to this upward flight gives

$$0 = (7.40\ \text{m/s})^2 + 2\left(-9.80\ \text{m/s}^2\right)\left(h_{\max} - 1.55\ \text{m}\right)$$

Solving for the maximum altitude of the rock gives

$$h_{\max} = 1.55\ \text{m} + \frac{(7.40\ \text{m/s})^2}{2\left(9.80\ \text{m/s}^2\right)} = 4.34\ \text{m}$$

Since $h_{\max} > 3.65\ \text{m}$ (height of the wall), $\boxed{\text{the rock does reach the top of the wall}}$.

(b) To find the velocity of the rock when it reaches the top of the wall, we use $v_f^2 = v_i^2 + 2a(\Delta y)$ and solve for v_f when $y_f = 3.65$ m (starting with $v_i = +7.40\ \text{m/s}$ at $y_i = 1.55$ m). This yields

$$v_f = \sqrt{v_i^2 + 2a\left(y_f - y_i\right)} = \sqrt{(7.40\ \text{m/s})^2 + 2\left(-9.80\ \text{m/s}^2\right)(3.65\ \text{m} - 1.55\ \text{m})} = \boxed{3.69\ \text{m/s}}$$

continued on next page

(c) A rock thrown *downward* at a speed of $7.40\ \text{m/s}\ \ (v_i = -7.40\ \text{m/s})$ from the top of the wall undergoes a displacement of $(\Delta y) = y_f - y_i = 1.55\ \text{m} - 3.65\ \text{m} = -2.10\ \text{m}$ before reaching the level of the attacker. Its velocity when it reaches the attacker is

$$v_f = -\sqrt{v_i^2 + 2a(\Delta y)} = -\sqrt{(-7.40\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(-2.10\ \text{m})} = -9.79\ \text{m/s}$$

so the change in speed of this rock as it goes between the 2 points located at the top of the wall and the attacker is given by

$$\Delta(speed)_{\text{down}} = \left\| v_f \right| - \left| v_i \right\| = \left\| -9.79\ \text{m/s} \right| - \left| -7.40\ \text{m/s} \right\| = \boxed{2.39\ \text{m/s}}$$

(d) Observe that the change in speed of the ball thrown upward as it went from the attacker to the top of the wall was

$$\Delta(speed)_{\text{up}} = \left\| v_f \right| - \left| v_i \right\| = \left| 3.69\ \text{m/s} - 7.40\ \text{m/s} \right| = 3.71\ \text{m/s}$$

$\boxed{\text{The two rocks do not undergo the same magnitude speed change.}}$ The rocks have the same acceleration, but the rock thrown downward has a higher average speed between the two levels, and is accelerated over a smaller time interval.

2.49 The velocity of the child's head just before impact (after falling a distance of 0.40 m, starting from rest) is given by $v^2 = v_0^2 + 2a(\Delta y)$ as

$$v_I = -\sqrt{v_0^2 + 2a(\Delta y)} = -\sqrt{0 + 2(-9.8\ \text{m/s}^2)(-0.40\ \text{m})} = -2.8\ \text{m/s}$$

If, upon impact, the child's head undergoes an additional displacement $\Delta y = -h$ before coming to rest, the acceleration during the impact can be found from $v^2 = v_0^2 + 2a(\Delta y)$ to be $a = \left(0 - v_I^2\right)/2(-h) = v_I^2/2h$. The duration of the impact is found from $v = v_0 + at$ as $t = \Delta v/a = -v_I/\left(v_I^2/2h\right)$, or $t = -2h/v_I$.

Applying these results to the two cases yields

Hardwood Floor ($h = 2.0\times10^{-3}$ m): $a = \dfrac{v_I^2}{2h} = \dfrac{(-2.8\ \text{m/s})^2}{2\left(2.0\times10^{-3}\ \text{m}\right)} = \boxed{2.0\times10^3\ \text{m/s}^2}$

and $\quad t = \dfrac{-2h}{v_I} = \dfrac{-2\left(2.0\times10^{-3}\ \text{m}\right)}{-2.8\ \text{m/s}} = 1.4\times10^{-3}\ \text{s} = \boxed{1.4\ \text{ms}}$

Carpeted Floor ($h = 1.0\times10^{-2}$ m): $a = \dfrac{v_I^2}{2h} = \dfrac{(-2.8\ \text{m/s})^2}{2\left(1.0\times10^{-2}\ \text{m}\right)} = \boxed{3.9\times10^2\ \text{m/s}^2}$

and $\quad t = \dfrac{-2h}{v_I} = \dfrac{-2\left(1.0\times10^{-2}\ \text{m}\right)}{-2.8\ \text{m/s}} = 7.1\times10^{-3}\ \text{s} = \boxed{7.1\ \text{ms}}$

2.50 (a) After 2.00 s, the velocity of the mailbag is

$$v_{\text{bag}} = v_0 + at = -1.50\ \text{m/s} + \left(-9.80\ \text{m/s}^2\right)(2.00\ \text{s}) = -21.1\ \text{m/s}$$

The negative sign tells us that the bag is moving downward and the magnitude of the velocity gives the speed as $\boxed{21.1\ \text{m/s}}$.

(b) The displacement of the mailbag after 2.00 s is

$$(\Delta y)_{\text{bag}} = \left(\frac{v + v_0}{2}\right)t = \left[\frac{-21.1\ \text{m/s} + (-1.50\ \text{m/s})}{2}\right](2.00\ \text{s}) = -22.6\ \text{m}$$

continued on next page

During this time, the helicopter, moving downward with constant velocity, undergoes a displacement of

$$(\Delta y)_{\text{copter}} = v_0 t + \frac{1}{2}at^2 = (-1.5 \text{ m/s})(2.00 \text{ s}) + 0 = -3.00 \text{ m}$$

The distance separating the package and the helicopter at this time is then

$$d = \left|(\Delta y)_p - (\Delta y)_h\right| = \left|-22.6 \text{ m} - (-3.00 \text{ m})\right| = \left|-19.6 \text{ m}\right| = \boxed{19.6 \text{ m}}$$

(c) Here, $(v_0)_{\text{bag}} = (v_0)_{\text{copter}} = +1.50 \text{ m/s}$ and $a_{bag} = -9.80 \text{ m/s}^2$ while $a_{\text{copter}} = 0$. After 2.00 s, the velocity of the mailbag is

$$v_{\text{bag}} = 1.50 \frac{\text{m}}{\text{s}} + \left(-9.80 \frac{\text{m}}{\text{s}^2}\right)(2.00 \text{ s}) = -18.1 \frac{\text{m}}{\text{s}} \text{ and its speed is } \left|v_{\text{bag}}\right| = \boxed{18.1 \frac{\text{m}}{\text{s}}}$$

In this case, the displacement of the helicopter during the 2.00 s interval is

$$\Delta y_{\text{copter}} = (+1.50 \text{ m/s})(2.00 \text{ s}) + 0 = +3.00 \text{ m}$$

Meanwhile, the mailbag has a displacement of

$$(\Delta y)_{\text{bag}} = \left(\frac{v_{\text{bag}} + v_0}{2}\right)t = \left[\frac{-18.1 \text{ m/s} + 1.50 \text{ m/s}}{2}\right](2.00 \text{ s}) = -16.6 \text{ m}$$

The distance separating the package and the helicopter at this time is then

$$d = \left|(\Delta y)_p - (\Delta y)_h\right| = \left|-16.6 \text{ m} - (+3.00 \text{ m})\right| = \left|-19.6 \text{ m}\right| = \boxed{19.6 \text{ m}}$$

2.51 (a) From the instant the ball leaves the player's hand until it is caught, the ball is a freely falling body with an acceleration of

$$a = -g = -9.80 \text{ m/s}^2 = \boxed{9.80 \text{ m/s}^2 \text{ (downward)}}$$

(b) At its maximum height, the ball comes to rest momentarily and then begins to fall back downward. Thus, $v_{\substack{\text{max}\\\text{height}}} = \boxed{0}$.

(c) Consider the relation $\Delta y = v_0 t + \frac{1}{2}at^2$ with $a = -g$. When the ball is at the thrower's hand, the displacement is $\Delta y = 0$, giving

$$0 = v_0 t - \tfrac{1}{2}gt^2$$

This equation has two solutions, $t = 0$, which corresponds to when the ball was thrown, and $t = 2v_0/g$, corresponding to when the ball is caught. Therefore, if the ball is caught at $t = 2.00$ s, the initial velocity must have been

$$v_0 = \frac{gt}{2} = \frac{(9.80 \text{ m/s}^2)(2.00 \text{ s})}{2} = \boxed{9.80 \text{ m/s}}$$

(d) From $v^2 = v_0^2 + 2a(\Delta y)$, with $v = 0$ at the maximum height,

$$(\Delta y)_{\text{max}} = \frac{v^2 - v_0^2}{2a} = \frac{0 - (9.80 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = \boxed{4.90 \text{ m}}$$

2.52 (a) Let $t = 0$ be the instant the package leaves the helicopter, so the package and the helicopter have a common initial velocity of $v_i = -v_0$ (choosing upward as positive).

At times $t > 0$, the velocity of the package (in free-fall with constant acceleration $a_p = -g$) is given by $v = v_i + at$ as $v_p = -v_0 - gt = -(v_0 + gt)$ and $speed = \boxed{|v_p| = v_0 + gt}$.

(b) After an elapsed time t, the downward displacement of the package from its point of release will be

$$(\Delta y)_p = v_i t + \frac{1}{2}a_p t^2 = -v_0 t - \frac{1}{2}gt^2 = -\left(v_0 t + \frac{1}{2}gt^2\right)$$

and the downward displacement of the helicopter (moving with constant velocity, or acceleration $a_h = 0$) from the release point at this time is

$$(\Delta y)_h = v_i t + \frac{1}{2}a_h t^2 = -v_0 t + 0 = -v_0 t$$

The distance separating the package and the helicopter at this time is then

$$d = \left|(\Delta y)_p - (\Delta y)_h\right| = \left|-\left(v_0 t + \frac{1}{2}gt^2\right) - (-v_0 t)\right| = \boxed{\frac{1}{2}gt^2}$$

(c) If the helicopter and package are moving upward at the instant of release, then the common initial velocity is $v_i = +v_0$. The accelerations of the helicopter (moving with constant velocity) and the package (a freely falling object) remain unchanged from the previous case $(a_p = -g \text{ and } a_h = 0)$.

In this case, the package speed at time $t > 0$ is $|v_p| = |v_i + a_p t| = \boxed{|v_0 - gt|}$.

At this time, the displacements from the release point of the package and the helicopter are given by

$$(\Delta y)_p = v_i t + \frac{1}{2}a_p t^2 = v_0 t - \frac{1}{2}gt^2 \qquad \text{and} \qquad (\Delta y)_h = v_i t + \frac{1}{2}a_h t^2 = v_0 t + 0 = +v_0 t$$

The distance separating the package and helicopter at time t is now given by

$$d = \left|(\Delta y)_p - (\Delta y)_h\right| = \left|v_0 t - \frac{1}{2}gt^2 - v_0 t\right| = \boxed{\frac{1}{2}gt^2} \quad \text{(the same as earlier!)}$$

2.53 (a) After its engines stop, the rocket is a freely falling body. It continues upward, slowing under the influence of gravity until it comes to rest momentarily at its maximum altitude. Then it falls back to Earth, gaining speed as it falls.

(b) When it reaches a height of 150 m, the speed of the rocket is

$$v = \sqrt{v_0^2 + 2a(\Delta y)} = \sqrt{(50.0 \text{ m/s})^2 + 2(2.00 \text{ m/s}^2)(150 \text{ m})} = 55.7 \text{ m/s}$$

After the engines stop, the rocket continues moving upward with an initial velocity of $v_0 = 55.7$ m/s and acceleration $a = -g = -9.80$ m/s^2. When the rocket reaches maximum height, $v = 0$. The displacement of the rocket above the point where the engines stopped (that is, above the 150 m level) is

$$\Delta y = \frac{v^2 - v_0^2}{2a} = \frac{0 - (55.7 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = 158 \text{ m}$$

The maximum height above ground that the rocket reaches is then given by

$h_{max} = 150 \text{ m} + 158 \text{ m} = \boxed{308 \text{ m}}$

continued on next page

(c) The total time of the upward motion of the rocket is the sum of two intervals. The first is the time for the rocket to go from $v_0 = 50.0$ m/s at the ground to a velocity of $v = 55.7$ m/s at an altitude of 150 m. This time is given by

$$t_1 = \frac{(\Delta y)_1}{\bar{v}_1} = \frac{(\Delta y)_1}{(v+v_0)/2} = \frac{2(150 \text{ m})}{(55.7+50.0) \text{ m/s}} = 2.84 \text{ s}$$

The second interval is the time to rise 158 m starting with $v_0 = 55.7$ m/s and ending with $v = 0$. This time is

$$t_2 = \frac{(\Delta y)_2}{\bar{v}_2} = \frac{(\Delta y)_2}{(v+v_0)/2} = \frac{2(158 \text{ m})}{0+55.7 \text{ m/s}} = 5.67 \text{ s}$$

The total time of the upward flight is then $\quad t_{up} = t_1 + t_2 = (2.84+5.67) \text{ s} = \boxed{8.51 \text{ s}}$

(d) The time for the rocket to fall 308 m back to the ground, with $v_0 = 0$ and acceleration $a = -g = -9.80 \text{ m/s}^2$, is found from $\Delta y = v_0 t + \frac{1}{2}at^2$ as

$$t_{down} = \sqrt{\frac{2(\Delta y)}{a}} = \sqrt{\frac{2(-308 \text{ m})}{-9.80 \text{ m/s}^2}} = 7.93 \text{ s}$$

so the total time of the flight is $\quad t_{flight} = t_{up} + t_{down} = (8.51+7.93) \text{ s} = \boxed{16.4 \text{ s}}$

2.54 (a) For the upward flight of the ball, we have $v_i = v_0$, $v_f = 0$, $a = -g$, and $\Delta t = 3.00$ s. Thus, $v_f = v_i + a(\Delta t)$ gives the initial velocity as

$$v_i = v_f - a(\Delta t) = v_f + g(\Delta t) \text{ or } v_0 = 0 + (9.80 \text{ m/s}^2)(3.00 \text{ s}) = \boxed{+29.4 \text{ m/s}}$$

(b) The vertical displacement of the ball during this 3.00-s upward flight is

$$(\Delta y)_{max} = h = \bar{v}(\Delta t) = \left(\frac{v_i + v_f}{2}\right)(\Delta t) = \left(\frac{29.4 \text{ m/s}+0}{2}\right)(3.00 \text{ s}) = \boxed{44.1 \text{ m}}$$

2.55 During the 0.600 s required for the rig to pass completely onto the bridge, the front bumper of the tractor moves a distance equal to the length of the rig at constant velocity of $v = 100$ km/h. Therefore the length of the rig is

$$L_{rig} = vt = \left[100 \frac{\text{km}}{\text{h}}\left(\frac{0.278 \text{ m/s}}{1 \text{ km/h}}\right)\right](0.600 \text{ s}) = 16.7 \text{ m}$$

While some part of the rig is on the bridge, the front bumper moves a distance $\Delta x = L_{bridge} + L_{rig} = 400 \text{ m} + 16.7 \text{ m}$. With a constant velocity of $v = 100$ km/h, the time for this to occur is

$$t = \frac{L_{bridge} + L_{rig}}{v} = \frac{400 \text{ m} + 16.7 \text{ m}}{100 \text{ km/h}}\left(\frac{1 \text{ km/h}}{0.278 \text{ m/s}}\right) = \boxed{15.0 \text{ s}}$$

2.56 (a) The acceleration experienced as he came to rest is given by $v = v_0 + at$ as

$$a = \frac{v - v_0}{t} = \frac{0 - \left(632 \frac{\text{mi}}{\text{h}}\right)\left(\frac{0.447 \text{ m/s}}{1 \text{ mi/h}}\right)}{1.40 \text{ s}} = \boxed{-202 \text{ m/s}^2}$$

continued on next page

(b) The distance traveled while stopping is found from

$$\Delta x = \bar{v}t = \left(\frac{v+v_0}{2}\right)t = \frac{\left[0+\left(632\ \frac{\text{mi}}{\text{h}}\right)\left(\frac{0.447\ \text{m/s}}{1\ \text{mi/h}}\right)\right]}{2}(1.40\ \text{s}) = \boxed{198\ \text{m}}$$

2.57 (a) The acceleration of the bullet is

$$a = \frac{v^2 - v_0^2}{2(\Delta x)} = \frac{(300\ \text{m/s})^2 - (400\ \text{m/s})^2}{2(0.100\ \text{m})} = \boxed{-3.50\times 10^5\ \text{m/s}^2}$$

(b) The time of contact with the board is

$$t = \frac{v - v_0}{a} = \frac{(300-400)\ \text{m/s}}{-3.50\times 10^5\ \text{m/s}^2} = \boxed{2.86\times 10^{-4}\ \text{s}}$$

2.58 (a) From $\Delta x = v_0 t + \frac{1}{2}at^2$, we have

$$100\ \text{m} = (30.0\ \text{m/s})t + \frac{1}{2}\left(-3.50\ \text{m/s}^2\right)t^2$$

This reduces to $3.50t^2 + (-60.0\ \text{s})t + \left(200\ \text{s}^2\right) = 0$, and the quadratic formula gives

$$t = \frac{-(-60.0\ \text{s}) \pm \sqrt{(-60.0\ \text{s})^2 - 4(3.50)\left(200\ \text{s}^2\right)}}{2(3.50)}$$

The desired time is the smaller solution of $t = \boxed{4.53\ \text{s}}$. The larger solution of $t = 12.6$ s is the time when the boat would pass the buoy moving backwards, assuming it maintained a constant acceleration.

(b) The velocity of the boat when it first reaches the buoy is

$$v = v_0 + at = 30.0\ \text{m/s} + \left(-3.50\ \text{m/s}^2\right)(4.53\ \text{s}) = \boxed{14.1\ \text{m/s}}$$

2.59 (a) The keys have acceleration $a = -g = -9.80\ \text{m/s}^2$ from the release point until they are caught 1.50 s later. Thus, $\Delta y = v_0 t + \frac{1}{2}at^2$ gives

$$v_0 = \frac{\Delta y - at^2/2}{t} = \frac{(+4.00\ \text{m}) - \left(-9.80\ \text{m/s}^2\right)(1.50\ \text{s})^2/2}{1.50\ \text{s}} = +10.0\ \text{m/s}$$

or $v_0 = \boxed{10.0\ \text{m/s upward}}$

(b) The velocity of the keys just before the catch was

$$v = v_0 + at = 10.0\ \text{m/s} + \left(-9.80\ \text{m/s}^2\right)(1.50\ \text{s}) = -4.70\ \text{m/s}$$

or $v = \boxed{4.70\ \text{m/s downward}}$

2.60 (a) The keys, moving freely under the influence of gravity $(a = -g)$, undergo a vertical displacement of $\Delta y = +h$ in time *t*. We use $\Delta y = v_i t + \frac{1}{2}at^2$ to find the initial velocity as

$$h = v_i t + \frac{1}{2}(-g)t^2 \qquad \text{giving} \qquad v_i = \frac{h + gt^2/2}{t} = \boxed{\frac{h}{t} + \frac{gt}{2}}$$

continued on next page

(b) The velocity of the keys just before they were caught (at time t) is given by $v = v_i + at$ as

$$v = \left(\frac{h}{t} + \frac{gt}{2}\right) + (-g)t = \frac{h}{t} + \frac{gt}{2} - gt = \boxed{\frac{h}{t} - \frac{gt}{2}}$$

2.61 (a) From $v^2 = v_0^2 + 2a(\Delta y)$, the insect's velocity after straightening its legs is

$$v = \sqrt{v_0^2 + 2a(\Delta y)} = \sqrt{0 + 2\left(4\ 000\ \text{m/s}^2\right)\left(2.0\times10^{-3}\ \text{m}\right)} = \boxed{4.0\ \text{m/s}}$$

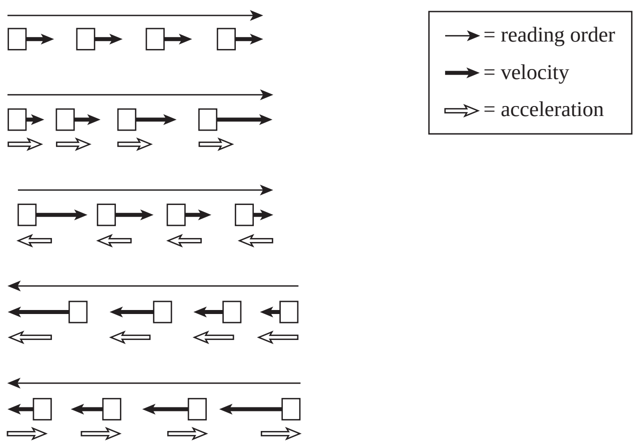
(b) The time to reach this velocity is

$$t = \frac{v - v_0}{a} = \frac{4.0\ \text{m/s} - 0}{4\ 000\ \text{m/s}^2} = 1.0\times10^{-3}\ \text{s} = \boxed{1.0\ \text{ms}}$$

(c) The upward displacement of the insect between when its feet leave the ground and it comes to rest momentarily at maximum altitude is

$$\Delta y = \frac{v^2 - v_0^2}{2a} = \frac{0 - v_0^2}{2(-g)} = \frac{-(4.0\ \text{m/s})^2}{2\left(-9.8\ \text{m/s}^2\right)} = \boxed{0.82\ \text{m}}$$

2.62 (a), (b), (c), (d), (e)



(f) If the speed did not change at a constant rate, the drawings would have less regularity than those given above.

2.63 The falling ball moves a distance of $(15\ \text{m} - h)$ before they meet, where h is the height above the ground where they meet. Apply $\Delta y = v_0 t + \frac{1}{2}at^2$, with $a = -g$, to obtain

$$-(15\ \text{m} - h) = 0 - \frac{1}{2}gt^2 \qquad \text{or} \qquad h = 15\ \text{m} - \frac{1}{2}gt^2 \qquad [1]$$

Applying $\Delta y = v_0 t + \frac{1}{2}at^2$ to the rising ball gives

$$h = (25\ \text{m/s})t - \frac{1}{2}gt^2 \qquad [2]$$

Combining Equations [1] and [2] gives

$$(25\ \text{m/s})t - \cancel{\frac{1}{2}gt^2} = 15\ \text{m} - \cancel{\frac{1}{2}gt^2}$$

or $$t = \frac{15\ \text{m}}{25\ \text{m/s}} = \boxed{0.60\ \text{s}}$$

2.64 The constant speed the student has maintained for the first 10 minutes, and hence her initial speed for the final 500 yard dash, is

$$v_0 = \frac{\Delta x_{10}}{\Delta t} = \frac{1.0\text{ mi} - 500\text{ yards}}{10\text{ min}} = \frac{(5280\text{ ft} - 1500\text{ ft})}{600\text{ s}}\left(\frac{1\text{ m}}{3.281\text{ ft}}\right) = 1.9\text{ m/s}$$

With an initial speed of $v_0 = 1.9$ m/s, the minimum constant acceleration that would be needed to complete the last 500 yards (1 500 ft) in the remaining 2.0 min (120 s) of her allotted time is found from $\Delta x = v_i t + \frac{1}{2}at^2$ as

$$a_{\text{min}} = \frac{2[\Delta x - v_0 t]}{t^2} = \frac{2[(1\,500\text{ ft})(1\text{ m}/3.281\text{ ft}) - (1.9\text{ m/s})(120\text{ s})]}{(120\text{ s})^2} = 0.032\text{ m/s}^2$$

Since this acceleration is considerably smaller than the acceleration of 0.15 m/s^2 that she is capable of producing, she should be able to $\boxed{\text{easily meet the requirement}}$ of running 1.0 mile in 12 minutes.

2.65 Once the gymnast's feet leave the ground, she is a freely falling body with constant acceleration $a = -g = -9.80\text{ m/s}^2$. Starting with an initial upward velocity of $v_0 = 2.80$ m/s, the vertical displacement of the gymnast's center of mass from its starting point is given as a function of time by $\Delta y = v_0 t + \frac{1}{2}at^2$.

(a) At $t = 0.100$ s, $\Delta y = (2.80\text{ m/s})(0.100\text{ s}) - (4.90\text{ m/s}^2)(0.100\text{ s})^2 = \boxed{0.231\text{ m}}$

(b) At $t = 0.200$ s, $\Delta y = (2.80\text{ m/s})(0.200\text{ s}) - (4.90\text{ m/s}^2)(0.200\text{ s})^2 = \boxed{0.364\text{ m}}$

(c) At $t = 0.300$ s, $\Delta y = (2.80\text{ m/s})(0.300\text{ s}) - (4.90\text{ m/s}^2)(0.300\text{ s})^2 = \boxed{0.399\text{ m}}$

(d) At $t = 0.500$ s, $\Delta y = (2.80\text{ m/s})(0.500\text{ s}) - (4.90\text{ m/s}^2)(0.500\text{ s})^2 = \boxed{0.175\text{ m}}$

2.66 (a) While in the air, both balls have acceleration $a_1 = a_2 = -g$ (where upward is taken as positive). Ball 1 (thrown downward) has initial velocity $v_{01} = -v_0$, while ball 2 (thrown upward) has initial velocity $v_{02} = +v_0$. Taking $y = 0$ at ground level, the initial y-coordinate of each ball is $y_{01} = y_{02} = +h$. Applying $\Delta y = y - y_i = v_i t + \frac{1}{2}at^2$ to each ball gives their y-coordinates at time t as:

Ball 1: $y_1 - h = -v_0 t + \frac{1}{2}(-g)t^2$ or $\boxed{y_1 = h - v_0 t - \frac{1}{2}gt^2}$

Ball 2: $y_2 - h = +v_0 t + \frac{1}{2}(-g)t^2$ or $\boxed{y_2 = h + v_0 t - \frac{1}{2}gt^2}$

(b) At ground level, $y = 0$. Thus, we equate each of the equations found above to zero and use the quadratic formula to solve for the times when each ball reaches the ground. This gives:

Ball 1: $0 = h - v_0 t_1 - \frac{1}{2}gt_1^2 \quad \rightarrow \quad gt_1^2 + (2v_0)t_1 + (-2h) = 0$

so $$t_1 = \frac{-2v_0 \pm \sqrt{(2v_0)^2 - 4(g)(-2h)}}{2g} = -\frac{v_0}{g} \pm \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Using only the *positive* solution gives $$t_1 = -\frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

continued on next page

Ball 2: $0 = h + v_0 t_2 - \frac{1}{2} g t_2^2 \quad \rightarrow \quad g t_2^2 + (-2v_0) t_2 + (-2h) = 0$

and $$t_2 = \frac{-(-2v_0) \pm \sqrt{(-2v_0)^2 - 4(g)(-2h)}}{2g} = +\frac{v_0}{g} \pm \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Again, using only the *positive* solution $$t_2 = \frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}$$

Thus, the difference in the times of flight of the two balls is

$$\Delta t = t_2 - t_1 = \frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}} - \left(-\frac{v_0}{g} + \sqrt{\left(\frac{v_0}{g}\right)^2 + \frac{2h}{g}}\right) = \boxed{\frac{2v_0}{g}}$$

(c) Realizing that the balls are going *downward* $(v < 0)$ as they near the ground, we use $v_f^2 = v_i^2 + 2a(\Delta y)$ with $\Delta y = -h$ to find the velocity of each ball just before it strikes the ground:

Ball 1: $v_{1f} = -\sqrt{v_{1i}^2 + 2a_1(-h)} = -\sqrt{(-v_0)^2 + 2(-g)(-h)} = \boxed{-\sqrt{v_0^2 + 2gh}}$

Ball 2: $v_{2f} = -\sqrt{v_{2i}^2 + 2a_2(-h)} = -\sqrt{(+v_0)^2 + 2(-g)(-h)} = \boxed{-\sqrt{v_0^2 + 2gh}}$

(d) While both balls are still in the air, the distance separating them is

$$d = y_2 - y_1 = \left(h + v_0 t - \frac{1}{2} g t^2\right) - \left(h - v_0 t - \frac{1}{2} g t^2\right) = \boxed{2v_0 t}$$

2.67 (a) The first ball is dropped from rest $(v_{01} = 0)$ from the height h of the window. Thus, $v_f^2 = v_0^2 + 2a(\Delta y)$ gives the speed of this ball as it reaches the ground (and hence the initial velocity of the second ball) as $|v_f| = \sqrt{v_{01}^2 + 2a_1(\Delta y_1)} = \sqrt{0 + 2(-g)(-h)} = \sqrt{2gh}$.

When ball 2 is thrown upward at the same time that ball 1 is dropped, their y-coordinates at time t during the flights are given by $y - y_o = v_0 t + \frac{1}{2} a t^2$ as:

Ball 1: $y_1 - h = (0)t + \frac{1}{2}(-g)t^2$ or $y_1 = h - \frac{1}{2} g t^2$

Ball 2: $y_2 - 0 = \left(\sqrt{2gh}\right)t + \frac{1}{2}(-g)t^2$ or $y_2 = \left(\sqrt{2gh}\right)t - \frac{1}{2} g t^2$

When the two balls pass, $y_1 = y_2$, or $h - \cancel{\frac{1}{2} g t^2} = \left(\sqrt{2gh}\right)t - \cancel{\frac{1}{2} g t^2}$

giving $$t = \frac{h}{\sqrt{2gh}} = \sqrt{\frac{h}{2g}} = \sqrt{\frac{28.7 \text{ m}}{2\left(9.80 \text{ m/s}^2\right)}} = \boxed{1.21 \text{ s}}$$

(b) When the balls meet,

$$t = \sqrt{\frac{h}{2g}} \quad \text{and} \quad y_1 = h - \frac{1}{2} g \left(\sqrt{\frac{h}{2g}}\right)^2 = h - \frac{h}{4} = \frac{3h}{4}$$

Thus, the distance below the window where this event occurs is

$$d = h - y_1 = h - \frac{3h}{4} = \frac{h}{4} = \frac{28.7 \text{ m}}{4} = \boxed{7.18 \text{ m}}$$

2.68 We do not know either the initial velocity or the final velocity (that is, velocity just before impact) for the truck. What we do know is that the truck skids 62.4 m in 4.20 s while accelerating at -5.60 m/s^2.

We have $v = v_0 + at$ and $\Delta x = \bar{v}t = \left[(v+v_0)/2\right]t$. Applied to the motion of the truck, these yield

$$v - v_0 = at = \left(-5.60 \text{ m/s}^2\right)(4.20 \text{ s}) \qquad \text{or} \qquad v - v_0 = -23.5 \text{ m/s} \qquad [1]$$

and

$$v + v_0 = \frac{2(\Delta x)}{t} = \frac{2(62.4 \text{ m})}{4.20 \text{ s}} \qquad \text{or} \qquad v + v_0 = 29.7 \text{ m/s} \qquad [2]$$

Adding Equations [1] and [2] gives the velocity just before impact as

$$2v = (-23.5 + 29.7) \text{ m/s} \qquad \text{or} \qquad v = \boxed{3.10 \text{ m/s}}$$

2.69 When released from rest $(v_0 = 0)$, the bill falls freely with a downward acceleration due to gravity $\left(a = -g = -9.80 \text{ m/s}^2\right)$. Thus, the magnitude of its downward displacement during David's 0.2 s reaction time will be

$$|\Delta y| = \left|v_0 t + \frac{1}{2}at^2\right| = \left|0 + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)(0.2 \text{ s})^2\right| = 0.2 \text{ m} = 20 \text{ cm}$$

This is over twice the distance from the center of the bill to its top edge $(\approx 8 \text{ cm})$, so $\boxed{\text{David will be unsuccessful}}$.

2.70 (a) The velocity with which the first stone hits the water is

$$v_1 = -\sqrt{v_{01}^2 + 2a(\Delta y)} = -\sqrt{\left(-2.00 \frac{\text{m}}{\text{s}}\right)^2 + 2\left(-9.80 \frac{\text{m}}{\text{s}^2}\right)(-50.0 \text{ m})} = -31.4 \frac{\text{m}}{\text{s}}$$

The time for this stone to hit the water is

$$t_1 = \frac{v_1 - v_{01}}{a} = \frac{\left[-31.4 \text{ m/s} - (-2.00 \text{ m/s})\right]}{-9.80 \text{ m/s}^2} = \boxed{3.00 \text{ s}}$$

(b) Since they hit simultaneously, the second stone, which is released 1.00 s later, will hit the water after an flight time of 2.00 s. Thus,

$$v_{02} = \frac{\Delta y - at_2^2/2}{t_2} = \frac{-50.0 \text{ m} - \left(-9.80 \text{ m/s}^2\right)(2.00 \text{ s})^2/2}{2.00 \text{ s}} = \boxed{-15.2 \text{ m/s}}$$

(c) From part (a), the final velocity of the first stone is $v_1 = \boxed{-31.4 \text{ m/s}}$.

The final velocity of the second stone is

$$v_2 = v_{02} + at_2 = -15.2 \text{ m/s} + \left(-9.80 \text{ m/s}^2\right)(2.00 \text{ s}) = \boxed{-34.8 \text{ m/s}}$$

2.71 (a) The sled's displacement, Δx_1, after accelerating at $a_1 = +40\text{ ft/s}^2$ for time t_1, is

$$\Delta x_1 = (0)t_1 + \tfrac{1}{2}a_1 t_1^2 = \left(20\text{ ft/s}^2\right)t_1^2 \qquad \text{or} \qquad \Delta x_1 = \left(20\text{ ft/s}^2\right)t_1^2 \qquad [1]$$

At the end of time t_1, the sled had achieved a velocity of

$$v = v_0 + a_1 t_1 = 0 + \left(40\text{ ft/s}^2\right)t_1 \qquad \text{or} \qquad v = \left(40\text{ ft/s}^2\right)t_1 \qquad [2]$$

The displacement of the sled while moving at constant velocity v for time t_2 is

$$\Delta x_2 = vt_2 = \left[\left(40\text{ ft/s}^2\right)t_1\right]t_2 \qquad \text{or} \qquad \Delta x_2 = \left(40\text{ ft/s}^2\right)t_1 t_2 \qquad [3]$$

It is known that $\Delta x_1 + \Delta x_2 = 17\,500\text{ ft}$, and substitutions from Equations [1] and [3] give

$$\left(20\text{ ft/s}^2\right)t_1^2 + \left(40\text{ ft/s}^2\right)t_1 t_2 = 17\,500\text{ ft} \qquad \text{or} \qquad t_1^2 + 2t_1 t_2 = 875\text{ s}^2 \qquad [4]$$

Also, it is known that $\qquad t_1 + t_2 = 90\text{ s} \qquad [5]$

Solving Equations [4] and [5] simultaneously yields

$$t_1^2 + 2t_1\left(90\text{ s} - t_1\right) = 875\text{ s}^2 \quad \text{or} \quad t_1^2 + (-180\text{ s})t_1 + 875\text{ s}^2 = 0$$

The quadratic formula then gives $\qquad t_1 = \dfrac{-(-180\text{ s}) \pm \sqrt{(-180\text{ s})^2 - 4(1)\left(875\text{ s}^2\right)}}{2(1)}$

with solutions $t_1 = 5.00\text{ s}$ $\left(\text{and } t_2 = 90\text{ s} - 5.0\text{ s} = 85\text{ s}\right)$ or $t_1 = 175\text{ s}$ $\left(\text{and } t_2 = -85\text{ s}\right)$

Since it is necessary that $t_2 > 0$, the valid solutions are $\boxed{t_1 = 5.0\text{ s and } t_2 = 85\text{ s}}$.

(b) From Equation [2] above, $\qquad v = \left(40\text{ ft/s}^2\right)t_1 = \left(40\text{ ft/s}^2\right)(5.0\text{ s}) = \boxed{200\text{ ft/s}}$

(c) The displacement Δx_3 of the sled as it comes to rest (with acceleration $a_3 = -20\text{ ft/s}^2$) is

$$\Delta x_3 = \frac{0 - v^2}{2a_3} = \frac{-(200\text{ ft/s})^2}{2\left(-20\text{ ft/s}^2\right)} = 1\,000\text{ ft}$$

Thus, the total displacement for the trip (measured from the starting point) is

$$\Delta x_{\text{total}} = \left(\Delta x_1 + \Delta x_2\right) + \Delta x_3 = 17\,500\text{ ft} + 1\,000\text{ ft} = \boxed{18\,500\text{ ft}}$$

(d) The time required to come to rest from velocity v (with acceleration a_3) is

$$t_3 = \frac{0 - v}{a_3} = \frac{-200\text{ ft/s}}{-20\text{ ft/s}^2} = \boxed{10\text{ s}}$$

so the duration of the entire trip is

$$t_{\text{total}} = t_1 + t_2 + t_3 = 5.0\text{ s} + 85\text{ s} + 10\text{ s} = \boxed{100\text{ s}}$$

2.72 (a) From $\Delta y = v_0 t + \dfrac{1}{2}at^2$ with $v_0 = 0$, we have

$$t = \sqrt{\frac{2(\Delta y)}{a}} = \sqrt{\frac{2(-23\text{ m})}{-9.80\text{ m/s}^2}} = \boxed{2.2\text{ s}}$$

continued on next page

(b) The final velocity is $v = 0 + \left(-9.80\ \text{m/s}^2\right)(2.2\text{s}) = \boxed{-22\ \text{m/s}}$

(c) The time it takes for the sound of the impact to reach the spectator is

$$t_{\text{sound}} = \frac{\Delta y}{v_{\text{sound}}} = \frac{23\ \text{m}}{340\ \text{m/s}} = 6.8\times10^{-2}\ \text{s}$$

so the total elapsed time is $t_{\text{total}} = 2.2\ \text{s} + 6.8\times10^{-2}\ \text{s} \approx \boxed{2.3\ \text{s}}$.

2.73 (a) Since the sound has constant velocity, the distance it traveled is

$$\Delta x = v_{\text{sound}}\, t = (1\,100\ \text{ft/s})(5.0\ \text{s}) = \boxed{5.5\times10^3\ \text{ft}}$$

(b) The plane travels this distance in a time of $5.0\ \text{s} + 10\ \text{s} = 15\ \text{s}$, so its velocity must be

$$v_{\text{plane}} = \frac{\Delta x}{t} = \frac{5.5\times10^3\ \text{ft}}{15\ \text{s}} = \boxed{3.7\times10^2\ \text{ft/s}}$$

(c) The time the light took to reach the observer was

$$t_{\text{light}} = \frac{\Delta x}{v_{\text{light}}} = \frac{5.5\times10^3\ \text{ft}}{3.00\times10^8\ \text{m/s}}\left(\frac{1\ \text{m/s}}{3.281\ \text{ft/s}}\right) = 5.6\times10^{-6}\ \text{s}$$

$\boxed{\text{During this time the plane would only travel a distance of 0.002 ft.}}$

2.74 The distance the glider moves during the time Δt_d is given by $\Delta x = \ell = v_0\left(\Delta t_d\right) + \frac{1}{2}a\left(\Delta t_d\right)^2$, where v_0 is the glider's velocity when the flag first enters the photogate and a is the glider's acceleration. Thus, the average velocity is

$$v_d = \frac{\ell}{\Delta t_d} = \frac{v_0\left(\Delta t_d\right) + \frac{1}{2}a\left(\Delta t_d\right)^2}{\Delta t_d} = v_0 + \frac{1}{2}a\left(\Delta t_d\right)$$

(a) The glider's velocity when it is halfway through the photogate in space (i.e., when $\Delta x = \ell/2$) is found from $v^2 = v_0^2 + 2a(\Delta x)$ as

$$v_1 = \sqrt{v_0^2 + 2a(\ell/2)} = \sqrt{v_0^2 + a\ell} = \sqrt{v_0^2 + a\left[v_d\left(\Delta t_d\right)\right]} = \sqrt{v_0^2 + a v_d\left(\Delta t_d\right)}$$

Note that this is $\boxed{\text{not equal to } v_d \text{ unless } a = 0}$, in which case $v_1 = v_d = v_0$.

(b) The speed v_2 when the glider is halfway through the photogate in time (i.e., when the elapsed time is $t_2 = \Delta t_d/2$) is given by $v = v_0 + at$ as

$$v = v_0 + at_2 = v_0 + a\left(\Delta t_d/2\right) = v_0 + \frac{1}{2}a\left(\Delta t_d\right)$$

which $\boxed{\text{is equal to } v_d}$ for all possible values of v_0 and a.

2.75 The time required for the stuntman to fall 3.00 m, starting from rest, is found from $\Delta y = v_0 t + \frac{1}{2}at^2$ as

$$-3.00\ \text{m} = 0 + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)t^2 \qquad \text{so} \qquad t = \sqrt{\frac{2(3.00\ \text{m})}{9.80\ \text{m/s}^2}} = 0.782\ \text{s}$$

(a) With the horse moving with constant velocity of 10.0 m/s, the horizontal distance is

$$\Delta x = v_{\text{horse}}\, t = (10.0\ \text{m/s})(0.782\ \text{s}) = \boxed{7.82\ \text{m}}$$

(b) The required time is $t = \boxed{0.782\ \text{s}}$ as calculated above.

3

Vectors and Two-Dimensional Motion

QUICK QUIZZES

1. Choice (c). The largest possible magnitude of the resultant occurs when the two vectors are in the same direction. In this case, the magnitude of the resultant is the sum of the magnitudes of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$: $R = A + B = 20$ units. The smallest possible magnitude of the resultant occurs when the two vectors are in opposite directions, and the magnitude is the difference of the magnitudes of $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$: $R = |A - B| = 4$ units.

2.

Vector	x-component	y-component
$\vec{\mathbf{A}}$	−	+
$\vec{\mathbf{B}}$	+	−
$\vec{\mathbf{A}} + \vec{\mathbf{B}}$	−	−

3. Vector $\vec{\mathbf{B}}$. The range of the inverse tangent function includes only the first and fourth quadrants (i.e., angles in the range $-\pi/2 < \theta < \pi/2$). Only vector $\vec{\mathbf{B}}$ has an orientation in this range.

4. Choice (b). If velocity is constant, the acceleration (rate of change in velocity) is zero. An object may have constant speed (magnitude of velocity) but still be accelerating due to a change in direction of the velocity. If an object is following a curved path, it is accelerating because the velocity is changing in direction.

5. Choice (a). Any change in the magnitude and/or direction of the velocity is an acceleration. The gas pedal and the brake produce accelerations by altering the magnitude of the velocity. The steering wheel produces accelerations by altering the direction of the velocity.

6. Choice (c). A projectile has constant horizontal velocity. Thus, if the runner throws the ball straight up into the air, the ball maintains the horizontal velocity it had before it was thrown (that is, the velocity of the runner). In the runner's frame of reference, the ball appears to go straight upward and come straight downward. To a stationary observer, the ball follows a parabolic trajectory, moving with the same horizontal velocity as the runner and staying above the runner's hand.

7. Choice (b). The velocity is always tangential to the path while the acceleration is always directed vertically downward. Thus, the velocity and acceleration are perpendicular only where the path is horizontal. This only occurs at the peak of the path.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. Choose coordinate system with north as the positive y-direction and east as the positive x-direction. The velocity of the cruise ship relative to Earth is $\vec{\mathbf{v}}_{CE} = 4.50$ m/s due north, with components of $(\vec{\mathbf{v}}_{CE})_x = 0$ and $(\vec{\mathbf{v}}_{CE})_y = 4.50\ \text{m/s}$. The velocity of the patrol boat relative to Earth is $\vec{\mathbf{v}}_{PE} = 5.20\ \text{m/s}$ at 45.0° north of west, with components of

$$(\vec{\mathbf{v}}_{PE})_x = -|\vec{\mathbf{v}}_{PE}|\cos 45.0° = -(5.20\ \text{m/s})(0.707) = -3.68\ \text{m/s}$$

and $$(\vec{\mathbf{v}}_{PE})_y = +|\vec{\mathbf{v}}_{PE}|\sin 45.0° = (5.20\ \text{m/s})(0.707) = 3.68\ \text{m/s}$$

Thus, the velocity of the cruise ship relative to the patrol boat is $\vec{\mathbf{v}}_{CP} = \vec{\mathbf{v}}_{CE} - \vec{\mathbf{v}}_{PE}$, which has components of

$$(\vec{\mathbf{v}}_{CP})_x = (\vec{\mathbf{v}}_{CE})_x - (\vec{\mathbf{v}}_{PE})_x = 0 - (-3.68\ \text{m/s}) = +3.68\ \text{m/s}$$

and $$(\vec{\mathbf{v}}_{CP})_y = (\vec{\mathbf{v}}_{CE})_y - (\vec{\mathbf{v}}_{PE})_y = 4.50\ \text{m/s} - 3.68\ \text{m/s} = +0.82\ \text{m/s}$$

Choice (a) is seen to be the correct answer.

2. The skier has zero initial velocity in the vertical direction $(v_{0y} = 0)$ and undergoes a vertical displacement of $\Delta y = -3.20\ \text{m}$. The constant acceleration in the vertical direction is $a_y = -g$, so we use $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ to find the time of flight as

$$-3.20\ \text{m} = 0 + \frac{1}{2}(-9.80\ \text{m/s}^2)t^2 \qquad \text{or} \qquad t = \sqrt{\frac{2(-3.20\ \text{m})}{-9.80\ \text{m/s}^2}} = 0.808\ \text{s}$$

During this time, the object moves with constant horizontal velocity $v_x = v_{0x} = 22.0\ \text{m/s}$. The horizontal distance traveled during the flight is

$$\Delta x = v_x t = (22.0\ \text{m/s})(0.808\ \text{s}) = 17.8\ \text{m}, \quad \text{which is choice (d).}$$

3. At maximum height $(\Delta y = h_{max})$, the vertical velocity of the stone will be zero. Thus, $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$ gives

$$h_{max} = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - v_0^2\sin^2\theta}{2(-g)} = \frac{-(45\ \text{m/s})^2\sin^2(55.0°)}{2(-9.80\ \text{m/s}^2)} = 69.3\ \text{m}$$

and we see that choice (c) is the correct answer.

4. For vectors in the *x*-*y* plane, their components have the signs indicated in the following table:

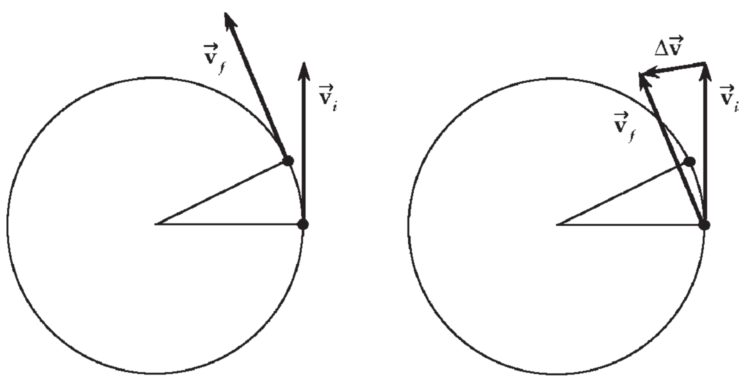
	Quadrant of Vector			
	First	Second	Third	Fourth
x-component	Positive	Negative	Negative	Positive
y-component	Positive	Positive	Negative	Negative

Thus, a vector having components of opposite sign must lie in either the second or fourth quadrants and choice (e) is the correct answer.

5. Whether on Earth or the Moon, a golf ball is in free fall with a constant downward acceleration of magnitude determined by local gravity from the time it leaves the tee until it strikes the ground or other object. Thus, the vertical component of velocity is constantly changing while the horizontal component of velocity is constant. Note that the speed (or magnitude of the velocity)

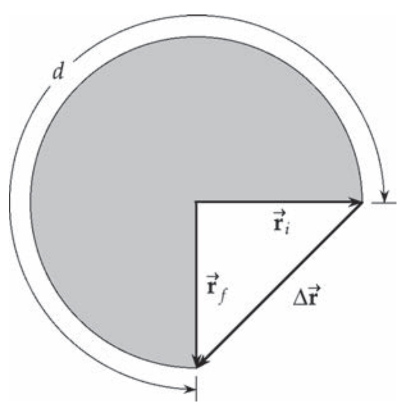
$v = |\vec{\mathbf{v}}| = \sqrt{v_x^2 + v_y^2}$ will change in time since v_y changes in time. Thus, the only correct choices for this question are (b) and (d).

6. Consider any two very closely spaced points on a circular path and draw vectors of the same length (to represent a constant velocity magnitude or speed) tangent to the path at each of these points as shown in the leftmost diagram below. Now carefully move the velocity vector $\vec{\mathbf{v}}_f$ at the second point down so its tail is at the first point as shown in the rightmost diagram. Then, draw the vector difference $\Delta\vec{\mathbf{v}} = \vec{\mathbf{v}}_f - \vec{\mathbf{v}}_i$, and observe that if the start of this vector were located on the circular path midway between the two points, its direction would be inward toward the center of the circle.



Thus, for an object following the circular path at constant speed, its instantaneous acceleration, $\vec{\mathbf{a}} = \lim_{\Delta t \to 0}\left(\Delta\vec{\mathbf{v}}/\Delta t\right)$, at the point midway between your initial and end points is directed toward the center of the circle, and the only correct choice for this question is (d).

7. The path followed (and distance traveled) by the athlete is shown in the sketch, along with the vectors for the initial position, final position, and change in position.



The average speed for the elapsed time interval Δt is

$$v_{\text{av}} = \frac{d}{\Delta t}$$

and the magnitude of the average velocity for this time interval is

$$|\vec{\mathbf{v}}_{\text{av}}| = \frac{|\Delta\vec{\mathbf{r}}|}{\Delta t}$$

The sketch clearly shows that $d > |\Delta\vec{\mathbf{r}}|$ in this case, meaning that $v_{\text{av}} > |\vec{\mathbf{v}}_{\text{av}}|$ and that (a) is the correct choice.

8. At maximum altitude, the projectile's vertical component of velocity is zero. The time for the projectile to reach its maximum height is found from $v_y = v_{0y} + a_y t$ as

$$t_{\text{max}} = \frac{v_y\big|_{\Delta y = h_{\text{max}}} - v_{0y}}{a_y} = \frac{0 - v_0 \sin\theta_0}{-g} = \frac{v_0 \sin\theta_0}{g}$$

Since the acceleration of gravity on the Moon is one-sixth that of Earth, we see that (with v_0 and θ_0 kept constant)

$$t_{\max}\big|_{\text{Moon}} = \frac{v_0 \sin\theta_0}{g_{\text{Moon}}} = \frac{v_0 \sin\theta_0}{g_{\text{Earth}}/6} = 6\left(\frac{v_0 \sin\theta_0}{g_{\text{Earth}}}\right) = 6\, t_{\max}\big|_{\text{Earth}}$$

and the correct answer for this question is (e).

9. The boat moves with a constant horizontal velocity (or its velocity relative to Earth has components of $(\vec{\mathbf{v}}_{\text{BE}})_x = v_0$ = constant, and $(\vec{\mathbf{v}}_{\text{BE}})_y = 0$), where the y-axis is vertical and the x-axis is parallel to the keel of the boat. Once the wrench is released, it is a projectile whose velocity relative to Earth has components of

$$(\vec{\mathbf{v}}_{\text{WE}})_x = v_{0x} + a_x t = v_0 + 0 = v_0 \qquad \text{and} \qquad (\vec{\mathbf{v}}_{\text{WE}})_y = v_{0y} + a_y t = 0 - gt = -gt$$

The velocity of the wrench relative to the boat ($\vec{\mathbf{v}}_{\text{WB}} = \vec{\mathbf{v}}_{\text{WE}} - \vec{\mathbf{v}}_{\text{BE}}$) has components of

$$(\vec{\mathbf{v}}_{\text{WB}})_x = (\vec{\mathbf{v}}_{\text{WE}})_x - (\vec{\mathbf{v}}_{\text{BE}})_x = v_0 - v_0 = 0 \qquad \text{and} \qquad (\vec{\mathbf{v}}_{\text{WB}})_y = (\vec{\mathbf{v}}_{\text{WE}})_y - (\vec{\mathbf{v}}_{\text{BE}})_y = -gt - 0 = -gt$$

Thus, the wrench has zero horizontal velocity relative to the boat and will land on the deck at a point directly below where it was released (i.e., at the base of the mast). The correct choice is (b).

10. While in the air, the baseball is a projectile whose velocity always has a constant horizontal component ($v_x = v_{0x}$) and a vertical component that changes at a constant rate ($\Delta v_y/\Delta t = a_y = -g$). At the highest point on the path, the vertical velocity of the ball is momentarily zero. Thus, at this point, the resultant velocity of the ball is horizontal and its acceleration continues to be directed downward ($a_x = 0,\ a_y = -g$). The only correct choice given for this question is (c).

11. Note that *for each* ball, $v_{0y} = 0$. Thus, the vertical velocity *of each ball* when it reaches the ground ($\Delta y = -h$) is given by $v_y^2 = v_{0u}^2 + 2a_y(\Delta y)$ as

$$v_y = -\sqrt{0 + 2(-g)(-h)} = -\sqrt{2gh}$$

and the time required *for each ball* to reach the ground is given by $v_y = v_{0y} + a_y t$ as

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-\sqrt{2gh} - 0}{-g} = \sqrt{\frac{2h}{g}}$$

The speeds (i.e., magnitudes of *total* velocities) of the balls at ground level are:

Red Ball: $\quad v_R = \sqrt{v_x^2 + v_y^2} = \sqrt{v_{0x}^2 + \left(-\sqrt{2gh}\right)^2} = \sqrt{v_0^2 + 2gh}$

Blue Ball: $\quad v_B = \sqrt{v_x^2 + v_y^2} = \sqrt{v_{0x}^2 + \left(-\sqrt{2gh}\right)^2} = \sqrt{0 + 2gh} = \sqrt{2gh}$

Therefore, we see that the two balls reach the ground at the same time but with different speeds ($v_R > v_B$), so only choice (b) is correct.

12. When the apple first comes off the tree, it is moving forward with the same horizontal velocity as the truck. Since, while in free fall, the apple has zero horizontal acceleration, it will maintain this constant horizontal velocity as it falls. Also, while in free fall, the apple has constant downward acceleration ($a_y = -g$), so its downward speed increases uniformly in time.

(i) As the truck moves left to right past an observer stationary on the ground, this observer will see both the constant velocity horizontal motion and the uniformly accelerated downward motion of the apple. The curve that best describes the path of the apple as seen by this observer is (a).

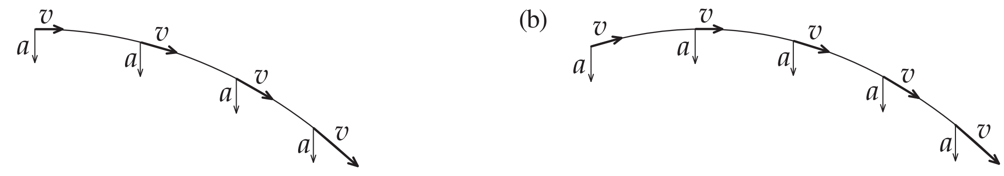
(ii) An observer on the truck moves with the same horizontal motion as does the apple. This observer does not detect any horizontal motion of the apple relative to him. However, this observer does detect the uniformly accelerated vertical motion of the apple. The curve best describing the path of the apple as seen by the observer on the truck is (b).

13. Of the choices listed, the quantities which have magnitude or size, but no direction, associated with them (i.e., scalar quantities) are (b) temperature, (c) volume, and (e) height. The other quantities, (a) velocity of a sports car and (d) displacement of a tennis player who moves from the court's backline to the net, have both magnitude and direction associated with them, and are both vector quantities.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The components of a vector will be equal in magnitude if the vector lies at a 45° angle with the two axes along which the components lie.

4. (a) (b)



6. (a) The balls will be closest at the instant the second ball is projected.

(b) The first ball will always be going faster than the second ball.

(c) There will be a one second time interval between their collisions with the ground.

(d) The two move with the same acceleration in the vertical direction. Thus, changing their horizontal velocities can never make them hit at the same time.

8. The equations of projectile motion are only valid for objects moving freely under the influence of gravity. The only acceleration such an object has is the free-fall acceleration, g, directed vertically downward. Of the objects listed, only a and d meet this requirement.

10. (a) The passenger sees the ball go into the air and come back in the same way he would if he were at rest on Earth. An observer by the tracks would see the ball follow the path of a projectile.

(b) If the train were accelerating, the ball would fall behind the position it would reach in the absence of the acceleration.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 6.1 units at $\theta = +113°$ **(b)** 15 units at $\theta = +23°$

4. **(a)** See Solution. **(b)** The sum does not depend on the order of adding.

6. **(a)** 484 km **(b)** 18.1° N of W

(c) Because of Earth's curvature, the plane does not follow straight lines.

8. $\vec{\mathbf{R}} = 9.5$ units at 57° above the $+x$-axis

10. 1.31 km northward, 2.81 km eastward

12. **(a)** 10.0 m **(b)** 15.7 m **(c)** 0

14. **(a)** $\vec{\mathbf{R}} = 358$ m at 2.00° south of east

(b) The vector sum is independent of the order in which they are added.

16. 42.7 yards

18. 788 mi at 48.0° N of E

20. **(a)** 185 N at 77.8° from the x-axis **(b)** 185 N at 258° from the x-axis

22. 2.68 ft

24. **(a)** 76.0° **(b)** $R_{\text{max}} = 2.13R$

(c) The result is independent of the acceleration of gravity and valid on all planets.

26. **(a)** 28.2 m/s **(b)** 4.07 s

(c) v_0 and t would increase

28. **(a)** $x = 0$, $y = y_0$ **(b)** $v_{0x} = 7.52$ m/s, $v_{0y} = -2.74$ m/s

(c) $x = (7.52\ \text{m/s})t,\ y = y_0 - (2.74\ \text{m/s})t - (4.90\ \text{m/s}^2)t^2$

(d) 22.6 m **(e)** $y_0 = 52.3$ m **(f)** 1.18 s

30. $x = 7.22$ km, $y = 1.69$ km

32. 18.6 m

34. **(a)** 18.1 m/s **(b)** 1.09 m **(c)** 2.8 m

36. **(a)** 57.7 km/h at 60.0° west of vertical

(b) 28.9 km/h vertically downward

38. **(a)** 27.7° east of north **(b)** 47.6 min

40. **(a)** 1.81 m/s **(b)** Yes, the required initial speed is < 6.26 m/s.

42. **(a)** $t_{\text{up}} = \dfrac{d}{v - v_s}$ **(b)** $t_{\text{down}} = \dfrac{d}{v + v_s}$ **(c)** See Solution.

(d) $t_b = \dfrac{2d}{v}$ **(e)** $t_a > t_b$ as long as $v_s > 0$

44. **(a)** $t_{\text{woman}} = L/v_1$ **(b)** $t_{\text{man}} = \dfrac{L}{v_1 + v_2}$

46. **(a)** $h_{\text{max}} = 57.4$ m, $t = 1.22$ s **(b)** Same as for part (a).

48. **(a)** $t = \sqrt{2h/g}$ **(b)** $v_i = d\sqrt{g/2h}$

(c) $v = \sqrt{d^2 g/2h + 2gh}$ **(d)** $\theta = \tan^{-1}(-2h/d)$

50. **(a)** 0.345 m **(b)** 2.35 m

52. **(a)** 0.85 m/s **(b)** 2.1 m/s

54. **(a)** 0° to the vertical **(b)** 8.25 m/s

(c) straight up and down **(d)** parabolic arc opening downward

(e) 12.6 m/s at 41.0° above the horizontal eastward line

56. **(a)** 14.7 m/s **(b)** 29.4 m/s

58. 10.7 m/s

60. **(a)** $\vec{\mathbf{d}}_{\text{male}} = 132\text{ cm}$ at 69.6° and $\vec{\mathbf{d}}_{\text{female}} = 111\text{ cm}$ at 70.0°

(b) $\vec{\mathbf{d}}'_{\text{male}} = 146\text{ cm}$ at 69.6° and $\vec{\mathbf{d}}'_{\text{female}} = 132\text{ cm}$ at 70.0°, $\Delta\vec{\mathbf{d}}' = 14$ cm at 66°

62. **(a)** $y = \left(-\dfrac{g}{2v_{0x}^2}\right)x^2 + \left(\dfrac{v_{0y}}{v_{0x}}\right)x + 0$

(b) $a = -g/2v_{0x}^2$, $b = v_{0y}/v_{0x}$, and $c = 0$

64. **(a)** 26.6° **(b)** $t_{1\text{ bounce}}/t_{\text{no bounce}} = 0.950$

66. 3.95 m/s

68. **(a)** 26 knots at 50° S of E **(b)** 20 knots due south

70. **(a)** 0.519 m, too narrow for a pedestrian walkway

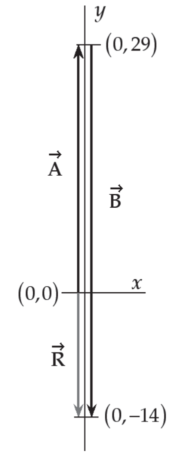
(b) 0.217 m/s

72. 4.14 m

74. $x = 18.8$ m, $y = -17.3$ m

PROBLEM SOLUTIONS

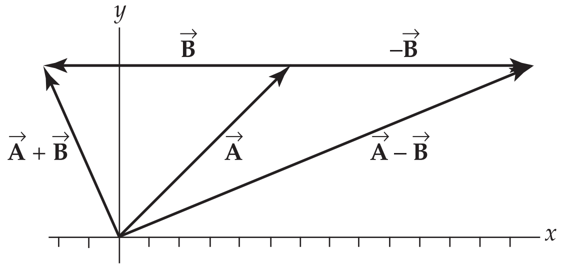
3.1 We are given that $\vec{\mathbf{R}} = \vec{\mathbf{A}} + \vec{\mathbf{B}}$. When two vectors are added graphically, the second vector is positioned with its tail at the tip of the first vector. The resultant then runs from the tail of the first vector to the tip of the second vector. In this case, vector $\vec{\mathbf{A}}$ will be positioned with its tail at the origin and its tip at the point (0, 29). The resultant is then drawn, starting at the origin (tail of first vector) and going 14 units in the negative y-direction to the point (0, −14). The second vector, $\vec{\mathbf{B}}$, must then start from the tip of $\vec{\mathbf{A}}$ at point (0, 29) and end on the tip of $\vec{\mathbf{R}}$ at point (0, −14) as shown in the sketch at the right. From this, it is seen that



$\boxed{\vec{\mathbf{B}} \text{ is 43 units in the negative } y\text{-direction}}$

continued on next page

3.2 (a) Using graphical methods, place the tail of vector $\vec{\mathbf{B}}$ at the head of vector $\vec{\mathbf{A}}$. The new vector $\vec{\mathbf{A}}+\vec{\mathbf{B}}$ has a magnitude of $\boxed{\text{6.1 units at 113°}}$ from the positive x-axis.



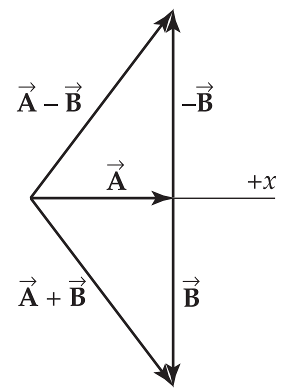
(b) The vector difference $\vec{\mathbf{A}}-\vec{\mathbf{B}}=\vec{\mathbf{A}}+(-\vec{\mathbf{B}})$ is found by placing the negative of vector $\vec{\mathbf{B}}$ (a vector of the same magnitude as $\vec{\mathbf{B}}$, but opposite direction) at the head of vector $\vec{\mathbf{A}}$. The resultant vector $\vec{\mathbf{A}}-\vec{\mathbf{B}}$ has magnitude $\boxed{\text{15 units at 23°}}$ from the positive x-axis.

3.3 (a) In your vector diagram, place the tail of vector $\vec{\mathbf{B}}$ at the tip of vector $\vec{\mathbf{A}}$. The vector sum, $\vec{\mathbf{A}}+\vec{\mathbf{B}}$, is then found as shown in the vector diagram and should be

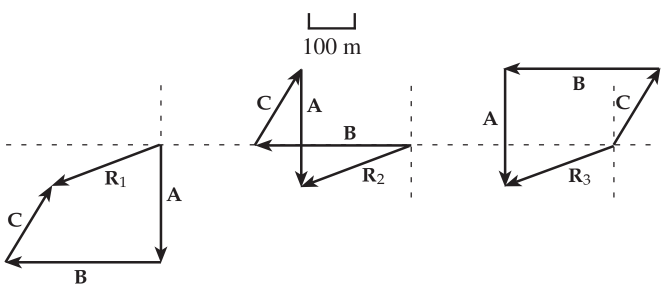
$$\boxed{\vec{\mathbf{A}}+\vec{\mathbf{B}}=5.0\text{ units at }-53°}$$

(b) To find the vector difference $\vec{\mathbf{A}}-\vec{\mathbf{B}}=\vec{\mathbf{A}}+(-\vec{\mathbf{B}})$, form the vector $-\vec{\mathbf{B}}$ (same magnitude as $\vec{\mathbf{B}}$, opposite direction) and add it to vector $\vec{\mathbf{A}}$ as shown in the diagram. You should find that

$$\boxed{\vec{\mathbf{A}}-\vec{\mathbf{B}}=5.0\text{ units at }+53°}$$

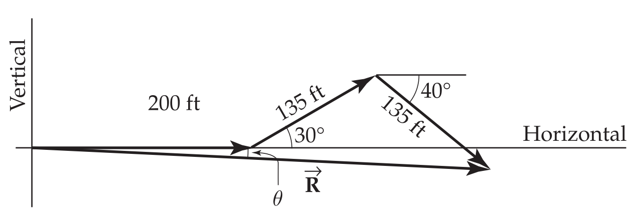


3.4 (a) The three diagrams shown below represent the graphical solutions for the three vector sums: $\vec{\mathbf{R}}_1=\vec{\mathbf{A}}+\vec{\mathbf{B}}+\vec{\mathbf{C}}$, $\vec{\mathbf{R}}_2=\vec{\mathbf{B}}+\vec{\mathbf{C}}+\vec{\mathbf{A}}$, and $\vec{\mathbf{R}}_3=\vec{\mathbf{C}}+\vec{\mathbf{B}}+\vec{\mathbf{A}}$.

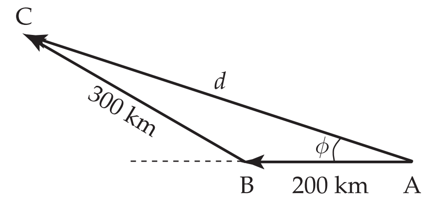


(b) We observe that $\vec{\mathbf{R}}_1=\vec{\mathbf{R}}_2=\vec{\mathbf{R}}_3$, illustrating that the sum of a set of vectors is not affected by the order in which the vectors are added.

3.5 Your sketch should be drawn to scale, and be similar to that pictured below. The length of $\vec{\mathbf{R}}$ and the angle θ can be measured to find, with use of your scale factor, the magnitude and direction of the resultant displacement. The result should be $\boxed{\text{approximately 421 ft at 3° below the horizontal}}$.



3.6 (a) The distance d from A to C is



$$d = \sqrt{x^2 + y^2}$$

where $x = 200\text{ km} + (300\text{ km})\cos 30.0° = 460\text{ km}$

and $y = 0 + (300\text{ km})\sin 30.0° = 150\text{ km}$

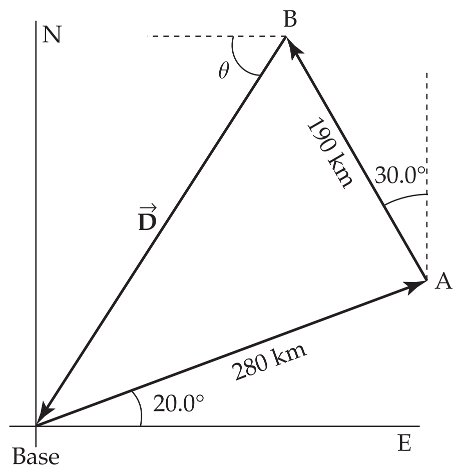
$$\therefore \quad d = \sqrt{(460\text{ km})^2 + (150\text{ km})^2} = \boxed{484\text{ km}}$$

(b) $\phi = \tan^{-1}\left(\dfrac{y}{x}\right) = \tan^{-1}\left(\dfrac{150\text{ km}}{460\text{ km}}\right) = \boxed{18.1°\text{ N of W}}$

(c) Because of the curvature of the Earth, the plane doesn't travel along straight lines. Thus, the answer computed above is only approximately correct.

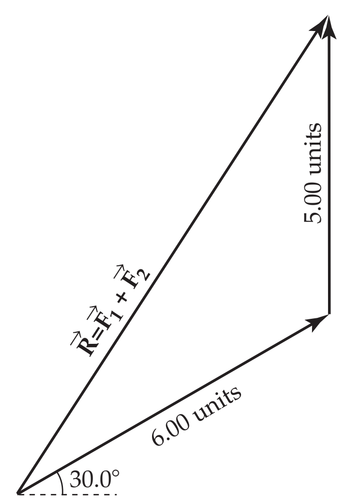
3.7 Using a vector diagram, drawn to scale, like that shown at the right, the displacement from Lake B back to base camp is given by the vector $\vec{\mathbf{D}}$. Measuring the length of this vector and multiplying by the chosen scale factor should give the magnitude of this displacement as 310 km. Measuring the angle θ should yield a value of 57°. Thus, the displacement from B to the base camp is

$$\vec{\mathbf{D}} = \boxed{310\text{ km at } \theta = 57° \text{ S of W}}$$



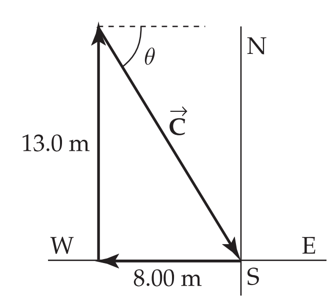
3.8 Find the resultant $\vec{\mathbf{R}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$ graphically by placing the tail of $\vec{\mathbf{F}}_2$ at the head of $\vec{\mathbf{F}}_1$ in a scale drawing as shown at the right. The resultant is the vector drawn from the tail of $\vec{\mathbf{F}}_1$ to the head of $\vec{\mathbf{F}}_2$ to close up the triangle. Measuring the length and orientation of this vector shows that

$$\vec{\mathbf{R}} = \boxed{9.5\text{ units at } 57° \text{ above the } +x\text{-axis}}$$



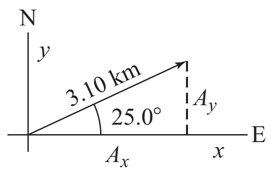
3.9 The displacement vectors $\vec{\mathbf{A}} = 8.00\text{ m westward}$ and $\vec{\mathbf{B}} = 13.0\text{ m north}$ can be drawn to scale as at the right. The vector $\vec{\mathbf{C}}$ represents the displacement that the man in the maze must undergo to return to his starting point. The scale used to draw the sketch can be used to find $\vec{\mathbf{C}}$ to be

$$\boxed{15\text{ m at } 58° \text{ S of E}}$$



3.10 The person undergoes a displacement $\vec{\mathbf{A}} = 3.10$ km at 25.0° north of east. Choose a coordinate system with origin at the starting point, positive x-direction oriented eastward, and positive y-direction oriented northward.

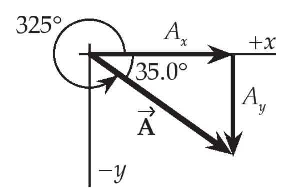
Then the components of her displacement are



$$A_x = A\cos\theta = (3.10\text{ km})\cos 25.0° = \boxed{2.81\text{ km eastward}}$$

and $$A_y = A\sin\theta = (3.10\text{ km})\sin 25.0° = \boxed{1.31\text{ km northward}}$$

3.11 The x- and y-components of vector $\vec{\mathbf{A}}$ are its projections on lines parallel to the x- and y-axes, respectively, as shown in the sketch. The magnitude of these components can be computed using the sine and cosine functions as shown below:

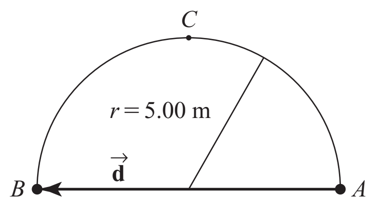


$$A_x = \left|\vec{\mathbf{A}}\right|\cos 325° = +\left|\vec{\mathbf{A}}\right|\cos 35° = (35.0)\cos 35° = \boxed{28.7\text{ units}}$$

and

$$A_y = \left|\vec{\mathbf{A}}\right|\sin 325° = -\left|\vec{\mathbf{A}}\right|\sin 35° = -(35.0)\sin 35° = \boxed{-20.1\text{ units}}$$

3.12 (a) The skater's displacement vector, $\vec{\mathbf{d}}$, extends in a straight line from her starting point A to the end point B. When she has coasted half way around a circular path as shown in the sketch at the right, the displacement vector coincides with the diameter of the circle and has magnitude



$$\left|\vec{\mathbf{d}}\right| = 2r = 2(5.00\text{ m}) = \boxed{10.0\text{ m}}$$

(b) The actual distance skated, s, is one half the circumference of the circular path of radius r. Thus

$$s = \frac{1}{2}(2\pi r) = \pi(5.00\text{ m}) = \boxed{15.7\text{ m}}$$

(c) When the skater skates all the way around the circular path, her end point, B, coincides with the start point, A. Thus, the displacement vector has zero length, or $\left|\vec{\mathbf{d}}\right| = \boxed{0}$.

3.13 (a) Her net x (east-west) displacement is $-3.00 + 0 + 6.00 = +3.00$ blocks, while her net y (north-south) displacement is $0 + 4.00 + 0 = +4.00$ blocks. The magnitude of the resultant displacement is

$$R = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \sqrt{(3.00)^2 + (4.00)^2} = 5.00\text{ blocks}$$

and the angle the resultant makes with the x-axis (eastward direction) is

$$\theta = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right) = \tan^{-1}\left(\frac{4.00}{3.00}\right) = \tan^{-1}(1.33) = 53.1°$$

The resultant displacement is then $\boxed{5.00\text{ blocks at }53.1°\text{ N of E}}$.

(b) The total distance traveled is $3.00 + 4.00 + 6.00 = \boxed{13.0\text{ blocks}}$.

3.14 (a) We choose a coordinate system with the positive x-axis eastward and the positive y-axis northward. The hiker then undergoes four successive displacements, given below along with their x- and y-components:

$\vec{\mathbf{A}} = 75.0$ m due north $\quad A_x = 0 \quad A_y = +75.0$ m

$\vec{\mathbf{B}} = 250.0$ m due east $\quad B_x = +250$ m $\quad B_y = 0$

$\vec{\mathbf{C}} = 125.0$ m at 30.0° N of E $\quad C_x = +108$ m $\quad C_y = +62.5$ m

$\vec{\mathbf{D}} = 150$ m due south $\quad D_x = 0 \quad D_y = -150$ m

The components of the resultant displacement are then

$$R_x = A_x + B_x + C_x + D_x = +358 \text{ m} \quad \text{and} \quad R_y = A_y + B_y + C_y + D_y = -12.5 \text{ m}$$

The magnitude of the resultant is $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(358 \text{ m})^2 + (-12.5 \text{ m})^2} = 358$ m

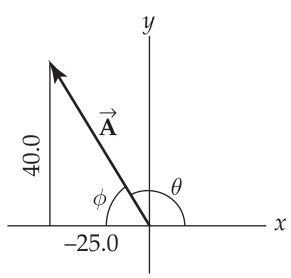
and its direction is $\qquad \theta = \tan^{-1}\left(\dfrac{R_y}{R_x}\right) = \tan^{-1}\left(\dfrac{-12.5 \text{ m}}{358 \text{ m}}\right) = -2.00°$

giving $\qquad \boxed{\vec{\mathbf{R}} = 358 \text{ m at } 2.00° \text{ south of east}}$

(b) $\boxed{\text{No.}}$ Because of the commutative property of vector addition, the net displacement is the same regardless of the order in which the individual displacements are executed.

3.15 $A_x = -25.0 \qquad A_y = 40.0$

$A = \sqrt{A_x^2 + A_y^2} = \sqrt{(-25.0)^2 + (40.0)^2} = 47.2$ units



From the triangle, we find that

$$\phi = \tan^{-1}\left(\frac{A_y}{|A_x|}\right) = \tan^{-1}\left(\frac{|40.0|}{25.0}\right) = 58.0°, \text{ so } \theta = 180° - \phi = 122°$$

Thus, $\boxed{\vec{\mathbf{A}} = 47.2 \text{ units at } 122° \text{ counterclockwise from the } +x\text{-axis}}$.

3.16 Let $\vec{\mathbf{A}}$ be the vector corresponding to the 10.0 yd run, $\vec{\mathbf{B}}$ to the 15.0 yd run, and $\vec{\mathbf{C}}$ to the 50.0 yd pass. Also, we choose a coordinate system with the $+y$-direction downfield, and the $+x$-direction toward the sideline to which the player runs.

The components of the vectors are then

$A_x = 0 \qquad A_y = -10.0$ yds

$B_x = 15.0$ yds $\qquad B_y = 0$

$C_x = 0 \qquad C_y = +50.0$ yds

From these, $R_x = \Sigma x = 15.0$ yds, and $R_y = \Sigma y = 40.0$ yds,

and $\qquad R = \sqrt{R_x^2 + R_y^2} = \sqrt{(15.0 \text{ yds})^2 + (40.0 \text{ yds})^2} = \boxed{42.7 \text{ yards}}$

3.17 After 3.00 h moving at 41.0 km/h, the hurricane is 123 km at 60.0° N of W from the island. In the next 1.50 h, it travels 37.5 km due north. The components of these two displacements are:

Displacement	x-component (eastward)	y-component (northward)
123 km	–61.5 km	+107 km
37.5 km	0	+37.5 km
Resultant	–61.5 km	144 km

Therefore, the eye of the hurricane is now

$$R=\sqrt{(-61.5\text{ km})^2+(144\text{ km})^2}=\boxed{157\text{ km from the island}}$$

3.18 Choose the positive x-direction to be eastward and positive y as northward. Then, the components of the resultant displacement from Dallas to Chicago are

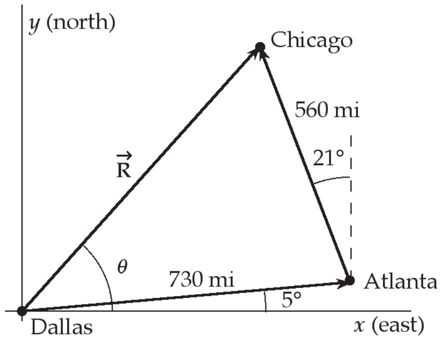
$$R_x=\Sigma x=(730\text{ mi})\cos 5.00°-(560\text{ mi})\sin 21.0°=527\text{ mi}$$

and $$R_y=\Sigma y=(730\text{ mi})\sin 5.00°+(560\text{ mi})\cos 21.0°=586\text{ mi}$$

$$R=\sqrt{R_x^2+R_y^2}=\sqrt{(527\text{ mi})^2+(586\text{ mi})^2}=788\text{ mi}$$

$$\theta=\tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right)=\tan^{-1}(1.11)=48.0°$$

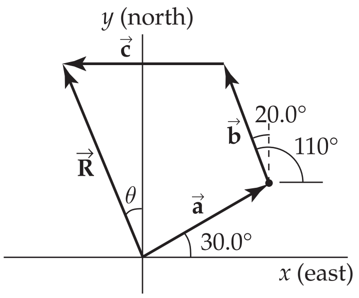
Thus, the displacement from Dallas to Chicago is $\vec{\mathbf{R}}=\boxed{788\text{ mi at }48.0°\text{ N of E}}$



3.19 The components of the displacements $\vec{\mathbf{a}}$, $\vec{\mathbf{b}}$, and $\vec{\mathbf{c}}$ are

$$a_x=a\cdot\cos 30.0°=+152\text{ km}$$
$$b_x=b\cdot\cos 110°=-51.3\text{ km}$$
$$c_x=c\cdot\cos 180°=-190\text{ km}$$

and
$$a_y=a\cdot\sin 30.0°=+87.5\text{ km}$$
$$b_y=b\cdot\sin 110°=+141\text{ km}$$
$$c_y=c\cdot\sin 180°=0$$



Thus, $R_x=a_x+b_x+c_x=-89.3\text{ km}$, and $R_y=a_y+b_y+c_y=+229\text{ km}$

so $R=\sqrt{R_x^2+R_y^2}=246\text{ km}$, and $\theta=\tan^{-1}\left(|R_x|/R_y\right)=\tan^{-1}(0.390)=21.3°$

City C is $\boxed{246\text{ km at }21.3°\text{ W of N}}$ from the starting point.

3.20 (a) $F_1=120\text{ N}$ $F_{1x}=(120\text{ N})\cos 60.0°=60.0\text{ N}$ $F_{1y}=(120\text{ N})\sin 60.0°=104\text{ N}$

$F_2=80.0\text{ N}$ $F_{2x}=-(80.0\text{ N})\cos 75.0°=-20.7\text{ N}$ $F_{2y}=(80.0\text{ N})\sin 75.0°=77.3\text{ N}$

$$F_R=\sqrt{(\Sigma F_x)^2+(\Sigma F_y)^2}=\sqrt{(39.3\text{ N})^2+(181\text{ N})^2}=185\text{ N}$$

and $\theta=\tan^{-1}\left(\dfrac{181\text{ N}}{39.3\text{ N}}\right)=\tan^{-1}(4.61)=77.8°$

The resultant force is $\vec{\mathbf{F}}_R=\boxed{185\text{ N at }77.8°\text{ from the }x\text{-axis}}$.

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(b) To have zero net force on the mule, the resultant above must be cancelled by a force equal in magnitude and oppositely directed. Thus, the required force is

$\boxed{185 \text{ N at } 258° \text{ from the } x\text{-axis}}$

3.21 The single displacement required to sink the putt in one stroke is equal to the resultant of the three actual putts used by the novice. Taking east as the positive x-direction and north as the positive y-direction, the components of the three individual putts and their resultant are

$$A_x = 0 \qquad A_y = +4.00 \text{ m}$$

$$B_x = (2.00 \text{ m})\cos 45.0° = +1.41 \text{ m} \qquad B_y = (2.00 \text{ m})\sin 45.0° = +1.41 \text{ m}$$

$$C_x = -(1.00 \text{ m})\sin 30.0° = -0.500 \text{ m} \qquad C_y = -(1.00 \text{ m})\cos 30.0° = -0.866 \text{ m}$$

$$R_x = A_x + B_x + C_x = +0.910 \text{ m} \qquad R_y = A_y + B_y + C_y = +4.55 \text{ m}$$

The magnitude and direction of the desired resultant is then

$$R = \sqrt{R_x^2 + R_y^2} = 4.64 \text{ m} \qquad \text{and} \qquad \theta = \tan^{-1}\left(\frac{R_y}{R_x}\right) = +78.7°$$

Thus, $\vec{\mathbf{R}} = \boxed{4.64 \text{ m at } 78.7° \text{ north of east}}$

3.22 $v_{0x} = (101 \text{ mi/h})\left(\dfrac{0.447 \text{ m/s}}{1 \text{ mi/h}}\right) = 45.1 \text{ m/s}$ and $\Delta x = (60.5 \text{ ft})\left(\dfrac{1 \text{ m}}{3.281 \text{ ft}}\right) = 18.4 \text{ m}$

The time to reach home plate is $t = \dfrac{\Delta x}{v_{0x}} = \dfrac{18.4 \text{ m}}{45.1 \text{ m/s}} = 0.408 \text{ s}$.

In this time interval, the vertical displacement is

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)(0.408 \text{ s})^2 = -0.817 \text{ m}$$

Thus, the ball drops vertically $0.817 \text{ m}(3.281 \text{ ft/1 m}) = \boxed{2.68 \text{ ft}}$.

3.23 (a) With the origin chosen at point O as shown in Figure P3.23, the coordinates of the original position of the stone are $\boxed{x_0 = 0 \text{ and } y_0 = +50.0 \text{ m}}$.

(b) The components of the initial velocity of the stone are $\boxed{v_{0x} = +18.0 \text{ m/s and } v_{0y} = 0}$.

(c) The components of the stone's velocity during its flight are given as functions of time by

$$v_x = v_{0x} + a_x t = 18.0 \text{ m/s} + (0)t \quad \text{or} \quad \boxed{v_x = 18.0 \text{ m/s}}$$

and $$v_y = v_{0y} + a_y t = 0 + (-g)t \quad \text{or} \quad \boxed{v_y = -\left(9.80 \text{ m/s}^2\right)t}$$

(d) The coordinates of the stone during its flight are

$$x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = 0 + (18.0 \text{ m/s})t + \frac{1}{2}(0)t^2 \quad \text{or} \quad \boxed{x = (18.0 \text{ m/s})t}$$

and $$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = 50.0 \text{ m} + (0)t + \frac{1}{2}(-g)t^2 \quad \text{or} \quad \boxed{y = 50.0 \text{ m} - \left(4.90 \text{ m/s}^2\right)t^2}$$

continued on next page

(e) We find the time of fall from $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ with $v_{0y} = 0$:

$$t = \sqrt{\frac{2(\Delta y)}{a}} = \sqrt{\frac{2(-50.0\ \text{m})}{-9.80\ \text{m/s}^2}} = \boxed{3.19\ \text{s}}$$

(f) At impact, $v_x = v_{0x} = 18.0$ m/s, and the vertical component is

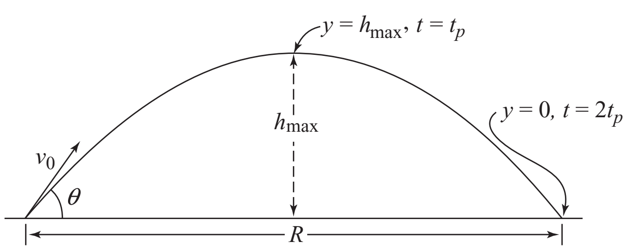
$$v_y = v_{0y} + a_y t = 0 + (-9.80\ \text{m/s}^2)(3.19\ \text{s}) = -31.3\ \text{m/s}$$

Thus, $v = \sqrt{v_x^2 + v_y^2} = \sqrt{(18.0\ \text{m/s})^2 + (-31.3\ \text{m/s})^2} = 36.1\ \text{m/s}$

and $\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-31.3}{18.0}\right) = -60.1°$

or $\vec{\mathbf{v}} = \boxed{36.1\ \text{m/s at } 60.1° \text{ below the horizontal}}$

3.24 (a) At $y = h_{\max}$, the vertical component of velocity is zero, so $v_y = v_{0y} + a_y t$ gives the time to reach the peak as



$$t_p = \frac{0 - v_{0y}}{a_y} = \frac{-v_0 \sin\theta}{-g} = \frac{v_0 \sin\theta}{g}$$

Then, $\Delta y = (v_y)_{\text{av}}\,t = \left[(v_y + v_{0y})/2\right]t$ gives the maximum vertical displacement as

$$h_{\max} = \left(\frac{0 + v_{0y}}{2}\right)t_p = \left(\frac{v_0 \sin\theta}{2}\right)\left(\frac{v_0 \sin\theta}{g}\right) = \frac{v_0^2 \sin^2\theta}{2g}$$

The total time of flight is $t_{\text{total}} = 2t_p = 2v_0 \sin\theta/g$, so the horizontal range is

$$R = v_{0x}t_{\text{total}} = (v_0 \cos\theta)\left(\frac{2v_0 \sin\theta}{g}\right) = \frac{2v_0^2 \sin\theta\cos\theta}{g}$$

If we are to have $h_{\max} = R$, it is necessary that $\dfrac{v_0^2 \sin^2\theta}{2g} = \dfrac{2v_0^2 \sin\theta\cos\theta}{g}$

This requirement reduces to $\dfrac{\sin\theta}{2} = 2\cos\theta$ or $\tan\theta = 4$

which gives the required launch angle as $\theta = \tan^{-1}(4) = \boxed{76.0°}$

(b) For maximum range, the launch angle would be $\theta = 45°$, so

$$R_{\max} = \frac{2v_0^2 \sin 45° \cos 45°}{g} = \frac{v_0^2}{g}$$

The ratio of $R_{\max}$ to the range in part (a) (where $\theta = 76.0°$) is

$$\frac{R_{\max}}{R} = \frac{v_0^2/g}{\left[2v_0^2 \sin(76.0°)\cos(76.0°)\right]/g} = \frac{1}{0.469} \qquad \text{or} \qquad \boxed{R_{\max} = 2.13R}$$

continued on next page

(c) Note that in the calculation of the answer to part (a), the acceleration of gravity canceled out. Thus, $\boxed{\text{the result is valid on every planet}}$ independent of the local acceleration of gravity.

3.25 At the maximum height $v_y = 0$, and the time to reach this height is found from

$$v_y = v_{0y} + a_y t \quad \text{as} \quad t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - v_{0y}}{-g} = \frac{v_{0y}}{g}$$

The vertical displacement that has occurred during this time is

$$(\Delta y)_{\max} = \left(v_y\right)_{\text{av}} t = \left(\frac{v_y + v_{0y}}{2}\right)t = \left(\frac{0 + v_{0y}}{2}\right)\left(\frac{v_{0y}}{g}\right) = \frac{v_{0y}^2}{2g}$$

If $(\Delta y)_{\max} = 3.7\text{ m}$, we find

$$v_{0y} = \sqrt{2g\,(\Delta y)_{\max}} = \sqrt{2\left(9.80\text{ m/s}^2\right)(3.7\text{ m})} = 8.5\text{ m/s}$$

and if the angle of projection is $\theta = 45°$, the launch speed is

$$v_0 = \frac{v_{0y}}{\sin\theta} = \frac{8.5\text{ m/s}}{\sin 45°} = \boxed{12\text{ m/s}}$$

3.26 (a) When a projectile is launched with speed v_0 at angle θ_0 above the horizontal, the initial velocity components are $v_{0x} = v_0\cos\theta_0$ and $v_{0y} = v_0\sin\theta_0$. Neglecting air resistance, the vertical velocity when the projectile returns to the level from which it was launched (in this case, the ground) will be $v_y = -v_{0y}$. From this information, the total time of flight is found from $v_y = v_{0y} + a_y t$ to be

$$t_{\text{total}} = \frac{v_{yf} - v_{0y}}{a_y} = \frac{-v_{0y} - v_{0y}}{-g} = \frac{2v_{0y}}{g} \quad \text{or} \quad t_{\text{total}} = \frac{2v_0\sin\theta_0}{g}$$

Since the horizontal velocity of a projectile with no air resistance is constant, the horizontal distance it will travel in this time (i.e., its range) is given by

$$R = v_{0x}t_{\text{total}} = \left(v_0\cos\theta_0\right)\left(\frac{2v_0\sin\theta_0}{g}\right) = \frac{v_0^2}{g}\left(2\sin\theta_0\cos\theta_0\right) = \frac{v_0^2\sin\left(2\theta_0\right)}{g}$$

Thus, if the projectile is to have a range of $R = 81.1\text{ m}$ when launched at an angle of $\theta_0 = 45.0°$, the required initial speed is

$$v_0 = \sqrt{\frac{Rg}{\sin\left(2\theta_0\right)}} = \sqrt{\frac{(81.1\text{ m})\left(9.80\text{ m/s}^2\right)}{\sin(90.0°)}} = \boxed{28.2\text{ m/s}}$$

(b) With $v_0 = 28.2\text{ m/s}$ and $\theta_0 = 45.0°$, the total time of flight (as found above) will be

$$t_{\text{total}} = \frac{2v_0\sin\theta_0}{g} = \frac{2(28.2\text{ m/s})\sin(45.0°)}{9.80\text{ m/s}^2} = \boxed{4.07\text{ s}}$$

(c) Note that at $\theta_0 = 45.0°$, $\sin\left(2\theta_0\right) = 1$ and that $\sin\left(2\theta_0\right)$ will decrease as θ_0 is increased above this optimum launch angle. Thus, if the range is to be kept constant while the launch angle is increased above 45.0°, we see from $v_0 = \sqrt{Rg/\sin\left(2\theta_0\right)}$ that $\boxed{\text{the required initial velocity will increase.}}$

continued on next page

Observe that for $\theta_0 < 90°$, the function $\sin\theta_0$ increases as θ_0 is increased. Thus, increasing the launch angle above 45.0° while keeping the range constant means that both v_0 and $\sin\theta_0$ will increase. Considering the expression for t_{total} given above, we see that $\boxed{\text{the total time of flight will increase.}}$

3.27 (a) The time required for the ball to travel $\Delta x = 36.0$ m horizontally to the goal is found from $\Delta x = v_{0x}t = (v_0\cos\theta)t$ as $t = \Delta x / v_0\cos\theta$.

At this time, the vertical displacement of the ball is given by $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$. The vertical distance by which the ball clears the bar is $d = \Delta y - h$, or

$$d = \cancel{v_0}\sin 53.0°\left(\frac{36.0\ \text{m}}{\cancel{v_0}\cos 53.0°}\right) + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)\left(\frac{36.0\ \text{m}}{(20.0\ \text{m/s})\cos 53.0°}\right)^2 - 3.05\ \text{m}$$

yielding $d = +0.89$ m. Thus, $\boxed{\text{the ball clears the crossbar by 0.89 m}}$.

(b) The ball reaches the plane of the goal post at $t = \Delta x / v_0\cos\theta$, or

$$t = \frac{36.0\ \text{m}}{(20.0\ \text{m/s})\cos 53.0°} = 2.99\ \text{s}$$

At this time, its vertical velocity is given by $v_y = v_{0y} + a_y t$ as

$$v_y = (20.0\ \text{m/s})\sin 53.0° + \left(-9.80\ \text{m/s}^2\right)(2.99\ \text{s}) = -13.3\ \text{m/s}$$

Since $v_y < 0$, the ball has passed the peak of its arc and is $\boxed{\text{descending}}$ when it crosses the crossbar.

3.28 (a) With the origin at ground level directly below the window, the original coordinates of the ball are $(x,\ y) = \boxed{(0,\ y_0)}$.

(b) $v_{0x} = v_0\cos\theta_0 = (8.00\ \text{m/s})\cos(-20.0°) = \boxed{+7.52\ \text{m/s}}$

$v_{0y} = v_0\sin\theta_0 = (8.00\ \text{m/s})\sin(-20.0°) = \boxed{-2.74\ \text{m/s}}$

(c) $x = x_0 + v_{0x}t + \frac{1}{2}a_x t^2 = 0 + (7.52\ \text{m/s})t + \frac{1}{2}(0)t^2$ or $\boxed{x = (7.52\ \text{m/s})t}$

$y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2 = y_0 + (-2.74\ \text{m/s})t + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)t^2$

or $\boxed{y = y_0 - (2.74\ \text{m/s})t - \left(4.90\ \text{m/s}^2\right)t^2}$

(d) Since the ball hits the ground at $t = 3.00$ s, the x-coordinate at the landing site is

$$x_{\text{landing}} = x\big|_{t=3.00\ \text{s}} = (7.52\ \text{m/s})(3.00\ \text{s}) = \boxed{22.6\ \text{m}}$$

(e) Since $y = 0$ when the ball reaches the ground at $t = 3.00$ s, the result of (c) above gives

$$y_0 = \left[y + \left(2.74\ \frac{\text{m}}{\text{s}}\right)t + \left(4.90\ \frac{\text{m}}{\text{s}^2}\right)t^2\right]\Bigg|_{t=3.00\ \text{s}} = 0 + \left(2.74\ \frac{\text{m}}{\text{s}}\right)(3.00\ \text{s}) + \left(4.90\ \frac{\text{m}}{\text{s}^2}\right)(3.00\ \text{s})^2$$

or $y_0 = \boxed{52.3\ \text{m}}$

continued on next page

(f) When the ball has a vertical displacement of $\Delta y = -10.0$ m, it will be moving downward with a velocity given by $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$ as

$$v_y = -\sqrt{v_{0y}^2 + 2a_y(\Delta y)} = -\sqrt{(-2.74\ \text{m/s})^2 + 2(-9.80\ \text{m/s}^2)(-10.0\ \text{m})} = -14.3\ \text{m/s}$$

The elapsed time at this point is then

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-14.3\ \text{m/s} - (-2.74\ \text{m/s})}{-9.80\ \text{m/s}^2} = \boxed{1.18\ \text{s}}$$

3.29 We choose our origin at the initial position of the projectile. After 3.0 s, it is at ground level, so the vertical displacement is $\Delta y = -H$.

To find H, we use $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, which becomes

$$-H = \left[(15\ \text{m/s})\sin 25°\right](3.0\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(3.0\ \text{s})^2, \text{ or } H = \boxed{25\ \text{m}}$$

3.30 The initial velocity components of the projectile are

$$v_{0x} = (300\ \text{m/s})\cos 55.0° = 172\ \text{m/s} \quad \text{and} \quad v_{0y} = (300\ \text{m/s})\sin 55.0° = 246\ \text{m/s}$$

while the constant acceleration components are $a_x = 0$ and $a_y = -g = -9.80\ \text{m/s}^2$

The coordinates of where the shell strikes the mountain at $t = 42.0$ s are

$$x = v_{0x}t + \frac{1}{2}a_x t^2 = (172\ \text{m/s})(42.0\ \text{s}) + 0 = 7.22 \times 10^3\ \text{m} = \boxed{7.22\ \text{km}}$$

and

$$y = v_{0y}t + \frac{1}{2}a_y t^2$$
$$= (246\ \text{m/s})(42.0\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(42.0\ \text{s})^2 = 1.69 \times 10^3\ \text{m} = \boxed{1.69\ \text{km}}$$

3.31 The speed of the car when it reaches the edge of the cliff is

$$v = \sqrt{v_0^2 + 2a(\Delta x)} = \sqrt{0 + 2(4.00\ \text{m/s}^2)(50.0\ \text{m})} = 20.0\ \text{m/s}$$

Now, consider the projectile phase of the car's motion. The vertical velocity of the car as it reaches the water is

$$v_y = -\sqrt{v_{0y}^2 + 2a_y(\Delta y)} = -\sqrt{\left[-(20.0\ \text{m/s})\sin 24.0°\right]^2 + 2(-9.80\ \text{m/s}^2)(-30.0\ \text{m})}$$

or $v_y = -25.6\ \text{m/s}$

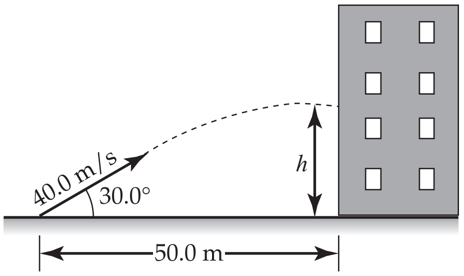
(b) The time of flight is

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-25.6\ \text{m/s} - \left[-(20.0\ \text{m/s})\sin 24.0°\right]}{-9.80\ \text{m/s}^2} = \boxed{1.78\ \text{s}}$$

(a) The horizontal displacement of the car during this time is

$$\Delta x = v_{0x}t = \left[(20.0\ \text{m/s})\cos 24.0°\right](1.78\ \text{s}) = \boxed{32.5\ \text{m}}$$

3.32 The components of the initial velocity are



$$v_{0x} = (40.0\ \text{m/s})\cos 30.0° = 34.6\ \text{m/s}$$

and

$$v_{0y} = (40.0\ \text{m/s})\sin 30.0° = 20.0\ \text{m/s}$$

The time for the water to reach the building is

$$t = \frac{\Delta x}{v_{0x}} = \frac{50.0\ \text{m}}{34.6\ \text{m}} = 1.44\ \text{s}$$

The height of the water at this time is $h = \Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, or

$$h = (20.0\ \text{m/s})(1.44\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(1.44\ \text{s})^2 = \boxed{18.6\ \text{m}}$$

3.33 (a) At the highest point of the trajectory, the projectile is moving horizontally with velocity components of $v_y = 0$ and

$$v_x = v_{0x} = v_0 \cos\theta = (60.0\ \text{m/s})\cos 30.0° = \boxed{52.0\ \text{m/s}}$$

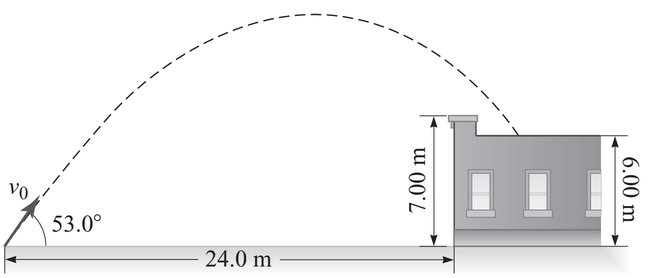
(b) The horizontal displacement is $\Delta x = v_{0x}t = (52.0\ \text{m/s})(4.00\ \text{s}) = 208\ \text{m}$ and, from $\Delta y = (v_0 \sin\theta)t + \frac{1}{2}a_y t^2$, the vertical displacement is

$$\Delta y = (60.0\ \text{m/s})(\sin 30.0°)(4.00\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(4.00\ \text{s})^2 = 41.6\ \text{m}$$

The straight line distance is

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(208\ \text{m})^2 + (41.6\ \text{m})^2} = \boxed{212\ \text{m}}$$

3.34 (a) The horizontal displacement of the ball at time t is $\Delta x = v_x t = (v_0 \cos 53.0°)t$. Thus, if the ball travels 24.0 m horizontally in 2.20 s, the initial speed of the ball must be



$$v_0 = \frac{\Delta x}{t \cdot \cos 53.0°} = \frac{24.0\ \text{m}}{(2.20\ \text{s})\cos 53.0°} = \boxed{18.1\ \text{m/s}}$$

(b) The vertical displacement of the ball at the instant it passes over the top of the wall (at $t = 2.20$ s, and 24.0 m horizontally from the launch point) is given by $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, with $v_{0y} = v_0 \sin 53.0°$, as

$$\Delta y = (18.1\ \text{m/s})\sin 53.0°(2.20\ \text{s}) + \frac{1}{2}(-9.80\ \text{m/s}^2)(2.20\ \text{s})^2 = 8.09\ \text{m}$$

Thus, the ball clears the top of the wall by: $d = 8.09\ \text{m} - 7.00\ \text{m} = \boxed{1.09\ \text{m}}$

(c) The times when the ball is 6.00 m above ground level are found from $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, using $\Delta y = 6.00$ m, $v_{0y} = v_0 \sin 53.0°$, and $a_y = -g$:

$$6.00\ \text{m} = [(18.1\ \text{m/s})\sin 53.0°]t - (4.90\ \text{m/s}^2)t^2$$

continued on next page

or $\quad (4.90)t^2 - (14.5\text{ s})t + (6.00\text{ s}^2) = 0$

This has solutions of $t = 0.497$ s and $t = 2.46$ s. The first solution is when the ball passes the 6.00 m level on the way up and the second solution is the time when it lands on the roof. Note that this is $\Delta t = 2.46\text{ s} - 2.20\text{ s} = 0.26$ s after it crosses over the wall. Thus, the ball lands a horizontal distance beyond the wall given by

$$\Delta x = v_x(\Delta t) = (v_0 \cos 53.0°)\Delta t = (18.1\text{ m/s})\cos 53.0°(0.26\text{ s}) = \boxed{2.8\text{ m}}$$

3.35 (a) The jet moves at 3.00×10^2 mi/h due east relative to the air. Choosing a coordinate system with the positive *x*-direction eastward and the positive *y*-direction northward, the components of this velocity are

$$\boxed{(\vec{\mathbf{v}}_{\text{JA}})_x = 3.00\times10^2\text{ mi/h} \quad \text{and} \quad (\vec{\mathbf{v}}_{\text{JA}})_y = 0}$$

(b) The velocity of the air relative to Earth is 1.00×10^2 mi/h at 30.0° north of east. Using the coordinate system adopted in (a) above, the components of this velocity are

$$(\vec{\mathbf{v}}_{\text{AE}})_x = |\vec{\mathbf{v}}_{\text{AE}}|\cos\theta = (1.00\times10^2\text{ mi/h})\cos 30.0° = \boxed{86.6\text{ mi/h}}$$

and $\quad (\vec{\mathbf{v}}_{\text{AE}})_y = |\vec{\mathbf{v}}_{\text{AE}}|\sin\theta = (1.00\times10^2\text{ mi/h})\sin 30.0° = \boxed{50.0\text{ mi/h}}$

(c) Carefully observe the pattern of the subscripts in Equation 3.16 of the textbook. There, two objects (cars A and B) both move relative to a third object (Earth, E). The velocity of object A relative to object B is given in terms of the velocities of these objects relative to E as $\vec{\mathbf{v}}_{\text{AB}} = \vec{\mathbf{v}}_{\text{AE}} - \vec{\mathbf{v}}_{\text{BE}}$. In the present case, we have two objects, a jet (J) and the air (A), both moving relative to a third object, Earth (E). Using the same pattern of subscripts as that in Equation 3.16, the velocity of the jet relative to the air is given by

$$\vec{\mathbf{v}}_{\text{JA}} = \vec{\mathbf{v}}_{\text{JE}} - \vec{\mathbf{v}}_{\text{AE}}$$

(d) From the expression for $\vec{\mathbf{v}}_{\text{JA}}$ found in (c) above, the velocity of the jet relative to the ground is $\vec{\mathbf{v}}_{\text{JE}} = \vec{\mathbf{v}}_{\text{JA}} + \vec{\mathbf{v}}_{\text{AE}}$. Its components are then

$$(\vec{\mathbf{v}}_{\text{JE}})_x = (\vec{\mathbf{v}}_{\text{JA}})_x + (\vec{\mathbf{v}}_{\text{AE}})_x = 3.00\times10^2\text{ mi/h} + 86.6\text{ mi/h} = 3.87\times10^2\text{ mi/h}$$

and $\quad (\vec{\mathbf{v}}_{\text{JE}})_y = (\vec{\mathbf{v}}_{\text{JA}})_y + (\vec{\mathbf{v}}_{\text{AE}})_y = 0 + 50.0\text{ mi/h} = 50.0\text{ mi/h}$

This gives the magnitude and direction of the jet's motion relative to Earth as

$$|\vec{\mathbf{v}}_{\text{JE}}| = \sqrt{|\vec{\mathbf{v}}_{\text{JE}}|_x^2 + |\vec{\mathbf{v}}_{\text{JE}}|_y^2} = \sqrt{(3.87\times10^2\text{ mi/h})^2 + (50.0\text{ mi/h})^2} = \boxed{3.90\times10^2\text{ mi/h}}$$

and $\quad \theta = \tan^{-1}\left(\dfrac{(\vec{\mathbf{v}}_{\text{JE}})_y}{(\vec{\mathbf{v}}_{\text{JE}})_x}\right) = \tan^{-1}\left(\dfrac{50.0\text{ mi/h}}{3.87\times10^2\text{ mi/h}}\right) = 7.36°$

Therefore, $\quad \boxed{\vec{\mathbf{v}}_{\text{JE}} = 3.90\times10^2\text{ mi/h at 7.36° north of east}}$

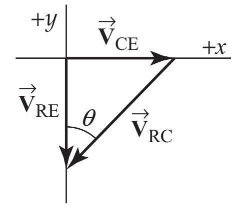
3.36 Choose a reference system with the positive *x*-axis in the eastward direction and the positive *y*-axis vertically upward. Then, the velocities of the car and the raindrops relative to Earth are: $\vec{\mathbf{v}}_{\text{CE}} = 50.0$ km/h in the $+x$-direction, and $\vec{\mathbf{v}}_{\text{RE}} = v_{\text{rain}}$ in the negative *y*-direction. We also know

continued on next page

that the velocity of the rain relative to the car, $\vec{\mathbf{v}}_{RC}$, is directed downward at $\theta = 60.0°$ from the vertical. From Equation 3.16 in the text, these relative velocities are related by

$$\vec{\mathbf{v}}_{RC} = \vec{\mathbf{v}}_{RE} - \vec{\mathbf{v}}_{CE} \qquad \text{or} \qquad \vec{\mathbf{v}}_{RE} = \vec{\mathbf{v}}_{CE} + \vec{\mathbf{v}}_{RC}$$

Thus, these relative velocities form a 90°-vector triangle as shown below:



We then have

$$\tan\theta = \frac{|\vec{\mathbf{v}}_{CE}|}{|\vec{\mathbf{v}}_{RE}|} = \frac{50.0 \text{ km/h}}{v_{rain}}$$

and $$v_{rain} = \frac{50.0 \text{ km/h}}{\tan 60.0°} = 28.9 \text{ km/h}$$

(a) $$|\vec{\mathbf{v}}_{RC}| = \sqrt{|\vec{\mathbf{v}}_{CE}|^2 + |\vec{\mathbf{v}}_{RE}|^2} = \sqrt{(50.0 \text{ km/h})^2 + (28.9 \text{ km/h})^2} = 57.7 \text{ km/h}$$

and $\boxed{\vec{\mathbf{v}}_{RC} = 57.7 \text{ km/h at } 60.0° \text{ west of vertical}}$

(b) $|\vec{\mathbf{v}}_{RE}| = v_{rain}$ in the negative y-direction

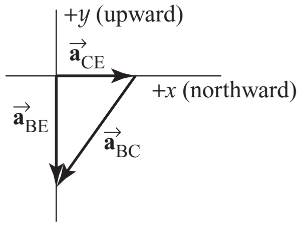
and $\boxed{\vec{\mathbf{v}}_{RE} = 28.9 \text{ km/h vertically downward}}$

3.37 Choose a reference system with the positive x-axis in the northward direction and the positive y-axis vertically upward. Then, the accelerations of the car and the bolt (in free-fall) relative to Earth are: $\vec{\mathbf{a}}_{CE} = 2.50 \text{ m/s}^2$ in the $+x$-direction, and $\vec{\mathbf{a}}_{BE} = 9.80 \text{ m/s}^2$ in the negative y-direction. Similar to Equation 3.16 in the text for relative velocities, these accelerations are related to the acceleration of the bolt relative to the car, $\vec{\mathbf{a}}_{BC}$, by

$$\vec{\mathbf{a}}_{BC} = \vec{\mathbf{a}}_{BE} - \vec{\mathbf{a}}_{CE} \qquad \text{or} \qquad \vec{\mathbf{a}}_{BE} = \vec{\mathbf{a}}_{CE} + \vec{\mathbf{a}}_{BC}$$

Thus, these relative accelerations form a 90°-vector triangle as shown below:

(a) We see that $\vec{\mathbf{a}}_{BC}$ has components of



$$(\vec{\mathbf{a}}_{BC})_x = -(\vec{\mathbf{a}}_{CE})_x = -2.50 \text{ m/s}^2$$

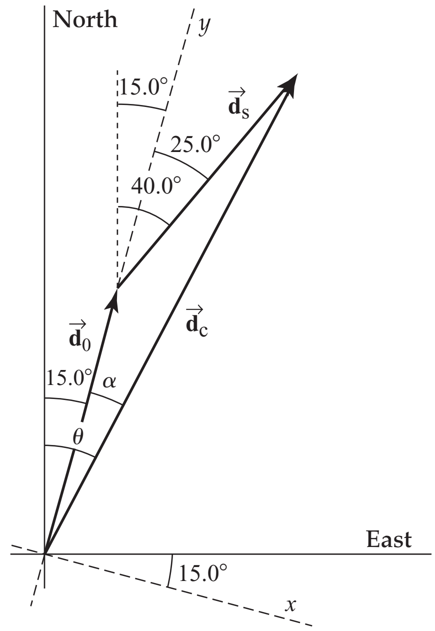
and $$(\vec{\mathbf{a}}_{BC})_y = -(\vec{\mathbf{a}}_{BE})_y = -9.80 \text{ m/s}^2$$

so $\boxed{\vec{\mathbf{a}}_{BC} = 2.50 \text{ m/s}^2 \text{ southward and } 9.80 \text{ m/s}^2 \text{ downward}}$

(b) The acceleration of the bolt relative to Earth is that of a freely falling body, namely $\boxed{9.80 \text{ m/s}^2 \text{ downward}}$.

(c) Observers fixed on Earth see the bolt follow a $\boxed{\text{parabolic path}}$ with $\boxed{\text{a vertical axis}}$, the same as any freely falling body having a horizontal initial velocity.

3.38 As shown in the figure at the right, we choose a coordinate system with the y-axis lying at 15.0° east of north (i.e., along the direction of the original displacement, $\vec{\mathbf{d}}_0$, of the ship from the cutter. During the time t required for the cutter to intercept the ship, the ship undergoes displacement $\vec{\mathbf{d}}_S$ (directed at 40.0° east of north and 25.0° east of our y-axis) and the cutter undergoes displacement $\vec{\mathbf{d}}_C$ (directed at angle θ east of north or angle α relative to our y-axis).



The magnitude of $\vec{\mathbf{d}}_0$ is given to be 20.0 km while the magnitudes of $\vec{\mathbf{d}}_S$ and $\vec{\mathbf{d}}_C$ will be $d_S = (26.0\ \text{km/h})t$ and $d_C = (50.0\ \text{km/h})t$, respectively.

(a) Equating the x-components in the vector triangle formed by the three displacements $\vec{\mathbf{d}}_0$, $\vec{\mathbf{d}}_S$, and $\vec{\mathbf{d}}_C$, we see that

$$d_C \sin\alpha = d_S \sin 25.0° \quad \text{or} \quad (50.0\ \text{km/h})t\sin\alpha = (26.0\ \text{km/h})t\sin 25.0°$$

yielding $$\sin\alpha = \frac{(26.0\ \text{km/h})\sin 25.0°}{50.0\ \text{km/h}} = 0.220 \quad \text{or} \quad \alpha = 12.7°$$

or to intercept the ship, the cutter must steer a course directed at

$$\theta = \alpha + 15.0° = 12.7° + 15.0° = \boxed{27.7°\ \text{east of north}}$$

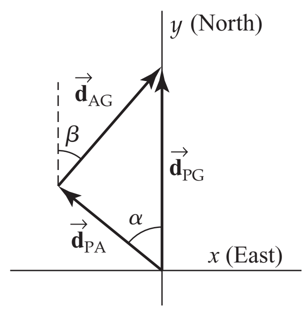
(b) To find the time required to intercept the ship, we equate the y-components in the vector triangle to find

$$\left[(50.0\ \text{km/h})\cos 12.7°\right]t = 20.0\ \text{km} + \left[(26.0\ \text{km/h})\cos 25.0°\right]t$$

or $$\left[(48.8 - 23.6)\ \text{km/h}\right]t = 20.0\ \text{km}$$

yielding $$t = \frac{20.0\ \text{km}}{(48.8 - 23.6)\ \text{km/h}} = 0.794\ \text{h} = \boxed{47.6\ \text{min}}$$

3.39 During the trip of duration t, the displacement of the plane relative to the ground, $\vec{\mathbf{d}}_{PG}$, is to have a magnitude of 750 km and be directed due north. We choose the positive y-axis to be directed northward and the positive x-axis directed eastward. During the trip, the plane's displacement relative to the air $\vec{\mathbf{d}}_{PA}$ has magnitude $\left|\vec{\mathbf{d}}_{PA}\right| = (630\ \text{km/h})t$ and is directed at some angle α relative to the y-axis. The displacement of the air relative to the ground, $\vec{\mathbf{d}}_{AG}$, has magnitude $\left|\vec{\mathbf{d}}_{AG}\right| = (35.0\ \text{km/h})t$ and is assumed to be at angle β from the y-axis.



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Since these relative displacements are related by $\vec{\mathbf{d}}_{PA} = \vec{\mathbf{d}}_{PG} - \vec{\mathbf{d}}_{AG}$, or $\vec{\mathbf{d}}_{PG} = \vec{\mathbf{d}}_{PA} + \vec{\mathbf{d}}_{AG}$, they form a vector triangle as shown above. Equating the x-components in the vector triangle gives $-\left|\vec{\mathbf{d}}_{PA}\right|\sin\alpha + \left|\vec{\mathbf{d}}_{AG}\right|\sin\beta = 0$, or

$$-(630 \text{ km/h})\cdot t\cdot\sin\alpha + (35.0 \text{ km/h})\cdot t\cdot\sin\beta = 0 \quad \text{and} \quad \sin\alpha = \left(\frac{35.0}{630}\right)\sin\beta \qquad \textbf{[1]}$$

Equating y-components in the vector triangle gives $\left|\vec{\mathbf{d}}_{PA}\right|\cos\alpha + \left|\vec{\mathbf{d}}_{AG}\right|\cos\beta = \left|\vec{\mathbf{d}}_{PG}\right|$, or

$$\left[(630 \text{ km/h})\cos\alpha + (35.0 \text{ km/h})\cos\beta\right]t = 750 \text{ km/h} \qquad \textbf{[2]}$$

(a) The wind blows toward the south $(\beta = 180°)$ and is a headwind for the plane $(\alpha = 0°)$. Then, Equation [2] gives

$$\left[630 \text{ km/h} - 35.0 \text{ km/h}\right]t = 750 \text{ km/h} \qquad \text{or} \qquad t = \frac{750 \text{ km/h}}{595 \text{ km/h}} = \boxed{1.26 \text{ h}}$$

(b) The wind blows northward as a tailwind $(\alpha = \beta = 0°)$, and Equation [2] yields

$$\left[630 \text{ km/h} + 35.0 \text{ km/h}\right]t = 750 \text{ km/h} \qquad \text{or} \qquad t = \frac{750 \text{ km/h}}{665 \text{ km/h}} = \boxed{1.13 \text{ h}}$$

(c) The wind blows due East, so $\beta = 90.0°$. Then Equation [1] requires that

$$\sin\alpha = \left(\frac{35.0}{630}\right)\sin 90.0° = 0.056 \qquad \text{and} \qquad \alpha = 3.18°$$

or the plane must fly 3.18° W of N relative to the air to maintain a due north heading relative to the ground.

Finally, Equation [2] for this case gives

$$\left[(630 \text{ km/h})\cos 3.18° + (35.0 \text{ km/h})\cos 90.0°\right]t = 750 \text{ km/h}$$

$$\text{or} \quad t = \frac{750 \text{ km/h}}{(629 \text{ km/h} + 0)} = \boxed{1.19 \text{ h}}$$

3.40 (a) If the salmon (a projectile) is to have $v_y = 0$ when $\Delta y = +1.50$ m, the required initial velocity in the vertical direction is given by $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ as

$$v_{0y} = +\sqrt{v_y^2 - 2a_y\Delta y} = \sqrt{0 - 2\left(-9.80 \text{ m/s}^2\right)(+1.50 \text{ m})} = \boxed{5.42 \text{ m/s}}$$

The elapsed time for the upward flight will be

$$\Delta t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - 5.42 \text{ m/s}}{-9.80 \text{ m/s}^2} = 0.553 \text{ s}$$

If the horizontal displacement at this time is to be $\Delta x = +1.00$ m, the required constant horizontal component of the salmon's velocity must be

$$v_{0x} = \frac{\Delta x}{\Delta t} = \frac{1.00 \text{ m}}{0.553 \text{ s}} = \boxed{1.81 \text{ m/s}}$$

(b) The speed with which the salmon must leave the water is then

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(1.81 \text{ m/s})^2 + (5.42 \text{ m/s})^2} = 5.71 \text{ m/s}$$

$\boxed{\text{Yes}}$, since $v_0 < 6.26$ m/s, the salmon is capable of making this jump.

3.41 (a) Both the student (S) and the water (W) move relative to Earth (E). The velocity of the student relative to the water is given by $\vec{\mathbf{v}}_{SW} = \vec{\mathbf{v}}_{SE} - \vec{\mathbf{v}}_{WE}$, where $\vec{\mathbf{v}}_{SE}$ and $\vec{\mathbf{v}}_{WE}$ are the velocities of the student relative to Earth and the water relative to Earth, respectively. If we choose downstream as the positive direction, then $\vec{\mathbf{v}}_{WE} = +0.500\ \text{m/s}$, $\vec{\mathbf{v}}_{SW} = -1.20\ \text{m/s}$ when the student is going up stream, and $\vec{\mathbf{v}}_{SW} = +1.20\ \text{m/s}$ when the student moves downstream.

The velocity of the student relative to Earth for each leg of the trip is

$$\left(\vec{\mathbf{v}}_{SE}\right)_{\text{upstream}} = \vec{\mathbf{v}}_{WE} + \left(\vec{\mathbf{v}}_{SW}\right)_{\text{upstream}} = 0.500\ \text{m/s} + (-1.20\ \text{m/s}) = -0.700\ \ \text{m/s}$$

and $$\left(\vec{\mathbf{v}}_{SE}\right)_{\text{downstream}} = \vec{\mathbf{v}}_{WE} + \left(\vec{\mathbf{v}}_{SW}\right)_{\text{downstream}} = 0.500\ \text{m/s} + (+1.20\ \text{m/s}) = +1.70\ \ \text{m/s}$$

The distance (measured relative to Earth) for each leg of the trip is $d = 1.00\ \text{km} = 1.00\times10^3\ \text{m}$. The times required for each of the two legs are

$$t_{\text{upstream}} = \frac{d}{\left|\vec{\mathbf{v}}_{SE}\right|_{\text{upstream}}} = \frac{1.00\times10^3\ \text{m}}{0.700\ \ \text{m/s}} = 1.43\times10^3\ \text{s}$$

and $$t_{\text{downstream}} = \frac{d}{\left|\vec{\mathbf{v}}_{SE}\right|_{\text{downstream}}} = \frac{1.00\times10^3\ \text{m}}{1.70\ \ \text{m/s}} = 5.88\times10^2\ \text{s}$$

so the time for the total trip is

$$t_{\text{total}} = t_{\text{upstream}} + t_{\text{downstream}} = 1.43\times10^3\ \text{s} + 5.88\times10^2\ \text{s} = \boxed{2.02\times10^3\ \text{s}}$$

(b) If the water had been still $\left(\left|\vec{\mathbf{v}}_{WE}\right| = 0\right)$, the speed of the student relative to Earth would have been the same for each leg of the trip, $\left|\vec{\mathbf{v}}_{SE}\right| = \left|\vec{\mathbf{v}}_{SE}\right|_{\text{upstream}} = \left|\vec{\mathbf{v}}_{SE}\right|_{\text{downstream}} = 1.20\ \text{m/s}$. In this case, the time for each leg and the total time would have been

$$t_{\text{leg}} = \frac{d}{\left|\vec{\mathbf{v}}_{SE}\right|} = \frac{1.00\times10^3\ \text{m}}{1.20\ \ \text{m/s}} = 8.33\times10^2\ \text{s}, \quad \text{and} \quad t_{\text{total}} = 2t_{\text{leg}} = \boxed{1.67\times10^3\ \text{s}}$$

(c) $\boxed{\text{The time savings going downstream with the current is always less than the extra time required to go the same distance against the current.}}$

3.42 (a) The speed of the student relative to shore is $v_{\text{up}} = v - v_s$ while swimming upstream and $v_{\text{down}} = v + v_s$ while swimming downstream. The time required to travel distance d upstream is then

$$t_{\text{up}} = \frac{d}{v_{\text{up}}} = \boxed{\frac{d}{v - v_s}}$$

(b) The time required to swim the same distance d downstream is

$$t_{\text{down}} = \frac{d}{v_{\text{down}}} = \boxed{\frac{d}{v + v_s}}$$

(c) The total time for the trip is therefore

$$t_a = t_{\text{up}} + t_{\text{down}} = \frac{d}{v - v_s} + \frac{d}{v + v_s} = \frac{d\left(v + v_s\right) + d\left(v - v_s\right)}{\left(v - v_s\right)\left(v + v_s\right)} = \frac{2dv}{v^2 - v_s^2} = \boxed{\frac{2d/v}{1 - v_s^2/v^2}}$$

(d) In still water, $v_s = 0$ and the time for the complete trip is seen to be

$$t_b = t_a\Big|_{v_s=0} = \frac{2d/v}{1 - 0/v^2} = \boxed{\frac{2d}{v}}$$

(e) Note that $t_b = \dfrac{2d}{v} = \dfrac{2dv}{v^2}$ and that $t_a = \dfrac{2dv}{v^2 - v_s^2}$. Thus, when there is a current $\left(v_s > 0\right)$, $\boxed{\text{it is always true that } t_a > t_b}$.

3.43 (a) The bomb starts its fall with $v_{0y} = 0$ and $v_{0x} = v_{\text{plane}} = 275 \text{ m/s}$. Choosing the origin at the location of the plane when the bomb is released and upward as positive, the y-coordinate of the bomb at ground level is $y = -h = -3.00 \times 10^3 \text{ m}$. The time required for the bomb to fall is given by $y = y_0 + v_{0y}t + \frac{1}{2}a_y t^2$ as $0 = y_0 + 0 + \frac{1}{2}(-g)t_{\text{fall}}^2$ or $t_{\text{fall}} = \sqrt{2y_0/g}$.

With $a_x = 0$, the horizontal distance the bomb travels during this time is

$$d = v_{0x}t_{\text{fall}} = v_{0x}\sqrt{\frac{2y_0}{g}} = (275 \text{ m/s})\sqrt{\frac{2(3.00 \times 10^3 \text{ m})}{9.80 \text{ m/s}^2}} = 6.80 \times 10^3 \text{ m} = \boxed{6.80 \text{ km}}$$

(b) While the bomb is falling, the plane travels in the same horizontal direction with the same constant horizontal speed, $v_x = v_{0x} = v_{\text{plane}}$, as the bomb. Thus, the plane remains directly above the bomb as the bomb falls to the ground. When impact occurs, the plane is $\boxed{\text{directly over the impact point, at an altitude of 3.00 km.}}$

(c) The angle, measured in the forward direction from the vertical, at which the bombsight must have been set is

$$\theta = \tan^{-1}\left(\frac{d}{h}\right) = \tan^{-1}\left(\frac{6.80 \text{ km}}{3.00 \text{ km}}\right) = \boxed{66.2^\circ}$$

3.44 (a) The time required for the woman, traveling at constant speed v_1 relative to the ground, to travel distance L relative to the ground is $t_{\text{woman}} = \boxed{L/v_1}$.

(b) With both the walkway (W) and the man (M) moving relative to Earth (E), we know that the velocity of the man relative to the moving walkway is $\vec{\mathbf{v}}_{\text{MW}} = \vec{\mathbf{v}}_{\text{ME}} - \vec{\mathbf{v}}_{\text{WE}}$. His velocity relative to Earth is then $\vec{\mathbf{v}}_{\text{ME}} = \vec{\mathbf{v}}_{\text{MW}} + \vec{\mathbf{v}}_{\text{WE}}$. Since all of these velocities are in the same direction, his speed relative to Earth is $|\vec{\mathbf{v}}_{\text{ME}}| = |\vec{\mathbf{v}}_{\text{MW}}| + |\vec{\mathbf{v}}_{\text{WE}}| = v_2 + v_1$. The time required for the man to travel distance L relative to the ground is then

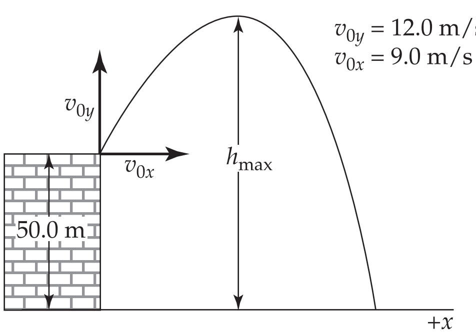
$$t_{\text{man}} = \frac{L}{|\vec{\mathbf{v}}_{\text{ME}}|} = \boxed{\frac{L}{v_1 + v_2}}$$

3.45 Choose the positive direction to be the direction of each car's motion relative to Earth. The velocity of the faster car relative to the slower car is given by $\vec{\mathbf{v}}_{\text{FS}} = \vec{\mathbf{v}}_{\text{FE}} - \vec{\mathbf{v}}_{\text{SE}}$, where $\vec{\mathbf{v}}_{\text{FE}} = +60.0 \text{ km/h}$ is the velocity of the faster car relative to Earth, and $\vec{\mathbf{v}}_{\text{SE}} = 40.0 \text{ km/h}$ is the velocity of the slower car relative to Earth.

Thus, $\vec{\mathbf{v}}_{\text{FS}} = +60.0 \text{ km/h} - 40.0 \text{ km/h} = +20.0 \text{ km/h}$ and the time required for the faster car to move 100 m (0.100 km) closer to the slower car is

$$t = \frac{d}{v_{\text{FS}}} = \frac{0.100 \text{ km}}{20.0 \text{ km/h}} = 5.00 \times 10^{-3} \text{ h}\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = \boxed{18.0 \text{ s}}$$

3.46 The vertical displacement from the launch point (top of the building) to the top of the arc may be found from $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ with $v_y = 0$ at the top of the arc. This yields



$$\Delta y = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (12.0 \text{ m/s})^2}{2(-9.80 \text{ m/s}^2)} = +7.35 \text{ m}$$

and $\Delta y = y_{\text{max}} - y_0$ gives

$$y_{\text{max}} = y_0 + \Delta y = y_0 + 7.35 \text{ m}$$

continued on next page

(a) If the origin is chosen at the top of the building, then $y_0 = 0$ and $y_{max} = 7.35$ m.

Thus, the maximum height above the ground is

$$h_{max} = 50.0 \text{ m} + y_{max} = 50.0 \text{ m} + 7.35 \text{ m} = \boxed{57.4 \text{ m}}$$

The elapsed time from the point of release to the top of the arc is found from $v_y = v_{0y} + a_y t$ as

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{0 - 12.0 \text{ m/s}}{-9.80 \text{ m/s}^2} = \boxed{1.22 \text{ s}}$$

(b) If the origin is chosen at the base of the building (ground level), then $y_0 = +50.0$ m and $h_{max} = y_{max}$, giving

$$h_{max} = y_0 + \Delta y = 50.0 \text{ m} + 7.35 \text{ m} = \boxed{57.4 \text{ m}}$$

The calculation for the time required to reach maximum height is exactly the same as that given above. Thus, $t = \boxed{1.22 \text{ s}}$

3.47 (a) The known parameters for this jump are: $\theta_0 = -10.0°$, $\Delta x = 108$ m, $\Delta y = -55.0$ m, $a_x = 0$, and $a_y = -g = -9.80$ m/s.

Since $a_x = 0$, the horizontal displacement is $\Delta x = v_{0x} t = (v_0 \cos\theta_0)t$, where t is the total time of the flight. Thus, $t = \Delta x / (v_0 \cos\theta_0)$.

The vertical displacement during the flight is given by

$$\Delta y = v_{0y} t + \frac{1}{2} a_y t^2 = (v_0 \sin\theta_0)t - \frac{gt^2}{2}$$

or $$\Delta y = (\cancel{v_0} \sin\theta_0)\left(\frac{\Delta x}{\cancel{v_0} \cos\theta_0}\right) - \frac{g}{2}\left(\frac{\Delta x}{v_0 \cos\theta_0}\right)^2 = (\Delta x)\tan\theta_0 - \left[\frac{g(\Delta x)^2}{2\cos^2\theta_0}\right]\frac{1}{v_0^2}$$

Thus, $$[\Delta y - (\Delta x)\tan\theta_0] = -\left[\frac{g(\Delta x)^2}{2\cos^2\theta_0}\right]\frac{1}{v_0^2}$$

or $$v_0 = \sqrt{\frac{-g(\Delta x)^2}{2[\Delta y - (\Delta x)\tan\theta_0]\cos^2\theta_0}} = \sqrt{\frac{-(9.80 \text{ m/s}^2)(108 \text{ m})^2}{2[-55.0 \text{ m} - (108 \text{ m})\tan(-10.0°)]\cos^2(-10.0°)}}$$

yielding $$v_0 = \sqrt{\frac{-1.143\times 10^5 \text{ m}^3/\text{s}^2}{-69.75 \text{ m}}} = \boxed{40.5 \text{ m/s}}$$

(b) Rather than falling like a rock, $\boxed{\text{the skier glides through the air}}$ much like a bird, prolonging the jump.

3.48 The cup leaves the counter with initial velocity components of $(v_{0x} = v_i,\ v_{0y} = 0)$, and has acceleration components of $(a_x = 0,\ a_y = -g)$ while in flight.

(a) Applying $\Delta y = v_{0y} t + \frac{1}{2} a_y t^2$ from when the cup leaves the counter until it reaches the floor gives

$$-h = 0 + \frac{(-g)}{2} t^2$$

so the time of the fall is $\boxed{t = \sqrt{2h/g}}$.

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(b) If the cup travels a horizontal distance d while falling to the floor, $\Delta x = v_{0x}t$ gives

$$v_i = v_{0x} = \frac{\Delta x}{t} = \frac{d}{\sqrt{2h/g}} \qquad \text{or} \qquad \boxed{v_i = d\sqrt{\frac{g}{2h}}}$$

(c) The components of the cup's velocity just before hitting the floor are

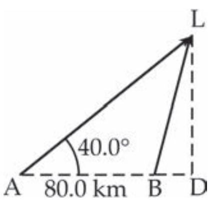
$$v_x = v_{0x} = v_i = d\sqrt{\frac{g}{2h}} \qquad \text{and} \qquad v_y = v_{0y} + a_y t = 0 - g\sqrt{\frac{2h}{g}} = -\sqrt{2gh}$$

Thus, the total speed at this point is $\quad v = \sqrt{v_x^2 + v_y^2} = \boxed{\sqrt{\frac{d^2 g}{2h} + 2gh}}$

(d) The direction of the cup's motion just before it hits the floor is

$$\theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}\left(\frac{-\sqrt{2gh}}{d\sqrt{g/2h}}\right) = \tan^{-1}\left(\frac{-1}{d}\sqrt{2\cancel{g}h\left(\frac{2h}{\cancel{g}}\right)}\right) = \boxed{\tan^{-1}\left(\frac{-2h}{d}\right)}$$

3.49 $\overline{AL} = v_1 t = (90.0 \text{ km/h})(2.50 \text{ h}) = 225 \text{ km}$



$\overline{BD} = \overline{AD} - \overline{AB} = \overline{AL}\cos 40.0° - 80.0 \text{ km} = 92.4 \text{ km}$

From the triangle BLD,

$$\overline{BL} = \sqrt{\left(\overline{BD}\right)^2 + \left(\overline{DL}\right)^2}$$

or $\quad \overline{BL} = \sqrt{(92.4 \text{ km})^2 + \left(\overline{AL}\sin 40.0°\right)^2} = 172 \text{ km}$

Since Car 2 travels this distance in 2.50 h, its constant speed is

$$v_2 = \frac{172 \text{ km}}{2.50 \text{ h}} = \boxed{68.8 \text{ km/h}}$$

3.50 (a) After leaving the ledge, the water has a constant horizontal component of velocity.

$$v_x = v_{0x} = 1.50 \text{ m/s}^2$$

Thus, when the speed of the water is $v = 3.00$ m/s, the vertical component of its velocity will be

$$v_y = -\sqrt{v^2 - v_x^2} = -\sqrt{(3.00 \text{ m/s})^2 - (1.50 \text{ m/s})^2} = -2.60 \text{ m/s}$$

The vertical displacement of the water at this point is

$$\Delta y = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{(-2.60 \text{ m/s})^2 - 0}{2(-9.80 \text{ m/s}^2)} = -0.345 \text{ m}$$

or the water is $\boxed{0.345 \text{ m}}$ below the ledge.

(b) If its speed leaving the water is 6.26 m/s, the maximum vertical leap of the salmon is

$$\Delta y_{\text{leap}} = \frac{0 - v_{0y}^2}{2a_y} = \frac{0 - (6.26 \text{ m/s})^2}{2(-9.80 \text{ m/s})} = 2.00 \text{ m}$$

Therefore, the maximum height waterfall the salmon can clear is

$$h_{\max} = \Delta y_{\text{leap}} + 0.345 \text{ m} = \boxed{2.35 \text{ m}}$$

3.51 The distance, s, moved in the first 3.00 seconds is given by

$$s = v_0 t + \frac{1}{2}at^2 = (100\ \text{m/s})(3.00\ \text{s}) + \frac{1}{2}(30.0\ \text{m/s}^2)(3.00\ \text{s})^2 = 435\ \text{m}$$

Choosing the origin at the point where the rocket was launched, the coordinates of the rocket at the end of powered flight are

$$x_1 = s(\cos 53.0°) = 262\ \text{m} \qquad \text{and} \qquad y_1 = s(\sin 53.0°) = 347\ \text{m}$$

The speed of the rocket at the end of powered flight is

$$v_1 = v_0 + at = 100\ \text{m/s} + (30.0\ \text{m/s}^2)(3.00\ \text{s}) = 190\ \text{m/s}$$

so the initial velocity components for the free-fall phase of the flight are

$$v_{0x} = v_1 \cos 53.0° = 114\ \text{m/s} \qquad \text{and} \qquad v_{0y} = v_1 \sin 53.0° = 152\ \text{m/s}$$

(a) When the rocket is at maximum altitude, $v_y = 0$. The rise time during the free-fall phase can be found from $v_y = v_{0y} + a_y t$ as

$$t_{\text{rise}} = \frac{0 - v_{0y}}{a_y} = \frac{0 - 152\ \text{m}}{-9.80\ \text{m/s}^2} = 15.5\ \text{s}$$

The vertical displacement occurring during this time is

$$\Delta y = \left(\frac{v_y + v_{0y}}{2}\right) t_{\text{rise}} = \left(\frac{0 + 152\ \text{m/s}}{2}\right)(15.5\ \text{s}) = 1.18 \times 10^3\ \text{m}$$

The maximum altitude reached is then

$$H = y_1 + \Delta y = 347\ \text{m} + 1.18 \times 10^3\ \text{m} = \boxed{1.53 \times 10^3\ \text{m}}$$

(b) After reaching the top of the arc, the rocket falls 1.53×10^3 m to the ground, starting with zero vertical velocity $(v_{0y} = 0)$. The time for this fall is found from $\Delta y = v_{0y}\,t + \frac{1}{2}a_y t^2$ as

$$t_{\text{fall}} = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-1.53 \times 10^3\ \text{m})}{-9.80\ \text{m/s}^2}} = 17.7\ \text{s}$$

The total time of flight is

$$t = t_{\text{powered}} + t_{\text{rise}} + t_{\text{fall}} = (3.00 + 15.5 + 17.7)\ \text{s} = \boxed{36.2\ \text{s}}$$

(c) The free-fall phase of the flight lasts for

$$t_2 = t_{\text{rise}} + t_{\text{fall}} = (15.5 + 17.7)\ \text{s} = 33.2\ \text{s}$$

The horizontal displacement occurring during this time is

$$\Delta x = v_{0x}\,t_2 = (114\ \text{m/s})(33.2\ \text{s}) = 3.78 \times 10^3\ \text{m}$$

and the full horizontal range is

$$R = x_1 + \Delta x = 262\ \text{m} + 3.78 \times 10^3\ \text{m} = \boxed{4.04 \times 10^3\ \text{m}}$$

3.52 Taking downstream as the positive direction, the velocity of the water relative to shore is $\vec{\mathbf{v}}_{\text{WS}} = +v_{\text{WS}}$, where v_{WS} is the speed of the flowing water. Also, if v_{CW} is the common *speed* of the two canoes relative to the water, their velocities relative to the water are

$$(\vec{\mathbf{v}}_{\text{CW}})_{\text{downstream}} = +v_{\text{CW}} \qquad \text{and} \qquad (\vec{\mathbf{v}}_{\text{CW}})_{\text{upstream}} = -v_{\text{CW}}$$

continued on next page

The velocity of a canoe relative to the water can also be expressed as $\vec{\mathbf{v}}_{\mathrm{CW}} = \vec{\mathbf{v}}_{\mathrm{CS}} - \vec{\mathbf{v}}_{\mathrm{WS}}$. Applying this to the canoe moving downstream gives

$$+v_{\mathrm{CW}} = +2.9 \text{ m/s} - v_{\mathrm{WS}} \qquad \textbf{[1]}$$

and for the canoe going upstream

$$-v_{\mathrm{CW}} = -1.2 \text{ m/s} - v_{\mathrm{WS}} \qquad \textbf{[2]}$$

(a) Adding Equations [1] and [2] gives

$$2v_{\mathrm{WS}} = 1.7 \text{ m/s}, \qquad \text{so} \qquad v_{\mathrm{WS}} = \boxed{0.85 \text{ m/s}}$$

(b) Subtracting Equation [2] from [1] yields

$$2v_{\mathrm{CW}} = 4.1 \text{ m/s}, \qquad \text{or} \qquad v_{\mathrm{CW}} = \boxed{2.1 \text{ m/s}}$$

3.53 (a) The time of flight is found from $\Delta y = v_{0y}\,t + \frac{1}{2}a_y t^2$ with $\Delta y = 0$, as $t = 2v_{0y}/g$.

This gives the range as $\quad R = v_{0x}t = \dfrac{2v_{0x}v_{0y}}{g}$

On Earth this becomes $\quad R_{\mathrm{Earth}} = 2v_{0x}v_{0y}/g_{\mathrm{Earth}}$

and on the Moon, $\quad R_{\mathrm{Moon}} = 2v_{0x}v_{0y}/g_{\mathrm{Moon}}$

Dividing R_{Moon} by R_{Earth}, we find $R_{\mathrm{Moon}} = \left(g_{\mathrm{Earth}}/g_{\mathrm{Moon}}\right)R_{\mathrm{Earth}}$. With $g_{\mathrm{Moon}} = g_{\mathrm{Earth}}/6$, this gives $R_{\mathrm{Moon}} = 6R_{\mathrm{Earth}} = 6(3.0 \text{ m}) = \boxed{18 \text{ m}}$

(b) Similarly, $R_{\mathrm{Mars}} = \left(g_{\mathrm{Earth}}/g_{\mathrm{Mars}}\right)R_{\mathrm{Earth}} = 3.0 \text{ m}/0.38 = \boxed{7.9 \text{ m}}$

3.54 (a) Since the can returns to the same spot on the truck bed that it was thrown from, the can must have zero horizontal velocity relative to the truck. This means that, in the reference frame of the truck, the can was thrown vertically upward or $\boxed{\text{at } 0° \text{ to the vertical}}$.

(b) While the can was in the air, the truck moved 16.0 m, at a constant speed of 9.50 m/s, relative to the ground. The time of flight of the can was therefore $t_{\mathrm{flight}} = (16.0 \text{ m})/(9.50 \text{ m/s})$. The time for the can to go from the truck bed to the top of its arc was then

$$t_{\mathrm{up}} = \frac{1}{2}t_{\mathrm{flight}} = \frac{1}{2}\left(\frac{16.0 \text{ m}}{9.50 \text{ m/s}}\right) = 0.842 \text{ s}$$

Thus, $v_y = v_{0y} + a_y t$ (with $v_y = 0$ at $t = t_{\mathrm{up}}$) gives the initial vertical velocity, and hence the initial speed of the can relative to the truck, as

$$v_{0y} = v_y - a_y t = 0 - \left(-9.80 \text{ m/s}^2\right)(0.842 \text{ s}) = \boxed{8.25 \text{ m/s}}$$

(c) Since the boy is stationary in the reference frame of the truck, he sees the can $\boxed{\text{go straight upward and fall straight back downward.}}$

(d) In a frame of reference fixed on the ground, the can has a constant horizontal velocity $v_x = v_{\mathrm{truck}} = 9.50 \text{ m/s}$ as it rises upward and falls back to the truck bed. Thus, $\boxed{\text{the can follows a parabolic path opening downward}}$ in this reference frame.

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(e) The initial velocity of the can as seen by an observer on the ground is

$$v_0 = \sqrt{v_{0x}^2 + v_{0y}^2} = \sqrt{(9.50)^2 + (8.25)^2}\ \text{m/s} = 12.6\ \text{m/s}$$

directed at $\theta = \tan^{-1}\left(v_{0y}/v_{0x}\right) = \tan^{-1}(8.25/9.50) = 41.0°$ above the horizontal

or $\vec{\mathbf{v}}_0 = \boxed{12.6\ \text{m/s at } 41.0° \text{ above the horizontal eastward line}}$

3.55 (a) The time for the ball to reach the fence is

$$t = \frac{\Delta x}{v_{0x}} = \frac{130\ \text{m}}{v_0 \cos 35°} = \frac{159\ \text{m}}{v_0}$$

At this time, the ball must be $\Delta y = 21\ \text{m} - 1.0\ \text{m} = 20\ \text{m}$ above its launch position, so $\Delta y = v_{0y}\,t + \frac{1}{2}a_y t^2$ gives

$$20\ \text{m} = \left(\cancel{v_0}\sin 35°\right)\left(\frac{159\ \text{m}}{\cancel{v_0}}\right) - \left(4.90\ \text{m/s}^2\right)\left(\frac{159\ \text{m}}{v_0}\right)^2$$

or $$(159\ \text{m})\sin 35° - 20\ \text{m} = \frac{\left(4.90\ \text{m/s}^2\right)(159\ \text{m})^2}{v_0^2}$$

From which, $$v_0 = \sqrt{\frac{\left(4.90\ \text{m/s}^2\right)(159\ \text{m})^2}{(159\ \text{m})\sin 35° - 20\ \text{m}}} = \boxed{42\ \text{m/s}}$$

(b) From above, $t = (159\ \text{m})/v_0 = (159\ \text{m})/(42\ \text{m/s}) = \boxed{3.8\ \text{s}}$

(c) When the ball reaches the wall (at $t = 3.8$ s),

$$v_x = v_{0x} = (42\ \text{m/s})\cos 35° = \boxed{34\ \text{m/s}}$$

$$v_y = v_{0y} + a_y t = (42\ \text{m/s})\sin 35° - \left(9.80\ \text{m/s}^2\right)(3.8\ \text{s}) = \boxed{-13\ \text{m/s}}$$

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(34\ \text{m/s})^2 + (-13\ \text{m/s})^2} = \boxed{36\ \text{m/s}}$$

3.56 (a) When a projectile returns to the level it was launched from, the time to reach the top of the arc is one half of the total time of flight. Thus, the elapsed time when the first ball reaches maximum height is $t = (3.00\ \text{s})/2 = 1.50$ s. Also, at this time, $v_y = 0$, and $v_y = v_{0y} + a_y t$ gives

$$0 = v_{0y} - \left(9.80\ \text{m/s}^2\right)(1.50\ \text{s}) \qquad \text{or} \qquad v_{0y} = \boxed{14.7\ \text{m/s}}$$

(b) In order for the second ball to reach the same vertical height as the first, the second must have the same initial vertical velocity as the first. Thus, we find v_0 as

$$v_0 = \frac{v_{0y}}{\sin 30.0°} = \frac{14.7\ \text{m/s}}{0.500} = \boxed{29.4\ \text{m/s}}$$

3.57 The time of flight of the ball is given by $\Delta y = v_{0y}\,t + \frac{1}{2}a_y t^2$, with $\Delta y = 0$, as

$$0 = \left[(20\ \text{m/s})\sin 30°\right]t + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)t^2$$

yielding a non-zero solution of $t = 2.0$ s

The horizontal distance the football moves in this time is

$$\Delta x = v_{0x}t = \left[(20\ \text{m/s})\cos 30°\right](2.0\ \text{s}) = 35\ \text{m}$$

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(a) Since $\Delta x > 20$ m, the receiver must run away from the quarterback, $\boxed{\text{in the direction the ball was thrown}}$, if he is to catch the ball.

(b) The receiver has a time of 2.0 s to run a distance of $d = \Delta x - 20 \text{ m} = 15 \text{ m}$, so the required speed is

$$v = \frac{d}{t} = \frac{15 \text{ m}}{2.0 \text{ s}} = \boxed{7.5 \text{ m/s}}$$

3.58 The horizontal component of the initial velocity is $v_{0x} = v_0 \cos 40° = 0.766\, v_0$ and the time required for the ball to move 10.0 m horizontally is

$$t = \frac{\Delta x}{v_{0x}} = \frac{10.0 \text{ m}}{0.766\, v_0} = \frac{13.1 \text{ m}}{v_0}$$

At this time, the vertical displacement of the ball must be

$$\Delta y = y - y_0 = 3.05 \text{ m} - 2.00 \text{ m} = 1.05 \text{ m}$$

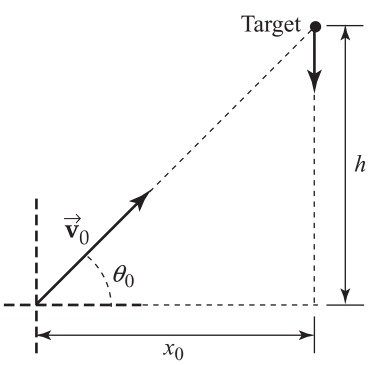
Thus, $\Delta y = v_{0y}\, t + \frac{1}{2} a_y t^2$ becomes

$$1.05 \text{ m} = \left(\cancel{v_0} \sin 40.0°\right) \frac{13.1 \text{ m}}{\cancel{v_0}} + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right) \frac{(13.1 \text{ m})^2}{v_0^2}$$

or $$1.05 \text{ m} = 8.42 \text{ m} - \frac{841 \text{ m}^3/\text{s}^2}{v_0^2}$$

which yields $v_0 = \sqrt{\dfrac{841 \text{ m}^3/\text{s}^2}{8.42 \text{ m} - 1.05 \text{ m}}} = \boxed{10.7 \text{ m/s}}$

3.59 Choose an origin where the projectile leaves the gun and let the y-coordinates of the projectile and the target at time t be labeled y_p and y_T, respectively.



Then, $(\Delta y)_p = y_p - 0 = (v_0 \sin \theta_0)\, t - (g/2)\, t^2$, and

$$(\Delta y)_T = y_T - h = 0 - (g/2)\, t^2 \text{ or } y_T = h - (g/2)\, t^2$$

The time when the projectile will have the same x-coordinate as the target is

$$t = \frac{\Delta x}{v_{0x}} = \frac{x_0}{v_0 \cos \theta_0}$$

For a collision to occur, it is necessary that $y_p = y_T$ at this time, or

$$\left(\cancel{v_0} \sin \theta_0\right)\left(\frac{x_0}{\cancel{v_0} \cos \theta_0}\right) - \cancel{\frac{g}{2} t^2} = h - \cancel{\frac{g}{2} t^2} \qquad \text{which reduces to} \qquad \tan \theta_0 = \frac{h}{x_0}$$

This requirement is satisfied provided that the gun is aimed at the initial location of the target. Thus, $\boxed{\text{a collision is guaranteed if the shooter aims the gun in this manner.}}$

3.60 (a) The components of the vectors are

Vector	x-component (cm)	y-component (cm)
$\vec{\mathbf{d}}_{1m}$	0	104
$\vec{\mathbf{d}}_{2m}$	46.0	19.5
$\vec{\mathbf{d}}_{1f}$	0	84.0
$\vec{\mathbf{d}}_{2f}$	38.0	20.2

The sums $\vec{\mathbf{d}}_m = \vec{\mathbf{d}}_{1m} + \vec{\mathbf{d}}_{2m}$ and $\vec{\mathbf{d}}_f = \vec{\mathbf{d}}_{1f} + \vec{\mathbf{d}}_{2f}$ are computed as

$$d_m = \sqrt{(0+46.0)^2 + (104+19.5)^2} = 132 \text{ cm} \text{ and } \theta = \tan^{-1}\left(\frac{104+19.5}{0+46.0}\right) = 69.6°$$

$$d_f = \sqrt{(0+38.0)^2 + (84.0+20.2)^2} = 111 \text{ cm and } \theta = \tan^{-1}\left(\frac{84.0+20.2}{0+38.0}\right) = 70.0°$$

or $\boxed{\vec{\mathbf{d}}_m = 132 \text{ cm at } 69.6° \text{ and } \vec{\mathbf{d}}_f = 111 \text{ cm at } 70.0°}$

(b) To normalize, multiply each component in the above calculation by the appropriate scale factor. The scale factor required for the components of $\vec{\mathbf{d}}_{1m}$ and $\vec{\mathbf{d}}_{2m}$ is

$$s_m = \frac{200 \text{ cm}}{180 \text{ cm}} = 1.11$$

and the scale factor needed for components of $\vec{\mathbf{d}}_{1f}$ and $\vec{\mathbf{d}}_{2f}$ is

$$s_f = \frac{200 \text{ cm}}{168 \text{ cm}} = 1.19$$

After using these scale factors and recalculating the vector sums, the results are

$$\boxed{\vec{\mathbf{d}}'_m = 146 \text{ cm at } 69.6° \text{ and } \vec{\mathbf{d}}'_f = 132 \text{ cm at } 70.0°}$$

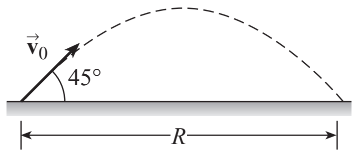
The difference in the normalized vector sums is $\Delta\vec{\mathbf{d}}' = \vec{\mathbf{d}}'_m - \vec{\mathbf{d}}'_f$.

Vector	x-component (cm)	y-component (cm)
$\vec{\mathbf{d}}'_m$	50.9	137
$-\vec{\mathbf{d}}'_f$	–45.1	–124
$\Delta\vec{\mathbf{d}}'$	$\Sigma x = 5.8$	$\Sigma y = 13$

Therefore, $\Delta d' = \sqrt{(\Sigma x)^2 + (\Sigma y)^2} = \sqrt{(5.8)^2 + (13)^2} \text{ cm} = 14 \text{ cm}$, and

$$\theta = \tan^{-1}\left(\frac{\Sigma y}{\Sigma x}\right) = \tan^{-1}\left(\frac{13}{5.8}\right) = 66°, \text{ or } \boxed{\Delta\vec{\mathbf{d}}' = 14 \text{ cm at } 66°}$$

3.61 To achieve maximum range, the projectile should be launched at 45° above the horizontal. In this case, the initial velocity components are



$$v_{0x} = v_{0y} = \frac{v_0}{\sqrt{2}}$$

The time of flight may be found from $v_y = v_{0y} - gt$ by recognizing that when the projectile returns to the original level, $v_y = -v_{0y}$.

Thus, the time of flight is

$$t = \frac{-v_{0y} - v_{0y}}{-g} = \frac{2v_{0y}}{g} = \frac{2}{g}\left(\frac{v_0}{\sqrt{2}}\right) = \frac{v_0\sqrt{2}}{g}$$

The maximum horizontal range is then $R = v_{0x}t = \left(\frac{v_0}{\sqrt{2}}\right)\left(\frac{v_0\sqrt{2}}{g}\right) = \frac{v_0^2}{g}$ **[1]**

Now, consider throwing the projectile straight upward at speed v_0. At maximum height, $v_y = 0$, and the time required to reach this height is found from $v_y = v_{0y} - gt$ as $v_{0y} = 0$, which yields $t = v_0/g$.

Therefore, the maximum height the projectile will reach is

$$(\Delta y)_{\text{max}} = (v_y)_{\text{av}}\, t = \left(\frac{0+v_0}{2}\right)\left(\frac{v_0}{g}\right) = \frac{v_0^2}{2g}$$

Comparing this result with the maximum range found in Equation [1] above reveals that $(\Delta y)_{\text{max}} = \boxed{R/2}$ provided the projectile is given the same initial speed in the two tosses.

If the girl takes a step when she makes the horizontal throw, she can likely give a higher initial speed for that throw than for the vertical throw.

3.62 (a) $x = v_{0x}t$, so the time may be written as $t = x/v_{0x}$.

Thus, $y = v_{0y}t - \frac{1}{2}gt^2$ becomes

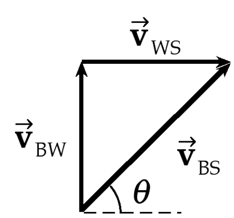
$$y = v_{0y}\left(\frac{x}{v_{0x}}\right) - \frac{1}{2}g\left(\frac{x}{v_{0x}}\right)^2$$

or $\boxed{y = \left(-\frac{g}{2v_{0x}^2}\right)x^2 + \left(\frac{v_{0y}}{v_{0x}}\right)x + 0}$

(b) Note that this result is of the general form $y = ax^2 + bx + c$ with

$$\boxed{a = \left(-\frac{g}{2v_{0x}^2}\right), \quad b = \left(\frac{v_{0y}}{v_{0x}}\right), \quad \text{and} \quad c = 0}$$

3.63 In order to cross the river in minimum time, the velocity of the boat relative to the water ($\vec{\mathbf{v}}_{\text{BW}}$) must be perpendicular to the banks (and hence perpendicular to the velocity $\vec{\mathbf{v}}_{\text{WS}}$ of the water relative to shore).



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The velocity of the boat relative to the water is $\vec{\mathbf{v}}_{BW} = \vec{\mathbf{v}}_{BS} - \vec{\mathbf{v}}_{WS}$, where $\vec{\mathbf{v}}_{BS}$ is the velocity of the boat relative to shore. Note that this vector equation can be rewritten as $\vec{\mathbf{v}}_{BS} = \vec{\mathbf{v}}_{BW} + \vec{\mathbf{v}}_{WS}$. Since $\vec{\mathbf{v}}_{BW}$ and $\vec{\mathbf{v}}_{WS}$ are to be perpendicular in this case, the vector diagram for this equation is a right triangle with $\vec{\mathbf{v}}_{BS}$ as the hypotenuse.

Hence, velocity of the boat relative to shore must have magnitude

$$v_{BS} = \sqrt{v_{BW}^2 + v_{WS}^2} = \sqrt{(12\ \text{km/h})^2 + (5.0\ \text{km/h})^2} = 13\ \text{km/h}$$

and be directed at

$$\theta = \tan^{-1}\left(\frac{v_{BW}}{v_{WS}}\right) = \tan^{-1}\left(\frac{12\ \text{km/h}}{5.0\ \text{km/h}}\right) = 67°$$

to the direction of the current in the river (which is the same as the line of the riverbank).

The minimum time to cross the river is

$$t = \frac{\text{width of river}}{v_{BW}} = \frac{1.5\ \text{km}}{12\ \text{km/h}}\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = \boxed{7.5\ \text{min}}$$

During this time, the boat drifts downstream a distance of

$$d = v_{WS}\,t = (5.0\ \text{km/h})(7.5\ \text{min})\left(\frac{1\ \text{h}}{60\ \text{min}}\right)\left(\frac{10^3\ \text{m}}{1\ \text{km}}\right) = 6.3\times10^2\ \text{m}$$

3.64 For the ball thrown at 45.0°, the time of flight is found from

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 \qquad \text{as} \qquad 0 = \left(\frac{v_0}{\sqrt{2}}\right)t_1 - \frac{g}{2}t_1^2$$

which has the single non-zero solution of

$$t_1 = \frac{v_0\sqrt{2}}{g}$$

The horizontal range of this ball is

$$R_1 = v_{0x}\,t_1 = \left(\frac{v_0}{\sqrt{2}}\right)\left(\frac{v_0\sqrt{2}}{g}\right) = \frac{v_0^2}{g}$$

Now consider the first arc in the motion of the second ball, started at angle θ with initial speed v_0. Applied to this arc, $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ becomes

$$0 = (v_0 \sin\theta)\,t_{21} - \frac{g}{2}t_{21}^2$$

with non-zero solution

$$t_{21} = \frac{2\,v_0 \sin\theta}{g}$$

Similarly, the time of flight for the second arc (started at angle θ with initial speed $v_0/2$) of this ball's motion is found to be

$$t_{22} = \frac{2(v_0/2)\sin\theta}{g} = \frac{v_0 \sin\theta}{g}$$

The horizontal displacement of the second ball during the first arc of its motion is

$$R_{21} = v_{0x}\,t_{21} = (v_0\cos\theta)\left(\frac{2\,v_0\sin\theta}{g}\right) = \frac{v_0^2\,(2\sin\theta\cos\theta)}{g} = \frac{v_0^2\sin(2\theta)}{g}$$

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Similarly, the horizontal displacement during the second arc of this motion is

$$R_{22} = \frac{(v_0/2)^2 \sin(2\theta)}{g} = \frac{1}{4}\frac{v_0^2 \sin(2\theta)}{g}$$

The total horizontal distance traveled in the two arcs is then

$$R_2 = R_{21} + R_{22} = \frac{5}{4}\frac{v_0^2 \sin(2\theta)}{g}$$

(a) Requiring that the two balls cover the same horizontal distance (that is, requiring that $R_2 = R_1$) gives

$$\frac{5}{4}\frac{v_0^2 \sin(2\theta)}{g} = \frac{v_0^2}{g}$$

This reduces to $\sin(2\theta) = 4/5$, which yields $2\theta = 53.1°$, so $\theta = \boxed{26.6°}$ is the required projection angle for the second ball.

(b) The total time of flight for the second ball is

$$t_2 = t_{21} + t_{22} = \frac{2 v_0 \sin\theta}{g} + \frac{v_0 \sin\theta}{g} = \frac{3 v_0 \sin\theta}{g}$$

Therefore, the ratio of the times of flight for the two balls is

$$\frac{t_2}{t_1} = \frac{(3 v_0 \sin\theta)/g}{\left(v_0 \sqrt{2}\right)/g} = \frac{3}{\sqrt{2}}\sin\theta$$

With $\theta = 26.6°$ as found in (a), this becomes

$$\frac{t_2}{t_1} = \frac{3}{\sqrt{2}}\sin(26.6°) = \boxed{0.950}$$

3.65 The initial velocity components for the daredevil are $v_{0x} = v_0 \cos 45°$ and $v_{0y} = v_0 \sin 45°$, or

$$v_{0x} = v_{0y} = \frac{v_0}{\sqrt{2}} = \frac{25.0 \text{ m/s}}{\sqrt{2}}$$

The time required to travel 50.0 m horizontally is

$$t = \frac{\Delta x}{v_{0x}} = \frac{(50.0 \text{ m})\sqrt{2}}{25.0 \text{ m/s}} = 2\sqrt{2} \text{ s}$$

The vertical displacement of the daredevil at this time, and the proper height above the level of the cannon to place the net, is

$$\Delta y = v_{0y}\, t + \frac{1}{2}a_y t^2 = \left(\frac{25.0 \text{ m/s}}{\sqrt{2}}\right)\left(2\sqrt{2} \text{ s}\right) - \frac{1}{2}\left(9.80 \text{ m/s}^2\right)\left(2\sqrt{2} \text{ s}\right)^2 = \boxed{10.8 \text{ m}}$$

3.66 The vertical component of the salmon's velocity as it leaves the water is

$$v_{0y} = +v_0 \sin\theta = +(6.26 \text{ m/s})\sin 45.0° = +4.43 \text{ m/s}$$

When the salmon returns to water level at the end of the leap, the vertical component of velocity will be $v_y = -v_{0y} = -4.43$ m/s.

The time the salmon is out of the water is given by

$$t_1 = \frac{v_y - v_{0y}}{a_y} = \frac{-4.43 \text{ m/s} - 4.43 \text{ m/s}}{-9.80 \text{ m/s}^2} = 0.904 \text{ s}$$

continued on next page

The horizontal distance traveled during the leap is

$$L = v_{0x}t_1 = (v_0 \cos\theta)t_1 = (6.26\ \text{m/s})\cos 45.0°(0.904\ \text{s}) = 4.00\ \text{m}$$

To travel this same distance underwater, at speed $v = 3.58\ \text{m/s}$, requires a time of

$$t_2 = \frac{L}{v} = \frac{4.00\ \text{m}}{3.58\ \text{m/s}} = 1.12\ \text{s}$$

The average horizontal speed for the full porpoising maneuver is then

$$v_{\text{av}} = \frac{\Delta x_{\text{total}}}{\Delta t_{\text{total}}} = \frac{2L}{t_1 + t_2} = \frac{2(4.00\ \text{m})}{0.904\ \text{s} + 1.12\ \text{s}} = \boxed{3.95\ \text{m/s}}$$

3.67 (a) and (b)

Since the shot leaves the gun horizontally, $v_{0x} = v_0$ and the time required to reach the target is

$$t = \frac{\Delta x}{v_{0x}} = \frac{x}{v_0}$$

The vertical displacement occurring in this time is

$$\Delta y = -y = v_{0y}t + \frac{1}{2}a_y t^2 = 0 - \frac{1}{2}g\left(\frac{x}{v_0}\right)^2$$

which gives the drop as

$$\boxed{y = \frac{1}{2}g\left(\frac{x}{v_0}\right)^2 = Ax^2 \text{ with } A = \frac{g}{2v_0^2}\text{, where } v_0 \text{ is the muzzle velocity}}$$

(c) If $x = 3.00$ m, and $y = 0.210$ m, then

$$A = \frac{y}{x^2} = \frac{0.210\ \text{m}}{(3.00\ \text{m})^2} = 2.33 \times 10^{-2}\ \text{m}^{-1}$$

and $$v_0 = \sqrt{\frac{g}{2A}} = \sqrt{\frac{9.80\ \text{m/s}^2}{2(2.33 \times 10^{-2}\ \text{m}^{-1})}} = \boxed{14.5\ \text{m/s}}$$

3.68 The velocity of the wind relative to the boat, $\vec{\mathbf{v}}_{\text{WB}}$, is given by $\vec{\mathbf{v}}_{\text{WB}} = \vec{\mathbf{v}}_{\text{WE}} - \vec{\mathbf{v}}_{\text{BE}}$ where $\vec{\mathbf{v}}_{\text{WE}}$ and $\vec{\mathbf{v}}_{\text{BE}}$ are the velocities of the wind and the boat relative to Earth, respectively. Choosing the positive x-direction as east and positive y as north, these relative velocities have components of

$$(\vec{\mathbf{v}}_{\text{WE}})_x = +17\ \text{knots} \qquad (\vec{\mathbf{v}}_{\text{WE}})_y = 0\ \text{knots}$$

$$(\vec{\mathbf{v}}_{\text{BE}})_x = 0 \qquad (\vec{\mathbf{v}}_{\text{BE}})_y = +20\ \text{knots}$$

so $$(\vec{\mathbf{v}}_{\text{WB}})_x = (\vec{\mathbf{v}}_{\text{WE}})_x - (\vec{\mathbf{v}}_{\text{BE}})_x = +17\ \text{knots} \qquad (\vec{\mathbf{v}}_{\text{WB}})_y = (\vec{\mathbf{v}}_{\text{WE}})_y - (\vec{\mathbf{v}}_{\text{BE}})_y = -20\ \text{knots}$$

(a) The velocity of the wind relative to the boat has magnitude and direction of

$$|\vec{\mathbf{v}}_{\text{WB}}| = \sqrt{(\vec{\mathbf{v}}_{\text{WB}})_x^2 + (\vec{\mathbf{v}}_{\text{WB}})_y^2} = \sqrt{(17\ \text{knots})^2 + (-20\ \text{knots})^2} = 26\ \text{knots}$$

and $$\theta = \tan^{-1}\left(\frac{(\vec{\mathbf{v}}_{\text{WB}})_y}{(\vec{\mathbf{v}}_{\text{WB}})_x}\right) = \tan^{-1}\left(\frac{-20\ \text{knots}}{17\ \text{knots}}\right) = -50°$$

or $$\vec{\mathbf{v}}_{\text{WB}} = \boxed{26\ \text{knots at } 50° \text{ south of east}}$$

continued on next page

(b) The component of this velocity parallel to the motion of the boat (that is, parallel to a north-south line) is $\left(\vec{\mathbf{v}}_{\mathrm{WB}}\right)_y = -20$ knots or $\boxed{20 \text{ knots south}}$.

3.69 (a) Take the origin at the point where the ball is launched. Then $x_0 = y_0 = 0$, and the coordinates of the ball at time t later are

$$x = v_{0x}t = \left(v_0 \cos\theta_0\right)t \qquad \text{and} \qquad y = v_{0y}t + \frac{1}{2}a_y t^2 = \left(v_0 \sin\theta_0\right)t - \left(\frac{g}{2}\right)t^2$$

When the ball lands at $x = R = 240$ m, the y-coordinate is $y = 0$ and the elapsed time is found from

$$0 = \left(v_0 \sin\theta_0\right)t - \left(\frac{g}{2}\right)t^2$$

for which the non-zero solution is

$$t = \frac{2v_0 \sin\theta_0}{g}$$

Substituting this time into the equation for the x-coordinate gives

$$x = 240 \text{ m} = \left(v_0 \cos\theta_0\right)\left(\frac{2v_0 \sin\theta_0}{g}\right) = \left(\frac{v_0^2}{g}\right)\left(2\sin\theta_0 \cos\theta_0\right) = \left(\frac{v_0^2}{g}\right)\sin\left(2\theta_0\right)$$

Thus, if $v_0 = 50.0$ m/s, we must have

$$\sin\left(2\theta_0\right) = \frac{(240 \text{ m})\,g}{v_0^2} = \frac{(240 \text{ m})\left(9.80 \text{ m/s}^2\right)}{(50.0 \text{ m/s})^2} = +0.941$$

with solutions of $\quad 2\theta_0 = 70.2° \quad$ and $\quad 2\theta_0 = 180° - 70.2° = 109.8°$

So, the two possible launch angles are $\boxed{\theta_0 = 35.1° \quad \text{and} \quad \theta_0 = 54.9°}$

(b) At maximum height, $v_y = 0$, and the elapsed time is given by

$$t_{\text{peak}} = \frac{\left(v_y\right)_{\text{peak}} - v_{0y}}{a_y} = \frac{0 - v_0 \sin\theta_0}{-g} \qquad \text{or} \qquad t_{\text{peak}} = \frac{v_0 \sin\theta_0}{g}$$

The y-coordinate of the ball at this time will be

$$y_{\max} = \left(v_0 \sin\theta_0\right)t_{\text{peak}} - \left(\frac{g}{2}\right)t_{\text{peak}}^2 = \left(v_0 \sin\theta_0\right)\left(\frac{v_0 \sin\theta_0}{g}\right) - \left(\frac{g}{2}\right)\frac{v_0^2 \sin^2\theta_0}{g^2} = \frac{v_0^2 \sin^2\theta_0}{2g}$$

The maximum heights corresponding to the two possible launch angles are

$$\left(y_{\max}\right)_1 = \frac{(50.0 \text{ m/s})^2 \sin^2(35.1°)}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{42.2 \text{ m}}$$

and $$\left(y_{\max}\right)_2 = \frac{(50.0 \text{ m/s})^2 \sin^2(54.9°)}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{85.4 \text{ m}}$$

3.70 (a) Consider the falling water to consist of droplets, each following a projectile trajectory. If the origin is chosen at the level of the pool and directly beneath the end of the channel, the parameters for these projectiles are:

$$x_0 = 0 \qquad y_0 = h = 2.35 \text{ m}$$

$$v_{0x} = 0.75 \text{ m/s} \qquad v_{0y} = 0$$

$$a_x = 0 \qquad a_y = -g$$

continued on next page

The elapsed time when the droplet reaches the pool is found from $y - y_0 = v_{0y}t + \frac{1}{2}a_y t^2$ as

$$0 - h = 0 - \frac{g}{2}t_p^2 \qquad \text{or} \qquad t_p = \sqrt{\frac{2h}{g}}$$

The distance from the wall where the water lands is then

$$R = x_{\max} = v_{0x}t_p = v_{0x}\sqrt{\frac{2h}{g}} = (0.750\ \text{m/s})\sqrt{\frac{2(2.35\ \text{m})}{9.80\ \text{m/s}^2}} = \boxed{0.519\ \text{m}}$$

This space is $\boxed{\text{too narrow for a pedestrian walkway.}}$

(b) It is desired to build a model in which linear dimensions, such as the height h_{model} and horizontal range of the water R_{model}, are one-twelfth the corresponding dimensions in the actual waterfall. If v_{model} is to be the speed of the water flow in the model, then we would have

$$R_{\text{model}} = v_{\text{model}}\left(t_p\right)_{\text{model}} = v_{\text{model}}\sqrt{\frac{2h_{\text{model}}}{g}}$$

or $$v_{\text{model}} = R_{\text{model}}\sqrt{\frac{g}{2h_{\text{model}}}} = \frac{R_{\text{actual}}}{12}\sqrt{\frac{g}{2\left(h_{\text{actual}}/12\right)}} = \frac{1}{\sqrt{12}}\left(R_{\text{actual}}\sqrt{\frac{g}{2h_{\text{actual}}}}\right) = \frac{v_{\text{actual}}}{\sqrt{12}}$$

and the needed speed of flow in the model is

$$v_{\text{model}} = \frac{v_{\text{actual}}}{\sqrt{12}} = \frac{0.750\ \text{m/s}}{\sqrt{12}} = \boxed{0.217\ \text{m/s}}$$

3.71 (a) Applying $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ to the vertical motion of the first snowball gives

$$0 = \left[(25.0\ \text{m/s})\sin 70.0°\right]t_1 + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)t_1^2$$

which has the non-zero solution of

$$t_1 = \frac{2(25.0\ \text{m/s})\sin 70.0°}{9.80\ \text{m/s}^2} = 4.79\ \text{s}$$

as the time of flight for this snowball.

The horizontal displacement this snowball achieves is

$$\Delta x = v_{0x}t_1 = \left[(25.0\ \text{m/s})\cos 70.0°\right](4.79\ \text{s}) = 41.0\ \text{m}$$

Now consider the second snowball, also given an initial speed of $v_0 = 25.0\ \text{m/s}$, thrown at angle θ, and in the air for time t_2. Applying $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ to its vertical motion yields

$$0 = \left[(25.0\ \text{m/s})\sin\theta\right]t_2 + \frac{1}{2}\left(-9.80\ \text{m/s}^2\right)t_2^2$$

which has a non-zero solution of

$$t_2 = \frac{2(25.0\ \text{m/s})\sin\theta}{9.80\ \text{m/s}^2} = (5.10\ \text{s})\sin\theta$$

We require the horizontal range of this snowball be the same as that of the first ball, namely

$$\Delta x = v_{0x}t_2 = \left[(25.0\ \text{m/s})\cos\theta\right]\left[(5.10\ \text{s})\sin\theta\right] = 41.0\ \text{m}$$

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This yields the equation

$$\sin\theta\cos\theta = \frac{41.0\text{ m}}{(25.0\text{ m/s})(5.10\text{ s})} = 0.321$$

From the trigonometric identity $\sin 2\theta = 2\sin\theta\cos\theta$, this result becomes

$$\sin 2\theta = 2(0.321) = 0.642 \qquad \text{so} \qquad 2\theta = 39.9°$$

and the required angle of projection for the second snowball is

$$\theta = \boxed{20.0° \text{ above the horizontal}}$$

(b) From above, the time of flight for the first snowball is $t_1 = 4.79$ s and that for the second snowball is

$$t_2 = (5.10\text{ s})\sin\theta = (5.10\text{ s})\sin 20.0° = 1.74\text{ s}$$

Thus, if they are to arrive simultaneously, the time delay between the first and second snowballs should be

$$\Delta t = t_1 - t_2 = 4.79\text{ s} - 1.74\text{ s} = \boxed{3.05\text{ s}}$$

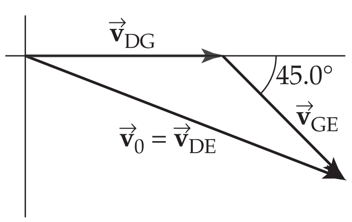
3.72 First, we determine the velocity with which the dart leaves the gun by using the data collected when the dart was fired horizontally ($v_{0y} = 0$) from a stationary gun. In this case, $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ gives the time of flight as

$$t = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-1.00\text{ m/s})}{-9.80\text{ m/s}^2}} = 0.452\text{ s}$$

Thus, the initial speed of the dart relative to the gun is

$$v_{\text{DG}} = v_{0x} = \frac{\Delta x}{t} = \frac{5.00\text{ m}}{0.452\text{ s}} = 11.1\text{ m/s}$$

At the instant when the dart is fired horizontally from a moving gun, the velocity of the dart relative to the gun may be expressed as $\vec{\mathbf{v}}_{\text{DG}} = \vec{\mathbf{v}}_{\text{DE}} - \vec{\mathbf{v}}_{\text{GE}}$, where $\vec{\mathbf{v}}_{\text{DE}}$ and $\vec{\mathbf{v}}_{\text{GE}}$ are the velocities of the dart and gun relative to Earth, respectively. The initial velocity of the dart relative to Earth is therefore



$$\vec{\mathbf{v}}_0 = \vec{\mathbf{v}}_{\text{DE}} = \vec{\mathbf{v}}_{\text{DG}} + \vec{\mathbf{v}}_{\text{GE}}$$

From the vector diagram, observe that

$$v_{0y} = -v_{\text{GE}}\sin 45.0° = -(2.00\text{ m/s})\sin 45.0° = -1.41\text{ m/s}$$

and $\quad v_{0x} = v_{\text{DG}} + v_{\text{GE}}\cos 45.0° = 11.1\text{ m/s} + (2.00\text{ m/s})\cos 45.0° = 12.5\text{ m/s}$

The vertical velocity of the dart after dropping 1.00 m to the ground is

$$v_y = -\sqrt{v_{0y}^2 + 2a_y\Delta y} = -\sqrt{(-1.41\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)(-1.00\text{ m})} = -4.65\text{ m/s}$$

and the time of flight is

$$t = \frac{v_y - v_{0y}}{a_y} = \frac{-4.65\text{ m/s} - (-1.41\text{ m/s})}{-9.80\text{ m/s}^2} = 0.331\text{ s}$$

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The displacement during the flight is

$$\Delta x = v_{0x}t = (12.5\ \text{m/s})(0.331\ \text{s}) = \boxed{4.14\ \text{m}}$$

3.73 (a) First, use $\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$ to find the time for the coyote to travel 70 m, starting from rest with constant acceleration $a_x = 15\ \text{m/s}^2$.

$$t_1 = \sqrt{\frac{2\Delta x}{a_x}} = \sqrt{\frac{2(70\ \text{m})}{15\ \text{m/s}^2}} = 3.1\ \text{s}$$

The minimum constant speed the roadrunner must have to reach the edge in this time is

$$v = \frac{\Delta x}{t_1} = \frac{70\ \text{m}}{3.1\ \text{s}} = \boxed{23\ \text{m/s}}$$

(b) The initial velocity of the coyote as it goes over the edge of the cliff is horizontal and equal to

$$v_0 = v_{0x} = 0 + a_x t_1 = (15\ \text{m/s}^2)(3.1\ \text{s}) = 46\ \text{m/s}$$

From $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, the time for the coyote to drop 100 m, with $v_{0y} = 0$, is

$$t_2 = \sqrt{\frac{2\Delta y}{a_y}} = \sqrt{\frac{2(-100\ \text{m})}{-9.80\ \text{m/s}^2}} = 4.52\ \text{s}$$

The horizontal displacement of the coyote during his fall is

$$\Delta x = v_{0x}t_2 + \tfrac{1}{2}a_x t_2^2 = (46\ \text{m/s})(4.52\ \text{s}) + \tfrac{1}{2}(15\ \text{m/s}^2)(4.52\ \text{s})^2 = \boxed{3.6\times10^2\ \text{m}}$$

3.74 The known parameters for the flight of the melon are:

$v_{0x} = v_0 = 10.0\ \text{m/s}$

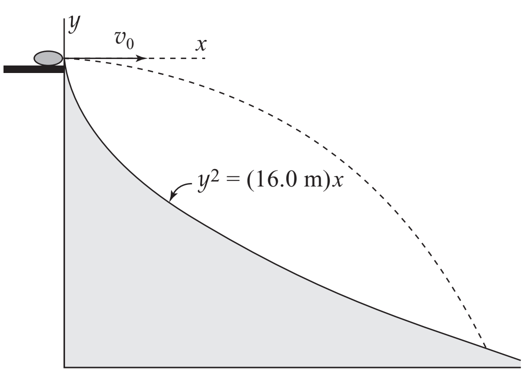
$v_{0y} = 0$

$a_x = 0$

$a_y = -g = -9.80\ \text{m/s}^2$

$x_0 = y_0 = 0$

At impact: $y^2 = (16.0\ \text{m})\,x$



The y-coordinate of the melon as a function of time is given by $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ as

$$y - 0 = 0 + \frac{1}{2}(-g)t^2 \qquad \text{or} \qquad y = -\frac{gt^2}{2} \qquad \textbf{[1]}$$

and the x-coordinate as a function of time is $x = v_{0x}t = v_0 t$. Thus, the elapsed time may be expressed in terms of the x-coordinate as $t = x/v_0$, so Equation [1] for the y-coordinate becomes

$$y = -\frac{g}{2}\left(\frac{x}{v_0}\right)^2 = -\left(\frac{g}{2v_0^2}\right)x^2 \qquad \text{and} \qquad y^2 = \left(\frac{g^2}{4v_0^4}\right)x^4 \qquad \textbf{[2]}$$

Since, at impact, we must have $y^2 = (16.0\ \text{m})\,x$, Equation [2] says that when impact occurs we must have

$$(16.0\ \text{m})\,x = \left(\frac{g^2}{4v_0^4}\right)x^4 \qquad \text{or} \qquad x\left(16.0\ \text{m} - \frac{g^2}{4v_0^4}x^3\right) = 0 \qquad \textbf{[3]}$$

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Equation [3] has a trivial solution $x = 0$, but the x-coordinate of the melon at impact is a solution of the equation

$$16.0\text{ m} - \frac{g^2}{4v_0^4}x^3 = 0 \qquad \text{or} \qquad x^3 = \frac{(16.0\text{ m})\left(4v_0^4\right)}{g^2}$$

Thus, at impact

$$x = \left[\frac{(64.0\text{ m})\,v_0^4}{g^2}\right]^{1/3} = \left[\frac{(64.0\text{ m})(10.0\text{ m/s})^4}{\left(9.80\text{ m/s}^2\right)^2}\right]^{1/3} = \boxed{18.8\text{ m}}$$

and the y-coordinate is $\quad y = -\sqrt{(16.0\text{ m})\,x} = -\sqrt{(16.0\text{ m})(18.8\text{ m})} = \boxed{-17.3\text{ m}}$

4

The Laws of Motion

QUICK QUIZZES

1. Newton's second law says that the acceleration of an object is directly proportional to the resultant (or net) force acting on. Recognizing this, consider the given statements one at a time.

(a) **True** – If the resultant force on an object is zero (either because no forces are present or the vector sum of the forces present is zero), the object can still move with constant velocity.

(b) **False** – An object that remains at rest has zero acceleration. However, any number of external forces could be acting on it, provided that the vector sum of these forces is zero.

(c) **True** – When a single force acts, the resultant force cannot be zero and the object must accelerate.

(d) **True** – When an object accelerates, a set containing one or more forces with a non-zero resultant must be acting on it.

(e) **False** – Many external forces could be acting on an object with zero acceleration, provided that the vector sum of these forces is zero.

(f) **False** – If the net force is in the positive x-direction, the *acceleration* will be in the positive x-direction. However, the velocity of an object does not have to be in the same direction as its acceleration (consider the motion of a projectile).

2. Choice (b). The newton is a unit of weight, and the quantity (or mass) of gold that weighs 1 newton is $m = (1\text{ N})/g$. Because the value of g is smaller on the Moon than on the Earth, someone possessing 1 newton of gold on the Moon has more gold than does a person having 1 newton of gold on Earth.

3. (a) **False** – When on an orbiting space station the astronaut is farther from the center of Earth than when on the surface of the Earth. Thus, the astronaut experiences a reduced gravitational force, but that force is not zero.

(b) **True** – On or above the surface of Earth, the acceleration of gravity is inversely proportional to the square of the distance from the center of Earth. Therefore, when this distance is increased by a factor of 3, the acceleration of gravity decreases by a factor of 9.

(c) **False** – The acceleration of gravity on the surface of a planet of mass M and volume V is $g = F_g/m = GM/R^2$, where $R = [3V/4\pi]^{1/3}$. If the mass and volume are doubled (as when two identical planets coalesce), the radius becomes $R' = [3(2V)/4\pi]^{1/3} = 2^{1/3}R$ and the surface gravity becomes

$$g' = GM'/R'^2 = G(2M)\big/\left(2^{1/3}R\right)^2 = 2^{2/3}g$$

(d) **False** – Unlike the weight of an object, its mass is independent of the local acceleration of gravity. Thus, one kilogram of gold contains the same amount of gold on the Moon as it does on Earth.

4. Choices (c) and (d). Newton's third law states that the car and truck will experience equal magnitude (but oppositely directed) forces. Newton's second law states that acceleration is inversely proportional to mass when the force is constant. Thus, the lower mass vehicle (the car) will experience the greater acceleration.

5. Choice (c). In case (i), the scale records the tension in the rope attached to its right end. The section of rope in the man's hands has zero acceleration, and hence, zero net force acting on it. This means that the tension in the rope pulling to the left on this section must equal the force *F* the man exerts toward the right on it. The scale reading in this case will then be *F*. In case (ii), the person on the left can be modeled as simply holding the rope tightly while the person on the right pulls. Thus, the person on the left is doing the same thing that the wall does in case (i). The resulting scale reading is the same whether there is a wall or a person holding the left side of the scale.

6. Choice (c). The tension in the rope has a vertical component that supports part of the total weight of the woman and sled. Thus, the upward normal force exerted by the ground is less than the total weight.

7. Choice (b). Friction forces are always parallel to the surfaces in contact, which, in this case, are the wall and the cover of the book. This tells us that the friction force is either upward or downward. Because the tendency of the book is to fall downward due to gravity, the friction force must be in the upward direction.

8. Choice (b). The static friction force between the bottom surface of the crate and the surface of the truck bed is the net horizontal force on the crate that causes it to accelerate. It is in the same direction as the acceleration, toward the east.

9. Choice (b). It is easier to attach the rope and pull. In this case, there is a component of your applied force that is upward. This reduces the normal force between the sled and the snow. In turn, this reduces the friction force between the sled and the snow, making it easier to move. If you push from behind, with a force having a downward component, the normal force is larger, the friction force is larger, and the sled is harder to move.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. Newton's second law gives the net force acting on the crate as

$$F_{\text{net}} = 95.0\text{ N} - f_k = (60.0\text{ kg})(1.20\text{ m/s}^2) = 72.0\text{ N}$$

This gives the kinetic friction force as $f_k = 23.0\text{ N}$, so choice (a) is correct.

2. As the block slides down the frictionless incline, there is a constant net force directed down the incline (i.e., the tangential component of the weight of the block) acting on it. This force will give the block a constant acceleration down the incline, meaning that its speed down the incline will increase at a constant rate. Thus, the only correct choice is (c).

3. From Newton's law of universal gravitation, the force Earth exerts on an object on its surface is $F_g = GM_E m/R_E^2 = mg$, or the acceleration of gravity at Earth's surface is $g = GM_E/R_E^2$. If both the mass and radius of the Earth should double, so $M_E' = 2M_E$ and $R_E' = 2R_E$, the acceleration of gravity at the surface would then be

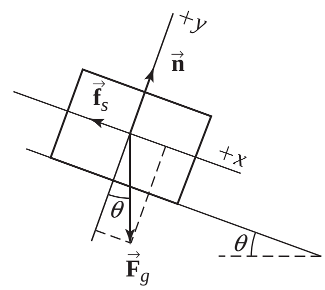
$$g' = G\frac{M_E'}{(R_E')^2} = G\left(\frac{2M_E}{4R_E^2}\right) = \frac{1}{2}\left(G\frac{M_E}{R_E^2}\right) = \frac{g}{2} = \frac{9.80\text{ m/s}^2}{2} = 4.90\text{ m/s}^2$$

meaning that (b) is the correct answer.

4. If a net external force acts on an object, Newton's second law ($\vec{\mathbf{F}}_{\text{net}} = m\vec{\mathbf{a}}$) tells us that the object will have a non-zero acceleration, and choice (d) is correct. Since the net force is constant, the acceleration will be constant and choice (c) is false. With a non-zero acceleration, the velocity of the object will undergo either a change in magnitude (could be an increase or a decrease) and/or a change in direction. Thus, neither choice (b) nor (e) is necessarily true. While an object with a non-zero acceleration may come to rest momentarily, it will immediately start moving again, so choice (a) is correct. Thus, we see that choices (a) and (d) are true and all others are false.

5. The tension, F_{upper}, in the vine at the point above the upper monkey must support the weight of both monkeys $\left(\text{i.e., } F_{\text{upper}} = 2\left(F_g\right)_{\substack{\text{single}\\\text{monkey}}}\right)$. However, the tension in the vine between the two monkeys supports only the weight of the lower monkey $\left(F_{\text{lower}} = \left(F_g\right)_{\substack{\text{single}\\\text{monkey}}}\right)$, meaning that $F_{\text{upper}}/F_{\text{lower}} = 2$ and choice (d) is correct.

6. When the crate is held in equilibrium on the incline as shown in the sketch, Newton's second law requires that $\Sigma F_x = \Sigma F_y = 0$. From $\Sigma F_x = +\left|\vec{\mathbf{F}}_g\right|_x - \left|\vec{\mathbf{f}}_s\right| = 0$, the magnitude of the friction force equals the component of gravitational force acting down the incline, or choice (e) is correct. Note that $\left|\vec{\mathbf{f}}_s\right| = \left|\vec{\mathbf{f}}_s\right|_{\text{max}} = \mu_s n$ only when the crate is on the verge of starting to slide.



7. As stated by Newton's third law of motion, "For every force of action, there is a force of reaction having equal magnitude but opposite direction." Thus, the force exerted on the locomotive by the wall has the same magnitude as the force exerted on the wall by the locomotive, and (b) is the correct choice.

8. Constant velocity means zero acceleration. From Newton's second law, $\Sigma\vec{\mathbf{F}} = m\vec{\mathbf{a}}$, so the total (or resultant) force acting on the object must be zero if it moves at constant velocity. This means that (d) is the correct choice.

9. Choose a coordinate system with the positive *x*-direction being east and the positive *y*-direction being north. Then the components of the four given forces are

$$A_x = +40 \text{ N}, A_y = 0 \qquad B_x = 0,\ B_y = +50 \text{ N}$$
$$C_x = -70 \text{ N}, C_y = 0 \qquad D_x = 0,\ D_y = -90 \text{ N}$$

The components of the resultant (or net) force are $R_x = A_x + B_x + C_x + D_x = -30$ N and $R_y = A_y + B_y + C_y + D_y = -40$ N. Therefore, the magnitude of the net force acting on the object is $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(-30\text{ N})^2 + (-40\text{ N})^2} = 50$ N, or choice (a) is correct.

10. An object in equilibrium has zero acceleration $\left(\vec{\mathbf{a}} = 0\right)$, so both the magnitude and direction of the object's velocity must be constant. Also, Newton's second law states that the net force acting on an object in equilibrium is zero. The only *untrue statement* among the given choices is (d), untrue because the value of the velocity's constant magnitude need not be zero.

11. The box will accelerate in the direction of the resultant force acting on it. The only horizontal forces present are the force exerted by the manager and the friction force. Since the acceleration is given to be in the direction of the force applied by the manager, the magnitude of this force must exceed that of the opposing friction force; however, the friction force is not necessarily zero. Thus, choice (b) is correct.

12. As the trailer leaks sand at a constant rate, the total mass of the vehicle (truck, trailer and remaining sand) decreases at a steady rate. Then, with a constant net force present, Newton's second law states that the magnitude of the vehicle's acceleration $\left(a = F_{\text{net}}/m\right)$ will *steadily increase*. Choice (b) is the correct answer.

13. When the truck accelerates forward, its natural tendency is to slip from beneath the crate, leaving the crate behind. However, friction between the crate and the bed of the truck acts in such a manner as to oppose this relative motion between truck and crate. Thus, the friction force acting on the crate will be in the forward horizontal direction and tend to accelerate the crate forward. The crate will slide only when the coefficient of static friction is inadequate to prevent slipping. The correct response to this question is (c).

14. Assuming that the cord connecting m_1 and m_2 has constant length, the two masses are a fixed distance (measured along the cord) apart. Thus, their speeds must always be the same, which means that their accelerations must have equal magnitudes. The magnitude of the downward acceleration of m_2 is given by Newton's second law as

$$a_2 = \frac{\Sigma F_y}{m_2} = \frac{m_2 g - T}{m_2} = g - \left(\frac{T}{m_2}\right) < g$$

where T is the tension in the cord, and downward has been chosen as the positive direction. Therefore, the only correct statement among the listed choices is (a).

15. The mass of an object is the same at all locations in space (e.g., on Earth, the Moon, or space station). However, the gravitational force the object experiences $\left(\text{weight, } w = mg\right)$ does vary, depending on the acceleration of gravity g at the object's current location in space. It is the gravitation attraction of the Earth that holds the space station (and all its contents, including astronauts) in orbit around Earth. The only correct choice listed is (b).

16. Only forces which act ***on*** the object should be included in the free-body diagram of the object. In this case, these forces are: (1) the gravitational force (acting vertically downward), (2) the normal force (perpendicular to the incline) exerted on the object by the incline, and (3) the friction force exerted on the object by the incline, and acting *up* the incline to oppose the motion of the object down the incline). Choices (d) and (f) are forces exerted *on the incline* by the object. Choice (b) is the resultant of forces (1), (2), and (3) listed above, and its inclusion in the free-body diagram would duplicate information already present. Thus, correct answers to this question are (b), (d), and (f).

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. If it has a large mass, it will take a large force to alter its motion even when floating in space. Thus, to avoid injuring himself, he should push it gently toward the storage compartment.

4. The coefficient of static friction is larger than that of kinetic friction. To start the box moving, you must counterbalance the maximum static friction force. This force exceeds the kinetic friction force that you must counterbalance to maintain constant velocity of the box once it starts moving.

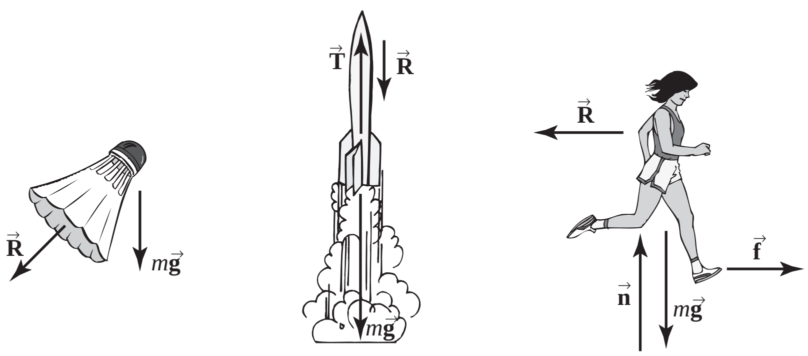
6. (a) The barbell always exerts a downward force on the lifter equal in magnitude to the upward force that she exerts on the barbell. Since the lifter is in equilibrium, the magnitude of the upward force exerted on her by the scale (that is, the scale reading) equals the sum of her weight and the downward force exerted by the barbell. As the barbell goes through the bottom of the cycle and is being lifted upward, the scale reading exceeds the combined weights of the lifter and the barbell. At the top of the motion and as the barbell is allowed to move back downward, the scale reading is less than the combined weights.

(b) If the barbell is moving upward, the lifter can declare she has thrown it just by letting go of it for a moment. Thus, the case is included in the previous answer.

8. In order for an object to be in equilibrium, the resultant force acting on it must be zero. Thus, it is not possible for an object to be in equilibrium when a single force of non-zero magnitude acts on it.

10. The net force acting on the object decreases as the resistive force increases. Eventually, the resistive force becomes equal to the weight of the object, and the net force goes to zero. In this condition, the object stops accelerating, and the velocity stays constant. The rock has reached its terminal velocity.

12.



(a) (b) (c)

In the free-body diagrams give above, $\vec{\mathbf{R}}$ represents a force due to air resistance, $\vec{\mathbf{T}}$ is a force due to the thrust of the rocket engine, $\vec{\mathbf{n}}$ is a normal force, $\vec{\mathbf{f}}$ is a friction force, and the forces labeled $m\vec{\mathbf{g}}$ are gravitational forces.

14. When a tire is rolling, the point on the tire in contact with the ground is momentarily at rest relative to the ground. Thus, static friction exists between the tire and the ground under these conditions. When the brakes lock, the tires begin to skid over the ground and kinetic friction now exists between tires and the ground. Since the kinetic friction force is less than the maximum static friction force ($\mu_k < \mu_s$), the friction force tending to slow the car is less with the brakes locked than while the tires continue to roll.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. 25 N

4. See Solution.

6. 7.4 min

8. See Solution.

10. 3.1×10^2 N

12. **(a)** 798 N at 8.79° to the right of the forward direction **(b)** 0.266 m/s^2

14. **(a)** $t = \sqrt{2h/g}$ **(b)** $a_x = F/m$ **(c)** $\Delta x = Fh/mg$

(d) $a = \sqrt{(F/m)^2 + g^2}$

16. $1.59\ \text{m/s}^2$ at $65.2°$ N of E

18. 8.71 N

20. $w_2 = 1.7\times10^2$ N, $\alpha = 61°$

22. **(a)** $T_1 = 2m(g+a),\ T_2 = m(g+a)$

(b) upper string breaks first **(c)** $T_1 = T_2 = 0$

24. **(a)** 49.0 N **(b)** 49.0 N **(c)** 98.0 N

(d) 24.5 N

26. $a_{\max} = 2.1\ \text{m/s}^2$

28. **(a)** $4.43\ \text{m/s}^2$ **(b)** 53.7 N

30. **(a)** $1.4\ \text{m/s}^2$ **(b)** $0.49\ \text{m/s}^2$

32. **(a)** See Solution. **(b)** $\Sigma F_x = F,\ \Sigma F_y = 0$

(c) $\Sigma F_x = F - P,\ \Sigma F_y = 0$ **(d)** $\Sigma F_x = P,\ \Sigma F_y = 0$

(e) For m_1: $F - P = m_1 a$; For m_2: $P = m_2 a$

(f) $a = F/(m_1+m_2)$; $P = [m_2/(m_1+m_2)]F$

(g) $P = [m_1/(m_1+m_2)]F$, larger because $m_1 > m_2$

34. **(a)** $T_A > T_B$ **(b)** $a_1 = a_2$

(c) Yes, by Newton's 3rd law.

36. **(a)** $a_1 = a_2 = 6.53\ \text{m/s}^2$ **(b)** $T = 32.7$ N

38. **(a)** 36.8 N **(b)** $2.44\ \text{m/s}^2$ **(c)** 1.22 m

40. **(a)** $a = 0$ **(b)** $0.70\ \text{m/s}^2$

42. **(a)** See Solution. **(b)** static friction between blocks

(c) $a = F/4m$

44. **(a)** $3.43\ \text{m/s}^2$ **(b)** $3.14\ \text{m/s}^2$

46. **(a)** $a_x = -\mu_k g$ **(b)** $\Delta x = v_0^2/2\mu_k g$

48. **(a)** $0.368\ \text{m/s}^2$ **(b)** $1.28\ \text{m/s}^2$ down the incline

50. **(a)** 98.6 m **(b)** 16.4 m

52. **(a)** 15.0 lb **(b)** 5.00 lb **(c)** zero

54. **(a)** 0.125 m/s^2 **(b)** 39.7 N **(c)** 0.235

56. 21.4 N

58. 2.6 m/s^2 northward

60. **(a)** 50.0 N **(b)** $\mu_s \geq 0.500$ **(c)** 25.0 N

62. **(a)** See Solution. **(b)** 55.2° **(c)** 167 N

64. **(a)** 1.63 m/s^2 **(b)** $T_1 = 57.0$ N, $T_2 = 24.5$ N

66. 1.18×10^3 N

68. **(a)** $a_1 = a_2/2$

(b) $T_1 = 4m_1m_2g/(m_1 + 4m_2)$, $T_2 = 2m_1m_2g/(m_1 + 4m_2)$

(c) $a_1 = m_1g/(m_1 + 4m_2)$, $a_2 = 2m_1g/(m_1 + 4m_2)$

70. **(a)** See Solution. **(b)** No. For $\theta \leq \theta_c$, $f \leq f_{s,\max} = \mu_s n$.

(c) See Solution. **(d)** $\mu_k = \tan\theta_c'$

72. **(a)** See Solution. **(b)** $a = F/(m_b + m_r)$

(c) $T_{\substack{\text{left}\\\text{end}}} = [m_b/(m_b + m_r)]F$

(d) The tension is uniform throughout a *light* cord.

74. 173 lb

76. **(a)** 7.25×10^3 N **(b)** 4.57 m/s^2

78. 104 N

80. 6.00 cm

82. 5.10×10^3 N

84. **(a)** $a_1 = 2a_2$

(b) $a_2 = m_2g/(4m_1 + m_2) \approx 12.7 \text{ N}/(4m_1 + 1.30 \text{ kg})$

(c) 9.80 m/s^2

(d) $a_2 \to 0$

(e) $T = m_2g/2 = 6.37$ N

(f) Yes. If $m_1 \ll m_2$, m_2 is in near free-fall; If $m_1 \gg m_2$, m_2 is held almost stationary.

PROBLEM SOLUTIONS

4.1 $w = (2\ \cancel{\text{tons}})\left(\dfrac{2000\ \cancel{\text{lbs}}}{1\ \cancel{\text{ton}}}\right)\left(\dfrac{4.448\ \text{N}}{1\ \cancel{\text{lb}}}\right) = \boxed{2\times10^4\ \text{N}}$

4.2 From $v = v_0 + at$, the acceleration given to the football is

$$a_{av} = \frac{v - v_0}{t} = \frac{10\ \text{m/s} - 0}{0.20\ \text{s}} = 50\ \text{m/s}^2$$

Then, from Newton's second law, we find

$$F_{av} = m\,a_{av} = (0.50\ \text{kg})(50\ \text{m/s}^2) = \boxed{25\ \text{N}}$$

4.3 (a) $\Sigma F_x = ma_x = (6.0\ \text{kg})(2.0\ \text{m/s}^2) = \boxed{12\ \text{N}}$

(b) $a_x = \dfrac{\Sigma F_x}{m} = \dfrac{12\ \text{N}}{4.0\ \text{kg}} = \boxed{3.0\ \text{m/s}^2}$

4.4 (a) Action: The hand exerts a force to the right on the spring. Reaction: The spring exerts an equal magnitude force to the left on the hand. Action: The wall exerts a force to the left on the spring. Reaction: The spring exerts an equal magnitude force to the right on the wall. Action: Earth exerts a downward gravitational force on the spring. Reaction: The spring exerts an equal magnitude gravitational force upward on the Earth.

(b) Action: The handle exerts a force upward to the right on the wagon. Reaction: The wagon exerts an equal magnitude force downward to the left on the handle. Action: Earth exerts an upward contact force on the wagon. Reaction: The wagon exerts an equal magnitude downward contact force on the Earth. Action: Earth exerts a downward gravitational force on the wagon. Reaction: The wagon exerts an equal magnitude gravitational force upward on the Earth.

(c) Action: The player exerts a force upward to the left on the ball. Reaction: The ball exerts an equal magnitude force downward to the right on the player. Action: Earth exerts an downward gravitational force on the ball. Reaction: The ball exerts an equal magnitude gravitational force upward on the Earth.

(d) Action: M exerts a gravitational force to the right on m. Reaction: m exerts an equal magnitude gravitational force to the left on M.

(e) Action: The charge $+Q$ exerts an electrostatic force to the right on the charge $-q$. Reaction: The charge $-q$ exerts an equal magnitude electrostatic force to the left on the charge $+Q$.

(f) Action: The magnet exerts a force to the right on the iron. Reaction: The iron exerts an equal magnitude force to the left on the magnet.

4.5 The weight of the bag of sugar on Earth is

$$w_E = mg_E = (5.00\ \text{lbs})\left(\frac{4.448\ \text{N}}{1\ \text{lb}}\right) = 22.2\ \text{N}$$

If g_M is the free-fall acceleration on the surface of the Moon, the ratio of the weight of an object on the Moon to its weight when on Earth is $w_M/w_E = mg_M/mg_E = g_M/g_E$, so $w_M = w_E\,(g_M/g_E)$. Hence, the weight of the bag of sugar on the Moon is

$$w_M = (22.2\ \text{N})\left(\frac{1}{6}\right) = \boxed{3.70\ \text{N}}$$

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On Jupiter, its weight would be

$$w_J = w_E\left(\frac{g_J}{g_E}\right) = (22.2\ \text{N})(2.64) = \boxed{58.6\ \text{N}}$$

The mass is the same at all three locations, and is given by

$$m = \frac{w_E}{g_E} = \frac{(5.00\ \text{lb})(4.448\ \text{N/lb})}{9.80\ \text{m/s}^2} = \boxed{2.27\ \text{kg}}$$

4.6 $$a = \frac{\Sigma F}{m} = \frac{7.5\times10^5\ \text{N}}{1.5\times10^7\ \text{kg}} = 5.0\times10^{-2}\ \text{m/s}^2$$

and $v = v_0 + at$ gives

$$t = \frac{v - v_0}{a} = \frac{80\ \text{km/h} - 0}{5.0\times10^{-2}\ \text{m/s}^2}\left(\frac{0.278\ \text{m/s}}{1\ \text{km/h}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{7.4\ \text{min}}$$

4.7 If $F = 825$ N is the upward force exerted on the man by the scales, the upward acceleration of the man (and hence, the acceleration of the elevator) is

$$a_y = \frac{\Sigma F_y}{m_{\text{man}}} = \frac{825\ \text{N} - m_{\text{man}}g}{m_{\text{man}}} = \frac{825\ \text{N}}{m_{\text{man}}} - g = \frac{825\ \text{N}}{75\ \text{kg}} - 9.8\ \text{m/s}^2 = \boxed{1.2\ \text{m/s}^2}$$

4.8 (a) The sphere has a larger mass than the feather. Hence, the sphere experiences a larger gravitational force $F_g = mg$ than does the feather.

(b) The time of fall is less for the sphere than for the feather. This is because air resistance affects the motion of the feather more than that of the sphere.

(c) In a vacuum, the time of fall is the same for the sphere and the feather. In the absence of air resistance, both objects have the free-fall acceleration g.

(d) In a vacuum, the total force on the sphere is greater than that on the feather. In the absence of air resistance, the total force is just the gravitational force, and the sphere weighs more than the feather.

4.9 (a) From $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$, with $\Delta y = (2/3)(1.50\ \text{m}) = 1.00$ m, we find

$$a_y = \frac{v_y^2 - v_{0y}^2}{2(\Delta y)} = \frac{(6.00\ \text{m/s})^2 - (3.00\ \text{m/s})^2}{2(1.00\ \text{m})} = \boxed{13.5\ \text{m/s}}$$

(b) We apply Newton's second law to the vertical motion of the fish with F being the upward force exerted by the tail fin. This gives

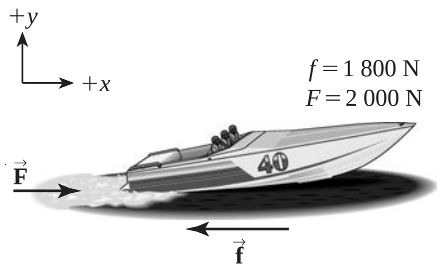
$$\Sigma F_y = ma_y \quad \Rightarrow \quad F - mg = ma_y$$

or $$F = m(a_y + g) = (61.0\ \text{kg})(13.5\ \text{m/s}^2 + 9.80\ \text{m/s}^2) = \boxed{1.42\times10^3\ \text{N}}$$

4.10 The acceleration of the bullet is given by $$a = \frac{v^2 - v_0^2}{2(\Delta x)} = \frac{(320\ \text{m/s})^2 - 0}{2(0.82\ \text{m})}$$

Then, $$\Sigma F = ma = (5.0\times10^{-3}\ \text{kg})\left[\frac{(320\ \text{m/s})^2}{2(0.82\ \text{m})}\right] = \boxed{3.1\times10^2\ \text{N}}$$

4.11 (a) From the second law, the acceleration of the boat is



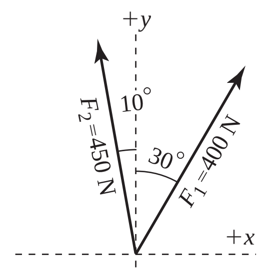
$$a = \frac{\Sigma F}{m} = \frac{2\,000\text{ N} - 1\,800\text{ N}}{1\,000\text{ kg}} = \boxed{0.200\text{ m/s}^2}$$

(b) The distance moved is

$$\Delta x = v_0 t + \frac{1}{2}at^2 = 0 + \frac{1}{2}\left(0.200\text{ m/s}^2\right)(10.0\text{ s})^2 = \boxed{10.0\text{ m}}$$

(c) The final velocity is $v = v_0 + at = 0 + \left(0.200\text{ m/s}^2\right)(10.0\text{ s}) = \boxed{2.00\text{ m/s}}$

4.12 (a) Choose the positive y-axis in the forward direction. We resolve the forces into their components as



Force	x-component	y-component
400 N	200 N	346 N
450 N	–78.1 N	443 N
Resultant	$\Sigma F_x = 122$ N	$\Sigma F_y = 789$ N

The magnitude and direction of the resultant force is

$$F_R = \sqrt{\left(\Sigma F_x\right)^2 + \left(\Sigma F_y\right)^2} = 798\text{ N} \qquad \theta = \tan^{-1}\left(\frac{\Sigma F_x}{\Sigma F_y}\right) = 8.79°\text{ to right of }y\text{-axis}$$

Thus, $\boxed{\vec{\mathbf{F}}_R = 798\text{ N at }8.79°\text{ to the right of the forward direction}}$.

(b) The acceleration is in the same direction as $\vec{\mathbf{F}}_R$ and has magnitude

$$a = \frac{F_R}{m} = \frac{798\text{ N}}{3\,000\text{ kg}} = \boxed{0.266\text{ m/s}^2}$$

4.13 Taking eastward as the positive x-direction, the average horizontal acceleration of the car is

$$a_x = \frac{v_x - v_{0x}}{\Delta t} = \frac{25.0\text{ m/s} - 0}{5.00\text{ s}} = +5.00\text{ m/s}^2$$

Thus, the average horizontal force acting on the car during this 5.00-s period is

$$\Sigma F_x = ma_x = (970\text{ kg})\left(+5.00\text{ m/s}^2\right) = +4.85\times10^3\text{ N} = \boxed{4.85\text{ kN eastward}}$$

4.14 (a) With the wind force being horizontal, the only vertical force acting on the object is its own weight, mg. This gives the object a downward acceleration of

$$a_y = \frac{\Sigma F_y}{m} = \frac{-mg}{m} = -g$$

The time required to undergo a vertical displacement $\Delta y = -h$, starting with initial vertical velocity $v_{0y} = 0$, is found from $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ as

$$-h = 0 - \frac{g}{2}t^2 \qquad \text{or} \qquad \boxed{t = \sqrt{\frac{2h}{g}}}$$

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(b) The only horizontal force acting on the object is that due to the wind, so $\Sigma F_x = F$ and the horizontal acceleration will be $a_x = \dfrac{\Sigma F_x}{m} = \boxed{\dfrac{F}{m}}$

(c) With $v_{0x} = 0$, the horizontal displacement the object undergoes while falling a vertical distance h is given by $\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$ as

$$\Delta x = 0 + \frac{1}{2}\left(\frac{F}{m}\right)\left(\sqrt{\frac{2h}{g}}\right)^2 = \boxed{\frac{Fh}{mg}}$$

(d) The total acceleration of this object while it is falling will be

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(F/m)^2 + (-g)^2} = \boxed{\sqrt{(F/m)^2 + g^2}}$$

4.15 Starting with $v_{0y} = 0$ and falling 30 m to the ground, the velocity of the ball just before it hits is

$$v_1 = -\sqrt{v_{0y}^2 + 2a_y\Delta y} = -\sqrt{0 + 2\left(-9.80\ \text{m/s}^2\right)(-30\ \text{m})} = -24\ \text{m/s}$$

On the rebound, the ball has $v_y = 0$ after a displacement $\Delta y = +20$ m. Its velocity as it left the ground must have been

$$v_2 = +\sqrt{v_y^2 - 2a_y\Delta y} = +\sqrt{0 - 2\left(-9.80\ \text{m/s}^2\right)(20\ \text{m})} = +20\ \text{m/s}$$

Thus, the average acceleration of the ball during the 2.0-ms contact with the ground was

$$a_{\text{av}} = \frac{v_2 - v_1}{\Delta t} = \frac{+20\ \text{m/s} - (-24\ \text{m/s})}{2.0\times10^{-3}\ \text{s}} = +2.2\times10^4\ \text{m/s}^2 \quad \text{upward}$$

The average resultant force acting on the ball during this time interval must have been

$$F_{\text{net}} = ma_{\text{av}} = (0.50\ \text{kg})\left(+2.2\times10^4\ \text{m/s}^2\right) = +1.1\times10^4\ \text{N}$$

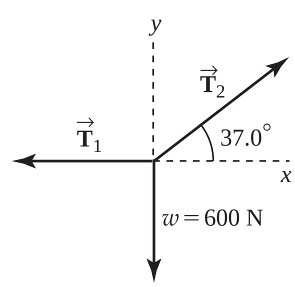
or $\vec{\mathbf{F}}_{\text{net}} = \boxed{1.1\times10^4\ \text{N upward}}$

4.16 Since the two forces are perpendicular to each other, their resultant is

$$F_R = \sqrt{(180\ \text{N})^2 + (390\ \text{N})^2} = 430\ \text{N, at} \qquad \theta = \tan^{-1}\left(\frac{390\ \text{N}}{180\ \text{N}}\right) = 65.2°\ \text{N of E}$$

Thus, $a = \dfrac{F_R}{m} = \dfrac{430\ \text{N}}{270\ \text{kg}} = 1.59\ \text{m/s}^2$ or $\vec{\mathbf{a}} = \boxed{1.59\ \text{m/s}^2\ \text{at}\ 65.2°\ \text{N of E}}$

4.17 (a) Since the burglar is held in equilibrium, the tension in the vertical cable equals the burglar's weight of $\boxed{600\ \text{N}}$.



Now, consider the junction in the three cables:

$\Sigma F_y = 0$, giving $T_2 \sin 37.0° - 600\ \text{N} = 0$

or $T_2 = \dfrac{600\ \text{N}}{\sin 37.0°} = \boxed{997\ \text{N in the inclined cable}}$

Also, $\Sigma F_x = 0$, which yields $T_2 \cos 37.0° - T_1 = 0$

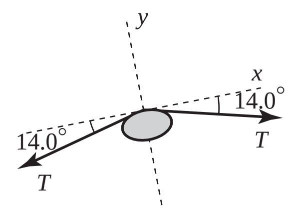
or $T_1 = (997\ \text{N})\cos 37.0° = \boxed{796\ \text{N in the horizontal cable}}$

continued on next page

(b) If the left end of the originally horizontal cable was attached to a point higher up the wall, the tension in this cable would then have an upward component. This upward component would support part of the weight of the cat burglar, thus

$\boxed{\text{decreasing the tension in the cable on the right}}$.

4.18 Using the reference axis shown in the sketch at the right, we see that



$$\Sigma F_x = T\cos 14.0° - T\cos 14.0° = 0$$

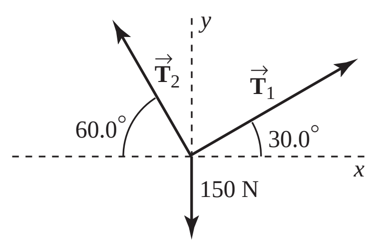
and $$\Sigma F_y = -T\sin 14.0° - T\sin 14.0° = -2T\sin 14.0°$$

Thus, the magnitude of the resultant force exerted on the tooth by the wire brace is

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{0 + (-2T\sin 14.0°)^2} = 2T\sin 14.0°$$

or $$R = 2(18.0\text{ N})\sin 14.0° = \boxed{8.71\text{ N}}$$

4.19 From $\Sigma F_x = 0$, $T_1\cos 30.0° - T_2\cos 60.0° = 0$



or $$T_2 = (1.73)T_1 \qquad \textbf{[1]}$$

The tension in the vertical cable is the full weight of the feeder, or $\boxed{T_{\text{vertical}} = 150\text{ N}}$

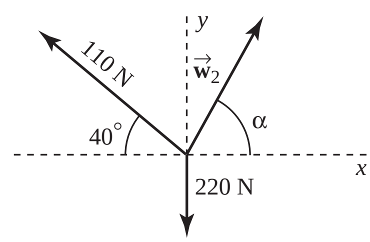
Then $\Sigma F_y = 0$ becomes

$$T_1\sin 30.0° + (1.73\ T_1)\sin 60.0° - 150\text{ N} = 0$$

which gives $T_1 = \boxed{75.1\text{ N in the right side cable}}$.

Finally, Equation [1] above gives $T_2 = \boxed{130\text{ N in the left side cable}}$.

4.20 If the hip exerts no force on the leg, the system must be in equilibrium with the three forces shown in the free-body diagram.



Thus $\Sigma F_x = 0$ becomes

$$w_2\cos\alpha = (110\text{ N})\cos 40° \qquad \textbf{[1]}$$

From $\Sigma F_y = 0$, we find

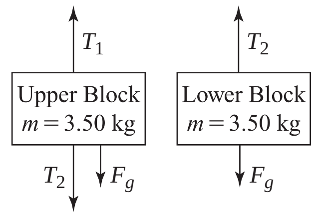
$$w_2\sin\alpha = 220\text{ N} - (110\text{ N})\sin 40° \qquad \textbf{[2]}$$

Dividing Equation [2] by Equation [1] yields

$$\alpha = \tan^{-1}\left(\frac{220\text{ N} - (110\text{ N})\sin 40°}{(110\text{ N})\cos 40°}\right) = \boxed{61°}$$

Then, from either Equation [1] or [2], $w_2 = \boxed{1.7\times10^2\text{ N}}$

4.21 (a) Force diagrams of the two blocks are shown at the right. Note that each block experiences a downward gravitational force



$$F_g = (3.50\text{ kg})(9.80\text{ m/s}^2) = 34.3\text{ N}$$

Also, each has the same upward acceleration as the elevator, in this case $a_y = +1.60\text{ m/s}^2$.

Applying Newton's second law to the lower block:

$$\Sigma F_y = ma_y \quad \Rightarrow \quad T_2 - F_g = ma_y$$

or $$T_2 = F_g + ma_y = 34.3\text{ N} + (3.50\text{ kg})(1.60\text{ m/s}^2) = \boxed{39.9\text{ N}}$$

Next, applying Newton's second law to the upper block:

$$\Sigma F_y = ma_y \quad \Rightarrow \quad T_1 - T_2 - F_g = ma_y$$

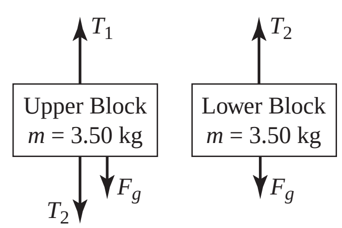
or $$T_1 = T_2 + F_g + ma_y = 39.9\text{ N} + 34.3\text{ N} + (3.50\text{ kg})(1.60\text{ m/s}^2) = \boxed{79.8\text{ N}}$$

(b) As the acceleration of the system increases, we wish to find the value of a_y when the upper string reaches its breaking point (i.e., when $T_1 = 85.0$ N). Making use of the general relationships derived in (a) above gives

$$T_1 = T_2 + F_g + ma_y = (F_g + ma_y) + F_g + ma_y = 2F_g + 2ma_y$$

or $$a_y = \frac{T_1 - 2F_g}{2m} = \frac{85.0\text{ N} - 2(34.3\text{ N})}{2(3.50\text{ kg})} = \boxed{2.34\text{ m/s}^2}$$

4.22 (a) Force diagrams of the two blocks are shown at the right. Note that each block experiences a downward gravitational force $F_g = mg$.



Also, each has the same upward acceleration as the elevator, $a_y = +a$.

Applying Newton's second law to the lower block:

$$\Sigma F_y = ma_y \quad \Rightarrow \quad T_2 - F_g = ma_y \quad \text{or} \quad T_2 = mg + ma = \boxed{m(g+a)}$$

Next, applying Newton's second law to the upper block:

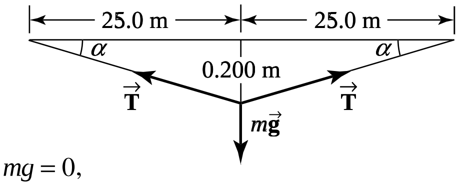
$$\Sigma F_y = ma_y \quad \Rightarrow \quad T_1 - T_2 - F_g = ma_y$$

or $$T_1 = T_2 + F_g + ma_y = (mg + ma) + mg + ma = 2(mg + ma) = \boxed{2T_2}$$

(b) Note that $\boxed{T_1 = 2T_2\text{, so the upper string breaks first}}$ as the acceleration of the system increases.

(c) When the upper string breaks, both blocks will be in free-fall with $a = -g$. Then, using the results of part (a), $T_2 = m(g+a) = m(g-g) = \boxed{0}$ and $T_1 = 2T_2 = \boxed{0}$.

4.23 $m = 1.00 \text{ kg}$ and $mg = 9.80 \text{ N}$

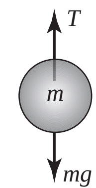


$$\alpha = \tan^{-1}\left(\frac{0.200 \text{ m}}{25.0 \text{ m}}\right) = 0.458°$$

Since $a_y = 0$, this requires that $\Sigma F_y = T\sin\alpha + T\sin\alpha - mg = 0$,

giving $2T\sin\alpha = mg$ or $T = \dfrac{9.80 \text{ N}}{2\sin\alpha} = \boxed{613 \text{ N}}$

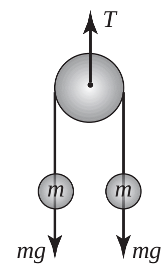
4.24 In each case, the scale is measuring the tension in the cord connecting to it. This tension can be determined by applying Newton's second law (with $\vec{\mathbf{a}} = 0$ for equilibrium) to the object attached to the end of this cord.



(a) $\Sigma F_y = 0 \quad \Rightarrow \quad T - mg = 0$

or $T = mg = (5.00 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{49.0 \text{ N}}$

(b) The solution to part (a) is also the solution to part (b).

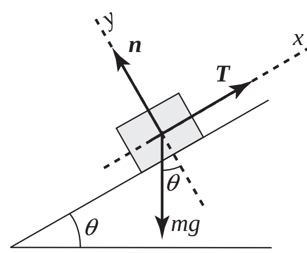


(c) Neglecting the mass of the pulley and the cord supporting the two objects of mass m, we find

$\Sigma F_y = 0 \quad \Rightarrow \quad T - mg - mg = 0$

or $T = 2mg = 2(5.00 \text{ kg})(9.80 \text{ m/s}^2) = \boxed{98.0 \text{ N}}$

(d) In the figure at the right, n is the normal force exerted on the block by the frictionless incline.

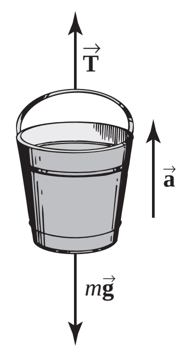


$\Sigma F_x = 0 \quad \Rightarrow \quad T - mg\sin\theta = 0$

or $T = mg\sin\theta = (5.00 \text{ kg})(9.80 \text{ m/s}^2)\sin 30.0°$

and $T = \boxed{24.5 \text{ N}}$

4.25 The forces on the bucket are the tension in the rope and the weight of the bucket, $mg = (5.0 \text{ kg})(9.80 \text{ m/s}^2) = 49 \text{ N}$. Choose the positive direction upward and use Newton's second law:

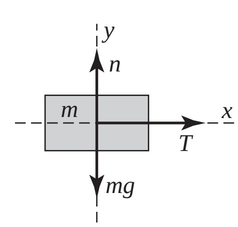


$\Sigma F_y = ma_y$

$T - 49 \text{ N} = (5.0 \text{ kg})(3.0 \text{ m/s}^2)$

$T = \boxed{64 \text{ N}}$

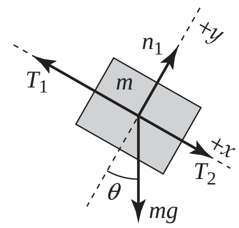
4.26 With the truck accelerating in a forward direction on a horizontal roadway, the acceleration of the crate is the same as that of the truck as long as the cord does not break. Applying Newton's second law to the horizontal motion of the block gives



$\Sigma F_x = ma_x \quad \Rightarrow \quad T = ma \quad \text{or} \quad a = T/m$

Thus $a_{max} = \dfrac{T_{max}}{m} = \dfrac{68 \text{ N}}{32 \text{ kg}} = \boxed{2.1 \text{ m/s}^2}$

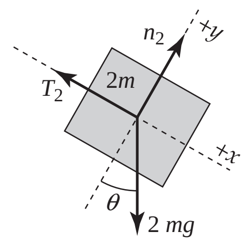
4.27 We choose reference axes that are parallel to and perpendicular to the incline as shown in the force diagrams at the right. Since both blocks are in equilibrium, $a_x = a_y = 0$ for each block. Then, applying Newton's second law to each block gives



For Block 1 (mass m):

$$\Sigma F_x = ma_x \quad \Rightarrow \quad -T_1 + T_2 + mg\sin\theta = 0$$

or $\quad T_1 = T_2 + mg\sin\theta$ **[1]**



For Block 2 (mass $2m$):

$$\Sigma F_x = ma_x \quad \Rightarrow \quad -T_2 + 2mg\sin\theta = 0$$

or $\quad T_2 = 2mg\sin\theta$ **[2]**

(a) Substituting Equation [2] into Equation [1] gives $\boxed{T_1 = 3mg\sin\theta}$

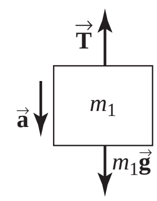
(b) From Equation [2] above, we have $\boxed{T_2 = 2mg\sin\theta}$

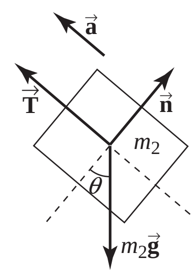
4.28 Let $m_1 = 10.0$ kg, $m_2 = 5.00$ kg, and $\theta = 40.0°$.

(a) Applying the second law to each object gives

$$m_1 a = m_1 g - T \qquad \textbf{[1]}$$

and $$m_2 a = T - m_2 g\sin\theta \qquad \textbf{[2]}$$





Adding these equations yields

$$m_1 a + m_2 a = m_1 g - \cancel{T} + \cancel{T} - m_2 g\sin\theta \quad \text{or} \quad a = \left(\frac{m_1 - m_2\sin\theta}{m_1 + m_2}\right)g$$

so $$a = \left(\frac{10.0\text{ kg} - (5.00\text{ kg})\sin 40.0°}{15.0\text{ kg}}\right)(9.80\text{ m/s}^2) = \boxed{4.43\text{ m/s}^2}$$

(b) Then, Equation [1] yields

$$T = m_1(g - a) = (10.0\text{ kg})\left[(9.80 - 4.43)\text{ m/s}^2\right] = \boxed{53.7\text{ N}}$$

4.29 (a) The resultant external force acting on this system, consisting of all three blocks having a total mass of 6.0 kg, is 42 N directed horizontally toward the right. Thus, the acceleration produced is

$$a = \frac{\Sigma F}{m} = \frac{42\text{ N}}{6.0\text{ kg}} = \boxed{7.0\text{ m/s}^2\text{ horizontally to the right}}$$

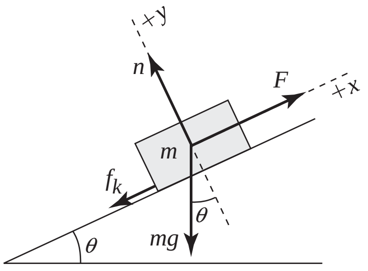
(b) Draw a free body diagram of the 3.0-kg block and apply Newton's second law to the horizontal forces acting on this block:

$$\Sigma F_x = ma_x \Rightarrow \quad 42\text{ N} - T = (3.0\text{ kg})(7.0\text{ m/s}^2),\text{ and therefore } T = \boxed{21\text{ N}}$$

(c) The force accelerating the 2.0-kg block is the force exerted on it by the 1.0-kg block. Therefore, this force is given by

$$F = ma = (2.0\text{ kg})(7.0\text{ m/s}^2), \quad \text{or} \quad \vec{\mathbf{F}} = \boxed{14\text{ N horizontally to the right}}$$

4.30 The figure at the right shows the forces acting on the block. The incline is tilted at $\theta = 25°$, the mass of the block is $m = 5.8$ kg, while the applied force pulling the block up the incline is $F = 32$ N. Since $a_y = 0$ for this block,



$$\Sigma F_y = n - mg\cos\theta = 0$$

and the normal force is $n = mg\cos\theta$

(a) Since the incline is considered frictionless for this part, we take the friction force to be $f_k = 0$ and find

$$\Sigma F_x = F - mg\sin\theta = ma_x \qquad \text{or} \qquad a_x = \frac{F}{m} - g\sin\theta$$

giving $a_x = \dfrac{32\text{ N}}{5.8\text{ kg}} - \left(9.8\text{ m/s}^2\right)\sin 25° = \boxed{1.4\text{ m/s}^2}$

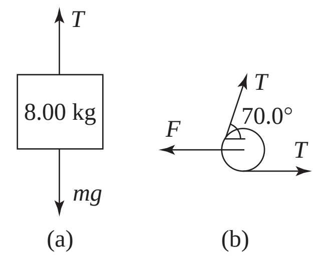
(b) If the coefficient of kinetic friction between the block and the incline is μ_k, then the friction force is $f_k = \mu_k n = \mu_k mg\cos\theta$, and

$$\Sigma F_x = F - f_k - mg\sin\theta = F - mg\left(\mu_k\cos\theta + \sin\theta\right) = ma_x$$

Thus, $a_x = \dfrac{F}{m} - g\left(\mu_k\cos\theta + \sin\theta\right)$

and $a_x = \dfrac{32\text{ N}}{5.8\text{ kg}} - \left(9.8\text{ m/s}^2\right)\left[(0.10)\cos 25° + \sin 25°\right] = \boxed{0.49\text{ m/s}^2}$

4.31 (a) Assuming frictionless pulleys, the tension is uniform through the entire length of the rope. Thus, the tension at the point where the rope attaches to the leg is the same as that at the 8.00 kg block. Part (a) of the sketch at the right gives a force diagram of the suspended block. Recognizing that the block has zero acceleration, Newton's second law gives



$$\Sigma F_y = T - mg = 0$$

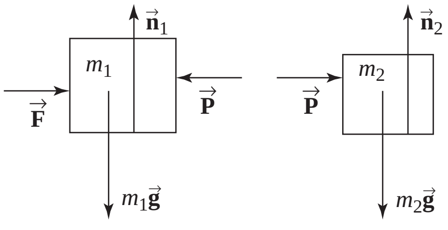
or $T = mg = (8.00\text{ kg})\left(9.80\text{ m/s}^2\right) = \boxed{78.4\text{ N}}$

(b) Part (b) of the sketch above gives a force diagram of the pulley near the foot. Here, F is the magnitude of the force the foot exerts on the pulley. By Newton's third law, this is the same as the magnitude of the force the pulley exerts on the foot. Applying the second law gives

$$\Sigma F_x = T + T\cos 70.0° - F = ma_x = 0$$

or $F = T(1 + \cos 70.0°) = (78.4\text{ N})(1 + \cos 70.0°) = \boxed{105\text{ N}}$

4.32 (a)



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(b) Note that the blocks move on a horizontal surface with $a_y = 0$. Thus, the net vertical force acting on each block and on the combined system of both blocks is zero. The net horizontal force acting on the combined system consisting of both m_1 and m_2 is $\Sigma F_x = F - P + P = \boxed{F}$.

(c) Looking at just m_1, $\Sigma F_y = 0$ as explained above, while $\Sigma F_x = \boxed{F - P}$.

(d) Looking at just m_2, we again have $\Sigma F_y = 0$, while $\Sigma F_x = \boxed{+P}$.

(e) For m_1: $\quad \Sigma F_x = ma_x \quad \Rightarrow \quad \boxed{F - P = m_1 a}$

For m_2: $\quad \Sigma F_x = ma_x \quad \Rightarrow \quad \boxed{P = m_2 a}$

(f) Substituting the second of the equations found in (e) above into the first gives

$$m_1 a = F - P = F - m_2 a \qquad \text{or} \qquad (m_1 + m_2)a = F \qquad \text{and} \qquad \boxed{a = F/(m_1 + m_2)}$$

Then substituting this result into the second equation from (e), we have

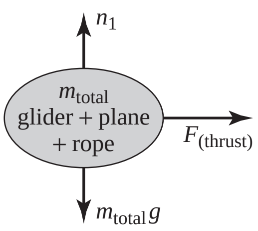
$$P = m_2 a = m_2\left(\frac{F}{m_1 + m_2}\right) \qquad \text{or} \qquad \boxed{P = \left(\frac{m_2}{m_1 + m_2}\right)F}$$

(g) Realize that applying the force to m_2 rather than m_1 would have the effect of interchanging the roles of m_1 and m_2. We may easily find the results for that case by simply interchanging the labels m_1 and m_2 in the results found in (f) above. This gives $a = F/(m_2 + m_1)$

(the same result as in the first case) and $\boxed{P = \left(\frac{m_1}{m_2 + m_1}\right)F}$.

We see that $\boxed{\text{the contact force, } P\text{, is larger in this case because } m_1 > m_2}$.

4.33 (a) First, we consider the glider, plane, and connecting rope to be a single unit having mass

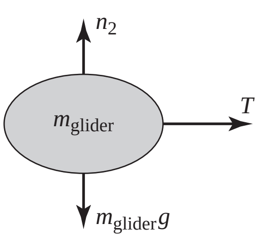


$$m_{total} = 276 \text{ kg} + 1\,950 \text{ kg} = 2\,226 \text{ kg}$$

For this system, the tension in the rope is an internal force and is not included in an application of Newton's second law. Applying the second law to the horizontal motion of this combined system gives

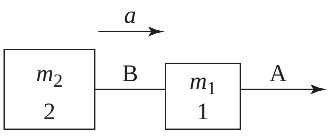
$$\Sigma F_x = m_{total} a_x \quad \Rightarrow \quad F = (2\,226 \text{ kg})(2.20 \text{ m/s}^2) = 4.90 \times 10^3 \text{ N} = \boxed{4.90 \text{ kN}}$$

(b) To determine the tension in the rope connecting the glider and the plane, we consider a system consisting of the glider alone. For this system, the rope is a external agent and the tension force it exerts on our system (glider) is included in a second law calculation.



$$\Sigma F_x = m_{glider} a_x \quad \Rightarrow \quad T = (276 \text{ kg})(2.20 \text{ m/s}^2) = \boxed{607 \text{ N}}$$

4.34 (a) First, consider a system consisting of the two blocks combined, with mass $m_1 + m_2$. For this system, the only external horizontal force is the tension in cord A pulling to the right. The tension in cord B is a force one part of our system exerts on another part of our system, and is therefore an internal force.



Applying Newton's second law to this system (including only external forces, as we should) gives

$$\Sigma F_x = ma \qquad \Rightarrow \qquad T_A = (m_1 + m_2)a \qquad \textbf{[1]}$$

Now, consider a system consisting of only m_2. For this system, the tension in cord B is an external force since it is a force exerted on block 2 by block 1 (which is not part of this system). Applying Newton's second law to this system gives

$$\Sigma F_x = ma \qquad \Rightarrow \qquad T_B = m_2 a \qquad \textbf{[2]}$$

Comparing Equations [1] and [2], and realizing that the acceleration is the same in both cases (see part (b) below), it is clear that

$\boxed{\text{Cord A exerts a larger force on block 1 than cord B exerts on block 2}}$.

(b) Since cord B connecting the two blocks is taut and unstretchable, the two blocks stay a fixed distance apart, and the velocities of the two blocks must be equal at all times. Thus, the rates at which the velocities of the two blocks change in time are equal, or $\boxed{\text{the two blocks must have equal accelerations}}$.

(c) $\boxed{\text{Yes. Block 1 exerts a forward force on Cord B, so Newton's third law tells us that Cord B exerts a force of equal magnitude in the backward direction on Block 1.}}$

4.35 (a) When the acceleration is upward, the total upward force $\boxed{T}$ must exceed the total downward force $w = mg = (1\,500 \text{ kg})(9.80 \text{ m/s}^2) = 1.47 \times 10^4 \text{ N}$.

(b) When the velocity is constant, the acceleration is zero. The total upward force T and the total downward force w must be $\boxed{\text{equal in magnitude}}$.

(c) If the acceleration is directed downward, the total downward force $\boxed{w}$ must exceed the total upward force T.

(d) $\Sigma F_y = ma_y \quad \Rightarrow \quad T = mg + ma_y = (1\,500 \text{ kg})(9.80 \text{ m/s}^2 + 2.50 \text{ m/s}^2) = \boxed{1.85 \times 10^4 \text{ N}}$

$\boxed{\text{Yes}}$, $T > w$.

(e) $\Sigma F_y = ma_y \quad \Rightarrow \quad T = mg + ma_y = (1\,500 \text{ kg})(9.80 \text{ m/s}^2 + 0) = \boxed{1.47 \times 10^4 \text{ N}}$

$\boxed{\text{Yes}}$, $T = w$.

(f) $\Sigma F_y = ma_y \quad \Rightarrow \quad T = mg + ma_y = (1\,500 \text{ kg})(9.80 \text{ m/s}^2 - 1.50 \text{ m/s}^2) = \boxed{1.25 \times 10^4 \text{ N}}$

$\boxed{\text{Yes}}$, $T < w$.

4.36 Note that if the cord connecting the two blocks has a fixed length, the accelerations of the blocks must have equal magnitudes, even though they differ in directions. Also, observe from the diagrams, we choose the positive direction for each block to be in its direction of motion.

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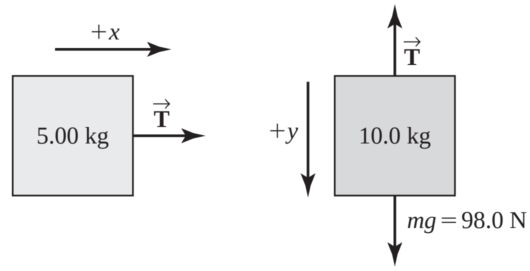
First consider the block moving along the horizontal. The only force in the direction of movement is T. Thus,

$$\Sigma F_x = ma_x \;\Rightarrow\; T = (5.00\text{ kg})a \quad \textbf{[1]}$$



(a) (b)

Next consider the block which moves vertically. The forces on it are the tension T and its weight, 98.0 N.

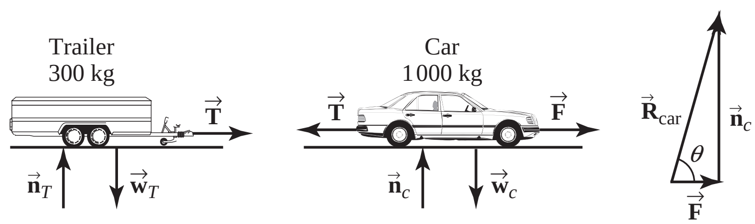
$$\Sigma F_y = ma_y \Rightarrow \; 98.0\text{ N} - T = (10.0\text{ kg})a \quad \textbf{[2]}$$

Equations [1] and [2] can be solved simultaneously to give

(a) $98.0\text{ N} - (5.00\text{ kg})a = (10.0\text{ kg})a$ or $a = \dfrac{98.0\text{ N}}{15.0\text{ kg}} = \boxed{6.53\text{ m/s}^2}$

(b) Then, Equation [1] yields $T = (5.00\text{ kg})(6.53\text{ m/s}^2) = \boxed{32.7\text{ N}}$

4.37



Choosing the +x-direction to be horizontal and forward, the +y-direction vertical and upward, the common acceleration of the car and trailer has components of $\boxed{a_x = +2.15\text{ m/s}^2}$ and $\boxed{a_y = 0}$.

(a) The net force on the car is horizontal and given by

$$(\Sigma F_x)_{\text{car}} = F - T = m_{\text{car}}a_x = (1\,000\text{ kg})(2.15\text{ m/s}^2) = \boxed{2.15\times10^3\text{ N forward}}$$

(b) The net force on the trailer is also horizontal and given by

$$(\Sigma F_x)_{\text{trailer}} = +T = m_{\text{trailer}}a_x = (300\text{ kg})(2.15\text{ m/s}^2) = \boxed{645\text{ N forward}}$$

(c) Consider the free-body diagrams of the car and trailer. The only horizontal force acting on the trailer is T = 645 N forward, and this is exerted on the trailer by the car. Newton's third law then states that the force the trailer exerts on the car is $\boxed{\text{645 N toward the rear}}$.

(d) The road exerts two forces on the car. These are F and n_c shown in the free-body diagram of the car.

From part (a), $F = T + 2.15\times10^3\text{ N} = 645\text{ N} + 2.15\times10^3\text{ N} = +2.80\times10^3\text{ N}$

Also, $(\Sigma F_y)_{\text{car}} = n_c - w_c = m_{\text{car}}a_y = 0$, so $n_c = w_c = m_{\text{car}}g = 9.80\times10^3\text{ N}$

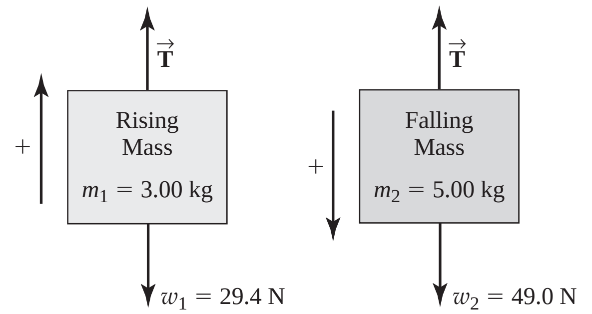
The resultant force exerted on the car by the road is then

$$R_{car} = \sqrt{F^2 + n_c^2} = \sqrt{(2.80\times10^3\text{ N})^2 + (9.80\times10^3\text{ N})^2} = 1.02\times10^4\text{ N}$$

at $\theta = \tan^{-1}(n_c/F) = \tan^{-1}(3.50) = 74.1°$ above the horizontal and forward. Newton's third law then states that the resultant force exerted on the road by the car is

$\boxed{1.02\times10^4\text{ N at }74.1°\text{ below the horizontal and rearward}}$.

4.38 First, consider the 3.00-kg rising mass. The forces on it are the tension, T, and its weight, 29.4 N. With the upward direction as positive, the second law becomes



$$T - 29.4 \text{ N} = (3.00 \text{ kg})a \qquad \textbf{[1]}$$

The forces on the falling 5.00-kg mass are its weight and T, and its acceleration has the same magnitude as that of the rising mass. Choosing the positive direction down for this mass, gives

$$49.0 \text{ N} - T = (5.00 \text{ kg})a \qquad \textbf{[2]}$$

(a) Solving Equation [2] for a and substituting into [1] gives

$$T - 29.4 \text{ N} = \left(\frac{3.00 \text{ kg}}{5.00 \text{ kg}}\right)(49.0 \text{ N} - T) \qquad \text{or} \qquad 1.60\,T = 58.8 \text{ N}$$

and the tension is $T = \boxed{36.8 \text{ N}}$

(b) Equation [2] then gives the acceleration as

$$a = \frac{49.0 \text{ N} - 36.8 \text{ N}}{5.00 \text{ kg}} = \boxed{2.44 \text{ m/s}^2}$$

(c) Consider the 3.00-kg mass. We have

$$\Delta y = v_{0y}t + \frac{1}{2}a_y t^2 = 0 + \frac{1}{2}(2.44 \text{ m/s}^2)(1.00 \text{ s})^2 = \boxed{1.22 \text{ m}}$$

4.39 When the block is on the verge of moving, the static friction force has a magnitude $f_s = (f_s)_{\text{max}} = \mu_s n$.

Since equilibrium still exists and the applied force is 75 N, we have

$$\Sigma F_x = 75 \text{ N} - f_s = 0 \qquad \text{or} \qquad (f_s)_{\text{max}} = 75 \text{ N}$$

In this case, the normal force is just the weight of the crate, or $n = mg$. Thus, the coefficient of static friction is

$$\mu_s = \frac{(f_s)_{\text{max}}}{n} = \frac{(f_s)_{\text{max}}}{mg} = \frac{75 \text{ N}}{(20 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{0.38}$$

After motion exists, the friction force is that of kinetic friction, $f_k = \mu_k n$.

Since the crate moves with constant velocity when the applied force is 60 N, we find that $\Sigma F_x = 60 \text{ N} - f_k = 0$ or $f_k = 60$ N. Therefore, the coefficient of kinetic friction is

$$\mu_k = \frac{f_k}{n} = \frac{f_k}{mg} = \frac{60 \text{ N}}{(20 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{0.31}$$

4.40 (a) The static friction force attempting to prevent motion may reach a maximum value of

$$\left(f_s\right)_{max} = \mu_s n_1 = \mu_s m_1 g = (0.50)(10\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 49\ \text{N}$$

This exceeds the force attempting to move the system, $w_2 = m_2 g = 39$ N. Hence, the system remains at rest and the acceleration is $a = \boxed{0}$.

(b) Once motion begins, the friction force retarding the motion is

$$f_k = \mu_k n_1 = \mu_k m_1 g = (0.30)(10\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 29\ \text{N}$$

This is less than the force trying to move the system, $w_2 = m_2 g$. Hence, the system gains speed at the rate

$$a = \frac{F_{net}}{m_{total}} = \frac{m_2 g - \mu_k m_1 g}{m_1 + m_2} = \frac{\left[4.0\ \text{kg} - 0.30(10\ \text{kg})\right]\left(9.80\ \text{m/s}^2\right)}{4.0\ \text{kg} + 10\ \text{kg}} = \boxed{0.70\ \text{m/s}^2}$$

4.41 (a) Since the crate has constant velocity, $a_x = a_y = 0$.

Applying Newton's second law:

$$\Sigma F_x = F\cos 20.0° - f_k = ma_x = 0 \quad \text{or} \quad f_k = (300\ \text{N})\cos 20.0° = 282\ \text{N}$$

and $\Sigma F_y = n - F\sin 20.0° - w = 0$ or $n = (300\ \text{N})\sin 20.0° + 1\,000\ \text{N} = 1.10 \times 10^3\ \text{N}$

The coefficient of friction is then $\mu_k = \dfrac{f_k}{n} = \dfrac{282\ \text{N}}{1.10 \times 10^3\ \text{N}} = \boxed{0.256}$.

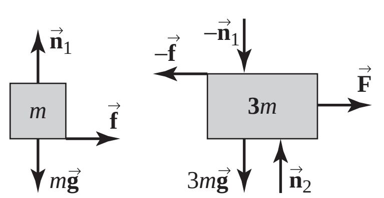
(b) In this case, $\Sigma F_y = n + F\sin 20.0° - w = 0$ so $n = w - F\sin 20.0° = 897$ N

The friction force now becomes $f_k = \mu_k n = (0.256)(897\ \text{N}) = 230\ \text{N}$

Therefore, $\Sigma F_x = F\cos 20.0° - f_k = ma_x = (w/g)a_x$ and the acceleration is

$$a = \frac{\left(F\cos 20.0° - f_k\right)g}{w} = \frac{\left[(300\ \text{N})\cos 20.0° - 230\ \text{N}\right]\left(9.80\ \text{m/s}^2\right)}{1\,000\ \text{N}} = \boxed{0.509\ \text{m/s}^2}$$

4.42 (a)



(b) $\boxed{\text{The static friction force, } \vec{\mathbf{f}},}$ exerted on the upper block (mass *m*) by the lower block causes the upper block to accelerate toward the right.

(c) As long as slipping does not occur between the two blocks, both have acceleration *a* directed toward the right. Apply Newton's second law to the horizontal motion of the upper block [see the leftmost diagram in part (a)] to find

$$\Sigma F_x = ma_x \quad \Rightarrow \quad f = ma$$

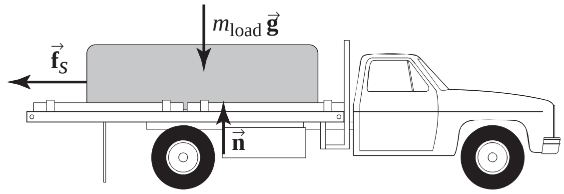
continued on next page

Now, we make use of this result as we apply Newton's second law to the horizontal motion of the lower block (mass $3m$):

$$\Sigma F_x = ma_x \quad \Rightarrow \quad F - f = (3m)a \qquad \text{or} \qquad F = f + 3ma$$

Thus, $\quad F = ma + 3ma = 4ma \qquad$ or $\qquad \boxed{a = F/4m}$

4.43 When the load is on the verge of sliding forward on the bed of the slowing truck, the rearward-directed static friction force has its maximum value



$$(f_s)_{\max} = \mu_s n = \mu_s m_{\text{load}} g$$

Since slipping is not yet occurring, this single horizontal force must give the load an acceleration equal to that the truck.

Thus, $\Sigma F_x = ma_x \quad \Rightarrow \quad -\mu_s \cancel{m_{\text{load}}} g = \cancel{m_{\text{load}}}(a_{\text{truck}})_{\max} \qquad$ or $\qquad (a_{\text{truck}})_{\max} = -\mu_s g$

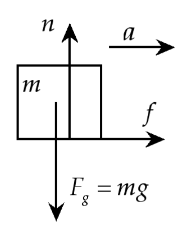
(a) If slipping is to be avoided, the maximum allowable rearward acceleration of the truck is seen to be $(a_{\text{truck}})_{\max} = -\mu_s g$, and $v_x^2 = v_{0x}^2 + 2a_x(\Delta x)$ gives the minimum stopping distance as

$$(\Delta x)_{\min} = \frac{0 - v_{0x}^2}{2(a_{\text{truck}})_{\max}} = \frac{v_{0x}^2}{2\mu_s g}$$

If $v_{0x} = 12\ \text{m/s}$ and $\mu_s = 0.500$, then $\quad (\Delta x)_{\min} = \dfrac{(12.0\ \text{m/s})^2}{2(0.500)(9.80\ \text{m/s}^2)} = \boxed{14.7\ \text{m}}$

(b) Examining the calculation of part (a) shows that $\boxed{\text{neither mass is necessary}}$.

4.44 (a) The free-body diagram of the crate is shown at the right. Since the crate has no vertical acceleration $(a_y = 0)$, we see that



$$\Sigma F_y = ma_y \qquad \Rightarrow \qquad n - mg = 0 \qquad \text{or} \qquad n = mg$$

The only horizontal force present is the friction force exerted on the crate by the truck bed. Thus,

$$\Sigma F_x = ma_x \qquad \Rightarrow \qquad f = ma$$

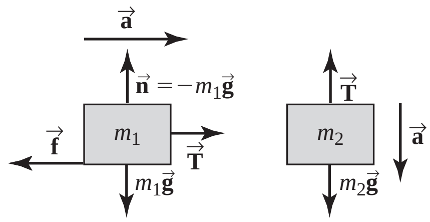
If the crate is not to slip (i.e., the static case is to prevail), it is necessary that the required friction force not exceed the maximum possible static friction force, $(f_s)_{\max} = \mu_s n$. From this, we find the maximum allowable acceleration as

$$a_{\max} = \frac{f_{\max}}{m} = \frac{(f_s)_{\max}}{m} = \frac{\mu_s n}{m} = \frac{\mu_s(mg)}{m} = \mu_s g = (0.350)(9.80\ \text{m/s}^2) = \boxed{3.43\ \text{m/s}^2}$$

(b) Once slipping has started, the kinetic friction case prevails and $f = f_k = \mu_k n$. The acceleration of the crate in this case will be

$$a = \frac{f}{m} = \frac{\mu_k n}{m} = \frac{\mu_k(mg)}{m} = \mu_k g = (0.320)(9.80\ \text{m/s}^2) = \boxed{3.14\ \text{m/s}^2}$$

4.45 The acceleration of the system is found from



$$\Delta y = v_{0y}t + \frac{1}{2}at^2, \text{ or } 1.00 \text{ m} = 0 + \frac{1}{2}a(1.20 \text{ s})^2$$

which gives $a = 1.39 \text{ m/s}^2$.

Using the force diagram of m_2, the second law gives

$$(5.00 \text{ kg})(9.80 \text{ m/s}^2) - T = (5.00 \text{ kg})(1.39 \text{ m/s}^2) \qquad \text{or} \qquad T = 42.1 \text{ N}$$

Then applying the second law to the horizontal motion of m_1,

$$42.1 \text{ N} - f = (10.0 \text{ kg})(1.39 \text{ m/s}^2) \qquad \text{or} \qquad f = 28.2 \text{ N}$$

Since $n = m_1 g = 98.0 \text{ N}$, we have $\mu_k = \dfrac{f}{n} = \dfrac{28.2 \text{ N}}{98.0 \text{ N}} = \boxed{0.288}$

4.46 (a) Since the puck is on a horizontal surface, the normal force is vertical. With $a_y = 0$, we see that

$$\Sigma F_y = ma_y \quad \Rightarrow \quad n - mg = 0 \qquad \text{or} \qquad n = mg$$

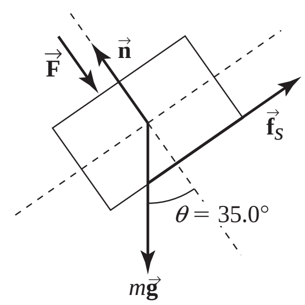
Once the puck leaves the stick, the only horizontal force is a friction force in the negative x-direction (to oppose the motion of the puck). The acceleration of the puck is

$$a_x = \frac{\Sigma F_x}{m} = \frac{-f_k}{m} = \frac{-\mu_k n}{m} = \frac{-\mu_k(mg)}{m} = \boxed{-\mu_k g}$$

(b) Then $v_x^2 = v_{0x}^2 + 2a_x(\Delta x)$ gives the horizontal displacement of the puck before coming to rest as

$$\Delta x = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0 - v_0^2}{2(-\mu_k g)} = \boxed{\frac{v_0^2}{2\mu_k g}}$$

4.47 The crate does not accelerate perpendicular to the incline. Thus,



$$\Sigma F_\perp = ma_\perp = 0 \quad \Rightarrow \quad n = F + mg\cos\theta$$

The net force tending to move the crate down the incline is $\Sigma F_\parallel = mg\sin\theta - f_s$, where f_s is the force of static friction between the crate and the incline. If the crate is in equilibrium, then

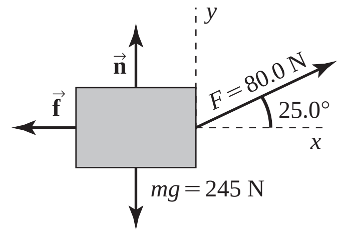
$$mg\sin\theta - f_s = 0 \qquad \text{so} \qquad f_s = F_g \sin\theta$$

But, we also know $f_s \le \mu_s n = \mu_s(F + mg\cos\theta)$

Therefore, we may write $\quad mg\sin\theta \le \mu_s(F + mg\cos\theta)$

or $\quad F \ge mg\left(\dfrac{\sin\theta}{\mu_s} - \cos\theta\right) = (3.00 \text{ kg})(9.80 \text{ m/s}^2)\left(\dfrac{\sin 35.0°}{0.300} - \cos 35.0°\right) = \boxed{32.1 \text{ N}}$

4.48 (a) Find the normal force $\vec{\mathbf{n}}$ on the 25.0 kg box:



$$\Sigma F_y = n + (80.0\text{ N})\sin 25.0° - 245\text{ N} = 0$$

or $\quad n = 211\text{ N}$

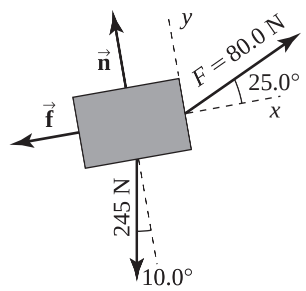
Now find the friction force, *f*, as

$$f = \mu_k n = 0.300(211\text{ N}) = 63.3\text{ N}$$

From the second law, we have $\Sigma F_x = ma$, or

$$(80.0\text{ N})\cos 25.0° - 63.3\text{ N} = (25.0\text{ kg})a \quad \text{which yields} \quad a = \boxed{0.368\ \text{m/s}^2}$$

(b) When the box is on the incline,



$$\Sigma F_y = n + (80.0\text{ N})\sin 25.0° - (245\text{ N})\cos 10.0° = 0$$

giving $\quad n = 207\text{ N}$

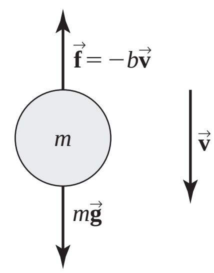
The friction force is $f = \mu_k n = 0.300(207\text{ N}) = 62.1\text{ N}$.

The net force parallel to the incline is then

$$\Sigma F_x = (80.0\text{ N})\cos 25.0° - (245\text{ N})\sin 10.0° - 62.1\text{ N} = -32.1\text{ N}$$

Thus, $a = \dfrac{\Sigma F_x}{m} = \dfrac{-32.1\text{ N}}{25.0\text{ kg}} = -1.28\ \text{m/s}^2$ $\quad$ or $\quad$ $\boxed{1.28\ \text{m/s}^2 \text{ down the incline}}$

4.49 (a) The object will fall so that $ma = mg - bv$, or $a = \dfrac{(mg - bv)}{m}$, where the downward direction is taken as positive.



Equilibrium $(a = 0)$ is reached when

$$v = v_{\text{terminal}} = \frac{mg}{b} = \frac{(50\text{ kg})(9.80\ \text{m/s}^2)}{15\ \text{kg/s}} = \boxed{33\ \text{m/s}}$$

(b) If the initial velocity is less than 33 m/s, then $a \geq 0$ and 33 m/s is the largest velocity attained by the object. On the other hand, if the initial velocity is *greater* than 33 m/s, then $a \leq 0$ and 33 m/s is the *smallest* velocity attained by the object. Note also that if the initial velocity is 33 m/s, then $a = 0$ and the object continues falling with a constant speed of 33 m/s.

4.50 (a) The force of friction is found as $\quad f = \mu_k n = \mu_k(mg)$

Choose the positive direction of the *x*-axis in the direction of motion and apply Newton's second law. We have

$$\Sigma F_x = -f = ma_x \qquad \text{or} \qquad a_x = \frac{-f}{m} = -\mu_k g$$

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From $v^2 = v_0^2 + 2a(\Delta x)$, with $v = 0$, $v_0 = 50.0\ \text{km/h} = 13.9\ \text{m/s}$, we find

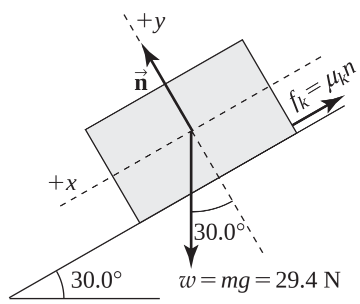
$$0 = (13.9\ \text{m/s})^2 + 2(-\mu_k g)(\Delta x) \qquad \text{or} \qquad \Delta x = \frac{(13.9\ \text{m/s})^2}{2\mu_k g} \qquad \textbf{[1]}$$

With $\mu_k = 0.100$, this gives $\Delta x = \dfrac{(13.9\ \text{m/s})^2}{2(0.100)(9.80\ \text{m/s}^2)} = \boxed{98.6\ \text{m}}$

(b) With $\mu_k = 0.600$, Equation [1] above gives $\Delta x = \dfrac{(13.9\ \text{m/s})^2}{2(0.600)(9.80\ \text{m/s}^2)} = \boxed{16.4\ \text{m}}$

4.51 (a) $\Delta x = v_0 t + \frac{1}{2} a_x t^2 = 0 + \frac{1}{2} a_x t^2$ gives $a_x = \dfrac{2(\Delta x)}{t^2} = \dfrac{2(2.00\ \text{m})}{(1.50\ \text{s})^2} = \boxed{1.78\ \text{m/s}^2}$

(b) Considering forces parallel to the incline, Newton's second law yields



$$\Sigma F_x = (29.4\ \text{N})\sin 30.0° - f_k = (3.00\ \text{kg})(1.78\ \text{m/s}^2)$$

or $\quad f_k = 9.36\ \text{N}$

Perpendicular to the plane, we have equilibrium, so

$$\Sigma F_y = n - (29.4\ \text{N})\cos 30.0° = 0 \qquad \text{or} \qquad n = 25.5\ \text{N}$$

Then, $\quad \mu_k = \dfrac{f_k}{n} = \dfrac{9.36\ \text{N}}{25.5\ \text{N}} = \boxed{0.367}$

(c) From part (b) above, $\quad f_k = \boxed{9.36\ \text{N}}$

(d) Finally, $v^2 = v_0^2 + 2a_x(\Delta x)$ gives

$$v = \sqrt{v_0^2 + 2a_x(\Delta x)} = \sqrt{0 + 2(1.78\ \text{m/s}^2)(2.00\ \text{m})} = \boxed{2.67\ \text{m/s}}$$

4.52 (a) When the block is resting in equilibrium $(a_y = 0)$ on a horizontal floor with only two vertical forces acting on it, the upward normal force exerted on the block by the floor must equal the downward gravitational force. That is,

$$\Sigma F_y = n - mg = 0 \qquad \Rightarrow \qquad n = mg = \boxed{15.0\ \text{lb}}$$

(b) When a 10.0-lb object hangs in equilibrium on one end of the rope, the tension in the rope must be $T = 10.0$ lb. Thus, the 15.0-lb block has now has three vertical forces acting on it: an upward-directed tension force, $T = 10.0$ lb, a downward gravitational force of magnitude $mg = 15.0$ lb, and the upward normal force, n, exerted by the floor. Since the block is in equilibrium, $a_y = 0$ and we find

$$\Sigma F_y = n + T - mg = 0 \qquad \text{or} \qquad n = mg - T = 15.0\ \text{lb} - 10.0\ \text{lb} = \boxed{5.00\ \text{lb}}$$

(c) If the hanging object on the end of the rope is heavier than the 15.0-lb block, the system will not remain in equilibrium. Rather, the hanging object will accelerate downward, lifting the 15.0-lb block off the floor. As soon as the block leaves the floor, the normal force exerted on the block by the floor becomes $\boxed{\text{zero}}$.

4.53 When a person walks in a forward direction on a level floor, the force propelling them is a forward reaction force equal in magnitude to the rearward static friction force, f_s, their foot exerts on the floor. The normal force exerted on the person by the level floor is $n = mg$. If the person is to have maximum acceleration (in order to travel distance $d = 3.00\ \text{m}$ in minimum time), the static friction force must have its maximum value, $(f_s)_{\max} = \mu_s n = \mu_s mg$, so the maximum acceleration possible is

$$a_{\max} = \frac{(f_s)_{\max}}{m} = \frac{\mu_s mg}{m} = \mu_s g$$

The minimum time required to travel distance d (starting from rest) is then given by $\Delta x = v_0 t + \frac{1}{2}at^2$ as

$$3.00\ \text{m} = 0 + \frac{1}{2}(\mu_s g)t_{\min}^2 \qquad \text{or} \qquad t_{\min} = \sqrt{\frac{6.00\ \text{m}}{\mu_s g}}$$

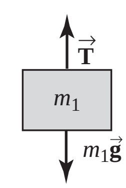
(a) If $\mu_s = 0.500$, $\quad t_{\min} = \sqrt{\dfrac{6.00\ \text{m}}{0.500(9.80\ \text{m/s}^2)}} = \boxed{1.11\ \text{s}}$

(b) If $\mu_s = 0.800$, $\quad t_{\min} = \sqrt{\dfrac{6.00\ \text{m}}{0.800(9.80\ \text{m/s}^2)}} = \boxed{0.875\ \text{s}}$

4.54 (a) Both objects start from rest and have accelerations of the same magnitude, a. This magnitude can be determined by applying $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ to the motion of m_1:

$$a = \frac{2(\Delta y)}{t^2} = \frac{2(1.00\ \text{m})}{(4.00\text{s})^2} = \boxed{0.125\ \text{m/s}^2}$$

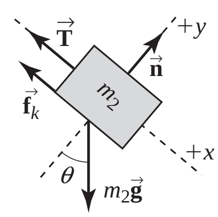
(b) Consider the free-body diagram of m_1 and apply Newton's second law:



$$\Sigma F_y = ma_y \quad \Rightarrow \quad T - m_1 g = m_1(+a)$$

or $\quad T = m_1(g + a) = (4.00\ \text{kg})(9.80\ \text{m/s}^2 + 0.125\ \text{m/s}^2) = \boxed{39.7\ \text{N}}$

(c) Considering the free-body diagram of m_2:



$$\Sigma F_y = ma_y \quad \Rightarrow \quad n - m_2 g\cos\theta = 0 \qquad \text{or} \qquad n = m_2 g\cos\theta$$

so $\quad n = (9.00\ \text{kg})(9.80\ \text{m/s}^2)\cos 40.0° = 67.6\ \text{N}$

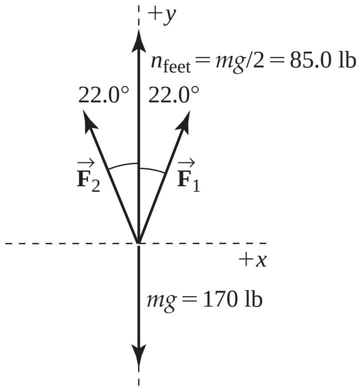
$$\Sigma F_x = ma_x \quad \Rightarrow \quad m_2 g\sin\theta - T - f_k = m_2(+a)$$

Then $\quad f_k = m_2(g\sin\theta - a) - T$

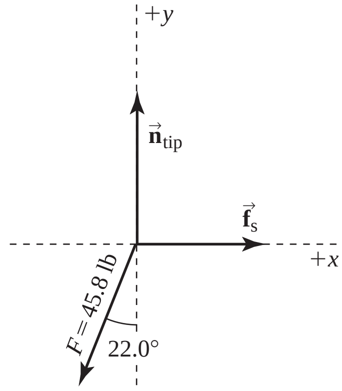
or $\quad f_k = (9.00\ \text{kg})\left[(9.80\ \text{m/s}^2)\sin 40.0° - 0.125\ \text{m/s}^2\right] - 39.7\ \text{N} = 15.9\ \text{N}$

The coefficient of kinetic friction is $\mu_k = \dfrac{f_k}{n} = \dfrac{15.9\ \text{N}}{67.6\ \text{N}} = \boxed{0.235}$.

4.55



Free-Body Diagram of Person



Free-Body Diagram of Crutch Tip

From the free-body diagram of the person,

$$\Sigma F_x = F_1 \sin(22.0°) - F_2 \sin(22.0°) = 0, \text{ which gives or } F_1 = F_2 = F$$

Then, $\Sigma F_y = 2F\cos 22.0° + 85.0 \text{ lbs} - 170 \text{ lbs} = 0$ yields $F = 45.8$ lb.

(a) Now consider the free-body diagram of a crutch tip.

$$\Sigma F_x = f_s - (45.8 \text{ lb})\sin 22.0° = 0, \text{ or } \qquad f_s = 17.2 \text{ lb}$$

$$\Sigma F_y = n_{\text{tip}} - (45.8 \text{ lb})\cos 22.0° = 0, \text{ which gives } \qquad n_{\text{tip}} = 42.5 \text{ lb}$$

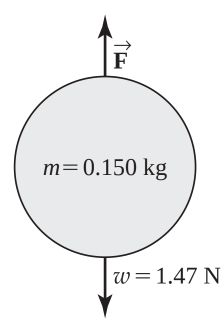
For minimum coefficient of friction, the crutch tip will be on the verge of slipping, so

$$f_s = (f_s)_{\max} = \mu_s n_{\text{tip}} \quad \text{and} \quad \mu_s = \frac{f_s}{n_{\text{tip}}} = \frac{17.2 \text{ lb}}{42.5 \text{ lb}} = \boxed{0.405}$$

(b) As found above, the compression force in each crutch is

$$F_1 = F_2 = F = \boxed{45.8 \text{ lb}}$$

4.56 The acceleration of the ball is found from

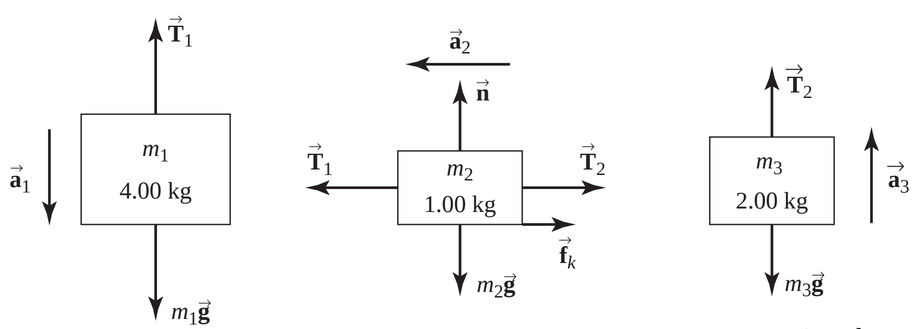


$$a = \frac{v^2 - v_0^2}{2(\Delta y)} = \frac{(20.0 \text{ m/s})^2 - 0}{2(1.50 \text{ m})} = 133 \text{ m/s}^2$$

From the second law, $\Sigma F_y = F - w = ma_y$, so

$$F = w + ma_y = 1.47 \text{ N} + (0.150 \text{ kg})(133 \text{ m/s}^2) = \boxed{21.4 \text{ N}}$$

4.57 (a)



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(b) Note that the suspended block on the left, m_1, is heavier than that on the right, m_3. Thus, if the system overcomes friction and moves, the center block will move right to left with each block's acceleration being $\boxed{\text{in the directions shown above}}$.

First, consider the center block, m_2, which has no vertical acceleration. Then,

$$\Sigma F_y = n - m_2 g = 0 \qquad \text{or} \qquad n = m_2 g = (1.00\ \text{kg})(9.80\ \text{m/s}^2) = 9.80\ \text{N}$$

This means the friction force is: $f_k = \mu_k n = (0.350)(9.80\ \text{N}) = 3.43\ \text{N}$

Assuming the cords do not stretch, the speeds of the three blocks must always be equal. Thus, the magnitudes of the blocks' accelerations must have a common value, *a*.

$$|\vec{\mathbf{a}}_1| = |\vec{\mathbf{a}}_2| = |\vec{\mathbf{a}}_3| = a$$

Taking the indicated direction of the acceleration as the positive direction of motion for each block, we apply Newton's second law to each block as follows:

For m_1: $\quad m_1 g - T_1 = m_1 a \quad$ or $\quad T_1 = m_1(g-a) = (4.00\ \text{kg})(g-a) \quad$ **[1]**

For m_2: $\quad T_1 - T_2 - f_k = m_2 a \quad$ or $\quad T_1 - T_2 = (1.00\ \text{kg})a + 3.43\ \text{N} \quad$ **[2]**

For m_3: $\quad T_2 - m_3 g = m_3 a \quad$ or $\quad T_2 = m_3(g+a) = (2.00\ \text{kg})(g+a) \quad$ **[3]**

Substituting Equations [1] and [3] into Equation [2], and solving for *a* yields

$$(4.00\ \text{kg})(g-a) - (2.00\ \text{kg})(g+a) = (1.00\ \text{kg})a + 3.43\ \text{N}$$

$$a = \frac{(4.00\ \text{kg} - 2.00\ \text{kg})(9.80\ \text{m/s}^2) - 3.43\ \text{N}}{4.00\ \text{kg} + 2.00\ \text{kg} + 1.00\ \text{kg}} = 2.31\ \text{m/s}^2$$

so $\boxed{\vec{\mathbf{a}}_1 = 2.31\ \text{m/s}^2\ \text{downward}}$, $\boxed{\vec{\mathbf{a}}_2 = 2.31\ \text{m/s}^2\ \text{to the left}}$,

and $\boxed{\vec{\mathbf{a}}_3 = 2.31\ \text{m/s}^2\ \text{upward}}$

(c) Using this result in Equations [1] and [3] gives the tensions in the two cords as

$$T_1 = (4.00\ \text{kg})(g-a) = (4.00\ \text{kg})(9.80 - 2.31)\ \text{m/s}^2 = \boxed{30.0\ \text{N}}$$

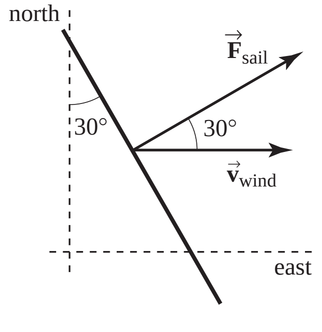
and $$T_2 = (2.00\ \text{kg})(g+a) = (2.00\ \text{kg})(9.80 + 2.31)\ \text{m/s}^2 = \boxed{24.2\ \text{N}}$$

(d) From the final calculation in part (b), observe that if the friction force had a value of zero (rather than 3.53 N), the acceleration of the system would increase in magnitude. Then, observe from Equations [1] and [3] that this would mean $\boxed{T_1\ \text{would decrease}}$ while $\boxed{T_2\ \text{would increase}}$.

4.58 The sketch at the right gives an edge view of the sail (heavy line) as seen from above. The velocity of the wind, $\vec{\mathbf{v}}_{\text{wind}}$, is directed to the east and the force the wind exerts on the sail is perpendicular to the sail. The magnitude of this force is

$$F_{\text{sail}} = \left(550\ \frac{\text{N}}{\text{m/s}}\right)|\vec{\mathbf{v}}_{\text{wind}}|_{\perp} \qquad \text{where } |\vec{\mathbf{v}}_{\text{wind}}|_{\perp}$$

is the component of the wind velocity perpendicular to the sail.



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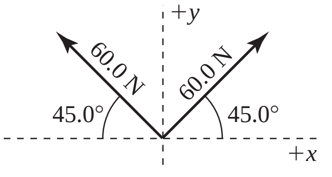
When the sail is oriented at 30° from the north-south line and the wind speed is $v_{\text{wind}} = 17$ knots, we have

$$F_{\text{sail}} = \left(550\ \frac{\text{N}}{\text{m/s}}\right)\left|\vec{\mathbf{v}}_{\text{wind}}\right|_{\perp} = \left(550\ \frac{\text{N}}{\text{m/s}}\right)\left[\left(17\ \cancel{\text{knots}}\right)\left(\frac{0.514\ \text{m/s}}{1\ \cancel{\text{knot}}}\right)\cos 30°\right] = 4.2\times10^{3}\ \text{N}$$

The eastward component of this force will be counterbalanced by the force of the water on the keel of the boat. Before the sailboat has significant speed (that is, before the drag force develops), its acceleration is provided by the northward component of $\vec{\mathbf{F}}_{\text{sail}}$. Thus, the initial acceleration is

$$a = \frac{\left|\vec{\mathbf{F}}_{\text{sail}}\right|_{\text{north}}}{m} = \frac{\left(4.2\times10^{3}\ \text{N}\right)\sin 30°}{800\ \text{kg}} = \boxed{2.6\ \text{m/s}^2}$$

4.59 (a) The horizontal component of the resultant force exerted on the light by the cables is



$$R_x = \Sigma F_x = (60.0\ \text{N})\cos 45.0° - (60.0\ \text{N})\cos 45.0° = 0$$

The resultant y-component is

$$R_y = \Sigma F_y = (60.0\ \text{N})\sin 45.0° + (60.0\ \text{N})\sin 45.0° = 84.9\ \text{N}$$

Hence, the resultant force is $\boxed{\text{84.9 N vertically upward}}$.

(b) The forces on the traffic light are the weight, directed downward, and the 84.9 N vertically upward force exerted by the cables. Since the light is in equilibrium, the resultant of these forces must be zero. Thus, $w = \boxed{\text{84.9 N downward}}$.

4.60 (a) For the suspended block, $\Sigma F_y = T - 50.0\ \text{N} = 0$, so the tension in the rope is $T = 50.0$ N. Then, considering the horizontal forces on the 100-N block, we find

$$\Sigma F_x = T - f_s = 0 \qquad \text{or} \qquad f_s = T = \boxed{50.0\ \text{N}}$$

(b) If the system is on the verge of slipping, $f_s = \left(f_s\right)_{\max} = \mu_s n$. Therefore, the minimum acceptable coefficient of friction is

$$\mu_s = \frac{f_s}{n} = \frac{50.0\ \text{N}}{100\ \text{N}} = \boxed{0.500}$$

(c) If $\mu_k = 0.250$, then the friction force acting on the 100-N block is

$$f_k = \mu_k n = (0.250)(100\ \text{N}) = 25.0\ \text{N}$$

Since the system is to move with constant velocity, the net horizontal force on the 100-N block must be zero, or $\Sigma F_x = T - f_k = T - 25.0\ \text{N} = 0$. The required tension in the rope is $T = 25.0$ N. Now, considering the forces acting on the suspended block when it moves with constant velocity, $\Sigma F_y = T - w = 0$, giving the required weight of this block as $w = T = \boxed{25.0\ \text{N}}$.

4.61 On the level surface, the normal force exerted on the sled by the ice equals the total weight, or $n = 600$ N. Thus, the friction force is

$$f_k = \mu_k n = (0.050)(600\ \text{N}) = 30\ \text{N}$$

continued on next page

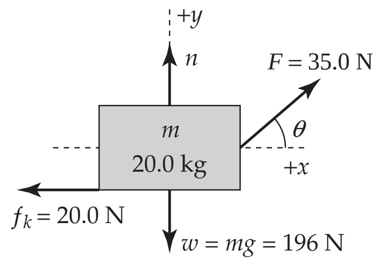
Hence, Newton's second law yields $\Sigma F_x = -f_k = ma_x$, or

$$a_x = \frac{-f_k}{m} = \frac{-f_k}{w/g} = \frac{-(30\text{ N})(9.80\text{ m/s}^2)}{600\text{ N}} = -0.49\text{ m/s}^2$$

The distance the sled travels on the level surface before coming to rest is

$$\Delta x = \frac{v_x^2 - v_{0x}^2}{2a_x} = \frac{0-(7.0\text{ m/s})^2}{2(-0.49\text{ m/s}^2)} = \boxed{50\text{ m}}$$

4.62 (a)



(b) Since the suitcase moves with constant velocity, $\vec{\mathbf{a}} = 0$ and $a_x = a_y = 0$. Applying Newton's second law to the horizontal motion of the suitcase gives

$$\Sigma F_x = ma_x = 0 \quad \Rightarrow \quad (35.0\text{ N})\cos\theta - 20.0\text{ N} = 0$$

or $\cos\theta = \dfrac{20.0\text{ N}}{35.0\text{ N}} = 0.571$ and $\boxed{\theta = 55.2°}$

(c) We find the normal force the ground exerts on the suitcase by applying Newton's second law to the vertical motion:

$$\Sigma F_y = ma_y = 0 \quad \Rightarrow \quad n + (35.0\text{ N})\sin\theta - w = 0$$

and $n = w - (35.0\text{ N})\sin\theta = 196\text{ N} - (35.0\text{ N})\sin 55.2° = \boxed{167\text{ N}}$

4.63 (a) The force that accelerates the box is $\boxed{\text{the friction force}}$ between the box and truck.

(b) We assume the truck is on level ground. Then, the normal force exerted on the box by the truck equals the weight of the box, $n = mg$. The maximum acceleration the truck can have before the box slides is found by considering the maximum static friction force the truck bed can exert on the box:

$$(f_s)_{\max} = \mu_s n = \mu_s(mg)$$

Thus, from Newton's second law,

$$a_{\max} = \frac{(f_s)_{\max}}{m} = \frac{\mu_s(mg)}{m} = \mu_s g = (0.300)(9.80\text{ m/s}^2) = \boxed{2.94\text{ m/s}^2}$$

4.64 Let $m_1 = 5.00$ kg, $m_2 = 4.00$ kg, and $m_3 = 3.00$ kg. Let T_1 be the tension in the string between m_1 and m_2, and T_2 be the tension in the string between m_2 and m_3. Note that the three objects have the same magnitude acceleration: $\vec{\mathbf{a}}_1 = a$ directed upward, $\vec{\mathbf{a}}_2 = \vec{\mathbf{a}}_3 = a$ directed downward. We take the positive direction for each object to be the direction of its acceleration.

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(a) We may apply Newton's second law to each of the masses.

for m_1: $m_1 a = T_1 - m_1 g$ **[1]**

for m_2: $m_2 a = T_2 + m_2 g - T_1$ **[2]**

for m_3: $m_3 a = m_3 g - T_2$ **[3]**

Adding these equations yields $(m_1 + m_2 + m_3)a = (-m_1 + m_2 + m_3)g$, so

$$a = \left(\frac{-m_1 + m_2 + m_3}{m_1 + m_2 + m_3}\right)g = \left(\frac{2.00\ \text{kg}}{12.0\ \text{kg}}\right)(9.80\ \text{m/s}^2) = \boxed{1.63\ \text{m/s}^2}$$

(b) From Equation [1], $T_1 = m_1(a+g) = (5.00\ \text{kg})(11.4\ \text{m/s}^2) = \boxed{57.0\ \text{N}}$, and

from Equation [3], $T_2 = m_3(g-a) = (3.00\ \text{kg})(8.17\ \text{m/s}^2) = \boxed{24.5\ \text{N}}$

4.65 When an object of mass m is on this frictionless incline, the only force acting parallel to the incline is the parallel component of weight, $mg\sin\theta$, directed down the incline. The acceleration is then

$$a = \frac{mg\sin\theta}{m} = g\sin\theta = (9.80\ \text{m/s}^2)\sin 35.0° = 5.62\ \text{m/s}^2 \quad \text{(directed down the incline)}$$

(a) Taking up the incline as positive, the time for the sled projected up the incline to come to rest is given by

$$t = \frac{v - v_0}{a} = \frac{0 - 5.00\ \text{m/s}}{-5.62\ \text{m/s}^2} = 0.890\ \text{s}$$

The distance the sled travels up the incline in this time is

$$\Delta s = v_{\text{av}}\,t = \left(\frac{v+v_0}{2}\right)t = \left(\frac{0+5.00\ \text{m/s}}{2}\right)(0.890\ \text{s}) = \boxed{2.23\ \text{m}}$$

(b) The time required for the first sled to return to the bottom of the incline is the same as the time needed to go up, that is, $t = 0.890\ \text{s}$. In this time, the second sled must travel down the entire 10.0 m length of the incline. The needed initial velocity is found from

$\Delta s = v_0 t + \frac{1}{2}at^2$ as

$$v_0 = \frac{\Delta s}{t} - \frac{at}{2} = \frac{-10.0\ \text{m}}{0.890\ \text{s}} - \frac{(-5.62\ \text{m/s}^2)(0.890\ \text{s})}{2} = -8.74\ \text{m/s}$$

or $\boxed{8.74\ \text{m/s down the incline}}$

4.66 Before she enters the water, the diver is in free-fall with an acceleration of $9.80\ \text{m/s}^2$ downward. Taking downward as the positive direction, her velocity when she reaches the water is given by

$$v = \sqrt{v_0^2 + 2a(\Delta y)} = \sqrt{0 + 2(9.80\ \text{m/s}^2)(10.0\ \text{m})} = 14.0\ \text{m/s}$$

This is also her initial velocity for the 2.00 s after hitting the water. Her average acceleration during this 2.00 s interval is

$$a_{\text{av}} = \frac{v - v_0}{t} = \frac{0 - (14.0\ \text{m/s})}{2.00\ \text{s}} = -7.00\ \text{m/s}^2$$

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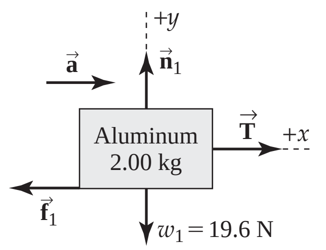
Continuing to take downward as the positive direction, the average upward force by the water is found as $\Sigma F_y = F_{av} + mg = ma_{av}$, or

$$F_{av} = m(a_{av} - g) = (70.0 \text{ kg})\left[(-7.00 \text{ m/s}^2) - 9.80 \text{ m/s}^2\right] = -1.18 \times 10^3 \text{ N}$$

or $\quad F_{av} = \boxed{1.18 \times 10^3 \text{ N upward}}$

4.67 (a) Free-body diagrams for the two blocks are given at the right. The coefficient of kinetic friction for aluminum on steel is $\mu_1 = 0.47$ while that for copper on steel is $\mu_2 = 0.36$. Since $a_y = 0$ for each block,

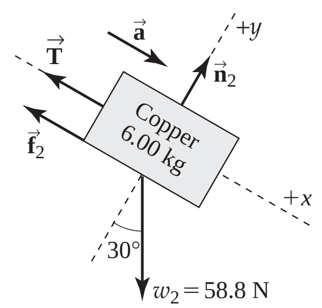


$$n_1 = w_1 \quad \text{and} \quad n_2 = w_2 \cos 30.0°$$

Thus, $\quad f_1 = \mu_1 n_1 = 0.47(19.6 \text{ N}) = 9.2 \text{ N}$

and $\quad f_2 = \mu_2 n_2 = 0.36(58.8 \text{ N})\cos 30.0° = 18 \text{ N}$

For the aluminum block:



$$\Sigma F_x = ma_x \quad \Rightarrow \quad T - f_1 = m(+a) \quad \text{or} \quad T = f_1 + ma$$

giving $\quad T = 9.2 \text{ N} + (2.00 \text{ kg})a \qquad$ **[1]**

For the copper block:

$$\Sigma F_x = ma_x \quad \Rightarrow \quad (58.8 \text{ N})\sin 30.0° - T - 18 \text{ N} = (6.00 \text{ kg})a$$

or $\quad 11 \text{ N} - T = (6.00 \text{ kg})a \qquad$ **[2]**

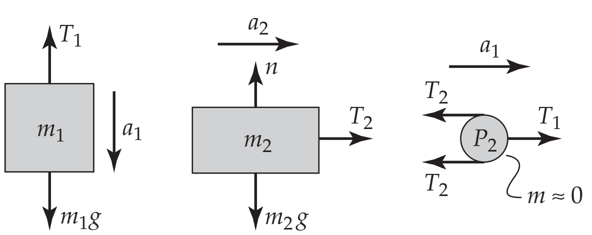
Substituting Equation [1] into Equation [2] gives

$$11 \text{ N} - 9.2 \text{ N} - (2.00 \text{ kg})a = (6.00 \text{ kg})a \quad \text{or} \quad a = \frac{1.8 \text{ N}}{8.00 \text{ kg}} = \boxed{0.23 \text{ m/s}^2}$$

(b) From Equation [1] above, $\quad T = 9.2 \text{ N} + (2.00 \text{ kg})(0.23 \text{ m/s}^2) = \boxed{9.7 \text{ N}}$

4.68 (a) If mass m_1 moves 1 unit downward, pulley P_2 must move 1 unit to the right. Also, mass m_2 must get 1 unit closer to P_2 (to provide an additional unit of length in the cord between P_2 and the wall). This means that during the time m_1 moved 1 unit, m_2 has moved 2 units. Therefore, m_2 is always moving twice as fast as m_1, giving $v_1 = v_2/2$ and $\boxed{a_1 = a_2/2}$.

(b) Draw force diagrams of m_1, m_2, and P_2 as shown below:



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Now, we apply Newton's second law to the motion of each object, using the fact that $a_1 = a_2/2$ from part (a), and taking the direction of each object's acceleration as the positive direction for that object.

For m_1: $m_1 g - T_1 = m_1 a_1 = m_1 (a_2/2)$ or $m_1 (g - a_2/2) = T_1$ **[1]**

For m_2: $T_2 = m_2 a_2$ or $a_2 = T_2/m_2$ **[2]**

For P_2: $T_1 - 2T_2 \approx 0$ or $T_1 = 2T_2$ **[3]**

Substitute Equations [2] and [3] into Equation [1] and simplify to obtain

$\boxed{T_2 = \dfrac{2m_1 m_2 g}{m_1 + 4m_2}}$ and, from Equation [3], $\boxed{T_1 = \dfrac{4m_1 m_2 g}{m_1 + 4m_2}}$

(c) From Equation [2], and the answer for T_2 from part (b), $\boxed{a_2 = \dfrac{2m_1 g}{m_1 + 4m_2}}$

And, from the answer of part (a), $\boxed{a_1 = \dfrac{m_1 g}{m_1 + 4m_2}}$

4.69 Figure 1 is a free-body diagram for the system consisting of both blocks. The friction forces are

$$f_1 = \mu_k n_1 = \mu_k (m_1 g) \quad \text{and} \quad f_2 = \mu_k (m_2 g)$$

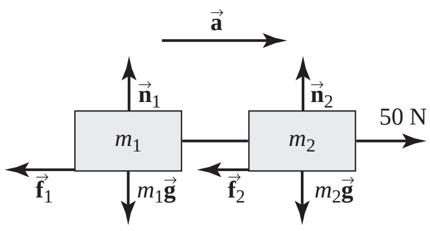


FIGURE 1

For this system, the tension in the connecting rope is an internal force and is not included in second law calculations. The second law gives

$$\Sigma F_x = 50\text{ N} - f_1 - f_2 = (m_1 + m_2)a$$

which reduces to $$a = \frac{50\text{ N}}{m_1 + m_2} - \mu_k g \qquad \textbf{[1]}$$

Figure 2 gives a free-body diagram of m_1 alone. For this system, the tension is an external force and must be included in the second law. We find $\Sigma F_x = T - f_1 = m_1 a$, or

$$T = m_1 (a + \mu_k g) \qquad \textbf{[2]}$$

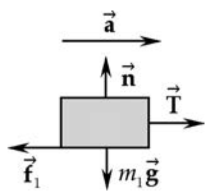


FIGURE 2

(a) If the surface is frictionless, $\mu_k = 0$. Then, Equation [1] gives

$$a = \frac{50\text{ N}}{m_1 + m_2} - 0 = \frac{50\text{ N}}{30\text{ kg}} = \boxed{1.7\text{ m/s}^2}$$

and Equation [2] yields $T = (10\text{ kg})(1.7\text{ m/s}^2 + 0) = \boxed{17\text{ N}}$

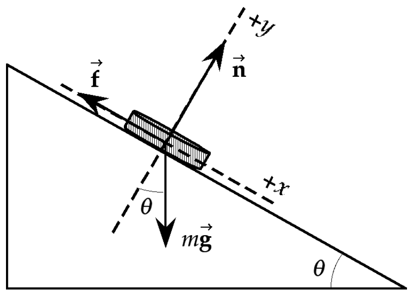
(b) If $\mu_k = 0.10$, Equation [1] gives the acceleration as

$$a = \frac{50\text{ N}}{30\text{ kg}} - (0.10)(9.80\text{ m/s}^2) = \boxed{0.69\text{ m/s}^2}$$

while Equation [2] gives the tension as

$$T = (10\text{ kg})\left[0.69\text{ m/s}^2 + (0.10)(9.80\text{ m/s}^2)\right] = \boxed{17\text{ N}}$$

4.70 (a)



(b) $\boxed{\text{No.}}$ In general, the static friction force is less than the maximum value of $\left(f_s\right)_{\max} = \mu_s n$. It is equal to this maximum value only when the coin is on the verge of slipping, or at the critical angle θ_c. $\boxed{\text{For } \theta \le \theta_c,\ f \le \left(f_s\right)_{\max} = \mu_s n}$.

(c) Recognize that when the *y*-axis is chosen perpendicular to the incline as shown above, $a_y = 0$ and we find

$$\Sigma F_y = n - mg\cos\theta = ma_y = 0 \qquad \text{or} \qquad n = mg\cos\theta$$

Also, when static conditions still prevail, but the coin is on the verge of slipping, we have $a_x = 0$, $\theta = \theta_c$, and $f = \left(f_s\right)_{\max} = \mu_s n = \mu_s mg\cos\theta_c$. Then, Newton's second law becomes

$$\Sigma F_x = mg\sin\theta_c - \mu_s mg\cos\theta_c = ma_x = 0$$

and $\quad \mu_s \cancel{mg}\cos\theta_c = \cancel{mg}\sin\theta_c \quad$ yielding $\quad \mu_s = \dfrac{\sin\theta_c}{\cos\theta_c} = \boxed{\tan\theta_c}$

(d) Once the coin starts to slide, kinetic conditions prevail and the friction force is

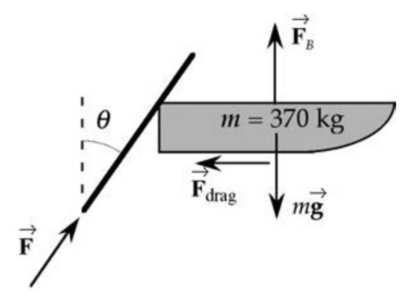
$$f = f_k = \mu_k n = \mu_k mg\cos\theta$$

At $\theta = \theta_c' < \theta_c$, the coin slides with constant velocity, and $a_x = 0$ again. Under these conditions, Newton's second law gives

$$\Sigma F_x = mg\sin\theta_c' - \mu_k mg\cos\theta_c' = ma_x = 0$$

and $\quad \mu_k \cancel{mg}\cos\theta_c' = \cancel{mg}\sin\theta_c' \quad$ yielding $\quad \mu_k = \dfrac{\sin\theta_c'}{\cos\theta_c'} = \boxed{\tan\theta_c'}$

4.71 (a) When the pole exerts a force downward and toward the rear on the lake bed, the lake bed exerts an oppositely directed force of equal magnitude, *F*, on the end of the pole.



As the boat floats on the surface of the lake, its vertical acceleration is $a_y = 0$. Thus, Newton's second law gives the magnitude of the buoyant force, F_B, as

$$\Sigma F_y = F_B + F\cos\theta - mg = 0$$

and, with $\theta = 35.0°$, $\quad F_B = mg - F\cos\theta = (370\text{ kg})\left(9.80\text{ m/s}^2\right) - (240\text{ N})\cos 35.0°$

or $\quad F_B = 3.43 \times 10^3\text{ N} = \boxed{3.43\text{ kN}}$

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(b) Applying Newton's second law to the horizontal motion of the boat gives

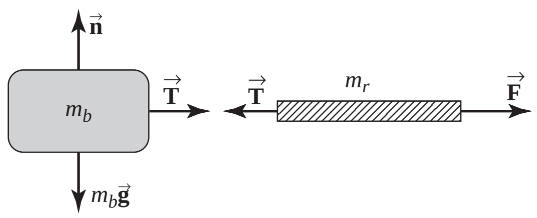
$$\Sigma F_x = F\sin\theta - F_{\text{drag}} = ma_x \qquad \text{or} \qquad a_x = \frac{(240\text{ N})\sin 35.0° - 47.5\text{ N}}{370\text{ kg}} = 0.244\text{ m/s}^2$$

After an elapsed time $t = 0.450$ s, $v_x = v_{0x} + a_x t$ gives the velocity of the boat as

$$v_x = 0.857\text{ m/s} + (0.244\text{ m/s}^2)(0.450\text{ s}) = \boxed{0.967\text{ m/s}}$$

(c) If angle θ increased while the magnitude of $\vec{\mathbf{F}}$ remained constant, the vertical component of this force would decrease. $\boxed{\text{The buoyant force would have to increase}}$ to support more of the weight of the boat and its contents. At the same time, the horizontal component of $\vec{\mathbf{F}}$ would increase, which would $\boxed{\text{increase the acceleration}}$ of the boat.

4.72 (a)



(b) Applying Newton's second law to the rope yields

$$\Sigma F_x = ma_x \qquad \Rightarrow \qquad F - T = m_r a \qquad \text{or} \qquad T = F - m_r a \qquad \textbf{[1]}$$

Then, applying Newton's second law to the block, we find

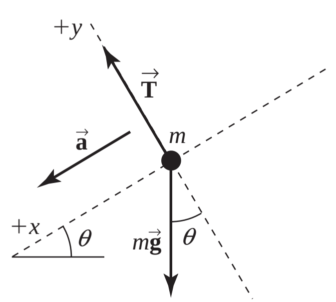
$$\Sigma F_x = ma_x \qquad \Rightarrow T = m_b a \qquad \text{or} \qquad F - m_r a = m_b a \text{ which gives } \boxed{a = \frac{F}{m_b + m_r}}$$

(c) Substituting the acceleration found above back into Equation [1] gives the tension at the left end of the rope as

$$T = F - m_r a = F - m_r\left(\frac{F}{m_b + m_r}\right) = F\left(\frac{m_b + \cancel{m_r} - \cancel{m_r}}{m_b + m_r}\right) \qquad \text{or} \qquad \boxed{T = \left(\frac{m_b}{m_b + m_r}\right)F}$$

(d) From the result of (c) above, we see that as m_r approaches zero, T approaches F. Thus, $\boxed{\text{the tension in a cord of negligible mass is constant along its length}}$.

4.73 Choose the positive x-axis to be down the incline and the y-axis perpendicular to this as shown in the free-body diagram of the toy. The acceleration of the toy then has components of



$$a_y = 0, \text{ and } a_x = \frac{\Delta v_x}{\Delta t} = \frac{+30.0\text{ m/s}}{6.00\text{ s}} = +5.00\text{ m/s}^2$$

Applying the second law to the toy gives

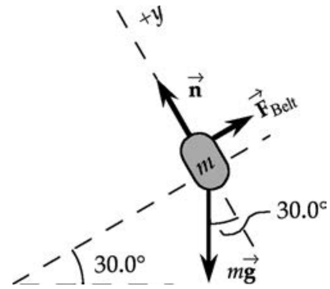
(a) $\Sigma F_x = mg\sin\theta = ma_x \qquad \Rightarrow \qquad \sin\theta = ma_x/mg = a_x/g,$

and $\qquad \theta = \sin^{-1}\left(\frac{a_x}{g}\right) = \sin^{-1}\left(\frac{5.00\text{ m/s}^2}{9.80\text{ m/s}^2}\right) = \boxed{30.7°}$

(b) $\Sigma F_y = T - mg\cos\theta = ma_y = 0$, or

$$T = mg\cos\theta = (0.100\text{ kg})(9.80\text{ m/s}^2)\cos 30.7° = \boxed{0.843\text{ N}}$$

4.74 The sketch at the right gives the force diagram of the person. The scale simply reads the magnitude of the normal force exerted on the student by the seat. From Newton's second law, we obtain



$$\Sigma F_y = ma_y = 0 \quad \Rightarrow \quad n - mg\cos 30.0° = 0$$

or $\quad n = mg\cos\theta = (200\ \text{lb})\cos 30.0° = \boxed{173\ \text{lb}}$

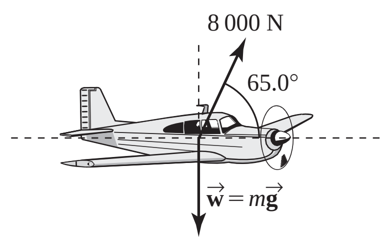
4.75 The acceleration the car has as it is coming to a stop is

$$a = \frac{v^2 - v_0^2}{2(\Delta x)} = \frac{0 - (35\ \text{m/s})^2}{2(1\,000\ \text{m})} = -0.61\ \text{m/s}^2$$

Thus, the magnitude of the total retarding force acting on the car is

$$F = m|a| = \left(\frac{w}{g}\right)|a| = \left(\frac{8\,820\ \text{N}}{9.80\ \text{m/s}^2}\right)(0.61\ \text{m/s}^2) = \boxed{5.5\times10^2\ \text{N}}$$

4.76 (a) In the vertical direction, we have



$$\Sigma F_y = (8\,000\ \text{N})\sin 65.0° - w = ma_y = 0$$

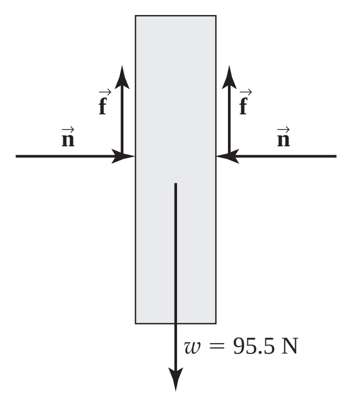
so $\quad w = (8\,000\ \text{N})\sin 65.0° = \boxed{7.25\times10^3\ \text{N}}$

(b) Along the horizontal, Newton's second law yields

$$\Sigma F_x = (8\,000\ \text{N})\cos 65.0° = ma_x = \left(\frac{w}{g}\right)a_x$$

or $\quad a_x = \dfrac{g\left[(8\,000\ \text{N})\cos 65.0°\right]}{w} = \dfrac{(9.80\ \text{m/s}^2)(8\,000\ \text{N})\cos 65.0°}{7.25\times10^3\ \text{N}} = \boxed{4.57\,\text{m/s}^2}$

4.77 Since the board is in equilibrium, $\Sigma F_x = 0$, and we see that the normal forces must have the same magnitudes on both sides of the board. Also, if the minimum normal forces (compression forces) are being applied, the board is on the verge of slipping and the friction force on each side is $f = (f_s)_{\max} = \mu_s n$.



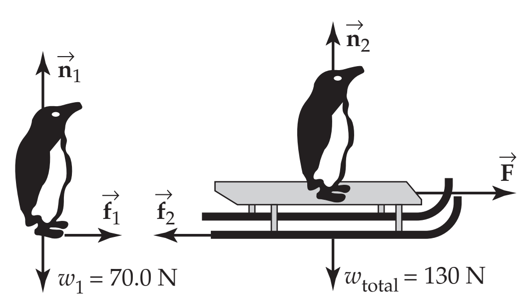
The board is also in equilibrium in the vertical direction, so

$$\Sigma F_y = 2f - w = 0, \ \text{ or } \ f = \frac{w}{2}$$

The minimum compression force needed is then

$$n = \frac{f}{\mu_s} = \frac{w}{2\,\mu_s} = \frac{95.5\ \text{N}}{2(0.663)} = \boxed{72.0\ \text{N}}$$

4.78 Consider the two force diagrams, one of the penguin alone and one of the combined system consisting of penguin plus sled.



The normal force exerted on the penguin by the sled is

$$n_1 = w_1 = m_1 g$$

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and the normal force exerted on the combined system by the ground is

$$n_2 = w_{\text{total}} = m_{\text{total}}g = 130 \text{ N}$$

The penguin is accelerated forward by the static friction force exerted on it by the sled. When the penguin is on the verge of slipping, this acceleration is

$$a_{\max} = \frac{(f_1)_{\max}}{m_1} = \frac{\mu_s n_1}{m_1} = \frac{\mu_s(\cancel{m_1}g)}{\cancel{m_1}} = \mu_s g = (0.700)(9.80 \text{ m/s}^2) = 6.86 \text{ m/s}^2$$

Since the penguin does not slip on the sled, the combined system must have the same acceleration as the penguin. Applying Newton's second law to this system gives

$$\Sigma F_x = F - f_2 = m_{\text{total}}\, a_{\max} \qquad \text{or} \qquad F = f_2 + m_{\text{total}}\, a_{\max} = \mu_k(w_{\text{total}}) + \left(\frac{w_{\text{total}}}{g}\right)a_{\max}$$

which yields $F = (0.100)(130 \text{ N}) + \left(\dfrac{130 \text{ N}}{9.80 \text{ m/s}^2}\right)(6.86 \text{ m/s}^2) = \boxed{104 \text{ N}}$

4.79 First, we will compute the needed accelerations:

(1) Before it starts to move: $a_y = 0$

(2) During the first 0.80 s: $a_y = \dfrac{v_y - v_{0y}}{t} = \dfrac{1.2 \text{ m/s} - 0}{0.80 \text{ s}} = 1.5 \text{ m/s}^2$

(3) While moving at constant velocity: $a_y = 0$

(4) During the last 1.5 s: $a_y = \dfrac{v_y - v_{0y}}{t} = \dfrac{0 - 1.2 \text{ m/s}}{1.5 \text{ s}} = -0.80 \text{ m/s}^2$

The spring scale reads the normal force the scale exerts on the man. Applying Newton's second law to the vertical motion of the man gives

$$\Sigma F_y = n - mg = ma_y \qquad \text{or} \qquad n = m(g + a_y)$$

(a) When $a_y = 0$, $n = (72 \text{ kg})(9.80 \text{ m/s}^2 + 0) = \boxed{7.1 \times 10^2 \text{ N}}$

(b) When $a_y = 1.5 \text{ m/s}^2$, $n = (72 \text{ kg})(9.80 \text{ m/s}^2 + 1.5 \text{ m/s}^2) = \boxed{8.1 \times 10^2 \text{ N}}$

(c) When $a_y = 0$, $n = (72 \text{ kg})(9.80 \text{ m/s}^2 + 0) = \boxed{7.1 \times 10^2 \text{ N}}$

(d) When $a_y = -0.80 \text{ m/s}^2$, $n = (72 \text{ kg})(9.80 \text{ m/s}^2 - 0.80 \text{ m/s}^2) = \boxed{6.5 \times 10^2 \text{ N}}$

4.80 The friction force exerted on the mug by the moving tablecloth is the only horizontal force the mug experiences during this process. Thus, the horizontal acceleration of the mug will be

$$a_{\text{mug}} = \frac{f_k}{m_{\text{mug}}} = \frac{0.100 \text{ N}}{0.200 \text{ kg}} = 0.500 \text{ m/s}^2$$

The cloth and the mug both start from rest ($v_{0x} = 0$) at time $t = 0$. Then, at time $t > 0$, the horizontal displacements of the mug and cloth are given by $\Delta x = v_{0x}t + \frac{1}{2}a_x t^2$ as

$$\Delta x_{\text{mug}} = 0 + \tfrac{1}{2}(0.500 \text{ m/s}^2)t^2 = (0.250 \text{ m/s}^2)t^2$$

and $\Delta x_{\text{cloth}} = 0 + \tfrac{1}{2}(3.00 \text{ m/s}^2)t^2 = (1.50 \text{ m/s}^2)t^2$

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In order for the edge of the cloth to slip under the mug, it is necessary that $\Delta x_{\text{cloth}} = \Delta x_{\text{mug}} + 0.300$ m, or

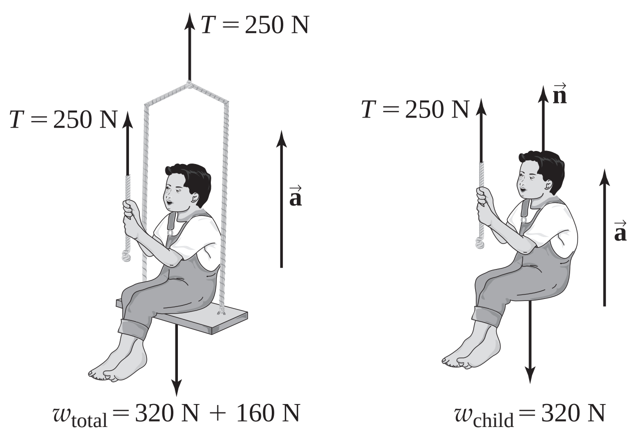
$$\left(1.50\ \text{m/s}^2\right)t^2 = \left(0.250\ \text{m/s}^2\right)t^2 + 0.300\ \text{m}$$

The elapsed time when this occurs is $\quad t = \sqrt{\dfrac{0.300\ \text{m}}{(1.50-0.250)\ \text{m/s}^2}} = 0.490\ \text{s}$

At this time, the mug has moved a distance of

$$\Delta x_{\text{mug}} = \left(0.250\ \text{m/s}^2\right)(0.490\ \text{s})^2 = 6.00\times10^{-2}\ \text{m} = \boxed{6.00\ \text{cm}}$$

4.81 (a) Consider the first free-body diagram in which the child and the chair are treated as a combined system. The weight of this system is $w_{\text{total}} = 480$ N, and its mass is



$$m_{\text{total}} = \frac{w_{\text{total}}}{g} = 49.0\ \text{kg}$$

Taking upward as positive, the acceleration of this system is found from Newton's second law as

$$\Sigma F_y = 2T - w_{\text{total}} = m_{\text{total}}\, a_y$$

Thus $\quad a_y = \dfrac{2(250\ \text{N}) - 480\ \text{N}}{49.0\ \text{kg}} = +0.408\ \text{m/s}^2 \quad$ or

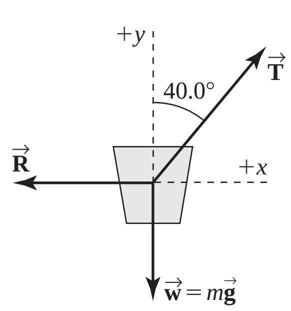
$$\boxed{0.408\ \text{m/s}^2\ \text{upward}}$$

(b) The downward force that the child exerts on the chair has the same magnitude as the upward normal force exerted on the child by the chair. This is found from the free-body diagram of the child alone as

$$\Sigma F_y = T + n - w_{\text{child}} = m_{\text{child}}\, a_y \qquad \text{so} \qquad n = m_{\text{child}}\, a_y + w_{\text{child}} - T$$

Hence, $\quad n = \left(\dfrac{320\ \text{N}}{9.80\ \text{m/s}^2}\right)\left(0.408\ \text{m/s}^2\right) + 320\ \text{N} - 250\ \text{N} = \boxed{83.3\ \text{N}}$

4.82 Let $\vec{\mathbf{R}}$ represent the horizontal force of air resistance. Since the helicopter and bucket move at constant velocity, $a_x = a_y = 0$. The second law then gives



$$\Sigma F_y = T\cos 40.0° - mg = 0 \qquad \text{or} \qquad T = \frac{mg}{\cos 40.0°}$$

Also, $\quad \Sigma F_x = T\sin 40.0° - R = 0 \qquad$ or $\qquad R = T\sin 40.0°$

Thus, $\quad R = \left(\dfrac{mg}{\cos 40.0°}\right)\sin 40.0° = (620\ \text{kg})\left(9.80\ \text{m/s}^2\right)\tan 40.0° = \boxed{5.10\times10^3\ \text{N}}$

4.83 (a) With the crate still in equilibrium, $a_x = a_y = 0$, and Newton's second law gives

$$\Sigma F_y = ma_y \quad \Rightarrow \quad n - P\sin\theta - F_g = 0$$

or $$n = F_g + P\sin\theta \qquad \textbf{[1]}$$

and $$\Sigma F_x = ma_x \quad \Rightarrow \quad P\cos\theta - f_s = 0$$

or $$P\cos\theta = f_s \qquad \textbf{[2]}$$

We now assume that the crate is on the verge of sliding and make use of Equation [1] to find

$$f_s = \left(f_s\right)_{\max} = \mu_s n = \mu_s\left(F_g + P\sin\theta\right)$$

Equation [2] then becomes $P\cos\theta = \mu_s F_g + \mu_s P\sin\theta$, and yields

$$P = \frac{\mu_s F_g}{\cos\theta - \mu_s\sin\theta} = \frac{\mu_s F_g\left(1/\cos\theta\right)}{1-\mu_s\left(\sin\theta/\cos\theta\right)} = \boxed{\frac{\mu_s F_g\sec\theta}{1-\mu_s\tan\theta}}$$

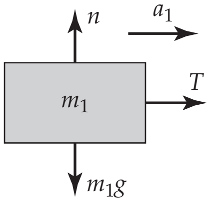
(c) Note that the answer to part (a) states that the magnitude of the applied force required to start the crate moving will approach infinity as $\mu_s\tan\theta$ approaches the value of 1. Thus, it is impossible to start the crate sliding if the angle equals or exceeds a critical value, θ_c, for which

$$\mu_s\tan\theta_c = 1 \qquad \text{or} \qquad \boxed{\theta_c = \tan^{-1}\left(1/\mu_s\right)}$$

4.84 (a) Observe that if block 2 moves downward one unit, two units of cord must pass over the pulley attached to the table. This is true because one unit of cord must be added to each of the vertical sections of cord above block 2. Thus, block 1 must always move at twice the speed of block 2. This means that $v_1 = 2v_2$ and $\boxed{a_1 = 2a_2}$ at all times.

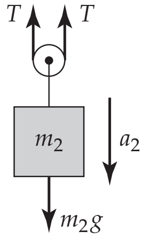
(b) Applying Newton's second law to block 1 gives

$$\Sigma F_x = ma_x \quad \Rightarrow \quad T = m_1 a_1 \qquad \textbf{[1]}$$



Taking downward as the positive direction for block 2, Newton's second law gives

$$\Sigma F_y = ma_y \quad \Rightarrow \quad m_2 g - 2T = m_2 a_2 \qquad \textbf{[2]}$$



Substitute Equation [1] and the result of part (a) into Equation [2] to obtain

$$m_2 g - 2m_1\left(2a_2\right) = m_2 a_2 \qquad \text{or} \qquad a_2 = \frac{m_2 g}{4m_1 + m_2}$$

If $m_2 = 1.30\text{ kg}$, then $m_2 g = 12.7\text{ N}$, and we have

$$\boxed{a_2 = \frac{12.7\text{ N}}{4m_1 + 1.30\text{ kg}}}$$

(c) If $m_1 << 1.30\text{ kg}$, then $$a_2 \approx \frac{12.7\text{ N}}{0 + 1.30\text{ kg}} = \boxed{9.80\text{ m/s}^2}$$

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(d) As $m_1 \to \infty$, $a_2 \to \dfrac{12.7 \text{ kg}}{\infty} = \boxed{0}$

(e) From Equation [2], if $a_2 = 0$, then $T = \dfrac{m_2 g}{2} = \dfrac{(1.30 \text{ kg})(9.80 \text{ m/s}^2)}{2} = \boxed{6.37 \text{ N}}$

(f) $\boxed{\text{Yes.}}$ When $m_1 \to 0$, block 2 is essentially a freely falling body and should have the free-fall acceleration as found in part (c). If $m_1 \to \infty$, then block 2 is essentially attached to an immovable object and we should expect to find $a_2 \approx 0$ as in part (d). Also if $m_1 \to \infty$, we should expect to find that $T = m_2 g/2$, as in part (e), because two strands of cord would be supporting the weight of block 2 and holding it stationary.

4.85 Note that the block of mass M moves horizontally across the floor. Therefore, if the blocks are to be stationary relative to each other, m_1 must have zero vertical acceleration. This means that the tension in the string must equal the weight of m_1, or $T = m_1 g$. This tension in the string accelerates m_2 toward the right at a rate $a = T/m_2 = (m_1 g)/m_2$. If m_2 is to remain stationary relative to M, then M must also accelerate to the right at the rate $a = (m_1 g)/m_2$. Hence, to keep all blocks stationary relative to each other, the force F must accelerate the entire system (of mass $m_{\text{total}} = M + m_1 + m_2$) toward the right at rate a. Newton's second law then gives the magnitude of F as

$$F = m_{\text{total}} a \qquad \text{or} \qquad \boxed{F = (M + m_1 + m_2)\left(\frac{m_1 g}{m_2}\right)}$$

5

Energy

QUICK QUIZZES

1. Choice (c). The work done by the force is $W = F(\Delta x)\cos\theta$, where θ is the angle between the direction of the force and the direction of the displacement (positive x-direction). Thus, the work has its largest positive value in (c) where $\theta = 0°$, the work done in (a) is zero since $\theta = 90°$, the work done in (d) is negative since $90° < \theta < 180°$, and the work done is most negative in (b) where $\theta = 180°$.

2. Choice (d). All three balls have the same speed the moment they hit the ground because all start with the same kinetic energy and undergo the same total change in gravitational potential energy.

3. Choice (c). They both start from rest, so the initial kinetic energy is zero for each of them. They have the same mass and start from the same height, so they have the same initial potential energy. Since neither spends energy overcoming friction, all of their original potential energy will be converted into kinetic energy as they move downward. Thus, they will have equal kinetic energies when they reach the ground.

4. Choice (c). The decrease in mechanical energy of the system is $f_k(\Delta x)$. This has a smaller value on the tilted surface for two reasons: (1) the force of kinetic friction f_k is smaller because the normal force is smaller, and (2) the displacement Δx is smaller because a component of the gravitational force is pulling on the book in the direction opposite to its velocity.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The net work done on the wheelbarrow is

$$W_{\text{net}} = W_{\substack{\text{applied}\\ \text{force}}} + W_{\text{friction}} = (F\cos 0°)\Delta x + (f\cos 180°)\Delta x$$

$$= (F - f)\Delta x = (50.0\text{ N} - 43\text{ N})(5.0\text{ m}) = +35\text{ J}$$

so choice (c) is the correct answer.

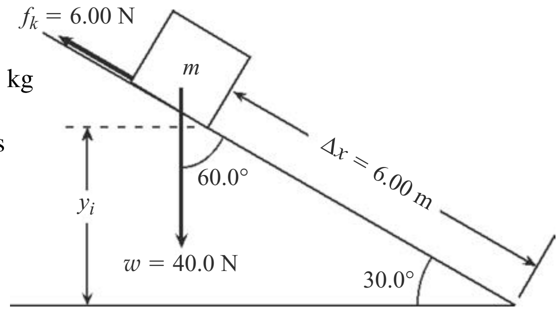
2. We assume the climber has negligible speed at both the beginning and the end of the climb. Then $KE_f = KE_i \approx 0$, and the work done by the muscles is

$$W_{nc} = 0 + \left(PE_f - PE_i\right) = mg\left(y_f - y_i\right) = (70.0\text{ kg})\left(9.80\text{ m/s}^2\right)(325\text{ m}) = 2.23\times10^5\text{ J}$$

The average power delivered is $\overline{P} = \dfrac{W_{nc}}{\Delta t} = \dfrac{2.23\times10^5\text{ J}}{95.0\text{ min}(60\text{ s/1 min})} = 39.1\text{ W}$

and the correct answer is choice (a).

3. The mass of the crate is



$$m = w/g = (40.0\text{ N})\big/\left(9.80\text{ m/s}^2\right) = 4.08\text{ kg}$$

and we may write the work-energy theorem as

$$W_{\text{net}} = W_{\text{friction}} + W_{\text{gravity}} = KE_f - KE_i$$

Since the crate starts from rest, $KE_i = \frac{1}{2}mv_i^2 = 0$ and we are left with

$$KE_f = W_{\text{friction}} + W_{\text{gravity}} = \left(f_k \cos 180°\right)\Delta x + \left(w\cos 60.0°\right)\Delta x$$

so

$$KE_f = (-6.00\text{ N})(6.00\text{ m}) + (40.0\text{ N})\cos 60.0°\ (6.00\text{ m}) = -36.0\text{ J} + 120\text{ J} = 84.0\text{ J}$$

and

$$v_f = \sqrt{\frac{2KE_f}{m}} = \sqrt{\frac{2(84.0\text{ J})}{4.08\text{ kg}}} = 6.42\text{ m/s}$$

making choice (d) the correct response.

4. In the absence of any air resistance, the work done by nonconservative forces is zero. The work-energy theorem then states that $KE_f + PE_f = KE_i + PE_i$, which becomes

$$\frac{1}{2}\cancel{m}v_f^2 + \cancel{m}gy_f = \frac{1}{2}\cancel{m}v_i^2 + \cancel{m}gy_i \qquad \text{or} \qquad v_f = \sqrt{v_i^2 + 2g\left(y_i - y_f\right)}$$

Choosing the initial point to be where the skier leaves the end of the jump and the final point where he reaches maximum height, this yields

$$v_f = \sqrt{(15.0\text{ m/s})^2 + 2\left(9.80\text{ m/s}^2\right)(-4.50\text{ m})} = 11.7\text{ m/s}$$

making (a) the correct answer.

5. The net work needed to accelerate the object from $v = 0$ to v is

$$W_1 = KE_f - KE_i = \tfrac{1}{2}mv^2 - \tfrac{1}{2}m(0)^2 = \tfrac{1}{2}mv^2$$

The work required to accelerate the object from speed v to speed $2v$ is

$$W_2 = KE_f - KE_i = \tfrac{1}{2}m(2v)^2 - \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\left(4v^2 - v^2\right) = 3\left(\tfrac{1}{2}mv^2\right) = 3W_1$$

Thus, the correct choice is (c).

6. Because the same slingshot is used in the same way for both the pebble and the bean, the work done on the projectile by the slingshot is the same in both cases. The work-energy theorem, $W_{\text{net}} = KE_f - KE_i = KE_f - 0$, then tells us that the projectile is given the same final kinetic energy in both cases. Thus,

$$\frac{1}{2}m_{\text{bean}}v_{\text{bean}}^2 = \frac{1}{2}m_{\text{pebble}}v_{\text{pebble}}^2 \qquad \text{or} \qquad \frac{m_{\text{bean}}}{m_{\text{pebble}}} = \frac{v_{\text{pebble}}^2}{v_{\text{bean}}^2} = \frac{(200\text{ cm/s})^2}{(600\text{ cm/s})^2} = \frac{1}{9}$$

and (a) is the correct choice.

7. Since the rollers on the ramp used by David were frictionless, he did not do any work overcoming nonconservative forces as he slid the block up the ramp. Neglecting any change in kinetic energy of the block (either because the speed was constant in the case of sliding the block, or, in the case of lifting the block, the speed at the ground and at the truck bed were both zero), the work done by either Mark or David equals the increase in the gravitational potential energy of the block as it is lifted from the ground to the truck bed. Because they lift identical blocks through the same vertical distance, they do equal amounts of work and the correct choice is (b).

8. The kinetic energy is proportional to the square of the speed of the particle. Thus, doubling the speed will increase the kinetic energy by a factor of 4. This is seen from

$$KE_f = \frac{1}{2}mv_f^2 = \frac{1}{2}m\left(2v_i\right)^2 = 4\left(\frac{1}{2}mv_i^2\right) = 4KE_i$$

and (a) is the correct response here.

9. $KE_{\text{car}} = \frac{1}{2}m_{\text{car}}v^2 = \frac{1}{2}\left(\frac{1}{2}m_{\text{truck}}\right)v^2 = \frac{1}{2}\left(\frac{1}{2}m_{\text{truck}}v^2\right) = \frac{1}{2}KE_{\text{truck}}$, so (b) is the correct answer.

10. Once the athlete leaves the surface of the trampoline, only a conservative force (her weight) acts on her. Therefore, her total mechanical energy is constant during her flight, or $KE_f + PE_f = KE_i + PE_i$. Taking the $y = 0$ at the surface of the trampoline, $PE_i = mgy_i = 0$. Also, her speed when she reaches maximum height is zero, or $KE_f = 0$. This leaves us with $PE_f = KE_i$, or $\cancel{m}gy_{\text{max}} = \frac{1}{2}\cancel{m}v_i^2$, which gives the maximum height as

$$y_{\text{max}} = \frac{v_i^2}{2g} = \frac{(8.5\ \text{m/s})^2}{2\left(9.80\ \text{m/s}^2\right)} = 3.7\ \text{m}$$

making (c) the correct choice.

11. The work-energy theorem states that $W_{\text{net}} = KE_f - KE_i$. Thus, if $W_{\text{net}} = 0$, then $KE_f = KE_i$ or $\frac{1}{2}mv_f^2 = \frac{1}{2}mv_i^2$, which leads to the conclusion that the speed is unchanged ($v_f = v_i$). The velocity of the particle involves both magnitude (speed) and direction. The work-energy theorem shows that the magnitude or speed is unchanged when $W_{\text{net}} = 0$, but makes no statement about the direction of the velocity. Therefore, choice (d) is correct but choice (c) is not necessarily true.

12. As the block falls freely, only the conservative gravitational force acts on it. Therefore, mechanical energy is conserved, or $KE_f + PE_f = KE_i + PE_i$. Assuming that the block is released from rest ($KE_i = 0$), and taking $y = 0$ at ground level ($PE_f = 0$), we have

$$KE_f = PE_i \qquad \text{or} \qquad \frac{1}{2}mv_f^2 = mgy_i \qquad \text{and} \qquad y_i = \frac{v_f^2}{2g}$$

Thus, to double the final speed, it is necessary to increase the initial height by a factor of four, and the correct choice for this question is (e).

13. If the car is to have uniform acceleration, a constant net force F must act on it. Since the instantaneous power delivered to the car is $P = Fv$, we see that maximum power is required just as the car reaches its maximum speed. The correct answer is (b).

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. (a) The effects are the same except for such features as possibly having to overcome air resistance if the wind is blowing outside.

(b) The person must lift his body slightly with each step on the tilted treadmill. Thus, the effect is that of running uphill.

4. (a) Kinetic energy is always positive. Mass and speed squared are both positive.

(b) Gravitational potential energy can be negative when the object is lower than the chosen reference level.

6. (a) The ball initially has gravitational potential energy mgh_i and zero kinetic energy. Because a small amount of energy is always spent overcoming air resistance and friction in the support as the ball swings, it will come to rest (i.e., have zero kinetic energy) on the return swing at a level slightly lower than its initial position.

(b) If anyone gives the ball a forward push anywhere along its path, positive work is done on the ball and it will tend to reach a level higher than its initial position before coming to rest again. In this case, the demonstrator may have to duck to avoid injury.

8. (a) The chicken does positive work on the ground.

(b) No work is done.

(c) The crane does positive work on the bucket.

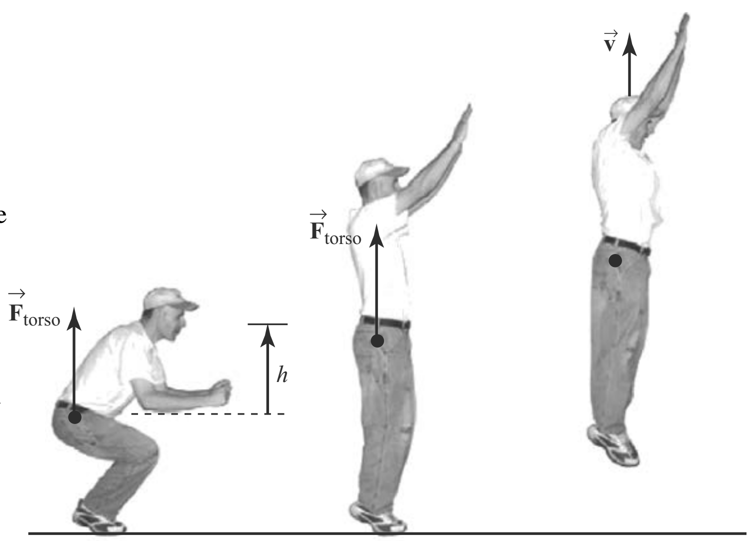
(d) The force of gravity does negative work on the bucket.

(e) The leg muscles do negative work on the individual.

10. The kinetic energy is converted to internal energy within the brake pads of the car, the roadway, and the tires.

12. If a crate is located on the bed of a truck, and the truck accelerates, the friction force exerted on the crate causes it to undergo the same acceleration as the truck, assuming that the crate doesn't slip. Another example is a car that accelerates because of the frictional forces between the road surface and its tires. This force is in the direction of the motion of the car and produces an increase in the car's kinetic energy.

14. Work is actually performed by the thigh bone (the femur) on the hips as the torso moves upwards a distance h. The force on the torso $\vec{\mathbf{F}}_{\text{torso}}$ is approximately the same as the normal force (since the legs are relatively light and are not moving much), and the work done by $\vec{\mathbf{F}}_{\text{torso}}$ minus the work done by gravity is equal to the change in kinetic energy of the torso.



At full extension the torso would continue upwards, leaving the legs behind on the ground (!), except that the torso now does work on the legs, increasing their speed (and decreasing the torso speed) so that both move upwards together.

Note: An alternative way to think about problems that involve internal motions of an object is to note that the net work done on an object is equal to the net force times the displacement of the center of mass. Using this idea, the effect of throwing the arms upwards during the extension phase is accounted for by noting that the position of the center of mass is higher on the body with the arms extended, so that total displacement of the center of mass is greater.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 472 J **(b)** 2.76 kN

4. **(a)** 1.6×10^{3} J **(b)** 0

(c) The applied force is smaller, the work done by the shopper is unchanged.

6. **(a)** 900 J **(b)** 0.383

8. **(a)** 31.9 J **(b)** 0 **(c)** 0

(d) 31.9 J

10. 160 m/s

12. **(a)** 29.2 N **(b)** The speed of the crate will increase.

(c) The crate would slow down and come to rest.

14. **(a)** 3.2×10^{4} J **(b)** 23 m/s

16. **(a)** 1.2 J **(b)** 5.0 m/s **(c)** 6.3 J

18. **(a)** 47.8 N directed opposite to the motion **(b)** –31 J

(c) 0.22 m/s

20. **(a)** 8.88×10^{2} N/m **(b)** 1.38 cm **(c)** 2.84 J

22. **(a)** The athlete and the Earth interact via the gravitational force.

(b) $KE = 2.4\times10^{3}$ J, $PE_g = 0$ **(c)** $KE = 0$, $PE_g = 2.4\times10^{3}$ J

(d) $y_{\text{max}} = 4.1$ m **(e)** $v\big|_{y=\frac{1}{2}y_{\text{max}}} = 6.4$ m/s

24. $d = 2(m_1 - m_2)g/k$

26. **(a)** 0.768 m **(b)** 1.68×10^{5} J

28. ≈ 0.5 m

30. **(a)** $W_g = mgh$ **(b)** $\Delta KE = mgh$ **(c)** $KE_f = \frac{1}{2}mv_0^2 + mgh$

(d) No, the previous answers are independent of the initial angle.

32. 5.1 m

34. **(a)** 1.13 kN/m **(b)** 51.8 cm

36. **(a)** $v_B = 5.94$ m/s, $v_C = 7.67$ m/s **(b)** 147 J

38. **(a)** $v_f = \sqrt{2(m_1 - m_2)gh/(m_1 + m_2)}$ **(b)** 3.7 m/s **(c)** 2.6 m/s

40. **(a)** No, $f_k = \mu_k(mg - F\sin\theta)$ **(b)** $W_{f_k} = -\mu_k(mg - F\sin\theta)x$, $W_F = Fx\cos\theta$

(c) Forces that do no work are $\vec{\mathbf{n}}$, $m\vec{\mathbf{g}}$, and the vertical component of $\vec{\mathbf{F}}$.

(d) 4.24 N, 47.9 J and −17.0 J

42. **(a)** No, mechanical energy is not conserved when friction forces are present.

(b) 77 m/s

44. **(a)** 2.29 m/s **(b)** 15.7 J

46. **(a)** Yes, the only nonconservative force is perpendicular to the motion.

(b) At the top, $KE = 0$, $PE_g = mgh$; at launch point, $KE = \frac{4}{5}mgh$, $PE_g = \frac{1}{5}mgh$;
at pool level, $KE = mgh$, $PE_g = 0$.

(c) $v_0 = \sqrt{8gh/5}$ **(d)** $y_{\max} = h - v_{0x}^2/2g$ **(e)** $y_{\max} = h\left(1 - \frac{4}{5}\cos^2\theta\right)$

(f) No, if energy is used overcoming friction, v_0, $y_{\max}$, and the final speed are all reduced.

48. 1.5 m along the incline or 0.17 m vertically

50. **(a)** 21 kJ **(b)** 0.92 hp

52. 2.9 m/s

54. 4.5×10^3 N

56. 2.03×10^8 s (or 6.43 yr)

58. **(a)** 7.92 hp **(b)** 14.9 hp

60. **(a)** 7.50 J **(b)** 15.0 J **(c)** 7.50 J

(d) at $x = 5.00$ m, $v = 2.29$ m/s; at $x = 15.0$ m, $v = 4.50$ m/s

62. 90.0 J

64. $H = h + d^2/4h$

66. **(a)** 3.57 m/s **(b)** 3.22 kN/m

68. 0.116 m

70. 1.4 m/s

72. **(a)** 21.0 m/s **(b)** 16.2 m/s

74. **(a)** 0.225 J **(b)** 0.363 J

(c) No, because the normal force (and hence the friction force) varies with position.

76. **(a)** $x = x_1 + x_2 = mg\left(1/k_1 + 1/k_2\right)$ **(b)** $k_{\text{effective}} = k_1 k_2/\left(k_1 + k_2\right)$

78. **(a)** 584 trips **(b)** 0.121 hp

80. **(a)** 3.0×10^2 J **(b)** -1.5×10^2 J **(c)** 0

(d) 1.5×10^2 J

82. 4.8 J

84. **(a)** $KE_{\text{Javelin}} = 3.8\times10^2$ J, $KE_{\text{Discus}} = 7.3\times10^2$ J, $KE_{\text{Shot}} = 8.1\times10^2$ J

(b) $F_{\text{av,Javelin}} = 1.9\times10^2$ N, $F_{\text{av,Discus}} = 3.6\times10^2$ N, $F_{\text{av,Shot}} = 4.1\times10^2$ N

(c) Yes. See Solution for explanation.

86. **(a)** 25.8 m **(b)** $27.1\ \text{m/s}^2\ (2.77\ g)$

88. **(a)** See Solution. **(b)** 2.06 m/s

90. **(a)** $-1.23\ \text{m/s}^2$ and $+0.616\ \text{m/s}^2$

(b) $-0.252\ \text{m/s}^2$ if the force of static friction is not too large, and 0

(c) 0 and 0 (friction prevents motion)

92. 3.9 kJ

PROBLEM SOLUTIONS

5.1 If the weights are to move at constant velocity, the net force on them must be zero. Thus, the force exerted on the weights is upward, parallel to the displacement, with magnitude 350 N. The work done by this force is

$$W = (F\cos\theta)s = \left[(350\ \text{N})\cos 0°\right](2.00\ \text{m}) = \boxed{700\ \text{J}}$$

5.2 (a) We assume the object moved upward with constant speed, so the kinetic energy did not change. Then, the work-energy theorem gives the work done on the object by the lifter as $W_{nc} = \Delta KE + \Delta PE = 0 + (mgy_f - mgy_i) = mg(\Delta y)$, or

$$W_{nc} = (281.5\ \text{kg})\left(9.80\ \text{m/s}^2\right)\left[(17.1\ \text{cm})\left(1\ \text{m}/10^2\ \text{cm}\right)\right] = \boxed{472\ \text{J}}$$

(b) If the object moved upward at constant speed, the net force acting on it was zero. Therefore, the magnitude of the upward force applied by the lifter must have been equal to the weight of the object:

$$F = mg = (281.5\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 2.76\times10^3\ \text{N} = \boxed{2.76\ \text{kN}}$$

5.3 Assuming the mass is lifted at constant velocity, the total upward force exerted by the two men equals the weight of the mass: $F_{\text{total}} = mg = (653.2\text{ kg})(9.80\text{ m/s}^2) = 6.40\times10^3\text{ N}$. They exert this upward force through a total upward displacement of 96 inches or 8 feet (4 inches per lift for each of 24 lifts). Thus, the total work done during the upward movements of the 24 lifts is

$$W = (F\cos\theta)\Delta x = (F_{\text{total}}\cos 0°)\Delta x = (6.40\times10^3\text{ N})(\cos 0°)\left[(8\text{ ft})\left(\frac{1\text{ m}}{3.281\text{ ft}}\right)\right] = \boxed{2\times10^4\text{ J}}$$

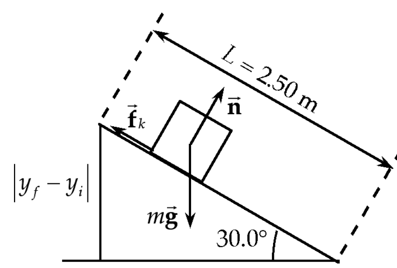
5.4 (a) The 35 N force applied by the shopper makes a 25° angle with the displacement of the cart (horizontal). The work done on the cart by the shopper is then

$$W_{\text{shopper}} = (F\cos\theta)\Delta x = (35\text{ N})(\cos 25°)(50.0\text{ m}) = \boxed{1.6\times10^3\text{ J}}$$

(b) Since the speed of the cart is constant, $KE_f = KE_i$ and $W_{\text{net}} = \Delta KE = \boxed{0}$.

(c) Since the cart continues to move at constant speed, the net work done on the cart in the second aisle is again zero. With both the net work and the work done by friction unchanged, the work done by the shopper $(W_{\text{shopper}} = W_{\text{net}} - W_{\text{friction}})$ is also $\boxed{\text{unchanged}}$. However, the shopper now pushes horizontally on the cart, making $\boxed{F' = W_{\text{shopper}}/(\Delta x\cdot\cos 0°) = W_{\text{shopper}}/\Delta x \text{ smaller than before}}$ when the force was $F = W_{\text{shopper}}/(\Delta x\cdot\cos 35°)$.

5.5 (a) The gravitational force acting on the object is



$$w = mg = (5.00\text{ kg})(9.80\text{ m/s}^2) = 49.0\text{ N}$$

and the work done by this force is

$$W_g = -\Delta PE_g = -mg(y_f - y_i) = +w(y_i - y_f)$$

or $\quad W_g = w(L\sin 30.0°) = (49.0\text{ N})(2.50\text{ m})\sin 30.0° = \boxed{61.3\text{ J}}$

(b) The normal force exerted on the block by the incline is $n = mg\cos 30.0°$, so the friction force is

$$f_k = \mu_k n = (0.436)(49.0\text{ N})\cos 30.0° = 18.5\text{ N}$$

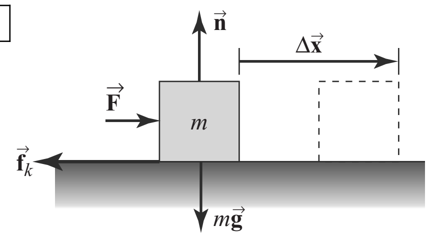
This force is directed opposite to the displacement (that is $\theta = 180°$), and the work it does is

$$W_f = (f_k\cos\theta)L = [(18.5\text{ N})\cos 180°](2.50\text{ m}) = \boxed{-46.3\text{ J}}$$

(c) Since the normal force is perpendicular to the displacement, so the work done by the normal force is $W_n = (n\cos 90.0°)L = \boxed{0}$.

(d) If a shorter ramp is used to increase the angle of inclination while maintaining the same vertical displacement $|y_f - y_i|$, the $\boxed{\text{work done by gravity will not change}}$, the $\boxed{\text{work done by the friction force will decrease}}$ (because the normal force, and hence the friction force, will decrease and also because the ramp length L decreases), and the $\boxed{\text{work done by the normal force remains zero}}$ (because the normal force remains perpendicular to the displacement).

5.6 (a) $W_F = F(\Delta x)\cos\theta = (150\text{ N})(6.00\text{ m})\cos 0° = \boxed{900\text{ J}}$



(b) Since the crate moves at constant velocity, $a_x = a_y = 0$. Thus,

$$\Sigma F_x = 0 \quad \Rightarrow \quad f_k = F = 150\text{ N}$$

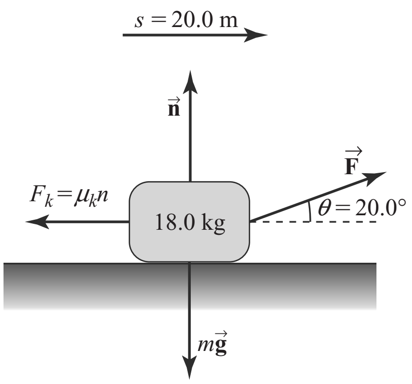
Also,

$$\Sigma F_y = 0 \quad \Rightarrow \quad n = mg = (40.0\text{ kg})\left(9.80\text{ m/s}^2\right) = 392\text{ N}$$

so

$$\mu_k = \frac{f_k}{n} = \frac{150\text{ N}}{392\text{ N}} = \boxed{0.383}$$

5.7 (a) $\Sigma F_y = F\sin\theta + n - mg = 0$



$$n = mg - F\sin\theta$$

$$\Sigma F_x = F\cos\theta - \mu_k n = 0$$

$$n = \frac{F\cos\theta}{\mu_k}$$

$$\therefore \quad mg - F\sin\theta = \frac{F\cos\theta}{\mu_k}$$

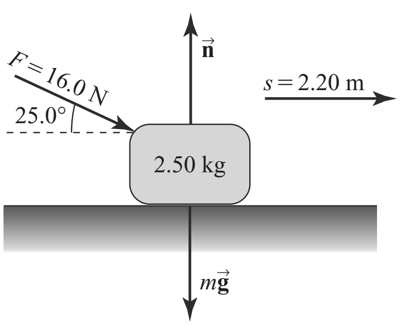
$$F = \frac{\mu_k mg}{\mu_k\sin\theta + \cos\theta} = \frac{(0.500)(18.0\text{ kg})\left(9.80\text{ m/s}^2\right)}{(0.500)\sin 20.0° + \cos 20.0°} = \boxed{79.4\text{ N}}$$

(b) $W_F = (F\cos\theta)s = [(79.4\text{ N})\cos 20.0°](20.0\text{ m}) = 1.49\times10^3\text{ J} = \boxed{1.49\text{ kJ}}$

(c) $f_k = F\cos\theta = (79.4\text{ N})\cos 20.0° = 74.6\text{ N}$

$$W_f = (f_k\cos\theta)s = [(74.6\text{ N})\cos 180°](20.0\text{ m}) = -1.49\times10^3\text{ J} = \boxed{-1.49\text{ kJ}}$$

5.8 (a) $W_F = (F\cos\theta)s = [(16.0\text{ N})\cos 25.0°](2.20\text{ m})$



$$W_F = \boxed{31.9\text{ J}}$$

(b) $W_n = (n\cos 90°)s = \boxed{0}$

(c) $W_g = (mg\cos 90°)s = \boxed{0}$

(d) $W_{net} = W_F + W_n + W_g = 31.9\text{ J} + 0 + 0 = \boxed{31.9\text{ J}}$

5.9 (a) The work-energy theorem, $W_{net} = KE_f - KE_i$, gives

$$5\,000\text{ J} = \frac{1}{2}\left(2.50\times10^3\text{ kg}\right)v^2 - 0, \text{ or } \quad v = \boxed{2.00\text{ m/s}}$$

(b) $W = (F\cos\theta)s = (F\cos 0°)(25.0\text{ m}) = 5\,000\text{ J}$, so $F = \boxed{200\text{ N}}$

5.10 Requiring that $KE_{\text{ping pong}} = KE_{\text{bowling}}$, with $KE = \frac{1}{2}mv^2$, we have

$$\frac{1}{2}\left(2.45\times10^{-3}\ \text{kg}\right)v^2 = \frac{1}{2}(7.00\ \text{kg})(3.00\ \text{m/s})^2$$

giving $v = \boxed{160\ \text{m/s}}$

5.11 (a) $KE = \dfrac{1}{2}mv^2 = \dfrac{1}{2}(65.0\ \text{kg})(5.20\ \text{m/s})^2 = \boxed{879\ \text{J}}$

(b) Since the kinetic energy of an object varies as the square of the speed, doubling the speed will increase the kinetic energy by a factor of 4 as shown below:

$$\frac{KE_f}{KE_i} = \frac{\frac{1}{2}mv_f^2}{\frac{1}{2}mv_i^2} = \left(\frac{v_f}{v_i}\right)^2 = (2)^2 = \boxed{4}$$

5.12 (a) Since the applied force is horizontal, it is in the direction of the displacement, giving $\theta = 0°$. The work done by this force is then $W_{F_0} = \left(F_0 \cos\theta\right)\Delta x = F_0\left(\cos 0°\right)\Delta x = F_0\left(\Delta x\right)$

and $F_0 = \dfrac{W_{F_0}}{\Delta x} = \dfrac{350\ \text{J}}{12.0\ \text{m}} = \boxed{29.2\ \text{N}}$

(b) Since the crate originally had zero acceleration, the original applied force was just enough to offset the retarding friction force. Therefore, when the applied force is increased, it has a magnitude greater than the friction force. This gives the crate a resultant force (and hence, an acceleration) in the direction of motion, meaning the $\boxed{\text{speed of the crate will increase with time}}$.

(c) If the applied force is made smaller than F_0, the magnitude of the friction force will be greater than that of the applied force. This means the crate has a resultant force, and acceleration, in the direction of the friction force (opposite to the direction of motion). $\boxed{\text{The crate will now slow down and come to rest.}}$

5.13 (a) We use the work-energy theorem to find the work.

$$W = \Delta KE = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2 = 0 - \frac{1}{2}(70\ \text{kg})(4.0\ \text{m/s})^2 = \boxed{-5.6\times10^2\ \text{J}}$$

(b) $W = (F\cos\theta)s = \left(f_k \cos 180°\right)s = \left(-\mu_k n\right)s = \left(-\mu_k\, mg\right)s,$

so $s = -\dfrac{W}{\mu_k\, mg} = -\dfrac{\left(-5.6\times10^2\ \text{J}\right)}{(0.70)(70\ \text{kg})\left(9.80\ \text{m/s}^2\right)} = \boxed{1.2\ \text{m}}$

5.14 (a) $KE_{\max} = \dfrac{1}{2}mv_{\max}^2 = \dfrac{1}{2}(62\ \text{kg})(32\ \text{m/s})^2 = \boxed{3.2\times10^4\ \text{J}}$

(b) Since $v = \sqrt{2(KE)/m}$, the speed of the cheetah when $KE = KE_{\max}/2$ is

so $v = \sqrt{\dfrac{2\left(KE_{\max}/2\right)}{m}} = \sqrt{\dfrac{KE_{\max}}{m}} = \sqrt{\dfrac{3.2\times10^4\ \text{J}}{62\ \text{kg}}} = \boxed{23\ \text{m/s}}$

5.15 (a) As the bullet penetrates the tree trunk, the only force doing work on it is the force of resistance exerted by the trunk. This force is directed opposite to the displacement, so the work done is $W_{\text{net}} = \left(f_{\text{av}} \cos 180°\right)\Delta x = KE_f - KE_i$, and the magnitude of the average resistance force is

$$f_{\text{av}} = \frac{KE_f - KE_i}{(\Delta x)\cos 180°} = \frac{0 - \frac{1}{2}\left(7.80\times10^{-3}\ \text{kg}\right)(575\ \text{m/s})^2}{-\left(5.50\times10^{-2}\ \text{m}\right)} = \boxed{2.34\times10^4\ \text{N}}$$

continued on next page

(b) If the friction force is constant, the bullet will have a constant acceleration and its average velocity while stopping is $\overline{v} = (v_f + v_i)/2$. The time required to stop is then

$$\Delta t = \frac{\Delta x}{\overline{v}} = \frac{2(\Delta x)}{v_f + v_i} = \frac{2(5.50\times10^{-2}\text{ m})}{0 + 575\text{ m/s}} = \boxed{1.91\times10^{-4}\text{ s}}$$

5.16 (a) $KE_A = \frac{1}{2}mv_A^2 = \frac{1}{2}(0.60\text{ kg})(2.0\text{ m/s})^2 = \boxed{1.2\text{ J}}$

(b) $KE_B = \frac{1}{2}mv_B^2$, so

$$v_B = \sqrt{\frac{2(KE_B)}{m}} = \sqrt{\frac{2(7.5\text{ J})}{0.60\text{ kg}}} = \boxed{5.0\text{ m/s}}$$

(c) $W_{\text{net}} = \Delta KE = KE_B - KE_A = (7.5 - 1.2)\text{ J} = \boxed{6.3\text{ J}}$

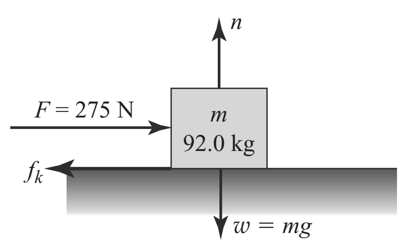
5.17 (a) $KE_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(6.50\times10^7\text{ kg})(12.0\text{ m/s})^2 = \boxed{4.68\times10^9\text{ J}}$

(b) $W_{\text{net}} = KE_f - KE_i = 0 - 4.68\times10^9\text{ J} = \boxed{-4.68\times10^9\text{ J}}$

(c) $W_{\text{net}} = (F_R\cos\theta)\Delta x$, and $\theta = 180°$ since the resultant force acting on the ship is a retarding frictional force (note that the normal force the water exerts on the ship simply cancels out the weight of the ship). Thus, if the ship comes to rest after a displacement of $\Delta x = 2.50$ km, the resultant force acting on the ship is

$$F_R = \frac{W_{\text{net}}}{(\Delta x)\cos\theta} = \frac{-4.68\times10^9\text{ J}}{(2.50\times10^3\text{ m})\cos180°} = \boxed{1.87\times10^6\text{ N}}$$

5.18 The crate has zero vertical acceleration, so $\Sigma F_y = n - mg = 0$, and the normal force is $n = mg$. Thus, the kinetic friction force is $f_k = \mu_k n = \mu_k mg$.



(a) Note that the two vertical forces acting on the crate cancel, meaning that the net force on the crate is horizontal. Choosing the $+x$-direction toward the right, we have

$$F_{\text{net}} = \Sigma F_x = F - f_k = F - \mu_k mg$$

or $$F_{\text{net}} = 275\text{ N} - (0.358)(92.0\text{ kg})(9.80\text{ m/s}^2) = -47.8\text{ N}$$

$$\boxed{F_{\text{net}} = 47.8\text{ N directed opposite to the motion of the crate}}$$

(b) $W_{\text{net}} = F_{\text{net}}(\Delta x)\cos\theta = (47.8\text{ N})(0.65\text{ m})\cos180° = \boxed{-31\text{ J}}$

(c) $W_{\text{net}} = KE_f - KE_i = \frac{1}{2}m(v_f^2 - v_i^2)$, so $v_f^2 = 2W_{\text{net}}/m + v_i^2$

or $$v_f = \sqrt{\frac{2(-31\text{ J})}{92.0\text{ kg}} + (0.850\text{ m/s}^2)^2} = \boxed{0.22\text{ m/s}}$$

5.19 (a) $PE_i = mgy_i = (0.20\text{ kg})(9.80\text{ m/s}^2)(1.3\text{ m}) = \boxed{2.5\text{ J}}$

(b) $PE_f = mgy_f = (0.20\text{ kg})(9.80\text{ m/s}^2)(-5.0\text{ m}) = \boxed{-9.8\text{ J}}$

(c) $\Delta PE = PE_f - PE_i = -9.8\text{ J} - 2.5\text{ J} = \boxed{-12\text{ J}}$

5.20 (a) The force stretching the spring is the weight of the suspended object. Therefore, the force constant of the spring is

$$k = \left|\frac{F_g}{\Delta x}\right| = \frac{mg}{|\Delta x|} = \frac{(2.50\ \text{kg})(9.80\ \text{m/s}^2)}{2.76\times 10^{-2}\ \text{m}} = \boxed{8.88\times 10^2\ \text{N/m}}$$

(b) If a 1.25 kg block replaces the original 2.50 kg suspended object, the force applied to the spring (weight of the suspended object) will be one-half the original stretching force. Since, for a spring obeying Hooke's law, the elongation is directly proportional to the stretching force, the amount the spring stretches now is

$$(\Delta x)_2 = \frac{1}{2}(\Delta x)_1 = \frac{1}{2}(2.76\ \text{cm}) = \boxed{1.38\ \text{cm}}$$

(c) The work an external agent must do *on* the initially unstretched spring to produce an elongation x_f is equal to the potential energy stored in the spring at this elongation:

$$W_{\substack{\text{done on}\\ \text{spring}}} = (PE_s)_f - (PE_s)_i = \frac{1}{2}kx_f^2 - 0 = \frac{1}{2}(8.88\times 10^2\ \text{N/m})(8.00\times 10^{-2}\ \text{m})^2 = \boxed{2.84\ \text{J}}$$

5.21 The magnitude of the force a spring must exert on an object of mass m to give it an acceleration of $a = 0.800g$ is $F = ma = 0.800mg$.

Then, by Newton's third law, this object exerts an oppositely directed force of equal magnitude on the spring. If this reaction force is to stretch the spring 0.500 cm, the required force constant of the spring is

$$k = \frac{F}{\Delta x} = \frac{0.800mg}{\Delta x} = \frac{0.800(4.70\times 10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)}{0.500\times 10^{-2}\ \text{m}} = \boxed{7.37\ \text{N/m}}$$

5.22 (a) While the athlete is in the air, the interacting objects are $\boxed{\text{the athlete and the Earth}}$. They interact $\boxed{\text{through the gravitational force}}$ that one exerts on the other.

(b) If the athlete leaves the trampoline (at the $y = 0$ level) with an initial speed of $v_i = 9.0\ \text{m/s}$, her initial kinetic energy is

$$KE_i = \frac{1}{2}mv_i^2 = \frac{1}{2}(60.0\ \text{kg})(9.0\ \text{m/s})^2 = \boxed{2.4\times 10^3\ \text{J}}$$

and her gravitational potential energy is $\quad (PE_g)_i = mgy_i = mg(0) = \boxed{0\ \text{J}}$

(c) When the athlete is at maximum height, she is momentarily at rest and $KE_f = \boxed{0\ \text{J}}$. Because the only force acting on the athlete during her flight is the conservative gravitation force, her total energy (kinetic plus potential) remains constant. Thus, the decrease in her kinetic energy as she goes from the launch point $(\text{where } PE_g = 0)$ to maximum height is matched by an equal size increase in the gravitational potential energy.

$$\Delta PE_g = -\Delta KE \quad \Rightarrow \quad PE_f - PE_i = -(KE_f - KE_i) \quad \text{or} \quad PE_f = PE_i + KE_i - KE_f$$

and $\quad PE_f = 0 + 2.4\times 10^3\ \text{J} - 0 = \boxed{2.4\times 10^3\ \text{J}}$

(d) The statement that the athlete's total energy is conserved is summarized by the equation $\Delta KE + \Delta PE = 0$ or $KE_2 + PE_2 = KE_1 + PE_1$. In terms of mass, speed, and height, this becomes $\boxed{\tfrac{1}{2}mv_2^2 + mgy_2 = \tfrac{1}{2}mv_1^2 + mgy_1}$. Solving for the final height gives

$$y_2 = \frac{mgy_1 + \frac{1}{2}mv_1^2 - \frac{1}{2}mv_2^2}{mg} \qquad \text{or} \qquad \boxed{y_2 = y_1 + \frac{(v_1^2 - v_2^2)}{2g}}$$

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The given numeric values for this case are $y_1 = 0,\ v_1 = 9.0\ \text{m/s}$ (at the trampoline level), and $v_2 = 0$ (at maximum height). The maximum height attained is then

$$y_2 = y_1 + \frac{\left(v_1^2 - v_2^2\right)}{2g} = 0 + \frac{(9.0\ \text{m/s})^2 - 0}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{4.1\ \text{m}}$$

(e) Solving the energy conservation equation given in part (d) for the final speed gives

$$v_2^2 = \frac{2}{m}\left(\frac{1}{2}mv_1^2 + mgy_1 - mgy_2\right) \qquad \text{or} \qquad \boxed{v_2 = \sqrt{v_1^2 + 2g\left(y_1 - y_2\right)}}$$

With $y_1 = 0$, $v_1 = 9.0\ \text{m/s}$, and $y_2 = y_{\max}/2 = (4.1\ \text{m})/2$, the speed at half the maximum height is given as

$$v_2 = \sqrt{(9.0\ \text{m/s})^2 + 2\left(9.80\ \text{m/s}^2\right)\left(0 - \frac{4.1\ \text{m}}{2}\right)} = \boxed{6.4\ \text{m/s}}$$

5.23 The work the beam does on the pile driver is given by

$$W_{nc} = (F\cos 180°)\Delta x = -F(0.120\ \text{m})$$

Here, the factor $\cos 180°$ is included because the force F exerted on the driver by the beam is directed upward, but the $\Delta x = 12.0\ \text{cm} = 0.120\ \text{m}$ displacement undergone by the driver while in contact with the beam is directed downward.

From the work-energy theorem, this work can also be expressed as

$$W_{nc} = \left(KE_f - KE_i\right) + \left(PE_f - PE_i\right) = \frac{1}{2}m\left(v_f^2 - v_i^2\right) + mg\left(y_f - y_i\right)$$

Choosing $y = 0$ at the level where the pile driver first contacts the top of the beam, the driver starts from rest $\left(v_i = 0\right)$ at $y_i = +5.00\ \text{m}$ and comes to rest again $\left(v_f = 0\right)$ at $y_f = -0.120\ \text{m}$. Therefore, we have

$$-F(0.120\ \text{m}) = \frac{1}{2}m(0-0) + (2\,100\ \text{kg})\left(9.80\ \text{m/s}^2\right)(-0.120\ \text{m} - 5.00\ \text{m})$$

yielding $\quad F = \boxed{8.78\times 10^5\ \text{N directed upward}}$

5.24 Note that the system is released from rest, and at the maximum upward displacement of m_2, the system is again at rest. Thus, the kinetic energy of the system is zero in both the initial and final states. Since only conservative forces (gravitational forces and a spring force) do work on this system, the total energy is constant. Therefore, the gravitational potential energy given up by m_1 as it drops down distance d must equal the sum of the gravitational potential energy gained by m_2 as it rises distance d and the elastic potential energy stored in the spring when it is stretched distance d. Mathematically, this is

$$m_1 g|-d| = m_2 g|d| + \frac{1}{2}k|d|^2 \qquad \text{or} \qquad m_1 g = m_2 g + \frac{1}{2}kd$$

yielding $\quad \boxed{d = \dfrac{2\left(m_1 - m_2\right)g}{k}}$

5.25 While the motorcycle is in the air, only the conservative gravitational force acts on cycle and rider. Thus, $\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$, which gives

$$h = y_f - y_i = \frac{v_i^2 - v_f^2}{2g} = \frac{(35.0\ \text{m/s})^2 - (33.0\ \text{m/s})^2}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{6.94\ \text{m}}$$

5.26 (a) When equilibrium is reached, the total spring force supporting the load equals the weight of the load, or $F_{s,\text{ total}} = F_{s,\text{ leaf}} + F_{s,\text{ helper}} = w_{\text{load}}$. Let k_ℓ and k_h represent the spring constants of the leaf spring and the helper spring, respectively. Then, if x_ℓ is the distance the leaf spring is compressed, the condition for equilibrium becomes

$$k_\ell x_\ell + k_h\left(x_\ell - y_0\right) = w_{\text{load}}$$

or $$x_\ell = \frac{w_{\text{load}} + k_h y_0}{k_\ell + k_h} = \frac{5.00\times10^5\ \text{N} + \left(3.60\times10^5\ \text{N/m}\right)(0.500\ \text{m})}{5.25\times10^5\ \text{N/m} + 3.60\times10^5\ \text{N/m}} = \boxed{0.768\ \text{m}}$$

(b) The work done compressing the springs equals the total elastic potential energy at equilibrium. Thus, $W = \frac{1}{2}k_\ell x_\ell^2 + \frac{1}{2}k_h\left(x_\ell - 0.500\ \text{m}\right)^2$, or

$$W = \frac{1}{2}\left(5.25\times10^5\ \text{N/m}\right)(0.768\ \text{m})^2 + \tfrac{1}{2}\left(3.60\times10^5\ \text{N/m}\right)(0.268\ \text{m})^2 = \boxed{1.68\times10^5\ \text{J}}$$

5.27 The total work done by the two bicep muscles as they contract is

$$W_{\text{biceps}} = 2F_{\text{av}}\Delta x = 2(800\ \text{N})(0.075\ \text{m}) = \boxed{1.2\times10^2\ \text{J}}$$

The total work done on the body as it is lifted 40 cm during a chin-up is

$$W_{\text{chin-up}} = mgh = (75\ \text{kg})\left(9.80\ \text{m/s}^2\right)(0.40\ \text{m}) = \boxed{2.9\times10^2\ \text{J}}$$

Since $W_{\text{chin-up}} > W_{\text{biceps}}$, it is clear that $\boxed{\text{additional muscles must be involved}}$.

5.28 Applying $W_{nc} = (KE + PE)_f - (KE + PE)_i$ to the jump of the "original" flea gives

$$F_m d = \left(0 + mgy_f\right) - (0 + 0) \qquad \text{or} \qquad y_f = \frac{F_m d}{mg}$$

where F_m is the force exerted by the muscle and d is the length of contraction.

If we scale the flea by a factor f, the muscle force increases by f^2 and the length of contraction increases by f. The mass, being proportional to the volume which increases by f^3, will also increase by f^3. Putting these factors into our expression for y_f gives

$$\left(y_f\right)_{\substack{\text{super}\\\text{flea}}} = \frac{\left(f^2F_m\right)(fd)}{\left(f^3m\right)g} = \frac{F_m d}{mg} = y_f \approx \boxed{0.5\ \text{m}}$$

so the "super flea" cannot jump any higher!

This analysis is used to argue that most animals should be able to jump to approximately the same height (~0.5 m). Data on mammals from elephants to shrews tend to support this.

5.29 (a) Taking $y = 0$, and hence $PE_g = mgy = 0$, at ground level, the initial total mechanical energy of the projectile is

$$\left(E_{\text{total}}\right)_i = KE_i + PE_i = \frac{1}{2}mv_i^2 + mgy_i$$

$$= \frac{1}{2}(50.0\ \text{kg})\left(1.20\times10^2\ \text{m/s}\right)^2 + (50.0\ \text{kg})\left(9.80\ \text{m/s}^2\right)(142\ \text{m}) = \boxed{4.30\times10^5\ \text{J}}$$

(b) The work done on the projectile is equal to the change in its total mechanical energy.

$$\left(W_{nc}\right)_{\text{rise}} = \left(KE_f + PE_f\right) - \left(KE_i + PE_i\right) = \frac{1}{2}m\left(v_f^2 - v_i^2\right) + mg\left(y_f - y_i\right)$$

$$= \frac{1}{2}(50.0\ \text{kg})\left[(85.0\ \text{m/s})^2 - (120\ \text{m/s})^2\right] + (50.0\ \text{kg})\left(9.80\ \text{m/s}^2\right)(427\ \text{m} - 142\ \text{m})$$

$$= \boxed{-3.97\times10^4\ \text{J}}$$

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(c) If, during the descent from the maximum height to the ground, air resistance does one and a half times as much work on the projectile as it did while the projectile was rising to the top of the arc, the total work done for the entire trip will be

$$\left(W_{nc}\right)_{\text{total}} = \left(W_{nc}\right)_{\text{rise}} + \left(W_{nc}\right)_{\text{descent}} = \left(W_{nc}\right)_{\text{rise}} + 1.50\left(W_{nc}\right)_{\text{rise}}$$
$$= 2.50\left(-3.97\times10^{4}\ \text{J}\right) = -9.93\times10^{4}\ \text{J}$$

Then, applying the work-energy theorem to the entire flight of the projectile gives

$$\left(W_{nc}\right)_{\text{total}} = (KE+PE)_{\substack{\text{just before}\\ \text{hitting ground}}} - (KE+PE)_{\text{at launch}} = \frac{1}{2}mv_f^2 + mgy_f - \frac{1}{2}mv_i^2 - mgy_i$$

and the speed of the projectile just before hitting the ground is

$$v_f = \sqrt{\frac{2\left(W_{nc}\right)_{\text{total}}}{m} + v_i^2 + 2g\left(y_i - y_f\right)}$$
$$= \sqrt{\frac{2\left(-9.93\times10^{4}\ \text{J}\right)}{50.0\ \text{kg}} + (120\ \text{m/s})^2 + 2\left(9.80\ \text{m/s}^2\right)(142\ \text{m} - 0)} = \boxed{115\ \text{m/s}}$$

5.30 (a) The work done by the gravitational force equals the decrease in the gravitational potential energy, or

$$W_g = -\left(PE_f - PE_i\right) = PE_i - PE_f = mg\left(y_i - y_f\right) = \boxed{mgh}$$

(b) The change in kinetic energy is equal to the net work done on the projectile, which, in the absence of air resistance, is just that done by the gravitational force. Thus,

$$W_{\text{net}} = W_g = \Delta KE \quad \Rightarrow \quad \Delta KE = \boxed{mgh}$$

(c) $\Delta KE = KE_f - KE_i = mgh$ so $KE_f = KE_i + mgh = \boxed{\frac{1}{2}mv_0^2 + mgh}$

(d) $\boxed{\text{No.}}$ None of the calculations in parts (a), (b), or (c) involve the initial angle.

5.31 (a) The system will consist of $\boxed{\text{the mass, the spring, and the Earth}}$. The parts of this system interact via $\boxed{\text{the spring force, the gravitational force, and a normal force}}$.

(b) The points of interest are $\boxed{\text{where the mass is released from rest (at } x = 6.00\text{ cm)}}$ and $\boxed{\text{the equilibrium point, } x = 0}$.

(c) The energy stored in the spring is the elastic potential energy, $PE_s = \frac{1}{2}kx^2$.
At $x = 6.00$ cm, $PE_s = \frac{1}{2}(850\ \text{N/m})\left(6.00\times10^{-2}\ \text{m}\right)^2 = \boxed{1.53\ \text{J}}$
and at the equilibrium position $(x = 0)$, $PE_s = \frac{1}{2}k(0)^2 = \boxed{0}$

(d) The only force doing work on the mass is the conservative spring force (the normal force and the gravitational force are both perpendicular to the motion). Thus, the total mechanical energy of the mass will be constant. Because we may choose $y = 0$, and hence $PE_g = 0$, at the level of the horizontal surface, the energy conservation equation becomes

$$KE_f + \left(PE_s\right)_f = KE_i + \left(PE_s\right)_i \quad \text{or} \quad \frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2$$

and solving for the final speed gives

$$\boxed{v_f = \sqrt{v_i^2 + \frac{k}{m}\left(x_i^2 - x_f^2\right)}}$$

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If the final position is the equilibrium position $(x_f = 0)$ and the object starts from rest $(v_i = 0)$ at $x_i = 6.00$ cm, the final speed is

$$v_f = \sqrt{0 + \frac{850 \text{ N/m}}{1.00 \text{ kg}}\left[\left(6.00\times10^{-2}\text{ m}\right)^2 - 0\right]} = \sqrt{3.06 \text{ m}^2/\text{s}^2} = \boxed{1.75 \text{ m/s}}$$

(e) When the object is halfway between the release point and the equilibrium position, we have $v_i = 0,\ x_i = 6.00$ cm, and $x_f = 3.00$ cm, giving

$$v_f = \sqrt{0 + \frac{850 \text{ N/m}}{1.00 \text{ kg}}\left[\left(6.00\times10^{-2}\text{ m}\right)^2 - \left(3.00\times10^{-2}\text{ m}\right)^2\right]} = \boxed{1.51 \text{ m/s}}$$

This is not half of the speed at equilibrium because the equation for final speed is $\boxed{\text{not a linear function of position}}$.

5.32 Using conservation of mechanical energy, we have

$$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + 0$$

or $$y_f = \frac{v_i^2 - v_f^2}{2g} = \frac{(10 \text{ m/s})^2 - (1.0 \text{ m/s})^2}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{5.1 \text{ m}}$$

5.33 Since no nonconservative forces do work, we use conservation of mechanical energy, with the zero of potential energy selected at the level of the base of the hill. Then,

$\frac{1}{2}mv_f^2 + mgy_f = \frac{1}{2}mv_i^2 + mgy_i$ with $y_f = 0$ yields

$$y_i = \frac{v_f^2 - v_i^2}{2g} = \frac{(3.00 \text{ m/s})^2 - 0}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{0.459 \text{ m}}$$

Note that this result is independent of the mass of the child and sled.

5.34 (a) The stretching force is the weight of the 7.50-kg object, and the elongation is $\Delta x = \ell_f - \ell_i$. Thus, the spring constant for this spring is

$$k = \frac{\left|\vec{\mathbf{F}}_s\right|}{\Delta x} = \frac{mg}{\ell_f - \ell_i} = \frac{(7.50 \text{ kg})\left(9.80 \text{ m/s}^2\right)}{0.415 \text{ m} - 0.350 \text{ m}} = 1.13\times10^3 \text{ N/m} = \boxed{1.13 \text{ kN/m}}$$

(b) The two men pulling on the ends of the spring create a tension $\left|\vec{\mathbf{F}}_s\right| = 190$ N in the spring. The elongation this will produce is

$$\Delta x = \frac{\left|\vec{\mathbf{F}}_s\right|}{k} = \frac{190 \text{ N}}{1.13\times10^3 \text{ N/m}} = 1.68\times10^{-1} \text{ m} = 16.8 \text{ cm}$$

The final length of the spring is then

$$\ell_f = \ell_i + \Delta x = 35.0 \text{ cm} + 16.8 \text{ cm} = \boxed{51.8 \text{ cm}}$$

5.35 (a) On a frictionless track, no external forces do work on the system consisting of the block and the spring as the spring is being compressed. Thus, the total mechanical energy of the system is constant, or $KE_f + (PE_g)_f + (PE_s)_f = KE_i + (PE_g)_i + (PE_s)_i$. Because the track is horizontal, the gravitational potential energy when the mass comes to rest is the same as just before it made contact with the spring, or $\left(PE_g\right)_f = \left(PE_g\right)_i$. This gives

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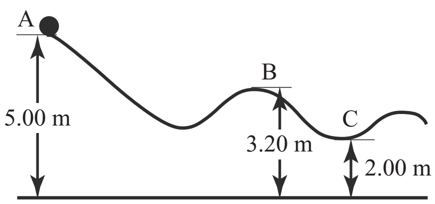
$$\frac{1}{2}mv_f^2 + \frac{1}{2}kx_f^2 = \frac{1}{2}mv_i^2 + \frac{1}{2}kx_i^2$$

Since $v_f = 0$ (the block comes to rest) and $x_i = 0$ (the spring is initially undistorted),

$$x_f = v_i\sqrt{\frac{m}{k}} = (1.50 \text{ m/s})\sqrt{\frac{0.250 \text{ kg}}{4.60 \text{ N/M}}} = \boxed{0.350 \text{ m}}$$

(b) If the track was not frictionless, some of the original kinetic energy would be spent overcoming friction between the block and track. This would mean that less energy would be stored as elastic potential energy in the spring when the block came to rest. Therefore, the maximum compression of the spring $\boxed{\text{would be less}}$ in this case.

5.36 (a) From conservation of mechanical energy,



$$\frac{1}{2}mv_B^2 + mgy_B = \frac{1}{2}mv_A^2 + mgy_A, \text{ or}$$

$$v_B = \sqrt{v_A^2 + 2g(y_A - y_B)}$$

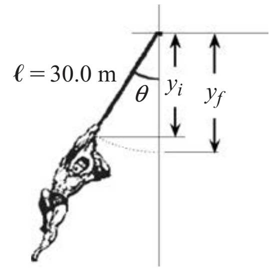
$$= \sqrt{0 + 2(9.80 \text{ m/s}^2)(1.80 \text{ m})} = \boxed{5.94 \text{ m/s}}$$

Similarly,

$$v_C = \sqrt{v_A^2 + 2g(y_A - y_C)} = \sqrt{0 + 2(9.80 \text{ m/s}^2)(5.00 \text{ m} - 2.00 \text{ m})} = \boxed{7.67 \text{ m/s}}$$

(b) $(W_g)_{A\to C} = (PE_g)_A - (PE_g)_C = mg(y_A - y_C) = (49.0 \text{ N})(3.00 \text{ m}) = \boxed{147 \text{ J}}$

5.37 (a) We choose the zero of potential energy at the level of the bottom of the arc. The initial height of Tarzan above this level is



$$y_i = (30.0 \text{ m})(1 - \cos 37.0°) = 6.04 \text{ m}$$

Then, using conservation of mechanical energy, we find

$$\frac{1}{2}mv_f^2 + 0 = \frac{1}{2}mv_i^2 + mgy_i$$

or $$v_f = \sqrt{v_i^2 + 2gy_i} = \sqrt{0 + 2(9.80 \text{ m/s}^2)(6.04 \text{ m})} = \boxed{10.9 \text{ m/s}}$$

(b) In this case, conservation of mechanical energy yields

$$v_f = \sqrt{v_i^2 + 2gy_i} = \sqrt{(4.00 \text{ m/s})^2 + 2(9.80 \text{ m/s}^2)(6.04 \text{ m})} = \boxed{11.6 \text{ m/s}}$$

5.38 (a) If the string does not stretch, the speeds of the two blocks must be equal at all times until m_1 reaches the floor. Also, while m_1 is falling, only conservative forces (the gravitational forces) do work on the system of two blocks. Thus, the total mechanical energy is constant, or

$$KE_{1,f} + KE_{2,f} + (PE_g)_{1,f} + (PE_g)_{2,f} = KE_{1,i} + KE_{2,i} + (PE_g)_{1,i} + (PE_g)_{2,i}$$

Choosing $y = 0$ at floor level, this becomes

$$\frac{1}{2}m_1v_f^2 + \frac{1}{2}m_2v_f^2 + 0 + m_2gh = 0 + 0 + m_1gh + 0$$

and yields $$\boxed{v_f = \sqrt{\frac{2(m_1 - m_2)gh}{m_1 + m_2}}}$$

continued on next page

(b) Using the provided data values, the answer from part (a) gives

$$v_f = \sqrt{\frac{2(6.5\text{ kg} - 4.2\text{ kg})(9.80\text{ m/s}^2)(3.2\text{ m})}{6.5\text{ kg} + 4.2\text{ kg}}} = \boxed{3.7\text{ m/s}}$$

(c) From conservation of energy,

$$\frac{1}{2}m_1 v_f^2 + \frac{1}{2}m_2 v_f^2 + m_1 g y_{1,f} + m_2 g y_{2,f} = 0 + 0 + m_1 g y_{1,i} + m_2 g y_{2,i}$$

or $$v_f = \sqrt{\frac{2g\left[m_1\left(y_{1,i} - y_{1,f}\right) + m_2\left(y_{2,i} - y_{2,f}\right)\right]}{m_1 + m_2}}$$

$$v_f = \sqrt{\frac{2\left(9.80\text{ m/s}^2\right)\left[(6.5\text{ kg})(1.6\text{ m}) + (4.2\text{ kg})(-1.6\text{ kg})\right]}{6.5\text{ kg} + 4.2\text{ kg}}} = \boxed{2.6\text{ m/s}}$$

5.39 (a) Initially, all the energy is stored as elastic potential energy within the spring. When the gun is fired, and as the projectile leaves the gun, most of the energy is in the form of kinetic energy along with a small amount of gravitational potential energy. When the projectile comes to rest momentarily at its maximum height, all of the energy is in the form of gravitational potential energy.

(b) Use conservation of mechanical energy from when the projectile is at rest within the gun $\left(v_i = 0,\ y_i = 0,\text{ and } x_i = -0.120\text{ m}\right)$ until it reaches maximum height where $v_f = 0$, $y_f = y_{\max} = 20.0$ m, and $x_f = 0$ (the spring is relaxed after the gun is fired).

Then, $\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$ becomes

$$0 + mgy_{\max} + 0 = 0 + 0 + \frac{1}{2}kx_i^2$$

or $$k = \frac{2mgy_{\max}}{x_i^2} = \frac{2\left(20.0\times10^{-3}\text{ kg}\right)\left(9.80\text{ m/s}^2\right)(20.0\text{ m})}{(-0.120\text{ m})^2} = \boxed{544\text{ N/m}}$$

(c) This time, we use conservation of mechanical energy from when the projectile is at rest within the gun ($v_i = 0,\ y_i = 0,\text{ and } x_i = -0.120$ m) until it reaches the equilibrium position of the spring ($y_f = +0.120$ m and $x_f = 0$). This gives

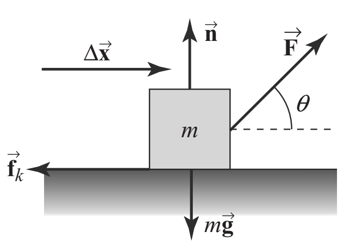
$$KE_f = \left(KE + PE_g + PE_s\right)_i - \left(PE_g + PE_s\right)_f \quad \text{or} \quad \frac{1}{2}mv_f^2 = \left(0 + 0 + \frac{1}{2}kx_i^2\right) - \left(mgy_f + 0\right)$$

$$v_f^2 = \left(\frac{k}{m}\right)x_i^2 - 2gy_f$$

$$= \left(\frac{544\text{ N/m}}{20.0\times10^{-3}\text{ kg}}\right)(-0.120\text{ m})^2 - 2\left(9.80\text{ m/s}^2\right)(0.120\text{ m})$$

yielding $v_f = \boxed{19.7\text{ m/s}}$

5.40 (a) $\Sigma F_y = 0 \quad \Rightarrow \quad n + F\sin\theta - mg = 0$



or $n = mg - F\sin\theta$

The friction force is then

$$f_k = \mu_k n = \boxed{\mu_k\left(mg - F\sin\theta\right)}$$

continued on next page

(b) The work done by the applied force is

$$W_F = F\left|\Delta\vec{\mathbf{x}}\right|\cos\theta = \boxed{Fx\cos\theta}$$

and the work done by the friction force is $W_{f_k} = f_k\left|\Delta\vec{\mathbf{x}}\right|\cos\phi$ where ϕ is the angle between the direction of $\vec{\mathbf{f}}_k$ and $\Delta\vec{\mathbf{x}}$. Thus, $W_{f_k} = f_k x\cos 180° = \boxed{-\mu_k\left(mg - F\sin\theta\right)x}$.

(c) The forces that do no work are those perpendicular to the direction of the displacement $\Delta\vec{\mathbf{x}}$. These are $\boxed{\vec{\mathbf{n}},\ m\vec{\mathbf{g}},\text{ and the vertical component of }\vec{\mathbf{F}}}$.

(d) For part (a): $n = mg - F\sin\theta = (2.00\text{ kg})\left(9.80\text{ m/s}^2\right) - (15.0\text{ N})\sin 37.0° = 10.6\text{ N}$

$$f_k = \mu_k n = (0.400)(10.6\text{ N}) = \boxed{4.24\text{ N}}$$

For part (b): $W_F = Fx\cos\theta = (15.0\text{ N})(4.00\text{ m})\cos 37.0° = \boxed{47.9\text{ J}}$

$$W_{f_k} = f_k x\cos\phi = (4.24\text{ N})(4.00\text{ m})\cos 180° = \boxed{-17.0\text{ J}}$$

5.41 (a) When the child slides down a frictionless surface, the only nonconservative force acting on the child is the normal force. At each instant, this force is perpendicular to the motion and, hence, does no work. Thus, conservation of mechanical energy $\boxed{\text{can be used}}$ in this case.

(b) The equation for conservation of mechanical energy, $(KE + PE)_f = (KE + PE)_i$, for this situation is $\frac{1}{2}\cancel{m}v_f^2 + \cancel{m}gy_f = \frac{1}{2}\cancel{m}v_i^2 + \cancel{m}gy_i$. Notice that the mass of the child cancels out of the equation, so $\boxed{\text{the mass of the child is not a factor}}$ in the frictionless case.

(c) Observe that solving the energy conservation equation from above for the final speed gives $v_f = \sqrt{v_i^2 + 2g\left(y_i - y_f\right)}$. Since the child starts with the same initial speed ($v_i = 0$) and has the same change in altitude in both cases, $\boxed{v_f\text{ is the same}}$ in the two cases.

(d) $\boxed{\text{Work done by a nonconservative force must be accounted for}}$ when friction is present. This is done by using the work-energy theorem rather than conservation of mechanical energy.

(e) From part (b), conservation of mechanical energy gives the final speed as

$$v_f = \sqrt{v_i^2 + 2g\left(y_i - y_f\right)} = \sqrt{0 + 2\left(9.80\text{ m/s}^2\right)(12.0\text{ m})} = \boxed{15.3\text{ m/s}}$$

5.42 (a) $\boxed{\text{No.}}$ The change in the kinetic energy of the plane is equal to the *net* work done by all forces doing work on it. In this case, there are two such forces, the thrust due to the engine and a resistive force due to the air. Since the work done by the air resistance force is negative, the net work done (and hence, the change in kinetic energy) is less than the positive work done by the engine thrust. Also, because the thrust from the engine and the air resistance force are nonconservative forces, $\boxed{\text{mechanical energy is not conserved in this case}}$.

(b) Since the plane is in level flight, $\left(PE_g\right)_f = \left(PE_g\right)_i$, and the work-energy theorem reduces to $W_{nc} = W_{\text{thrust}} + W_{\text{resistance}} = KE_f - KE_i$, or

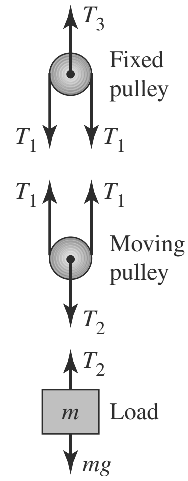
$$\left(F\cos 0°\right)s + \left(f\cos 180°\right)s = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

continued on next page

This gives

$$v_f = \sqrt{v_i^2 + \frac{2(F-f)s}{m}} = \sqrt{(60\ \text{m/s})^2 + \frac{2\left[(7.5-4.0)\times10^4\ \text{N}\right](500\ \text{m})}{1.5\times10^4\ \text{kg}}} = \boxed{77\ \text{m/s}}$$

5.43 Neglecting the mass of the rope and any friction in the pulleys, the tension T_1 is uniform throughout the length of the rope and equal to the magnitude of the force F applied to the loose end of the rope. Also, since the load (and hence the moving pulley) have constant velocity, applying Newton's second law to the three force diagrams at the right gives



Load: $T_2 - mg = 0$ or $T_2 = mg$ **[1]**

moving pulley: $2T_1 - T_2 = 0$ or $T_1 = \dfrac{T_2}{2} = \dfrac{mg}{2}$ **[2]**

fixed pulley: $T_3 - 2T_1 = 0$ or $T_3 = 2T_1 = mg$ **[3]**

(a) If $m = 76.0$ kg, Equation [2] gives

$$F = T_1 = \frac{(76.0\ \text{kg})(9.80\ \text{m/s}^2)}{2} = \boxed{372\ \text{N}}$$

(b) From above, $\boxed{T_1 = 372\ \text{N}}$. From Equations [1] and [3],

$$T_2 = T_3 = mg = (76.0\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{745\ \text{N}}$$

(c) Observe that if the load is raised 1.80 m, this length of rope must be removed from each of the two vertical segments of rope supporting the moving pulley. Thus, the loose end of the rope must be pulled downward a distance $d = 3.60$ m. The work done by the applied force is then

$$W_F = F\cdot d\cdot\cos\theta = (372\ \text{N})(3.60\ \text{m})\cos 0° = 1.34\times10^3\ \text{J} = \boxed{1.34\ \text{kJ}}$$

5.44 (a) Choose $PE_g = 0$ at the level of the bottom of the arc. The child's initial vertical displacement from this level is

$$y_i = (2.00\ \text{m})(1-\cos 30.0°) = 0.268\ \text{m}$$

In the absence of friction, we use conservation of mechanical energy as

$(KE + PE_g)_f = (KE + PE_g)_i$, or $\dfrac{1}{2}mv_f^2 + 0 = 0 + mgy_i$, which gives

$$v_f = \sqrt{2gy_i} = \sqrt{2(9.80\ \text{m/s}^2)(0.268\ \text{m})} = \boxed{2.29\ \text{m/s}}$$

(b) With a nonconservative force present, we use

$W_{nc} = (KE + PE_g)_f - (KE + PE_g)_i = \left(\dfrac{1}{2}mv_f^2 + 0\right) - (0 + mgy_i)$, or

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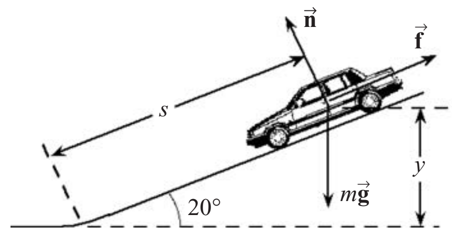
$$W_{nc} = m\left(\frac{v_f^2}{2} - gy_i\right)$$

$$= (25.0 \text{ kg})\left[\frac{(2.00 \text{ m/s})^2}{2} - (9.80 \text{ m/s}^2)(0.268 \text{ m})\right] = -15.7 \text{ J}$$

Thus, $\boxed{15.7 \text{ J}}$ of energy is spent overcoming friction.

5.45 Choose $PE_g = 0$ at the level of the bottom of the driveway.

Then $W_{nc} = (KE + PE_g)_f - (KE + PE_g)_i$ becomes



$$(f\cos 180°)s = \left[\frac{1}{2}mv_f^2 + 0\right] - \left[0 + mg(s\sin 20°)\right]$$

Solving for the final speed gives

$$v_f = \sqrt{(2gs)\sin 20° - \frac{2fs}{m}}$$

or $$v_f = \sqrt{2(9.80 \text{ m/s}^2)(5.0 \text{ m})\sin 20° - \frac{2(4.0\times 10^3 \text{ N})(5.0 \text{ m})}{2.10\times 10^3 \text{ kg}}} = \boxed{3.8 \text{ m/s}}$$

5.46 (a) $\boxed{\text{Yes.}}$ Two forces, a conservative gravitational force and a nonconservative normal force, act on the child as she goes down the slide. However, the normal force is perpendicular to the child's motion at each point on the path and does no work. In the absence of work done by nonconservative forces, mechanical energy is conserved.

(b) We choose the level of the pool to be the $y = 0$ (and hence, $PE_g = 0$) level. Then, when the child is at rest at the top of the slide, $PE_g = \boxed{mgh}$ and $KE = \boxed{0}$. Note that this gives the constant total mechanical energy of the child as $E_{total} = KE + PE_g = mgh$. At the launching point (where $y = h/5$), we have $PE_g = mgy = \boxed{mgh/5}$ and $KE = E_{total} - PE_g = \boxed{4mgh/5}$. At the pool level, $PE_g = \boxed{0}$ and $KE = \boxed{mgh}$.

(c) At the launching point (i.e., where the child leaves the end of the slide),

$$KE = \frac{1}{2}mv_0^2 = \frac{4mgh}{5}$$

meaning that

$$\boxed{v_0 = \sqrt{\frac{8gh}{5}}}$$

(d) After the child leaves the slide and becomes a projectile, energy conservation gives $KE + PE_g = \frac{1}{2}mv^2 + mgy = E_{total} = mgh$ where $v^2 = v_x^2 + v_y^2$. Here, $v_x = v_{0x}$ is constant, but v_y varies with time. At maximum height, $y = y_{max}$ and $v_y = 0$, yielding

$$\frac{1}{2}m(v_{0x}^2 + 0) + mgy_{max} = mgh \qquad \text{and} \qquad \boxed{y_{max} = h - \frac{v_{0x}^2}{2g}}$$

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(e) If the child's launch angle leaving the slide is θ, then $v_{0x} = v_0 \cos\theta$. Substituting this into the result from part (d) and making use of the result from part (c) gives

$$y_{max} = h - \frac{v_0^2}{2g}\cos^2\theta = h - \frac{1}{2g}\left(\frac{8gh}{5}\right)\cos^2\theta \qquad \text{or} \qquad \boxed{y_{max} = h\left(1 - \frac{4}{5}\cos^2\theta\right)}$$

(f) $\boxed{\text{No.}}$ If friction is present, mechanical energy would *not* be conserved, so her kinetic energy at all points after leaving the top of the waterslide would be reduced when compared with the frictionless case. Consequently, her launch speed, maximum height reached, and final speed would be reduced as well.

5.47 Choose $PE_g = 0$ at the level of the base of the hill and let *x* represent the distance the skier moves along the horizontal portion before coming to rest. The normal force exerted on the skier by the snow while on the hill is $n_1 = mg\cos 10.5°$ and, while on the horizontal portion, $n_2 = mg$.

Consider the entire trip, starting from rest at the top of the hill until the skier comes to rest on the horizontal portion. The work done by friction forces is

$$W_{nc} = \left[(f_k)_1 \cos 180°\right](200 \text{ m}) + \left[(f_k)_2 \cos 180°\right]x$$

$$= -\mu_k(mg\cos 10.5°)(200 \text{ m}) - \mu_k(mg)x$$

Applying $W_{nc} = (KE + PE_g)_f - (KE + PE_g)_i$ to this complete trip gives

$$-\mu_k(mg\cos 10.5°)(200 \text{ m}) - \mu_k(mg)x = [0+0] - [0 + mg(200 \text{ m})\sin 10.5°]$$

or $x = \left(\dfrac{\sin 10.5°}{\mu_k} - \cos 10.5°\right)(200 \text{ m})$. If $\mu_k = 0.0750$, then $x = \boxed{289 \text{ m}}$.

5.48 The normal force exerted on the sled by the track is $n = mg\cos\theta$ and the friction force is $f_k = \mu_k n = \mu_k\, mg\cos\theta$.

If *s* is the distance measured along the incline that the sled travels, applying $W_{nc} = (KE + PE_g)_f - (KE + PE_g)_i$ to the entire trip gives

$$\left[(\mu_k\, mg\cos\theta)\cos 180°\right]s = [0 + mg\,s(\sin\theta)] - \left[\frac{1}{2}mv_i^2 + 0\right]$$

or $$s = \frac{v_i^2}{2g(\sin\theta + \mu_k\cos\theta)} = \frac{(4.0 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)(\sin 20° + 0.20\cos 20°)} = \boxed{1.5 \text{ m}}$$

5.49 (a) Consider the entire trip and apply $W_{nc} = (KE + PE_g)_f - (KE + PE_g)_i$ to obtain

$$(f_1\cos 180°)d_1 + (f_2\cos 180°)d_2 = \left(\frac{1}{2}mv_f^2 + 0\right) - (0 + mgy_i)$$

or $$v_f = \sqrt{2\left(g\,y_i - \frac{f_1 d_1 + f_2 d_2}{m}\right)}$$

$$= \sqrt{2\left((9.80 \text{ m/s}^2)(1000 \text{ m}) - \frac{(50.0 \text{ N})(800 \text{ m}) + (3600 \text{ N})(200 \text{ m})}{80.0 \text{ kg}}\right)}$$

which yields $v_f = \boxed{24.5 \text{ m/s}}$

(b) $\boxed{\text{Yes}}$, this is too fast for safety.

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(c) Again, apply $W_{nc} = \left(KE + PE_g\right)_f - \left(KE + PE_g\right)_i$, now with d_2 considered to be a variable, $d_1 = 1000\text{ m} - d_2$, and $v_f = 5.00\text{ m/s}$. This gives

$$\left(f_1 \cos 180°\right)\left(1000\text{ m} - d_2\right) + \left(f_2 \cos 180°\right)d_2 = \left(\frac{1}{2}mv_f^2 + 0\right) - \left(0 + mgy_i\right)$$

which reduces to $-(1000\text{ m})f_1 + f_1 d_2 - f_2 d_2 = \frac{1}{2}mv_f^2 - mgy_i$. Therefore,

$$d_2 = \frac{(mg)y_i - (1000\text{ m})f_1 - \frac{1}{2}mv_f^2}{f_2 - f_1}$$

$$= \frac{(784\text{ N})(1000\text{ m}) - (1000\text{ m})(50.0\text{ N}) - \frac{1}{2}(80.0\text{ kg})(5.00\text{ m/s})^2}{3600\text{ N} - 50.0\text{ N}} = \boxed{206\text{ m}}$$

(d) In reality, the air drag will depend on the skydiver's speed. It will be larger than her 784 N weight only after the chute is opened. It will be nearly equal to 784 N before she opens the chute and again before she touches down, whenever she moves near terminal speed.

5.50 (a) $W_{nc} = \Delta KE + \Delta PE$, but $\Delta KE = 0$ because the speed is constant. The skier rises a vertical distance of $\Delta y = (60\text{ m})\sin 30° = 30\text{ m}$. Thus,

$$W_{nc} = (70\text{ kg})\left(9.80\text{ m/s}^2\right)(30\text{ m}) = 2.1\times10^4\text{ J} = \boxed{21\text{ kJ}}$$

(b) The time to travel 60 m at a constant speed of 2.0 m/s is 30 s. Thus, the required power input is

$$P = \frac{W_{nc}}{\Delta t} = \frac{mg(\Delta y)}{\Delta t} = \frac{(70\text{ kg})\left(9.80\text{ m/s}^2\right)(30\text{ m})}{30\text{ s}} = (686\text{ W})\left(\frac{1\text{ hp}}{746\text{ W}}\right) = \boxed{0.92\text{ hp}}$$

5.51 As the piano is lifted at constant speed up to the apartment, the total work that must be done on it is

$$W_{nc} = \Delta KE + \Delta PE_g = 0 + mg\left(y_f - y_i\right) = \left(3.50\times10^3\text{ N}\right)(25.0\text{ m}) = 8.75\times10^4\text{ J}$$

The three workmen (using a pulley system with an efficiency of 0.750) do work on the piano at a rate of

$$P_{\text{net}} = 0.750\left(3P_{\substack{\text{single}\\\text{worker}}}\right) = 0.750\left[3(165\text{ W})\right] = 371\text{ W} = 371\text{ J/s}$$

so the time required to do the necessary work on the piano is

$$\Delta t = \frac{W_{nc}}{P_{\text{net}}} = \frac{8.75\times10^4\text{ J}}{371\text{ J/s}} = \boxed{236\text{ s}} = (236\text{ s})\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{3.93\text{ min}}$$

5.52 Let ΔN be the number of steps taken in time Δt. We determine the number of steps per unit time by

$$\text{Power} = \frac{\text{work done}}{\Delta t} = \frac{(\text{work per step per unit mass})(\text{mass})(\#\text{ steps})}{\Delta t}$$

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or $\quad 70 \text{ W} = \left(0.60 \ \frac{\text{J/step}}{\text{kg}}\right)(60 \text{ kg})\left(\frac{\Delta N}{\Delta t}\right)$, giving $\frac{\Delta N}{\Delta t} = 1.9 \text{ steps/s}$

The running speed is then

$$v_{\text{av}} = \frac{\Delta x}{\Delta t} = \left(\frac{\Delta N}{\Delta t}\right)(\text{distance traveled per step}) = \left(1.9 \ \frac{\text{step}}{\text{s}}\right)\left(1.5 \ \frac{\text{m}}{\text{step}}\right) = \boxed{2.9 \text{ m/s}}$$

5.53 Assuming a level track, $PE_f = PE_i$, and the work done on the train is

$$W_{nc} = (KE + PE)_f - (KE + PE)_i$$

$$= \frac{1}{2}m\left(v_f^2 - v_i^2\right) = \frac{1}{2}(0.875 \text{ kg})\left[(0.620 \text{ m/s})^2 - 0\right] = 0.168 \text{ J}$$

The power delivered by the motor is then

$$P = \frac{W_{nc}}{\Delta t} = \frac{0.168 \text{ J}}{21.0 \times 10^{-3} \text{ s}} = \boxed{8.00 \text{ W}}$$

5.54 When the car moves at constant speed on a level roadway, the power used to overcome the total frictional force equals the power input from the engine, or $P_{\text{output}} = f_{\text{total}} v = P_{\text{input}}$. This gives

$$f_{\text{total}} = \frac{P_{\text{input}}}{v} = \frac{175 \text{ hp}}{29 \text{ m/s}}\left(\frac{746 \text{ W}}{1 \text{ hp}}\right) = \boxed{4.5 \times 10^3 \text{ N}}$$

5.55 The work done on the older car is $\left(W_{\text{net}}\right)_{\text{old}} = \left(KE_f - KE_i\right)_{\text{old}} = \frac{1}{2}mv^2 - 0 = \frac{1}{2}mv^2$

The work done on the newer car is

$$\left(W_{\text{net}}\right)_{\text{new}} = \left(KE_f - KE_i\right)_{\text{new}} = \frac{1}{2}m(2v)^2 - 0 = 4\left(\frac{1}{2}mv^2\right) = 4\left(W_{\text{net}}\right)_{\text{old}}$$

and the power input to this car is $P_{\text{new}} = \frac{\left(W_{\text{net}}\right)_{\text{new}}}{\Delta t} = \frac{4\left(W_{\text{net}}\right)_{\text{old}}}{\Delta t} = 4P_{\text{old}}$

or $\boxed{\text{the power of the newer car is 4 times that of the older car}}$.

5.56 Neglecting any variation of gravity with altitude, the work required to lift a $3.20 \times 10^7 \text{ kg}$ load at constant speed to an altitude of $\Delta y = 1.75 \text{ km}$ is

$$W = \Delta PE_g = mg(\Delta y) = \left(3.20 \times 10^7 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\left(1.75 \times 10^3 \text{ m}\right) = 5.49 \times 10^{11} \text{ J}$$

The time required to do this work using a $P = 2.70 \text{ kW} = 2.70 \times 10^3 \text{ J/s}$ pump is

$$\Delta t = \frac{W}{P} = \frac{5.49 \times 10^{11} \text{ J}}{2.70 \times 10^3 \text{ J/s}} = \boxed{2.03 \times 10^8 \text{ s}} = \left(2.03 \times 10^8 \text{ s}\right)\left(\frac{1 \text{ yr}}{3.156 \times 10^7 \text{ s}}\right) = \boxed{6.43 \text{ yr}}$$

5.57 (a) The acceleration of the car is

$$a = \frac{v - v_0}{t} = \frac{18.0 \text{ m/s} - 0}{12.0 \text{ s}} = 1.50 \text{ m/s}^2$$

continued on next page

Thus, the constant forward force due to the engine is found from $\Sigma F = F_{\text{engine}} - F_{\text{air}} = ma$ as

$$F_{\text{engine}} = F_{\text{air}} + ma = 400\ \text{N} + \left(1.50\times10^{3}\ \text{kg}\right)\left(1.50\ \text{m/s}^2\right) = 2.65\times10^{3}\ \text{N}$$

The average velocity of the car during this interval is $v_{\text{av}} = (v+v_0)/2 = 9.00\ \text{m/s}$, so the average power input from the engine during this time is

$$P_{\text{av}} = F_{\text{engine}}\, v_{\text{av}} = \left(2.65\times10^{3}\ \text{N}\right)\left(9.00\ \text{m/s}\right) = \left(2.39\times10^{4}\ \text{W}\right)\left(\frac{1\ \text{hp}}{746\ \text{W}}\right) = \boxed{32.0\ \text{hp}}$$

(b) At $t = 12.0$ s, the instantaneous velocity of the car is $v = 18.0\ \text{m/s}$ and the instantaneous power input from the engine is

$$P = F_{\text{engine}}\, v = \left(2.65\times10^{3}\ \text{N}\right)\left(18.0\ \text{m/s}\right) = \left(4.77\times10^{4}\ \text{W}\right)\left(\frac{1\ \text{hp}}{746\ \text{W}}\right) = \boxed{63.9\ \text{hp}}$$

5.58 (a) The acceleration of the elevator during the first 3.00 s is

$$a = \frac{v - v_0}{t} = \frac{1.75\ \text{m/s} - 0}{3.00\ \text{s}} = 0.583\ \text{m/s}^2$$

so $F_{\text{net}} = F_{\text{motor}} - mg = ma$ gives the force exerted by the motor as

$$F_{\text{motor}} = m(a+g) = (650\ \text{kg})\left[(0.583+9.80)\ \text{m/s}^2\right] = 6.75\times10^{3}\ \text{N}$$

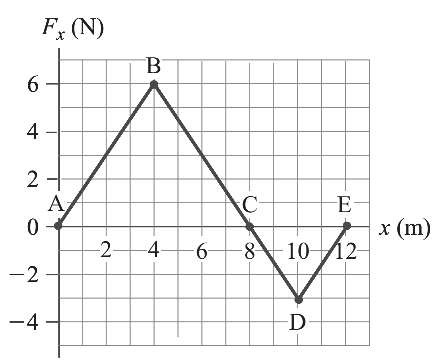
The average velocity during this interval is $v_{\text{av}} = (v+v_0)/2 = 0.875\ \text{m/s}$ so the average power input from the motor during this time is

$$P_{\text{av}} = F_{\text{motor}}\, v_{\text{av}} = \left(6.75\times10^{3}\ \text{N}\right)\left(0.875\ \text{m/s}\right)\left(\frac{1\ \text{hp}}{746\ \text{W}}\right) = \boxed{7.92\ \text{hp}}$$

(b) When the elevator moves upward with a constant speed of $v = 1.75$ m/s, the upward force exerted by the motor is $F_{\text{motor}} = mg$ and the instantaneous power input from the motor is

$$P = (mg)v = (650\ \text{kg})\left(9.80\ \text{m/s}^2\right)\left(1.75\ \text{m/s}\right)\left(\frac{1\ \text{hp}}{746\ \text{W}}\right) = \boxed{14.9\ \text{hp}}$$

5.59 The work done on the particle by the force F as the particle moves from $x = x_i$ to $x = x_f$ is the area under the curve from x_i to x_f.



(a) For $x = 0$ to $x = 8.00$ m,

$$W = \text{area of triangle } ABC = \frac{1}{2}\overline{AC}\times\text{altitude}$$

$$W_{0\to8} = \frac{1}{2}(8.00\ \text{m})(6.00\ \text{N}) = \boxed{24.0\ \text{J}}$$

(b) For $x = 8.00$ m to $x = 10.0$ m,

$$W_{8\to10} = \text{area of triangle } CDE = \frac{1}{2}\overline{CE}\times\text{altitude}$$

$$= \frac{1}{2}(2.00\ \text{m})(-3.00\ \text{N}) = \boxed{-3.00\ \text{J}}$$

(c) $W_{0\to10} = W_{0\to8} + W_{8\to10} = 24.0\ \text{J} + (-3.00\ \text{J}) = \boxed{21.0\ \text{J}}$

5.60 The work done by a force equals the area under the Force versus Displacement curve.

(a) For the region $0 \le x \le 5.00$ m,

$$W_{0 \text{ to } 5} = \frac{(3.00 \text{ N})(5.00 \text{ m})}{2} = \boxed{7.50 \text{ J}}$$

(b) For the region $5.00 \text{ m} \le x \le 10.0$ m,

$$W_{5 \text{ to } 10} = (3.00 \text{ N})(5.00 \text{ m}) = \boxed{15.0 \text{ J}}$$

(c) For the region $10.0 \text{ m} \le x \le 15.0$ m, $W_{10 \text{ to } 15} = \dfrac{(3.00 \text{ N})(5.00 \text{ m})}{2} = \boxed{7.50 \text{ J}}$

(d) $KE|_{x=x_f} - KE|_{x=0} = W_{0 \text{ to } x_f}$ = area under F vs. x curve from $x = 0$ m to $x = x_f$, or

$$\frac{1}{2}mv_f^2 = \frac{1}{2}mv_0^2 + W_{0 \text{ to } x_f} \qquad \text{giving} \qquad v_f = \sqrt{v_0^2 + \left(\frac{2}{m}\right)W_{0 \text{ to } x_f}}$$

For $x_f = 5.00$ m:

$$v_f = \sqrt{v_0^2 + \left(\frac{2}{m}\right)W_{0 \text{ to } 5}} = \sqrt{(0.500 \text{ m/s})^2 + \left(\frac{2}{3.00 \text{ kg}}\right)(7.50 \text{ J})} = \boxed{2.29 \text{ m/s}}$$

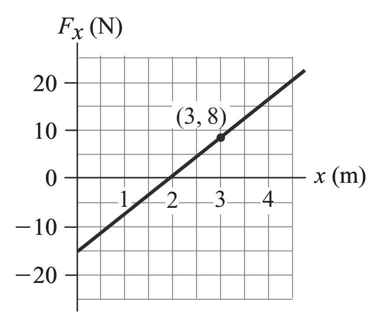
For $x_f = 15.0$ m:

$$v_f = \sqrt{v_0^2 + \left(\frac{2}{m}\right)W_{0 \text{ to } 15}} = \sqrt{v_0^2 + \left(\frac{2}{m}\right)\left(W_{0 \text{ to } 5} + W_{5 \text{ to } 10} + W_{10 \text{ to } 15}\right)}$$

or

$$v_f = \sqrt{(0.500 \text{ m/s})^2 + \left(\frac{2}{3.00 \text{ kg}}\right)(7.50 \text{ J} + 15.0 \text{ J} + 7.50 \text{ J})} = \boxed{4.50 \text{ m/s}}$$

5.61 (a) $F_x = (8x - 16)$ N; see the graph at the right.



(b) The net work done is the total area under the graph from $x = 0$ to $x = 3.00$ m. This consists of two triangular shapes, one below the axis (negative area) and one above the axis (positive). The net work is then

$$W_{\text{net}} = \frac{(2.00 \text{ m})(-16.0 \text{ N})}{2} + \frac{(1.00 \text{ m})(8.00 \text{ N})}{2} = \boxed{-12.0 \text{ J}}$$

5.62 At the top of the arc, $v_y = 0$, and $v_x = v_{0x} = v_0 \cos 30.0°$

Therefore $v^2 = v_x^2 + v_y^2 = v_{0x}^2 + 0 = (v_0 \cos 30.0°)^2$, and

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(0.150 \text{ kg})\left[(40.0 \text{ m/s})^2 \cos^2(30.0°)\right] = \boxed{90.0 \text{ J}}$$

5.63 The person's mass is $m = \dfrac{w}{g} = \dfrac{700 \text{ N}}{9.80 \text{ m/s}^2} = 71.4$ kg. The net upward force acting on the body is $F_{\text{net}} = 2(355 \text{ N}) - 700 \text{ N} = 10.0$ N. The final upward velocity can then be calculated from the work-energy theorem as

$$W_{\text{net}} = KE_f - KE_i = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2$$

or $$\left(F_{\text{net}}\cos\theta\right)s = \left[(10.0 \text{ N})\cos 0°\right](0.250 \text{ m}) = \frac{1}{2}(71.4 \text{ kg})v^2 - 0$$

which gives $v = \boxed{0.265 \text{ m/s upward}}$

5.64 Taking $y = 0$ at ground level, and using conservation of energy from when the boy starts from rest $(v_i = 0)$ at the top of the slide $(y_i = H)$ to the instant he leaves the lower end $(y_f = h)$ of the frictionless slide with a horizontal velocity $(v_{0x} = v_f,\ v_{0y} = 0)$, yields

$$\frac{1}{2}mv_f^2 + mgh = 0 + mgH \qquad \text{or} \qquad v_f^2 = 2g(H - h) \qquad \textbf{[1]}$$

Considering his flight as a projectile after leaving the end of the slide, $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ gives the time to drop distance *h* to the ground as

$$-h = 0 + \frac{1}{2}(-g)t^2 \qquad \text{or} \qquad t = \sqrt{\frac{2h}{g}}$$

The horizontal distance traveled (at constant horizontal velocity) during this time is *d*, so

$$d = v_{0x}t = v_f\sqrt{\frac{2h}{g}} \qquad \text{and} \qquad v_f = d\sqrt{\frac{g}{2h}} = \sqrt{\frac{gd^2}{2h}}$$

Substituting this result into Equation [1] above gives

$$\frac{gd^2}{2h} = 2g(H - h) \qquad \text{or} \qquad \boxed{H = h + \frac{d^2}{4h}}$$

5.65 (a) If $y = 0$ at point B, then $y_A = (35.0 \text{ m})\sin 50.0° = 26.8$ m and $y_B = 0$. Thus,

$$PE_A = mgy_A = \left(1.50\times10^3 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)(26.8 \text{ m}) = \boxed{3.94\times10^5 \text{ J}}$$

$$PE_B = mgy_B = \boxed{0} \qquad \text{and} \qquad \Delta PE_{A\to B} = PE_B - PE_A = 0 - 3.94\times10^5 \text{ J} = \boxed{-3.94\times10^5 \text{ J}}$$

(b) If $y = 0$ at point C, then $y_A = (50.0 \text{ m})\sin 50.0° = 38.3$ m and $y_B = (15.0 \text{ m})\sin 50.0° = 11.5$ m. In this case,

$$PE_A = mgy_A = \left(1.50\times10^3 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)(38.3 \text{ m}) = \boxed{5.63\times10^5 \text{ J}}$$

$$PE_B = mgy_B = \left(1.50\times10^3 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)(11.5 \text{ m}) = \boxed{1.69\times10^5 \text{ J}}$$

and $$\Delta PE_{A\to B} = PE_B - PE_A = 1.69\times10^5 \text{ J} - 5.63\times10^5 \text{ J} = \boxed{-3.94\times10^5 \text{ J}}$$

5.66 (a) Taking $y = 0$ at the initial level of the upper end of the spring, and applying conservation of energy from the instant the ball is released from rest to the instant just before it contacts the spring gives

continued on next page

$$KE_f = KE_i + \left(PE_{gi} - PE_{gf}\right) + \left(PE_{si} - PE_{sf}\right)$$

or $\quad \frac{1}{2}mv_f^2 = 0 + \left(mgh - 0\right) + \left(0 - 0\right)$

and $\quad v_f = \sqrt{2gh} = \sqrt{2\left(9.80 \text{ m/s}^2\right)\left(0.650 \text{ m}\right)} = \boxed{3.57 \text{ m/s}}$

(b) Since the spring is very light, we neglect any energy loss during the collision between the ball and spring, and apply conservation of energy from the instant the ball is released from rest at $y = +h$ to the instant the ball comes to rest momentarily at $y = -d$. This yields

$$KE_f = KE_i + \left(PE_{gi} - PE_{gf}\right) + \left(PE_{si} - PE_{sf}\right)$$

or $\quad 0 = 0 + \left[mgh - mg(-d)\right] + \left(0 - \frac{1}{2}kd^2\right)$

Thus, the force constant of the spring is

$$k = \frac{2mg\left(h+d\right)}{d^2} = \frac{2\left(1.80 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\left(0.650 \text{ m} + 0.0900 \text{ m}\right)}{\left(0.0900 \text{ m}\right)^2}$$

or $\quad k = 3.22 \times 10^3 \text{ N/m} = \boxed{3.22 \text{ kN/m}}$

5.67 (a) The equivalent spring constant of the bow is given by $F = kx$ as

$$k = \frac{F_f}{x_f} = \frac{230 \text{ N}}{0.400 \text{ m}} = \boxed{575 \text{ N/m}}$$

(b) The work done pulling the bow is equal to the elastic potential energy stored in the bow in its final configuration, or

$$W = \frac{1}{2}kx_f^2 = \frac{1}{2}\left(575 \text{ N/m}\right)\left(0.400 \text{ m}\right)^2 = \boxed{46.0 \text{ J}}$$

5.68 Choose $PE_g = 0$ at the level where the block comes to rest against the spring. Then, in the absence of work done by nonconservative forces, the conservation of mechanical energy gives

$$\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$$

or $\quad 0 + 0 + \frac{1}{2}kx_f^2 = 0 + mg\,L\sin\theta + 0$

Thus, $\quad x_f = \sqrt{\frac{2\,mg\,L\sin\theta}{k}} = \sqrt{\frac{2\left(12.0 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)\left(3.00 \text{ m}\right)\sin 35.0^\circ}{3.00 \times 10^4 \text{ N/m}}} = \boxed{0.116 \text{ m}}$

5.69 (a) From $v^2 = v_0^2 + 2a_y\left(\Delta y\right)$, we find the speed just before touching the ground as

$$v = \sqrt{0 + 2\left(9.80 \text{ m/s}^2\right)\left(1.0 \text{ m}\right)} = \boxed{4.4 \text{ m/s}}$$

(b) Choose $PE_g = 0$ at the level where the feet come to rest. Then $W_{nc} = \left(KE + PE_g\right)_f - \left(KE + PE_g\right)_i$ becomes

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$$\left(F_{\text{av}}\cos 180°\right)s = (0+0) - \left(\frac{1}{2}mv_i^2 + mg\,s\right)$$

or $$F_{\text{av}} = \frac{mv_i^2}{2s} + mg = \frac{(75\text{ kg})(4.4\text{ m/s})^2}{2\left(5.0\times10^{-3}\text{ m}\right)} + (75\text{ kg})\left(9.80\text{ m/s}^2\right) = \boxed{1.5\times10^5\text{ N}}$$

5.70 From the work-energy theorem,

$$W_{nc} = \left(KE + PE_g + PE_s\right)_f - \left(KE + PE_g + PE_s\right)_i$$

we have

$$\left(f_k\cos 180°\right)s = \left(\frac{1}{2}mv_f^2 + 0 + 0\right) - \left(0 + 0 + \frac{1}{2}kx_i^2\right)$$

or $$v_f = \sqrt{\frac{kx_i^2 - 2f_k\,s}{m}} = \sqrt{\frac{(8.0\text{ N/m})\left(5.0\times10^{-2}\text{ m}\right)^2 - 2(0.032\text{ N})(0.15\text{ m})}{5.3\times10^{-3}\text{ kg}}} = \boxed{1.4\text{ m/s}}$$

5.71 (a) The two masses will pass when both are at $y_f = 2.00\text{ m}$ above the table. From conservation of energy, $(KE + PE_g + PE_s)_f = (KE + PE_g + PE_s)_i$

$$\frac{1}{2}\left(m_1 + m_2\right)v_f^2 + \left(m_1 + m_2\right)gy_f + 0 = 0 + m_1 g y_{1i} + 0, \text{ or}$$

$$v_f = \sqrt{\frac{2m_1\,g\,y_{1i}}{m_1 + m_2} - 2\,g\,y_f}$$

$$= \sqrt{\frac{2(5.00\text{ kg})\left(9.80\text{ m/s}^2\right)(4.00\text{ m})}{8.00\text{ kg}} - 2\left(9.80\text{ m/s}^2\right)(2.00\text{ m})}$$

This yields the passing speed as $v_f = \boxed{3.13\text{ m/s}}$.

(b) When $m_1 = 5.00\text{ kg}$ reaches the table, $m_2 = 3.00\text{ kg}$ is $y_{2f} = 4.00\text{ m}$ above the table. Thus, $(KE + PE_g + PE_s)_f = (KE + PE_g + PE_s)_i$ becomes

$$\frac{1}{2}\left(m_1 + m_2\right)v_f^2 + m_2 g\,y_{2f} + 0 = 0 + m_1 g y_{1i} + 0, \text{ or } v_f = \sqrt{\frac{2\,g\left(m_1 y_{1i} - m_2 y_{2f}\right)}{m_1 + m_2}}$$

Thus,

$$v_f = \sqrt{\frac{2\left(9.80\text{ m/s}^2\right)\left[(5.00\text{ kg})(4.00\text{ m}) - (3.00\text{ kg})(4.00\text{ m})\right]}{8.00\text{ kg}}} = \boxed{4.43\text{ m/s}}$$

(c) When the 5.00-kg object hits the table, the string goes slack and the 3.00-kg object becomes a projectile launched straight upward with initial speed $v_{0y} = 4.43\text{ m/s}$. At the top of its arc, $v_y = 0$ and $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$ gives

$$\Delta y = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - (4.43\text{ m/s})^2}{2\left(-9.80\text{ m/s}^2\right)} = \boxed{1.00\text{ m}}$$

5.72 (a) The needle has maximum speed during the interval between when the spring returns to normal length and the needle tip first contacts the skin. During this interval, the kinetic energy of the needle equals the original elastic potential energy of the spring, or $\frac{1}{2}mv_{\max}^2 = \frac{1}{2}kx_i^2$. This gives

$$v_{\max} = x_i\sqrt{\frac{k}{m}} = \left(8.10\times10^{-2}\text{ m}\right)\sqrt{\frac{375\text{ N/m}}{5.60\times10^{-3}\text{ kg}}} = \boxed{21.0\text{ m/s}}$$

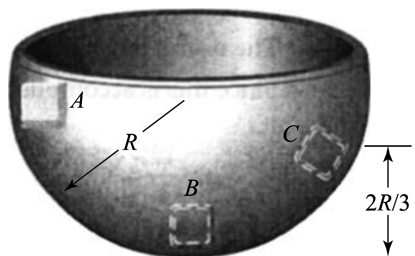
(b) If F_1 is the force the needle must overcome as it penetrates a thickness x_1 of skin and soft tissue while F_2 is the force overcame while penetrating thickness x_2 of organ material, application of the work-energy theorem from the instant before skin contact until the instant before hitting the stop gives

$$W_{\text{net}} = -F_1x_1 - F_2x_2 = \tfrac{1}{2}mv_f^2 - \tfrac{1}{2}mv_{\max}^2$$

or $$v_f = \sqrt{v_{\max}^2 - \frac{2\left(F_1x_1 + F_2x_2\right)}{m}}$$

$$v_f = \sqrt{\left(21.0\text{ m/s}\right)^2 - \frac{2\left[(7.60\text{ N})\left(2.40\times10^{-2}\text{ m}\right)+(9.20\text{ N})\left(3.50\times10^{-2}\text{ m}\right)\right]}{5.60\times10^{-3}\text{ kg}}} = \boxed{16.2\text{ m/s}}$$

5.73 Since the bowl is smooth (that is, frictionless), mechanical energy is conserved or $(KE+PE)_f = (KE+PE)_i$. Also, if we choose $y=0$ (and hence, $PE_g = 0$) at the lowest point in the bowl, then $y_A = +R$, $y_B = 0$, and $y_C = 2R/3$.



(a) $\left(PE_g\right)_A = mgy_A = mgR$

or $\left(PE_g\right)_A = (0.200\text{ kg})\left(9.80\text{ m/s}^2\right)(0.300\text{ m}) = \boxed{0.588\text{ J}}$

(b) $KE_B = KE_A + PE_A - PE_B = 0 + mgy_A - mgy_B = 0.588\text{ J} - 0 = \boxed{0.588\text{ J}}$

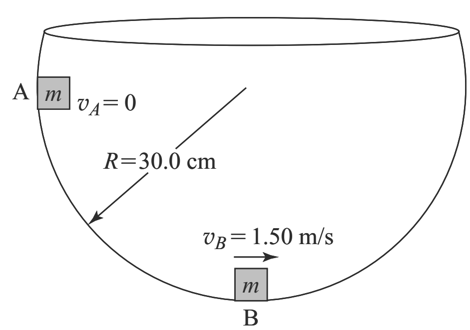
(c) $KE_B = \frac{1}{2}mv_B^2 \Rightarrow v_B = \sqrt{\frac{2\,KE_B}{m}} = \sqrt{\frac{2(0.588\text{ J})}{0.200\text{ kg}}} = \boxed{2.42\text{ m/s}}$

(d) $\left(PE_g\right)_C = mgy_C = (0.200\text{ kg})\left(9.80\text{ m/s}^2\right)\left[\frac{2(0.300\text{ m})}{3}\right] = \boxed{0.392\text{ J}}$

(e) $KE_C = KE_B + PE_B - PE_C = 0.588\text{ J} + 0 - 0.392\text{ J} = \boxed{0.196\text{ J}}$

5.74 (a) $KE_B = \dfrac{1}{2}mv_B^2 = \dfrac{1}{2}(0.200\text{ kg})(1.50\text{ m/s})^2 = \boxed{0.225\text{ J}}$

(b) The change in the altitude of the particle as it goes from A to B is $y_A - y_B = R$, where $R = 0.300\text{ m}$ is the radius of the bowl. Therefore, the work-energy theorem gives



$$\begin{aligned} W_{nc} &= \left(KE_B - KE_A\right) + \left(PE_B - PE_A\right) \\ &= KE_B - 0 + mg\left(y_B - y_A\right) = KE_B + mg(-R) \end{aligned}$$

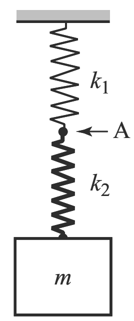
continued on next page

or $\quad W_{nc} = 0.225\text{ J} + (0.200\text{ kg})(9.80\text{ m/s}^2)(-0.300\text{ m}) = -0.363\text{ J}$

The loss of mechanical energy as a result of friction is then $\boxed{0.363\text{ J}}$.

(c) $\boxed{\text{No. Because the normal force, and hence the friction force, varies}}$ with the position of the particle on its path, it is not possible to use the result from part (b) to determine the coefficient of friction without using calculus.

5.75 (a) Consider the sketch at the right. When the mass $m = 1.50\text{ kg}$ is in equilibrium, the upward spring force exerted on it by the lower spring (i.e., the tension in this spring) must equal the weight of the object, or $F_{S2} = mg$. Hooke's law then gives the elongation of this spring as



$$x_2 = \frac{F_{S2}}{k_2} = \frac{mg}{k_2}$$

Now, consider point A where the two springs join. Because this point is in equilibrium, the upward spring force exerted on A by the upper spring must have the same magnitude as the downward spring force exerted on A by the lower spring (that is, the tensions in the two springs must be equal).

The elongation of the upper spring must be

$$x_1 = \frac{F_{S1}}{k_1} = \frac{F_{S2}}{k_1} = \frac{mg}{k_1}$$

and the total elongation of the spring system is

$$x = x_1 + x_2 = \frac{mg}{k_1} + \frac{mg}{k_2} = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)$$

$$= (1.50\text{ kg})(9.80\text{ m/s}^2)\left(\frac{1}{1.20\times 10^3\text{ N/m}} + \frac{1}{1.80\times 10^3\text{ N/m}}\right) = \boxed{2.04\times 10^{-2}\text{ m}}$$

(b) The spring system exerts an upward spring force of $F_S = mg$ on the suspended object and undergoes an elongation of x. The effective spring constant is then

$$k_{\text{effective}} = \frac{F_S}{x} = \frac{mg}{x} = \frac{(1.50\text{ kg})(9.80\text{ m/s}^2)}{2.04\times 10^{-2}\text{ m}} = \boxed{7.20\times 10^2\text{ N/m}}$$

5.76 Refer to the sketch given in the solution of Problem 5.75.

(a) Because the object of mass m is in equilibrium, the tension in the lower spring, F_{S2}, must equal the weight of the object. Therefore, from Hooke's law, the elongation of the lower spring is

$$x_2 = \frac{F_{S2}}{k_2} = \frac{mg}{k_2}$$

From the fact that the point where the springs join (A) is in equilibrium, we conclude that the tensions in the two springs must be equal, $F_{S1} = F_{S2} = mg$. The elongation of the upper spring is then

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$$x_1 = \frac{F_{S1}}{k_1} = \frac{mg}{k_1}$$

and the total elongation of the spring system is

$$\boxed{x = x_1 + x_2 = mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)}$$

(b) The two-spring system undergoes a total elongation of x and exerts an upward spring force $F_S = mg$ on the suspended mass. The effective spring constant of the two springs in series is then

$$k_{\text{effective}} = \frac{F_S}{x} = \frac{mg}{x} = \frac{mg}{mg\left(\frac{1}{k_1} + \frac{1}{k_2}\right)} = \left(\frac{1}{k_1} + \frac{1}{k_2}\right)^{-1}$$

or $\boxed{k_{\text{effective}} = \dfrac{k_1 k_2}{k_1 + k_2}}$

5.77 (a) The person walking uses $E_w = (220\text{ kcal})(4\,186\text{ J/1 kcal}) = 9.21\times 10^5\text{ J}$ of energy while going 3.00 miles. The quantity of gasoline which could furnish this much energy is

$$V_1 = \frac{9.21\times 10^5\text{ J}}{1.30\times 10^8\text{ J/gal}} = 7.08\times 10^{-3}\text{ gal}$$

This means that the walker's fuel economy in equivalent miles per gallon is

$$\textit{fuel economy} = \frac{3.00\text{ mi}}{7.08\times 10^{-3}\text{ gal}} = \boxed{424\text{ mi/gal}}$$

(b) In 1 hour, the bicyclist travels 10.0 miles and uses

$$E_B = (400\text{ kcal})\left(\frac{4\,186\text{ J}}{1\text{ kcal}}\right) = 1.67\times 10^6\text{ J}$$

which is equal to the energy available in

$$V_2 = \frac{1.67\times 10^6\text{ J}}{1.30\times 10^8\text{ J/gal}} = 1.29\times 10^{-2}\text{ gal}$$

of gasoline. Thus, the equivalent fuel economy for the bicyclist is

$$\frac{10.0\text{ mi}}{1.29\times 10^{-2}\text{ gal}} = \boxed{775\text{ mi/gal}}$$

5.78 When 1 pound (454 grams) of fat is metabolized, the energy released is $E = (454\text{ g})(9.00\text{ kcal/g}) = 4.09\times 10^3\text{ kcal}$. Of this, 20.0% goes into mechanical energy (climbing stairs in this case). Thus, the mechanical energy generated by metabolizing 1 pound of fat is

$$E_m = (0.200)(4.09\times 10^3\text{ kcal}) = 818\text{ kcal}$$

Each time the student climbs the stairs, she raises her body a vertical distance of $\Delta y = (80\text{ steps})(0.150\text{ m/step}) = 12.0\text{ m}$. The mechanical energy required to do this is $\Delta PE_g = mg(\Delta y)$, or

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$$\Delta PE_g = (50.0 \text{ kg})(9.80 \text{ m/s}^2)(12.0 \text{ m}) = (5.88\times 10^3 \text{ J})\left(\frac{1 \text{ kcal}}{4186 \text{ J}}\right) = 1.40 \text{ kcal}$$

(a) The number of times the student must climb the stairs to metabolize 1 pound of fat is

$$N = \frac{E_m}{\Delta PE_g} = \frac{818 \text{ kcal}}{1.40 \text{ kcal/trip}} = \boxed{584 \text{ trips}}$$

It would be more practical for her to reduce food intake.

(b) The useful work done each time the student climbs the stairs is

$$W = \Delta PE_g = 5.88\times 10^3 \text{ J}$$

Since this is accomplished in 65.0 s, the average power output is

$$P_{\text{av}} = \frac{W}{t} = \frac{5.88\times 10^3 \text{ J}}{65.0 \text{ s}} = \boxed{90.5 \text{ W}} = (90.5 \text{ W})\left(\frac{1 \text{ hp}}{746 \text{ W}}\right) = \boxed{0.121 \text{ hp}}$$

5.79 (a) Use conservation of mechanical energy, $(KE+PE_g)_f = (KE+PE_g)_i$, from the start to the end of the track to find the speed of the skier as she leaves the track. This gives $\frac{1}{2}mv^2 + mgy_f = 0 + mgy_i$, or

$$v = \sqrt{2g(y_i - y_f)} = \sqrt{2(9.80 \text{ m/s}^2)(40.0 \text{ m})} = \boxed{28.0 \text{ m/s}}$$

(b) At the top of the parabolic arc the skier follows after leaving the track, $v_y = 0$ and $v_x = (28.0 \text{ m/s})\cos 45.0° = 19.8 \text{ m/s}$. Thus, $v_{top} = \sqrt{v_x^2 + v_y^2} = 19.8 \text{ m/s}$. Applying conservation of mechanical energy from the end of the track to the top of the arc gives $\frac{1}{2}m(19.8 \text{ m/s})^2 + mgy_{\max} = \frac{1}{2}m(28.0 \text{ m/s})^2 + mg(10.0 \text{ m})$, or

$$y_{\max} = 10.0 \text{ m} + \frac{(28.0 \text{ m/s})^2 - (19.8 \text{ m/s})^2}{2(9.80 \text{ m/s}^2)} = \boxed{30.0 \text{ m}}$$

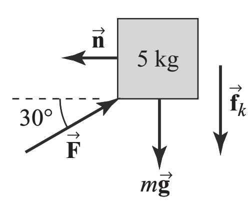
(c) Using $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ for the flight from the end of the track to the ground gives

$$-10.0 \text{ m} = \left[(28.0 \text{ m/s})\sin 45.0°\right]t + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)t^2$$

The positive solution of this equation gives the total time of flight as $t = 4.49$ s. During this time, the skier has a horizontal displacement of

$$\Delta x = v_{0x}t = \left[(28.0 \text{ m/s})\cos 45.0°\right](4.49 \text{ s}) = \boxed{88.9 \text{ m}}$$

5.80 First, determine the magnitude of the applied force by considering a free-body diagram of the block. Since the block moves with constant velocity, $\Sigma F_x = \Sigma F_y = 0$.



From $\Sigma F_x = 0$, we see that $n = F\cos 30°$.

Thus, $f_k = \mu_k n = \mu_k F\cos 30°$, and $\Sigma F_y = 0$ becomes

$$F\sin 30° = mg + \mu_k F\cos 30°, \text{ or}$$

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$$F = \frac{mg}{\sin 30° - \mu_k \cos 30°} = \frac{(5.0\text{ kg})(9.80\text{ m/s}^2)}{\sin 30° - (0.30)\cos 30°} = 2.0 \times 10^2\text{ N}$$

(a) The applied force makes a 60° angle with the displacement up the wall. Therefore,

$$W_F = (F\cos 60°)s = \left[(2.0\times10^2\text{ N})\cos 60°\right](3.0\text{ m}) = \boxed{3.0\times10^2\text{ J}}$$

(b) $W_g = (mg\cos 180°)s = (49\text{ N})(-1.0)(3.0\text{ m}) = \boxed{-1.5\times10^2\text{ J}}$

(c) $W_n = (n\cos 90°)s = \boxed{0}$

(d) $PE_g = mg(\Delta y) = (49\text{ N})(3.0\text{ m}) = \boxed{1.5\times10^2\text{ J}}$

5.81 We choose $PE_g = 0$ at the level where the spring is relaxed $(x = 0)$, or at the level of position B.

(a) At position A, $KE = 0$ and the total energy of the system is given by

$$E = \left(0 + PE_g + PE_s\right)_A = mg\,x_1 + \frac{1}{2}k\,x_1^2$$

or $E = (25.0\text{ kg})(9.80\text{ m/s}^2)(-0.100\text{ m}) + \frac{1}{2}(2.50\times10^4\text{ N/m})(-0.100\text{ m})^2 = \boxed{101\text{ J}}$

(b) In position C, $KE = 0$ and the spring is uncompressed, so $PE_s = 0$. Hence,

$$E = (0 + PE_g + 0)_C = mg\,x_2$$

or $x_2 = \frac{E}{mg} = \frac{101\text{ J}}{(25.0\text{ kg})(9.80\text{ m/s}^2)} = \boxed{0.412\text{ m}}$

(c) At Position B, $PE_g = PE_s = 0$ and $E = (KE + 0 + 0)_B = \frac{1}{2}mv_B^2$

Therefore, $v_B = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(101\text{ J})}{25.0\text{ kg}}} = \boxed{2.84\text{ m/s}}$

(d) Where the velocity (and hence the kinetic energy) is a maximum, the acceleration (slope of the velocity versus time graph) is zero. Thus, $\Sigma F_y = -kx - mg = 0$ and we find

$$x = -\frac{mg}{k} = -\frac{245\text{ kg}}{2.50\times10^4\text{ N/m}} = -9.80\times10^{-3}\text{ m} = \boxed{-9.80\text{ mm}}$$

(e) From the total energy, $E = KE + PE_g + PE_s = \frac{1}{2}mv^2 + mg\,x + \frac{1}{2}k\,x^2$, we find

$$v = \sqrt{\frac{2E}{m} - 2g\,x - \frac{k}{m}x^2}$$

Where the speed, and hence kinetic energy, is a maximum (that is, at $x = -9.80\text{ mm}$), this gives

$$v_{\max} = \sqrt{\frac{2(101\text{ J})}{25.0\text{ kg}} - 2(9.80\text{ m/s}^2)(-9.80\times10^{-3}\text{ m}) - \frac{(2.50\times10^4\text{ N/m})}{25.0\text{ kg}}(-9.80\times10^{-3}\text{ m})^2}$$

or $v_{\max} = \boxed{2.86\text{ m/s}}$

5.82 When the hummingbird is hovering, the magnitude of the average upward force exerted by the air on the wings (and hence, the average downward force the wings exert on the air) must be $F_{av} = mg$, where mg is the weight of the bird. Thus, if the wings move downward distance d during a wing stroke, the work done each beat of the wings is

$$W_{beat} = F_{av}d = mgd = \left(3.0\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\left(3.5\times10^{-2}\ \text{m}\right) = 1.0\times10^{-3}\ \text{J}$$

In one minute, the number of beats of the wings that occur is

$$N = \left(80\ \text{beats/s}\right)\left(60\ \text{s/min}\right) = 4.8\times10^{3}\ \text{beats/min}$$

so the total work preformed in one minute is

$$W_{total} = NW_{beat}\,(1\ \text{min}) = \left(4.8\times10^{3}\ \frac{\text{beats}}{\text{min}}\right)\left(1.0\times10^{-3}\ \frac{\text{J}}{\text{beat}}\right)(1\ \text{min}) = \boxed{4.8\ \text{J}}$$

5.83 Choose $PE_g = 0$ at the level of the river. Then $y_i = 36.0$ m, $y_f = 4.00$, the jumper falls 32.0 m, and the cord stretches 7.00 m. Between the balloon and the level where the diver stops momentarily, $(KE + PE_g + PE_s)_f = (KE + PE_g + PE_s)_i$ gives

$$0 + (700\ \text{N})(4.00\ \text{m}) + \frac{1}{2}k(7.00\ \text{m})^2 = 0 + (700\ \text{N})(36.0\ \text{m}) + 0$$

or $k = \boxed{914\ \text{N/m}}$

5.84 If a projectile is launched, in the absence of air resistance, with speed v_0 at angle θ above the horizontal, the time required to return to the original level is found from $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ as $0 = (v_0\sin\theta)t - (g/2)t^2$, which gives $t = (2v_0\sin\theta)/g$. The range is the horizontal displacement occurring in this time. Thus,

$$R = v_{0x}\,t = \left(v_0\cos\theta\right)\left(\frac{2v_0\sin\theta}{g}\right) = \frac{v_0^2\left(2\sin\theta\cos\theta\right)}{g} = \frac{v_0^2\sin\left(2\theta\right)}{g}$$

Maximum range occurs when $\theta = 45°$, giving $R_{max} = v_0^2/g$ or $v_0^2 = g\,R_{max}$. The minimum kinetic energy required to reach a given maximum range is then

$$KE = \frac{1}{2}mv_0^2 = \frac{1}{2}mg\,R_{max}$$

(a) The minimum kinetic energy needed in the record throw of each object is

Javelin: $KE = \frac{1}{2}(0.80\ \text{kg})\left(9.80\ \text{m/s}^2\right)(98\ \text{m}) = \boxed{3.8\times10^2\ \text{J}}$

Discus: $KE = \frac{1}{2}(2.0\ \text{kg})\left(9.80\ \text{m/s}^2\right)(74\ \text{m}) = \boxed{7.3\times10^2\ \text{J}}$

Shot: $KE = \frac{1}{2}(7.2\ \text{kg})\left(9.80\ \text{m/s}^2\right)(23\ \text{m}) = \boxed{8.1\times10^2\ \text{J}}$

(b) The average force exerted on an object during launch, when it starts from rest and is given the kinetic energy found above, is computed from $W_{net} = F_{av}\,s = \Delta KE$ as $F_{av} = \dfrac{KE - 0}{s}$. Thus, the required force for each object is

Javelin: $F_{av} = \dfrac{3.8\times10^2\ \text{J}}{2.00\ \text{m}} = \boxed{1.9\times10^2\ \text{N}}$

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Discus: $F_{av} = \dfrac{7.3\times10^2 \text{ J}}{2.00 \text{ m}} = \boxed{3.6\times10^2 \text{ N}}$

Shot: $F_{av} = \dfrac{8.1\times10^2 \text{ J}}{2.00 \text{ m}} = \boxed{4.1\times10^2 \text{ N}}$

(c) $\boxed{\text{Yes}}$. If the muscles are capable of exerting 4.1×10^2 N on an object and giving that object a kinetic energy of 8.1×10^2 J, as in the case of the shot, those same muscles should be able to give the javelin a launch speed of

$$v_0 = \sqrt{\frac{2KE}{m}} = \sqrt{\frac{2\left(8.1\times10^2 \text{ J}\right)}{0.80 \text{ kg}}} = 45 \text{ m/s}$$

with a resulting range of

$$R_{max} = \frac{v_0^2}{g} = \frac{(45 \text{ m/s})^2}{9.80 \text{ m/s}^2} = 2.1\times10^2 \text{ m}$$

Since this far exceeds the record range for the javelin, one must conclude that air resistance plays a very significant role in these events.

5.85 (a) From the work-energy theorem, $W_{net} = KE_f - KE_i$. Since the package moves with constant velocity, $KE_f = KE_i$, giving $W_{net} = \boxed{0}$.

Note that the above result can also be obtained by the following reasoning:
Since the object has zero acceleration, the net (or resultant) force acting on it must be zero. The net work done is $W_{net} = F_{net}d = \boxed{0}$.

(b) The work done by the conservative gravitational force is

$$W_{gravity} = -\Delta PE_g = -mg\left(y_f - y_i\right) = -mg(d\sin\theta)$$

or $$W_{gravity} = -(50 \text{ kg})\left(9.80 \text{ m/s}^2\right)(340 \text{ m})\sin 7.0° = \boxed{-2.0\times10^4 \text{ J}}$$

(c) The normal force is perpendicular to the displacement. The work it does is

$$W_{normal} = |\vec{\mathbf{n}}|d\cos 90° = \boxed{0}$$

(d) Since the package moves up the incline at constant speed, the net force parallel to the incline is zero. Thus, $\Sigma F_{\|} = 0 \;\Rightarrow\; f_s - mg\sin\theta = 0$ or $f_s = mg\sin\theta$.

The work done by the friction force in moving the package distance d up the incline is

$$W_{friction} = f_k d = (mg\sin\theta)d = \left[(50 \text{ kg})\left(9.80 \text{ m/s}^2\right)\sin 7.0°\right](340 \text{ m}) = \boxed{2.0\times10^4 \text{ J}}$$

5.86 Each 5.00-m length of the cord will stretch 1.50 m when the tension in the cord equals the weight of the jumper (that is, when $F_s = w = mg$). Thus, the elongation in a cord of original length L when $F_s = w = mg$ will be

$$x = \left(\frac{L}{5.00 \text{ m}}\right)(1.50 \text{ m}) = 0.300L$$

and the force constant for the cord of length L is $k = \dfrac{F_s}{x} = \dfrac{mg}{0.300L}$

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(a) In the bungee-jump from the balloon, the daredevil drops $y_i - y_f = 55.0$ m.

The stretch of the cord at the start of the jump is $x_i = 0$, and that at the lowest point is $x_f = 55.0\text{ m} - L$. Since $KE_i = KE_f = 0$ for the fall, conservation of mechanical energy gives

$$0 + \left(PE_g\right)_f + \left(PE_s\right)_f = 0 + \left(PE_g\right)_i + \left(PE_s\right)_i \quad \Rightarrow \quad \frac{1}{2}k\left(x_f^2 - x_i^2\right) = mg\left(y_i - y_f\right)$$

giving

$$\frac{1}{2}\left(\frac{\cancel{mg}}{0.300L}\right)(55.0\text{ m} - L)^2 = \cancel{mg}\,(55.0\text{ m}) \quad \text{and} \quad (55.0\text{ m} - L)^2 = (33.0\text{ m})L$$

which reduces to

$$(55.0\text{ m})^2 - (110\text{ m})L + L^2 = (33.0\text{ m})L$$

or $\quad L^2 - (143\text{ m})L + (55.0\text{ m})^2 = 0$

and has solutions of

$$L = \frac{-(-143\text{ m}) \pm \sqrt{(-143\text{ m})^2 - 4(1)(55.0\text{ m})^2}}{2(1)}$$

This yields

$$L = \frac{143\text{ m} \pm 91.4\text{ m}}{2} \quad \text{and} \quad L = 117\text{ m} \quad \text{or} \quad L = 25.8\text{ m}$$

Only the $L = \boxed{25.8\text{ m}}$ solution is physically acceptable!

(b) During the jump, $\Sigma F_y = ma_y \quad \Rightarrow \quad kx - mg = ma_y \quad$ or $\quad \left(\frac{\cancel{m}g}{0.300L}\right)x - \cancel{m}g = \cancel{m}a_y$

Thus, $\qquad a_y = \left(\frac{x}{0.300L} - 1\right)g$

which has maximum value at $x = x_{\max} = 55.0\text{ m} - L = 29.2$ m

$$\left(a_y\right)_{\max} = \left[\frac{29.2\text{ m}}{0.300(25.8\text{ m})} - 1\right]g = \boxed{2.77g} = \boxed{27.1\text{ m/s}^2}$$

5.87 (a) While the car moves at constant speed, the tension in the cable is $F = mg\sin\theta$, and the power input is $P = Fv = mgv\sin\theta$ or

$$P = (950\text{ kg})\left(9.80\text{ m/s}^2\right)(2.20\text{ m/s})\sin 30.0° = 1.02\times10^4\text{ W} = \boxed{10.2\text{ kW}}$$

(b) While the car is accelerating, the tension in the cable is

$$F_a = mg\sin\theta + ma = m\left(g\sin\theta + \frac{\Delta v}{\Delta t}\right)$$

$$= (950\text{ kg})\left[\left(9.80\text{ m/s}^2\right)\sin 30.0° + \frac{2.20\text{ m/s} - 0}{12.0\text{ s}}\right] = 4.83\times10^3\text{ N}$$

continued on next page

Maximum power input occurs the last instant of the acceleration phase. Thus,

$$P_{\max} = F_a v_{\max} = \left(4.83\times10^3\ \text{N}\right)\left(2.20\ \text{m/s}\right) = \boxed{10.6\ \text{kW}}$$

(c) The work done by the motor in moving the car up the frictionless track is

$$W_{nc} = (KE+PE)_f - (KE+PE)_i = KE_f + \left(PE_g\right)_f - 0 = \frac{1}{2}mv_f^2 + mg\left(L\sin\theta\right)$$

or $W_{nc} = (950\ \text{kg})\left[\frac{1}{2}(2.20\ \text{m/s})^2 + \left(9.80\ \text{m/s}^2\right)\left(1\,250\ \text{m}\right)\sin 30.0°\right] = \boxed{5.82\times10^6\ \text{J}}$

5.88 (a) Since the tension in the string is always perpendicular to the motion of the object, the string does no work on the object. Then, mechanical energy is conserved:

$$\left(KE + PE_g\right)_f = \left(KE + PE_g\right)_i$$

Choosing $PE_g = 0$ at the level where the string attaches to the cart, this gives

$$0 + mg\left(-L\cos\theta\right) = \frac{1}{2}mv_0^2 + mg(-L), \text{ or } \boxed{v_0 = \sqrt{2gL\left(1-\cos\theta\right)}}$$

(b) If $L = 1.20$ m and $\theta = 35.0°$, the result of part (a) gives

$$v_0 = \sqrt{2\left(9.80\ \text{m/s}^2\right)(1.20\ \text{m})\left(1-\cos 35.0°\right)} = \boxed{2.06\ \text{m/s}}$$

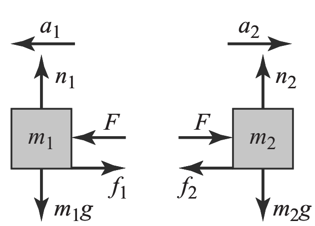
5.89 Observe that when m_3 moves downward distance d, m_1 must move upward distance d and m_2 must slide distance d to the right across the horizontal tabletop. Also, each block must always have the same speed as each of the other blocks. Therefore, if the system starts from rest, and f is the friction force the table exerts on m_2, the work energy theorem $(W_{nc} = \Delta KE + \Delta PE_g)$ gives

$$-fd = \frac{1}{2}\left(m_1+m_2+m_3\right)\left(v_f^2 - v_i^2\right) + \left[m_1 g(+d) + m_2 g(0) + m_3 g(-d)\right]$$

With $v_i = 0$, $m_1 = 5.0$ kg, $m_2 = 10$ kg, $m_3 = 15$ kg, $f = 30$ N, and $d = 4.0$ m, this yields a final speed of

$$v_f = \sqrt{\frac{2d\left[\left(m_3-m_1\right)g - f\right]}{m_1+m_2+m_3}} = \sqrt{\frac{2(4.0\ \text{m})\left[(10\ \text{kg})\left(9.8\ \text{m/s}^2\right) - 30\ \text{N}\right]}{30\ \text{kg}}} = \boxed{4.3\ \text{m/s}}$$

5.90 Call the object on the left Block 1 and that on the right Block 2. Then, $m_1 = 0.250$ kg and $m_2 = 0.500$ kg. The spring force exerted on each block has magnitude



$$F = kx = \left(3.85\ \text{N/m}\right)\left(8.00\times10^{-2}\ \text{m}\right) = 0.308\ \text{N}$$

The magnitudes of normal forces are $n_1 = m_1 g = 2.45$ N and $n_2 = m_2 g = 4.90$ N.

(a) If the surface is frictionless $(\mu = 0)$, then $f_1 = f_2 = 0$ and the net horizontal force acting on each block is $F = 0.308$ N. Taking to the right as the positive x-direction, the acceleration of each block is

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For m_1: $$a_1 = \frac{\Sigma F_{1,x}}{m_1} = \frac{-F}{m_1} = \frac{-0.308\text{ N}}{0.250\text{ kg}} = \boxed{-1.23\text{ m/s}^2}$$

For m_2: $$a_2 = \frac{\Sigma F_{2,x}}{m_2} = \frac{+F}{m_2} = \frac{+0.308\text{ N}}{0.500\text{ kg}} = \boxed{+0.616\text{ m/s}^2}$$

(b) If $\mu_k = 0.100$ for each block, the kinetic friction forces would be

$$f_{k,1} = \mu_k n_1 = (0.100)(2.45\text{ n}) = 0.245\text{ N}$$

and $$f_{k,2} = \mu_k n_2 = (0.100)(4.90\text{ n}) = 0.490\text{ N}$$

Note that $f_{k,1} < F$, so if the coefficient of static friction is low enough to allow the spring force to start m_1 moving, this block will have a net horizontal force acting on it and its acceleration will be

$$a_1 = \frac{-F + f_{1,k}}{m_1} = \frac{-0.308\text{ N} + 0.245\text{ N}}{0.250\text{ kg}} = \boxed{-0.252\text{ m/s}^2}$$

However, $f_{k,2} > F$, and the maximum static friction force is even larger. Thus, the spring force will not be able to overcome static friction and start m_2 moving. The acceleration of this block is $\boxed{a_2 = 0}$ in this case.

(c) For $\mu_k = 0.462$, $f_{k,1} = (0.462)n_1 = 1.13\text{ N}$ and $f_{k,2} = (0.462)n_2 = 2.26\text{ N}$, with even larger static friction forces possible. In this case, the spring force is unable to start either block, meaning that $\boxed{a_1 = a_2 = 0}$.

5.91 When the cyclist travels at constant speed, the magnitude of the forward static friction force on the drive wheel equals that of the retarding air resistance force. Hence, the friction force is proportional to the square of the speed, and her power output may be written as

$$P = f_s v = \left(kv^2\right)v = kv^3$$

where *k* is a proportionality constant.

If the heart rate *R* is proportional to the power output, then $R = k'P = k'(kv^3) = k'kv^3$, where k' is also a proportionality constant.

The ratio of the heart rate R_2 at speed v_2 to the rate R_1 at speed v_1 is then

$$\frac{R_2}{R_1} = \frac{k'kv_2^3}{k'kv_1^3} = \left(\frac{v_2}{v_1}\right)^3 \qquad \text{giving} \qquad v_2 = v_1\left(\frac{R_2}{R_1}\right)^{1/3}$$

Thus, if $R = 90.0$ beats/min at $v = 22.0$ km/h, the speed at which the rate would be 136 beats/min is

$$v = (22.0\text{ km/h})\left(\frac{136\text{ beats/min}}{90.0\text{ beats/min}}\right)^{1/3} = \boxed{25.2\text{ km/h}}$$

and the speed at which the rate would be 166 beats/min is

$$v = (22.0\text{ km/h})\left(\frac{166\text{ beats/min}}{90.0\text{ beats/min}}\right)^{1/3} = \boxed{27.0\text{ km/h}}$$

5.92 The normal force the incline exerts on block A is $n_A = (m_A g)\cos 37°$, and the friction force is $f_k = \mu_k n_A = \mu_k m_A g \cos 37°$. The vertical distance block A rises is $\Delta y_A = (20\text{ m})\sin 37° = 12\text{ m}$, while the vertical displacement of block B is $\Delta y_B = -20\text{ m}$.

We find the common final speed of the two blocks by use of

$$W_{nc} = \left(KE + PE_g\right)_f - \left(KE + PE_g\right)_i = \Delta KE + \Delta PE_g$$

This gives $-\left(\mu_k m_A g \cos 37°\right)s = \left[\frac{1}{2}\left(m_A + m_B\right)v_f^2 - 0\right] + \left[m_A g\left(\Delta y_A\right) + m_B g\left(\Delta y_B\right)\right]$

or $$v_f^2 = \frac{2g\left[-m_B\left(\Delta y_B\right) - m_A\left(\Delta y_A\right) - \left(\mu_k m_A \cos 37°\right)s\right]}{m_A + m_B}$$

$$= \frac{2\left(9.80\text{ m/s}^2\right)\left[-(100\text{ kg})(-20\text{ m}) - (50\text{ kg})(12\text{ m}) - 0.25(50\text{ kg})(20\text{ m})\cos 37°\right]}{150\text{ kg}}$$

which yields $v_f^2 = 157\text{ m}^2/\text{s}^2$.

The change in the kinetic energy of block A is then

$$\Delta KE_A = \frac{1}{2}m_A v_f^2 - 0 = \frac{1}{2}(50\text{ kg})\left(157\text{ m}^2/\text{s}^2\right) = 3.9\times 10^3\text{ J} = \boxed{3.9\text{ kJ}}$$

6

Momentum and Collisions

QUICK QUIZZES

1. Choice (b). The relation between the kinetic energy of an object and the magnitude of the momentum of that object is $KE = p^2/2m$. Thus, when two objects having masses $m_1 < m_2$ have equal kinetic energies, we may write

$$\frac{p_1^2}{2m_1} = \frac{p_2^2}{2m_2} \qquad \text{so} \qquad \frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} < 1 \qquad \text{and} \qquad p_1 < p_2$$

2. Choice (c). Because the momentum of the system (boy + raft) remains constant with zero magnitude, the raft moves towards the shore as the boy walks away from the shore.

3. Choice (c). The total momentum of the car-truck system is conserved. Hence, any change in momentum of the truck must be counterbalanced by an equal magnitude change of opposite sign in the momentum of the car.

4. Choice (a). The total momentum of the two-object system is zero before collision. To conserve momentum, the momentum of the combined object must be zero after the collision. Thus, the combined object must be at rest after the collision.

5. (a) Perfectly inelastic. Any collision in which the two objects stick together afterwards is perfectly inelastic.

(b) Inelastic. Both the Frisbee and the skater lose speed (and hence, kinetic energy) in this collision. Thus, the total kinetic energy of the system is not conserved.

(c) Inelastic. The kinetic energy of the Frisbee is conserved. However, the skater loses speed (and hence, kinetic energy) in this collision. Thus, the total kinetic energy of the system is not conserved.

6. Choice (a). If all of the initial kinetic energy is transformed, then nothing is moving after the collision. Consequently, the final momentum of the system is necessarily zero. Because momentum of the system is conserved, the initial momentum of the system must be zero, meaning that the two objects must have had equal magnitude momenta in opposite directions before the collision.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The magnitude of the impulse is

$$I = \Delta p = p_f - p_i = mv_f - mv_i = m\left(v_f - v_i\right)$$

or $$I = (0.450\text{ kg})(12.8\text{ m/s} - 3.20\text{ m/s}) = 4.32\text{ kg}\cdot\text{m/s}$$

making (b) the correct choice.

2. The mass in motion after the rice ball is added to the bowl is twice the original moving mass. Therefore, to conserve momentum, the speed of the (rice ball + bowl) after the event must be one half of the initial speed of the bowl (i.e., $v_f = v_i/2$). The final kinetic energy is then

$$KE_f = \frac{1}{2}\left(m_{\text{ball}} + m_{\text{bowl}}\right)v_f^2 = \frac{1}{2}\left(2m_{\text{bowl}}\right)\left(\frac{v_i}{2}\right)^2 = \frac{1}{2}\left(\frac{1}{2}m_{\text{bowl}}v_i^2\right) = \frac{E}{2}$$

and the correct choice is (c).

3. Assuming that the collision was head-on so that, after impact, the wreckage moves in the original direction of the car's motion, conservation of momentum during the impact gives

$$\left(m_c + m_t\right)v_f = m_c v_{0c} + m_t v_{0t} = m_c v + m_t(0)$$

or $$v_f = \left(\frac{m_c}{m_c + m_t}\right)v = \left(\frac{m}{m + 2m}\right)v = \frac{v}{3}$$

showing that (c) is the correct choice.

4. The impulse given to the ball is $I = F_{\text{av}}(\Delta t) = mv_f - mv_i = m\left(v_f - v_i\right)$. Choosing the direction of the final velocity of the ball as the positive direction, this gives

$$F_{\text{av}} = \frac{m\left(v_f - v_i\right)}{(\Delta t)} = \frac{\left(57.0\times10^{-3}\text{ kg}\right)\left[+25.0\text{ m/s} - (-21.0\text{ m/s})\right]}{0.060\text{ s}} = 43.7\text{ kg}\cdot\text{m/s}^2 = 43.7\text{ N}$$

and the correct choice is (c).

5. Billiard balls all have the same mass and collisions between them may be considered to be elastic. The dual requirements of conservation of momentum and conservation of kinetic energy in a one-dimensional, elastic, collision are summarized by the two relations:

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \qquad \textbf{[1]}$$

and $$v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f}\right) \qquad \textbf{[2]}$$

In this case, $m_1 = m_2$, and the masses cancel out of the first equation. Call the cue ball #1 and the red ball #2 so that $v_{1i} = -3v$, $v_{2i} = +v$, $v_{1f} = v_{\text{cue}}$, and $v_{2f} = v_{\text{red}}$. Then, the two equations become

$$-3v + v = v_{\text{cue}} + v_{\text{red}} \quad \text{or} \quad v_{\text{cue}} + v_{\text{red}} = -2v \qquad \textbf{[1]}$$

and $$-3v - v = -\left(v_{\text{cue}} - v_{\text{red}}\right) \quad \text{or} \quad v_{\text{cue}} - v_{\text{red}} = 4v \qquad \textbf{[2]}$$

Adding the final versions of these equations yields $2v_{\text{cue}} = 2v$, or $v_{\text{cue}} = v$. Substituting this result into either Equation [1] or [2] above yields $v_{\text{red}} = -3v$. Thus, the correct response for this question is (c).

6. We choose the original direction of motion of the cart as the positive direction. Then, $v_i = 6\text{ m/s}$ and $v_f = -2\text{ m/s}$. The change in the momentum of the cart is

$$\Delta p = mv_f - mv_i = m\left(v_f - v_i\right) = (5\text{ kg})(-2\text{ m/s} - 6\text{ m/s}) = -40\text{ kg}\cdot\text{m/s}$$

and choice (c) is the correct answer.

7. The requirements of conserving both momentum and kinetic energy in a head-on elastic collision are summarized by the equations

$$m_1 v_{1i} + m_2 v_{2i} = m_1 v_{1f} + m_2 v_{2f} \quad \text{and} \quad v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f}\right)$$

With $m_1 = 2$ kg, $v_{1i} = +4\text{ m/s}$, $m_2 = 1$ kg, and $v_{2i} = 0$, these requirements become

$$8\text{ m/s} + 0 = 2v_{1f} + v_{2f} \quad \text{and} \quad 4\text{ m/s} - 0 = -\left(v_{1f} - v_{2f}\right)$$

Solving these equations simultaneously yields

$$v_{1f} = +\frac{4}{3}\ \text{m/s} \qquad \text{and} \qquad v_{2f} = +\frac{16}{3}\ \text{m/s}$$

Thus, choice (a) is the correct answer.

8. Conserving momentum in this inelastic one-dimensional collision gives

$$(5\ \text{kg})v_f = (3\ \text{kg})(+2\ \text{m/s}) + (2\ \text{kg})(-4\ \text{m/s}) \qquad \text{and} \qquad v_f = -0.4\ \text{m/s}$$

The final kinetic energy is then $KE_f = \frac{1}{2}m_{\text{total}}v_f^2 = 0.4$ J, so both choices (a) and (c) are false. The total momentum both before and after the collision is $p_f = p_i = m_{\text{total}}v_f = -2\ \text{kg}\cdot\text{m/s}$. Thus, choices (b) and (e) are both false, and the correct answer is choice (d).

9. Expressing the kinetic energy as $KE = p^2/2m$ (see Questions 10 and 11), we see that the ratio of the magnitudes of the momenta of two particles is

$$\frac{p_2}{p_1} = \frac{\sqrt{2m_2(KE)_2}}{\sqrt{2m_1(KE)_1}} = \sqrt{\left(\frac{m_2}{m_1}\right)\frac{(KE)_2}{(KE)_1}}$$

Thus, we see that if the particles have equal kinetic energies, the magnitudes of their momenta are equal only if the masses are also equal. However, momentum is a *vector quantity* and we can say the two particles have equal momenta only it both the magnitudes and directions are equal, making choice (d) the correct answer.

10. The kinetic energy of a particle may be written as

$$KE = \frac{mv^2}{2} = \frac{m^2v^2}{2m} = \frac{(mv)^2}{2m} = \frac{p^2}{2m}$$

The ratio of the kinetic energies of two particles is then

$$\frac{(KE)_2}{(KE)_1} = \frac{p_2^2/2m_2}{p_1^2/2m_1} = \left(\frac{p_2}{p_1}\right)^2\left(\frac{m_1}{m_2}\right)$$

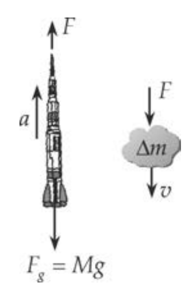
We see that, if the magnitudes of the momenta are equal ($p_2 = p_1$), the kinetic energies will be equal only if the masses are also equal. The correct response is then (c).

11. With the kinetic energy written as $KE = p^2/2m$, we solve for the magnitude of the momentum as $p = \sqrt{2m(KE)}$. The ratio of the final momentum of the rocket to its initial momentum is then given by

$$\frac{p_f}{p_i} = \frac{\sqrt{2m_f(KE)_f}}{\sqrt{2m_i(KE)_i}} = \sqrt{\left(\frac{m_f}{m_i}\right)\frac{(KE)_f}{(KE)_i}} = \sqrt{\left(\frac{1}{2}\right)8} = 2 \qquad \text{or} \qquad p_f = 2p_i$$

and the correct choice is (a).

12. Consider the sketches at the right. The leftmost sketch shows the rocket immediately after the engine is fired (while the rocket's velocity is still essentially zero). It has two forces acting on it, an upward thrust F exerted by the burnt fuel being ejected from the engine, and a downward force of gravity. These forces produce the upward acceleration a of the rocket according to Newton's second law:



$$\Sigma F_y = F - F_g = Ma$$

Since $F_g = Mg$, the thrust exerted on the rocket by the ejected fuel is

$$F = F_g + Ma = M(a+g)$$

The rightmost part of the sketch shows a quantity of burnt fuel that was initially at rest within the rocket, but a very short time Δt later is moving downward at speed v. As this material is ejected, it exerts the upward thrust F on the rocket. By Newton's third law, the rocket exerts a downward force of equal magnitude on this burnt fuel. This force imparts an impulse $I = F(\Delta t) = \Delta p = \Delta m(v - 0)$ to the ejected material. Thus, the rate the rocket is burning and ejecting fuel must be

$$\frac{\Delta m}{\Delta t} = \frac{F}{v-0} = \frac{M(a+g)}{v} = \frac{(3.00\times 10^5\text{ kg})\left[(36.0+9.80)\text{ m/s}^2\right]}{4.50\times 10^3\text{ m/s}} = 3.05\times 10^3\text{ kg/s}$$

and we see that choice (a) is the correct response.

Note: Failure to include the gravitational force in this analysis will lead some students to incorrectly select choice (b) as their answer.

13. Since the same net force acts on the two particles through equal displacements, they have equal amounts of work done on them. From the work-energy theorem, we see that the two particles, both having started with zero kinetic energy, will have equal final kinetic energies. Choice (c) is the correct response.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. (a) No. One of the objects was in motion before collision, so the system consisting of the two particles had a nonzero momentum before impact. Since momentum is always conserved in collisions, the system must have nonzero momentum after impact, meaning that at least one of the particles must be in motion.

(b) Yes. It is possible for one of the particles to be at rest after collision, provided the other particle leaves the collision with a momentum equal to the total momentum of the two-particle system before impact.

4. The glass, concrete, and steel were part of a rigid structure that shattered upon impact of the airplanes with the towers and upon collapse of the buildings as the steel support structures weakened due to high temperatures of the burning fuel. The sheets of paper floating down were probably not in the vicinity of the direct impact, where they would have burned after being exposed to very high temperatures. The papers were most likely situated on desktops or open file cabinets and were blown out of the buildings as they collapsed.

6. Since the total momentum of the skater-Frisbee system is conserved, the momentum transferred to the skater equals the magnitude of the change in the Frisbee's momentum. This is greatest when the skater throws the Frisbee back after catching it.

8. (a) No. In most collisions, there is some loss of kinetic energy (meaning the collision is not elastic), but the objects fail to stick together as they would in a completely inelastic collision (a collision in which the maximum possible kinetic energy loss, consistent with conservation of momentum, occurs).

(b) A greater portion of the incident kinetic energy is transformed to other forms of energy in a head-on collision than in a glancing collision. Thus, the expectation of damage to passengers is greatest in head-on collisions.

10. A certain impulse is required to stop the egg. But, if the time during which the momentum change of the egg occurs is increased, the resulting force on the egg is reduced. The time is increased when the sheet billows out as the egg is brought to a stop. The force is reduced low enough so that the egg will not break.

12. The passenger must undergo a certain momentum change in the collision. This means that an impulse, $I = F_{av} \cdot \Delta t = \Delta p$, must be imparted the passenger by the steering wheel, the window, an air bag, or something. By increasing the time Δt during which this momentum change occurs, the resulting force on the passenger can be decreased.

14. Its speed decreases as its mass increases. No external horizontal forces act on the box-rainwater system, so its horizontal momentum cannot change as the box moves along the surface. Because the product mv_x must be constant, and because the mass of the box (m) is increasing as it slowly fills with water, the horizontal speed of the box must decrease.

16. No. The change in kinetic energy of an object is equal to the net work done on it. This net work is the product of the net force acting on the object and the displacement in the direction of the force. Thus, a small magnitude force acting through a large distance may do more work (and hence produce a greater change in kinetic energy) than a large force acting through a small distance.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. 1.7 kN

4. **(a)** $h_{max} = v_0^2/2g$ **(b)** $mv_0/\sqrt{2}$

6. See Solution.

8. **(a)** $13.5\ \text{N}\cdot\text{s}$ **(b)** 9.00 kN

10. **(a)** $F_{av} = 6.4\times10^3\ \text{N} = 1.4\times10^3\ \text{lb}$

(b) It is unlikely that the man has sufficient arm strength to guarantee the safety of the child during a collision. The violent forces during the collision would tear the child from his arms.

(c) The laws are soundly based on physical principles: always wear a seat belt when in a car.

12. **(a)** $5.40\ \text{N}\cdot\text{s}$ in the direction of the final velocity **(b)** –27.0 J

14. **(a)** $117\ \text{N}\cdot\text{s}$ upward **(b)** 637 N upward **(c)** 897 N upward

16. **(a)** $12.0\ \text{N}\cdot\text{s}$ **(b)** 6.00 m/s **(c)** 4.00 m/s

18. 260 N in the $-x$-direction or perpendicular to the wall

20. **(a)** $13\ \text{N}\cdot\text{s}$ **(b)** 2.5 kN toward the pitcher

(c) The impulsive force is much larger than the ball's weight of 1.4 N.

22. **(a)** 0.49 m/s **(b)** 2.0×10^{-2} m/s

24. **(a)** $\vec{\mathbf{v}}_{PI} = -\left(\frac{m_G}{m_G + m_P}\right)\vec{\mathbf{v}}_{GP}$ **(b)** $\vec{\mathbf{v}}_{GI} = \left(\frac{m_P}{m_G + m_P}\right)\vec{\mathbf{v}}_{GP}$

26. 0.34 m/s toward the left

28. **(a)** $\vec{\mathbf{v}}_{\text{girl}} = -\left(\dfrac{m}{M-m}\right)\vec{\mathbf{v}}_{\text{gloves}}$

(b) As she throws the gloves and exerts a force on them, the gloves exert a force of equal magnitude in the opposite direction on her (Newton's third law) that causes her to accelerate from rest to reach the velocity $\vec{\mathbf{v}}_{\text{girl}}$.

30. 1.67 m/s

32. No. The average force exerted is 3.75×10^3 N, which is less than 4 500 N.

34. **(a)** $v_f = (v_1 + 2v_2)/3$ **(b)** $KE_i - KE_f = (M/3)(v_1 - v_2)^2$

36. **(a)** $v_f = (v_1 + 2v_2)/3$ **(b)** $\Delta KE = -m\left(v_1^2 - 2v_1v_2 + v_2^2\right)/3$

38. $v_{Of} = 3.99$ m/s, $v_{Gf} = 3.01$ m/s

40. 143 m/s

42. **(a)** 20.9 m/s eastward **(b)** 1.50×10^4 J, which becomes internal energy

44. **(a)** See Solution.

(b) x-direction: $m_1v_1\cos0° + m_2v_2\cos105° + m_3v_3\cos\theta = 0$

y-direction: $m_1v_1\sin0° + m_2v_2\sin105° + m_3v_3\sin\theta = 0$

(c) $p_{1x} = 576$ kg·m/s, $p_{2x} = -241$ kg·m/s

(d) $p_{1y} = 0$, $p_{2y} = 898$ m/s

(e) x-direction: $576\ \text{kg}\cdot\text{m/s} - 241\ \text{kg}\cdot\text{m/s} + (112\ \text{kg})v_3\cos\theta = 0$

y-direction: $0 + 898\ \text{kg}\cdot\text{m/s} + (112\ \text{kg})v_3\sin\theta = 0$

(f) $v_3\cos\theta = -2.99$ m/s; $v_3\sin\theta = -8.02$ m/s; $v_3 = 8.56$ m/s

(g) $\theta = 250°$

(h) Because the third fragment must have a momentum equal in magnitude and opposite direction to the resultant of the other two fragments momenta, all three pieces must travel in the same plane.

46. **(a)** $v_{1f} = 0$; $v_{2f} = 1.50$ m/s **(b)** $v_{1f} = -1.00$ m/s; $v_{2f} = 1.50$ m/s

(c) $v_{1f} = 1.00$ m/s; $v_{2f} = 1.50$ m/s

48. $|v_{Af}| = 0.655$ m/s; $v_{Bf} = 0.537$ m/s

50. No, his initial speed was 41.5 mi/h.

52. 40.5 g

54. 0.556 m

56. $v = 4M\sqrt{g\ell}/m$

58. 0.961 m

60. $\vec{\mathbf{v}}_3 = 1.3 \times 10^7 \text{ m/s}$ at $220°$ counterclockwise from the $+x$-axis

62. **(a)** $v_{1i} = +9.90 \text{ m/s};\ v_{2i} = -9.90 \text{ m/s}$

(b) $v_{1f} = -16.5 \text{ m/s};\ v_{2f} = +3.30 \text{ m/s}$

(c) $h_{1f} = 13.9 \text{ m};\ h_{2f} = 0.556 \text{ m}$

64. **(a)** $v_m = v_0\sqrt{2}\ ;\ v_{3m} = v_0\sqrt{2/3}$ **(b)** $\theta_{3m} = 35.3°$

66. **(a)** $90.0°$ **(b)** $v_{\text{cue ball}} = 3.46 \text{ m/s}\ ;\ v_{\text{target ball}} = 2.00 \text{ m/s}$

68. 0.32 m

70. **(a)** See Solution.

(b) From Newton's third law, the two horizontal forces are equal in magnitude and opposite in direction.

(c) $\Delta\vec{\mathbf{p}}_A = -2Mv/3,\ \Delta\vec{\mathbf{p}}_B = +2Mv/3,\ \Delta\vec{\mathbf{p}}_C = 0$

(d) Kinetic energy is not conserved in this inelastic collision.

72. **(a)** 33 m/s **(b)** $2.9 \times 10^3 \text{ m/s}^2$

74. 6.15 m/s

76. **(a)** 7.1 m/s **(b)** 2.6 m

78. **(a)** Conservation of mechanical energy of the bullet-block-Earth system from just after impact until maximum height is reached may be used to relate the speed of the block and bullet just after collision to the maximum height. Then, conservation of momentum from just before to just after impact can be used to relate the initial speed of the bullet to the speed of the block and bullet just after collision.

(b) $v_i = \left[(M+m)/m\right]\sqrt{2gh}$

PROBLEM SOLUTIONS

6.1 Use $p = mv$

(a) $p = \left(1.67 \times 10^{-27} \text{ kg}\right)\left(5.00 \times 10^6 \text{ m/s}\right) = \boxed{8.35 \times 10^{-21} \text{ kg} \cdot \text{m/s}}$

(b) $p = \left(1.50 \times 10^{-2} \text{ kg}\right)\left(3.00 \times 10^2 \text{ m/s}\right) = \boxed{4.50 \text{ kg} \cdot \text{m/s}}$

continued on next page

(c) $p = (75.0\text{ kg})(10.0\text{ m/s}) = \boxed{750\text{ kg}\cdot\text{m/s}}$

(d) $p = (5.98\times10^{24}\text{ kg})(2.98\times10^{4}\text{ m/s}) = \boxed{1.78\times10^{29}\text{ kg}\cdot\text{m/s}}$

6.2 From the impulse-momentum theorem, $F_{\text{av}}(\Delta t) = \Delta p = mv_f - mv_i$, we find the average force to be

$$F_{\text{av}} = \frac{m(v_f - v_i)}{(\Delta t)} = \frac{(55\times10^{-3}\text{ kg})(2.0\times10^{2}\text{ ft/s} - 0)}{0.0020\text{ s} - 0}\left(\frac{1\text{ m/s}}{3.281\text{ ft/s}}\right) = \boxed{1.7\text{ kN}}$$

6.3 (a) If $p_{\text{ball}} = p_{\text{bullet}}$, then

$$v_{\text{ball}} = \frac{m_{\text{bullet}}v_{\text{bullet}}}{m_{\text{ball}}} = \frac{(3.00\times10^{-3}\text{ kg})(1.50\times10^{3}\text{ m/s})}{0.145\text{ kg}} = \boxed{31.0\text{ m/s}}$$

(b) The kinetic energy of the bullet is

$$KE_{\text{bullet}} = \frac{1}{2}m_{\text{bullet}}v_{\text{bullet}}^2 = \frac{(3.00\times10^{-3}\text{ kg})(1.50\times10^{3}\text{ m/s})^2}{2} = 3.38\times10^{3}\text{ J}$$

while that of the baseball is

$$KE_{\text{ball}} = \frac{1}{2}m_{\text{ball}}v_{\text{ball}}^2 = \frac{(0.145\text{ kg})(31.0\text{ m/s})^2}{2} = 69.7\text{ J}$$

The $\boxed{\text{bullet has the larger kinetic energy}}$ by a factor of 48.5.

6.4 (a) We find the maximum height from $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$, with $v_y = 0$ at $\Delta y = h_{\text{max}}$.

$$h_{\text{max}} = \frac{0 - v_0^2}{2(-g)} = \boxed{v_0^2/2g}$$

(b) We use conservation of energy to find the velocity of the ball at $y_f = h_{\text{max}}/2 = v_0^2/4g$. Taking $PE_g = 0$ at $y_i = 0$, we have

$$KE_f + PE_{g,f} = KE_i + PE_{g,i}$$

or $$\frac{1}{2}\cancel{m}v^2 + \cancel{m}\cancel{g}\left(\frac{v_0^2}{4\cancel{g}}\right) = \frac{1}{2}\cancel{m}v_0^2 + 0$$

giving $v^2 = v_0^2/2$, or $v = v_0/\sqrt{2}$. The momentum at this height is then

$$p = mv = \boxed{mv_0/\sqrt{2}}$$

6.5 (a) If Δm is the mass of rain hitting the roof in time Δt, the impulse imparted to the rain by the roof is

$$\vec{\mathbf{I}}_{\text{rain}} = \left(\vec{\mathbf{F}}_{\text{av}}\right)_{\text{rain}}\Delta t = (\Delta m)\vec{\mathbf{v}}_f - (\Delta m)\vec{\mathbf{v}}_i$$

or (taking upward as positive)

$$\left(\vec{\mathbf{F}}_{\text{av}}\right)_{\text{rain}} = \frac{0 - (\Delta m)\vec{\mathbf{v}}_i}{\Delta t} = (0.035\text{ kg/s})\left[0 - (-12\text{ m/s})\right] = +0.42\text{ N}$$

continued on next page

From Newton's third law, the average force the rain exerts on the roof is

$$\left(\vec{\mathbf{F}}_{\text{av}}\right)_{\text{roof}} = -\left(\vec{\mathbf{F}}_{\text{av}}\right)_{\text{rain}} = -0.42\text{ N} = \boxed{0.42\text{ N downward}}$$

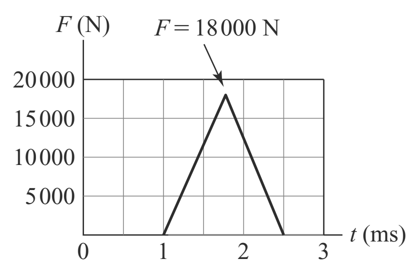
(b) Hailstones striking the roof would rebound upward, and hence experience a greater change in momentum than that experienced by an equal mass of liquid water which strikes the roof without rebounding. Thus, the impulse-momentum theorem, $\vec{\mathbf{F}} = \Delta\vec{\mathbf{p}}/\Delta t$, tells us that the hail will experience a greater average force than that experienced by an equal mass of water striking the roof. Newton's third law then tells us that $\boxed{\text{the hailstones will exert the greater force on the roof.}}$

6.6 $$KE = \frac{mv^2}{2} = \frac{m^2v^2}{2m} = \frac{(mv)^2}{2m} = \boxed{\frac{p^2}{2m}}$$

6.7 (a) $\dfrac{KE}{p} = \dfrac{\frac{1}{2}mv^2}{mv} = \dfrac{v}{2}$ so $v = \dfrac{2(KE)}{p} = \dfrac{2(275\text{ J})}{25.0\text{ kg}\cdot\text{m/s}} = \boxed{22.0\text{ m/s}}$

(b) $m = \dfrac{p}{v} = \dfrac{25.0\text{ kg}\cdot\text{m/s}}{22.0\text{ m/s}} = \boxed{1.14\text{ kg}}$

6.8 (a) The impulse delivered by a force is equal to the area under the Force versus Time curve. From the figure at the right, this is seen to be a triangular area having a base of $1.50\text{ ms} = 1.50 \times 10^{-3}\text{ s}$ and altitude of 18 000 N. Thus,



$$I = \frac{1}{2}\left(1.50 \times 10^{-3}\text{ s}\right)\left(18\,000\text{ N}\right) = \boxed{13.5\text{ N}\cdot\text{s}}$$

(b) $F_{\text{av}} = \dfrac{I}{\Delta t} = \dfrac{13.5\text{ N}\cdot\text{s}}{1.50 \times 10^{-3}\text{ s}} = 9.00 \times 10^3\text{ N} = \boxed{9.00\text{ kN}}$

6.9 (a) We choose the positive direction to be the direction of the final velocity of the ball.

$$I = \Delta p = m\left(v_f - v_i\right) = (0.280\text{ kg})\left[+22.0\text{ m/s} - (-15.0\text{ m/s})\right]$$

or $I = +10.4\text{ kg}\cdot\text{m/s} = \boxed{10.4\text{ kg}\cdot\text{m/s in the direction of the final velocity}}$

(b) The average force the player exerts on the ball is

$$F_{\text{av}} = \frac{I}{\Delta t} = \frac{10.4\text{ kg}\cdot\text{m/s}}{0.060\,0\text{ s}} = \boxed{173\text{ N}}$$

By Newton's third law, the ball exerts a force of equal magnitude back on the player's fist.

6.10 (a) $\left|F_{\text{av}}\right| = \dfrac{|I|}{\Delta t}$, where I is the impulse the man must deliver to the child: $|I| = m_{\text{child}}\left|v_f - v_0\right|$

$$\left|F_{\text{av}}\right| = \frac{m_{\text{child}}\left|v_f - v_0\right|}{\Delta t} = \frac{(12.0\text{ kg})\left|0 - 120\text{ mi/h}\right|}{0.10\text{ s}}\left(\frac{0.447\text{ m/s}}{1\text{ mi/h}}\right) = \boxed{6.4 \times 10^3\text{ N}}$$

or $\left|F_{\text{av}}\right| = \left(6.4 \times 10^3\text{ N}\right)\left(\dfrac{0.224\,8\text{ lb}}{1\text{ N}}\right) = \boxed{1.4 \times 10^3\text{ lb}}$

continued on next page

(b) It is unlikely that the man has sufficient arm strength to guarantee the safety of the child during a collision. The violent forces during the collision would tear the child from his arms.

(c) The laws are soundly based on physical principles: always wear a seat belt when in a car.

6.11 The velocity of the ball just before impact is found from $v_y^2 = v_{0y}^2 + 2a_y \Delta y$ as

$$v_1 = -\sqrt{v_{0y}^2 + 2a_y \Delta y} = -\sqrt{0 + 2\left(-9.80 \text{ m/s}^2\right)\left(-1.25 \text{ m}\right)} = -4.95 \text{ m/s}$$

and the rebound velocity with which it leaves the floor is

$$v_2 = +\sqrt{v_f^2 - 2a_y \Delta y} = +\sqrt{0 - 2\left(-9.80 \text{ m/s}^2\right)\left(+0.960 \text{ m}\right)} = +4.34 \text{ m/s}$$

The impulse given the ball by the floor is then

$$\vec{\mathbf{I}} = \vec{\mathbf{F}}\Delta t = \Delta\left(m\vec{\mathbf{v}}\right) = m\left(\vec{\mathbf{v}}_2 - \vec{\mathbf{v}}_1\right)$$

$$= \left(0.150 \text{ kg}\right)\left[+4.34 \text{ m/s} - \left(-4.95 \text{ m/s}\right)\right] = +1.39 \text{ N}\cdot\text{s} = \boxed{1.39 \text{ N/s upward}}$$

6.12 Take the direction of the ball's final velocity (toward the net) to be the $+x$-direction.

(a) $I = \Delta p = m\left(v_f - v_i\right) = \left(0.060\,0 \text{ kg}\right)\left[40.0 \text{ m/s} - \left(-50.0 \text{ m/s}\right)\right]$

giving $I = +5.40 \text{ kg}\cdot\text{m/s} = \boxed{5.40 \text{ N}\cdot\text{s in direction of final velocity}}$.

(b) $W_{\text{net}} = \Delta KE = \dfrac{1}{2}m\left(v_f^2 - v_i^2\right)$

$$= \frac{\left(0.060\,0 \text{ kg}\right)\left[\left(40.0 \text{ m/s}\right)^2 - \left(50.0 \text{ m/s}\right)^2\right]}{2} = \boxed{-27.0 \text{ J}}$$

6.13 (a) Taking forward as the positive direction,

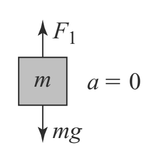
$$I = m\left(\Delta v\right) = \left(70.0 \text{ kg}\right)\left(5.20 \text{ m/s} - 0\right) = +364 \text{ kg}\cdot\text{m/s} = \boxed{364 \text{ N}\cdot\text{s forward}}$$

(b) $F_{\text{av}} = \dfrac{I}{\Delta t} = \dfrac{+364 \text{ kg}\cdot\text{m/s}}{0.832 \text{ s}} = +438 \text{ kg}\cdot\text{m/s}^2 = \boxed{438 \text{ N forward}}$

6.14 (a) Choose upward as the positive direction:

$$I = m\left(v_f - v_i\right) = \left(65.0 \text{ kg}\right)\left(+1.80 \text{ m/s} - 0\right) = +117 \text{ kg}\cdot\text{m/s} = \boxed{117 \text{ kg}\cdot\text{m/s upward}}$$

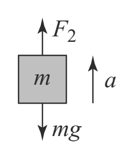
(b) Before the jump, the player is in equilibrium, so



$$\Sigma F_y = ma_y = 0 \quad \Rightarrow \quad F_1 = mg$$

or $F_1 = \left(65.0 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right) = +637 \text{ N} = \boxed{637 \text{ N upward}}$

(c) From the impulse-momentum theorem, the net force the player experiences during the jump is



$$F_{\text{net}} = \frac{I}{\Delta t} = \frac{+117 \text{ kg}\cdot\text{m/s}}{0.450 \text{ s}} = +260$$

continued on next page

But $F_{\text{net}} = F_2 - mg = F_2 - F_1$, where F_2 is the upward force the floor exerts on the player during the jump and F_1 is the force exerted by the floor before the jump. Thus,

$$F_2 = F_{\text{net}} + F_1 = +260\text{ N} + 637\text{ N} = +897\text{ N} = \boxed{897\text{ N upward}}$$

6.15 (a) The impulse equals the area under the F versus t graph. This area is the sum of the area of the rectangle plus the area of the triangle. Thus,

$$I = (2.0\text{ N})(3.0\text{ s}) + \frac{1}{2}(2.0\text{ N})(2.0\text{ s}) = \boxed{8.0\text{ N}\cdot\text{s}}$$

(b) $I = F_{\text{av}}(\Delta t) = \Delta p = m(v_f - v_i)$

$$8.0\text{ N}\cdot\text{s} = (1.5\text{ kg})v_f - 0, \text{ giving } \; v_f = \boxed{5.3\text{ m/s}}$$

(c) $I = F_{\text{av}}(\Delta t) = \Delta p = m(v_f - v_i)$, so $\; v_f = v_i + \dfrac{I}{m}$

$$v_f = -2.0\text{ m/s} + \frac{8.0\text{ N}\cdot\text{s}}{1.5\text{ kg}} = \boxed{3.3\text{ m/s}}$$

6.16 (a) Impulse = area under curve = (two triangular areas of altitude 4.00 N and base 2.00 s) + (one rectangular area of width 1.00 s and height of 4.00 N). Thus,

$$I = 2\left[\frac{(4.00\text{ N})(2.00\text{ s})}{2}\right] + (4.00\text{ N})(1.00\text{ s}) = \boxed{12.0\text{ N}\cdot\text{s}}$$

(b) $I = F_{\text{av}}(\Delta t) = \Delta p = m(v_f - v_i)$, so $\; v_f = v_i + I/m$

$$v_f = 0 + \frac{12.0\text{ N}\cdot\text{s}}{2.00\text{ kg}} = \boxed{6.00\text{ m/s}}$$

(c) $v_f = v_i + \dfrac{I}{m} = -2.00\text{ m/s} + \dfrac{12.0\text{ N}\cdot\text{s}}{2.00\text{ kg}} = \boxed{4.00\text{ m/s}}$

6.17 (a) The impulse is the area under the curve between 0 and 3.0 s.

This is: $\quad I = (4.0\text{ N})(3.0\text{ s}) = \boxed{12\text{ N}\cdot\text{s}}$

(b) The area under the curve between 0 and 5.0 s is

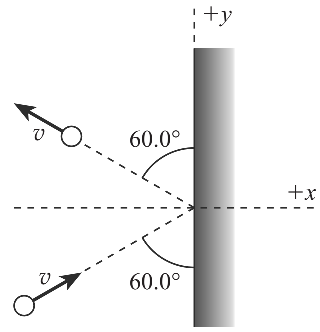
$$I = (4.0\text{ N})(3.0\text{ s}) + (-2.0\text{ N})(2.0\text{ s}) = \boxed{8.0\text{ N}\cdot\text{s}}$$

For parts (c) and (d), we use $I = F_{\text{av}}(\Delta t) = \Delta p = m(v_f - v_i)$, giving $\; v_f = v_i + I/m$

(c) At 3.0 s: $\quad v_f = v_i + \dfrac{I}{m} = 0 + \dfrac{12\text{ N}\cdot\text{s}}{1.50\text{ kg}} = \boxed{8.0\text{ m/s}}$

(d) At 5.0 s: $\quad v_f = v_i + \dfrac{I}{m} = 0 + \dfrac{8.0\text{ N}\cdot\text{s}}{1.50\text{ kg}} = \boxed{5.3\text{ m/s}}$

6.18 $\vec{\mathbf{F}}_{av} = \dfrac{\Delta\vec{\mathbf{p}}}{\Delta t}$ so $\left(F_{av}\right)_x = \dfrac{\Delta p_x}{\Delta t}$ and $\left(F_{av}\right)_y = \dfrac{\Delta p_y}{\Delta t}$



$$\left(F_{av}\right)_y = \frac{m\left[\left(v_y\right)_f - \left(v_y\right)_i\right]}{\Delta t} = \frac{m\left[v\cos 60.0° - v\cos 60.0°\right]}{\Delta t} = 0$$

$$\left(F_{av}\right)_x = \frac{m\left[\left(v_x\right)_f - \left(v_x\right)_i\right]}{\Delta t} = \frac{m\left[\left(-v\sin 60.0°\right) - \left(+v\sin 60.0°\right)\right]}{\Delta t}$$

$$= \frac{-2mv\sin 60.0°}{\Delta t} = \frac{-2\left(3.00\text{ kg}\right)\left(10.0\text{ m/s}\right)\sin 60.0°}{0.200\text{ s}} = -260\text{ N}$$

Thus, $\vec{\mathbf{F}}_{av} = \boxed{\text{260 N in the negative } x\text{-direction or perpendicular to the wall}}$

6.19 (a) $\Delta t = \dfrac{\Delta x}{v_{av}} = \dfrac{2\left(\Delta x\right)}{v_f + v_i} = \dfrac{2\left(1.20\text{ m}\right)}{0 + 25.0\text{ m/s}} = \boxed{9.60 \times 10^{-2}\text{ s}}$

(b) $F_{av} = \dfrac{\Delta p}{\Delta t} = \dfrac{m\left(\Delta v\right)}{\Delta t} = \dfrac{\left(1\,400\text{ kg}\right)\left(25.0\text{ m/s}\right)}{9.60 \times 10^{-2}\text{ s}} = \boxed{3.65 \times 10^{5}\text{ N}}$

(c) $a_{av} = \dfrac{\Delta v}{\Delta t} = \dfrac{25.0\text{ m/s}}{9.60 \times 10^{-2}\text{ s}} = 260\text{ m/s}^2 = \left(260\text{ m/s}^2\right)\left(\dfrac{1\,g}{9.80\text{ m/s}^2}\right) = \boxed{26.5\,g}$

6.20 We shall take toward the pitcher as the positive direction. Then, the velocity of the ball just before contact with the bat is $v_i = -42\text{ m/s}$, and its velocity just as it leaves the bat is $v_f = +48\text{ m/s}$

(a) $I = \Delta p = m\left(\Delta v\right) = \left(0.14\text{ kg}\right)\left[48\text{ m/s} - \left(-42\text{ m/s}\right)\right] = \left(0.14\text{ kg}\right)\left(90\text{ m/s}\right)$

yielding $I = \boxed{13\text{ kg}\cdot\text{m/s} = 13\text{ N}\cdot\text{s}}$

(b) Also, $I = F_{av} \cdot \Delta t$, so

$$F_{av} = \frac{I}{\Delta t} = \frac{+13\text{ N}\cdot\text{s}}{0.005\,0\text{ s}} = +2.6 \times 10^{3}\text{ N} = \boxed{\text{2.6 kN toward the pitcher}}$$

(c) The magnitude of the impulsive force is $\boxed{\text{much larger than the weight}}$, with $F_{av} = 2.6\text{ kN}$ and the weight being $w = mg = \left(0.14\text{ kg}\right)\left(9.80\text{ m/s}^2\right) = 1.4\text{ N}$.

6.21 Requiring that total momentum be conserved gives

$$\left(m_{club}v_{club} + m_{ball}v_{ball}\right)_f = \left(m_{club}v_{club} + m_{ball}v_{ball}\right)_i$$

or $\left(200\text{ g}\right)\left(40\text{ m/s}\right) + \left(46\text{ g}\right)v_{ball} = \left(200\text{ g}\right)\left(55\text{ m/s}\right) + 0$

and $v_{ball} = \boxed{65\text{ m/s}}$

6.22 (a) The mass of the rifle is

$$m = \frac{w}{g} = \frac{30\text{ N}}{9.80\text{ m/s}^2} = \left(\frac{30}{9.8}\right)\text{ kg}$$

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We choose the direction of the bullet's motion to be negative. Then, conservation of momentum gives

$$\left(m_{\text{rifle}}v_{\text{rifle}} + m_{\text{bullet}}v_{\text{bullet}}\right)_f = \left(m_{\text{rifle}}v_{\text{rifle}} + m_{\text{bullet}}v_{\text{bullet}}\right)_i$$

or $$\left[(30/9.8)\ \text{kg}\right]v_{\text{rifle}} + \left(5.0\times10^{-3}\ \text{kg}\right)\left(-300\ \text{m/s}\right) = 0 + 0$$

and $$v_{\text{rifle}} = \frac{9.8\left(5.0\times10^{-3}\ \text{kg}\right)\left(300\ \text{m/s}\right)}{30\ \text{kg}} = \boxed{0.49\ \text{m/s}}$$

(b) The mass of the man plus rifle is

$$m = \frac{730\ \text{N}}{9.80\ \text{m/s}^2} = 74.5\ \text{kg}$$

We use the same approach as in (a), to find

$$v = \left(\frac{5.0\times10^{-3}\ \text{kg}}{74.5\ \text{kg}}\right)\left(300\ \text{m/s}\right) = \boxed{2.0\times10^{-2}\ \text{m/s}}$$

6.23 The velocity of the girl relative to the ice, v_{GI}, is $v_{\text{GI}} = v_{\text{GP}} + v_{\text{PI}}$, where v_{GP} = velocity of girl relative to plank, and v_{PI} = velocity of plank relative to ice. Since we are given that $v_{\text{GP}} = 1.50\ \text{m/s}$, this becomes

$$v_{\text{GI}} = 1.50\ \text{m/s} + v_{\text{PI}} \qquad \textbf{[1]}$$

(a) Conservation of momentum gives $m_{\text{G}}v_{\text{GI}} + m_{\text{P}}v_{\text{PI}} = 0$, or $v_{\text{PI}} = -\left(m_{\text{G}}/m_{\text{P}}\right)v_{\text{GI}}$ **[2]**

Then, Equation [1] becomes

$$v_{\text{GI}} = 1.50\ \text{m/s} - \left(\frac{m_{\text{G}}}{m_{\text{P}}}\right)v_{\text{GI}} \qquad \text{or} \qquad \left(1 + \frac{m_{\text{G}}}{m_{\text{P}}}\right)v_{\text{GI}} = 1.50\ \text{m/s}$$

giving

$$v_{\text{GI}} = \frac{1.50\ \text{m/s}}{1 + \left(\dfrac{45.0\ \text{kg}}{150\ \text{kg}}\right)} = \boxed{1.15\ \text{m/s}}$$

(b) Then, using Equation [2] above,

$$v_{\text{PI}} = -\left(\frac{45.0\ \text{kg}}{150\ \text{kg}}\right)\left(1.15\ \text{m/s}\right) = -0.345\ \text{m/s}$$

or $$v_{\text{PI}} = \boxed{0.345\ \text{m/s directed opposite to the girl's motion}}$$

6.24 Originally, with both girl and plank at rest, the total momentum of the girl-plank system is zero. With negligible friction between the plank and the ice, the total momentum of the girl-plank system is conserved.

(a) The velocity of the girl relative to the ice is given by $\vec{\mathbf{v}}_{\text{GI}} = \vec{\mathbf{v}}_{\text{GP}} + \vec{\mathbf{v}}_{\text{PI}}$, where $\vec{\mathbf{v}}_{\text{GP}}$ is the velocity of the girl relative to the plank and $\vec{\mathbf{v}}_{\text{PI}}$ is the velocity of the plank relative to the ice.

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Conservation of momentum of the girl-plank system then gives

$$0 = m_G \vec{\mathbf{v}}_{GI} + m_P \vec{\mathbf{v}}_{PI} = m_G \left(\vec{\mathbf{v}}_{GP} + \vec{\mathbf{v}}_{PI} \right) + m_P \vec{\mathbf{v}}_{PI}$$

or $\quad 0 = m_G \vec{\mathbf{v}}_{GP} + \left(m_G + m_P \right) \vec{\mathbf{v}}_{PI} \quad$ and $\quad \boxed{\vec{\mathbf{v}}_{PI} = -\left(\dfrac{m_G}{m_G + m_P} \right) \vec{\mathbf{v}}_{GP}}$

(b) The velocity of the girl relative to the ice is

$$\vec{\mathbf{v}}_{GI} = \vec{\mathbf{v}}_{GP} + \vec{\mathbf{v}}_{PI} = \vec{\mathbf{v}}_{GP} - \left(\frac{m_G}{m_G + m_P} \right) \vec{\mathbf{v}}_{GP} = \left(\frac{m_G + m_P - m_G}{m_G + m_P} \right) \vec{\mathbf{v}}_{GP}$$

or $\quad \boxed{\vec{\mathbf{v}}_{GI} = \left(\dfrac{m_P}{m_G + m_P} \right) \vec{\mathbf{v}}_{GP}}$

6.25 (a) Using a subscript *a* for the astronaut and *t* for the tank, conservation of momentum gives $m_a v_{af} + m_t v_{tf} = m_a v_{ai} + m_t v_{ti}$. Since both astronaut and tank were initially at rest, this becomes

$$m_a v_{af} + m_t v_{tf} = 0 + 0 \qquad \text{or} \qquad v_{af} = -\left(\frac{m_t}{m_a} \right) v_{tf}$$

The mass of the astronaut alone (after the oxygen tank has been discarded) is $m_a = 75.0$ kg. Taking toward the spacecraft as the positive direction, the velocity imparted to the astronaut is

$$v_{af} = -\left(\frac{12.0 \text{ kg}}{75.0 \text{ kg}} \right) \left(-8.00 \text{ m/s} \right) = +1.28 \text{ m/s}$$

and the distance she will move in 2.00 min is

$$d = v_{af} t = \left(1.28 \text{ m/s} \right) \left(120 \text{ s} \right) = \boxed{154 \text{ m}}$$

(b) By Newton's third law, when the astronaut exerts a force on the tank, the tank exerts a force back on the astronaut. This reaction force accelerates the astronaut towards the spacecraft.

6.26 The boat and fisherman move as a single unit having mass

$$m_{BF} = m_B + m_F = 125 \text{ kg} + 75 \text{ kg} = 200 \text{ kg}$$

Before the package is thrown, all parts of the system, (boat + fisherman) and package, are at rest, so the total initial momentum is zero. Neglecting water resistance, the final momentum of the system must also be zero, or

$$m_{BF} v_{BF} + m_p v_p = 0$$

giving $\quad v_{BF} = -\left(\dfrac{m_p}{m_{BF}} \right) v_p = -\left(\dfrac{15 \text{ kg}}{200 \text{ kg}} \right) \left(+4.5 \text{ m/s} \right)$

and $\quad v_{BF} = -0.34 \text{ m/s} \quad$ or $\quad \boxed{0.34 \text{ m/s toward the left}}$

6.27 Consider the thrower first, with velocity after the throw of v_{thrower}. Applying conservation of momentum yields

$$(65.0\text{ kg})v_{\text{thrower}} + (0.045\,0\text{ kg})(30.0\text{ m/s}) = (65.0\text{ kg} + 0.045\,0\text{ kg})(2.50\text{ m/s})$$

or $\quad v_{\text{thrower}} = \boxed{2.48\text{ m/s}}$

Now, consider the (catcher + ball), with velocity of v_{catcher} after the catch. From momentum conservation,

$$(60.0\text{ kg} + 0.045\,0\text{ kg})v_{\text{catcher}} = (0.045\,0\text{ kg})(30.0\text{ m/s}) + (60.0\text{ kg})(0)$$

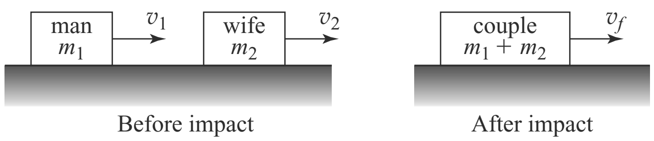
or $\quad v_{\text{catcher}} = \boxed{2.25 \times 10^{-2}\text{ m/s}}$

6.28 (a) The total momentum of the system (girl plus gloves) is zero before the gloves are thrown. Neglecting friction between the girl and the ice, the total momentum is also zero after the gloves are thrown, giving

$$(M - m)\vec{\mathbf{v}}_{\text{girl}} + m\vec{\mathbf{v}}_{\text{gloves}} = 0 \qquad \text{and} \qquad \boxed{\vec{\mathbf{v}}_{\text{girl}} = -\left(\frac{m}{M-m}\right)\vec{\mathbf{v}}_{\text{gloves}}}$$

(b) As she throws the gloves, she exerts a force on them. As described by Newton's third law, the gloves exert a force of equal magnitude in the opposite direction on the girl. This force causes her to accelerate from rest to reach the velocity $\vec{\mathbf{v}}_{\text{girl}}$.

6.29 (a)



(b) The collision is best described as $\boxed{\text{perfectly inelastic}}$, because the skaters remain in contact after the collision.

(c) $\boxed{m_1v_1 + m_2v_2 = (m_1 + m_2)v_f}$

(d) $\boxed{v_f = \dfrac{m_1v_1 + m_2v_2}{m_1 + m_2}}$

(e) $$v_f = \frac{(70.0\text{ kg})(8.00\text{ m/s}) + (50.0\text{ kg})(4.00\text{ m/s})}{70.0\text{ kg} + 50.0\text{ kg}} = \boxed{6.33\text{ m/s}}$$

6.30 Consider a system consisting of arrow and target from the instant just before impact until the instant after the arrow emerges from the target. No external horizontal forces act on the system, so total horizontal momentum must be conserved, or

$$(m_av_a + m_tv_t)_f = (m_av_a + m_tv_t)_i$$

Thus, $$(v_a)_f = \frac{m_a(v_a)_i + m_t(v_t)_i - m_t(v_t)_f}{m_a}$$

$$= \frac{(22.5\text{ g})(+35.0\text{ m/s}) + (300\text{ g})(-2.50\text{ m/s}) - 0}{22.5\text{ g}} = \boxed{1.67\text{ m/s}}$$

6.31 When Gayle jumps on the sled, conservation of momentum gives

$$(50.0 \text{ kg} + 5.00 \text{ kg})v_2 = (50.0 \text{ kg})(4.00 \text{ m/s}) + 0$$

or the speed of Gayle and the sled as they start down the hill is $v_2 = 3.64 \text{ m/s}$.

After Gayle and the sled glide down 5.00 m, conservation of mechanical energy (taking $y = 0$ at the level of the top of the hill) gives

$$\frac{1}{2}\left(\cancel{55.0 \text{ kg}}\right)v_3^2 + \left(\cancel{55.0 \text{ kg}}\right)\left(9.80 \text{ m/s}^2\right)(-5.00 \text{ m}) = \frac{1}{2}\left(\cancel{55.0 \text{ kg}}\right)(3.64 \text{ m/s})^2 + 0$$

so Gayle's speed just before the brother hops on is $v_3 = \sqrt{111} \text{ m/s}$.

After her brother jumps on, conservation of momentum yields

$$(55.0 \text{ kg} + 30.0 \text{ kg})v_4 = (55.0 \text{ kg})\left(\sqrt{111} \text{ m/s}\right) + 0$$

and the speed of Gayle, brother, and sled just after her brother hops on is $v_4 = 6.82 \text{ m/s}$.

After all slide an additional 10.0 m down (to a level 15.0 m below the level of the hilltop), conservation of mechanical energy from just after her brother hops on to the end gives the final speed as

$$\frac{1}{2}\cancel{(85.0 \text{ kg})}\, v_5^2 + \cancel{(85.0 \text{ kg})}\left(9.80 \text{ m/s}^2\right)(-15.0 \text{ m})$$
$$= \frac{1}{2}\cancel{(85.0 \text{ kg})}(6.82 \text{ m/s})^2 + \cancel{(85.0 \text{ kg})}\left(9.80 \text{ m/s}^2\right)(-5.00 \text{ m})$$

or $\quad v_5 = \boxed{15.6 \text{ m/s}}$

6.32 For *each* skater, the impulse-momentum theorem gives

$$\left|F_{av}\right| = \frac{|\Delta p|}{\Delta t} = \frac{m|\Delta v|}{\Delta t} = \frac{(75.0 \text{ kg})(5.00 \text{ m/s})}{0.100 \text{ s}} = \boxed{3.75 \times 10^3 \text{ N}}$$

Since $F_{av} < 4\ 500$ N, there are $\boxed{\text{no broken bones}}$.

6.33 (a) If M is the mass of a single car, conservation of momentum gives

$$(3M)v_f = M(3.00 \text{ m/s}) + (2M)(1.20 \text{ m/s})$$

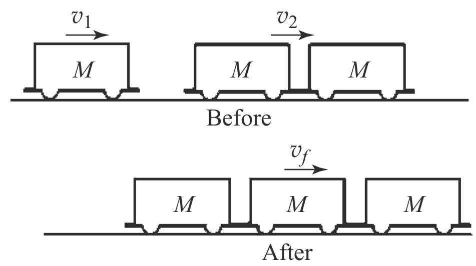
or $\quad v_f = \boxed{1.80 \text{ m/s}}$

(b) The kinetic energy lost is $KE_{\text{lost}} = KE_i - KE_f$, or

$$KE_{\text{lost}} = \frac{1}{2}M(3.00 \text{ m/s})^2 + \frac{1}{2}(2M)(1.20 \text{ m/s})^2 - \frac{1}{2}(3M)(1.80 \text{ m/s})^2$$

With $M = 2.00 \times 10^4$ kg, this yields $KE_{\text{lost}} = \boxed{2.16 \times 10^4 \text{ J}}$.

6.34 (a) From conservation of momentum,



$$(3M)v_f = Mv_1 + (2M)v_2$$

or $$\boxed{v_f = \frac{1}{3}(v_1 + 2v_2)}$$

(b) The kinetic energy before is

$$KE_i = \frac{1}{2}Mv_1^2 + \frac{1}{2}(2M)v_2^2 = \frac{M}{2}(v_1^2 + 2v_2^2)$$

and after collision

$$KE_f = \frac{1}{2}(3M)v_f^2 = \frac{3M}{2}\left[\frac{(v_1 + 2v_2)^2}{9}\right] = \frac{M}{6}(v_1^2 + 4v_1v_2 + 4v_2^2)$$

or $$KE_f = \frac{M}{6}v_1^2 + \frac{2M}{3}v_1v_2 + \frac{2M}{3}v_2^2$$

The kinetic energy lost is

$$KE_i - KE_f = \left(\frac{1}{2} - \frac{1}{6}\right)Mv_1^2 + \left(1 - \frac{2}{3}\right)Mv_2^2 - \frac{2}{3}Mv_1v_2$$

or $$\boxed{KE_i - KE_f = \frac{M}{3}(v_1^2 + v_2^2 - 2v_1v_2) = \frac{M}{3}(v_1 - v_2)^2}$$

6.35 (a) Because momentum is conserved even in a perfectly inelastic collision such as this, the ratio is $\boxed{p_f/p_i = 1}$.

(b) $$p_f = p_i \quad \Rightarrow \quad (m_1 + m_2)v_f = m_1v_{1i} + m_2(0) \quad \text{or} \quad v_f = \frac{m_1v_{1i}}{m_1 + m_2}$$

$$KE_i = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2(0) = \frac{1}{2}m_1v_{1i}^2 \quad \text{and} \quad KE_f = \frac{1}{2}(m_1 + m_2)v_f^2$$

so $$\frac{KE_f}{KE_i} = \frac{(m_1 + m_2)v_f^2}{m_1v_{1i}^2} = \frac{(m_1 + m_2)}{m_1\cancel{v_{1i}^2}}\frac{m_1^2\cancel{v_{1i}^2}}{(m_1 + m_2)^2} = \boxed{\frac{m_1}{m_1 + m_2}}$$

6.36 (a) Momentum is conserved, even in a perfectly inelastic collision. Thus, $p_f = p_i$ or

$$(m + 2m)v_f = mv_1 + 2mv_2 \quad \text{giving} \quad \boxed{v_f = \frac{1}{3}(v_1 + 2v_2)}$$

(b) The kinetic energy before impact is

$$KE_i = \frac{1}{2}mv_1^2 + \frac{1}{2}(2m)v_2^2 = \frac{m}{2}(v_1^2 + 2v_2^2)$$

After impact, the total kinetic energy is

$$KE_f = \frac{1}{2}(3m)v_f^2 = \frac{3}{2}m\left[\frac{1}{9}(v_1^2 + 4v_1v_2 + 4v_2^2)\right] = \frac{m}{2}\left[\frac{1}{3}v_1^2 + \frac{4}{3}v_1v_2 + \frac{4}{3}v_2^2\right]$$

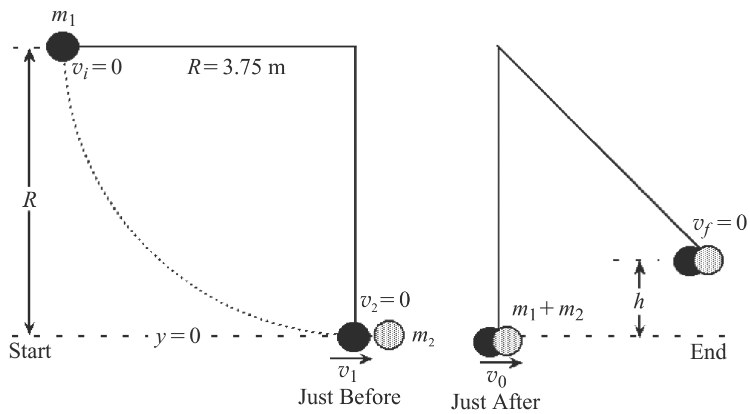
continued on next page

The change that has occurred in the kinetic energy is then

$$\Delta KE = KE_f - KE_i = \frac{m}{2}\left[\left(\frac{1}{3}-1\right)v_1^2 + \frac{4}{3}v_1v_2 + \left(\frac{4}{3}-2\right)v_2^2\right] = \frac{m}{2}\left[-\frac{2}{3}v_1^2 + \frac{4}{3}v_1v_2 - \frac{2}{3}v_2^2\right]$$

or $$\Delta KE = -\frac{m}{3}\left(v_1^2 - 2v_1v_2 + v_2^2\right) = \boxed{-\frac{m}{3}\left(v_1 - v_2\right)^2}$$

6.37 The leftmost part of the sketch depicts the situation from when the actor starts from rest until just before he makes contact with his costar. Using conservation of energy over this period gives



$$\left(KE + PE\right)_1 = \left(KE + PE\right)_i$$

or $$\frac{1}{2}m_1v_1^2 + 0 = 0 + mgR$$

so his speed just before impact is

$$v_1 = \sqrt{2gR} = \sqrt{2\left(9.80 \text{ m/s}^2\right)\left(3.75 \text{ m}\right)} = 8.57 \text{ m/s}$$

Now, employing conservation of momentum from just before to just after impact gives

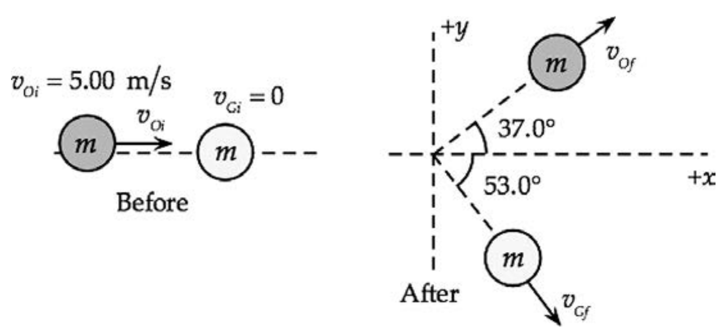
$$\left(m_1 + m_2\right)v_0 = m_1v_1 + m_2\left(0\right) \quad \text{or} \quad v_0 = \frac{m_1v_1}{m_1 + m_2} = \frac{\left(80.0 \text{ kg}\right)\left(8.57 \text{ m/s}\right)}{80.0 \text{ kg} + 55.0 \text{ kg}} = 5.08 \text{ m/s}$$

Finally, using conservation of energy from just after impact to the end yields

$$\left(KE + PE\right)_f = \left(KE + PE\right)_0 \quad \text{or} \quad 0 + \cancel{\left(m_1 + m_2\right)}gh = \frac{1}{2}\cancel{\left(m_1 + m_2\right)}v_0^2$$

and $$h = \frac{v_0^2}{2g} = \frac{\left(5.08 \text{ m/s}\right)^2}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{1.32 \text{ m}}$$

6.38



Consider the sketches above, which show the situation just before and just after collision.

Conserving momentum in y-direction: $p_{yf} = p_{yi} \Rightarrow \cancel{m}v_{Of}\sin 37.0° - \cancel{m}v_{Gf}\sin 53.0° = 0$

or $$v_{Gf} = \left(\frac{\sin 37.0°}{\sin 53.0°}\right)v_{Of} = 0.754\,v_{Of}$$

continued on next page

Now, conserving momentum in the x-direction:

$$p_{xf} = p_{xi} \;\Rightarrow\; \cancel{m} v_{Of} \cos 37.0° + \cancel{m} v_{Gf} \cos 53.0° = \cancel{m} v_{Oi} + 0$$

or $\quad v_{Of} \cos 37.0° + \left(0.754\, v_{Of}\right) \cos 53.0° = v_{Oi}$

and $\quad v_{Of} = \dfrac{v_{Oi}}{\cos 37.0° + (0.754)\cos 53.0°} = \dfrac{5.00 \text{ m/s}}{\cos 37.0° + (0.754)\cos 53.0°} = \boxed{3.99 \text{ m/s}}$

Then, $\quad v_{Gf} = 0.754\, v_{Of} = 0.754(3.99 \text{ m/s}) = \boxed{3.01 \text{ m/s}}$

Now, we can verify that this collision was indeed an elastic collision:

$$KE_i = \frac{1}{2} m v_{Oi}^2 = \frac{m}{2}(5.00 \text{ m/s})^2 = m\left(12.5 \text{ m}^2/s^2\right)$$

and $\quad KE_f = \dfrac{1}{2} m v_{Of}^2 + \dfrac{1}{2} m v_{Gf}^2 = \dfrac{m}{2}(3.99 \text{ m/s})^2 + \dfrac{m}{2}(3.01 \text{ m/s})^2 = m\left(12.5 \text{ m}^2/s^2\right)$

so $\quad KE_f = KE_i$, which is the criterion for an elastic collision.

6.39 Let M = mass of ball, m = mass of bullet, v = velocity of bullet, and V = the initial velocity of the ball-bullet combination. Then, using conservation of momentum from just before to just after collision gives

$$(M+m)V = mv + 0 \quad \text{or} \quad V = \left(\frac{m}{M+m}\right)v$$

Now, we use conservation of mechanical energy from just after the collision until the ball reaches maximum height to find

$$0 + (M+m)g\,h_{\text{max}} = \frac{1}{2}(M+m)V^2 + 0 \;\text{ or }\; h_{\text{max}} = \frac{V^2}{2g} = \frac{1}{2g}\left(\frac{m}{M+m}\right)^2 v^2$$

With the data values provided, this becomes

$$h_{\text{max}} = \frac{1}{2\left(9.80 \text{ m/s}^2\right)}\left(\frac{0.030 \text{ kg}}{0.15 \text{ kg} + 0.030 \text{ kg}}\right)^2 (200 \text{ m/s})^2 = \boxed{57 \text{ m}}$$

6.40 First, we will find the horizontal speed, v_{0x}, of the block and embedded bullet just after impact. After this instant, the block-bullet combination is a projectile, and we find the time to reach the floor by use of $\Delta y = v_{0y} t + \frac{1}{2} a_y t^2$, which becomes

$$-1.00 \text{ m} = 0 + \frac{1}{2}\left(-9.80 \text{ m/s}^2\right)t^2, \text{ giving} \qquad t = 0.452 \text{ s}$$

Thus,

$$v_{0x} = \frac{\Delta x}{t} = \frac{2.00 \text{ m}}{0.452 \text{ s}} = 4.42 \text{ m/s}$$

Now use conservation of momentum for the collision, with v_b = speed of incoming bullet:

$$\left(8.00\times10^{-3} \text{ kg}\right)v_b + 0 = \left(258\times10^{-3} \text{ kg}\right)(4.42 \text{ m/s})$$

so $\quad v_b = \boxed{143 \text{ m/s}} \qquad$ (about 320 mph)

6.41 First, we use conservation of mechanical energy to find the speed of the block and embedded bullet just after impact:

$$\left(KE+PE_s\right)_f=\left(KE+PE_s\right)_i \text{ becomes } \frac{1}{2}(m+M)V^2+0=0+\frac{1}{2}kx^2$$

and yields

$$V=\sqrt{\frac{kx^2}{m+M}}=\sqrt{\frac{(150\ \text{N/m})(0.800\ \text{m})^2}{(0.0120+0.100)\ \text{kg}}}=29.3\ \text{m/s}$$

Now, employ conservation of momentum to find the speed of the bullet just before impact: $mv+M(0)=(m+M)V$, or

$$v=\left(\frac{m+M}{m}\right)V=\left(\frac{0.112\ \text{kg}}{0.0120\ \text{kg}}\right)(29.3\ \text{m/s})=\boxed{273\ \text{m/s}}$$

6.42 (a) Conservation of momentum gives $m_T v_{fT}+m_c v_{fc}=m_T v_{iT}+m_c v_{ic}$, or

$$v_{fT}=\frac{m_T v_{iT}+m_c\left(v_{ic}-v_{fc}\right)}{m_T}$$

$$=\frac{(9\,000\ \text{kg})(20.0\ \text{m/s})+(1\,200\ \text{kg})\left[(25.0-18.0)\ \text{m/s}\right]}{9\,000\ \text{kg}}$$

$$v_{fT}=\boxed{20.9\ \text{m/s east}}$$

(b) $$KE_{\text{lost}}=KE_i-KE_f=\left[\frac{1}{2}m_c v_{ic}^2+\frac{1}{2}m_T v_{iT}^2\right]-\left[\frac{1}{2}m_c v_{fc}^2+\frac{1}{2}m_T v_{fT}^2\right]$$

$$=\frac{1}{2}\left[m_c\left(v_{ic}^2-v_{fc}^2\right)+m_T\left(v_{iT}^2-v_{fT}^2\right)\right]$$

$$=\frac{1}{2}\ (1\,200\ \text{kg})\left[(25.0)^2-(18.0)^2\right]\left(\text{m}^2/\text{s}^2\right)+(9\,000\ \text{kg})\left[(20.0)^2-(20.9)^2\right]\left(\text{m}^2/\text{s}^2\right)$$

$$KE_{\text{lost}}=\boxed{1.50\times10^4\ \text{J, which becomes internal energy}}$$

6.43 (a) We choose east (the direction of the girl's velocity) to be the positive direction. Since momentum is conserved in the event and both skaters were initially at rest, it is necessary that

$$m_b v_b+m_g v_g=0 \qquad \text{giving} \qquad v_b=-\left(m_g/m_b\right)v_g$$

Thus, $\boxed{\text{the boy will recoil toward the west with speed } |v_b|=\left(m_g/m_b\right)v_g}$.

(b) The girl's kinetic energy is $\boxed{KE_g=\tfrac{1}{2}m_g v_g^2}$

For the boy: $KE_b=\dfrac{1}{2}m_b v_b^2=\dfrac{1}{2}m_b\left(\dfrac{m_g^2}{m_b^2}v_g^2\right)$ or $\boxed{KE_b=\left(m_g^2/2m_b\right)v_g^2}$

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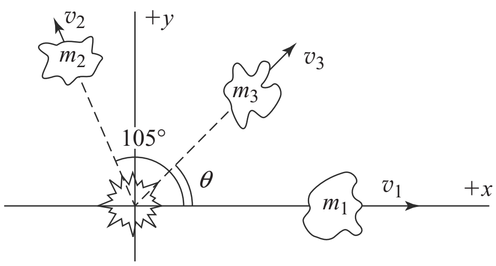
Therefore, $\dfrac{KE_g}{KE_b} = \dfrac{\frac{1}{2} m_g v_g^2}{\left(m_g^2/2m_b\right) v_g^2}$ or $\boxed{\dfrac{KE_g}{KE_b} = \dfrac{m_b}{m_g} > 1, \text{ since } m_b > m_g}$

(c) Mechanical energy is gained because $\boxed{\text{work is done by the skater's muscles}}$ as they push each other apart. The $\boxed{\text{origin of this work is chemical energy}}$ within their bodies.

6.44 (a)



$m_1 = 48.0$ kg
$v_1 = 12.0$ m/s
$m_2 = 62.0$ kg
$v_2 = 15.0$ m/s
$m_3 = 112$ kg

(b) x-direction: $\Sigma p_{xf} = \Sigma p_{xi} \quad \Rightarrow \quad \boxed{m_1 v_1 \cos 0° + m_2 v_2 \cos 105° + m_3 v_3 \cos\theta = 0}$

y-direction: $\Sigma p_{yf} = \Sigma p_{yi} \quad \Rightarrow \quad \boxed{m_1 v_1 \sin 0° + m_2 v_2 \sin 105° + m_3 v_3 \sin\theta = 0}$

(c) $p_{1x} = m_1 v_1 \cos 0° = (48.0 \text{ kg})(12.0 \text{ m/s})(1) = \boxed{576 \text{ kg}\cdot\text{m/s}}$

$p_{2x} = m_2 v_2 \cos 105° = (62.0 \text{ kg})(15.0 \text{ m/s})(-0.259) = \boxed{-241 \text{ kg}\cdot\text{m/s}}$

(d) $p_{1y} = m_1 v_1 \sin 0° = (48.0 \text{ kg})(12.0 \text{ m/s})(0) = \boxed{0}$

$p_{2y} = m_2 v_2 \sin 105° = (62.0 \text{ kg})(15.0 \text{ m/s})(+0.966) = \boxed{898 \text{ kg}\cdot\text{m/s}}$

(e) x-direction: $\boxed{576 \text{ kg}\cdot\text{m/s} - 241 \text{ kg}\cdot\text{m/s} + (112 \text{ kg}) v_3 \cos\theta = 0}$

y-direction: $\boxed{0 + 898 \text{ kg}\cdot\text{m/s} + (112 \text{ kg}) v_3 \sin\theta = 0}$

(f) x-direction: $v_3 \cos\theta = \dfrac{-576 \text{ kg}\cdot\text{m/s} + 241 \text{ kg}\cdot\text{m/s}}{112 \text{ kg}}$ or $\boxed{v_3 \cos\theta = -2.99 \text{ m/s}}$

y-direction: $v_3 \sin\theta = \dfrac{-898 \text{ kg}\cdot\text{m/s}}{112 \text{ kg}}$ or $\boxed{v_3 \sin\theta = -8.02 \text{ m/s}}$

Then, squaring and adding these results, recognizing that $\cos^2\theta + \sin^2\theta = 1$, gives

$v_3^2\left(\cos^2\theta + \sin^2\theta\right) = (-2.99 \text{ m/s})^2 + (-8.02 \text{ m/s})^2$ and $v_3 = \sqrt{73.3 \text{ m}^2/s^2} = \boxed{8.56 \text{ m/}s}$

(g) $\dfrac{\cancel{v_3}\sin\theta}{\cancel{v_3}\cos\theta} = \tan\theta = \dfrac{-8.02 \text{ m/s}}{-2.99 \text{ m/s}} = 2.68$ so $\theta = \tan^{-1}(2.68) + 180° = \boxed{250°}$

Note that the factor of 180° was included in the last calculation because it was recognized that both the sine and cosine of angle θ were negative. This meant that θ had to be a third quadrant angle. Use of the inverse tangent function alone yields only the principle angles $(-90° \le \theta \le +90°)$ that have the given value for the tangent function.

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(h) Because the third fragment must have a momentum equal in magnitude and opposite direction to the resultant of the other two fragments momenta, $\boxed{\text{all three pieces must travel in the same plane}}$.

6.45 Conservation of momentum gives

$$(25.0\text{ g})v_{1f} + (10.0\text{ g})v_{2f} = (25.0\text{ g})(20.0\text{ cm/s}) + (10.0\text{ g})(15.0\text{ cm/s})$$

or $2.50v_{1f} + v_{2f} = 65.0\text{ cm/s}$ **[1]**

For head-on, elastic collisions, we know that $v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$.

Thus,

$$20.0\text{ cm/s} - 15.0\text{ cm/s} = -v_{1f} + v_{2f} \text{ or } v_{2f} = v_{1f} + 5.00\text{ cm/s} \quad \textbf{[2]}$$

Substituting Equation [2] into [1] yields $3.50v_{1f} = 60.0\text{ cm/s}$, or $v_{1f} = \boxed{17.1\text{ cm/s}}$.

Equation [2] then gives $v_{2f} = 17.1\text{ cm/s} + 5.00\text{ cm/s} = \boxed{22.1\text{ cm/s}}$.

6.46 First, consider conservation of momentum and write

$$m_1v_{1i} + m_2v_{2i} = m_1v_{1f} + m_2v_{2f}$$

Since $m_1 = m_2$, this becomes

$$v_{1i} + v_{2i} = v_{1f} + v_{2f} \quad \textbf{[1]}$$

For an elastic head-on collision, we also have $v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$, which may be written as

$$v_{1i} - v_{2i} = -v_{1f} + v_{2f} \quad \textbf{[2]}$$

Adding Equations [1] and [2] yields

$$v_{2f} = v_{1i} \quad \textbf{[3]}$$

Subtracting Equation [2] from [1] gives

$$v_{1f} = v_{2i} \quad \textbf{[4]}$$

Equations [3] and [4] show us that, under the conditions of equal mass objects striking one another in a head-on, elastic collision, *the two objects simply exchange velocities*. Thus, we may write the results of the various collisions as

(a) $v_{1f} = \boxed{0}$, $v_{2f} = \boxed{1.50\text{ m/s}}$

(b) $v_{1f} = \boxed{-1.00\text{ m/s}}$, $v_{2f} = \boxed{1.50\text{ m/s}}$

(c) $v_{1f} = \boxed{1.00\text{ m/s}}$, $v_{2f} = \boxed{1.50\text{ m/s}}$

6.47 (a) Over a the short time interval of the collision, external forces have no time to impart significant impulse to the players. $\boxed{\text{The two players move together after the tackle}}$, so the collision is completely inelastic.

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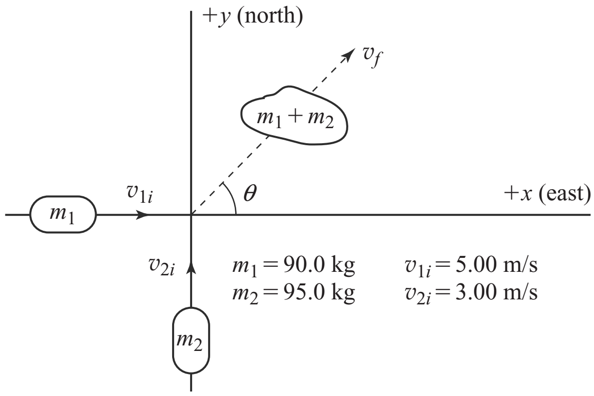
(b)



$$p_{xf} = \Sigma p_{xi} \quad \Rightarrow \quad (m_1 + m_2) v_f \cos\theta = m_1 v_{1i} + 0$$

or $v_f \cos\theta = \dfrac{m_1 v_{1i}}{(m_1 + m_2)} = \dfrac{(90.0\text{ kg})(5.00\text{ m/s})}{90.0\text{ kg} + 95.0\text{ kg}}$

and $\quad v_f \cos\theta = 2.43\text{ m/s}$

$$p_{yf} = \Sigma p_{yi} \quad \Rightarrow \quad (m_1 + m_2) v_f \sin\theta = 0 + m_2 v_{2i}$$

giving $\quad v_f \sin\theta = \dfrac{m_2 v_{2i}}{(m_1 + m_2)} = \dfrac{(95.0\text{ kg})(3.00\text{ m/s})}{90.0\text{ kg} + 95.0\text{ kg}}$ $\quad$ and $\quad v_f \sin\theta = 1.54\text{ m/s}$

Therefore, $v_f^2 (\sin^2\theta + \cos^2\theta) = v_f^2 = (1.54\text{ m/s})^2 + (2.43\text{ m/s})^2$, and

$$v_f = \sqrt{8.28\text{ m}^2/s^2} = 2.88\text{ m/s}$$

Also,

$$\tan\theta = \frac{\cancel{v_f}\sin\theta}{\cancel{v_f}\cos\theta} = \frac{1.54\text{ m/s}}{2.43\text{ m/s}} = 0.634 \quad \text{and} \quad \theta = \tan^{-1}(0.634) = 32.4°$$

Thus, $\boxed{\vec{\mathbf{v}}_f = 2.88\text{ m/s at } 32.4° \text{ north of east}}$

(c) $KE_{\text{lost}} = KE_i - KE_f = \dfrac{1}{2} m_1 v_{1i}^2 + \dfrac{1}{2} m_2 v_{2i}^2 - \dfrac{1}{2}(m_1 + m_2) v_f^2$

$$= \frac{1}{2}\left[(90.0\text{ kg})(5.00\text{ m/s})^2 + (95.0\text{ kg})(3.00\text{ m/s})^2\right] - \frac{1}{2}(185\text{ kg})(2.88\text{ m/s})^2 = \boxed{785\text{ J}}$$

(d) The lost kinetic energy is transformed into other forms of energy, such as thermal energy and sound.

6.48 Consider conservation of momentum in the first event (twin A tossing the pack), taking the direction of the velocity given the backpack as positive. This yields

$$m_A v_{Af} + m_{\text{pack}} v_{\text{pack}} = (m_A + m_{\text{pack}})(0) = 0$$

or

$$v_{Af} = \frac{-m_{\text{pack}} v_{\text{pack}}}{m_A} = -\left(\frac{12.0\text{ kg}}{55.0\text{ kg}}\right)(+3.00\text{ m/s}) = -0.655\text{ m/s} \quad \text{and} \quad |v_{Af}| = \boxed{0.655\text{ m/s}}$$

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Conservation of momentum when twin B catches and holds onto the backpack yields

$$\left(m_B + m_{\text{pack}}\right)v_{Bf} = m_B\left(0\right) + m_{\text{pack}}v_{\text{pack}}$$

or $$v_{Bf} = \frac{m_{\text{pack}}v_{\text{pack}}}{m_B + m_{\text{pack}}} = \frac{\left(12.0\text{ kg}\right)\left(+3.00\text{ m/s}\right)}{55.0\text{ kg} + 12.0\text{ kg}} = \boxed{0.537\text{ m/s}}$$

6.49 Choose the +x-axis to be eastward and the +y-axis northward.

If v_i is the initial northward speed of the 3 000-kg car, conservation of momentum in the y-direction gives

$$0 + \left(3\,000\text{ kg}\right)v_i = \left(3\,000\text{ kg} + 2\,000\text{ kg}\right)\left[\left(5.22\text{ m/s}\right)\sin 40.0°\right]$$

or $$v_i = \boxed{5.59\text{ m/s}}$$

Observe that knowledge of the initial speed of the 2 000-kg car was unnecessary for this solution.

6.50 We use conservation of momentum for both eastward and northward components.

For the eastward direction: $M\left(13.0\text{ m/s}\right) = 2\,M\,V_f\cos 55.0°$

For the northward direction: $M\,v_{2i} = 2\,M\,V_f\sin 55.0°$

Divide the northward equation by the eastward equation to find

$$\frac{\cancel{M}v_{2i}}{\cancel{M}\left(13.0\text{ m/s}\right)} = \frac{\cancel{2MV_f}\sin 55.0°}{\cancel{2MV_f}\cos 55.0°} \qquad \text{or} \qquad v_{2i} = \left(13.0\text{ m/s}\right)\tan 55.0°$$

yielding $$v_{2i} = \left[\left(13.0\text{ m/s}\right)\left(\frac{2.237\text{ mi/h}}{1\text{ m/s}}\right)\right]\tan 55.0° = \boxed{41.5\text{ mi/h}}$$

Thus, $\boxed{\text{the driver of the northbound car was untruthful}}$.

6.51 Choose the x-axis to be along the original line of motion.

(a) From conservation of momentum in the x-direction,

$$m\left(5.00\text{ m/s}\right) + 0 = m\left(4.33\text{ m/s}\right)\cos 30.0° + m\,v_{2f}\cos\theta$$

or $$v_{2f}\cos\theta = 1.25\text{ m/s} \qquad \textbf{[1]}$$

Conservation of momentum in the y-direction gives

$$0 = m\left(4.33\text{ m/s}\right)\sin 30.0° + m\,v_{2f}\sin\theta,\text{ or } v_{2f}\sin\theta = -2.16\text{ m/s} \qquad \textbf{[2]}$$

Dividing Equation [2] by [1] gives $\tan\theta = \dfrac{-2.16}{1.25} = -1.73$ and $\theta = -60.0°$

Then, either Equation [1] or [2] gives $v_{2f} = 2.50\text{ m/s}$, so the final velocity of the second ball is $\vec{\mathbf{v}}_{2f} = \boxed{2.50\text{ m/s at }-60.0°}$.

continued on next page

(b) $KE_i = \frac{1}{2}m v_{1i}^2 + 0 = \frac{1}{2}m(5.00\text{ m/s})^2 = m(12.5\text{ m}^2/\text{s}^2)$

$$KE_f = \frac{1}{2}m v_{1f}^2 + \frac{1}{2}m v_{2f}^2$$

$$= \frac{1}{2}m(4.33\text{ m/s})^2 + \frac{1}{2}m(2.50\text{ m/s})^2 = m(12.5\text{ m}^2/\text{s}^2)$$

Since $KE_f = KE_i$, this is an $\boxed{\text{elastic collision}}$.

6.52 The recoil speed of the subject plus pallet after a heartbeat is

$$V = \frac{\Delta x}{\Delta t} = \frac{6.00\times10^{-5}\text{ m}}{0.160\text{ s}} = 3.75\times10^{-4}\text{ m/s}$$

From conservation of momentum, $mv - MV = 0 + 0$, so the mass of blood leaving the heart is

$$m = M\left(\frac{V}{v}\right) = (54.0\text{ kg})\left(\frac{3.75\times10^{-4}\text{ m/s}}{0.500\text{ m/s}}\right) = 4.05\times10^{-2}\text{ kg} = \boxed{40.5\text{ g}}$$

6.53 Choose the positive direction to be the direction of the truck's initial velocity.

Apply conservation of momentum to find the velocity of the combined vehicles after collision:

$$(4\,000\text{ kg} + 800\text{ kg})v_f = (4\,000\text{ kg})(+8.00\text{ m/s}) + (800\text{ kg})(-8.00\text{ m/s})$$

which yields $v_f = +5.33\text{ m/s}$.

Use the impulse-momentum theorem, $I = F_{av}(\Delta t) = \Delta p = m(v_f - v_i)$, to find the magnitude of the average force exerted on each driver during the collision.

Truck Driver:

$$|F_{av}| = \frac{m|v_f - v_i|_{\text{truck}}}{\Delta t} = \frac{(80.0\text{ kg})|5.33\text{ m/s} - 8.00\text{ m/s}|}{0.120\text{ s}} = \boxed{1.78\times10^3\text{ N}}$$

Car Driver:

$$|F_{av}| = \frac{m|v_f - v_i|_{\text{car}}}{\Delta t} = \frac{(80.0\text{ kg})|5.33\text{ m/s} - (-8.00\text{ m/s})|}{0.120\text{ s}} = \boxed{8.89\times10^3\text{ N}}$$

6.54 First, we use conservation of mechanical energy to find the speed of m_1 at B just before collision. This gives $\frac{1}{2}m_1 v_1^2 + 0 = 0 + m_1 g h_i$,

or $\quad v_1^2 = \sqrt{2gh_i} = \sqrt{2(9.80\text{ m/s}^2)(5.00\text{ m})} = 9.90\text{ m/s}$

Next, we apply conservation of momentum and knowledge of elastic collisions to find the velocity of m_1 at B just after collision.

continued on next page

From conservation of momentum, with the second object initially at rest, we have

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + 0, \qquad \text{or} \qquad v_{2f} = \frac{m_1}{m_2}\left(v_{1i} - v_{1f}\right) \qquad \textbf{[1]}$$

For head-on elastic collisions, $v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$. Since $v_{2i} = 0$ in this case, this becomes $v_{2f} = v_{1f} + v_{1i}$, and, combining this with Equation [1] above, we obtain

$$v_{1f} + v_{1i} = \frac{m_1}{m_2}\left(v_{1i} - v_{1f}\right) \qquad \text{or} \qquad \left(m_1 + m_2\right) v_{1f} = \left(m_1 - m_2\right) v_{1i}$$

so $$v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} = \left(\frac{5.00 - 10.0}{5.00 + 10.0}\right)\left(9.90 \text{ m/s}\right) = -3.30 \text{ m/s}$$

Finally, use conservation of mechanical energy for m_1 after the collision to find the maximum rebound height. This gives $(KE + PE_g)_f = (KE + PE_g)_i$

or $$0 + m_1 g h_{\max} = \frac{1}{2} m_1 v_{1f}^2 + 0 \qquad \text{and} \qquad h_{\max} = \frac{v_{1f}^2}{2g} = \frac{\left(-3.30 \text{ m/s}\right)^2}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{0.556 \text{ m}}$$

6.55 Note that the initial velocity of the target particle is zero (that is, $v_{2i} = 0$).

From conservation of momentum

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + 0 \qquad \textbf{[1]}$$

For head-on elastic collisions, $v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f}\right)$, and with $v_{2i} = 0$, this gives

$$v_{2f} = v_{1i} + v_{1f} \qquad \textbf{[2]}$$

Substituting Equation [2] into [1] yields

$$m_1 v_{1f} + m_2\left(v_{1i} + v_{1f}\right) = m_1 v_{1i}$$

or

$$\left(m_1 + m_2\right) v_{1f} = \left(m_1 - m_2\right) v_{1i} \qquad \text{and} \qquad v_{1f} = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} \qquad \textbf{[3]}$$

Now, we substitute Equation [3] into [2] to obtain

$$v_{2f} = v_{1i} + \left(\frac{m_1 - m_2}{m_1 + m_2}\right) v_{1i} \qquad \text{or} \qquad v_{2f} = \left(\frac{2m_1}{m_1 + m_2}\right) v_{1i} \qquad \textbf{[4]}$$

Equations [3] and [4] can now be used to answer both parts (a) and (b).

(a) If $m_1 = 2.0$ g, $m_2 = 1.0$ g, and $v_{1i} = 8.0$ m/s, then

$$v_{1f} = \boxed{\frac{8}{3} \text{ m/s}} \quad \text{and} \quad v_{2f} = \boxed{\frac{32}{3} \text{ m/s}}$$

continued on next page

(b) If $m_1 = 2.0$ g, $m_2 = 10$ g, and $v_{1i} = 8.0$ m/s, we find

$$v_{1f} = \boxed{-\frac{16}{3}\text{ m/s}} \text{ and } v_{2f} = \boxed{\frac{8}{3}\text{ m/s}}$$

(c) The final kinetic energy of the 2.0 g particle in each case is:

Case (a): $KE_{1f} = \frac{1}{2}m_1 v_{1f}^2 = \frac{1}{2}\left(2.0\times10^{-3}\text{ kg}\right)\left(\frac{8}{3}\text{ m/s}\right)^2 = \boxed{7.1\times10^{-3}\text{ J}}$

Case (b): $KE_{1f} = \frac{1}{2}m_1 v_{1f}^2 = \frac{1}{2}\left(2.0\times10^{-3}\text{ kg}\right)\left(-\frac{16}{3}\text{ m/s}\right)^2 = \boxed{2.8\times10^{-2}\text{ J}}$

Since the incident kinetic energy is the same in cases (a) and (b), we observe that $\boxed{\text{the incident particle loses more kinetic energy in case (a)}}$.

6.56 If the pendulum bob barely swings through a complete circle, it arrives at the top of the arc (having risen a vertical distance of 2ℓ) with essentially zero velocity.

From conservation of mechanical energy, we find the minimum velocity of the bob at the bottom of the arc as $(KE+PE_g)_{\text{bottom}} = (KE+PE_g)_{\text{top}}$, or $\frac{1}{2}MV^2 = 0 + Mg(2\ell)$. This gives $V = 2\sqrt{g\ell}$ as the needed velocity of the bob just after the collision.

Conserving momentum through the collision then gives the minimum initial velocity of the bullet as

$$m\left(\frac{v}{2}\right) + M\left(2\sqrt{g\ell}\right) = mv + 0 \qquad \text{or} \qquad v = \boxed{\frac{4M}{m}\sqrt{g\ell}}$$

6.57 Note: We consider the spring to have negligible mass and ignore any energy or momentum it may possess after being released. Also, we take toward the right as the positive direction.

Conservation of momentum, $\vec{\mathbf{p}}_f = \vec{\mathbf{p}}_i$, gives

$$m_1v_1 + m_2v_2 = 0 \qquad \text{or} \qquad v_2 = -\left(\frac{m_1}{m_2}\right)v_1 \qquad \textbf{[1]}$$

Since the surface is frictionless, conservation of energy,

$$KE_{1f} + KE_{2f} + PE_{s,f} = KE_{1i} + KE_{2i} + PE_{s,i}$$

with $KE_{1i} = KE_{2i} = PE_{s,f} = 0$ gives

$$\frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 = \frac{1}{2}kd^2 \qquad \textbf{[2]}$$

where $d = 9.8$ cm is the initial compression of the spring. Substituting Equation [1] into Equation [2] gives:

$$\cancel{\frac{1}{2}}m_1v_1^2 + \cancel{\frac{1}{2}}m_2\left(\frac{m_1^2}{m_2^2}v_1^2\right) = \cancel{\frac{1}{2}}kd^2 \qquad \text{or} \qquad m_1\left(1+\frac{m_1}{m_2}\right)v_1^2 = kd^2$$

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Choosing the negative sign (since m_1 will recoil to the left), this result yields

$$v_1 = -d\sqrt{\frac{k}{m_1\left(1+m_1/m_2\right)}} = -\left(9.8\times10^{-2}\ \text{m}\right)\sqrt{\frac{280\ \text{N/m}}{(0.56\ \text{kg})(1+0.56/0.88)}}$$

and $v_1 = -1.7\ \text{m/s}$ or $\boxed{1.7\ \text{m/s to the left}}$

Then, Equation [1] gives the velocity of the second object as

$$v_2 = -\left(\frac{0.56}{0.88}\right)(-1.7\ \text{m/s}) = +1.1\ \text{m/s} \quad \text{or} \quad \boxed{1.1\ \text{m/s to the right}}$$

6.58 Use conservation of mechanical energy, $\left(KE+PE_g\right)_B = \left(KE+PE_g\right)_A$, to find the speed of the blue bead at point B just before it collides with the green bead. This gives $\frac{1}{2}m\,v_{1i}^2 + 0 = 0 + m\,g\,y_A$, or

$$v_{1i} = \sqrt{2\,g\,y_A} = \sqrt{2\left(9.80\ \text{m/s}^2\right)(1.50\ \text{m})} = 5.42\ \text{m/s}$$

Conservation of momentum during the collision gives

$$(0.400\ \text{kg})\,v_{1f} + (0.600\ \text{kg})\,v_{2f} = (0.400\ \text{kg})(5.42\ \text{m/s}) + 0$$

or $\quad v_{1f} + 1.50\,v_{2f} = 5.42\ \text{m/s}$ **[1]**

For a head-on elastic collision, we have $v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f}\right)$, and with $v_{2i} = 0$, this becomes

$$v_{1f} = v_{2f} - v_{1i} \quad \text{or} \quad v_{1f} = v_{2f} - 5.42\ \text{m/s}$$ **[2]**

Substitute Equation [2] into [1] to find the speed of the green bead just after collision as

$$v_{2f} - 5.42\ \text{m/s} + 1.50\,v_{2f} = 5.42\ \text{m/s} \quad \text{or} \quad v_{2f} = \frac{2(5.42\ \text{m/s})}{2.50} = 4.34\ \text{m/s}$$

Now, we use conservation of energy for the green bead after collision to find the maximum height it will achieve. This gives

$$0 + m_2\,g\,y_{\max} = \frac{1}{2}m_2 v_{2f}^2 + 0 \quad \text{or} \quad y_{\max} = \frac{v_{2f}^2}{2\,g} = \frac{(4.34\ \text{m/s})^2}{2\left(9.80\ \text{m/s}^2\right)} = \boxed{0.961\ \text{m}}$$

6.59 We shall choose southward as the positive direction. The mass of the man is

$$m = \frac{w}{g} = \frac{730\ \text{N}}{9.80\ \text{m/s}^2} = 74.5\ \text{kg}$$

Then, from conservation of momentum, we find

$$\left(m_{\text{man}}v_{\text{man}} + m_{\text{book}}v_{\text{book}}\right)_f = \left(m_{\text{man}}v_{\text{man}} + m_{\text{book}}v_{\text{book}}\right)_i$$

or

$$(74.5\ \text{kg})\,v_{\text{man}} + (1.2\ \text{kg})(-5.0\ \text{m/s}) = 0 + 0 \quad \text{and} \quad v_{\text{man}} = 8.1\times10^{-2}\ \text{m/s}$$

continued on next page

Therefore, the time required to travel the 5.0 m to shore is

$$t = \frac{\Delta x}{v_{\text{man}}} = \frac{5.0 \text{ m}}{8.1\times10^{-2} \text{ m/s}} = \boxed{62 \text{ s}}$$

6.60 The mass of the third fragment must be

$$m_3 = m_{\text{nucleus}} - m_1 - m_2 = (17 - 5.0 - 8.4)\times10^{-27} \text{ kg} = 3.6\times10^{-27} \text{ kg}$$

Conserving momentum in both the *x*- and *y*-directions gives

y-direction: $m_1 v_{1y} + m_2 v_{2y} + m_3 v_{3y} = 0$

or $$v_{3y} = -\frac{m_1 v_{1y} + m_2 v_{2y}}{m_3} = -\frac{(5.0\times10^{-27} \text{ kg})(6.0\times10^{6} \text{ m/s}) + 0}{3.6\times10^{-27} \text{ kg}} = -\frac{30}{3.6}\times10^{6} \text{ m/s}$$

x-direction: $m_1 v_{1x} + m_2 v_{2x} + m_3 v_{3x} = 0$

or $$v_{3x} = -\frac{m_1 v_{1x} + m_2 v_{2x}}{m_3} = -\frac{0 + (8.4\times10^{-27} \text{ kg})(4.0\times10^{6} \text{ m/s})}{3.6\times10^{-27} \text{ kg}} = -\frac{34}{3.6}\times10^{6} \text{ m/s}$$

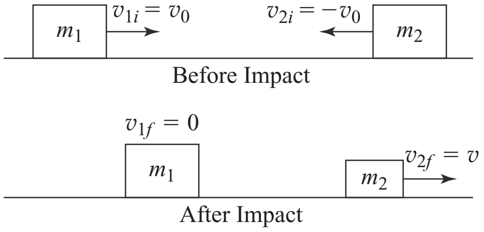
and $$v_3 = \sqrt{v_{3x}^2 + v_{3y}^2} = \sqrt{\left(-(34/3.6)\times10^{6} \text{ m/s}\right)^2 + \left(-(30/3.6)\times10^{6} \text{ m/s}\right)^2} = 1.3\times10^{7} \text{ m/s}$$

Also, $$\theta = \tan^{-1}\left(\frac{v_{3y}}{v_{3x}}\right) + 180° = \tan^{-1}\left(\frac{30}{34}\right) + 180° = 2.2\times10^{2} \text{ degrees} = 220°$$

Therefore, $\boxed{\vec{\mathbf{v}}_3 = 1.3\times10^{7} \text{ m/s at } 220° \text{ counterclockwise from the } +x\text{-axis}}$.

Note that the factor of 180° was included in the calculation for θ because it was recognized that both v_{3x} and v_{3y} were negative. This meant that θ had to be a third quadrant angle. Use of the inverse tangent function alone yields only the principal angles $(-90° \le \theta \le +90°)$ that have the given value for the tangent function.

6.61 The sketch at the right gives before and after views of the collision between these two objects. Since the collision is elastic, both kinetic energy and momentum must be conserved.



Conservation of Momentum:

$$m_1 v_{1f} + m_2 v_{2f} = m_1 v_{1i} + m_2 v_{2i}$$

$$m_1(0) + m_2 v = m_1 v_0 + m_2(-v_0)$$

or $$v = \left(\frac{m_1}{m_2} - 1\right)v_0 \qquad \textbf{[1]}$$

Since this is a perfectly elastic collision, $v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$, and with the given velocities this becomes

$$v_0 - (-v_0) = -(0 - v) \qquad \text{or} \qquad v = 2v_0 \qquad \textbf{[2]}$$

continued on next page

(a) Substituting Equation [2] into [1] gives $2\cancel{v_0} = \left(\frac{m_1}{m_2} - 1\right)\cancel{v_0}$ or $\boxed{m_1/m_2 = 3}$

(b) From Equation [2] above, we have $\boxed{v/v_0 = 2}$

6.62 (a) Let v_{1i} and v_{2i} be the velocities of m_1 and m_2 just before the collision. Then, using conservation of mechanical energy, $\left(KE + PE_g\right)_i = \left(KE + PE_g\right)_0$ or $\frac{1}{2}mv_i^2 + 0 = 0 + mgh_0$,

gives $$v_{1i} = -v_{2i} = \sqrt{2gh_0} = \sqrt{2\left(9.80\text{ m/s}^2\right)\left(5.00\text{ m}\right)} = 9.90\text{ m/s}$$

$$\boxed{v_{1i} = +9.90\text{ m/s}} \text{ and } \boxed{v_{2i} = -9.90\text{ m/s}}$$

(b) From conservation of momentum:

$$\left(2.00\text{ g}\right)v_{1f} + \left(4.00\text{ g}\right)v_{2f} = \left(2.00\text{ g}\right)\left(9.90\text{ m/s}\right) + \left(4.00\text{ g}\right)\left(-9.90\text{ m/s}\right)$$

or $$v_{1f} + \left(2.00\right)v_{2f} = -9.90\text{ m/s} \qquad \textbf{[1]}$$

For a perfectly elastic, head-on collision, $v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f}\right)$, giving

$$+9.90\text{ m/s} - \left(-9.90\text{ m/s}\right) = -v_{1f} + v_{2f} \quad \text{or} \quad v_{2f} = v_{1f} + 19.8\text{ m/s} \qquad \textbf{[2]}$$

Substituting Equation [2] into [1] gives $v_{1f} + \left(2.00\right)\left(v_{1f} + 19.8\text{ m/s}\right) = -9.90\text{ m/s}$

or $$v_{1f} = \frac{-9.90\text{ m/s} - 39.6\text{ m/s}}{3.00} = \boxed{-16.5\text{ m/s}}$$

Then, Equation [2] yields $v_{2f} = -16.5\text{ m/s} + 19.8\text{ m/s} = \boxed{+3.30\text{ m/s}}$.

(c) Applying conservation of energy to each block after the collision gives

$$\frac{1}{2}m\left(0\right)^2 + mgh_{\max} = \frac{1}{2}mv_f^2 + mg\left(0\right) \quad \text{or} \quad h_{\max} = \frac{v_f^2}{2g}$$

Thus, $$h_{1f} = \frac{v_{1f}^2}{2g} = \frac{\left(-16.5\text{ m/s}\right)^2}{2\left(9.80\text{ m/s}^2\right)} = \boxed{13.9\text{ m}}$$

and $$h_{2f} = \frac{v_{2f}^2}{2g} = \frac{\left(3.30\text{ m/s}\right)^2}{2\left(9.80\text{ m/s}^2\right)} = \boxed{0.556\text{ m}}$$

6.63 (a) Use conservation of mechanical energy to find the speed of m_1 just before collision. Taking $y = 0$ at the tabletop level, this gives $\frac{1}{2}m_1v_{1i}^2 + mg\left(0\right) = \frac{1}{2}m_1\left(0\right) + m_1gh_1$, or

$$v_{1i} = \sqrt{2gh_1} = \sqrt{2\left(9.80\text{ m/s}^2\right)\left(2.50\text{ m}\right)} = 7.00\text{ m/s}$$

Apply conservation of momentum from just before to just after the collision:

$$\left(0.500\text{ kg}\right)v_{1f} + \left(1.00\text{ kg}\right)v_{2f} = \left(0.500\text{ kg}\right)\left(7.00\text{ m/s}\right) + 0$$

or $$v_{1f} + 2v_{2f} = 7.00\text{ m/s} \qquad \textbf{[1]}$$

continued on next page

For a perfectly elastic head-on collision, $v_{1i} - v_{2i} = -(v_{1f} - v_{2f})$. With $v_{2i} = 0$, this becomes

$$v_{2f} = v_{1f} + v_{1i} \qquad \text{or} \qquad v_{2f} = v_{1f} + 7.00 \text{ m/s} \qquad \textbf{[2]}$$

Substituting Equation [2] into [1] yields

$$v_{1f} + 2\left(v_{1f} + 7.00 \text{ m/s}\right) = 7.00 \text{ m/s} \qquad \text{and} \qquad v_{1f} = \frac{-7.00 \text{ m/s}}{3} = \boxed{-2.33 \text{ m/s}}$$

Then, from Equation [2],

$$v_{2f} = -2.33 \text{ m/s} + 7.00 \text{ m/s} = \boxed{4.67 \text{ m/s}}$$

(b) Apply conservation of mechanical energy to m_1 after the collision to find the rebound height of this object.

$$\frac{1}{2}m_1(0) + m_1 g h_1' = \frac{1}{2}m_1 v_{1f}^2 + mg(0) \qquad \text{or} \qquad h_1' = \frac{v_{1f}^2}{2g} = \frac{(-2.33 \text{ m/s})^2}{2\left(9.80 \text{ m/s}^2\right)} = \boxed{0.277 \text{ m}}$$

(c) From $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, with $v_{0y} = 0$, the time for m_2 to reach the floor after it flies horizontally off the table is

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-2.00 \text{ m})}{-9.80 \text{ m/s}^2}} = 0.639 \text{ s}$$

During this time it travels a horizontal distance

$$\Delta x = v_{0x}t = (4.67 \text{ m/s})(0.639 \text{ s}) = \boxed{2.98 \text{ m}}$$

(d) After the 0.500 kg mass comes back down the incline, it flies off the table with a horizontal velocity of 2.33 m/s. The time of the flight to the floor is 0.639 s as found above and the horizontal distance traveled is

$$\Delta x = v_{0x}t = (2.33 \text{ m/s})(0.639 \text{ s}) = \boxed{1.49 \text{ m}}$$

6.64 We label the two objects such that object 1 has mass m while object 2 has mass $3m$. Conservation of the x-component of momentum gives

$$(3m)v_{2x} + 0 = -mv_0 + (3m)v_0 \qquad \text{or} \qquad v_{2x} = \frac{2}{3}v_0 \qquad \textbf{[1]}$$

Likewise, conservation of the y-component of momentum gives

$$-m v_{1y} + (3m)v_{2y} = 0 \qquad \text{and} \qquad v_{1y} = 3v_{2y} \qquad \textbf{[2]}$$

Since the collision is elastic, $(KE)_f = (KE)_i$,

or $$\frac{1}{2}m v_{1y}^2 + \frac{1}{2}(3m)\left(v_{2x}^2 + v_{2y}^2\right) = \frac{1}{2}m v_0^2 + \frac{1}{2}(3m)v_0^2$$

which reduces to

$$v_{1y}^2 + 3\left(v_{2x}^2 + v_{2y}^2\right) = 4v_0^2 \qquad \textbf{[3]}$$

continued on next page

Substituting Equations [1] and [2] into [3] yields

$$9\,v_{2y}^2 + 3\left(\frac{4}{9}v_0^2 + v_{2y}^2\right) = 4\,v_0^2 \qquad \text{or} \qquad v_{2y} = v_0\,\frac{\sqrt{2}}{3}$$

(a) From Equation [2], the particle of mass *m* has final speed $v_{1y} = 3\,v_{2y} = \boxed{v_0\sqrt{2}}$, and the particle of mass 3*m* moves at

$$v_2 = \sqrt{v_{2x}^2 + v_{2y}^2} = \sqrt{\frac{4}{9}v_0^2 + \frac{2}{9}v_0^2} = \boxed{v_0\sqrt{\frac{2}{3}}}$$

(b) $\theta = \tan^{-1}\left(\dfrac{v_{2y}}{v_{2x}}\right) = \tan^{-1}\left(\dfrac{v_0\sqrt{2}/3}{2v_0/3}\right) = \tan^{-1}\left(\dfrac{1}{\sqrt{2}}\right) = \boxed{35.3°}$

6.65 (a) The momentum of the system is initially zero and remains constant throughout the motion. Therefore, when m_1 leaves the wedge, we must have $m_2v_2 + m_1v_1 = 0$, or

$$v_2 = -\left(\frac{m_1}{m_2}\right)v_f = -\left(\frac{0.500}{3.00}\right)(4.00\ \text{m/s}) = -\frac{2.00}{3.00}\ \text{m/s}$$

meaning $\boxed{\vec{\mathbf{v}}_2 = 0.667\ \text{m/s to the left}}$

(b) Using conservation of energy as the block slides down the wedge, we have

$$\left(KE + PE_g\right)_i = \left(KE + PE_g\right)_f \qquad \text{or} \qquad 0 + m_1gh = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 + 0$$

Thus, $h = \dfrac{1}{2g}\left[v_1^2 + \left(\dfrac{m_2}{m_1}\right)v_2^2\right]$

$$= \frac{1}{19.6\ \text{m/s}^2}\left[(4.00\ \text{m/s})^2 + \left(\frac{3.00}{0.500}\right)\left(-\frac{2.00}{3.00}\ \text{m/s}\right)^2\right] = \boxed{0.952\ \text{m}}$$

6.66 Choose the positive *x*-axis in the direction of the initial velocity of the cue ball. Let v_{ci} be the initial speed of the cue ball, v_{cf} be the final speed of the cue ball, v_{Tf} be the final speed of the target, and θ be the angle the target's final velocity makes with the *x*-axis.

Conservation of momentum in the *x*-direction, recognizing that all billiard balls have the same mass, gives

$$m\,v_{Tf}\cos\theta + m\,v_{cf}\cos 30.0° = 0 + mv_{ci} \qquad \text{or} \qquad v_{Tf}\cos\theta = v_{ci} - v_{cf}\cos 30.0° \qquad \textbf{[1]}$$

The conservation equation for momentum in the *y*-direction is

$$m\,v_{Tf}\sin\theta + m\,v_{cf}\sin 30.0° = 0 + 0 \qquad \text{or} \qquad v_{Tf}\sin\theta = -v_{cf}\sin 30.0° \qquad \textbf{[2]}$$

Since this is an elastic collision, kinetic energy is conserved, giving

$$\frac{1}{2}m\,v_{Tf}^2 + \frac{1}{2}m\,v_{cf}^2 = \frac{1}{2}m\,v_{ci}^2 \qquad \text{or} \qquad v_{Tf}^2 = v_{ci}^2 - v_{cf}^2 \qquad \textbf{[3]}$$

continued on next page

(b) To solve, square Equations [1] and [2] and add the results to obtain

$$v_{Tf}^2\left(\cos^2\theta+\sin^2\theta\right)=v_{ci}^2-2\,v_{ci}v_{cf}\cos 30.0°+v_{cf}^2\left(\cos^2 30.0°+\sin^2 30.0°\right)$$

or $$v_{Tf}^2=v_{ci}^2-2\,v_{ci}v_{cf}\cos 30.0°+v_{cf}^2$$

Now, substitute this result into Equation [3] to get

$$\cancel{v_{ci}^2}-2\,v_{ci}v_{cf}\cos 30.0°+v_{cf}^2=\cancel{v_{ci}^2}-v_{cf}^2 \qquad \text{or} \qquad 2v_{cf}\left(v_{cf}-v_{ci}\cos 30.0°\right)=0$$

Since $v_{cf}\neq 0$, it is necessary that

$$v_{cf}=v_{ci}\cos 30.0°=\left(4.00\ \text{m/s}\right)\cos 30.0°=\boxed{3.46\ \text{m/s}}$$

Then, Equation [3] yields $v_{Tf}=\sqrt{v_{ci}^2-v_{cf}^2}$, or

$$v_{Tf}=\sqrt{\left(4.00\ \text{m/s}\right)^2-\left(3.46\ \text{m/s}\right)^2}=\boxed{2.00\ \text{m/s}}$$

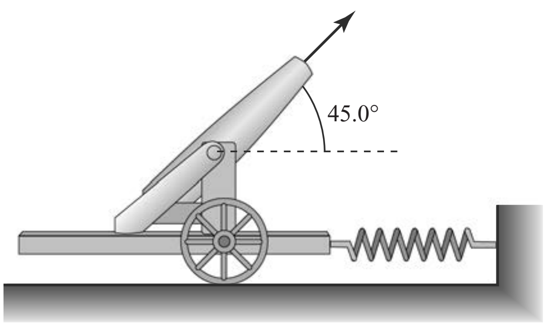
(a) With the results found above, Equation [2] gives

$$\sin\theta=-\left(\frac{v_{cf}}{v_{Tf}}\right)\sin 30.0°=-\left(\frac{3.46\ \text{m/s}}{2.00\ \text{m/s}}\right)\sin 30.0°=-0.866,\ \text{or}\ \theta=-60.0°$$

Thus, the angle between the velocity vectors after collision is

$$\phi=30.0°-\left(-60.0°\right)=\boxed{90.0°}$$

6.67 (a) Use conservation of the horizontal component of momentum from just before to just after the cannon is fired.



$\left(\Sigma p_x\right)_f=\left(\Sigma p_x\right)_i$ gives

$$m_{\text{shell}}\left(v_{\text{shell}}\cos 45.0°\right)+m_{\text{cannon}}v_{\text{recoil}}=0,$$

or

$$v_{\text{recoil}}=-\left(\frac{m_{\text{shell}}}{m_{\text{cannon}}}\right)v_{\text{shell}}\cos 45.0°=-\left(\frac{200\ \text{kg}}{5\,000\ \text{kg}}\right)\left(125\ \text{m/s}\right)\cos 45.0°=\boxed{-3.54\ \text{m/s}}$$

(b) Use conservation of mechanical energy for the cannon-spring system from right after the cannon is fired to the instant when the cannon comes to rest.

$$\left(KE+PE_g+PE_s\right)_f=\left(KE+PE_g+PE_s\right)_i$$

$$0+0+\frac{1}{2}kx_{\max}^2=\frac{1}{2}m_{\text{cannon}}v_{\text{recoil}}^2+0+0$$

$$x_{\max}=\sqrt{\frac{m_{\text{cannon}}v_{\text{recoil}}^2}{k}}=\sqrt{\frac{\left(5\,000\ \text{kg}\right)\left(-3.54\ \text{m/s}\right)^2}{2.00\times 10^4\ \text{N/m}}}=\boxed{1.77\ \text{m}}$$

continued on next page

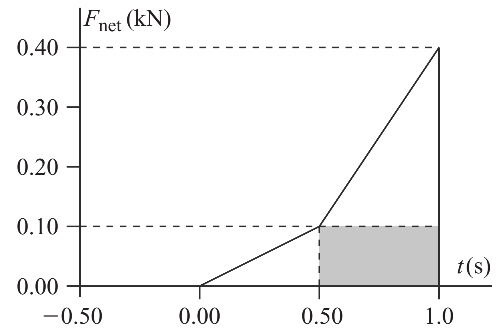
(c) $\left|F_{\max}\right| = k\,x_{\max} = \left(2.00\times10^{4}\ \text{N/m}\right)(1.77\ \text{m}) = \boxed{3.54\times10^{4}\ \text{N}}$

(d) No. The rail exerts a vertical external force (the normal force) on the cannon and prevents it from recoiling vertically. Momentum is not conserved in the vertical direction. The spring does not have time to stretch during the cannon firing. Thus, no external horizontal force is exerted on the system (cannon plus shell) from just before to just after firing. Momentum is conserved in the horizontal direction during this interval.

6.68 Observe from Figure P6.68, the platform exerts a 0.60-kN to support the weight of the standing athlete prior to $t = 0.00$ s. From this, we determine the mass of the athlete:

$$m = \frac{w}{g} = \frac{0.60\ \text{kN}}{g} = \frac{600\ \text{N}}{9.8\ \text{m/s}^2} = 61\ \text{kg}$$

For the interval $t = 0.00$ s to $t = 1.0$ s, we subtract the 0.60 kN force used to counterbalance the weight to get the *net* upward force exerted on the athlete by the platform during the jump. The result is shown in the force-versus-time graph at the right. The net impulse imparted to the athlete is given by the area under this graph. Note that this area can be broken into two triangular areas plus a rectangular area.



The net upward impulse is then

$$I = \frac{1}{2}(0.50\ \text{s})(100\ \text{N}) + \frac{1}{2}(0.50\ \text{s})(300\ \text{N}) + (0.50\ \text{N})(100\ \text{N}) = 150\ \text{N}\cdot\text{s}$$

The upward velocity v_i of the athlete as he lifts off of the platform (at $t = 1.0$ s) is found from

$$I = \Delta p = mv_i - mv_0 = mv_i - 0 \quad \Rightarrow \quad v_i = \frac{I}{m} = \frac{150\ \text{N}\cdot\text{s}}{61\ \text{kg}} = 2.5\ \text{m/s}$$

The height of the jump can then be found from $v_f^2 = v_i^2 + 2a_y\Delta y$ (with $v_f = 0$) to be

$$\Delta y = \frac{v_f^2 - v_i^2}{2a_y} = \frac{0 - (2.5\ \text{m/s})^2}{2\left(-9.8\ \text{m/s}^2\right)} = \boxed{0.32\ \text{m}}$$

6.69 Let particle 1 be the neutron and particle 2 be the carbon nucleus. Then, we are given that $m_2 = 12\,m_1$.

(a) From conservation of momentum, $m_2 v_{2f} + m_1 v_{1f} = m_1 v_{1i} + 0$.

Since $m_2 = 12\,m_1$, this reduces to $\quad 12v_{2f} + v_{1f} = v_{1i}$ **[1]**

For a head-on elastic collision, $\quad v_{1i} - v_{2i} = -\left(v_{1f} - v_{2f}\right)$

Since $v_{2i} = 0$, this becomes $\quad v_{2f} = v_{1i} + v_{1f}$ **[2]**

Substitute Equation [2] into [1] to obtain $12\left(v_{1i} + v_{1f}\right) + v_{1f} = v_{1i}$

or $\quad 13v_{1f} = -11\,v_{1i} \quad$ and $\quad v_{1f} = -\dfrac{11}{13}v_{1i}$

continued on next page

Then, Equation [2] yields $v_{2f} = (2/13)v_{1i}$.

The initial kinetic energy of the neutron is $KE_{1i} = \frac{1}{2}m_1 v_{1i}^2$, and the final kinetic energy of the carbon nucleus is

$$KE_{2f} = \frac{1}{2}m_2 v_{2f}^2 = \frac{1}{2}(12\,m_1)\left(\frac{4}{169}v_{1i}^2\right) = \frac{48}{169}\left(\frac{1}{2}m_1 v_{1i}^2\right) = \frac{48}{169}KE_{1i}$$

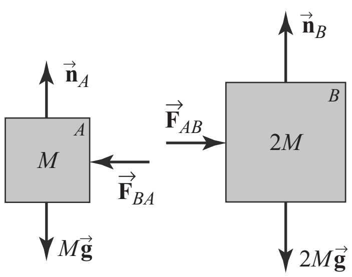
The fraction of kinetic energy transferred is $\dfrac{KE_{2f}}{KE_{1i}} = \dfrac{48}{169} = \boxed{0.28}$

(b) If $KE_{1i} = 1.6\times10^{-13}$ J, then

$$KE_{2f} = \frac{48}{169}KE_{1i} = \frac{48}{169}\left(1.6\times10^{-13}\text{ J}\right) = \boxed{4.5\times10^{-14}\text{ J}}$$

The remaining energy, $(1-48/169)KE_{1i} = (121/169)\left(1.6\times10^{-13}\text{ J}\right) = \boxed{1.1\times10^{-13}\text{ J}}$, stays with the neutron.

6.70 (a)



(b) From Newton's third law, the force $\vec{\mathbf{F}}_{BA}$ exerted by B on A is at each instant equal in magnitude and opposite in direction to the force $\vec{\mathbf{F}}_{AB}$ exerted by A on B.

(c) There are no horizontal external forces acting on System C, which consists of both blocks. The forces $\vec{\mathbf{F}}_{BA}$ and $\vec{\mathbf{F}}_{AB}$ are internal forces exerted on one part of System C by another part of System C.

Thus, $\quad \Sigma\vec{\mathbf{F}}_{\text{external}} = \dfrac{\Delta\vec{\mathbf{p}}_C}{\Delta t} = 0 \;\Rightarrow\; \boxed{\Delta\vec{\mathbf{p}}_C = 0}$

This gives

$$\left(\vec{\mathbf{p}}_C\right)_f = \left(\vec{\mathbf{p}}_C\right)_i = \left(\vec{\mathbf{p}}_A\right)_i + \left(\vec{\mathbf{p}}_B\right)_i \quad \text{or} \quad (M+2M)V = M(+v)+0$$

so the velocity of the combined blocks after collision is $V = +v/3$.

The change in momentum of A is then

$$\Delta\vec{\mathbf{p}}_A = \left(\vec{\mathbf{p}}_A\right)_f - \left(\vec{\mathbf{p}}_A\right)_i = MV - Mv = M\left(\frac{v}{3}-v\right) = \boxed{-2Mv/3}$$

and the change in momentum for B is

$$\Delta\vec{\mathbf{p}}_B = \left(\vec{\mathbf{p}}_B\right)_f - \left(\vec{\mathbf{p}}_B\right)_i = 2MV - 0 = 2M\left(\frac{+v}{3}\right) = \boxed{+2Mv/3}$$

continued on next page

(d) $$\Delta KE = \left(KE_C\right)_f - \left[\left(KE_A\right)+\left(KE_B\right)\right]_i = \frac{1}{2}(3M)\left(\frac{v}{3}\right)^2 - \left[\frac{1}{2}Mv^2 + 0\right] = -\frac{1}{3}Mv^2$$

Thus, $\boxed{\text{kinetic energy is not conserved in this inelastic collision}}$.

6.71 (a) The owner's $\boxed{\text{claim should be denied}}$. Immediately prior to impact, the total momentum of the two-car system had a northward component and an eastward component. Thus, after impact, the wreckage moved in a northeasterly direction and could not possibly have damaged the owner's property on the southeast corner.

(b) Choose east as the positive x-direction and north as the positive y-direction. From conservation of momentum:

$$\left(p_x\right)_{\text{after}} = \left(p_x\right)_{\text{before}} \quad \Rightarrow \quad \left(m_1 + m_2\right)v_x = m_1\left(v_{1i}\right)_x + m_2\left(v_{2i}\right)_x$$

or $$v_x = \frac{m_1\left(v_{1i}\right)_x + m_2\left(v_{2i}\right)_x}{m_1 + m_2} = \frac{(1\,300\text{ kg})(30.0\text{ km/h}) + 0}{1\,300\text{ kg} + 1\,100\text{ kg}} = \boxed{16.3\text{ km/h}}$$

$$\left(p_y\right)_{\text{after}} = \left(p_y\right)_{\text{before}} \quad \Rightarrow \quad \left(m_1 + m_2\right)v_y = m_1\left(v_{1i}\right)_y + m_2\left(v_{2i}\right)_y$$

or $$v_y = \frac{m_1\left(v_{1i}\right)_y + m_2\left(v_{2i}\right)_y}{m_1 + m_2} = \frac{0 + (1\,100\text{ kg})(20.0\text{ km/h})}{1\,300\text{ kg} + 1\,100\text{ kg}} = \boxed{9.17\text{ km/h}}$$

Thus, the velocity of the wreckage immediately after impact is

$$v = \sqrt{v_x^2 + v_y^2} = 18.7\text{ km/h} \quad \text{and} \quad \theta = \tan^{-1}\left(\frac{v_y}{v_x}\right) = \tan^{-1}(0.563) = 29.4°$$

or $\vec{\mathbf{v}} = \boxed{18.7\text{ km/h at }29.4°\text{ north of east, consistent with part (a)}}$.

6.72 (a) Ignore any change in velocity due to the force of gravity during the brief collision time, and let v_{bf} denote the velocity of the ball just after impact while v_{pf} is that of the player. We use the conservation of momentum to obtain

$$(0.45\text{ kg})v_{bf} + (60\text{ kg})v_{pf} = (0.45\text{ kg})(-25\text{ m/s}) + (60\text{ kg})(4.0\text{ m/s})$$

or $$v_{pf} = 3.8\text{ m/s} - \left(7.5\times10^{-3}\right)v_{bf} \qquad \textbf{[1]}$$

Also, for an elastic collision $\Rightarrow$ $v_{bf} - v_{pf} = -\left(v_{bi} - v_{pi}\right) = -(-25\text{ m/s} - 4.0\text{ m/s})$

or $$v_{bf} = 29\text{ m/s} + v_{pf} \qquad \textbf{[2]}$$

Substituting Equation [1] into [2] yields

$$v_{bf} = \frac{29\text{ m/s} + 3.8\text{ m/s}}{1 + 7.5\times10^{-3}} = \boxed{33\text{ m/s}}$$

(b) The average acceleration of the ball during the collision is

$$a_{\text{av}} = \frac{v_{bf} - v_{bi}}{\Delta t} = \frac{33\text{ m/s} - (-25\text{ m/s})}{20\times10^{-3}\text{ s}} = \boxed{2.9\times10^3\text{ m/s}^2}$$

6.73 (a) The speed v of both balls just before the basketball reaches the ground may be found from $v_y^2 = v_{0y}^2 + 2a_y \Delta y$ as

$$v = \sqrt{v_{0y}^2 + 2a_y \Delta y} = \sqrt{0 + 2(-g)(-h)} = \sqrt{2gh} = \sqrt{2(9.80 \text{ m/s}^2)(1.20 \text{ m})} = \boxed{4.85 \text{ m/s}}$$

(b) Immediately after the basketball rebounds from the floor, it and the tennis ball meet in an elastic collision. The velocities of the two balls just before collision are

for the tennis ball: $v_{ti} = -v$ and for the basketball: $v_{bi} = +v$

We determine the velocity of the tennis ball immediately after this elastic collision as follows:

Momentum conservation gives

$$m_t v_{tf} + m_b v_{bf} = m_t v_{ti} + m_b v_{bi} \quad \text{or} \quad m_t v_{tf} + m_b v_{bf} = (m_b - m_t)v \qquad \textbf{[1]}$$

From the criteria for a perfectly elastic collision:

$$v_{ti} - v_{bi} = -(v_{tf} - v_{bf}) \quad \text{or} \quad v_{bf} = v_{tf} + v_{ti} - v_{bi} = v_{tf} - 2v \qquad \textbf{[2]}$$

Substituting Equation [2] into [1] gives

$$m_t v_{tf} + m_b (v_{tf} - 2v) = (m_b - m_t)v$$

or the upward speed of the tennis ball immediately after the collision is

$$v_{tf} = \left(\frac{3m_b - m_t}{m_t + m_b}\right)v = \left(\frac{3m_b - m_t}{m_t + m_b}\right)\sqrt{2gh}$$

The vertical displacement of the tennis ball during its rebound following the collision is given by $v_y^2 = v_{0y}^2 + 2a_y \Delta y$ as

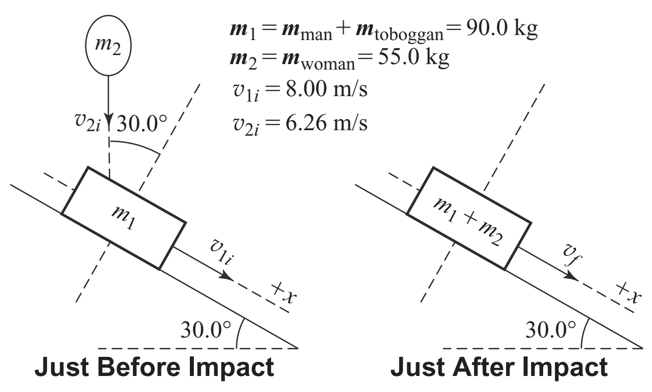
$$\Delta y = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - v_{tf}^2}{2(-g)} = \frac{1}{\cancel{2g}}\left(\frac{3m_b - m_t}{m_t + m_b}\right)^2 (\cancel{2g}h) = \left(\frac{3m_b - m_t}{m_t + m_b}\right)^2 h$$

or $$\Delta y = \left[\frac{3(590 \text{ g}) - (57.0 \text{ g})}{57.0 \text{ g} + 590 \text{ g}}\right]^2 (1.20 \text{ m}) = \boxed{8.41 \text{ m}}$$

6.74 The woman starts from rest ($v_{0y} = 0$) and drops freely with $a_y = -g$ for 2.00 m before the impact with the toboggan. Then, $v_{2i}^2 = v_{0y}^2 + 2a_y(\Delta y)$ gives her speed just before impact as

$$v_{2i} = \sqrt{v_{0y}^2 + 2a_y(\Delta y)} = \sqrt{0 + 2(-9.80 \text{ m/s}^2)(-2.00 \text{ m})} = 6.26 \text{ m/s}$$

The sketches at the right show the situation just before and just after the woman's impact with the toboggan. Since no external forces impart any significant impulse directed parallel to the incline (+x-direction) to the system consisting of man, woman, and toboggan during the very brief duration of the impact,



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we will consider the total momentum parallel to the incline to be conserved. That is,

$$(m_1 + m_2)v_f = m_1 v_{1i} + m_2 (v_{2i})_x = m_1 v_{1i} + m_2 v_{2i} \sin 30.0°$$

or the speed of the system *immediately* after impact is

$$v_f = \frac{m_1 v_{1i} + m_2 v_{2i} \sin 30.0°}{m_1 + m_2} = \frac{(90.0 \text{ kg})(8.00 \text{ m/s}) + (55.0 \text{ kg})(6.26 \text{ m/s})\sin 30.0°}{90.0 \text{ kg} + 55.0 \text{ kg}} = \boxed{6.15 \text{ m/s}}$$

6.75 First consider the motion of the block and embedded bullet from immediately after impact until the block comes to rest after sliding distance d across the horizontal table. During this time, a kinetic friction force $f_k = \mu_k n = \mu_k (M + m) g$, directed opposite to the motion, acts on the block. The net work done on the block and embedded bullet during this time is

$$W_{\text{net}} = (f_k \cos 180°)d = KE_f - KE_i = 0 - \frac{1}{2}(M + m)V^2$$

so the speed, V, of the block and embedded bullet immediately after impact is

$$V = \sqrt{\frac{-2 f_k d}{-(M + m)}} = \sqrt{\frac{2\mu_k \cancel{(M + m)} gd}{\cancel{M + m}}} = \sqrt{2\mu_k g d}$$

Now, make use of conservation of momentum from just before to just after impact to obtain

$$p_{xi} = p_{xf} \qquad \Rightarrow \qquad m v_0 = (M + m)V = (M + m)\sqrt{2\mu_k g d}$$

and the initial velocity of the bullet was

$$\boxed{v_0 = \left(\frac{M + m}{m}\right)\sqrt{2\mu_k g d}}$$

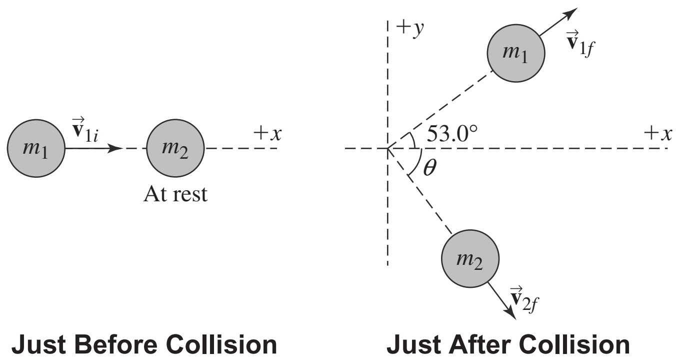
6.76 (a) Apply conservation of momentum in the vertical direction to the squid-water system from the instant before to the instant after the water is ejected. This gives

$$m_s v_s + m_w v_w = (m_s + m_w)(0) \text{ or } v_s = -\left(\frac{m_w}{m_s}\right)v_w = -\left(\frac{0.30 \text{ kg}}{0.85 \text{ kg}}\right)(-20 \text{ m/s}) = \boxed{7.1 \text{ m/s}}$$

(b) Apply conservation of mechanical energy to the squid from the instant after the water is ejected until the squid reaches maximum height to find

$$0 + m_s g y_f = \frac{1}{2} m_s v_s^2 + m g y_i \qquad \text{or} \qquad \Delta y = y_f - y_i = \frac{v_s^2}{2g} = \frac{(7.1 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = \boxed{2.6 \text{ m}}$$

6.77 (a)



The situations just before and just after the collision are shown above. Conserving momentum in both the x- and y-directions gives

$$(p_y)_f = (p_y)_i \Rightarrow m_1 v_{1f}\sin 53° - m_2 v_{2f}\sin\phi = 0 \quad \text{or} \quad m_2 v_{2f}\sin\phi = m_1 v_{1f}\sin 53° \quad \textbf{[1]}$$

$$(p_x)_f = (p_x)_i \Rightarrow m_1 v_{1f}\cos 53° + m_2 v_{2f}\cos\phi = m_1 v_{1i} + 0$$

or
$$m_2 v_{2f}\cos\phi = m_1 v_{1i} - m_1 v_{1f}\cos 53° \quad \textbf{[2]}$$

Dividing Equation [1] by [2] yields

$$\tan\phi = \frac{v_{1f}\sin 53°}{v_{1i} - v_{1f}\cos 53°} = \frac{(1.0\ \text{m/s})\sin 53°}{(2.0\ \text{m/s}) - (1.0\ \text{m/s})\cos 53°} = 0.57 \quad \text{or} \quad \boxed{\phi = 30°}$$

Equation [1] then gives
$$v_{2f} = \frac{m_1 v_{1f}\sin 53°}{m_2\sin\phi} = \frac{(0.20\ \text{kg})(1.0\ \text{m/s})\sin 53°}{(0.30\ \text{kg})\sin 30°} = \boxed{1.1\ \text{m/s}}$$

(b) The fraction of the incident kinetic energy lost in this collision is

$$\frac{|\Delta KE|}{KE_i} = \frac{KE_i - KE_f}{KE_i} = 1 - \frac{KE_f}{KE_i} = 1 - \frac{\frac{1}{2}(0.20\ \text{kg})(1.0\ \text{m/s})^2 + \frac{1}{2}(0.30\ \text{kg})(1.1\ \text{m/s})^2}{\frac{1}{2}(0.20\ \text{kg})(2.0\ \text{m/s})^2}$$

$$\frac{|\Delta KE|}{KE_i} = \boxed{0.30} \text{ or } \boxed{30\%}$$

6.78 (a) Conservation of mechanical energy of the bullet-block-Earth system from just after impact until maximum height is reached may be used to relate the speed of the block and bullet just after collision to the maximum height. Then, conservation of momentum from just before to just after impact can be used to relate the initial speed of the bullet to the speed of the block and bullet just after collision.

(b) Conservation of energy from just after impact until the block and embedded bullet come to rest momentarily at height h gives

$$KE_i + PE_{g,i} = KE_f + PE_{g,f} \quad \text{or} \quad \frac{1}{2}(M+m)V^2 + 0 = 0 + (M+m)gh$$

continued on next page

and the speed of the block+bullet just after collision is $V = \sqrt{2gh}$.

Now, using conservation of momentum during the collision gives

$$mv_i + 0 = (M+m)V \qquad \text{or} \qquad v_i = \left(\frac{M+m}{m}\right)V$$

The initial speed of the bullet is then

$$\boxed{v_i = \left(\frac{M+m}{m}\right)\sqrt{2gh}}$$

6.79 (a) Conservation of mechanical energy of the bullet-block-Earth system from just after impact until maximum height is reached may be used to relate the speed of the block and bullet just after collision to the maximum height. Then, conservation of momentum from just before to just after impact can be used to relate the initial speed of the bullet to the speed of the block and bullet just after collision.

(b) Conservation of energy from just after impact until the block and embedded bullet come to rest momentarily at height $h = 22.0\text{ cm}$ gives

$$KE_i + PE_{g,i} = KE_f + PE_{g,f} \qquad \text{or} \qquad \frac{1}{2}(M+m)V^2 + 0 = 0 + (M+m)gh$$

and the speed of the block+bullet just after collision is $V = \sqrt{2gh}$.

Now, using conservation of momentum during the collision gives

$$mv_i + 0 = (M+m)V \qquad \text{or} \qquad v_i = \left(\frac{M+m}{m}\right)V = \left(\frac{M+m}{m}\right)\sqrt{2gh}$$

The initial speed of the bullet is then

$$v_i = \left(\frac{1.25\text{ kg} + 5.00\times10^{-3}\text{ kg}}{5.00\times10^{-3}\text{ kg}}\right)\sqrt{2\left(9.80\text{ m/s}^2\right)(0.220\text{ m})} = 521\text{ m/s}$$

and the initial velocity of the bullet is $\boxed{\vec{\mathbf{v}}_i = 521\text{ m/s upward}}$.

7

Rotational Motion and the Law of Gravity

QUICK QUIZZES

1. Choice (c). For a rotation of more than 180°, the angular displacement must be larger than $\pi = 3.14$ rad. The angular displacements in the three choices are (a) $6 \text{ rad} - 3 \text{ rad} = 3 \text{ rad}$, (b) $1 \text{ rad} - (-1) \text{ rad} = 2 \text{ rad}$, (c) $5 \text{ rad} - 1 \text{ rad} = 4 \text{ rad}$.

2. Choice (b). Because all angular displacements occurred in the same time interval, the displacement with the lowest value will be associated with the lowest average angular speed.

3. Choice (b). From

$$\alpha = \frac{\omega^2 - \omega_0^2}{2\,\Delta\theta} = \frac{\omega^2 - 0}{2\,\Delta\theta} = \frac{\omega^2}{2\,\Delta\theta}$$

it is seen that the case with the smallest angular displacement involves the highest angular acceleration.

4. Choice (b). All points in a rotating rigid body have the same angular speed.

5. Choice (a). Andrea and Chuck have the same angular speed ω, but Andrea moves in a circle with twice the radius of the circle followed by Chuck. Thus, from $v_t = r\omega$, it is seen that Andrea's tangential speed is twice Chuck's.

6.
1. Choice (e). Since the tangential speed is constant, the tangential acceleration is zero.
2. Choice (a). The centripetal acceleration, $a_c = v_t^2/r$, is inversely proportional to the radius when the tangential speed is constant.
3. Choice (b). The angular speed, $\omega = v_t/r$, is inversely proportional to the radius when the tangential speed is constant.

7. Choice (c). Both the velocity and acceleration are changing in direction, so neither of these vector quantities is constant.

8. Choices (b) and (c). According to Newton's law of universal gravitation, the force between the ball and the Earth depends on the product of their masses, so both forces, that of the ball on the Earth, and that of the Earth on the ball, are equal in magnitude. This follows also, of course, from Newton's third law. The ball has large motion compared to the Earth because, according to Newton's second law, the force gives a much greater acceleration to the small mass of the ball.

9. Choice (e). From $F = G\,Mm/r^2$, the gravitational force is inversely proportional to the square of the radius of the orbit.

10. Choice (d). The semimajor axis of the asteroid's orbit is 4 times the size of Earth's orbit. Thus, Kepler's third law ($T^2/r^3 = constant$) indicates that its orbital period is 8 times that of Earth.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. Earth moves 2π radians around the Sun in 1 year. The average angular speed is then

$$\omega_{\text{av}} = \frac{2\pi \text{ rad}}{1 \text{ y}}\left(\frac{1 \text{ y}}{3.156\times 10^{7} \text{ s}}\right) = 1.99\times 10^{-7} \text{ rad/s}$$

which is choice (e).

2. At the top of the circular path, both the tension in the string and the gravitational force act downward, toward the center of the circle, and together supply the needed centripetal force. Thus, $F_c = T + mg = mr\omega^2$ or

$$T = m\left(r\omega^2 - g\right) = (0.400 \text{ kg})\left[(0.500 \text{ m})(8.00 \text{ rad/s})^2 - 9.80 \text{ m/s}^2\right] = 8.88 \text{ N}$$

making (a) the correct choice for this question.

3. The wheel has a radius of 0.500 m and made 320 revolutions. The distance traveled is

$$s = r\theta = (0.500 \text{ m})\left(3.2\times 10^{2} \text{ rev}\right)\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = 1.0\times 10^{3} \text{ m} = 1.0 \text{ km}$$

so choice (c) is the correct answer.

4. The angular displacement will be

$$\Delta\theta = \omega_{\text{av}}\cdot\Delta t = \left(\frac{\omega_f + \omega_i}{2}\right)\Delta t = \left(\frac{12.00 \text{ rad/s} + 4.00 \text{ rad/s}}{2}\right)(4.00 \text{ s}) = 32.0 \text{ rad}$$

which matches choice (d).

5. According to Newton's law of universal gravitation, the gravitational force one body exerts on the other decreases as the distance separating the two bodies increases. When on Earth's surface, the astronaut's distance from the center of the Earth is Earth's radius $r_0 = R_E$. If h is the altitude at which the station orbits above the surface, her distance from Earth's center when on the station is $r' = R_E + h > r_0$. Thus, she experiences a smaller force while on the space station and (c) is the correct choice.

6. Any object moving in a circular path undergoes a constant change in the direction of its velocity. This change in the direction of velocity is an acceleration, always directed toward the center of the path, called the centripetal acceleration, $a_c = v^2/r = r\omega^2$. The tangential speed of the object is $v_t = r\omega$, where ω is the angular velocity. If ω is not constant, the object will have both an angular acceleration, $\alpha_{\text{av}} = \Delta\omega/\Delta t$, and a tangential acceleration, $a_t = r\alpha$. The only untrue statement among the listed choices is (b). Even when ω is constant, the object still has centripetal acceleration.

7. The required centripetal force is $F_c = ma_c = mv^2/r = mr\omega^2$. When m and ω are both constant, the centripetal force is directly proportional to the radius of the circular path. Thus, as the rider moves toward the center of the merry-go-round, the centripetal force decreases and the correct choice is (c).

8. The mass of a spherical body of radius R and density ρ is $M = \rho V = \rho\left(4\pi R^3/3\right)$. The escape velocity from the surface of this body may then be written in either of the following equivalent forms:

$$v_{\text{esc}} = \sqrt{\frac{2GM}{R}} \qquad \text{and} \qquad v_{\text{esc}} = \sqrt{\frac{2G}{R}\left(\frac{4\pi\rho R^3}{3}\right)} = \sqrt{\frac{8\pi\rho G R^2}{3}}$$

We see that the escape velocity depends on the three properties (mass, density, and radius) of the planet. Also, the weight of an object on the surface of the planet is $F_g = mg = GMm/R^2$, giving

$$g = GM/R^2 = \frac{G}{R^2}\left[\rho\left(\frac{4\pi R^3}{3}\right)\right] = \frac{4}{3}\pi\rho GR$$

The acceleration of gravity at the planet surface then depends on the same properties as does the escape velocity. Changing the value of g would necessarily change the escape velocity. Of the listed quantities, the only one that does not affect the escape velocity is choice (e), the mass of the object.

9. The satellite experiences a gravitational force, always directed toward the center of its orbit, and supplying the centripetal force required to hold it in its orbit. This force gives the satellite a centripetal acceleration, even if it is moving with constant angular speed. At each point on the circular orbit, the gravitational force is directed along a radius line of the path, and is perpendicular to the motion of the satellite, so this force does no work on the satellite. Therefore, the only true statement among the listed choices is (d).

10. The total gravitational potential energy of this set of 4 particles is the sum of the gravitational energies of each distinct pair of particles in the set of four. There are six distinct pairs in a set of four particles, which are: 1 & 2, 1 & 3, 1 & 4, 2 & 3, 2 & 4, and 3 & 4. Therefore, the correct answer to this question is (b).

11. The weight of an object of mass m at the surface of a spherical body of mass M and radius R is $F_g = mg = GMm/R^2$. Thus, the acceleration of gravity at the surface is $g = GM/R^2$. For Earth, $g_E = GM_E/R_E^2$ and for the planet,

$$g_p = \frac{GM_p}{R_p^2} = \frac{G(2M_E)}{(2R_E)^2} = \frac{1}{2}\left(\frac{GM_E}{R_E^2}\right) = \frac{1}{2}g_E = 0.5g_E$$

meaning that choice (b) is the correct response.

12. In a circular orbit, the gravity force is always directed along a radius line of the circle, and hence, perpendicular to the object's velocity which is tangential to the circle. In an elliptical orbit, the gravity force is always directed toward the center of the Earth, located at one of the foci of the orbit. This means that it is perpendicular to the velocity, which is always tangential to the orbit, only at the two points where the object crosses the major axis of the ellipse. These are the points where the object is nearest to and farthest from Earth. Since the gravity force is a conservative force, the total energy (kinetic plus gravitational potential energy) of the object is constant as it moves around the orbit. This means that it has maximum kinetic energy (and hence, greatest speed) when its potential energy is lowest (i.e., when it is closest to Earth. The only true statements among the listed choices are (a) and (b).

13. We assume that the elliptical orbit is so elongated that the Sun, at one of the foci, is almost at one end of the major axis. If the period, T, is expressed in years and the semimajor axis, a, in astronomical units (AU), Kepler's third law states that $T^2 = a^3$. Thus, for Halley's comet, with a period of $T = 76$ y, the semimajor axis of its orbit is $a = \sqrt[3]{(76)^2} = 18$ AU. The length of the major axis, and the approximate maximum distance from the Sun, is $2a = 36$ AU, making the correct answer for this question choice (e).

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The statement is wrong for two reasons. First, gravity is the centripetal force that keeps the astronauts and their spacecraft in orbit around the Earth, so the astronauts aren't beyond gravity's influence. Second, weight is the magnitude of the gravitational force, so astronauts in Earth orbit have weight, although it's lower than on Earth's surface. Because they are freely falling around the Earth along with their spacecraft environment, gravity doesn't press them against the cabin's walls or floor, giving rise to the feeling of weightlessness. The same feeling of weightlessness would occur in a freely falling elevator.

4. To a good first approximation, your bathroom scale reading is unaffected because you, the Earth, and the scale are all in free fall in the Sun's gravitational field, in orbit around the Sun. To a precise second approximation, you weigh slightly less at noon and at midnight than you do at sunrise or sunset. The Sun's gravitational field is a little weaker at the center of the Earth than at the surface sub-solar point, and a little weaker still on the far side of the planet. When the Sun is high in your sky, its gravity pulls up on you a little more strongly than on the Earth as a whole. At midnight the Sun pulls down on you a little less strongly than it does on the Earth below you. So you can have another doughnut with lunch, and your bedsprings will still last a little longer.

6. Consider one end of a string connected to a spring scale and the other end connected to an object, of true weight w. The tension T in the string will be measured by the scale and construed as the apparent weight. We have $w - T = ma_c$. This gives $T = w - m\,a_c$. Thus, the apparent weight is less than the actual weight by the term ma_c. At the poles the centripetal acceleration is zero, and $T = w$. However, at the equator the term containing the centripetal acceleration is non-zero, and the apparent weight is less than the true weight.

8. (a) If the acceleration is constant in magnitude and perpendicular to the velocity, the object is moving in a circular path at constant speed.

(b) If the acceleration is parallel to the velocity, the object moves in a straight line, and is either speeding up (v and a in same direction) or slowing down (v and a in opposite directions).

10. Kepler's second law says that equal areas are swept out in equal times by a line drawn from the Sun to the planet. For this to be so, the planet must move fastest when it is closest to the Sun. This occurs during the winter season in the northern hemisphere.

12. (a) Velocity is north at A, west at B, and south at C.

(b) The acceleration is west at A, nonexistent at B, east at C, to be radially inward.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 2.1 m **(b)** 1.2×10^2 m **(c)** 7.7×10^2 m

4. **(a)** $0.209\ \text{rad/s}^2$ **(b)** Yes.

6. $-226\ \text{rad/s}^2$

8. **(a)** The tire is slowing down, so the second drop was released with a smaller initial velocity.

(b) $-0.322\ \text{rad/s}^2$

10. 51 revolutions

12. **(a)** $5.75\ \text{rad/s}$ **(b)** $v_t = 1.29\ \text{m/s},\ a_t = 0.563\ \text{m/s}^2$

(c) 76° counterclockwise from the $+x$-axis

14. **(a)** 5.25 s **(b)** 27.6 rad

16. 4.9×10^{-2} rad/s

18. No. The required maximum tension in the vine is 1.38×10^3 N.

20. **(a)** the static friction force; slipping starts when $mr\omega^2 > f_{s,\max} = \mu_s mg$.

(b) 6.55 s

22. **(a)** 4.81 m/s **(b)** 700 N

24. 1.5×10^2 rev/s

26. **(a)** a normal force exerted by the "floor" of the cabin

(b) $\Sigma F_c = n = m v_t^2/r$ **(c)** 294 N **(d)** 7.00 m/s

(e) 0.700 rad/s **(f)** 8.98 s **(g)** slower; 5.74 m/s

28. **(a)** $T = m_2 g$ **(b)** $T = m_1 v_t^2/R$ **(c)** $v_t = \sqrt{m_2 g R/m_1}$

(d) $T = 9.8$ m, $v_t = 6.3$ m/s

30. **(a)** the gravitational force and a contact force exerted by the pail

(b) the contact force (the gravitational force alone would produce projectile motion)

(c) 3.13 m/s; No, the motion would be identical to projectile motion.

32. **(a)** 25 kN **(b)** 12 m/s

34. **(a)** -4.76×10^9 J **(b)** 568 N

36. **(a)** 1.83×10^9 kg/m^3 **(b)** 3.26×10^6 m/s^2 **(c)** -2.08×10^{13} J

38. 2.59×10^8 m from the center of the Earth

40. The two masses are 2.00 kg and 3.00 kg.

42. **(a)** 1.23×10^6 m **(b)** 6.89 m/s^2

44. **(a)** 5.59×10^3 m/s **(b)** 3.98 h **(c)** 1.47×10^3 N

46. 8.91×10^7 m

48. 1.63×10^4 rad/s

50. **(a)** 56.5 rad/s **(b)** 22.4 rad/s **(c)** -7.62×10^{-3} rad/s^2

(d) 1.77×10^5 rad **(e)** 5.81 km

52. **(a)** $v_{\min} = \sqrt{Rg\left(\dfrac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}$; $v_{\max} = \sqrt{Rg\left(\dfrac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}$ **(b)** 8.6 m/s ; 17 m/s

54. **(a)** 8.42 N **(b)** 64.8° **(c)** 1.67 N

56. See Solution.

58. **(a)** $n_A = 15\text{ N}$; $n_B = 30\text{ N}$ **(b)** $\mu_k = 0.17$

60. **(a)** See Solution. **(b)** See Solution.

(c) The seat exerts the greatest force on the passenger at the bottom of the loop.

(d) $n_{\text{top}} = 546\text{ N}$; $n_{\text{bottom}} = 826\text{ N}$

62. 12.8 N

64. $2.06\times 10^3\ \text{rev/min}$

66. **(a)** 0 **(b)** 1.3 kN **(c)** 2.1 kN

68. **(a)** $1.62\times 10^3\ \text{N}$ **(b)** 196 N

(c) With the right speed, the needed centripetal force at the top of the loop can be made exactly equal to the gravitational force. At this speed, the normal force exerted on the pilot by the seat (his apparent weight) will be zero, and the pilot will have the sensation of weightlessness.

(d) 177 m/s

70. **(a)** a downward gravitation force and a tension force that is directed toward the center of the circular path

(b) See Solution. **(c)** 6.05 N **(d)** 7.82 m/s

72. **(a)** 4.1 m/s **(b)** 80° **(c)** 1.7 m

(d) No. Flying slower increases the minimum radius of achievable turns.

74. **(a)** 2.1 m/s **(b)** 54° **(c)** 4.7 m/s

76. 0.75 m

PROBLEM SOLUTIONS

7.1 (a) Earth rotates 2π radians (360°) on its axis in 1 day. Thus,

$$\omega = \frac{\Delta\theta}{\Delta t} = \frac{2\pi\text{ rad}}{1\ \cancel{\text{day}}}\left(\frac{1\ \cancel{\text{day}}}{8.64\times 10^4\ \text{s}}\right) = \boxed{7.27\times 10^{-5}\ \text{rad/s}}$$

(b) Because of its rotation about its axis, $\boxed{\text{Earth bulges at the equator.}}$

7.2 The distance traveled is $s = r\theta$, where θ is in radians.

(a) For 30°, $s = r\theta = (4.1\text{ m})\left[30°\left(\frac{\pi\text{ rad}}{180°}\right)\right] = \boxed{2.1\text{ m}}$

continued on next page

(b) For 30 radians, $s = r\theta = (4.1\text{ m})(30\text{ rad}) = \boxed{1.2\times10^{2}\text{ m}}$

(c) For 30 revolutions, $s = r\theta = (4.1\text{ m})\left[30\text{ rev}\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\right] = \boxed{7.7\times10^{2}\text{ m}}$

7.3 (a) $\theta = \dfrac{s}{r} = \dfrac{60\,000\text{ mi}}{1.0\text{ ft}}\left(\dfrac{5\,280\text{ ft}}{1\text{ mi}}\right) = \boxed{3.2\times10^{8}\text{ rad}}$

(b) The car travels a distance equal to the circumference of the tire for every revolution the tire makes if there is no slipping of the tire on the roadway. Thus, the number of revolutions made during the warranty period is

$$n = \frac{S}{2\pi r} = \frac{60\,000\text{ miles}}{2\pi(1.0\text{ ft})}\left(\frac{5\,280\text{ ft}}{1\text{ mile}}\right) = \boxed{5.0\times10^{7}\text{ rev}}$$

7.4 (a) $\alpha = \dfrac{\Delta\omega}{\Delta t} = \dfrac{1.00\text{ rev/s} - 0}{30.0\text{ s}} = \left(3.33\times10^{-2}\ \dfrac{\cancel{\text{rev}}}{\text{s}^2}\right)\left(\dfrac{2\pi\text{ rad}}{1\ \cancel{\text{rev}}}\right) = \boxed{0.209\text{ rad/s}^2}$

(b) $\boxed{\text{Yes.}}$ When an object starts from rest, its angular speed is related to the angular acceleration and time by the equation $\omega = \alpha(\Delta t)$. Thus, the angular speed is directly proportional to both the angular acceleration and the time interval. If the time interval is held constant, doubling the angular acceleration will double the angular speed attained during the interval.

7.5 (a) $\alpha = \dfrac{(2.51\times10^{4}\text{ rev/min} - 0)}{3.20\text{ s}}\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\dfrac{1\text{ min}}{60.0\text{ s}}\right) = \boxed{821\text{ rad/s}^2}$

(b) $\theta = \bar{\omega}t = \left(\dfrac{\omega_f + \omega_0}{2}\right)t = \left[\dfrac{(2.51\times10^{4}\text{ rev/min})(2\pi\text{ rad/1 rev})(1\text{ min/60.0 s}) + 0}{2}\right](3.20\text{ s})$

$= \boxed{4.21\times10^{3}\text{ rad}}$

7.6 $\omega_i = 3\,600\ \dfrac{\text{rev}}{\text{min}}\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\dfrac{1\text{ min}}{60.0\text{ s}}\right) = 377\text{ rad/s}$

$\Delta\theta = 50.0\text{ rev}\left(\dfrac{2\pi\text{ rad}}{1\text{ rev}}\right) = 314\text{ rad}$

Thus, $\alpha = \dfrac{w^2 - \omega_i^2}{2\Delta\theta} = \dfrac{0 - (377\text{ rad/s})^2}{2(314\text{ rad})} = \boxed{-226\text{ rad/s}^2}$

7.7 (a) From $\omega^2 = \omega_0^2 + 2\alpha(\Delta\theta)$, the angular displacement is

$$\Delta\theta = \frac{\omega^2 - \omega_0^2}{2\alpha} = \frac{(2.2\text{ rad/s})^2 - (0.06\text{ rad/s})^2}{2(0.70\text{ rad/s}^2)} = \boxed{3.5\text{ rad}}$$

(b) From the equation given above for $\Delta\theta$, observe that when the angular acceleration is constant, the displacement is proportional to the difference in the *squares* of the final and initial angular speeds. Thus, the angular displacement would $\boxed{\text{increase by a factor of 4}}$ if both of these speeds were doubled.

7.8 (a) The maximum height *h* depends on the drop's vertical speed at the instant it leaves the tire and becomes a projectile. The vertical speed at this instant is the same as the tangential speed, $v_t = r\omega$, of points on the tire. Since the second drop rose to a lesser height, $\boxed{\text{the tangential speed decreased}}$ during the intervening rotation of the tire.

(b) From $v^2 = v_0^2 + 2a_y\left(\Delta y\right)$, with $v_0 = v_t$, $a_y = -g$, and $v = 0$ when $\Delta y = h$, the relation between the tangential speed of the tire and the maximum height *h* is found to be

$$0 = v_t^2 + 2\left(-g\right)h \qquad \text{or} \qquad v_t = \sqrt{2gh}$$

Thus, the angular speed of the tire when the first drop left was $\omega_1 = \dfrac{\left(v_t\right)_1}{r} = \dfrac{\sqrt{2gh_1}}{r}$,

and when the second drop left, the angular speed was $\omega_2 = \dfrac{\left(v_t\right)_2}{r} = \dfrac{\sqrt{2gh_2}}{r}$. From $\omega^2 = \omega_0^2 + 2\alpha\left(\Delta\theta\right)$, with $\Delta\theta = 2\pi$ rad, the angular acceleration is found to be

$$\alpha = \frac{\omega_2^2 - \omega_1^2}{2\left(\Delta\theta\right)} = \frac{2gh_2/r^2 - 2gh_1/r^2}{2\left(\Delta\theta\right)} = \frac{g}{r^2\left(\Delta\theta\right)}\left(h_2 - h_2\right)$$

or $$\alpha = \frac{\left(9.80\ \text{m/s}^2\right)}{\left(0.381\ \text{m}\right)^2\left(2\pi\ \text{rad}\right)}\left(0.510\ \text{m} - 0.540\ \text{m}\right) = \boxed{-0.322\ \text{rad/s}^2}$$

7.9 Main Rotor: $v = r\omega = \left(3.80\ \text{m}\right)\left(450\ \dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{179\ \text{m/s}}$

$$v = \left(179\ \frac{\text{m}}{\text{s}}\right)\left(\frac{v_{\text{sound}}}{343\ \text{m/s}}\right) = \boxed{0.522\, v_{\text{sound}}}$$

Tail Rotor: $v = r\omega = \left(0.510\ \text{m}\right)\left(4138\ \dfrac{\text{rev}}{\text{min}}\right)\left(\dfrac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{221\ \text{m/s}}$

$$v = \left(221\ \frac{\text{m}}{\text{s}}\right)\left(\frac{v_{\text{sound}}}{343\ \text{m/s}}\right) = \boxed{0.644\, v_{\text{sound}}}$$

7.10 We will break the motion into two stages: (1) an acceleration period and (2) a deceleration period.

The angular displacement during the acceleration period is

$$\theta_1 = \omega_{\text{av}}t = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left[\frac{\left(5.0\ \text{rev/s}\right)\left(2\pi\ \text{rad/1 rev}\right) + 0}{2}\right]\left(8.0\ \text{s}\right) = 1.3\times10^2\ \text{rad}$$

and while decelerating,

$$\theta_2 = \left(\frac{\omega_f + \omega_i}{2}\right)t = \left[\frac{0 + \left(5.0\ \text{rev/s}\right)\left(2\pi\ \text{rad/1 rev}\right)}{2}\right]\left(12\ \text{s}\right) = 1.9\times10^2\ \text{rad}$$

The total displacement is $\theta = \theta_1 + \theta_2 = \left[\left(1.3+1.9\right)\times10^2\ \text{rad}\right]\left(\dfrac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = \boxed{51\ \text{rev}}$

7.11 (a) The linear distance the car travels in coming to rest is given by $v_f^2 = v_0^2 + 2a\left(\Delta x\right)$ as

$$\Delta x = \frac{v_f^2 - v_0^2}{2a} = \frac{0 - \left(29.0\ \text{m/s}\right)^2}{2\left(-1.75\ \text{m/s}^2\right)} = 240\ \text{m}$$

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Since the car does not skid, the linear displacement of the car and the angular displacement of the tires are related by $\Delta x = r(\Delta\theta)$. Thus, the angular displacement of the tires is

$$\Delta\theta = \frac{\Delta x}{r} = \frac{240 \text{ m}}{0.330 \text{ m}} = (727 \text{ rad})\left(\frac{1 \text{ rev}}{2\pi \text{ rad}}\right) = \boxed{116 \text{ rev}}$$

(b) When the car has traveled 120 m (one half of the total distance), the linear speed of the car is

$$v = \sqrt{v_0^2 + 2a(\Delta x)} = \sqrt{(29.0 \text{ m/s})^2 + 2(-1.75 \text{ m/s}^2)(120 \text{ m})} = 20.5 \text{ m/s}$$

and the angular speed of the tires is

$$\omega = \frac{v}{r} = \frac{20.5 \text{ m/s}}{0.330 \text{ m}} = \boxed{62.1 \text{ rad/s}}$$

7.12 (a) The angular speed is $\omega = \omega_0 + \alpha t = 0 + (2.50 \text{ rad/s}^2)(2.30 \text{ s}) = \boxed{5.75 \text{ rad/s}}$

(b) Since the disk has a diameter of 45.0 cm, its radius is $r = (0.450 \text{ m})/2 = 0.225 \text{ m}$. Thus,

$$v_t = r\omega = (0.225 \text{ m})(5.75 \text{ rad/s}) = \boxed{1.29 \text{ m/s}}$$

and $$a_t = r\alpha = (0.225 \text{ m})(2.50 \text{ rad/s}^2) = \boxed{0.563 \text{ m/s}^2}$$

(c) The angular displacement of the disk is

$$\Delta\theta = \theta_f - \theta_0 = \frac{\omega_f^2 - \omega_0^2}{2\alpha} = \frac{(5.75 \text{ rad/s})^2 - 0}{2(2.50 \text{ rad/s}^2)} = (6.61 \text{ rad})\left(\frac{360°}{2\pi \text{ rad}}\right) = 379°$$

and the final angular position of the radius line through point P is

$$\theta_f = \theta_0 + \Delta\theta = 57.3° + 379° = 436°$$

or it is at $\boxed{76° \text{ counterclockwise from the } +x\text{-axis}}$ after turning 19° beyond one full revolution.

7.13 From $\Delta\theta = \omega_{av} t = \left[(\omega_f + \omega_i)/2\right]t$, we find the initial angular speed to be

$$\omega_i = \frac{2\,\Delta\theta}{t} - \omega_f = \frac{2(37.0 \text{ rev})(2\pi \text{ rad/1 rev})}{3.00 \text{ s}} - 98.0 \text{ rad/s} = 57.0 \text{ rad/s}$$

The angular acceleration is then

$$\alpha = \frac{\omega_f - \omega_i}{t} = \frac{98.0 \text{ rad/s} - 57.0 \text{ rad/s}}{3.00 \text{ s}} = \boxed{13.7 \text{ rad/s}^2}$$

7.14 (a) The initial angular speed is

$$\omega_0 = 1.00 \times 10^2 \ \frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\left(\frac{2\pi \text{ rad}}{1 \ \cancel{\text{rev}}}\right)\left(\frac{1 \ \cancel{\text{min}}}{60.0 \text{ s}}\right) = 10.5 \text{ rad/s}$$

The time to stop (i.e., reach a speed of $\omega = 0$) with $\alpha = -2.00 \text{ rad/s}^2$ is

continued on next page

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{0 - 10.5\ \text{rad/s}}{-2.00\ \text{rad/s}^2} = \boxed{5.25\ \text{s}}$$

(b) $$\Delta\theta = \omega_{\text{av}} t = \left(\frac{\omega + \omega_0}{2}\right) t = \left(\frac{0 + 10.5\ \text{rad/s}}{2}\right)(5.25\ \text{s}) = \boxed{27.6\ \text{rad}}$$

7.15 (a) The car travels 235 m at constant speed in an elapsed time of 36.0 s. Its constant speed is therefore

$$v = \frac{\Delta s}{\Delta t} = \frac{235\ \text{m}}{36.0\ \text{s}} = \boxed{6.53\ \text{m/s}}$$

(b) The angular displacement of the car during the 36.0 s time interval is one-fourth of a full circle or $\pi/2$ radians. Thus, the radius of the circular path is

$$r = \frac{\Delta s}{\Delta\theta} = \frac{235\ \text{m}}{\pi/2\ \text{rad}} = \frac{470}{\pi}\ \text{m}$$

During the 36.0 s interval, the car has zero tangential acceleration, but does have a centripetal acceleration of constant magnitude

$$a_c = \frac{v^2}{r} = \frac{(6.53\ \text{m/s})^2}{(470/\pi)\ \text{m}} = \frac{\pi(6.53\ \text{m/s})^2}{470\ \text{m}} = 0.285\ \text{m/s}^2$$

This acceleration is always directed $\boxed{\text{toward the center of the circle}}$. Therefore, when the car is at point B, the vector expression for the car's acceleration is

$$\boxed{\vec{\mathbf{a}}_c = 0.285\ \text{m/s}^2\ \text{at } 35.0°\ \text{north of west}}$$

7.16 The radius of the cylinder is $r = 2.5\ \text{mi}\left(\frac{1609\ \text{m}}{1\ \text{mi}}\right) = 4.0 \times 10^3\ \text{m}$. Thus, from $a_c = r\omega^2$, the required angular velocity is

$$\omega = \sqrt{\frac{a_c}{r}} = \sqrt{\frac{9.80\ \text{m/s}^2}{4.0 \times 10^3\ \text{m}}} = \boxed{4.9 \times 10^{-2}\ \text{rad/s}}$$

7.17 (a) The tangential acceleration of the bug as the disk speeds up is

$$a_t = r\alpha = r\left(\frac{\omega_f - \omega_0}{\Delta t}\right)$$
$$= (5.00\ \cancel{\text{in}})\left(\frac{78.0\ \cancel{\text{rev}}/\cancel{\text{min}} - 0}{3.00\ \text{s}}\right)\left(\frac{1\ \text{m}}{39.37\ \cancel{\text{in}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \cancel{\text{rev}}}\right) = \boxed{0.346\ \text{m/s}^2}$$

(b) The final tangential speed of the bug is

$$v_t = r\omega_f = (5.00\ \cancel{\text{in}})\left(78.0\ \frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\right)\left(\frac{1\ \text{m}}{39.37\ \cancel{\text{in}}}\right)\left(\frac{1\ \cancel{\text{min}}}{60\ \text{s}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \cancel{\text{rev}}}\right) = \boxed{1.04\ \text{m/s}}$$

(c) Since the bug has constant angular acceleration, and hence constant tangential acceleration ($a_t = r\alpha$), the tangential acceleration at $t = 1.00$ is $a_t = \boxed{0.346\ \text{m/s}^2}$ as above.

(d) At $t = 1.00$ s, the tangential velocity of the bug is

$$v_t = v_0 + a_t t = 0 + (0.346\ \text{m/s}^2)(1.00\ \text{s}) = 0.346\ \text{m/s}$$

continued on next page

and the radial or centripetal acceleration is

$$a_c = \frac{v_t^2}{r} = \frac{(0.346\ \text{m/s})^2}{(5.00\ \text{in})(1\ \text{m}/39.37\ \text{in})} = \boxed{0.943\ \text{m/s}^2}$$

(e) The total acceleration is $a = \sqrt{a_c^2 + a_t^2} = \boxed{1.00\ \text{m/s}^2}$, and the angle this acceleration makes with the direction of $\vec{\mathbf{a}}_c$ is

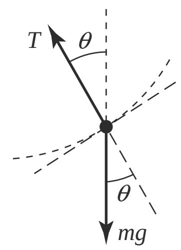
$$\theta = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{0.346}{0.943}\right) = \boxed{20.1°}$$

7.18 In order for the archeologist to make it safely across the river, the vine must be capable of giving him a net acceleration of $a_c = v_{\text{max}}^2/r$ upward as he passes through the lowest point on the swing with a speed of $v_{\text{max}} = 8.00\ \text{m/s}$. Thus, with T being the tension in the vine, the net force acting on the archeologist at the lowest point is $\Sigma F_y = T - mg = ma_c$, giving the required minimum tensile strength of the vine as

$$T = mg + ma_c = m\left(g + \frac{v_{\text{max}}^2}{r}\right) = (85.0\ \text{kg})\left(9.80\ \frac{\text{m}}{\text{s}^2} + \frac{(8.00\ \text{m/s})^2}{10.0\ \text{m}}\right) = 1.38 \times 10^3\ \text{N}$$

$\boxed{\text{Since he chose a vine with breaking strength of 1 000 N, he does not make it across.}}$

7.19 (a) The tension in the string must counteract the radial component of the object's weight, and also supply the needed centripetal acceleration.



$$\Sigma F_c = T - mg\cos\theta = ma_c = \frac{mv^2}{r}$$

or $$T = m\left(v^2/r + g\cos\theta\right)$$

$$= (0.500\ \text{kg})\left[\frac{(8.00\ \text{m/s})^2}{2.00\ \text{m}} + (9.80\ \text{m/s}^2)\cos 20.0°\right]$$

$$= \boxed{20.6\ \text{N}}$$

(b) The net tangential force acting on the object is $F_t = mg\sin\theta$, so the tangential acceleration has magnitude

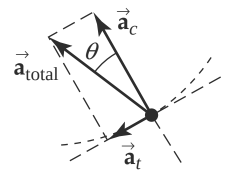
$$a_t = \frac{F_t}{m} = g\sin\theta = (9.80\ \text{m/s}^2)\sin 20.0° = \boxed{3.35\ \text{m/s}^2}$$

and is directed $\boxed{\text{downward, tangential to the circular path.}}$

The radial component of the acceleration is

$$a_c = \frac{v^2}{r} = \frac{(8.00\ \text{m/s})^2}{2.00\ \text{m}} = \boxed{32.0\ \text{m/s}^2\ \text{toward the center of the path}}$$

(c) The total acceleration has magnitude



$$a_{\text{total}} = \sqrt{a_t^2 + a_c^2} = \sqrt{(3.35\ \text{m/s}^2)^2 + (32.0\ \text{m/s}^2)^2}$$

or $$a_{\text{total}} = 32.2\ \text{m/s}^2$$

continued on next page

at $$\theta = \tan^{-1}\left(\frac{a_t}{a_c}\right) = \tan^{-1}\left(\frac{3.35}{32.0}\right) = 5.98°$$

Thus, $\boxed{\vec{\mathbf{a}}_{\text{total}} = 32.2 \text{ m/s}^2 \text{ at } 5.98° \text{ to the cord and pointing below the center of the circular path}}$

(d) $\boxed{\text{No change}}$ in answers if the object is swinging toward the equilibrium point instead of away from it.

(e) If the object is swinging toward the equilibrium position, it is gaining speed, whereas it is losing speed if it is swinging away from the equilibrium position. In both cases, when the cord is 20.0° from the vertical, the tangential, centripetal, and total accelerations have the magnitudes and directions calculated in parts (a) through (c).

7.20 (a) The natural tendency of the coin is to move in a straight line (tangent to the circular path of radius 15.0 cm), and hence, go farther from the center of the turntable. To prevent this, $\boxed{\text{the force of static friction}}$ must act toward the center of the turntable and supply the needed centripetal force. When the necessary centripetal force exceeds the maximum value of the static friction force, $(f_s)_{\text{max}} = \mu_s n = \mu_s mg$, the coin begins to slip.

(b) When the turntable has angular speed ω, the required centripetal force is $F_c = mr\omega^2$. Thus, if the coin is not to slip, it is necessary that $mr\omega^2 \leq \mu_s mg$

or $$\omega \leq \sqrt{\frac{\mu_s g}{r}} = \sqrt{\frac{(0.350)(9.80 \text{ m/s}^2)}{0.150 \text{ m}}} = 4.78 \text{ rad/s}$$

With a constant angular acceleration of $\alpha = 0.730 \text{ rad/s}^2$, the time required to reach the critical angular speed is

$$t = \frac{\omega - \omega_0}{\alpha} = \frac{4.78 \text{ rad/s} - 0}{0.730 \text{ rad/s}^2} = \boxed{6.55 \text{ s}}$$

7.21 (a) From $\Sigma F_r = ma_c$, we have

$$T = m\left(\frac{v_t^2}{r}\right) = \frac{(55.0 \text{ kg})(4.00 \text{ m/s})^2}{0.800 \text{ m}} = 1.10 \times 10^3 \text{ N} = \boxed{1.10 \text{ kN}}$$

(b) The tension is $\boxed{\text{larger than her weight}}$ by a factor of

$$\frac{T}{mg} = \frac{1.10 \times 10^3 \text{ N}}{(55.0 \text{ kg})(9.80 \text{ m/s}^2)} = \boxed{2.04 \text{ times}}$$

7.22 (a) If T is the tension in each of the two support chains, the net force acting on the child at the lowest point on the circular path is

$$\Sigma F_y = 2T - mg = ma_c = m\left(\frac{v^2}{r}\right)$$

so the speed at this point is

$$v = \sqrt{r\left(\frac{2T}{m} - g\right)} = \sqrt{(3.00 \text{ m})\left(\frac{2(350 \text{ N})}{40.0 \text{ kg}} - 9.80 \text{ m/s}^2\right)} = \boxed{4.81 \text{ m/s}}$$

continued on next page

(b) The upward force the seat exerts on the child at this lowest point is

$$F_{\text{seat}} = 2T = 2(350\text{ N}) = \boxed{700\text{ N}}$$

7.23 Friction between the tires and the roadway is capable of giving the truck a maximum centripetal acceleration of

$$a_{c,\,\max} = \frac{v_{t,\,\max}^2}{r} = \frac{(32.0\ \text{m/s})^2}{150\text{ m}} = 6.83\ \text{m/s}^2$$

If the radius of the curve changes to 75.0 m, the maximum safe speed will be

$$v_{t,\,\max} = \sqrt{r\,a_{c,\,\max}} = \sqrt{(75.0\text{ m})(6.83\ \text{m/s}^2)} = \boxed{22.6\ \text{m/s}}$$

7.24 Since $F_c = m\dfrac{v_t^2}{r} = m\,r\,\omega^2$, the needed angular velocity is

$$\omega = \sqrt{\frac{F_c}{m\,r}} = \sqrt{\frac{4.0\times10^{-11}\text{ N}}{(3.0\times10^{-16}\text{ kg})(0.150\text{ m})}}$$

$$= (9.4\times10^2\ \text{rad/s})\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{1.5\times10^2\ \text{rev/s}}$$

7.25 (a) $a_c = r\omega^2 = (2.00\text{ m})(3.00\ \text{rad/s})^2 = \boxed{18.0\ \text{m/s}^2}$

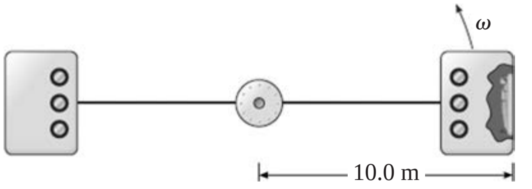
(b) $F_c = ma_c = (50.0\text{ kg})(18.0\ \text{m/s}^2) = \boxed{900\text{ N}}$

(c) We know the centripetal acceleration is produced by the force of friction. Therefore, the needed static friction force is $f_s = 900$ N. Also, the normal force is $n = mg = 490$ N. Thus, the minimum coefficient of friction required is

$$\mu_s = \frac{(f_s)_{\max}}{n} = \frac{900\text{ N}}{490\text{ N}} = \boxed{1.84}$$

Such a large coefficient of friction is unrealistic, and she will not be able to stay on the merry-go-round.

7.26 (a) The only force acting on the astronaut is $\boxed{\text{the normal force}}$ exerted on him by the "floor" of the cabin.



(b) $\Sigma F = n = ma_c$ or $\boxed{n = mv_t^2/r}$

(c) If $n = \frac{1}{2}mg_E$, then

$$n = \frac{1}{2}(60.0\text{ kg})(9.80\ \text{m/s}^2) = \boxed{294\text{ N}}$$

(d) From the equation in Part (b), $v_t = \sqrt{\dfrac{nr}{m}} = \sqrt{\dfrac{(294\text{ N})(10.0\text{ m})}{60.0\text{ kg}}} = \boxed{7.00\ \text{m/s}}$

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(e) Since $v_t = r\omega$, we have

$$\omega = \frac{v_t}{r} = \frac{7.00 \text{ m/s}}{10.0 \text{ m}} = \boxed{0.700 \text{ rad/s}}$$

(f) The period of rotation is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{0.700 \text{ rad/s}} = \boxed{8.98 \text{ s}}$$

(g) Upon standing, the astronaut's head is moving slower than his feet because his head is closer to the axis of rotation. When standing, the radius of the circular path followed by the head is $r_{\text{head}} = 10.0 \text{ m} - 1.80 \text{ m} = 8.20 \text{ m}$, and the tangential speed of the head is

$$(v_t)_{\text{head}} = r_{\text{head}}\omega = (8.20 \text{ m})(0.700 \text{ rad/s}) = \boxed{5.74 \text{ m/s}}$$

7.27 (a) Since the 1.0-kg mass is in equilibrium, the tension in the string is

$$T = mg = (1.0 \text{ kg})(9.8 \text{ m/s}^2) = \boxed{9.8 \text{ N}}$$

(b) The tension in the string must produce the centripetal acceleration of the puck. Hence, $F_c = T = \boxed{9.8 \text{ N}}$.

(c) From $F_c = m_{\text{puck}}\left(v_t^2/R\right)$, we find

$$v_t = \sqrt{\frac{R F_c}{m_{\text{puck}}}} = \sqrt{\frac{(1.0 \text{ m})(9.8 \text{ N})}{0.25 \text{ kg}}} = \boxed{6.3 \text{ m/s}}$$

7.28 (a) Since the mass m_2 hangs in equilibrium on the end of the string,

$$\Sigma F_y = T - m_2 g = 0 \qquad \text{or} \qquad \boxed{T = m_2 g}$$

(b) The puck moves in a circular path of radius R and must have an acceleration directed toward the center equal to $a_c = v_t^2/R$. The only force acting on the puck and directed toward the center is the tension in the string. Newton's second law requires

$$\Sigma F_{\substack{\text{toward}\\ \text{center}}} = m_1 a_c \qquad \text{giving} \qquad \boxed{T = m_1\left(v_t^2/R\right)}$$

(c) Combining the results from (a) and (b) gives

$$m_1 \frac{v_t^2}{R} = m_2 g \qquad \text{or} \qquad \boxed{v_t = \sqrt{m_2 g R/m_1}}$$

(d) Substitution of the numeric data from Problem 7.27 into the results for (a) and (c) shown above will yield the answers given for that problem.

7.29 (a) The $\boxed{\text{force of static friction}}$ acting toward the road's center of curvature must supply the briefcase's required centripetal acceleration. The condition that it be able to meet this need is that $F_c = m\,v_t^2/r \le (f_s)_{\max} = \mu_s mg$, or $\mu_s \ge v_t^2/rg$. When the tangential $\boxed{\text{speed becomes large enough that } \mu_s = v_t^2/rg}$, the briefcase will begin to slide.

(b) As discussed above, the briefcase starts to slide when $\mu_s = v_t^2/rg$. If this occurs at the speed, $v_t = 15.0 \text{ m/s}$, the coefficient of static friction must be

$$\mu_s = \frac{(15.0 \text{ m/s})^2}{(62.0 \text{ m})(9.80 \text{ m/s}^2)} = \boxed{0.370}$$

7.30 (a) The external forces acting on the water are $\boxed{\text{the gravitational force}}$ and $\boxed{\text{the contact force exerted on the water by the pail}}$.

(b) The $\boxed{\text{contact force exerted by the pail}}$ is the most important in causing the water to move in a circle. If the gravitational force acted alone, the water would follow the parabolic path of a projectile.

(c) When the pail is inverted at the top of the circular path, it cannot hold the water up to prevent it from falling out. If the water is not to spill, the pail must be moving fast enough that the required centripetal force is at least as large as the gravitational force. That is, we must have

$$m\frac{v^2}{r} \geq mg \qquad \text{or} \qquad v \geq \sqrt{rg} = \sqrt{(1.00 \text{ m})(9.80 \text{ m/s}^2)} = \boxed{3.13 \text{ m/s}}$$

If the pail were to suddenly disappear when is it at the top of the circle and moving at 3.13 m/s, $\boxed{\text{the water would follow the parabolic arc of a projectile}}$ launched with initial velocity components of $v_{0x} = 3.13 \text{ m/s}$, $v_{0y} = 0$.

7.31 (a) The centripetal acceleration is

$$a_c = r\omega^2 = (9.00 \text{ m})\left[\left(4.00 \frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\right)\left(\frac{2\pi \text{ rad}}{1 \cancel{\text{rev}}}\right)\left(\frac{1 \cancel{\text{min}}}{60 \text{ s}}\right)\right]^2 = \boxed{1.58 \text{ m/s}^2}$$

(b) At the bottom of the circular path, we take upward as positive and apply Newton's second law. This yields $\Sigma F_y = n - mg = m(+a_c)$, or

$$n = m(g + a_c) = (40.0 \text{ kg})\left[(9.80 + 1.58) \text{ m/s}^2\right] = \boxed{455 \text{ N upward}}$$

(c) At the top of the path, we again take upward as positive and apply Newton's second law to find $\Sigma F_y = n - mg = m(-a_c)$, or

$$n = m(g - a_c) = (40.0 \text{ kg})\left[(9.80 - 1.58) \text{ m/s}^2\right] = \boxed{329 \text{ N upward}}$$

(d) At a point halfway up, the seat exerts an upward vertical component equal to the child's weight (392 N) and a component toward the center having magnitude $F_c = ma_c = (40.0 \text{ kg})(1.58 \text{ m/s}^2) = 63.2 \text{ N}$. The total force exerted by the seat is

$$F_R = \sqrt{(392 \text{ N})^2 + (63.2 \text{ N})^2} = \boxed{397 \text{ N}} \text{ directed inward and at}$$

$$\theta = \tan^{-1}\left(\frac{392 \text{ N}}{63.2 \text{ N}}\right) = \boxed{80.8° \text{ above the horizontal}}$$

7.32 (a) At A, taking upward as positive, Newton's second law gives $\Sigma F_y = n - mg = m(+a_c)$. Thus,

$$n = mg + ma_c = m\left(g + \frac{v_t^2}{r}\right) = (500 \text{ kg})\left[9.80 \text{ m/s}^2 + \frac{(20.0 \text{ m/s})^2}{10 \text{ m}}\right] = \boxed{25 \text{ kN}}$$

(b) At B, still taking upward as positive, Newton's second law yields $\Sigma F_y = n - mg = m(-a_c)$, or $mg = n + ma_c = n + mv_t^2/r$. If the car is on the verge of leaving the track, then $n = 0$ and $mg = mv_t^2/r$, giving

$$v_t = \sqrt{rg} = \sqrt{(15 \text{ m})(9.80 \text{ m/s}^2)} = \boxed{12 \text{ m/s}}$$

7.33 At the half-way point the spaceship is 1.92×10^8 m from both bodies. The force exerted on the ship by the Earth is directed toward the Earth and has magnitude

$$F_E=\frac{Gm_Em_s}{r^2}$$
$$=\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\ \text{kg}\right)\left(3.00\times10^4\ \text{kg}\right)}{\left(1.92\times10^8\ \text{m}\right)^2}=325\ \text{N}$$

The force exerted on the ship by the Moon is directed toward the Moon and has a magnitude of

$$F_M=\frac{Gm_Mm_s}{r^2}$$
$$=\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(7.36\times10^{22}\ \text{kg}\right)\left(3.00\times10^4\ \text{kg}\right)}{\left(1.92\times10^8\ \text{m}\right)^2}=4.00\ \text{N}$$

The resultant force is $\left(325\ \text{N}-4.00\ \text{N}\right)=\boxed{321\ \text{N directed toward Earth}}$.

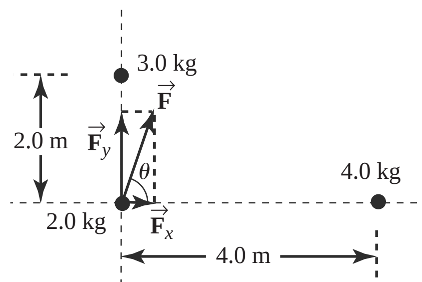
7.34 The radius of the satellite's orbit is

$$r=R_E+h=6.38\times10^6\ \text{m}+2.00\times10^6\ \text{m}=8.38\times10^6\ \text{m}$$

(a) $$PE_g=-\frac{GM_E\,m}{r}$$
$$=-\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98\times10^{24}\ \text{kg}\right)\left(100\ \text{kg}\right)}{8.38\times10^6\ \text{m}}=\boxed{-4.76\times10^9\ \text{J}}$$

(b) $$F=\frac{GM_E\,m}{r^2}=\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98\times10^{24}\ \text{kg}\right)\left(100\ \text{kg}\right)}{\left(8.38\times10^6\ \text{m}\right)^2}=\boxed{568\ \text{N}}$$

7.35 The forces exerted on the 2.0-kg by the other bodies are F_x and F_y as shown in the diagram at the right. The magnitudes of these forces are



$$F_x=\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.0\ \text{kg}\right)\left(4.0\ \text{kg}\right)}{\left(4.0\ \text{m}\right)^2}$$
$$=3.3\times10^{-11}\ \text{N}$$

and $$F_y=\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.0\ \text{kg}\right)\left(3.0\ \text{kg}\right)}{\left(2.0\ \text{m}\right)^2}=1.0\times10^{-10}\ \text{N}$$

The resultant force exerted on the 2.0-kg is $F=\sqrt{F_x^2+F_y^2}=\boxed{1.1\times10^{-10}\ \text{N}}$

directed at $\theta=\tan^{-1}\left(\dfrac{F_y}{F_x}\right)=\tan^{-1}(3.0)=\boxed{72^\circ\text{ above the }+x\text{-axis}}$

7.36 (a) The density of the white dwarf would be

$$\rho=\frac{M}{V}=\frac{M_{\text{Sun}}}{V_{\text{Earth}}}=\frac{M_{\text{Sun}}}{4\pi R_E^3/3}=\frac{3M_{\text{Sun}}}{4\pi R_E^3}$$

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Using data from Table 7.3,

$$\rho = \frac{3\left(1.991\times10^{30}\ \text{kg}\right)}{4\pi\left(6.38\times10^{6}\ \text{m}\right)^3} = \boxed{1.83\times10^{9}\ \text{kg/m}^3}$$

(b) $F_g = mg = GMm/r^2$, so the acceleration of gravity on the surface of the white dwarf would be

$$g = \frac{GM_{\text{Sun}}}{R_E^2} = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\ \text{kg}\right)}{\left(6.38\times10^{6}\ \text{m}\right)^2} = \boxed{3.26\times10^{6}\ \text{m/s}^2}$$

(c) The general expression for the gravitational potential energy of an object of mass m at distance r from the center of a spherical mass M is $PE = -GMm/r$. Thus, the potential energy of a 1.00 kg mass on the surface of the white dwarf would be

$$PE = -\frac{GM_{\text{Sun}}(1.00\ \text{kg})}{R_E}$$

$$= -\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\ \text{kg}\right)(1.00\ \text{kg})}{6.38\times10^{6}\ \text{m}} = \boxed{-2.08\times10^{13}\ \text{J}}$$

7.37 (a) At the midpoint between the two masses, the forces exerted by the 200-kg and 500-kg masses are oppositely directed, so from $F = GMm/r^2$ and $r_1 = r_2 = r$, we have

$$\Sigma F = \frac{GM_1m}{r_1^2} - \frac{GM_2m}{r_2^2} = \frac{Gm}{r^2}\left(M_1 - M_2\right)$$

or

$$\Sigma F = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)(50.0\ \text{kg})(500\ \text{kg} - 200\ \text{kg})}{(0.200\ \text{m})^2}$$

$$= \boxed{2.50\times10^{-5}\ \text{N toward the 500-kg}}$$

(b) At a point between the two masses and distance d from the 500-kg mass, the net force will be zero when

$$\frac{G(50.0\ \text{kg})(200\ \text{kg})}{(0.400\ \text{m} - d)^2} = \frac{G(50.0\ \text{kg})(500\ \text{kg})}{d^2} \qquad \text{or} \qquad d = \boxed{0.245\ \text{m}}$$

Note that the above equation yields a second solution $d = 1.09$ m. At that point, the two gravitational forces do have equal magnitudes, but are in the same direction and cannot add to zero.

7.38 The equilibrium position lies between the Earth and the Sun on the line connecting their centers. At this point, the gravitational forces exerted on the object by the Earth and Sun have equal magnitudes and opposite directions. Let this point be located distance r from the center of the Earth. Then, its distance from the Sun is $\left(1.496\times10^{11}\ \text{m} - r\right)$, and we may determine the value of r by requiring that

$$\frac{Gm_Em}{r^2} = \frac{Gm_Sm}{\left(1.496\times10^{11}\ \text{m} - r\right)^2}$$

where m_E and m_S are the masses of the Earth and Sun, respectively. This reduces to

$$\frac{\left(1.496\times10^{11}\ \text{m} - r\right)}{r} = \sqrt{\frac{m_S}{m_E}} = 577$$

or $1.496\times10^{11}\ \text{m} = 577r$, which yields $r = \boxed{2.59\times10^{8}\ \text{m from center of the Earth}}$.

7.39 (a) If air resistance is ignored, the only force acting on the projectile during its flight is the gravitational force. Since the gravitational force is a conservative force, the total energy of the projectile remains constant. At $r = R_E$, the projectile has speed $v = v_{esc}/3 = \sqrt{2GM_E/R_E}/3$, and its total energy is

$$E = KE + PE_g = \frac{1}{2}mv^2 + \left(-\frac{GM_E m}{R_E}\right) = \frac{1}{\cancel{2}}m\left(\frac{1}{9}\cdot\frac{\cancel{2}GM_E}{R_E}\right) - \frac{GM_E m}{R_E}$$

or $$E = -\frac{8}{9}\cdot\frac{GM_E m}{R_E}$$

When the projectile reaches maximum height at $r = r_{max}$, and is momentarily at rest, the kinetic energy is zero and we have

$$E = KE + PE_g = 0 - \frac{GM_E m}{r_{max}} = -\frac{8}{9}\cdot\frac{GM_E m}{R_E}$$

or $$r_{max} = \frac{9}{8}R_E = \frac{9}{8}\left(6.38\times10^6 \text{ m}\right) = \boxed{7.18\times10^6 \text{ m}}$$

(b) The altitude of the projectile when at $r = r_{max}$ is

$$h = r_{max} - R_E = \frac{9}{8}R_E - R_E = \frac{R_E}{8} = \frac{6.38\times10^6 \text{ m}}{8} = \boxed{7.98\times10^5 \text{ m}}$$

7.40 We know that $m_1 + m_2 = 5.00$ kg, or $m_2 = 5.00 \text{ kg} - m_1$.

$$F = \frac{Gm_1m_2}{r^2} \Rightarrow 1.00\times10^{-8} \text{ N} = \left(6.67\times10^{-11} \ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{m_1\left(5.00 \text{ kg} - m_1\right)}{\left(0.200 \text{ m}\right)^2}$$

$$\left(5.00 \text{ kg}\right)m_1 - m_1^2 = \frac{\left(1.00\times10^{-8} \text{ N}\right)\left(0.200 \text{ m}\right)^2}{6.67\times10^{-11} \ \text{N}\cdot\text{m}^2/\text{kg}^2} = 6.00 \text{ kg}^2$$

Thus, $m_1^2 - \left(5.00 \text{ kg}\right)m_1 + 6.00 \text{ kg}^2 = 0$, or $\left(m_1 - 3.00 \text{ kg}\right)\left(m_1 - 2.00 \text{ kg}\right) = 0$.

This yields $\boxed{m_1 = 3.00 \text{ kg, so } m_2 = 2.00 \text{ kg}}$. The answer $m_1 = 2.00$ kg and $m_2 = 3.00$ kg is physically equivalent.

7.41 (a) The radius of the satellite's orbit is

$$r = h + R_E = 2.80\times10^6 \text{ m} + 6.38\times10^6 \text{ m} = 9.18\times10^6 \text{ m}$$

Then, modifying Equation 7.23 (Kepler's third law) for orbital motion about the Earth rather than the Sun, we have

$$T^2 = \left(\frac{4\pi^2}{GM_E}\right)r^3 = \frac{4\pi^2\left(9.18\times10^6 \text{ m}\right)^3}{\left(6.67\times10^{-11} \ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24} \text{ kg}\right)}$$

yielding $T^2 = 7.66\times10^7 \text{ s}^2$ and $T = 8.75\times10^3 \text{ s}\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right) = \boxed{2.43 \text{ h}}$

continued on next page

(b) The constant tangential speed of the satellite is

$$v_t = \frac{\text{circumference of orbit}}{\text{period}} = \frac{2\pi r}{T} = \frac{2\pi\left(9.18\times10^6\text{ m}\right)}{8.75\times10^3\text{ s}}$$

or $v_t = 6.59\times10^3\text{ m/s} = \boxed{6.59\text{ km/s}}$

(c) The satellite's only acceleration is centripetal acceleration, so

$$a = a_c = \frac{v_t^2}{r} = \frac{\left(6.59\times10^3\text{ km/s}\right)^2}{9.18\times10^6\text{ m}} = \boxed{4.73\text{ m/s}^2\text{ toward center of Earth}}$$

7.42 (a) The satellite's period is $T = 110\text{ min}\left(60.0\text{ s}/1.00\text{ min}\right) = 6.60\times10^3\text{ s}$.

Using the form of Kepler's third law (Equation 7.23) suitable for objects orbiting the Earth, we have

$$T^2 = \left(\frac{4\pi^2}{GM_E}\right)r^3 \qquad \text{or} \qquad r = \left(\frac{T^2 GM_E}{4\pi^2}\right)^{\frac{1}{3}}$$

Thus, the radius of the orbit must be

$$r = \left(\frac{\left(6.60\times10^3\text{ s}\right)^2\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)}{4\pi^2}\right)^{\frac{1}{3}} = 7.61\times10^6\text{ m}$$

and the altitude of the satellite is

$$h = r - R_E = 7.61\times10^6\text{ m} - 6.38\times10^6\text{ m} = \boxed{1.23\times10^6\text{ m}}$$

(b) The gravitational force is

$$F_g = \left(\frac{GM_E}{r^2}\right)m = mg \qquad \text{so} \qquad g = \frac{GM_E}{r^2}$$

Thus, at the altitude of the satellite, the acceleration due to gravity is

$$g = \frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(5.98\times10^{24}\text{ kg}\right)}{\left(7.61\times10^6\text{ m}\right)^2} = \boxed{6.89\text{ m/s}^2}$$

7.43 From Kepler's third law (Equation 7.23), written in the form suitable for bodies orbiting Mars, we have $T^2 = \left(4\pi^2/GM_{\text{Mars}}\right)r^3$, so the mass of Mars, computed from the given data, must be

$$M_{\text{Mars}} = \left(\frac{4\pi^2}{GT^2}\right)r^3 = \left(\frac{4\pi^2}{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(2.8\times10^4\text{ s}\right)^2}\right)\left(9.4\times10^6\text{ m}\right)^3 = \boxed{6.3\times10^{23}\text{ kg}}$$

7.44 (a) The satellite moves in an orbit of radius $r = 2\,R_E$ and the gravitational force supplies the required centripetal acceleration. Hence, $m\left(v_t^2/2\,R_E\right) = Gm_E m/\left(2\,R_E\right)^2$, or

$$v_t = \sqrt{\frac{Gm_E}{2\,R_E}} = \sqrt{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98\times10^{24}\text{ kg}\right)}{2\left(6.38\times10^6\text{ m}\right)}} = \boxed{5.59\times10^3\text{ m/s}}$$

continued on next page

(b) The period of the satellite's motion is

$$T = \frac{2\pi r}{v_t} = \frac{2\pi\left[2\left(6.38\times10^{6}\text{ m}\right)\right]}{5.59\times10^{3}\text{ m/s}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = \boxed{3.98\text{ h}}$$

(c) The gravitational force acting on the satellite is $F = Gm_E m/r^2$ or

$$F = \left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{\left(5.98\times10^{24}\text{ kg}\right)(600\text{ kg})}{\left[2\left(6.38\times10^{6}\text{ m}\right)\right]^2} = \boxed{1.47\times10^{3}\text{ N}}$$

7.45 The radii of the orbits of the two satellites are

$$r_A = h_A + R_E = R_E + R_E = 2R_E \qquad \text{and} \qquad r_B = h_B + R_E = 2R_E + R_E = 3R_E$$

From Kepler's third law, the ratio of the squares of the periods of the two satellites is

$$\frac{T_B^2}{T_A^2} = \left(\frac{4\pi^2 r_B^3}{GM_E}\right)\cdot\left(\frac{GM_E}{4\pi^2 r_A^3}\right) = \frac{r_B^3}{r_A^3} = \left(\frac{3R_E}{2R_E}\right)^3 = \frac{27}{8}$$

Thus, the ratio of their periods is

$$\frac{T_B}{T_A} = \sqrt{\frac{T_B^2}{T_A^2}} = \sqrt{\frac{27}{8}} = \boxed{1.84}$$

7.46 A synchronous satellite will have an orbital period equal to Jupiter's rotation period, so the satellite can have the red spot in sight at all times. Thus, the desired orbital period is

$$T = 9.84\text{ h}\left(\frac{3\,600\text{ s}}{1\text{ h}}\right) = 3.54\times10^{4}\text{ s}$$

Kepler's third law gives the period of a satellite in orbit around Jupiter as $T^2 = \left(4\pi^2/GM_{\text{Jupiter}}\right)r^3$. The required radius of the circular orbit is therefore

$$r = \left(\frac{GM_{\text{Jupiter}}T^2}{4\pi^2}\right)^{1/3} = \left[\frac{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.90\times10^{27}\text{ kg}\right)\left(3.54\times10^{4}\text{ s}\right)^2}{4\pi^2}\right]^{1/3}$$

or $\qquad r = 1.59\times10^{8}\text{ m}$

The altitude of the satellite above Jupiter's surface should be

$$h = r - R_{\text{Jupiter}} = 1.59\times10^{8}\text{ m} - 6.99\times10^{7}\text{ m} = \boxed{8.91\times10^{7}\text{ m}}$$

7.47 From Kepler's third law, the mass of Jupiter can be expressed in terms of one of its satellite's orbital radius and period as $M_{\text{Jupiter}} = \left(4\pi^2/GT^2\right)r^3$.

(a) For Io,

$$r = 4.22\times10^{8}\text{ m} \qquad \text{and} \qquad T = 1.77\text{ days}\left[(8.64\times10^{4}\text{ s})/1\text{ day}\right] = 1.53\times10^{5}\text{ s},$$

giving $$M_{\text{Jupiter}} = \frac{4\pi^2\left(4.22\times10^{8}\text{ m}\right)^3}{\left(6.67\times10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.53\times10^{5}\text{ s}\right)^2} = \boxed{1.90\times10^{27}\text{ kg}}$$

continued on next page

(b) For Ganymede,

$$r = 1.07\times10^{9}\ \text{m} \qquad \text{and} \qquad T = 7.16\ \text{days}\left[(8.64\times10^{4}\ \text{s})/1\ \text{day}\right] = 6.19\times10^{5}\ \text{s},$$

giving $$M_{\text{Jupiter}} = \frac{4\pi^2\left(1.07\times10^{9}\ \text{m}\right)^3}{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(6.19\times10^{5}\ \text{s}\right)^2} = \boxed{1.89\times10^{27}\ \text{kg}}$$

(c) $\boxed{\text{Yes.}}$ The results of parts (a) and (b) are consistent. They predict the same mass within the limits of uncertainty of the data used to compute these results.

7.48 The gravitational force on a small parcel of material at the star's equator supplies the centripetal acceleration, or $GM_s m/R_s^2 = m\left(v_t^2/R_s\right) = m\left(R_s\,\omega^2\right)$. Hence, $\omega = \sqrt{GM_s/R_s^3}$.

$$\omega = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left[2\left(1.99\times10^{30}\ \text{kg}\right)\right]}{\left(10.0\times10^{3}\ \text{m}\right)^3}} = \boxed{1.63\times10^{4}\ \text{rad/s}}$$

7.49 (a) $$\omega = \frac{v_t}{r} = \frac{(98.0\ \text{mi/h})}{0.742\ \text{m}}\left(\frac{0.447\ \text{m/s}}{1\ \text{mi/h}}\right)\left(\frac{1\ \text{rev}}{2\pi\ \text{rad}}\right) = \boxed{9.40\ \text{rev/s}}$$

(b) $$\alpha = \frac{\omega^2-\omega_i^2}{2\,\Delta\theta} = \frac{\left(9.40\ \text{rev/s}\right)^2-0}{2\left(1\ \text{rev}\right)} = \boxed{44.2\ \text{rev/s}^2}$$

$$a_r = \frac{v_t^2}{r} = \frac{\left[\left(98.0\ \text{mi/h}\right)\left(\dfrac{0.447\ \text{m/s}}{1\ \text{mi/h}}\right)\right]^2}{0.742\ \text{m}} = \boxed{2.59\times10^{3}\ \text{m/s}^2}$$

$$a_t = r\alpha = \left(0.742\ \text{m}\right)\left[44.2\ \frac{\text{rev}}{\text{s}^2}\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\right] = \boxed{206\ \text{m/s}^2}$$

(c) In the radial direction at the release point, the hand supports the weight of the ball and also supplies the centripetal acceleration. Thus, $F_r = mg + ma_r = m\left(g + a_r\right)$ or

$$F_r = \left(0.198\ \text{kg}\right)\left(9.80\ \text{m/s}^2 + 2.59\times10^{3}\ \text{m/s}^2\right) = \boxed{515\ \text{N}}$$

In the tangential direction, the hand supplies only the tangential acceleration, so

$$F_t = ma_t = \left(0.198\ \text{kg}\right)\left(206\ \text{m/s}^2\right) = \boxed{40.8\ \text{N}}$$

7.50 (a) $$\omega_i = \frac{v_t}{r_i} = \frac{1.30\ \text{m/s}}{2.30\times10^{-2}\ \text{m}} = \boxed{56.5\ \text{rad/s}}$$

(b) $$\omega_f = \frac{v_t}{r_f} = \frac{1.30\ \text{m/s}}{5.80\times10^{-2}\ \text{m}} = \boxed{22.4\ \text{rad/s}}$$

(c) The duration of the recording is

$$\Delta t = \left(74\ \text{min}\right)\left(60\ \text{s/min}\right) + 33\ \text{s} = 4\ 473\ \text{s}$$

Thus, $$\alpha_{\text{av}} = \frac{\omega_f-\omega_i}{\Delta t} = \frac{\left(22.4-56.5\right)\ \text{rad/s}}{4\ 473\ \text{s}} = \boxed{-7.62\times10^{-3}\ \text{rad/s}^2}$$

continued on next page

(d) $\Delta\theta = \dfrac{\omega_f^2 - \omega_i^2}{2\alpha} = \dfrac{(22.4\ \text{rad/s})^2 - (56.5\ \text{rad/s})^2}{2(-7.62\times10^{-3}\ \text{rad/s}^2)} = \boxed{1.77\times10^5\ \text{rad}}$

(e) The track moves past the lens at a constant speed of $v_t = 1.30\ \text{m/s}$ for 4 473 seconds. Therefore, the length of the spiral track is

$$\Delta s = v_t(\Delta t) = (1.30\ \text{m/s})(4\ 473\ \text{s}) = 5.81\times10^3\ \text{m} = \boxed{5.81\ \text{km}}$$

7.51 The angular velocity of the ball is $\omega = 0.500\ \text{rev/s} = \pi\ \text{rad/s}$.

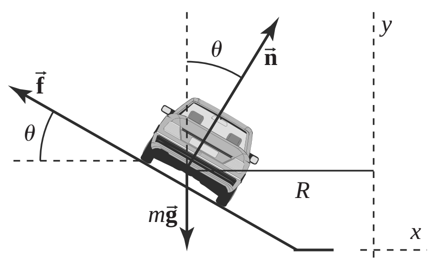
(a) $v_t = r\omega = (0.800\ \text{m})(\pi\ \text{rad/s}) = \boxed{2.51\ \text{m/s}}$

(b) $a_c = \dfrac{v_t^2}{r} = r\omega^2 = (0.800\ \text{m})(\pi\ \text{rad/s})^2 = \boxed{7.90\ \text{m/s}^2}$

(c) We imagine that the weight of the ball is supported by a frictionless platform. Then, the rope tension need only produce the centripetal acceleration. The force required to produce the needed centripetal acceleration is $F = m(v_t^2/r)$. Thus, if the maximum force the rope can exert is 100 N, the maximum tangential speed of the ball is

$$(v_t)_{\max} = \sqrt{\frac{rF_{\max}}{m}} = \sqrt{\frac{(0.800\ \text{m})(100\ \text{N})}{5.00\ \text{kg}}} = \boxed{4.00\ \text{m/s}}$$

7.52 (a) When the car is about to slip down the incline, the friction force, $\vec{\mathbf{f}}$, is directed up the incline as shown and has the magnitude $f = \mu n$. Thus,



$$\Sigma F_y = n\cos\theta + \mu n\sin\theta - mg = 0$$

or $$n = \frac{mg}{\cos\theta + \mu\sin\theta} \quad \textbf{[1]}$$

Also, $\Sigma F_x = n\sin\theta - \mu n\cos\theta = m\left(\dfrac{v_{\min}^2}{R}\right)$

or $$v_{\min} = \sqrt{\frac{nR}{m}(\sin\theta - \mu\cos\theta)} \quad \textbf{[2]}$$

Substituting Equation [1] into [2] gives

$$v_{\min} = \sqrt{Rg\left(\frac{\sin\theta - \mu\cos\theta}{\cos\theta + \mu\sin\theta}\right)} = \boxed{\sqrt{Rg\left(\frac{\tan\theta - \mu}{1 + \mu\tan\theta}\right)}}$$

If the car is about to slip up the incline, $f = \mu n$ is directed down the slope (opposite to what is shown in the sketch). Then,

$$\Sigma F_y = n\cos\theta - \mu n\sin\theta - mg = 0,\ \text{or}\ n = \frac{mg}{\cos\theta - \mu\sin\theta} \quad \textbf{[3]}$$

Also, $\Sigma F_x = n\sin\theta + \mu n\cos\theta = m\left(\dfrac{v_{\max}^2}{R}\right)$, or

$$v_{\max} = \sqrt{\frac{nR}{m}(\sin\theta + \mu\cos\theta)} \quad \textbf{[4]}$$

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Combining Equations [3] and [4] gives

$$v_{\max} = \sqrt{Rg\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)} = \boxed{\sqrt{Rg\left(\frac{\tan\theta + \mu}{1 - \mu\tan\theta}\right)}}$$

(b) If $R = 100$ m, $\theta = 10°$, and $\mu = 0.10$, the lower and upper limits of safe speeds are

$$v_{\min} = \sqrt{(100\text{ m})(9.8\text{ m/s}^2)\left(\frac{\tan 10° - 0.10}{1 + 0.10\tan 10°}\right)} = \boxed{8.6\text{ m/s}}$$

and $$v_{\max} = \sqrt{(100\text{ m})(9.8\text{ m/s}^2)\left(\frac{\tan 10° + 0.10}{1 - 0.10\tan 10°}\right)} = \boxed{17\text{ m/s}}$$

7.53 The radius of the satellite's orbit is

$$r = R_E + h = 6.38\times10^6\text{ m} + (1.50\times10^2\text{ mi})(1\,609\text{ m/1 mi}) = 6.62\times10^6\text{ m}$$

(a) The required centripetal acceleration is produced by the gravitational force, so $m(v_t^2/r) = GM_E m/r^2$, which gives $v_t = \sqrt{GM_E/r}$.

$$v_t = \sqrt{\left(6.67\times10^{-11}\ \frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\frac{(5.98\times10^{24}\text{ kg})}{6.62\times10^6\text{ m}}} = \boxed{7.76\times10^3\text{ m/s}}$$

(b) The time for one complete revolution is

$$T = \frac{2\pi r}{v_t} = \frac{2\pi(6.62\times10^6\text{ m})}{7.76\times10^3\text{ m/s}} = 5.36\times10^3\text{ s} = \boxed{89.3\text{ min}}$$

7.54 (a) At the lowest point on the path, the net upward force (i.e., the force directed toward the center of the path and supplying the centripetal acceleration) is $\Sigma F_{\text{up}} = T - mg = m(v_t^2/r)$, so the tension in the cable is

$$T = m\left(g + \frac{v_t^2}{r}\right) = (0.400\text{ kg})\left(9.80\text{ m/s}^2 + \frac{(3.00\text{ m/s})^2}{0.800\text{ m}}\right) = \boxed{8.42\text{ N}}$$

(b) Using conservation of mechanical energy, $(KE + PE_g)_f = (KE + PE_g)_i$, as the bob goes from the lowest to the highest point on the path, gives

$$0 + mg\left[L(1 - \cos\theta_{\max})\right] = \frac{1}{2}mv_i^2 + 0, \text{ or } \cos\theta_{\max} = 1 - \frac{v_i^2}{2gL}$$

$$\theta_{\max} = \cos^{-1}\left(1 - \frac{v_i^2}{2gL}\right) = \cos^{-1}\left(1 - \frac{(3.00\text{ m/s})^2}{2(9.80\text{ m/s}^2)(0.800\text{ m})}\right) = \boxed{64.8°}$$

(c) At the highest point on the path, the bob is at rest and the net radial force is

$$\Sigma F_r = T - mg\cos\theta_{\max} = m\left(\frac{v_t^2}{r}\right) = 0$$

Therefore,

$$T = mg\cos\theta_{\max} = (0.400\text{ kg})(9.80\text{ m/s}^2)\cos(64.8°) = \boxed{1.67\text{ N}}$$

7.55 (a) When the car is at the top of the arc, the normal force is upward and the weight downward. The net force directed downward, toward the center of the circular path and hence supplying the centripetal acceleration, is $\Sigma F_{\text{down}} = mg - n = m\left(v_t^2/r\right)$. Thus, the normal force is $\boxed{n = m\left(g - v_t^2/r\right)}$.

(b) If $r = 30.0\text{ m}$ and $n \to 0$, then $g - v_t^2/r \to 0$. For this to be true, the speed of the car must be

$$v_t = \sqrt{rg} = \sqrt{(30.0\text{ m})\left(9.80\text{ m/s}^2\right)} = \boxed{17.1\text{ m/s}}$$

7.56 The escape speed from the surface of a planet of radius R and mass M is given by $v_e = \sqrt{2GM/R}$. If the planet has uniform density, ρ, the mass is given by

$$M = \rho(\textit{volume}) = \rho\left(4\pi R^3/3\right) = 4\pi\rho R^3/3$$

The expression for the escape speed then becomes

$$v_e = \sqrt{\frac{2G}{R}\left(\frac{4\pi\rho R^3}{3}\right)} = \left(\sqrt{\frac{8\pi\rho G}{3}}\right)R = \boxed{(\textit{constant})R}$$

or the escape speed is directly proportional to the radius of the planet.

7.57 The speed the person has due to the rotation of the Earth is $v_t = r\omega$, where r is the distance from the rotation axis and ω is the angular velocity of rotation.

The person's apparent weight, $F_{g,\text{ apparent}}$, equals the magnitude of the upward normal force exerted on him by the scales. The true weight, $F_{g,\text{ true}} = mg$, is directed downward. The net downward force produces the needed centripetal acceleration, or

$$\Sigma F_{\text{down}} = -n + F_{g,\text{true}} = -F_{g,\text{apparent}} + F_{g,\text{true}} = m\left(\frac{v_t^2}{r}\right) = mr\omega^2$$

(a) At the equator, $r = R_E$, so $\boxed{F_{g,\text{true}} = F_{g,\text{apparent}} + mR_E\omega^2} > F_{g,\text{apparent}}$

(b) At the equator, it is given that $a_c = R_E\omega^2 = 0.0340\text{ m/s}^2$, so the apparent weight is

$$F_{g,\text{apparent}} = F_{g,\text{true}} - mR_E\omega^2 = (75.0\text{ kg})\left[(9.80 - 0.0340)\text{ m/s}^2\right] = \boxed{732\text{ N}}$$

At either pole, $r = 0$ (the person is on the rotation axis) and

$$F_{g,\text{apparent}} = F_{g,\text{true}} = mg = (75.0\text{ kg})\left(9.80\text{ m/s}^2\right) = \boxed{735\text{ N}}$$

7.58 Choosing $y = 0$ and $PE_g = 0$ at the level of point B, applying the work-energy theorem $W_{nc} = (KE + PE_g)_f - (KE + PE_g)_i$ to the block's motion gives

$$W_{nc} = \frac{1}{2}mv^2 + mgy - \frac{1}{2}mv_0^2 - mg(2R), \qquad \text{or} \qquad v^2 = v_0^2 + \frac{2W_{nc}}{m} + 2g(2R - y) \qquad \textbf{[1]}$$

(a) At point A, $y = R$ and $W_{nc} = 0$ (no non-conservative force has done work on the block yet). Thus, $v_A^2 = v_0^2 + 2gR$. The normal force exerted on the block by the track must supply the centripetal acceleration at point A, so

$$n_A = m\left(\frac{v_A^2}{R}\right) = m\left(\frac{v_0^2 + 2gR}{R}\right) = m\left(\frac{v_0^2}{R} + 2g\right)$$

continued on next page

or $$n_A = (0.50\text{ kg})\left(\frac{(4.0\text{ m/s})^2}{1.5\text{ m}} + 2(9.8\text{ m/s}^2)\right) = \boxed{15\text{ N}}$$

At point B, $y = 0$ and W_{nc} is still zero. Thus, $v_B^2 = v_0^2 + 4\,gR$. Here, the normal force must supply the centripetal acceleration *and* support the weight of the block. Therefore,

$$n_B = m\left(\frac{v_B^2}{R}\right) + mg = m\left(\frac{v_0^2 + 4\,gR}{R}\right) + mg = m\left(\frac{v_0^2}{R} + 5g\right)$$

or $$n_B = (0.50\text{ kg})\left(\frac{(4.0\text{ m/s})^2}{1.5\text{ m}} + 5(9.8\text{ m/s}^2)\right) = \boxed{30\text{ N}}$$

(b) When the block reaches point C, $y = 2R$ and $W_{nc} = -f_k L = -\mu_k (mg) L$. At this point, the normal force is to be zero, so the weight alone must supply the centripetal acceleration. Thus, $m\left(v_C^2/R\right) = mg$, or the required speed at point C is $v_C^2 = Rg$. Substituting this into Equation [1] yields $Rg = v_0^2 - 2\,\mu_k gL + 0$, or

$$\mu_k = \frac{v_0^2 - Rg}{2\,gL} = \frac{(4.0\text{ m/s})^2 - (1.5\text{ m})(9.8\text{ m/s}^2)}{2(9.8\text{ m/s}^2)(0.40\text{ m})} = \boxed{0.17}$$

7.59 Define the following symbols: $M_\text{M} =$ mass of the Moon, $M_\text{E} =$ mass of the Earth, $R_\text{M} =$ radius of the Moon, R_E = radius of the Earth, and r = radius of the Moon's orbit around the Earth.

We interpret "lunar escape speed" to be the escape speed from the surface of a stationary Moon alone in the universe. Then,

$$v_\text{launch} = 2\,v_\text{escape} = 2\sqrt{\frac{2\,GM_\text{M}}{R_\text{M}}}, \text{ or } v_\text{launch}^2 = \frac{8\,GM_\text{M}}{R_\text{M}}$$

Applying conservation of mechanical energy from launch to impact gives

$$\tfrac{1}{2}m\,v_\text{impact}^2 + (PE_g)_f = \tfrac{1}{2}m\,v_\text{launch}^2 + (PE_g)_i, \text{ or } v_\text{impact} = \sqrt{v_\text{launch}^2 + \frac{2}{m}\left[(PE_g)_i - (PE_g)_f\right]}$$

The needed potential energies are

$$(PE_g)_i = -\frac{GM_\text{M}\,m}{R_\text{M}} - \frac{GM_\text{E}\,m}{r} \text{ and } (PE_g)_f = -\frac{GM_\text{E}\,m}{R_\text{E}} - \frac{GM_\text{M}\,m}{r}$$

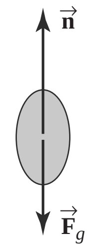
Using these potential energies and the expression for v_launch^2 from above, the equation for the impact speed reduces to

$$v_\text{impact} = \sqrt{2\,G\left(\frac{3\,M_\text{M}}{R_\text{M}} + \frac{M_\text{E}}{R_\text{E}} - \frac{(M_\text{E} - M_\text{M})}{r}\right)}$$

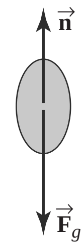
With numeric values of $G = 6.67\times10^{-11}\ \dfrac{\text{N}\cdot\text{m}^2}{\text{kg}^2}$, $M_\text{E} = 5.98\times10^{24}$ kg, $M_\text{M} = 7.36\times10^{22}$ kg, $R_\text{M} = 1.74\times10^{6}$ m, $R_\text{E} = 6.38\times10^{6}$ m, and $r = 3.84\times10^{8}$ m, we find

$$v_\text{impact} = 1.18\times10^{4}\text{ m/s} = \boxed{11.8\text{ km/s}}$$

7.60 (a) When the passenger is at the top, the radial forces producing the centripetal acceleration are the upward force of the seat and the downward force of gravity. The downward force must exceed the upward force to yield a net force toward the center of the circular path.



(b) At the lowest point on the path, the radial forces contributing to the centripetal acceleration are again the upward force of the seat and the downward force of gravity. However, the upward force must now exceed the downward force to yield a net force directed toward the center of the circular path.



(c) The seat must exert the greatest force on the passenger $\boxed{\text{at the lowest point}}$ on the circular path.

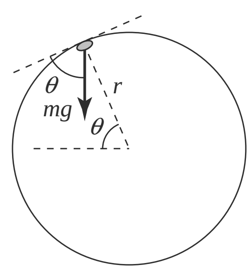
(d) At the top of the loop, $\Sigma F_r = F_g - n = m\,v^2/r$, or

$$n = F_g - m\frac{v^2}{r} = m\left(g - \frac{v^2}{r}\right) = (70.0\text{ kg})\left(9.80\ \text{m/s}^2 - \frac{(4.00\ \text{m/s})^2}{8.00\text{ m}}\right) = \boxed{546\text{ N}}$$

At the bottom of the loop, $\Sigma F_r = n - F_g = mv^2/r$, or

$$n = F_g + m\frac{v^2}{r} = m\left(g + \frac{v^2}{r}\right) = (70.0\text{ kg})\left(9.80\ \text{m/s}^2 + \frac{(4.00\ \text{m/s})^2}{8.00\text{ m}}\right) = \boxed{826\text{ N}}$$

7.61 The item of clothing will fall away from the rotating drum when the component of its weight directed toward the center of the drum exceeds the needed centripetal force. That is, when



$$mg\sin\theta \geq m\left(v_t^2/r\right) = mr\omega^2$$

Thus, if the clothing is to lose contact with the drum

at $\theta = 68.0°$, we must have $mr\omega^2 = mg\sin 68.0°$.

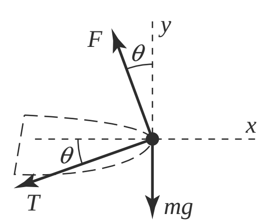
The required angular velocity, expressed in revolutions per second, is therefore

$$\omega = \sqrt{\frac{g\sin 68.0°}{r}}\ \text{rad/s} = \left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\sqrt{\frac{(9.80\ \text{m/s}^2)\sin 68.0°}{0.330\text{ m}}} = \boxed{0.835\ \text{rev/s}}$$

7.62 Since the airplane flies in a horizontal circle, its vertical acceleration is zero, and

$$\Sigma F_y = ma_y \quad\Rightarrow\quad F\cos\theta - T\sin\theta - mg = 0$$

or $$F = \frac{mg}{\cos\theta} + T\tan\theta \qquad \textbf{[1]}$$



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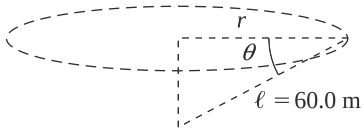
Also, the components of the lift force and the tension in the wire directed toward the center of the circular path must supply the required centripetal acceleration. Hence,

$$\Sigma F_{\text{radial}} = ma_c = m\left(\frac{v_t^2}{r}\right) \qquad \text{or} \qquad F\sin\theta + T\cos\theta = \frac{mv_t^2}{r} \qquad \textbf{[2]}$$

Substituting Equation [1] into [2] yields

$$\left(mg\tan\theta + T\sin\theta\tan\theta\right) + T\cos\theta = \frac{mv_t^2}{r}$$



or the tension in the wire is

$$T = \frac{m\left(v_t^2/r\right) - mg\tan\theta}{\sin\theta\tan\theta + \cos\theta} \qquad \text{where} \qquad r = \ell\cos\theta$$

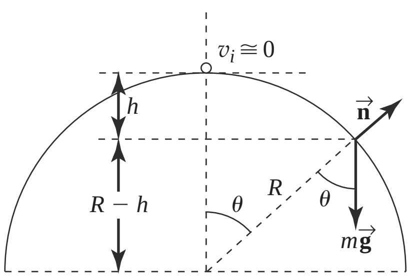
If $\theta = 20.0°$, $m = 0.750$ kg, $v_t = 35.0$ m/s, and $\ell = 60.0$ m, this gives

$$T = (0.750\text{ kg})\left[\frac{(35.0\text{ m/s})^2/\left[(60.0\text{ m})\cos 20.0°\right] - \left(9.80\text{ m/s}^2\right)\tan 20.0°}{\sin 20.0°\tan 20.0° + \cos 20.0°}\right]$$

or $\boxed{T = 12.8\text{ N}}$

7.63 Choosing $PE_g = 0$ at the top of the hill, the speed of the skier after dropping distance *h* is found using conservation of mechanical energy as

$$\tfrac{1}{2}mv_t^2 - mgh = 0 + 0 \text{ , or } v_t^2 = 2gh$$



The net force directed toward the center of the circular path, and providing the centripetal acceleration, is

$$\Sigma F_r = mg\cos\theta - n = m\left(v_t^2/R\right)$$

Solving for the normal force, after making the substitutions $v_t^2 = 2gh$ and $\cos\theta = (R-h)/R = 1-(h/R)$, gives

$$n = mg(1 - h/R) - m(2gh/R) = mg(1 - 3h/R)$$

The skier leaves the hill when $n \to 0$. This occurs when

$$1 - \frac{3h}{R} = 0 \text{ or } \boxed{h = R/3}$$

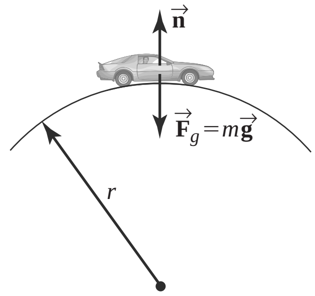
7.64 The centripetal acceleration of a particle at distance *r* from the axis is $a_c = v_t^2/r = r\omega^2$. If we are to have $a_c = 100g$, then it is necessary that

$$r\omega^2 = 100g \qquad \text{or} \qquad \omega = \sqrt{100g/r}$$

The required rotation rate increases as *r* decreases. In order to maintain the required acceleration for all particles in the casting, we use the minimum value of *r* and find

$$\omega = \sqrt{\frac{100g}{r_{\min}}} = \sqrt{\frac{100\left(9.80\text{ m/s}^2\right)}{2.10\times10^{-2}\text{ m}}} = 216\ \frac{\text{rad}}{\text{s}}\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right)\left(\frac{60.0\text{ s}}{1\text{ min}}\right) = \boxed{2.06\times10^3\ \frac{\text{rev}}{\text{min}}}$$

7.65 The sketch at the right shows the car as it passes the highest point on the bump. Taking upward as positive, we have



$$\Sigma F_y = ma_y \Rightarrow n - mg = m\left(-v^2/r\right)$$

or $$n = m\left(g - v^2/r\right)$$

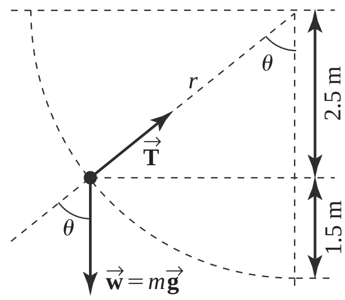
(a) If $v = 8.94$ m/s, the normal force exerted by the road is

$$n = (1\,800 \text{ kg})\left[9.80 \ \frac{\text{m}}{\text{s}^2} - \frac{(8.94 \text{ m/s})^2}{20.4 \text{ m}}\right] = 1.06 \times 10^4 \text{ N} = \boxed{10.6 \text{ kN}}$$

(b) When the car is on the verge of losing contact with the road, $n = 0$. This gives $g = v^2/r$ and the speed must be

$$v = \sqrt{rg} = \sqrt{(20.4 \text{ m})(9.80 \text{ m/s}^2)} = \boxed{14.1 \text{ m/s}}$$

7.66 When the rope makes angle θ with the vertical, the net force directed toward the center of the circular path is $\Sigma F_r = T - mg\cos\theta$ as shown in the sketch. This force supplies the needed centripetal acceleration, so



$$T - mg\cos\theta = m\left(\frac{v_t^2}{r}\right), \text{ or } T = m\left(g\cos\theta + \frac{v_t^2}{r}\right)$$

Using conservation of mechanical energy, $(KE + PE_g)_f = (KE + PE_g)_i$, with $KE = 0$ at $\theta = 90°$ and $PE_g = 0$ at the bottom of the arc, the speed when the rope is at angle θ from the vertical is given by $\frac{1}{2}mv_t^2 + mg(r - r\cos\theta) = 0 + mgr$, or $v_t^2 = 2gr\cos\theta$. The expression for the tension in the rope at angle θ then reduces to $T = 3mg\cos\theta$.

(a) At the beginning of the motion, $\theta = 90°$ and $T = \boxed{0}$.

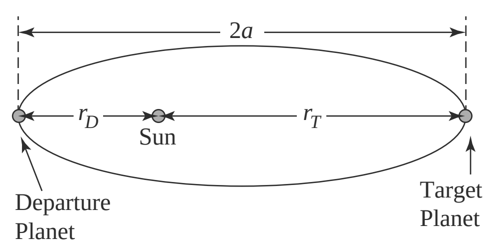
(b) At 1.5 m from the bottom of the arc, $\cos\theta = 2.5 \text{ m}/r = 2.5 \text{ m}/4.0 \text{ m} = 0.63$ and the tension is

$$T = 3(70 \text{ kg})(9.8 \text{ m/s}^2)(0.63) = 1.3 \times 10^3 \text{ N} = \boxed{1.3 \text{ kN}}$$

(c) At the bottom of the arc, $\theta = 0°$ and $\cos\theta = 1.0$, so the tension is

$$T = 3(70 \text{ kg})(9.8 \text{ m/s}^2)(1.0) = 2.1 \times 10^3 \text{ N} = \boxed{2.1 \text{ kN}}$$

7.67 (a) The desired path is an elliptical trajectory with the Sun at one of the foci, the departure planet at the perihelion, and the target planet at the aphelion. The perihelion distance r_D is the radius of the departure planet's orbit, while the aphelion distance r_T is the radius of the target planet's orbit. The semimajor axis of the desired trajectory is then $a = (r_D + r_T)/2$.



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If Earth is the departure planet, $r_D = 1.496 \times 10^{11}\ \text{m} = 1.00\ \text{AU}$.

With Mars as the target planet, $r_T = 2.28 \times 10^{11}\ \text{m}\left(\dfrac{1\ \text{AU}}{1.496 \times 10^{11}\ \text{m}}\right) = 1.52\ \text{AU}$

Thus, the semimajor axis of the minimum energy trajectory is

$$a = \frac{r_D + r_T}{2} = \frac{1.00\ \text{AU} + 1.52\ \text{AU}}{2} = 1.26\ \text{AU}$$

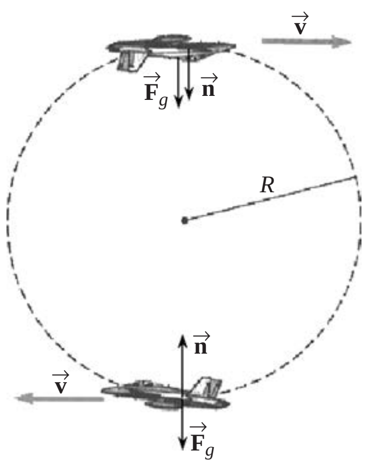
Kepler's third law, $T^2 = a^3$, then gives the time for a full trip around this path as

$$T = \sqrt{a^3} = \sqrt{(1.26\ \text{AU})^3} = 1.41\ \text{y}$$

so the time for a one-way trip from Earth to Mars is $\Delta t = T/2 = 1.41\ \text{y}/2 = \boxed{0.71\ \text{y}}$.

(b) $\boxed{\text{This trip cannot be taken at just any time.}}$ The departure must be timed so that the spacecraft arrives at the aphelion when the target planet is located there.

7.68 (a) Consider the sketch at the right. At the bottom of the loop, the net force toward the center (i.e., the centripetal force) is



$$F_c = \frac{mv^2}{R} = n - F_g$$

so the pilot's apparent weight (normal force) is

$$n = F_g + \frac{mv^2}{R} = F_g + \frac{(F_g/g)v^2}{R} = F_g\left(1 + \frac{v^2}{gR}\right)$$

or $$n = (712\ \text{N})\left(1 + \frac{(2.00 \times 10^2\ \text{m/s})^2}{(9.80\ \text{m/s}^2)(3.20 \times 10^3\ \text{m/s}^2)}\right)$$

$$= \boxed{1.62 \times 10^{13}\ \text{N}}$$

(b) At the top of the loop, the centripetal force is $F_c = mv^2/R = n + F_g$, so the apparent weight is

$$n = \frac{mv^2}{R} - F_g = \frac{(F_g/g)v^2}{R} - F_g = F_g\left(\frac{v^2}{gR} - 1\right)$$

$$= (712\ \text{N})\left(\frac{(2.00 \times 10^2\ \text{m/s})^2}{(9.80\ \text{m/s}^2)(3.20 \times 10^3\ \text{m/s}^2)} - 1\right) = \boxed{196\ \text{N}}$$

(c) With the right speed, the needed centripetal force at the top of the loop can be made exactly equal to the gravitational force. At this speed, the normal force exerted on the pilot by the seat (his apparent weight) will be zero, and the pilot will have the sensation of weightlessness.

(d) When $n = 0$ at the top of the loop, $F_c = mv^2/R = mg = F_g$, and the speed will be

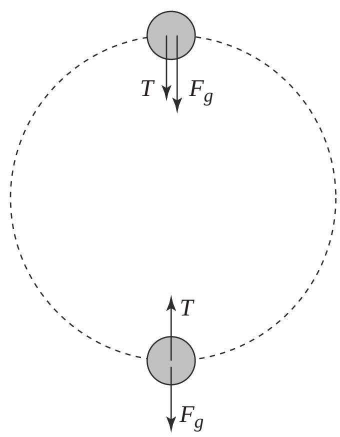
$$v = \sqrt{\frac{mg}{m/R}} = \sqrt{Rg} = \sqrt{(3.20 \times 10^3\ \text{m})(9.80\ \text{m/s}^2)} = \boxed{177\ \text{m/s}}$$

7.69 (a) At the instant the mud leaves the tire and becomes a projectile, its velocity components are $v_{0x} = 0$, $v_{0y} = v_t = R\omega$. From $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ with $a_y = -g$, the time required for the mud to return to its starting point (with $\Delta y = 0$) is given by $0 = t(R\omega - gt/2)$, for which the non-zero solution is $\boxed{t = 2R\omega/g}$.

(b) The angular displacement of the wheel (turning at constant angular speed ω) in time t is $\Delta\theta = \omega t$. If the displacement is $\Delta\theta = 1$ rev $= 2\pi$ rad at $t = 2R\omega/g$, then 2π rad $= \omega(2R\omega/g)$, or $\omega^2 = \pi g/R$ and $\boxed{\omega = \sqrt{\pi g/R}}$.

7.70 (a) At each point on the vertical circular path, two forces are acting on the ball. These are:
(1) $\boxed{\text{the downward gravitational force}}$ with constant magnitude $F_g = mg$, and
(2) $\boxed{\text{the tension force in the string, always directed toward the center of the path}}$.

(b) The sketch at the right shows the forces acting on the ball when it is at the bottom of the circular path, and when it is at the highest point on the path. Note that the gravitational force has the same magnitude and direction at each point on the circular path. The tension force varies in magnitude at different points and is always directed toward the center of the path.



(c) At the top of the circle, $F_c = T + F_g = mv^2/r$, or

$$T = mv^2/r - F_g = mv^2/r - mg = m(v^2/r - g)$$

$$= (0.275 \text{ kg})\left[\frac{(5.20 \text{ m/s})^2}{0.850 \text{ m}} - 9.80 \text{ m/s}^2\right] = \boxed{6.05 \text{ N}}$$

(d) At the bottom of the circle, $F_c = T - F_g = T - mg = mv^2/r$, and solving for the speed gives

$$v^2 = \frac{r}{m}(T - mg) = r\left(\frac{T}{m} - g\right) \qquad \text{and} \qquad v = \sqrt{r\left(\frac{T}{m} - g\right)}$$

If the string is at the breaking point at the bottom of the circle, then $T = 22.5$ N, and the speed of the object at this point must be

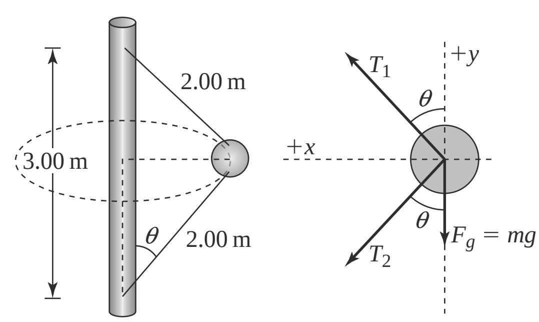
$$v = \sqrt{(0.850 \text{ m})\left(\frac{22.5 \text{ N}}{0.275 \text{ kg}} - 9.80 \text{ m/s}^2\right)} = \boxed{7.82 \text{ m/s}}$$

7.71 From Figure (a) at the right, observe that the angle the strings make with the vertical is

$$\theta = \cos^{-1}\left(\frac{1.50 \text{ m}}{2.00 \text{ m}}\right) = 41.4°$$

Also, the radius of the circular path is

$$r = \sqrt{(2.00 \text{ m})^2 - (1.50 \text{ m})^2} = 1.32 \text{ m}$$



(a) (b)

Figure (b) gives a force diagram of the object with the +y-axis vertical and the +x-axis directed toward the center of the circular path.

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(a) Since the object has zero vertical acceleration, Newton's second law gives

$$\Sigma F_y = T_1 \cos\theta - T_2 \cos\theta - mg = 0 \qquad \text{or} \qquad T_1 - T_2 = \frac{mg}{\cos\theta} \qquad \textbf{[1]}$$

In the horizontal direction, the object has the centripetal acceleration $a_c = v^2/r$ directed in the $+x$-direction (toward the center of the circular path). Thus,

$$\Sigma F_x = T_1 \sin\theta + T_2 \sin\theta = \frac{mv^2}{r} \qquad \text{or} \qquad T_1 + T_2 = \frac{mv^2}{r\sin\theta} \qquad \textbf{[2]}$$

Adding Equations [1] and [2] gives $2T_1 = m\left(\dfrac{g}{\cos\theta} + \dfrac{v^2}{r\sin\theta}\right)$, so the tension in the upper string is

$$T_1 = \frac{(4.00\text{ kg})}{2}\left[\frac{9.80\text{ m/s}^2}{\cos 41.4^\circ} + \frac{(6.00\text{ m/s}^2)^2}{(1.32\text{ m})\sin 41.4^\circ}\right] = \boxed{109\text{ N}}$$

(b) To compute the tension T_2 in the lower string, subtract Equation [1] above from Equation [2] to obtain $2T_2 = m\left(v^2/r\sin\theta - g/\cos\theta\right)$. Thus,

$$T_2 = \frac{(4.00\text{ kg})}{2}\left[\frac{(6.00\text{ m/s}^2)^2}{(1.32\text{ m})\sin 41.4^\circ} - \frac{9.80\text{ m/s}^2}{\cos 41.4^\circ}\right] = \boxed{56.4\text{ N}}$$

7.72 The maximum lift force is $(F_L)_{\max} = Cv^2$, where $C = 0.018\ \text{N}\cdot\text{s}^2/\text{m}^2$ and v is the flying speed. For the bat to stay aloft, the vertical component of the lift force must equal the weight, or $F_L \cos\theta = mg$, where θ is the banking angle. The horizontal component of this force supplies the centripetal acceleration needed to make a turn, or $F_L \sin\theta = m v^2/r$, where r is the radius of the turn.

(a) To stay aloft while flying at minimum speed, the bat must have $\theta = 0$ (to give $\cos\theta = (\cos\theta)_{\max} = 1$) and also use the maximum lift force possible at that speed. That is, we need

$$(F_L)_{\max}(\cos\theta)_{\max} = mg, \qquad \text{or} \qquad Cv_{\min}^2(1) = mg$$

Thus, we see that minimum flying speed is

$$v_{\min} = \sqrt{\frac{mg}{C}} = \sqrt{\frac{(0.031\text{ kg})(9.8\text{ m/s}^2)}{0.018\ \text{N}\cdot\text{s}^2/\text{m}^2}} = \boxed{4.1\text{ m/s}}$$

(b) To maintain horizontal flight while banking at the maximum possible angle, we must have $(F_L)_{\max}\cos\theta_{\max} = mg$, or $Cv^2\cos\theta_{\max} = mg$. For $v = 10\text{ m/s}$, this yields

$$\cos\theta_{\max} = \frac{mg}{Cv^2} = \frac{(0.031\text{ kg})(9.8\text{ m/s}^2)}{(0.018\ \text{N}\cdot\text{s}^2/\text{m}^2)(10\text{ m/s})^2} = 0.17 \qquad \text{or} \qquad \theta_{\max} = \boxed{80^\circ}$$

(c) The horizontal component of the lift force supplies the centripetal acceleration in a turn, $F_L \sin\theta = mv^2/r$. Thus, the minimum radius turn possible is given by

$$r_{\min} = \frac{mv^2}{(F_L)_{\max}(\sin\theta)_{\max}} = \frac{m\cancel{v^2}}{C\cancel{v^2}\sin\theta_{\max}} = \frac{m}{C\sin\theta_{\max}}$$

continued on next page

where we have recognized that $\sin\theta$ has its maximum value at the largest allowable value of θ. For a flying speed of $v = 10\ \text{m/s}$, the maximum allowable bank angle is $\theta_{max} = 80°$ as found in part (b). The minimum radius turn possible at this flying speed is then

$$r_{min} = \frac{0.031\ \text{kg}}{\left(0.018\ \text{N}\cdot\text{s}^2/\text{m}^2\right)\sin 80.0°} = \boxed{1.7\ \text{m}}$$

(d) $\boxed{\text{No}}$. Flying slower actually increases the minimum radius of the achievable turns. As found in part (c), $r_{min} = m/C\sin\theta_{max}$. To see how this depends on the flying speed, recall that the vertical component of the lift force must equal the weight, or $F_L\cos\theta = mg$. At the maximum allowable bank angle, $\cos\theta$ will be a minimum. This occurs when $F_L = \left(F_L\right)_{max} = Cv^2$. Thus, $\cos\theta_{max} = mg/Cv^2$ and

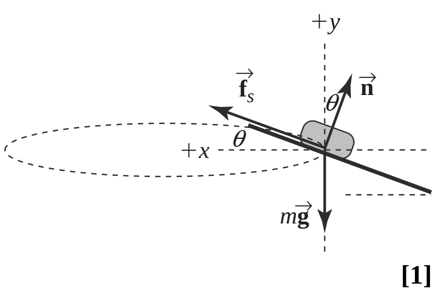
$$\sin\theta_{max} = \sqrt{1-\cos^2\theta_{max}} = \sqrt{1-\left(\frac{mg}{Cv^2}\right)^2}$$

This gives the minimum radius turn possible at flying speed v as

$$r_{min} = \frac{m}{C\sqrt{1-\left(\dfrac{mg}{Cv^2}\right)^2}}$$

Decreasing the flying speed v will decrease the denominator of this expression, yielding a larger value for the minimum radius of achievable turns.

7.73 The angular speed of the luggage is $\omega = 2\pi/T$, where T is the time for one complete rotation of the carousel. The resultant force acting on the luggage must be directed toward the center of the horizontal circular path (that is, in the $+x$-direction) and supply the needed centripetal force $F_c = ma_c = mr\omega^2 = 4\pi^2 mr/T^2$.



Thus, $\Sigma F_x = ma_x \Rightarrow f_s\cos\theta - n\sin\theta = 4\pi^2 mr/T^2$ **[1]**

and $\Sigma F_y = ma_y \Rightarrow f_s\sin\theta + n\cos\theta - mg = 0$

or $n = \dfrac{mg - f_s\sin\theta}{\cos\theta}$ **[2]**

Substituting Equation [2] into Equation [1] gives

$$f_s\left(\cos\theta + \frac{\sin^2\theta}{\cos\theta}\right) = m\left(g\tan\theta + \frac{4\pi^2 r}{T^2}\right)$$

or, multiplying by $\cos\theta$ and using the identity $\cos^2\theta + \sin^2\theta = 1$, we have

$$f_s = m\left(g\sin\theta + \frac{4\pi^2 r\cos\theta}{T^2}\right) \qquad \textbf{[3]}$$

(a) With $r = 7.46$ m and $T = 38.0$ s, Equation [3] gives the required friction force as

$$f_s = (30.0\ \text{kg})\left[\left(9.80\ \text{m/s}^2\right)\sin 20.0° + \frac{4\pi^2(7.46\ \text{m})\cos 20.0°}{(38.0\ \text{s})^2}\right] = \boxed{106\ \text{N}}$$

continued on next page

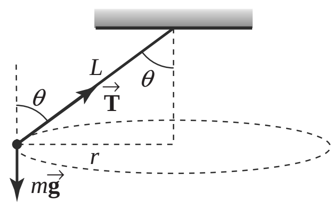
(b) If the bag is on the verge of slipping when $r = 7.94\ \text{m}$ and $T = 34.0\ \text{s}$, then $f_s = (f_s)_{\max} = \mu_s n$ and (using Equations [2] and [3]) the coefficient of static friction is

$$\mu_s = \frac{f_s}{n} = \frac{f_s \cos\theta}{mg - f_s \sin\theta} = \frac{\cos\theta}{(mg/f_s) - \sin\theta} = \frac{\cos\theta}{\left[g/\left(g\sin\theta + 4\pi^2 r\cos\theta/T^2\right)\right] - \sin\theta}$$

or $$\mu_s = \frac{\cos 20.0^\circ}{\left[\dfrac{9.80\ \text{m/s}^2}{\left(9.80\ \text{m/s}^2\right)\sin 20.0^\circ + 4\pi^2(7.94\ \text{m})\cos 20.0^\circ/(34.0\ \text{s})^2}\right] - \sin 20.0^\circ}$$

giving $\mu_s = \boxed{0.396}$

7.74 The horizontal component of the tension in the cord is the only force directed toward the center of the circular path, so it must supply the centripetal acceleration. Thus,



$$T\sin\theta = m\left(\frac{v_t^2}{r}\right) = m\left(\frac{v_t^2}{L\sin\theta}\right)$$

or $$T\sin^2\theta = \frac{mv_t^2}{L} \qquad \textbf{[1]}$$

Also, the vertical component of the tension must support the weight of the ball, or

$$T\cos\theta = mg \qquad \textbf{[2]}$$

(a) Dividing Equation [1] by [2] gives

$$\frac{\sin^2\theta}{\cos\theta} = \frac{v_t^2}{Lg} \qquad \text{or} \qquad v_t = \sin\theta\sqrt{\frac{Lg}{\cos\theta}} \qquad \textbf{[3]}$$

With $L = 1.5\ \text{m/s}$ and $\theta = 30^\circ$,

$$v_t = \sin 30^\circ\sqrt{\frac{(1.5\ \text{m})\left(9.8\ \text{m/s}^2\right)}{\cos 30^\circ}} = \boxed{2.1\ \text{m/s}}$$

(b) From Equation [3], with $\sin^2\theta = 1 - \cos^2\theta$, we find

$$\frac{1-\cos^2\theta}{\cos\theta} = \frac{v_t^2}{Lg} \qquad \text{or} \qquad \cos^2\theta + \left(\frac{v_t^2}{Lg}\right)\cos\theta - 1 = 0$$

Solving this quadratic equation for $\cos\theta$ gives

$$\cos\theta = -\left(\frac{v_t^2}{2\,Lg}\right) \pm \sqrt{\left(\frac{v_t^2}{2\,Lg}\right)^2 + 1}$$

If $L = 1.5\ \text{m}$ and $v_t = 4.0\ \text{m/s}$, this yields solutions $\cos\theta = -1.7$ (which is impossible), and $\cos\theta = +0.59$ (which is possible). Thus, $\theta = \cos^{-1}(0.59) = \boxed{54^\circ}$.

(c) From Equation [2], when $T = 9.8\ \text{N}$ and the cord is about to break, the angle is

$$\theta = \cos^{-1}\left(\frac{mg}{T}\right) = \cos^{-1}\left(\frac{(0.50\ \text{kg})\left(9.8\ \text{m/s}^2\right)}{9.8\ \text{N}}\right) = 60^\circ$$

continued on next page

Then Equation [3] gives

$$v_t = \sin\theta\sqrt{\frac{Lg}{\cos\theta}} = \sin 60°\sqrt{\frac{(1.5\text{ m})\left(9.8\text{ m/s}^2\right)}{\cos 60°}} = \boxed{4.7\text{ m/s}}$$

7.75 The normal force exerted on the person by the cylindrical wall must provide the centripetal acceleration, so $n = m\left(r\,\omega^2\right)$.

If the minimum acceptable coefficient of friction is present, the person is on the verge of slipping and the maximum static friction force equals the person's weight, or $\left(f_s\right)_{\max} = \left(\mu_s\right)_{\min} n = mg$.

Thus, $\left(\mu_s\right)_{\min} = \dfrac{mg}{n} = \dfrac{g}{r\,\omega^2} = \dfrac{9.80\text{ m/s}^2}{(3.00\text{ m})(5.00\text{ rad/s})^2} = \boxed{0.131}$

7.76 If the block will just make it through the top of the loop, the force required to produce the centripetal acceleration at point C must equal the block's weight, or $m\left(v_C^2/R\right) = mg$.

This gives $v_C = \sqrt{Rg}$ as the required speed of the block at point C.

We apply the work-energy theorem in the form

$$W_{nc} = \left(KE + PE_g + PE_s\right)_f - \left(KE + PE_g + PE_s\right)_i$$

from when the block is first released until it reaches point C to obtain

$$f_k\left(\overline{AB}\right)\cos 180° = \frac{1}{2}mv_C^2 + mg(2R) + 0 - 0 - 0 - \frac{1}{2}kd^2$$

The friction force between points A and B is $f_k = u_k\,(mg)$, and for minimum initial compression of the spring, $v_C^2 = Rg$ as found above. Thus, the work-energy equation reduces to

$$d_{\min} = \sqrt{\frac{2\,\mu_k mg\left(\overline{AB}\right) + mRg + 2\,mg\,(2R)}{k}} = \sqrt{\frac{mg\left(2\,\mu_k\,\overline{AB} + 5R\right)}{k}}$$

$$d_{\min} = \sqrt{\frac{(0.50\text{ kg})\left(9.8\text{ m/s}^2\right)\left[2(0.30)(2.5\text{ m}) + 5(1.5\text{ m})\right]}{78.4\text{ N/m}}} = \boxed{0.75\text{ m}}$$

8

Rotational Equilibrium and Rotational Dynamics

QUICK QUIZZES

1. Choice (d). A larger torque is needed to turn the screw. Increasing the radius of the screwdriver handle provides a greater lever arm and hence an increased torque.

2. Choice (b). Since the object has a constant net torque acting on it, it will experience a constant angular acceleration. Thus, the angular velocity will change at a constant rate.

3. Choice (b). The hollow cylinder has the larger moment of inertia, so it will be given the smaller angular acceleration and take longer to stop.

4. Choice (a). The hollow sphere has the larger moment of inertia, so it will have the higher rotational kinetic energy, $KE_r = \frac{1}{2} I\omega^2$.

5. Choice (c). Apply conservation of angular momentum to the system (the two disks) before and after the second disk is added to get the result: $I_1\omega_1 = (I_1 + I_2)\omega$.

6. Choice (a). Earth already bulges slightly at the Equator, and is slightly flat at the poles. If more mass moved towards the Equator, it would essentially move the mass to a greater distance from the axis of rotation, and increase the moment of inertia. Because conservation of angular momentum requires that $I_z\omega_z =$ constant, an increase in the moment of inertia would decrease the angular velocity, and slow down the spinning of Earth. Thus, the length of each day would increase.

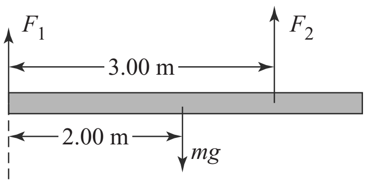
ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. Assuming a uniform, solid disk, its moment of inertia about a perpendicular axis through its center is $I = MR^2/2$, so $\tau = I\alpha$ gives

$$\alpha = \frac{2\tau}{MR^2} = \frac{2(40.0\ \text{N}\cdot\text{m})}{(25.0\ \text{kg})(0.800\ \text{m})^2} = 5.00\ \text{rad/s}^2$$

and the correct answer is (b).

2. Using the left end of the plank as a pivot and requiring that $\Sigma\tau = 0$ gives $-mg(2.00\ \text{m}) + F_2(3.00\ \text{m}) = 0$, or



$$F_2 = \frac{2mg}{3} = \frac{2(20.0\ \text{kg})(9.80\ \text{m/s}^2)}{3} = 131\ \text{N}$$

and choice (d) is the correct response.

3. $\tau = rF\sin\theta = (0.500\ \text{m})(80.0\ \text{N})\sin(60.0°) = 34.6\ \text{N}\cdot\text{m}$ which is choice (a).

4. The reel will be given a greater angular acceleration when a constant tension of 50 N is maintained in the cord as in choice (a). When an object weighing 50 N is attached to the end of the cord and allowed to fall, the tension in the cord will be less than 50 N, with the difference between the 50 N weight of the object and the tension being the net downward force accelerating the object.

5. In order for an object to be in equilibrium, it must be in both translational equilibrium and rotational equilibrium. Thus, it must meet two conditions of equilibrium, namely $\vec{\mathbf{F}}_{\text{net}} = 0$ and $\vec{\boldsymbol{\tau}}_{\text{net}} = 0$. The correct answer is therefore choice (d).

6. When objects travel down ramps of the same length, the one with the greatest translational kinetic energy at the bottom will have the greatest final translational speed (and, hence, greatest average translational speed). This means that it will require less time to travel the length of the ramp. Of the objects listed, all will have the same *total* kinetic energy at the bottom, since they have the same decrease in gravitational potential energy (due to the ramps having the same vertical drop) and no energy has been spent overcoming friction. All of the block's kinetic energy is in the form of translational kinetic energy. Of the rolling bodies, the fraction of their total kinetic energy that is in the translational form is

$$f = \frac{KE_t}{KE_t + KE_r} = \frac{\frac{1}{2}Mv^2}{\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2} = \frac{1}{1+(I/M)(\omega/v)^2} = \frac{1}{1+I/(MR^2)}$$

Since the ratio I/MR^2 equals $2/5$ for a solid ball and $2/3$ for a hollow sphere, the solid ball has the larger translational kinetic energy at the bottom and will arrive before the hollow sphere. The correct rankings of arrival times, from shortest to longest, is then block, solid ball, hollow sphere, and choice (e) is the correct response.

7. The moment of inertia of a body is determined by its mass and the way that mass is distributed about the rotation axis. Also, the location of the body's center of mass is determined by how its mass is distributed. As long as these qualities do not change, both the moment of inertia and the center of mass are constant. From $\tau = I\alpha$, we see that when a body experiences a constant, non-zero torque, it will have a constant, non-zero angular acceleration. However, since the angular acceleration is non-zero, the angular velocity ω (and hence the angular momentum, $L = I\omega$) will vary in time. The correct responses to this question are then (b) and (e).

8. In a rigid, rotating body, all points in that body rotate about the axis at the same rate (or have the same angular velocity). The centripetal acceleration, tangential acceleration, linear velocity, and total acceleration of a point in the body all vary with the distance that point is from the axis of rotation. Thus, the only correct choice is (b).

9. Please read the answer to Question 6 above, since most of what is said there also applies to this question. The total kinetic energy of either the disk or the hoop at the bottom of the ramp will be $KE_{\text{total}} = Mgh$, where M is the mass of the body in question and h is the vertical drop of the ramp. The translational kinetic energy of this body will then be $KE_t = f\,KE_{\text{total}} = fMgh$, where f is the fraction discussed in Question 6. Hence, $\cancel{M}v^2/2 = f\cancel{M}gh$ and the translational speed at the bottom is $v = \sqrt{2fgh}$. Since $f = 1/(1+1/2) = 2/3$ for the disk and $f = 1/(1+1) = 1/2$ for the hoop, we see that the disk will have the greater translational speed at the bottom, and hence, will arrive first. Notice that both the mass and radius of the object have canceled in the calculation. Our conclusion is then independent of the object's mass and/or radius. Therefore, the only correct response is choice (d).

10. The ratio of rotational kinetic energy to the total kinetic energy for an object that rolls without slipping is

$$\frac{KE_r}{KE_{\text{total}}} = \frac{KE_r}{KE_t + KE_r} = \frac{\frac{1}{2}I\omega^2}{\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2} = \frac{1}{\dfrac{M}{I}\left(\dfrac{v}{\omega}\right)^2 + 1} = \frac{1}{\dfrac{MR^2}{I} + 1}$$

For a solid cylinder, $I = MR^2/2$ and this ratio becomes

$$\frac{KE_r}{KE_{\text{total}}} = \frac{1}{2+1} = \frac{1}{3}$$

and the correct answer is (c).

11. Since the axle of the turntable is frictionless, no external agent exerts a torque about this vertical axis of the mouse-turntable system. This means that the total angular momentum of the mouse-turntable system will remain constant at its initial value of zero. Thus, as the mouse starts walking around the axis (and developing an angular momentum, $L_{\text{mouse}} = I_m\omega_m$, in the direction of its angular velocity), the turntable must start to turn in the opposite direction so it will possess an angular momentum, $L_{\text{table}} = I_t\omega_t$, such that $L_{\text{total}} = L_{\text{mouse}} + L_{\text{table}} = I_m\omega_m + I_t\omega_t = 0$. Thus, the angular velocity of the table will be $\omega_t = -\left(I_m/I_t\right)\omega_m$. The negative sign means that if the mouse is walking around the axis in a clockwise direction, the turntable will be rotating in the opposite direction, or counterclockwise. The correct choice for this question is (d).

12. Please review the answers given above for Questions 6 and 9. In the answer to Question 9, it is shown that the translational speed at the bottom of the hill of an object that rolls without slipping is $v = \sqrt{2fgh}$, where h is the vertical drop of the hill and f is the ratio of the translational kinetic energy to the total kinetic energy of the rolling body. For a solid sphere, $I = 2MR^2/5$, so the ratio f is

$$f = \frac{1}{1+I/\left(MR^2\right)} = \frac{1}{1+2/5} = \frac{1}{1.4}$$

and the translational speed at the bottom of the hill is $v = \sqrt{2gh/1.4}$. Notice that this result is the same for *all uniform, solid, spheres*. Thus, the two spheres have the same translational speed at the bottom of the hill. This also means that they have the same average speed for the trip, and hence, both make the trip in the same time. The correct answer to this question is (d).

13. If a car is to reach the bottom of the hill in the shortest time, it must have the greatest translational speed at the bottom (and hence, greatest average speed for the trip). To maximize its final translational speed, the car should be designed so as much as possible of the car's total kinetic energy is in the form of translational kinetic energy. This means that the rotating parts of the car (i.e., the wheels) should have as little kinetic energy as possible. Therefore, the mass of these parts should be kept small, and the mass they do have should be concentrated near the axle in order to keep the moment of inertia as small as possible. The correct response to this question is (e).

14. (i) As the ponies walk inward toward the center of the turntable, the moment of inertia of the rotating system decreases. Thus, the angular speed of the system must increase to conserve angular momentum. Choice (a) is the correct answer.

(ii) No. The mechanical energy of the system increases. The ponies must do work, converting chemical energy into additional mechanical energy, as they walk toward the center of the turntable.

(iii) Yes. The center of gravity of this system is located at the center of the turntable and is stationary. Thus, the linear momentum of the system has a constant value of zero.

(iv) Yes. Since no external torque acts on the ponies-turntable system, the angular momentum of this system is constant.

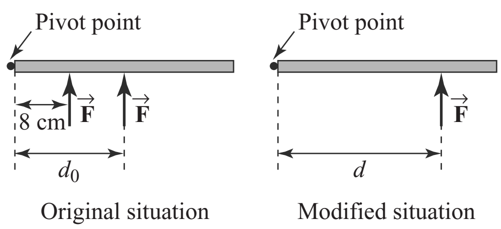
ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The moment of inertia depends on the distribution of mass with respect to a given axis. If the axis is changed, then each bit of mass that makes up the object is at a different distance from the axis than before. Compare the moments of inertia of a uniform rigid rod about axes perpendicular to the rod, first passing through its center of mass, then passing through an end. For example, if you wiggle repeatedly a meter stick back and forth about an axis passing through its center of mass, you will find it does not take much effort to reverse the direction of rotation. However, if you move the axis to an end, you will find it more difficult to wiggle the stick back and forth. The moment of inertia about the end is much larger, because much of the mass of the stick is farther from the axis.

4. (a) The lever arm of a particular force is found with respect to some reference point or axis. Thus, an origin must be chosen to compute the torque of a force.

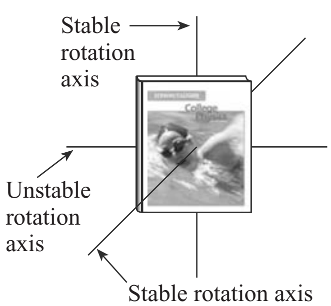
(b) If the object is in translational equilibrium $\left(\vec{\mathbf{F}}_{\text{net}} = \Sigma\vec{\mathbf{F}}_i = 0\right)$, the net torque acting on the system is independent of the origin or axis considered. However, if the resultant force acting on the object is not zero, the net torque has different values for different axis of rotation.

6. The critical factor is the total torque being exerted about the line of the hinges. For simplicity, we assume that the paleontologist and the botanist exert equal magnitude forces. The free body diagram of the original situation is shown on the left and that for the modified situation is shown on the right in the sketches below:



In order for the torque exerted on the door in the modified situation to equal that of the original situation, it is necessary that $Fd = Fd_0 + F(8\text{ cm})$ or $d = d_0 + 8\text{ cm}$. Thus, the paleontologist would need to relocate about 8 cm farther from the hinge.

8.



10. After the head crosses the bar, the jumper should arch his back so the head and legs are lower than the midsection of the body. In this position, the center of gravity may pass under the bar while the midsection of the body is still above the bar. As the feet approach the bar, the legs should be straightened to avoid hitting the bar.

12. (a) Consider two people, at the ends of a long table, pushing with equal magnitude forces directed in opposite directions perpendicular to the length of the table. The net force will be zero, yet the net torque is not zero.

(b) Consider a falling body. The net force acting on it is its weight, yet the net torque about the center of gravity is zero.

14. As the cat falls, angular momentum must be conserved. Thus, if the upper half of the body twists in one direction, something must get an equal angular momentum in the opposite direction. Rotating the lower half of the body in the opposite direction satisfies the law of conservation of angular momentum.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. $3.55\ \text{N}\cdot\text{m}$ clockwise

4. $0.642\ \text{N}\cdot\text{m}$ counterclockwise

6. $\Sigma F_x = F_x - R_x = 0,\ \Sigma F_y = F_y + R_y - F_g = 0,\ \Sigma\tau_O = +F_y(\ell\cos\theta) - F_x(\ell\sin\theta) - F_g(\ell/2)\cos\theta = 0$

8. **(a)** 226 N **(b)** 117 N upward

10. 139 g

12. **(a)** See Solution. **(b)** at $x = 0$ **(c)** $n_1 = 0$

(d) $n_2 = 1.42\times10^3$ N **(e)** 5.64 m **(f)** yes

14. **(a)** $n_2 = (m+M)g$ **(b)** $x = (1+M/m)\ell - (M/2m)L$

(c) $\ell = \left(\dfrac{m+M/2}{m+M}\right)L$

16. $x_{\text{cg}} = 0.459$ m, $y_{\text{cg}} = 0.103$ m

18. $T = 1.68\times10^3$ N, $R = 2.33\times10^3$ N, $\theta = 21.0°$

20. 567 N, 333 N

22. **(a)** See Solution.

(b) $T = 343$ N, 172 N to the right, 683 N upward

(c) $x_{\max} = 5.14$ m

24. **(a)** See Solution.

(b) To simplify equations by eliminating 2 unknowns.

(c) $\Sigma\tau)_{\text{hinge}} = 0 + 0 - mg\left(\dfrac{L}{2}\cos\theta\right) + T(L\sin\theta) = 0$ **(d)** $T = 136$ N

(e) $\Sigma F_x = F_x - T = 0,\ \Sigma F_y = F_y - mg = 0$

(f) $F_x = 136$ N, $F_y = 157$ N

(g) Yes, this would reduce tension in the cable and stress on the hinge.

26. **(a)** See Solution. **(b)** $T = \frac{1}{2}mg\cot\theta$

(c) $T = \mu_s mg$ **(d)** $\mu_s = \frac{1}{2}\cot\theta$

(e) At smaller angles, the beam slips and the equation no longer applies. At larger angles, there is no longer impending motion and the equation is invalid.

28. **(a)** $T = 1.47\text{ kN}$ **(b)** 1.33 kN to the right, 2.58 kN upward

30. $x_{\text{min}} = 2.8\text{ m}$

32. $\tau_x = 149\text{ N}\cdot\text{m},\ \tau_y = 66.0\text{ N}\cdot\text{m},\ \tau_O = 215\text{ N}\cdot\text{m}$

34. **(a)** $I = MR^2 + \frac{1}{2}mr^2$ **(b)** 0 **(c)** $\tau_T > 0,\ \alpha > 0,\ a < 0$

(d) $\alpha = -a/r$ **(e)** $T - (2M+m)g = (2M+m)a$

(f) $rT = I\alpha$ **(g)** $a = \dfrac{-(2M+m)g}{2M + M(R/r)^2 + 3m/2}$

(h) $a = -2.72\ \text{m/s}^2$ **(i)** $T = 35.4\text{ N}$ **(j)** $t = 0.857\text{ s}$

36. $\mu_k = 0.30$

38. **(a)** 872 N **(b)** 1.40 kN

40. **(a)** To provide the net clockwise torque that accelerates the pulley.

(b) $2.88\ \text{m/s}^2$ **(c)** $T_1 = 127\text{ N},\ T_2 = 138\text{ N}$

42. **(a)** $1.37 \times 10^8\text{ J}$ **(b)** 5.11 h

44. **(a)** $I_{\text{hoop}} = 0.254\text{ kg}\cdot\text{m}^2,\ I_{\text{cyl}} = 0.127\text{ kg}\cdot\text{m}^2,\ I_{\text{sphere}} = 0.102\text{ kg}\cdot\text{m}^2,\ I_{\text{shell}} = 0.169\text{ kg}\cdot\text{m}^2$

(b) solid sphere; solid cylinder; thin spherical shell; and hoop

(c) hoop; thin spherical shell; solid cylinder; and solid sphere

46. 36 rad/s

48. **(a)** rotational KE, $\frac{1}{2}I\omega^2$; translational KE, $\frac{1}{2}mv^2$; gravitational PE, mgy

(b) static friction **(c)** 2/7

50. 30.2 rev/s

52. 10.9 rad/s

54. **(a)** $2.72\ \text{kg}\cdot\text{m}^2/\text{s}$ **(b)** $1.36\ \text{kg}\cdot\text{m}^2/\text{s}$

(c) $1.09\ \text{kg}\cdot\text{m}^2/\text{s}$ **(d)** $1.81\ \text{kg}\cdot\text{m}^2/\text{s}$

56. **(a)** Yes, the bullet effectively moves in a circle around the hinge just before impact.

(b) No, the bullet undergoes an inelastic collision with the door.

(c) 0.749 rad/s **(d)** $KE_f = 1.68\text{ J} \ll KE_i = 2.50 \times 10^3\text{ J}$

58. $0.91\ \text{km/s}$

60. **(a)** $0.360\ \text{rad/s}$, counterclockwise **(b)** 99.9 J

62. **(a)** $1.9\ \text{rad/s}$

(b) $KE_i = 2.5\ \text{J}$, $KE_f = 6.3\ \text{J}$

64. $12.3\ \text{m/s}^2$

66. **(a)** $0.433\ \text{kg}\cdot\text{m}^2/\text{s}$ **(b)** $1.73\ \text{kg}\cdot\text{m}^2/\text{s}$

68. **(a)** 780 N

(b) $\vec{\mathbf{R}} = 716\ \text{N}$ at 70.4° above the horizontal to the right

70. **(a)** 46.8 N **(b)** $0.234\ \text{kg}\cdot\text{m}^2$ **(c)** $40.0\ \text{rad/s}$

72. **(a)** $3.75\times10^3\ \text{kg}\cdot\text{m}^2/\text{s}$ **(b)** 1.88 kJ **(c)** $3.75\times10^3\ \text{kg}\cdot\text{m}^2/\text{s}$

(d) $10.0\ \text{m/s}$ **(e)** 7.50 kJ **(f)** 5.62 kJ

74. $T_{\text{left}} = 1.01\ \text{kN}$, $T_{\text{right}} = 1.59\ \text{kN}$

76. **(a)** $E = (m_1 - m_2)gL$

(b) $E = \frac{1}{2}(m_1 + m_2)L^2\omega^2 + (m_1 - m_2)gL\sin\theta$

(c) Equate the results of parts (a) and (b) and solve for ω.

(d) $\tau_{\text{net}}\big|_{\text{vertical}} = 0$; $\tau_{\text{net}}\big|_{\text{rotated}} = (m_1 - m_2)gL\cos\theta$; Since $\Delta L/\Delta t = \tau_{\text{net}}$, the angular momentum changes at a non-uniform rate.

(e) $\alpha = (m_1 - m_2)g\cos\theta/(m_1 + m_2)L$; Yes, $\alpha \to 0$ as $\theta \to 90°$ and α is a maximum at $\theta = 0°$.

78. 22 N

80. **(a)** $v_{\text{cg}} = \sqrt{3gL}/2$ **(b)** $v_{\substack{\text{lower}\\\text{end}}} = \sqrt{3gL} = 2v_{\text{cg}}$

82. **(a)** A frictionless wall cannot exert a component of force parallel to its surface.

(b) $L\sin\theta$ **(c)** $\frac{1}{2}L\sin\theta$ **(d)** 2.5 m

84. Answers are given in the problem statement.

86. **(a)** $T = 10.2\ \text{N}$ **(b)** $R_x = 6.56\ \text{N}$, $R_y = 7.84\ \text{N}$

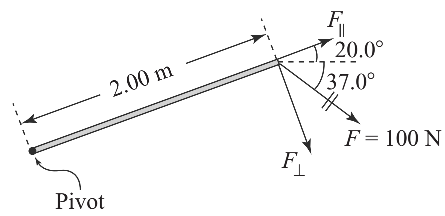
88. **(a)** See Solution. **(b)** 218 N

(c) 72.4 N **(d)** 2.41 m

(e) The analysis would need to include a horizontal friction force at the lower end of the ladder. The coefficient of static friction between the ladder and the floor would have to be known.

PROBLEM SOLUTIONS

8.1 Resolve the 100-N force into components parallel to and perpendicular to the rod, as



$$F_{\parallel} = F\cos(20.0° + 37.0°) = F\cos 57.0°$$

and

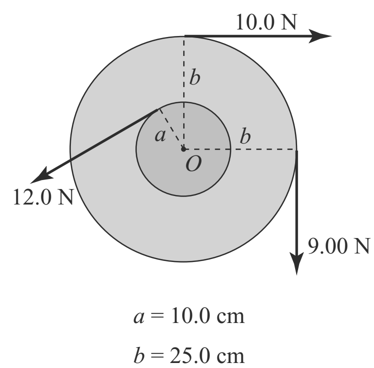
$$F_{\perp} = F\sin(20.0° + 37.0°) = F\sin 57.0°$$

The lever arm of $F_{\perp}$ about the indicated pivot is 2.00 m, while that of $F_{\parallel}$ is zero. The torque due to the 100-N force may be computed as the sum of the torques of its components, giving

$$\tau = F_{\parallel}(0) - F_{\perp}(2.00\text{ m}) = 0 - [(100\text{ N})\sin 57.0°](2.00\text{ m}) = -168\text{ N}\cdot\text{m}$$

or $\boxed{\tau = 168\text{ N}\cdot\text{m clockwise}}$

8.2 Note that each of the forces is perpendicular to the radius line of the wheel at the point where the force is tangent to the wheel. Thus, when considering torques about the center of the wheel, the radius line is the lever arm of the force. Taking counterclockwise torques as positive,

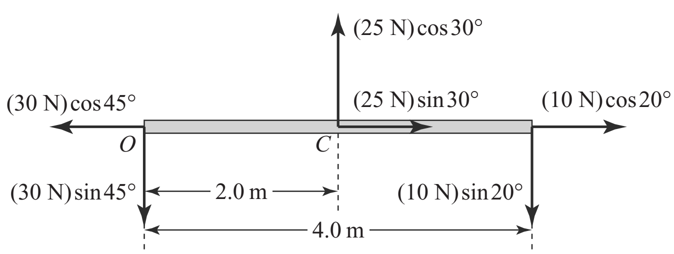


$$\Sigma\tau_O = +(12.0\text{ N})a - (10.0\text{ N})b - (9.00\text{ N})b$$

$$= +(12.0\text{ N})(0.100\text{ m}) - (19.0\text{ N})(0.250\text{ m})$$

$$= -3.55\text{ N}\cdot\text{m} \quad \text{or} \quad \boxed{3.55\text{ N}\cdot\text{m clockwise}}$$

8.3 First resolve all of the forces shown in Figure P8.3 into components parallel to and perpendicular to the beam as shown in the sketch below.



(a) $\tau_O = +[(25\text{ N})\cos 30°](2.0\text{ m}) - [(10\text{ N})\sin 20°](4.0\text{ m}) = \boxed{+30\text{ N}\cdot\text{m}}$

or $\tau_O = \boxed{30\text{ N}\cdot\text{m counterclockwise}}$

(b) $\tau_C = +[(30\text{ N})\sin 45°](2.0\text{ m}) - [(10\text{ N})\sin 20°](2.0\text{ m}) = \boxed{+36\text{ N}\cdot\text{m}}$

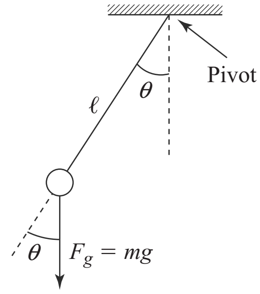
or $\tau_C = \boxed{36\text{ N}\cdot\text{m counterclockwise}}$

8.4 The lever arm is $d = (1.20\times10^{-2}\text{ m})\cos 48.0° = 8.03\times10^{-3}$ m, and the torque is

$$\tau = Fd = (80.0\text{ N})(8.03\times10^{-3}\text{ m}) = \boxed{0.642\text{ N}\cdot\text{m counterclockwise}}$$

8.5 (a) $|\tau| = F_g \cdot (\textit{lever arm}) = (mg)\cdot[\ell \sin\theta]$

$$= (3.0\ \text{kg})(9.8\ \text{m/s}^2)\cdot[(2.0\ \text{m})\sin 5.0°] = \boxed{5.1\ \text{N}\cdot\text{m}}$$



(b) The magnitude of the torque is proportional to $\sin\theta$, where θ is the angle between the direction of the force and the line from the pivot to the point where the force acts. Note from the sketch that this is the same as the angle the pendulum string makes with the vertical.

$\boxed{\text{Since } \sin\theta \text{ increases as } \theta \text{ increases, the torque also increases with the angle.}}$

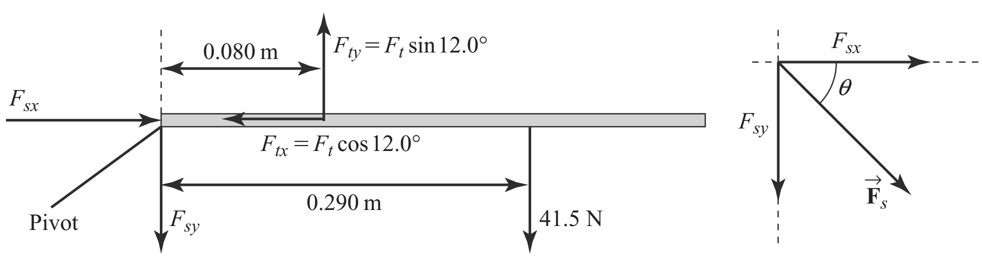
8.6 The object is in both translational and rotational equilibrium. Thus, we may write

$$\Sigma F_x = 0 \quad \Rightarrow \quad \boxed{F_x - R_x = 0}$$

$$\Sigma F_y = 0 \quad \Rightarrow \quad \boxed{F_y + R_y - F_g = 0}$$

and $$\Sigma \tau_O = 0 \quad \Rightarrow \quad \boxed{F_y(\ell\cos\theta) - F_x(\ell\sin\theta) - F_g\left(\frac{\ell}{2}\cos\theta\right) = 0}$$

8.7



Requiring that $\Sigma\tau = 0$, using the shoulder joint at point O as a pivot, gives

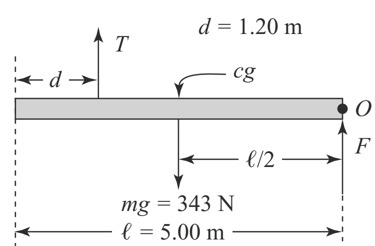
$$\Sigma\tau = (F_t \sin 12.0°)(0.080\ \text{m}) - (41.5\ \text{N})(0.290\ \text{m}) = 0, \text{ or } F_t = \boxed{724\ \text{N}}$$

Then $\Sigma F_y = 0 \Rightarrow -F_{sy} + (724\ \text{N})\sin 12.0° - 41.5\ \text{N} = 0$ yielding $F_{sy} = 109\ \text{N}$.

$\Sigma Fx = 0$ gives $\quad F_{sx} - (724\ \text{N})\cos 12.0° = 0$, or $F_{sx} = 708\ \text{N}$

Therefore, $\quad F_s = \sqrt{F_{sx}^2 + F_{sy}^2} = \sqrt{(708\ \text{N})^2 + (109\ \text{N})^2} = \boxed{716\ \text{N}}$

8.8 Since the beam is uniform, its center of gravity is at its geometric center.



Requiring that $\Sigma\tau = 0$ about an axis through point O and perpendicular to the page gives

$$F(0) + mg(\ell/2) - T(\ell - d) = 0$$

(a) The tension in the rope must then be

$$T = \frac{mg(\ell/2)}{\ell - d} = \frac{(343\ \text{N})(2.50\ \text{m})}{5.00\ \text{m} - 1.20\ \text{m}} = \boxed{226\ \text{N}}$$

continued on next page

(b) The force the column exerts is found from

$$\Sigma F_y = 0 \quad \Rightarrow \quad F + T - mg = 0$$

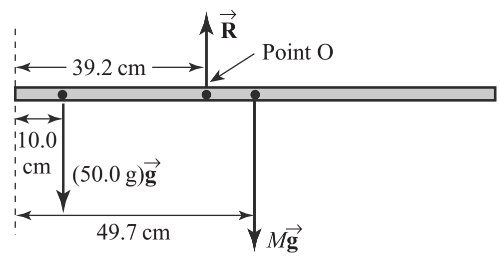
or $$F = mg - T = 343 \text{ N} - 226 \text{ N} = \boxed{717 \text{ N upward}}$$

8.9 Require that $\Sigma\tau = 0$ about an axis through the elbow and perpendicular to the page. This gives

$$\Sigma\tau = +\left[(2.00 \text{ kg})\left(9.80 \text{ m/s}^2\right)\right](25.0 \text{ cm} + 8.00 \text{ cm}) - \left(F_B \cos 75.0°\right)(8.00 \text{ cm}) = 0$$

or $$F_B = \frac{(19.6 \text{ N})(33.0 \text{ cm})}{(8.00 \text{ cm})\cos 75.0°} = \boxed{312 \text{ N}}$$

8.10 Since the bare meter stick balances at the 49.7 cm mark when placed on the fulcrum, the center of gravity of the meter stick is located 49.7 cm from the zero end. Thus, the entire weight of the meter stick may be considered to be concentrated at this point. The force diagram of the stick when it is balanced with the 50.0-g mass attached at the 10.0 cm mark is as given at the right.

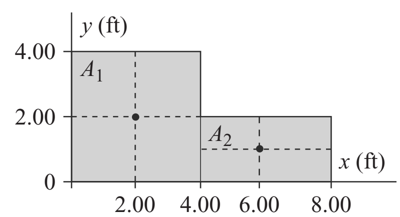


Requiring that the sum of the torques about point O be zero yields

$$+\left[(50.0 \text{ g})\cancel{g}\right](39.2 \text{ cm} - 10.0 \text{ cm}) - M\cancel{g}(49.7 \text{ cm} - 39.2 \text{ cm}) = 0$$

or $$M = (50.0 \text{ g})\left(\frac{39.2 \text{ cm} - 10.0 \text{ cm}}{49.7 \text{ cm} - 39.2 \text{ cm}}\right) = \boxed{139 \text{ g}}$$

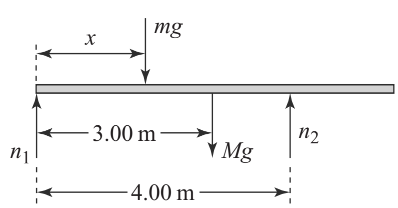
8.11 Consider the remaining plywood to consist of two parts: A_1 is a 4.00-ft by 4.00-ft section with center of gravity located at $(2.00 \text{ ft}, 2.00 \text{ ft})$, while A_2 is a 2.00-ft by 4.00-ft section with center of gravity at $(6.00 \text{ ft}, 1.00 \text{ ft})$. Since the plywood is uniform, its mass per area σ is constant and the mass of a section having area A is $m = \sigma A$. The center of gravity of the remaining plywood has coordinates given by



$$x_{cg} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{\cancel{\sigma} A_1 x_1 + \cancel{\sigma} A_2 x_2}{\cancel{\sigma} A_1 + \cancel{\sigma} A_2} = \frac{\left(16.0 \text{ ft}^2\right)(2.00 \text{ ft}) + \left(8.00 \text{ ft}^2\right)(6.00 \text{ ft})}{\left(16.0 \text{ ft}^2\right) + \left(8.00 \text{ ft}^2\right)} = \boxed{3.33 \text{ ft}}$$

and $$y_{cg} = \frac{\Sigma m_i y_i}{\Sigma m_i} = \frac{\cancel{\sigma} A_1 y_1 + \cancel{\sigma} A_2 y_2}{\cancel{\sigma} A_1 + \cancel{\sigma} A_2} = \frac{\left(16.0 \text{ ft}^2\right)(2.00 \text{ ft}) + \left(8.00 \text{ ft}^2\right)(1.00 \text{ ft})}{\left(16.0 \text{ ft}^2\right) + \left(8.00 \text{ ft}^2\right)} = \boxed{1.67 \text{ ft}}$$

8.12 (a)



$$Mg = (90.0 \text{ kg})\left(9.80 \text{ m/s}^2\right) = 882 \text{ N} \qquad mg = (55.0 \text{ kg})\left(9.80 \text{ m/s}^2\right) = 539 \text{ N}$$

continued on next page

(b) $\boxed{\text{The woman is at } x = 0 \text{ when } n_1 \text{ is greatest.}}$ With this location of the woman, she exerts her maximum possible counterclockwise torque about the center of the beam. Thus, n_1 must be exerting its maximum clockwise torque about the center to hold the beam in rotational equilibrium.

(c) $\boxed{n_1 = 0}$ As the woman walks to the right along the beam, she will eventually reach a point where the beam will start to rotate clockwise about the rightmost pivot. At this point, the beam is starting to lift up off of the leftmost pivot and the normal force exerted by that pivot will have diminished to zero.

(d) When the beam is about to tip, $n_1 = 0$, and $\Sigma F_y = 0$ gives

$$0 + n_2 - Mg - mg = 0$$

or $\quad n_2 = Mg + mg = 882 \text{ N} + 539 \text{ N} = \boxed{1.42 \times 10^3 \text{ N}}$

(e) Requiring that $\Sigma\tau)_{\substack{\text{rightmost}\\ \text{pivot}}} = 0$ when the beam is about to tip ($n_1 = 0$) gives

$$-(4.00 \text{ m} - x)mg - (4.00 \text{ m} - 3.00 \text{ m})Mg = 0$$

or $\quad (mg)x = (1.00 \text{ m})Mg + (4.00 \text{ m})mg$

and $\quad x = (1.00 \text{ m})\dfrac{M}{m} + 4.00 \text{ m}$

Thus, $\quad x = (1.00 \text{ m})\dfrac{(90.0 \text{ kg})}{(55.0 \text{ kg})} + 4.00 \text{ m} = \boxed{5.64 \text{ m}}$

(f) When $n_1 = 0$ and $n_2 = 1.42 \times 10^3$ N, requiring that $\Sigma\tau)_{\substack{\text{left}\\ \text{end}}} = 0$ gives

$$0 - (539 \text{ N})x - (882 \text{ N})(3.00 \text{ m}) + (1.42 \times 10^3 \text{ N})(4.00 \text{ m}) = 0$$

or $\quad x = \dfrac{-3.03 \times 10^3 \text{ N} \cdot \text{m}}{-539 \text{ N}} = \boxed{5.62 \text{ N}}$

which, within limits of rounding errors, is $\boxed{\text{the same as the answer to part (e)}}$.

8.13 Requiring that $x_{cg} = \dfrac{\Sigma m_i x_i}{\Sigma m_i} = 0$ gives

$$\frac{(5.0 \text{ kg})(0) + (3.0 \text{ kg})(0) + (4.0 \text{ kg})(3.0 \text{ m}) + (8.0 \text{ kg})x}{(5.0 + 3.0 + 4.0 + 8.0) \text{ kg}} = 0$$

or $8.0\,x + 12 \text{ m} = 0$ which yields $x = -1.5$ m

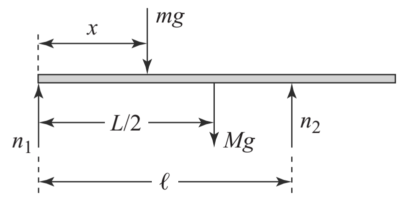
Also, requiring that $y_{cg} = \Sigma m_i y_i / \Sigma m_i = 0$ gives

$$\frac{(5.0 \text{ kg})(0) + (3.0 \text{ kg})(4.0 \text{ m}) + (4.0 \text{ kg})(0) + (8.0 \text{ kg})y}{(5.0 + 3.0 + 4.0 + 8.0) \text{ kg}} = 0$$

or $8.0\,y + 12 \text{ m} = 0$ yielding $y = -1.5$ m

Thus, the 8.0-kg object should be placed at coordinates $\boxed{(-1.5 \text{ m}, -1.5 \text{ m})}$.

8.14 (a) As the woman walks to the right along the beam, she will eventually reach a point where the beam will start to rotate clockwise about the rightmost pivot. At this point, the beam is starting to lift up off of the leftmost pivot and the normal force, n_1, exerted by that pivot will have diminished to zero.



Then, $\quad \Sigma F_y = 0 \quad \Rightarrow \quad 0 - mg - Mg + n_2 = 0 \qquad$ or $\qquad \boxed{n_2 = (m+M)g}$

(b) When $n_1 = 0$ and $n_2 = (m+M)g$, requiring that $\Sigma\tau)_{\substack{\text{left}\\\text{end}}} = 0$ gives

$$0 - (mg)x - (Mg)\frac{L}{2} + (mg + Mg)\ell = 0 \qquad \text{or} \qquad \boxed{x = \left(1 + \frac{M}{m}\right)\ell - \left(\frac{M}{2m}\right)\text{L}}$$

(c) If the woman is to just reach the right end of the beam ($x = L$) when $n_1 = 0$ and $n_2 = (m+M)g$ (i.e., the beam is ready to tip), then the result from part (b) requires that

$$L = \left(1 + \frac{M}{m}\right)\ell - \left(\frac{M}{2m}\right)L \qquad \text{or} \qquad \ell = \frac{(1 + M/2m)}{(1 + M/m)} = \boxed{\left(\frac{m + M/2}{m + M}\right)L}$$

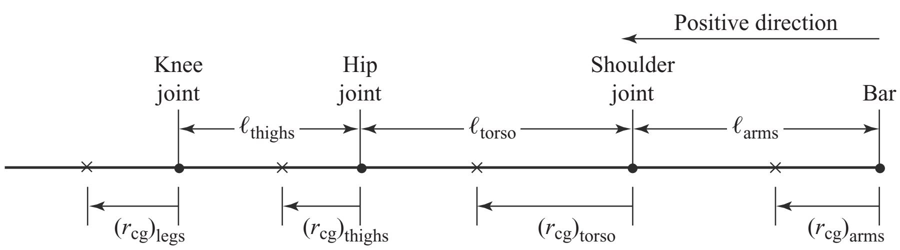
8.15 In each case, the distance from the bar to the center of mass of the body is given by

$$x_{cg} = \frac{\Sigma m_i x_i}{\Sigma m_i} = \frac{m_{\text{arms}}x_{\text{arms}} + m_{\text{torso}}x_{\text{torso}} + m_{\text{thighs}}x_{\text{thighs}} + m_{\text{legs}}x_{\text{legs}}}{m_{\text{arms}} + m_{\text{torso}} + m_{\text{thighs}} + m_{\text{legs}}}$$

where the distance x for any body part is the distance from the bar to the center of gravity of that body part. *In each case, we shall take the positive direction for distances to run from the bar toward the location of the head*.

Note that $\Sigma m_i = (6.87 + 33.57 + 14.07 + 7.54)\ \text{kg} = 62.05\ \text{kg}$

With the body positioned as shown in Figure P8.15b, the distances x for each body part is computed using the sketch given below:



$$x_{\text{arms}} = +\left(r_{cg}\right)_{\text{arms}} = +0.239\ \text{m}$$

$$x_{\text{torso}} = \ell_{\text{arms}} + \left(r_{cg}\right)_{\text{torso}} = +0.548\ \text{m} + 0.337\ \text{m} = 0.885\ \text{m}$$

$$x_{\text{thighs}} = \ell_{\text{arms}} + \ell_{\text{torso}} + \left(r_{cg}\right)_{\text{thighs}} = (+0.548 + 0.601 + 0.151)\ \text{m} = 1.30\ \text{m}$$

$$x_{\text{legs}} = \ell_{\text{arms}} + \ell_{\text{torso}} + \ell_{\text{thighs}} + \left(r_{cg}\right)_{\text{legs}} = (+0.548 + 0.601 + 0.374 + 0.227)\ \text{m} = 1.75\ \text{m}$$

With these distances and the given masses we find

$$x_{cg} = \frac{+62.8\ \text{kg}\cdot\text{m}}{62.05\ \text{kg}} = \boxed{+1.01\ \text{m}}$$

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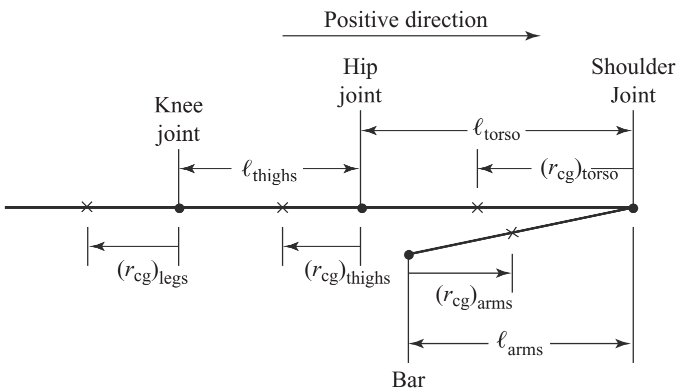
With the body positioned as shown in Figure P8.15c, we use the following sketch to determine the distance x for each body part:



$$x_{arms} = +\left(r_{cg}\right)_{arms} = +0.239\ \text{m}$$

$$x_{torso} = \ell_{arms} - \left(r_{cg}\right)_{torso} = +0.548\ \text{m} - 0.337\ \text{m} = +0.211\ \text{m}$$

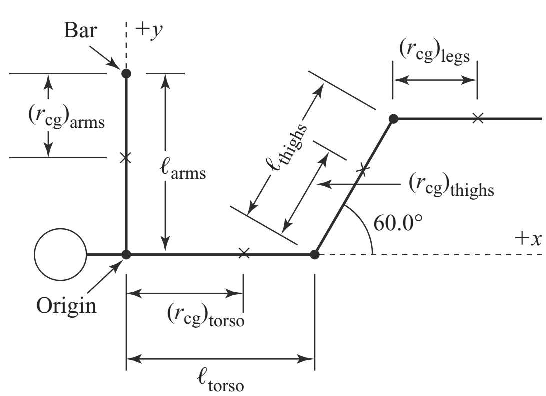
$$x_{thighs} = \ell_{arms} - \ell_{torso} - \left(r_{cg}\right)_{thighs} = \left(+0.548 - 0.601 - 0.151\right)\ \text{m} = -0.204\ \text{m}$$

$$x_{legs} = \ell_{arms} - \ell_{torso} - \ell_{thighs} - \left(r_{cg}\right)_{legs} = \left(+0.548 - 0.601 - 0.374 - 0.227\right)\ \text{m} = -0.654\ \text{m}$$

With these distances, the location (relative to the bar) of the center of gravity of the body is

$$x_{cg} = \frac{+0.924\ \text{kg}\cdot\text{m}}{62.05\ \text{kg}} = \boxed{+0.015\ \text{m}} = \boxed{0.015\ \text{m towards the head}}$$

8.16 With the coordinate system shown at the right, the coordinates of the center of gravity of each body part may be computed:



$$x_{cg,\ arms} = 0 \qquad y_{cg,\ arms} = \ell_{arms} - \left(r_{cg}\right)_{arms} = 0.309\ \text{m}$$

$$x_{cg,\ torso} = \left(r_{cg}\right)_{torso} = 0.337\ \text{m} \qquad y_{cg,\ torso} = 0$$

$$x_{cg,\ thighs} = \ell_{torso} + \left(r_{cg}\right)_{thighs} \cos 60.0° = 0.677\ \text{m} \qquad y_{cg,\ thighs} = \left(r_{cg}\right)_{thighs} \sin 60.0° = 0.131\ \text{m}$$

$$x_{cg,\ legs} = \ell_{torso} + \ell_{thighs} \cos 60.0° + \left(r_{cg}\right)_{legs} = 1.02\ \text{m} \qquad y_{cg,\ legs} = \ell_{thighs} \sin 60.0° = 0.324\ \text{m}$$

With these coordinates for individual body parts and the masses given in Problem 8.15, the coordinates of the center of mass for the entire body are found to be

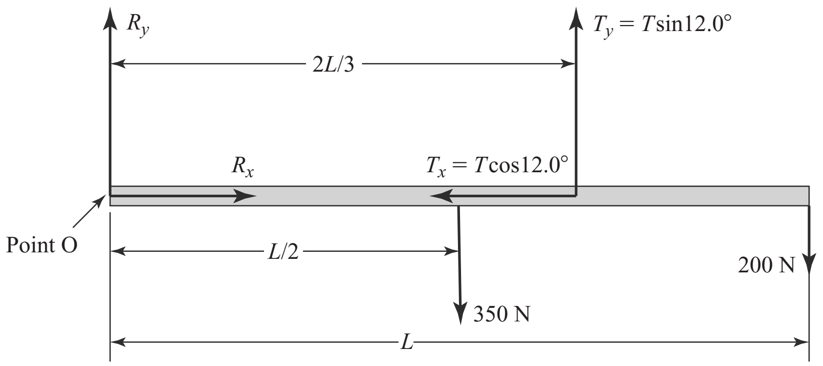
continued on next page

$$x_{cg} = \frac{m_{arms}x_{cg,\,arms} + m_{torso}x_{cg,\,torso} + m_{thighs}x_{cg,\,thighs} + m_{legs}x_{cg,\,legs}}{m_{arms} + m_{torso} + m_{thighs} + m_{legs}} = \frac{28.5\text{ kg}\cdot\text{m}}{62.05\text{ kg}} = \boxed{0.459\text{ m}}$$

and

$$y_{cg} = \frac{m_{arms}y_{cg,\,arms} + m_{torso}y_{cg,\,torso} + m_{thighs}y_{cg,\,thighs} + m_{legs}y_{cg,\,legs}}{m_{arms} + m_{torso} + m_{thighs} + m_{legs}} = \frac{6.41\text{ kg}\cdot\text{m}}{62.05\text{ kg}} = \boxed{0.103\text{ m}}$$

8.17 The force diagram for the spine is shown below.



(a) When the spine is in rotational equilibrium, the sum of the torques about the left end (point O) must be zero. Thus,

$$+T_y\left(\frac{2L}{3}\right) - (350\text{ N})\left(\frac{L}{2}\right) - (200\text{ N})(L) = 0$$

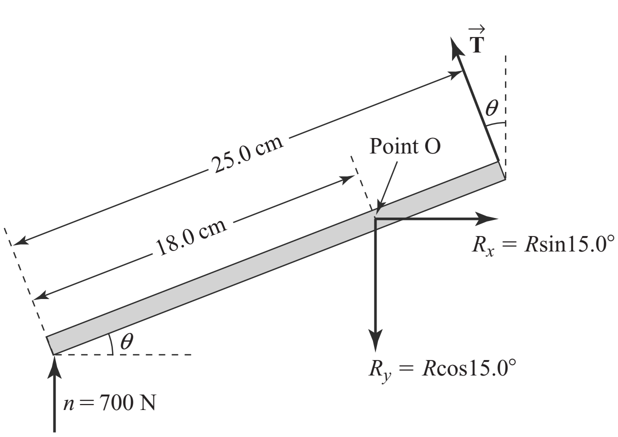
yielding $T_y = T\sin 12.0° = 563\text{ N}$

The tension in the back muscle is then $T = \dfrac{563\text{ N}}{\sin 12.0°} = 2.71\times 10^3\text{ N} = \boxed{2.71\text{ kN}}$.

(b) The spine is also in translational equilibrium, so $\Sigma F_x = 0 \Rightarrow R_x - T_x = 0$, and the compression force in the spine is

$$R_x = T_x = T\cos 12.0° = (2.71\text{ kN})\cos 12.0° = \boxed{2.65\text{ kN}}$$

8.18 In the force diagram of the foot given below, note that the force $\vec{\mathbf{R}}$ (exerted on the foot by the tibia) has been replaced by its horizontal and vertical components. Employing both conditions of equilibrium (using point O as the pivot point) gives the following three equations:



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$$\Sigma F_x = 0 \;\Rightarrow\; R\sin 15.0° - T\sin\theta = 0$$

or $$R = \frac{T\sin\theta}{\sin 15.0°} \qquad \textbf{[1]}$$

$$\Sigma F_y = 0 \;\Rightarrow\; 700\text{ N} - R\cos 15.0° + T\cos\theta = 0 \qquad \textbf{[2]}$$

$$\Sigma\tau_o = 0 \;\Rightarrow\; -(700\text{ N})\left[(18.0\text{ cm})\cos\theta\right] + T(25.0\text{ cm} - 18.0\text{ cm}) = 0$$

or $$T = (1\,800\text{ N})\cos\theta \qquad \textbf{[3]}$$

Substituting Equation [3] into Equation [1] gives

$$R = \left(\frac{1\,800\text{ N}}{\sin 15.0°}\right)\sin\theta\cos\theta \qquad \textbf{[4]}$$

Substituting Equations [3] and [4] into Equation [2] yields

$$\left(\frac{1\,800\text{ N}}{\tan 15.0°}\right)\sin\theta\cos\theta - (1\,800\text{ N})\cos^2\theta = 700\text{ N}$$

which reduces to: $\sin\theta\cos\theta = (\tan 15.0°)\cos^2\theta + 0.104$

Squaring this result and using the identity $\sin^2\theta = 1 - \cos^2\theta$ gives

$$\left[\tan^2(15.0°) + 1\right]\cos^4\theta + \left[(2\tan 15.0°)(0.104) - 1\right]\cos^2 + (0.104)^2 = 0$$

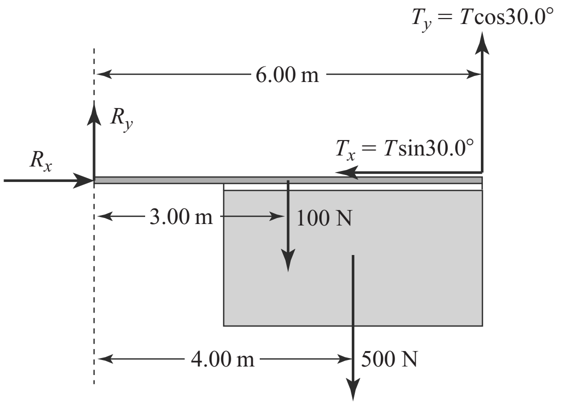
In this last result, let $u = \cos^2\theta$ and evaluate the constants to obtain the quadratic equation: $(1.07)u^2 - (0.944)u + (0.010\,8) = 0$. The quadratic formula yields the solutions $u = 0.871$ and $u = 0.011\,6$.

Thus $\theta = \cos^{-1}\left(\sqrt{0.871}\right) = 21.0°$ or $\theta = \cos^{-1}\left(\sqrt{0.011\,6}\right) = 83.8°$. We ignore the second solution since it is physically impossible for the human foot to stand with the sole inclined at 83.8° to the floor. We are the left with $\theta = \boxed{21.0°}$.

Equation [3] then yields: $$T = (1\,800\text{ N})\cos 21.0° = \boxed{1.68\times10^3\text{ N}}$$

and Equation [1] gives: $$R = \frac{(1.68\times10^3\text{ N})\sin 21.0°}{\sin 15.0°} = \boxed{2.33\times10^3\text{ N}}$$

8.19 Consider the torques about an axis perpendicular to the page through the left end of the rod.



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$$\Sigma\tau = 0 \;\Rightarrow\; T = \frac{(100\text{ N})(3.00\text{ m}) + (500\text{ N})(4.00\text{ m})}{(6.00\text{ m})\cos 30.0°}$$

$$T = \boxed{443\text{ N}}$$

$$\Sigma F_x = 0 \;\Rightarrow\; R_x = T\sin 30.0° = (443\text{ N})\sin 30.0°$$

$$R_x = \boxed{222\text{ N toward the right}}$$

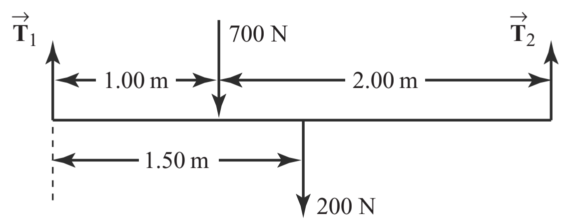
$$\Sigma F_y = 0 \;\Rightarrow\; R_y + T\cos 30.0° - 100\text{ N} - 500\text{ N} = 0$$

$$R_y = 600\text{ N} - (443\text{ N})\cos 30.0° = \boxed{216\text{ N upward}}$$

8.20 Consider the torques about an axis perpendicular to the page through the left end of the scaffold.

$$\Sigma\tau = 0 \Rightarrow T_1(0) - (700\text{ N})(1.00\text{ m}) - (200\text{ N})(1.50\text{ m}) + T_2(3.00\text{ m}) = 0$$

From which, $T_2 = \boxed{333\text{ N}}$



Then, from $\Sigma F_y = 0$, we have

$$T_1 + T_2 - 700\text{ N} - 200\text{ N} = 0$$

or

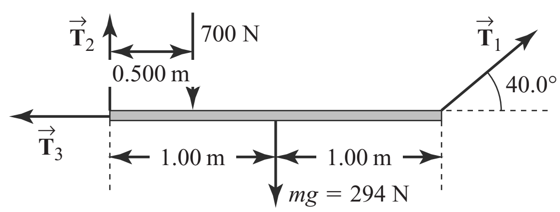
$$T_1 = 900\text{ N} - T_2 = 900\text{ N} - 333\text{ N} = \boxed{567\text{ N}}$$

8.21 Consider the torques about an axis perpendicular to the page and through the left end of the plank.

$\Sigma\tau = 0$ gives

$$-(700\text{ N})(0.500\text{ m}) - (294\text{ N})(1.00\text{ m}) + (T_1\sin 40.0°)(2.00\text{ m}) = 0$$

or $T_1 = \boxed{501\text{ N}}$



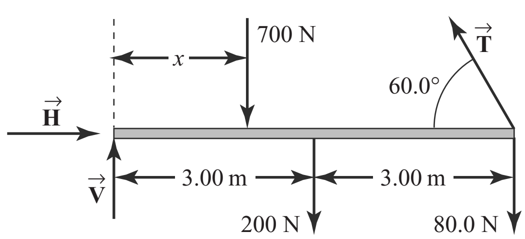
Then, $\Sigma F_x = 0$ gives $-T_3 + T_1\cos 40.0° = 0$,

or $T_3 = (501\text{ N})\cos 40.0° = \boxed{384\text{ N}}$

From $\Sigma F_y = 0$, $T_2 - 994\text{ N} + T_1\sin 40.0° = 0$,

or $T_2 = 994\text{ N} - (501\text{ N})\sin 40.0° = \boxed{672\text{ N}}$

8.22 (a) See the diagram below:



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(b) If $x = 1.00$ m, then

$$\Sigma\tau)_{\text{left end}} = 0 \Rightarrow -(700\text{ N})(1.00\text{ m}) - (200\text{ N})(3.00\text{ m}) - (80.0\text{ N})(6.00\text{ m}) + (T\sin 60.0°)(6.00\text{ m}) = 0$$

giving $T = \boxed{343\text{ N}}$

Then, $\Sigma F_x = 0 \Rightarrow H - T\cos 60.0° = 0,$ or $H = (343\text{ N})\cos 60.0° = \boxed{172\text{ N to the right}}$

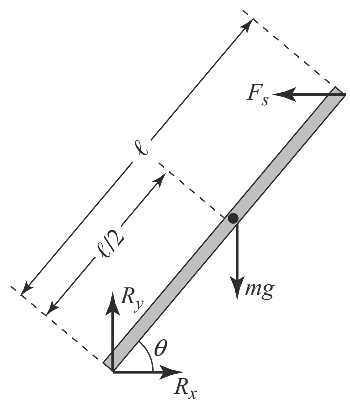
and $\Sigma F_y = 0 \Rightarrow V - 980\text{ N} + (343\text{ N})\sin 60.0° = 0$, or $V = \boxed{683\text{ N upward}}$

(c) When the wire is on the verge of breaking, $T = 900$ N and

$$\Sigma\tau)_{\text{left end}} = -(700\text{ N})x_{\text{max}} - (200\text{ N})(3.00\text{ m}) - (80.0\text{ N})(6.00\text{ m}) + [(900\text{ N})\sin 60.0°](6.00\text{ m}) = 0$$

which gives $x_{\text{max}} = \boxed{5.14\text{ m}}$

8.23 (a) Considering a pivot at the lower end of the beam, we get



$$\Sigma\tau)_{\substack{\text{lower}\\\text{end}}} = 0 \quad\Rightarrow\quad +F_S\ell\sin\theta - mg\left(\frac{\ell}{2}\cos\theta\right) = 0$$

and the spring force is

$$F_S = \frac{mg(\ell\cos\theta/2)}{\ell\sin\theta} = \frac{mg}{2\tan\theta}$$

From Hooke's law, $F_S = kd$, the distance the spring is stretched is then

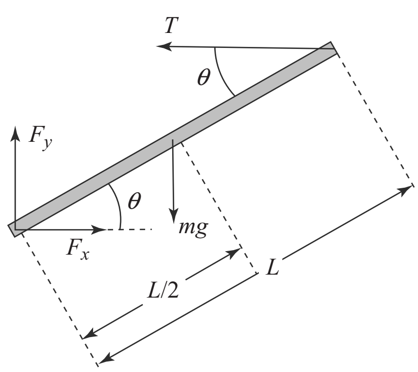
$$d = \frac{F_S}{k} \qquad \text{or} \qquad \boxed{d = \frac{mg}{2k\tan\theta}}$$

(b) From the first condition of equilibrium,

$$\Sigma F_x = 0 \quad\Rightarrow\quad R_x - F_S = 0 \qquad \text{or} \qquad \boxed{R_x = F_S = \frac{mg}{2\tan\theta}}$$

and $\quad \Sigma F_y = 0 \quad\Rightarrow\quad R_y - mg = 0 \qquad \text{or} \qquad \boxed{R_y = mg}$

8.24 (a)



(b) The point of intersection of two unknown forces is always a good choice as the pivot point in a torque calculation. Doing this eliminates these two unknowns from the calculation (since they have zero lever arms about the chosen pivot) and makes it $\boxed{\text{easier to solve the resulting equilibrium equation}}$.

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(c) $\Sigma\tau)_{\text{hinge}} = 0 \quad \Rightarrow \quad \boxed{0 + 0 - mg\left(\dfrac{L}{2}\cos\theta\right) + T\left(L\sin\theta\right) = 0}$

(d) Solving the above result for the tension in the cable gives

$$T = \frac{(mg/2)L\cos\theta}{L\sin\theta} = \frac{mg}{2\tan\theta}$$

or $$T = \frac{(16.0\ \text{kg})(9.80\ \text{m/s}^2)}{2\tan 30.0^\circ} = \boxed{136\ \text{N}}$$

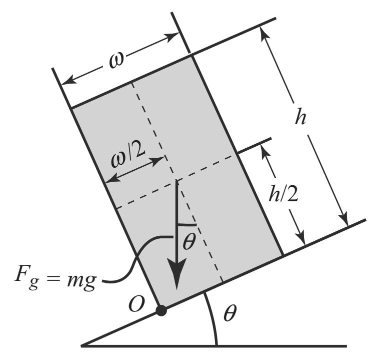
(e) $\Sigma F_x = 0 \quad \Rightarrow \quad \boxed{F_x - T = 0}$ and $\Sigma F_y = 0 \quad \Rightarrow \quad \boxed{F_y - mg = 0}$

(f) Solving the above results for the components of the hinge force gives

$$F_x = T = \boxed{136\ \text{N}} \qquad \text{and} \qquad F_y = mg = (16.0\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{157\ \text{N}}$$

(g) Attaching the cable higher up would allow the cable to bear some of the weight, thereby reducing the stress on the hinge. It would also reduce the tension in the cable.

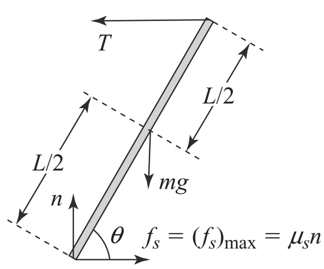
8.25 When the refrigerator is on the verge of tipping (i.e., rotating counterclockwise about point O in the sketch at the right), the center of gravity of the refrigerator will be directly above point O and the line of action of the gravitational force will pass through point O. When this is true, increasing angle θ by any amount will cause the line of action of F_g to pass to the left of point O, producing a counterclockwise torque with no force available to produce a counterbalancing clockwise torque about this point.



At this critical value of angle θ, we have

$$\tan\theta = \frac{w/2}{h/2} \qquad \text{or} \qquad \boxed{\theta = \tan^{-1}(w/h)}$$

8.26 (a)



(b) $\Sigma\tau)_{\substack{\text{lower}\\ \text{end}}} = 0 \quad \Rightarrow \quad 0 + 0 - mg\left(\dfrac{L}{2}\cos\theta\right) + T\left(L\sin\theta\right) = 0$

or $$T = \frac{mg}{2}\left(\frac{\cos\theta}{\sin\theta}\right) = \boxed{\frac{mg}{2}\cot\theta}$$

(c) $\Sigma F_x = 0 \quad \Rightarrow \quad -T + \mu_s n = 0 \qquad \text{or} \qquad T = \mu_s n$ **[1]**

$\Sigma F_y = 0 \quad \Rightarrow \quad n - mg = 0 \qquad \text{or} \qquad n = mg$ **[2]**

Substitute Equation [2] into [1] to obtain $\boxed{T = \mu_s mg}$

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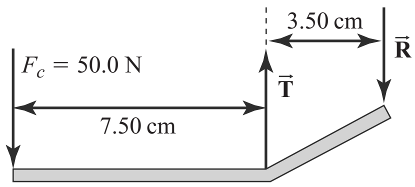
(d) Equate the results of parts (b) and (c) to obtain $\boxed{\mu_s = \frac{1}{2}\cot\theta}$. This result is valid only at the critical angle θ where the beam is on the verge of slipping (i.e., where $f_s = (f_s)_{\text{max}}$ is valid).

(e) At angles below the critical angle (where $f_s = (f_s)_{\text{max}}$ is valid), the beam will slip. At larger angles, the static friction force is reduced below the maximum value, and it is no longer appropriate to use μ_s in the calculation.

8.27 Consider the torques about an axis perpendicular to the page and through the point where the force $\vec{\mathbf{T}}$ acts on the jawbone.



$$\Sigma\tau = 0 \Rightarrow (50.0\text{ N})(7.50\text{ cm}) - R(3.50\text{ cm}) = 0$$

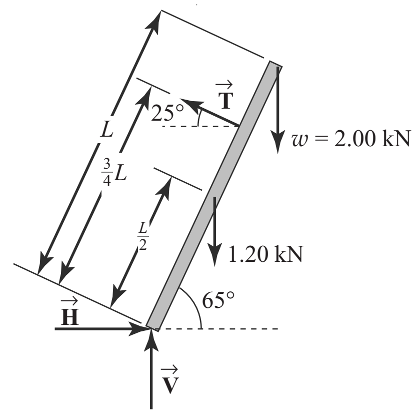
which yields $R = \boxed{107\text{ N}}$

Then, $\Sigma F_y = 0 \Rightarrow -(50.0\text{ N}) + T - 107\text{ N} = 0$, or $T = \boxed{157\text{ N}}$

8.28 (a) Observe that the cable is perpendicular to the boom. Then, using $\Sigma\tau = 0$ for an axis perpendicular to the page and through the lower end of the boom gives

$$-(1.20\text{ kN})\left(\frac{L}{2}\cos 65.0°\right) + T\left(\frac{3}{4}L\right) - (2.00\text{ kN})(L\cos 65.0°) = 0$$

or $T = \boxed{1.47\text{ kN}}$



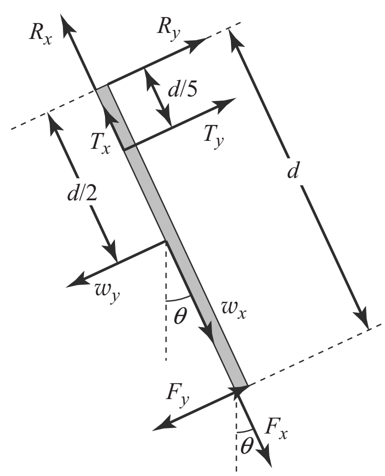
(b) From $\Sigma F_x = 0$,

$$H = T\cos 25.0° = \boxed{1.33\text{ kN to the right}}$$

and $\Sigma F_y = 0$ gives

$$V = 3.20\text{ kN} - T\sin 25.0° = \boxed{2.58\text{ kN upward}}$$

8.29 Choose a reference frame with the x-axis parallel to the tibia and the y-axis perpendicular to it. Then, resolve all forces into their x- and y-components, as shown. Note that $\theta = 40.0°$ and



$$w_y = (30.0\text{ N})\sin 40.0° = 19.3\text{ N}$$

$$F_y = (12.5\text{ N})\sin 40.0° = 8.03\text{ N}$$

and $T_y = T\sin 25.0°$

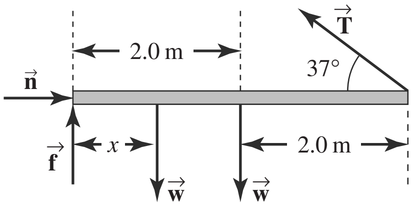
Using $\Sigma\tau = 0$ for an axis perpendicular to the page and through the upper end of the tibia gives

$$(T\sin 25.0°)\frac{\cancel{d}}{5} - (19.3\text{ N})\frac{\cancel{d}}{2} - (8.03\text{ N})\cancel{d} = 0,$$

or $T = \boxed{209\text{ N}}$

8.30 When $x = x_{\text{min}}$, the rod is on the verge of slipping, so

$$f = (f_s)_{\text{max}} = \mu_s n = 0.50n$$



From $\Sigma F_x = 0$, $n - T\cos 37° = 0$, or $n = 0.80T$
Thus, $f = 0.50(0.80T) = 0.40T$.

From $\Sigma F_y = 0$, $f + T\sin 37° - 2w = 0$, or $0.40T + 0.60T - 2w = 0$, giving $T = 2w$.

Using $\Sigma\tau = 0$ for an axis perpendicular to the page and through the left end of the beam gives $-w \cdot x_{\text{min}} - w(2.0\text{ m}) + [(2w)\sin 37°](4.0\text{ m}) = 0$, which reduces to $x_{\text{min}} = \boxed{2.8\text{ m}}$.

8.31 The moment of inertia for rotations about an axis is $I = \Sigma m_i r_i^2$, where r_i is the distance mass m_i is from that axis.

(a) For rotation about the x-axis,

$$I_x = (3.00\text{ kg})(3.00\text{ m})^2 + (2.00\text{ kg})(3.00\text{ m})^2 + (2.00\text{ kg})(3.00\text{ m})^2 + (4.00\text{ kg})(3.00\text{ m})^2 = \boxed{99.0\text{ kg}\cdot\text{m}^2}$$

(b) When rotating about the y-axis,

$$I_y = (3.00\text{ kg})(2.00\text{ m})^2 + (2.00\text{ kg})(2.00\text{ m})^2 + (2.00\text{ kg})(2.00\text{ m})^2 + (4.00\text{ kg})(2.00\text{ m})^2 = \boxed{44.0\text{ kg}\cdot\text{m}^2}$$

(c) For rotations about an axis perpendicular to the page through point O, the distance r_i for each mass is $r_i = \sqrt{(2.00\text{ m})^2 + (3.00\text{ m})^2} = \sqrt{13.0}$ m. Thus,

$$I_O = [(3.00 + 2.00 + 2.00 + 4.00)\text{ kg}](13.0\text{ m}^2) = \boxed{143\text{ kg}\cdot\text{m}^2}$$

8.32 The required torque in each case is $\tau = I\alpha$. Thus,

$$\tau_x = I_x\alpha = (99.0\text{ kg}\cdot\text{m}^2)(1.50\text{ rad/s}^2) = \boxed{149\text{ N}\cdot\text{m}}$$

$$\tau_y = I_y\alpha = (44.0\text{ kg}\cdot\text{m}^2)(1.50\text{ rad/s}^2) = \boxed{66.0\text{ N}\cdot\text{m}}$$

and $$\tau_O = I_O\alpha = (143\text{ kg}\cdot\text{m}^2)(1.50\text{ rad/s}^2) = \boxed{215\text{ N}\cdot\text{m}}$$

8.33 (a) $$\tau_{\text{net}} = I\alpha \quad\Rightarrow\quad I = \frac{\tau_{\text{net}}}{\alpha} = \frac{rF\sin 90°}{\alpha} = \frac{(0.330\text{ m})(250\text{ N})}{0.940\text{ rad/s}^2} = \boxed{87.8\text{ kg}\cdot\text{m}^2}$$

(b) For a solid cylinder, $I = Mr^2/2$, so

$$M = \frac{2I}{r^2} = \frac{2(87.8\text{ kg}\cdot\text{m}^2)}{(0.330\text{ m})^2} = \boxed{1.61\times10^3\text{ kg}}$$

(c) $$\omega = \omega_0 + \alpha t = 0 + (0.940\text{ rad/s}^2)(5.00\text{ s}) = \boxed{4.70\text{ rad/s}}$$

8.34 (a) $I = 2I_{\text{disk}} + I_{\text{cylinder}} = 2(MR^2/2) + mr^2/2$ or $\boxed{I = MR^2 + mr^2/2}$

continued on next page

(b) $\boxed{\tau_g = 0}$ Since the line of action of the gravitational force passes through the rotation axis, it has zero lever arm about this axis and zero torque.

(c) The torque due to the tension force is $\boxed{\text{positive.}}$ Imagine gripping the cylinder with your right hand so your fingers on the front side of the cylinder point upward in the direction of the tension force. The thumb of your right hand then points toward the left (positive direction) along the rotation axis. Because $\vec{\tau} = I\vec{\alpha}$, the torque and angular acceleration have the same direction. Thus, a positive torque produces a $\boxed{\text{positive angular acceleration.}}$ When released, the center of mass of the yoyo drops downward, in the negative direction. $\boxed{\text{The translational acceleration is negative.}}$

(d) Since, with the chosen sign convention, the translational acceleration is negative when the angular acceleration is positive, we must include a negative sign in the proportionality between these two quantities. Thus, we write: $a = -r\alpha$ or $\boxed{\alpha = -a/r}$.

(e) Translation: $\quad \Sigma F_y = m_{\text{total}}a \quad \Rightarrow \quad \boxed{T - (2M + m)g = (2M + m)a}$ **[1]**

(f) Rotational: $\quad \Sigma\tau = I\alpha \quad \Rightarrow \quad rT\sin 90° = I\alpha \quad$ or $\quad \boxed{rT = I\alpha}$ **[2]**

(g) Substitute the results of (d) and (a) into Equation [2] to obtain

$$T = I\left(\frac{\alpha}{r}\right) = I\left(\frac{-a/r}{r}\right) = -\left(MR^2 + mr^2/2\right)\frac{a}{r^2} \text{ or } T = -\left[M\left(R/r\right)^2 + m/2\right]a \qquad \textbf{[3]}$$

Substituting Equation [3] into [1] yields

$$-\left[M\left(R/r\right)^2 + m/2\right]a - (2M + m)g = (2M + m)a$$

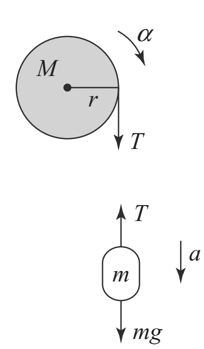
or $$\boxed{a = \frac{-(2M + m)g}{2M + M\left(R/r\right)^2 + 3m/2}}$$

(h) $$a = \frac{-\left[2(2.00\text{ kg}) + 1.00\text{ kg}\right]\left(9.80\ \text{m/s}^2\right)}{2(2.00\text{ kg}) + (2.00\text{ kg})(10.0/4.00)^2 + 3(1.00\text{ kg})/2} = \boxed{-2.72\ \text{m/s}^2}$$

(i) From Equation [1], $T = (2M + m)(g + a) = (5.00\text{ kg})\left(9.80\ \text{m/s}^2 - 2.72\ \text{m/s}^2\right) = \boxed{35.4\text{ N}}$

(j) $\Delta y = (0)t + at^2/2 \quad \Rightarrow \quad t = \sqrt{\dfrac{2(\Delta y)}{a}} = \sqrt{\dfrac{2(-1.00\text{ m})}{-2.72\ \text{m/s}^2}} = \boxed{0.857\text{ s}}$

8.35 (a) Consider the force diagrams of the cylinder and man given at the right. Note that we shall adopt a sign convention with clockwise and downward as the positive directions. Thus, both a and α are positive in the indicated directions and $a = r\alpha$. We apply the appropriate form of Newton's second law to each diagram to obtain:



Rotation of Cylinder: $\tau = I\alpha \quad \Rightarrow \quad rT\sin 90° = I\alpha \quad$ or $\quad T = I\alpha/r$

so

$$T = \frac{1}{r}\left(\frac{1}{2}Mr^2\right)\left(\frac{a}{r}\right) \qquad \text{giving} \qquad T = \frac{1}{2}Ma \qquad \textbf{[1]}$$

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Translation of man:

$$\Sigma F_y = ma \quad \Rightarrow \quad mg - T = ma \qquad \text{or} \qquad T = m(g-a) \qquad \textbf{[2]}$$

Equating Equations [1] and [2] gives $\frac{1}{2}Ma = m(g-a)$, or

$$a = \frac{mg}{m + M/2} = \frac{(75.0\text{ kg})(9.80\text{ m/s}^2)}{75.0\text{ kg} + (225\text{ kg}/2)} = \boxed{3.92\text{ m/s}^2}$$

(b) From $a = r\alpha$, we have $\alpha = \dfrac{a}{r} = \dfrac{3.92\text{ m/s}^2}{0.400\text{ m}} = \boxed{9.80\text{ rad/s}^2}$

(c) As the rope leaves the cylinder, the mass of the cylinder decreases, thereby decreasing the moment of inertia. At the same time, the weight of the rope leaving the cylinder would increase the downward force acting tangential to the cylinder, and hence increase the torque exerted on the cylinder. Both of these effects $\boxed{\text{will cause the acceleration of the system to increase with time.}}$ (The increase would be slight in this case, given the large mass of the cylinder.)

8.36 The angular acceleration is $\alpha = (\omega_f - \omega_i)/\Delta t = -(\omega_i/\Delta t)$ since $\omega_f = 0$.

The torque is $\tau = I\alpha = -(I\omega_i/\Delta t)$. But the torque is also $\tau = -fr$, so the magnitude of the required friction force is

$$f = \frac{I\omega_i}{r(\Delta t)} = \frac{(12\text{ kg}\cdot\text{m}^2)(50\text{ rev/min})}{(0.50\text{ m})(6.0\text{ s})}\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = 21\text{ N}$$

Therefore, the coefficient of friction is

$$\mu_k = \frac{f}{n} = \frac{21\text{ N}}{70\text{ N}} = \boxed{0.30}$$

8.37 (a) $\tau = F\cdot r\sin\theta = (0.800\text{ N})(30.0\text{ m})\sin 90.0° = \boxed{24.0\text{ N}\cdot\text{m}}$

(b) $\alpha = \dfrac{\tau}{I} = \dfrac{\tau}{mr^2} = \dfrac{24.0\text{ N}\cdot\text{m}}{(0.750\text{ kg})(30.0\text{ m})^2} = \boxed{0.035\,6\text{ rad/s}^2}$

(c) $a_t = r\alpha = (30.0\text{ m})(0.035\,6\text{ rad/s}^2) = \boxed{1.07\text{ m/s}^2}$

8.38 $I = MR^2 = (1.80\text{ kg})(0.320\text{ m})^2 = 0.184\text{ kg}\cdot\text{m}^2$

$$\tau_{\text{net}} = \tau_{\text{applied}} - \tau_{\text{resistive}} = I\alpha, \quad \text{or} \quad F\cdot r - f\cdot R = I\alpha$$

yielding $F = (I\alpha + f\cdot R)/r$

(a) $F = \dfrac{(0.184\text{ kg}\cdot\text{m}^2)(4.50\text{ rad/s}^2) + (120\text{ N})(0.320\text{ m})}{4.50\times 10^{-2}\text{ m}} = \boxed{872\text{ N}}$

(b) $F = \dfrac{(0.184\text{ kg}\cdot\text{m}^2)(4.50\text{ rad/s}^2) + (120\text{ N})(0.320\text{ m})}{2.80\times 10^{-2}\text{ m}} = \boxed{1.40\text{ kN}}$

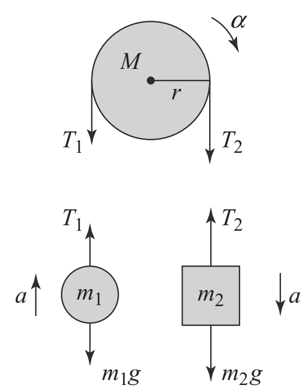
8.39 $I = \frac{1}{2}MR^2 = \frac{1}{2}(150 \text{ kg})(1.50 \text{ m})^2 = 169 \text{ kg} \cdot \text{m}^2$

and $\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{(0.500 \text{ rev/s} - 0)}{2.00 \text{ s}}\left(\frac{2\pi \text{ rad}}{1 \text{ rev}}\right) = \frac{\pi}{2} \text{ rad/s}^2$

Thus, $\tau = F \cdot r = I\alpha$ gives

$$F = \frac{I\alpha}{r} = \frac{(169 \text{ kg} \cdot \text{m}^2)\left(\frac{\pi}{2} \text{ rad/s}^2\right)}{1.50 \text{ m}} = \boxed{177 \text{ N}}$$

8.40 (a) It is necessary that the tensions T_1 and T_2 be different in order $\boxed{\text{to provide a net torque}}$ about the axis of the pulley $\boxed{\text{and produce an angular acceleration}}$ of the pulley. Since intuition tells us that the system will accelerate in the directions shown in the diagrams at the right when $m_2 > m_1$, it is necessary that $T_2 > T_1$.



(b) We adopt a sign convention for each object with the positive direction being the indicated direction of the acceleration of that object in the diagrams at the right. Then, apply Newton's second law to each object:

For m_1: $\Sigma F_y = m_1 a \;\Rightarrow\; T_1 - m_1 g = m_1 a$ or $T_1 = m_1(g + a)$ **[1]**

For m_2: $\Sigma F_y = m_2 a \;\Rightarrow\; m_2 g - T_2 = m_2 a$ or $T_2 = m_2(g - a)$ **[2]**

For M: $\Sigma\tau = I\alpha \;\Rightarrow\; rT_2 - rT_1 = I\alpha$ or $T_2 - T_1 = I\alpha/r$ **[3]**

Substitute Equations [1] and [2], along with the relations $I = Mr^2/2$ and $\alpha = a/r$, into Equation [3] to obtain

$$m_2(g - a) - m_1(g + a) = \frac{Mr^2}{2r}\left(\frac{a}{r}\right) = \frac{Ma}{2} \quad \text{or} \quad \left(m_1 + m_2 + \frac{M}{2}\right)a = (m_2 - m_1)g$$

and $$a = \frac{(m_2 - m_1)g}{m_1 + m_2 + M/2} = \frac{(20.0 \text{ kg} - 10.0 \text{ kg})(9.80 \text{ m/s}^2)}{20.0 \text{ kg} + 10.0 \text{ kg} + (8.00 \text{ kg})/2} = \boxed{2.88 \text{ m/s}^2}$$

(c) From Equation [1]: $T_1 = (10.0 \text{ kg})(9.80 \text{ m/s}^2 + 2.88 \text{ m/s}^2) = \boxed{127 \text{ N}}$

From Equation [2]: $T_2 = (20.0 \text{ kg})(9.80 \text{ m/s}^2 - 2.88 \text{ m/s}^2) = \boxed{138 \text{ N}}$

8.41 The initial angular velocity of the wheel is zero, and the final angular velocity is

$$\omega_f = \frac{v}{r} = \frac{50.0 \text{ m/s}}{1.25 \text{ m}} = 40.0 \text{ rad/s}$$

Hence, the angular acceleration is

$$\alpha = \frac{\omega_f - \omega_i}{\Delta t} = \frac{40.0 \text{ rad/s} - 0}{0.480 \text{ s}} = 83.3 \text{ rad/s}^2$$

continued on next page

The torque acting on the wheel is $\tau = f_k \cdot r$, so $\tau = I\alpha$ gives

$$f_k = \frac{I\alpha}{r} = \frac{(110\ \text{kg}\cdot\text{m}^2)(83.3\ \text{rad/s}^2)}{1.25\ \text{m}} = 7.33\times10^3\ \text{N}$$

Thus, the coefficient of friction is

$$\mu_k = \frac{f_k}{n} = \frac{7.33\times10^3\ \text{N}}{1.40\times10^4\ \text{N}} = \boxed{0.524}$$

8.42 (a) The moment of inertia of the flywheel is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}(500\ \text{kg})(2.00\ \text{m})^2 = 1.00\times10^3\ \text{kg}\cdot\text{m}^2$$

and the angular velocity is

$$\omega = \left(5000\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 524\ \text{rad/s}$$

Therefore, the stored kinetic energy is

$$KE_{\text{stored}} = \frac{1}{2}I\omega^2 = \frac{1}{2}(1.00\times10^3\ \text{kg}\cdot\text{m}^2)(524\ \text{rad/s})^2 = \boxed{1.37\times10^8\ \text{J}}$$

(b) A 10.0-hp motor supplies energy at the rate of

$$P = (10.0\ \text{hp})\left(\frac{746\ \text{W}}{1\ \text{hp}}\right) = 7.46\times10^3\ \text{J/s}$$

The time the flywheel could supply energy at this rate is

$$t = \frac{KE_{\text{stored}}}{P} = \frac{1.37\times10^8\ \text{J}}{7.46\times10^3\ \text{J/s}} = 1.84\times10^4\ \text{s}\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{5.11\ \text{h}}$$

8.43 The moment of inertia of the cylinder is

$$I = \frac{1}{2}MR^2 = \frac{1}{2}\left(\frac{w}{g}\right)R^2 = \frac{1}{2}\left(\frac{800\ \text{N}}{9.80\ \text{m/s}^2}\right)(1.50\ \text{m})^2 = 91.8\ \text{kg}\cdot\text{m}^2$$

The angular acceleration is given by

$$\alpha = \frac{\tau}{I} = \frac{F\cdot R}{I} = \frac{(50.0\ \text{N})(1.50\ \text{m})}{91.8\ \text{kg}\cdot\text{m}^2} = 0.817\ \text{rad/s}^2$$

At $t = 3.00$ s, the angular velocity is

$$\omega = \omega_i + \alpha t = 0 + (0.817\ \text{rad/s}^2)(3.00\ \text{s}) = 2.45\ \text{rad/s}$$

and the kinetic energy is

$$KE_r = \frac{1}{2}I\omega^2 = \frac{1}{2}(91.8\ \text{kg}\cdot\text{m}^2)(2.45\ \text{rad/s})^2 = \boxed{276\ \text{J}}$$

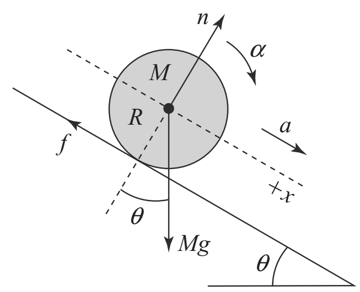
8.44 (a) Hoop: $I = MR^2 = (4.80\text{ kg})(0.230\text{ m})^2 = \boxed{0.254\text{ kg}\cdot\text{m}^2}$

Solid Cylinder: $I = \dfrac{1}{2}MR^2 = \dfrac{1}{2}(4.80\text{ kg})(0.230\text{ m})^2 = \boxed{0.127\text{ kg}\cdot\text{m}^2}$

Solid Sphere: $I = \dfrac{2}{5}MR^2 = \dfrac{2}{5}(4.80\text{ kg})(0.230\text{ m})^2 = \boxed{0.102\text{ kg}\cdot\text{m}^2}$

Thin Spherical Shell: $I = \dfrac{2}{3}MR^2 = \dfrac{2}{3}(4.80\text{ kg})(0.230\text{ m})^2 = \boxed{0.169\text{ kg}\cdot\text{m}^2}$

(b) When different objects of mass M and radius R roll without slipping $(\Rightarrow\ a = R\alpha)$ down a ramp, the one with the largest translational acceleration a will have the highest translational speed at the bottom. To determine the translational acceleration for the various objects, consider the force diagram at the right:



$$\Sigma F_x = Ma \quad \Rightarrow \quad Mg\sin\theta - f = Ma \qquad \textbf{[1]}$$

$$\tau = I\alpha \quad \Rightarrow \quad f\,R = I(a/R) \quad \text{or} \quad f = Ia/R^2 \qquad \textbf{[2]}$$

Substitute Equation [2] into [1] to obtain

$$Mg\sin\theta - Ia/R^2 = Ma \qquad \text{or} \qquad a = \frac{Mg\sin\theta}{M + I/R^2}$$

Since M, R, g, and θ are the same for all of the objects, we see that the translational acceleration (and hence, the translational speed) increases as the moment of inertia decreases. Thus, the proper rankings from highest to lowest by translational speed will be:

$\boxed{\text{solid sphere; solid cylinder; thin spherical shell; and hoop}}$

(c) When an object rolls down the ramp without slipping, the friction force does no work and mechanical energy is conserved. Then, the total kinetic energy gained equals the gravitational potential energy given up: $KE_r + KE_t = -\Delta PE_g = Mgh$ and $KE_r = Mgh - \frac{1}{2}Mv^2$, where h is the vertical drop of the ramp and v is the translational speed at the bottom. Since M, g, and h are the same for all of the objects, the rotational kinetic energy decreases as the translational speed increases. Using this fact, along with the result of part (b), we rank the object's final rotational kinetic energies, from highest to lowest, as:

$\boxed{\text{hoop; thin spherical shell; solid cylinder; and solid sphere}}$

8.45 (a) Treating the particles on the ends of the rod as point masses, the total moment of inertia of the rotating system is $I = I_{\text{rod}} + I_1 + I_2 = m_{\text{rod}}\ell^2/12 + m_1(\ell/2)^2 + m_2(\ell/2)^2$. If the mass of the rod can be ignored, this reduces to $I = 0 + (m_1 + m_2)(\ell/2)^2$, and the rotational kinetic energy is

$$KE_r = \frac{1}{2}I\omega^2 = \frac{1}{2}\left[(3.00\text{ kg} + 4.00\text{ kg})(0.500\text{ m})^2\right](2.50\text{ rad/s})^2 = \boxed{5.47\text{ J}}$$

continued on next page

(b) If the rod has mass $m_{\text{rod}} = 2.00$ kg, the rotational kinetic energy is

$$KE_r = \frac{1}{2}I\omega^2 = \frac{1}{2}\left[\frac{1}{12}(2.00\text{ kg})(1.00\text{ m})^2 + (3.00\text{ kg} + 4.00\text{ kg})(0.500\text{ m})^2\right](2.50\text{ rad/s})^2$$

or $KE_r = \boxed{5.99\text{ J}}$.

8.46 Using conservation of mechanical energy,

$$\left(KE_t + KE_r + PE_g\right)_f = \left(KE_t + KE_r + PE_g\right)_i$$

or $$\frac{1}{2}Mv_t^2 + \frac{1}{2}I\omega^2 + 0 = 0 + 0 + Mg(L\sin\theta)$$

Since $I = \frac{2}{5}MR^2$ for a solid sphere and $v_t = R\omega$ when rolling without slipping, this becomes

$$\frac{1}{2}MR^2\omega^2 + \frac{1}{5}MR^2\omega^2 = Mg(L\sin\theta)$$

and reduces to

$$\omega = \sqrt{\frac{10gL\sin\theta}{7R^2}} = \sqrt{\frac{10(9.8\text{ m/s}^2)(6.0\text{ m})\sin 37°}{7(0.20\text{ m})^2}} = \boxed{36\text{ rad/s}}$$

8.47 (a) Assuming the disk rolls without slipping, the angular speed of the disk is $\omega = v/R$, where v is the translational speed of the center of the disk. Also, if the disk does not slip, the friction force between disk and ramp does no work and total mechanical energy is conserved. Hence, $(KE_t + KE_r + PE_g)_f = (KE_t + KE_r + PE_g)_i$, or

$$\frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2 + 0 = 0 + 0 + mgh_i$$

Since $I = MR^2/2$, and $h_i = L\sin\theta = (4.50\text{ m})\sin 15.0°$, we have

$$\frac{1}{2}Mv^2 + \frac{1}{2}\left(\frac{M\cancel{R^2}}{2}\right)\left(\frac{v^2}{\cancel{R^2}}\right) = mgL\sin\theta$$

and $$v = \sqrt{\frac{4gL\sin\theta}{3}} = \sqrt{\frac{4(9.80\text{ m/s}^2)(4.50\text{ m})\sin 15.0°}{3}} = \boxed{3.90\text{ m/s}}$$

(b) The angular speed of the disk at the bottom is

$$\omega = \frac{v}{R} = \frac{3.90\text{ m/s}}{0.250\text{ m}} = \boxed{15.6\text{ rad/s}}$$

8.48 (a) Assuming the solid sphere starts from rest, and taking $y = 0$ at the level of the bottom of the incline, the total mechanical energy $E = \left(PE_g\right)_i = mgh$ will be split among three distinct forms of energy as the sphere rolls down the incline. These are $\boxed{\text{rotational kinetic energy, } \frac{1}{2}I\omega^2}$; $\boxed{\text{translational kinetic energy, } \frac{1}{2}mv^2}$; and $\boxed{\text{gravitational potential energy, } mgy}$, where y is the current height of the center of mass of the sphere above the level of the bottom of the incline.

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(b) The $\boxed{\text{force of static friction, exerted on the sphere by the incline}}$ and directed up the incline, exerts a torque about the center of mass, giving the sphere an angular acceleration.

(c) $KE_t = \frac{1}{2}Mv^2$ and $KE_r = \frac{1}{2}I\omega^2$, where $v = R\omega$ (since the sphere rolls without slipping), and $I = \frac{2}{5}MR^2$ for a solid sphere. Therefore,

$$\frac{KE_r}{KE_t + KE_r} = \frac{I\omega^2/2}{Mv^2/2 + I\omega^2/2} = \frac{\left(2MR^2/5\right)\omega^2}{M(R\omega)^2 + \left(2MR^2/5\right)\omega^2} = \frac{2\cancel{MR^2\omega^2}}{5\cancel{MR^2\omega^2} + 2\cancel{MR^2\omega^2}} = \boxed{\frac{2}{7}}$$

8.49 Using $W_{\text{net}} = KE_f - KE_i = \frac{1}{2}I\omega_f^2 - 0$, we have

$$\omega_f = \sqrt{\frac{2W_{\text{net}}}{I}} = \sqrt{\frac{2F\cdot s}{I}} = \sqrt{\frac{2(5.57\text{ N})(0.800\text{ m})}{4.00\times 10^{-4}\text{ kg}\cdot\text{m}^2}} = \boxed{149\text{ rad/s}}$$

8.50 The work done on the grindstone is $W_{\text{net}} = F\cdot s = F\cdot(r\theta) = (F\cdot r)\theta = \tau\cdot\theta$

Thus, $W_{\text{net}} = \Delta KE$ becomes $\tau\cdot\theta = \frac{1}{2}I\omega_f^2 - \frac{1}{2}I\omega_i^2$, or

$$(25.0\text{ N}\cdot\text{m})(15.0\text{ rev})\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = \frac{1}{2}\left(0.130\text{ kg}\cdot\text{m}^2\right)\omega_f^2 - 0$$

This yields

$$\omega_f = \left(190\ \frac{\text{rad}}{\text{s}}\right)\left(\frac{1\text{ rev}}{2\pi\text{ rad}}\right) = \boxed{30.2\text{ rev/s}}$$

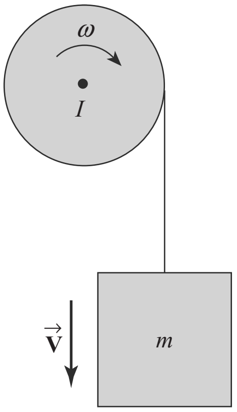
8.51 (a) $KE_t = \frac{1}{2}mv_t^2 = \frac{1}{2}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{500\text{ J}}$

(b) $KE_r = \frac{1}{2}I\omega^2 = \frac{1}{2}\left(\frac{1}{2}m\cancel{R^2}\right)\left(\frac{v_t^2}{\cancel{R^2}}\right)$

$$= \frac{1}{4}mv_t^2 = \frac{1}{4}(10.0\text{ kg})(10.0\text{ m/s})^2 = \boxed{250\text{ J}}$$

(c) $KE_{\text{total}} = KE_t + KE_r = \boxed{750\text{ J}}$

8.52 As the bucket drops, it loses gravitational potential energy. The spool gains rotational kinetic energy and the bucket gains translational kinetic energy. Since the string does not slip on the spool, $v = r\omega$ where r is the radius of the spool. The moment of inertia of the spool is $I = \frac{1}{2}Mr^2$, where M is the mass of the spool. Conservation of energy gives



$$\left(KE_t + KE_r + PE_g\right)_f = \left(KE_t + KE_r + PE_g\right)_i$$

$$\frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 + mgy_f = 0 + 0 + mgy_i$$

or $$\frac{1}{2}m(r\omega)^2 + \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\omega^2 = mg\left(y_i - y_f\right)$$

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This gives

$$\omega = \sqrt{\frac{2mg\left(y_i - y_f\right)}{\left(m + \frac{1}{2}M\right)r^2}} = \sqrt{\frac{2(3.00\text{ kg})\left(9.80\text{ m/s}^2\right)(4.00\text{ m})}{\left[3.00\text{ kg} + \frac{1}{2}(5.00\text{ kg})\right](0.600\text{ m})^2}} = \boxed{10.9\text{ rad/s}}$$

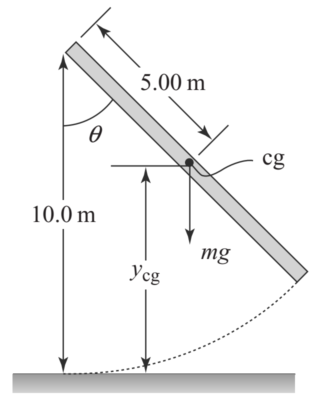
8.53 (a) The arm consists of a uniform rod of 10.0 m length and the mass of the seats at the lower end is negligible. The center of gravity of this system is then located at the $\boxed{\text{geometric center of the arm, located 5.00 m from the upper end.}}$

From the sketch below, the height of the center of gravity above the zero level (chosen to be 10.0 m below the axis) is $y_{cg} = 10.0\text{ m} - (5.00\text{ m})\cos\theta$.

(b) When $\theta = 45.0°$, $y_{cg} = 10.0\text{ m} - (5.00\text{ m})\cos 45.0°$ and $PE_g = mgy_{cg}$ gives

$$PE_g = (365\text{ kg})\left(9.80\text{ m/s}^2\right)\left[10.0\text{ m} - (5.00\text{ m})\cos 45.0°\right]$$

$$= \boxed{2.31 \times 10^4\text{ J}}$$



(c) In the vertical orientation, $\theta = 0°$ and $\cos\theta = 1$, giving $y_{cg} = 10.0\text{ m} - 5.00\text{ m} = 5.00\text{ m}$. Then,

$$PE_g = mgy_{cg} = (365\text{ kg})\left(9.80\text{ m/s}^2\right)(5.00\text{ m}) = \boxed{1.79 \times 10^4\text{ J}}$$

(d) Using conservation of mechanical energy as the arm starts from rest in the 45° orientation and rotates about the upper end to the vertical orientation gives

$$\frac{1}{2}I_{\text{end}}\omega_f^2 + mg\left(y_{cg}\right)_f = 0 + mg\left(y_{cg}\right)_i \quad \text{or} \quad \omega_f = \sqrt{\frac{2mg\left[\left(y_{cg}\right)_i - \left(y_{cg}\right)_f\right]}{I_{\text{end}}}} \qquad \textbf{[1]}$$

For a long, thin rod, $I_{\text{end}} = mL^2/3$, and Equation [1] becomes

$$\omega_f = \sqrt{\frac{2\cancel{m}g\left[\left(y_{cg}\right)_i - \left(y_{cg}\right)_f\right]}{\cancel{m}L^2/3}} = \sqrt{\frac{6g\left[\left(y_{cg}\right)_i - \left(y_{cg}\right)_f\right]}{L^2}}$$

Then, from $v = r\omega$ with $r = L$, the translational speed of the seats at the lower end of the rod is

$$v = \cancel{L}\sqrt{\frac{6g\left[\left(y_{cg}\right)_i - \left(y_{cg}\right)_f\right]}{\cancel{L^2}}}$$

$$= \sqrt{6\left(9.80\text{ m/s}^2\right)\left[(10.0 - 5.00\cos 45.0°)\text{ m} - 5.00\text{ m}\right]} = \boxed{9.28\text{ m/s}}$$

8.54 (a) $L = I\omega = \left(MR^2\right)\omega = (2.40\text{ kg})(0.180\text{ m})^2(35.0\text{ rad/s}) = \boxed{2.72\text{ kg}\cdot\text{m}^2/\text{s}}$

(b) $L = I\omega = \left(\frac{1}{2}MR^2\right)\omega = \frac{1}{2}(2.40\text{ kg})(0.180\text{ m})^2(35.0\text{ rad/s}) = \boxed{1.36\text{ kg}\cdot\text{m}^2/\text{s}}$

(c) $L = I\omega = \left(\frac{2}{5}MR^2\right)\omega = \frac{2}{5}(2.40\text{ kg})(0.180\text{ m})^2(35.0\text{ rad/s}) = \boxed{1.09\text{ kg}\cdot\text{m}^2/\text{s}}$

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(d) $$L = I\omega = \left(\frac{2}{3}MR^2\right)\omega = \frac{2}{3}(2.40\ \text{kg})(0.180\ \text{m})^2(35.0\ \text{rad/s}) = \boxed{1.81\ \text{kg}\cdot\text{m}^2/\text{s}}$$

8.55 (a) The rotational speed of Earth is $\omega_E = \dfrac{2\pi\ \text{rad}}{1\ \text{d}}\left(\dfrac{1\ \text{d}}{8.64\times10^4\ \text{s}}\right) = 7.27\times10^{-5}\ \text{rad/s}$

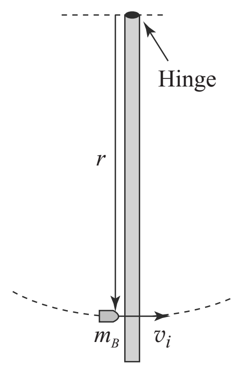
$$L_{\text{spin}} = I_{\text{sphere}}\omega_E = \left(\frac{2}{5}M_E R_E^2\right)\omega_E$$

$$= \left[\frac{2}{5}\left(5.98\times10^{24}\ \text{kg}\right)\left(6.38\times10^{6}\ \text{m}\right)^2\right]\left(7.27\times10^{-5}\ \text{rad/s}\right) = \boxed{7.08\times10^{33}\ \text{J}\cdot\text{s}}$$

(b) For Earth's orbital motion, $\omega_{\text{orbit}} = 2\pi\ \text{rad/y}$ and $L_{\text{orbit}} = I_{\text{point}}\omega_{\text{orbit}} = \left(M_E R_{\text{orbit}}^2\right)\omega_{\text{orbit}}$. Using data from Table 7.3, we find

$$L_{\text{orbit}} = \left(5.98\times10^{24}\ \text{kg}\right)\left(1.496\times10^{11}\ \text{m}\right)^2\left(2\pi\ \text{rad/y}\right)\left(\frac{1\ \text{y}}{3.156\times10^{7}\ \text{s}}\right) = \boxed{2.66\times10^{40}\ \text{J}\cdot\text{s}}$$

8.56 (a) $\boxed{\text{Yes}}$, the bullet has angular momentum about an axis through the hinges of the door before the collision. Consider the sketch at the right, showing the bullet the instant before it hits the door. The physical situation is identical to that of a point mass m_B moving in a circular path of radius r with tangential speed $v_t = v_i$. For that situation the angular momentum is



$$L_i = I_i\omega_i = \left(m_B r^2\right)\left(\frac{v_i}{r}\right) = m_B r v_i$$

and this is also the angular momentum of the bullet about the axis through the hinge at the instant just before impact.

(b) $\boxed{\text{No, mechanical energy is not conserved}}$ in the collision. The bullet embeds itself in the door with the two moving as a unit after impact. This is a perfectly inelastic collision in which a significant amount of mechanical energy is converted to other forms, notably thermal energy.

(c) Apply conservation of angular momentum with $L_i = m_B r v_i$ as discussed in part (a). After impact, $L_f = I_f\omega_f = \left(I_{\text{door}} + I_{\text{bullet}}\right)\omega_f = \left(\frac{1}{3}M_{\text{door}}L^2 + m_B r^2\right)\omega_f$ where $L = 1.00\ \text{m}$ = the width of the door and $r = L - 10.0\ \text{cm} = 0.900\ \text{m}$. Then,

$$L_f = L_i \quad\Rightarrow\quad \omega_f = \frac{m_B r v_i}{\frac{1}{3}\left(M_{\text{door}}L^2\right) + m_B r^2} = \frac{(0.005\ 00\ \text{kg})(0.900\ \text{m})\left(1.00\times10^3\ \text{m/s}\right)}{\frac{1}{3}(18.0\ \text{kg})(1.00\ \text{m})^2 + (0.005\ \text{kg})(0.900\ \text{m})^2}$$

yielding $\boxed{\omega_f = 0.749\ \text{rad/s}}$

(d) The kinetic energy of the door-bullet system immediately after impact is

$$KE_f = \frac{1}{2}I_f\omega_f^2 = \frac{1}{2}\left[\frac{1}{3}(18.0\ \text{kg})(1.00\ \text{m})^2 + (0.005\ 00\ \text{kg})(0.900\ \text{m})^2\right](0.749\ \text{rad/s})^2$$

or $\boxed{KE_f = 1.68\ \text{J}}$

The kinetic energy (of the bullet) just before impact was

$$KE_i = \frac{1}{2}m_B v_i^2 = \frac{1}{2}(0.005\ 00\ \text{kg})\left(1.00\times10^3\ \text{m/s}\right)^2 = \boxed{2.50\times10^3\ \text{J}} \approx \boxed{1\,490\cdot KE_f}$$

8.57 Each mass moves in a circular path of radius $r = 0.500\ \text{m/s}$ about the center of the connecting rod. Their angular speed is

$$\omega = \frac{v}{r} = \frac{5.00\ \text{m/s}}{0.500\ \text{m}} = 10.0\ \text{m/s}$$

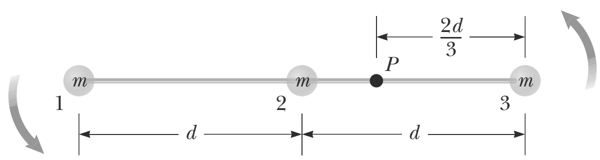
Neglecting the moment of inertia of the light connecting rod, the angular momentum of this rotating system is

$$L = I\omega = \left[m_1 r^2 + m_2 r^2\right]\omega = (4.00\ \text{kg} + 3.00\ \text{kg})(0.500\ \text{m})^2(10.0\ \text{rad/s}) = \boxed{17.5\ \text{J}\cdot\text{s}}$$

8.58 Using conservation of angular momentum, $L_{\text{aphelion}} = L_{\text{perihelion}}$. Thus, $\left(mr_a^2\right)\omega_a = \left(mr_p^2\right)\omega_p$. Since $\omega = v_t/r$ at both aphelion and perihelion, this is equivalent to $\left(mr_a^2\right)v_a/r_a = \left(mr_p^2\right)v_p/r_p$, giving

$$v_a = \left(\frac{r_p}{r_a}\right)v_p = \left(\frac{0.59\ \text{AU}}{35\ \text{AU}}\right)(54\ \text{km/s}) = \boxed{0.91\ \text{km/s}}$$

8.59 (a) $$I_P = m(d + d/3)^2 + m(d/3)^2 + m(2d/3)^2$$
$$= md^2\left(\frac{16}{9} + \frac{1}{9} + \frac{4}{9}\right) = \frac{21}{9}md^2 = \boxed{\frac{7}{3}md^2}$$



(b) $$\Sigma\tau)_P = mg\left(\frac{4}{3}d\right) + mg\left(\frac{1}{3}d\right) - mg\left(\frac{2}{3}d\right) = +\,mgd$$

or $$\Sigma\tau)_P = \boxed{mgd\ \text{counterclockwise}}$$

(c) From $\Sigma\tau)_P = I_P\alpha$, we have

$$\alpha = \frac{\Sigma\tau)_P}{I_P} = \frac{+mgd}{7md^2/3} = +\frac{3g}{7d} = \boxed{\frac{3g}{7d}\ \text{counterclockwise}}$$

(d) $$a_t = r\alpha = \left(\frac{2}{3}d\right)\left(\frac{3g}{7d}\right) = \boxed{\frac{2g}{7}\ \text{upward}}$$

(e) Maximum kinetic energy occurs when the system's gravitational potential energy is a minimum (i.e., when the center of gravity is at its lowest point). This occurs when the rod is vertical, where the y-coordinate (taking $y = 0$ at the level of point P) of the center of gravity is

$$y_{\text{cg}} = \frac{\Sigma(m_i g y_i)}{\Sigma m_i g} = m\left(-\frac{4}{3}d\right) + m\left(-\frac{d}{3}\right) + m\left(+\frac{2}{3}d\right) = -\frac{d}{3}$$

Applying conservation of energy from when the rod is released from rest in the horizontal position until it reaches the vertical position gives

continued on next page

$$KE_f = KE_i + \left(PE_i - PE_f\right) = 0 + \left(m_{\text{total}}g\right)\left(y_{\text{cg},i} - y_{\text{cg},f}\right)$$

or $$KE_f = 3mg\left[0 - (-d/3)\right] = \boxed{mgd}$$

(f) $KE_f = \frac{1}{2}I_P\omega_{\max}^2$ so $\omega_{\max} = \sqrt{\frac{2\,KE_f}{I_P}} = \sqrt{\frac{2(mgd)}{7md^2/3}} = \boxed{\sqrt{6g/7d}}$

(g) $L_{\max} = I_P\omega_{\max} = \left(\frac{7}{3}md^2\right)\sqrt{\frac{6g}{7d}} = \sqrt{\left(\frac{49}{9}m^2d^4\right)\frac{6g}{7d}} = \sqrt{m^2\left(\frac{7}{3}d^3\right)2g} = \boxed{m\sqrt{14gd^3/3}}$

(h) $(v_2)_{\max} = r_2\omega_{\max} = \frac{d}{3}\sqrt{\frac{6g}{7d}} = \sqrt{\left(\frac{d^2}{9}\right)\frac{6g}{7d}} = \sqrt{\left(\frac{d}{3}\right)\frac{2g}{7}} = \boxed{\sqrt{2gd/21}}$

8.60 (a) The table turns counterclockwise, opposite to the way the woman walks. Its angular momentum cancels that of the woman so the total angular momentum maintains a constant value of $L_{\text{total}} = L_{\text{woman}} + L_{\text{table}} = 0$.

Since the final angular momentum is $L_{\text{total}} = I_w\,\omega_w + I_t\,\omega_t = 0$, we have

$$\omega_t = -\left(\frac{I_w}{I_t}\right)\omega_w = -\left(\frac{m_w r^2}{I_t}\right)\left(\frac{v_w}{r}\right) = -\left(\frac{m_w r}{I_t}\right)v_w$$

or (taking counterclockwise as positive),

$$\omega_t = -\left[\frac{(60.0\text{ kg})(2.00\text{ m})}{500\text{ kg}\cdot\text{m}^2}\right](-1.50\text{ m/s}) = +0.360\text{ rad/s}$$

Hence, $\omega_{\text{table}} = \boxed{0.360\text{ rad/s counterclockwise}}$

(b) $W_{\text{net}} = \Delta KE = KE_f - 0 = \frac{1}{2}mv_w^2 + \frac{1}{2}I_t\,\omega_t^2$

$$W_{\text{net}} = \frac{1}{2}(60.0\text{ kg})(1.50\text{ m/s})^2 + \frac{1}{2}\left(500\text{ kg}\cdot\text{m}^2\right)(0.360\text{ rad/s})^2 = \boxed{99.9\text{ J}}$$

8.61 The moment of inertia of the cylinder before the putty arrives is

$$I_i = \frac{1}{2}MR^2 = \frac{1}{2}(10.0\text{ kg})(1.00\text{ m})^2 = 5.00\text{ kg}\cdot\text{m}^2$$

After the putty sticks to the cylinder, the moment of inertia is

$$I_f = I_i + mr^2 = 5.00\text{ kg}\cdot\text{m}^2 + (0.250\text{ kg})(0.900\text{ m})^2 = 5.20\text{ kg}\cdot\text{m}^2$$

Conservation of angular momentum gives $I_f\,\omega_f = I_i\,\omega_i$,

or $$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{5.00\text{ kg}\cdot\text{m}^2}{5.20\text{ kg}\cdot\text{m}^2}\right)(7.00\text{ rad/s}) = \boxed{6.73\text{ rad/s}}$$

8.62 The total moment of inertia of the system is

$$I_{\text{total}} = I_{\text{masses}} + I_{\substack{\text{student}\\\text{plus stool}}} = 2\left(mr^2\right) + 3.0\text{ kg}\cdot\text{m}^2$$

continued on next page

Initially, $r = 1.0$ m, and $I_i = 2\left[(3.0\text{ kg})(1.0\text{ m})^2\right] + 3.0\text{ kg}\cdot\text{m}^2 = 9.0\text{ kg}\cdot\text{m}^2$

Afterward, $r = 0.30$ m, so

$$I_f = 2\left[(3.0\text{ kg})(0.30\text{ m})^2\right] + 3.0\text{ kg}\cdot\text{m}^2 = 3.5\text{ kg}\cdot\text{m}^2$$

(a) From conservation of angular momentum, $I_f\,\omega_f = I_i\,\omega_i$, or

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{9.0\text{ kg}\cdot\text{m}^2}{3.5\text{ kg}\cdot\text{m}^2}\right)(0.75\text{ rad/s}) = \boxed{1.9\text{ rad/s}}$$

(b) $KE_i = \frac{1}{2}I_i\,\omega_i^2 = \frac{1}{2}(9.0\text{ kg}\cdot\text{m}^2)(0.75\text{ rad/s})^2 = \boxed{2.5\text{ J}}$

$$KE_f = \frac{1}{2}I_f\,\omega_f^2 = \frac{1}{2}(3.5\text{ kg}\cdot\text{m}^2)(1.9\text{ rad/s})^2 = \boxed{6.3\text{ J}}$$

8.63 The initial angular velocity of the puck is

$$\omega_i = \frac{(v_t)_i}{r_i} = \frac{0.800\text{ m/s}}{0.400\text{ m}} = 2.00\ \frac{\text{rad}}{\text{s}}$$

Since the tension in the string does not exert a torque about the axis of revolution, the angular momentum of the puck is conserved, or $I_f\,\omega_f = I_i\,\omega_i$. Thus,

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{mr_i^2}{mr_f^2}\right)\omega_i = \left(\frac{r_i}{r_f}\right)^2\omega_i = \left(\frac{0.400\text{ m}}{0.250\text{ m}}\right)^2(2.00\text{ rad/s}) = 5.12\text{ rad/s}$$

The net work done on the puck is

$$W_{\text{net}} = KE_f - KE_i = \frac{1}{2}I_f\,\omega_f^2 - \frac{1}{2}I_i\,\omega_i^2 = \frac{1}{2}\left[\left(mr_f^2\right)\omega_f^2 - \left(mr_i^2\right)\omega_i^2\right] = \frac{m}{2}\left[r_f^2\omega_f^2 - r_i^2\omega_i^2\right]$$

or $$W_{\text{net}} = \frac{(0.120\text{ kg})}{2}\left[(0.250\text{ m})^2(5.12\text{ rad/s})^2 - (0.400\text{ m})^2(2.00\text{ rad/s})^2\right]$$

This yields $W_{\text{net}} = \boxed{5.99\times10^{-2}\text{ J}}$.

8.64 With all crew members on the rim of the station, the apparent acceleration experienced is the centripetal acceleration, $a_c = r\omega^2 = g$. Thus, the initial angular velocity of the station is $\omega_i = \sqrt{g/r}$.

The initial moment of inertia of the rotating system is

$$I_i = I_{\text{crew}} + I_{\text{station}} = 150mr^2 + I_{\text{station}}$$

After most of the crew move to the rotation axis, leaving only the managers on the rim, the moment of inertia is

$$I_f = I_{\text{managers}} + I_{\text{station}} = 50mr^2 + I_{\text{station}}$$

Thus, conservation of angular momentum $\left(I_f\omega_f = I_i\omega_i\right)$ gives the angular velocity during the union meeting as

$$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \left(\frac{150(65.0\text{ kg})(100\text{ m})^2 + 5.00\times10^8\text{ kg}\cdot\text{m}^2}{50(65.0\text{ kg})(100\text{ m})^2 + 5.00\times10^8\text{ kg}\cdot\text{m}^2}\right)\sqrt{\frac{g}{r}} = 1.12\sqrt{\frac{g}{r}}$$

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The centripetal acceleration experienced by the managers still on the rim is

$$a_c = r\omega_f^2 = r(1.12)^2 \frac{g}{r} = (1.12)^2 \left(9.80 \text{ m/s}^2\right) = \boxed{12.3 \text{ m/s}^2}$$

8.65 (a) From conservation of angular momentum, $I_f \omega_f = I_i \omega_i$,

so $$\omega_f = \left(\frac{I_i}{I_f}\right)\omega_i = \boxed{\left(\frac{I_1}{I_1 + I_2}\right)\omega_o}$$

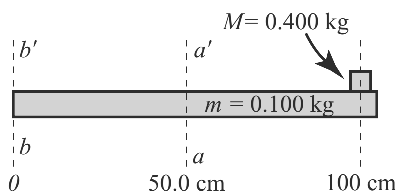
(b) $$KE_f = \frac{1}{2} I_f \omega_f^2 = \frac{1}{2}(I_1 + I_2)\left(\frac{I_1}{I_1+I_2}\right)^2 \omega_o^2 = \left(\frac{I_1}{I_1+I_2}\right)\left[\frac{1}{2} I_1 \omega_o^2\right] = \left(\frac{I_1}{I_1+I_2}\right) KE_i$$

or

$$\boxed{\frac{KE_f}{KE_i} = \frac{I_1}{I_1 + I_2} < 1}$$

Since this is less than 1.0, kinetic energy was lost.

8.66 (a) When rotating about axis $a\text{-}a'$:



$$I_{a\text{-}a'} = I_{\text{stick}} + I_{\text{particle}}$$
$$= \frac{1}{12} mL^2 + M(L - L/2)^2 = \left(\frac{m}{12} + \frac{M}{4}\right)L^2$$

Thus,

$$L_{a\text{-}a'} = I_{a\text{-}a'}\omega = \left[\frac{(0.100 \text{ kg})}{12} + \frac{(0.400 \text{ kg})}{4}\right](1.00 \text{ m})^2 (4.00 \text{ rad/s}) = \boxed{0.433 \text{ kg}\cdot\text{m}^2/\text{s}}$$

(b) When rotating about axis $b\text{-}b'$:

$$I_{b\text{-}b'} = I_{\text{stick}} + I_{\text{particle}} = \frac{1}{3} mL^2 + M(L - 0)^2 = \left(\frac{m}{3} + M\right)L^2$$

and

$$L_{b\text{-}b'} = I_{b\text{-}b'}\omega = \left[\frac{(0.100 \text{ kg})}{3} + 0.400 \text{ kg}\right](1.00 \text{ m})^2 (4.00 \text{ rad/s}) = \boxed{1.73 \text{ kg}\cdot\text{m}^2/\text{s}}$$

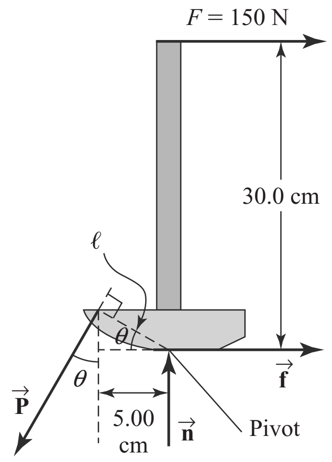
8.67 (a) $$\omega = 25 \frac{\cancel{\text{rev}}}{\cancel{\text{min}}}\left(\frac{2\pi \text{ rad}}{1 \cancel{\text{rev}}}\right)\left(\frac{1 \cancel{\text{min}}}{60 \text{ s}}\right) = \boxed{2.6 \text{ rad/s}}$$

(b) Treat each blade as a long, thin rod rotating about an axis perpendicular to its length and passing through its end. Then, $I_{\text{blade}} = mL^2/3$ and

$$I = 3I_{\text{blade}} = 3\left(\frac{mL^2}{3}\right) = (420 \text{ kg})(35 \text{ m})^2 = \boxed{5.1\times 10^5 \text{ kg}\cdot\text{m}^2}$$

(c) $$KE_r = \frac{1}{2} I\omega^2 = \frac{1}{2}\left(5.1\times 10^5 \text{ kg}\cdot\text{m}^2\right)(2.6 \text{ rad/s})^2 = \boxed{1.7\times 10^6 \text{ J}}$$

8.68 (a) In the sketch at the right, the force $\vec{\mathbf{P}}$ is the force the nail exerts on the claws of the hammer. It is equal in magnitude and oppositely directed to the force the claws exert on the nail. Choose an axis perpendicular to the page and passing through the indicated pivot. Then, with $\theta = 30.0°$, the lever arm of the force $\vec{\mathbf{P}}$ is



$$\ell = \frac{5.00 \text{ cm}}{\cos\theta} = \frac{5.00 \text{ cm}}{\cos 30.0°} = 5.77 \text{ cm}$$

and $\Sigma\tau = 0$ gives

$$+P(5.77 \text{ cm}) - (150 \text{ N})(30.0 \text{ cm}) = 0$$

so $$P = \frac{(150 \text{ N})(30.0 \text{ cm})}{5.77 \text{ cm}} = \boxed{780 \text{ N}}$$

(b) $\Sigma F_y = 0 \Rightarrow n - P\cos 30.0° = 0$, giving

$$n = P\cos 30.0° = (780 \text{ N})\cos 30.0° = 675 \text{ N}$$

$\Sigma F_x = 0 \Rightarrow f + F - P\sin 30.0° = 0$, or

$$f = P\sin 30.0° - F = (780 \text{ N})\sin 30.0° - 150 \text{ N} = 240 \text{ N}$$

The resultant force exerted on the hammer at the pivot is

$$R = \sqrt{f^2 + n^2} = \sqrt{(240 \text{ N})^2 + (675 \text{ N})^2} = 716 \text{ N}$$

at $\theta = \tan^{-1}(n/f) = \tan^{-1}(675 \text{ N}/240 \text{ N}) = 70.4°$, or

$$\vec{\mathbf{R}} = \boxed{716 \text{ N at } 70.4° \text{ above the horizontal to the right}}$$

8.69 (a) Since no horizontal force acts on the child-boat system, the center of gravity of this system will remain stationary, or

$$x_{\text{cg}} = \frac{m_{\text{child}}x_{\text{child}} + m_{\text{boat}}x_{\text{boat}}}{m_{\text{child}} + m_{\text{boat}}} = \text{constant}$$

The masses do not change, so this result becomes $m_{\text{child}}x_{\text{child}} + m_{\text{boat}}x_{\text{boat}} =$ constant.

Thus, $\boxed{\text{as the child walks to the right, the boat will move to the left.}}$

(b) Measuring distances from the stationary pier, with away from the pier being positive, the child is initially at $(x_{\text{child}})_i = 3.00$ m and the center of gravity of the boat is at $(x_{\text{boat}})_i = 5.00$ m. At the end, the child is at the right end of the boat, so $(x_{\text{child}})_f = (x_{\text{boat}})_f + 2.00$ m. Since the center of gravity of the system does not move, we have $(m_{\text{child}}x_{\text{child}} + m_{\text{boat}}x_{\text{boat}})_f = (m_{\text{child}}x_{\text{child}} + m_{\text{boat}}x_{\text{boat}})_i$, or

$$m_{\text{child}}(x_{\text{child}})_f + m_{\text{boat}}\left[(x_{\text{child}})_f - 2.00 \text{ m}\right] = m_{\text{child}}(3.00 \text{ m}) + m_{\text{boat}}(5.00 \text{ m})$$

continued on next page

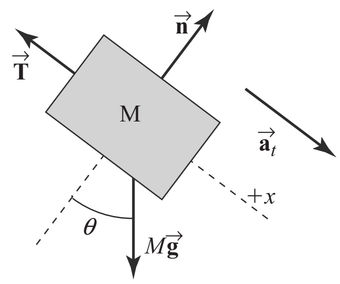
and

$$\left(x_{\text{child}}\right)_f = \frac{m_{\text{child}}\left(3.00\text{ m}\right) + m_{\text{boat}}\left(5.00\text{ m} + 2.00\text{ m}\right)}{m_{\text{child}} + m_{\text{boat}}}$$

$$\left(x_{\text{child}}\right)_f = \frac{\left(40.0\text{ kg}\right)\left(3.00\text{ m}\right) + \left(70.0\text{ kg}\right)\left(5.00\text{ m} + 2.00\text{ m}\right)}{40.0\text{ kg} + 70.0\text{ kg}} = \boxed{5.55\text{ m}}$$

(c) When the child arrives at the right end of the boat, the greatest distance from the pier that he can reach is $x_{\max} = \left(x_{\text{child}}\right)_f + 1.00\text{ m} = 5.55\text{ m} + 1.00\text{ m} = 6.55\text{ m}$. This leaves him $\boxed{\text{0.45 m short of reaching the turtle}}$.

8.70 (a) Consider the force diagram of the block given at the right. If the $+x$-axis is directed down the incline, $\Sigma F_x = ma_x$ gives

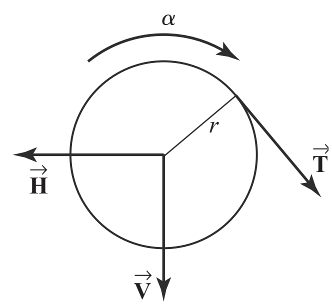


$$Mg\sin\theta° - T = Ma_t, \text{ or } T = M\left(g\sin\theta - a_t\right)$$

$$T = \left(12.0\text{ kg}\right)\left[\left(9.80\text{ m/s}^2\right)\sin 37.0° - 2.00\text{ m/s}^2\right]$$

$$= \boxed{46.8\text{ N}}$$

(b) Now, consider the force diagram of the pulley. Choose an axis perpendicular to the page and passing through the center of the pulley,

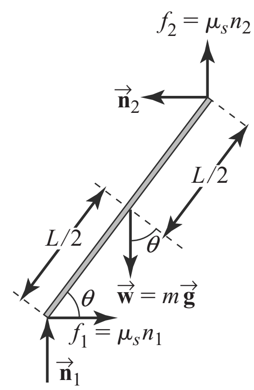


$$\Sigma\tau = I\alpha \text{ gives } T\cdot r = I\left(\frac{a_t}{r}\right), \text{ or}$$

$$I = \frac{T\cdot r^2}{a_t} = \frac{\left(46.8\text{ N}\right)\left(0.100\text{ m}\right)^2}{2.00\text{ m/s}^2} = \boxed{0.234\text{ kg}\cdot\text{m}^2}$$

(c) $\omega = \omega_i + \alpha t = 0 + \left(\frac{a_t}{r}\right)t = \left(\frac{2.00\text{ m/s}^2}{0.100\text{ m}}\right)\left(2.00\text{ s}\right) = \boxed{40.0\text{ rad/s}}$

8.71 If the ladder is on the verge of slipping, $f = \left(f_s\right)_{\max} = \mu_s n$ at both the floor and the wall.



From $\Sigma F_x = 0$, we find $f_1 - n_2 = 0$,

or $\quad n_2 = \mu_s n_1$ **[1]**

Also, $\Sigma F_y = 0$ gives $n_1 - w + \mu_s n_2 = 0$

Using Equation [1], this becomes

$$n_1 - w + \mu_s\left(\mu_s n_1\right) = 0$$

or $\quad n_1 = \dfrac{w}{1+\mu_s^2} = \dfrac{w}{1+\left(0.500\right)^2} = 0.800\,w$ **[2]**

Thus, Equation [1] gives $n_2 = 0.500\left(0.800\,w\right) = 0.400\,w.$ **[3]**

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Choose an axis perpendicular to the page and passing through the lower end of the ladder. Then, $\Sigma\tau = 0$ yields

$$-w\left(\frac{L}{2}\cos\theta\right)+n_2\left(L\sin\theta\right)+f_2\left(L\cos\theta\right)=0$$

Making the substitutions $n_2 = 0.400\,w$ and $f_2 = \mu_s n_2 = 0.200\,w$, this becomes

$$-w\left(\frac{L}{2}\cos\theta\right)+\left(0.400\,w\right)\left(L\sin\theta\right)+\left(0.200\,w\right)\left(L\cos\theta\right)=0$$

and reduces to $\sin\theta = \left(\dfrac{0.500-0.200}{0.400}\right)\cos\theta$

Hence, $\tan\theta = 0.750$ and $\theta = \boxed{36.9°}$

8.72 We treat each astronaut as a point object, of mass M, moving at speed v in a circle of radius $r = d/2$. Then the total angular momentum is

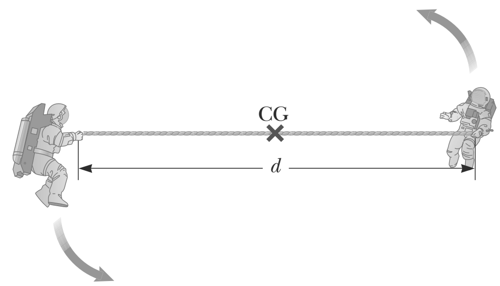


FIGURE P8.72

$$L = I_1\omega + I_2\omega = 2\left[\left(Mr^2\right)\left(\frac{v}{r}\right)\right]$$

$$= 2Mvr = 2Mv\left(\frac{d}{2}\right) = Mvd$$

(a) $L_i = Mv_id_i = (75.0\text{ kg})(5.00\text{ m/s})(10.0\text{ m}) = \boxed{3.75\times10^3\ \text{kg}\cdot\text{m}^2/\text{s}}$

(b) $KE_i = \frac{1}{2}m_1v_{1i}^2 + \frac{1}{2}m_2v_{2i}^2 = 2\left(\frac{1}{2}Mv_i^2\right) = (75.0\text{ kg})(5.00\text{ m/s})^2 = 1.88\times10^3\text{ J} = \boxed{1.88\text{ kJ}}$

(c) Angular momentum is conserved: $L_f = L_i = \boxed{3.75\times10^3\ \text{kg}\cdot\text{m}^2/\text{s}}$

(d) $v_f = \dfrac{L_f}{Md_f} = \dfrac{3.75\times10^3\ \text{kg}\cdot\text{m}^2/\text{s}}{(75.0\text{ kg})(5.00\text{ m})} = \boxed{10.0\text{ m/s}}$

(e) $KE_f = 2\left(\frac{1}{2}Mv_f^2\right) = (75.0\text{ kg})(10.0\text{ m/s})^2 = 7.50\times10^3\text{ J} = \boxed{7.50\text{ kJ}}$

(f) $W_{\text{net}} = KE_f - KE_i = \boxed{5.62\text{ kJ}}$

8.73 (a) $L_i = 2\left[Mv\left(\frac{d}{2}\right)\right] = \boxed{Mvd}$

(b) $KE_i = 2\left(\frac{1}{2}Mv_i^2\right) = \boxed{Mv^2}$

(c) $L_f = L_i = \boxed{Mvd}$

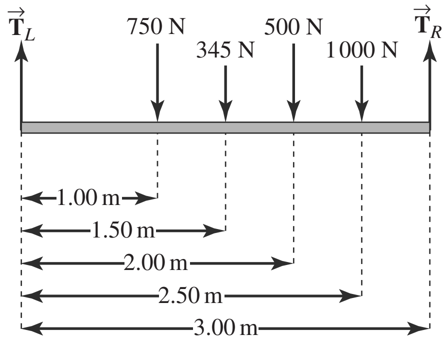
(d) $v_f = \dfrac{L_f}{2\left(Mr_f\right)} = \dfrac{Mvd}{2M(d/4)} = \boxed{2v}$

continued on next page

(e) $KE_f = 2\left(\frac{1}{2}Mv_f^2\right) = M(2v)^2 = \boxed{4Mv^2}$

(f) $W_{\text{net}} = KE_f - KE_i = \boxed{3Mv^2}$

8.74 Choose an axis that is perpendicular to the page and passing through the left end of the scaffold. Then $\Sigma\tau = 0$ gives



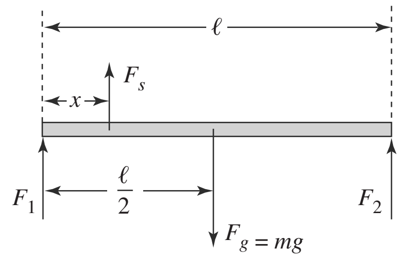
$$\begin{aligned}&-(750\text{ N})(1.00\text{ m})-(345\text{ N})(1.50\text{ m})\\&\quad-(500\text{ N})(2.00\text{ m})-(1000\text{ N})(2.50\text{ m})\\&\qquad+T_R(3.00\text{ m})=0\end{aligned}$$

or $T_R = 1.59\times10^3\text{ N} = \boxed{1.59\text{ kN}}$

Then,

$$\Sigma F_y = 0 \Rightarrow T_L = (750+345+500+1000)\text{ N} - 1.59\times10^3\text{ N} = \boxed{1.01\text{ kN}}$$

8.75 (a) Since the bar is in equilibrium, $\Sigma F_y = 0$ giving



$$F_s = mg - F_1 - F_2$$

and $F_s = (2.35\text{ kg})(9.80\text{ m/s}^2) - 6.80\text{ N} - 9.50\text{ N}$

or $F_s = \boxed{6.73\text{ N upward}}$

(b) We require the sum of the torques about an axis perpendicular to the page and passing through the left end of the bar be zero. This gives

$$\Sigma\tau_{\substack{\text{left}\\\text{end}}} = 0 + F_s\cdot x - (mg)(\ell/2) + F_2\cdot\ell = 0$$

or $x = \dfrac{[(mg/2)-F_2]\cdot\ell}{F_s} = \dfrac{[(2.35\text{ kg})(9.80\text{ m/s}^2)/2 - 9.50\text{ N}](1.30\text{ m})}{6.73\text{ N}}$

and $x = \boxed{0.389\text{ m}} = 38.9\text{ cm}$

8.76 (a) Taking $PE_g = 0$ at the level of the horizontal axis passing through the center of the rod, the total energy of the rod in the vertical position is

$$E = KE + PE_g$$
$$= 0 + m_1g(+L) + m_2g(-L) = \boxed{(m_1 - m_2)gL}$$

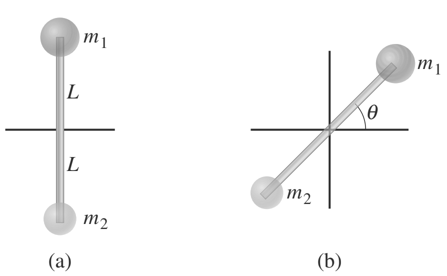


FIGURE P8.76

(b) In the rotated position of Figure P8.76b, the rod is in motion and the total energy is

$$E = KE_r + PE_g = \frac{1}{2}I_{\text{total}}\omega^2 + m_1gy_1 + m_2gy_2$$

$$= \frac{1}{2}\left(m_1L^2 + m_2L^2\right)\omega^2 + m_1g(+L\sin\theta) + m_2g(-L\sin\theta)$$

or $E = \boxed{\dfrac{(m_1+m_2)L^2\omega^2}{2} + (m_1 - m_2)gL\sin\theta}$

continued on next page

(c) In the absence of any nonconservative forces that do work on the rotating system, the total mechanical energy of the system is constant. Thus, the results of parts (a) and (b) may be equated to yield

$$\frac{(m_1+m_2)L^2\omega^2}{2}+(m_1-m_2)gL\sin\theta=(m_1-m_2)gL$$

and $$\omega=\boxed{\sqrt{\frac{2(m_1-m_2)g(1-\sin\theta)}{(m_1+m_2)L}}}$$

(d) In the vertical position, the net torque acting on the system is zero, $\boxed{\tau_{\text{net}}=0}$. This is because the lines of action of both external gravitational forces $(m_1g$ and $m_2g)$ pass through the pivot, and hence have zero lever arms about the rotation axis. In the rotated position, the net torque (taking clockwise as positive) is

$$\tau_{\text{net}}=\Sigma\tau=m_1g(L\cos\theta)-m_2g(L\cos\theta)=\boxed{(m_1-m_2)gL\cos\theta}$$

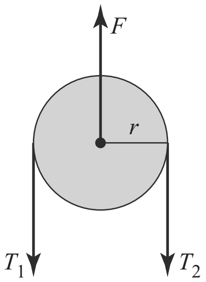
Note that the net torque is not constant as the system rotates. Thus, the angular momentum $\boxed{\text{will change at a non-uniform rate, } \Delta L/\Delta t=\tau_{\text{net}}}$.

(e) In the rotated position, the angular acceleration is

$$\alpha=\frac{\tau_{\text{net}}}{I}=\frac{(m_1-m_2)gL\cos\theta}{m_1L^2+m_2L^2}=\boxed{\frac{(m_1-m_2)g\cos\theta}{(m_1+m_2)L}}$$

The angular acceleration is a maximum in the horizontal position ($\theta=0°$), where the gravitational forces have maximum lever arms and exert the maximum torque on the system. Also, note that $\alpha=0$ at $\theta=90°$. This is understandable since the vertical orientation is a position of unstable equilibrium ($\tau_{\text{net}}=0$).

8.77 (a) Since the pulley is very light (so $I\approx0$) and rotates without friction, the net torque about the axis of the pulley is



$$\Sigma\tau=T_1r-T_2r=I\alpha\approx(0)\alpha=\boxed{0}$$

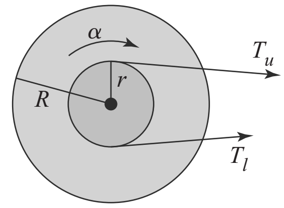
From this, we see that $T_1=T_2$, or the tension in the rope has the same value T on both sides of the pulley.

(b) From the rotational form of Newton's second law, $\Sigma\tau=\Delta L/\Delta t$, we see that if $\Sigma\tau=0$ then $\Delta L/\Delta t=0$. This means that the angular momentum of the system will be constant at its initial value of zero at all times. Thus, the upward speeds of the monkey and the bananas must always be equal.

Another way of arriving at this conclusion is to realize that the monkey and the bananas have the same net upward force, $\Sigma F_y=T-Mg$, acting on them. Thus, they have the same upward acceleration and, both having started from rest, will have the same upward speeds at all times.

(c) $\boxed{\text{No, the monkey will not reach the bananas.}}$ The monkey and the bananas move upward at the same speed, staying a fixed distance apart (at least until the bananas become tangled in the pulley).

8.78 Consider the sketch of the flywheel at the right and compute the net torque (taking counterclockwise as positive) about the center of this wheel:



$$\left.\Sigma\tau\right)_{\text{center}} = -T_u r + T_l r = r\left(T_l - T_u\right)$$

Because the mass of the pulley is very small in comparison to that of the flywheel, we neglect the moment of inertia of the pulley.

Since the flywheel is a solid cylinder, its moment of inertia about the axis through its center is $I = MR^2/2$ and the rotational form of Newton's second law, $\left.\Sigma\tau\right)_{\text{center}} = I\alpha$, gives

$$r\left(T_l - T_u\right) = \left(\frac{1}{2}MR^2\right)\alpha$$

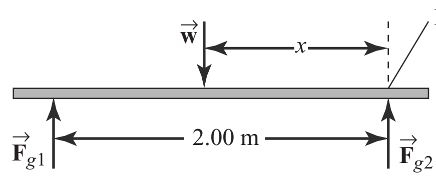
The tension in the lower segment of the belt is then

$$T_l = T_u + \left(\frac{mR^2}{2r}\right)\alpha$$

Since $\alpha = 1.67\ \text{rad/s}^2$ clockwise $\left(\text{or } \alpha = -1.67\ \text{rad/s}^2\right)$, $T_u = 135$ N, $M = 80.0$ kg, $R = 0.625$ m, and $r = 0.230$ m, we have

$$T_l = 135\ \text{N} + \frac{(80.0\ \text{kg})(0.625\ \text{m})^2}{2(0.230\ \text{m})}\left(-1.67\ \text{rad/s}^2\right) = \boxed{22\ \text{N}}$$

8.79 We neglect the weight of the board and assume that the woman's feet are directly above the point of support by the rightmost scale. Then, the force diagram for the situation is as shown at the right.



From $\Sigma F_y = 0$, we have $F_{g1} + F_{g2} - w = 0$, or $w = 380\ \text{N} + 320\ \text{N} = 700$ N.

Choose an axis perpendicular to the page and passing through point P.

Then $\Sigma\tau = 0$ gives $w \cdot x - F_{g1}(2.00\ \text{m}) = 0$, or

$$x = \frac{F_{g1}(2.00\ \text{m})}{w} = \frac{(380\ \text{N})(2.00\ \text{m})}{700\ \text{N}} = \boxed{1.09\ \text{m}}$$

8.80 (a) Since only conservative forces do work on the long rod, we use conservation of energy for this pure rotation about the fixed point O. The rod starts from rest $(\omega_i = 0)$ with the center of gravity at the level of point O. Choosing this level as the reference level for gravitational potential energy, we have

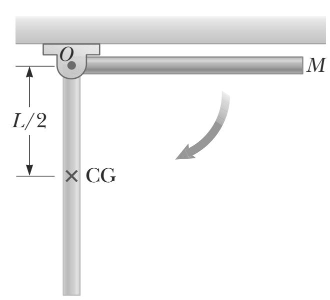


FIGURE P8.80

$$KE_f = KE_i + PE_{g,i} - PE_{g,f}$$

or $$\frac{1}{2}I_O\omega_f^2 = 0 + 0 - Mg\left(-\frac{L}{2}\right)$$

and $$\omega_f = \sqrt{\frac{MgL}{I_O}} = \sqrt{\frac{MgL}{\frac{1}{3}ML^2}} = \sqrt{\frac{3g}{L}}$$

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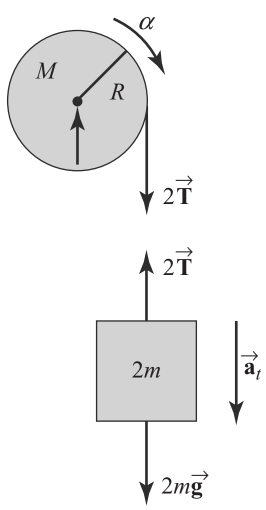
The tangential speed of the center of mass when the rod reaches the vertical position is then

$$v_{cg} = r_{cg}\omega_f = \left(\frac{L}{2}\right)\sqrt{\frac{3g}{L}} = \sqrt{\left(\frac{L}{2}\right)^2 \frac{3g}{L}} \quad \text{or} \quad \boxed{v_{cg} = \sqrt{3gL/4} = \sqrt{3gL}/2}$$

(b) The tangential speed of the lowest point on the rod when it reaches the vertical position is

$$v_{\substack{\text{lower}\\ \text{end}}} = r_{\substack{\text{lower}\\ \text{end}}}\omega_f = (L)\sqrt{\frac{3g}{L}} = \sqrt{(L)^2 \frac{3g}{L}} \quad \text{and} \quad \boxed{v_{\substack{\text{lower}\\ \text{end}}} = \sqrt{3gL} = 2v_{cg}}$$

8.81 Choose an axis perpendicular to the page and passing through the center of the cylinder. Then, applying $\Sigma\tau = I\alpha$ to the cylinder gives $(2T)\cdot R = \left(\frac{1}{2}MR^2\right)\alpha = \left(\frac{1}{2}MR^2\right)(a_t/R)$, or



$$T = \frac{1}{4}Ma_t \qquad \textbf{[1]}$$

Now apply $\Sigma F_y = ma_y$ to the falling objects to obtain $(2m)g - 2T = (2m)a_t$, or

$$a_t = g - \frac{T}{m} \qquad \textbf{[2]}$$

(a) Substituting Equation [2] into [1] yields

$$T = \frac{Mg}{4} - \left(\frac{M}{4m}\right)T$$

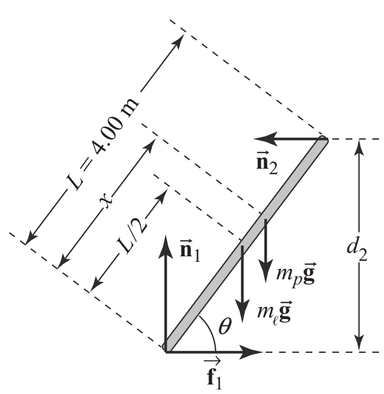
which reduces to $\boxed{T = \dfrac{Mmg}{M+4m}}$.

(b) From Equation [2] above,

$$a_t = g - \frac{1}{m}\left(\frac{Mmg}{M+4m}\right) = g - \frac{Mg}{M+4m} = \boxed{\frac{4mg}{M+4m}}$$

8.82 (a) A smooth (that is, frictionless) wall cannot exert a force parallel to its surface. Thus, the only force the vertical wall can exert on the upper end of the ladder is a horizontal normal force.

(b) Consider the force diagram of the ladder given at the right. If the rotation axis is perpendicular to the page and passing through the lower end of the ladder, the lever arm of the normal force $\vec{\mathbf{n}}_2$ that the wall exerts on the upper end of the ladder is



$$d_2 = \boxed{L\sin\theta}$$

(c) The lever arm of the force of gravity, $m_\ell \vec{\mathbf{g}}$, acting on the ladder is

$$d_\ell = (L/2)\cos\theta = \boxed{(L\cos\theta)/2}$$

continued on next page

(d) Refer to the force diagram given in part (b) of this solution and make use of the fact that the ladder is in both translational and rotational equilibrium.

$$\Sigma F_y = 0 \;\Rightarrow\; n_1 - m_\ell g - m_p g = 0\text{, or } n_1 = \left(m_\ell + m_p\right)g$$

When the ladder is on the verge of slipping, $f_1 = \left(f_1\right)_{\max} = \mu_s n_1 = \mu_s\left(m_\ell + m_p\right)g$.

Then, $\Sigma F_x = 0 \;\Rightarrow\; n_2 = f_1$, or $n_2 = \mu_s\left(m_\ell + m_p\right)g$.

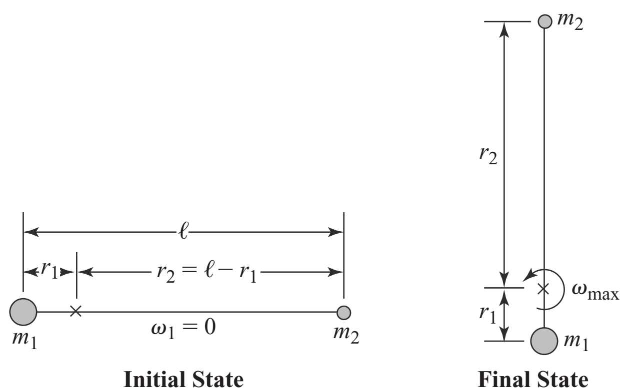
Finally, $\Sigma\tau = 0 \;\Rightarrow\; n_2\left(L\sin\theta\right) - m_\ell g\left(L/2\right)\cos\theta - m_p g x\cos\theta = 0$, where x is the maximum distance the painter can go up the ladder before it will start to slip. Solving for x gives

$$x = \frac{n_2\left(L\sin\theta\right) - m_\ell g\left(L/2\right)\cos\theta}{m_p g\cos\theta} = \mu_s\left(\frac{m_\ell}{m_p} + 1\right)L\tan\theta - \left(\frac{m_\ell}{2m_p}\right)L$$

and using the given numerical data, we find

$$x = (0.45)\left(\frac{30\text{ kg}}{80\text{ kg}} + 1\right)(4.0\text{ m})\tan 53^\circ - \left[\frac{30\text{ kg}}{2(80\text{ kg})}\right](4.0\text{ m}) = \boxed{2.5\text{ m}}$$

8.83 The large mass ($m_1 = 60.0$ kg) moves in a circular path of radius $r_1 = 0.140$ m, while the radius of the path for the small mass ($m_2 = 0.120$ kg) is $r_2 = \ell - r_1 = 3.00\text{ m} - 0.140\text{ m} = 2.86\text{ m}$.



The system has maximum angular speed when the rod is in the vertical position as shown above.

We take $PE_g = 0$ at the level of the horizontal rotation axis and use conservation of energy to find

$$KE_f + \left(PE_g\right)_f = KE_i + \left(PE_g\right)_i \quad\Rightarrow\quad \left(\frac{1}{2}I_1\omega_{\max}^2 + \frac{1}{2}I_2\omega_{\max}^2\right) + \left(m_2 g r_2 - m_1 g r_1\right) = 0 + 0$$

Approximating the two objects as point masses, we have $I_1 = m_1 r_1^2$ and $I_2 = m_2 r_2^2$. The energy conservation equation then becomes $\frac{1}{2}\left(m_1 r_1^2 + m_2 r_2^2\right)\omega_{\max}^2 = \left(m_1 r_1 - m_2 r_2\right)g$ and yields

$$\omega_{\max} = \sqrt{\frac{2\left(m_1 r_1 - m_2 r_2\right)g}{m_1 r_1^2 + m_2 r_2^2}} = \sqrt{\frac{2\left[(60.0\text{ kg})(0.140\text{ m}) - (0.120\text{ kg})(2.86\text{ m})\right]\left(9.80\text{ m/s}^2\right)}{(60.0\text{ kg})(0.140\text{ m})^2 + (0.120\text{ kg})(2.86\text{ m})^2}}$$

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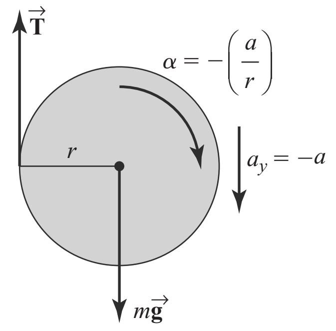
or $\omega_{\max} = 8.56$ rad/s. The maximum tangential speed of the small mass object is then

$$(v_2)_{\max} = r_2\omega_{\max} = (2.86 \text{ m})(8.56 \text{ rad/s}) = \boxed{24.5 \text{ m/s}}$$

8.84 (a) Note that the cylinder has both translational and rotational motion. The center of gravity accelerates downward while the cylinder rotates around the center of gravity. Thus, we apply both the translational and the rotational forms of Newton's second law to the cylinder:



$$\Sigma F_y = ma_y \;\Rightarrow\; T - mg = m(-a)$$

or $$T = m(g - a) \qquad \textbf{[1]}$$

$$\Sigma\tau = I\alpha \;\Rightarrow\; -Tr = I(-a/r)$$

For a uniform, solid cylinder, $I = \frac{1}{2}mr^2$ so our last result becomes

$$Tr = \left(\frac{mr^2}{2}\right)\left(\frac{a}{r}\right) \quad \text{or} \quad a = \frac{2T}{m} \qquad \textbf{[2]}$$

Substituting Equation [2] into Equation [1] gives $T = mg - 2T$, and solving for T yields $T = \boxed{mg/3}$.

(b) From Equation [2] above,

$$a = \frac{2T}{m} = \frac{2}{m}\left(\frac{mg}{3}\right) = \boxed{2g/3}$$

(c) Considering the translational motion of the center of gravity, $v_y^2 = v_{0y}^2 + 2a_y\Delta y$ gives

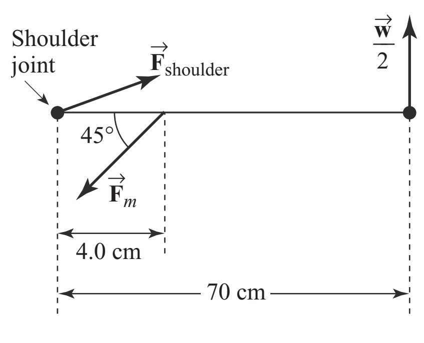
$$v_y = \sqrt{0 + 2\left(-\frac{2g}{3}\right)(-h)} = \boxed{\sqrt{4gh/3}}$$

Using conservation of energy, with $PE_g = 0$ at the final level of the cylinder, gives

$$\left(KE_t + KE_r + PE_g\right)_f = \left(KE_t + KE_r + PE_g\right)_i, \text{ or } \tfrac{1}{2}mv_y^2 + \tfrac{1}{2}I\omega^2 + 0 = 0 + 0 + mgh$$

Since $\omega = v_y/r$ and $I = \frac{1}{2}mr^2$, this becomes $\frac{1}{2}mv_y^2 + \frac{1}{2}\left(\frac{1}{2}m\cancel{r^2}\right)\left(v_y^2/\cancel{r^2}\right) = mgh$, or $\frac{3}{4}mv_y^2 = mgh$ yielding $v_y = \boxed{\sqrt{4gh/3}}$.

8.85 Considering the shoulder joint as the pivot, the second condition of equilibrium gives

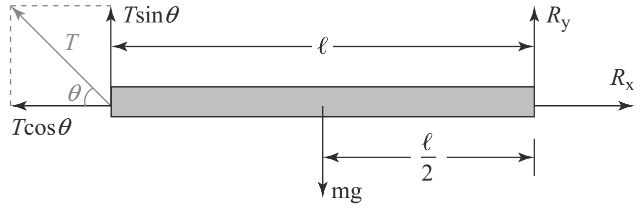


$$\Sigma\tau = 0 \;\Rightarrow\; \frac{w}{2}(70 \text{ cm}) - \left(F_m \sin 45°\right)(4.0 \text{ cm}) = 0$$

or $$F_m = \frac{w(70 \text{ cm})}{2(4.0 \text{ cm})\sin 45°}$$

Thus, $$F_m = \frac{(750 \text{ N})(70 \text{ cm})}{(8.0 \text{ cm})\sin 45°} = 9.3\times 10^3 \text{ N} = \boxed{9.3 \text{ kN}}$$

8.86 (a) Choosing the elbow (right end of the forearm) as the pivot, the second condition of equilibrium gives



$$\Sigma\tau = 0 + 0 + mg\left(\frac{\ell}{2}\right) + 0 - (T\sin\theta)\ell = 0$$

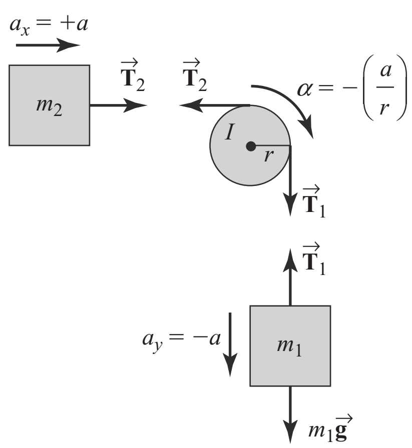
or $$T = \frac{mg}{2\sin\theta} = \frac{(1.60\text{ kg})(9.80\text{ m/s}^2)}{2\sin 50.0°} = \boxed{10.2\text{ N}}$$

(b) $\Sigma F_x = 0 \quad \Rightarrow \quad R_x = T\cos\theta = (10.2\text{ N})\cos 50.0° = \boxed{6.56\text{ N}}$

and $\Sigma F_y = 0$ gives

$$R_y = mg - T\sin\theta = mg - \left(\frac{mg}{2\sin\theta}\right)\sin\theta = \frac{mg}{2} = \frac{(1.60\text{ kg})(9.80\text{ m/s}^2)}{2} = \boxed{7.84\text{ N}}$$

8.87 (a) Force diagrams for each block and the pulley are given at the right. Observe that the angular acceleration of the pulley will be clockwise in direction and has been given a negative sign. Since $\Sigma\tau = I\alpha$, the positive sense for torques and angular acceleration must be the same (counterclockwise).



For m_1: $\Sigma F_y = ma_y \;\Rightarrow\; T_1 - m_1 g = m_1(-a)$, or

$$T_1 = m_1(g - a) \qquad \textbf{[1]}$$

For m_2: $\Sigma F_x = ma_x \;\Rightarrow\; T_2 = m_2 a$ **[2]**

For the pulley: $\Sigma\tau = I\alpha \;\Rightarrow\; T_2 r - T_1 r = I(-a/r)$, or

$$T_1 - T_2 = \left(\frac{I}{r^2}\right)a \qquad \textbf{[3]}$$

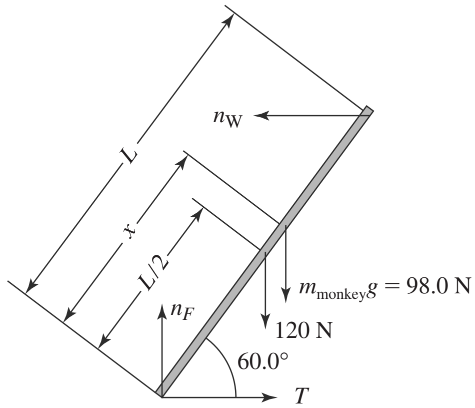
Substitute Equations [1] and [2] into Equation [3] and solve for a to obtain $a = m_1 g/\left[(I/r^2) + m_1 + m_2\right]$, or

$$a = \frac{(4.00\text{ kg})(9.80\text{ m/s}^2)}{(0.500\text{ kg}\cdot\text{m}^2)/(0.300\text{ m})^2 + 4.00\text{ kg} + 3.00\text{ kg}} = \boxed{3.12\text{ m/s}^2}$$

(b) Equation [1] above gives: $T_1 = (4.00\text{ kg})(9.80\text{ m/s}^2 - 3.12\text{ m/s}^2) = \boxed{26.7\text{ N}}$

and Equation [2] yields: $T_2 = (3.00\text{ kg})(3.12\text{ m/s}^2) = \boxed{9.36\text{ N}}$.

8.88 (a)



(b) $\Sigma F_y = 0 \quad \Rightarrow \quad n_F - 120\text{ N} - m_{\text{monkey}}g = 0$, so

$$n_F = 120\text{ N} + (10.0\text{ kg})(9.80\text{ m/s}^2) = \boxed{218\text{ N}}$$

(c) When $x = 2L/3$, we consider the lower end of the ladder as our pivot and obtain

$$\Sigma\tau)_{\substack{\text{lower}\\\text{end}}} = 0 \;\Rightarrow\; -(120\text{ N})\left(\frac{\cancel{L}}{2}\cos 60.0°\right) - (98.0\text{ N})\left(\frac{2\cancel{L}}{3}\cos 60.0°\right) + n_{\text{W}}\left(\cancel{L}\sin 60.0°\right) = 0$$

or

$$n_{\text{W}} = \frac{[60.0\text{ N} + (196/3)\text{ N}]\cos 60.0°}{\sin 60.0°} = 72.4\text{ N}$$

Then,

$$\Sigma F_x = 0 \quad \Rightarrow \quad T - n_{\text{W}} = 0 \qquad \text{or} \qquad T = n_{\text{W}} = \boxed{72.4\text{ N}}$$

(d) When the rope is ready to break, $T = n_{\text{W}} = 80.0$ N. Then $\Sigma\tau)_{\substack{\text{lower}\\\text{end}}} = 0$ yields

$$-(120\text{ N})\left(\frac{L}{2}\cos 60.0°\right) - (98.0\text{ N})x\cos 60.0° + (80.0\text{ N})(L\sin 60.0°) = 0$$

or

$$x = \frac{[(80.0\text{ N})\sin 60.0° - (60.0\text{ N})\cos 60.0°]L}{(98.0\text{ N})\cos 60.0°} = 0.802\,L = 0.802(3.00\text{ m}) = \boxed{2.41\text{ m}}$$

(e) If the horizontal surface were rough and the rope removed, a horizontal static friction force directed toward the wall would act on the lower end of the ladder. Otherwise, the analysis would be much as what is done above. The maximum distance the monkey could climb would correspond to the condition that the friction force have its maximum value, $\mu_s n_{\text{F}}$, so you would need to know the coefficient of static friction to solve part (d).

8.89 (a) Choose the initial position of the block as the zero gravitational potential energy level. Then, conservation of energy from when the block is released from rest until it comes to rest momentarily after falling a distance h gives

$$KE_{t,f} + KE_{r,f} + PE_{g,f} + PE_{s,f} = KE_{t,i} + KE_{r,i} + PE_{g,i} + PE_{s,i}$$

Since $v_f = v_i = 0$ and $\omega_f = \omega_i = 0$, this becomes $0 + 0 - mgh + \frac{1}{2}kh^2 = 0 + 0 + 0 + 0$, or

$$h = \frac{2mg}{k} = \frac{2(3.2\text{ kg})(9.80\text{ m/s}^2)}{86\text{ N/m}} = \boxed{0.73\text{ m}}$$

continued on next page

(b) Here, we use conservation of energy from when the block is released from rest until it has dropped a distance $h = 25\text{ cm} = 0.25\text{ m}$. Recognize that if the string does not slip on the pulley, the angular speed of the pulley is given by $\omega = v_t/r$, where v_t is the translational speed of the block and r is the radius of the pulley.

$$KE_{t,f} + KE_{r,f} + PE_{g,f} + PE_{s,f} = KE_{t,i} + KE_{r,i} + PE_{g,i} + PE_{s,i}$$

$$\frac{1}{2}mv_t^2 + \frac{1}{2}\left(\frac{1}{2}Mr^2\right)\left(\frac{v_t}{r}\right)^2 + mg(-h) + \frac{1}{2}kh^2 = 0 + 0 + 0 + 0$$

which reduces to $\frac{1}{2}(m + M/2)v_t^2 = mgh - \frac{1}{2}kh^2$, and

$$v_t = \sqrt{\frac{2mgh - kh^2}{m + M/2}} = \sqrt{\frac{2(3.2\text{ kg})(9.8\text{ m/s}^2)(0.25\text{ m}) - (86\text{ N/m})(0.25\text{ m})^2}{3.2\text{ kg} + (1.8\text{ kg})/2}}$$

yielding $\boxed{v_t = 1.6\text{ m/s}}$.

9

Solids and Fluids

QUICK QUIZZES

1. Choice (c). The mass that you have of each element is:

$$m_{\text{gold}} = \rho_{\text{gold}} V_{\text{gold}} = \left(19.3 \times 10^3 \text{ kg/m}^3\right)\left(1 \text{ m}^3\right) = 19.3 \times 10^3 \text{ kg}$$

$$m_{\text{silver}} = \rho_{\text{silver}} V_{\text{silver}} = \left(10.5 \times 10^3 \text{ kg/m}^3\right)\left(2 \text{ m}^3\right) = 21.0 \times 10^3 \text{ kg}$$

$$m_{\text{aluminum}} = \rho_{\text{aluminum}} V_{\text{aluminum}} = \left(2.70 \times 10^3 \text{ kg/m}^3\right)\left(6 \text{ m}^3\right) = 16.2 \times 10^3 \text{ kg}$$

2. Choice (a). At a fixed depth, the pressure in a fluid is directly proportional to the density of the fluid. Since ethyl alcohol is less dense than water, the pressure is smaller than P when the glass is filled with alcohol.

3. Choice (c). For a fixed pressure, the height of the fluid in a barometer is inversely proportional to the density of the fluid. Of the fluids listed in the selection, ethyl alcohol is the least dense.

4. Choice (b). The blood pressure measured at the calf would be larger than that measured at the arm. If we imagine the vascular system of the body to be a vessel containing a liquid (blood), the pressure in the liquid will increase with depth. The blood at the calf is deeper in the liquid than that at the arm and is at a higher pressure.

Blood pressures are normally taken at the arm because that is approximately the same height as the heart. If blood pressures at the calf were used as a standard, adjustments would need to be made for the height of the person, and the blood pressure would be different if the person were lying down.

5. Choice (c). The level of a floating ship is unaffected by the atmospheric pressure. The buoyant force results from the pressure differential in the fluid. On a high-pressure day, the pressure at all points in the water is higher than on a low-pressure day. Because water is almost incompressible, however, the rate of change of pressure with depth is the same, resulting in no change in the buoyant force.

6. Choice (b). Since both lead and iron are denser than water, both objects will be fully submerged and (since they have the same dimensions) will displace equal volumes of water. Hence, the buoyant forces acting on the two objects will be equal.

7. Choice (a). When there is a moving air stream in the region between the balloons, the pressure in this region will be less than on the opposite sides of the balloons where the air is not moving. The pressure differential will cause the balloons to move toward each other. This is a demonstration of Bernoulli's principle in action.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. From Pascal's principle, $F_1/A_1 = F_2/A_2$, so if the output force is to be $F_2 = 1.2\times10^3$ N, the required input force is $F_1 = \left(A_1/A_2\right)F_2 = \left(0.050\text{ m}^2/0.70\text{ m}^2\right)\left(1.2\times10^3\text{ N}\right) = 86$ N, making (c) the correct answer.

2. On average, the support force each nail exerts on the body is

$$\overline{F}_1 = \frac{mg}{1\,208} = \frac{(66.0\text{ kg})\left(9.80\text{ m/s}^2\right)}{1\,208} = 0.535\text{ N}$$

so the average pressure exerted on the body by each nail is

$$P_{\text{av}} = \frac{\overline{F}_1}{A_{\substack{\text{nail}\\\text{end}}}} = \frac{0.535\text{ N}}{1.00\times10^{-6}\text{ m}^2} = 5.35\times10^5\text{ Pa}$$

and (d) is the correct choice.

3. $m = \rho_{\text{gold}}V = \left(19.3\times10^3\text{ kg/m}^3\right)\left(4.50\times10^{-2}\text{ m}\right)\left(11.0\times10^{-2}\text{ m}\right)\left(26.0\times10^{-2}\text{ m}\right) = 24.8\text{ kg}$, and choice (a) is the correct response.

4. If the bullet is to float, the buoyant force must equal the weight of the bullet. Thus, the bullet will sink until the weight of the displaced mercury equals the weight of the bullet, or $\rho_{\text{mercury}}V_{\text{submerged}}g = \rho_{\text{lead}}V_{\text{bullet}}g$, and

$$\frac{V_{\text{submerged}}}{V_{\text{bullet}}} = \frac{\rho_{\text{lead}}}{\rho_{\text{mercury}}} = \frac{11.3\times10^3\text{ kg/m}^3}{13.6\times10^3\text{ kg/m}^3} = 0.831$$

so the correct response is (d).

5. The absolute pressure at depth h below the surface of a liquid with density ρ, and with pressure P_0 at its surface, is $P = P_0 + \rho gh$. Thus, at a depth of 754 ft in the waters of Loch Ness,

$$P = 1.013\times10^5\text{ Pa} + \left(1.00\times10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)\left[(754\text{ ft})\left(\frac{1\text{ m}}{3.281\text{ ft}}\right)\right] = 2.35\times10^6\text{ Pa}$$

and (c) is the correct response.

6. Both the block and the steel object are in equilibrium. Thus, the sum of the buoyant force on the steel object and the tension in the string must equal the weight of the steel object, so this buoyant force and the tension in the string must each be less than the weight of the object. The buoyant force on the block equals the weight of the water displaced by the block. This buoyant force must equal the sum of the weight of the block and the tension in the string, and hence, exceeds the magnitude of each of these individual forces. Therefore, the only correct answers to this question are choices (d) and (e).

7. From the equation of continuity, $A_1v_1 = A_2v_2$, the speed of the water in the smaller pipe is

$$v_2 = \left(\frac{A_1}{A_2}\right)v_1 = \left[\frac{\pi(0.250\text{ m})^2}{\pi(0.100\text{ m})^2}\right](1.00\text{ m/s}) = 6.25\text{ m/s}$$

so (d) is the correct answer.

8. Since water is more dense than the air-filled ball, the buoyant force acting on the ball exceeds the weight of the fully submerged ball. This means that choice (b) is true and choice (d) is false. Neglecting a very small variation in the density of water with depth, the weight of the displaced water (i.e., the buoyant force on the ball) remains constant as long as the ball is totally submerged and its size does not change. Therefore, both choices (a) and (c) are false. That choice (e) is true follows directly from Archimedes' principle, meaning the correct responses to this question are choices (b) and (e).

9. The boat, even after it sinks, experiences a buoyant force, B, equal to the weight of whatever water it is displacing. This force will support part of the weight, w, of the boat. The force exerted on the boat by the bottom of the lake will be $F_{\text{bottom}} = w - B < w$ and will support the balance of the boat's weight. The correct response is (c).

10. The absolute pressure at depth h below the surface of a fluid having density ρ is $P = P_0 + \rho g h$, where P_0 is the pressure at the upper surface of that fluid. The fluid in each of the three vessels has density $\rho = \rho_{\text{water}}$, the top of each vessel is open to the atmosphere so that $P_0 = P_{\text{atmo}}$ in each case, and the bottom is at the same depth h below the upper surface for the three vessels. Thus, the pressure P at the bottom of each vessel is the same and (c) is the correct choice.

11. The two spheres displace equal volumes (and hence, equal weights) of water. Therefore, they experience the same magnitude buoyant forces. This means that choice (a) is true while choices (b) and (d) are false. The tension in the string attached to each sphere must equal the difference between the weight of that sphere and the buoyant force. Since the lead sphere has the greater weight, and the two buoyant forces are equal, the tension in the string attached to the lead sphere is greater than the tension in the string attached to the iron sphere. Thus, choice (c) is also true and the correct responses are choices (a) and (c).

12. As the ball moves to a greater depth in the pool, the pressure exerted due to the water increases significantly. The thin plastic wall of the ball is not rigid enough to prevent the air in the ball from being compressed into a smaller volume as the water pressure increases. Since the buoyant force is the weight of the displaced water, and this weight decreases because the volume of water displaced by the ball decreases while the density of the water is essentially constant, the buoyant force exerted on the ball by the water decreases. Hence, choice (c) is the correct answer.

13. When the anchor was in the boat, sufficient water was displaced to fully support the total weight of the person, boat, and anchor. After the anchor is thrown overboard, the bottom of the lake supports most of the anchor's weight. Thus, less water must be displaced to keep the person plus boat afloat and also support the small remainder of the anchor's weight. The correct response is choice (b).

14. Since most of the ice at the south pole is supported by land, it does not displace any seawater, and hence, does not contribute to the water level in the oceans. However, after this ice melts and flows into the sea, it will significantly add to the water level in the oceans. On the contrary, the ice at the north pole is currently displacing its own weight in water, just as it will after melting. Thus, the ice at the south pole will have the greater impact on sea levels as it melts and the correct choice is (b).

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. We approximate the thickness of the atmosphere by using $P = P_0 + \rho g h$ with $P_0 = 0$ at the top of the atmosphere and $P = 1 \text{ atm}$ at sea level. This gives an approximation of

$$h = \frac{P - P_0}{\rho g} \sim \frac{10^5 \text{ Pa} - 0}{\left(1 \text{ kg/m}^3\right)\left(10^1 \text{ m/s}^2\right)} = 10^4 \text{ m} \qquad \text{or} \qquad h \sim 10 \text{ km}$$

Because both the density of the air, ρ, and the acceleration of gravity, g, decrease with altitude, the actual thickness of the atmosphere will be greater than our estimate.

4. The two dams must have the same strength. The force on the back of each dam is the average pressure of the water times the area of the dam. If the two reservoirs are equally deep, the forces exerted on the two dams by the water have equal magnitudes.

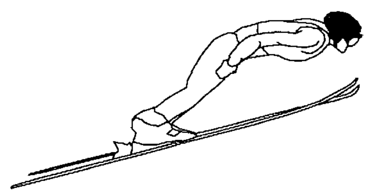
6. A fan driven by the motor removes air, and hence decreases the pressure inside the cleaner. The greater air pressure outside the cleaner pushes air in through the nozzle toward this region of lower pressure. This inward rush of air pushes or carries the dirt along with it.

8. The external pressure exerted on the chest by the water makes it difficult to expand the chest cavity and take a breath while under water. Thus, a snorkel will not work in deep water.

10. The water level on the side of the glass stays the same. The floating ice cube displaces its own weight of liquid water, and so does the liquid water into which it melts.

12. The higher the density of a fluid, the higher an object will float in it. Thus, an object will float lower in low-density alcohol.

14. The ski jumper gives her body the shape of an airfoil. She deflects the air stream downward as it rushes past and it deflects her upward in agreement with Newton's third law. Thus, the air exerts a lift force on her, giving a higher and longer trajectory.



ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 7.322×10^{-3} kg **(b)** $V_{gold} = 3.79 \times 10^{-7}\ \text{m}^3$, $V_{copper} = 7.46 \times 10^{-8}\ \text{m}^3$

(c) $1.76 \times 10^{4}\ \text{kg/m}^3$

4. 24.8 kg

6. 1.9×10^{4} N

8. 1.00×10^{11} Pa

10. **(a)** 1.77×10^{6} N **(b)** The superhero would be thrown backward by a reaction force of 1.77×10^{6} N exerted on him by the wall according to Newton's third law.

12. **(a)** 3.14×10^{4} N **(b)** 6.28×10^{4} N

14. 22 N toward the bottom of the page in Figure P9.14

16. **(a)** 2.5 mm **(b)** 0.70 mm **(c)** 6.9×10^{3} kg

18. Yes, the average stress is 5.5×10^{7} Pa, considerably less than 16×10^{7} Pa.

20. 2.11 m

22. **(a)** 20.0 cm **(b)** 0.490 cm

24. **(a)** $\Delta V = -0.054\ \text{m}^3$ **(b)** $1.1 \times 10^3\ \text{kg/m}^3$

(c) With only a 5.4% change in volume in this extreme case, liquid water is indeed nearly incompressible in biological and student laboratory situations.

26. **(a)** 10.5 m

(b) No. Since water and alcohol are more volatile than mercury, more liquid will evaporate and degrade the vacuum above the liquid column inside the tube of this barometer.

28. 2.3 lb

30. **(a)** 23.2% after inhaling, 17.1% after exhaling

(b) In general, "sinkers" would be expected to be thinner with heavier bones, whereas "floaters" would have lighter bones and more fat.

32. **(a)** See Solution. **(b)** $\Sigma F_y = B - w - w_r = 0$ **(c)** 964 N

(d) 356 N **(e)** $101\ \text{kg/m}^3$ **(f)** $3.62 \times 10^3\ \text{N}$

(g) 333 kg

34. **(a)** See Solution. **(b)** $4.11 \times 10^3\ \text{N}$

(c) $\Sigma F_y = +1.33 \times 10^3\ \text{N}$, the balloon rises when released **(d)** 136 kg

(e) The balloon and its load accelerate upward.

(f) As the balloon rises, decreasing atmospheric density decreases the buoyancy force.

36. **(a)** $\Sigma F_y = \rho_w g V - mg = ma_y$ **(b)** $a_y = \left[(\rho_w/\rho) - 1\right]g$ **(c)** $0.467\ \text{m/s}^2$ down

(d) 5.85 s

38. $3.33 \times 10^3\ \text{kg/m}^3$

40. 16.5 cm

42. **(a)** $8.57 \times 10^3\ \text{kg/m}^3$ **(b)** $715\ \text{kg/m}^3$

44. **(a)** $0.471\ \text{m/s}$ **(b)** $4.24\ \text{m/s}$

46. **(a)** $11.0\ \text{m/s}$ **(b)** $2.64 \times 10^4\ \text{Pa}$

48. $4.4 \times 10^{-2}\ \text{Pa}$

50. $P_{\substack{\text{upper}\\\text{surface}}} = P_{\substack{\text{lower}\\\text{surface}}} - Mg/A_{\text{wings}}$

52. **(a)** $2.02\ \text{m/s}$ **(b)** $8.08\ \text{m/s}$ **(c)** $5.71 \times 10^{-3}\ \text{m}^3/\text{s}$

54. **(a)** $17.7\ \text{m/s}$ **(b)** 1.73 mm

56. **(a)** 15.1 MPa **(b)** $2.95\ \text{m/s}$ **(c)** 4.35 kPa

58. **(a)** $1.91\ \text{m/s}$

(b) $8.64\times10^{-4}\ \text{m}^3/\text{s}$

60. $7.32\times10^{-2}\ \text{N/m}$

62. 5.6 m

64. 0.12 N

66. $1.5\ \text{m/s}$

68. $1.5\times10^{5}\ \text{Pa}$

70. 455 kPa

72. $8.0\ \text{cm/s}$

74. $9.5\times10^{-10}\ \text{m}^2/\text{s}$

76. $1.02\times10^{3}\ \text{kg/m}^3$

78. See Solution.

80. **(a)** See Solution. **(b)** $1.23\times10^{4}\ \text{Pa}$

82. **(a)** 10.3 m **(b)** 0

84. See Solution.

86. 1.9 m

88. **(a)** 1.25 cm **(b)** $13.8\ \text{m/s}$

90. 1.71 cm

PROBLEM SOLUTIONS

9.1 The average density of either of the two original worlds was

$$\rho_0 = \frac{M}{V} = \frac{M}{4\pi R^3/3} = \frac{3M}{4\pi R^3}$$

The average density of the combined world is

$$\rho = \frac{M_{\text{total}}}{V'} = \frac{2M}{\frac{4\pi}{3}\left(\frac{3}{4}R\right)^3} = \frac{4^2(2M)}{\pi\left(3^2\right)R^3} = \frac{32M}{9\pi R^3}$$

so $\quad \dfrac{\rho}{\rho_0} = \left(\dfrac{32M}{9\pi R^3}\right)\left(\dfrac{4\pi R^3}{3M}\right) = \dfrac{128}{27} = 4.74 \quad$ or $\quad \boxed{\rho = 4.74\rho_0}$

9.2 (a) The mass of gold in the coin is

$$m_{\text{Au}} = \frac{(\#\ \text{karats})\, m_{\text{total}}}{24} = \frac{22}{24} m_{\text{total}} = \frac{11}{12}\left(7.988\times10^{-3}\ \text{kg}\right) = \boxed{7.322\times10^{-3}\ \text{kg}}$$

and the mass of copper is $m_{\text{Cu}} = m_{\text{total}}/12 = \left(7.988\times10^{-3}\ \text{kg}\right)/12 = 6.657\times10^{-4}\ \text{kg}$.

(b) The volume of the gold present is

$$V_{\text{Au}} = \frac{m_{\text{Au}}}{\rho_{\text{Au}}} = \frac{7.322\times10^{-3}\ \text{kg}}{19.3\times10^{3}\ \text{kg/m}^3} = \boxed{3.79\times10^{-7}\ \text{m}^3}$$

and the volume of the copper is

$$V_{\text{Cu}} = \frac{m_{\text{Cu}}}{\rho_{\text{Cu}}} = \frac{6.657\times10^{-4}\ \text{kg}}{8.92\times10^{3}\ \text{kg/m}^3} = \boxed{7.46\times10^{-8}\ \text{m}^3}$$

(c) The average density of the British sovereign coin is

$$\rho_{\text{av}} = \frac{m_{\text{total}}}{V_{\text{total}}} = \frac{m_{\text{total}}}{V_{\text{Au}} + V_{\text{Cu}}} = \frac{7.988\times10^{-3}\ \text{kg}}{3.79\times10^{-7}\ \text{m}^3 + 7.46\times10^{-8}\ \text{m}^3} = \boxed{1.76\times10^{4}\ \text{kg/m}^3}$$

9.3 (a) The total normal force exerted on the bottom acrobat's shoes by the floor equals the total weight of the acrobats in the tower. That is

$$n = m_{\text{total}}g = \left[(75.0+68.0+62.0+55.0)\ \text{kg}\right]\left(9.80\ \text{m/s}^2\right) = \boxed{2.55\times10^{3}\ \text{N}}$$

(b) $$P = \frac{n}{A_{\text{total}}} = \frac{n}{2A_{\substack{\text{shoe}\\ \text{sole}}}} = \frac{2.55\times10^{3}\ \text{N}}{2\left[425\ \text{cm}^2\left(1\ \text{m}^2/10^4\ \text{cm}^2\right)\right]} = \boxed{3.00\times10^{4}\ \text{Pa}}$$

(c) If the acrobats are rearranged so different ones are at the bottom of the tower, the total weight supported, and hence the total normal force n, will be unchanged. However, the total area $A_{\text{total}} = 2A_{\text{shoe sole}}$, and hence the pressure, will change unless all the acrobats wear the same size shoes.

9.4 $$M_{\text{bar}} = \rho_{\text{Au}}V_{\text{bar}} = \rho_{\text{Au}}(\ell\times w\times h) = \left(19.3\times10^{3}\ \text{kg/m}^3\right)\left[(0.045\,0\ \text{m})(0.110\ \text{m})(0.260\ \text{m})\right]$$

or $M_{\text{bar}} = \boxed{24.8\ \text{kg}}$

9.5 (a) If the particles in the nucleus are closely packed with negligible space between them, the average nuclear density should be approximately that of a proton or neutron. That is

$$\rho_{\text{nucleus}} \approx \frac{m_{\text{proton}}}{V_{\text{proton}}} = \frac{m_{\text{proton}}}{4\pi r^3/3} \sim \frac{3\left(1.67\times10^{-27}\ \text{kg}\right)}{4\pi\left(1\times10^{-15}\ \text{m}\right)^3} \quad \boxed{\sim 4\times10^{17}\ \text{kg/m}^3}$$

(b) The density of iron is $\rho_{\text{Fe}} = 7.86\times10^{3}\ \text{kg/m}^3$ and the densities of other solids and liquids are on the order of $10^3\ \text{kg/m}^3$. Thus, the nuclear density is about 10^{14} times greater than that of common solids and liquids, which suggests that atoms must be mostly empty space. Solids and liquids, as well as gases, are mostly empty space.

9.6 Let the weight of the car be W. Then, each tire supports $W/4$, and the gauge pressure is $P = F/A = (W/4)/A = W/4A$. Thus,

$$W = 4\,A\,P = 4\left(0.024\ \text{m}^2\right)\left(2.0\times10^{5}\ \text{Pa}\right) = \boxed{1.9\times10^{4}\ \text{N}}$$

9.7 (a) $F_{\text{atm}} = PA = P_{\text{atm}}\left(\pi r^2\right) = \left(8.04\times10^4\text{ Pa}\right)\pi(2.00\text{ m})^2 = \boxed{1.01\times10^6\text{ N}}$

(b) $F_g = mg = (\rho V)g = \rho\left[\left(\pi r^2\right)h\right]g$

$$= \left(415\text{ kg/m}^3\right)\left[\pi(2.00\text{ m})^2(10.0\text{ m})\right]\left(7.44\text{ m/s}^2\right) = \boxed{3.88\times10^5\text{ N}}$$

(c) Now, consider the thin disk-shaped region 2.00 m in radius at the bottom end of the column of methane. The total downward force on it is the weight of the 10.0-meter tall column of methane plus the downward force exerted on the upper end of the column by the atmosphere. Thus, the pressure (force per unit area) on the disk-shaped region located 10.0 meters below the ocean surface is

$$P = \frac{F_{\text{total}}}{A} = \frac{F_{\text{atm}} + F_g}{\pi r^2} = \frac{1.01\times10^6\text{ N} + 3.88\times10^5\text{ N}}{\pi(2.00\text{ m})^2} = \boxed{1.11\times10^5\text{ Pa}}$$

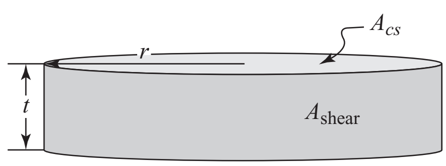
9.8 By definition, Young's modulus is the ratio of the tensile stress to the tensile strain in an elastic material. Thus, Young's modulus for this material is the slope of the linear portion of the graph in the "elastic behavior" region. This is

$$Y = \frac{\Delta(\mathit{stress})}{\Delta(\mathit{strain})} = \frac{200\text{ MPa} - 0}{0.002 - 0} = 1.00\times10^5\text{ MPa} = 1.00\times10^5\left(10^6\text{ Pa}\right) = \boxed{1.00\times10^{11}\text{ Pa}}$$

9.9 Young's modulus is defined as $Y = \mathit{stress}/\mathit{strain} = (F/A)/(\Delta L/L_0) = (F\cdot L_0)/(A\cdot\Delta L)$. Thus, the elongation of the wire is

$$\Delta L = \frac{F\cdot L_0}{A\cdot Y} = \frac{\left[(200\text{ kg})\left(9.80\text{ m/s}^2\right)\right]\cdot(4.00\text{ m})}{\left(0.200\times10^{-4}\text{ m}^2\right)\left(8.00\times10^{10}\text{ N/m}^2\right)} = 4.90\times10^{-3}\text{ m} = \boxed{4.90\text{ mm}}$$

9.10 (a) In order to punch a hole in the steel plate, the superhero must punch out a plug with cross-sectional area, A_{cs}, equal to that of his fist and a height t equal to the thickness of the steel plate. The area A_{shear} of the face that is sheared as the plug is removed is the cylindrical surface with radius r and height t as shown in the sketch. Since $A_{cs} = \pi r^2$, then $r = \sqrt{A_{cs}/\pi}$ and



$$A_{\text{shear}} = (2\pi r)t = 2\pi t\sqrt{\frac{A_{cs}}{\pi}} = 2\pi(2.00\text{ cm})\sqrt{\frac{1.00\times10^2\text{ cm}^2}{\pi}} = 70.9\text{ cm}^2$$

If the ultimate shear strength of steel (i.e., the maximum shear stress it can withstand before shearing) is $2.50\times10^8\text{ Pa} = 2.50\times10^8\text{ N/m}^2$, the minimum force required to punch out this plug is

$$F = A_{\text{shear}}\cdot\mathit{stress} = \left[70.9\ \cancel{\text{cm}^2}\left(\frac{1\ \cancel{\text{m}^2}}{10^4\ \cancel{\text{cm}^2}}\right)\right]\left(2.50\times10^8\ \frac{\text{N}}{\cancel{\text{m}^2}}\right) = \boxed{1.77\times10^6\text{ N}}$$

(b) By $\boxed{\text{Newton's third law}}$, the wall would exert a force of equal magnitude in the opposite direction on the superhero, $\boxed{\text{who would be thrown backward}}$ at a very high recoil speed.

9.11 Two cross-sectional areas in the plank, with one directly above the rail and one at the outer end of the plank, separated by distance $h = 2.00\text{ m}$ and each with area $A = (2.00\text{ cm})(15.0\text{ cm}) = 30.0\text{ cm}^2$,

continued on next page

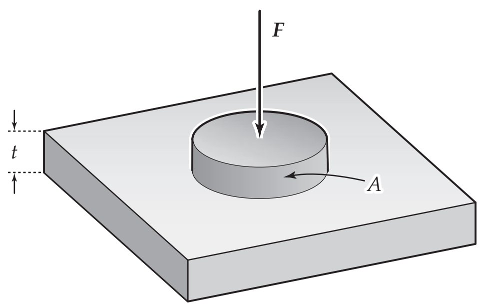
move a distance $\Delta x = 5.00 \times 10^{-2}$ m parallel to each other. The force causing this shearing effect in the plank is the weight of the man $F = mg$ applied perpendicular to the length of the plank at its outer end. Since the shear modulus is given by $S = \textit{shear stress}/\textit{shear strain} = (F/A)/(\Delta x/h) = Fh/(\Delta x)A$, the shear modulus for the wood in this plank must be

$$S = \frac{(80.0\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})}{(5.00\times10^{-2}\text{ m})\left[(30.0\text{ cm}^2)(1\text{ m}^2/10^4\text{ cm}^2)\right]} = \boxed{1.05\times10^7\text{ Pa}}$$

9.12 (a) The force needed to shear the bolt through its cross-sectional area is $F = (A)(\textit{stress})$, or

$$F = \pi(5.00\times10^{-3}\text{ m})^2(4.00\times10^8\text{ N/m}^2) = \boxed{3.14\times10^4\text{ N}}$$

(b) The area over which the shear occurs is equal to the circumference of the hole times its thickness. Thus,



$$A = (2\pi r)t = \left[2\pi(5.00\times10^{-3}\text{ m})\right](5.00\times10^{-3}\text{ m}) = 1.57\times10^{-4}\text{ m}^2$$

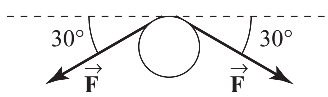
The force required to punch a hole of this area in the 0.500-cm thick steel plate is then

$$F = (A)(\textit{stress}) = (1.57\times10^{-4}\text{ m}^2)(4.00\times10^8\text{ N/m}^2) = \boxed{6.28\times10^4\text{ N}}$$

9.13 Using $Y = \dfrac{F L_0}{A(\Delta L)}$ with $A = \pi d^2/4$ and $F = mg$, we get

$$Y = \frac{4\left[(90\text{ kg})(9.80\text{ m/s}^2)\right](50\text{ m})}{\pi(1.0\times10^{-2}\text{ m})^2(1.6\text{ m})} = \boxed{3.5\times10^8\text{ Pa}}$$

9.14 From $Y = F L_0/A(\Delta L)$, the tension needed to stretch the wire by 0.10 mm is



$$F = \frac{Y A(\Delta L)}{L_0} = \frac{Y(\pi r^2)(\Delta L)}{L_0}$$

$$= \frac{(18\times10^{10}\text{ Pa})\pi(0.11\times10^{-3}\text{ m})^2(0.10\times10^{-3}\text{ m})}{(3.1\times10^{-2}\text{ m})} = 22\text{ N}$$

The tension in the wire exerts a force of magnitude F on the tooth in each direction along the length of the wire as shown in the above sketch. The resultant force exerted on the tooth has an x-component of $R_x = \Sigma F_x = -F\cos30° + F\cos30° = 0$, and a y-component of $R_y = \Sigma F_y = -F\sin30° - F\sin30° = -F = -22$ N.

Thus, the resultant force is

$$\vec{\mathbf{R}} = \boxed{22\text{ N directed down the page in the diagram}}.$$

9.15 From $Y = stress/strain = (stress)(L_0/\Delta L)$, the maximum compression the femur can withstand before breaking is

$$\Delta L_{\max} = \frac{(stress)_{\max}(L_0)}{Y} = \frac{(160\times 10^6 \text{ Pa})(0.50 \text{ m})}{18\times 10^9 \text{ Pa}} = 4.4\times 10^{-3} \text{ m} = \boxed{4.4 \text{ mm}}$$

9.16 (a) When at rest, the tension in the cable equals the weight of the 800-kg object, 7.84×10^3 N. Thus, from $Y = \dfrac{F\,L_0}{A(\Delta L)}$, the initial elongation of the cable is

$$\Delta L = \frac{F\cdot L_0}{A\cdot Y} = \frac{(7.84\times 10^3 \text{ N})(25.0 \text{ m})}{(4.00\times 10^{-4} \text{ m}^2)(20\times 10^{10} \text{ Pa})} = 2.5\times 10^{-3} \text{ m} = \boxed{2.5 \text{ mm}}$$

(b) When the load is accelerating upward, Newton's second law gives the tension in the cable as

$$F - mg = ma_y, \quad \text{or} \quad F = m(g + a_y) \qquad \textbf{[1]}$$

If $m = 800$ kg and $a_y = +3.0 \text{ m/s}^2$, the elongation of the cable will be

$$\Delta L = \frac{F\cdot L_0}{A\cdot Y} = \frac{\left[(800 \text{ kg})(9.80+3.0) \text{ m/s}^2\right](25.0 \text{ m})}{(4.00\times 10^{-4} \text{ m}^2)(20\times 10^{10} \text{ Pa})} = 3.2\times 10^{-3} \text{ m} = 3.2 \text{ mm}$$

Thus, the increase in the elongation has been

$$increase = (\Delta L) - (\Delta L)_{\text{initial}} = 3.2 \text{ mm} - 2.5 \text{ mm} = \boxed{0.70 \text{ mm}}$$

(c) From the definition of the tensile stress, $stress = F/A$, the maximum tension the cable can withstand is

$$F_{\max} = A\cdot(stress)_{\max} = (4.00\times 10^{-4} \text{ m}^2)(2.2\times 10^8 \text{ Pa}) = 8.8\times 10^4 \text{ N}$$

Then, Equation [1] above gives the mass of the maximum load as

$$m_{\max} = \frac{F_{\max}}{g+a} = \frac{8.8\times 10^4 \text{ N}}{(9.8+3.0) \text{ m/s}^2} = \boxed{6.9\times 10^3 \text{ kg}}$$

9.17 The upward force supporting the 8 500 N load is the sum of the compression force exerted by the column and the tension force exerted by the cable.

$$F_{\text{column}} + F_{\text{cable}} = F_{g,\text{ load}} = 8\,500 \text{ N}$$

Since the magnitude of the change in length is the same for the column and the cable, this becomes

$$Y_{\text{Al}}A_{\text{column}}\left(\frac{\Delta L}{L_{0,\text{ column}}}\right) + Y_{\text{steel}}A_{\text{cable}}\left(\frac{\Delta L}{L_{0,\text{ cable}}}\right) = 8.50\times 10^3 \text{ N}$$

yielding
$$\Delta L = \frac{8.50\times 10^3 \text{ N}}{\dfrac{Y_{\text{Al}}}{L_{0,\text{ column}}}\left(\dfrac{\pi d_{\text{outer}}^2}{4} - \dfrac{\pi d_{\text{inner}}^2}{4}\right) + \dfrac{Y_{\text{steel}}}{L_{0,\text{ cable}}}\left(\dfrac{\pi d_{\text{cable}}^2}{4}\right)}$$

continued on next page

or $$\Delta L = \frac{8.50\times10^3\ \text{N}}{\dfrac{7.0\times10^{10}\ \text{Pa}}{3.25\ \text{m}}\left[\dfrac{\pi(0.1624\ \text{m})^2}{4}-\dfrac{\pi(0.1614\ \text{m})^2}{4}\right]+\dfrac{20\times10^{10}\ \text{Pa}}{5.75\ \text{m}}\left[\dfrac{\pi(0.012\,7\ \text{m})^2}{4}\right]}$$

$$\Delta L = 8.6\times10^{-4}\ \text{m} = \boxed{0.86\ \text{mm}}$$

9.18 The acceleration of the forearm has magnitude

$$a = \frac{|\Delta v|}{\Delta t} = \frac{80\ \text{km/h}\left(10^3\ \text{m/1 km}\right)\left(1\ \text{h/3\,600 s}\right)}{5.0\times10^{-3}\ \text{s}} = 4.4\times10^3\ \text{m/s}^2$$

The compression force exerted on the arm is $F = ma$ and the compressional stress on the bone material is

$$Stress = \frac{F}{A} = \frac{(3.0\ \text{kg})\left(4.4\times10^3\ \text{m/s}^2\right)}{2.4\ \text{cm}^2\left(10^{-4}\ \text{m}^2/1\ \text{cm}^2\right)} = \boxed{5.5\times10^7\ \text{Pa}}$$

Since the calculated stress is less than the maximum stress bone material can withstand without breaking, $\boxed{\text{the arm should survive}}$.

9.19 The tension and cross-sectional area are constant through the entire length of the rod, and the total elongation is the sum of that of the aluminum section and that of the copper section.

$$\Delta L_{\text{rod}} = \Delta L_{\text{Al}} + \Delta L_{\text{Cu}} = \frac{F(L_0)_{\text{Al}}}{AY_{\text{Al}}} + \frac{F(L_0)_{\text{Cu}}}{AY_{\text{Cu}}} = \frac{F}{A}\left[\frac{(L_0)_{\text{Al}}}{Y_{\text{Al}}} + \frac{(L_0)_{\text{Cu}}}{Y_{\text{Cu}}}\right]$$

where $A = \pi r^2$ with $r = 0.20\ \text{cm} = 2.0\times10^{-3}\ \text{m}$. Thus,

$$\Delta L_{\text{rod}} = \frac{\left(5.8\times10^3\ \text{N}\right)}{\pi\left(2.0\times10^{-3}\ \text{m}\right)^2}\left[\frac{1.3\ \text{m}}{7.0\times10^{10}\ \text{Pa}} + \frac{2.6\ \text{m}}{11\times10^{10}\ \text{Pa}}\right] = 1.9\times10^{-2}\ \text{m} = \boxed{1.9\ \text{cm}}$$

9.20 Assuming the spring obeys Hooke's law, the increase in force on the piston required to compress the spring an additional amount Δx is $\Delta F = F - F_0 = (P - P_0)A = k(\Delta x)$. The gauge pressure at depth h beneath the surface of a fluid is $P - P_0 = \rho g h$, so we have $\rho g h A = k(\Delta x)$, or the required depth is $h = k(\Delta x)/\rho g A$. If $k = 1\,250\ \text{N/m}$, $A = \pi r^2$ with $r = 1.20\times10^{-2}\ \text{m}$, and the fluid is water ($\rho = 1.00\times10^3\ \text{kg/m}^3$), the depth required to compress the spring an additional $0.750\ \text{cm} = 7.50\times10^{-3}\ \text{m}$ is

$$h = \frac{\left(1\,250\ \text{N/m}\right)\left(7.50\times10^{-3}\ \text{m}\right)}{\left(1.00\times10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left[\pi\left(1.20\times10^{-2}\ \text{m}\right)^2\right]} = \boxed{2.11\ \text{m}}$$

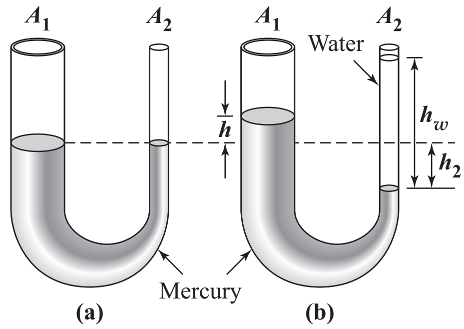
9.21 (a) $P = P_0 + \rho g h = 101.3\times10^3\ \text{Pa} + \left(1.00\times10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(27.5\ \text{m})$

$$= \boxed{3.71\times10^5\ \text{Pa}}$$

(b) The inward force the water will exert on the window is

$$F = PA = P\left(\pi r^2\right) = \left(3.71\times10^5\ \text{Pa}\right)\pi\left(\frac{35.0\times10^{-2}\ \text{m}}{2}\right)^2 = \boxed{3.57\times10^4\ \text{N}}$$

9.22 (a) Consider the tube as shown in part (b) of the sketch at the right. The volume of water in the right arm is



$$V_w = \frac{m_w}{\rho_w} = \frac{100 \text{ g}}{1.00 \text{ g/cm}^3} = 100 \text{ cm}^3$$

But, we also know that $V_w = A_2 h_w$, giving

$$h_w = \frac{V_w}{A_2} = \frac{100 \text{ cm}^3}{5.00 \text{ cm}^2} = \boxed{20.0 \text{ cm}}$$

(b) From the sketch above, observe that the mercury that has been forced out of the right arm into the left arm is $V_{\text{displaced}} = A_1 h = A_2 h_2$, so $h_2 = (A_1/A_2)h$

or $$h_2 = \left(\frac{10.0 \text{ cm}^2}{5.00 \text{ cm}^2}\right)h = 2.00\,h$$

The absolute pressure at the water-mercury interface in the right arm is

$$P = P_0 + \rho_w g h_w \qquad \textbf{[1]}$$

The absolute pressure at the same level in the left arm is

$$P = P_0 + \rho_{Hg} g(h + h_2) = P_0 + \rho_{Hg} g(h + 2.00h)$$

or $$P = P_0 + 3.00\rho_{Hg} g h \qquad \textbf{[2]}$$

Since the pressure is the same at all points at a given level in a static fluid, we equate the pressures in Equations [1] and [2] to obtain $P_0 + 3.00\rho_{Hg} g h = P_0 + \rho_w g h_w$, which yields

$$h = \left(\frac{\rho_w}{3.00\rho_{Hg}}\right)h_w = \left[\frac{1.00\times10^3 \text{ kg/m}^3}{(3.00)(13.6\times10^3 \text{ kg/m}^3)}\right](20.0 \text{ cm}) = \boxed{0.490 \text{ cm}}$$

9.23 The density of the solution is $\rho = 1.02\,\rho_{\text{water}} = 1.02\times10^3 \text{ kg/m}^3$. If the glucose solution is to flow into the vein, the minimum required gauge pressure of the fluid at the level of the needle is equal to the gauge pressure in the vein, giving

$$P_{\text{gauge}} = P - P_0 = \rho g h_{\min} = 1.33\times10^3 \text{ Pa}$$

and $$h_{\min} = \frac{1.33\times10^3 \text{ Pa}}{\rho g} = \frac{1.33\times10^3 \text{ Pa}}{(1.02\times10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = \boxed{0.133 \text{ m}}$$

9.24 (a) From the definition of bulk modulus, $B = -\Delta P/(\Delta V/V_0)$, the change in volume of the 1.00 m^3 of seawater will be

$$\Delta V = -\frac{V_0(\Delta P)}{B_{\text{water}}} = -\frac{(1.00 \text{ m}^3)(1.13\times10^8 \text{ Pa} - 1.013\times10^5 \text{ Pa})}{0.21\times10^{10} \text{ Pa}} = \boxed{-0.054 \text{ m}^3}$$

(b) The quantity of seawater that had volume $V_0 = 1.00 \text{ m}^3$ at the surface has a mass of 1 030 kg. Thus, the density of this water at the ocean floor is

$$\rho = \frac{m}{V} = \frac{m}{V_0 + \Delta V} = \frac{1\,030 \text{ kg}}{(1.00 - 0.054) \text{ m}^3} = \boxed{1.1\times10^3 \text{ kg/m}^3}$$

continued on next page

(c) With only a 5.4% change in volume in this extreme case, liquid water is indeed nearly incompressible in biological and student-laboratory situations.

9.25 We first find the absolute pressure at the interface between oil and water.

$$P_1 = P_0 + \rho_{\text{oil}}\, g h_{\text{oil}}$$

$$= 1.013\times10^5\ \text{Pa} + \left(700\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(0.300\ \text{m}\right) = 1.03\times10^5\ \text{Pa}$$

This is the pressure at the top of the water. To find the absolute pressure at the bottom, we use $P_2 = P_1 + \rho_{\text{water}}\, g h_{\text{water}}$, or

$$P_2 = 1.03\times10^5\ \text{Pa} + \left(10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(0.200\ \text{m}\right) = \boxed{1.05\times10^5\ \text{Pa}}$$

9.26 (a) If we assume a vacuum $(P = 0)$ inside the tube above the wine column and atmospheric pressure at the base of the column (that is, at the level of the wine in the open container), we start at the top of the liquid in the tube and calculate the pressure at depth h in the wine as $P_{\text{atmo}} = 0 + \rho g h = \rho g h$. Thus,

$$h = \frac{P_{\text{atmo}}}{\rho g} = \frac{1.013\times10^5\ \text{Pa}}{\left(984\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)} = \boxed{10.5\ \text{m}}$$

(b) $\boxed{\text{No.}}$ Since water and alcohol are more volatile than mercury, more liquid will evaporate and degrade the vacuum above the liquid column inside the tube of this barometer.

9.27 Pascal's principle, $F_1/A_1 = F_2/A_2$, or $F_{\text{pedal}}\big/A_{\substack{\text{Master}\\\text{cylinder}}} = F_{\text{brake}}\big/A_{\substack{\text{brake}\\\text{cylinder}}}$, gives

$$F_{\text{brake}} = \left(\frac{A_{\text{brake cylinder}}}{A_{\text{master cylinder}}}\right)F_{\text{pedal}}$$

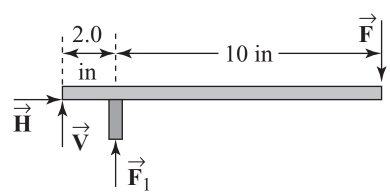
This is the normal force exerted on the brake shoe. The frictional force is

$$f = \mu_k\, n = 0.50\, F_{\text{brake}} = 0.50\left(\frac{6.4\ \text{cm}^2}{1.8\ \text{cm}^2}\right)\left(44\ \text{N}\right) = 78\ \text{N}$$

and the torque is

$$\tau = f\cdot r_{\text{drum}} = \left(78\ \text{N}\right)\left(0.34\ \text{m}\right) = \boxed{27\ \text{N}\cdot\text{m}}$$

9.28 First, use Pascal's principle, $F_1/A_1 = F_2/A_2$, to find the force piston 1 will exert on the handle when a 500-lb force pushes downward on piston 2.



Free-Body Diagram of Handle

$$F_1 = \left(\frac{A_1}{A_2}\right)F_2 = \left(\frac{\pi\, d_1^2/4}{\pi\, d_2^2/4}\right)F_2 = \left(\frac{d_1^2}{d_2^2}\right)F_2$$

$$= \frac{\left(0.25\ \text{in}\right)^2}{\left(1.5\ \text{in}\right)^2}\left(500\ \text{lb}\right) = 14\ \text{lb}$$

Now, consider an axis perpendicular to the page, passing through the left end of the jack handle. $\Sigma\tau = 0$ yields

$$+\left(14\ \text{lb}\right)\left(2.0\ \text{in}\right) - F\cdot\left(12\ \text{in}\right) = 0, \qquad \text{or} \qquad F = \boxed{2.3\ \text{lb}}$$

9.29 When held underwater, the ball will have three forces acting on it: a downward gravitational force, $mg = \rho_{\text{ball}} V g = \rho_{\text{ball}}\left(4\pi r^3/3\right)g$; an upward buoyant force, $B = \rho_{\text{water}} V g = \rho_{\text{water}}\left(4\pi r^3/3\right)g$; and an applied force, F. If the ball is to be in equilibrium, we have (taking upward as positive)

$$\Sigma F_y = F + B - mg = 0$$

or $$F = mg - B = \left[\rho_{\text{ball}}\left(\frac{4\pi r^3}{3}\right)\right]g - \rho_{\text{water}}\left(\frac{4\pi r^3}{3}\right)g = \left(\rho_{\text{ball}} - \rho_{\text{water}}\right)\left(\frac{4\pi r^3}{3}\right)g$$

giving

$$F = \left[\left(0.084\,0 - 1.00\right)\times 10^3 \text{ kg/m}^3\right]\frac{4\pi}{3}\left(\frac{0.038\,0 \text{ m}}{2}\right)^3\left(9.80 \text{ m/s}^2\right)$$
$$= -0.258 \text{ N}$$

so the required applied force is $\boxed{\vec{\mathbf{F}} = 0.258 \text{ N directed downward}}$.

9.30 (a) To float, the buoyant force acting on the person must equal the weight of that person, or the weight of the water displaced by the person must equal the person's own weight. Thus,

$$B = mg \quad \Rightarrow \quad \left(\rho_{\text{sea}} V_{\text{submerged}}\right)g = \left(\rho_{\text{body}}\, g V_{\text{total}}\right)g \qquad \text{or} \qquad \frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{\rho_{\text{body}}}{\rho_{\text{sea}}}$$

After inhaling,

$$\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{945 \text{ kg/m}^3}{1\,230 \text{ kg/m}^3} = 0.768 = 76.8\%$$

leaving $\boxed{23.2\% \text{ above surface}}$

After exhaling,

$$\frac{V_{\text{submerged}}}{V_{\text{total}}} = \frac{1\,020 \text{ kg/m}^3}{1\,230 \text{ kg/m}^3} = 0.829 = 82.9\%$$

leaving $\boxed{17.1\% \text{ above surface}}$

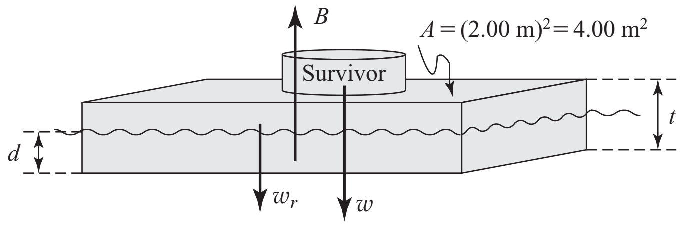
(b) In general, "sinkers" would be expected to be thinner with heavier bones, whereas "floaters" would have lighter bones and more fat.

9.31 The boat sinks until the weight of the additional water displaced equals the weight of the truck. Thus,

$$w_{\text{truck}} = \left[\rho_{\text{water}}\left(\Delta V\right)\right]g$$
$$= \left(10^3 \ \frac{\text{kg}}{\text{m}^3}\right)\left[\left(4.00 \text{ m}\right)\left(6.00 \text{ m}\right)\left(4.00\times 10^{-2} \text{ m}\right)\right]\left(9.80 \ \frac{\text{m}}{\text{s}^2}\right),$$

or $$w_{\text{truck}} = 9.41\times 10^3 \text{ N} = \boxed{9.41 \text{ kN}}$$

9.32 (a)



(b) Since the system is in equilibrium, $\boxed{\Sigma F_y = B - w - w_r = 0}$

(c) $B = \rho_w g V_{\text{submerged}} = \rho_w g(d \cdot A)$

$$= (1\,025\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.024\,0\ \text{m})(4.00\ \text{m}^2) = \boxed{964\ \text{N}}$$

(d) From $B - w - w_r = 0$,

$$w_r = B - w = B - mg = 964\ \text{N} - (62.0\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{356\ \text{N}}$$

(e) $$\rho_{\text{foam}} = \frac{m_r}{V_r} = \frac{w_r/g}{t \cdot A} = \frac{356\ \text{N}}{(9.80\ \text{m/s}^2)(0.090\ \text{m})(4.00\ \text{m}^2)} = \boxed{101\ \text{kg/m}^3}$$

(f) $B_{\max} = \rho_w g V_r = \rho_w g(t \cdot A)$

$$= (1\,025\ \text{kg/m}^3)(9.80\ \text{m/s}^2)(0.090\,0\ \text{m})(4.00\ \text{m}^2) = \boxed{3.62 \times 10^3\ \text{N}}$$

(g) The maximum weight of survivors the raft can support is $w_{\max} = m_{\max} g = B_{\max} - w_r$, so

$$m_{\max} = \frac{B_{\max} - w_r}{g} = \frac{3.62 \times 10^3\ \text{N} - 356\ \text{N}}{9.80\ \text{m/s}^2} = \boxed{333\ \text{kg}}$$

9.33 (a) While the system floats, $B = w_{\text{total}} = w_{\text{block}} + w_{\text{steel}}$, or

$$\rho_{\text{wood}} \cancel{g} V_{\text{submerged}} = \rho_{\text{block}} \cancel{g} V_{\text{block}} + m_{\text{steel}} \cancel{g}$$

When $m_{\text{steel}} = 0.310$ kg, $V_{\text{submerged}} = V_{\text{block}} = 5.24 \times 10^{-4}\ \text{m}^3$, giving

$$\rho_{\text{block}} = \frac{\rho_{\text{wood}} V_{\text{block}} - m_{\text{steel}}}{V_{\text{block}}} = \rho_{\text{wood}} - \frac{m_{\text{steel}}}{V_{\text{block}}}$$

$$= 1.00 \times 10^3\ \text{kg/m}^3 - \frac{0.310\ \text{kg}}{5.24 \times 10^{-4}\ \text{m}^3} = \boxed{408\ \text{kg/m}^3}$$

(b) If the total weight of the block+steel system is reduced, by having $m_{\text{steel}} < 0.310$ kg, a smaller buoyant force is needed to allow the system to float in equilibrium. Thus, the block will displace a smaller volume of water and will be only partially submerged.

The block is fully submerged when $m_{\text{steel}} = 0.310$ kg. The mass of the steel object can increase slightly above this value without causing it and the block to sink to the bottom. As the mass of the steel object is gradually increased above 0.310 kg, the steel object begins to submerge, displacing additional water, and providing a slight increase in the buoyant force. With a density of about eight times that of water, the steel object will be able to displace approximately 0.310 kg/8 = 0.039 kg of additional water before it becomes fully submerged. At this point, the steel object will have a mass of about 0.349 kg and will be unable to displace any additional water. Any further increase in

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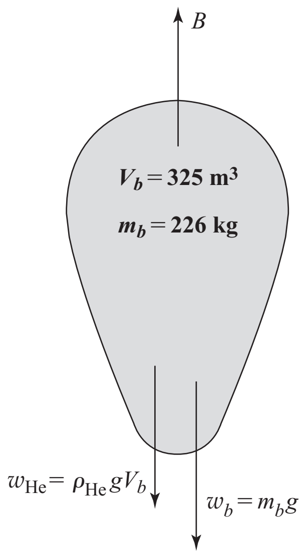
the mass of the object causes it and the block to sink to the bottom. In conclusion, $\boxed{\text{the block + steel system will sink if } m_{\text{steel}} \geq 0.350 \text{ kg.}}$

9.34 (a)



(b) Since the balloon is fully submerged in air, $V_{\text{submerged}} = V_b = 325 \text{ m}^3$, and

$$B = \rho_{\text{air}} g V_b = \left(1.29 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)\left(325 \text{ m}^3\right) = \boxed{4.11 \times 10^3 \text{ N}}$$

(c) $\Sigma F_y = B - w_b - w_{\text{He}} = B - m_b g - \rho_{\text{He}} g V_b = B - \left(m_b + \rho_{\text{He}} V_b\right) g$

$$= 4.11 \times 10^3 \text{ N} - \left[226 \text{ kg} + \left(0.179 \text{ kg/m}^3\right)\left(325 \text{ m}^3\right)\right]\left(9.80 \text{ m/s}^2\right) = \boxed{+1.33 \times 10^3 \text{ N}}$$

Since $\Sigma F_y = ma_y > 0$, a_y will be positive (upward), and $\boxed{\text{the balloon rises}}$.

(d) If the balloon and load are in equilibrium, $\Sigma F_y = \left(B - w_b - w_{\text{He}}\right) - w_{\text{load}} = 0$ and $w_{\text{load}} = \left(B - w_b - w_{\text{He}}\right) = 1.33 \times 10^3$ N. Thus, the mass of the load is

$$m_{\text{load}} = \frac{w_{\text{load}}}{g} = \frac{1.33 \times 10^3 \text{ N}}{9.80 \text{ m/s}^2} = \boxed{136 \text{ kg}}$$

(e) If $m_{\text{load}} < 136$ kg, the net force acting on the balloon+load system is upward and $\boxed{\text{the balloon and its load will accelerate upward}}$.

(f) $\boxed{\text{As the balloon rises, decreasing atmospheric density decreases the buoyancy force.}}$ At some height the balloon will come to equilibrium and go no higher.

9.35 (a) $B = \rho_{\text{air}} g V_{\text{balloon}} = \rho_{\text{air}} g \left(\dfrac{4\pi r^3}{3}\right) = \left(1.29 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right)\left(\dfrac{4\pi}{3}\right)(3.00 \text{ m})^3$

$$= 1.43 \times 10^3 \text{ N} = \boxed{1.43 \text{ kN}}$$

(b) $\Sigma F_y = B - w_{\text{total}} = 1.43 \times 10^3 \text{ N} - (15.0 \text{ kg})\left(9.80 \text{ m/s}^2\right)$

$$= +1.28 \times 10^3 \text{ N} = \boxed{1.28 \text{ kN upward}}$$

(c) The balloon expands as it rises because the external pressure (atmospheric pressure) decreases with increasing altitude.

9.36 (a) Taking upward as positive, $\Sigma F_y = B - mg = ma_y$, or $\boxed{ma_y = \rho_w gV - mg}$.

(b) Since $m = \rho V$, we have $\rho \cancel{V} a_y = \rho_w g \cancel{V} - \rho \cancel{V} g$

or $$\boxed{a_y = \left(\frac{\rho_w}{\rho} - 1\right)g}$$

(c) $$a_y = \left(\frac{1.00\times 10^3\ \text{m/kg}^3}{1\,050\ \text{m/kg}^3} - 1\right)\left(9.80\ \text{m/s}^2\right) = \boxed{-0.467\ \text{m/s}^2 = 0.467\ \text{m/s}^2\ \text{downward}}$$

(d) From $\Delta y = v_{0y}t + a_y t^2/2$, with $v_{0y} = 0$, we find

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-8.00\ \text{m})}{-0.467\ \text{m/s}^2}} = \boxed{5.85\ \text{s}}$$

9.37 (a) $$B_{\text{total}} = 600 \cdot B_{\substack{\text{single}\\ \text{balloon}}} = 600\left(\rho_{\text{air}} g V_{\text{balloon}}\right) = 600\left[\rho_{\text{air}} g\left(\frac{4\pi}{3}r^3\right)\right]$$

$$= 600\left[\left(1.29\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\frac{4\pi}{3}\left(0.50\ \text{m}\right)^3\right] = 4.0\times 10^3\ \text{N} = \boxed{4.0\ \text{kN}}$$

(b) $$\Sigma F_y = B_{\text{total}} - m_{\text{total}}g = 4.0\times 10^3\ \text{N} - 600(0.30\ \text{kg})\left(9.8\ \text{m/s}^2\right) = 2.2\times 10^3\ \text{N} = \boxed{2.2\ \text{kN}}$$

(c) $\boxed{\text{Atmospheric pressure at this high altitude is much lower than at Earth's surface}}$, so the balloons expanded and eventually burst.

9.38 The actual weight of the object is $F_{g,\text{ actual}} = m_{\text{object}}g = 5.00$ N, and its mass is $m_{\text{object}} = 5.00\ \text{N}/g$. When fully submerged, the upward buoyant force (equal to the weight of the displaced water) and the upward force exerted on the object by the scale $\left(F_{g,\text{ apparent}} = 3.50\ \text{N}\right)$ together support the actual weight of the object. That is,

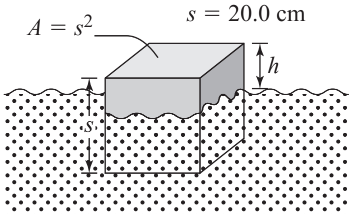
$$\Sigma F_y = 0 \quad \Rightarrow \quad B + F_{g,\text{ apparent}} - F_{g,\text{ actual}} = 0$$

and $$B = F_{g,\text{ actual}} - F_{g,\text{ apparent}} = 5.00\ \text{N} - 3.50\ \text{N} = 1.50\ \text{N}$$

Thus, $B = \rho_{\text{water}} g V_{\text{object}}$ gives $V_{\text{object}} = B/\left(\rho_{\text{water}} g\right)$ and the density of the object is

$$\rho_{\text{object}} = \frac{m_{\text{object}}}{V_{\text{object}}} = \left(\frac{5.00\ \text{N}}{g}\right)\left(\frac{\rho_{\text{water}} g}{1.50\ \text{N}}\right) = 3.33\rho_{\text{water}} = \boxed{3.33\times 10^3\ \text{kg/m}^3}$$

9.39 (a) The wooden block sinks until the buoyant force (weight of the displaced water) equals the weight of the block. That is, when equilibrium is reached, $B = \rho_{\text{water}} g\left[(s-h)s^2\right] = \rho_{\text{wood}} g \cdot s^3$, giving



$$s - h = \left(\frac{\rho_{\text{wood}}}{\rho_{\text{water}}}\right)\cdot s$$

or $$h = s\cdot\left(1 - \frac{\rho_{\text{wood}}}{\rho_{\text{water}}}\right) = (20.0\ \text{cm})\left(1 - \frac{650\ \text{kg/m}^3}{1000\ \text{kg/m}^3}\right) = \boxed{7.00\ \text{cm}}$$

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(b) When the upper surface of the block is level with the water surface, the buoyant force is

$$B = \rho_{\text{water}} g V_{\text{block}} = \rho_{\text{water}} g \cdot s^3$$

This must equal the weight of the block plus the weight of the added lead, or $m_{\text{Pb}} g + m_{\text{block}} g = B$, and

$$m_{\text{Pb}} = \frac{B}{g} - m_{\text{block}} = \rho_{\text{water}} V_{\text{block}} - \rho_{\text{wood}} V_{\text{block}} = \left(\rho_{\text{water}} - \rho_{\text{wood}}\right) \cdot s^3$$

giving $\quad m_{\text{Pb}} = \left(1000\ \text{kg/m}^3 - 650\ \text{kg/m}^3\right)(0.200\ \text{m})^3 = \boxed{2.80\ \text{kg}}$

9.40 At equilibrium, $\Sigma F_y = B - F_{\text{spring}} - mg = 0$, so the spring force is $F_{\text{spring}} = B - mg = \left[\left(\rho_{\text{water}} V_{\text{block}}\right) - m\right] g$, where

$$V_{\text{block}} = \frac{m}{\rho_{\text{wood}}} = \frac{5.00\ \text{kg}}{650\ \text{kg/m}^3} = 7.69 \times 10^{-3}\ \text{m}^3$$

Thus, $F_{\text{spring}} = \left[\left(10^3\ \text{kg/m}^3\right)\left(7.69 \times 10^{-3}\ \text{m}^3\right) - 5.00\ \text{kg}\right]\left(9.80\ \text{m/s}^2\right) = 26.4\ \text{N}$.

The elongation of the spring is then

$$\Delta x = \frac{F_{\text{spring}}}{k} = \frac{26.4\ \text{N}}{160\ \text{N/m}} = 0.165\ \text{m} = \boxed{16.5\ \text{cm}}$$

9.41 (a) The buoyant force is the difference between the weight in air and the apparent weight when immersed in the alcohol, or $B = 300\ \text{N} - 200\ \text{N} = 100\ \text{N}$. But, from Archimedes' principle, this is also the weight of the displaced alcohol, so $B = \left(\rho_{\text{alcohol}} V\right) g$. Since the sample is fully submerged, the volume of the displaced alcohol is the same as the volume of the sample. This volume is

$$V = \frac{B}{\rho_{\text{alcohol}}\, g} = \frac{100\ \text{N}}{\left(700\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)} = \boxed{1.46 \times 10^{-2}\ \text{m}^3}$$

(b) The mass of the sample is

$$m = \frac{\textit{weight in air}}{g} = \frac{300\ \text{N}}{9.80\ \text{m/s}^2} = 30.6\ \text{kg}$$

and its density is

$$\rho = \frac{m}{V} = \frac{30.6\ \text{kg}}{1.46 \times 10^{-2}\ \text{m}^3} = \boxed{2.10 \times 10^3\ \text{kg/m}^3}$$

9.42 The difference between the weight in air and the apparent weight when immersed is the buoyant force exerted on the object by the fluid.

(a) The mass of the object is

$$m = \frac{\textit{weight in air}}{g} = \frac{300\ \text{N}}{9.80\ \text{m/s}^2} = 30.6\ \text{kg}$$

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The buoyant force when immersed in water is the weight of a volume of water equal to the volume of the object, or $B_{\text{water}} = (\rho_{\text{water}} V)g$. Thus, the volume of the object is

$$V = \frac{B_{\text{water}}}{\rho_{\text{water}}\, g} = \frac{300 \text{ N} - 265 \text{ N}}{(10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 3.57 \times 10^{-3} \text{ m}^3$$

and its density is

$$\rho_{\text{object}} = \frac{m}{V} = \frac{30.6 \text{ kg}}{3.57 \times 10^{-3} \text{ m}^3} = \boxed{8.57 \times 10^3 \text{ kg/m}^3}$$

(b) The buoyant force when immersed in oil is equal to the weight of a volume $V = 3.57 \times 10^{-3} \text{ m}^3$ of oil. Hence, $B_{\text{oil}} = (\rho_{\text{oil}} V)g$, or the density of the oil is

$$\rho_{\text{oil}} = \frac{B_{\text{oil}}}{V g} = \frac{300 \text{ N} - 275 \text{ N}}{(3.57 \times 10^{-3} \text{ m}^3)(9.80 \text{ m/s}^2)} = \boxed{715 \text{ kg/m}^3}$$

9.43 The volume of the iron block is

$$V = \frac{m_{\text{iron}}}{\rho_{\text{iron}}} = \frac{2.00 \text{ kg}}{7.86 \times 10^3 \text{ kg/m}^3} = 2.54 \times 10^{-4} \text{ m}^3$$

and the buoyant force exerted on the iron by the oil is

$$B = (\rho_{\text{oil}} V)g = (916 \text{ kg/m}^3)(2.54 \times 10^{-4} \text{ m}^3)(9.80 \text{ m/s}^2) = 2.28 \text{ N}$$

Applying $\Sigma F_y = 0$ to the iron block gives the support force exerted by the upper scale (and hence the reading on that scale) as

$$F_{\text{upper}} = m_{\text{iron}}\, g - B = 19.6 \text{ N} - 2.28 \text{ N} = \boxed{17.3 \text{ N}}$$

From Newton's third law, the iron exerts force *B* downward on the oil (and hence the beaker). Applying $\Sigma F_y = 0$ to the system consisting of the beaker and the oil gives

$$F_{\text{lower}} - B - (m_{\text{oil}} + m_{\text{beaker}})g = 0$$

The support force exerted by the lower scale (and the lower scale reading) is then

$$F_{\text{lower}} = B + (m_{\text{oil}} + m_{\text{beaker}})\, g = 2.28 \text{ N} + [(2.00 + 1.00) \text{ kg}](9.80 \text{ m/s}^2) = \boxed{31.7 \text{ N}}$$

9.44 (a) The cross-sectional area of the hose is $A = \pi r^2 = \pi d^2/4 = \pi(2.74 \text{ cm})^2/4$, and the volume flow rate (volume per unit time) is $Av = 25.0 \text{ L}/1.50 \text{ min}$. Thus,

$$v = \frac{25.0 \text{ L}/1.50 \text{ min}}{A} = \left(\frac{25.0 \cancel{\text{L}}}{1.50 \cancel{\text{min}}}\right)\left[\frac{4}{\pi \cdot (2.74)^2 \text{ cm}^2}\right]\left(\frac{1 \cancel{\text{min}}}{60 \text{ s}}\right)\left(\frac{10^3 \text{ cm}^3}{1 \cancel{\text{L}}}\right)$$

$$= (47.1 \text{ cm/s})\left(\frac{1 \text{ m}}{10^2 \text{ cm}}\right) = \boxed{0.471 \text{ m/s}}$$

(b) $$\frac{A_2}{A_1} = \left(\frac{\pi d_2^2}{4}\right)\left(\frac{4}{\pi d_1^2}\right) = \left(\frac{d_2}{d_1}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9} \qquad \text{or} \qquad A_2 = \frac{A_1}{9}$$

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Then, from the equation of continuity, $A_2 v_2 = A_1 v_1$ and we find

$$v_2 = \left(\frac{A_1}{A_2}\right) v_1 = 9(0.471 \text{ m/s}) = \boxed{4.24 \text{ m/s}}$$

9.45 (a) The volume flow rate is Av, and the mass flow rate is

$$\rho A v = (1.0 \text{ g/cm}^3)(2.0 \text{ cm}^2)(40 \text{ cm/s}) = \boxed{80 \text{ g/s}}$$

(b) From the equation of continuity, the speed in the capillaries is

$$v_{\text{capiliaries}} = \left(\frac{A_{\text{aorta}}}{A_{\text{capillaries}}}\right) v_{\text{aorta}} = \left(\frac{2.0 \text{ cm}^2}{3.0 \times 10^3 \text{ cm}^2}\right)(40 \text{ cm/s})$$

or $\quad v_{\text{capiliaries}} = 2.7 \times 10^{-2} \text{ cm/s} = \boxed{0.27 \text{ mm/s}}$

9.46 (a) From the equation of continuity, the flow speed in the second section of the pipe is

$$v_2 = \left(\frac{A_1}{A_2}\right) v_1 = \left(\frac{10.0 \text{ cm}^2}{2.50 \text{ cm}^2}\right)(2.75 \text{ m/s}) = \boxed{11.0 \text{ m/s}}$$

(b) Using Bernoulli's equation and choosing $y = 0$ along the centerline of the pipe gives

$$P_2 = P_1 + \frac{1}{2}\rho\left(v_1^2 - v_2^2\right)$$

$$= 1.20 \times 10^5 \text{ Pa} + \frac{1}{2}\left(1.65 \times 10^3 \text{ kg/m}^3\right)\left[(2.75 \text{ m/s})^2 - (11.0 \text{ m/s})^2\right]$$

or $\quad P_2 = \boxed{2.64 \times 10^4 \text{ Pa}}$

9.47 From Bernoulli's equation, choosing $y = 0$ at the level of the syringe and needle, $P_2 + \frac{1}{2}\rho v_2^2 = P_1 + \frac{1}{2}\rho v_1^2$, so the flow speed in the needle is

$$v_2 = \sqrt{v_1^2 + \frac{2(P_1 - P_2)}{\rho}}$$

In this situation,

$$P_1 - P_2 = P_1 - P_{\text{atm}} = (P_1)_{\text{gauge}} = \frac{F}{A_1} = \frac{2.00 \text{ N}}{2.50 \times 10^{-5} \text{ m}^2} = 8.00 \times 10^4 \text{ Pa}$$

Thus, assuming $v_1 \approx 0$,

$$v_2 = \sqrt{0 + \frac{2\left(8.00 \times 10^4 \text{ Pa}\right)}{1.00 \times 10^3 \text{ kg/m}^3}} = \boxed{12.6 \text{ m/s}}$$

9.48 We apply Bernoulli's equation, ignoring the very small change in vertical position, to obtain $P_1 - P_2 = \frac{1}{2}\rho\left(v_2^2 - v_1^2\right) = \frac{1}{2}\rho\left[(2v_1)^2 - v_1^2\right] = \frac{3}{2}\rho v_1^2$, or

$$\Delta P = \frac{3}{2}\left(1.29 \text{ kg/m}^3\right)\left(15 \times 10^{-2} \text{ m/s}\right)^2 = \boxed{4.4 \times 10^{-2} \text{ Pa}}$$

9.49 (a) Assuming the airplane is in level flight, the net lift (the difference in the upward and downward forces exerted on the wings by the air flowing over them) must equal the weight of the plane, or $(P_{\substack{\text{lower}\\\text{surface}}} - P_{\substack{\text{upper}\\\text{surface}}})A_{\text{wings}} = mg$. This yields

$$P_{\substack{\text{lower}\\\text{surface}}} - P_{\substack{\text{upper}\\\text{surface}}} = \frac{mg}{A_{\text{wings}}} = \frac{\left(8.66\times10^4 \text{ kg}\right)\left(9.80 \text{ m/s}^2\right)}{90.0 \text{ m}^2} = \boxed{9.43\times10^3 \text{ Pa}}$$

(b) Neglecting the small difference in altitude between the upper and lower surfaces of the wings, and applying Bernoulli's equation, yields

$$P_{\text{lower}} + \frac{1}{2}\rho_{\text{air}} v_{\text{lower}}^2 = P_{\text{upper}} + \frac{1}{2}\rho_{\text{air}} v_{\text{upper}}^2$$

so $$v_{\text{upper}} = \sqrt{v_{\text{lower}}^2 + \frac{2\left(P_{\text{lower}} - P_{\text{upper}}\right)}{\rho_{\text{air}}}} = \sqrt{\left(225 \text{ m/s}\right)^2 + \frac{2\left(9.43\times10^3 \text{ Pa}\right)}{1.29 \text{ kg/m}^3}} = \boxed{255 \text{ m/s}}$$

(c) $\boxed{\text{The density of air decreases with increasing height}}$, resulting in a smaller pressure difference, $\Delta P = \frac{1}{2}\rho_{\text{air}}(v_{\text{upper}}^2 - v_{\text{lower}}^2)$. Beyond the maximum operational altitude, the pressure difference can no longer support the aircraft.

9.50 For level flight, the net lift (difference between the upward and downward forces exerted on the wing surfaces by air flowing over them) must equal the weight of the aircraft, or $(P_{\substack{\text{lower}\\\text{surface}}} - P_{\substack{\text{upper}\\\text{surface}}})A_{\text{wings}} = Mg$. This gives the air pressure at the upper surface as $\boxed{P_{\substack{\text{upper}\\\text{surface}}} = P_{\substack{\text{lower}\\\text{surface}}} - Mg/A_{\text{wings}}}$.

9.51 (a) Since the pistol is fired horizontally, the emerging water stream has initial velocity components of $(v_{0x} = v_{\text{nozzle}},\ v_{0y} = 0)$. Then, $\Delta y = v_{0y}t + a_y t^2/2$, with $a_y = -g$, gives the time of flight as

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-1.50 \text{ m})}{-9.80 \text{ m/s}^2}} = \boxed{0.553 \text{ s}}$$

(b) With $a_x = 0$ and $v_{0x} = v_{\text{nozzle}}$, the horizontal range of the emergent stream is $\Delta x = v_{\text{nozzle}}t$, where t is the time of flight from above. Thus, the speed of the water emerging from the nozzle must be

$$v_{\text{nozzle}} = \frac{\Delta x}{t} = \frac{8.00 \text{ m}}{0.553 \text{ s}} = \boxed{14.5 \text{ m/s}}$$

(c) From the equation of continuity, $A_1 v_1 = A_2 v_2$, the speed of the water in the larger cylinder is $v_1 = \left(A_2/A_1\right)v_2 = \left(A_2/A_1\right)v_{\text{nozzle}}$, or

$$v_1 = \left(\frac{\pi r_2^2}{\pi r_1^2}\right)v_{\text{nozzle}} = \left(\frac{r_2}{r_1}\right)^2 v_{\text{nozzle}} = \left(\frac{1.00 \text{ mm}}{10.0 \text{ mm}}\right)^2 \left(14.5 \text{ m/s}\right) = \boxed{0.145 \text{ m/s}}$$

(d) The pressure at the nozzle is atmospheric pressure, or $\boxed{P_2 = 1.013\times10^5 \text{ Pa}}$.

(e) With the two cylinders horizontal, $y_1 \approx y_2$ and gravity terms from Bernoulli's equation $\boxed{\text{can be neglected}}$, leaving $P_1 + \rho_{\text{water}}v_1^2/2 = P_2 + \rho_{\text{water}}v_2^2/2$, so the needed pressure in the larger cylinder is

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$$P_1 = P_2 + \frac{\rho_{\text{water}}}{2}\left(v_2^2 - v_1^2\right)$$

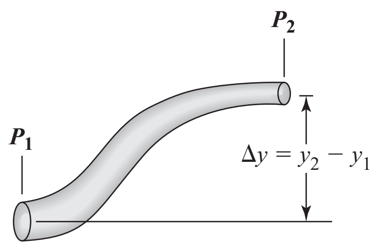
$$= 1.013\times10^5\ \text{Pa} + \frac{1.00\times10^3\ \text{kg/m}^3}{2}\left[\left(14.5\ \text{m/s}\right)^2 - \left(0.145\ \text{m/s}\right)^2\right]$$

or $\quad P_1 = \boxed{2.06\times10^5\ \text{Pa}}$

(f) To create an overpressure of $\Delta P = 2.06\times10^5\ \text{Pa} - 1.013\times10^5\ \text{Pa} = 1.05\times10^5\ \text{Pa}$ in the larger cylinder, the force that must be exerted on the piston is

$$F_1 = (\Delta P)A_1 = (\Delta P)\left(\pi r_1^2\right) = \left(1.05\times10^5\ \text{Pa}\right)\pi\left(1.00\times10^{-2}\ \text{m}\right)^2 = \boxed{33.0\ \text{N}}$$

9.52 (a) From Bernoulli's equation,



$$P_1 + \frac{\rho_{\text{water}}v_1^2}{2} + \rho_{\text{water}}gy_1 = P_2 + \frac{\rho_{\text{water}}v_2^2}{2} + \rho_{\text{water}}gy_2$$

or $\quad v_2^2 - v_1^2 = 2\left[\dfrac{P_1 - P_2}{\rho_{\text{water}}} - g\left(y_2 - y_1\right)\right]$

and using the given data values, we obtain

$$v_2^2 - v_1^2 = 2\left[\frac{1.75\times10^5\ \text{Pa} - 1.20\times10^5\ \text{Pa}}{1.00\times10^3\ \text{kg/m}^3} - \left(9.80\ \text{m/s}^2\right)\left(2.50\ \text{m}\right)\right]$$

and $\quad v_2^2 - v_1^2 = 61.0\ \text{m}^2/\text{s}^2$ **[1]**

From the equation of continuity,

$$v_2 = \left(\frac{A_1}{A_2}\right)v_1 = \left(\frac{\pi r_1^2}{\pi r_2^2}\right)v_1 = \left(\frac{r_1}{r_2}\right)^2 v_1 = \left(\frac{3.00\ \text{cm}}{1.50\ \text{cm}}\right)^2 v_1$$

or $\quad v_2 = 4v_1$ **[2]**

Substituting Equation [2] into [1] gives $(16-1)v_1^2 = 61.0\ \text{m}^2/\text{s}^2$, or

$$v_1 = \sqrt{\frac{61.0\ \text{m}^2/\text{s}^2}{15}} = \boxed{2.02\ \text{m/s}}$$

(b) Equation [2] above now yields $v_2 = 4\left(2.02\ \text{m/s}\right) = \boxed{8.08\ \text{m/s}}$.

(c) The volume flow rate through the pipe is $\textit{flow rate} = A_1v_1 = A_2v_2$

Looking at the lower point:

$$\textit{flow rate} = \left(\pi r_1^2\right)v_1 = \pi\left(3.00\times10^{-2}\ \text{m}\right)^2\left(2.02\ \text{m/s}\right) = \boxed{5.71\times10^{-3}\ \text{m}^3/\text{s}}$$

9.53 First, consider the path from the viewpoint of projectile motion to find the speed at which the water emerges from the tank. From $\Delta y = v_{0y}t + \frac{1}{2}a_yt^2$ with $v_{0y} = 0$, $\Delta y = -1.00\ \text{m}$, and $a_y = -g$, we find the time of flight as

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2.00\ \text{m}}{g}}$$

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From the horizontal motion, the speed of the water coming out of the hole is

$$v_2 = v_{0x} = \frac{\Delta x}{t} = (0.600\ \text{m})\sqrt{\frac{g}{2.00\ \text{m}}} = \sqrt{\frac{(0.600\ \text{m})^2 g}{2.00\ \text{m}}} = \sqrt{(1.80\times 10^{-1}\ \text{m})g}$$

We now use Bernoulli's equation, with point 1 at the top of the tank and point 2 at the level of the hole. With $P_1 = P_2 = P_{\text{atm}}$ and $v_1 \approx 0$, this gives $\rho g y_1 = \frac{1}{2}\rho v_2^2 + \rho g y_2$, or

$$h = y_1 - y_2 = \frac{v_2^2}{2g} = \frac{(1.80\times 10^{-1}\ \text{m})g}{2g} = 9.00\times 10^{-2}\ \text{m} = \boxed{9.00\ \text{cm}}$$

9.54 (a) Apply Bernoulli's equation with point 1 at the open top of the tank and point 2 at the opening of the hole. Then, $P_1 = P_2 = P_{\text{atm}}$ and we assume $v_1 \approx 0$. This gives $\frac{1}{2}\rho v_2^2 + \rho g y_2 = \rho g y_1$, or

$$v_2 = \sqrt{2g(y_1 - y_2)} = \sqrt{2(9.80\ \text{m/s}^2)(16.0\ \text{m})} = \boxed{17.7\ \text{m/s}}$$

(b) The area of the hole is found from

$$A_2 = \frac{\textit{flow rate}}{v_2} = \frac{2.50\times 10^{-3}\ \text{m}^3/\text{min}}{17.7\ \text{m/s}}\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 2.35\times 10^{-6}\ \text{m}^2$$

But, $A_2 = \pi d_2^2/4$ and the diameter of the hole must be

$$d_2 = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{4(2.35\times 10^{-6}\ \text{m}^2)}{\pi}} = 1.73\times 10^{-3}\ \text{m} = \boxed{1.73\ \text{mm}}$$

9.55 First, determine the flow speed inside the larger section from

$$v_1 = \frac{\textit{flow rate}}{A_1} = \frac{1.80\times 10^{-4}\ \text{m}^3/\text{s}}{\pi(2.50\times 10^{-2}\ \text{m})^2/4} = 0.367\ \text{m/s}$$

The absolute pressure inside the large section on the left is $P_1 = P_{\text{atm}} + \rho g h_1$, where h_1 is the height of the water in the leftmost standpipe. The absolute pressure in the constriction is $P_2 = P_{\text{atm}} + \rho g h_2$, so

$$P_1 - P_2 = \rho g(h_1 - h_2) = \rho g(5.00\ \text{cm})$$

The flow speed inside the constriction is found from Bernoulli's equation with $y_1 = y_2$ (since the pipe is horizontal). This gives

$$v_2^2 = v_1^2 + \frac{2}{\rho}(P_1 - P_2) = v_1^2 + 2g(h_1 - h_2)$$

or $$v_2 = \sqrt{(0.367\ \text{m/s})^2 + 2(9.80\ \text{m/s})(5.00\times 10^{-2}\ \text{m})} = 1.06\ \text{m/s}$$

The cross-sectional area of the constriction is then

$$A_2 = \frac{\textit{flow rate}}{v_2} = \frac{1.80\times 10^{-4}\ \text{m}^3/\text{s}}{1.06\ \text{m/s}} = 1.70\times 10^{-4}\ \text{m}^2$$

and the diameter is

$$d_2 = \sqrt{\frac{4A_2}{\pi}} = \sqrt{\frac{4(1.70\times 10^{-4}\ \text{m}^2)}{\pi}} = 1.47\times 10^{-2}\ \text{m} = \boxed{1.47\ \text{cm}}$$

9.56 (a) For minimum input pressure so the water will just reach the level of the rim, the gauge pressure at the upper end is zero (i.e., the absolute pressure inside the upper end of the pipe is atmospheric pressure), and the flow rate is zero. Thus, Bernoulli's equation, $\left(P+\frac{1}{2}\rho v^2+\rho g y\right)_{\text{river}}=\left(P+\frac{1}{2}\rho v^2+\rho g y\right)_{\text{rim}}$, becomes

$$\left(P_{\text{river}}\right)_{\min}+0=1\text{ atm}+0+\rho g\left(y_{\text{rim}}-y_{\text{river}}\right)=1\text{ atm}+\rho g\left(y_{\text{rim}}-y_{\text{river}}\right) \qquad \textbf{[1]}$$

or, $$\left(P_{\text{river}}\right)_{\min}=1.013\times10^5\text{ Pa}+\left(10^3\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.80\ \frac{\text{m}}{\text{s}^2}\right)(2096\text{ m}-564\text{ m})$$

$$\left(P_{\text{river}}\right)_{\min}=\left(1.013\times10^5+1.50\times10^7\right)\text{ Pa}=1.51\times10^7\text{ Pa}=\boxed{15.1\text{ MPa}}$$

(b) When volume flow rate is

$$\textit{flow rate}=Av=\left(\frac{\pi d^2}{4}\right)v=4\,500\ \text{m}^3/\text{d}$$

the velocity in the pipe is

$$v=\frac{4(\textit{flow rate})}{\pi d^2}=\frac{4\left(4\,500\ \text{m}^3/\text{d}\right)}{\pi\,(0.150\text{ m})^2}\left(\frac{1\text{ d}}{86\,400\text{ s}}\right)=\boxed{2.95\ \text{m/s}}$$

(c) We imagine the pressure being applied to stationary water at river level, so Bernoulli's equation becomes

$$P_{\text{river}}+0=\left[1\text{ atm}+\rho g\left(y_{\text{rim}}-y_{\text{river}}\right)\right]+\frac{1}{2}\rho v_{\text{rim}}^2$$

or, using Equation [1] from above,

$$P_{\text{river}}=\left(P_{\text{river}}\right)_{\min}+\frac{1}{2}\rho v_{\text{rim}}^2=\left(P_{\text{river}}\right)_{\min}+\frac{1}{2}\left(10^3\ \frac{\text{kg}}{\text{m3}}\right)\left(2.95\ \frac{\text{m}}{\text{s}}\right)^2$$

$$=\left(P_{\text{river}}\right)_{\min}+4.35\text{ kPa}$$

The additional pressure required to achieve the desired flow rate is

$$\Delta P=\boxed{4.35\text{ kPa}}$$

9.57 (a) For the upward flight of a water-drop projectile from geyser vent to fountain-top, $v_y^2=v_{0y}^2+2a_y(\Delta y)$, with $v_y=0$ when $\Delta y=\Delta y_{\max}$, gives

$$v_{0y}=\sqrt{0-2a_y(\Delta y)_{\max}}=\sqrt{-2\left(-9.80\ \text{m/s}^2\right)(40.0\text{ m})}=\boxed{28.0\ \text{m/s}}$$

(b) Because of the low density of air and the small change in altitude, atmospheric pressure at the fountain top will be considered equal to that at the geyser vent. Bernoulli's equation, with $v_{\text{top}}=0$, then gives $\frac{1}{2}\rho v_{\text{vent}}^2=0+\rho g(y_{\text{top}}-y_{\text{vent}})$, or

$$v_{\text{vent}}=\sqrt{2g\left(y_{\text{top}}-y_{\text{vent}}\right)}=\sqrt{2\left(9.80\ \text{m/s}^2\right)(40.0\text{ m})}=\boxed{28.0\ \text{m/s}}$$

(c) Between the chamber and the geyser vent, Bernoulli's equation with $v_{\text{chamber}}\approx0$ yields

$$\left(P+0+\rho g y\right)_{\text{chamber}}=P_{\text{atm}}+\frac{1}{2}\rho v_{\text{vent}}^2+\rho g y_{\text{vent}},\text{ or}$$

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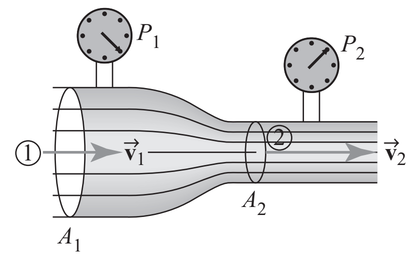
$$P - P_{\text{atm}} = \rho\left[\frac{1}{2}v_{\text{vent}}^2 + g\left(y_{\text{vent}} - y_{\text{chamber}}\right)\right]$$

$$= \left(10^3\ \frac{\text{kg}}{\text{m}^3}\right)\left[\frac{(28.0\ \text{m/s})^2}{2} + \left(9.80\ \frac{\text{m}}{\text{s}^2}\right)(175\ \text{m})\right] = 2.11\times 10^6\ \text{Pa}$$

or $\quad P_{\text{gauge}} = P - P_{\text{atm}} = \boxed{2.11\ \text{MPa}} = 20.8$ atmospheres

9.58 (a) Since the tube is horizontal, $y_1 = y_2$ and the gravity terms in Bernoulli's equation cancel, leaving



$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$

or

$$v_2^2 - v_1^2 = \frac{2(P_1 - P_2)}{\rho} = \frac{2\left(1.20\times 10^3\ \text{Pa}\right)}{7.00\times 10^2\ \text{kg/m}^3}$$

and

$$v_2^2 - v_1^2 = 3.43\ \text{m}^2/\text{s}^2 \qquad \textbf{[1]}$$

From the continuity equation, $A_1 v_1 = A_2 v_2$, we find

$$v_2 = \left(\frac{A_1}{A_2}\right)v_1 = \left(\frac{r_1}{r_2}\right)^2 v_1 = \left(\frac{2.40\ \text{cm}}{1.20\ \text{cm}}\right)^2 v_1$$

or

$$v_2 = 4v_1 \qquad \textbf{[2]}$$

Substituting Equation [2] into [1] yields $15v_1^2 = 3.43\ \text{m}^2/\text{s}^2$, and $v_1 = 0.478\ \text{m/s}$.

Then, Equation [2] gives $v_2 = 4(0.478\ \text{m/s}) = \boxed{1.91\ \text{m/s}}$.

(b) The volume flow rate is

$$A_1 v_1 = A_2 v_2 = \left(\pi r_2^2\right)v_2 = \pi\left(1.20\times 10^{-2}\ \text{m}\right)^2(1.91\ \text{m/s}) = \boxed{8.64\times 10^{-4}\ \text{m}^3/\text{s}}$$

9.59 From $\Sigma F_y = T - mg - F_y = 0$, the balance reading is found to be $T = mg + F_y$, where F_y is the vertical component of the surface tension force. Since this is a two-sided surface, the surface tension force is $F = \gamma(2L)$ and its vertical component is $F_y = \gamma(2L)\cos\phi$, where ϕ is the contact angle. Thus, $T = mg + 2\gamma L\cos\phi$.

$$T = 0.40\ \text{N when } \phi = 0° \Rightarrow \qquad mg + 2\gamma L = 0.40\ \text{N} \qquad \textbf{[1]}$$

$$T = 0.39\ \text{N when } \phi = 180° \Rightarrow \qquad mg - 2\gamma L = 0.39\ \text{N} \qquad \textbf{[2]}$$

Subtracting Equation [2] from [1] gives

$$\gamma = \frac{0.40\ \text{N} - 0.39\ \text{N}}{4L} = \frac{0.40\ \text{N} - 0.39\ \text{N}}{4\left(3.0\times 10^{-2}\ \text{m}\right)} = \boxed{8.3\times 10^{-2}\ \text{N/m}}$$

9.60 Because there are two edges (the inside and outside of the ring), we have

$$\gamma = \frac{F}{L_{\text{total}}} = \frac{F}{2(\textit{circumference})}$$

$$= \frac{F}{4\pi r} = \frac{1.61\times10^{-2}\ \text{N}}{4\pi\left(1.75\times10^{-2}\ \text{m}\right)} = \boxed{7.32\times10^{-2}\ \text{N/m}}$$

9.61 The total vertical component of the surface tension force must equal the weight of the column of fluid, or $F\cos\phi = \gamma(2\pi r)\cdot\cos\phi = \rho(\pi r^2)h\cdot g$. Thus,

$$\gamma = \frac{h\rho g r}{2\cos\phi} = \frac{\left(2.1\times10^{-2}\ \text{m}\right)\left(1\,080\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)\left(5.0\times10^{-4}\ \text{m}\right)}{2\cos0^\circ} = \boxed{5.6\times10^{-2}\ \text{N/m}}$$

9.62 The blood will rise in the capillary until the weight of the fluid column equals the total vertical component of the surface tension force, or until

$$\rho(\pi r^2)h\cdot g = F\cos\phi = \gamma(2\pi r)\cdot\cos\phi$$

This gives

$$h = \frac{2\gamma\cos\phi}{\rho g r} = \frac{2\left(0.058\ \text{N/m}\right)\cos0^\circ}{\left(1\,050\ \text{kg/m}^2\right)\left(9.80\ \text{m/s}^2\right)\left(2.0\times10^{-6}\ \text{m}\right)} = \boxed{5.6\ \text{m}}$$

9.63 From the definition of the coefficient of viscosity, $\eta = F\,d/A\,v$, the required force is

$$F = \frac{\eta A v}{d} = \frac{\left(1.79\times10^{-3}\ \text{N}\cdot\text{s/m}^2\right)\left[\left(0.800\ \text{m}\right)\left(1.20\ \text{m}\right)\right]\left(0.50\ \text{m/s}\right)}{0.10\times10^{-3}\ \text{m}} = \boxed{8.6\ \text{N}}$$

9.64 From the definition of the coefficient of viscosity, $\eta = F\,d/A\,v$, the required force is

$$F = \frac{\eta A v}{d} = \frac{\left(1\,500\times10^{-3}\ \text{N}\cdot\text{s/m}^2\right)\left[\left(0.010\ \text{m}\right)\left(0.040\ \text{m}\right)\right]\left(0.30\ \text{m/s}\right)}{1.5\times10^{-3}\ \text{m}} = \boxed{0.12\ \text{N}}$$

9.65 Poiseuille's law gives $\textit{flow rate} = \left(P_1 - P_2\right)\pi R^4/8\eta L$, and $P_2 = P_{\text{atm}}$ in this case. Thus, the desired gauge pressure is

$$P_1 - P_{\text{atm}} = \frac{8\eta L(\textit{flow rate})}{\pi R^4} = \frac{8\left(0.12\ \text{N}\cdot\text{s/m}^2\right)\left(50\ \text{m}\right)\left(8.6\times10^{-5}\ \text{m}^3/\text{s}\right)}{\pi\left(0.50\times10^{-2}\ \text{m}\right)^4}$$

or $\quad P_1 - P_{\text{atm}} = 2.1\times10^6\ \text{Pa} = \boxed{2.1\ \text{MPa}}$

9.66 From Poiseuille's law, the flow rate in the artery is

$$\textit{flow rate} = \frac{(\Delta P)\pi R^4}{8\eta L} = \frac{(400\ \text{Pa})\pi\left(2.6\times10^{-3}\ \text{m}\right)^4}{8\left(2.7\times10^{-3}\ \text{N}\cdot\text{s/m}^2\right)\left(8.4\times10^{-2}\ \text{m}\right)} = 3.2\times10^{-5}\ \text{m}^3/\text{s}$$

Thus, the flow speed is

$$v = \frac{\textit{flow rate}}{A} = \frac{3.2\times10^{-5}\ \text{m}^3/\text{s}}{\pi\left(2.6\times10^{-3}\ \text{m}\right)^2} = \boxed{1.5\ \text{m/s}}$$

9.67 If a particle is still in suspension after 1 hour, its terminal velocity must be less than

$$(v_t)_{\max} = \left(5.0\ \frac{\text{cm}}{\text{h}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right)\left(\frac{1\ \text{m}}{100\ \text{cm}}\right) = 1.4\times10^{-5}\ \text{m/s}$$

Thus, from $v_t = 2r^2 g(\rho - \rho_f)/9\eta$, we find the maximum radius of the particle:

$$r_{\max} = \sqrt{\frac{9\eta_{\text{water}}(v_t)_{\max}}{2g\left(\rho_{\text{protein}} - \rho_{\text{water}}\right)}}$$

$$= \sqrt{\frac{9\left(1.00\times10^{-3}\ \text{N}\cdot\text{s/m}^2\right)\left(1.4\times10^{-5}\ \text{m/s}\right)}{2\left(9.80\ \text{m/s}^2\right)\left[(1\,800-1\,000)\ \text{kg/m}^3\right]}} = 2.8\times10^{-6}\ \text{m} = \boxed{2.8\ \mu\text{m}}$$

9.68 From Poiseuille's law, the pressure difference required to produce a given volume flow rate of fluid with viscosity η through a tube of radius R and length L is

$$\Delta P = \frac{8\eta L(\Delta V/\Delta t)}{\pi R^4}$$

If the mass flow rate is $(\Delta m/\Delta t) = 1.0\times10^{-3}\ \text{kg/s}$, the volume flow rate of the water is

$$\frac{\Delta V}{\Delta t} = \frac{\Delta m/\Delta t}{\rho} = \frac{1.0\times10^{-3}\ \text{kg/s}}{1.0\times10^{3}\ \text{kg/m}^3} = 1.0\times10^{-6}\ \text{m}^3/\text{s}$$

and the required pressure difference is

$$\Delta P = \frac{8\left(1.0\times10^{-3}\ \text{Pa}\cdot\text{s}\right)\left(3.0\times10^{-2}\ \text{m}\right)\left(1.0\times10^{-6}\ \text{m}^3/\text{s}\right)}{\pi\left(0.15\times10^{-3}\ \text{m}\right)^4} = \boxed{1.5\times10^{5}\ \text{Pa}}$$

9.69 With the IV bag elevated 1.0 m above the needle and atmospheric pressure in the vein, the pressure difference between the input and output points of the needle is

$$\Delta P = \left(P_{\text{atm}} + \rho g h\right) - P_{\text{atm}} = \rho g h = \left(1.0\times10^{3}\ \text{kg/m}^3\right)\left(9.8\ \text{m/s}^2\right)(1.0\ \text{m}) = 9.8\times10^{3}\ \text{Pa}$$

The desired flow rate is

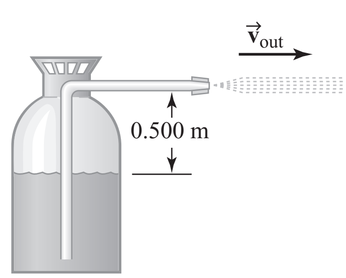
$$\frac{\Delta V}{\Delta t} = \frac{500\ \text{cm}^3\left(1\ \text{m}^3/10^6\ \text{cm}^3\right)}{30\ \text{min}(60\ \text{s}/1\ \text{min})} = 2.8\times10^{-7}\ \text{m}^3/\text{s}$$

Poiseuille's law then gives the required radius of the needle as

$$R = \left[\frac{8\eta L(\Delta V/\Delta t)}{\pi(\Delta P)}\right]^{1/4} = \left[\frac{8\left(1.0\times10^{-3}\ \text{Pa}\cdot\text{s}\right)\left(2.5\times10^{-2}\ \text{m}\right)\left(2.8\times10^{-7}\ \text{m}^3/\text{s}\right)}{\pi\left(9.8\times10^{3}\ \text{Pa}\right)}\right]^{1/4}$$

or $\quad R = 2.1\times10^{-4}\ \text{m} = \boxed{0.21\ \text{mm}}$

9.70 We write Bernoulli's equation as



$$P_{\text{out}} + \tfrac{1}{2}\rho v_{\text{out}}^2 + \rho g y_{\text{out}} = P_{\text{in}} + \tfrac{1}{2}\rho v_{\text{in}}^2 + \rho g y_{\text{in}}$$

or $\quad P_{\text{gauge}} = P_{\text{in}} - P_{\text{out}} = \rho\left[\tfrac{1}{2}\left(v_{\text{out}}^2 - v_{\text{in}}^2\right) + g\left(y_{\text{out}} - y_{\text{in}}\right)\right]$

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Approximating the speed of the fluid inside the tank as $v_{\text{in}} \approx 0$, we find

$$P_{\text{gauge}} = \left(1.00\times10^{3}\ \text{kg/m}^{3}\right)\left[\frac{1}{2}\left(30.0\ \text{m/s}\right)^{2} - 0 + \left(9.80\ \text{m/s}^{2}\right)\left(0.500\ \text{m}\right)\right]$$

or $P_{\text{gauge}} = 4.55\times10^{5}\ \text{Pa} = \boxed{455\ \text{kPa}}$

9.71 The Reynolds number is

$$RN = \frac{\rho v d}{\eta} = \frac{\left(1\,050\ \text{kg/m}^{3}\right)\left(0.55\ \text{m/s}\right)\left(2.0\times10^{-2}\ \text{m}\right)}{2.7\times10^{-3}\ \text{N}\cdot\text{s/m}^{2}} = 4.3\times10^{3}$$

In this region ($RN > 3\,000$), the flow is $\boxed{\text{turbulent}}$.

9.72 From the definition of the Reynolds number, the maximum flow speed for streamlined (or laminar) flow in this pipe is

$$v_{\max} = \frac{\eta\cdot(RN)_{\max}}{\rho\, d} = \frac{\left(1.0\times10^{-3}\ \text{N}\cdot\text{s/m}^{2}\right)\left(2\,000\right)}{\left(1\,000\ \text{kg/m}^{3}\right)\left(2.5\times10^{-2}\ \text{m}\right)} = 0.080\ \text{m/s} = \boxed{8.0\ \text{cm/s}}$$

9.73 The observed diffusion rate is $(8.0\times10^{-14}\ \text{kg})/(15\ \text{s}) = 5.3\times10^{-15}\ \text{kg/s}$. Then, from Fick's law, the difference in concentration levels is found to be

$$C_2 - C_1 = \frac{\left(\textit{Diffusion rate}\right)L}{DA}$$

$$= \frac{\left(5.3\times10^{-15}\ \text{kg/s}\right)\left(0.10\ \text{m}\right)}{\left(5.0\times10^{-10}\ \text{m}^{2}\text{/s}\right)\left(6.0\times10^{-4}\ \text{m}^{2}\right)} = \boxed{1.8\times10^{-3}\ \text{kg/m}^{3}}$$

9.74 Fick's law gives the diffusion coefficient as $D = (\textit{Diffusion rate})/[A\cdot(\Delta C/L)]$, where $\Delta C/L$ is the concentration gradient.

Thus, $$D = \frac{5.7\times10^{-15}\ \text{kg/s}}{\left(2.0\times10^{-4}\ \text{m}^{2}\right)\cdot\left(3.0\times10^{-2}\ \text{kg/m}^{4}\right)} = \boxed{9.5\times10^{-10}\ \text{m}^{2}\text{/s}}$$

9.75 Stokes's law gives the viscosity of the air as

$$\eta = \frac{F_r}{6\pi r v} = \frac{3.0\times10^{-13}\ \text{N}}{6\pi\left(2.5\times10^{-6}\ \text{m}\right)\left(4.5\times10^{-4}\ \text{m/s}\right)} = \boxed{1.4\times10^{-5}\ \text{N}\cdot\text{s/m}^{2}}$$

9.76 Using $v_t = 2r^2 g\left(\rho - \rho_f\right)/9\eta$, the density of the sphere is found to be $\rho_{\text{sphere}} = \rho_{\text{water}} + 9\eta_{\text{water}} v_t/2r^2 g$. Thus, if $r = d/2 = 0.500\times10^{-3}\ \text{m}$ and $v_t = 1.10\times10^{-2}\ \text{m/s}$ when falling through 20°C water,

$$\rho_{\text{sphere}} = 1.00\times10^{3}\ \frac{\text{kg}}{\text{m}^{3}} + \frac{9\left(1.00\times10^{-3}\ \text{N}\cdot\text{s/m}^{2}\right)\left(1.10\times10^{-2}\ \text{m/s}\right)}{2\left(5.00\times10^{-4}\ \text{m}\right)^{2}\left(9.80\ \text{m/s}^{2}\right)} = \boxed{1.02\times10^{3}\ \text{kg/m}^{3}}$$

9.77 (a) Both iron and aluminum are denser than water, so both blocks will be fully submerged. Since the two blocks have the same volume, they displace equal amounts of water and $\boxed{\text{the buoyant forces acting on the two blocks are equal.}}$

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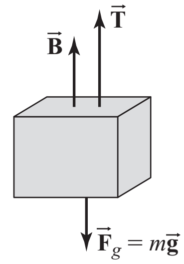
(b) Since the block is held in equilibrium, the force diagram at the right shows that



$$\Sigma F_y = 0 \;\Rightarrow\; T = mg - B$$

The buoyant force $\vec{\mathbf{B}}$ is the same for the two blocks, so the spring scale reading $\vec{\mathbf{T}}$ is $\boxed{\text{largest for the iron block}}$, which has a higher density, and hence weight, than the aluminum block.

(c) The buoyant force in each case is

$$B = (\rho_{\text{water}}V)g = (1.00\times10^3 \text{ kg/m}^3)(0.20 \text{ m}^3)(9.80 \text{ m/s}^2) = \boxed{2.0\times10^3 \text{ N}}$$

For the iron block:

$$T_{\text{iron}} = (\rho_{\text{iron}}V)g - B = (7.86\times10^3 \text{ kg/m}^3)(0.20 \text{ m}^3)(9.8 \text{ m/s}^2) - B$$

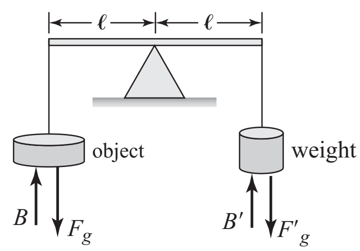
or $\quad T_{\text{iron}} = 1.5\times10^4 \text{ N} - 2.0\times10^3 \text{ N} = \boxed{13\times10^3 \text{ N}}$

For the aluminum block:

$$T_{\text{aluminum}} = (\rho_{\text{aluminum}}V)g - B = (2.70\times10^3 \text{ kg/m}^3)(0.20 \text{ m}^3)(9.8 \text{ m/s}^2) - B$$

or $\quad T_{\text{aluminum}} = 5.3\times10^3 \text{ N} - 2.0\times10^3 \text{ N} = \boxed{3.3\times10^3 \text{ N}}$

9.78 The object has volume V and true weight F_g. It also experiences a buoyant force $B = (\rho_{\text{air}}V)g$ exerted on it by the surrounding air. The counterweight, of density ρ, on the opposite end of the balance has a true weight F_g' and experiences a buoyant force



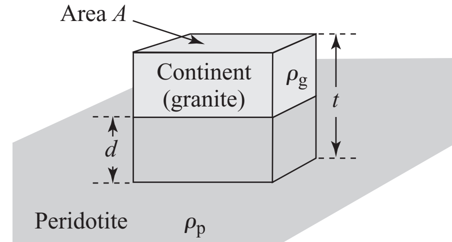
$$B' = (\rho_{\text{air}}V')g = \rho_{\text{air}}\left(\frac{m'}{\rho}\right)g = \left(\frac{\rho_{\text{air}}}{\rho}\right)m'g = \left(\frac{\rho_{\text{air}}}{\rho}\right)F_g'$$

The balance is in equilibrium when the net downward forces acting on its two ends are equal, that is, when $F_g - B = F_g' - B'$. Thus, the true weight of the object may be expressed as

$$F_g = F_g' + B - B' = F_g' + (\rho_{\text{air}}V)g - \left(\frac{\rho_{\text{air}}}{\rho}\right)F_g'$$

or $$\boxed{F_g = F_g' + \left(V - \frac{F_g'}{\rho g}\right)\rho_{\text{air}}g}$$

9.79 (a) From Archimedes' principle, the granite continent will sink down into the peridotite layer until the weight of the displaced peridotite equals the weight of the continent. Thus, at equilibrium,



$$[\rho_g(At)]g = [\rho_p(Ad)]g$$

or $\quad \boxed{\rho_g t = \rho_p d}$

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(b) If the continent sinks 5.0 km below the surface of the peridotite, then $d = 5.0$ km, and the result of part (a) gives the first approximation of the thickness of the continent as

$$t = \left(\frac{\rho_p}{\rho_g}\right)d = \left(\frac{3.3\times10^3\ \text{kg/m}^3}{2.8\times10^3\ \text{kg/m}^3}\right)(5.0\ \text{km}) = \boxed{5.9\ \text{km}}$$

9.80 (a) Starting with $P = P_0 + \rho g h$, we choose the reference level at the level of the heart, so $P_0 = P_H$. The pressure at the feet, a depth h_H below the reference level in the pool of blood in the body, is $P_F = P_H + \rho g h_H$. The pressure difference between feet and heart is then $\boxed{P_F - P_H = \rho g h_H}$.

(b) Using the result of part (a),

$$P_F - P_H = \left(1.05\times10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)(1.20\ \text{m}) = \boxed{1.23\times10^4\ \text{Pa}}$$

9.81 The cross-sectional area of the aorta is $A_1 = \pi d_1^2/4$ and that of a single capillary is $A_c = \pi d_2^2/4$. If the circulatory system has N such capillaries, the total cross-sectional area carrying blood from the aorta is $A_2 = NA_c = N\pi d_2^2/4$.

From the equation of continuity, $A_2 = (v_1/v_2)A_1$, or

$$\frac{N\pi d_2^2}{4} = \left(\frac{v_1}{v_2}\right)\frac{\pi d_1^2}{4}$$

yielding

$$N = \left(\frac{v_1}{v_2}\right)\left(\frac{d_1}{d_2}\right)^2 = \left(\frac{1.0\ \text{m/s}}{1.0\times10^{-2}\ \text{m/s}}\right)\left(\frac{0.50\times10^{-2}\ \text{m}}{10\times10^{-6}\ \text{m}}\right)^2 = \boxed{2.5\times10^7}$$

9.82 (a) We imagine that a superhero is capable of producing a perfect vacuum above the water in the straw. Then $P = P_0 + \rho g h$, with the reference level at the water surface inside the straw and P being atmospheric pressure on the water in the cup outside the straw, gives the maximum height of the water in the straw as

$$h_{\text{max}} = \frac{P_{\text{atm}} - 0}{\rho_{\text{water}}\, g} = \frac{P_{\text{atm}}}{\rho_{\text{water}}\, g} = \frac{1.013\times10^5\ \text{N/m}^2}{\left(1.00\times10^3\ \text{kg/m}^3\right)\left(9.80\ \text{m/s}^2\right)} = \boxed{10.3\ \text{m}}$$

(b) The Moon has no atmosphere so $P_{\text{atm}} = 0$, which yields $h_{\text{max}} = \boxed{0}$.

9.83 (a) $P = 160\ \text{mm of H}_2\text{O} = \rho_{\text{H}_2\text{O}}\, g(160\ \text{mm})$

$$= \left(10^3\ \frac{\text{kg}}{\text{m}^3}\right)\left(9.80\ \frac{\text{m}}{\text{s}^2}\right)(0.160\ \text{m}) = \boxed{1.57\ \text{kPa}}$$

$$P = \left(1.57\times10^3\ \text{Pa}\right)\left(\frac{1\ \text{atm}}{1.013\times10^5\ \text{Pa}}\right) = \boxed{1.55\times10^{-2}\ \text{atm}}$$

The pressure is $P = \rho_{\text{H}_2\text{O}}\, g h_{\text{H}_2\text{O}} = \rho_{\text{Hg}}\, g h_{\text{Hg}}$, so

$$h_{\text{Hg}} = \left(\frac{\rho_{\text{H}_2\text{O}}}{\rho_{\text{Hg}}}\right)h_{\text{H}_2\text{O}} = \left(\frac{10^3\ \text{kg/m}^3}{13.6\times10^3\ \text{kg/m}^3}\right)(160\ \text{mm}) = \boxed{11.8\ \text{mm of Hg}}$$

continued on next page

(b) The fluid level in the tap should rise.

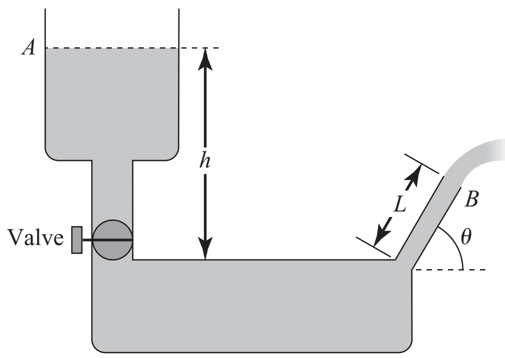
(c) Blockage of flow of the cerebrospinal fluid

9.84 When the rod floats, the weight of the displaced fluid equals the weight of the rod, or $(\rho_f V_{\text{displaced}})g = (\rho_0 V_{\text{rod}})g$. Assuming a cylindrical rod, $V_{\text{rod}} = \pi r^2 L$. The volume of fluid displaced is the same as the volume of the rod that is submerged

or $V_{\text{displaced}} = \pi r^2 (L-h)$.

Thus, $\rho_f \left[\pi r^2 (L-h)\right] g = \rho_0 \left[\pi r^2 L\right] g$ which reduces to $\boxed{\rho_f = \rho_0 L/(L-h)}$.

9.85 Consider the diagram and apply Bernoulli's equation to points A and B, taking $y = 0$ at the level of point B, and recognizing that $v_A \approx 0$. This gives



$$P_A + 0 + \rho_w g(h - L\sin\theta) = P_B + \frac{1}{2}\rho_w v_B^2 + 0$$

Recognize that $P_A = P_B = P_{\text{atm}}$ since both points are open to the atmosphere. Thus, we obtain $v_B = \sqrt{2g(h - L\sin\theta)}$.

Now the problem reduces to one of projectile motion with $v_{0y} = v_B \sin\theta$. At the top of the arc $v_y = 0$, $y = y_{\max}$, and $v_y^2 = v_{0y}^2 + 2a_y(\Delta y)$ gives

$$y_{\max} - 0 = \frac{v_y^2 - v_{0y}^2}{2a_y} = \frac{0 - v_B^2 \sin^2\theta}{2(-g)} = \frac{\cancel{2g}\,(h - L\sin\theta)\sin^2\theta}{\cancel{2g}}$$

or

$$y_{\max} = \left[10.0\text{ m} - (2.00\text{ m})\sin 30.0°\right]\sin^2 30.0° = \boxed{2.25\text{ m above the level of point } B}$$

9.86 When the balloon comes into equilibrium, the weight of the displaced air equals the weight of the filled balloon plus the weight of string that is above ground level. If m_s and L are the total mass and length of the string, the mass of string that is above ground level is $(h/L)m_s$. Thus, $(\rho_{\text{air}} V_{\text{balloon}})g = m_{\text{balloon}}\, g + (\rho_{\text{helium}} V_{\text{balloon}})g + (h/L)m_s\, g$, which reduces to

$$h = \left[\frac{(\rho_{\text{air}} - \rho_{\text{helium}})V_{\text{balloon}} - m_{\text{balloon}}}{m_s}\right]L$$

This yields

$$h = \left[\frac{(1.29\text{ kg/m}^3 - 0.179\text{ kg/m}^3)\left[4\pi(0.40\text{ m})^3/3\right] - 0.25\text{ kg}}{0.050\text{ kg}}\right](2.0\text{ m}) = \boxed{1.9\text{ m}}$$

9.87 Four forces are acting on the balloon: an upward buoyant force exerted by the surrounding air, $B = (\rho_{\text{air}} V_{\text{balloon}})g$; the downward weight of the balloon envelope, $F_{g,\text{ balloon}} = mg$; the downward weight of the helium filling the balloon, $F_{g,\text{ He}} = (\rho_{\text{He}} V_{\text{balloon}})g$; and the downward spring force, $F_s = k|\Delta x|$. At equilibrium, $|\Delta x| = L$, and we have

$$\Sigma F_y = 0 \quad \Rightarrow \quad B - F_s - F_{g,\text{ balloon}} - F_{g,\text{ He}} = 0$$

continued on next page

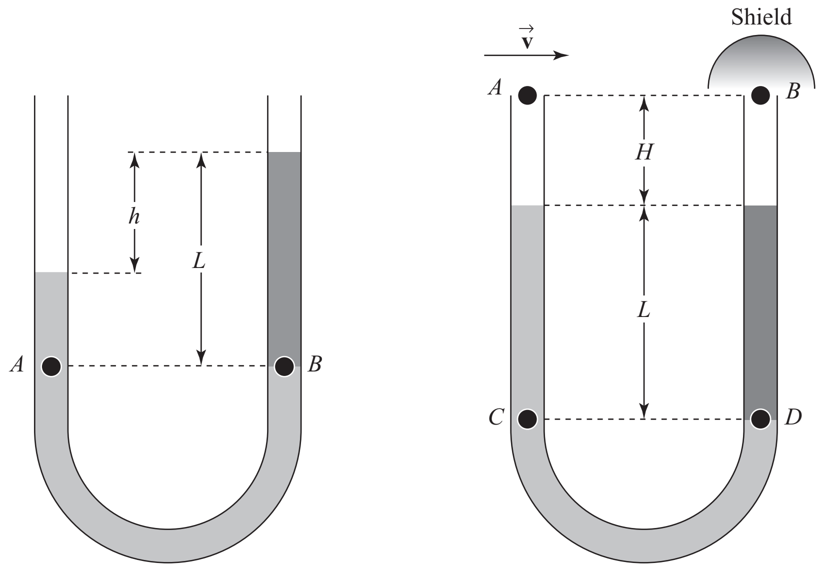
or $$F_s = kL = B - F_{g,\,\text{balloon}} - F_{g,\,\text{He}} = \left(\rho_{\text{air}} V_{\text{balloon}}\right) g - mg - \left(\rho_{\text{He}} V_{\text{balloon}}\right) g$$

and $$L = \frac{\left[\left(\rho_{\text{air}} - \rho_{\text{He}}\right) V_{\text{balloon}} - m\right] g}{k}$$

This yields $$L = \frac{\left\{\left[(1.29 - 0.179)\ \text{kg/m}^3\right]\left(5.00\ \text{m}^3\right) - 2.00\times10^{-3}\ \text{kg}\right\}\left(9.80\ \text{m/s}^2\right)}{90.0\ \text{N/m}}$$

or $$\boxed{L = 0.605\ \text{m}}$$

9.88



(b) **(c)**

(a) Consider the pressure at points A and B in part (b) of the figure by applying $P = P_0 + \rho_f gh$. Looking at the left tube gives $P_A = P_{\text{atm}} + \rho_{\text{water}}\, g(L - h)$, and looking at the tube on the right, $P_B = P_{\text{atm}} + \rho_{\text{oil}}\, gL$. Pascal's principle says that $P_B = P_A$. Therefore, $P_{\text{atm}} + \rho_{\text{oil}}\, gL = P_{\text{atm}} + \rho_{\text{water}}\, g(L - h)$, giving

$$h = \left(1 - \frac{\rho_{\text{oil}}}{\rho_{\text{water}}}\right) L = \left(1 - \frac{750\ \text{kg/m}^3}{1\,000\ \text{kg/m}^3}\right)(5.00\ \text{cm}) = \boxed{1.25\ \text{cm}}$$

(b) Consider part (c) of the diagram showing the situation when the air flow over the left tube equalizes the fluid levels in the two tubes. First, apply Bernoulli's equation to points A and B. This gives $P_A + \frac{1}{2}\rho_{\text{air}} v_A^2 + \rho_{\text{air}}\, g y_A = P_B + \frac{1}{2}\rho_{\text{air}} v_B^2 + \rho_{\text{air}}\, g y_B$. Since $y_A = y_B$, $v_A = v$, and $v_B = 0$, this reduces to

$$P_B - P_A = \frac{1}{2}\rho_{\text{air}} v^2 \qquad \textbf{[1]}$$

Now use $P = P_0 + \rho_f gh$ to find the pressure at points C and D, both at the level of the oil–water interface in the right tube. From the left tube, $P_C = P_A + \rho_{\text{water}}\, gL$, and from the right tube, $P_D = P_B + \rho_{\text{oil}}\, gL$.

Pascal's principle says that $P_D = P_C$, and equating these two gives $P_B + \rho_{\text{oil}}\, gL = P_A + \rho_{\text{water}}\, gL$, or

$$P_B - P_A = \left(\rho_{\text{water}} - \rho_{\text{oil}}\right) gL \qquad \textbf{[2]}$$

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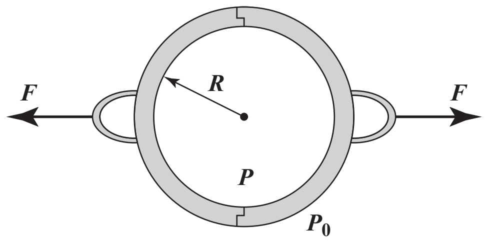
Combining Equations [1] and [2] yields

$$v = \sqrt{\frac{2(\rho_{\text{water}} - \rho_{\text{oil}})gL}{\rho_{\text{air}}}} = \sqrt{\frac{2(1\,000 - 750)(9.80\ \text{m/s}^2)(5.00 \times 10^{-2}\ \text{m})}{1.29}} = \boxed{13.8\ \text{m/s}}$$

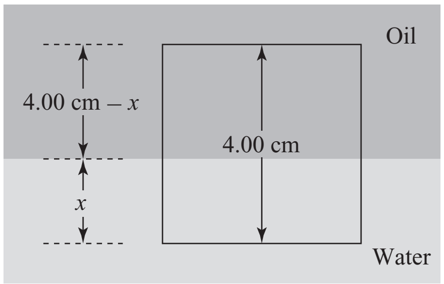
9.89 The pressure on the surface of the two hemispheres is constant at all points, and the force on each element of surface area is directed along the radius of the hemispheres. The applied force along the horizontal axis must balance the net force on the "effective" area, which is the cross-sectional area of the sphere,



$$A_{\text{effective}} = \pi R^2$$

and $$F = P_{\text{gauge}} A_{\text{effective}} = \boxed{(P_0 - P)\pi R^2}$$

9.90 Since the block is floating, the total buoyant force must equal the weight of the block. Thus,



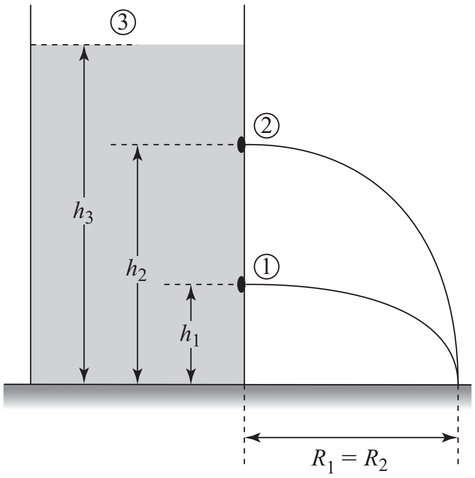
$$\rho_{\text{oil}}\left[A(4.00\ \text{cm} - x)\right]g + \rho_{\text{water}}\left[A \cdot x\right]g$$
$$= \rho_{\text{wood}}\left[A(4.00\ \text{cm})\right]g$$

where A is the surface area of the top or bottom of the rectangular block.

Solving for the distance x gives

$$x = \left(\frac{\rho_{\text{wood}} - \rho_{\text{oil}}}{\rho_{\text{water}} - \rho_{\text{oil}}}\right)(4.00\ \text{cm}) = \left(\frac{960 - 930}{1\,000 - 930}\right)(4.00\ \text{cm}) = \boxed{1.71\ \text{cm}}$$

9.91 A water droplet emerging from one of the holes becomes a projectile with $v_{0y} = 0$ and $v_{0x} = v$. The time for this droplet to fall distance h to the floor is found from $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$ to be $t = \sqrt{2h/g}$.



The horizontal range is $R = vt = v\sqrt{2h/g}$.

If the two streams hit the floor at the same spot, it is necessary that $R_1 = R_2$, or

$$v_1\sqrt{\frac{2h_1}{g}} = v_2\sqrt{\frac{2h_2}{g}}$$

With $h_1 = 5.00$ cm and $h_2 = 12.0$ cm, this reduces to $v_1 = v_2\sqrt{h_2/h_1} = v_2\sqrt{12.0\ \text{cm}/5.00\ \text{cm}}$, or

$$v_1 = v_2\sqrt{2.40} \qquad \textbf{[1]}$$

Apply Bernoulli's equation to points 1 (the lower hole) and 3 (the surface of the water). The pressure is atmospheric pressure at both points and, if the tank is large in comparison to the size of the holes, $v_3 \approx 0$. Thus, $P_{\text{atm}} + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_{\text{atm}} + 0 + \rho g h_3$, or

$$v_1^2 = 2g(h_3 - h_1) \qquad \textbf{[2]}$$

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Similarly, applying Bernoulli's equation to point 2 (the upper hole) and point 3 gives $P_{atm} + \frac{1}{2}\rho v_2^2 + \rho g h_2 = \rho_{atm} + 0 + \rho g h_3$, or

$$v_2^2 = 2g\left(h_3 - h_2\right) \tag{3}$$

Square Equation [1] and substitute from Equations [2] and [3] to obtain

$$2g\left(h_3 - h_1\right) = 2.40\left[2g\left(h_3 - h_2\right)\right]$$

Solving for h_3 yields

$$h_3 = \frac{2.40\,h_2 - h_1}{1.40} = \frac{2.40\left(12.0\text{ cm}\right) - 5.00\text{ cm}}{1.40} = 17.0\text{ cm}$$

so the surface of the water in the tank is $\boxed{\text{17.0 cm above floor level}}$.

10

Thermal Physics

QUICK QUIZZES

1. Choice (c). When two objects having different temperatures are in thermal contact, energy is transferred from the higher temperature object to the lower temperature object. As a result, the temperature of the hotter object decreases and that of the cooler object increases until thermal equilibrium is reached at some intermediate temperature.

2. Choice (b). The glass surrounding the mercury expands before the mercury does, causing the level of the mercury to drop slightly. The mercury rises after it begins to get warmer and approaches the temperature of the hot water, because its coefficient of expansion is greater than that for glass.

3. Choice (c). Gasoline has the highest coefficient of expansion so it undergoes the greatest change in volume per degree change in temperature.

4. Choice (c). A cavity in a material expands in exactly the same way as if the cavity were filled with the surrounding material. Thus, both spheres will expand by the same amount.

5. Unlike land-based ice, ice floating in the ocean already displaces a quantity of liquid water whose weight equals the weight of the ice. This is the same situation as will exist after the ice melts, so the melting ocean-based ice will not change ocean levels much.

6. Choice (b). Since the two containers are at the same temperature, the average kinetic energy per molecule is the same for the argon and helium gases. However, helium has a lower molar mass than does argon, so the rms speed of the helium atoms must be higher than that of the argon atoms.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. $T_F = \frac{9}{5}T_C + 32 = \frac{9}{5}(-25°) + 32° = -13°$ F, and the correct response is choice (e).

2. The correct choice is (b). When an object, containing a cavity, is heated, the cavity expands in the same way as it would if filled with the material making up the rest of the object.

3. $\Delta L = \alpha_{\text{Cu}} L_0 (\Delta T) = \left[17\times10^{-6}\ (°\text{C})^{-1}\right](93\text{ m})(5°\text{C}) = 8\times10^{-3}\text{ m} \sim 10^{-2}\text{ m} = 1\text{ cm}$ and choice (c) is the correct order of magnitude.

4. $T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(162 - 32) = 72.2°\text{C}$, then $T_K = T_C + 273 = 72.2 + 273 = 345\text{ K}$, so choice (c) is the correct answer.

5. Remember that one must use absolute temperatures and pressures in the ideal gas law. Thus, the original temperature is $T_K = T_C + 273.13 = 25.0 + 273.15 = 298.2\text{ K}$, and with the mass of the gas constant, the ideal gas law gives

$$T_2 = \left(\frac{P_2}{P_1}\right)\left(\frac{V_2}{V_1}\right)T_1 = \left(\frac{1.07\times10^6\text{ Pa}}{5.00\times10^6\text{ Pa}}\right)(3.00)(298.2\text{ K}) = 191\text{ K}$$

and (d) is the best choice.

6. From the ideal gas law, with the mass of the gas constant, $P_2V_2/T_2 = P_1V_1/T_1$. Thus,

$$P_2 = \left(\frac{V_1}{V_2}\right)\left(\frac{T_2}{T_1}\right)P_1 = \left(\frac{1}{2}\right)(4)P_1 = 2P_1$$

and (d) is the correct choice.

7. From the ideal gas law, with the mass of the gas constant, $P_2V_2/T_2 = P_1V_1/T_1$. Thus,

$$V_2 = \left(\frac{P_1}{P_2}\right)\left(\frac{T_2}{T_1}\right)V_1 = (4)(1)\left(0.50 \text{ m}^3\right) = 2.0 \text{ m}^3$$

and (c) is the correct choice.

8. The internal energy of n moles of a monatomic ideal gas is $U = \frac{3}{2}nRT$, where R is the universal gas constant and T is the absolute temperature of the gas. For the given neon sample, $T = T_C + 273.15 = (152 + 273.15)$ K $= 425$ K, and

$$n = \frac{m}{molar\ mass} = \frac{26.0 \text{ g}}{20.18 \text{ g/mol}} = 1.29 \text{ mol}$$

Thus, $U = \frac{3}{2}(1.29 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(425 \text{ K}) = 6.83\times10^3 \text{ J}$ and (b) is correct answer.

9. If two objects at different temperatures are in thermal contact, energy flows from the warmer object to the cooler object. Thermal equilibrium occurs only when the two objects are at the same temperature. Thus, since the contents of the bowl have reached thermal equilibrium, choice (b) is the correct answer.

10. The kinetic theory of gases does assume that the molecules in a pure substance obey Newton's laws and undergo elastic collisions, and the average distance between molecules is very large in comparison to molecular sizes. However, it also assumes that the number of molecules in the sample is large so that statistical averages are meaningful. The untrue statement included in the list of choices is (a).

11. In a head-on, elastic collision with a wall, the change in momentum of a gas molecule is $\Delta p = m\left(v_f - v_0\right) = m\left(-v_0 - v_0\right) = -2mv_0$. If the molecule should stick to the wall instead of rebounding, the change in the molecule's momentum would be $\Delta p = m\left(0 - v_0\right) = -mv_0$, which is half that in the elastic collision. Since a gas exerts a pressure on its container by molecules imparting impulses to the walls during collisions, and the impulse imparted equals the magnitude of the change in the molecular momentum, decreasing the change in momentum during the collisions by a factor of 2 would reduce the pressure by a factor of 2. Thus, the correct response is choice (b).

12. The rms speed of molecules in the gas is $v_{\text{rms}} = \sqrt{3RT/M}$. Thus, the ratio of the final speed to the original speed would be

$$\frac{\left(v_{\text{rms}}\right)_f}{\left(v_{\text{rms}}\right)_0} = \frac{\sqrt{3RT_f/M}}{\sqrt{3RT_0/M}} = \sqrt{\frac{T_f}{T_0}} = \sqrt{\frac{600 \text{ K}}{200 \text{ K}}} = \sqrt{3}$$

Therefore, the correct answer to this question is choice (d).

13. Since, when the balloon is fully inflated, the inside and outside pressures are nearly equal at one atmosphere, the walls of the balloon must be very easily stretched. As the air cools, the walls will contract, just maintaining equality of the pressure inside and outside. Thus, this is an isobaric cooling process. Since, even at 100 K, the temperature is far above the liquefaction point, we treat the air as an ideal gas under constant pressure and find:

(i) $V_f = \left(T_f/T_i\right)V_i = \left(100\text{ K}/300\text{ K}\right)\left(1\text{ L}\right) = \frac{1}{3}\text{L}$, and choice (b) is the correct answer.

(ii) As noted above, the pressure will remain constant at nearly one atmosphere, so the correct answer is choice (d).

14. Consider the ideal gas law, $PV = nRT$, recognizing that the two cylinders contain equal quantities of gas ($n_A = n_B$) at equal temperatures ($T_A = T_B$). Thus, $P_AV_A = P_BV_B$ and we have $P_A = \left(V_B/V_A\right)P_B = \left(1/3\right)P_B$, meaning that the correct answer is choice (d).

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The lower temperature will make the power line decrease in length. This increases the tension in the line so it is closer to the breaking point.

4. The pressure inside the balloon is greater than the ambient atmospheric pressure because the pressure inside must not only resist the external pressure, but also the force exerted by the elastic material of the balloon.

6. At high temperature and pressure, the steam inside exerts large forces on the pot and cover. Strong latches hold them together, but they would explode apart if you tried to open the hot cooker.

8. The measurements are too short. At 22°C the tape would read the width of the object accurately, but an increase in temperature causes the divisions ruled on the tape to be farther apart than they should be. This “too long” ruler will, then, measure objects to be shorter than they really are.

10. The existence of an atmosphere on a planet is due to the gravitational force holding the gas of the atmosphere to the planet. On a small planet, the gravitational force is very small, and the escape speed is correspondingly small. If a small planet starts its existence with atmosphere, the molecules of the gas will have a distribution of speeds, according to kinetic theory. Some of these molecules will have speeds higher than the escape speed of the planet and will leave the atmosphere. As the remaining atmosphere is warmed by radiation from the Sun, more molecules will attain speeds high enough to escape. As a result, the atmosphere bleeds off into space.

12. Doubling the volume while reducing the pressure by half results in no change in the quantity PV that appears in the ideal gas law equation. Consequently, the temperature and hence the internal energy remain the same.

14. (a) The sphere, which initially could just barely fit through the ring, expands when it is heated and thus becomes too large to fit through the unheated ring.

(b) If the ring is heated, every linear dimension of the ring (including the inner circumference, and hence the diameter of the hole), increases. This occurs because, when a material is heated, the atoms in that material push against each other and get farther apart. The only way atoms on the inner circumference of the ring can get farther apart is for the circumference—and corresponding diameter—to increase. Thus, if the ring is heated while the sphere remains cool, the ring fits over the sphere with additional room to spare.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** $-251°C$ **(b)** 1.36 atm

4. **(a)** $56.7°C, \ -62.1°C$ **(b)** 330 K, 211 K

6. **(a)** $-270°C$ **(b)** 1.27 atm **(c)** 1.74 atm

8. **(a)** 31.7°C **(b)** 31.7 K

10. **(a)** $T_R = T_F + 459.67$ **(b)** $T_K = \frac{5}{9} T_R$

12. **(a)** $L = 1.3 \text{ m} - 0.49 \text{ mm}$ **(b)** The clock will run fast.

14. **(a)** 2.542 cm **(b)** $3.0 \times 10^2 \ °C$

16. See Solution.

18. **(a)** 396 N **(b)** $-101°C$

(c) The initial length of the wire cancels during the calculation, so the results are not affected by doubling the length of the bridge.

20. See Solution.

22. 18.702 m

24. **(a)** 1.5 km

(b) Accordion-like expansion joints are placed in the pipeline at periodic intervals.

26. **(a)** 27.7 kg **(b)** 1.02 m^3 **(c)** $716 \ \text{kg/m}^3$

(d) 27.2 kg **(e)** 0.5 kg

28. 1.2 cm

30. **(a)** 292 K **(b)** 7.83 mol **(c)** $44.0 \ \text{g/mol}$

(d) 345 g **(e)** 516 K, 5.98 mol **(f)** $P_f = \left(n_f T_f / n_i T_i\right) P_i$

(g) $1.28 \times 10^6 \ \text{Pa}$

32. **(a)** $3.95 \text{ atm} = 400 \text{ kPa}$ **(b)** $4.43 \text{ atm} = 449 \text{ kPa}$

34. **(a)** $3.00 \ \text{mol}$ **(b)** 1.81×10^{24} molecules

36. $0.131 \ \text{kg/m}^3$

38. **(a)** $\rho = nM/V$ **(b)** See Solution. **(c)** 69 kg/m^3

(d) The shell would rise because $\rho_{\text{shell}} = 47.7 \text{ kg/m}^3 < \rho_{\text{atmosphere}}$.

40. 36.5 kN

42. **(a)** $\dfrac{v_{\text{rms}}\left({}^{35}\text{Cl}\right)}{v_{\text{rms}}\left({}^{37}\text{Cl}\right)} = 1.03$ **(b)** The less massive atom, ${}^{35}\text{Cl}$, moves faster.

44. **(a)** 385 K **(b)** 7.97×10^{-21} J

(c) The molar mass, from which the mass of a molecule can be computed.

46. 18 kPa

48. **(a)** 0.176 mm **(b)** $8.78\ \mu\text{m}$ **(c)** $9.30 \times 10^{-8} \text{ m}^3$

50. **(a)** 0.34% **(b)** 0.48%

(c) The moment of inertia for each of the shapes has the same mathematical form: the product of a constant, the mass, and the square of a length.

52. 28 m

54. **(a)** See Solution. **(b)** greater than atmospheric pressure

(c) The equilibrium value of h increases as the temperature T increases.

56. See Solution.

58. **(a)** 343 K **(b)** 12.5%

60. **(a)** 16.9 cm **(b)** 1.35×10^5 Pa

62. **(a)** 6.0 cm **(b)** See Solution.

(c) The bridge does not crumble. The stress developed is $4.8 \times 10^6 \text{ Pa} < 2.0 \times 10^7 \text{ Pa}$.

64. 1.15 atm

PROBLEM SOLUTIONS

10.1 (a) $T_F = \dfrac{9}{5}T_C + 32 = \dfrac{9}{5}(-273.15) + 32 = \boxed{-460°\text{F}}$

(b) $T_C = \dfrac{5}{9}\left(T_F - 32\right) = \dfrac{5}{9}(98.6 - 32) = \boxed{37°\text{C}}$

(c) $T_F = \dfrac{9}{5}T_C + 32 = \dfrac{9}{5}\left(T_K - 273.15\right) + 32 = \dfrac{9}{5}(-173.15) + 32 = \boxed{-280°\text{F}}$

10.2 When the volume of a low-density gas is held constant, pressure and temperature are related by a linear equation $P = AT + B$, where A and B are constants to be determined. For the given constant-volume gas thermometer,

$$P = 0.700 \text{ atm when } T = 100°\text{C} \quad \Rightarrow \quad 0.700 \text{ atm} = A(100°\text{C}) + B \qquad \textbf{[1]}$$

$$P = 0.512 \text{ atm when } T = 0°\text{C} \quad \Rightarrow \quad 0.512 \text{ atm} = A(0) + B \qquad \textbf{[2]}$$

From Equation [2], $B = 0.512$ atm. Substituting this result into Equation [1] yields

$$A = \frac{0.700 \text{ atm} - 0.512 \text{ atm}}{100°\text{C}} = 1.88 \times 10^{-3} \text{ atm/°C}$$

so, the linear equation for this thermometer is: $P = \left(1.88 \times 10^{-3} \text{ atm/°C}\right)T + 0.512 \text{ atm}$

(a) If $P = 0.0400$ atm, then $T = \dfrac{P - B}{A} = \dfrac{0.0400 \text{ atm} - 0.512 \text{ atm}}{1.88 \times 10^{-3} \text{ atm/°C}} = \boxed{-251°\text{C}}$

(b) If $T = 450°\text{C}$, then $P = \left(1.88 \times 10^{-3} \text{ atm/°C}\right)(450°\text{C}) + 0.512 \text{ atm} = \boxed{1.36 \text{ atm}}$

10.3 (a) $T_C = T_K - 273.15 = 20.3 - 273.15 = \boxed{-253°\text{C}}$

(b) $T_F = \dfrac{9}{5}T_C + 32 = \dfrac{9}{5}(-253) + 32 = \boxed{-423°\text{F}}$

10.4 (a) $T_C = \dfrac{5}{9}\left(T_F - 32\right) = \dfrac{5}{9}(134 - 32) = \boxed{56.7°\text{C}}$, and

$$T_C = \frac{5}{9}(-79.8 - 32) = \boxed{-62.1°\text{C}}$$

(b) $T = T_C + 273.15 = 56.7 + 273.15 = \boxed{330 \text{ K}}$, and

$$T_K = T_C + 273.15 = -62.1 + 273.15 = \boxed{211 \text{ K}}$$

10.5 Start with $T_F = -40°\text{F}$ and convert to Celsius.

$$T_C = \frac{5}{9}\left(T_F - 32\right) = \frac{5}{9}(-40 - 32) = \boxed{-40°\text{C}}$$

Since Celsius and Fahrenheit degrees of temperature change are different sizes (1 Celsius degree $= 1.8$ Fahrenheit degrees), this is the only temperature with the same numeric value on both scales.

10.6 Since we have a linear graph, we know that the pressure is related to the temperature as $P = A + BT_C$, where A and B are constants. To find A and B, we use the given data:

$$0.900 \text{ atm} = A + B(-78.5°\text{C}) \qquad \textbf{[1]}$$

and

$$1.635 \text{ atm} = A + B(78.0°\text{C}) \qquad \textbf{[2]}$$

Solving Equations [1] and [2] simultaneously, we find:

$$A = 1.27 \text{ atm, and} \qquad B = 4.70 \times 10^{-3} \text{ atm/°C}$$

continued on next page

Therefore, $P = 1.27 \text{ atm} + \left(4.70 \times 10^{-3} \text{ atm/°C}\right) T_C$

(a) At absolute zero the gas exerts zero pressure $(P = 0)$, so

$$T_C = \frac{-1.27 \text{ atm}}{4.70 \times 10^{-3} \text{ atm/°C}} = \boxed{-270\text{°C}}$$

(b) At the freezing point of water, $T_C = 0\text{°C}$ and

$$P = 1.27 \text{ atm} + 0 = \boxed{1.27 \text{ atm}}$$

(c) At the boiling point of water, $T_C = 100\text{°C}$, so

$$P = 1.27 \text{ atm} + \left(4.70 \times 10^{-3} \text{ atm/°C}\right)(100\text{°C}) = \boxed{1.74 \text{ atm}}$$

10.7 Apply $T_F = \frac{9}{5} T_C + 32$ to two different Celsius temperatures, $\left(T_C\right)_1$ and $\left(T_C\right)_2$,

to obtain $$\left(T_F\right)_1 = \frac{9}{5}\left(T_C\right)_1 + 32 \qquad \textbf{[1]}$$

and $$\left(T_F\right)_2 = \frac{9}{5}\left(T_C\right)_2 + 32 \qquad \textbf{[2]}$$

Subtracting Equation [1] from [2] yields

$$\left(T_F\right)_2 - \left(T_F\right)_1 = \frac{9}{5}\left[\left(T_C\right)_2 - \left(T_C\right)_1\right]$$

or $\boxed{\Delta T_F = (9/5)\Delta T_C}$

10.8 (a) Using the result of Problem 10.7 above, $\Delta T_C = \frac{5}{9}\left(\Delta T_F\right) = \frac{5}{9}(57.0)\text{°C} = \boxed{31.7\text{°C}}$

(b) $\Delta T_K = \left(T_{C,\,\text{out}} + 273.15\right) - \left(T_{C,\,\text{in}} + 273.15\right) = \left(T_{C,\,\text{out}} - T_{C,\,\text{in}}\right) = \Delta T_C = \boxed{31.7 \text{ K}}$

10.9 (a) $T_F = \frac{9}{5} T_C + 32 = \frac{9}{5}(41.5) + 32 = \boxed{107\text{°F}}$

(b) $\boxed{\text{Yes.}}$ The normal body temperature is 98.6°F, so this patient has a high fever and needs immediate attention.

10.10 (a) Since temperature differences on the Rankine and Fahrenheit scales are identical, the temperature readings on the two thermometers must differ by no more than an additive constant (i.e., $T_R = T_F + constant$). To evaluate this constant, consider the temperature readings on the two scales at absolute zero. We have $T_R = 0\text{°R}$ at absolute zero, and

$$T_F = \frac{9}{5} T_C + 32.00 = \frac{9}{5}(-273.15) + 32.00 = -459.67\text{°F}$$

Substituting these temperatures in our Fahrenheit to Rankine conversion gives

$$0° = -459.67° + constant \qquad \text{or} \qquad constant = 459.67°$$

giving $\boxed{T_R = T_F + 459.67}$

continued on next page

(b) We start with the Kelvin temperature and convert to the Rankine temperature in several stages, using the Fahrenheit to Rankine conversion from part (a) above.

$$T_K = T_C + 273.15 = \frac{5}{9}\left(T_F - 32.00\right) + 273.15 = \frac{5}{9}\left[\left(T_R - 459.67\right) - 32.00\right] + 273.15$$

$$= \frac{5}{9}\left(T_R - 491.67\right) + 273.15 = \frac{5}{9}T_R - \frac{5}{9}(491.67) + 273.15 = \frac{5}{9}T_R - \cancel{273.15} + \cancel{273.15}$$

or $\boxed{T_K = \frac{5}{9}T_R}$

10.11 The increase in temperature is $\Delta T = 35°\text{C} - (-20°\text{C}) = 55°\text{C}$.

Thus, $\Delta L = \alpha L_0(\Delta T) = \left[11\times10^{-6}\ (°\text{C})^{-1}\right](518\text{ m})(55°\text{C}) = 0.31\text{ m} = \boxed{31\text{ cm}}$

10.12 (a) As the temperature drops by 20°C, the length of the pendulum changes by

$$\Delta L = \alpha L_0(\Delta T) = \left[19\times10^{-6}\ (°\text{C})^{-1}\right](1.3\text{ m})(-20°\text{C})$$

or $\Delta L = -4.9\times10^{-4}\text{ m} = -0.49\text{ mm}$

Thus, the final length of the rod is $\boxed{L = 1.3\text{ m} - 0.49\text{ mm}}$.

(b) From the expression for the period, $T = 2\pi\sqrt{L/g}$, we see that as the length decreases the period decreases. Thus, the pendulum will swing too rapidly and the clock will run $\boxed{\text{fast}}$.

10.13 We choose the radius as our linear dimension. Then, from $\Delta L = \alpha L_0(\Delta T)$,

$$\Delta T = T_C - 20.0°\text{C} = \frac{L - L_0}{\alpha L_0} = \frac{2.21\text{ cm} - 2.20\text{ cm}}{\left[1.30\times10^{-4}\ (°\text{C})^{-1}\right](2.20\text{ cm})} = 35.0°\text{C}$$

or $T_C = \boxed{55.0°\text{C}}$

10.14 (a) The diameter is a linear dimension, so we consider the linear expansion of steel:

$$d = d_0\left[1 + \alpha(\Delta T)\right] = (2.540\text{ cm})\left[1 + \left(11\times10^{-6}\ (°\text{C})^{-1}\right)(100.0°\text{C} - 25.00°\text{C})\right] = \boxed{2.542\text{ cm}}$$

(b) If the volume increases by 1.000%, then $\Delta V = \left(1.000\times10^{-2}\right)V_0$. Then, using $\Delta V = \beta V_0(\Delta T)$, where $\beta = 3\alpha$ is the volume expansion coefficient, we find

$$\Delta T = \frac{\Delta V/V_0}{\beta} = \frac{1.000\times10^{-2}}{3\left[11\times10^{-6}\ (°\text{C})^{-1}\right]} = \boxed{3.0\times10^2\ °\text{C}}$$

10.15 From $\Delta L = L - L_0 = \alpha L_0(\Delta T)$, the final value of the linear dimension is $L = L_0 + \alpha L_0(\Delta T)$. To remove the ring from the rod, the diameter of the ring must be at least as large as the diameter of the rod. Thus, we require that

$$L_{\text{Brass}} = L_{\text{Al}},\text{ or }\left(L_0\right)_{\text{Brass}} + \alpha_{\text{Brass}}\left(L_0\right)_{\text{Brass}}(\Delta T) = \left(L_0\right)_{\text{Al}} + \alpha_{\text{Al}}\left(L_0\right)_{\text{Al}}(\Delta T)$$

continued on next page

This gives $$\Delta T = \frac{(L_0)_{\text{Al}} - (L_0)_{\text{Brass}}}{\alpha_{\text{Brass}}(L_0)_{\text{Brass}} - \alpha_{\text{Al}}(L_0)_{\text{Al}}}$$

(a) If $(L_0)_{\text{Al}} = 10.01$ cm,

$$\Delta T = \frac{10.01 - 10.00}{\left[19\times10^{-6}\ (^\circ\text{C})^{-1}\right](10.00) - \left[24\times10^{-6}\ (^\circ\text{C})^{-1}\right](10.01)} = -199^\circ\text{C}$$

so $T = T_0 + \Delta T = 20.0^\circ\text{C} - 199^\circ\text{C} = \boxed{-179^\circ\text{C}\ \text{ which is attainable}}$

(b) If $(L_0)_{\text{Al}} = 10.02$ cm

$$\Delta T = \frac{10.02 - 10.00}{\left[19\times10^{-6}\ (^\circ\text{C})^{-1}\right](10.00) - \left[24\times10^{-6}\ (^\circ\text{C})^{-1}\right](10.02)} = -396^\circ\text{C}$$

and

$$T = T_0 + \Delta T = \boxed{-376^\circ\text{C}\ \text{ which is below absolute zero and unattainable}}$$

10.16 $$\rho = \frac{m}{V} = \frac{m}{V_0 + \Delta V} = \frac{m}{V_0 + \beta V_0(\Delta T)} = \frac{m/V_0}{1+\beta(\Delta T)} = \boxed{\frac{\rho_0}{1+\beta(\Delta T)}}$$

10.17 (a) Using the result of Problem 10.16, with $\beta = 3\alpha$, gives

$$\rho = \frac{\rho_0}{1+\beta(\Delta T)} = \frac{11.3\times10^3\ \text{kg/m}^3}{1+3\left[29\times10^{-6}\ (^\circ\text{C})^{-1}\right](90^\circ\text{C} - 0^\circ\text{C})} = \boxed{11.2\times10^3\ \text{kg/m}^3}$$

(b) $\boxed{\text{No}}$. Although the density of gold would be less on a warm day, the mass of the bar would be the same, regardless of its temperature, and that is what you are paying for. (Note that the volume of the bar increases with increasing temperature, whereas its density decreases. Its mass, however, remains constant.)

10.18 (a) When a wire undergoes a decrease in temperature of magnitude $|\Delta T|$, it will attempt to contract by an amount $|\Delta L| = \alpha L_0 |\Delta T|$. If the ends of the wire are held fixed, not allowing it to contract, the wire will develop a tension sufficient to stretch it by the amount of its normal contraction, $|\Delta L|$. This tension is

$$F = YA\left(\frac{\Delta L}{L_0}\right) = YA\left(\frac{\alpha \cancel{L_0}|\Delta T|}{\cancel{L_0}}\right) = YA\alpha|\Delta T|$$

where Y is Young's modulus for the wire material and A is the cross-sectional area of the wire. For the given steel wire, with $|\Delta T| = |-10.0 - 35.0|^\circ\text{C}$, the tension that develops in the wire is

$$F = \left(20.0\times10^{10}\ \text{N/m}^2\right)\left(4.00\times10^{-6}\ \text{m}^2\right)\left[11.0\times10^{-6}\ (^\circ\text{C})^{-1}\right](45.0^\circ\text{C}) = \boxed{396\ \text{N}}$$

(b) As the temperature decreases while the wire is prevented from contracting, the stress that develops in the wire is $stress = (F/A) = Y\alpha|\Delta T|$. The decrease in temperature required to reach the elastic limit is

$$\Delta T = -\frac{(stress)_{\text{limit}}}{Y\alpha} = -\frac{3.00\times10^8\ \text{N/m}^2}{\left(20.0\times10^{10}\ \text{N/m}^2\right)\left[11\times10^{-6}\ (^\circ\text{C})^{-1}\right]} = -136^\circ\text{C}$$

continued on next page

Thus, the temperature of the wire when it reaches its elastic limit is

$$T = T_0 + \Delta T = 35.0°\text{C} + (-136°\text{C}) = \boxed{-101°\text{C}}$$

(c) Observe that $\boxed{\text{the initial length of the wire cancels out}}$ in the calculations of parts (a) and (b). Thus, the results obtained would not be changed if the initial length of the wire were doubled.

10.19 The difference in Celsius temperature in the underground tank and the tanker truck is

$$\Delta T_C = \frac{5}{9}(\Delta T_F) = \frac{5}{9}(95.0 - 52.0) = 23.9°\text{C}$$

If $V_{52°\text{F}}$ is the volume of gasoline that fills the tank at 52.0°F, the volume this quantity of gas would occupy on the tanker truck at 95.0°F is

$$V_{95°\text{F}} = V_{52°\text{F}} + \Delta V = V_{52°\text{F}} + \beta V_{52°\text{F}}(\Delta T) = V_{52°\text{F}}\left[1 + \beta(\Delta T)\right]$$

$$= \left(1.00 \times 10^3 \text{ gal}\right)\left[1 + \left(9.6 \times 10^{-4} \ (°\text{C})^{-1}\right)(23.9°\text{C})\right] = \boxed{1.02 \times 10^3 \text{ gal}}$$

10.20 Consider a regular solid with initial volume given by $V_0 = A_0 L_0$ at temperature T_0. Here, A is the cross-sectional area and L is the length of the regular solid.

When temperature undergoes a change $\Delta T = T - T_0$, the change in the cross-sectional area is $\Delta A = A - A_0 = \gamma A_0(\Delta T) = 2\alpha A_0(\Delta T)$, giving $A = A_0 + 2\alpha A_0(\Delta T)$. Similarly, the new length will be $L = L_0 + \alpha L_0(\Delta T)$, so the new volume is

$$V = \left[A_0 + 2\alpha A_0(\Delta T)\right]\left[L_0 + \alpha L_0(\Delta T)\right] = A_0 L_0 + 3\alpha A_0 L_0(\Delta T) + 2\alpha^2 A_0 L_0(\Delta T)^2$$

The term involving α^2 is negligibly small in comparison to the other terms, so

$$V \approx A_0 L_0 + 3\alpha A_0 L_0(\Delta T) = V_0 + 3\alpha V_0(\Delta T)$$

This is of the form $\Delta V = V - V_0 = \beta V_0(\Delta T)$ where $\boxed{\beta = 3\alpha}$.

10.21 (a) The volume of turpentine that overflows as the temperature rises equals the difference in the increase in the volume of the turpentine and the increase in the volume of the aluminum container.

$$V_{\text{overflow}} = \Delta V_{\text{T}} - \Delta V_{\text{Al}} = \beta_{\text{T}} V_0(\Delta T) - \beta_{\text{Al}} V_0(\Delta T) = \left(\beta_{\text{T}} - \beta_{\text{Al}}\right)V_0(\Delta T) = \left(\beta_{\text{T}} - 3\alpha_{\text{Al}}\right)V_0(\Delta T)$$

$$= \left[9.0 \times 10^{-4} \ °\text{C}^{-1} - 3\left(24 \times 10^{-6} \ °\text{C}^{-1}\right)\right](2.000 \text{ L})(80.0°\text{C} - 20.0°\text{C})$$

or $V_{\text{overflow}} = 9.9 \times 10^{-2} \text{ L} = \boxed{99 \text{ mL}}$

(b) The volume of turpentine remaining in the cylinder at 80.0°C is the same as the volume of the aluminum cylinder at 80.0°C. This is

$$V_1 = V_0\left[1 + 3\alpha_{\text{Al}}(\Delta T)\right] = (2.000 \text{ L})\left[1 + 3\left(24 \times 10^{-6} \ °\text{C}^{-1}\right)(80.0 - 20.0)°\text{C}\right]$$

$$= 2.000 \text{ L} + 0.008\,6 \text{ L} = 2.009 \text{ L}$$

or $V_1 = \boxed{V_{\substack{\text{turpentine}\\ \text{remaining}}} = 2.009 \text{ L}}$

continued on next page

(c) The volume of the aluminum cylinder when cooled back to 20.0°C will be $V_0 = 2.000$ L. The volume of the remaining turpentine when cooled to 20.0°C will be

$$V_2 = V_1\left[1+\beta_T(\Delta T)\right] = (2.009\text{ L})\left[1+\left(9.0\times10^{-4}\ {}^\circ\text{C}^{-1}\right)(20.0-80.0)^\circ\text{C}\right]$$
$$= 2.009\text{ L} - 0.11\text{ L} = 1.90\text{ L}$$

The fraction of the cylinder's volume that is now empty will be

$$\text{fraction empty} = \frac{V_0 - V_2}{V_0} = \frac{2.000\text{ L} - 1.90\text{ L}}{2.000\text{ L}} = 5.00\times10^{-2}$$

so, the empty height above the remaining turpentine at 20.0°C is

$$h_{\text{empty}} = h(\text{fraction empty}) = (20.0\text{ cm})\left(5.00\times10^{-2}\right) = \boxed{1.00\text{ cm}}$$

10.22 [Note that some rules concerning significant figures are deliberately violated in this solution to better illustrate the method of solution.]

Let L be the final length of the aluminum column. This will also be the final length of the quantity of tape now stretching from one end of the column to the other. In order to determine what the scale reading now is, we need to find the initial length this quantity of tape had at 21.2°C (when the scale markings were presumably put on the tape).

Thus, we let this initial length of tape be $\left(L_0\right)_{\text{tape}}$ and require that

$$L = \left(L_0\right)_{\text{tape}}\left[1+\alpha_{\text{steel}}(\Delta T)\right] = \left(L_0\right)_{\text{column}}\left[1+\alpha_{\text{Al}}(\Delta T)\right]\text{, which gives}$$

$$\left(L_0\right)_{\text{tape}} = \frac{\left(L_0\right)_{\text{column}}\left[1+\alpha_{\text{Al}}(\Delta T)\right]}{1+\alpha_{\text{steel}}(\Delta T)}$$

or $$\left(L_0\right)_{\text{tape}} = \frac{(18.700\text{ m})\left[1+\left(24\times10^{-6}\ (^\circ\text{C})^{-1}\right)(29.4^\circ\text{C}-21.2^\circ\text{C})\right]}{1+\left(11\times10^{-6}\ (^\circ\text{C})^{-1}\right)(29.4^\circ\text{C}-21.2^\circ\text{C})} = 18.702\text{ m}$$

The measured length of the column, according to the markings on the tape, at 29.4°C is therefore $\boxed{18.702\text{ m}}$.

10.23 If allowed to do so, the amount the band (with initial length L_0) would contract as it cools to 37°C is $\Delta L = \alpha L_0|\Delta T|$. Since the band is not allowed to contract, it will develop a tensile stress given by

$$Stress = Y\left(\frac{\Delta L}{L_0}\right) = Y\left(\frac{\alpha \cancel{L_0}|\Delta T|}{\cancel{L_0}}\right) = Y\alpha|\Delta T|$$

If A = (height · thickness) = (4.0 mm)(0.50 mm) = $2.0\times10^{-6}\text{ m}^2$ is the cross-sectional area of the band, the tension in the band will be

$$F = A\cdot(Stress) = \left(2.0\times10^{-6}\text{ m}^2\right)\left(18\times10^{10}\ \frac{\text{N}}{\text{m}^2}\right)\left(17.3\times10^{-6}\ {}^\circ\text{C}^{-1}\right)(80^\circ\text{C}-37^\circ\text{C}) = \boxed{2.7\times10^2\text{ N}}$$

10.24 (a) The expansion of the pipeline will be $\Delta L = \alpha L_0(\Delta T)$, or

$$\Delta L = \left[11\times10^{-6}\ (^\circ\text{C})^{-1}\right](1300\text{ km})\left[35^\circ\text{C}-(-73^\circ\text{C})\right] = \boxed{1.5\text{ km}}$$

(b) This is accommodated by $\boxed{\text{accordion-like expansion joints}}$ placed in the pipeline at periodic intervals.

10.25 The drum and the carbon tetrachloride, both having an initial volume of $V_0 = 50.0$ gal, expand at different rates as the temperature rises by $\Delta T = 20.0°\text{C}$. From $\Delta V = \beta V_0(\Delta T)$, with $\beta = 3\alpha$ as the coefficient of volume expansion for the steel drum, we obtain

$$V_{\text{spillage}} = \Delta V_{\substack{\text{carbon}\\ \text{tetrachloride}}} - \Delta V_{\substack{\text{steel}\\ \text{drum}}} = \left(\beta_{\substack{\text{carbon}\\ \text{tetrachloride}}} - 3\alpha_{\text{steel}}\right)V_0(\Delta T)$$

or $$V_{\text{spillage}} = \left[5.81\times10^{-4}\ (°\text{C})^{-1} - 3\left(11\times10^{-6}\ (°\text{C})^{-1}\right)\right](50.0\text{ gal})(20.0°\text{C}) = \boxed{0.548\text{ gal}}$$

10.26 (a) $$m_0 = \rho_0 V = \left(7.30\times10^2\ \text{kg/m}^3\right)(10.0\text{ gal})\left(\frac{3.80\times10^{-3}\ \text{m}^3}{1\text{ gal}}\right) = \boxed{27.7\text{ kg}}$$

(b) $$V = V_0 + \Delta V = V_0 + \beta V_0(\Delta T) = V_0\left[1 + \beta(\Delta T)\right]$$

or $$V = \left(1.000\text{ m}^3\right)\left[1 + \left(9.60\times10^{-4}\ (°\text{C})^{-1}\right)(20.0°\text{C})\right] = \boxed{1.02\text{ m}^3}$$

(c) Gasoline having a mass of $m = 7.30\times10^2$ kg occupies a volume of $V_0 = 1.000\text{ m}^3$ at 0°C and a volume of $V = 1.02\text{ m}^3$ at 20.0°C. The density of gasoline at 20.0°C is then

$$\rho_{20} = \frac{m}{V} = \frac{7.30\times10^2\text{ kg}}{1.02\text{ m}^3} = \boxed{716\ \text{kg/m}^3}$$

(d) $$m_{20} = \rho_{20} V = \left(7.16\times10^2\ \text{kg/m}^3\right)(10.0\text{ gal})\left(\frac{3.80\times10^{-3}\ \text{m}^3}{1\text{ gal}}\right) = \boxed{27.2\text{ kg}}$$

(e) $$\Delta m = m_0 - m_{20} = 27.7\text{ kg} - 27.2\text{ kg} = \boxed{0.5\text{ kg}}$$

10.27 (a) The gap width is a linear dimension, so it $\boxed{\text{increases}}$ in "thermal enlargement" as the temperature goes up. The gap expands in the same way the material removed to create the gap would have expanded.

(b) At 190°C, the length of the piece of steel that is missing, or has been removed to create the gap, is $L = L_0 + \Delta L = L_0\left[1 + \alpha(\Delta T)\right]$. This gives

$$L = (1.600\text{ cm})\left[1 + \left(11\times10^{-6}\ °\text{C}^{-1}\right)(190°\text{C} - 30.0°\text{C})\right] = \boxed{1.603\text{ cm}}$$

10.28 Each slab will undergo an increase in length of $\Delta L = \alpha L_0(\Delta T)$, and the gap between successive slabs must be at least this wide to accommodate this expansion. Thus, the minimum gap size should be

$$\Delta L = \alpha_{\text{concrete}} L_0(T - T_0) = \left(12\times10^{-6}\ °\text{C}^{-1}\right)(25.0\text{ m})(50.0°\text{C} - 10.0°\text{C})$$

or $$\Delta L = 1.2\times10^{-2}\text{ m} = \boxed{1.2\text{ cm}}$$

10.29 (a) From the ideal gas law, $PV = nRT$, we find $P/T = nR/V$. Thus, if both n and V are constant as the gas is heated, the ratio P/T is constant, giving $P_f/T_f = P_i/T_i$, or

$$T_f = T_i\left(\frac{P_f}{P_i}\right) = (300\text{ K})\left(\frac{3P_i}{P_i}\right) = 900\text{ K} = \boxed{627°\text{C}}$$

(b) If both pressure and volume double as n is held constant, the ideal gas law gives

$$T_f = T_i\left(\frac{P_f V_f}{P_i V_i}\right) = T_i\left(\frac{(2P_i)(2V_i)}{P_i V_i}\right) = 4T_i = 4(300\text{ K}) = 1\,200\text{ K} = \boxed{927°\text{C}}$$

10.30 (a) $T_i = T_C + 273.15 = (19.0 + 273.15)\ \text{K} = \boxed{292\ \text{K}}$

(b) $$n_i = \frac{P_i V_i}{RT_i} = \frac{(9.50\times10^5\ \text{Pa})\left[(20.0\ \text{L})(10^3\ \text{cm}^3/1\ \text{L})(1\ \text{m}^3/10^6\ \text{cm}^3)\right]}{(8.31\ \text{J/mol}\cdot\text{K})(292\ \text{K})} = \boxed{7.83\ \text{mol}}$$

(c) $$M_{CO_2} = [12.0 + 2(16.0)]\ \frac{\text{g}}{\text{mol}} = \boxed{44.0\ \text{g/mol}}$$

(d) $m_i = n_i M_{CO_2} = (7.83\ \text{mol})(44.0\ \text{g/mol}) = \boxed{345\ \text{g}}$

(e) $T_f = T_i + \Delta T = 292\ \text{K} + 224\ \text{K} = \boxed{516\ \text{K}}$

$$n_f = \frac{m_f}{M_{CO_2}} = \frac{m_i - \Delta m}{M_{CO_2}} = \frac{345\ \text{g} - 82.0\ \text{g}}{44.0\ \text{g/mol}} = \boxed{5.98\ \text{mol}}$$

(f) Neglecting any change in volume of the tank, $V_f \approx V_i$, and we have

$$\frac{P_f \cancel{V_f}}{P_i \cancel{V_i}} = \frac{n_f \cancel{R} T_f}{n_i \cancel{R} T_i} \quad\Rightarrow\quad \boxed{P_f = \left(\frac{n_f T_f}{n_i T_i}\right) P_i}$$

(g) $$P_f = \left(\frac{n_f}{n_i}\right)\left(\frac{T_f}{T_i}\right) P_i = \left(\frac{5.98\ \text{mol}}{7.83\ \text{mol}}\right)\left(\frac{516\ \text{K}}{292\ \text{K}}\right)(9.50\times10^5\ \text{Pa}) = \boxed{1.28\times10^6\ \text{Pa}}$$

10.31 (a) $$n = \frac{PV}{RT} = \frac{(1.013\times10^5\ \text{Pa/atm})(1.0\times10^{-6}\ \text{m}^3)}{(8.31\ \text{J/mol}\cdot\text{K})(293\ \text{K})} = 4.2\times10^{-5}\ \text{mol}$$

Thus, $N = n\cdot N_A = (4.2\times10^{-5}\ \text{mol})\left(6.02\times10^{23}\ \frac{\text{molecules}}{\text{mol}}\right) = \boxed{2.5\times10^{19}\ \text{molecules}}$.

(b) Since both V and T are constant, $n_2/n_1 = \left(P_2 \cancel{V_2}/\cancel{RT_2}\right)\big/\left(P_1 \cancel{V_1}/\cancel{RT_1}\right) = P_2/P_1$, or

$$n_2 = \left(\frac{P_2}{P_1}\right) n_1 = \left(\frac{1.0\times10^{-11}\ \text{Pa}}{1.013\times10^5\ \text{Pa}}\right)(4.2\times10^{-5}\ \text{mol}) = \boxed{4.1\times10^{-21}\ \text{mol}}$$

10.32 (a) With $P_0 = 1.00$ atm, $T_0 = 10.0°\text{C} = 283$ K, $T_1 = 40.0°\text{C} = 313$ K, and $V_1 = 0.280V_0$, we find

$$\frac{P_1V_1}{P_0V_0} = \frac{\cancel{n_1}RT_1}{\cancel{n_0}RT_0} \quad\Rightarrow\quad P_1 = \left(\frac{V_0}{V_1}\right)\left(\frac{T_1}{T_0}\right)P_0 = \left(\frac{1}{0.280}\right)\left(\frac{313\ \text{K}}{283\ \text{K}}\right)(1.00\ \text{atm}) = \boxed{3.95\ \text{atm}}$$

and $P_1 = (3.95\ \text{atm})(1.013\times10^5\ \text{Pa}/1\ \text{atm}) = 4.00\times10^5\ \text{Pa} = \boxed{400\ \text{kPa}}$.

(b) If now, conditions inside the tire change so that $V_f = 1.02V_1$ and $T_f = 85.0°\text{C} = 358$ K, we form a new ratio to find

$$\frac{P_fV_f}{P_1V_1} = \frac{\cancel{n_f}RT_f}{\cancel{n_1}RT_1} \quad\Rightarrow\quad P_f = \left(\frac{V_1}{V_f}\right)\left(\frac{T_f}{T_1}\right)P_1 = \left(\frac{1}{1.02}\right)\left(\frac{358\ \text{K}}{313\ \text{K}}\right)(3.95\ \text{atm}) = \boxed{4.43\ \text{atm}}$$

and $$P_f = (4.43\ \text{atm})\left(\frac{1.013\times10^5\ \text{Pa}}{1\ \text{atm}}\right) = 4.49\times10^5\ \text{Pa} = \boxed{449\ \text{kPa}}$$

10.33 The initial and final absolute temperatures are

$$T_i = T_{C,i} + 273 = (25.0 + 273)\text{ K} = 298\text{ K} \quad \text{and} \quad T_f = T_{C,f} + 273 = (75.0 + 273)\text{ K} = 348\text{ K}$$

The volume of the tank is assumed to be unchanged, or $V_f = V_i$. Also, two-thirds of the gas is withdrawn, so $n_f = n_i/3$. Thus, from the ideal gas law,

$$\frac{P_f \cancel{V_f}}{P_i \cancel{V_i}} = \frac{n_f \cancel{R} T_f}{n_i \cancel{R} T_i} \quad \Rightarrow \quad P_f = \left(\frac{n_f}{n_i}\right)\left(\frac{T_f}{T_i}\right)P_i = \left(\frac{1}{3}\right)\left(\frac{348\text{ K}}{298\text{ K}}\right)(11.0\text{ atm}) = \boxed{4.28\text{ atm}}$$

10.34 (a) The volume of the gas is $V = 8.00\text{ L} = 8.00 \times 10^3\text{ cm}^3 = 8.00 \times 10^{-3}\text{ m}^3$, and the absolute temperature is $T = (20.0 + 273)\text{ K} = 293\text{ K}$. The ideal gas law then gives the number of moles present as

$$n = \frac{PV}{RT} = \frac{\left[9.00\text{ atm}\left(1.013 \times 10^5\text{ Pa/1 atm}\right)\right]\left(8.00 \times 10^{-3}\text{ m}^3\right)}{(8.31\text{ J/mol}\cdot\text{K})(293\text{ K})} = \boxed{3.00\text{ mol}}$$

(b) The number of molecules present in the container is

$$N = n \cdot N_A = (3.00\text{ mol})\left(6.02 \times 10^{23}\text{ molecules/mol}\right) = \boxed{1.81 \times 10^{24}\text{ molecules}}$$

10.35 With n held constant, the ideal gas law gives

$$\frac{V_1}{V_2} = \left(\frac{P_2}{P_1}\right)\left(\frac{T_1}{T_2}\right) = \left(\frac{0.030\text{ atm}}{1.0\text{ atm}}\right)\left(\frac{300\text{ K}}{200\text{ K}}\right) = 4.5 \times 10^{-2}$$

Since the volume of a sphere is $V = (4\pi/3)r^3$, $V_1/V_2 = (r_1/r_2)^3$.

Thus, $$r_1 = \left(\frac{V_1}{V_2}\right)^{1/3} r_2 = \left(4.5 \times 10^{-2}\right)^{1/3}(20\text{ m}) = \boxed{7.1\text{ m}}$$

10.36 The mass of the gas in the balloon does not change as the temperature increases. Thus,

$$\frac{\rho_f}{\rho_i} = \frac{(m/V_f)}{(m/V_i)} = \frac{V_i}{V_f} \quad \text{or} \quad \rho_f = \rho_i\left(\frac{V_i}{V_f}\right)$$

From the ideal gas law with both n and P constant, we find $V_i/V_f = T_i/T_f$, and now have

$$\rho_f = \rho_i\left(\frac{T_i}{T_f}\right) = \left(0.179\text{ kg/m}^3\right)\left(\frac{273\text{ K}}{373\text{ K}}\right) = \boxed{0.131\text{ kg/m}^3}$$

10.37 The pressure 100 m below the surface is found, using $P_1 = P_{\text{atm}} + \rho g h$, to be

$$P_1 = 1.013 \times 10^5\text{ Pa} + \left(10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)(100\text{ m}) = 1.08 \times 10^6\text{ Pa}$$

The ideal gas law, with both n and T constant, gives the volume at the surface as

$$V_2 = \left(\frac{P_1}{P_2}\right)V_1 = \left(\frac{P_1}{P_{\text{atm}}}\right)V = \left(\frac{1.08 \times 10^6\text{ Pa}}{1.013 \times 10^5\text{ Pa}}\right)\left(1.50\text{ cm}^3\right) = \boxed{16.0\text{ cm}^3}$$

10.38 (a) We assume the density ρ can be written as $\rho = n^a V^b M^c$, where n is the number of moles, V is the volume, and M is the molar mass in kilograms per mole, while a, b, and c are constants to be determined by dimensional analysis. In terms of mass (M), length (L), time (T), and number of moles (N), the fundamental units of density are $[\rho] = [\textit{mass / volume}] = \text{ML}^{-3}$, those of n are $[n] = \text{N}$, for volume $[V] = \text{L}^3$, and molar mass $[M] = [kg/mol] = \text{MN}^{-1}$. In terms of basic units, our assumed equation for density becomes

$$[\rho] = [n]^a [V]^b [M]^c \quad \text{or} \quad \text{M}^1\text{L}^{-3} = \text{N}^a\left(\text{L}^3\right)^b\left(\text{MN}^{-1}\right)^c = \text{N}^{a-c}\text{L}^{3b}\text{M}^c$$

continued on next page

and equating the powers of each of the basic units on the two sides of the equation gives:

$$1 = c \;\Rightarrow\; c = 1; \qquad -3 = 3b \;\Rightarrow\; b = -1; \qquad 0 = a - c \;\Rightarrow\; a = c = 1$$

so our expression for density, derived by dimensional analysis is $\rho = n^1 V^{-1} M^1$, or $\boxed{\rho = nM/V}$ where M is expressed in kilograms per mole.

(b) From the ideal gas law, $PV = nRT$, or $P = (n/V)RT$. From the result of part (a), we may write $n/V = \rho/M$, so the ideal gas law may be written in terms of the density of the gas as $\boxed{P = (\rho/M)RT}$, where M is again expressed in kilograms per mole.

(c) For carbon dioxide, $M = 44\ \text{g/mol} = 44 \times 10^{-3}\ \text{kg/mol}$. Then, if the pressure is $P = (90.0\ \text{atm})(1.013 \times 10^5\ \text{Pa/1 atm}) = 9.12 \times 10^6\ \text{Pa}$, and $T = 7.00 \times 10^2\ \text{K}$, the density of the atmosphere on Venus is

$$\rho = \frac{PM}{RT} = \frac{(9.12 \times 10^6\ \text{Pa})(44 \times 10^{-3}\ \text{kg/mol})}{(8.31\ \text{J/mol} \cdot \text{K})(7.00 \times 10^2\ \text{K})} = \boxed{69\ \text{kg/m}^3}$$

(d) The density of the evacuated steel shell would be

$$\rho_{\text{shell}} = \frac{m_{\text{shell}}}{V_{\text{shell}}} = \frac{2.00 \times 10^2\ \text{kg}}{4\pi(1.00\ \text{m})^3/3} = 47.7\ \text{kg/m}^3$$

$\boxed{\text{Since } \rho_{\text{shell}} < \rho_{\text{atmosphere}}\text{, this shell would rise in the atmosphere on Venus.}}$

10.39 The average kinetic energy of the molecules of *any* ideal gas at 300 K is

$$\overline{KE} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}k_{\text{B}}T = \frac{3}{2}\left(1.38 \times 10^{-23}\ \frac{\text{J}}{\text{K}}\right)(300\ \text{K}) = \boxed{6.21 \times 10^{-21}\ \text{J}}$$

10.40 Since the sample contains three times Avogadro's number of molecules, there must be 3 moles of gas present. The ideal gas law then gives

$$P = \frac{nRT}{V} = \frac{(3\ \text{mol})(8.31\ \text{J/mol} \cdot \text{K})(293\ \text{K})}{(0.200\ \text{m})^3} = 9.13 \times 10^5\ \text{Pa}$$

The force this gas will exert on one face of the cubical container is

$$F = PA = (9.13 \times 10^5\ \text{Pa})(0.200\ \text{m})^2 = 3.65 \times 10^4\ \text{N} = \boxed{36.5\ \text{kN}}$$

10.41 One mole of any substance contains Avogadro's number of molecules and has a mass equal to the molar mass, M. Thus, the mass of a single molecule is $m = M/N_{\text{A}}$.

For helium, $M = 4.00\ \text{g/mol} = 4.00 \times 10^{-3}\ \text{kg/mol}$, and the mass of a helium molecule is

$$m = \frac{4.00 \times 10^{-3}\ \text{kg/mol}}{6.02 \times 10^{23}\ \text{molecule/mol}} = 6.64 \times 10^{-27}\ \text{kg/molecule}$$

Since a helium molecule contains a single helium atom, the mass of a helium atom is

$$m_{\text{atom}} = \boxed{6.64 \times 10^{-27}\ \text{kg}}$$

10.42 (a) The rms speed of molecules in a gas of molar mass M and absolute temperature T is $v_{\text{rms}} = \sqrt{3RT/M}$. Since the molar masses of ${}^{35}\text{Cl}$ and ${}^{37}\text{Cl}$ are 35.0×10^{-3} kg/mol and 37.0×10^{-3} kg/mol, respectively, the desired ratio is

$$\frac{v_{\text{rms}}({}^{35}\text{Cl})}{v_{\text{rms}}({}^{37}\text{Cl})} = \frac{\sqrt{3RT/35.0 \times 10^{-3}\ \text{kg/mol}}}{\sqrt{3RT/37.0 \times 10^{-3}\ \text{kg/mol}}} = \sqrt{\frac{37.0}{35.0}} = \boxed{1.03}$$

(b) Since the above ratio is larger than 1, $\boxed{\text{the less massive atom, } {}^{35}\text{Cl, moves faster}}$.

10.43 If $v_{\text{rms}} = v_{\text{esc}}$, we must have $v_{\text{rms}} = \sqrt{3k_{\text{B}}T/m} = v_{\text{esc}}$, where $k_{\text{B}} = 1.38 \times 10^{-23}$ J/K is Boltzmann's constant and m is the mass of a molecule (for helium, $m = 6.64 \times 10^{-27}$ kg). Thus, the required absolute temperature is $T = mv_{\text{esc}}^2/3k_{\text{B}}$.

(a) To have $v_{\text{rms}} = v_{\text{esc}}$ on Earth where $v_{\text{esc}} = 1.12 \times 10^4$ m/s, the required temperature for the helium gas is

$$T = \frac{(6.64 \times 10^{-27}\ \text{kg})(1.12 \times 10^4\ \text{m/s})^2}{3(1.38 \times 10^{-23}\ \text{J/K})} = \boxed{2.01 \times 10^4\ \text{K}}$$

(b) If $v_{\text{rms}} = v_{\text{esc}}$ on the Moon where $v_{\text{esc}} = 2.37 \times 10^3$ m/s, the temperature must be

$$T = \frac{(6.64 \times 10^{-27}\ \text{kg})(2.37 \times 10^3\ \text{m/s})^2}{3(1.38 \times 10^{-23}\ \text{J/K})} = \boxed{901\ \text{K}}$$

10.44 (a) The volume occupied by this gas is

$$V = 7.00\ \text{L} = 7.00 \times 10^3\ \text{cm}^3 = 7.00 \times 10^{-3}\ \text{m}^3$$

Then, the ideal gas law gives

$$T = \frac{PV}{nR} = \frac{(1.60 \times 10^6\ \text{Pa})(7.00 \times 10^{-3}\ \text{m}^3)}{(3.50\ \text{mol})(8.31\ \text{J/mol} \cdot \text{K})} = \boxed{385\ \text{K}}$$

(b) The average kinetic energy per molecule in this gas is

$$\overline{KE}_{\text{molecule}} = \frac{3}{2}k_B T = \frac{3}{2}(1.38 \times 10^{-23}\ \text{J/K})(385\ \text{K}) = \boxed{7.97 \times 10^{-21}\ \text{J}}$$

(c) You would need to know the mass of the gas molecule to find its rms speed, which in turn requires knowledge of $\boxed{\text{the molar mass of the gas.}}$

10.45 Consider a time interval of 1.0 min = 60 s, during which 150 bullets bounce off Superman's chest. From the impulse-momentum theorem, the magnitude of the average force exerted on Superman is

$$F_{\text{av}} = \frac{I}{\Delta t} = \frac{150|\Delta p|_{\text{bullet}}}{\Delta t} = \frac{150|m(v - v_0)|}{\Delta t}$$

$$= \frac{150(8.0 \times 10^{-3}\ \text{kg})[(400\ \text{m/s}) - (-400\ \text{m/s})]}{60\ \text{s}} = \boxed{16\ \text{N}}$$

10.46 From the impulse-momentum theorem, the average force exerted on the wall is

$$F_{\text{av}} = \frac{I}{\Delta t} = \frac{N|\Delta p|_{\text{molecule}}}{\Delta t} = \frac{N|m(v - v_0)|}{\Delta t}$$

or $$F_{\text{av}} = \frac{(5.0 \times 10^{23})(4.68 \times 10^{-26}\ \text{kg})[(300\ \text{m/s}) - (-300\ \text{m/s})]}{1.0\ \text{s}} = 14\ \text{N}$$

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The pressure on the wall is then

$$P = \frac{F_{av}}{A} = \frac{14 \text{ N}}{8.0 \text{ cm}^2}\left(\frac{10^4 \text{ cm}^2}{1 \text{ m}^2}\right) = 1.8\times10^4 \text{ N/m}^2 = \boxed{18 \text{ kPa}}$$

10.47 As the pipe undergoes a temperature change $\Delta T = 46.5°\text{C} - 18.0°\text{C} = 28.5°\text{C}$, the expansion toward the right of the horizontal segment is

$$\Delta L_x = \alpha L_{0x}(\Delta T)$$

$$= \left[17\times10^{-6} \ (°\text{C})^{-1}\right](28.0 \text{ cm})(28.5°\text{C}) = 1.4\times10^{-2} \text{ cm} = 0.14 \text{ mm}$$

The downward expansion of the vertical section is

$$\Delta L_y = \alpha L_{0y}(\Delta T) = \left[17\times10^{-6} \ (°\text{C})^{-1}\right](134 \text{ cm})(28.5°\text{C}) = 6.5\times10^{-2} \text{ cm} = 0.65 \text{ mm}$$

The total displacement of the pipe elbow is

$$\Delta L = \sqrt{\Delta L_x^2 + \Delta L_y^2} = \sqrt{(0.14 \text{ mm})^2 + (0.65 \text{ mm})^2} = 0.66 \text{ mm}$$

at $\theta = \tan^{-1}\left(\frac{\Delta L_y}{\Delta L_x}\right) = \tan^{-1}\left(\frac{0.65 \text{ mm}}{0.14 \text{ mm}}\right) = 78°$

or $\Delta\vec{\mathbf{L}} = \boxed{\text{0.66 mm toward the right at 78° below the horizontal}}$

10.48 (a) $\Delta L = \alpha L_0(\Delta T) = \left[9.00\times10^{-6} \ (°\text{C})^{-1}\right](30.0 \text{ cm})(65.0°\text{C}) = 1.76\times10^{-2} \text{ cm} = \boxed{0.176 \text{ mm}}$

(b) $\Delta D = \alpha D_0(\Delta T) = \left[9.00\times10^{-6} \ (°\text{C})^{-1}\right](1.50 \text{ cm})(65.0°\text{C}) = 8.78\times10^{-4} \text{ cm} = \boxed{8.78 \ \mu\text{m}}$

(c) The initial volume is $V_0 = \left(\frac{\pi D_0^2}{4}\right)L_0 = \frac{\pi}{4}\left(1.50\times10^{-2} \text{ m}\right)^2(0.300 \text{ m}) = 5.30\times10^{-5} \text{ m}^3$

$$\Delta V = \beta V_0(\Delta T)$$

$$= 3\alpha V_0(\Delta T) = 3\left[9.00\times10^{-6} \ (°\text{C})^{-1}\right]\left(5.30\times10^{-5} \text{ m}^3\right)(65.0°\text{C}) = \boxed{9.30\times10^{-8} \text{ m}^3}$$

10.49 The number of moles of CO_2 present is $n = m/M$, where $m = 6.50$ g and $M = 44.0$ g/mol. Thus, at the given temperature ($20.0°\text{C} = 293$ K) and pressure ($1.00 \text{ atm} = 1.013\times10^5$ Pa), the volume will be

$$V = \frac{nRT}{P} = \frac{mRT}{MP} = \frac{(6.50 \text{ g})(8.31 \text{ J/mol}\cdot\text{K})(293 \text{ K})}{(44.0 \text{ g/mol})\left(1.013\times10^5 \text{ Pa}\right)} = 3.55\times10^{-3} \text{ m}^3 = \boxed{3.55 \text{ L}}$$

10.50 Note that the moment of inertia of each object in Table 8.1 is of the form $I = CM\ell^2$, where C is a constant, M is the mass of the object, and ℓ is some length. As the temperature increases, the factors C and M are unchanged, while the value of the length measurement at temperature T is $\ell = \ell_0(1+\alpha\cdot\Delta T)$. Therefore, the percentage increase in the moment of inertia of the object is

$$\% \text{ change} = \frac{I - I_0}{I_0}\times100\% = \left\{\frac{CM\left[\ell_0(1+\alpha\cdot\Delta T)\right]^2 - CM\ell_0^2}{CM\ell_0^2}\right\}\times100\%$$

or $\% \text{ change} = \left[(1+\alpha\cdot\Delta T)^2 - 1\right]\times100\%$

continued on next page

(a) For copper, with $\alpha = 17\times10^{-6}\ ^\circ\text{C}^{-1}$ and $\Delta T = 100^\circ\text{C}$,

$$\%\text{ change} = \left\{\left[1+\left(17\times10^{-6}\ ^\circ\text{C}^{-1}\right)(100^\circ\text{C})\right]^2 - 1\right\}\times 100\% = \boxed{0.34\%}$$

(b) For aluminum, with $\alpha = 24\times10^{-6}\ ^\circ\text{C}^{-1}$ and $\Delta T = 100^\circ\text{C}$,

$$\%\text{ change} = \left\{\left[1+\left(24\times10^{-6}\ ^\circ\text{C}^{-1}\right)(100^\circ\text{C})\right]^2 - 1\right\}\times 100\% = \boxed{0.48\%}$$

(c) The moment of inertia for each of the shapes has the same mathematical form: the product of a constant, the mass, and the square of a length.

10.51 For a temperature change $\Delta T_F = T_F - T_{F,0}$ on the Fahrenheit scale, the corresponding temperature change on the Celsius scale is

$$\Delta T_C = T_C - T_{C,0} = \frac{5}{9}\left(T_F - 32\right) - \frac{5}{9}\left(T_{F,0} - 32\right) = \frac{5}{9}\left(T_F - T_{F,0}\right) = \frac{5}{9}\left(\Delta T_F\right)$$

Therefore, if $L_0 = 35.000$ m and $\Delta T_F = 90.000^\circ\text{F} - 15.000^\circ\text{F} = 75.000^\circ\text{F}$, the final length of the beam is

$$L = L_0\left[1+\alpha(\Delta T)\right] = (35.000\text{ m})\left[1+\left(11\times10^{-6}\ ^\circ\text{C}^{-1}\right)\frac{5}{9}(75.000)^\circ\text{C}\right] = \boxed{35.016\text{ m}}$$

10.52 When air trapped in the tube is compressed, at constant temperature, into a cylindrical volume 0.40 m long, the ideal gas law gives its pressure as

$$P_2 = \left(\frac{V_1}{V_2}\right)P_1 = \left(\frac{L_1}{L_2}\right)P_1 = \left(\frac{1.5\text{ m}}{0.40\text{ m}}\right)\left(1.013\times10^5\text{ Pa}\right) = 3.8\times10^5\text{ Pa}$$

This is also the water pressure at the bottom of the lake. Thus, $P = P_{\text{atm}} + \rho g h$ gives the depth of the lake as

$$h = \frac{P_2 - P_{\text{atm}}}{\rho g} = \frac{(3.8-1.013)\times10^5\text{ Pa}}{\left(10^3\text{ kg/m}^3\right)\left(9.80\text{ m/s}^2\right)} = \boxed{28\text{ m}}$$

10.53 The mass of CO_2 produced by three astronauts in 7.00 days is $m = 3(1.09\text{ kg/d})(7.00\text{ d}) = 22.9$ kg, and the number of moles of CO_2 available is

$$n = \frac{m}{M} = \frac{22.9\text{ kg}}{44.0\times10^{-3}\text{ kg/mol}} = 520\text{ mol}$$

The recycling process will generate 520 moles of methane to be stored. In a volume of $V = 150\text{ L} = 0.150\text{ m}^3$ and at temperature $T = -45.0^\circ\text{C} = 228$ K, the pressure of the stored methane is

$$P = \frac{nRT}{V} = \frac{(520\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(228\text{ K})}{0.150\text{ m}^3} = 6.57\times10^6\text{ Pa} = \boxed{6.57\text{ MPa}}$$

10.54 (a) The piston in this vertical cylinder has three forces acting on it. These are: (1) a downward gravitational force, mg, the piston's own weight; (2) a downward pressure force, $F_d = P_0A$, due to the atmospheric pressure above the piston; and (3) an upward pressure force, $F_u = PA$, due to the absolute pressure of the gas trapped inside the cylinder. Since the piston is in equilibrium, Newton's second law requires

$$\Sigma F_y = 0 \quad\Rightarrow\quad F_u - mg - F_d = 0 \qquad \text{or} \qquad PA = mg + P_0A \qquad \textbf{[1]}$$

continued on next page

From the ideal gas law, the absolute pressure of the trapped gas is

$$P = \frac{nRT}{V} = \frac{nRT}{Ah} \qquad \textbf{[2]}$$

Substituting Equation [2] into [1] yields

$$\left(\frac{nRT}{\cancel{A}h}\right)\cancel{A} = mg + P_0 A \qquad \text{or} \qquad \boxed{h = \frac{nRT}{mg + P_0 A}}$$

(b) From Equation [1] above, the absolute pressure inside the cylinder is $P = \dfrac{mg}{A} + P_0$ where P_0 is atmospheric pressure. This is $\boxed{\text{greater than atmospheric pressure}}$ because $mg/A > 0$.

(c) Observe from the result of part (a) above, if the absolute temperature *T* increases, $\boxed{\text{the equilibrium value of } h \text{ also increases.}}$

10.55 (a) As the acetone undergoes a change in temperature $\Delta T = (20.0 - 35.0)\ °\text{C} = -15.0\ °\text{C}$, the final volume will be

$$V_f = V_0 + \Delta V = V_0 + \beta V_0(\Delta T) = V_0\left[1 + \beta(\Delta T)\right]$$
$$= (100\ \text{mL})\left[1 + \left(1.50\times 10^{-4}\ (°\text{C})^{-1}\right)(-15.0\ °\text{C})\right] = \boxed{99.8\ \text{mL}}$$

(b) When acetone at 35°C is poured into the Pyrex flask that was calibrated at 20°C, the volume of the flask temporarily expands to be larger than its calibration markings indicate. However, the coefficient of volume expansion for Pyrex $\left[\beta = 3\alpha = 9.6\times 10^{-6}\ (°\text{C})^{-1}\right]$ is much smaller than that of acetone $\left[\beta = 1.5\times 10^{-4}\ (°\text{C})^{-1}\right]$. Hence, the temporary increase in the volume of the flask will be much smaller than the change in volume of the acetone as the materials cool back to 20°C, and this change in volume of the flask $\boxed{\text{has negligible effect on the answer.}}$

10.56 If P_i is the initial gauge pressure of the gas in the cylinder, the initial absolute pressure is $P_{i,\,\text{abs}} = P_i + P_0$, where P_0 is atmospheric pressure. Likewise, the final absolute pressure in the cylinder is $P_{f,\,\text{abs}} = P_f + P_0$, where P_f is the final gauge pressure. The initial and final masses of gas in the cylinder are $m_i = n_i M$ and $m_f = n_f M$, where n is the number of moles of gas present and *M* is the molar mass of this gas. Thus, $m_f/m_i = n_f/n_i$.

We assume the cylinder is a rigid container whose volume does not vary with internal pressure. Also, since the temperature of the cylinder is constant, its volume does not expand nor contract. Then, the ideal gas law (using absolute pressures) with both temperature and volume constant gives

$$\frac{P_{f,\,\text{abs}}\cancel{V}}{P_{i,\,\text{abs}}\cancel{V}} = \frac{n_f\cancel{RT}}{n_i\cancel{RT}} = \frac{m_f}{m_i} \qquad \text{or} \qquad m_f = \left(\frac{P_{f,\,\text{abs}}}{P_{i,\,\text{abs}}}\right)m_i$$

and in terms of gauge pressures,

$$\boxed{m_f = \left(\frac{P_f + P_0}{P_i + P_0}\right)m_i}$$

10.57 (a) The volume of the liquid expands by $\Delta V_{\text{liquid}} = \beta V_0(\Delta T)$ and the volume of the glass flask expands by $\Delta V_{\text{flask}} = (3\alpha)V_0(\Delta T)$. The amount of liquid that must overflow into the capillary is $V_{\text{overflow}} = \Delta V_{\text{liquid}} - \Delta V_{\text{flask}} = V_0(\beta - 3\alpha)(\Delta T)$. The distance the liquid will rise into the capillary is then

$$\Delta h = \frac{V_{\text{overflow}}}{A} = \boxed{\left(\frac{V_0}{A}\right)(\beta - 3\alpha)(\Delta T)}$$

(b) For a mercury thermometer, $\beta_{\text{Hg}} = 1.82 \times 10^{-4}\ (°\text{C})^{-1}$ and (assuming Pyrex glass), $3\alpha_{\text{glass}} = 3\left(3.2 \times 10^{-6}\ (°\text{C})^{-1}\right) = 9.6 \times 10^{-6}\ (°\text{C})^{-1}$. Thus, the expansion of the mercury is $\boxed{\text{almost 20 times the expansion of the flask}}$, making it a rather good approximation to neglect the expansion of the flask.

10.58 (a) The initial absolute pressure in the tire is $P_1 = P_{\text{atm}} + P_{1,\text{ gauge}} = 2.80$ atm, and the final absolute pressure is $P_2 = P_{\text{atm}} + P_{2,\text{ gauge}} = 3.20$ atm.

The ideal gas law, with both n and V constant, gives

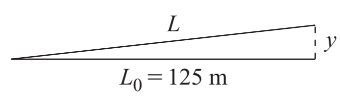
$$T_2 = \left(\frac{P_2}{P_1}\right)T_1 = \left(\frac{3.20 \text{ atm}}{2.80 \text{ atm}}\right)(300 \text{ K}) = \boxed{343 \text{ K}}$$

(b) When the quantity of gas varies, while volume and temperature are constant, the ideal gas law gives $\dfrac{n_3}{n_2} = \dfrac{P_3}{P_2}$. Thus, when air is released to lower the absolute pressure back to 2.80 atm, we have

$$\frac{n_3}{n_2} = \frac{2.80 \text{ atm}}{3.20 \text{ atm}} = 0.875$$

At the end, we have 87.5% of the original mass of air remaining, or $\boxed{\text{12.5\% of the original mass}}$ was released.

10.59 After expansion, the increase in the length of one span is



$$\Delta L = \alpha L_{,0}(\Delta T)$$

$$= \left[12 \times 10^{-6}\ (°\text{C})^{-1}\right](125 \text{ m})(20.0°\text{C}) = 3.0 \times 10^{-2} \text{ m}$$

giving a final length of $L = L_0 + \Delta L = 125 \text{ m} + 3.0 \times 10^{-2} \text{ m}$

From the Pythagorean theorem,

$$y = \sqrt{L^2 - L_0^2} = \sqrt{\left[(125 + 0.030) \text{ m}\right]^2 - (125 \text{ m})^2} = \boxed{2.7 \text{ m}}$$

10.60 (a) From the ideal gas law, $\dfrac{P_2V_2}{T_2} = \dfrac{P_1V_1}{T_1}$, or $\left(\dfrac{P_2}{P_1}\right)\left(\dfrac{V_2}{V_1}\right) = \left(\dfrac{T_2}{T_1}\right)$

The initial conditions are:

$$P_1 = 1 \text{ atm},\ V_1 = 5.00 \text{ L} = 5.00 \times 10^{-3} \text{ m}^3, \text{ and } T_1 = 20.0°\text{C} = 293 \text{ K}$$

The final conditions are:

$$P_2 = 1 \text{ atm} + \frac{F}{A} = 1 \text{ atm} + \frac{k \cdot h}{A},\ V_2 = V_1 + A \cdot h, \text{ and } T_2 = 250\ °\text{C} = 523 \text{ K}$$

continued on next page

Thus, $$\left(1+\frac{k\cdot h}{A(1\text{ atm})}\right)\left(1+\frac{A\cdot h}{V_1}\right)=\left(\frac{523\text{ K}}{293\text{ K}}\right)$$

or $$\left(1+\frac{\left(2.00\times10^3\text{ N/m}\right)\cdot h}{\left(0.010\,0\text{ m}^2\right)\left(1.013\times10^5\text{ N/m}^2\right)}\right)\left(1+\frac{\left(0.010\,0\text{ m}^2\right)\cdot h}{\left(5.00\times10^{-3}\text{ m}^3\right)}\right)=\left(\frac{523}{293}\right)$$

Simplifying and using the quadratic formula yields

$$h=0.169\text{ m}=\boxed{16.9\text{ cm}}$$

(b) $$P_2=1\text{ atm}+\frac{k\cdot h}{A}=1.013\times10^5\text{ Pa}+\frac{\left(2.00\times10^3\text{ N/m}\right)(0.169\text{ m})}{0.0100\text{ m}^2}=\boxed{1.35\times10^5\text{ Pa}}$$

10.61 (a) The two metallic strips have the same length L_0 at the initial temperature T_0. After the temperature has changed by $\Delta T=T-T_0$, the lengths of the two strips are

$$L_1=L_0\left[1+\alpha_1(\Delta T)\right]\qquad\text{and}\qquad L_2=L_0\left[1+\alpha_2(\Delta T)\right]$$

The lengths of the circular arcs are related to their radii by $L_1=r_1\theta$ and $L_2=r_2\theta$, where θ is measured in radians.

Thus, $$\Delta r=r_2-r_1=\frac{L_2}{\theta}-\frac{L_1}{\theta}=\frac{(\alpha_2-\alpha_1)L_0(\Delta T)}{\theta},\text{ or }\boxed{\theta=\frac{(\alpha_2-\alpha_1)L_0(\Delta T)}{\Delta r}}$$

(b) As seen in the above result, $\boxed{\theta=0\text{ if either }\Delta T=0\text{ or }\alpha_1=\alpha_2}$.

(c) If $\Delta T<0$, then θ is negative so $\boxed{\text{the bar bends in the opposite direction}}$.

10.62 (a) If the bridge were free to expand as the temperature increased by $\Delta T=20°\text{C}$, the increase in length would be

$$\Delta L=\alpha L_0(\Delta T)=\left(12\times10^{-6}\ °\text{C}^{-1}\right)(250\text{ m})(20°\text{C})=6.0\times10^{-2}\text{ m}=\boxed{6.0\text{ cm}}$$

(b) When the bridge is not allowed to expand naturally, stress builds up in the bridge, effectively compressing it the distance ΔL that it would normally have expanded. Combining the defining equation for Young's modulus,

$$Y=Stress/Strain=Stress/(\Delta L/L)$$

with the expression, $\Delta L=\alpha L(\Delta T)$, for the linear expansion when the temperature changes by ΔT yields

$$Stress=Y\left(\frac{\Delta L}{L}\right)=Y\left(\frac{\alpha L(\Delta T)}{L}\right)=\boxed{\alpha Y(\Delta T)}$$

(c) When $\Delta T=20°\text{C}$, the stress in the specified bridge would be

$$Stress=\alpha Y(\Delta T)=\left(12\times10^{-6}\ °\text{C}^{-1}\right)\left(2.0\times10^{10}\text{ Pa}\right)(20°\text{C})=\boxed{4.8\times10^6\text{ Pa}}$$

Since this is considerably less than the maximum stress, 2.0×10^7 Pa, that concrete can withstand, $\boxed{\text{the bridge will not crumble}}$.

10.63 (a) $\boxed{\text{Yes, the angular speed will increase as the disk cools.}}$ Since no external torque acts on the disk, the angular momentum of the disk will be conserved. As the disk cools, its radius, and hence its moment of inertia will decrease. Then, in order to keep angular momentum ($L = I\omega$) constant, the angular speed must increase.

(b) Since angular momentum is conserved, $I\omega = I_0\omega_0$ or $\omega = \left(I_0/I\right)\omega_0$. Thus,

$$\omega = \left(\frac{\frac{1}{2}MR_0^2}{\frac{1}{2}MR^2}\right)\omega_0 = \left(\frac{R_0}{R}\right)^2\omega_0 = \left(\frac{\cancel{R_0}}{\cancel{R_0}\left[1+\alpha\left(\Delta T\right)\right]}\right)^2\omega_0$$

or

$$\omega = \frac{\omega_0}{\left[1+\alpha\left(\Delta T\right)\right]^2} = \frac{25.0 \text{ rad/s}}{\left[1+\left(17\times10^{-6}\ {}^\circ\text{C}^{-1}\right)\left(20.0-850\right){}^\circ\text{C}\right]^2} = \boxed{25.7 \text{ rad/s}}$$

10.64 Let container 1 be maintained at $T_1 = T_0 = 0^\circ\text{C} = 273$ K, while the temperature of container 2 is raised to $T_2 = 100^\circ\text{C} = 373$ K. Both containers have the same constant volume, V, and the same initial pressures, $\left(P_0\right)_2 = \left(P_0\right)_1 = P_0$. As the temperature of container 2 is raised, gas flows from one container to the other until the final pressures are again equal, $P_2 = P_1 = P$. The total mass of gas is constant, so

$$n_2 + n_1 = \left(n_0\right)_2 + \left(n_0\right)_1 \qquad \textbf{[1]}$$

From the ideal gas law, $n = \dfrac{PV}{RT}$, so Equation [1] becomes

$$\frac{PV}{RT_2} + \frac{PV}{RT_1} = \frac{P_0V}{RT_0} + \frac{P_0V}{RT_0}, \qquad \text{or} \qquad P\left(\frac{1}{T_2} + \frac{1}{T_1}\right) = \frac{2P_0}{T_0}$$

Thus,

$$P = \frac{2P_0}{T_0}\left(\frac{T_1T_2}{T_1+T_2}\right) = \frac{2\left(1.00 \text{ atm}\right)}{\cancel{273}}\left(\frac{\cancel{273}\cdot 373}{273+373}\right) = \boxed{1.15 \text{ atm}}$$

11

Energy in Thermal Processes

QUICK QUIZZES

1. (a) Water, glass, iron. Because it has the highest specific heat $(4186\ \text{J/kg}\cdot{}^\circ\text{C})$, water has the smallest change in temperature. Glass is next $(837\ \text{J/kg}\cdot{}^\circ\text{C})$, and iron $(448\ \text{J/kg}\cdot{}^\circ\text{C})$ is last.
(b) Iron, glass, water. For a given temperature increase, the energy transfer by heat is proportional to the specific heat.

2. Choice (b). The slopes are proportional to the reciprocal of the specific heat, so larger specific heat results in a smaller slope, meaning more energy to achieve a given change in temperature.

3. Choice (c). The blanket acts as a thermal insulator, slowing the transfer of energy by heat from the air into the cube.

4. Choice (b). The rate of energy transfer by conduction through a rod is proportional to the difference in the temperatures of the ends of the rod. When the rods are in parallel, each rod experiences the full difference in the temperatures of the two regions. If the rods are connected in series, neither rod will experience the full temperature difference between the two regions, and hence neither will conduct energy as rapidly as it did in the parallel connection.

5. (a) $P_\text{A}/P_\text{B} = 4$. From Stefan's law, the power radiated from an object at absolute temperature T is proportional to the surface area of that object. Star A has twice the radius and four times the surface area of star B. (b) $P_\text{A}/P_\text{B} = 16$. From Stefan's law, the power radiated from an object having surface area A is proportional to the fourth power of the absolute temperature. Thus, $P_\text{A} = \sigma Ae\left(2T_\text{B}\right)^4 = 2^4\left(\sigma AeT_\text{B}^4\right) = 16P_\text{B}$. (c) $P_\text{A}/P_\text{B} = 64$. When star A has both twice the radius and twice the absolute temperature of star B, the ratio of the radiated powers is

$$\frac{P_\text{A}}{P_\text{B}} = \frac{\sigma A_\text{A} eT_\text{A}^4}{\sigma A_\text{B} eT_\text{B}^4} = \frac{\sigma\left(4\pi R_\text{A}^2\right)(1)T_\text{A}^4}{\sigma\left(4\pi R_\text{B}^2\right)(1)T_\text{B}^4} = \frac{\left(2R_\text{B}\right)^2\left(2T_\text{B}\right)^4}{R_\text{B}^2 T_\text{B}^4} = \left(2^2\right)\left(2^4\right) = 64$$

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. From the mechanical equivalent of heat, 1 cal = 4.186 J. Therefore,

$$3.50\times10^3\ \text{cal} = \left(3.50\times10^3\ \text{cal}\right)\left(\frac{4.186\ \text{J}}{1\ \text{cal}}\right) = 1.47\times10^4\ \text{J}$$

and (b) is the correct choice for this question.

2. From $Q = mc(\Delta T)$, we see that when equal-mass samples have equal amounts of energy transferred to them by heat, the expected temperature rise is inversely proportional to the specific heat:

$$\Delta T = \frac{(Q/m)}{c}$$

Thus, the alcohol sample, with a specific heat that is about one-half that of water, should experience a temperature rise that is approximately twice that of the water sample. The correct choice is (c).

3. The rate of energy transfer by conduction through a wall of area A and thickness L is $P = kA\left(T_h - T_c\right)/L$, where k is the thermal conductivity of the material making up the wall, while T_h and T_c are the temperatures on the hotter and cooler sides of the wall, respectively. For the case given, the transfer rate will be

$$P = \left(0.08\ \frac{\text{J}}{\text{s}\cdot\text{m}\cdot{}^\circ\text{C}}\right)\left(48.0\ \text{m}^2\right)\frac{\left(25^\circ\text{C} - 14^\circ\text{C}\right)}{\left(4.00\times 10^{-2}\ \text{m}\right)} = 1.1\times 10^3\ \text{J/s} = 1.1\times 10^3\ \text{W}$$

and (d) is the correct answer.

4. The energy which must be added to the 0°C ice to melt it, leaving liquid at 0°C, is

$$Q_1 = mL_f = \left(2.00\ \text{kg}\right)\left(3.33\times 10^5\ \text{J/kg}\right) = 6.66\times 10^5\ \text{J}$$

Once this is done, there is $Q_2 = Q_{\text{total}} - Q_1 = 9.30\times 10^5\ \text{J} - 6.66\times 10^5\ \text{J} = 2.64\times 10^5\ \text{J}$ of energy still available to raise the temperature of the liquid. The change in temperature this produces is

$$\Delta T = T_f - 0^\circ\text{C} = \frac{Q_2}{mc_{\text{water}}} = \frac{2.64\times 10^5\ \text{J}}{\left(2.00\ \text{kg}\right)\left(4\,186\ \text{J/kg}\cdot{}^\circ\text{C}\right)} = 31.5^\circ\text{C}$$

so the final temperature is $T_f = 0^\circ\text{C} + 31.5^\circ\text{C} = 31.5^\circ\text{C}$ and the correct choice is (c).

5. The required energy input is

$$Q = mc\left(\Delta T\right) = \left(5.00\ \text{kg}\right)\left(128\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(327^\circ\text{C} - 20.0^\circ\text{C}\right) = 1.96\times 10^5\ \text{J}$$

and the correct response is (e).

6. The power radiated by an object with emissivity e, surface area A, and absolute temperature T, in a location with absolute ambient temperature T_0, is given by $P = \sigma Ae\left(T^4 - T_0^4\right)$, where $\sigma = 5.669\,6\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4$ is a constant. Thus, for the given spherical object $\left(A = 4\pi r^2\right)$ with $T = 273 + 135 = 408\ \text{K}$ and $T_0 = 273 + 25 = 298\ \text{K}$, we have

$$P = \left(5.669\,6\times 10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)4\pi\left(2.00\ \text{m}\right)^2\left(0.450\right)\left[\left(408\ \text{K}\right)^4 - \left(298\ \text{K}\right)^4\right]$$

yielding $P = 2.54\times 10^4\ \text{W}$, so (e) is the correct choice.

7. The temperature of the ice must be raised to the melting point, $\Delta T = +20.0^\circ\text{C}$, before it will start to melt. The total energy input required to melt the 2.00-kg of ice is

$$Q = mc\left(\Delta T\right) + mL_f = \left(2.00\ \text{kg}\right)\left[\left(2\,090\ \text{J/kg}\cdot{}^\circ\text{C}\right)\left(20.0^\circ\text{C}\right) + 3.33\times 10^5\ \text{J/kg}\right] = 7.50\times 10^5\ \text{J}$$

The time the heating element will need to supply this quantity of energy is

$$\Delta t = \frac{Q}{P} = \frac{7.50\times 10^5\ \text{J}}{1.00\times 10^3\ \text{J/s}} = 750\ \text{s}\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = 12.5\ \text{min}$$

making (d) the correct choice.

8. If energy transfer between the environment and the contents of the calorimeter cannot be avoided, one would like the initial temperature to be such that the contents of the calorimeter would gain as much energy from the environment in one part of the process as it loses to the environment in another part of the process. Thus, we would like (after a few trial runs) to choose an initial temperature such that room temperature will be about halfway between the initial and final temperatures of the calorimeter contents. The best response is therefore choice (a).

9. Since less energy was required to produce a 5°C rise in the temperature of the ice than was required to produce a 5°C rise in temperature of an equal mass of water, we conclude that the specific heat of ice $\left(c = \dfrac{Q}{m(\Delta T)}\right)$ is less than that of water. Thus, choice (c) is correct.

10. One would like the poker to be capable of absorbing a large amount of energy, but undergo a small rise in temperature. This means it should be made of a material with a high specific heat capacity. Also, it is desirable that energy absorbed by the end of the poker in the fire be conducted to the person holding the other end very slowly. Thus, the material should have a low thermal conductivity. The correct choice is (d).

11. With $e_A = e_B$, $r_A = 2r_B$, and $T_A = 2T_B$, the ratio of the power output of A to that of B is

$$\frac{P_A}{P_B} = \frac{\cancel{\sigma} A_A \cancel{e_A} T_A^4}{\cancel{\sigma} A_B \cancel{e_B} T_B^4} = \frac{\cancel{4\pi} r_A^2 T_A^4}{\cancel{4\pi} r_B^2 T_B^4} = \left(\frac{r_A}{r_B}\right)^2 \left(\frac{T_A}{T_B}\right)^4 = (2)^2 (2)^4 = (2)^6 = 64$$

making (e) the correct choice.

12. By agitating the coffee inside this sealed, insulated container, the person is raising the internal energy of the coffee, which will result is a rise in the temperature of the coffee. However, doing this for only a few minutes, the temperature rise will be quite small. The correct response to this question is (d).

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The high thermal capacity of the barrel of water and its high heat of fusion mean that a large amount of energy would have to leak out of the cellar before the water and produce froze solid. Also, evaporation of the water keeps the relative humidity high to protect foodstuffs from drying out.

4. (a) Yes, wrap the blanket around the ice chest. The environment is warmer than the ice, so the low thermal conductivity of the blanket slows energy transfer by heat from the environment to the ice.

(b) Explain to your little sister that her body is warmer than the environment and requires energy transfer by heat into the air to remain at a fixed temperature. The blanket will slow this conduction and cause her to feel warmer, not cool like the ice.

6. Yes, if you know the specific heat of zinc and copper, you can determine the relative fraction of each by heating a known weight of pennies to a specific initial temperature, say 100°C, then dump them into a known quantity of water, at say 20°C. The equation for conservation of energy will be

$$m_{\text{pennies}}\left[x \cdot c_{\text{Cu}} + (1-x)c_{\text{Zn}}\right](100°\text{C} - T) = m_{\text{water}} c_{\text{water}} (T - 20°\text{C})$$

The equilibrium temperature, T, and the masses will be measured. The specific heats are known, so the fraction of metal that is copper, x, can be computed.

8. Write $m_{\text{water}}c_{\text{water}}(1°\text{C}) = (\rho_{\text{air}}V)c_{\text{air}}(1°\text{C})$, to find

$$V = \frac{m_{\text{water}}c_{\text{water}}}{\rho_{\text{air}}c_{\text{air}}} = \frac{(1.0\times10^3\ \text{kg})(4\,186\ \text{J/kg}\cdot°\text{C})}{(1.3\ \text{kg/m}^3)(1.0\times10^3\ \text{J/kg}\cdot°\text{C})} = 3.2\times10^3\ \text{m}^3$$

10. The black car absorbs more of the incoming energy from the Sun than does the white car, making it more likely to cook the egg.

12. Keep them dry. The air pockets in the pad conduct energy slowly. Wet pads absorb some energy in warming up themselves, but the pot would still be hot and the water would quickly conduct a lot of energy to your hand.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. $0.234\ \text{kJ/kg}\cdot°\text{C}$

4. 0.15 mm

6. **(a)** $2.3\times10^6\ \text{J}$ **(b)** 2.8×10^4 stairs **(c)** 7.0×10^3 stairs

8. **(a)** $P = Fv$ **(b)** $P = ma^2t$ **(c)** $2.20\ \text{m/s}^2$

(d) $P = (363\ \text{W/s})\cdot t$ **(e)** $1.74\ \text{Cal/s}$

10. 0.105°C

12. **(a)** $9.9\times10^{-3}\ °\text{C}$ **(b)** It is absorbed by the rough horizontal surface.

14. **(a)** $Stress = \dfrac{F}{A} = Y[\alpha(\Delta T)]$ **(b)** $Q = \dfrac{mc}{Y\alpha}\left(\dfrac{F}{A}\right)$ **(c)** 96.0 kg

(d) $6.7\times10^6\ \text{J}$ **(e)** 79°C **(f)** ~ 4 h

16. 467

18. The copper bullet wins: $T_{\text{copper}} = 89.7°\text{C}$, $T_{\text{silver}} = 89.8°\text{C}$

20. 29.6°C

22. 47°C

24. $1.18\times10^3\ \text{J/kg}\cdot°\text{C}$

26. 49 kJ

28. 0.12 MJ

30. **(a)** ice at –10.0°C to ice at 0°C; ice at 0°C to liquid water at 0°C; water at 0°C to water at T; aluminum at 20.0°C to aluminum at T; ethyl alcohol at 30.0°C to ethyl alcohol at T.

(b) See Solution.

(c) $m_{\text{ice}}c_{\text{ice}}(10.0°\text{C}) + m_{\text{ice}}L_f + m_{\text{ice}}c_{\text{water}}(T-0) + m_{\text{Al}}c_{\text{Al}}[T-20.0°\text{C}] + m_{\text{alc}}c_{\text{alc}}[T-30.0°\text{C}] = 0$

(d) 4.81°C

32. 0.33 kg, 0.067 or 6.7%

34. 403 cm^3

36. 11.1 W

38. **(a)** 6×10^3 W **(b)** 5×10^8 J

40. **(a)** $R_{\text{skin}} = 5.0\times10^{-2}\ \text{m}^2\cdot\text{K/W}$, $R_{\text{fat}} = 2.5\times10^{-2}\ \text{m}^2\cdot\text{K/W}$, $R_{\text{tissue}} = 6.4\times10^{-2}\ \text{m}^2\cdot\text{K/W}$, $R_{\text{total}} = 0.14\ \text{m}^2\cdot\text{K/W}$

(b) 5.3×10^2 W

42. 39 m^3

44. **(a)** 52 W **(b)** 2 kW

46. $7.3\times10^{-2}\ \text{W/m}\cdot°\text{C}$

48. 330 K

50. $1.1\times10^{-5}\ \text{m}^2$

52. 1.83 h

54. **(a)** 1.1×10^2 W

(b) The positive sign indicates that the body is radiating energy away faster than it absorbs energy from the environment.

56. 1.8 kg

58. **(a)** 1.6×10^2 W **(b)** 2.7×10^2 W **(c)** 11 W

(d) 1.2×10^2 W

60. 45°C

62. **(a)** 2.0 kW **(b)** 4.5°C

64. 28°C

66. **(a)** 2.03×10^3 J/s **(b)** $7.84\ \text{ft}^2\cdot°\text{F}\cdot\text{h/Btu}$

68. **(a)** 0.457 kg

(b) If the samples and inner surface of the insulation are preheated, nothing undergoes a temperature change during the test. Therefore, only the mass of the wax, which undergoes a change of phase, needs to be known.

70. 0.9 kg

72. **(a)** The processes involved are the removal of energy to: (1) cool liquid water from 20.0 °C to 0°C, (2) convert liquid water at 0°C to solid water (ice) at 0°C, and (3) cool ice from 0°C to −8.00°C.

(b) 32.5 kJ

PROBLEM SOLUTIONS

11.1 As mass m of water drops from the top to the bottom of the falls, the gravitational potential energy given up (and hence, the kinetic energy gained) is $Q = mgh$. If all of this goes into raising the temperature, the rise in temperature will be

$$\Delta T = \frac{Q}{mc_{\text{water}}} = \frac{\cancel{m}gh}{\cancel{m}c_{\text{water}}} = \frac{\left(9.80\ \text{m/s}^2\right)(807\ \text{m})}{4\,186\ \text{J/kg}\cdot{}^\circ\text{C}} = 1.89^\circ\text{C}$$

and the final temperature is $T_f = T_i + \Delta T = 15.0^\circ\text{C} + 1.89^\circ\text{C} = \boxed{16.9^\circ\text{C}}$

11.2 $$c = \frac{Q}{m(\Delta T)} = \frac{1.23\times10^3\ \text{J}}{(0.525\ \text{kg})(10.0^\circ\text{C})} = 234\ \text{J/kg}\cdot{}^\circ\text{C} = \boxed{0.234\ \text{kJ/kg}\cdot{}^\circ\text{C}}$$

11.3 The mass of water involved is

$$m = \rho V = \left(10^3\ \frac{\text{kg}}{\text{m}^3}\right)\left(4.00\times10^{11}\ \text{m}^3\right) = 4.00\times10^{14}\ \text{kg}$$

(a) $Q = mc(\Delta T) = \left(4.00\times10^{14}\ \text{kg}\right)\left(4\,186\ \text{J/kg}\cdot{}^\circ\text{C}\right)(1.00^\circ\text{C}) = \boxed{1.67\times10^{18}\ \text{J}}$

(b) The power input is $P = 1\,000\ \text{MW} = 1.00\times10^9\ \text{J/s}$,

so $$t = \frac{Q}{P} = \frac{1.67\times10^{18}\ \text{J}}{1.00\times10^9\ \text{J/s}}\left(\frac{1\ \text{yr}}{3.156\times10^7\ \text{s}}\right) = \boxed{52.9\ \text{yr}}$$

11.4 The change in temperature of the rod is

$$\Delta T = \frac{Q}{mc} = \frac{1.00\times10^4\ \text{J}}{(0.350\ \text{kg})(900\ \text{J/kg}^\circ\text{C})} = 31.7^\circ\text{C}$$

and the change in the length is

$$\Delta L = \alpha L_0(\Delta T)$$
$$= \left[24\times10^{-6}\,(^\circ\text{C})^{-1}\right](20.0\ \text{cm})(31.7^\circ\text{C}) = 1.5\times10^{-2}\ \text{cm} = \boxed{0.15\ \text{mm}}$$

11.5 (a) $Q = 0.600\left|\Delta PE_g\right| = 0.600\left(mg|h|\right) = 0.600\cdot m\left(9.80\ \text{m/s}^2\right)(50.0\ \text{m})$

or $Q = \left(294\ \text{m}^2/\text{s}^2\right)\cdot m$

continued on next page

From $Q = mc(\Delta T) = mc(T_f - T_i)$, we find the final temperature as

$$T_f = T_i + \frac{Q}{mc} = 25.0°\text{C} + \frac{(294\ \text{m}^2/\text{s}^2)\cdot \cancel{m}}{\cancel{m}(387\ \text{J/kg}\cdot°\text{C})} = \boxed{25.8°\text{C}}$$

(b) Observe that the mass of the coin cancels out in the calculation of part (a). Hence, the $\boxed{\text{result is independent of the mass of the coin}}$.

11.6 (a) $Q = 540\ \cancel{\text{Cal}}\left(\frac{10^3\ \cancel{\text{cal}}}{1\ \cancel{\text{Cal}}}\right)\left(\frac{4.186\ \text{J}}{1\ \cancel{\text{cal}}}\right) = \boxed{2.3\times10^6\ \text{J}}$

(b) The work done lifting her weight mg up one stair of height h is $W_1 = mgh$. Thus, the total work done in climbing N stairs is $W = Nmgh$, and we have $W = Nmgh = Q$ or

$$N = \frac{Q}{mgh} = \frac{2.3\times10^6\ \text{J}}{(55\ \text{kg})(9.80\ \text{m/s}^2)(0.15\ \text{m})} = \boxed{2.8\times10^4\ \text{stairs}}$$

(c) If only 25% of the energy from the donut goes into mechanical energy, we have

$$N = \frac{0.25Q}{mgh} = 0.25\left(\frac{Q}{mgh}\right) = 0.25(2.8\times10^4\ \text{stairs}) = \boxed{7.0\times10^3\ \text{stairs}}$$

11.7 (a) $W_{\text{net}} = \Delta KE = \frac{1}{2}m(v_f^2 - v_0^2) = \frac{1}{2}(75\ \text{kg})\left[(11.0\ \text{m/s})^2 - 0\right] = 4.54\times10^3\ \text{J} \rightarrow \boxed{4.5\times10^3\ \text{J}}$

(b) $\overline{P} = \frac{W_{\text{net}}}{\Delta t} = \frac{4.54\times10^3\ \text{J}}{5.0\ \text{s}} = 9.1\times10^2\ \text{J/s} = \boxed{910\ \text{W}}$

(c) If the mechanical energy is 25% of the energy gained from converting food energy, then $W_{\text{net}} = 0.25(\Delta Q)$ and $\overline{P} = 0.25(\Delta Q)/\Delta t$, so the food energy conversion rate is

$$\frac{\Delta Q}{\Delta t} = \frac{\overline{P}}{0.25} = \left(\frac{910\ \text{J/s}}{0.25}\right)\left(\frac{1\ \text{Cal}}{4\,186\ \text{J}}\right) = \boxed{0.87\ \text{Cal/s}}$$

(d) The excess thermal energy is transported by conduction and convection to the surface of the skin and disposed of through the evaporation of perspiration.

11.8 (a) The instantaneous power is $\boxed{P = Fv}$, where F is the applied force and v is the instantaneous velocity.

(b) From Newton's second law, $F_{\text{net}} = ma$, and the kinematics equation $v = v_0 + at$ with $v_0 = 0$, the instantaneous power expression given above may be written as

$$P = Fv = (ma)(0 + at) \qquad \text{or} \qquad \boxed{P = ma^2t}$$

(c) $a = \frac{\Delta v}{\Delta t} = \frac{v - 0}{t - 0} = \frac{11.0\ \text{m/s}}{5.00\ \text{s}} = \boxed{2.20\ \text{m/s}^2}$

continued on next page

(d) $P = ma^2t = (75.0\text{ kg})(2.20\text{ m/s}^2)^2 t = (363\text{ kg}\cdot\text{m}^2/\text{s}^4)\cdot t = \boxed{(363\text{ W/s})\cdot t}$

(e) Maximum instantaneous power occurs when $t = t_{\text{max}} = 5.00$ s, so

$$P_{\text{max}} = (363\text{ J/s}^2)(5.00\text{ s}) = 1.82\times10^3\text{ J/s}$$

If this corresponds to 25.0% of the rate of using food energy, that rate must be

$$\frac{\Delta Q}{\Delta t} = \frac{P_{\text{max}}}{0.250} = \frac{1.82\times10^3\text{ J/s}}{0.250}\left(\frac{1\text{ Cal}}{4\,186\text{ J}}\right) = \boxed{1.74\text{ Cal/s}}$$

11.9 The mechanical energy transformed into internal energy of the bullet is $Q = \frac{1}{2}(KE_i) = \frac{1}{2}\left(\frac{1}{2}mv_i^2\right) = \frac{1}{4}mv_i^2$. Thus, the change in temperature of the bullet is

$$\Delta T = \frac{Q}{mc} = \frac{\frac{1}{4}\cancel{m}v_i^2}{\cancel{m}c_{\text{lead}}} = \frac{(300\text{ m/s})^2}{4(128\text{ J/kg}\cdot{}^\circ\text{C})} = \boxed{176^\circ\text{C}}$$

11.10 The internal energy added to the system equals the gravitational potential energy given up by the 2 falling blocks, or $Q = \Delta PE_g = 2m_b gh$. Thus,

$$\Delta T = \frac{Q}{m_w c_w} = \frac{2m_b gh}{m_w c_w} = \frac{2(1.50\text{ kg})(9.80\text{ m/s}^2)(3.00\text{ m})}{(0.200\text{ kg})(4\,186\text{ J/kg}\cdot{}^\circ\text{C})} = \boxed{0.105^\circ\text{C}}$$

11.11 The quantity of energy transferred from the water-cup combination in a time interval of 1 minute is

$$Q = \left[(mc)_{\text{water}} + (mc)_{\text{cup}}\right](\Delta T)$$

$$= \left[(0.800\text{ kg})\left(4\,186\ \frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right) + (0.200\text{ kg})\left(900\ \frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)\right](1.5^\circ\text{C}) = 5.3\times10^3\text{ J}$$

The rate of energy transfer is

$$P = \frac{Q}{\Delta t} = \frac{5.3\times10^3\text{ J}}{60\text{ s}} = 88\ \frac{\text{J}}{\text{s}} = \boxed{88\text{ W}}$$

11.12 (a) The mechanical energy converted into internal energy of the block is $Q = 0.85(KE_i) = 0.85\left(\frac{1}{2}mv_i^2\right)$. The change in temperature of the block will be

$$\Delta T = \frac{Q}{mc_{\text{Cu}}} = \frac{0.85\left(\frac{1}{2}\cancel{m}v_i^2\right)}{\cancel{m}c_{\text{Cu}}} = \frac{0.85(3.0\text{ m/s})^2}{2(387\text{ J/kg}\cdot{}^\circ\text{C})} = \boxed{9.9\times10^{-3}\,{}^\circ\text{C}}$$

(b) The remaining energy is $\boxed{\text{absorbed by the horizontal surface}}$ on which the block slides.

11.13 From $\Delta L = \alpha L_0(\Delta T)$, the required increase in temperature is found, using Table 10.1, as

$$\Delta T = \frac{\Delta L}{\alpha_{\text{steel}}L_0} = \frac{3.0\times10^{-3}\ \cancel{\text{m}}}{\left(11\times10^{-6}\ ({}^\circ\text{C})^{-1}\right)(13\ \cancel{\text{yd}})}\left(\frac{1\ \cancel{\text{yd}}}{3.0\ \cancel{\text{ft}}}\right)\left(\frac{3.281\ \cancel{\text{ft}}}{1\ \cancel{\text{m}}}\right) = 23^\circ\text{C}$$

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The mass of the rail is

$$m = \frac{w}{g} = \frac{(70\ \cancel{\text{lb}}/\cancel{\text{yd}})(13\ \cancel{\text{yd}})}{9.80\ \text{m/s}^2}\left(\frac{4.448\ \text{N}}{1\ \cancel{\text{lb}}}\right) = 4.1\times10^2\ \text{kg}$$

so the required thermal energy (assuming that $c_{\text{steel}} = c_{\text{iron}}$) is

$$Q = mc_{\text{steel}}(\Delta T) = (4.1\times10^2\ \text{kg})(448\ \text{J/kg}\cdot{}^\circ\text{C})(23^\circ\text{C}) = \boxed{4.2\times10^6\ \text{J}}$$

11.14 (a) From the relation between compressive stress and strain, $F/A = Y(\Delta L/L_0)$, where Y is Young's modulus of the material. From the discussion on linear expansion, the strain due to thermal expansion can be written as $(\Delta L/L_0) = \alpha(\Delta T)$, where α is the coefficient of linear expansion. Thus, the stress becomes $\boxed{F/A = Y[\alpha(\Delta T)]}$.

(b) If the concrete slab has mass m, the thermal energy required to produce a change in temperature ΔT is $Q = mc(\Delta T)$, where c is the specific heat of concrete. Using the result from part (a), the absorbed thermal energy required to produce compressive stress F/A is

$$Q = mc\left(\frac{F/A}{Y\alpha}\right) \qquad \text{or} \qquad \boxed{Q = \frac{mc}{Y\alpha}\left(\frac{F}{A}\right)}$$

(c) The mass of the given concrete slab is

$$m = \rho V = (2.40\times10^3\ \text{kg/m}^3)\left[(4.00\times10^{-2}\ \text{m})(1.00\ \text{m})(1.00\ \text{m})\right] = \boxed{96.0\ \text{kg}}$$

(d) If the maximum compressive stress concrete can withstand is $F/A = 2.00\times10^7\ \text{Pa}$, the maximum thermal energy this slab can absorb before starting to break up is found, using Table 10.1, to be

$$Q_{\max} = \frac{mc}{Y\alpha}\left(\frac{F}{A}\right)_{\max} = \frac{(96.0\ \text{kg})(880\ \text{J/kg}\cdot{}^\circ\text{C})}{(2.1\times10^{10}\ \text{Pa})(12\times10^{-6}\ ({}^\circ\text{C})^{-1})}(2.00\times10^7\ \text{Pa}) = \boxed{6.7\times10^6\ \text{J}}$$

(e) The change in temperature of the slab as it absorbs the thermal energy computed above is

$$\Delta T = \frac{Q}{mc} = \frac{6.7\times10^6\ \text{J}}{(96.0\ \text{kg})(880\ \text{J/kg}\cdot{}^\circ\text{C})} = \boxed{79^\circ\text{C}}$$

(f) The rate the slab absorbs solar energy is

$$P_{\text{absorbed}} = 0.5P_{\text{solar}} = 0.5(1.00\times10^3\ \text{W}) = 5\times10^2\ \text{J/s}$$

so the time required to absorb the thermal energy computed in (d) above is

$$t = \frac{Q}{P_{\text{absorbed}}} = \frac{6.7\times10^6\ \text{J}}{5\times10^2\ \text{J/s}}\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) \sim \boxed{4\ \text{h}}$$

11.15 When thermal equilibrium is reached, the water and aluminum will have a common temperature of $T_f = 65.0^\circ\text{C}$. Assuming that the water-aluminum system is thermally isolated from the environment, $Q_{\text{cold}} = -Q_{\text{hot}}$, so $m_{\text{w}}c_{\text{w}}(T_f - T_{i,\text{w}}) = -m_{\text{Al}}c_{\text{Al}}(T_f - T_{i,\text{Al}})$, or

$$m_{\text{w}} = \frac{-m_{\text{Al}}c_{\text{Al}}(T_f - T_{i,\text{Al}})}{c_{\text{w}}(T_f - T_{i,\text{w}})} = \frac{-(1.85\ \text{kg})(900\ \text{J/kg}\cdot{}^\circ\text{C})(65.0^\circ\text{C} - 150^\circ\text{C})}{(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(65.0^\circ\text{C} - 25.0^\circ\text{C})} = \boxed{0.845\ \text{kg}}$$

11.16 If N pellets are used, the mass of the lead is Nm_{pellet}. Since the energy lost by the lead must equal the energy absorbed by the water,

$$\left|Nm_{\text{pellet}}c(\Delta T)\right|_{\text{lead}} = \left[mc(\Delta T)\right]_{\text{water}}$$

or the number of pellets required is

$$N = \frac{m_{\text{w}}c_{\text{w}}(\Delta T)_{\text{w}}}{m_{\text{pellet}}c_{\text{lead}}\left|\Delta T\right|_{\text{lead}}}$$

$$= \frac{(0.500\ \text{kg})(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(25.0^\circ\text{C} - 20.0^\circ\text{C})}{\left(1.00\times10^{-3}\ \text{kg}\right)(128\ \text{J/kg}\cdot{}^\circ\text{C})(200^\circ\text{C} - 25.0^\circ\text{C})} = \boxed{467}$$

11.17 The total energy absorbed by the cup, stirrer, and water equals the energy given up by the silver sample. Thus,

$$\left[m_{\text{c}}c_{\text{Al}} + m_{\text{s}}c_{\text{Cu}} + m_{\text{w}}c_{\text{w}}\right](\Delta T)_{\text{w}} = \left[mc\left|\Delta T\right|\right]_{\text{Ag}}$$

Solving for the mass of the cup gives

$$m_{\text{c}} = \frac{1}{c_{\text{Al}}}\left[\left(m_{\text{Ag}}c_{\text{Ag}}\right)\frac{\left|\Delta T\right|_{\text{Ag}}}{(\Delta T)_{\text{w}}} - m_{\text{s}}c_{\text{Cu}} - m_{\text{w}}c_{\text{w}}\right],$$

or $$m_{\text{c}} = \frac{1}{900}\left[(400\ \text{g})(234)\frac{(87-32)}{(32-27)} - (40\ \text{g})(387) - (225\ \text{g})(4\,186)\right] = \boxed{80\ \text{g}}$$

11.18 The mass of water is

$$m_{\text{w}} = \rho_{\text{w}}V_{\text{w}} = \left(1.00\ \text{g/cm}^3\right)\left(100\ \text{cm}^3\right) = 100\ \text{g} = 0.100\ \text{kg}$$

For each bullet, the energy absorbed by the bullet equals the energy given up by the water, so $m_{\text{b}}c_{\text{b}}(T - 20.0^\circ\text{C}) = m_{\text{w}}c_{\text{w}}(90.0^\circ\text{C} - T)$. Solving for the final temperature gives

$$T = \frac{m_{\text{w}}c_{\text{w}}(90.0^\circ\text{C}) + m_{\text{b}}c_{\text{b}}(20.0^\circ\text{C})}{m_{\text{w}}c_{\text{w}} + m_{\text{b}}c_{\text{b}}}$$

For the silver bullet, $m_{\text{b}} = 5.00\times10^{-3}\ \text{kg}$ and $c_{\text{b}} = 234\ \text{J/kg}\cdot{}^\circ\text{C}$, giving

$$T_{\text{silver}} = \frac{(0.100)(4\,186)(90.0^\circ\text{C}) + \left(5.00\times10^{-3}\right)(234)(20.0^\circ\text{C})}{(0.100)(4\,186) + \left(5.00\times10^{-3}\right)(234)} = \boxed{89.8^\circ\text{C}}$$

For the copper bullet, $m_{\text{b}} = 5.00\times10^{-3}\ \text{kg}$ and $c_{\text{b}} = 387\ \text{J/kg}\cdot{}^\circ\text{C}$, which yields

$$T_{\text{copper}} = \frac{(0.100)(4\,186)(90.0^\circ\text{C}) + \left(5.00\times10^{-3}\right)(387)(20.0^\circ\text{C})}{(0.100)(4\,186) + \left(5.00\times10^{-3}\right)(387)} = \boxed{89.7^\circ\text{C}}$$

Thus, $\boxed{\text{the copper bullet wins}}$ the showdown of the water cups.

11.19 (a) The total energy given up by the copper and the unknown sample equals the total energy absorbed by the calorimeter and water. Hence,

$$m_{\text{Cu}}c_{\text{Cu}}\left|\Delta T\right|_{\text{Cu}} + m_{\text{unk}}c_{\text{unk}}\left|\Delta T\right|_{\text{unk}} = \left[m_{\text{c}}c_{\text{Al}} + m_{\text{w}}c_{\text{w}}\right](\Delta T)_{\text{w}}$$

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Solving for the specific heat of the unknown material gives

$$c_{\text{unk}} = \frac{\left[m_c c_{\text{Al}} + m_w c_w\right](\Delta T)_w - m_{\text{Cu}} c_{\text{Cu}} |\Delta T|_{\text{Cu}}}{m_{\text{unk}} |\Delta T|_{\text{unk}}}, \text{ or}$$

$$c_{\text{unk}} = \frac{1}{(70.0\text{ g})(80.0°\text{C})}\left\{\left[(100\text{ g})(900\text{ J/kg}\cdot°\text{C}) + (250\text{ g})(4\,186\text{ J/kg}\cdot°\text{C})\right](10.0°\text{C})\right.$$

$$\left. - (50.0\text{ g})(387\text{ J/kg}\cdot°\text{C})(60.0°\text{C})\right\} = \boxed{1.82\times 10^3\text{ J/kg}\cdot°\text{C}}$$

(b) $\boxed{\text{The unknown could be beryllium,}}$ but other possibilities also exist.

(c) $\boxed{\text{The material could be an unknown alloy or a material not listed in the table.}}$

11.20 $Q_{\text{cold}} = -Q_{\text{hot}} \quad\Rightarrow\quad m_w c_w \left(T_f - T_{i,w}\right) = -m_{\text{Fe}} c_{\text{Fe}} \left(T_f - T_{i,\text{Fe}}\right)$

or $$T_f = \frac{m_w c_w T_{i,w} + m_{\text{Fe}} c_{\text{Fe}} T_{i,\text{Fe}}}{m_w c_w + m_{\text{Fe}} c_{\text{Fe}}}$$

$$= \frac{(20.0\text{ kg})(4\,186\text{ J/kg}\cdot°\text{C})(25.0°\text{C}) + (1.50\text{ kg})(448\text{ J/kg}\cdot°\text{C})(600°\text{C})}{(20.0\text{ kg})(4\,186\text{ J/kg}\cdot°\text{C}) + (1.50\text{ kg})(448\text{ J/kg}\cdot°\text{C})}$$

and $T_f = \boxed{29.6°\text{C}}$

11.21 Since the temperature of the water and the steel container is unchanged, and neither substance undergoes a phase change, the internal energy of these materials is constant. Thus, all the energy given up by the copper is absorbed by the aluminum, giving $m_{\text{Al}} c_{\text{Al}} (\Delta T)_{\text{Al}} = m_{\text{Cu}} c_{\text{Cu}} |\Delta T|_{\text{Cu}}$, or

$$m_{\text{Al}} = \left(\frac{c_{\text{Cu}}}{c_{\text{Al}}}\right)\left[\frac{|\Delta T|_{\text{Cu}}}{(\Delta T)_{\text{Al}}}\right] m_{\text{Cu}}$$

$$= \left(\frac{387}{900}\right)\left(\frac{85°\text{C} - 25°\text{C}}{25°\text{C} - 5.0°\text{C}}\right)(200\text{ g}) = 2.6\times 10^2\text{ g} = \boxed{0.26\text{ kg}}$$

11.22 The kinetic energy given up by the car is absorbed as internal energy by the four brake drums (a total mass of 32 kg of iron). Thus, $\Delta KE = Q = m_{\text{drums}} c_{\text{Fe}} (\Delta T)$, or

$$\Delta T = \frac{\frac{1}{2} m_{\text{car}} v_i^2}{m_{\text{drums}} c_{\text{Fe}}} = \frac{\frac{1}{2}(1\,500\text{ kg})(30\text{ m/s})^2}{(32\text{ kg})(448\text{ J/kg}\cdot°\text{C})} = \boxed{47°\text{C}}$$

11.23 (a) Assuming that the tin-lead-water mixture is thermally isolated from the environment, we have

$$Q_{\text{cold}} = -Q_{\text{hot}} \qquad \text{or} \qquad m_w c_w \left(T_f - T_{i,w}\right) = -m_{\text{Sn}} c_{\text{Sn}} \left(T_f - T_{i,\text{Sn}}\right) - m_{\text{Pb}} c_{\text{Pb}} \left(T_f - T_{i,\text{Pb}}\right)$$

and since $m_{\text{Sn}} = m_{\text{Pb}} = m_{\text{metal}} = 0.400\text{ kg}$ and $T_{i,\text{Sn}} = T_{i,\text{Pb}} = T_{\text{hot}} = 60.0°\text{C}$, this yields

$$T_f = \frac{m_w c_w T_{i,w} + m_{\text{metal}}\left(c_{\text{Sn}} + c_{\text{Pb}}\right) T_{\text{hot}}}{m_w c_w + m_{\text{metal}}\left(c_{\text{Sn}} + c_{\text{Pb}}\right)}$$

$$= \frac{(1.00\text{ kg})(4\,186\text{ J/kg}\cdot°\text{C})(20.0°\text{C}) + (0.400\text{ kg})(227\text{ J/kg}\cdot°\text{C} + 128\text{ J/kg}\cdot°\text{C})(60.0°\text{C})}{(1.00\text{ kg})(4\,186\text{ J/kg}\cdot°\text{C}) + (0.400\text{ kg})(227\text{ J/kg}\cdot°\text{C} + 128\text{ J/kg}\cdot°\text{C})}$$

yielding $\boxed{T_f = 21.3°\text{C}}$

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(b) If an alloy containing a mass m_{Sn} of tin and a mass m_{Pb} of lead undergoes a rise in temperature ΔT, the thermal energy absorbed would be $Q = Q_{Sn} + Q_{Pb}$, or

$$(m_{Sn} + m_{Pb})c_{alloy}\left(\cancel{\Delta T}\right) = m_{Sn}c_{Sn}\left(\cancel{\Delta T}\right) + m_{Pb}c_{Pb}\left(\cancel{\Delta T}\right), \text{ giving } \quad c_{alloy} = \frac{m_{Sn}c_{Sn} + m_{Pb}c_{Pb}}{m_{Sn} + m_{Pb}}$$

If the alloy is a half-and-half mixture, $m_{Sn} = m_{Pb}$, then this reduces to $c_{alloy} = \dfrac{c_{Sn} + c_{Pb}}{2}$

and yields $\quad c_{alloy} = \dfrac{227\ \text{J/kg}\cdot°\text{C} + 128\ \text{J/kg}\cdot°\text{C}}{2} = \boxed{178\ \text{J/kg}\cdot°\text{C}}$

(c) For a substance forming monatomic molecules, the number of atoms in a mass equal to the molecular weight of that material is Avogadro's number, N_A. Thus, the number of tin atoms in $m_{Sn} = 0.400\ \text{kg} = 400\ \text{g}$ of tin with a molecular weight of $M_{Sn} = 118.7\ \text{g/mol}$ is

$$N_{Sn} = \left(\frac{m_{Sn}}{M_{Sn}}\right)N_A = \left(\frac{400\ \text{g}}{118.7\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{mol}^{-1}\right) = \boxed{2.03\times10^{24}}$$

and, for the lead, $N_{Pb} = \left(\dfrac{m_{Pb}}{M_{Pb}}\right)N_A = \left(\dfrac{400\ \text{g}}{207.2\ \text{g/mol}}\right)\left(6.02\times10^{23}\ \text{mol}^{-1}\right) = \boxed{1.16\times10^{24}}$

(d) We have $\quad \dfrac{N_{Sn}}{N_{Pb}} = \dfrac{2.03\times10^{24}}{1.16\times10^{24}} = \boxed{1.75}$

and observe that $\quad \dfrac{c_{Sn}}{c_{Pb}} = \dfrac{227\ \text{J/kg}\cdot°\text{C}}{128\ \text{J/kg}\cdot°\text{C}} = \boxed{1.77}$

from which we conclude that the specific heat of an element is proportional to the number of atoms per unit mass of that element.

11.24 Assuming that the unknown-water-calorimeter system is thermally isolated from the environment, $-Q_{hot} = Q_{cold}$, or $-m_x c_x\left(T_f - T_{i,x}\right) = m_w c_w\left(T_f - T_{i,w}\right) + m_{Al}c_{Al}\left(T_f - T_{i,Al}\right)$ and, since $T_{i,w} = T_{i,Al} = T_{cold} = 25.0°\text{C}$, we have

$$c_x = \left(m_w c_w + m_{Al}c_{Al}\right)\left(T_f - T_{cold}\right)\big/ m_x\left(T_{i,x} - T_f\right)$$

or $\quad c_x = \dfrac{\left[(0.285\ \text{kg})(4\,186\ \text{J/kg}\cdot°\text{C}) + (0.150\ \text{kg})(900\ \text{J/kg}\cdot°\text{C})\right](32.0 - 25.0)°\text{C}}{(0.125\ \text{kg})(95.0°\text{C} - 32.0°\text{C})}$

yielding $\quad c_x = \boxed{1.18\times10^{3}\ \text{J/kg}\cdot°\text{C}}$

11.25 Remember that energy must be supplied to melt the ice before its temperature will begin to rise. Then, assuming a thermally isolated system, $Q_{cold} = -Q_{hot}$, or

$$m_{ice}L_f + m_{ice}c_{water}\left(T_f - 0°\text{C}\right) = -m_{water}c_{water}\left(T_f - 25°\text{C}\right)$$

and

$$T_f = \frac{m_{water}c_{water}(25°\text{C}) - m_{ice}L_f}{\left(m_{ice} + m_{water}\right)c_{water}} = \frac{\left(825\ \cancel{\text{g}}\right)(4\,186\ \text{J/kg}\cdot°\text{C})(25°\text{C}) - \left(75\ \cancel{\text{g}}\right)\left(3.33\times10^{5}\ \text{J/kg}\right)}{\left(75\ \cancel{\text{g}} + 825\ \cancel{\text{g}}\right)(4\,186\ \text{J/kg}\cdot°\text{C})}$$

yielding $\quad \boxed{T_f = 16°\text{C}}$

11.26 The total energy input required is

$$\begin{aligned} Q &= (\text{energy to melt 50 g of ice}) \\ &\qquad + (\text{energy to warm 50 g of water to } 100°\text{C}) \\ &\qquad\qquad + (\text{energy to vaporize 5.0 g water}) \\ &= (50\text{ g})L_f + (50\text{ g})c_{\text{water}}(100°\text{C} - 0°\text{C}) + (5.0\text{ g})L_v \end{aligned}$$

Thus,
$$\begin{aligned} Q &= (0.050\text{ kg})\left(3.33\times10^5\ \frac{\text{J}}{\text{kg}}\right) \\ &\quad + (0.050\text{ kg})\left(4\,186\ \frac{\text{J}}{\text{kg}\cdot°\text{C}}\right)(100°\text{C} - 0°\text{C}) \\ &\quad + (5.0\times10^{-3}\text{ kg})\left(2.26\times10^6\ \frac{\text{J}}{\text{kg}}\right) \end{aligned}$$

which gives $Q = 4.9\times10^4\text{ J} = \boxed{49\text{ kJ}}$

11.27 The conservation of energy equation for this process is

$$(\text{energy to melt ice}) + (\text{energy to warm melted ice to } T) = (\text{energy to cool water to } T)$$

or $\quad m_{\text{ice}}L_f + m_{\text{ice}}c_{\text{w}}(T - 0°\text{C}) = m_{\text{w}}c_{\text{w}}(80°\text{C} - T)$

This yields $T = \dfrac{m_{\text{w}}c_{\text{w}}(80°\text{C}) - m_{\text{ice}}L_f}{(m_{\text{ice}} + m_{\text{w}})c_{\text{w}}}$

so

$$T = \frac{(1.0\text{ kg})(4\,186\ \text{J/kg}\cdot°\text{C})(80°\text{C}) - (0.100\text{ kg})(3.33\times10^5\ \text{J/kg})}{(1.1\text{ kg})(4\,186\ \text{J/kg}\cdot°\text{C})} = \boxed{65\,°\text{C}}$$

11.28 The energy required is the following sum of terms:

$$\begin{aligned} Q &= (\text{energy to reach melting point}) \\ &\quad + (\text{energy to melt}) + (\text{energy to reach boiling point}) \\ &\qquad + (\text{energy to vaporize}) + (\text{energy to reach } 110°\text{C}) \end{aligned}$$

Mathematically,

$$Q = m\Big[c_{\text{ice}}\left[0°\text{C} - (-10°\text{C})\right] + L_f + c_{\text{water}}(100°\text{C} - 0°\text{C}) + L_v + c_{\text{steam}}(110°\text{C} - 100°\text{C})\Big]$$

This yields

$$\begin{aligned} Q = (40\times10^{-3}\text{ kg})\Bigg[&\left(2\,090\ \frac{\text{J}}{\text{kg}\cdot°\text{C}}\right)(10°\text{C}) + \left(3.33\times10^5\ \frac{\text{J}}{\text{kg}}\right) \\ &+ \left(4\,186\ \frac{\text{J}}{\text{kg}\cdot°\text{C}}\right)(100°\text{C}) + \left(2.26\times10^6\ \frac{\text{J}}{\text{kg}}\right) + \left(2\,010\ \frac{\text{J}}{\text{kg}\cdot°\text{C}}\right)(10°\text{C})\Bigg] \end{aligned}$$

or $\quad Q = 1.2\times10^5\text{ J} = \boxed{0.12\text{ MJ}}$

11.29 Assuming all work done against friction is used to melt snow, the energy balance equation is $f \cdot s = m_{\text{snow}} L_f$. Since $f = \mu_k \left(m_{\text{skier}} g\right)$, the distance traveled is

$$s = \frac{m_{\text{snow}} L_f}{\mu_k \left(m_{\text{skier}} g\right)} = \frac{(1.0\ \text{kg})\left(3.33\times 10^5\ \text{J/kg}\right)}{0.20(75\ \text{kg})\left(9.80\ \text{m/s}^2\right)} = 2.3\times 10^3\ \text{m} = \boxed{2.3\ \text{km}}$$

11.30 (a) Observe that the equilibrium temperature will lie between the two extreme temperatures $(-10.0°\text{C and } +30.0°\text{C})$ of the mixed materials. Also, observe that a water-ice change of phase can be expected in this temperature range, but that neither aluminum nor ethyl alcohol undergoes a change of phase in this temperature range. The thermal energy transfers we can anticipate as the system comes to an equilibrium temperature are:

ice at $-10.0°$C to ice at 0°C; ice at 0°C to liquid water at 0°C; water at 0°C to water at T; aluminum at 20.0°C to aluminum at T; ethyl alcohol at 30.0°C to ethyl alcohol at T.

(b)

Q	m (kg)	c (j/kg·°C)	L (j/kg)	T_f(°C)	T_i(°C)	Expression
Q_{ice}	1.00	2 090		0	–10.0	$m_{\text{ice}} c_{\text{ice}} \left[0-(-10.0°\text{C})\right]$
Q_{melt}	1.00		3.33×10^5	0	0	$m_{\text{ice}} L_f$
Q_{water}	1.00	4 186		T	0	$m_{\text{ice}} c_{\text{water}} (T-0)$
Q_{Al}	0.500	900		T	20.0	$m_{\text{Al}} c_{\text{Al}} [T-20.0°\text{C}]$
Q_{alc}	6.00	2 430		T	30.0	$m_{\text{alc}} c_{\text{alc}} [T-30.0°\text{C}]$

(c) $\boxed{m_{\text{ice}} c_{\text{ice}} (10.0°\text{C}) + m_{\text{ice}} L_f + m_{\text{ice}} c_{\text{water}} (T-0) + m_{\text{Al}} c_{\text{Al}} [T-20.0°\text{C}] + m_{\text{alc}} c_{\text{alc}} [T-30.0°\text{C}] = 0}$

(d) $$T = \frac{m_{\text{Al}} c_{\text{Al}} (20.0°\text{C}) + m_{\text{alc}} c_{\text{alc}} (30.0°\text{C}) - m_{\text{ice}} \left[c_{\text{ice}} (10.0°\text{C}) + L_f\right]}{m_{\text{ice}} c_{\text{water}} + m_{\text{Al}} c_{\text{Al}} + m_{\text{alc}} c_{\text{alc}}}$$

Substituting in numeric values from the table in (b) above gives

$$T = \frac{(0.500)(900)(20.0) + (6.00)(2\,430)(30.0) - (1.00)\left[(2\,090)(10.0) + 3.33\times 10^5\right]}{(1.00)(4\,186) + (0.500)(900) + (6.00)(2\,430)}$$

and yields $\boxed{T = 4.81°\text{C}}$

11.31 Assume that all the ice melts. If this yields a result $T > 0$, the assumption is valid, otherwise the problem must be solved again based on a different premise. If all ice melts, energy conservation $\left(Q_{\text{cold}} = -Q_{\text{hot}}\right)$ yields

$$m_{\text{ice}} \left[c_{\text{ice}} \left[0°\text{C} - (-78°\text{C})\right] + L_f + c_{\text{w}} (T - 0°\text{C})\right] = -\left(m_{\text{w}} c_{\text{w}} + m_{\text{cal}} c_{\text{Cu}}\right)(T - 25°\text{C})$$

or $$T = \frac{\left(m_{\text{w}} c_{\text{w}} + m_{\text{cal}} c_{\text{Cu}}\right)(25°\text{C}) - m_{\text{ice}} \left[c_{\text{ice}} (78°\text{C}) + L_f\right]}{\left(m_{\text{w}} + m_{\text{ice}}\right) c_{\text{w}} + m_{\text{cal}} c_{\text{Cu}}}$$

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With $m_w = 0.560 \text{ kg}, m_{cal} = 0.080 \text{ g}, m_{ice} = 0.040 \text{ g}, c_w = 4\,186 \text{ J/kg}\cdot°\text{C},$

$c_{Cu} = 387 \text{ J/kg}\cdot°\text{C},\ c_{ice} = 2\,090 \text{ J/kg}\cdot°\text{C}$, and $L_f = 3.33\times10^5 \text{ J/kg}$,

this gives

$$T = \frac{\left[(0.560)(4\,186)+(0.080)(387)\right](25°\text{C})-(0.040)\left[(2\,090)(78°\text{C})+3.33\times10^5\right]}{(0.560+0.040)(4\,186)+0.080(387)}$$

or $T = \boxed{16°\text{C}}$ and the assumption that all ice melts is seen to be valid.

11.32 At a rate of 400 kcal/h, the excess internal energy that must be eliminated in a half-hour run is

$$Q = \left(400\times10^3 \ \frac{\text{cal}}{\text{h}}\right)\left(\frac{4.186 \text{ J}}{1 \text{ cal}}\right)(0.500 \text{ h}) = 8.37\times10^5 \text{ J}$$

The mass of water that will be evaporated by this amount of excess energy is

$$m_{\text{evaporated}} = \frac{Q}{L_v} = \frac{8.37\times10^5 \text{ J}}{2.5\times10^6 \text{ J/kg}} = \boxed{0.33 \text{ kg}}$$

The mass of fat burned (and thus, the mass of water produced at a rate of 1 gram of water per gram of fat burned) is

$$m_{\text{produced}} = \frac{(400 \text{ kcal/h})(0.500 \text{ h})}{9.0 \text{ kcal/gram of fat}} = 22 \text{ g} = 22\times10^{-3} \text{ kg}$$

so the fraction of water needs provided by burning fat is

$$f = \frac{m_{\text{produced}}}{m_{\text{evaporated}}} = \frac{22\times10^{-3} \text{ kg}}{0.33 \text{ kg}} = \boxed{0.067 \text{ or } 6.7\%}$$

11.33 (a) The mass of 2.0 liters of water is $m_w = \rho V = (10^3 \text{ kg/m}^3)(2.0\times10^{-3} \text{ m}^3) = 2.0 \text{ kg}$.

The energy required to raise the temperature of the water (and pot) up to the boiling point of water is

$$Q_{\text{boil}} = (m_w c_w + m_{Al} c_{Al})(\Delta T)$$

or $$Q_{\text{boil}} = \left[(2.0 \text{ kg})\left(4\,186 \ \frac{\text{J}}{\text{kg}}\right)+(0.25 \text{ kg})\left(900 \ \frac{\text{J}}{\text{kg}}\right)\right](100°\text{C}-20°\text{C}) = 6.9\times10^5 \text{ J}$$

The time required for the 14 000 Btu/h burner to produce this much energy is

$$t_{\text{boil}} = \frac{Q_{\text{boil}}}{14\,000 \text{ Btu/h}} = \frac{6.9\times10^5 \text{ J}}{14\,000 \text{ Btu/h}}\left(\frac{1 \text{ Btu}}{1.054\times10^3 \text{ J}}\right) = 4.7\times10^{-2} \text{ h} = \boxed{2.8 \text{ min}}$$

(b) Once the boiling temperature is reached, the additional energy required to evaporate all of the water is

$$Q_{\text{evaporate}} = m_w L_v = (2.0 \text{ kg})(2.26\times10^6 \text{ J/kg}) = 4.5\times10^6 \text{ J}$$

and the time required for the burner to produce this energy is

$$t_{\text{evaporate}} = \frac{Q_{\text{evaporate}}}{14\,000 \text{ Btu/h}} = \frac{4.5\times10^6 \text{ J}}{14\,000 \text{ Btu/h}}\left(\frac{1 \text{ Btu}}{1.054\times10^3 \text{ J}}\right) = 0.30 \text{ h} = \boxed{18 \text{ min}}$$

11.34 In one hour, the energy dissipated by the runner is

$$\Delta E = P \cdot t = (300 \text{ J/s})(3\,600 \text{ s}) = 1.08 \times 10^6 \text{ J}$$

Ninety percent, or $Q = 0.900(1.08 \times 10^6 \text{ J}) = 9.72 \times 10^5 \text{ J}$, of this is used to evaporate bodily fluids. The mass of fluid evaporated is

$$m = \frac{Q}{L_v} = \frac{9.72 \times 10^5 \text{ J}}{2.41 \times 10^6 \text{ J/kg}} = 0.403 \text{ kg}$$

Assuming the fluid is primarily water, the volume of fluid evaporated in one hour is

$$V = \frac{m}{\rho} = \frac{0.403 \text{ kg}}{1\,000 \text{ kg/m}^3} = (4.03 \times 10^{-4} \text{ m}^3)\left(\frac{10^6 \text{ cm}^3}{1 \text{ m}^3}\right) = \boxed{403 \text{ cm}^3}$$

11.35 The energy required to melt 50 g of ice is

$$Q_1 = m_{\text{ice}} L_f = (0.050 \text{ kg})(333 \text{ kJ/kg}) = 17 \text{ kJ}$$

The energy needed to warm 50 g of melted ice from 0°C to 100°C is

$$Q_2 = m_{\text{ice}} c_{\text{w}} (\Delta T) = (0.050 \text{ kg})(4.186 \text{ kJ/kg} \cdot °\text{C})(100°\text{C}) = 21 \text{ kJ}$$

(a) If 10 g of steam is used, the energy it will give up as it condenses is

$$Q_3 = m_{\text{steam}} L_v = (0.010 \text{ kg})(2\,260 \text{ kJ/kg}) = 23 \text{ kJ}$$

Since $Q_3 > Q_1$, all of the ice will melt. However, $Q_3 < Q_1 + Q_2$, so the final temperature is less than 100°C. From conservation of energy, we find

$$Q_{\text{cold}} = -Q_{\text{hot}}$$

$$m_{\text{ice}}\left[L_f + c_{\text{w}}(T - 0°\text{C})\right] = -m_{\text{steam}}\left[-L_v + c_{\text{w}}(T - 100°\text{C})\right]$$

or $$T = \frac{m_{\text{steam}}\left[L_v + c_{\text{w}}(100°\text{C})\right] - m_{\text{ice}} L_f}{(m_{\text{ice}} + m_{\text{steam}}) c_{\text{w}}},$$

giving $$T = \frac{(10 \text{ g})\left[2.26 \times 10^6 + (4\,186)(100)\right] - (50 \text{ g})(3.33 \times 10^5)}{(50 \text{ g} + 10 \text{ g})(4\,186)} = \boxed{40\,°\text{C}}$$

(b) If only 1.0 g of steam is used, then $Q_3' = m_{\text{steam}} L_v = 2.26$ kJ. The energy 1.0 g of condensed steam can give up as it cools from 100°C to 0°C is

$$Q_4 = m_{\text{steam}} c_{\text{w}} (\Delta T) = (1.0 \times 10^{-3} \text{ kg})(4.186 \text{ kJ/kg} \cdot °\text{C})(100°\text{C}) = 0.42 \text{ kJ}$$

Since $Q_3' + Q_4$ is less than Q_1, not all of the 50 g of ice will melt, so the final temperature will be $\boxed{0\,°\text{C}}$. The mass of ice which melts as the steam condenses and the condensate cools to 0°C is

$$m = \frac{Q_3' + Q_4}{L_f} = \frac{(2.26 + 0.42) \text{ kJ}}{333 \text{ kJ/kg}} = 8.0 \times 10^{-3} \text{ kg} = \boxed{8.0 \text{ g}}$$

11.36 First, we use the ideal gas law (with $V = 0.600\text{ L} = 0.600 \times 10^{-3}\text{ m}^3$ and $T = 37.0°\text{C} = 310\text{ K}$) to determine the quantity of water vapor in each exhaled breath:

$$PV = nRT \;\Rightarrow\; n = \frac{PV}{RT} = \frac{\left(3.20 \times 10^3\text{ Pa}\right)\left(0.600 \times 10^{-3}\text{ m}^3\right)}{(8.31\text{ J/mol}\cdot\text{K})(310\text{ K})} = 7.45 \times 10^{-4}\text{ mol}$$

or $$m = nM_{\text{water}} = \left(7.45 \times 10^{-4}\ \cancel{\text{mol}}\right)\left(18.0\ \frac{\cancel{\text{g}}}{\cancel{\text{mol}}}\right)\left(\frac{1\text{ kg}}{10^3\ \cancel{\text{g}}}\right) = 1.34 \times 10^{-5}\text{ kg}$$

The energy required to vaporize this much water, and hence the energy carried from the body with each breath, is

$$Q = mL_v = \left(1.34 \times 10^{-5}\text{ kg}\right)\left(2.26 \times 10^6\text{ J/kg}\right) = 30.3\text{ J}$$

The rate of losing energy by exhaling humid air is then

$$P = Q \cdot (\textit{respiration rate}) = \left(30.3\ \frac{\text{J}}{\text{breath}}\right)\left(22.0\ \frac{\text{breaths}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right) = \boxed{11.1\text{ W}}$$

11.37 (a) The bullet loses all of its kinetic energy as it is stopped by the ice. Also, thermal energy is transferred from the bullet to the ice as the bullet cools from 30.0°C to the final temperature. The sum of these two quantities of energy equals the energy required to melt part of the ice. The $\boxed{\text{final temperature is }0°\text{C}}$ because not all of the ice melts.

(b) The total energy transferred from the bullet to the ice is

$$Q = KE_i + m_{\text{bullet}}c_{\text{lead}}\left|0°\text{C} - 30.0°\text{C}\right| = \frac{1}{2}m_{\text{bullet}}v_i^2 + m_{\text{bullet}}c_{\text{lead}}(30.0°\text{C})$$

$$= \left(3.00 \times 10^{-3}\text{ kg}\right)\left[\frac{\left(2.40 \times 10^2\text{ m/s}\right)^2}{2} + (128\text{ J/kg}\cdot°\text{C})(30.0°\text{C})\right] = 97.9\text{ J}$$

The mass of ice that melts when this quantity of thermal energy is absorbed is

$$m = \frac{Q}{\left(L_f\right)_{\text{water}}} = \frac{97.9\text{ J}}{3.33 \times 10^5\text{ J/kg}} = 2.94 \times 10^{-4}\text{ kg}\left(\frac{10^3\text{ g}}{1\text{ kg}}\right) = \boxed{0.294\text{ g}}$$

11.38 (a) The rate of energy transfer by conduction through a material of area *A*, thickness *L*, with thermal conductivity *k*, and temperatures $T_h > T_c$ on opposite sides is $P = kA\left(T_h - T_c\right)/L$. For the given windowpane, this is

$$P = \left(0.8\ \frac{\text{J}}{\text{s}\cdot\text{m}\cdot°\text{C}}\right)\left[(1.0\text{ m})(2.0\text{ m})\right]\frac{(25°\text{C} - 0°\text{C})}{0.62 \times 10^{-2}\text{ m}} = 6 \times 10^3\text{ J/s} = \boxed{6 \times 10^3\text{ W}}$$

(b) The total energy lost per day is

$$E = P \cdot \Delta t = \left(6 \times 10^3\text{ J/s}\right)\left(8.64 \times 10^4\text{ s}\right) = \boxed{5 \times 10^8\text{ J}}$$

11.39 The rate of energy transfer by conduction through a material having thermal conductivity κ, cross-sectional area A, thickness L and a temperature change of $T_h - T_c$ across it is $P = \kappa A\left(T_h - T_c\right)/L$. Hence, with $\kappa = 0.6\text{ J/s}\cdot\text{m}\cdot°\text{C}$ for water, the rate of energy transfer by conduction to the bottom of the pond is

$$P = \frac{(0.6\text{ J/s}\cdot\text{m}\cdot°\text{C})\left(820\text{ m}^2\right)(25°\text{C} - 12°\text{C})}{2.0\text{ m}} = 3 \times 10^3\text{ J/s} = \boxed{3 \times 10^3\text{ W}}$$

11.40 (a) The R value of a material is $R = L/\kappa$, where L is its thickness and κ is the thermal conductivity. The R values of the three layers covering the core tissues in this body are:

$$R_{\text{skin}} = \frac{1.0\times10^{-3}\ \text{m}}{0.020\ \text{W/m}\cdot\text{K}} = \boxed{5.0\times10^{-2}\ \text{m}^2\cdot\text{K/W}}$$

$$R_{\text{fat}} = \frac{0.50\times10^{-2}\ \text{m}}{0.020\ \text{W/m}\cdot\text{K}} = \boxed{2.5\times10^{-2}\ \text{m}^2\cdot\text{K/W}}$$

and $$R_{\text{tissue}} = \frac{3.2\times10^{-2}\ \text{m}}{0.50\ \text{W/m}\cdot\text{K}} = \boxed{6.4\times10^{-2}\ \text{m}^2\cdot\text{K/W}}$$

so the total R value of the three layers taken together is

$$R_{\text{total}} = \sum_{i=1}^{3} R_i = (5.0+2.5+6.4)\times10^{-2}\ \frac{\text{m}^2\cdot\text{K}}{\text{W}} = 14\times10^{-2}\ \frac{\text{m}^2\cdot\text{K}}{\text{W}} = \boxed{0.14\ \frac{\text{m}^2\cdot\text{K}}{\text{W}}}$$

(b) The rate of energy transfer by conduction through these three layers with a surface area of $A = 2.0\ \text{m}^2$ and temperature difference of $\Delta T = (37-0)°\text{C} = 37°\text{C} = 37\ \text{K}$ is

$$P = \frac{A(\Delta T)}{R_{\text{total}}} = \frac{(2.0\ \text{m}^2)(37\ \text{K})}{0.14\ \text{m}^2\cdot\text{K/W}} = \boxed{5.3\times10^2\ \text{W}}$$

11.41 $$P = \kappa A\left(\frac{\Delta T}{L}\right),\ \text{with}\ \kappa = 0.200\ \frac{\text{cal}}{\text{cm}\cdot°\text{C}\cdot\text{s}}\left(\frac{10^2\ \text{cm}}{1\ \text{m}}\right)\left(\frac{4.186\ \text{J}}{1\ \text{cal}}\right) = 83.7\ \frac{\text{J}}{\text{s}\cdot\text{m}\cdot°\text{C}}$$

Thus, the energy transfer rate is

$$P = \left(83.7\ \frac{\text{J}}{\text{s}\cdot\text{m}\cdot°\text{C}}\right)\left[(8.00\ \text{m})(50.0\ \text{m})\right]\left(\frac{200°\text{C}-20.0°\text{C}}{1.50\times10^{-2}\ \text{m}}\right)$$

$$= 4.02\times10^8\ \text{J/s} = 4.02\times10^8\ \text{W} = \boxed{402\ \text{MW}}$$

11.42 The total surface area of the house is

$$A = A_{\text{side walls}} + A_{\text{end walls}} + A_{\text{gables}} + A_{\text{roof}}$$

where $$A_{\text{side walls}} = 2\left[(5.00\ \text{m})\times(10.0\ \text{m})\right] = 100\ \text{m}^2$$

$$A_{\text{end walls}} = 2\left[(5.00\ \text{m})\times(8.00\ \text{m})\right] = 80.0\ \text{m}^2$$

$$A_{\text{gables}} = 2\left[\tfrac{1}{2}(\textit{base})\times(\textit{altitude})\right] = 2\left[\tfrac{1}{2}(8.00\ \text{m})\times(4.00\ \text{m})\tan 37.0°\right] = 24.1\ \text{m}^2$$

$$A_{\text{roof}} = 2\left[(10.0\ \text{m})\times(4.00\ \text{m}/\cos 37.0°)\right] = 100\ \text{m}^2$$

Thus, $$A = 100\ \text{m}^2 + 80.0\ \text{m}^2 + 24.1\ \text{m}^2 + 100\ \text{m}^2 = 304\ \text{m}^2$$

With an average thickness of 0.210 m, average thermal conductivity of $4.8\times10^{-4}\ \text{kW/m}\cdot°\text{C}$, and a 25.0°C difference between inside and outside temperatures, the energy transfer from the house to the outside air each day is

$$E = P(\Delta t) = \left[\frac{\kappa A(\Delta T)}{L}\right](\Delta t) = \left[\frac{(4.8\times10^{-4}\ \text{kW/m}\cdot°\text{C})(304\ \text{m}^2)(25.0°\text{C})}{0.210\ \text{m}}\right](86\,400\ \text{s})$$

continued on next page

or $\quad E = 1.5\times10^6 \text{ kJ} = 1.5\times10^9 \text{ J}$

The volume of gas that must be burned to replace this energy is

$$V = \frac{E}{\textit{heat of combustion}} = \frac{1.5\times10^9 \text{ J}}{\left(9\,300 \text{ kcal/m}^3\right)\left(4\,186 \text{ J/kcal}\right)} = \boxed{39 \text{ m}^3}$$

11.43 Because the two pots hold the same quantity of water at the same initial temperature, the same amount, Q, of thermal energy is required to boil away the water in each pot. The time required to do this for one of the pots is $t = Q/P$, where P is the rate of energy conduction from the stove to the water through the bottom of the pot. The ratio of the times required for the two pots is

$$\frac{t_{\text{Al}}}{t_{\text{Cu}}} = \left(\frac{\cancel{Q}}{P_{\text{Al}}}\right)\left(\frac{P_{\text{Cu}}}{\cancel{Q}}\right) = \frac{\kappa_{\text{Cu}}\cancel{A}\left(\cancel{\Delta T}\right)/\cancel{L}}{\kappa_{\text{Al}}\cancel{A}\left(\cancel{\Delta T}\right)/\cancel{L}} = \frac{\kappa_{\text{Cu}}}{\kappa_{\text{Al}}}$$

Note that in the above calculation everything except the thermal conductivities canceled because the two pots are identical except for the material making up the bottoms. Thus, the time required to boil away the water in the aluminum bottomed pot is

$$t_{\text{Al}} = \left(\frac{\kappa_{\text{Cu}}}{\kappa_{\text{Al}}}\right)t_{\text{Cu}} = \left(\frac{397 \text{ W/m}\cdot{}^\circ\text{C}}{238 \text{ W/m}\cdot{}^\circ\text{C}}\right)(425 \text{ s}) = \boxed{709 \text{ s}}$$

11.44 The rate of energy transfer through a compound slab is

$$P = \frac{A(\Delta T)}{R}, \text{ where } R = \Sigma L_i/\kappa_i$$

(a) For the thermopane, $R = R_{\text{pane}} + R_{\text{trapped air}} + R_{\text{pane}} = 2R_{\text{pane}} + R_{\text{trapped air}}$

Thus, $$R = 2\left(\frac{0.50\times10^{-2} \text{ m}}{0.8 \text{ W/m}\cdot{}^\circ\text{C}}\right) + \frac{1.0\times10^{-2} \text{ m}}{0.023\,4 \text{ W/m}\cdot{}^\circ\text{C}} = (0.01+0.43)\ \frac{\text{m}^2\cdot{}^\circ\text{C}}{\text{W}} = 0.44\ \frac{\text{m}^2\cdot{}^\circ\text{C}}{\text{W}}$$

and $$P = \frac{\left(1.0 \text{ m}^2\right)(23^\circ\text{C})}{0.44 \text{ m}^2\cdot{}^\circ\text{C/W}} = \boxed{52 \text{ W}}$$

(b) For the 1.0 cm thick pane of glass,

$$R = \frac{1.0\times10^{-2} \text{ m}}{0.8 \text{ W/m}\cdot{}^\circ\text{C}} = 1\times10^{-2}\ \frac{\text{m}^2\cdot{}^\circ\text{C}}{\text{W}}$$

so $$P = \frac{\left(1.0 \text{ m}^2\right)(23^\circ\text{C})}{1\times10^{-2} \text{ m}^2\cdot{}^\circ\text{C/W}} = 2\times10^3 \text{ W} = \boxed{2 \text{ kW}}\text{, about 38 times greater}$$

11.45 When the temperature of the junction stabilizes, the energy transfer rate must be the same for each of the rods, or $P_{\text{Cu}} = P_{\text{Al}}$. The cross-sectional areas of the rods are equal, and if the temperature of the junction is 50°C, the temperature difference is $\Delta T = 50^\circ\text{C}$ for each rod.

Thus, $$P_{\text{Cu}} = \kappa_{\text{Cu}}A\left(\frac{\Delta T}{L_{\text{Cu}}}\right) = \kappa_{\text{Al}}A\left(\frac{\Delta T}{L_{\text{Al}}}\right) = P_{\text{Al}}\text{, which gives}$$

$$L_{\text{Al}} = \left(\frac{\kappa_{\text{Al}}}{\kappa_{\text{Cu}}}\right)L_{\text{Cu}} = \left(\frac{238 \text{ W/m}\cdot{}^\circ\text{C}}{397 \text{ W/m}\cdot{}^\circ\text{C}}\right)(15 \text{ cm}) = \boxed{9.0 \text{ cm}}$$

11.46 The energy transfer rate is $P = \dfrac{\Delta Q}{\Delta t} = \dfrac{m_{\text{ice}} L_f}{\Delta t} = \dfrac{(5.0\text{ kg})(3.33\times 10^5\text{ J/kg})}{(8.0\text{ h})(3\,600\text{ s/1 h})} = 58\text{ W}$

Thus, $P = \kappa A\left(\dfrac{\Delta T}{L}\right)$ gives the thermal conductivity as

$$\kappa = \frac{P\cdot L}{A(\Delta T)} = \frac{(58\text{ W})(2.0\times 10^{-2}\text{ m})}{(0.80\text{ m}^2)(25°\text{C} - 5.0°\text{C})} = \boxed{7.3\times 10^{-2}\text{ W/m}\cdot°\text{C}}$$

11.47 The window will consist of the glass pane and a stagnant air layer on each side (see Example 11.10 in text). From Tables 11.3 and 11.4, the *R*-values for these layers are

$$R_{\text{pane}} = \frac{L}{\kappa_{\text{glass}}} = \frac{0.40\times 10^{-2}\text{ m}}{0.80\text{ W/m}\cdot°\text{C}} = 5.0\times 10^{-3}\ \frac{\text{m}^2\cdot°\text{C}}{\text{W}}$$

and $$R_{\substack{\text{air}\\\text{layer}}} = 0.17\ \frac{\text{ft}^2\cdot°\text{F}}{\text{Btu/h}}\cdot\left(\frac{1\text{ Btu/h}}{0.293\text{ W}}\right)\left(\frac{1\text{ m}}{3.281\text{ ft}}\right)^2\left(\frac{1\ °\text{C}}{9/5\ °\text{F}}\right) = 0.030\ \frac{\text{m}^2\cdot°\text{C}}{\text{W}}$$

Thus, $$R_{\text{total}} = \Sigma R_i = \left(0.030 + 5.0\times 10^{-3} + 0.030\right)\ \frac{\text{m}^2\cdot°\text{C}}{\text{W}} = 0.065\ \frac{\text{m}^2\cdot°\text{C}}{\text{W}}$$

The energy loss through the window in a 12 hour interval is then

$$Q = P\cdot t = \frac{A(T_h - T_c)}{\Sigma R_i}\cdot t = \frac{(2.0\text{ m}^2)(20°\text{C})}{0.065\text{ m}^2\cdot°\text{C/W}}\cdot(12\text{ h})\left(\frac{3\,600\text{ s}}{1\text{ h}}\right) = \boxed{2.7\times 10^7\text{ J}}$$

11.48 Since 97% of the incident energy is reflected, the rate of energy absorption from the sunlight is $P_{\text{absorbed}} = 3.0\%\times(I\cdot A) = 0.030(I\cdot A)$, where I is the intensity of the solar radiation.

$$P_{\text{absorbed}} = 0.030\left(1.40\times 10^3\text{ W/m}^2\right)\left(1.00\times 10^3\text{ m}\right)^2 = 4.2\times 10^7\text{ W}$$

Assuming the sail radiates equally from both sides (so $A = 2(1.00\text{ km})^2 = 2.00\times 10^6\text{ m}^2$), the rate at which it will radiate energy to a 0 K environment when it has absolute temperature T is

$$\begin{aligned}P_{\text{radiated}} &= \sigma A e\left(T^4 - 0\right)\\ &= \left(5.669\,6\times 10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)\left(2.00\times 10^6\text{ m}^2\right)(0.03)\cdot T^4 = \left(3.4\times 10^{-3}\ \frac{\text{W}}{\text{K}^4}\right)\cdot T^4\end{aligned}$$

At the equilibrium temperature, where $P_{\text{radiated}} = P_{\text{absorbed}}$, we then have

$$\left(3.4\times 10^{-3}\ \frac{\text{W}}{\text{K}^4}\right)\cdot T^4 = 4.2\times 10^7\text{ W} \qquad \text{or} \qquad T = \left[\frac{4.2\times 10^7\text{ W}}{3.4\times 10^{-3}\text{ W/K}^4}\right]^{1/4} = \boxed{330\text{ K}}$$

11.49 The absolute temperatures of the two stars are $T_X = 5\,727 + 273 = 6\,000\text{ K}$ and $T_Y = 11\,727 + 273 = 12\,000\text{ K}$. Thus, the ratio of their radiated powers is

$$\frac{P_Y}{P_X} = \frac{\sigma A e T_Y^4}{\sigma A e T_X^4} = \left(\frac{T_Y}{T_X}\right)^4 = (2)^4 = \boxed{16}$$

11.50 From Stefan's law, the power radiated by an object at absolute temperature T and surface area A is $P = \sigma A e T^4$, where $\sigma = 5.669\,6\times 10^{-8}\text{ W/m}^2\cdot\text{K}$ and e is the emissivity. Thus, the surface area of the filament must be

$$A = \frac{P}{\sigma e T^4} = \frac{75\text{ W}}{\left(5.669\,6\times 10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)(1.0)(3\,300\text{ K})^4} = \boxed{1.1\times 10^{-5}\text{ m}^2}$$

11.51 At a pressure of 1 atm, water boils at 100°C. Thus, the temperature on the interior of the copper kettle is 100°C and the energy transfer rate through the bottom is

$$P = \kappa A\left(\frac{\Delta T}{L}\right) = \left(397\ \frac{\text{W}}{\text{m}\cdot{}^\circ\text{C}}\right)\left[\pi(0.10\text{ m})^2\right]\left(\frac{102^\circ\text{C} - 100^\circ\text{C}}{2.0\times10^{-3}\text{ m}}\right)$$

$$= 1.2\times10^4\text{ W} = \boxed{12\text{ kW}}$$

11.52 The mass of the water in the heater is

$$m = \rho V = \left(10^3\ \frac{\text{kg}}{\text{m}^3}\right)(50.0\text{ gal})\left(\frac{3.786\text{ L}}{1\text{ gal}}\right)\left(\frac{1\text{ m}^3}{10^3\text{ L}}\right) = 189\text{ kg}$$

The energy required to raise the temperature of the water from 20.0°C to 60.0°C is

$$Q = mc(\Delta T) = (189\text{ kg})(4\,186\text{ J/kg})(60.0^\circ\text{C} - 20.0^\circ\text{C}) = 3.16\times10^7\text{ J}$$

The time required for the water heater to transfer this energy is

$$t = \frac{Q}{P} = \frac{3.16\times10^7\text{ J}}{4\,800\text{ J/s}}\left(\frac{1\text{ h}}{3\,600\text{ s}}\right) = \boxed{1.83\text{ h}}$$

11.53 The energy conservation equation is $Q_{\text{cold}} = -Q_{\text{hot}}$, or

$$m_{\text{ice}}L_f + \left[(m_{\text{ice}} + m_{\text{w}})c_{\text{w}} + m_{\text{cup}}c_{\text{Cu}}\right](12^\circ\text{C} - 0^\circ\text{C}) = -m_{\text{Pb}}c_{\text{Pb}}(12^\circ\text{C} - 98^\circ\text{C})$$

This gives

$$m_{\text{Pb}}\left(128\ \frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)(86^\circ\text{C}) = (0.040\text{ kg})(3.33\times10^5\text{ J/kg})$$
$$+\left[(0.24\text{ kg})(4\,186\text{ J/kg}\cdot{}^\circ\text{C}) + (0.100\text{ kg})(387\text{ J/kg}\cdot{}^\circ\text{C})\right](12^\circ\text{C})$$

or $m_{\text{Pb}} = \boxed{2.3\text{ kg}}$

11.54 (a) The net rate of energy transfer by radiation between a body at absolute temperature T and its surroundings at absolute temperature T_0 is $P_{\text{net}} = \sigma Ae(T^4 - T_0^4)$. Hence, with $T = 33.0^\circ\text{C} = 306\text{ K}$, $T_0 = 20.0^\circ\text{C} = 293\text{ K}$, emissivity $= e = 0.95$, and surface area $A = 1.50\text{ m}^2$, the net power radiated is

$$P_{\text{net}} = \left(5.669\,6\times10^{-8}\ \frac{\text{W}}{\text{m}^2\cdot\text{K}^4}\right)(1.50\text{ m}^2)(0.95)\left[(306\text{ K})^4 - (293\text{ K})^4\right] = \boxed{+1.1\times10^2\text{ W}}$$

(b) The positive sign on the net power radiated means that the body is radiating energy away faster than it is absorbing energy from the environment.

11.55 The conservation of energy equation is $Q_{\text{cold}} = -Q_{\text{hot}}$, or

$$(m_{\text{w}}c_{\text{w}} + m_{\text{cup}}c_{\text{glass}})(T - 27^\circ\text{C}) = -m_{\text{Cu}}c_{\text{Cu}}(T - 90^\circ\text{C})$$

This gives $$T = \frac{m_{\text{Cu}}c_{\text{Cu}}(90^\circ\text{C}) + (m_{\text{w}}c_{\text{w}} + m_{\text{cup}}c_{\text{glass}})(27^\circ\text{C})}{m_{\text{w}}c_{\text{w}} + m_{\text{cup}}c_{\text{glass}} + m_{\text{Cu}}c_{\text{Cu}}},$$ or

$$T = \frac{(0.200)(387)(90^\circ\text{C}) + \left[(0.400)(4\,186) + (0.300)(837)\right](27^\circ\text{C})}{(0.400)(4\,186) + (0.300)(837) + (0.200)(387)} = \boxed{29^\circ\text{C}}$$

11.56 (a) The energy delivered to the heating element (a resistor) is transferred to the liquid nitrogen, causing part of it to vaporize in a liquid-to-gas phase transition. The total energy delivered to the element equals the product of the power and the time interval of 4.0 h.

(b) The mass of nitrogen vaporized in a 4.0 h period is

$$m = \frac{Q}{L_f} = \frac{P \cdot (\Delta t)}{L_f} = \frac{(25 \text{ J/s})(4.0 \text{ h})(3\,600 \text{ s/h})}{2.01 \times 10^5 \text{ J/kg}} = \boxed{1.8 \text{ kg}}$$

11.57 Assuming the aluminum-water-calorimeter system is thermally isolated from the environment, $Q_{\text{cold}} = -Q_{\text{hot}}$, or

$$m_{\text{Al}} c_{\text{Al}} \left(T_f - T_{i,\text{Al}}\right) = -m_{\text{w}} c_{\text{w}} \left(T_f - T_{i,\text{w}}\right) - m_{\text{cal}} c_{\text{cal}} \left(T_f - T_{i,\text{cal}}\right)$$

Since $T_f = 66.3°\text{C}$ and $T_{i,\text{cal}} = T_{i,\text{w}} = 70.0°\text{C}$, this gives $c_{\text{Al}} = \dfrac{\left(m_{\text{w}} c_{\text{w}} + m_{\text{cal}} c_{\text{cal}}\right)\left(T_{i,\text{w}} - T_f\right)}{m_{\text{Al}}\left(T_f - T_{i,\text{Al}}\right)}$, or

$$c_{\text{Al}} = \frac{\left[(0.400 \text{ kg})\left(4\,186 \ \frac{\text{J}}{\text{kg} \cdot °\text{C}}\right) + (0.040 \text{ kg})\left(630 \ \frac{\text{J}}{\text{kg} \cdot °\text{C}}\right)\right](70.0 - 66.3)°\text{C}}{(0.200 \text{ kg})(66.3 - 27.0)°\text{C}} = \boxed{8.00 \times 10^2 \ \frac{\text{J}}{\text{kg} \cdot °\text{C}}}$$

The variation between this result and the value from Table 11.1 is

$$\% = \left(\frac{|\text{variation}|}{\text{accepted value}}\right) \times 100\% = \left(\frac{|800 - 900| \ \text{J/kg} \cdot °\text{C}}{900 \ \text{J/kg} \cdot °\text{C}}\right) \times 100\% = \boxed{11.1\%}$$

which is within the 15% tolerance.

11.58 (a) With a body temperature of $T = 37°\text{C} + 273 = 310 \text{ K}$ and surroundings at temperature $T_0 = 24°\text{C} + 273 = 297 \text{ K}$, the rate of energy transfer by radiation is

$$P_{\text{radiation}} = \sigma A e\left(T^4 - T_0^4\right)$$

$$= \left(5.669\,6 \times 10^{-8} \ \frac{\text{W}}{\text{m}^2 \cdot \text{K}^4}\right)\left(2.0 \text{ m}^2\right)(0.97)\left[(310 \text{ K})^4 - (297 \text{ K})^4\right] = \boxed{1.6 \times 10^2 \text{ W}}$$

(b) The rate of energy transfer by evaporation of perspiration is

$$P_{\text{perspiration}} = \frac{Q}{\Delta t} = \frac{m L_{v,\text{perspiration}}}{\Delta t} = \frac{(0.40 \text{ kg})\left(2.43 \times 10^3 \text{ kJ/kg}\right)\left(10^3 \text{ J/kJ}\right)}{3\,600 \text{ s}} = \boxed{2.7 \times 10^2 \text{ W}}$$

(c) The rate of energy transfer by evaporation from the lungs is

$$P_{\text{lungs}} = \left(38 \ \frac{\text{kJ}}{\text{h}}\right)\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right)\left(\frac{10^3 \text{ J}}{1 \text{ kJ}}\right) = \boxed{11 \text{ W}}$$

(d) The excess thermal energy that must be dissipated is

$$P_{\text{excess}} = 0.80 P_{\text{metabolic}} = 0.80\left(2.50 \times 10^3 \ \frac{\text{kJ}}{\text{h}}\right)\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right)\left(\frac{10^3 \text{ J}}{1 \text{ kJ}}\right) = 5.6 \times 10^2 \text{ W}$$

so the rate energy must be transferred by conduction and convection is

$$P_{\text{c\&c}} = P_{\text{excess}} - \left(P_{\text{radiation}} + P_{\text{perspiration}} + P_{\text{lungs}}\right) = (5.6 - 1.6 - 2.7 - .11) \times 10^2 \text{ W} = \boxed{1.2 \times 10^2 \text{ W}}$$

11.59 The total energy needed is

$$Q = mL_v = (2.00\ \text{kg})(2.00\times10^4\ \text{J/kg}) = 4.00\times10^4\ \text{J}$$

and the time required to supply this energy is

$$t = \frac{Q}{P} = \frac{4.00\times10^4\ \text{J}}{10.0\ \text{J/s}} = 4.00\times10^3\ \text{s}\left(\frac{1\ \text{min}}{60.0\ \text{s}}\right) = \boxed{66.7\ \text{min}}$$

11.60 The energy added to the air in one hour is

$$Q = (P_{\text{total}})t = [10(200\ \text{W})](3\,600\ \text{s}) = 7.20\times10^6\ \text{J}$$

and the mass of air in the room is

$$m = \rho V = (1.3\ \text{kg/m}^3)[(6.0\ \text{m})(15.0\ \text{m})(3.0\ \text{m})] = 3.5\times10^2\ \text{kg}$$

The change in temperature is $\Delta T = \dfrac{Q}{mc} = \dfrac{7.2\times10^6\ \text{J}}{(3.5\times10^2\ \text{kg})(837\ \text{J/kg}\cdot{}^\circ\text{C})} = 25^\circ\text{C}$

giving $T = T_0 + \Delta T = 20^\circ\text{C} + 25^\circ\text{C} = \boxed{45^\circ\text{C}}$

11.61 In the steady state, $P_{\text{Au}} = P_{\text{Ag}}$, or $\kappa_{\text{Au}} A\left(\dfrac{80.0^\circ\text{C} - T}{L}\right) = \kappa_{\text{Ag}} A\left(\dfrac{T - 30.0^\circ\text{C}}{L}\right)$

This gives

$$T = \frac{\kappa_{\text{Au}}(80.0^\circ\text{C}) + \kappa_{\text{Ag}}(30.0^\circ\text{C})}{\kappa_{\text{Au}} + \kappa_{\text{Ag}}} = \frac{314(80.0^\circ\text{C}) + 427(30.0^\circ\text{C})}{314 + 427} = \boxed{51.2^\circ\text{C}}$$

11.62 (a) The rate work is done against friction is

$$P = f\cdot v = (50\ \text{N})(40\ \text{m/s}) = 2.0\times10^3\ \text{J/s} = \boxed{2.0\ \text{kW}}$$

(b) In a time interval of 10 s, the energy added to the 10 kg of iron is

$$Q = P\cdot t = (2.0\times10^3\ \text{J/s})(10\ \text{s}) = 2.0\times10^4\ \text{J}$$

and the change in temperature is

$$\Delta T = \frac{Q}{mc} = \frac{2.0\times10^4\ \text{J}}{(10\ \text{kg})(448\ \text{J/kg}\cdot{}^\circ\text{C})} = \boxed{4.5^\circ\text{C}}$$

11.63 (a) The energy required to raise the temperature of the brakes to the melting point at 660°C is

$$Q = mc(\Delta T) = (6.00\ \text{kg})(900\ \text{J/kg}\cdot{}^\circ\text{C})(660^\circ\text{C} - 20.0^\circ\text{C}) = 3.46\times10^6\ \text{J}$$

The internal energy added to the brakes on each stop is

$$Q_1 = \Delta KE = \frac{1}{2}m_{\text{car}}v_i^2 = \frac{1}{2}(1\,500\ \text{kg})(25.0\ \text{m/s})^2 = 4.69\times10^5\ \text{J}$$

The number of stops before reaching the melting point is

$$N = \frac{Q}{Q_1} = \frac{3.46\times10^6\ \text{J}}{4.69\times10^5\ \text{J}} = \boxed{7\ \text{stops}}$$

continued on next page

(b) As the car stops, it transforms part of its kinetic energy into internal energy due to air resistance. As soon as the brakes rise above air temperature, they transfer energy by heat to the air. If they reach a high temperature, they transfer energy to the air very quickly.

11.64 When liquids 1 and 2 are mixed, the conservation of energy equation is

$$mc_1(17°\text{C} - 10°\text{C}) = mc_2(20°\text{C} - 17°\text{C}) \qquad \text{or} \qquad c_2 = \left(\frac{7}{3}\right)c_1$$

When liquids 2 and 3 are mixed, energy conservation yields

$$mc_3(30°\text{C} - 28°\text{C}) = mc_2(28°\text{C} - 20°\text{C}) \qquad \text{or} \qquad c_3 = 4\,c_2$$

Thus, we now know that $c_3 = 4(7c_1/3)$ or $c_3/c_1 = 28/3$

Mixing liquids 1 and 3 will give $mc_1(T - 10°\text{C}) = mc_3(30°\text{C} - T)$

or $$T = \frac{c_1(10°\text{C}) + c_3(30°\text{C})}{c_1 + c_3} = \frac{10°\text{C} + (28/3)(30°\text{C})}{1 + (28/3)} = \boxed{28\,°\text{C}}$$

11.65 (a) The internal energy ΔQ added to the volume ΔV of liquid that flows through the calorimeter in time Δt is $\Delta Q = (\Delta m)c(\Delta T) = \rho(\Delta V)c(\Delta T)$. Thus, the rate of adding energy is

$$\boxed{\frac{\Delta Q}{\Delta t} = \rho c(\Delta T)\left(\frac{\Delta V}{\Delta t}\right)}$$

where $\left(\dfrac{\Delta V}{\Delta t}\right)$ is the flow rate through the calorimeter.

(b) From the result of part (a), the specific heat is

$$c = \frac{\Delta Q/\Delta t}{\rho(\Delta T)(\Delta V/\Delta t)} = \frac{40\ \text{J/s}}{(0.72\ \text{g/cm}^3)(5.8°\text{C})(3.5\ \text{cm}^3/\text{s})}$$

$$= \left(2.7\ \frac{\text{J}}{\text{g}\cdot°\text{C}}\right)\left(\frac{10^3\ \text{g}}{1\ \text{kg}}\right) = \boxed{2.7\times10^3\ \text{J/kg}\cdot°\text{C}}$$

11.66 (a) The surface area of the stove is $A_{\text{stove}} = A_{\text{ends}} + A_{\substack{\text{cylindrical}\\\text{side}}} = 2(\pi r^2) + (2\pi r h)$, or

$$A_{\text{stove}} = 2\pi(0.200\ \text{m})^2 + 2\pi(0.200\ \text{m})(0.500\ \text{m}) = 0.880\ \text{m}^2$$

The temperature of the stove is $T_{\text{stove}} = \frac{5}{9}(400°\text{F} - 32.0°\text{F}) = 204°\text{C} = 477\ \text{K}$ while that of the air in the room is $T_{\text{room}} = \frac{5}{9}(70.0°\text{F} - 32.0°\text{F}) = 21.1°\text{C} = 294\ \text{K}$. If the emissivity of the stove is $e = 0.920$, the net power radiated to the room is

$$P = \sigma A_{\text{stove}} e\left(T_{\text{stove}}^4 - T_{\text{room}}^4\right)$$

$$= \left(5.669\,6\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(0.880\ \text{m}^2\right)(0.920)\left[(477\ \text{K})^4 - (294\ \text{K})^4\right]$$

or $$P = 2.03\times10^3\ \text{W} = \boxed{2.03\times10^3\ \text{J/s}}$$

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(b) The total surface area of the walls and ceiling of the room is

$$A = 4A_{\text{wall}} + A_{\text{ceiling}} = 4\left[(8.00 \text{ ft})(25.0 \text{ ft})\right] + (25.0 \text{ ft})^2 = 1.43\times 10^3 \text{ ft}^2$$

If the temperature of the room is constant, the power lost by conduction through the walls and ceiling must equal the power radiated by the stove. Thus, from thermal conduction equation, $P = A\left(T_h - T_c\right)/\Sigma R_i$, the net R value needed in the walls and ceiling is

$$\Sigma R_i = \frac{A\left(T_h - T_c\right)}{P} = \frac{\left(1.43\times 10^3 \text{ ft}^2\right)(70.0°\text{F} - 32.0°\text{F})}{2.03\times 10^3 \text{ J/s}}\left(\frac{1\,054 \text{ J}}{1 \text{ Btu}}\right)\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right)$$

or $\Sigma R_i = \boxed{7.84 \text{ ft}^2\cdot°\text{F}\cdot\text{h/Btu}}$

11.67 A volume of 1.0 L of water has a mass of $m = \rho V = \left(10^3 \text{ kg/m}^3\right)\left(1.0\times 10^{-3} \text{ m}^3\right) = 1.0 \text{ kg}$
The energy required to raise the temperature of the water to 100°C and then completely evaporate it is $Q = mc(\Delta T) + mL_v$, or

$$Q = (1.0 \text{ kg})(4\,186 \text{ J/kg}\cdot°\text{C})(100°\text{C} - 20°\text{C}) + (1.0 \text{ kg})\left(2.26\times 10^6 \text{ J/kg}\right) = 2.59\times 10^6 \text{ J}$$

The power input to the water from the solar cooker is

$$P = (\textit{efficiency})IA = (0.50)\left(600 \text{ W/m}^2\right)\left[\frac{\pi(0.50 \text{ m})^2}{4}\right] = 59 \text{ W}$$

so the time required to evaporate the water is

$$t = \frac{Q}{P} = \frac{2.59\times 10^6 \text{ J}}{59 \text{ J/s}} = \left(4.4\times 10^4 \text{ s}\right)\left(\frac{1 \text{ h}}{3\,600 \text{ s}}\right) = \boxed{12 \text{ h}}$$

11.68 (a) From the thermal conductivity equation, $P = \kappa A\left[\left(T_h - T_c\right)/L\right]$, the total energy lost by conduction through the insulation during the 24-h period will be

$$Q = P_1(12.0 \text{ h}) + P_2(12.0 \text{ h}) = \frac{\kappa A}{L}\left[(37.0°\text{C} - 23.0°\text{C}) + (37.0°\text{C} - 16.0°\text{C})\right](12.0 \text{ h})$$

or $$Q = \frac{(0.012\,0 \text{ J/s}\cdot\text{m}°\text{C})\left(0.490 \text{ m}^2\right)}{9.50\times 10^{-2} \text{ m}}[14.0°\text{C} + 21.0°\text{C}](12.0 \text{ h})\left(\frac{3\,600 \text{ s}}{1 \text{ h}}\right) = 9.36\times 10^4 \text{ J}$$

The mass of molten wax which will give off this much energy as it solidifies (all at 37°C) is

$$m = \frac{Q}{L_f} = \frac{9.36\times 10^4 \text{ J}}{205\times 10^3 \text{ J/kg}} = \boxed{0.457 \text{ kg}}$$

(b) If the test samples and the inner surface of the insulation are preheated to 37.0°C during the assembly of the box, $\boxed{\text{nothing undergoes a temperature change}}$ during the test period. Thus, the masses of the samples and insulation do not enter into the calculation. Only the duration of the test, inside and outside temperatures, along with the surface area, thickness, and thermal conductivity of the insulation need to be known.

11.69 The total power radiated by the Sun is $P = \sigma AeT^4$, where $\sigma = 5.669\,6\times 10^{-8} \text{ W/m}^2\cdot\text{K}^4$, the emissivity is $e = 0.986$, the surface area (a sphere) is $A = 4\pi r^2$, and the absolute temperature is $T = 5\,800$ K. Thus,

$$P = \left(5.669\,6\times 10^{-8} \text{ W/m}^2\cdot\text{K}^4\right)4\pi\left(6.96\times 10^8 \text{ m}\right)^2(0.986)(5\,800 \text{ K})^4$$

continued on next page

or $P = 3.85 \times 10^{26}$ W. Thus, the energy radiated each second is

$$E = P \cdot \Delta t = \left(3.85 \times 10^{26} \text{ J/s}\right)(1.00 \text{ s}) = \boxed{3.85 \times 10^{26} \text{ J}}$$

11.70 We approximate the latent heat of vaporization of water on the skin (at 37°C) by asking how much energy would be needed to raise the temperature of 1.0 kg of water to the boiling point and evaporate it. The answer is

$$L_v^{37°\text{C}} \approx c_{\text{water}}(\Delta T) + L_v^{100°\text{C}} = (4\,186 \text{ J/kg}\cdot°\text{C})(100°\text{C} - 37°\text{C}) + 2.26 \times 10^6 \text{ J/kg}$$

or $\quad L_v^{37°\text{C}} \approx 2.5 \times 10^6 \text{ J/kg}$

Assuming that you are approximately 2.0 m tall and 0.30 m wide, you will cover an area of $A = (2.0 \text{ m})(0.30 \text{ m}) = 0.60 \text{ m}^2$ of the beach, and the energy you receive from the sunlight in one hour is

$$Q = IA(\Delta t) = \left(1\,000 \text{ W/m}^2\right)\left(0.60 \text{ m}^2\right)(3\,600 \text{ s}) = 2.2 \times 10^6 \text{ J}$$

The quantity of water this much energy could evaporate from your body is

$$m = \frac{Q}{L_v^{37°\text{C}}} \approx \frac{2.2 \times 10^6 \text{ J}}{2.5 \times 10^6 \text{ J/kg}} = \boxed{0.9 \text{ kg}}$$

The volume of this quantity of water is $V = \dfrac{m}{\rho} = \dfrac{0.9 \text{ kg}}{10^3 \text{ kg/m}^3} \approx 10^{-3} \text{ m}^3 = 1 \text{ L}$

Thus, you will need to drink almost a liter of water each hour to stay hydrated. Note, of course, that any perspiration that drips off your body does not contribute to the cooling process, so drink up!

11.71 During the first 50 minutes, the energy input is used converting m kilograms of ice at 0°C into liquid water at 0°C. The energy required is $Q_1 = mL_f = m\left(3.33 \times 10^5 \text{ J/kg}\right)$, so the constant power input must be

$$P = \frac{Q_1}{(\Delta t)_1} = \frac{m\left(3.33 \times 10^5 \text{ J/kg}\right)}{50 \text{ min}}$$

During the last 10 minutes, the same constant power input raises the temperature of water having a total mass of $(m + 10 \text{ kg})$ by 2.0°C. The power input needed to do this is

$$P = \frac{Q_2}{(\Delta t)_2} = \frac{(m + 10 \text{ kg})c(\Delta T)}{(\Delta t)_2} = \frac{(m + 10 \text{ kg})(4\,186 \text{ J/kg}\cdot°\text{C})(2.0°\text{C})}{10 \text{ min}}$$

Since the power input is the same in the two periods, we have

$$\frac{m\left(3.33 \times 10^5 \text{ J/kg}\right)}{50 \text{ min}} = \frac{(m + 10 \text{ kg})(4\,186 \text{ J/kg}\cdot°\text{C})(2.0°\text{C})}{10 \text{ min}}$$

which simplifies to $(8.0)m = m + 10 \text{ kg}$ or $\quad m = \dfrac{10 \text{ kg}}{7.0} = \boxed{1.4 \text{ kg}}$

11.72 (a) First, energy must be removed from the liquid water to cool it to 0°C. Next, energy must be removed from the water at 0°C to freeze it, which corresponds to a liquid-to-solid phase transition. Finally, once all the water has frozen, additional energy must be removed from the ice to cool it from 0°C to –8.00°C.

(b) The total energy that must be removed is

$$Q = \left|Q_{\substack{\text{cool water}\\ \text{to } 0°\text{C}}}\right| + \left|Q_{\substack{\text{freeze}\\ \text{at } 0°\text{C}}}\right| + \left|Q_{\substack{\text{cool ice}\\ \text{to } -8.00°\text{C}}}\right| = m_w c_w \left|0°\text{C} - T_i\right| + m_w L_f + m_w c_{\text{ice}} \left|T_f - 0°\text{C}\right|$$

or

$$Q = \left(75.0\times10^{-3}\ \text{kg}\right)\left[\left(4\,186\ \frac{\text{J}}{\text{kg}\cdot°\text{C}}\right)\left|-20.0\ °\text{C}\right| + 3.33\times10^{5}\ \frac{\text{J}}{\text{kg}} + \left(2\,090\ \frac{\text{J}}{\text{kg}\cdot°\text{C}}\right)\left|-8.00\ °\text{C}\right|\right]$$

$$= 3.25\times10^{4}\ \text{J} = \boxed{32.5\ \text{kJ}}$$

11.73 (a) In steady state, the energy transfer rate is the same for each of the rods, or $P_{\text{Al}} = P_{\text{Fe}}$.

Thus,

$$\kappa_{\text{Al}} A\left(\frac{100°\text{C} - T}{L}\right) = \kappa_{\text{Fe}} A\left(\frac{T - 0°\text{C}}{L}\right)$$

giving

$$T = \left(\frac{\kappa_{\text{Al}}}{\kappa_{\text{Al}} + \kappa_{\text{Fe}}}\right)(100°\text{C}) = \left(\frac{238}{238 + 79.5}\right)(100°\text{C}) = \boxed{75.0°\text{C}}$$

(b) If $L = 15$ cm and $A = 5.0\ \text{cm}^2$, the energy conducted in 30 min is

$$Q = P_{\text{Al}}\cdot t = \left[\left(238\ \frac{\text{W}}{\text{m}\cdot°\text{C}}\right)\left(5.0\times10^{-4}\ \text{m}^2\right)\left(\frac{100°\text{C} - 75.0°\text{C}}{0.15\ \text{m}}\right)\right]\left(1\,800\ \text{s}\right)$$

$$= 3.6\times10^{4}\ \text{J} = \boxed{36\ \text{kJ}}$$

12

The Laws of Thermodynamics

QUICK QUIZZES

1. Choice (b). The magnitude of the work done on a gas during a thermodynamic process is equal to the area under the curve on a *PV* diagram. Processes in which the volume decreases do positive work on the gas, while processes in which the volume increases do negative work on the gas. The work done on the gas in each of the four processes shown is

$$W_a = -4.00 \times 10^5 \text{ J},\ W_b = +3.00 \times 10^5 \text{ J},\ W_c = -3.00 \times 10^5 \text{ J, and } W_d = +4.00 \times 10^5 \text{ J}$$

Thus, the correct ranking (from most negative to most positive) is a, c, b, d.

2. *A* is isovolumetric, *B* is adiabatic, *C* is isothermal, *D* is isobaric.

3. Choice (c). The highest theoretical efficiency of an engine is the Carnot efficiency given by $e_C = 1 - T_c/T_h$. Hence, the theoretically possible efficiencies of the given engines are

$$e_\text{A} = 1 - \frac{700 \text{ K}}{1\,000 \text{ K}} = 0.300,\ e_\text{B} = 1 - \frac{500 \text{ K}}{800 \text{ K}} = 0.375, \quad \text{and} \quad e_\text{C} = 1 - \frac{300 \text{ K}}{600 \text{ K}} = 0.500$$

and the correct ranking (from highest to lowest) is C, B, A.

4. Choice (b). $\Delta S = \dfrac{Q_r}{T} = 0$ and $Q = 0$ in an adiabatic process. If the process was reversible, but not adiabatic, the entropy of the system could undergo a non-zero change. However, in that case, the entropy of the system's surroundings would undergo a change of equal magnitude but opposite sign, and the total change of entropy in the universe would be zero. If the process was irreversible, the total entropy of the universe would increase.

5. The number 7 is the most probable outcome because there are six ways this could occur: 1-6, 2-5, 3-4, 4-3, 5-2, and 6-1. The numbers 2 and 12 are the least probable because they could only occur one way each: either 1-1, or 6-6. Thus, you are six times more likely to throw a 7 than a 2 or 12.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. For a monatomic ideal gas, the internal energy is $U = \frac{3}{2}nRT$ and the change in internal energy is $\Delta U = \frac{3}{2}nR(\Delta T)$. From the ideal gas law, $PV = nRT$, observe that $nR(\Delta T) = nRT_f - nRT_i = P_fV_f - P_iV_i$. When the pressure is constant, $P_f = P_i = P$, this reduces to $\Delta U = \frac{3}{2}\left[P\left(V_f - V_i\right)\right] = \frac{3}{2}P(\Delta V)$, so the change in internal energy for this gas is $\Delta U = \frac{3}{2}\left(2.00 \times 10^5 \text{ Pa}\right)\left(+1.50 \text{ m}^3\right) = 4.50 \times 10^5$ J, making (c) the correct answer.

2. When volume is constant, the work done on the gas is zero, so the first law of thermodynamics gives the change in internal energy as $\Delta U = Q + W = 100 \text{ J} + 0 = 100$ J, and (d) is the correct answer for this question.

3. The work done by the system on the environment is

$$W_{\text{env}} = +P(\Delta V) = (70.0\times10^3\ \text{Pa})(-0.20\ \text{m}^3) = -14\times10^3\ \text{J} = -14\ \text{kJ}$$

and (c) is the correct choice.

4. In a cyclic process, the net work done equals the area enclosed by the process curve in a *PV* diagram. Thus,

$$W_{\text{net}} = \left[(4.00-1.00)\times10^5\ \text{Pa}\right]\left[(2.00-1.00)\ \text{m}^3\right] + \left[(2.00-1.00)\times10^5\ \text{Pa}\right]\left[(3.00-2.00)\ \text{m}^3\right] = 4.00\times10^5\ \text{J}$$

and (d) is the correct answer.

5. For an adiabatic process, $PV^\gamma = \text{constant}$, where $\gamma = c_p/c_v = 1.4$ for diatomic gases. (See Table 12.1 in the textbook.) Thus, $P_f V_f^{1.4} = P_i V_i^{1.4}$, or the final pressure will be

$$P_f = P_i\left(\frac{V_i}{V_f}\right)^{1.4} = (1.00\times10^5\ \text{Pa})\left(\frac{1.00\ \text{m}^3}{3.50\ \text{m}^3}\right)^{1.4} = 1.73\times10^4\ \text{Pa}$$

and the correct response is (e).

6. The work an ideal gas does on the environment during an isothermal expansion is $W_{\text{env}} = nRT\ln(V_f/V_i)$, so for the given process,

$$W_{\text{env}} = (3.9\times10^2\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})(850\ \text{K})\ln(2) = 1.9\times10^6\ \text{J}$$

and (a) is the correct choice.

7. From conservation of energy, the energy input to the engine must be

$$Q_h = W_{\text{eng}} + Q_c = 15\ \text{kJ} + 37\ \text{kJ} = 52\ \text{kJ}$$

so the efficiency is

$$e = \frac{W_{\text{eng}}}{Q_h} = \frac{15\ \text{kJ}}{52\ \text{kJ}} = 0.29\ \text{or}\ 29\%$$

and the correct choice is (b).

8. The coefficient of performance of this refrigerator is

$$\text{COP} = \frac{|Q_c|}{W} = \frac{115\ \text{kJ}}{18\ \text{kJ}} = 6.4 \quad \text{which is choice (d).}$$

9. The internal energy of an ideal gas is a function of the temperature alone. Thus, for the isothermal compression, the internal energy remains constant or choice (d) is true. Work must be done on the gas to compress it into a smaller volume (see Equation 12.10 in the text), so choice (b) is false. If work is done on the gas in compression, but the internal energy is constant, the first law of thermodynamics says that energy must be transferred *from* of the gas by heat, making choice (a) false. Also, since the process is isothermal, choice (c) is false. Thus, the only true choice listed is (d).

10. At a pressure of 1.0 atm, ice melts at absolute temperature $T_K = 0° + 273.15 = 273.15\ \text{K}$. The thermal energy this block of ice must absorb to fully melt is

$$Q_r = mL_f = (1.00\ \text{kg})(3.33\times10^5\ \text{J/kg}) = 3.33\times10^5\ \text{J}$$

so the change in entropy of the ice is

$$\Delta S = \frac{Q_r}{T_K} = \frac{+3.33\times10^5 \text{ J}}{273.15 \text{ K}} = +1.22\times10^3 \text{ J/K} = +1.22 \text{ kJ/K}$$

and (d) is the correct choice.

11. The maximum theoretical efficiency (the Carnot efficiency) of a device operating between absolute temperatures $T_c < T_h$ is $e_C = 1 - T_c/T_h$. For the given steam turbine, this is

$$e_C = 1 - \frac{3.0\times10^2 \text{ K}}{450 \text{ K}} = 0.33 \text{ or } 33\% \quad \text{and (c) is the correct answer.}$$

12. By definition, in an adiabatic process, no energy is transferred to or from the gas by heat. Thus, (c) is a true statement. All other choices are false. In an expansion process, the gas does work on the environment. Since there is no energy input by heat, the first law of thermodynamics says that the internal energy of the ideal gas must decrease, meaning the temperature will decrease. Also, in an adiabatic process, $PV^\gamma =$ constant, meaning that the pressure must decrease as the volume increases in this adiabatic expansion.

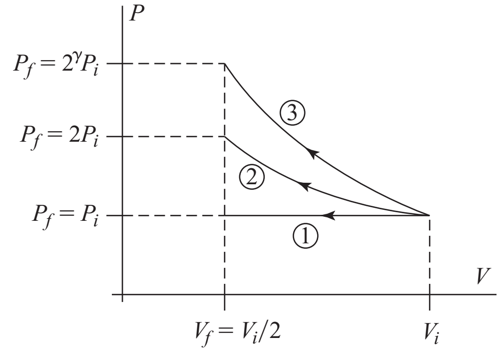
13. In an isobaric process on an ideal gas, pressure is constant while the gas either expands or is compressed. Since the volume of the gas is changing, work is done either on or by the gas, so choice (b) is a true statement. Also, from the ideal gas law with pressure constant, $P(\Delta V) = nR(\Delta T)$. Thus, the gas must undergo a change in temperature having the same sign as the change in volume. If $\Delta V > 0$, then both ΔT and the change in the internal energy of the gas are positive $(\Delta U > 0)$. However, when $\Delta V > 0$, the work done *on the gas* is negative $(W < 0)$, and the first law of thermodynamics says that there must be a positive transfer of energy by heat to the gas $(Q = \Delta U - W > 0)$. When $\Delta V < 0$, a similar argument shows that $\Delta U < 0$, $W > 0$, and $Q = \Delta U - W < 0$. Thus, all of the other listed choices are false statements.

14. We know that $T_{A,i} = T_{B,i}$, $P_{A,i} = P_{B,i}$, and that $V_A = V_B =$ constant. From the ideal gas law, $n = PV/RT$, we see that $n_A = n_B$. Thus, both choices (c) and (d) are false. Also from the ideal gas law, $\Delta T = (\Delta P)V/nR$ with $(\Delta P)_A > (\Delta P)_B$, we see that $(\Delta T)_A > (\Delta T)_B$. However, the first law of thermodynamics $(\Delta U = Q + W = Q + 0 = Q)$ tells us that $(\Delta U)_A = (\Delta U)_B$ since the two gases received equal amounts of thermal energy. Since, for an ideal gas, $\Delta U = nC_v\Delta T$, the ratio of the molar heat capacities of the two gases must be

$$\frac{C_{v,A}}{C_{v,B}} = \frac{\Delta U_A/n_A\Delta T_A}{\Delta U_B/n_B\Delta T_B} = \frac{\Delta T_B}{\Delta T_A} < 1 \qquad \text{or} \qquad C_{v,A} < C_{v,B}$$

This difference in the molar heat capacities would be explained if gas *A* was monatomic while gas *B* was diatomic. Hence, we see that the only true statements among the listed choices are (a) and (e).

15. The work done on the gas equals the area under the process curve in a PV diagram. In an isobaric process, the pressure is constant, so $P_f = P_i$ and the work done is the area under curve 1 in the sketch at the right. For an isothermal process, the ideal gas law gives $P_fV_f = P_iV_i$, so $P_f = (V_i/V_f)P_i = 2P_i$ and the work done is the area under curve 2 in the sketch. Finally, for an adiabatic process, $P_fV_f^\gamma = P_iV_i^\gamma =$ constant, so $P_f = (V_i/V_f)^\gamma P_i$ and $P_f = 2^\gamma P_i > 2P_i$ since $\gamma > 1$ for all ideal gases (see Table 12.1 in the textbook). The work done in an adiabatic process is the area under curve 3, which exceeds that done in either of the other processes. Thus, the correct choice is (b), the adiabatic process involves the most work.



16. The Clausius statement of the second law of thermodynamics says that in any thermodynamics process, reversible or irreversible, the total entropy of the universe must either remain constant (reversible process) or increase (irreversible process). Thus, if in a thermodynamics process, the entropy of a system changes by $-6\ \text{J/K}$, the entropy of the environment (i.e., the rest of the universe) must increase by $+6\ \text{J/K}$ or more. The correct choice here is (e).

17. Melting is a constant temperature process, so the change in entropy as the substance melts is given by $\Delta S = Q_r/T$, where $T > 0$ is the absolute temperature at which the process occurs and Q_r is the energy transferred as heat during the process. Since the substance must absorb energy ($Q_r > 0$) as it melts, we see that $\Delta S > 0$ and the correct answer is choice (b).

18. With this method of using an air conditioner, the average temperature in the room will increase, and choice (a) is the correct answer. The air conditioner operates on a cyclic process so the change in the internal energy of the refrigerant is zero. Then the conservation of energy gives the thermal energy exhausted to the room as $Q_h = Q_c + W_{\text{eng}}$, where Q_c is the thermal energy the air conditioner removes from the room and W_{eng} is the work done to operate the device. Since $W_{\text{eng}} > 0$, the air conditioner is returning more thermal energy to the room than it is removing, so the average temperature in the room will increase.

ANSWERS TO EVEN-NUMBERED CONCEPTUAL QUESTIONS

2. (a) Shaking opens up spaces between the jelly beans. The smaller ones have a chance of falling down into spaces below them. The accumulation of larger ones on top and smaller ones on the bottom implies an increase in order and a decrease in one contribution to the total entropy.

(b) The second law is not violated and the total entropy of the system increases. The increase in the internal energy of the system comes from the work required to shake the jar of beans (that is, work your muscles must do, with an increase in entropy accompanying the biological process) and also from the small loss of gravitational potential energy as the beans settle together more compactly.

4. Temperature = a measure of molecular motion. Heat = the process through which energy is transferred between objects by means of random collisions of molecules. Internal energy = energy associated with random molecular motions plus chemical energy, strain potential energy, and an object's other energy not associated with center of mass motion or location.

6. A higher steam temperature means that more energy can be extracted from the steam. For a constant temperature heat sink at T_c and steam at T_h, the maximum efficiency of the power plant varies as

$$e_C = \frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h}$$

and is maximized for high T_h.

8. Assuming an air temperature of 20°C above the surface of the pond, the difference in temperature between the lower layer of the pond and the atmosphere is

$$\Delta T = T_h - T_c = (100° + 273) - (20° + 273) = 80\ \text{K}$$

and the Carnot efficiency is $$e_{\max} = e_C = \frac{T_h - T_c}{T_h} = \frac{80\ \text{K}}{373\ \text{K}} \approx 21\%$$

10. A slice of hot pizza cools off. Road friction brings a skidding car to a stop. A cup falls to the floor and shatters. Any process is irreversible if it looks funny or frightening when shown in a videotape running backward. At fairly low speeds, air resistance is small and the flight of a projectile is nearly reversible.

12. While the entropy of the sugar solution decreases by some amount, the second law of thermodynamics is not violated if the entropy of the environment increases by an equal or greater amount. Even at essentially constant temperature, energy must be transferred by heat out of the solidifying sugar into the surroundings. This action will increase the entropy of the environment. The water molecules become less ordered as they leave the liquid in the container to mix with the entire atmosphere.

14. Heat is not a form of energy. Rather, it is the process of transferring energy through microscopic collisions between atoms or molecules. An object having a large mass and/or a high specific heat may be at a low temperature and still contain more internal energy than does a higher temperature, low-mass object.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 6.1×10^{2} J **(b)** 0 **(c)** -4.1×10^{2} J

(d) 0 **(e)** $+2.0\times10^{2}$ J

4. **(a)** 31 m/s **(b)** 0.17

6. **(a)** See Solution. **(b)** See Solution.

(c) The higher pressure during the expansion phase of the process results in more work being done in the process of part (a).

8. **(a)** 1.25 MJ **(b)** -1.25 MJ

10. **(a)** –12 MJ **(b)** +12 MJ

12. **(a)** 8.12×10^{4} Pa **(b)** 3.65×10^{4} J **(c)** -5.68×10^{4} J

(d) 977 K **(e)** 1.22×10^{5} J **(f)** 8.55×10^{4} J

(g) 1.42×10^{5} J **(h)** additional energy must be transferred to the gas

(i) 1.42×10^{5} J **(j)** $Q=\Delta U-W$

14. **(a)** -2.7×10^{5} J

(b) energy is transferred from the sprinter to the environment by heat

16. $6P_0V_0$

18. $W=+\frac{3}{2}P_0V_0>0,\quad \Delta U=0,\quad Q=-\frac{3}{2}P_0V_0<0$

20. **(a)** –395 J **(b)** 285 K; at point A **(c)** 37 J

22. **(a)** See Solution. **(b)** See Solution. **(c)** See Solution.

(d) No. From part (c), $Q = \frac{5}{2} W_{\text{env}}$, and W_{env} is positive for an isobaric expansion process. Thus, the energy transfer by heat must be directed into the gas.

24. $W_{BC} = 0,\ Q_{BC} < 0,\ \Delta U_{BC} < 0;\ W_{CA} > 0,\ Q_{CA} < 0,\ \Delta U_{CA} < 0;\ W_{AB} < 0,\ Q_{AB} > 0,\ \Delta U_{AB} > 0$

26. **(a)** $W_{IAF} = -76.0$ J, $W_{IBF} = -101$ J, $W_{IF} = -88.7$ J

(b) $Q_{IAF} = 167$ J, $Q_{IBF} = 192$ J, $Q_{IF} = 180$ J

28. **(a)** $1.66 \times 10^{-24}\ \text{mol/m}^3$ **(b)** 3.7×10^{-23} Pa **(c)** 6.2×10^{116} Pa

30. 600 kJ

32. **(a)** 564°C

(b) No. Any real heat engine operates in an irreversible manner and has an efficiency less than the Carnot efficiency.

34. **(a)** 4 200 J **(b)** 0.43 (or 43%)

36. 13.7°C

38. **(a)** 7.69×10^8 J **(b)** 5.67×10^8 J

40. **(a)** $\text{COP} = T_h / (T_h - T_c)$ **(b)** smaller **(c)** 17.2

42. **(a)** 0.051 2 (or 5.12%) **(b)** 5.27×10^{12} J

(c) As fossil-fuel prices rise, this could be an attractive way to use solar energy. However, the potential environmental impact of such an engine would require serious study. The energy output, $|Q_c| = |Q_h| - W_{\text{eng}} = |Q_h|(1-e)$, to the low temperature reservoir (cool water deep in the ocean) could raise the temperature of over a million cubic meters of water by 1°C every hour.

44. **(a)** 9.09 kW **(b)** 11.9 kJ

46. **(a)** $+79$ J/K **(b)** -79 J/K

48. 6.06 kJ/K

50. **(a)** one way; 6+6 **(b)** 6 ways; 1+6, 2+5, 3+4, 4+3, 5+2, and 6+1

52. **(a)** -3.45 J/K **(b)** $+8.06$ J/K **(c)** $+4.61$ J/K

(d) Thermal energy always flows from $(Q_r < 0)$ the hot reservoir at T_h and into $(Q_r > 0)$ the cooler reservoir at T_c. Thus, in the expression for the total change in entropy, $\Delta S = Q_r/T_c + Q_r/T_h = |Q_r|/T_c - |Q_r|/T_h$, the positive term has the smaller denominator and dominates the sum.

54. **(a)** $\Delta S_h = -|Q_h|/T_h$ **(b)** $\Delta S_c = +|Q_h|/T_c$

(c) $\Delta S_{\text{Universe}} = -|Q_h|/T_h + |Q_h|/T_c$

56. **(a)** 6.54 h **(b)** 0.717 h **(c)** 2.35×10^{3} J

(d) 16.6 lifts **(e)** No, the human body is only about 25% efficient in converting chemical energy to mechanical energy.

58. 3.01

60. 0.55 kg

62. **(a)** 251 J **(b)** 314 J **(c)** 104 J done *by* the gas

(d) 104 J done *on* the gas **(e)** $\Delta U = 0$ (cyclic process)

64. **(a)** $4P_0V_0$ **(b)** $4P_0V_0$ **(c)** 9.07 kJ

66. 8×10^{6} J/K·s

68. **(a)** 1.00×10^{5} Pa **(b)** 1.31×10^{5} J **(c)** 3.48×10^{6} J

(d) 9.28×10^{3} K; 2.12×10^{6} Pa **(e)** 0.259 kg; 0.494 kg/m^3 **(f)** 2.93×10^{3} m/s

70. **(a)** 2.4×10^{6} J **(b)** 1.6×10^{6} J **(c)** 2.8×10^{2} J

72. **(a)** -6.00×10^{5} J **(b)** 5.50×10^{2} K **(c)** 1.00×10^{6} J

(d) 1.60×10^{6} J

PROBLEM SOLUTIONS

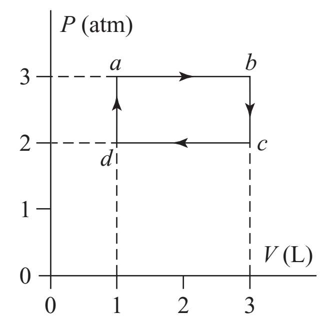
12.1 (a) The work done on the gas in this constant pressure process is

$$W = -P(\Delta V) = -P\left(\frac{nRT_f}{P} - \frac{nRT_i}{P}\right) = -nR\left(T_f - T_i\right)$$

or $W = -(0.200\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(573\text{ K} - 293\text{ K}) = \boxed{-465\text{ J}}$

(b) The negative sign for work done on the gas indicates that $\boxed{\text{the gas does positive work on its surroundings}}$.

12.2 (a) $\left(W_{\text{env}}\right)_{ab} = P_a\left(V_b - V_a\right)$



$$= \left[3\left(1.013\times10^{5}\text{ Pa}\right)\right](3.0\text{ L} - 1.0\text{ L})\left(\frac{10^{-3}\text{ m}^3}{1\text{ L}}\right)$$

$$= \boxed{6.1\times10^{2}\text{ J}}$$

(b) $\left(W_{\text{env}}\right)_{bc} = P\left(V_c - V_b\right) = \boxed{0}$

(c) $\left(W_{\text{env}}\right)_{cd} = P_c\left(V_d - V_c\right)$

$$= \left[2\left(1.013\times10^{5}\text{ Pa}\right)\right](1.0\text{ L} - 3.0\text{ L})\left(\frac{10^{-3}\text{ m}^3}{1\text{ L}}\right) = \boxed{-4.1\times10^{2}\text{ J}}$$

continued on next page

(d) $\left(W_{\text{env}}\right)_{da} = P\left(V_a - V_d\right) = \boxed{0}$

(e) $\left(W_{\text{env}}\right)_{\text{net}} = \left(W_{\text{env}}\right)_{ab} + \left(W_{\text{env}}\right)_{bc} + \left(W_{\text{env}}\right)_{cd} + \left(W_{\text{env}}\right)_{da}$

$$= +6.1\times10^2\ \text{J} + 0 - 4.1\times10^2\ \text{J} + 0 = \boxed{+2.0\times10^2\ \text{J}}$$

Note that, to 2 significant figures, this equals the area enclosed within the process diagram given above.

12.3 The constant pressure is $P = (1.5\ \text{atm})\left(1.013\times10^5\ \text{Pa/atm}\right)$ and the work done on the gas is $W = -P(\Delta V)$.

(a) $\Delta V = +4.0\ \text{m}^3$ and

$$W = -P(\Delta V) = -(1.5\ \text{atm})\left(1.013\times10^5\ \text{Pa/atm}\right)\left(+4.0\ \text{m}^3\right) = \boxed{-6.1\times10^5\ \text{J}}$$

(b) $\Delta V = -3.0\ \text{m}^3$, so

$$W = -P(\Delta V) = -(1.5\ \text{atm})\left(1.013\times10^5\ \text{Pa/atm}\right)\left(-3.0\ \text{m}^3\right) = \boxed{+4.6\times10^5\ \text{J}}$$

12.4 (a) The work done by the gas on the projectile is given by the area under the curve in the *PV* diagram. This is

$$W_{\text{env}} = (\text{triangular area}) + (\text{rectangular area})$$

$$= \frac{1}{2}\left(P_0 - P_f\right)\left(V_f - V_0\right) + P_f\left(V_f - V_0\right) = \frac{1}{2}\left(P_0 + P_f\right)\left(V_f - V_0\right)$$

$$= \frac{1}{2}\left[(11+1.0)\times10^5\ \text{Pa}\right]\left[(40.0-8.0)\ \text{cm}^3\right]\left(\frac{1\ \text{m}^3}{10^6\ \text{cm}^3}\right) = 19\ \text{J}$$

From the work-energy theorem, $W_{\text{env}} = \Delta KE = \frac{1}{2}mv^2 - 0$, where W_{env} is the work done on the projectile by the gas. Thus, the speed of the emerging projectile is

$$v = \sqrt{\frac{2W_{\text{env}}}{m}} = \sqrt{\frac{2(19\ \text{J})}{40.0\times10^{-3}\ \text{kg}}} = \boxed{31\ \text{m/s}}$$

(b) The air in front of the projectile would exert a retarding force of

$$F_r = P_{\text{air}}A = \left(1.0\times10^5\ \text{Pa}\right)\left[\left(1.0\ \text{cm}^2\right)\left(1\ \text{m}^2/10^4\ \text{cm}^2\right)\right] = 10\ \text{N}$$

on the projectile as it moves down the launch tube. The energy spent overcoming this retarding force would be

$$W_{\text{spent}} = F_r \cdot s = (10\ \text{N})(0.32\ \text{m}) = 3.2\ \text{J}$$

and the needed fraction is $\dfrac{W_{\text{spent}}}{W_{\text{env}}} = \dfrac{3.2\ \text{J}}{19\ \text{J}} = \boxed{0.17}$

12.5 In each case, the work done on the gas is given by the negative of the area under the path on the *PV* diagram. Along those parts of the path where volume is constant, no work is done. Note that $1\ \text{atm} = 1.013\times10^5\ \text{Pa}$ and $1\ \text{Liter} = 10^{-3}\ \text{m}^3$.

continued on next page

(a) $W_{IAF} = W_{IA} + W_{AF} = -P_I\left(V_A - V_I\right) + 0$

$$= -\left[4.00\left(1.013\times10^5\ \text{Pa}\right)\right]\left[(4.00-2.00)\times10^{-3}\ \text{m}^3\right] = \boxed{-810\ \text{J}}$$

(b) $W_{IF} = -(\text{triangular area}) - (\text{rectangular area})$

$$= -\frac{1}{2}\left(P_I - P_B\right)\left(V_F - V_B\right) - P_B\left(V_F - V_B\right) = -\frac{1}{2}\left(P_I + P_B\right)\left(V_F - V_B\right)$$

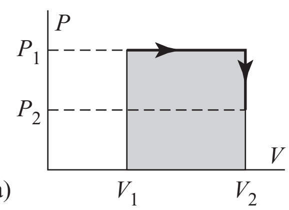
$$= -\frac{1}{2}\left[(4.00+1.00)\left(1.013\times10^5\ \text{Pa}\right)\right](4.00-2.00)\times10^{-3}\ \text{m}^3$$

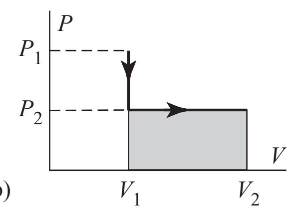
$$= \boxed{-507\ \text{J}}$$

(c) $W_{IBF} = W_{IB} + W_{BF} = 0 - P_B\left(V_F - V_B\right)$

$$= -\left[1.00\left(1.013\times10^5\ \text{Pa}\right)\right]\left[(4.00-2.00)\times10^{-3}\ \text{m}^3\right] = \boxed{-203\ \text{J}}$$

12.6 The sketches for (a) and (b) are shown below:





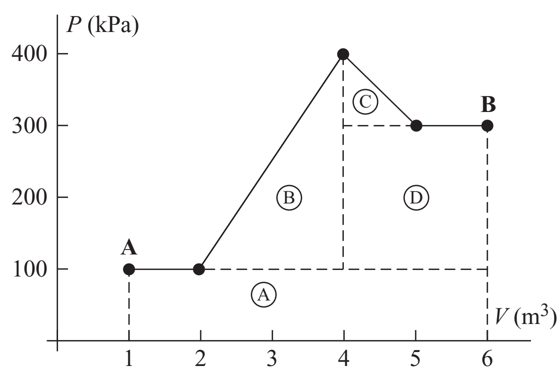
(c) As seen from the areas under the paths in the *PV* diagrams above, the higher pressure during the expansion phase of the process results in more work done *by* the gas in (a) than in (b).

12.7 With $P_f = P_i = P$, the ideal gas law gives $P_f V_f - P_i V_i = P(\Delta V) = nR(\Delta T)$, so the work done by the gas is $W_{\text{env}} = +P(\Delta V) = nR(\Delta T) = \left(\dfrac{m}{M_{\text{He}}}\right)R\left(T_f - T_i\right)$

If $W_{\text{env}} = 20.0$ J, the mass of helium in the gas sample is

$$m = \frac{W_{\text{env}}\left(M_{\text{He}}\right)}{R\left(T_f - T_i\right)} = \frac{(20.0\ \text{J})(4.00\ \text{g/mol})}{(8.31\ \text{J/mol}\cdot\text{K})(373\ \text{K} - 273\ \text{K})} = 0.0963\ \text{g} = \boxed{96.3\ \text{mg}}$$

12.8 (a) The work done by the gas as it expands from point A to point B is given by the area under the *PV* diagram between these points. Consider the sketch given at the right and observe that this area can be broken into two rectangular areas and two triangular areas. The total area is given by



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$$W_{\text{env}} = (100 \text{ kPa})(5.00 \text{ m}^3) + \frac{1}{2}(300 \text{ kPa})(2.00 \text{ m}^3)$$

$$+\frac{1}{2}(100 \text{ kPa})(1.00 \text{ m}^3) + (200 \text{ kPa})(2.00 \text{ m}^3)$$

or

$$W_{\text{env}} = 1250 \ (\text{kPa})(\text{m}^3) = 1.25\times10^6 \ (\text{N/m}^2)(\text{m}^3) = \boxed{1.25 \text{ MJ}}$$

(b) When the volume is decreasing, the work done by the gas is the negative of the area under the PV diagram. Thus, if the gas is compressed from point B to point A along the same path,

$$W_{\text{env}} = \boxed{-1.25 \text{ MJ}}$$

12.9 (a) From the ideal gas law, $nR = PV_f/T_f = PV_i/T_i$. With pressure constant this gives

$$T_f = T_i\left(\frac{V_f}{V_i}\right) = (273 \text{ K})(4) = \boxed{1.09\times10^3 \text{ K}}$$

(b) The work done on the gas is

$$W = -P(\Delta V) = -(PV_f - PV_i) = -nR(T_f - T_i) = -nR(4T_i - T_i)$$

$$= -(1.00 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})[3(273 \text{ K})] = -6.81\times10^3 \text{ J} = \boxed{-6.81 \text{ kJ}}$$

12.10 (a) The work done *on* the fluid is the negative $(\text{since } V_f > V_i)$ of the area under the curve on the PV diagram. Thus,

$$W_{if} = -\left\{(6\times10^6 \text{ Pa} - 0)(2 \text{ m}^3 - 1 \text{ m}^3) + \frac{1}{2}\left[(6-2)\times10^6 \text{ Pa}\right](2 \text{ m}^3 - 1 \text{ m}^3)\right.$$

$$\left. + (2.00\times10^6 \text{ Pa} - 0)(4 \text{ m}^3 - 2 \text{ m}^3)\right\}$$

$$W_{if} = -12\times10^6 \text{ J} = \boxed{-12 \text{ MJ}}$$

(b) When the system follows the process curve in the reverse direction $(\text{with } V_f < V_i)$, the work done *on* the fluid equals the area under the process curve, which is the negative of that computed in (a). Thus,

$$W_{fi} = -W_{if} = \boxed{+12 \text{ MJ}}$$

12.11 From kinetic theory, the average kinetic energy per molecule is

$$\overline{KE}_{\text{molecule}} = \frac{3}{2}k_{\text{B}}T = \frac{3}{2}\left(\frac{R}{N_{\text{A}}}\right)T$$

For a monatomic ideal gas containing N molecules, the total energy associated with random molecular motions is

$$U = N\cdot\overline{KE}_{\text{molecule}} = \frac{3}{2}\left(\frac{N}{N_{\text{A}}}\right)RT = \frac{3}{2}nRT$$

Since $PV = nRT$ for an ideal gas, the internal energy of a monatomic ideal gas is found to be given by $\boxed{U = \frac{3}{2}PV}$.

12.12 (a) The initial absolute temperature is $T_i = 20.0° + 273.15 = 293\text{ K}$, so the initial pressure is

$$P_i = \frac{nRT_i}{V_i} = \frac{(10.0\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(293\text{ K})}{0.300\text{ m}^3} = \boxed{8.12\times10^4\text{ Pa}}$$

(b) For a monatomic ideal gas, the internal energy is $U = 3nRT/2$. Thus,

$$U_i = \frac{3}{2}nRT_i = \frac{3}{2}(10.0\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(293\text{ K}) = \boxed{3.65\times10^4\text{ J}}$$

(c) The work done on the gas in this isobaric expansion is

$$W = -P(\Delta V) = -(8.12\times10^4\text{ Pa})(1.000\text{ m}^3 - 0.300\text{ m}^3) = \boxed{-5.68\times10^4\text{ J}}$$

(d) $$T_f = \frac{P_f V_f}{nR} = \frac{(8.12\times10^4\text{ Pa})(1.00\text{ m}^3)}{(10.0\text{ mol})(8.31\text{ J/mol}\cdot\text{K})} = \boxed{977\text{ K}}$$

(e) $$U_f = \frac{3}{2}nRT_f = \frac{3}{2}(10.0\text{ mol})(8.31\text{ J/mol}\cdot\text{K})(977\text{ K}) = \boxed{1.22\times10^5\text{ J}}$$

(f) $$\Delta U = U_f - U_i = 1.22\times10^5\text{ J} - 3.65\times10^4\text{ J} = \boxed{8.55\times10^4\text{ J}}$$

(g) $$\Delta U - W = 8.55\times10^4\text{ J} - (-5.68\times10^4\text{ J}) = \boxed{+1.42\times10^5\text{ J}}$$

(h) Since $\Delta U - W > 0$, the increase in the internal energy of the gas exceeds the energy transferred to the gas by work. Thus, $\boxed{\text{additional energy must be transferred to the gas by heat.}}$

(i) The additional energy that must be transferred to the gas by heat is

$$\Delta U - W = \boxed{+1.42\times10^5\text{ J}} \qquad \text{[See part (g) above.]}$$

(j) The suggested relationship is $\boxed{Q = \Delta U - W}$, which is a statement of the first law of thermodynamics.

12.13 (a) Along the direct path *IF*, the work done on the gas is

$$W = -(\textit{area under curve})$$

$$= -\left[(1.00\text{ atm})(4.00\text{ L} - 2.00\text{ L}) + \frac{1}{2}(4.00\text{ atm} - 1.00\text{ atm})(4.00\text{ L} - 2.00\text{ L})\right]$$

$$W = -5.00\text{ atm}\cdot\text{L}$$

Thus, $\Delta U = Q + W = 418\text{ J} - 5.00\text{ atm}\cdot\text{L}$

$$= 418\text{ J} - (5.00\text{ atm}\cdot\text{L})\left(\frac{1.013\times10^5\text{ Pa}}{1\text{ atm}}\right)\left(\frac{10^{-3}\text{ m}^3}{1\text{ L}}\right) = \boxed{-88.5\text{ J}}$$

(b) Along path *IAF*, the work done on the gas is

$$W = -(4.00\text{ atm})(4.00\text{ L} - 2.00\text{ L})\left(\frac{1.013\times10^5\text{ Pa}}{1\text{ atm}}\right)\left(\frac{10^{-3}\text{ m}^3}{1\text{ L}}\right) = -810\text{ J}$$

From the first law, $Q = \Delta U - W = -88.5\text{ J} - (-810\text{ J}) = \boxed{722\text{ J}}$

12.14 (a) We treat the sprinter's body as a thermodynamic system and apply the first law of thermodynamics, $\Delta U = Q + W$. Then, with $\Delta U = -7.5\times10^5$ J and $W = -4.8\times10^5$ J (negative because the sprinter does work on the environment), the energy absorbed as heat is

$$Q = \Delta U - W = -7.5\times10^5 \text{ J} - \left(-4.8\times10^5 \text{ J}\right) = \boxed{-2.7\times10^5 \text{ J}}$$

(b) The negative sign in the answer to part (a) means that $\boxed{\text{energy is transferred from the sprinter to the environment by heat}}$.

12.15 (a) $W = -P(\Delta V) = -(0.800 \text{ atm})(-7.00 \text{ L})\left(\dfrac{1.013\times10^5 \text{ Pa}}{1 \text{ atm}}\right)\left(\dfrac{10^{-3} \text{ m}^3}{1 \text{ L}}\right) = \boxed{567 \text{ J}}$

(b) $\Delta U = Q + W = -400 \text{ J} + 567 \text{ J} = \boxed{167 \text{ J}}$

12.16 The work done on the gas is the negative $\left(\text{since } V_f > V_i\right)$ of the area under the curve on the *PV* diagram, or

$$W = -\left[P_0\left(2V_0 - V_0\right) + \frac{1}{2}\left(2P_0 - P_0\right)\left(2V_0 - V_0\right)\right] = -\frac{3}{2}P_0V_0$$

From the result of Problem 11 (given as a hint in this problem),

$$\Delta U = \frac{3}{2}P_fV_f - \frac{3}{2}P_iV_i = \frac{3}{2}\left(2P_0\right)\left(2V_0\right) - \frac{3}{2}P_0V_0 = \frac{9}{2}P_0V_0$$

Thus, from the first law, $Q = \Delta U - W = \dfrac{9}{2}P_0V_0 - \left(-\dfrac{3}{2}P_0V_0\right) = \boxed{6P_0V_0}$

12.17 (a) The change in the volume occupied by the gas is

$$\Delta V = V_f - V_i = A\left(L_f - L_i\right) = \left(0.150 \text{ m}^2\right)(-0.200 \text{ m}) = -3.00\times10^{-2} \text{ m}^3$$

and the work done by the gas is

$$W_{\text{env}} = +P(\Delta V) = (6\,000 \text{ Pa})\left(-3.00\times10^{-2} \text{ m}^3\right) = \boxed{-180 \text{ J}}$$

(b) The first law of thermodynamics is $\Delta U = Q + W = -Q_{\text{output}} - W_{\text{env}}$. Thus, if $\Delta U = -8.00$ J, the energy transferred out of the system by heat is

$$Q_{\text{output}} = -\Delta U - W_{\text{env}} = -(-8.00 \text{ J}) - (-180 \text{ J}) = \boxed{+188 \text{ J}}$$

12.18 The work done on the gas is the negative $\left(\text{since } V_f < V_i\right)$ of the area under the curve on the *PV* diagram,

so $W = -\left[P_0\left(V_0 - 2V_0\right) + \dfrac{1}{2}\left(2P_0 - P_0\right)\left(V_0 - 2V_0\right)\right] = +\dfrac{3}{2}P_0V_0$, or $\boxed{W > 0}$

The change in the internal energy of this monatomic ideal gas is

$$\Delta U = \frac{3}{2}P_fV_f - \frac{3}{2}P_iV_i = \frac{3}{2}\left(2P_0\right)\left(V_0\right) - \frac{3}{2}\left(P_0\right)\left(2V_0\right) = \boxed{0}$$

Then, from the first law, $Q = \Delta U - W = 0 - \dfrac{3}{2}P_0V_0 = -\dfrac{3}{2}P_0V_0$, or $\boxed{Q < 0}$

12.19 (a) $W = \bar{F} \cdot d = \left(25.0 \times 10^3 \text{ N}\right) \cdot \left(0.130 \text{ m}\right) = 3.25 \times 10^3 \text{ J} = \boxed{3.25 \text{ kJ}}$

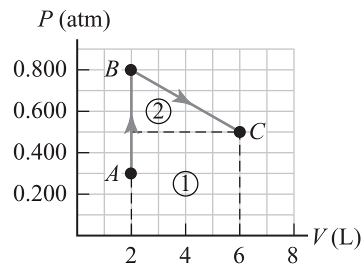
(b) Since the internal energy of an ideal gas is a function of temperature alone, the change in the internal energy in this isothermal process is $\boxed{\Delta U = 0}$.

(c) From the first law of thermodynamics,

$$Q = \Delta U - W = 0 - (3.25 \text{ kJ}) = \boxed{-3.25 \text{ kJ}}$$

(d) If the energy exchanged as heat is $Q = 0$ while the work done on the gas is positive ($W > 0$), the first law of thermodynamics, $\Delta U = Q + W = 0 + W > 0$, tells us that the internal energy of the system must increase. Since the internal energy of an ideal gas is directly proportional to the absolute temperature, $\boxed{\text{the temperature must increase}}$.

12.20 (a) The work done on the gas as it goes from point A to point C is the negative of the area under the *PV* diagram between these points. Consider the sketch given at the right and observe that this area can be broken into a rectangular area and a triangular area. The work done on the gas is



$$\begin{aligned} W &= -\left[(\text{Area 1}) + (\text{Area 2})\right] \\ &= -(0.500 \text{ atm})(6.00 \text{ L}) - \frac{1}{2}(0.300 \text{ atm})(6.00 \text{ L}) \\ &= -3.90 \text{ atm} \cdot \text{L}\left(\frac{1.013 \times 10^5 \text{ N/m}^2}{1 \text{ atm}}\right)\left(\frac{10^{-3} \text{ m}^3}{1 \text{ L}}\right) = \boxed{-395 \text{ J}} \end{aligned}$$

(b) The absolute temperature of an ideal gas is given by $T = PV/nR$. Thus, the lowest temperature occurs where the product *PV* is the smallest. This is seen to be $\boxed{\text{at point A}}$, and the temperature at this point is

$$T_A = \frac{P_A V_A}{nR} = \frac{(0.300 \text{ atm})(2.00 \text{ L})}{(0.025\,6 \text{ mol})(0.082\,1 \text{ L·atm/mol·K})} = \boxed{285 \text{ K}}$$

(c) From the first law of thermodynamics,

$$\Delta U = Q + W = +432 \text{ J} - 395 \text{ J} = \boxed{37 \text{ J}}$$

12.21 (a) In an isothermal process involving an ideal gas, the work done on the gas is $W = -W_{\text{env}} = -nRT \ln\left(V_f/V_i\right)$. But, when temperature is constant, the ideal gas law gives $P_i V_i = P_f V_f = nRT$ and we may write the work done on the gas as

$$W = -P_i V_i \ln\left(\frac{V_f}{V_i}\right) = -\left(1.00 \times 10^5 \text{ Pa}\right)\left(0.500 \text{ m}^3\right) \ln\left(\frac{1.25 \text{ m}^3}{0.500 \text{ m}^3}\right) = \boxed{-4.58 \times 10^4 \text{ J}}$$

(b) The change in the internal energy of an ideal gas is $\Delta U = nC_v(\Delta T)$, and for an isothermal process, we have $\Delta U = 0$. Thus, from the first law of thermodynamics, the energy transfer by heat in this isothermal expansion is

$$Q = \Delta U - W = 0 - \left(-4.58 \times 10^4 \text{ J}\right) = \boxed{+4.58 \times 10^4 \text{ J}}$$

(c) $\boxed{\Delta U = 0}$ [See part (b) above.]

12.22 (a) From the ideal gas law, $P_iV_i = nRT_i$ and $P_fV_f = nRT_f$. Thus, if $P_i = P_f = P$, subtracting these two expressions gives $PV_f - PV_i = nRT_f - nRT_i$, or $\boxed{P(\Delta V) = nR(\Delta T)}$.

(b) For a monatomic, ideal gas containing *N* gas atoms, the internal energy is $U = N\left(\frac{1}{2}m\overline{v^2}\right) = (nN_A)\left(\frac{3}{2}k_BT\right) = \frac{3}{2}nRT$. Thus, the change in internal energy of this gas in a thermodynamic process is $\Delta U = \frac{3}{2}nR(\Delta T)$. But, using the result of part (a) above, we have, for an isobaric process involving an monatomic ideal gas,

$$\Delta U = \frac{3}{2}nR(\Delta T) = \frac{3}{2}P(\Delta V) = \boxed{\frac{3}{2}W_{\text{env}}}$$

(c) We recall that the work done on the gas is $W = -W_{\text{env}}$, and use the first law of thermodynamics to find that the energy transferred to the gas by heat to be

$$Q = \Delta U - W = \Delta U + W_{\text{env}} = \frac{3}{2}W_{\text{env}} + W_{\text{env}} \quad \text{or} \quad \boxed{Q = \frac{5}{2}W_{\text{env}}}$$

(d) In an isobaric *expansion* $(\Rightarrow \ \Delta V > 0)$, the work done on the environment is $W_{\text{env}} = P(\Delta V) > 0$. Thus, from the result of part (c) above, the energy transfer as heat is $Q > 0$, meaning that the energy flow is *into* the gas. Therefore, it is

$\boxed{\text{impossible for the gas to exhaust thermal energy in an isobaric expansion.}}$

12.23 (a) From $PV = nRT$, and with $V_f = V_i = V = 0.200 \text{ m}^3$, we have

$$n = \frac{P_iV}{RT_i} = \frac{5.00 \text{ atm}\left(\dfrac{1.013\times10^5 \text{ N/m}^2}{1 \text{ atm}}\right)(0.200 \text{ m}^3)}{(8.31 \text{ J/mol·K})(300 \text{ K})} = \boxed{40.6 \text{ mol}}$$

(b) The total heat capacity of the gas is

$$C_{\text{total}} = nC_v = n(3R/2) = (40.6 \text{ mol})(1.50)(8.31 \text{ J/mol·K}) = \boxed{506 \text{ J/K}}$$

(c) The work done by the gas during this constant volume process is $\boxed{W = 0}$.

(d) From the first law of thermodynamics, $\Delta U = Q + W = 16.0 \text{ kJ} + 0 = \boxed{16.0 \text{ kJ}}$.

(e) The change in internal energy of a monatomic ideal gas is

$$\Delta U = nC_v(\Delta T) = C_{\text{total}}\Delta T$$

so the change in temperature is

$$\Delta T = \frac{\Delta U}{C_{\text{total}}} = \frac{16.0 \text{ kJ}}{506 \text{ J/K}} = \frac{16.0\times10^3 \text{ J}}{506 \text{ J/K}} = \boxed{31.6 \text{ K}}$$

(f) The final temperature is $T_f = T_i + \Delta T = 300 \text{ K} + 31.6 \text{ K} = \boxed{332 \text{ K}}$.

(g) From the ideal gas law, the final pressure of the gas is

$$P_f = \frac{nRT_f}{V} = \frac{(40.6 \text{ mol})(8.31 \text{ J/mol·K})(332 \text{ K})}{0.200 \text{ m}^3}$$

$$= 5.60\times10^5 \text{ Pa}\left(\frac{1 \text{ atm}}{1.013\times10^5 \text{ Pa}}\right) = \boxed{5.53 \text{ atm}}$$

12.24 Volume is constant in process *BC*, so $\boxed{W_{BC} = 0}$. Given that $\boxed{Q_{BC} < 0}$, the first law shows that $\Delta U_{BC} = Q_{BC} + W_{BC} = Q_{BC} + 0$. Thus, $\boxed{\Delta U_{BC} < 0}$.

For process *CA*, $\Delta V_{CA} = V_A - V_C < 0$, so $W = -P(\Delta V)$ shows that $\boxed{W_{CA} > 0}$. Then, given that $\boxed{\Delta U_{CA} < 0}$, the first law gives $Q_{CA} = \Delta U_{CA} - W_{CA}$ and $\boxed{Q_{CA} < 0}$.

In process *AB*, the work done on the system is $W = -(\text{area under curve } AB)$ where

$$(\text{area under curve } AB) = P_A\left(V_B - V_A\right) + \frac{1}{2}\left(P_B - P_A\right)\left(V_B - V_A\right) > 0$$

Hence, $\boxed{W_{AB} < 0}$. For the cyclic process, $\Delta U = \Delta U_{AB} + \Delta U_{BC} + \Delta U_{CA} = 0$, so $\Delta U_{AB} = -\left(\Delta U_{BC} + \Delta U_{CA}\right)$. This gives $\boxed{\Delta U_{AB} > 0}$, since both ΔU_{BC} and ΔU_{CA} are negative. Finally, from the first law, $Q = \Delta U - W$ shows that $\boxed{Q_{AB} > 0}$, since both ΔU_{AB} and $-\mathrm{W}_{AB}$ are positive.

12.25 (a) The original volume of the aluminum is $V_0 = m/\rho_{\mathrm{Al}}$, and the change in volume is $\Delta V = \beta V_0(\Delta T) = \left(3\alpha_{\mathrm{Al}}\right)\left(m/\rho_{\mathrm{Al}}\right)(\Delta T)$.

The work done by the aluminum is then

$$\begin{aligned} W_{\mathrm{env}} &= +P(\Delta V) = P\left(3\alpha_{\mathrm{Al}}\right)\left(m/\rho_{\mathrm{Al}}\right)(\Delta T) \\ &= \left(1.013\times10^5\ \mathrm{Pa}\right)3\left[24\times10^{-6}\ (^\circ\mathrm{C})^{-1}\right]\left(\frac{5.0\ \mathrm{kg}}{2.70\times10^3\ \mathrm{kg/m^3}}\right)(70^\circ\mathrm{C}) = \boxed{0.95\ \mathrm{J}} \end{aligned}$$

(b) The energy transferred by heat to the aluminum is

$$Q = mc_{\mathrm{Al}}(\Delta T) = (5.0\ \mathrm{kg})(900\ \mathrm{J/kg\cdot{}^\circ C})(70^\circ\mathrm{C}) = \boxed{3.2\times10^5\ \mathrm{J}}$$

(c) The work done on the aluminum is $W = -W_{\mathrm{env}} = -0.95$ J, so the first law gives

$$\Delta U = Q + W = 3.2\times10^5\ \mathrm{J} - 0.95\ \mathrm{J} = \boxed{3.2\times10^5\ \mathrm{J}}$$

12.26 (a) The work done on the gas in each process is the negative of the area under the process curve on the *PV* diagram.

For path *IAF*, $W_{IAF} = W_{IA} + W_{AF} = 0 + W_{AF}$, or

$$W_{IAF} = -\left[(1.50\ \mathrm{atm})\left(\frac{1.013\times10^5\ \mathrm{Pa}}{1\ \mathrm{atm}}\right)\right]\left[(0.500\ \mathrm{L})\left(\frac{10^{-3}\ \mathrm{m^3}}{1\ \mathrm{L}}\right)\right] = \boxed{-76.0\ \mathrm{J}}$$

For path *IBF*, $W_{IBF} = W_{IB} + W_{BF} = W_{IB} + 0$, or

$$\mathrm{W}_{IBF} = -\left[(2.00\ \mathrm{atm})\left(\frac{1.013\times10^5\ \mathrm{Pa}}{1\ \mathrm{atm}}\right)\right]\left[(0.500\ \mathrm{L})\left(\frac{10^{-3}\ \mathrm{m^3}}{1\ \mathrm{L}}\right)\right] = \boxed{-101\ \mathrm{J}}$$

For path *IF*, $W_{IF} = W_{AF} - (\textit{triangular area})$, or

$$W_{IF} = -76.0\ \mathrm{J} - \frac{1}{2}\left[\left(0.500\ \mathrm{m^3}\right)\left(\frac{1.013\times10^5\ \mathrm{Pa}}{1\ \mathrm{atm}}\right)\right]\left[(0.500\ \mathrm{L})\left(\frac{10^{-3}\ \mathrm{m^3}}{1\ \mathrm{L}}\right)\right] = \boxed{-88.7\ \mathrm{J}}$$

continued on next page

(b) Using the first law, with $\Delta U = U_F - U_A = (182 - 91.0)\text{ J} = 91.0\text{ J}$ for each process gives

$$Q_{IAF} = \Delta U - W_{IAF} = 91.0\text{ J} - (-76.0\text{ J}) = \boxed{167\text{ J}}$$

$$Q_{IBF} = \Delta U - W_{IBF} = 91.0\text{ J} - (-101\text{ J}) = \boxed{192\text{ J}}$$

$$Q_{IF} = \Delta U - W_{IF} = 91.0\text{ J} - (-88.7\text{ J}) = \boxed{180\text{ J}}$$

12.27 (a) For adiabatic processes in ideal gases, $P_f V_f^\gamma = P_i V_i^\gamma = \text{constant}$. From the ideal gas law, $P = nRT/V$, so the above expression becomes

$$\left(\frac{\cancel{nR}T_f}{V_f}\right)V_f^\gamma = \left(\frac{\cancel{nR}T_i}{V_i}\right)V_i^\gamma \qquad \text{or} \qquad T_f V_f^{\gamma-1} = T_i V_i^{\gamma-1}$$

which can be summarized as $\boxed{TV^{\gamma-1} = C}$, where C is a constant.

(b) The current radius of the Universe is assumed to be $r_f = 1.4 \times 10^{26}$ m and the temperature is $T_f = 2.7$ K. Since $\gamma = 1.67 = 5/3$ for monatomic ideal gases (see Table 12.1 in the text), the temperature T_i of the Universe when its radius was $r_i = 2.0\text{ cm} = 2.0 \times 10^{-2}$ m must have been

$$T_i = T_f\left(\frac{V_f^{\gamma-1}}{V_i^{\gamma-1}}\right) = T_f\left(\frac{V_f}{V_i}\right)^{\gamma-1} = T_f\left(\frac{\cancel{\tfrac{4}{3}\pi} r_f^3}{\cancel{\tfrac{4}{3}\pi} r_i^3}\right)^{\gamma-1} = T_f\left(\frac{r_f}{r_i}\right)^{3\gamma-3}$$

or $$T_i = (2.7\text{ K})\left(\frac{1.4 \times 10^{26}\text{ m}}{2 \times 10^{-2}\text{ m}}\right)^{3(5/3)-3} = (2.7\text{ K})\left(\frac{1.4 \times 10^{26}\text{ m}}{2 \times 10^{-2}\text{ m}}\right)^2 = \boxed{1 \times 10^{56}\text{ K}}$$

12.28 (a) The number of atoms per mole in any monatomic gas is Avogadro's number $N_A = 6.02 \times 10^{23}$ atoms/mol. Thus, if the density of gas in the Universe is 1 hydrogen atom per cubic meter, the number of moles per unit volume is

$$\frac{n}{V} = \frac{1\text{ atom/m}^3}{N_A} == \frac{1\text{ atom/m}^3}{6.02 \times 10^{23}\text{ atoms/mol}} = \boxed{1.66 \times 10^{-24}\text{ mol/m}^3}$$

(b) With the density of gas found in part (a) and an absolute temperature of $T = 2.7$ K, the ideal gas law gives the pressure of the Universe as

$$P = \left(\frac{n}{V}\right)RT = (1.66 \times 10^{-24}\text{ mol/m}^3)(8.31\text{ J/mol}\cdot\text{K})(2.7\text{ K}) = \boxed{3.7 \times 10^{-23}\text{ Pa}}$$

(c) For an adiabatic expansion, $P_i V_i^\gamma = P_f V_f^\gamma$ with $\gamma = 1.67 = 5/3$ for monatomic ideal gases (see Table 12.1 in the text), so the initial pressure of the Universe is estimated to be

$$P_i = P_f\left(\frac{V_f^\gamma}{V_i^\gamma}\right) = P_f\left(\frac{V_f}{V_i}\right)^\gamma = P_f\left(\frac{\cancel{\tfrac{4}{3}\pi} r_f^3}{\cancel{\tfrac{4}{3}\pi} r_f^3}\right)^\gamma = P_f\left(\frac{r_f}{r_i}\right)^{3\gamma}$$

or $$P_i = (3.7 \times 10^{-23}\text{ Pa})\left(\frac{1.4 \times 10^{26}\text{ m}}{2.0 \times 10^{-2}\text{ m}}\right)^{3(5/3)} = (3.7 \times 10^{-23}\text{ Pa})(7.0 \times 10^{27})^{5.0}$$

giving $$P_i = \left[(3.7 \times 10^{-23}\text{ Pa})(7.0)^{5.0}\right](10^{27})^{5.0} = \left[6.2 \times 10^{-19}\text{ Pa}\right](10^{135}) = \boxed{6.2 \times 10^{116}\text{ Pa}}$$

12.29 The net work done by a heat engine operating on the cyclic process shown in Figure P12.29 equals the triangular area enclosed by this process curve. Thus,

$$W_{\text{eng}} = \frac{1}{2}(6.00\text{ atm} - 2.00\text{ atm})(3.00\text{ m}^3 - 1.00\text{ m}^3)$$

$$= 4.00\ \cancel{\text{atm}} \cdot \text{m}^3\left(\frac{1.013 \times 10^5\text{ Pa}}{1\ \cancel{\text{atm}}}\right) = 4.05 \times 10^5\text{ J}$$

$$= 405 \times 10^3\text{ J} = \boxed{405\text{ kJ}}$$

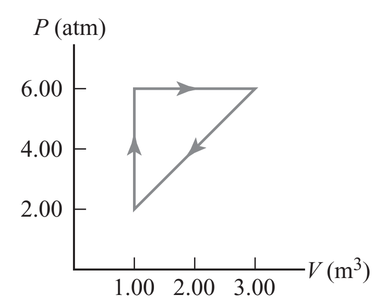


FIGURE P12.29

12.30 The net work done by a heat engine operating on the cyclic process shown in Figure P12.30 equals the triangular area enclosed by this process curve. This is

$$W_{eng} = \frac{1}{2}(base)(altitude)$$

$$= \frac{1}{2}\left[(4.00-1.00)\ \text{m}^3\right]\left[(6.00-2.00)\times 10^5\ \text{Pa}\right]$$

$$= 6.00\times 10^5\ \text{Pa} = \boxed{600\ \text{kJ}}$$

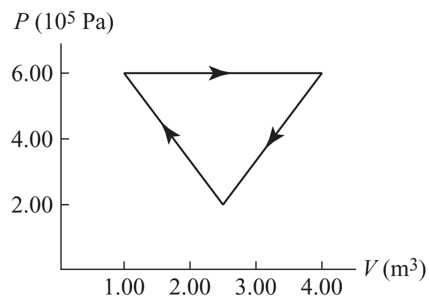


FIGURE P12.30

12.31 The maximum possible efficiency for a heat engine operating between reservoirs with absolute temperatures of $T_c = 25° + 273 = 298\ \text{K}$ and $T_h = 375° + 273 = 648\ \text{K}$ is the Carnot efficiency

$$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{298\ \text{K}}{648\ \text{K}} = \boxed{0.540\ \text{or}\ 54.0\%}$$

12.32 (a) The absolute temperature of the cold reservoir is $T_c = 20° + 273 = 293\ \text{K}$. If the Carnot efficiency is to be $e_C = 0.65$, it is necessary that

$$1 - \frac{T_c}{T_h} = 0.65 \qquad \text{or} \qquad \frac{T_c}{T_h} = 0.35 \qquad \text{and} \qquad T_h = \frac{T_c}{0.35}$$

Thus, $$T_h = \frac{293\ \text{K}}{0.35} = 837\ \text{K} \qquad \text{or} \qquad T_h = 837 - 273 = \boxed{564°\text{C}}$$

(b) $\boxed{\text{No.}}$ Any real heat engine will have an efficiency *less* that the Carnot efficiency because it operates in an irreversible manner.

12.33 (a) The efficiency of a heat engine is $e = W_{eng}/|Q_h|$, where W_{eng} is the work done by the engine and $|Q_h|$ is the energy absorbed from the higher temperature reservoir. Thus, if $W_{eng} = |Q_h|/4$, the efficiency is $e = 1/4 = \boxed{0.25\ \text{or}\ 25\%}$.

(b) From conservation of energy, the energy exhausted to the lower temperature reservoir is $|Q_c| = |Q_h| - W_{eng}$. Therefore, if $W_{eng} = |Q_h|/4$, we have $|Q_c| = 3|Q_h|/4$ or $\boxed{|Q_c|/|Q_h| = 3/4}$.

12.34 (a) The work done by a heat engine equals the net energy absorbed by the engine, or $W_{eng} = |Q_h| - |Q_c|$. Thus, the energy absorbed from the high temperature reservoir is

$$|Q_h| = W_{eng} + |Q_c| = 1\,800\ \text{J} + 2\,400\ \text{J} = \boxed{4\,200\ \text{J}}$$

(b) The efficiency of the heat engine is

$$e \equiv \frac{W_{eng}}{|Q_h|} = \frac{1\,800\ \text{J}}{4\,200\ \text{J}} = 0.43 \qquad \text{or} \qquad \boxed{43\%}$$

12.35 (a) The maximum efficiency possible is that of a Carnot engine operating between reservoirs having absolute temperatures of $T_h = 1\,870 + 273 = 2\,143\ \text{K}$ and $T_c = 430 + 273 = 703\ \text{K}$.

$$e_C = \frac{T_h - T_c}{T_h} = 1 - \frac{T_c}{T_h} = 1 - \frac{703\ \text{K}}{2\,143\ \text{K}} = \boxed{0.672\ \ (\text{or}\ 67.2\%)}$$

(b) From $e = \dfrac{W_{eng}}{|Q_h|}$, we find $W_{eng} = e|Q_h| = 0.420\left(1.40\times 10^5\ \text{J}\right) = 5.88\times 10^4\ \text{J}$

so $$P = \frac{W_{eng}}{t} = \frac{5.88\times 10^4\ \text{J}}{1.00\ \text{s}} = 5.88\times 10^4\ \text{W} = \boxed{58.8\ \text{kW}}$$

12.36 The work done by the engine equals the change in the kinetic energy of the bullet, or

$$W_{\text{eng}} = \frac{1}{2}m_b v_f^2 - 0 = \frac{1}{2}\left(2.40\times10^{-3}\ \text{kg}\right)\left(320\ \text{m/s}\right)^2 = 123\ \text{J}$$

Since the efficiency of an engine may be written as

$$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{W_{\text{eng}} + |Q_c|}$$

where $|Q_c|$ is the exhaust energy from the engine, we find that $|Q_c| = W_{\text{eng}}\left(1/e - 1\right)$. This exhaust energy is absorbed by the 1.80-kg iron body of the gun, so the rise in temperature is

$$\Delta T = \frac{|Q_c|}{m_{\text{gun}}c_{\text{iron}}} = \frac{(123\ \text{J})\left(1/0.011\,0 - 1\right)}{(1.80\ \text{kg})\left(448\ \text{J/kg}\cdot°\text{C}\right)} = \boxed{13.7°\text{C}}$$

12.37 (a) $e \equiv \dfrac{W_{\text{eng}}}{|Q_h|} = 1 - \dfrac{|Q_c|}{|Q_h|} = 1 - \dfrac{1.20\times10^3\ \text{J}}{1.70\times10^3\ \text{J}} = \boxed{0.294\ \ (\text{or } 29.4\%)}$

(b) $W_{\text{eng}} = |Q_h| - |Q_c| = 1.70\times10^3\ \text{J} - 1.20\times10^3\ \text{J} = \boxed{5.00\times10^2\ \text{J}}$

(c) $P = \dfrac{W_{\text{eng}}}{t} = \dfrac{5.00\times10^2\ \text{J}}{0.300\ \text{s}} = 1.67\times10^3\ \text{W} = \boxed{1.67\ \text{kW}}$

12.38 (a) The coefficient of performance of a heat pump is $\text{COP} = |Q_h|/W$, where $|Q_h|$ is the thermal energy delivered to the warm space and W is the work input required to operate the heat pump. Therefore,

$$|Q_h| = W\cdot\text{COP} = (P\cdot\Delta t)\cdot\text{COP} = \left[\left(7.03\times10^3\ \frac{\text{J}}{\cancel{\text{s}}}\right)\left(8.00\ \cancel{\text{h}}\right)\left(\frac{3\,600\ \cancel{\text{s}}}{1\ \cancel{\text{h}}}\right)\right]3.80 = \boxed{7.69\times10^8\ \text{J}}$$

(b) The energy extracted from the cold space (outside air) is

$$|Q_c| = |Q_h| - W = |Q_h| - \frac{|Q_h|}{\text{COP}} = |Q_h|\left(1 - \frac{1}{\text{COP}}\right)$$

or $\quad |Q_c| = \left(7.69\times10^8\ \text{J}\right)\left(1 - \dfrac{1}{3.80}\right) = \boxed{5.67\times10^8\ \text{J}}$

12.39 (a) $W = P\cdot\Delta t = \left(457\ \dfrac{\cancel{\text{kWh}}}{\cancel{\text{y}}}\right)\left(\dfrac{3.60\times10^6\ \text{J}}{1\ \cancel{\text{kWh}}}\right)\left(\dfrac{1\ \cancel{\text{y}}}{365.242\ \text{d}}\right)\cdot(1\ \text{d}) = \boxed{4.50\times10^6\ \text{J}}$

(b) From the definition of the coefficient of performance for a refrigerator, $(\text{COP})_{\text{R}} = |Q_c|/W$, the thermal energy removed from the cold space each day is

$$|Q_c| = (\text{COP})_{\text{R}}\cdot W = 6.30\left(4.50\times10^6\ \text{J}\right) = \boxed{2.84\times10^7\ \text{J}}$$

(c) The water must be cooled 20.0°C before it will start to freeze, so the thermal energy that must be removed from mass m of water to freeze it is $|Q_c| = mc_{\text{w}}|\Delta T| + mL_f$. The mass of water that can be frozen each day is then

$$m = \frac{|Q_c|}{c_{\text{w}}|\Delta T| + L_f} = \frac{2.84\times10^7\ \text{J}}{\left(4\,186\ \text{J/kg}\cdot°\text{C}\right)(20.0°\text{C}) + 3.33\times10^5\ \text{J/kg}} = \boxed{68.2\ \text{kg}}$$

12.40 (a) The coefficient of performance of a heat pump is defined as

$$(\text{COP})_{\text{hp}} = \frac{|Q_h|}{W} = \frac{|Q_h|}{|Q_h| - |Q_c|} = \frac{1}{1 - |Q_c|/|Q_h|}$$

But when a device operates on the Carnot cycle, $|Q_c|/|Q_h| = T_c/T_h$. Thus, the coefficient of performance for a Carnot heat pump would be

$$(\text{COP})_{\text{hp,C}} = \frac{1}{1 - T_c/T_h} = \boxed{\frac{T_h}{T_h - T_c}}$$

continued on next page

(b) From the result of part (a) above, we observe that the COP of a Carnot heat pump would increase if the temperature difference $T_h - T_c$ became $\boxed{\text{smaller.}}$

(c) If $T_c = 50° + 273 = 323$ K and $T_h = 70° + 273 = 343$ K, the COP of a Carnot heat pump would be

$$(\text{COP})_{\text{hp,C}} = \frac{T_h}{T_h - T_c} = \frac{343 \text{ K}}{343 \text{ K} - 323 \text{ K}} = \boxed{17.2}$$

12.41 The actual efficiency of the engine is

$$e = 1 - \frac{|Q_c|}{|Q_h|} = 1 - \frac{300 \text{ J}}{500 \text{ J}} = 0.400$$

If this is 60.0% of the Carnot efficiency, then

$$e_C = \frac{e}{0.600} = \frac{0.400}{0.600} = \frac{2}{3}$$

Thus, from $e_C = 1 - T_c/T_h$, we find

$$\frac{T_c}{T_h} = 1 - e_C = 1 - \frac{2}{3} = \boxed{\frac{1}{3}}$$

12.42 (a) The Carnot efficiency represents the maximum possible efficiency. With $T_h = 20.0°\text{C} = 293$ K and $T_c = 5.00°\text{C} = 278$ K, this efficiency is given by

$$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{278 \text{ K}}{293 \text{ K}} = \boxed{0.051\,2 \ \ (\text{or } 5.12\%)}$$

(b) The efficiency of an engine is $e = W_{\text{eng}} / |Q_h|$, so the minimum energy input by heat each hour is

$$|Q_h|_{\text{min}} = \frac{W_{\text{eng}}}{e_{\text{max}}} = \frac{P \cdot \Delta t}{e_{\text{max}}} = \frac{(75.0 \times 10^6 \text{ J/s})(3\,600 \text{ s})}{0.0512} = \boxed{5.27 \times 10^{12} \text{ J}}$$

(c) As fossil-fuel prices rise, this could be an attractive way to use solar energy. However, the potential environmental impact of such an engine would require serious study. The energy output, $|Q_c| = |Q_h| - W_{\text{eng}} = |Q_h|(1-e)$, to the low temperature reservoir (cool water deep in the ocean) could raise the temperature of over a million cubic meters of water by 1°C every hour.

12.43 (a) We treat the power plant as a heat engine and compute its efficiency as

$$e \equiv \frac{W_{\text{eng}}}{|Q_h|} = \frac{435 \text{ MW}}{1\,420 \text{ MW}} = 0.306 \quad \text{or} \quad \boxed{30.6\%}$$

(b) The work done by a heat engine equals the net energy absorbed by the engine, or $W_{\text{eng}} = |Q_h| - |Q_c|$. Thus, the energy expelled to the low temperature reservoir is

$$|Q_c| = |Q_h| - W_{\text{eng}} = 1\,420 \text{ MW} - 435 \text{ MW} = \boxed{985 \text{ MW}}$$

12.44 (a) With reservoirs at absolute temperatures of $T_c = 80.0 + 273 = 353$ K and $T_h = 350 + 273 = 623$ K, the Carnot efficiency is

$$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{353 \text{ K}}{623 \text{ K}} = 0.433 \quad (\text{or } 43.3\%)$$

continued on next page

so the maximum power output is

$$P_{\max} = \frac{\left(W_{\text{eng}}\right)_{\max}}{t} = \frac{e_C\left|Q_h\right|}{t} = \frac{0.433(21.0\text{ kJ})}{1.00\text{ s}} = \boxed{9.09\text{ kW}}$$

(b) From $e = 1 - |Q_c|/|Q_h|$, the energy expelled by heat each cycle is

$$|Q_c| = |Q_h|(1-e) = (21.0\text{ kJ})(1-0.433) = \boxed{11.9\text{ kJ}}$$

12.45 The thermal energy transferred to the room by the water as the water cools from 1.00×10^2 °C to 20.0°C is

$$Q = mc_{\text{w}}|\Delta T| = (0.125\text{ kg})(4\,186\text{ J/kg}\cdot°\text{C})(80.0°\text{C}) = 4.19\times10^4\text{ J}$$

If the room has a constant absolute temperature of $T = 20.0° + 273 = 293$ K, the increase in the entropy of the room is

$$\Delta S = \frac{Q}{T} = \frac{4.19\times10^4\text{ J}}{293\text{ K}} = \boxed{143\text{ J/K}}$$

12.46 (a) The energy transferred to the ice cube as it melts at a constant temperature of $T = 0.0\text{ °C} = 273$ K is $Q_{\text{ice}} = +m_{\text{ice}}L_f$ and the change in entropy of the ice cube is

$$\Delta S_{\text{ice}} = \frac{Q_{\text{ice}}}{T} = \frac{m_{\text{ice}}L_f}{T} = \frac{(0.065\text{ kg})(3.33\times10^5\text{ J/kg})}{273\text{ K}} = \boxed{+79\text{ J/K}}$$

(b) The energy transferred to the environment during this melting process is $Q_{\text{env}} = -Q_{\text{ice}} = -m_{\text{ice}}L_f$ and the change in entropy of the environment is

$$\Delta S_{\text{env}} = \frac{Q_{\text{env}}}{T} = \frac{-m_{\text{ice}}L_f}{T} = \boxed{-79\text{ J/K}}$$

12.47 The energy transferred from the water by heat, and absorbed by the freezer, is

$$Q = mL_f = (\rho V)L_f = \left[\left(10^3\text{ kg/m}^3\right)\left(1.0\times10^{-3}\text{ m}^3\right)\right]\left(3.33\times10^5\ \frac{\text{J}}{\text{kg}}\right) = 3.3\times10^5\text{ J}$$

Thus, the change in entropy of the water is

(a) $$\Delta S_{\text{water}} = \frac{\left(\Delta Q_r\right)_{\text{water}}}{T} = \frac{-3.3\times10^5\text{ J}}{273\text{ K}} = -1.2\times10^3\ \frac{\text{J}}{\text{K}} = \boxed{-1.2\text{ kJ/K}}$$

and that of the freezer is

(b) $$\Delta S_{\text{freezer}} = \frac{\left(\Delta Q_r\right)_{\text{freezer}}}{T} = \frac{+3.3\times10^5\text{ J}}{273\text{ K}} = \boxed{+1.2\text{ kJ/K}}$$

12.48 The energy added to the water by heat is

$$\Delta Q_r = mL_v = (1.00\text{ kg})\left(2.26\times10^6\text{ J/kg}\right) = 2.26\times10^6\text{ J}$$

so the change in entropy is

$$\Delta S = \frac{\Delta Q_r}{T} = \frac{2.26\times10^6\text{ J}}{373\text{ K}} = 6.06\times10^3\ \frac{\text{J}}{\text{K}} = \boxed{6.06\text{ kJ/K}}$$

12.49 The potential energy lost by the log is transferred away by heat, so the energy transferred from the log to the reservoir is $\Delta Q_r = mgh$. The change in entropy of the reservoir (universe) is then

$$\Delta S = \frac{\Delta Q_r}{T} = \frac{mgh}{T} = \frac{(70.0\text{ kg})\left(9.80\text{ m/s}^2\right)(25.0\text{ m})}{300\text{ K}} = \boxed{57.2\text{ J/K}}$$

12.50 (a) In a game of dice, there is only $\boxed{\text{one way}}$ you can roll a 12. You must have a 6 on each die.

(b) There are $\boxed{\text{six ways}}$ to obtain a 7 with a pair of dice. The combinations that yield a 7 are: 1 + 6, 2 + 5, 3 + 4, 4 + 3, 5 + 2, and 6 + 1. Note that 1 + 6 and 6 + 1 are different combinations in that the 6 occurs on different members of the pair of dice in the two combinations.

12.51 A quantity of energy, of magnitude Q, is transferred from the Sun and added to Earth. Thus, $\Delta S_{\text{Sun}} = \dfrac{-Q}{T_{\text{Sun}}}$ and $\Delta S_{\text{Earth}} = \dfrac{+Q}{T_{\text{Earth}}}$, so the total change in entropy is

$$\Delta S_{\text{total}} = \Delta S_{\text{Earth}} + \Delta S_{\text{Sun}} = \frac{Q}{T_{\text{Earth}}} - \frac{Q}{T_{\text{Sun}}}$$

$$= (1\,000 \text{ J})\left(\frac{1}{290 \text{ K}} - \frac{1}{5\,700 \text{ K}}\right) = \boxed{+3.27 \text{ J/K}}$$

12.52 The change in entropy of a reservoir is $\Delta S = Q_r/T$, where Q_r is the energy absorbed $(Q_r > 0)$ or expelled $(Q_r < 0)$ by the reservoir, and T is the absolute temperature of the reservoir.

(a) For the hot reservoir: $\Delta S_h = \dfrac{-2.50 \times 10^3 \text{ J}}{725 \text{ K}} = \boxed{-3.45 \text{ J/K}}$

(b) For the cold reservoir: $\Delta S_c = \dfrac{+2.50 \times 10^3 \text{ J}}{310 \text{ K}} = \boxed{+8.06 \text{ J/K}}$

(c) For the Universe: $\Delta S_U = \Delta S_h + \Delta S_c = -3.45 \text{ J/K} + 8.06 \text{ J/K} = \boxed{+4.61 \text{ J/K}}$

(d) The magnitudes of the thermal energy transfers, appearing in the numerators, are the same for the two reservoirs, but the cold reservoir necessarily has a smaller denominator. Hence, its positive change dominates.

12.53 (a) The table is shown below. On the basis of the table, the most probable result of a toss is $\boxed{\text{2 H and 2 T}}$.

End Result	Possible Tosses	Total Number of Same Result
All H	HHHH	1
1T, 3H	HHHT, HHTH, HTHH, THHH	4
2T, 2H	HHTT, HTHT, THHT, HTTH, THTH, TTHH	6
3T, 1H	TTTH, TTHT, THTT, HTTT	4
All T	TTTT	1

(b) The most ordered state is the least likely. This is seen to be $\boxed{\text{all H or all T}}$.

(c) The least ordered state is the most likely. This is seen to be $\boxed{\text{2H and 2T}}$.

12.54 The change in entropy of a reservoir is $\Delta S = Q_r/T$, where Q_r is the energy absorbed $(Q_r > 0)$ or expelled $(Q_r < 0)$ by the reservoir, and T is the absolute temperature of the reservoir.

(a) For the hot reservoir, $Q_r = -|Q_h|$, and $\boxed{\Delta S_h = \dfrac{-|Q_h|}{T_h}}$

(b) For the cold reservoir, $Q_r = +|Q_h|$, and $\boxed{\Delta S_c = \dfrac{+|Q_h|}{T_c}}$

(c) For the Universe: $\Delta S_U = \Delta S_h + \Delta S_c = \boxed{-\dfrac{|Q_h|}{T_h} + \dfrac{|Q_h|}{T_c}}$

12.55 Using the metabolic rates from Table 12.4, we find the change in the body's internal energy (i.e., the energy consumed) for each activity, and the total change for the day, as:

Activity	Metabolic Rate (W)	Internal Energy Change
Sleeping – 8.0 h	80	$\Delta U_1 = -(80\ \text{J/s})(8.0\ \text{h})(3\,600\ \text{s/1 h}) = -2.3\times10^6\ \text{J}$
Light Chores – 3.0 h	230	$\Delta U_2 = -(230\ \text{J/s})(3.0\ \text{h})(3\,600\ \text{s/1 h}) = -2.5\times10^6\ \text{J}$
Slow Walk – 1.0 h	230	$\Delta U_3 = -(230\ \text{J/s})(1.0\ \text{h})(3\,600\ \text{s/1 h}) = -8.3\times10^5\ \text{J}$
Running – 0.5 h	465	$\Delta U_4 = -(465\ \text{J/s})(0.50\ \text{h})(3\,600\ \text{s/1 h}) = -8.4\times10^5\ \text{J}$
Total		$\Delta U_1 + \Delta U_2 + \Delta U_3 + \Delta U_4 = -6.5\times10^6\ \text{J} = \boxed{-6.5\ \text{MJ}}$

12.56 (a) At the sleeping rate of 80.0 W, the time required for the body to use the 450 Cal of energy supplied by the bagel is

$$\Delta t = \frac{\Delta U}{P} = \frac{450\ \text{Cal}}{80.0\ \text{J/s}}\left(\frac{4\,186\ \text{J}}{1\ \text{Cal}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{6.54\ \text{h}}$$

(b) The metabolic rate while working out is $P = 80.0\ \text{W} + 650\ \text{W} = 7.30\times10^2\ \text{W}$, and the time to use the energy from the bagel at this rate is

$$\Delta t = \frac{\Delta U}{P} = \frac{450\ \text{Cal}}{7.30\times10^2\ \text{J/s}}\left(\frac{4\,186\ \text{J}}{1\ \text{Cal}}\right)\left(\frac{1\ \text{h}}{3\,600\ \text{s}}\right) = \boxed{0.717\ \text{h}}$$

(c) $W_{\substack{\text{per}\\\text{lift}}} = F\cdot\Delta x = mgh = (120\ \text{kg})(9.80\ \text{m/s}^2)(2.00\ \text{m}) = \boxed{2.35\times10^3\ \text{J}}$

(d) The *additional* energy consumed in 1 minute while working out (instead of sleeping) is $\Delta U = P_{\text{increase}}(1\ \text{min}) = (650\ \text{J/s})(60.0\ \text{s}) = 3.90\times10^4\ \text{J}$. The number of barbell lifts this should allow in 1 minute is

$$N = \frac{\Delta U}{W_{\substack{\text{per}\\\text{lift}}}} = \frac{3.90\times10^4\ \text{J}}{2.35\times10^3\ \text{J}} = \boxed{16.6}$$

(e) $\boxed{\text{No.}}$ The body is only about 25% efficient in converting chemical energy to mechanical energy.

12.57 The maximum rate at which the body can dissipate waste heat by sweating is

$$\frac{\Delta Q}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)L_v = \left(1.5\ \frac{\cancel{\text{kg}}}{\cancel{\text{h}}}\right)\left(2\,430\times10^3\ \frac{\text{J}}{\cancel{\text{kg}}}\right)\left(\frac{1\ \cancel{\text{h}}}{3\,600\ \text{s}}\right) = 1.0\times10^3\ \text{W}$$

If this represents 80% of the maximum sustainable metabolic rate $\left[\text{i.e., } \Delta Q/\Delta t = 0.80(\Delta U/\Delta t)_{\text{max}}\right]$, then that maximum rate is

$$\left(\frac{\Delta U}{\Delta t}\right)_{\text{max}} = \frac{\Delta Q/\Delta t}{0.80} = \frac{1.0\times10^3\ \text{W}}{0.80} = \boxed{1.3\times10^3\ \text{W}}$$

12.58 Operating between reservoirs having temperatures of $T_h = 100°\text{C} = 373\ \text{K}$ and $T_c = 20°\text{C} = 293\ \text{K}$, the theoretical efficiency of a Carnot engine is

$$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{293\ \text{K}}{373\ \text{K}} = 0.214$$

If the temperature of the hotter reservoir is changed to $T_h' = 550°\text{C} = 823\ \text{K}$, the theoretical efficiency of the Carnot engine increases to

$$e_C' = 1 - \frac{T_c}{T_h'} = 1 - \frac{293\ \text{K}}{823\ \text{K}} = 0.644$$

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The factor by which the efficiency has increased is

$$\frac{e'_C}{e_C} = \frac{0.644}{0.214} = \boxed{3.01}$$

12.59 The work output from the engine in an interval of one second is $W_{\text{eng}} = 1\,500\text{ kJ}$. Since the efficiency of an engine may be expressed as

$$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{W_{\text{eng}}}{W_{\text{eng}} + |Q_c|}$$

the exhaust energy each second is $|Q_c| = W_{\text{eng}}\left(\frac{1}{e} - 1\right) = (1\,500\text{ kJ})\left(\frac{1}{0.25} - 1\right) = 4.5\times10^{3}\text{ kJ}$

The mass of water flowing through the cooling coils each second is

$$m = \rho V = \left(10^{3}\text{ kg/m}^{3}\right)(60\text{ L})\left(10^{-3}\text{ m}^{3}/1\text{ L}\right) = 60\text{ kg}$$

so the rise in the temperature of the water is

$$\Delta T = \frac{|Q_c|}{mc_{\text{water}}} = \frac{4.5\times10^{6}\text{ J}}{(60\text{ kg})(4\,186\text{ J/kg}\cdot{}^\circ\text{C})} = \boxed{18^\circ\text{C}}$$

12.60 The energy exhausted from a heat engine is

$$Q_c = Q_h - W_{\text{eng}} = \frac{W_{\text{eng}}}{e} - W_{\text{eng}} = W_{\text{eng}}\left(\frac{1}{e} - 1\right)$$

where Q_h is the energy input from the high temperature reservoir, W_{eng} is the useful work done, and $e = W_{\text{eng}}/Q_h$ is the efficiency of the engine.

For a Carnot engine, the efficiency is $e_C = 1 - T_c/T_h = (T_h - T_c)/T_h$

so we now have $$Q_c = W_{\text{eng}}\left(\frac{T_h}{T_h - T_c} - 1\right) = W_{\text{eng}}\left(\frac{T_c}{T_h - T_c}\right)$$

Thus, if $T_h = 100^\circ\text{C} = 373\text{ K}$ and $T_c = 20^\circ\text{C} = 293\text{ K}$, the energy exhausted when the engine has done $5.0\times10^{4}\text{ J}$ of work is

$$Q_c = \left(5.0\times10^{4}\text{ J}\right)\left(\frac{293\text{ K}}{373\text{ K} - 293\text{ K}}\right) = 1.83\times10^{5}\text{ J}$$

The mass of ice (at 0°C) this exhaust energy could melt is

$$m = \frac{Q_c}{L_{f,\text{ water}}} = \frac{1.83\times10^{5}\text{ J}}{3.33\times10^{5}\text{ J/kg}} = \boxed{0.55\text{ kg}}$$

12.61 (a) The work done by the system in process *AB* equals the area under this curve on the *PV* diagram. Thus,

$$W_{\text{env}} = (\text{triangular area}) + (\text{rectangular area}),\text{ or}$$

$$W_{\text{env}} = \left[\tfrac{1}{2}(4.00\text{ atm})(40.0\text{ L}) + (1.00\text{ atm})(40.0\text{ L})\right]\left(1.013\times10^{5}\ \frac{\text{Pa}}{\text{atm}}\right)\left(\frac{10^{-3}\text{ m}^{3}}{\text{L}}\right)$$

$$= 1.22\times10^{4}\text{ J} = \boxed{12.2\text{ kJ}}$$

Note that the work done on the system is $W_{AB} = -W_{\text{env}} = -12.2\text{ kJ}$ for this process.

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(b) The work done on the system (that is, the work input) for process *BC* is the negative of the area under the curve on the *PV* diagram, or

$$W_{BC} = -\left[(1.00 \text{ atm})(10.0 \text{ L} - 50.0 \text{ L})\right]\left(1.013\times10^{5} \ \frac{\text{Pa}}{\text{atm}}\right)\left(\frac{10^{-3} \text{ m}^{3}}{1 \text{ L}}\right)$$

$$= \boxed{4.05 \text{ kJ}}$$

(c) The change in internal energy is zero for any full cycle, so the first law gives

$$Q_{\text{cycle}} = \Delta U_{\text{cycle}} - W_{\text{cycle}} = 0 - \left(W_{AB} + W_{BC} + W_{CA}\right)$$

$$= 0 - \left(-12.2 \text{ kJ} + 4.05 \text{ kJ} + 0\right) = \boxed{8.15 \text{ kJ}}$$

12.62 (a) From the first law, $\Delta U_{1\to3} = Q_{123} + W_{123} = +418 \text{ J} + (-167 \text{ J}) = \boxed{251 \text{ J}}$.

(b) The difference in internal energy between states 1 and 3 is independent of the path used to get from state 1 to state 3.

Thus, $\Delta U_{1\to3} = Q_{143} + W_{143} = 251 \text{ J}$,

and $Q_{143} = 251 \text{ J} - W_{143} = 251 \text{ J} - (-63.0 \text{ J}) = \boxed{314 \text{ J}}$

(c) $W_{12341} = W_{123} + W_{341} = W_{123} + (-W_{143}) = -167 \text{ J} - (-63.0 \text{ J}) = -104 \text{ J}$

or $\boxed{\text{104 J of work is done by the gas}}$ in the cyclic process 12341.

(d) $W_{14321} = W_{143} + W_{321} = W_{143} + (-W_{123}) = -63.0 \text{ J} - (-167 \text{ J}) = +104 \text{ J}$

or $\boxed{\text{104 J of work is done on the gas}}$ in the cyclic process 14321.

(e) The change in internal energy is $\boxed{\text{zero}}$ for both parts (c) and (d) since both are cyclic processes.

12.63 (a) The change in length, due to linear expansion, of the rod is

$$\Delta L = \alpha L_0 (\Delta T) = \left[11\times10^{-6} \ (^\circ\text{C})^{-1}\right](2.0 \text{ m})(40^\circ\text{C} - 20^\circ\text{C}) = 4.4\times10^{-4} \text{ m}$$

The load exerts a force $F = mg = (6\,000 \text{ kg})(9.80 \text{ m/s}^2) = 5.88\times10^{4} \text{ N}$ on the end of the rod in the direction of movement of that end. Thus, the work done on the rod is

$$W = F\cdot\Delta L = \left(5.88\times10^{4} \text{ N}\right)\left(4.4\times10^{-4} \text{ m}\right) = \boxed{26 \text{ J}}$$

(b) The energy added by heat is

$$Q = mc(\Delta T) = (100 \text{ kg})\left(448 \ \frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)(20\,^\circ\text{C}) = \boxed{9.0\times10^{5} \text{ J}}$$

(c) From the first law, $\Delta U = Q + W = 9.0\times10^{5} \text{ J} + 26 \text{ J} = \boxed{9.0\times10^{5} \text{ J}}$.

12.64 (a) The work done by the gas during each full cycle equals the area enclosed by the cycle on the *PV* diagram. Thus

$$W_{\text{env}} = (3P_0 - P_0)(3V_0 - V_0) = \boxed{4P_0V_0}$$

(b) Since the work done on the gas is $W = -W_{\text{env}} = -4P_0V_0$ and $\Delta U = 0$ for any cyclic process, the first law gives

$$Q = \Delta U - W = 0 - (-4P_0V_0) = \boxed{4P_0V_0}$$

(c) From the ideal gas law, $P_0V_0 = nRT_0$, so the work done by the gas each cycle is

$$W_{\text{env}} = 4nRT_0 = 4(1.00 \text{ mol})\left(8.31 \ \frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(273 \text{ K})$$

$$= 9.07\times10^3 \text{ J} = \boxed{9.07 \text{ kJ}}$$

12.65 (a) The energy transferred to the gas by heat is

$$Q = mc(\Delta T) = (1.00 \text{ mol})\left(20.79 \ \frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(120 \text{ K}) = 2.49\times10^3 \text{ J} = \boxed{2.49 \text{ kJ}}$$

(b) Treating the neon as a monatomic ideal gas, Equation 12.3b gives the change in internal energy as $\Delta U = \frac{3}{2}nR(\Delta T)$, or

$$\Delta U = \frac{3}{2}(1.00 \text{ mol})\left(8.31 \ \frac{\text{J}}{\text{mol}\cdot\text{K}}\right)(120 \text{ K}) = 1.50\times10^3 \text{ J} = \boxed{1.50 \text{ kJ}}$$

(c) From the first law, the work done on the gas is

$$W = \Delta U - Q = 1.50\times10^3 \text{ J} - 2.49\times10^3 \text{ J} = \boxed{-990 \text{ J}}$$

12.66 Assuming the gravitational potential energy given up by the falling water is transformed into thermal energy when the water hits the bottom of the falls, the rate of thermal energy production is

$$\frac{\Delta Q}{\Delta t} = \left(\frac{\Delta m}{\Delta t}\right)gh = \rho_{\text{w}}\left(\frac{\Delta V}{\Delta t}\right)gh$$

Then, if the absolute temperature of the environment is $T_{\text{K}} = 20.0° + 273 = 293$ K, the rate of entropy production is

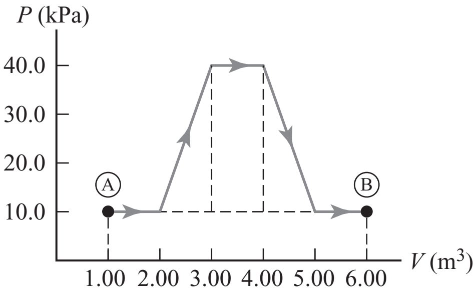
$$\frac{\Delta S}{\Delta t} = \frac{\Delta Q/\Delta t}{T_{\text{K}}} = \frac{\rho_{\text{w}}(\Delta V/\Delta t)gh}{T_{\text{K}}}$$

or $$\frac{\Delta S}{\Delta t} = \frac{1}{293 \text{ K}}\left(10^3 \ \frac{\text{kg}}{\cancel{\text{m}^3}}\right)\left(5\times10^3 \ \frac{\cancel{\text{m}^3}}{\text{s}}\right)\left(9.8 \ \frac{\text{m}}{\text{s}^2}\right)(50.0 \text{ m}) = 8\times10^6 \ \frac{(\text{kg}\cdot\text{m/s}^2)\cdot\text{m}}{\text{K}\cdot\text{s}}$$

and $$\frac{\Delta S}{\Delta t} = 8\times10^6 \ \frac{(\text{N}\cdot\text{m})}{\text{K}\cdot\text{s}} = \boxed{8\times10^6 \ \text{J/K}\cdot\text{s}}$$

12.67 (a) $T_A = \dfrac{P_A V_A}{nR} = \dfrac{(10.0\times10^3\ \text{Pa})(1.00\ \text{m}^3)}{(10.0\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})} = \boxed{1.20\times10^2\ \text{K}}$

$T_B = \dfrac{P_B V_B}{nR} = \dfrac{(10.0\times10^3\ \text{Pa})(6.00\ \text{m}^3)}{(10.0\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})} = \boxed{722\ \text{K}}$



(b) As it goes from A to B, the gas is expanding and hence, doing work on the environment. The magnitude of the work done equals the area under the process curve from A to B. We subdivide this area into 2 rectangular and 2 triangular parts:

$$W_{\text{env}} = \left[(10.0\times10^3\ \text{Pa})(6.00-1.00)\ \text{m}^3\right] + \left[(40.0-10.0)\times10^3\ \text{Pa}\right](1.00\ \text{m}^3) + 2\left[\frac{1}{2}(1.00\ \text{m}^3)(40.0-10.0)\times10^3\ \text{Pa}\right] = \boxed{1.10\times10^5\ \text{J}}$$

(c) The change in the internal energy of a monatomic, ideal gas is $\Delta U = \frac{3}{2}nR(\Delta T)$, so

$$\Delta U_{A\to B} = \frac{3}{2}nR(T_B - T_A) = \frac{3}{2}(10.0\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})(722\ \text{K} - 120\ \text{K}) = \boxed{7.50\times10^4\ \text{J}}$$

(d) From the first law of thermodynamics, $Q = \Delta U - W$, where W is the work done *on the gas*. In this case, $W = -W_{\text{env}} = -1.10\times10^5\ \text{J}$, and

$$Q = \Delta U - W = 7.50\times10^4\ \text{J} - (-1.10\times10^5\ \text{J}) = \boxed{1.85\times10^5\ \text{J}}$$

12.68 (a) The constant volume occupied by the gases is $V = \frac{4}{3}\pi r^3 = 4\pi(0.500\ \text{m})^3/3 = 0.524\ \text{m}^3$ and the initial absolute temperature is $T_i = 20.0° + 273 = 293\ \text{K}$.

To determine the initial pressure, we treat each component of the mixture as an ideal gas and compute the pressure it would exert if it occupied the entire volume of the container. The total pressure exerted by the mixture is then the sum of the partial pressures exerted by the components of the mixture. This gives

$$P_{H_2} = \frac{n_{H_2}RT}{V} \qquad P_{O_2} = \frac{n_{O_2}RT}{V}$$

and $$P_i = P_{H_2} + P_{O_2} = \frac{n_{H_2}RT}{V} + \frac{n_{O_2}RT}{V} = \frac{(n_{H_2} + n_{O_2})RT}{V}$$

or $$P_i = \frac{(14.4\ \text{mol} + 7.2\ \text{mol})(8.31\ \text{J/mol}\cdot\text{K})(293\ \text{K})}{0.524\ \text{m}^3} = \boxed{1.00\times10^5\ \text{Pa}}$$

(b) Treating both the hydrogen and oxygen as ideal gases, each with internal energy $U = nC_vT$, where C_v is the molar specific heat at constant volume, we use Table 12.1 and find the initial internal energy of the mixture as

$$U_i = U_{H_2,i} + U_{O_2,i} = \left(n_{H_2}C_{v,H_2} + n_{O_2}C_{v,O_2}\right)T_i$$

or $$U_i = \left[(14.4\ \text{mol})(20.4\ \text{J/mol}\cdot\text{K}) + (7.2\ \text{mol})(21.1\ \text{J/mol}\cdot\text{K})\right](293\ \text{K}) = \boxed{1.31\times10^5\ \text{J}}$$

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(c) During combustion, this mixture produces 14.4 moles of water (1 mole of water for each mole of hydrogen used) with a conversion of 241.8 kJ of chemical potential energy per mole. Since the volume is constant, no work is done and the additional internal energy generated in the combustion is

$$\Delta U = (14.4 \text{ mol})(241.8 \text{ kJ/mol}) = 3.48\times10^3 \text{ kJ} = \boxed{3.48\times10^6 \text{ J}}$$

(d) After combustion, the internal energy of the system is

$$U_f = U_i + \Delta U = 1.31\times10^5 \text{ J} + 3.48\times10^6 \text{ J} = 3.61\times10^6 \text{ J}$$

Treating the steam as an ideal gas, so $U = nC_v T_K$, and obtaining the molar heat capacity for water vapor (a polyatomic gas) from Table 12.1, we find

$$T_f = \frac{U_f}{n_{\text{water}} C_{v,\text{water}}} = \frac{3.61\times10^6 \text{ J}}{(14.4 \text{ mol})(27.0 \text{ J/mol}\cdot\text{K})} = \boxed{9.28\times10^3 \text{ K}}$$

and the final pressure is

$$P_f = \frac{n_{\text{water}} R T_f}{V} = \frac{(14.4 \text{ mol})(8.31 \text{ J/mol}\cdot\text{K})(9.28\times10^3 \text{ K})}{0.524 \text{ m}^3} = \boxed{2.12\times10^6 \text{ Pa}}$$

(e) The total mass of steam present after combustion is

$$m_{\text{steam}} = n_{\text{steam}} M_{\text{water}} = (14.4 \text{ mol})(18.0\times10^{-3} \text{ kg/mol}) = \boxed{0.259 \text{ kg}}$$

and its density is $$\rho_s = \frac{m_{\text{steam}}}{V} = \frac{0.259 \text{ kg}}{0.524 \text{ m}^3} = \boxed{0.494 \text{ kg/m}^3}$$

(f) Assuming the steam is essentially at rest within the container $(v_1 \approx 0)$, $P_2 = 0$ (since the steam spews into a vacuum), and $y_2 = y_1$, we use the pressure from part (d) above and Bernoulli's equation $\left(P_2 + \frac{1}{2}\rho_s v_2^2 + \rho_s g y_2 = P_1 + \frac{1}{2}\rho_s v_1^2 + \rho_s g y_1\right)$ to find the exhaust speed as

$$v_2 = \sqrt{\frac{2P_1}{\rho_s}} = \sqrt{\frac{2(2.12\times10^6 \text{ Pa})}{0.494 \text{ kg/m}^3}} = \boxed{2.93\times10^3 \text{ m/s}}$$

12.69 The work that you have done is

$$W_{\text{eng}} = mg(\Delta h) = \left[(150 \text{ lb})\left(\frac{4.448\text{N}}{1 \text{ lb}}\right)\right]\left[\left(90.0 \ \frac{\text{step}}{\text{min}}\right)(30.0 \text{ min})\left(8.00 \ \frac{\text{in}}{\text{step}}\right)\left(\frac{2.54\times10^{-2} \text{ m}}{1 \text{ in}}\right)\right]$$

or $$W_{\text{eng}} = 3.66\times10^5 \text{ J}$$

If the energy input by heat was $|Q_h| = (600 \text{ kcal})\left(\frac{4\,186 \text{ J}}{1 \text{ kcal}}\right) = 2.51\times10^6 \text{ J}$, your efficiency has been

$$e = \frac{W_{\text{eng}}}{|Q_h|} = \frac{3.66\times10^5 \text{ J}}{2.51\times10^6 \text{ J}} = \boxed{0.146 \text{ or } 14.6\%}$$

If the actual efficiency was $e = 0.180$ or 18.0%, the actual energy input was

$$|Q_h|_{\text{actual}} = \frac{W_{\text{eng}}}{e_{\text{actual}}} = \frac{3.66\times10^5 \text{ J}}{0.180} = (2.03\times10^6 \text{ J})\left(\frac{1 \text{ kcal}}{4\,186 \text{ J}}\right) = \boxed{485 \text{ kcal}}$$

12.70 (a) The energy transferred from the water by heat as it cools is

$$|Q_h| = mc|\Delta T| = (\rho V)c|\Delta T|$$

$$= \left[\left(1.0\ \frac{\text{g}}{\text{cm}^3}\right)(1.0\ \text{L})\left(\frac{10^3\ \text{cm}^3}{1\ \text{L}}\right)\right]\left(1.0\ \frac{\text{cal}}{\text{g}\cdot{}^\circ\text{C}}\right)\left(\frac{4.186\ \text{J}}{1\ \text{cal}}\right)(570^\circ\text{C} - 4.0^\circ\text{C})$$

or $|Q_h| = \boxed{2.4\times10^6\ \text{J}}$

(b) The maximum efficiency of a heat engine is the Carnot efficiency. Thus,

$$e_C = 1 - \frac{T_c}{T_h} = 1 - \frac{(4.0+273)\ \text{K}}{(570+273)\ \text{K}} = 1 - \frac{277\ \text{K}}{843\ \text{K}} = 0.67$$

The maximum useful work output is then

$$\left(W_{\text{eng}}\right)_{\max} = e_C|Q_h| = (0.67)(2.4\times10^6\ \text{J}) = \boxed{1.6\times10^6\ \text{J}}$$

(c) The energy available from oxidation of the hydrogen sulfide in 1.0 L of this water is

$$U = n(310\ \text{kJ/mol}) = \left[\left(0.90\times10^{-3}\ \frac{\text{mol}}{\text{L}}\right)(1.0\ \text{L})\right]\left(310\times10^3\ \frac{\text{J}}{\text{mol}}\right) = \boxed{2.8\times10^2\ \text{J}}$$

12.71 (a) With an overall efficiency of $e = 0.15$ and a power output of $P_{\text{out}} = 150$ MW, the required power input (from burning coal) is

$$P_{\text{in}} = \frac{P_{\text{out}}}{e} = \frac{150\times10^6\ \text{W}}{0.15} = 1.0\times10^9\ \text{J/s}$$

The coal used each day is

$$\frac{\Delta m}{\Delta t} = \frac{P_{\text{in}}}{\textit{heat of combustion}} = \frac{(1.0\times10^9\ \text{J/s})(8.64\times10^4\ \text{s/d})}{\left(7.8\times10^3\ \frac{\text{cal}}{\text{g}}\right)\left(\frac{10^3\ \text{g}}{1\ \text{kg}}\right)\left(\frac{4.186\ \text{J}}{1\ \text{cal}}\right)} = 2.6\times10^6\ \frac{\text{kg}}{\text{d}}$$

or $\dfrac{\Delta m}{\Delta t} = \left(2.6\times10^6\ \dfrac{\text{kg}}{\text{d}}\right)\left(\dfrac{1\ \text{metric ton}}{10^3\ \text{kg}}\right) = \boxed{2.6\times10^3\ \text{metric ton/d}}$

(b) The annual fuel cost is $\textit{cost} = (\textit{coal used yearly})\cdot(\textit{rate})$, or

$$\textit{cost} = (2.6\times10^3\ \text{ton/d})(365.242\ \text{d/y})(\$8.0/\text{ton}) = \boxed{\$7.6\times10^6/\text{y}}$$

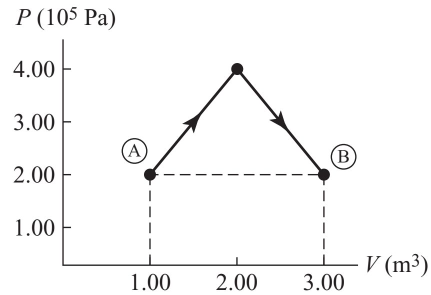
(c) The rate of energy transfer to the river by heat is

$$P_{\text{exhaust}} = P_{\text{in}} - P_{\text{out}} = 1.0\times10^9\ \text{W} - 150\times10^6\ \text{W} = 8.5\times10^8\ \text{W}$$

Thus, the flow required if the maximum rise in temperature is 5.0°C is

$$\textit{flow rate} = \frac{\Delta m_{\text{water}}}{\Delta t} = \frac{P_{\text{exhaust}}}{c_{\text{water}}(\Delta T)} = \frac{8.5\times10^8\ \text{J/s}}{(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(5.0^\circ\text{C})} = \boxed{4.1\times10^4\ \text{kg/s}}$$

12.72 (a) The work done on the gas is the negative of the area under the process curve in a PV diagram. From the sketch at the right, observe that this area consists of a triangular area sitting atop a rectangular area, or



$$W = -\left[\tfrac{1}{2}(base)(altitude) + (width)(height)\right]$$

Thus,

$$W = -\left[\frac{1}{2}(3.00-1.00)\text{ m}^3(4.00-2.00)\times10^5\text{ Pa} + (3.00-1.00)\text{ m}^3(4.00-2.00)\times10^5\text{ Pa}\right]$$

$$= \boxed{-6.00\times10^5\text{ J}}$$

(b) $$\Delta T = T_B - T_A = \frac{P_B V_B}{nR} - \frac{P_A V_A}{nR} = \frac{P_A(\Delta V)}{nR} = \frac{(2.00\times10^5\text{ Pa})(3.00-1.00)\text{ m}^3}{(87.5\text{ mol})(8.31\text{ J/mol}\cdot\text{K})} = \boxed{5.50\times10^2\text{ K}}$$

(c) The change in internal energy of an ideal gas is $\Delta U = nC_v\Delta T$. From Table 12.1, which gives the molar specific heats of diatomic gases, we take the average of the first four entries to obtain $C_v \approx 20.8\text{ J/mol}\cdot\text{K}$. This then yields

$$\Delta U = U_B - U_A = nC_v\Delta T = (87.5\text{ mol})(20.8\text{ J/mol}\cdot\text{K})(550\text{ K}) = \boxed{1.00\times10^6\text{ J}}$$

(d) From the first law of thermodynamics, $\Delta U = Q + W$, we find

$$Q = \Delta U - W = 1.00\times10^6\text{ J} - (-6.00\times10^5\text{ J}) = \boxed{1.60\times10^6\text{ J}}$$

13

Vibrations and Waves

QUICK QUIZZES

1. Choice (d). To complete a full cycle of oscillation, the object must travel distance $2A$ to position $x = -A$ and then travel an additional distance $2A$ returning to the original position at $x = +A$.

2. Choice (c). The force producing harmonic oscillation is always directed toward the equilibrium position, and hence, directed opposite to the displacement from equilibrium. The acceleration is in the direction of the force. Thus, it is also always directed opposite to the displacement from equilibrium.

3. Choice (b). In simple harmonic motion, the force (and hence, the acceleration) is directly proportional to the displacement from equilibrium. Therefore, force and acceleration are both at a maximum when the displacement is a maximum.

4. Choice (a). The period of an object-spring system is $T = 2\pi\sqrt{m/k}$. Thus, increasing the mass by a factor of 4 will double the period of oscillation.

5. Choice (c). The total energy of the oscillating system is equal to $\frac{1}{2}kA^2$, where A is the amplitude of oscillation. Since the object starts from rest at displacement A in both cases, it has the same amplitude of oscillation in both cases.

6. Choice (d). The expressions for the total energy, maximum speed, and maximum acceleration are $E = \frac{1}{2}kA^2$, $v_{\text{max}} = A\sqrt{k/m}$, and $a_{\text{max}} = A(k/m)$, where A is the amplitude. Thus, all are changed by a change in amplitude. The period of oscillation is $T = 2\pi\sqrt{m/k}$ and is unchanged by altering the amplitude.

7. Choices (c) and (b). An accelerating elevator is equivalent to a gravitational field. Thus, if the elevator is accelerating upward, this is equivalent to an increased effective gravitational field magnitude g, and the period will decrease. Similarly, if the elevator is accelerating downward, the effective value of g is reduced and the period increases. If the elevator moves with constant velocity, the period of the pendulum is the same as that in the stationary elevator.

8. Choice (a). The clock will run *slow*. With a longer length, the period of the pendulum will increase. Thus, it will take longer to execute each swing, so that each second according to the clock will take longer than an actual second.

9. Choice (b). Greater. The value of g on the Moon is about one-sixth the value of g on Earth, so the period of the pendulum on the Moon will be greater than the period on Earth.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The wavelength of a wave is the distance from crest to the following crest. Thus, the distance between a crest and the following trough is a half wavelength, giving $\lambda = 2(2\text{ m}) = 4\text{ m}$. The speed of the wave is then $v = \lambda f = (4\text{ m})(2\text{ Hz}) = 8\text{ m/s}$, and (c) is the correct choice.

2. When an object undergoes simple harmonic motion, the position as a function of time may be written as $x = A\cos\omega t = A\cos(2\pi ft)$. Comparing this to the given relation, we see that the frequency of vibration is $f = 3$ Hz, and the period is $T = 1/f = 1/3$ s, so the correct answer is (c).

3. In this spring-mass system, the total energy equals the elastic potential energy at the moment the mass is temporarily at rest at $x = A = 6$ cm (i.e., at the extreme ends of the simple harmonic motion). Thus, $E = \frac{1}{2}kA^2$ and we see that as long as the spring constant k and the amplitude A remain unchanged, the total energy is unchanged. Hence, the energy is still 12 J and (a) is the correct choice.

4. The energy given the vibratory system equals the elastic potential energy at the extremes of the motion, $x = \pm A$. Thus, $E = \frac{1}{2}kA^2$ and this energy will all be in the form of kinetic energy as the body passes through the equilibrium position, giving $\frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2$ or

$$v_{\text{max}} = A\sqrt{\frac{k}{m}} = (0.10\text{ m})\sqrt{\frac{80.0\text{ N/m}}{0.40\text{ kg}}} = 1.4\text{ m/s}$$

and (b) is the correct choice.

5. The frequency of vibration is

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

Thus, increasing the mass by a factor of 9 will decrease the frequency to $\frac{1}{3}$ of its original value, and the correct answer is (b).

6. When the object is at its maximum displacement, the net force (directed back toward the equilibrium position) acting on it has magnitude

$$F_{\text{net}} = k|x_{\text{max}}| = (8.0\text{ N/m})(0.10\text{ m}) = 0.80\text{ N}$$

This force will give the mass an acceleration of $a = F_{\text{net}}/m = 0.80\text{ N}/0.40\text{ kg} = 2.0\text{ m/s}^2$, making (d) the correct choice.

7. The car will continue to compress the spring until all of the car's original kinetic energy has been converted into elastic potential energy within the spring, i.e., until $\frac{1}{2}kx^2 = \frac{1}{2}mv_i^2$ or

$$x = v_i\sqrt{\frac{m}{k}} = (2.0\text{ m/s})\sqrt{\frac{3.0\times10^5\text{ kg}}{2.0\times10^6\text{ N/m}}} = 0.77\text{ m}$$

The correct choice is seen to be (a).

8. The period of a simple pendulum is $T = 2\pi\sqrt{\ell/g}$, and its frequency is $f = 1/T = (1/2\pi)\sqrt{g/\ell}$. Thus, if the length is doubled so $\ell' = 2\ell$, the new frequency is

$$f' = \frac{1}{2\pi}\sqrt{\frac{g}{\ell'}} = \frac{1}{2\pi}\sqrt{\frac{g}{2\ell}} = \frac{1}{\sqrt{2}}\left(\frac{1}{2\pi}\sqrt{\frac{g}{\ell}}\right) = \frac{f}{\sqrt{2}}$$

and we see that (d) is the correct response.

9. The period of a simple pendulum is $T = 2\pi\sqrt{\ell/g}$. If the length is changed to $\ell' = 4\ell$, the new period will be

$$T' = 2\pi\sqrt{\frac{\ell'}{g}} = 2\pi\sqrt{\frac{4\ell}{g}} = 2\left(2\pi\sqrt{\frac{\ell}{g}}\right) = 2T$$

or the period will be doubled. The correct choice is (e).

10. For a particle executing simple harmonic motion about an equilibrium point x_0, its position as a function of time is given by $x - x_0 = A\cos(\omega t)$, and the turning points (i.e., the extremes of the position) are at $x = x_0 \pm A$. That is, the equilibrium position is midway between the turning points, so the correct response is choice (c).

11. The only false statement among the listed choices is choice (d). At the equilibrium position, $x = 0$, the elastic potential energy $\left(PE_s = kx^2/2\right)$ is a minimum, and the kinetic energy is a maximum.

12. In a vertical mass-spring system, the equilibrium position is the point at which the mass will hang at rest on the lower end of the spring. If the mass is raised distance A above this position and released from rest, it will undergo simple harmonic motion, with amplitude A, about the equilibrium position. The upper turning point of the motion is at the point of release, and the lower turning point is distance A below the equilibrium position or distance $2A$ below the release point. Thus, if the release point is 15 cm above the equilibrium position, the mass drops 30 cm before stopping momentarily and reversing direction. The correct answer is choice (c).

13. The total energy of the system is constant and equal to the elastic potential energy at $x = \pm A$ (where the velocity is zero). That is, $KE + PE_s = \frac{1}{2}kA^2$. Thus, at a location where $KE = 2PE_s$, we have $3PE_s = \frac{1}{2}kA^2$ or

$$3\left(\frac{1}{2}kx^2\right) = \frac{1}{2}kA^2 \qquad \text{and} \qquad x^2 = \frac{A^2}{3} \qquad \text{or} \qquad x = \pm\frac{A}{\sqrt{3}}$$

and the correct choice is (c).

14. When it supports your weight, the center of the diving board flexes down less than the end does when it supports your weight—this is similar to a spring that stretches a smaller distance for the same force: its spring constant is greater because the displacement is smaller. Therefore, the stiffness constant describing the center of the board is greater than the stiffness constant describing the end. Thus, the frequency, given by

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}}$$

is greater when you bounce at the center of the board. Choice (a) is correct.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. Friction. This includes both air resistance and damping within the spring.

4. Each half-spring will have twice the spring constant of the full spring, as shown by the following argument. The force exerted by a spring is proportional to the separation of the coils as the spring is extended. Imagine that we extend a spring by a given distance and measure the distance between coils. We then cut the spring in half. If one of the half-springs is now extended by the same distance, the coils will be twice as far apart as they were for the complete spring. Thus, it takes twice as much force to stretch the half-spring, from which we conclude that the half-spring has a spring constant which is twice that of the complete spring.

6. No. The period of vibration is $T = 2\pi\sqrt{\ell/g}$ and g is smaller at high altitude. Therefore, the period is longer on the mountain top and the clock will run slower.

8. Shorten the pendulum to decrease the period between ticks.

10. The speed of the pulse is $v = \sqrt{F/\mu}$, so increasing the tension F in the hose increases the speed of the pulse. Filling the hose with water increases the mass per unit length μ, and will decrease the speed of the pulse.

12. As the temperature increases, the length of the pendulum will increase due to thermal expansion, and with a greater length, the period of the pendulum increases. Thus, it takes longer to execute each swing, so that each second according to the clock will take longer than an actual second. Consequently, the clock will run slow.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. 1.54 cm

4. **(a)** 1.1×10^{2} N

(b) The graph should be a straight line passing through the origin with a positive slope of 1.0×10^{3} N/m.

6. **(a)** 327 N **(b)** 1.25×10^{3} N

8. **(a)** 0.625 J **(b)** 0.791 m/s

10. **(a)** 575 N/m **(b)** 46.0 J

12. 2.23 m/s

14. **(a)** $E = \frac{1}{2}kA^2$ **(b)** $\frac{1}{2}mv^2 = kx^2$ **(c)** $x = \pm A/\sqrt{3}$

16. **(a)** 4.58 N **(b)** 0.125 J **(c)** 18.3 m/s^2

(d) 1.00 m/s **(e)** the speed would be lower

(f) The numeric value of the coefficient of kinetic friction would be required.

18. **(a)** 0.15 J **(b)** 0.78 m/s **(c)** 18 m/s^2

20. 3.06 m/s

22. **(a)** 0.628 m/s **(b)** 0.500 Hz **(c)** 3.14 rad/s

24. **(a)** 1.89 Hz **(b)** 33.7 N/m **(c)** 0.118 m

26. **(a)** 3.9×10^{5} N/m **(b)** 2.2 Hz

28. **(a)** 0.25 s **(b)** 4.0 Hz **(c)** 5.2 cm

(d) 21 ms

30. **(a)** $\pm A\sqrt{3}/2$ **(b)** $\pm A/\sqrt{2}$

32. **(a)** 250 N/m **(b)** 22.4 rad/s, 3.56 Hz, 0.281 s

(c) 0.313 J **(d)** 5.00 cm **(e)** $1.12 \text{ m/s}, 25.0 \text{ m/s}^2$

(f) 0.919 cm **(g)** $+1.10 \text{ m/s}, -4.59 \text{ m/s}^2$

34. **(a)** 59.6 m **(b)** 37.5 s

36. 1.001 5

38. $1.66\times10^{-2} \text{ kg}\cdot\text{m}^2$

40. **(a)** 3.65 s **(b)** 6.41 s **(c)** 4.24 s

42. **(a)** 2.00 cm **(b)** 4.00 s **(c)** $\pi/2 \text{ rad/s}$

(d) $\pi \text{ cm/s}$ **(e)** 4.93 cm/s^2 **(f)** $x = (2.00 \text{ cm})\sin(\pi t/2)$

44. **(a)** 0.357 Hz **(b)** 0.985 m/s

46. 5.72 mm

48. **(a)** 0.20 Hz **(b)** 0.25 Hz

50. 219 N

52. **(a)** The units of the first T are seconds, the units of the second are newtons.

(b) The first T is period of time; the second is force of tension.

54. 1.64 m/s^2

56. 7.07 m/s

58. 586 m/s

60. **(a)** $v = 2nL/t$ **(b)** $F = 4n^2ML/t^2$

62. **(a)** 0.25 m **(b)** 0.47 N/m **(c)** 0.23 m

(d) 0.12 m/s

64. **(a)** 5.10 ms **(b)** 1.75 m

66. 0.990 m

68. **(a)** 100 m/s **(b)** 374 J

70. **(a)** 19.8 m/s **(b)** 8.95 m

72. **(a)** and **(b)** See Solution for proofs.

74. 1.3 cm/s

76. **(a)** $\Sigma F_y = -ky_f - mg = mv^2/(L - y_f)$

(b) $mv^2 = 2mg\left(L - y_f\right) - ky_f^2$ **(c)** $y_f = -0.110$ m **(d)** greater than

PROBLEM SOLUTIONS

13.1 (a) Taking to the right as positive, the spring force acting on the block at the instant of release is

$$F_s = -kA = -(130\ \text{N/m})(+0.13\ \text{m}) = -17\ \text{N} \quad \text{or} \quad \boxed{17\ \text{N to the left}}$$

(b) At this instant, the acceleration is

$$a = \frac{F_s}{m} = \frac{-17\ \text{N}}{0.60\ \text{kg}} = -28\ \text{m/s}^2 \qquad \text{or} \qquad \boxed{a = 28\ \text{m/s}^2\ \text{to the left}}$$

13.2 The force compressing the spring is the weight of the object. Thus, the spring will be compressed a distance of

$$|x| = \frac{|F|}{k} = \frac{mg}{k} = \frac{(2.30\ \text{kg})(9.80\ \text{m/s}^2)}{1.46\times10^3\ \text{N/m}} = 1.54\times10^{-2}\ \text{m} = \boxed{1.54\ \text{cm}}$$

13.3 Assuming the spring obeys Hooke's law, the magnitude of the force required to displace the end a distance $|x|$ from the equilibrium position (by either compressing or stretching the spring) is $|F| = k|x|$, where k is the force constant of the spring.

(a) If $x = -4.80$ cm, the required force is $|F| = k|x| = (137\ \text{N/m})(4.80\times10^{-2}\ \text{m}) = \boxed{6.58\ \text{N}}$

(b) If $x = +7.36$ cm, the required force is $|F| = k|x| = (137\ \text{N/m})(7.36\times10^{-2}\ \text{m}) = \boxed{10.1\ \text{N}}$

13.4 (a) The spring constant is $k = \dfrac{|F_s|}{x} = \dfrac{mg}{x} = \dfrac{50\ \text{N}}{5.0\times10^{-2}\ \text{m}} = 1.0\times10^3\ \text{N/m}$

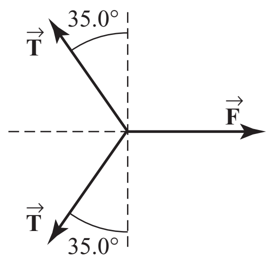
$$F = |F_s| = kx = (1.0\times10^3\ \text{N/m})(0.11\ \text{m}) = \boxed{1.1\times10^2\ \text{N}}$$

(b) The graph will be a $\boxed{\text{straight line passing through the origin}}$ with a slope equal to $k = 1.0\times10^3$ N/m.

13.5 When the system is in equilibrium, the tension in the spring $F = k|x|$ must equal the weight of the object. Thus,

$$k|x| = mg \text{ giving } m = \frac{k|x|}{g} = \frac{(47.5\ \text{N/m})(5.00\times10^{-2}\ \text{m})}{9.80\ \text{m/s}^2} = \boxed{0.242\ \text{kg}}$$

13.6 (a) The free-body diagram of the point in the center of the string is given at the right. From this, we see that



$$\Sigma F_x = 0 \;\Rightarrow\; F - 2T\sin 35.0° = 0$$

or $$T = \frac{F}{2\sin 35.0°} = \frac{375\ \text{N}}{2\sin 35.0°} = \boxed{327\ \text{N}}$$

(b) Since the bow requires an applied horizontal force of 375 N to hold the string at 35.0° from the vertical, the tension in the spring must be 375 N when the spring is stretched 30.0 cm. Thus, the spring constant is

$$k = \frac{F}{x} = \frac{375\ \text{N}}{0.300\ \text{m}} = \boxed{1.25\times 10^{3}\ \ \text{N/m}}$$

13.7 (a) When the block comes to equilibrium, $\Sigma F_y = -ky_0 - mg = 0$, giving

$$y_0 = -\frac{mg}{k} = -\frac{(10.0\ \text{kg})(9.80\ \ \text{m/s}^2)}{475\ \ \text{N/m}} = -0.206\ \text{m}$$

or the equilibrium position is $\boxed{0.206\ \text{m}}$ *below* the unstretched position of the lower end of the spring.

(b) When the elevator (and everything in it) has an upward acceleration of $a = 2.00\ \ \text{m/s}^2$, applying Newton's second law to the block gives

$$\Sigma F_y = -k(y_0 + y) - mg = ma_y \quad \text{or} \quad \Sigma F_y = (-ky_0 - mg) - ky = ma_y$$

where $y = 0$ at the equilibrium position of the block. Since $-ky_0 - mg = 0$ [see part (a)], this becomes $-ky = ma$ and the new position of the block is

$$y = \frac{ma_y}{-k} = \frac{(10.0\ \text{kg})(+2.00\ \ \text{m/s}^2)}{-475\ \ \text{N/m}} = -4.21\times 10^{-2}\ \text{m} = \boxed{-4.21\ \text{cm}}$$

or $\boxed{\text{4.21 cm below the equilibrium position}}$.

(c) When the cable breaks, the elevator and its contents will be in free-fall with $a_y = -g$. The new "equilibrium" position of the block is found from $\Sigma F_y = -ky_0' - mg = m(-g)$, which yields $y_0' = 0$. When the cable snapped, the block was at rest relative to the elevator at distance $y_0 + y = 0.206\ \text{m} + 0.042\,1\ \text{m} = 0.248\ \text{m}$ below the new "equilibrium" position. Thus, while the elevator is in free-fall, the block will oscillate with $\boxed{\text{amplitude} = 0.248\ \text{m}}$ about the new "equilibrium" position, which is the unstretched position of the spring's lower end.

13.8 (a) The work required to stretch the spring equals the elastic potential energy of the spring in the stretched condition, or

$$W = \frac{1}{2}kx^2 = \frac{1}{2}(5.00\times 10^{2}\ \ \text{N/m})(5.00\times 10^{-2}\ \text{m})^2 = \boxed{0.625\ \text{J}}$$

(b) In the initial condition, the spring-block system is at rest $(KE_i = 0)$ with elastic potential energy of $PE_{s,i} = 0.625$ J. Since the spring force is conservative, conservation of energy gives $KE_f + PE_{s,f} = KE_i + PE_{s,i} = 0.625$ J. Thus, when the block is at the equilibrium position $(PE_{s,f} = 0)$, we have $KE_f = \frac{1}{2}mv_f^2 = 0.625$ J, or

$$v_f = \sqrt{\frac{2(0.625\ \text{J})}{m}} = \sqrt{\frac{2(0.625\ \text{J})}{2.00\ \text{kg}}} = \boxed{0.791\ \ \text{m/s}}$$

13.9 (a) Assume the rubber bands obey Hooke's law. Then, the force constant of each band is

$$k = \frac{F_s}{x} = \frac{15\ \text{N}}{1.0\times10^{-2}\ \text{m}} = 1.5\times10^{3}\ \text{N/m}$$

Thus, when both bands are stretched 0.20 m, the total elastic potential energy is

$$PE_s = 2\left(\frac{1}{2}kx^2\right) = \left(1.5\times10^{3}\ \text{N/m}\right)\left(0.20\ \text{m}\right)^2 = \boxed{60\ \text{J}}$$

(b) Conservation of mechanical energy gives $\left(KE+PE_s\right)_f = \left(KE+PE_s\right)_i$, or

$$\frac{1}{2}mv^2 + 0 = 0 + 60\ \text{J} \qquad \text{so} \qquad v = \sqrt{\frac{2(60\ \text{J})}{50\times10^{-3}\ \text{kg}}} = \boxed{49\ \text{m/s}}$$

13.10 (a) $k = \dfrac{F_{\max}}{x_{\max}} = \dfrac{230\ \text{N}}{0.400\ \text{m}} = \boxed{575\ \text{N/m}}$

(b) $\textit{work done} = PE_s = \dfrac{1}{2}kx^2 = \dfrac{1}{2}\left(575\ \text{N/m}\right)\left(0.400\right)^2 = \boxed{46.0\ \text{J}}$

13.11 From conservation of mechanical energy,

$$\left(KE+PE_g+PE_s\right)_f = \left(KE+PE_g+PE_s\right)_i \ \text{ or } \ 0 + mgh_f + 0 = 0 + 0 + \frac{1}{2}kx_i^2,$$

giving

$$k = \frac{2mgh_f}{x_i^2} = \frac{2\left(0.100\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\left(0.600\ \text{m}\right)}{\left(2.00\times10^{-2}\ \text{m}\right)^2} = \boxed{2.94\times10^{3}\ \text{N/m}}$$

13.12 Conservation of mechanical energy, $\left(KE+PE_g+PE_s\right)_i = \left(KE+PE_g+PE_s\right)_f$, gives

$$\frac{1}{2}mv_i^2 + 0 + 0 = 0 + 0 + \frac{1}{2}kx_f^2,$$

or $\quad v_i = \sqrt{\dfrac{k}{m}}\,x_i = \sqrt{\dfrac{5.00\times10^{6}\ \text{N/m}}{1000\ \text{kg}}}\left(3.16\times10^{-2}\ \text{m}\right) = \boxed{2.23\ \text{m/s}}$

13.13 An unknown quantity of mechanical energy is converted into internal energy during the collision. Thus, we apply conservation of momentum from just before to just after the collision and obtain $mv_i + M(0) = (M+m)V$, or the speed of the block and embedded bullet just after collision is $V = (m/M+m)v_i$. We now use conservation of mechanical energy, $(KE+PE_s)_f = (KE+PE_s)_i$, from just after collision until the block comes to rest. This gives $0 + \frac{1}{2}kx_f^2 = \frac{1}{2}(M+m)V^2 + 0$, or

$$x_f = V\sqrt{\frac{M+m}{k}} = v_i\left(\frac{m}{M+m}\right)\sqrt{\frac{M+m}{k}} = \frac{mv_i}{\sqrt{(M+m)k}}$$

yielding $\quad x_f = \dfrac{\left(10.0\times10^{-3}\ \text{kg}\right)\left(300\ \text{m/s}\right)}{\sqrt{\left(2.01\ \text{kg}\right)\left(19.6\ \text{N/m}\right)}} = \boxed{0.478\ \text{m}}$

13.14 (a) At either of the turning points, $x = \pm A$, the constant total energy of the system is momentarily stored as elastic potential energy in the spring. Thus, $\boxed{E = \frac{1}{2}kA^2}$.

(b) When the object is distance x from the equilibrium position, the elastic potential energy is $PE_s = kx^2/2$ and the kinetic energy is $KE = mv^2/2$. At the position where $KE = 2PE_s$, it is necessary that

$$\frac{1}{2}mv^2 = 2\left(\frac{1}{2}kx^2\right) \qquad \text{or} \qquad \boxed{\frac{1}{2}mv^2 = kx^2}$$

continued on next page

(c) When $KE = 2PE_s$, conservation of energy gives $E = KE + PE_s = 2(PE_s) + PE_s = 3PE_s$, or

$$\frac{1}{2}kA^2 = 3\left(\frac{1}{2}kx^2\right) \quad \Rightarrow \quad x = \pm\sqrt{\frac{\cancel{k}A^2/\cancel{2}}{3\cancel{k}/\cancel{2}}} \quad \text{or} \quad \boxed{x = \pm\frac{A}{\sqrt{3}}}$$

13.15 (a) At maximum displacement from equilibrium, all of the energy is in the form of elastic potential energy, giving $E = \frac{1}{2}kx_{max}^2$, and

$$k = \frac{2E}{x_{max}^2} = \frac{2(47.0\text{ J})}{(0.240\text{ m})^2} = \boxed{1.63 \times 10^3\ \text{N/m}}$$

(b) At the equilibrium position $(x = 0)$, the spring is momentarily in its relaxed state and $PE_s = 0$, so all of the energy is in the form of kinetic energy. This gives

$$KE\big|_{x=0} = \frac{1}{2}mv_{max}^2 = E = \boxed{47.0\text{ J}}$$

(c) If, at the equilibrium position, $v = v_{max} = 3.45\ \text{m/s}$, the mass of the block is

$$m = \frac{2E}{v_{max}^2} = \frac{2(47.0\text{ J})}{(3.45\ \text{m/s})^2} = \boxed{7.90\text{ kg}}$$

(d) At any position, the constant total energy is $E = KE + PE_s = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$, so at $x = 0.160$ m

$$v = \sqrt{\frac{2E - kx^2}{m}} = \sqrt{\frac{2(47.0\text{ J}) - (1.63 \times 10^3\ \text{N/m})(0.160\text{ m})^2}{7.90\text{ kg}}} = \boxed{2.57\ \text{m/s}}$$

(e) At $x = 0.160$ m, where $v = 2.57\ \text{m/s}$, the kinetic energy is

$$KE = \frac{1}{2}mv^2 = \frac{1}{2}(7.90\text{ kg})(2.57\ \text{m/s})^2 = \boxed{26.1\text{ J}}$$

(f) At $x = 0.160$ m, where $KE = 26.1$ J, the elastic potential energy is

$$PE_s = E - KE = 47.0\text{ J} - 26.1\text{ J} = \boxed{20.9\text{ J}}$$

or alternately: $$PE_s = \frac{1}{2}kx^2 = \frac{1}{2}(1.63 \times 10^3\ \text{N/m})(0.160\text{ m})^2 = \boxed{20.9\text{ J}}$$

(g) At the first turning point (for which $x < 0$ since the block started from rest at $x = +0.240$ m and has passed through the equilibrium at $x = 0$), all of the remaining energy is in the form of elastic potential energy, so

$$\frac{1}{2}kx^2 = E - E_{loss} = 47.0\text{ J} - 14.0\text{ J} = 33.0\text{ J}$$

and $$x = -\sqrt{\frac{2(33.0\text{ J})}{k}} = -\sqrt{\frac{2(33.0\text{ J})}{1.63 \times 10^3\ \text{N/m}}} = \boxed{-0.201\text{ m}}$$

13.16 (a) $F = k|x| = (83.8\ \text{N/m})(5.46 \times 10^{-2}\ \text{m}) = \boxed{4.58\text{ N}}$

(b) $E = PE_s = \frac{1}{2}kx^2 = \frac{1}{2}(83.8\ \text{N/m})(5.46 \times 10^{-2}\ \text{m})^2 = \boxed{0.125\text{ J}}$

(c) While the block was held stationary at $x = 5.46$ cm, $\Sigma F_x = -F_s + F = 0$, or the spring force was equal in magnitude and oppositely directed to the applied force. When the applied force is suddenly removed, there is a net force $F_s = 4.58$ N directed toward the equilibrium position acting on the block. This gives the block an acceleration having magnitude

$$|a| = \frac{F_s}{m} = \frac{4.58\text{ N}}{0.250\text{ kg}} = \boxed{18.3\ \text{m/s}^2}$$

continued on next page

(d) At the equilibrium position, $PE_s = 0$, so the block has kinetic energy $KE = E = 0.125$ J and speed

$$v = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(0.125 \text{ J})}{0.250 \text{ kg}}} = \boxed{1.00 \text{ m/s}}$$

(e) If the surface was rough, the block would spend energy overcoming a retarding friction force as it moved toward the equilibrium position, causing it to arrive at that position with $\boxed{\text{a lower speed}}$ than that computed above.

(f) Computing a numeric value for this lower speed requires knowledge of the $\boxed{\text{coefficient of kinetic friction}}$ between the block and surface.

13.17 From conservation of mechanical energy, $\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i$, we have $\frac{1}{2}mv^2 + 0 + \frac{1}{2}kx^2 = 0 + 0 + \frac{1}{2}kA^2$, or

$$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)}$$

(a) The speed is a maximum at the equilibrium position, $x = 0$.

$$v_{\max} = \sqrt{\frac{k}{m}A^2} = \sqrt{\frac{(19.6 \text{ N/m})}{(0.40 \text{ kg})}(0.040 \text{ m})^2} = \boxed{0.28 \text{ m/s}}$$

(b) When $x = -0.015$ m,

$$v = \sqrt{\frac{(19.6 \text{ N/m})}{(0.40 \text{ kg})}\left[(0.040 \text{ m})^2 - (-0.015 \text{ m})^2\right]} = \boxed{0.26 \text{ m/s}}$$

(c) When $x = +0.015$ m,

$$v = \sqrt{\frac{(19.6 \text{ N/m})}{(0.40 \text{ kg})}\left[(0.040 \text{ m})^2 - (+0.015 \text{ m})^2\right]} = \boxed{0.26 \text{ m/s}}$$

(d) If $v = \frac{1}{2}v_{\max}$, then

$$\sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \frac{1}{2}\sqrt{\frac{k}{m}A^2}$$

This gives $A^2 - x^2 = \frac{1}{4}A^2$, or

$$x = A\frac{\sqrt{3}}{2} = (4.0 \text{ cm})\frac{\sqrt{3}}{2} = \boxed{3.5 \text{ cm}}$$

13.18 (a) $KE = 0$ at $x = A$, so $E = KE + PE_s = 0 + \frac{1}{2}kA^2$, or the total energy is

$$E = \frac{1}{2}kA^2 = \frac{1}{2}(250 \text{ N/m})(0.035 \text{ m})^2 = \boxed{0.15 \text{ J}}$$

(b) The maximum speed occurs at the equilibrium position where $PE_s = 0$. Thus, $E = \frac{1}{2}mv_{\max}^2$, or

$$v_{\max} = \sqrt{\frac{2E}{m}} = A\sqrt{\frac{k}{m}} = (0.035 \text{ m})\sqrt{\frac{250 \text{ N/m}}{0.50 \text{ kg}}} = \boxed{0.78 \text{ m/s}}$$

(c) The acceleration is $a = \Sigma F/m = -kx/m$. Thus, $a = a_{\max}$ at $x = -x_{\max} = -A$.

$$a_{\max} = \frac{-k(-A)}{m} = \left(\frac{k}{m}\right)A = \left(\frac{250 \text{ N/m}}{0.50 \text{ kg}}\right)(0.035 \text{ m}) = \boxed{18 \text{ m/s}^2}$$

13.19 The maximum speed occurs at the equilibrium position and is $v_{max} = A\sqrt{k/m}$. Thus,

$$m = \frac{kA^2}{v_{max}^2} = \frac{(16.0\ \text{N/m})(0.200\ \text{m})^2}{(0.400\ \text{m/s})^2} = 4.00\ \text{kg}$$

and

$$F_g = mg = (4.00\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{39.2\ \text{N}}$$

13.20 $$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \sqrt{\left(\frac{10.0\ \text{N/m}}{50.0\times10^{-3}\ \text{kg}}\right)\left[(0.250\ \text{m})^2 - (0.125\ \text{m})^2\right]} = \boxed{3.06\ \text{m/s}}$$

13.21 (a) $$PE_{s,i} = \frac{1}{2}kx_i^2 = \frac{1}{2}(850\ \text{N/m})\left(6.00\times10^{-2}\ \text{m}\right)^2 = \boxed{1.53\ \text{J}}$$

(b) Since the surface is frictionless, the total energy of the block-spring system is constant. Thus, $KE + PE_s = KE_i + PE_{s,i} = 0 + 1.53$ J. At the equilibrium position, $PE_s = 0$, so the kinetic energy must be $KE_{max} = \frac{1}{2}mv_{max}^2 = 1.53$ J, which yields

$$v_{max} = \sqrt{\frac{2KE_{max}}{m}} = \sqrt{\frac{2(1.53\ \text{J})}{1.00\ \text{kg}}} = \boxed{1.75\ \text{m/s}}$$

(c) At $x = x_i/2 = 3.00$ cm, the elastic potential energy is $PE_s = \frac{1}{2}kx^2$, and the conservation of energy gives $KE + PE_s = E$, or $\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E$ and

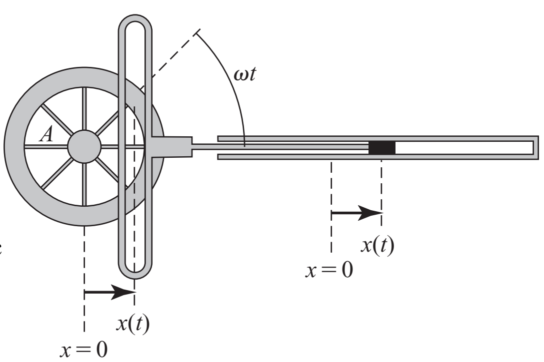
$$v = \sqrt{\frac{2E - kx^2}{m}} = \sqrt{\frac{2(1.53\ \text{J}) - (850\ \text{N/m})\left(3.00\times10^{-2}\ \text{m}\right)^2}{1.00\ \text{kg}}} = \boxed{1.51\ \text{m/s}}$$

13.22 (a) $$v_t = \frac{2\pi r}{T} = \frac{2\pi(0.200\ \text{m})}{2.00\ \text{s}} = \boxed{0.628\ \text{m/s}}$$

(b) $$f = \frac{1}{T} = \frac{1}{2.00\ \text{s}} = \boxed{0.500\ \text{Hz}}$$

(c) $$\omega = \frac{2\pi}{T} = \frac{2\pi}{2.00\ \text{s}} = \boxed{3.14\ \text{rad/s}}$$

13.23 The angle of the crank pin is $\theta = \omega t$. Its x-coordinate is $x = A\cos\theta = A\cos\omega t$, where A is the distance from the center of the wheel to the crank pin. The displacement of the piston from its zero position (i.e., its location when $\theta = \omega t = \pi/2$) is the same as that of the crankpin, $x(t) = A\cos\omega t$. This is of the correct form to describe simple harmonic motion. Hence, one must conclude that the motion is indeed simple harmonic.



13.24 (a) $$f = \frac{1}{T} = \frac{1}{0.528\ \text{s}} = \boxed{1.89\ \text{Hz}}$$

(b) The period of oscillation of an object-spring system is $T = 2\pi\sqrt{m/k}$, so the force constant is

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(0.238\ \text{kg})}{(0.528\ \text{s})^2} = \boxed{33.7\ \text{N/m}}$$

(c) At the turning points ($x = \pm A$) in the oscillation, all of the energy is temporarily stored as elastic potential energy, or $E = kA^2/2$. Thus,

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(0.234\ \text{J})}{33.7\ \text{N/m}}} = \boxed{0.118\ \text{m}}$$

13.25 The spring constant is found from

$$k = \frac{F_s}{x} = \frac{mg}{x} = \frac{(0.010\ \text{kg})(9.80\ \text{m/s}^2)}{3.9 \times 10^{-2}\ \text{m}} = 2.5\ \text{N/m}$$

When the object attached to the spring has mass $m = 25$ g, the period of oscillation is

$$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{0.025\ \text{kg}}{2.5\ \text{N/m}}} = \boxed{0.63\ \text{s}}$$

13.26 (a) The springs compress 0.80 cm when supporting an additional load of $m = 320$ kg. Thus, the spring constant is

$$k = \frac{\Delta F_s}{\Delta x} = \frac{mg}{\Delta x} = \frac{(320\ \text{kg})(9.80\ \text{m/s}^2)}{0.80 \times 10^{-2}\ \text{m}} = \boxed{3.9 \times 10^5\ \text{N/m}}$$

(b) When the empty car, $M = 2.0 \times 10^3$ kg, oscillates on the springs, the frequency will be $f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{M}} = \frac{1}{2\pi}\sqrt{\frac{3.9 \times 10^5\ \text{N/m}}{2.0 \times 10^3\ \text{kg}}} = \boxed{2.2\ \text{Hz}}$

13.27 (a) The period of oscillation is $T = 2\pi\sqrt{m/k}$, where k is the spring constant and m is the mass of the object attached to the end of the spring. Hence,

$$T = 2\pi\sqrt{\frac{0.250\ \text{kg}}{9.5\ \text{N/m}}} = \boxed{1.0\ \text{s}}$$

(b) If the cart is released from rest when it is 4.5 cm from the equilibrium position, the amplitude of oscillation will be $A = 4.5\ \text{cm} = 4.5 \times 10^{-2}\ \text{m}$. The maximum speed is then given by

$$v_{\text{max}} = A\omega = A\sqrt{\frac{k}{m}} = \left(4.5 \times 10^{-2}\ \text{m}\right)\sqrt{\frac{9.5\ \text{N/m}}{0.250\ \text{kg}}} = \boxed{0.28\ \text{m/s}}$$

(c) When the cart has a displacement of $x = 2.0$ cm from the equilibrium position, its speed will be

$$v = \sqrt{\frac{k}{m}\left(A^2 - x^2\right)} = \sqrt{\frac{9.5\ \text{N/m}}{0.250\ \text{kg}}\left[(0.045\ \text{m})^2 - (0.020\ \text{m})^2\right]} = \boxed{0.25\ \text{m/s}}$$

13.28 The general expression for the position as a function of time for an object undergoing simple harmonic motion with $x = 0$ at $t = 0$ is $x = A\sin(\omega t)$. Thus, if $x = (5.2\ \text{cm})\sin(8.0\pi \cdot t)$, we have that the amplitude is $A = 5.2$ cm and the angular frequency is $\omega = 8.0\pi$ rad/s.

(a) The period is

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{8.0\pi\ \text{s}^{-1}} = \boxed{0.25\ \text{s}}$$

(b) The frequency of motion is

$$f = \frac{1}{T} = \frac{1}{0.25\ \text{s}} = 4.0\ \text{s}^{-1} = \boxed{4.0\ \text{Hz}}$$

(c) As discussed above, the amplitude of the motion is $\boxed{A = 5.2\ \text{cm}}$.

(d) **Note:** For this part, your calculator should be set to operate in *radians mode*. If $x = 2.6$ cm, then

$$\omega t = \sin^{-1}\left(\frac{x}{A}\right) = \sin^{-1}\left(\frac{2.6\ \text{cm}}{5.2\ \text{cm}}\right) = \sin^{-1}(0.50) = 0.52\ \text{radians}$$

continued on next page

and

$$t = \frac{0.52 \text{ rad}}{\omega} = \frac{0.52 \text{ rad}}{8.0\pi \text{ rad/s}} = 2.1\times10^{-2} \text{ s} = 21\times10^{-3} \text{ s} = \boxed{21 \text{ ms}}$$

13.29 (a) At the equilibrium position, the total energy of the system is in the form of kinetic energy and $\frac{1}{2}mv_{max}^2 = E$, so the maximum speed is

$$v_{max} = \sqrt{\frac{2E}{m}} = \sqrt{\frac{2(5.83 \text{ J})}{0.326 \text{ kg}}} = \boxed{5.98 \text{ m/s}}$$

(b) The period of an object-spring system is $T = 2\pi\sqrt{m/k}$, so the force constant of the spring is

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(0.326 \text{ kg})}{(0.250 \text{ s})^2} = \boxed{206 \text{ N/m}}$$

(c) At the turning points, $x = \pm A$, the total energy of the system is in the form of elastic potential energy, or $E = \frac{1}{2}kA^2$, giving the amplitude as

$$A = \sqrt{\frac{2E}{k}} = \sqrt{\frac{2(5.83 \text{ J})}{206 \text{ N/m}}} = \boxed{0.238 \text{ m}}$$

13.30 For a system executing simple harmonic motion, the total energy may be written as $E = KE + PE_s = \frac{1}{2}mv_{max}^2 = \frac{1}{2}kA^2$, where A is the amplitude and v_{max} is the speed at the equilibrium position. Observe from this expression, that we may write $v_{max}^2 = kA^2/m$.

(a) If $v = \frac{1}{2}v_{max}$, then $E = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}mv_{max}^2$ becomes

$$\cancel{\frac{1}{2}}m\left(\frac{v_{max}^2}{4}\right) + \cancel{\frac{1}{2}}kx^2 = \cancel{\frac{1}{2}}mv_{max}^2$$

and gives

$$x^2 = \frac{3}{4}\left(\frac{m}{k}\right)v_{max}^2 = \frac{3}{4}\left(\frac{m}{k}\right)\left[\frac{k}{m}A^2\right] = \frac{3A^2}{4} \qquad \text{or} \qquad \boxed{x = \pm\frac{A\sqrt{3}}{2}}$$

(b) If the elastic potential energy is $PE_s = \frac{1}{2}E$, we have

$$\frac{1}{2}kx^2 = \frac{1}{2}\left(\frac{1}{2}kA^2\right) \qquad \text{or} \qquad x^2 = \frac{A^2}{2} \qquad \text{and} \qquad \boxed{x = \pm\frac{A}{\sqrt{2}}}$$

13.31 **Note**: Your calculator must be in radians mode for part (a) of this problem.

(a) The angular frequency of this oscillation is $\omega = \sqrt{k/m}$ and the displacement at time t is $x = A\cos\omega t$. At $t = 3.50$ s, the spring force will be $F = -kx = -kA\cos(\omega t)$, or

$$F = -\left(5.00 \frac{\text{N}}{\text{m}}\right)(3.00 \text{ m})\cos\left[\left(\sqrt{\frac{5.00 \text{ N/m}}{2.00 \text{ kg}}}\right)(3.50 \text{ s})\right] = -11.0 \text{ N},$$

or $F = \boxed{11.0 \text{ N directed to the left}}$

(b) The time required for one complete oscillation is $T = 2\pi/\omega = 2\pi\sqrt{m/k}$. Hence, the number of oscillations made in 3.50 s is

$$N = \frac{\Delta t}{T} = \frac{\Delta t}{2\pi\sqrt{m/k}} = \frac{3.50 \text{ s}}{2\pi}\sqrt{\frac{5.00 \text{ N/m}}{2.00 \text{ kg}}} = \boxed{0.881}$$

13.32 (a) $k = \dfrac{F}{x} = \dfrac{7.50 \text{ N}}{3.00 \times 10^{-2} \text{ m}} = \boxed{250 \text{ N/m}}$

(b) $\omega = \sqrt{\dfrac{k}{m}} = \sqrt{\dfrac{250 \text{ N/m}}{0.500 \text{ kg}}} = \boxed{22.4 \text{ rad/s}}$,

$$f = \frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{250 \text{ N/m}}{0.500 \text{ kg}}} = \boxed{3.56 \text{ Hz}},$$

and $T = \dfrac{1}{f} = \dfrac{1}{3.56 \text{ Hz}} = \boxed{0.281 \text{ s}}$

(c) At $t = 0$, $v = 0$ and $x = 5.00 \times 10^{-2}$ m, so the total energy of the oscillator is

$$E = KE + PE_s = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = 0 + \frac{1}{2}(250 \text{ N/m})(5.00 \times 10^{-2} \text{ m})^2 = \boxed{0.313 \text{ J}}$$

(d) When $x = A$, $v = 0$ so $E = KE + PE_s = 0 + \dfrac{1}{2}kA^2$.

Thus, $A = \sqrt{\dfrac{2E}{k}} = \sqrt{\dfrac{2(0.313 \text{ J})}{250 \text{ N/m}}} = 5.00 \times 10^{-2} \text{ m} = \boxed{5.00 \text{ cm}}$

(e) At $x = 0$, $KE = \dfrac{1}{2}mv_{\max}^2 = E$, or $v_{\max} = \sqrt{\dfrac{2E}{m}} = \sqrt{\dfrac{2(0.313 \text{ J})}{0.500 \text{ kg}}} = \boxed{1.12 \text{ m/s}}$

$$a_{\max} = \frac{F_{\max}}{m} = \frac{kA}{m} = \frac{(250 \text{ N/m})(5.00 \times 10^{-2} \text{ m})}{0.500 \text{ kg}} = \boxed{25.0 \text{ m/s}^2}$$

Note: To solve parts (f) and (g), your calculator should be set in *radians mode*.

(f) At $t = 0.500$ s, Equation 13.14a gives the displacement as

$$x = A\cos(\omega t) = A\cos\left(t\sqrt{k/m}\right) = (5.00 \text{ cm})\cos\left[(0.500 \text{ s})\sqrt{\frac{250 \text{ N/m}}{0.500 \text{ kg}}}\right] = \boxed{0.919 \text{ cm}}$$

(g) From Equation 13.14b, the velocity at $t = 0.500$ s is

$$v = -A\omega\sin(\omega t) = -A\sqrt{k/m}\sin\left(t\sqrt{k/m}\right)$$

$$= -\left(5.00 \times 10^{-2} \text{ m}\right)\sqrt{\frac{250 \text{ N/m}}{0.500 \text{ kg}}}\sin\left[(0.500 \text{ s})\sqrt{\frac{250 \text{ N/m}}{0.500 \text{ kg}}}\right] = \boxed{+1.10 \text{ m/s}}$$

and from Equation 13.14c, the acceleration at this time is

$$a = -A\omega^2\cos(\omega t) = -A(k/m)\cos\left(t\sqrt{k/m}\right)$$

$$= -\left(5.00 \times 10^{-2} \text{ m}\right)\left(\frac{250 \text{ N/m}}{0.500 \text{ kg}}\right)\cos\left[(0.500 \text{ s})\sqrt{\frac{250 \text{ N/m}}{0.500 \text{ kg}}}\right] = \boxed{-4.59 \text{ m/s}^2}$$

13.33 From Equation 13.6, $v = \pm\sqrt{\dfrac{k}{m}\left(A^2 - x^2\right)} = \pm\sqrt{\omega^2\left(A^2 - x^2\right)}$

Hence, $v = \pm\omega\sqrt{A^2 - A^2\cos^2(\omega t)} = \pm\omega A\sqrt{1 - \cos^2(\omega t)} = \boxed{\pm\omega A\sin(\omega t)}$

From Equation 13.2, $a = -\dfrac{k}{m}x = -\omega^2\left[A\cos(\omega t)\right] = \boxed{-\omega^2 A\cos(\omega t)}$

13.34 (a) The height of the tower is almost the same as the length of the pendulum. From $T = 2\pi\sqrt{L/g}$, we obtain

$$L = \frac{gT^2}{4\pi^2} = \frac{(9.80\ \text{m/s}^2)(15.5\ \text{s})^2}{4\pi^2} = \boxed{59.6\ \text{m}}$$

(b) On the Moon, where $g = 1.67\ \text{m/s}^2$, the period will be

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{59.6\ \text{m}}{1.67\ \text{m/s}^2}} = \boxed{37.5\ \text{s}}$$

13.35 (a) The period is the time for one complete oscillation. Hence,

$$T = \frac{2.00\ \text{min}}{82}\left(\frac{60\ \text{s}}{1\ \text{min}}\right) = \frac{120\ \text{s}}{82.0} \qquad \text{or} \qquad T = \boxed{1.46\ \text{s}}$$

(b) The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\ell/g}$, so the local acceleration of gravity must be

$$g = \frac{4\pi^2\ell}{T^2} = \frac{4\pi^2(0.520\ \text{m})}{(120\ \text{s}/82.0)^2} = \boxed{9.59\ \text{m/s}^2}$$

13.36 The period in Tokyo is $T_T = 2\pi\sqrt{\frac{L_T}{g_T}}$ and the period in Cambridge is $T_C = 2\pi\sqrt{\frac{L_C}{g_C}}$.

We know that $T_T = T_C = 2.000\ \text{s}$, from which we see that

$$\frac{L_T}{g_T} = \frac{L_C}{g_C} \qquad \text{or} \qquad \frac{g_C}{g_T} = \frac{L_C}{L_T} = \frac{0.994\,2}{0.992\,7} = \boxed{1.001\,5}$$

13.37 (a) The period of the pendulum is $T = 2\pi\sqrt{L/g}$. Thus, on the Moon where the free-fall acceleration is smaller, the period will be longer and the clock will run $\boxed{\text{slow}}$.

(b) The ratio of the pendulum's period on the Moon to that on Earth is

$$\frac{T_{\text{Moon}}}{T_{\text{Earth}}} = \frac{2\pi\sqrt{L/g_{\text{Moon}}}}{2\pi\sqrt{L/g_{\text{Earth}}}} = \sqrt{\frac{g_{\text{Earth}}}{g_{\text{Moon}}}}$$

Hence, the pendulum of the clock on Earth makes $\sqrt{g_{\text{Earth}}/g_{\text{Moon}}}$ "ticks" while the clock on the Moon is making 1.00 "tick." After the Earth clock has ticked off 24.0 h and again reads 12:00 midnight, the Moon clock will have ticked off

$$(24.0\ \text{h})\sqrt{\frac{g_{\text{Moon}}}{g_{\text{Earth}}}} = (24.0\ \text{h})\sqrt{\frac{1.63\ \text{m/s}^2}{9.80\ \text{m/s}^2}} = 9.79\ \text{h} = 9\ \text{h} + (0.79\ \text{h})\left(\frac{60\ \text{min}}{1\ \text{h}}\right) = 9\ \text{h} + 47\ \text{min}$$

and will read $\boxed{9:47\ \text{AM}}$.

13.38 The coat hanger acts as a physical pendulum and its period of oscillation is $T = 2\pi\sqrt{I/mgd}$, where d is the distance from the pivot to the center of mass. Thus, the moment of inertia about the axis perpendicular to the plane of oscillation and passing through the pivot must be

$$I = mgd\left(\frac{T}{2\pi}\right)^2 = (0.238\ \text{kg})(9.80\ \text{m/s}^2)(0.180\ \text{m})\left(\frac{1.25\ \text{s}}{2\pi}\right)^2 = \boxed{1.66\times10^{-2}\ \text{kg}\cdot\text{m}^2}$$

13.39 From $T = 2\pi\sqrt{L/g}$, the length of a pendulum with period T is $L = \dfrac{gT^2}{4\pi^2}$.

(a) On Earth, with $T = 1.0 \text{ s}$, $L = \dfrac{(9.8 \text{ m/s}^2)(1.0 \text{ s})^2}{4\pi^2} = 0.25 \text{ m} = \boxed{25 \text{ cm}}$

(b) If $T = 1.0 \text{ s}$ on Mars, $L = \dfrac{(3.7 \text{ m/s}^2)(1.0 \text{ s})^2}{4\pi^2} = 0.094 \text{ m} = \boxed{9.4 \text{ cm}}$

(c) and (d) The period of an object on a spring is $T = 2\pi\sqrt{m/k}$, which is independent of the local free-fall acceleration. Thus, the same mass will work on Earth and on Mars. This mass is

$$m = \frac{kT^2}{4\pi^2} = \frac{(10 \text{ N/m})(1.0 \text{ s})^2}{4\pi^2} = \boxed{0.25 \text{ kg}}$$

13.40 The apparent free-fall acceleration is the vector sum of the actual free-fall acceleration and the negative of the elevator's acceleration. To see this, consider an object that is hanging from a vertical string in the elevator **and appears to be at rest to the elevator passengers**. These passengers believe the tension in the string is the negative of the object's weight, or $\vec{\mathbf{T}} = -m\vec{\mathbf{g}}_{\text{apparent}}$, where $\vec{\mathbf{g}}_{\text{apparent}}$ is the apparent free-fall acceleration in the elevator.

An observer located outside the elevator applies Newton's second law to this object by writing $\Sigma\vec{\mathbf{F}} = \vec{\mathbf{T}} + m\vec{\mathbf{g}} = m\vec{\mathbf{a}}_e$, where $\vec{\mathbf{a}}_e$ is the acceleration of the elevator and all its contents. Thus, $\vec{\mathbf{T}} = m(\vec{\mathbf{a}}_e - \vec{\mathbf{g}}) = -m\vec{\mathbf{g}}_{\text{apparent}}$, which gives $\vec{\mathbf{g}}_{\text{apparent}} = \vec{\mathbf{g}} - \vec{\mathbf{a}}_e$.

(a) If we choose downward as the positive direction, then $\vec{\mathbf{a}}_e = -5.00 \text{ m/s}^2$ in this case and $\vec{\mathbf{g}}_{\text{apparent}} = (9.80 + 5.00) \text{ m/s}^2 = +14.8 \text{ m/s}^2$ (downward). The period of the pendulum is

$$T = 2\pi\sqrt{\frac{L}{g_{\text{apparent}}}} = 2\pi\sqrt{\frac{5.00 \text{ m}}{14.8 \text{ m/s}^2}} = \boxed{3.65 \text{ s}}$$

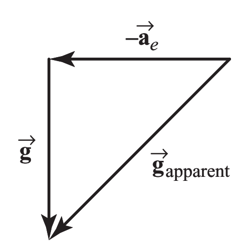
(b) Again choosing downward as positive, $\vec{\mathbf{a}}_e = 5.00 \text{ m/s}^2$ and

$$\vec{\mathbf{g}}_{\text{apparent}} = (9.80 - 5.00) \text{ m/s}^2 = +4.80 \text{ m/s}^2 \text{ (downward)}$$

in this case. The period is now given by

$$T = 2\pi\sqrt{\frac{L}{g_{\text{apparent}}}} = 2\pi\sqrt{\frac{5.00 \text{ m}}{4.80 \text{ m/s}^2}} = \boxed{6.41 \text{ s}}$$

(c) If $\vec{\mathbf{a}}_e = 5.00 \text{ m/s}^2$ horizontally, the vector sum $\vec{\mathbf{g}}_{\text{apparent}} = \vec{\mathbf{g}} - \vec{\mathbf{a}}_e$ is as shown in the sketch at the right. The magnitude is



$$g_{\text{apparent}} = \sqrt{(5.00 \text{ m/s}^2)^2 + (9.80 \text{ m/s}^2)^2} = 11.0 \text{ m/s}^2,$$

and the period of the pendulum is

$$T = 2\pi\sqrt{\frac{L}{g_{\text{apparent}}}} = 2\pi\sqrt{\frac{5.00 \text{ m}}{11.0 \text{ m/s}^2}} = \boxed{4.24 \text{ s}}$$

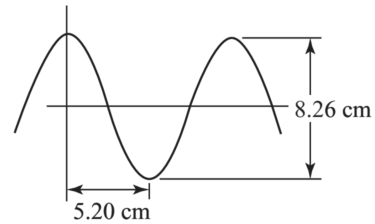


FIGURE P13.41

13.41 (a) The distance from the bottom of a trough to the top of a crest is twice the amplitude of the wave. Thus, $2A = 8.26$ cm and $\boxed{A = 4.13 \text{ cm}}$.

(b) The horizontal distance from a crest to a trough is a half wavelength. Hence,

$$\lambda/2 = 5.20 \text{ cm} \quad \text{and} \quad \boxed{\lambda = 10.4 \text{ cm}}$$

(c) The period is

$$T = \frac{1}{f} = \frac{1}{18.0 \text{ s}^{-1}} = \boxed{5.56 \times 10^{-2} \text{ s}}$$

(d) The wave speed is

$$v = \lambda f = (10.4 \text{ cm})(18.0 \text{ s}^{-1}) = \boxed{187 \text{ cm/s} = 1.87 \text{ m/s}}$$

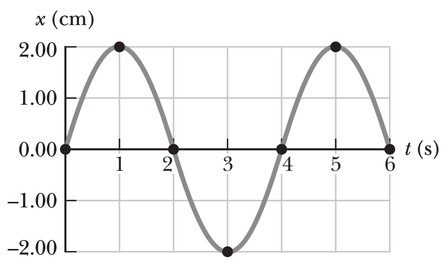


FIGURE P13.42

13.42 (a) The amplitude is the magnitude of the maximum displacement from equilibrium $(\text{at } x = 0)$. Thus, $\boxed{A = 2.00 \text{ cm}}$.

(b) The period is the time for one full cycle of the motion. Therefore, $\boxed{T = 4.00 \text{ s}}$.

(c) The period may be written as $T = 2\pi/\omega$, so the angular frequency is

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{4.00 \text{ s}} = \boxed{\frac{\pi}{2} \text{ rad/s}}$$

(d) The total energy may be expressed as $E = \frac{1}{2}mv_{\text{max}}^2 = \frac{1}{2}kA^2$. Thus, $v_{\text{max}} = A\sqrt{k/m}$, and since $\omega = \sqrt{k/m}$, this becomes $v_{\text{max}} = \omega A$ and yields

$$v_{\text{max}} = \omega A = \left(\frac{\pi}{2} \text{ rad/s}\right)(2.00 \text{ cm}) = \boxed{\pi \text{ cm/s}}$$

(e) The spring exerts maximum force, $|F| = k|x|$, when the object is at maximum distance from equilibrium, i.e., at $|x| = A = 2.00$ cm. Thus, the maximum acceleration of the object is

$$a_{\text{max}} = \frac{|F_{\text{max}}|}{m} = \frac{kA}{m} = \omega^2 A = \left(\frac{\pi}{2} \text{ rad/s}\right)^2 (2.00 \text{ cm}) = \boxed{4.93 \text{ cm/s}^2}$$

(f) The general equation for position as a function of time for an object undergoing simple harmonic motion with $t = 0$ when $x = 0$ is $x = A\sin(\omega t)$. For this oscillator, this becomes

$$\boxed{x = (2.00 \text{ cm})\sin\left(\frac{\pi}{2}t\right)}$$

13.43 (a) The speed of propagation for a wave is the product of its frequency and its wavelength, $v = \lambda f$. Thus, the frequency must be

$$f = \frac{v}{\lambda} = \frac{3.00 \times 10^{8} \text{ m/s}}{5.50 \times 10^{-7} \text{ m}} = \boxed{5.45 \times 10^{14} \text{ Hz}}$$

(b) The period is $T = \dfrac{1}{f} = \dfrac{1}{5.45 \times 10^{14} \text{ Hz}} = \boxed{1.83 \times 10^{-15} \text{ s}}$

13.44 (a) The frequency of a transverse wave is the number of crests that pass a given point each second. Thus, if 5.00 crests pass in 14.0 seconds, the frequency is

$$f = \frac{5.00}{14.0 \text{ s}} = 0.357 \text{ s}^{-1} = \boxed{0.357 \text{ Hz}}$$

(b) The wavelength of the wave is the distance between successive maxima or successive minima. Thus, $\lambda = 2.76$ m and the wave speed is

$$v = \lambda f = (2.76 \text{ m})(0.357 \text{ s}^{-1}) = \boxed{0.985 \text{ m/s}}$$

13.45 The speed of the wave is

$$v = \frac{\Delta x}{\Delta t} = \frac{425 \text{ cm}}{10.0 \text{ s}} = 42.5 \text{ cm/s}$$

and the frequency is number of vibrations occurring each second, or $f = 40.0 \text{ vib}/30.0 \text{ s}$.

Thus, $\lambda = \dfrac{v}{f} = \dfrac{42.5 \text{ cm/s}}{40.0 \text{ vib}/30.0 \text{ s}} = \dfrac{(42.5 \text{ cm/s})(30.0 \text{ s})}{40.0 \text{ vib}} = \boxed{31.9 \text{ cm}}$

13.46 From $v = \lambda f$, the wavelength (and size of smallest detectable insect) is

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{60.0 \times 10^{3} \text{ Hz}} = 5.72 \times 10^{-3} \text{ m} = \boxed{5.72 \text{ mm}}$$

13.47 The frequency of the wave (that is, the number of crests passing the cork each second) is $f = 2.00 \text{ s}^{-1}$ and the wavelength (distance between successive crests) is $\lambda = 8.50$ cm. Thus, the wave speed is

$$v = \lambda f = (8.50 \text{ cm})(2.00 \text{ s}^{-1}) = 17.0 \text{ cm/s} = 0.170 \text{ m/s}$$

and the time required for the ripples to travel 10.0 m over the surface of the water is

$$\Delta t = \frac{\Delta x}{v} = \frac{10.0 \text{ m}}{0.170 \text{ m/s}} = \boxed{58.8 \text{ s}}$$

13.48 (a) When the boat is at rest in the water, the speed of the wave relative to the boat is the same as the speed of the wave relative to the water, $v = 4.0 \text{ m/s}$. The frequency detected in this case is

$$f = \frac{v}{\lambda} = \frac{4.0 \text{ m/s}}{20 \text{ m}} = \boxed{0.20 \text{ Hz}}$$

continued on next page

(b) Taking eastward as positive, $\vec{\mathbf{v}}_{\text{wave,boat}} = \vec{\mathbf{v}}_{\text{wave,water}} - \vec{\mathbf{v}}_{\text{boat,water}}$ (see the discussion of relative velocity in Chapter 3 of the textbook) gives

$$\vec{\mathbf{v}}_{\text{wave,boat}} = +4.0\ \text{m/s} - (-1.0\ \text{m/s}) = +5.0\ \text{m/s} \quad \text{and} \quad v_{\text{boat,wave}} = \left|\vec{\mathbf{v}}_{\text{wave,boat}}\right| = 5.0\ \text{m/s}$$

Thus,

$$f = \frac{v_{\text{boat,wave}}}{\lambda} = \frac{5.0\ \text{m/s}}{20\ \text{m}} = \boxed{0.25\ \text{Hz}}$$

13.49 The down and back distance is $4.00\ \text{m} + 4.00\ \text{m} = 8.00\ \text{m}$. The speed is then

$$v = \frac{d_{\text{total}}}{t} = \frac{4(8.00\ \text{m})}{0.800\ \text{s}} = 40.0\ \text{m/s} = \sqrt{F/\mu}$$

Now,

$$\mu = \frac{m}{L} = \frac{0.200\ \text{kg}}{4.00\ \text{m}} = 5.00 \times 10^{-2}\ \text{kg/m}$$

so

$$F = \mu v^2 = \left(5.00 \times 10^{-2}\ \text{kg/m}\right)\left(40.0\ \text{m/s}\right)^2 = \boxed{80.0\ \text{N}}$$

13.50 The speed of the wave is

$$v = \frac{\Delta x}{\Delta t} = \frac{20.0\ \text{m}}{0.800\ \text{s}} = 25.0\ \text{m/s}$$

and the mass per unit length of the rope is $\mu = m/L = 0.350\ \text{kg/m}$. Thus, from $v = \sqrt{F/\mu}$, we obtain

$$F = v^2\mu = (25.0\ \text{m/s})^2\,(0.350\ \text{kg/m}) = \boxed{219\ \text{N}}$$

13.51 $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{1\,350\ \text{N}}{5.00 \times 10^{-3}\ \text{kg/m}}} = \boxed{5.20 \times 10^2\ \text{m/s}}$

13.52 (a) $f = \dfrac{1}{T} \quad \rightarrow \quad T = \dfrac{1}{f} \quad \rightarrow \quad [T] = \dfrac{1}{[f]} = \dfrac{1}{\text{T}^{-1}} = \text{T} \quad \boxed{\text{units are seconds}}$

$$v = \sqrt{\frac{T}{\mu}} \quad \rightarrow \quad T = \mu v^2 \quad \rightarrow \quad [T] = [\mu][v^2] = \frac{\text{M}}{\text{L}} \cdot \frac{\text{L}^2}{\text{T}^2} = \frac{\text{ML}}{\text{T}^2} \quad \boxed{\text{units are newtons}}$$

(b) $\boxed{\text{The first } T \text{ is period of time; the second is force of tension.}}$

13.53 (a) The mass per unit length is

$$\mu = \frac{m}{L} = \frac{0.0600\ \text{kg}}{5.00\ \text{m}} = 1.20 \times 10^{-2}\ \text{kg/m}$$

From $v = \sqrt{F/\mu}$, the required tension in the string is

$$F = v^2\mu = (50.0\ \text{m/s})^2\left(1.20 \times 10^{-2}\ \text{kg/m}\right) = \boxed{30.0\ \text{N}}$$

(b) $v = \sqrt{\dfrac{F}{\mu}} = \sqrt{\dfrac{8.00\ \text{N}}{1.20 \times 10^{-2}\ \text{kg/m}}} = \boxed{25.8\ \text{m/s}}$

13.54 The mass per unit length of the wire is

$$\mu = \frac{m}{L} = \frac{4.00\times10^{-3}\ \text{kg}}{1.60\ \text{m}} = 2.50\times10^{-3}\ \text{kg/m},$$

and the speed of the pulse is

$$v = \frac{L}{\Delta t} = \frac{1.60\ \text{m}}{0.0361\ \text{s}} = 44.3\ \text{m/s}$$

The tension in the wire is $F = mg = \mu v^2$, so the lunar acceleration of gravity must be

$$g = \frac{v^2\mu}{m} = \frac{(44.3\ \text{m/s})^2\left(2.50\times10^{-3}\ \text{kg/m}\right)}{3.00\ \text{kg}} = \boxed{1.64\ \text{m/s}^2}$$

13.55 The period of the pendulum is $T = 2\pi\sqrt{L/g}$, so the length of the string is

$$L = \frac{gT^2}{4\pi^2} = \frac{\left(9.80\ \text{m/s}^2\right)(2.00\ \text{s})^2}{4\pi^2} = 0.993\ \text{m}$$

The mass per unit length of the string is then

$$\mu = \frac{m}{L} = \frac{0.060\ 0\ \text{kg}}{0.993\ \text{m}} = 6.04\times10^{-2}\ \frac{\text{kg}}{\text{m}}$$

When the pendulum is vertical and stationary, the tension in the string is

$$F = M_{\text{ball}}\,g = (5.00\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 49.0\ \text{N}$$

and the speed of transverse waves in it is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{49.0\ \text{N}}{6.04\times10^{-2}\ \text{kg/m}}} = \boxed{28.5\ \text{m/s}}$$

13.56 If $\mu_1 = m_1/L$ is the mass per unit length for the first string, then $\mu_2 = m_2/L = m_1/2L = \mu_1/2$ is that of the second string. Thus, with $F_2 = F_1 = F$, the speed of waves in the second string is

$$v_2 = \sqrt{\frac{F}{\mu_2}} = \sqrt{\frac{2F}{\mu_1}} = \sqrt{2}\left(\sqrt{\frac{F}{\mu_1}}\right) = \sqrt{2}\,v_1 = \sqrt{2}\,(5.00\ \text{m/s}) = \boxed{7.07\ \text{m/s}}$$

13.57 (a) The tension in the string is $F = mg = (3.00\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 29.4$ N. Then, from $v = \sqrt{F/\mu}$, the mass per unit length is

$$\mu = \frac{F}{v^2} = \frac{29.4\ \text{N}}{(24.0\ \text{m/s})^2} = \boxed{5.10\times10^{-2}\ \text{kg/m}}$$

(b) When $m = 2.00$ kg, the tension is

$$F = mg = (2.00\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 19.6\ \text{N}$$

and the speed of transverse waves in the string is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{19.6\ \text{N}}{5.10\times10^{-2}\ \text{kg/m}}} = \boxed{19.6\ \text{m/s}}$$

13.58 If the tension in the wire is F, the tensile stress is $Stress = F/A$, so the speed of transverse waves in the wire may be written as

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{A \cdot Stress}{m/L}} = \sqrt{\frac{Stress}{m/(A \cdot L)}}$$

But $A \cdot L = V =$ volume, so $m/(A \cdot L) = \rho =$ density. Thus, $v = \sqrt{Stress/\rho}$.

Taking the density of steel to be equal to that of iron, the maximum speed of waves in the wire is

$$v_{\text{max}} = \sqrt{\frac{(Stress)_{\text{max}}}{\rho_{\text{steel}}}} = \sqrt{\frac{2.70 \times 10^9 \text{ Pa}}{7.86 \times 10^3 \text{ kg/m}^3}} = \boxed{586 \text{ m/s}}$$

13.59 (a) The speed of transverse waves in the line is $v = \sqrt{F/\mu}$, with $\mu = m/L$ being the mass per unit length. Therefore,

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{FL}{m}} = \sqrt{\frac{(12.5 \text{ N})(38.0 \text{ m})}{2.65 \text{ kg}}} = \boxed{13.4 \text{ m/s}}$$

(b) The worker could throw an object, such as a snowball, at one end of the line to set up a pulse, and use a stopwatch to measure the time it takes a pulse to travel the length of the line. From this measurement, the worker would have an estimate of the wave speed, which in turn can be used to estimate the tension.

13.60 (a) In making n round trips along the length of the line, the total distance traveled by the pulse is $\Delta x = n(2L) = 2nL$. The wave speed is then

$$v = \frac{\Delta x}{t} = \boxed{\frac{2nL}{t}}$$

(b) From $v = \sqrt{F/\mu}$ as the speed of transverse waves in the line, the tension is

$$F = \mu v^2 = \left(\frac{M}{L}\right)\left(\frac{2nL}{t}\right)^2 = \left(\frac{M}{L}\right)\left(\frac{4n^2L^2}{t^2}\right) = \boxed{\frac{4n^2ML}{t^2}}$$

13.61 (a) $\boxed{\text{Constructive interference}}$ produces the maximum amplitude

$$A'_{\text{max}} = A_1 + A_2 = 0.30 \text{ m} + 0.20 \text{ m} = \boxed{0.50 \text{ m}}$$

(b) $\boxed{\text{Destructive interference}}$ produces the minimum amplitude

$$A'_{\text{min}} = A_1 - A_2 = 0.30 \text{ m} - 0.20 \text{ m} = \boxed{0.10 \text{ m}}$$

13.62 We are given that $x = A\cos(\omega t) = (0.25 \text{ m})\cos(0.4\pi t)$.

(a) By inspection, the amplitude is seen to be $A = \boxed{0.25 \text{ m}}$.

(b) The angular frequency is $\omega = 0.4\pi$ rad/s. But $\omega = \sqrt{k/m}$, so the spring constant is

$$k = m\omega^2 = (0.30 \text{ kg})(0.4\pi \text{ rad/s})^2 = \boxed{0.47 \text{ N/m}}$$

continued on next page

(c) **Note**: Your calculator must be in *radians mode* for part (c).

At $t = 0.30$ s, $x = (0.25\text{ m})\cos\left[(0.4\pi\ \text{rad/ s})(0.30\text{ s})\right] = \boxed{0.23\text{ m}}$

(d) From conservation of mechanical energy, the speed at displacement x is given by $v = \omega\sqrt{A^2 - x^2}$. Thus, at $t = 0.30$ s, when $x = 0.23$ m, the speed is

$$v = (0.4\pi\ \text{rad/s})\sqrt{(0.25\text{ m})^2 - (0.23\text{ m})^2} = \boxed{0.12\text{ m/s}}$$

13.63 (a) The period of a vibrating object-spring system is $T = 2\pi/\omega = 2\pi\sqrt{m/k}$, so the spring constant is

$$k = \frac{4\pi^2 m}{T^2} = \frac{4\pi^2(2.00\text{ kg})}{(0.600\text{ s})^2} = \boxed{219\text{ N/m}}$$

(b) If $T = 1.05$ s for mass m_2, this mass is

$$m_2 = \frac{kT^2}{4\pi^2} = \frac{(219\text{ N/m})(1.05\text{ s})^2}{4\pi^2} = \boxed{6.12\text{ kg}}$$

13.64 (a) The period is the reciprocal of the frequency, or

$$T = \frac{1}{f} = \frac{1}{196\text{ s}^{-1}} = 5.10\times10^{-3}\text{ s} = \boxed{5.10\text{ ms}}$$

(b) $\lambda = \frac{v_{\text{sound}}}{f} = \frac{343\text{ m/s}}{196\text{ s}^{-1}} = \boxed{1.75\text{ m}}$

13.65 (a) The period of a simple pendulum is $T = 2\pi\sqrt{\ell/g}$, so the period of the first system is

$$T_1 = 2\pi\sqrt{\frac{\ell}{g}} = 2\pi\sqrt{\frac{0.700\text{ m}}{9.80\text{ m/s}^2}} = \boxed{1.68\text{ s}}$$

(b) The period of an object-spring system is $T = 2\pi\sqrt{m/k}$, so if the period of the second system is $T_2 = T_1$, then $2\pi\sqrt{m/k} = 2\pi\sqrt{\ell/g}$ and the spring constant is

$$k = \frac{mg}{\ell} = \frac{(1.20\text{ kg})(9.80\text{ m/s}^2)}{0.700\text{ m}} = \boxed{16.8\text{ N/m}}$$

13.66 Since the spring is "light," we neglect any small amount of energy lost in the collision with the spring, and apply conservation of mechanical energy from when the block first starts until it comes to rest again. This gives

$$\left(KE + PE_g + PE_s\right)_f = \left(KE + PE_g + PE_s\right)_i,\text{ or } 0 + 0 + \frac{1}{2}kx_{\max}^2 = 0 + 0 + mgh_i$$

Thus, $$x_{\max} = \sqrt{\frac{2mgh_i}{k}} = \sqrt{\frac{2(0.500\text{ kg})(9.80\text{ m/s}^2)(2.00\text{ m})}{20.0\text{ N/m}}} = \boxed{0.990\text{ m}}$$

13.67 Choosing $PE_g = 0$ at the initial height of the 3.00-kg object, conservation of mechanical energy gives $(KE + PE_g + PE_s)_f = (KE + PE_g + PE_s)_i$, or $\frac{1}{2}mv^2 + mg(-x) + \frac{1}{2}kx^2 = 0$, where v is the speed of the object after falling distance x.

(a) When $v = 0$, the non-zero solution to the energy equation from above gives $\frac{1}{2}kx_{\max}^2 = mgx_{\max}$, or

$$k = \frac{2mg}{x_{\max}} = \frac{2(3.00\text{ kg})(9.80\text{ m/s}^2)}{0.100\text{ m}} = \boxed{588\text{ N/m}}$$

(b) When $x = 5.00\text{ cm} = 0.050\,0\text{ m}$, the energy equation gives $v = \sqrt{2gx - kx^2/m}$, or

$$v = \sqrt{2(9.80\text{ m/s}^2)(0.050\,0\text{ m}) - \frac{(588\text{ N/m})(0.050\,0\text{ m})^2}{3.00\text{ kg}}} = \boxed{0.700\text{ m/s}}$$

13.68 (a) We apply conservation of mechanical energy from *just after* the collision until the block comes to rest. Conservation of energy gives $(KE + PE_s)_f = (KE + PE_s)_i$ or $0 + \frac{1}{2}kx_f^2 = \frac{1}{2}MV^2 + 0$. The speed of the block just after the collision is then

$$V = \sqrt{\frac{kx_f^2}{M}} = \sqrt{\frac{(900\text{ N/m})(0.050\,0\text{ m})^2}{1.00\text{ kg}}} = 1.50\text{ m/s}$$

Now, we apply conservation of momentum from just before impact to immediately after the collision. This gives $m(v_{\text{bullet}})_i + 0 = m(v_{\text{bullet}})_f + MV$, or

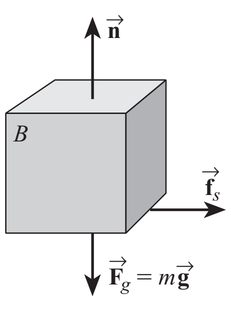
$$(v_{\text{bullet}})_f = (v_{\text{bullet}})_i - \left(\frac{M}{m}\right)V = 400\text{ m/s} - \left(\frac{1.00\text{ kg}}{5.00\times10^{-3}\text{ kg}}\right)(1.5\text{ m/s}) = \boxed{100\text{ m/s}}$$

(b) The mechanical energy converted into internal energy during the collision is $\Delta E = KE_i - \Sigma KE_f = \frac{1}{2}m(v_{\text{bullet}})_i^2 - \frac{1}{2}m(v_{\text{bullet}})_f^2 - \frac{1}{2}MV^2$, or

$$\Delta E = \frac{1}{2}(5.00\times10^{-3}\text{ kg})\left[(400\text{ m/s})^2 - (100\text{ m/s})^2\right] - \frac{1}{2}(1.00\text{ kg})(1.50\text{ m/s})^2$$

$$\Delta E = \boxed{374\text{ J}}$$

13.69 The maximum acceleration of the oscillating system is



$$a_{\max} = \omega^2 A = (2\pi f)^2 A$$

The friction force, f_s, acting between the two blocks must be capable of accelerating block B at this rate. When block B is on the verge of slipping, $f_s = (f_s)_{\max} = \mu_s n = \mu_s mg = ma_{\max}$ and we must have

$$a_{\max} = (2\pi f)^2 A = \mu_s g$$

Thus, $A = \dfrac{\mu_s g}{(2\pi f)^2} = \dfrac{(0.600)(9.80\text{ m/s}^2)}{[2\pi(1.50\text{ Hz})]^2} = 6.62\times10^{-2}\text{ m} = \boxed{6.62\text{ cm}}$

13.70 (a) When the gun is fired, the energy initially stored as elastic potential energy in the spring is transformed into kinetic energy of the bullet. Assuming no loss of energy, we have $\frac{1}{2}mv^2 = \frac{1}{2}kx_i^2$, or

$$v = x_i\sqrt{\frac{k}{m}} = (0.200\ \text{m})\sqrt{\frac{9.80\ \text{N/m}}{1.00\times10^{-3}\ \text{kg}}} = \boxed{19.8\ \text{m/s}}$$

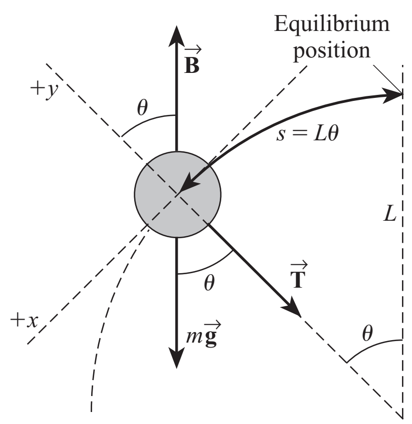
(b) From $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, the time required for the pellet to drop 1.00 m to the floor, starting with $v_{0y} = 0$, is

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-1.00\ \text{m})}{-9.80\ \text{m/s}^2}} = 0.452\ \text{s}$$

The range (horizontal distance traveled during the flight) is then

$$\Delta x = v_{0x}t = (19.8\ \text{m/s})(0.452\ \text{s}) = \boxed{8.95\ \text{m}}$$

13.71 (a) The force diagram at the right shows the forces acting on the balloon when it is displaced distance $s = L\theta$ along the circular arc it follows. The net force tangential to this path is



$$F_{\text{net}} = \Sigma F_x = -B\sin\theta + mg\sin\theta = -(B - mg)\sin\theta$$

For small angles, $\sin\theta \approx \theta = s/L$. Also, $mg = (\rho_{\text{He}}V)g$ and the buoyant force is $B = (\rho_{\text{air}}V)g$. Thus, the net restoring force acting on the balloon is

$$\boxed{F_{\text{net}} \approx -\left[\frac{(\rho_{\text{air}} - \rho_{\text{He}})Vg}{L}\right]s}$$

Observe that this is in the form of Hooke's law, $F = -ks$, with $k = (\rho_{\text{air}} - \rho_{\text{He}})Vg/L$. Thus, the motion will be $\boxed{\text{simple harmonic}}$.

(b) The period of this simple harmonic motion is given by

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{\rho_{\text{He}}V}{(\rho_{\text{air}} - \rho_{\text{He}})Vg/L}} = 2\pi\sqrt{\left(\frac{\rho_{\text{He}}}{\rho_{\text{air}} - \rho_{\text{He}}}\right)\frac{L}{g}}$$

This yields

$$T = 2\pi\sqrt{\left(\frac{0.179\ \text{kg/m}^3}{1.29\ \text{kg/m}^3 - 0.179\ \text{kg/m}^3}\right)\frac{(3.00\ \text{m})}{(9.80\ \text{m/s}^2)}} = \boxed{1.40\ \text{s}}$$

13.72 (a) When the object is given some small upward displacement, the net restoring force exerted on it by the rubber bands is

$$F_{\text{net}} = \Sigma F_y = -2F\sin\theta\text{, where }\tan\theta = \frac{y}{L}$$

For small displacements, the angle θ will be very small. Then $\sin\theta \approx \tan\theta = y/L$, and the net restoring force is

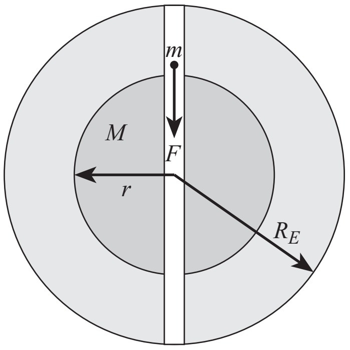
$$F_{\text{net}} = -2F\left(\frac{y}{L}\right) = \boxed{-\left(\frac{2F}{L}\right)y}$$

continued on next page

(b) The net restoring force found in part (a) is in the form of Hooke's law $F = -ky$, with $k = 2F/L$. Thus, the motion will be simple harmonic, and the angular frequency is

$$\omega = \sqrt{\frac{k}{m}} = \boxed{\sqrt{\frac{2F}{mL}}}$$

13.73 Newton's law of gravitation is



$$F = -\frac{GMm}{r^2}, \quad \text{where } M = \rho\left(\frac{4}{3}\pi r^3\right)$$

Thus, $\boxed{F = -\left(\frac{4}{3}\pi\rho G m\right) r}$

which is of Hooke's law form, $F = -k\,r$, with

$$\boxed{k = \frac{4}{3}\pi\rho G m}$$

13.74 The inner tip of the wing is attached to the end of the spring and always moves with the same speed as the end of the vibrating spring. Thus, its maximum speed is

$$v_{\text{inner, max}} = v_{\text{spring, max}} = A\sqrt{\frac{k}{m}} = (0.20\ \text{cm})\sqrt{\frac{4.7\times10^{-4}\ \text{N/m}}{0.30\times10^{-3}\ \text{kg}}} = 0.25\ \text{cm/s}$$

Treating the wing as a rigid bar, all points in the wing have the same angular velocity at any instant in time. As the wing rocks on the fulcrum, the inner tip and outer tips follow circular paths of different radii. Since the angular velocities of the tips are always equal, we may write $\omega = \frac{v_{\text{outer}}}{r_{\text{outer}}} = \frac{v_{\text{inner}}}{r_{\text{inner}}}$. The maximum speed of the outer tip is then

$$v_{\text{outer, max}} = \left(\frac{r_{\text{outer}}}{r_{\text{inner}}}\right) v_{\text{inner, max}} = \left(\frac{15.0\ \text{mm}}{3.00\ \text{mm}}\right)(0.25\ \text{cm/s}) = \boxed{1.3\ \text{cm/s}}$$

13.75 (a) $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{500\ \text{N/m}}{2.00\ \text{kg}}} = \boxed{15.8\ \text{rad/s}}$

(b) Apply Newton's second law to the block while the elevator is accelerating:

$$\Sigma F_y = F_s - mg = ma_y$$

With $F_s = kx$ and $a_y = g/3$, this gives $kx = m(g + g/3)$, or

$$x = \frac{4mg}{3k} = \frac{4(2.00\ \text{kg})(9.80\ \text{m/s}^2)}{3(500\ \text{N/m})} = 5.23\times10^{-2}\ \text{m} = \boxed{5.23\ \text{cm}}$$

13.76 (a) Note that as the spring passes through the vertical position, the object is moving in a circular arc of radius $L - y_f$. Also, observe that the y-coordinate of the object at this point must be negative $(y_f < 0)$, so the spring is stretched and exerting an upward tension force of magnitude greater than the object's weight. This is necessary so the object experiences a net

continued on next page

force toward the pivot to supply the needed centripetal acceleration in this position. This is summarized by Newton's second law applied to the object at this point, stating

$$\boxed{\Sigma F_y = -ky_f - mg = \frac{mv^2}{L - y_f}}$$

(b) Conservation of energy requires that $E = KE_i + PE_{g,i} + PE_{s,i} = KE_f + PE_{g,f} + PE_{s,f}$, or

$$E = 0 + mgL + 0 = \frac{1}{2}mv^2 + mgy_f + \frac{1}{2}ky_f^2$$

reducing to $\boxed{mv^2 = 2mg\left(L - y_f\right) - ky_f^2}$

(c) From the result of part (a), observe that

$$mv^2 = -\left(L - y_f\right)\left(ky_f + mg\right)$$

Substituting this into the result from part (b) gives

$$2mg\left(L - y_f\right) = -\left(L - y_f\right)\left(ky_f + mg\right) + ky_f^2$$

After expanding and regrouping terms, this becomes

$$(2k)\,y_f^2 + (3mg - kL)\,y_f + (-3mgL) = 0$$

which is a quadratic equation $ay_f^2 + by_f + c = 0$, with

$$a = 2k = 2(1\,250\ \text{N/m}) = 2.50 \times 10^3\ \text{N/m}$$

$$b = 3mg - kL = 3(5.00\ \text{kg})\left(9.80\ \text{m/s}^2\right) - (1\,250\ \text{N/m})(1.50\ \text{m}) = -1.73 \times 10^3\ \text{N}$$

and $c = -3mgL = -3(5.00\ \text{kg})\left(9.80\ \text{m/s}^2\right)(1.50\ \text{m}) = -221\ \text{N}\cdot\text{m}$

Applying the quadratic formula, keeping only the negative solution [see the discussion in part (a)] gives

$$y_f = \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-\left(-1.73 \times 10^3\right) - \sqrt{\left(-1.73 \times 10^3\right)^2 - 4\left(2.50 \times 10^3\right)(-221)}}{2\left(2.50 \times 10^3\right)}$$

or $\boxed{y_f = -0.110\ \text{m}}$

(d) Because the length of this pendulum varies and is longer throughout its motion than a simple pendulum of length L, $\boxed{\text{its period will be greater than}}$ that of a simple pendulum.

14

Sound

QUICK QUIZZES

1. Choice (c). The speed of sound in air is given by $v = (331\ \text{m/s})\sqrt{T/273\ \text{K}}$. Thus, increasing the absolute temperature, T, will increase the speed of sound. Changes in frequency, amplitude, or air pressure have no effect on the speed of sound.

2. Choice (c). The distance between you and the buzzer is increasing. Therefore, the intensity at your location is decreasing. As the buzzer falls, it moves away from you with increasing speed. This causes the detected frequency to decrease.

3. Choice (b). The speed of sound increases in the warmer air, while the speed of the sound source (the plane) remains constant. Therefore, the ratio of the speed of the source to that of sound (that is, the Mach number) decreases.

4. Choices (b) and (e). A string fastened at both ends can resonate at any integer multiple of the fundamental frequency. Of the choices listed, only 300 Hz and 600 Hz are integer multiples of the 150 Hz fundamental frequency.

5. Choice (d). In the fundamental mode, an open pipe has a node at the center and antinodes at each end. The fundamental wavelength of the open pipe is then twice the length of the pipe and the fundamental frequency is $f_{\text{open}} = v/2L$. When one end of the pipe is closed, the fundamental mode has a node at the closed end and an antinode at the open end. In this case, the fundamental wavelength is four times the length of the pipe and the fundamental frequency is $f_{\text{close}} = v/4L$.

6. Choice (a). The change in the length of the pipe, and hence the fundamental wavelength, is negligible. As the temperature increases, the speed of sound in air increases and this causes an increase in the fundamental frequency, $f_0 = v/\lambda_0$.

7. Choice (b). Since the beat frequency is steadily increasing, you are increasing the difference between the frequency of the string and the frequency of the tuning fork. Thus, your action is counterproductive and you should reverse course by loosening the string.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. All sound waves travel at the same speed in air of a given temperature. Thus, $v = \lambda f = \lambda_2 f_2$, giving

$$f_2 = \left(\frac{\lambda}{\lambda_2}\right) f = \left(\frac{\lambda}{\lambda/2}\right) f = 2f$$

and the correct choice is (b).

2. The Celsius temperature on this day was $T_C = \frac{5}{9}(T_F - 32) = \frac{5}{9}(134 - 32) = 56.7°\text{C}$. The speed of sound in the air was

$$v = (331 \text{ m/s})\sqrt{1+\frac{T_C}{273}} = (331 \text{ m/s})\sqrt{1+\frac{56.7}{273}} = 364 \text{ m/s}$$

and the correct answer is (c).

3. The speed of sound in a fluid is given in Equation 14.1 as $v = \sqrt{B/\rho}$, where B is the bulk modulus of the fluid and ρ is its density. The speed of sound in ethyl alcohol is found to be

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{1.0 \times 10^9 \text{ Pa}}{0.806 \times 10^3 \text{ kg/m}^3}} = 1.1 \times 10^3 \text{ m/s}$$

and (a) is the correct choice.

4. Note that the value given for the speed of sound in aluminum in Table 14.1 is for "bulk media" only and does not apply to the thin, solid rod at issue here. Thus, we turn to Equation 14.3, and use data from Tables 9.1 and 9.2 to obtain

$$v_{\text{Al}} = \sqrt{\frac{Y_{\text{Al}}}{\rho_{\text{Al}}}} = \sqrt{\frac{7.0 \times 10^{10} \text{ Pa}}{2.7 \times 10^3 \text{ kg/m}^3}} = 5.1 \times 10^3 \text{ m/s}$$

This shows choice (e) to be the correct answer.

5. The relation between the decibel level and the sound intensity is $\beta = 10 \cdot \log(I/I_0)$, where the reference intensity is $I_0 = 1.0 \times 10^{-12} \text{ W/m}^2$ and log refers to the base 10 logarithm. Thus, if $\beta = 105$ dB, the sound intensity is

$$I = I_0 \cdot 10^{\beta/10} = (1.0 \times 10^{-12} \text{ W/m}^2) \cdot 10^{10.5} = 3.16 \times 10^{-2} \text{ W/m}^2$$

and (d) is the correct answer.

6. In a uniform medium, the intensity of sound varies inversely with the square of the distance from the source. (See Equation 14.8 in the textbook.) Thus, if the distance from the source is tripled, the new sound intensity will be one-ninth of its original value, making (a) the correct choice.

7. The apparent frequency f_O detected by an observer from a source emitting sound of frequency f_S is given by

$$f_O = f_S\left(\frac{v + v_O}{v - v_S}\right)$$

where v is the speed of sound in air, v_O is the velocity of the observer relative to the air and is positive if the observer moves toward the source, while v_S is the velocity of the source relative to the air and is positive if the source moves toward the observer. In this case, we have

$$f_O = (1.00 \times 10^3 \text{ Hz})\left[\frac{343 \text{ m/s} + (-30.0 \text{ m/s})}{343 \text{ m/s} - (+50.0 \text{ m/s})}\right] = 1.07 \times 10^3 \text{ m/s}$$

so the correct choice is (d).

8. When a sound wave travels from air into water, several properties will change. The wave speed will increase as the wave crosses the boundary into the water, the spacing between crests (the wavelength) will increase since crests move away from the boundary faster than they move up to the boundary, and the sound intensity in the water will be less than it was in air because some sound is reflected by the water surface. However, the frequency (number of crests passing each second) will be unchanged, since a crest moves away from the boundary every time a crest arrives at the boundary. Among the listed choices, the only correct statement is choice (d).

9. The number of beats per second (the beat frequency) equals the difference in the frequencies of the two tuning forks. Thus, if the beat frequency is 5 Hz and one fork is known to have a frequency of 245 Hz, the frequency of the second fork could be either $f_2 = 245\text{ Hz} - 5\text{ Hz} = 240\text{ Hz}$ or $f_2 = 245\text{ Hz} + 5\text{ Hz} = 250\text{ Hz}$. This means that the best answer for the question is choice (e), since choices (a) and (d) are both possibly correct.

10. At resonance, a tube closed at one end and open at the other forms a standing wave pattern with a node at the closed end and antinode at the open end. In the fundamental mode (or first harmonic), the length of the tube closed at one end is a quarter wavelength $\left(L = \lambda_1/4 \text{ or } \lambda_1 = 4L\right)$. Therefore, for the given tube, $\lambda_1 = 4(0.580\text{ m}) = 2.32\text{ m}$ and the fundamental frequency is

$$f_1 = \frac{v}{\lambda_1} = \frac{343\text{ m/s}}{2.32\text{ m}} = 148\text{ Hz}$$

and the correct answer is choice (a).

11. When the two ends of a pipe are alike (either both open or both closed), all harmonics (integer multiples of the fundamental frequency) are present among the resonant frequencies of the pipe. However, in a pipe closed at one end and open at the other, only the odd harmonics (i.e., only *odd integer* multiples of the fundamental frequency) are resonant frequencies for the pipe. In the case of the given pipe, the fundamental frequency is $f_1 = 150\text{ Hz}$, and both $f_2 = 2f_1 = 300\text{ Hz}$ (an even multiple) and $f_3 = 3f_1 = 450\text{ Hz}$ (an odd multiple) are resonant frequencies. Thus, the pipe must have either two open ends or two closed ends, and the correct choice is (b).

12. The ambulance driver, sitting at a fixed distance from the siren, hears the actual frequency emitted by the siren. However, the distance between you and the siren is decreasing, so you will detect a frequency higher than the actual 500 Hz. Choice (c) is the correct answer.

13. The speed of sound in air, at atmospheric pressure, is determined by the temperature of the air and does not depend on the frequency of the sound. Sound from siren *A* will have a wavelength that is half the wavelength of the sound from *B*, but the speed of the sound (the product of frequency times wavelength) will be the same for the two sirens. The correct choice is (e).

14. In the fundamental mode (first harmonic), a pipe open at both ends has antinodes at each end and a node at the center. The wavelength of this harmonic is $\lambda_1 = 2L$ and the resonant frequency is $f_{\text{open}} = v/2L$. If one end of the pipe is now closed, the fundamental mode will have a node at the closed end with an antinode at the open end. The wavelength of the first harmonic is $\lambda_1' = 4L$ and the resonant frequency is $f_{\text{closed}} = v/4L$. Thus, we see that $f_{\text{open}} = 2f_{\text{closed}}$ and the correct choice is (b).

15. Doubling the power output of the source will double the intensity of the sound at the observer's location. The original decibel level of the sound is $\beta = 10\cdot\log\left(I/I_0\right)$. After doubling the power output and intensity, the new decibel level will be

$$\beta' = 10\cdot\log\left(2I/I_0\right) = 10\cdot\log\left[2\left(I/I_0\right)\right] = 10\cdot\left[\log 2 + \log\left(I/I_0\right)\right] = 10\cdot\log 2 + \beta$$

so the increase in decibel level is $\beta' - \beta = 10\cdot\log 2 = 3.0\text{ dB}$, making (c) the correct answer.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The resonant frequency depends on the length of the pipe. Thus, changing the length of the pipe will cause different frequencies to be emphasized in the resulting sound. The shorter the pipe, the higher the fundamental resonance frequency.

4. The speed of light is so high that the arrival of the flash is practically simultaneous with the lightning discharge. Thus, the delay between the flash and the arrival of the sound of thunder is the time sound takes to travel the distance separating the lightning from you. By counting the seconds between the flash and thunder and knowing the approximate speed of sound in air, you have a rough measure of the distance to the lightning bolt.

6. A vibrating string is not able to set very much air into motion when vibrated alone. Thus it will not be very loud. If it is placed on the instrument, however, the string's vibration sets the sounding board of the guitar into vibration. A vibrating piece of wood is able to move a lot of air, and the note is louder.

8. A beam of electromagnetic waves of known frequency is sent toward a speeding car, which reflects the beam back to a detector in the police car. The amount the returning frequency has been shifted depends on the velocity of the oncoming car.

10. Consider the level of fluid in the bottle to be adjusted so that the air column above it resonates at the first harmonic. This is given by $f = v/4L$. This equation indicates that as the length L of the column increases (fluid level decreases), the resonant frequency decreases.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. 1×10^{11} Pa

4. 0.196 s

6. $\lambda = 17$ m for $f = 20$ Hz; $\lambda = 1.7$ cm for $f = 20\,000$ Hz

8. 18.6 m

10. **(a)** $1.0\times10^{3}\ \text{W/m}^2$

(b) The intensity level is 1 000 times that at the threshold of pain.

12. 150 dB

14. **(a)** 5.0×10^{-17} W **(b)** 5.0×10^{-5} W

16. **(a)** $0.316\ \text{W/m}^2$ **(b)** $1.58\ \text{W/m}^2$ **(c)** $2.47\times10^{-2}\ \text{W/m}^2$

(d) 104 dB **(e)** 1.26×10^{6} m

(f) The sound intensity falls as the sound wave travels farther from the source until it is much lower than the ambient noise level and is drowned out.

18. **(a)** $1.32\times10^{-4}\ \text{W/m}^2$ **(b)** 81.2 dB

20. **(a)** 7.96×10^{-2} W/m^2 **(b)** 109 dB **(c)** 2.82 m

22. **(a)** $I_A/I_B = 2$ **(b)** $I_A/I_C = 5$

24. **(a)** 10.0 kHz **(b)** 3.33 kHz

26. 32.0 m/s

28. **(a)** 3.29 m/s **(b)** Yes, the bat gains on the insect at a rate of 1.71 m/s.

30. **(a)** 2.16×10^{-2} m/s **(b)** 2 000 029 Hz **(c)** 2 000 058 Hz

32. **(a)** $f_O = f_S\left[(v+v_O)/(v-v_S)\right]$ **(b)** the yellow submarine **(c)** the red submarine

(d) increases the time (period), decreases the frequency **(e)** negative

(f) decreases the time (period), increases the frequency **(g)** positive

(h) 5.30×10^3 Hz

34. **(a)** 0.227 m **(b)** 0.454 m

36. 1.43 m

38. 823.8 N

40. 1.00 cm toward the nut

42. 120 Hz

44. **(a)** $f_A = \dfrac{n_A}{2L_A}\sqrt{\dfrac{T_A}{\mu_A}}$ **(b)** $f_B = f_A/2$ **(c)** $T_B = \left(\dfrac{n_A}{n_A+1}\right)^2 T_A$

(d) $T_B/T_A = 9/16$

46. **(a)** $\mu = 4.9 \times 10^{-3}$ kg/m **(b)** 2

(c) no standing wave will form

48. 9.00 kHz

50. **(a)** 536 Hz **(b)** 4.29 cm

52. **(a)** $f_n = n(0.085\,8 \text{ Hz})$ $n = 1, 2, 3, \ldots$

(b) Yes. The tunnel can resonate at many closely spaced frequencies, and the sound would be greatly amplified.

54. **(a)** $f_1 = 50.0$ Hz **(b)** open at only one end **(c)** 1.72 m

56. 29.7 cm

58. 3.98 Hz

60. 2.95 cm

62. ~ 1 000 mosquitoes

64. **(a)** 65.0 dB **(b)** 67.8 dB **(c)** 69.6 dB

66. 21.4 m

68. 1 204 Hz

70. 7.8 m

72. **(a)** 0.655 m **(b)** 13.4°C

74. 1.95 m/s

76. **(a)** 32.2°C **(b)** 3.6×10^{2} Hz

PROBLEM SOLUTIONS

14.1 (a) We ignore the time required for the lightning flash to arrive. Then, the distance to the lightning stroke is

$$d \approx v_{\text{sound}}\cdot\Delta t = (343\ \text{m/s})(16.2\ \text{s}) = 5.56\times10^{3}\ \text{m} = \boxed{5.56\ \text{km}}$$

(b) $\boxed{\text{No.}}$ Since $v_{\text{light}} >> v_{\text{sound}}$, the time required for the flash of light to reach the observer is negligible in comparison to the time required for the sound to arrive, and knowledge of the actual value of the speed of light is not needed.

14.2 The speed of longitudinal waves in a fluid is $v=\sqrt{B/\rho}$. Considering the Earth's crust to consist of a very viscous fluid, our estimate of the average bulk modulus of the material in Earth's crust is

$$B=\rho v^{2} = \left(2\,500\ \text{kg/m}^{3}\right)\left(7\times10^{3}\ \text{m/s}\right)^{2} = \boxed{1\times10^{11}\ \text{Pa}}$$

14.3 The Celsius temperature is $T_{\text{C}} = \frac{5}{9}\left(T_{\text{F}}-32\right) = \frac{5}{9}(114-32) = 45.6°\text{C}$ and the speed of sound in the air is

$$v=(331\ \text{m/s})\sqrt{1+\frac{T_C}{273}} = (331\ \text{m/s})\sqrt{1+\frac{45.6}{273}} = \boxed{358\ \text{m/s}}$$

14.4 The speed of sound in seawater at 25°C is 1 533 m/s. Therefore, the time for the sound to reach the sea floor and return is

$$t=\frac{2d}{v}=\frac{2(150\ \text{m})}{1\,533\ \text{m/s}}=\boxed{0.196\ \text{s}}$$

14.5 Since the sound had to travel the distance between the hikers and the mountain twice, the time required for a one-way trip was 1.50 s. The distance the sound traveled to the mountain was

$$d=(343\ \text{m/s})(1.50\ \text{s}) = \boxed{515\ \text{m}}$$

14.6 At $T = 27°C$, the speed of sound in air is

$$v = (331\ \text{m/s})\sqrt{1+\frac{T_C}{273}} = (331\ \text{m/s})\sqrt{1+\frac{27}{273}} = 347\ \text{m/s}$$

The wavelength of the 20 Hz sound is

$$\lambda = \frac{v}{f} = \frac{347\ \text{m/s}}{20\ \text{Hz}} = 17\ \text{m}$$

and that of the 20 000 Hz is

$$\lambda = \frac{347\ \text{m/s}}{20\ 000\ \text{Hz}} = 1.7\times10^{-2}\ \text{m} = 1.7\ \text{cm}$$

Thus, range of wavelengths of audible sounds at 27°C is $\boxed{1.7\ \text{cm to } 17\ \text{m}}$.

14.7 At $T = 27.0°C$, the speed of sound in air is

$$v_{27} = (331\ \text{m/s})\sqrt{1+\frac{T_C}{273}} = (331\ \text{m/s})\sqrt{1+\frac{27.0}{273}} = 347\ \text{m/s}$$

and at $T = 10.0°C$, the speed is

$$v_{10} = (331\ \text{m/s})\sqrt{1+\frac{T_C}{273}} = (331\ \text{m/s})\sqrt{1+\frac{10.0}{273}} = 337\ \text{m/s}$$

Since $v = \lambda f$, the change in wavelength will be

$$\Delta\lambda = \frac{v_{10}}{f} - \frac{v_{27}}{f} = \frac{v_{10}-v_{27}}{f} = \frac{(337-347)\ \text{m/s}}{4.00\times10^{3}\ \text{Hz}} = -2.5\times10^{-3}\ \text{m} = \boxed{-2.5\ \text{mm}}$$

14.8 At a temperature of $T = 10.0°C$, the speed of sound in air is

$$v = (331\ \text{m/s})\sqrt{1+\frac{T_C}{273}} = (331\ \text{m/s})\sqrt{1+\frac{10.0}{273}} = 337\ \text{m/s}$$

The elapsed time between when the stone was released and when the sound is heard is the sum of the time t_1 required for the stone to fall distance h and the time t_2 required for sound to travel distance h in air on the return up the well. That is, $t_1 + t_2 = 2.00$ s. The distance the stone falls, starting from rest, in time t_1 is $\quad h = \dfrac{gt_1^2}{2}$

Also, the time for the sound to travel back up the well is $\quad t_2 = \dfrac{h}{v} = 2.00\ \text{s} - t_1$

Combining these two equations yields $\quad (g/2v)t_1^2 = 2.00\ \text{s} - t_1$

With $v = 337\ \text{m/s}$ and $g = 9.80\ \text{m/s}^2$, this becomes $\quad (1.45\times10^{-2}\ \text{s}^{-1})t_1^2 + t_1 - 2.00\ \text{s} = 0$

Applying the quadratic formula yields one positive solution of $t_1 = 1.95$ s, so the depth of the well is

$$h = \frac{gt_1^2}{2} = \frac{(9.80\ \text{m/s}^2)(1.95\ \text{s})^2}{2} = \boxed{18.6\ \text{m}}$$

14.9 (a) Because the speed of sound in air is $v_{\text{air}} = 343\ \text{m/s}$ while its speed in the steel rail is $v_{\text{steel}} = 5\,950\ \text{m/s}$, $\boxed{\text{the pulse traveling in the steel rail arrives first}}$.

(b) The difference in times when the two pulses reach the microphone at the opposite end of the rail is

$$\Delta t = \frac{L}{v_{\text{air}}} - \frac{L}{v_{\text{steel}}} = (8.50\ \text{m})\left(\frac{1}{343\ \text{m/s}} - \frac{1}{5\,950\ \text{m/s}}\right) = 2.34\times10^{-2}\ \text{s} = \boxed{23.4\ \text{ms}}$$

14.10 (a) The decibel level, β, of a sound is given $\beta = 10\log\left(I/I_0\right)$, where I is the intensity of the sound, and $I_0 = 1.0\times10^{-12}\ \text{W/m}^2$ is the reference intensity. Therefore, if $\beta = 150$ dB, the intensity is

$$I = I_0\times10^{\beta/10} = \left(1.0\times10^{-12}\ \text{W/m}^2\right)\times10^{15} = \boxed{1.0\times10^{3}\ \text{W/m}^2}$$

(b) The threshold of pain is $I = 1\ \text{W/m}^2$ and the answer to part (a) is 1 000 times greater than this, explaining why some airport employees must wear hearing protection equipment.

14.11 If the intensity of this sound varied inversely with the square of the distance from the source $\left(I = \text{constant}/r^2\right)$, the ratio of the intensities at distances $r_1 = 161$ km and $r_2 = 4\,800$ km from the source is given by

$$\frac{I_2}{I_1} = \left(\frac{\cancel{\text{constant}}}{r_2^2}\right)\left(\frac{r_1^2}{\cancel{\text{constant}}}\right) = \left(\frac{r_1}{r_2}\right)^2 = \left(\frac{161\ \text{km}}{4\,800\ \text{km}}\right)^2$$

The difference in the decibel levels at distances r_1 and r_2 from this source was then

$$\beta_2 - \beta_1 = 10\cdot\log\left(\frac{I_2}{I_0}\right) - 10\cdot\log\left(\frac{I_1}{I_0}\right) = 10\cdot\log\left(\frac{I_2}{\cancel{I_0}}\cdot\frac{\cancel{I_0}}{I_1}\right) = 10\cdot\log\left(\frac{I_2}{I_1}\right) = 10\cdot\log\left(\frac{161\ \text{km}}{4\,800\ \text{km}}\right)^2$$

or $\beta_2 - \beta_1 = -29.5$ dB. This gives the decibel level on Rodriguez Island as

$$\beta_2 = \beta_1 - 29.5\ \text{dB} = 180\ \text{dB} - 29.5\ \text{dB} \approx \boxed{151\ \text{dB}}$$

14.12 The decibel level due to the first siren is

$$\beta_1 = 10\cdot\log\left(\frac{100.0\ \text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2}\right) = 140\ \text{dB}$$

Thus, the decibel level of the sound from the ambulance is

$$\beta_2 = \beta_1 + 10\ \text{dB} = 140\ \text{dB} + 10\ \text{dB} = \boxed{150\ \text{dB}}$$

14.13 In terms of their intensities, the difference in the decibel level of two sounds is

$$\beta_2 - \beta_1 = 10\cdot\log\left(\frac{I_2}{I_0}\right) - 10\cdot\log\left(\frac{I_1}{I_0}\right) = 10\cdot\log\left(\frac{I_2}{\cancel{I_0}}\cdot\frac{\cancel{I_0}}{I_1}\right) = 10\cdot\log\left(\frac{I_2}{I_1}\right)$$

Thus, $\dfrac{I_2}{I_1} = 10^{(\beta_2-\beta_1)/10}$ or $I_2 = I_1\times10^{(\beta_2-\beta_1)/10}$

If $\beta_2 - \beta_1 = 30.0$ dB and $I_1 = 3.0\times10^{-11}\ \text{W/m}^2$, then

$$I_2 = \left(3.0\times10^{-11}\ \text{W/m}^2\right)\times10^{3.00} = \boxed{3.0\times10^{-8}\ \text{W/m}^2}$$

14.14 The sound power incident on the eardrum is $P = IA$, where I is the intensity of the sound and $A = 5.0\times10^{-5}\ \text{m}^2$ is the area of the eardrum.

(a) At the threshold of hearing, $I = 1.0\times10^{-12}\ \text{W/m}^2$, and

$$P = \left(1.0\times10^{-12}\ \text{W/m}^2\right)\left(5.0\times10^{-5}\ \text{m}^2\right) = \boxed{5.0\times10^{-17}\ \text{W}}$$

(b) At the threshold of pain, $I = 1.0\ \text{W/m}^2$, and

$$P = \left(1.0\ \text{W/m}^2\right)\left(5.0\times10^{-5}\ \text{m}^2\right) = \boxed{5.0\times10^{-5}\ \text{W}}$$

14.15 The decibel level $\beta = 10\log\left(I/I_0\right)$, where $I_0 = 1.00\times10^{-12}\ \text{W/m}^2$.

(a) If $\beta = 100$ dB, then $\log\left(I/I_0\right) = 10$, giving $I = 10^{10}\ I_0 = \boxed{1.00\times10^{-2}\ \text{W/m}^2}$.

(b) If all three toadfish sound at the same time, the total intensity of the sound produced is $I' = 3I = 3.00\times10^{-2}\ \text{W/m}^2$, and the decibel level is

$$\beta' = 10\log\left(\frac{3.00\times10^{-2}\ \text{W/m}^2}{1.00\times10^{-12}\ \text{W/m}^2}\right)$$

$$= 10\log\left[(3.00)\left(10^{10}\right)\right] = 10\left[\log(3.00)+10\right] = \boxed{105\ \text{dB}}$$

14.16 (a) From the defining equation of the decibel level, $\beta = 10\cdot\log\left(I/I_0\right)$, we solve for the intensity as $I = I_0\cdot10^{\beta/10}$ and find that

$$I = \left(1.0\times10^{-12}\ \text{W/m}^2\right)\cdot10^{115/10} = 1.0\times10^{-12+11.5}\ \text{W/m}^2 = 10^{-0.5}\ \text{W/m}^2 = \boxed{0.316\ \text{W/m}^2}$$

(b) If 5 trumpets are sounded together, the total intensity of the sound is

$$I_5 = 5I_1 = 5\left(0.316\ \text{W/m}^2\right) = \boxed{1.58\ \text{W/m}^2}$$

(c) If the sound propagates uniformly in all directions, the intensity varies inversely as the square of the distance from the source, $I = \text{constant}/r^2$, and we find that

$$\frac{I_2}{I_1} = \left(\frac{r_1}{r_2}\right)^2 \quad \text{or} \quad I_2 = I_1\left(\frac{r_1}{r_2}\right)^2 = \left(1.58\ \text{W/m}^2\right)\left(\frac{1.0\ \text{m}}{8.0\ \text{m}}\right)^2 = \boxed{2.47\times10^{-2}\ \text{W/m}^2}$$

(d) $$\beta_{\text{row 1}} = 10\cdot\log\left(\frac{I_{\text{row 1}}}{I_0}\right) = 10\cdot\log\left(\frac{2.47\times10^{-2}\ \text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2}\right) = \boxed{104\ \text{dB}}$$

(e) The intensity of sound at the threshold of hearing is $I_0 = 1.0\times10^{-12}\ \text{W/m}^2$, and from the discussion and result of part (c), we have $I_0/I_2 = \left(r_2/r_0\right)^2$, and with the intensity being $I_2 = 2.47\times10^{-2}\ \text{W/m}^2$ at distance $r_2 = 8.0$ m, the distance at which the intensity would be $I_0 = 1.0\times10^{-12}\ \text{W/m}^2$ is

$$r_0 = r_2\sqrt{\frac{I_2}{I_0}} = (8.0\ \text{m})\sqrt{\frac{2.47\times10^{-2}\ \text{W/m}^2}{1.0\times10^{-12}\ \text{W/m}^2}} = \boxed{1.26\times10^{6}\ \text{m}}$$

(f) The sound intensity level falls as the sound wave travels farther from the source until it is much lower than the ambient noise level and is drowned out.

14.17 The intensity of a spherical sound wave at distance r from a point source is $I = P_{av}/4\pi r^2$, where P_{av} is the average power radiated by the source. Thus, at distances $r_1 = 5.0$ m and $r_2 = 10\text{ km} = 10^4$ m, the intensities of the sound wave radiating out from the elephant are $I_1 = P_{av}/4\pi r_1^2$ and $I_2 = P_{av}/4\pi r_2^2$ giving $I_2 = (r_1/r_2)^2 I_1$. From the defining equation, $\beta = 10\log(I/I_o)$, the intensity level of the sound at distance r_2 from the elephant is seen to be

$$\beta_2 = 10\log\left(\frac{I_2}{I_0}\right) = 10\log\left[\left(\frac{r_1}{r_2}\right)^2 \frac{I_1}{I_0}\right] = 10\log\left(\frac{r_1}{r_2}\right)^2 + 10\log\left(\frac{I_1}{I_0}\right) = 20\log\left(\frac{r_1}{r_2}\right) + 10\log\left(\frac{I_1}{I_0}\right)$$

or $$\beta_2 = 20\log\left(\frac{5.0\text{ m}}{10^4\text{ m}}\right) + \beta_1 = -66\text{ dB} + 103\text{ dB} = \boxed{37\text{ dB}}$$

14.18 (a) The intensity of the musical sound (β = 80 dB) is $I_{music} = I_0 10^{\beta/10} = I_0\left(10^{8.0}\right)$, and that produced by the crying baby (β = 75 dB) is $I_{baby} = I_0\left(10^{7.5}\right)$. Thus, the total intensity of the sound engulfing you is

$$I = I_{music} + I_{baby} = I_0\left(10^{8.0} + 10^{7.5}\right)$$

$$= \left(1.0\times10^{-12}\text{ W/m}^2\right)\left(1.32\times10^8\right) = \boxed{1.32\times10^{-4}\text{ W/m}^2}$$

(b) The combined sound level is

$$\beta = 10\log\left(I/I_0\right) = 10\log\left(1.32\times10^8\right) = \boxed{81.2\text{ dB}}$$

14.19 (a) The intensity of sound at 10 km from the horn (where β = 50 dB) is

$$I = I_0 10^{\beta/10} = \left(1.0\times10^{-12}\text{ W/m}^2\right)10^{5.0} = 1.0\times10^{-7}\text{ W/m}^2$$

Thus, from $I = P/4\pi r^2$, the power emitted by the source is

$$P = 4\pi r^2 I = 4\pi\left(10\times10^3\text{ m}\right)^2\left(1.0\times10^{-7}\text{ W/m}^2\right) = 4\pi\times10^1\text{ W} = \boxed{1.3\times10^2\text{ W}}$$

(b) At $r = 50$ m, the intensity of the sound will be

$$I = \frac{P}{4\pi r^2} = \frac{1.3\times10^2\text{ W}}{4\pi(50\text{ m})^2} = 4.1\times10^{-3}\text{ W/m}^2$$

and the sound level is

$$\beta = 10\log\left(\frac{I}{I_0}\right) = 10\log\left(\frac{4.1\times10^{-3}\text{ W/m}^2}{1.0\times10^{-12}\text{ W/m}^2}\right) = 10\log\left(4.1\times10^9\right) = \boxed{96\text{ dB}}$$

14.20 (a) $$I = \frac{P}{4\pi r^2} = \frac{100\text{ W}}{4\pi(10.0\text{ m})^2} = \boxed{7.96\times10^{-2}\text{ W/m}^2}$$

(b) $$\beta = 10\log\left(\frac{I}{I_0}\right) = 10\log\left(\frac{7.96\times10^{-2}\text{ W/m}^2}{1.00\times10^{-12}\text{ W/m}^2}\right)$$

$$= 10\log\left(7.96\times10^{10}\right) = \boxed{109\text{ dB}}$$

continued on next page

(c) At the threshold of pain ($\beta = 120$ dB), the intensity is $I = 1.00\ \text{W/m}^2$. Thus, from $I = P/4\pi r^2$, the distance from the speaker is

$$r = \sqrt{\frac{P}{4\pi I}} = \sqrt{\frac{100\ \text{W}}{4\pi\left(1.00\ \text{W/m}^2\right)}} = \boxed{2.82\ \text{m}}$$

14.21 The sound level for intensity I is $\beta = 10\log\left(I/I_0\right)$. Therefore,

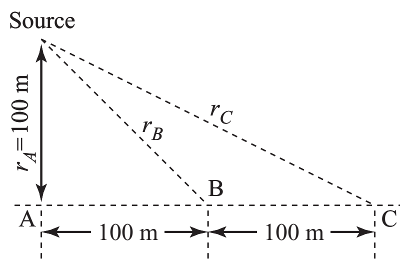
$$\beta_2 - \beta_1 = 10\log\left(\frac{I_2}{I_0}\right) - 10\log\left(\frac{I_1}{I_0}\right) = 10\log\left(\frac{I_2}{\cancel{I_0}}\cdot\frac{\cancel{I_0}}{I_1}\right) = 10\log\left(\frac{I_2}{I_1}\right)$$

Since $I = P/4\pi r^2 = \left(P/4\pi\right)/r^2$, the ratio of intensities is

$$\frac{I_2}{I_1} = \left(\frac{P/4\pi}{r_2^2}\right)\left(\frac{r_1^2}{P/4\pi}\right) = \frac{r_1^2}{r_2^2}$$

Thus, $$\beta_2 - \beta_1 = 10\log\left(\frac{r_1^2}{r_2^2}\right) = 10\log\left(\frac{r_1}{r_2}\right)^2 = \boxed{20\log\left(\frac{r_1}{r_2}\right)}$$

14.22 The intensity at distance r from the source is $I = \dfrac{P}{4\pi r^2} = \dfrac{\left(P/4\pi\right)}{r^2}$



(a) $$\frac{I_A}{I_B} = \frac{r_B^2}{r_A^2} = \frac{(100\ \text{m})^2 + (100\ \text{m})^2}{(100\ \text{m})^2} = \boxed{2}$$

(b) $$\frac{I_A}{I_C} = \frac{r_C^2}{r_A^2} = \frac{(100\ \text{m})^2 + (200\ \text{m})^2}{(100\ \text{m})^2} = \boxed{5}$$

14.23 When a stationary observer $\left(v_O = 0\right)$ hears a moving source, the observed frequency is

$$f_O = f_S\left(\frac{v + v_O}{v - v_S}\right) = f_S\left(\frac{v}{v - v_S}\right)$$

(a) When the train is approaching, $v_S = +40.0\ \text{m/s}$, and

$$\left(f_O\right)_{\text{approach}} = (320\ \text{Hz})\left(\frac{343\ \text{m/s}}{343\ \text{m/s} - 40.0\ \text{m/s}}\right) = 362\ \text{Hz}$$

After the train passes and is receding, $v_S = -40.0\ \text{m/s}$, and

$$\left(f_O\right)_{\text{recede}} = (320\ \text{Hz})\left[\frac{343\ \text{m/s}}{343\ \text{m/s} - (-40.0\ \text{m/s})}\right] = 287\ \text{Hz}$$

Thus, the frequency shift that occurs as the train passes is

$$\Delta f_O = \left(f_O\right)_{\text{recede}} - \left(f_O\right)_{\text{approach}} = -75\ \text{Hz, or it is a } \boxed{75\ \text{Hz drop}}$$

(b) As the train approaches, the observed wavelength is

$$\lambda = \frac{v}{\left(f_O\right)_{\text{approach}}} = \frac{343\ \text{m/s}}{362\ \text{Hz}} = \boxed{0.948\ \text{m}}$$

14.24 The general expression for the observed frequency of a sound when the source and/or the observer are in motion is

$$f_o = f_s\left(\frac{v+v_o}{v-v_s}\right)$$

Here, v is the velocity of sound in air, v_o is the velocity of the observer, v_s is the velocity of the source, and f_s is the frequency that would be detected if both the source and observer were stationary.

(a) If $f_s = 5.00$ kHz and the observer is stationary $(v_o = 0)$, the frequency detected when the source moves toward the observer at half the speed of sound $(v_s = +v/2)$ is

$$f_o = (5.00\text{ kHz})\left(\frac{v+0}{v-v/2}\right) = (5.00\text{ kHz})(2) = \boxed{10.0\text{ kHz}}$$

(b) When $f_s = 5.00$ kHz and the source moves away from a stationary observer at half the speed of sound $(v_s = -v/2)$, the observed frequency is

$$f_o = (5.00\text{ kHz})\left(\frac{v+0}{v+v/2}\right) = (5.00\text{ kHz})\left(\frac{2}{3}\right) = \boxed{3.33\text{ kHz}}$$

14.25 Both source and observer are in motion, so $f_o = f_s\left(\frac{v+v_o}{v-v_s}\right)$. Since each train moves *toward* the other, $v_o > 0$ and $v_s > 0$. The speed of the source (train 2) is

$$v_s = 90.0\ \frac{\text{km}}{\text{h}}\left(\frac{1\ 000\text{ m}}{1\text{ km}}\right)\left(\frac{1\text{ h}}{3\ 600\text{ s}}\right) = 25.0\text{ m/s}$$

and that of the observer (train 1) is $v_o = 130\text{ km/h} = 36.1\text{ m/s}$. Thus, the observed frequency is

$$f_o = (500\text{ Hz})\left(\frac{343\text{ m/s} + 36.1\text{ m/s}}{343\text{ m/s} - 25.0\text{ m/s}}\right) = \boxed{596\text{ Hz}}$$

14.26 (a) Since the observer hears a reduced frequency, the source and observer are getting farther apart. Hence, the cyclist is $\boxed{\text{behind the car}}$.

(b) With the cyclist (observer) behind the car (source) and both moving in the same direction, the observer moves *toward* the source $(v_o > 0)$ while the source moves *away from* the observer $(v_s < 0)$. Thus, $v_o = +|v_{\text{cyclist}}| = +|v_{\text{car}}|/3$ and $v_s = -|v_{\text{car}}|$, where $|v_{\text{car}}|$ is the speed of the car.

The observed frequency is

$$f_o = f_s\left(\frac{v+v_o}{v-v_s}\right) = f_s\left[\frac{v+|v_{\text{car}}|/3}{v-(-|v_{\text{car}}|)}\right] = f_s\left(\frac{v+|v_{\text{car}}|/3}{v+|v_{\text{car}}|}\right),$$

giving

$$415\text{ Hz} = (440\text{ Hz})\left(\frac{343\text{ m/s} + |v_{\text{car}}|/3}{343\text{ m/s} + |v_{\text{car}}|}\right) \quad \text{and} \quad |v_{\text{car}}| = \boxed{32.0\text{ m/s}}$$

14.27 With the train *approaching* the stationary observer ($v_O = 0$) at speed $|v_t|$, the source velocity is $v_S = +|v_t|$ and the observed frequency is

$$465\text{ Hz} = f_S\left(\frac{343\text{ m/s}}{343\text{ m/s} - |v_t|}\right) \qquad [1]$$

As the train *recedes*, the source velocity is $v_S = -|v_t|$ and the observed frequency is

$$441\text{ Hz} = f_S\left(\frac{343\text{ m/s}}{343\text{ m/s} + |v_t|}\right) \qquad [2]$$

Dividing Equation [1] by [2] gives

$$\frac{465}{441} = \frac{343\text{ m/s} + |v_t|}{343\text{ m/s} - |v_t|}$$

and solving for the speed of the train yields $|v_t| = \boxed{9.09\text{ m/s}}$.

14.28 (a) We let the speed of the insect be $|v_{\text{bug}}|$ and the speed of the bat be $|v_{\text{bat}}| = 5.00\text{ m/s}$, and break the action into two steps. In the first step, the bat is the sound source flying *toward* the observer (the insect), so $v_S = +|v_{\text{bat}}|$, while the insect (observer) is flying *away* from the source, making $v_O = -|v_{\text{bug}}|$. If f_0 is the actual frequency sound emitted by the bat, the frequency detected (and reflected) by the moving insect is

$$f_{\text{reflect}} = f_0\left(\frac{v+v_O}{v-v_S}\right) = f_0\left[\frac{v+\left(-|v_{\text{bug}}|\right)}{v-\left(+|v_{\text{bat}}|\right)}\right] \quad \text{or} \quad f_{\text{reflect}} = f_0\left(\frac{v-|v_{\text{bug}}|}{v-|v_{\text{bat}}|}\right)$$

In the second step of the action, the insect acts as a sound source, reflecting a wave of frequency f_{reflect} back to the bat which acts as a moving observer. Since the source (insect) is moving *away* from the observer, $v_S = -|v_{\text{bug}}|$, and the observer (bat) is moving *toward* the source (insect) giving $v_O = +|v_{\text{bat}}|$. The frequency of the return sound received by the bat is then

$$f_{\text{return}} = f_{\text{reflect}}\left(\frac{v+v_O}{v-v_S}\right) = f_{\text{reflect}}\left[\frac{v+\left(+|v_{\text{bat}}|\right)}{v-\left(-|v_{\text{bug}}|\right)}\right] \quad \text{or} \quad f_{\text{return}} = f_{\text{reflect}}\left(\frac{v+|v_{\text{bat}}|}{v+|v_{\text{bug}}|}\right)$$

Combing the results of the two steps gives

$$f_{\text{return}} = f_0\left(\frac{v-|v_{\text{bug}}|}{v-|v_{\text{bat}}|}\right)\left(\frac{v+|v_{\text{bat}}|}{v+|v_{\text{bug}}|}\right)$$

or

$$40.4\text{ kHz} = (40.0\text{ kHz})\left(\frac{343\text{ m/s} - |v_{\text{bug}}|}{343 - 5.00}\right)\left(\frac{343+5.00}{343\text{ m/s} + |v_{\text{bug}}|}\right)$$

This reduces to

$$343\text{ m/s} + |v_{\text{bug}}| = \left(\frac{40.0}{40.4}\right)\left(\frac{348}{338}\right)\left(343\text{ m/s} - |v_{\text{bug}}|\right)$$

continued on next page

or

$$\left[\left(\frac{40.0}{40.4}\right)\left(\frac{348}{338}\right)+1\right]\left|v_{\text{bug}}\right|=(343\ \text{m/s})\left[\left(\frac{40.0}{40.4}\right)\left(\frac{348}{338}\right)-1\right]$$

and yields $\left|v_{\text{bug}}\right|=\boxed{3.29\ \text{m/s}}$.

(b) $\boxed{\text{Yes, the bat is gaining on the insect at a rate of } 5.00\ \text{m/s}-3.29\ \text{m/s}=1.71\ \text{m/s}}$.

14.29 For a source *receding* from a stationary observer,

$$f_O = f_S\left(\frac{v}{v-\left(-\left|v_S\right|\right)}\right)=f_S\left(\frac{v}{v+\left|v_S\right|}\right)$$

Thus, the speed the falling tuning fork must reach is

$$\left|v_S\right|=v\left(\frac{f_S}{f_O}-1\right)=(343\ \text{m/s})\left(\frac{512\ \text{Hz}}{485\ \text{Hz}}-1\right)=19.1\ \text{m/s}$$

The distance it has fallen from rest before reaching this speed is

$$\Delta y_1=\frac{v_S^2-0}{2a_y}=\frac{(19.1\ \text{m/s})^2-0}{2\left(9.80\ \text{m/s}^2\right)}=18.6\ \text{m}$$

The time required for the 485 Hz sound to reach the observer is

$$t=\frac{\Delta y_1}{v}=\frac{18.6\ \text{m}}{343\ \text{m/s}}=0.054\ 2\ \text{s}$$

During this time the fork falls an additional distance

$$\Delta y_2=v_S t+\frac{1}{2}a_y t^2=(19.1\ \text{m/s})(0.054\ 2\ \text{s})+\frac{1}{2}\left(9.80\ \text{m/s}^2\right)(0.054\ 2\ \text{s})^2=1.05\ \text{m}$$

The total distance fallen before the 485 Hz sound reaches the observer is

$$\Delta y=\Delta y_1+\Delta y_2=18.6\ \text{m}+1.05\ \text{m}=\boxed{19.7\ \text{m}}$$

14.30 (a) $\omega=2\pi f=2\pi\left(\dfrac{115/\text{min}}{60.0\ \text{s/min}}\right)=12.0\ \text{rad/s}$

and for harmonic motion,

$$v_{\max}=\omega A=(12.0\ \text{rad/s})\left(1.80\times10^{-3}\ \text{m}\right)=\boxed{2.16\times10^{-2}\ \text{m/s}}$$

(b) The heart wall is a moving observer ($v_O=+\left|v_{\max}\right|$) and the detector a stationary source, so the maximum frequency reflected by the heart wall is

$$\left(f_{\text{wall}}\right)_{\max}=f_S\left(\frac{v+\left|v_{\max}\right|}{v}\right)=(2\ 000\ 000\ \text{Hz})\left(\frac{1\ 500+0.021\ 6}{1\ 500}\right)=\boxed{2\ 000\ 029\ \text{Hz}}$$

continued on next page

(c) Now, the heart wall is a moving source ($v_S = +|v_{max}|$) and the detector a stationary observer. The observed frequency of the returning echo is

$$f_{echo} = \left(f_{wall}\right)_{max}\left(\frac{v}{v - |v_{max}|}\right) = (2\ 000\ 029\ \text{Hz})\left(\frac{1\ 500}{1\ 500 - 0.021\ 6}\right) = \boxed{2\ 000\ 058\ \text{Hz}}$$

14.31 (a) For a plane traveling at Mach 3.00, the half-angle of the conical wave front is

$$\theta = \sin^{-1}\left(\frac{v_{sound}}{v_{plane}}\right) = \sin^{-1}\left(\frac{1}{3.00}\right)$$

The distance the plane has moved when the wave front reaches the observer is $x = h/\tan\theta$, or

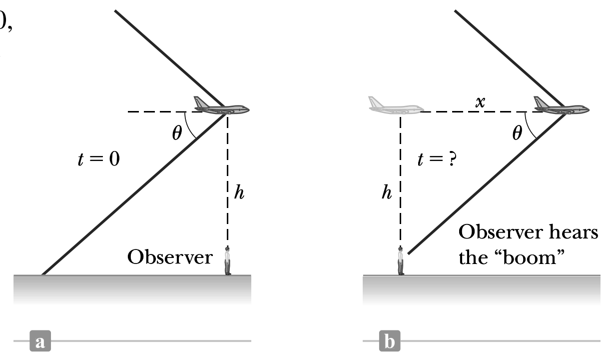


FIGURE P14.31

$$x = \frac{20.0\ \text{km}}{\tan\left[\sin^{-1}(1/3.00)\right]} = 56.6\ \text{km}$$

The time required for the plane to travel this distance, and hence the time when the shock wave reaches the observer, is

$$t = \frac{x}{v_{plane}} = \frac{x}{3.00v_{sound}} = \frac{56.6\times 10^3\ \text{m}}{3.00(335\ \text{m/s})} = \boxed{56.3\ \text{s}}$$

(b) The plane is $\boxed{\text{56.6 km farther along}}$ as computed above.

14.32 (a) From Equation 14.12 in the textbook,

$$\boxed{f_O = f_S\left(\frac{v + v_O}{v - v_S}\right)}$$

where f_S is the frequency emitted by the source, f_O is the frequency detected by the observer, v is the speed of the wave in the propagating medium, v_O is the velocity of the observer relative to the medium, and v_S is the velocity of the source relative to the propagating medium.

(b) The $\boxed{\text{yellow submarine}}$ is the source or emitter of the sound waves.

(c) The $\boxed{\text{red submarine}}$ is the observer or receiver of the sound waves.

(d) The motion of the observer away from the source tends to increase the time observed between arrivals of successive pressure maxima. This effect tends to cause an $\boxed{\text{increase in the observed period}}$ and a $\boxed{\text{decrease in the observed frequency.}}$

(e) In this case, the sign of v_O should be $\boxed{\text{negative}}$ to decrease the numerator in Equation 14.12, and thereby decrease the calculated observed frequency.

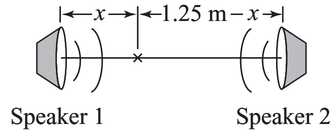
(f) The motion of the source toward the observer tends to decrease the time between the arrival of successive pressure maxima, $\boxed{\text{decreasing the observed period}}$, and $\boxed{\text{increasing the observed frequency.}}$

continued on next page

(g) In this case, the sign of v_s should be $\boxed{\text{positive}}$ to decrease the denominator in Equation 14.12, and thereby increase the calculated observed frequency.

(h) $$f_o = f_s\left(\frac{v+v_o}{v-v_s}\right) = \left(5.27\times10^3\ \text{Hz}\right)\left[\frac{1\,533\ \text{m/s}+\left(-3.00\ \text{m/s}\right)}{1\,533\ \text{m/s}-\left(+11.0\ \text{m/s}\right)}\right] = \boxed{5.30\times10^3\ \text{Hz}}$$

14.33 The wavelength of the waves generated by the speakers is



$$\lambda = \frac{v}{f} = \frac{343\ \text{m/s}}{800\ \text{Hz}} = 0.429\ \text{m}$$

For the waves from the two speakers to interfere destructively at some point, the difference in the path lengths from the speakers to that point must be an odd multiple of a half-wavelength. Thus, along the line connecting the two speakers, destructive interference (and minima in amplitude) occur where

$$(1.25\ \text{m}-x)-x = (2n+1)\frac{\lambda}{2} \qquad \text{where } n \text{ is any integer}$$

or where $$x = 0.625\ \text{m}-(2n+1)\frac{\lambda}{4} = 0.625\ \text{m}-\left(\frac{2n+1}{4}\right)(0.429\ \text{m})$$

This gives: $n=0 \Rightarrow x=0.518\ \text{m}$ $\qquad n=-1 \Rightarrow x=0.732\ \text{m}$

$n=1 \Rightarrow x=0.303\ \text{m}$ $\qquad n=-2 \Rightarrow x=0.947\ \text{m}$

$n=2 \Rightarrow x=0.089\ \text{m}$ $\qquad n=-3 \Rightarrow x=1.16\ \text{m}$

Thus, minima occur at distances of

$$\boxed{0.089\ \text{m},\ 0.303\ \text{m},\ 0.518\ \text{m},\ 0.732\ \text{m},\ 0.947\ \text{m, and } 1.16\ \text{m}}$$

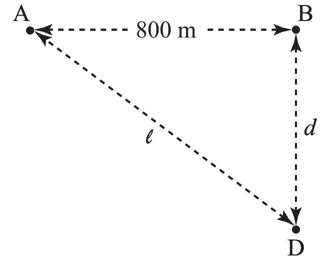
from either speaker.

14.34 The wavelength of the sound emitted by the speaker is $\lambda = \dfrac{v}{f} = \dfrac{343\ \text{m/s}}{756\ \text{Hz}} = 0.454\ \text{m}$, and a half wavelength is $\lambda/2 = 0.227\ \text{m}$.

(a) If a condition of constructive interference currently exists, this can be changed to a case of destructive interference by adding a distance of $\boxed{\lambda/2 = 0.227\ \text{m}}$ to the path length through the upper arm.

(b) To move from a case of constructive interference to the next occurrence of constructive interference, one should increase the path length through the upper arm by a full wavelength, or by $\boxed{\lambda = 0.454\ \text{m}}$.

14.35 At point D, the distance of the ship from point A is

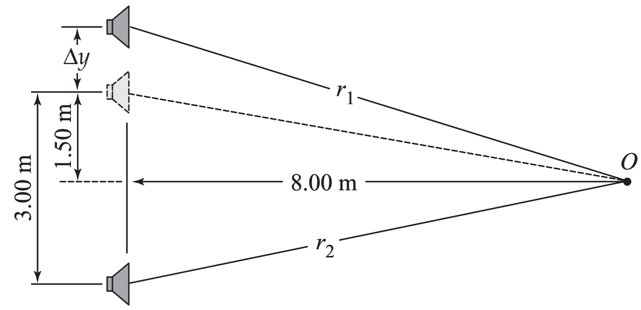


$$\ell = \sqrt{d^2+(800\ \text{m})^2} = \sqrt{(600\ \text{m})^2+(800\ \text{m})^2} = 1\ 000\ \text{m}$$

Since destructive interference occurs for the first time when the ship reaches D, it is necessary that $\ell - d = \lambda/2$, or

$$l = 2(\ell-d) = 2(1\ 000\ \text{m}-600\ \text{m}) = \boxed{800\ \text{m}}$$

14.36 The speakers emit sound of wavelength



$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{450 \text{ Hz}} = 0.762 \text{ m}$$

so $\lambda/2 = 0.381$ m

Initially, $\Delta y = 0$, and

$$r_1 = r_2 = \sqrt{(1.50 \text{ m})^2 + (8.00 \text{ m})^2} = 8.14 \text{ m}$$

To create destructive interference at point O, we move the top speaker upward distance Δy from its original location until we have $r_1 - r_2 = \lambda/2$. Since this did not change r_2, we must now have

$$r_1 = r_2 + \lambda/2 = 8.14 \text{ m} + 0.381 \text{ m} = 8.52 \text{ m}$$

But, after moving the speaker, this gives

$$r_1 = \sqrt{(1.50 \text{ m} + \Delta y)^2 + (8.00 \text{ m})^2} = 8.52 \text{ m}$$

or $$(1.50 \text{ m} + \Delta y)^2 = (8.52 \text{ m})^2 - (8.00 \text{ m})^2 = 8.59 \text{ m}^2$$

Thus, $\Delta y = \sqrt{8.59 \text{ m}^2} - 1.50 \text{ m} = \boxed{1.43 \text{ m}}$

14.37 The wavelength of the sound is

$$\lambda = \frac{v}{f} = \frac{343 \text{ m/s}}{686 \text{ Hz}} = 0.500 \text{ m}$$

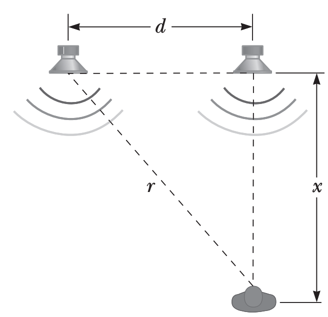


FIGURE P14.37 (modified)

(a) At the first relative maximum (constructive interference),

$$r = x + \lambda = x + 0.500 \text{ m}$$

Using the Pythagorean theorem, $r^2 = x^2 + d^2$, or

$$(x + 0.500 \text{ m})^2 = x^2 + (0.700 \text{ m})^2$$

giving $x = \boxed{0.240 \text{ m}}$.

(b) At the first relative minimum (destructive interference),

$$r = x + \lambda/2 = x + 0.250 \text{ m}$$

Therefore, the Pythagorean theorem yields

$$(x + 0.250 \text{ m})^2 = x^2 + (0.700 \text{ m})^2$$

or $x = \boxed{0.855 \text{ m}}$

14.38 In the fundamental mode of vibration, the wavelength of waves in the wire is

$$\lambda = 2L = 2(0.700\,0 \text{ m}) = 1.400 \text{ m}$$

If the wire is to vibrate at $f = 261.6$ Hz, the speed of the waves must be

$$v = \lambda f = (1.400 \text{ m})(261.6 \text{ Hz}) = 366.2 \text{ m/s}$$

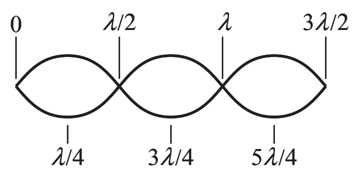
The mass per unit length of the wire is

$$\mu = \frac{m}{L} = \frac{4.300 \times 10^{-3} \text{ kg}}{0.700\,0 \text{ m}} = 6.143 \times 10^{-3} \text{ kg/m}$$

and the required tension is given by $v = \sqrt{F/\mu}$ as

$$F = v^2 \mu = (366.2 \text{ m/s})^2 (6.143 \times 10^{-3} \text{ kg/m}) = \boxed{823.8 \text{ N}}$$

14.39 In the third harmonic, the string of length L forms a standing wave of three loops, each of length $\lambda/2 = L/3$. The wavelength of the wave is then



$$\lambda = \frac{2L}{3} = \frac{16.0 \text{ m}}{3} \approx 5.33 \text{ m}$$

(a) The nodes in this string, fixed at each end, will occur at distances of $\boxed{0,\ \lambda/2 = 2.67 \text{ m},\ \lambda = 5.33 \text{ m, and } 3\lambda/2 = 8.00 \text{ m}}$ from the end.

Antinodes occur halfway between each pair of adjacent nodes, or at distances of $\boxed{\lambda/4 = 1.33 \text{ m},\ 3\lambda/4 = 4.00 \text{ m, and } 5\lambda/4 = 6.67 \text{ m}}$ from the end.

(b) The linear density is

$$\mu = \frac{m}{L} = \frac{40.0 \times 10^{-3} \text{ kg}}{8.00 \text{ m}} = 5.00 \times 10^{-3} \text{ kg/m}$$

and the wave speed is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{49.0 \text{ N}}{5.00 \times 10^{-3} \text{ kg/m}}} = 99.0 \text{ m/s}$$

Thus, the frequency is $f = \dfrac{v}{\lambda} = \dfrac{99.0 \text{ m/s}}{5.33 \text{ m}} = \boxed{18.6 \text{ Hz}}$

14.40 In the fundamental mode, the distance from the finger of the cellist to the far end of the string is one-half of the wavelength for the transverse waves in the string. Thus, when the string resonates at 449 Hz,

$$\lambda = 2(68.0 \text{ cm} - 20.0 \text{ cm}) = 96.0 \text{ cm}$$

The speed of transverse waves in the string is therefore

$$v = \lambda f = (0.960 \text{ m})(449 \text{ Hz}) = 431 \text{ m/s}$$

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For a resonance frequency of 440 Hz, the wavelength would be

$$\lambda' = \frac{v}{f'} = \frac{431\ \text{m/s}}{440\ \text{Hz}} = 0.980\ \text{m} = 98.0\ \text{cm}$$

To produce this tone, the cellist should position her finger at a distance of

$$x = L - \frac{\lambda}{2} = 68.0\ \text{cm} - \frac{98.0\ \text{cm}}{2} = 19.0\ \text{cm}$$

from the nut. Thus, she should $\boxed{\text{move her finger 1.00 cm toward the nut.}}$

14.41 When the string vibrates in the fifth harmonic (i.e., in five equal segments) at a frequency of $f_5 = 630$ Hz, we have $L = 5(\lambda_5/2)$, or the wavelength is $\lambda_5 = 2L/5$. The speed of transverse waves in the string is then

$$v = \lambda_5 f_5 = (2L/5) f_5$$

For the string to vibrate in three segments (i.e., third harmonic), the wavelength must be such that $L = 3(\lambda_3/2)$ or $\lambda_3 = 2L/3$. The new frequency would then be

$$f_3 = \frac{v}{\lambda_3} = \frac{(2L/5) f_5}{2L/3} = \frac{3}{5} f_5 = \frac{3}{5}(630\ \text{Hz}) = \boxed{378\ \text{Hz}}$$

14.42 If a wire of length ℓ is fixed at both ends, the wavelength of the fundamental mode of vibration is $\lambda_1 = 2\ell$. The speed of transverse waves in the wire is $v = \sqrt{F/\mu}$, where F is the tension in the wire and μ is the mass per unit length of the wire. The fundamental frequency for the wire is then

$$f_1 = \frac{v}{\lambda_1} = \frac{1}{2\ell}\sqrt{\frac{F}{\mu}} = \frac{1}{2}\left(\frac{1}{\ell}\right)\sqrt{\frac{F}{\mu}}$$

If we have two wires with the same mass per unit length, one of length L and under tension F while the second has length $2L$ and tension $4F$, the ratio of the fundamental frequencies of the two wires is

$$\frac{f_{1,\,\text{long}}}{f_{1,\,\text{short}}} = \frac{\frac{1}{2}(1/2L)\sqrt{4F/\mu}}{\frac{1}{2}(1/L)\sqrt{F/\mu}} = \frac{1}{2}\sqrt{4} = 1$$

or the two wires have the same fundamental frequency of vibration. If this frequency is $f_1 = 60$ Hz, then the frequency of the second harmonic for both wires is

$$f_2 = 2 f_1 = 2(60\ \text{Hz}) = \boxed{120\ \text{Hz}}$$

14.43 (a) The linear density is $\mu = \dfrac{m}{L} = \dfrac{25.0\times10^{-3}\ \text{kg}}{1.35\ \text{m}} = \boxed{1.85\times10^{-2}\ \text{kg/m}}$

(b) In a string fixed at both ends, the fundamental mode has a node at each end and a single antinode in the center, so that $L = \lambda/2$, or $\lambda = 2L = 2(1.10\ \text{m}) = 2.20$ m.

Then, the desired wave speed in the wire is $v = \lambda f = (2.20\ \text{m})(41.2\ \text{Hz}) = \boxed{90.6\ \text{m/s}}$

(c) The speed of transverse waves in a string is $v = \sqrt{F/\mu}$, so the required tension is

$$F = \mu v^2 = \left(1.85\times10^{-2}\ \text{kg/m}\right)(90.6\ \text{m/s})^2 = \boxed{152\ \text{N}}$$

continued on next page

(d) $\lambda = 2L = 2(1.10 \text{ m}) = \boxed{2.20 \text{ m}}$ [See part (b) above.]

(e) The wavelength of the longitudinal sound waves produced in air by the vibrating string is

$$\lambda_{\text{air}} = \frac{v_{\text{air}}}{f} = \frac{343 \text{ m/s}}{41.2 \text{ Hz}} = \boxed{8.33 \text{ m}}$$

14.44 (a) A string fixed at each end forms standing wave patterns with a node at each end and an integer number of loops, each loop of length $\lambda/2$, with an antinode at its center. Thus, $L = n(\lambda/2)$ or $\lambda = 2L/n$.

If the string has tension T and mass per unit length μ, the speed of transverse waves is $v = \lambda f = \sqrt{T/\mu}$. Thus, when the string forms a standing wave of n loops (and hence n antinodes), the frequency of vibration is

$$f = \frac{v}{\lambda} = \frac{\sqrt{T/\mu}}{2L/n} = \frac{n}{2L}\sqrt{\frac{T}{\mu}} \quad \Rightarrow \quad \boxed{f_A = \frac{n_A}{2L_A}\sqrt{\frac{T_A}{\mu_A}}}$$

(b) Assume the length is doubled, $L_B = 2L_A$, and a new standing wave is formed having $n_B = n_A$ and $T_B = T_A$. Then

$$f_B = \frac{n_B}{2L_B}\sqrt{\frac{T_B}{\mu_A}} = \frac{n_A}{2(2L_A)}\sqrt{\frac{T_A}{\mu_A}} = \frac{1}{2}\left(\frac{n_A}{2L_A}\sqrt{\frac{T_A}{\mu_A}}\right) = \boxed{\frac{f_A}{2}}$$

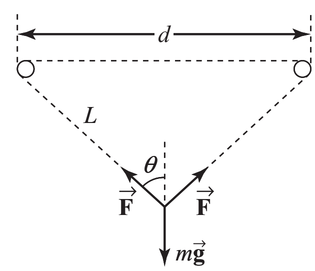
(c) Solving the general result obtained in part (a) for the tension in the string gives $T = 4\mu f^2 L^2/n^2$. Thus, if $f_B = f_A$, $L_B = L_A$, and $n_B = n_A + 1$, we find

$$T_B = \frac{4\mu_A f_B^2 L_B^2}{n_B^2} = \frac{4\mu_A f_A^2 L_A^2}{(n_A+1)^2} = \frac{n_A^2}{(n_A+1)^2}\left(\frac{4\mu_A f_A^2 L_A^2}{n_A^2}\right) = \frac{n_A^2}{(n_A+1)^2} T_A = \boxed{\left(\frac{n_A}{n_A+1}\right)^2 T_A}$$

(d) If now we have $f_B = 3f_A$, $L_B = L_A/2$, and $n_B = 2n_A$, then

$$T_B = \frac{4\mu_A f_B^2 L_B^2}{n_B^2} = \frac{4\mu_A (9f_A^2)(L_A^2/4)}{(4n_A^2)} = \frac{9}{16}\left(\frac{4\mu_A f_A^2 L_A^2}{n_A^2}\right) = \frac{9}{16}T_A \quad \text{or} \quad \boxed{\frac{T_B}{T_A} = \frac{9}{16}}$$

14.45 (a) From the sketch at the right, notice that when $d = 2.00$ m,



$$L = \frac{5.00 \text{ m} - d}{2} = 1.50 \text{ m},$$

and

$$\theta = \sin^{-1}\left(\frac{d/2}{L}\right) = 41.8°$$

Then evaluating the net vertical force on the lowest bit of string, $\Sigma F_y = 2F\cos\theta - mg = 0$ gives the tension in the string as

$$F = \frac{mg}{2\cos\theta} = \frac{(12.0 \text{ kg})(9.80 \text{ m/s}^2)}{2\cos(41.8°)} = \boxed{78.9 \text{ N}}$$

(b) The speed of transverse waves in the string is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{78.9 \text{ N}}{0.001\,00 \text{ kg/m}}} = 2.81 \times 10^2 \text{ m/s}$$

continued on next page

For the pattern shown,

$$3(\lambda/2) = d\text{, so } \lambda = \frac{2d}{3} = \frac{4.00\ \text{m}}{3}$$

Thus, the frequency is

$$f = \frac{v}{\lambda} = \frac{3\left(2.81\times 10^{2}\ \text{m/s}\right)}{4.00\ \text{m}} = \boxed{2.11\times 10^{2}\ \text{Hz}}$$

14.46 (a) For a standing wave of 6 loops, $6(\lambda/2) = L$, or $\lambda = L/3 = (2.0\ \text{m})/3$.

The speed of the waves in the string is then

$$v = \lambda f = \left(\frac{2.0\ \text{m}}{3}\right)(150\ \text{Hz}) = 1.0\times 10^{2}\ \text{m/s}$$

Since the tension in the string is $F = mg = (5.0\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 49\ \text{N}$, $v = \sqrt{F/\mu}$ gives

$$\mu = \frac{F}{v^2} = \frac{49\ \text{N}}{\left(1.0\times 10^{2}\ \text{m}\right)^2} = \boxed{4.9\times 10^{-3}\ \text{kg/m}}$$

(b) If $m = 45$ kg, then $F = (45\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 4.4\times 10^{2}\ \text{N}$, and

$$v = \sqrt{\frac{4.4\times 10^{2}\ \text{N}}{4.9\times 10^{-3}\ \text{kg/m}}} = 3.0\times 10^{2}\ \text{m/s}$$

Thus, the wavelength will be

$$\lambda = \frac{v}{f} = \frac{3.0\times 10^{2}\ \text{m/s}}{150\ \text{Hz}} = 2.0\ \text{m}$$

and the number of loops is $n = \dfrac{L}{\lambda/2} = \dfrac{2.0\ \text{m}}{1.0\ \text{m}} = \boxed{2}$

(c) If $m = 10$ kg, the tension is $F = (10\ \text{kg})\left(9.80\ \text{m/s}^2\right) = 98\ \text{N}$, and

$$v = \sqrt{\frac{98\ \text{N}}{4.9\times 10^{-3}\ \text{kg/m}}} = 1.4\times 10^{2}\ \text{m/s}$$

Then, $\lambda = \dfrac{v}{f} = \dfrac{1.4\times 10^{2}\ \text{m/s}}{150\ \text{Hz}} = 0.93\ \text{m}$

and $n = \dfrac{L}{\lambda/2} = \dfrac{2.0\ \text{m}}{0.47\ \text{m}}$ is not an integer,

so $\boxed{\text{no standing wave will form}}$.

14.47 The speed of transverse waves in the string is

$$v = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{50.000\ \text{N}}{1.000\,0\times 10^{-2}\ \text{kg/m}}} = 70.711\ \text{m/s}$$

continued on next page

The fundamental wavelength is $\lambda_1 = 2L = 1.200\,0$ m and its frequency is

$$f_1 = \frac{v}{\lambda_1} = \frac{70.711 \text{ m/s}}{1.200\,0 \text{ m}} = 58.926 \text{ Hz}$$

The harmonic frequencies are then

$f_n = nf_1 = n(58.926 \text{ Hz})$, with n being an integer

The largest one under 20 000 Hz is $f_{339} = 19\,976 \text{ Hz} = \boxed{19.976 \text{ kHz}}$.

14.48 The distance between adjacent nodes (and between adjacent antinodes) is one-quarter of the circumference.

$$d_{\text{NN}} = d_{\text{AA}} = \frac{\lambda}{2} = \frac{20.0 \text{ cm}}{4} = 5.00 \text{ cm}$$

so $\lambda = 10.0 \text{ cm} = 0.100 \text{ m}$,

and $f = \dfrac{v}{\lambda} = \dfrac{900 \text{ m/s}}{0.100 \text{ m}} = 9.00 \times 10^3 \text{ Hz} = \boxed{9.00 \text{ kHz}}$

The singer must match this frequency quite precisely for some interval of time to feed enough energy into the glass to crack it.

14.49 Assuming an air temperature of $T = 37°\text{C} = 310$ K, the speed of sound inside the pipe is

$$v = (331 \text{ m/s})\sqrt{\frac{T_K}{273 \text{ K}}} = (331 \text{ m/s})\sqrt{\frac{310}{273}} = 353 \text{ m/s}$$

In the fundamental resonant mode, the wavelength of sound waves in a pipe closed at one end is $\lambda = 4L$. Thus, for the whooping crane

$$\lambda = 4(5.0 \text{ ft}) = 2.0 \times 10^1 \text{ ft}$$

and $f = \dfrac{v}{\lambda} = \dfrac{(353 \text{ m/s})}{2.0 \times 10^1 \text{ ft}}\left(\dfrac{3.281 \text{ ft}}{1 \text{ m}}\right) = \boxed{58 \text{ Hz}}$

14.50 (a) In the fundamental resonant mode of a pipe open at both ends, the distance between antinodes is $d_{\text{AA}} = \lambda/2 = L$.

Thus, $\lambda = 2L = 2(0.320 \text{ m}) = 0.640 \text{ m}$

and $f = \dfrac{v}{\lambda} = \dfrac{343 \text{ m/s}}{0.640 \text{ m}} = \boxed{536 \text{ Hz}}$

(b) $d_{\text{AA}} = \dfrac{\lambda}{2} = \dfrac{1}{2}\left(\dfrac{v}{f}\right) = \dfrac{1}{2}\left(\dfrac{343 \text{ m/s}}{4\,000 \text{ Hz}}\right) = 4.29 \times 10^{-2} \text{ m} = \boxed{4.29 \text{ cm}}$

14.51 Hearing would be best at the fundamental resonance, so $\lambda = 4L = 4(2.8 \text{ cm})$

and $f = \dfrac{v}{\lambda} = \dfrac{343 \text{ m/s}}{4(2.8 \text{ cm})}\left(\dfrac{100 \text{ cm}}{1 \text{ m}}\right) = 3.1 \times 10^3 \text{ Hz} = \boxed{3.1 \text{ kHz}}$

14.52 (a) To form a standing wave in the tunnel, open at both ends, one must have an antinode at each end, a node at the middle of the tunnel, and the length of the tunnel must be equal to an integral number of half-wavelengths [$L = n(\lambda/2)$ or $\lambda = 2L/n$]. The resonance frequencies of the tunnel are then

$$f_n = \frac{v_{\substack{\text{sound}\\ \text{in air}}}}{\lambda_n} = \frac{343\ \text{m/s}}{2L/n} = n\left(\frac{343\ \text{m/s}}{2\left(2.00\times 10^{3}\ \text{m}\right)}\right) = \boxed{n\left(0.085\,8\ \text{Hz}\right)} \quad n = 1,\ 2,\ 3,\ \ldots$$

(b) It would be good to make such a rule. Any car horn would produce several closely spaced resonance frequencies of the air in the tunnel, so the sound would be greatly amplified. Other drivers might be frightened directly into dangerous behavior or might blow their horns also.

14.53 (a) The fundamental wavelength of the pipe open at both ends is $\lambda_1 = 2L = v/f_1$. Since the speed of sound is $331\ \text{m/s}$ at 0°C, the length of the pipe is

$$L = \frac{v}{2f_1} = \frac{331\ \text{m/s}}{2(300\ \text{Hz})} = \boxed{0.552\ \text{m}}$$

(b) At $T = 30°\text{C} = 303\ \text{K}$,

$$v = (331\ \text{m/s})\sqrt{\frac{T_K}{273}} = (331\ \text{m/s})\sqrt{\frac{303}{273}} = 349\ \text{m/s}$$

and

$$f_1 = \frac{v}{\lambda_1} = \frac{v}{2L} = \frac{349\ \text{m/s}}{2(0.552\ \text{m})} = \boxed{316\ \text{Hz}}$$

14.54 (a) Observe from Equations 14.18 and 14.19 in the textbook that the difference between successive resonance frequencies is constant, regardless of whether the pipe is open at both ends or is closed at one end. Thus, the resonance frequencies of 650 Hz or less for this pipe must be 650 Hz, 550 Hz, 450 Hz, 350 Hz, 250 Hz, 150 Hz, and 50.0 Hz, with the lowest or fundamental frequency being $\boxed{f_1 = 50.0\ \text{Hz}}$.

(b) Note, from the list given above, the resonance frequencies are only the *odd* multiples of the fundamental frequency. This is a characteristic of a pipe that is $\boxed{\text{open at only one end}}$ and closed at the other.

(c) The length of a pipe with an antinode at the open end and a node at the closed end is one-quarter of the wavelength of the fundamental frequency, so the length of this pipe must be

$$L = \frac{\lambda_1}{4} = \frac{v_{\text{sound}}}{4f_1} = \frac{343\ \text{m/s}}{4(50.0\ \text{Hz})} = \boxed{1.72\ \text{m}}$$

14.55 In a string fixed at both ends, the length of the string is equal to a half-wavelength of the fundamental resonance frequency, so $\lambda_1 = 2L$. The fundamental frequency may then be written as

$$f_1 = \frac{v}{\lambda_1} = \frac{1}{2L}\sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{4L^2\mu}}$$

If a second identical string with tension $F' < F$ is struck, the fundamental frequency of vibration would be

$$f_1' = \sqrt{\frac{F'}{4L^2\mu}} = \sqrt{\left(\frac{F}{4L^2\mu}\right)\frac{F'}{F}} = f_1\sqrt{\frac{F'}{F}}$$

continued on next page

When the two strings are sounded together, the beat frequency heard will be

$$f_{\text{beat}} = f_1 - f_1' = f_1\left(1-\sqrt{\frac{F'}{F}}\right) = \left(1.10\times10^2\ \text{Hz}\right)\left(1-\sqrt{\frac{5.40\times10^2\ \text{N}}{6.00\times10^2\ \text{N}}}\right) = \boxed{5.64\ \text{beats/s}}$$

14.56 By shortening her string, the second violinist increases its fundamental frequency. Thus, $f_1' = f_1 + f_{\text{beat}} = (196+2.00)\ \text{Hz} = 198\ \text{Hz}$. Since the tension and the linear density are both identical for the two strings, the speed of transverse waves, $v=\sqrt{F/\mu}$, has the same value for both strings. Therefore, $\lambda_1' f_1' = \lambda_1 f_1$, or $\lambda_1' = \lambda_1\left(f_1/f_1'\right)$. The fundamental wavelength of a string fixed at both ends is $\lambda = 2L$, and this yields

$$L' = L\left(\frac{f_1}{f_1'}\right) = (30.0\ \text{cm})\left(\frac{196}{198}\right) = \boxed{29.7\ \text{cm}}$$

14.57 The commuter, stationary relative to the station and the first train, hears the actual source frequency $\left(f_{O,1} = f_S = 180\ \text{Hz}\right)$ from the first train. The frequency the commuter hears from the second train, moving relative to the station and commuter, is given by

$$f_{O,2} = f_S \pm f_{\text{beat}} = 180\ \text{Hz} \pm 2\ \text{Hz} = 178\ \text{Hz or } 182\ \text{Hz}$$

This stationary observer $\left(v_O = 0\right)$ hears the lower frequency $\left(f_{O,2} = 178\ \text{Hz}\right)$ if the second train is moving *away* from the station $\left(v_S = -|v_S|\right)$, so $f_O = f_S[(v+v_O)/(v-v_S)]$ gives the speed of the receding second train as

$$178\ \text{Hz} = (180\ \text{Hz})\left(\frac{343\ \text{m/s}+0}{343\ \text{m/s}-\left(-|v_S|\right)}\right) = (180\ \text{Hz})\left(\frac{343\ \text{m/s}+0}{343\ \text{m/s}+|v_S|}\right)$$

or

$$343\ \text{m/s}+|v_S| = (343\ \text{m/s})\left(\frac{180\ \text{Hz}}{178\ \text{Hz}}\right) \qquad \text{and} \qquad |v_S| = (343\ \text{m/s})\left[\left(\frac{180\ \text{Hz}}{178\ \text{Hz}}\right)-1\right] = 3.85\ \text{m/s}$$

so one possibility for the second train is $\boxed{v_S = 3.85\ \text{m/s away from the station}}$.

The other possibility is that the second train is moving toward the station $\left(v_S = +|v_S|\right)$ and the commuter is detecting the higher of the possible frequencies $\left(f_{O,2} = 182\ \text{Hz}\right)$. In this case, $f_O = f_S[(v+v_O)/(v-v_S)]$ yields

$$182\ \text{Hz} = (180\ \text{Hz})\left(\frac{343\ \text{m/s}+0}{343\ \text{m/s}-|v_S|}\right) \qquad \text{and} \qquad 343\ \text{m/s}-|v_S| = (343\ \text{m/s})\left(\frac{180\ \text{Hz}}{182\ \text{Hz}}\right)$$

or $$|v_S| = (343\ \text{m/s})\left[1-\left(\frac{180\ \text{Hz}}{182\ \text{Hz}}\right)\right] = 3.77\ \text{m/s}$$

In this case, the velocity of the second train is $\boxed{v_S = 3.77\ \text{m/s toward the station}}$.

14.58 The temperatures of the air in the two pipes are $T_1 = 27°\text{C} = 300\ \text{K}$ and $T_2 = 32°\text{C} = 305\ \text{K}$. The speed of sound in the two pipes is

$$v_1 = (331\ \text{m/s})\sqrt{\frac{T_1}{273\ \text{K}}} \quad \text{and} \quad v_2 = (331\ \text{m/s})\sqrt{\frac{T_2}{273\ \text{K}}}$$

Since the pipes have the same length, the fundamental wavelength, $\lambda = 4L$, is the same for them. Thus, from $f = v/\lambda$, the ratio of their fundamental frequencies is seen to be $f_2/f_1 = v_2/v_1$, which gives $f_2 = f_1\left(v_2/v_1\right)$.

continued on next page

The beat frequency produced is then

$$f_{\text{beat}} = f_2 - f_1 = f_1\left(\frac{v_2}{v_1} - 1\right) = f_1\left(\sqrt{\frac{T_2}{T_1}} - 1\right)$$

or $$f_{\text{beat}} = (480\text{ Hz})\left(\sqrt{\frac{305\text{ K}}{300\text{ K}}} - 1\right) = \boxed{3.98\text{ Hz}}$$

14.59 (a) First consider the wall a stationary observer receiving sound from an *approaching* source having velocity v_a. The frequency received and reflected by the wall is $f_{\text{reflect}} = f_S\,[v/(v - v_a)]$.

Now consider the wall as a stationary source emitting sound of frequency f_{reflect} to an observer *approaching* at velocity v_a. The frequency of the echo heard by the observer is

$$f_{\text{echo}} = f_{\text{reflect}}\left(\frac{v + v_a}{v}\right) = f_S\left(\frac{v}{v - v_a}\right)\left(\frac{v + v_a}{v}\right) = f_S\left(\frac{v + v_a}{v - v_a}\right)$$

Thus, the beat frequency between the tuning fork and its echo is

$$f_{\text{beat}} = f_{\text{echo}} - f_S = f_S\left(\frac{v + v_a}{v - v_a} - 1\right) = f_S\left(\frac{2v_a}{v - v_a}\right) = (256\text{ Hz})\left(\frac{2(1.33)}{343 - 1.33}\right) = \boxed{1.99\text{ Hz}}$$

(b) When the student moves away from the wall, v_a changes sign so the frequency of the echo heard is $f_{\text{echo}} = f_S\,[(v - |v_a|)/(v + |v_a|)]$. The beat frequency is then

$$f_{\text{beat}} = f_S - f_{\text{echo}} = f_S\left(1 - \frac{v - |v_a|}{v + |v_a|}\right) = f_S\left(\frac{2|v_a|}{v + |v_a|}\right)$$

giving $$|v_a| = \frac{v\,f_{\text{beat}}}{2\,f_S - f_{\text{beat}}}$$

The receding speed needed to observe a beat frequency of 5.00 Hz is

$$|v_a| = \frac{(343\text{ m/s})(5.00\text{ Hz})}{2(256\text{ Hz}) - 5.00\text{ Hz}} = \boxed{3.38\text{ m/s}}$$

14.60 The extra sensitivity of the ear at 3 000 Hz appears as downward dimples on the curves in Figure 14.29 of the textbook.

At $T = 37°\text{C} = 310\text{ K}$, the speed of sound in air is

$$v = (331\text{ m/s})\sqrt{\frac{T_K}{273}} = (331\text{ m/s})\sqrt{\frac{310}{273}} = 353\text{ m/s}$$

Thus, the wavelength of 3 000 Hz sound is

$$\lambda = \frac{v}{f} = \frac{353\text{ m/s}}{3\,000\text{ Hz}} = 0.118\text{ m}$$

For the fundamental resonant mode in a pipe closed at one end, the length required is

$$L = \frac{\lambda}{4} = \frac{0.118\text{ m}}{4} = 0.0295\text{ m} = \boxed{2.95\text{ cm}}$$

14.61 At normal body temperature of $T = 37.0°\text{C}$, the speed of sound in air is

$$v = (331\ \text{m/s})\sqrt{1+\frac{T_C}{273}} = (331\ \text{m/s})\sqrt{1+\frac{37.0}{273}}$$

and the wavelength of sound having a frequency of $f = 20\,000\ \text{Hz}$ is

$$\lambda = \frac{v}{f} = \frac{(331\ \text{m/s})}{(20\,000\ \text{Hz})}\sqrt{1+\frac{37.0}{273}} = 1.76\times10^{-2}\ \text{m} = 1.76\ \text{cm}$$

Thus, the diameter of the eardrum is $\boxed{1.76\ \text{cm}}$.

14.62 From the defining equation for the decibel level, $\beta = 10\log(I/I_0)$, the intensity of sound having a decibel level β is

$$I = \left(10^{\beta/10}\right)I_0$$

Thus, the intensity of a 40 dB sound is $I_{40} = (10^{4.0})I_0$, while that of a 70 dB sound is $I_{70} = (10^{7.0})I_0$. Since the combined intensity of sound from a swarm of n mosquitoes is $I_{\text{swarm}} = nI_{40}$, we must require that

$$I_{\text{swarm}} = nI_{40} = I_{70}$$

or $$n = \frac{I_{70}}{I_{40}} = \frac{\left(10^{7.0}\right)I_0}{\left(10^{4.0}\right)I_0} = 10^{3.0} = 1\,000$$

We conclude that the swarm should contain $\boxed{\sim 1\,000\ \text{mosquitoes}}$ to yield a 70 dB sound.

14.63 (a) With a decibel level of 103 dB, the intensity of the sound at 1.60 m from the speaker is found from $\beta = 10\cdot\log\left(I/I_0\right)$ as

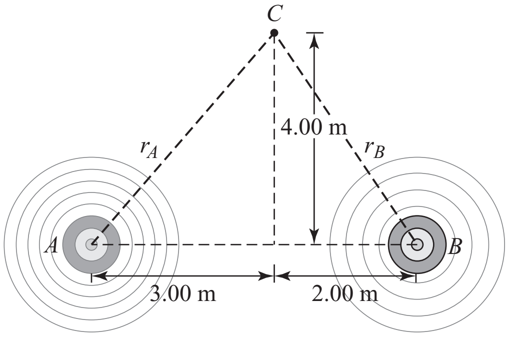
$$I = I_0\cdot10^{\beta/10} = \left(1.00\times10^{-12}\ \text{W/m}^2\right)\cdot10^{10.3} = 1.00\times10^{-1.7}\ \text{W/m}^2$$

If the speaker broadcasts equally well in all directions, the intensity (power per unit area) at 1.60 m from the speaker is uniformly distributed over a spherical wave front of radius $r = 1.60\ \text{m}$ centered on the speaker. Thus, the power radiated is

$$P = IA = I\left(4\pi r^2\right) = \left(1.00\times10^{-1.7}\ \text{W/m}^2\right)4\pi(1.60\ \text{m})^2 = \boxed{0.642\ \text{W}}$$

(b) $$\textit{efficiency} = \frac{P_{\text{output}}}{P_{\text{input}}} = \frac{0.642\ \text{W}}{150\ \text{W}} = \boxed{0.004\,3\ \text{ or }\ 0.43\%}$$

14.64 (a) At point C, the distance from speaker A is



$$r_A = \sqrt{(3.00\ \text{m})^2 + (4.00\ \text{m})^2} = 5.00\ \text{m}$$

and the intensity of the sound from this speaker is

$$I_A = \frac{P_A}{4\pi r_A^2} = \frac{1.00\times10^{-3}\ \text{W}}{4\pi(5.00\ \text{m})^2} = 3.18\times10^{-6}\ \text{W/m}^2$$

The sound level at C due to speaker A alone is then

$$\beta_A = 10\cdot\log\left(\frac{I_A}{I_0}\right) = 10\cdot\log\left(\frac{3.18\times10^{-6}\ \text{W/m}^2}{1.00\times10^{-12}\ \text{W/m}^2}\right) = \boxed{65.0\ \text{dB}}$$

continued on next page

(b) The distance from point C to speaker B is $r_B = \sqrt{(2.00\text{ m})^2 + (4.00\text{ m})^2} = 4.47\text{ m}$ and the intensity of the sound from this speaker alone is

$$I_B = \frac{P_B}{4\pi r_B^2} = \frac{1.50\times10^{-3}\text{ W}}{4\pi(4.47\text{ m})^2} = 5.97\times10^{-6}\text{ W/m}^2$$

The sound level at C due to speaker B alone is therefore

$$\beta_B = 10\cdot\log\left(\frac{I_B}{I_0}\right) = 10\cdot\log\left(\frac{5.97\times10^{-6}\text{ W/m}^2}{1.00\times10^{-12}\text{ W/m}^2}\right) = \boxed{67.8\text{ dB}}$$

(c) If both speakers are sounded together, the total sound intensity at point C is

$$I_{AB} = I_A + I_B = 3.18\times10^{-6}\text{ W/m}^2 + 5.97\times10^{-6}\text{ W/m}^2 = 9.15\times10^{-6}\text{ W/m}^2$$

and the total sound level in decibels is

$$\beta_{AB} = 10\cdot\log\left(\frac{I_{AB}}{I_0}\right) = 10\cdot\log\left(\frac{9.15\times10^{-6}\text{ W/m}^2}{1.00\times10^{-12}\text{ W/m}^2}\right) = \boxed{69.6\text{ dB}}$$

14.65 We assume that the average intensity of the sound is directly proportional to the number of cars passing each minute. If the sound level in decibels is $\beta = 10\cdot\log(I/I_0)$, the intensity of the sound is $I = I_0\cdot10^{\beta/10}$, so the average intensity in the afternoon, when 100 cars per minute are passing, is

$$I_{100} = I_0\cdot10^{80.0/10} = \left(1.00\times10^{-12}\text{ W/m}^2\right)\cdot10^{8.00} = 1.00\times10^{-4}\text{ W/m}^2$$

The expected average intensity at night, when only 5 cars pass per minute, is given by the ratio $I_5/I_{100} = 5/100 = 1/20$, or

$$I_5 = \frac{I_{100}}{20} = \frac{1.00\times10^{-4}\text{ W/m}^2}{20} = 5.00\times10^{-6}\text{ W/m}^2$$

and the expected sound level in decibels is

$$\beta_5 = 10\cdot\log\left(\frac{I_5}{I_0}\right) = 10\cdot\log\left(\frac{5.00\times10^{-6}\text{ W/m}^2}{1.00\times10^{-12}\text{ W/m}^2}\right) = \boxed{67.0\text{ dB}}$$

14.66 The well will act as a pipe closed at one end (the bottom) and open at the other (the top). The resonant frequencies are the *odd integer multiples* of the fundamental frequency, or $f_n = (2n-1)f_1$, where $n = 1, 2, 3, \ldots$. Thus, if f_n and f_{n+1} are two successive resonant frequencies, their difference is

$$f_{n+1} - f_n = [2(n+1)-1]f_1 - (2n-1)f_1 = \left(\cancel{2n} + 2 - \cancel{1} - \cancel{2n} + \cancel{1}\right)f_1 = 2f_1$$

In this case, we have $60.0\text{ Hz} - 52.0\text{ Hz} = 2f_1$, giving the fundamental frequency for the well as $f_1 = 4.00\text{ Hz}$. In the fundamental mode, the well (pipe closed at one end) forms a standing wave pattern with a node at the bottom and the first antinode at the top, making the depth of the well

$$d = \frac{\lambda_1}{4} = \frac{1}{4}\left(\frac{v_{\text{sound}}}{f_1}\right) = \frac{1}{4}\left(\frac{343\text{ m/s}}{4.00\text{ Hz}}\right) = \boxed{21.4\text{ m}}$$

14.67 If r_1 and r_2 are the distances of the two observers from the speaker, the intensities of the sound at their locations are

$$I_1 = \frac{P}{4\pi r_1^2} \qquad \text{and} \qquad I_2 = \frac{P}{4\pi r_2^2}$$

where P is the power output of the speaker. The difference in the decibel levels for the two observers is

$$\beta_1 - \beta_2 = 10\log\left(\frac{I_1}{I_0}\right) - 10\log\left(\frac{I_2}{I_0}\right) = 10\log\left(\frac{I_2}{I_1}\right) = 10\log\left(\frac{r_2^2}{r_1^2}\right) = 10\log\left(\frac{r_2}{r_1}\right)^2 = 20\log\left(\frac{r_2}{r_1}\right)$$

Since $\beta_1 = 80$ dB and $\beta_2 = 60$ dB, we find that $80 - 60 = 20\log(r_2/r_1)$. This yields

$$\log(r_2/r_1) = 1.0 \qquad \text{and} \qquad r_2/r_1 = 10 \qquad \text{or} \qquad r_2 = 10r_1 \qquad \textbf{[1]}$$

We also know that $r_1 + r_2 = 36.0\text{ m}$ **[2]**

Substituting Equation [1] into [2] gives: $11r_1 = 36.0\text{ m}$ or $r_1 = \boxed{36.0\text{ m}/11 \approx 3.3\text{ m}}$

Then, Equation [2] yields $r_2 = 36.0\text{ m} - r_1 = 36.0\text{ m} - 3.3\text{ m} = \boxed{32.7\text{ m}}$

14.68 We take toward the east as the positive direction, so the velocity of the sea water relative to Earth is $\vec{\mathbf{v}}_{\text{WE}} = -10.0\text{ km/h}$. The velocity of the trailing ship, which is the sound source (S), relative the propagation medium (sea water) is then

$$\vec{\mathbf{v}}_{\text{SW}} = \vec{\mathbf{v}}_{\text{SE}} - \vec{\mathbf{v}}_{\text{WE}} = +64.0\text{ km/h} - (-10.0\text{ km/h}) = +74.0\text{ km/h}\left(\frac{0.278\text{ m/s}}{1\text{ km/h}}\right) = 20.6\text{ m/s}$$

The velocity of the leading ship, the observer (O) in this case, relative to the water is

$$\vec{\mathbf{v}}_{\text{OW}} = \vec{\mathbf{v}}_{\text{OE}} - \vec{\mathbf{v}}_{\text{WE}} = +45.0\text{ km/h} - (-10.0\text{ km/h}) = +55.0\text{ km/h}\left(\frac{0.278\text{ m/s}}{1\text{ km/h}}\right) = 15.3\text{ m/s}$$

With the source moving toward the observer, but the observer moving away from the source, the frequency detected by the observer is given by Equation 14.12 as

$$f_0 = f_S\left(\frac{v + v_O}{v - v_S}\right) = f_S\left(\frac{v + (-|\vec{\mathbf{v}}_{\text{OW}}|)}{v - (+|\vec{\mathbf{v}}_{\text{SW}}|)}\right)$$

The speed of sound in sea water is $v = 1\,533\text{ m/s}$ (Table 14.1) and the frequency emitted by the source is $f_S = 1\,200.0$ Hz, so the observed frequency should be

$$f_0 = (1\,200.0\text{ Hz})\left(\frac{1\,533\text{ m/s} - 15.3\text{ m/s}}{1\,533\text{ m/s} - 20.6\text{ m/s}}\right) = \boxed{1\,204\text{ Hz}}$$

14.69 This situation is very similar to the fundamental resonance of an organ pipe that is open at both ends. The wavelength of the standing waves in the crystal is $\lambda = 2t$, where t is the thickness of the crystal, and the frequency is

$$f = \frac{v}{\lambda} = \frac{3.70\times10^3\text{ m/s}}{2(7.05\times10^{-3}\text{ m})} = 2.62\times10^5\text{ Hz} = \boxed{262\text{ kHz}}$$

14.70 The distance from the window ledge to the man's head is

$$\Delta y = d - h = 20.0\ \text{m} - 1.75\ \text{m} = 18.3\ \text{m}$$

The time for a warning to travel this distance is $t_1 = (18.3\ \text{m})/(343\ \text{m/s}) = 0.0534\ \text{s}$, so the total time needed to receive the warning and react is $t_1' = t_1 + 0.300\ \text{s} = 0.353\ \text{s}$.

The elapsed time when the pot, starting from rest, reaches the level of the man's head is

$$t_2 = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-18.3\ \text{m})}{-9.80\ \text{m/s}^2}} = 1.93\ \text{s}$$

Thus, the latest the warning should be sent is at

$$t = t_2 - t_1' = 1.93\ \text{s} - 0.353\ \text{s} = 1.58\ \text{s}$$

into the fall. At this time, the pot has fallen

$$\Delta y = \frac{1}{2}gt^2 = \frac{1}{2}\left(9.80\ \text{m/s}^2\right)\left(1.58\ \text{s}\right)^2 = 12.2\ \text{m}$$

and is $20.0\ \text{m} - 12.2\ \text{m} = \boxed{7.8\ \text{m}}$ above the sidewalk.

14.71 On the weekend, there are one-fourth as many cars passing per minute as on a weekday. Thus, the intensity, I_2, of the sound on the weekend is one-fourth of that, I_1, on a weekday. The difference in the decibel levels is therefore

$$\beta_1 - \beta_2 = 10\log\left(\frac{I_1}{I_o}\right) - 10\log\left(\frac{I_2}{I_o}\right) = 10\log\left(\frac{I_1}{I_2}\right) = 10\log(4) = 6\ \text{dB}$$

so, $\beta_2 = \beta_1 - 6\ \text{dB} = 70\ \text{dB} - 6\ \text{dB} = \boxed{64\ \text{dB}}$

14.72 (a) At $T = 20°\text{C} = 293\ \text{K}$, the speed of sound in air is

$$v = (331\ \text{m/s})\sqrt{1 + \frac{T_C}{273}} = (331\ \text{m/s})\sqrt{1 + \frac{20.0}{273}} = 343\ \text{m/s}$$

The first harmonic or fundamental of the flute (a pipe open at both ends) is given by

$$\lambda_1 = 2L = \frac{v}{f_1} = \frac{343\ \text{m/s}}{261.6\ \text{Hz}} = 1.31\ \text{m}$$

Therefore, the length of the flute is

$$L = \frac{\lambda_1}{2} = \frac{1.31\ \text{m}}{2} = \boxed{0.655\ \text{m}}$$

(b) In the colder room, the length of the flute, and hence its fundamental wavelength, is essentially unchanged (that is, $\lambda_1' = \lambda_1 = 1.31\ \text{m}$). However, the speed of sound, and thus the frequency of the fundamental, will be lowered. At this lower temperature, the frequency must be

$$f_1' = f_1 - f_{\text{beat}} = 261.6\ \text{Hz} - 3.00\ \text{Hz} = 258.6\ \text{Hz}$$

The speed of sound in this room is

$$v' = \lambda_1' f_1' = (1.31\ \text{m})(258.6\ \text{Hz}) = 339\ \text{m/s}$$

From $v = (331\ \text{m/s})\sqrt{1 + T_C/273}$, the temperature in the colder room is given by

$$T = (273°\text{C})\left[\left(\frac{v}{331\ \text{m/s}}\right)^2 - 1\right] = (273°\text{C})\left[\left(\frac{339\ \text{m/s}}{331\ \text{m/s}}\right)^2 - 1\right] = \boxed{13.4°\text{C}}$$

14.73 The maximum speed of the oscillating block and speaker is

$$v_{\max} = A\omega = A\sqrt{\frac{k}{m}} = (0.500\ \text{m})\sqrt{\frac{20.0\ \text{N/m}}{5.00\ \text{kg}}} = 1.00\ \text{m/s}$$

(a) When the speaker moves *away from* the stationary observer, the source velocity is $v_S = -v_{\max}$ and the minimum frequency heard is

$$(f_O)_{\min} = f_S\left(\frac{v}{v+v_{\max}}\right) = (440\ \text{Hz})\left(\frac{343\ \text{m/s}}{343\ \text{m/s}+1.00\ \text{m/s}}\right) = \boxed{439\ \text{Hz}}$$

(b) When the speaker (sound source) moves *toward* the stationary observer, then $v_S = +v_{\max}$ and the maximum frequency heard is

$$(f_O)_{\max} = f_S\left(\frac{v}{v-v_{\max}}\right) = (440\ \text{Hz})\left(\frac{343\ \text{m/s}}{343\ \text{m/s}-1.00\ \text{m/s}}\right) = \boxed{441\ \text{Hz}}$$

14.74 The speed of transverse waves in the wire is

$$v_T = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F\cdot L}{m}} = \sqrt{\frac{(400\ \text{N})(0.750\ \text{m})}{2.25\times10^{-3}\ \text{kg}}} = 365\ \text{m/s}$$

When the wire vibrates in its third harmonic, $\lambda = 2L/3 = 0.500$ m, so the frequency of the vibrating wire and the sound produced by the wire is

$$f_S = \frac{v_T}{\lambda} = \frac{365\ \text{m/s}}{0.500\ \text{m}} = 730\ \text{Hz}$$

Since both the wire and the wall are stationary, the frequency of the wave reflected from the wall matches that of the waves emitted by the wire. Thus, as the student approaches the wall at speed $|v_O|$, he approaches one stationary source and recedes from another stationary source, both emitting frequency $f_S = 730$ Hz. The two frequencies that will be observed are

$$(f_O)_1 = f_S\left(\frac{v+|v_O|}{v}\right) \text{ and } (f_O)_2 = f_S\left(\frac{v-|v_O|}{v}\right)$$

The beat frequency is $$f_{\text{beat}} = (f_O)_1 - (f_O)_2 = f_S\left(\frac{v+|v_O|-(v-|v_O|)}{v}\right) = \frac{2f_S|v_O|}{v}$$

so $$|v_O| = \left(\frac{f_{\text{beat}}}{2f_S}\right)v = \left[\frac{8.30\ \text{Hz}}{2(730\ \text{Hz})}\right](343\ \text{m/s}) = \boxed{1.95\ \text{m/s}}$$

14.75 The speeds of the two types of waves in the rod are

$$v_{\text{long}} = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{Y}{m/V}} = \sqrt{\frac{Y(A\cdot L)}{m}} \text{ and } v_{\text{trans}} = \sqrt{\frac{F}{\mu}} = \sqrt{\frac{F}{m/L}} = \sqrt{\frac{F\cdot L}{m}}$$

Thus, if $v_{\text{long}} = 8v_{\text{trans}}$, we have $\frac{Y(A\cdot L)}{m} = 64\left(\frac{F\cdot L}{m}\right)$, or the required tension is

$$F = \frac{Y\cdot A}{64} = \frac{(6.80\times10^{10}\ \text{N/m}^2)\left[\pi(0.200\times10^{-2}\ \text{m})^2\right]}{64} = \boxed{1.34\times10^4\ \text{N}}$$

14.76 (a) For the fundamental mode of a pipe open at both ends, $L = \lambda_1/2$ or the wavelength of the waves traveling through the air in the pipe is $\lambda_1 = 2L = 2(0.500\text{ m}) = 1.00\text{ m}$.

If the frequency of this fundamental mode is $f_1 = 350$ Hz, the speed of sound waves within the pipe must be

$$v = \lambda_1 f_1 = (1.00\text{ m})(350\text{ Hz}) = 350\text{ m/s}$$

From $v = (331\text{ m/s})\sqrt{1+T_C/273}$, the Celsius temperature of the air in the pipe is

$$T_C = (273°\text{C})\left[\left(\frac{v}{331\text{ m/s}}\right)^2 - 1\right] = (273°\text{C})\left[\left(\frac{350\text{ m/s}}{331\text{ m/s}}\right)^2 - 1\right] = \boxed{32.2°\text{C}}$$

(b) If the temperature rises to $T' = T + 20.0°\text{C} = 52.2°\text{C}$, the speed of sound in the air will be $v' = (331\text{ m/s})\sqrt{1+T_C'/273} = (331\text{ m/s})\sqrt{1+52.2/273}$, and the new length of the pipe will be

$$L' = L_0[1+\alpha(\Delta T)] = (0.500\text{ m})[1+(19\times10^{-6}\ °\text{C}^{-1})(20.0°\text{C})]$$

The new fundamental wavelength is $\lambda_1' = 2L'$, and the new fundamental resonance frequency will be

$$f_1' = \frac{v'}{\lambda_1'} = \frac{(331\text{ m/s})\sqrt{1+52.2/273}}{2(0.500\text{ m})[1+(19\times10^{-6}\ °\text{C}^{-1})(20.0°\text{C})]} = \boxed{3.6\times10^2\text{ Hz}}$$

15

Electric Forces and Electric Fields

QUICK QUIZZES

1. Choice (b). Object A must have a net charge because two neutral objects do not attract each other. Since object A is attracted to positively-charged object B, the net charge on A must be negative.

2. Choice (b). By Newton's third law, the two objects will exert forces having equal magnitudes but opposite directions on each other.

3. Choice (c). The electric field at point P is due to charges *other* than the test charge. Thus, it is unchanged when the test charge is altered. However, the direction of the force this field exerts on the test change is reversed when the sign of the test charge is changed.

4. Choice (a). If a test charge is at the center of the ring, the force exerted on the test charge by charge on any small segment of the ring will be balanced by the force exerted by charge on the diametrically opposite segment of the ring. The net force on the test charge, and hence the electric field at this location, must then be zero.

5. Choices (c) and (d). The electron and the proton have equal magnitude charges of opposite signs. The forces exerted on these particles by the electric field have equal magnitude and opposite directions. The electron experiences an acceleration of greater magnitude than does the proton because the electron's mass is much smaller than that of the proton.

6. Choice (a). The field is greatest at point A because this is where the field lines are closest together. The absence of lines at point C indicates that the electric field there is zero.

7. Choice (c). When a plane area A is in a uniform electric field E, the flux through that area is $\Phi_E = EA\cos\theta$, where θ is the angle the electric field makes with the line normal to the plane of A. If A lies in the xy-plane and E is in the z-direction, then $\theta = 0°$ and $\Phi_E = EA = (5.00\ \text{N/C})(4.00\ \text{m}^2) = 20.0\ \text{N}\cdot\text{m}^2/\text{C}$.

8. Choice (b). If $\theta = 60°$ in Quick Quiz 15.7 above, then $\Phi_E = EA\cos\theta$ which yields $\Phi_E = (5.00\ \text{N/C})(4.00\ \text{m}^2)\cos(60°) = 10.0\ \text{N}\cdot\text{m}^2/\text{C}$.

9. Choice (d). Gauss's law states that the electric flux through any closed surface is equal to the net enclosed charge divided by the permittivity of free space. For the surface shown in Figure 15.28, the net enclosed charge is $Q = -6$ C, which gives $\Phi_E = Q/\epsilon_0 = -(6\ \text{C})/\epsilon_0$.

10. Choices (b) and (d). Since the net flux through the surface is zero, Gauss's law says that the net change enclosed by that surface must be zero as stated in (b). Statement (d) must be true because there would be a net flux through the surface if more lines entered the surface than left it (or vise-versa).

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. To balance the weight of the ball, the magnitude of the upward electric force must equal the magnitude of the downward gravitation force, or $qE = mg$, which gives

$$E = \frac{mg}{q} = \frac{\left(5.0\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}{4.0\times10^{-6}\ \text{C}} = 1.2\times10^{4}\ \text{N/C}$$

and the correct choice is (b).

2. The magnitude of the electric field at distance r from a point charge q is $E = k_e q/r^2$, so

$$E = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)}{\left(5.29\times10^{-11}\ \text{m}\right)^2} = 5.14\times10^{11}\ \text{N/C} \sim 10^{12}\ \text{N/C}$$

making (e) the best choice for this question.

3. The magnitude of the electric force between two protons separated by distance r is $F = k_e e^2/r^2$, so the distance of separation must be

$$r = \sqrt{\frac{k_e e^2}{F}} = \sqrt{\frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{2.3\times10^{-26}\ \text{N}}} = 0.10\ \text{m}$$

and (a) is the correct choice.

4. The ball is made of a metal, so free charges within the ball will very quickly rearrange themselves to produce electrostatic equilibrium at all points within the ball. As soon as electrostatic equilibrium exists inside the ball, the electric field is zero at all points within the ball. Thus, the correct choice is (c).

5. Choosing the surface of the box as the closed surface of interest and applying Gauss's law, the net electric flux through the surface of the box is found to be

$$\Phi_E = \frac{Q_{\text{inside}}}{\epsilon_0} = \frac{(3.0-2.0-7.0+1.0)\times10^{-9}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = -5.6\times10^{2}\ \text{N}\cdot\text{m}^2/\text{C}$$

meaning that (b) is the correct choice.

6. From Newton's second law, the acceleration of the electron will be

$$a_x = \frac{F_x}{m} = \frac{qE_x}{m} = \frac{\left(-1.60\times10^{-19}\ \text{C}\right)\left(1.00\times10^{3}\ \text{N/C}\right)}{9.11\times10^{-31}\ \text{kg}} = -1.76\times10^{14}\ \text{m/s}^2$$

The kinematics equation $v_x^2 = v_{0x}^2 + 2a_x(\Delta x)$, with $v_x = 0$, gives the stopping distance as

$$\Delta x = \frac{-v_{0x}^2}{2a_x} = \frac{-\left(3.00\times10^{6}\ \text{m/s}\right)^2}{2\left(-1.76\times10^{14}\ \text{m/s}^2\right)} = 2.56\times10^{-2}\ \text{m} = 2.56\ \text{cm}$$

so (a) is the correct response for this question.

7. The displacement from the –4.00 nC charge at point $(0,\ 1.00)$ m to the point $(4.00,\ -2.00)$ m has components $r_x = (x_f - x_i) = +4.00$ m and $r_y = (y_f - y_i) = -3.00$ m, so the magnitude of this displacement is $r = \sqrt{r_x^2 + r_y^2} = 5.00$ m and its direction is $\theta = \tan^{-1}(r_y/r_x) = -36.9°$. The x-component of the electric field at point $(4.00,\ -2.00)$ m is then

$$E_x = E\cos\theta = \frac{k_e q}{r^2}\cos\theta = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(-4.00\times10^{-9}\ \text{C})}{(5.00\ \text{m})^2}\cos(-36.9°) = -1.15\ \text{N/C}$$

and the correct response is (d).

8. The magnitude of the electric force between charges Q_{1i} and Q_{2i}, separated by distance r_i, is $F_i = k_e Q_{1i} Q_{2i}/r_i^2$. If changes are made so $Q_{1f} = Q_{1i}$, $Q_{2f} = Q_{2i}/3$, and $r_f = 2r_i$, the magnitude of the new force will be

$$F_f = \frac{k_e Q_{1f} Q_{2f}}{r_f^2} = \frac{k_e Q_{1i}(Q_{2i}/3)}{(2r_i)^2} = \frac{1}{3(2)^2}\left(\frac{k_e Q_{1i} Q_{2i}}{r_i^2}\right) = \frac{1}{12}F_i$$

so choice (a) is the correct answer for this question.

9. Each of the situations described in choices (a) through (d) displays a high degree of symmetry, and as such, readily lends itself to the use of Gauss's law to determine the electric fields generated. Thus, the best answer for this question is choice (e), stating that Gauss's law can be readily applied to find the electric field in all of these contexts.

10. When a charged insulator is brought near a metallic object, free charges within the metal move around, causing the metallic object to become polarized. Within the metallic object, the center of charge for the type of charge opposite to that on the insulator will be located closer to the charged insulator than will the center of charge for the same type as that on the insulator. This causes the attractive force between the charged insulator and the opposite type of charge in the metal to exceed the magnitude of the repulsive force between the insulator and the same type of charge in the metal. Thus, the net electric force between the insulator and the metallic object is one of attraction, and choice (b) is the correct answer.

11. The outer regions of the atoms in your body and the atoms making up the ground both contain negatively charged electrons. When your body is in close proximity to the ground, these negatively charged regions exert repulsive forces on each other. Since the atoms in the solid ground are rigidly locked in position and cannot move away from your body, this repulsive force prevents your body from penetrating the ground. The best response for this question is choice (e).

12. The positive charge $+2Q$ makes a contribution to the electric field at the upper right corner that is directed away from this charge in the direction of the arrow labeled (a). The magnitude of this contribution is $E_+ = k_e(2Q)/2s^2$, where s is the length of a side of the square. Each of the negative charges makes a contribution of magnitude $E_{-Q} = k_e Q/s^2$ directed back toward that charge. The vector sum of these two contributions due to negative charges has magnitude

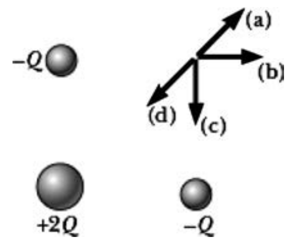


FIGURE MCQ15.12

$$E_- = 2E_{-Q}\cos 45° = \sqrt{2}k_e Q/s^2$$

and is directed along the diagonal of the square in the direction of the arrow labeled (d). Since $E_- > E_+$, the resultant electric field at the upper right corner of the square is in the direction of arrow (d) and has magnitude $E = E_- - E_+ = (\sqrt{2} - 1)k_e Q/s^2$. The correct answer to the question is choice (d).

13. If the positive charge $+2Q$ at the lower left corner of the square in the above figure were removed, the field contribution E_+ discussed above would be eliminated. This would leave only $E_- = \sqrt{2}k_e Q/s^2$ as the resultant field at the upper right corner. This has a larger magnitude than the resultant field E found above, making choice (a) the correct answer.

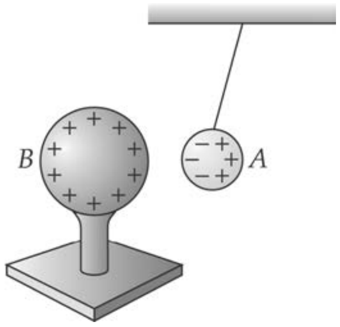
14. Metal objects normally contain equal amounts of positive and negative charge and are electrically neutral. The positive charges in both metals and nonmetals are bound up in the nuclei of the atoms and cannot move about or be easily removed. However, in metals, some of the negative charges (the outer or valence electrons in the atoms) are quite loosely bound, can move about rather freely, and are easily removed from the metal. When a metal object is given a positive charge, this is accomplished by removing loosely bound electrons from the metal rather than by adding positive charge to it. Taking away the electrons to leave a net positive charge behind very slightly decreases the mass of the coin. Thus, choice (d) is the best choice for this question.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. Electrons are more mobile than protons and are more easily freed from atoms than the protons which are tightly bound within the nuclei of the atoms.

4. Conducting shoes are worn to avoid the build up of a static charge on them as the wearer walks. Rubber-soled shoes acquire a charge by friction with the floor and could discharge with a spark, possibly causing an explosive burning situation, where the burning is enhanced by the oxygen.

6. No. Object *A* might have a charge opposite in sign to that of *B*, but it also might be neutral. In this latter case, object *B* causes object *A* to be polarized, pulling charge of the sign opposite the change on *B* toward the near face of *A* and pushing an equal amount of charge of the same sign as that on *B* toward the far face. Then, due to difference in distances, the force of attraction exerted by *B* on the induced charge of opposite sign is slightly larger than the repulsive force exerted by *B* on the induced charge of like sign. Therefore, the net force on *A* is toward *B*.



8. (a) Yes. The positive charges create electric fields that extend in all directions from those charges. The total field at point *A* is the vector sum of the individual fields produced by the charges at that point.

(b) No, because there are no field lines emanating from or converging on point *A*.

(c) No. There must be a charged object present to experience a force.

10. Electric field lines start on positive charges and end on negative charges. Thus, if the fair-weather field is directed into the ground, the ground must have a negative charge.

12. To some extent, a television antenna will act as a lightning rod on the house. If the antenna is connected to the Earth by a heavy wire, a lightning discharge striking the house may pass through the metal support rod and be safely carried to the Earth by the ground wire.

14. (a) If the charge is tripled, the flux through the surface is also tripled because the net flux is proportional to the charge inside the surface.

(b) The flux remains constant when the volume changes because the surface surrounds the same amount of charge, regardless of its volume.

(c) The flux does not change when the shape of the closed surface changes.

(d) The flux through the closed surface remains unchanged as the charge inside the surface is moved to another location inside that surface.

(e) The flux is zero because the charge inside the surface is zero. All of these conclusions are arrived at through an understanding of Gauss's law.

16. All of the constituents of air are nonpolar except for water. The polar water molecules in the air quite readily "steal" charge from a charged object, as any physics teacher trying to perform electrostatics demonstrations in the summer well knows. As a result—it is difficult to accumulate large amounts of excess charge on an object in a humid climate. During a North American winter, the cold, dry air allows accumulation of significant excess charge, giving the possibility for shocks caused by static electricity sparks.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. 1.57 N directed to the left

4. **(a)** 0.115 N **(b)** 1.25 cm

6. 2.25×10^{-9} N/m

8. $4.33k_e\,q^2/a^2$ to the right and 45° above the horizontal

10. $F_{6\,\mu\text{C}} = 46.7$ N to the left; $F_{1.5\,\mu\text{C}} = 157$ N to the right; $F_{-2\,\mu\text{C}} = 111$ N to the left

12. 5.15×10^{3} N/m

14. 16.7 μC

16. **(a)** 0 **(b)** 30.0 N **(c)** 21.6 N

(d) 17.3 N **(e)** –13.0 N **(f)** 17.3 N

(g) 17.0 N **(h)** $\vec{\mathbf{F}}_R = 24.3$ N at 44.5° above the $+x$-axis

18. **(a)** 2.00×10^{7} N/C to the right **(b)** 40.0 N to the left

20. **(a)** 5.27×10^{13} m/s^2 **(b)** 5.27×10^{5} m/s

22. **(a)** See Solution. **(b)** 1.4×10^{3} N/C **(c)** 7.5×10^{-2} N

24. **(a)** 2.19×10^{5} N/C at 85.2° below the $+x$-axis

(b) The electric field would be unchanged, but the force would double in magnitude.

26. **(a)** 1.80×10^4 N/C to the right **(b)** 9.00×10^{-5} N to the left

28. 1.88×10^3 N/C at 4.40° below the $+x$-axis

30. **(a)** $q_1/q_2 = -1/3$ **(b)** $q_1 < 0,\ q_2 > 0$

32. **(a), (b)**, and **(c)** See Solution for Sketches.

34. **(a)** See Solution. **(b)** $\vec{\mathbf{E}} = 0$ at the center of the triangle.

(c) $1.73 k_e q/a^2$ **(b)** positive y-direction

36. **(a)** zero **(b)** 1.8×10^6 N/m **(c)** 1.1×10^5 N/m

38. $r \sim 10^{-6}$ m

40. $1.38 \times 10^3\ \mathrm{N \cdot m^2/C}$

42. **(a)** 3.38 nC **(b)** The sphere contains a net positive charge in a spherically symmetric distribution.

44. **(a)** $6.55 \times 10^5\ \mathrm{N \cdot m^2/C}$ **(b)** $1.64 \times 10^5\ \mathrm{N \cdot m^2/C}$

46. **(a)** $1.92 \times 10^7\ \mathrm{N \cdot m^2/C}$ **(b)** $3.20 \times 10^6\ \mathrm{N \cdot m^2/C}$

(c) The answer to (b) would change because of the loss of symmetry; however, the answer to (a) would be unchanged since Q_{inside} is unaltered.

48. The electric field is everywhere parallel to the z-axis with the following magnitudes:

(a) $E_z = +\sigma/2\epsilon_0$ **(b)** $E_z = +3\sigma/2\epsilon_0$ **(c)** $E_z = -\sigma/2\epsilon_0$

50. **(a)** 2.94×10^5 N/C

(b) The magnitude of the field is independent of distance from the sheet, provided that distance is small compared to the dimensions of the sheet.

52. $+5.25\ \mu$C

54. $E = K/ed$ in the direction of the electron's motion.

56. **(a)** 0.307 s **(b)** Yes, neglecting gravity causes a 2.28% error.

58. **(a)** For $0 < x < 1.00$ m, the field contributions made by the two charges are in the same direction and cannot add to zero.

(b) Any point for $x < 0$ is nearer the larger charge, so the field contributions made by the two charges cannot have equal magnitudes and add to zero.

(c) Points for $x > 1.00$ m are nearer the smaller charge and the field contributions by the two charges are in opposite directions. Thus, it is possible for them to add to zero.

(d) $E = 0$ at $x = +9.47$ m

60. 0.951 m

62. 1.98 μC

64. **(a)** 37.0° or 53.0° **(b)** 1.67×10^{-7} s for $\theta=37.0°$; 2.21×10^{-7} s for $\theta=53.0°$

PROBLEM SOLUTIONS

15.1 (a) From Coulomb's law, $F=k_e\dfrac{Q_1Q_2}{r^2}$, we have

$$F=\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(7.50\times10^{-9}\ \text{C}\right)\left(4.20\times10^{-9}\ \text{C}\right)}{(1.80\ \text{m})^2}=\boxed{8.74\times10^{-8}\ \text{N}}$$

(b) Since these are like charges (both positive), the force is $\boxed{\text{repulsive}}$.

15.2 Particle *A* exerts a force toward the right on particle *B*. By Newton's third law, particle *B* will then exert a of equal magnitude force toward the left on particle *A*. The ratio of the final magnitude of the force to the original magnitude of the force is

$$\frac{F_f}{F_i}=\frac{k_eq_1q_2/r_f^2}{k_eq_1q_2/r_i^2}=\left(\frac{r_i}{r_f}\right)^2 \quad\text{so}\quad F_f=F_i\left(\frac{r_i}{r_f}\right)^2=(2.62\ \text{N})\left(\frac{13.7\ \text{mm}}{17.7\ \text{mm}}\right)^2=1.57\ \text{N}$$

The final vector force that *B* exerts on *A* is $\boxed{\text{1.57 N directed to the left}}$.

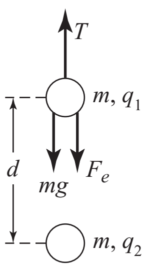
15.3 (a) When the balls are an equilibrium distance apart, the tension in the string equals the magnitude of the repulsive electric force between the balls. Thus,

$$F=\frac{k_eq(2q)}{r^2}=2.50\ \text{N}\quad\Rightarrow\quad q^2=\frac{(2.50\ \text{N})r^2}{2k_e}$$

or $$q=\sqrt{\frac{(2.50\ \text{N})(2.00\ \text{m})^2}{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}}=2.36\times10^{-5}\ \text{C}=\boxed{23.6\ \mu\text{C}}$$

(b) The charges induce opposite charges in the bulkheads, but the induced charge in the bulkhead near ball *B* is greater, due to *B*'s greater charge. Therefore, the system $\boxed{\text{moves slowly towards the bulkhead closer to ball } B}$.

15.4 (a) The gravitational force exerted on the upper sphere by the lower one is negligible in comparison to the gravitational force exerted by the Earth and the downward electrical force exerted by the lower sphere. Therefore,



$$\Sigma F_y=0\ \Rightarrow\ T-mg-F_e=0$$

or $$T=mg+\frac{k_e|q_1||q_2|}{d^2}$$

$$T=\left(7.50\times10^{-3}\ \text{kg}\right)\left(9.80\ \frac{\text{m}}{\text{s}^2}\right)+\frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(32.0\times10^{-9}\ \text{C}\right)\left(58.0\times10^{-9}\ \text{C}\right)}{\left(2.00\times10^{-2}\ \text{m}\right)^2}$$

giving $T=\boxed{0.115\ \text{N}}$

continued on next page

(b) $\Sigma F_y = 0 \rightarrow F_e = \dfrac{k_e|q_1||q_2|}{d^2} = T - mg$, and $d = \sqrt{\dfrac{k_e|q_1||q_2|}{T - mg}}$

Thus, if $T = 0.180$ N,

$$d = \sqrt{\frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(32.0\times10^{-9}\ \text{C})(58.0\times10^{-9}\ \text{C})}{0.180\ \text{N} - (7.50\times10^{-3}\ \text{kg})\left(9.80\ \dfrac{\text{m}}{\text{s}^2}\right)}} = 1.25\times10^{-2}\ \text{m} = \boxed{1.25\ \text{cm}}$$

15.5 (a) $$F = \frac{k_e(2e)^2}{r^2} = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\left[4(1.60\times10^{-19}\ \text{C})^2\right]}{(5.00\times10^{-15}\ \text{m})^2} = \boxed{36.8\ \text{N}}$$

(b) The mass of an alpha particle is $m = 4.002\ 6$ u, where 1 u $= 1.66\times10^{-27}$ kg is the unified mass unit. The acceleration of either alpha particle is then

$$a = \frac{F}{m} = \frac{36.8\ \text{N}}{4.002\,6(1.66\times10^{-27}\ \text{kg})} = \boxed{5.54\times10^{27}\ \text{m/s}^2}$$

15.6 The attractive force between the charged ends tends to compress the molecule. Its magnitude is

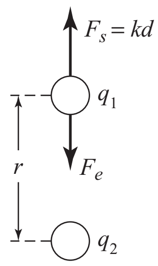
$$F = \frac{k_e(1e)^2}{r^2} = \left(8.99\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{(1.60\times10^{-19}\ \text{C})^2}{(2.17\times10^{-6}\ \text{m})^2} = 4.89\times10^{-17}\ \text{N}$$

The compression of the "spring" is

$$x = (0.010\ 0)r = (0.010\ 0)(2.17\times10^{-6}\ \text{m}) = 2.17\times10^{-8}\ \text{m},$$

so the spring constant is $k = \dfrac{F}{x} = \dfrac{4.89\times10^{-17}\ \text{N}}{2.17\times10^{-8}\ \text{m}} = \boxed{2.25\times10^{-9}\ \text{N/m}}$.

15.7 In the new equilibrium position, $\Sigma F_y = F_s - F_e = kd - F_e = 0$.

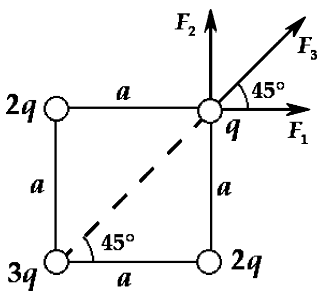


Thus, $$k = \frac{|F_s|}{d} = \frac{k_e|q_1q_2|/r^2}{d} = \frac{k_e|q_1q_2|}{d\cdot r^2}$$

or $$k = \frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)\left|(0.800\times10^{-6}\ \text{C})(-0.600\times10^{-6}\ \text{C})\right|}{(3.50\times10^{-2}\ \text{m})(5.00\times10^{-2}\ \text{m})^2}$$

$$k = \boxed{49.3\ \text{N/m}}$$

15.8 See the sketch at the right. The magnitudes of the forces are



$$F_1 = F_2 = k_e\frac{q(2q)}{a^2} = 2k_e\frac{q^2}{a^2}\ \text{ and }\ F_3 = k_e\frac{q(3q)}{(a\sqrt{2})^2} = 1.50k_e\frac{q^2}{a^2}$$

The components of the resultant force on charge q are

$$F_x = F_1 + F_3\cos45° = (2 + 1.50\cos45°)k_e\frac{q^2}{a^2} = 3.06k_e\frac{q^2}{a^2}$$

continued on next page

and $F_y = F_1 + F_3 \sin 45° = (2 + 1.50 \sin 45°) k_e \dfrac{q^2}{a^2} = 3.06 k_e \dfrac{q^2}{a^2}$

The magnitude of the resultant force is $F_e = \sqrt{F_x^2 + F_y^2} = \sqrt{2}\left(3.06 k_e \dfrac{q^2}{a^2}\right) = \boxed{4.33 k_e \dfrac{q^2}{a^2}}$

and it is directed at $\theta = \tan^{-1}\left(\dfrac{F_y}{F_x}\right) = \tan^{-1}(1.00) = \boxed{45° \text{ above the horizontal}}$.

15.9 (a) The spherically symmetric charge distributions behave as if all charge was located at the centers of the spheres. Therefore, the magnitude of the $\boxed{\text{attractive}}$ force is

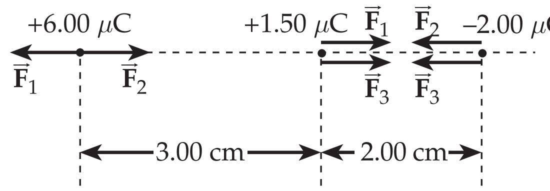
$$F = \frac{k_e q_1 |q_2|}{r^2} = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2\right) \frac{\left(12 \times 10^{-9} \text{ C}\right)\left(18 \times 10^{-9} \text{ C}\right)}{(0.30 \text{ m})^2} = \boxed{2.2 \times 10^{-5} \text{ N}}$$

(b) When the spheres are connected by a conducting wire, the net charge $q_{\text{net}} = q_1 + q_2 = -6.0 \times 10^{-9}$ C will divide equally between the two identical spheres. Thus, the force is now

$$F = \frac{k_e \left(q_{\text{net}}/2\right)^2}{r^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{\left(-6.0 \times 10^{-9} \text{ C}\right)^2}{4(0.30 \text{ m})^2}$$

or $F = \boxed{9.0 \times 10^{-7} \text{ N (repulsion)}}$

15.10 The forces are as shown in the sketch below.



$$F_1 = \frac{k_e q_1 q_2}{r_{12}^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{\left(6.00 \times 10^{-6} \text{ C}\right)\left(1.50 \times 10^{-6} \text{ C}\right)}{\left(3.00 \times 10^{-2} \text{ m}\right)^2} = 89.9 \text{ N}$$

$$F_2 = \frac{k_e q_1 |q_3|}{r_{13}^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{\left(6.00 \times 10^{-6} \text{ C}\right)\left(2.00 \times 10^{-6} \text{ C}\right)}{\left(5.00 \times 10^{-2} \text{ m}\right)^2} = 43.2 \text{ N}$$

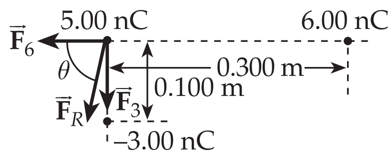
$$F_3 = \frac{k_e q_2 |q_3|}{r_{23}^2} = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right) \frac{\left(1.50 \times 10^{-6} \text{ C}\right)\left(2.00 \times 10^{-6} \text{ C}\right)}{\left(2.00 \times 10^{-2} \text{ m}\right)^2} = 67.4 \text{ N}$$

The net force on the $6\,\mu$C charge is $F_{6\,\mu\text{C}} = F_1 - F_2 = \boxed{46.7 \text{ N to the left}}$.

The net force on the $1.5\,\mu$C charge is $F_{1.5\,\mu\text{C}} = F_1 + F_3 = \boxed{157 \text{ N to the right}}$.

The net force on the $-2\,\mu$C charge is $F_{-2\,\mu\text{C}} = F_2 + F_3 = \boxed{111 \text{ N to the left}}$.

15.11 In the sketch at the right, F_R is the resultant of the forces F_6 and F_3 that are exerted on the charge at the origin by the 6.00 nC and the –3.00 nC charges, respectively.



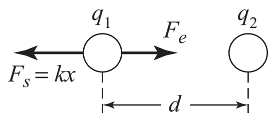
$$F_6 = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(6.00\times10^{-9}\ \text{C}\right)\left(5.00\times10^{-9}\ \text{C}\right)}{(0.300\ \text{m})^2} = 3.00\times10^{-6}\ \text{N}$$

$$F_3 = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\frac{\left(3.00\times10^{-9}\ \text{C}\right)\left(5.00\times10^{-9}\ \text{C}\right)}{(0.100\ \text{m})^2} = 1.35\times10^{-5}\ \text{N}$$

The resultant is $F_R = \sqrt{(F_6)^2+(F_3)^2} = 1.38\times10^{-5}$ N at $\theta = \tan^{-1}\left(\frac{F_3}{F_6}\right) = 77.5°$

or $\vec{\mathbf{F}}_R = \boxed{1.38\times10^{-5}\ \text{N at } 77.5°\text{ below the } -x\text{-axis}}$

15.12 At equilibrium, $\Sigma F_x = F_e - F_s = 0$ or $F_s = F_e$



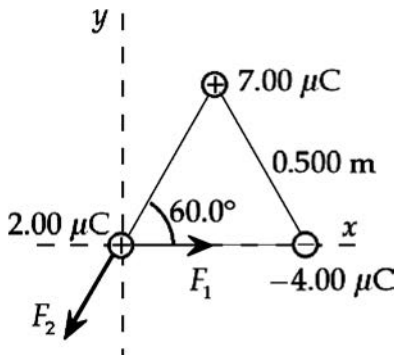
Thus, $kx = \dfrac{k_e|q_1q_2|}{d^2}$

and the force constant of the spring is

$$k = \frac{k_e|q_1q_2|}{xd^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.70\times10^{-6}\ \text{C}\right)\left(8.60\times10^{-6}\ \text{C}\right)}{\left(5.00\times10^{-3}\ \text{m}\right)\left(9.00\times10^{-2}\ \text{m}\right)^2}$$

$$k = \boxed{5.15\times10^3\ \text{N/m}}$$

15.13 Please see the sketch at the right.



$$F_1 = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.00\times10^{-6}\ \text{C}\right)\left(4.00\times10^{-6}\ \text{C}\right)}{(0.500\ \text{m})^2}$$

or $F_1 = 0.288$ N

$$F_2 = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.00\times10^{-6}\ \text{C}\right)\left(7.00\times10^{-6}\ \text{C}\right)}{(0.500\ \text{m})^2}$$

or $F_2 = 0.503$ N

The components of the resultant force acting on the 2.00 μC charge are:

$$F_x = F_1 - F_2\cos60.0° = 0.288\ \text{N} - (0.503\ \text{N})\cos60.0° = 3.65\times10^{-2}\ \text{N}$$

and $F_y = -F_2\sin60.0° = -(0.503\ \text{N})\sin60.0° = -0.436$ N

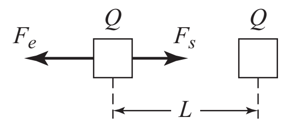
The magnitude and direction of this resultant force are

continued on next page

$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(0.036\,5\text{ N})^2 + (0.436\text{ N})^2} = \boxed{0.438\text{ N}}$$

at $\theta = \tan^{-1}\left(\dfrac{F_y}{F_x}\right) = \tan^{-1}\left(\dfrac{-0.436\text{ N}}{0.036\,5\text{ N}}\right) = -85.2°$ or $\boxed{85.2° \text{ below the } +x\text{-axis}}$

15.14 At equilibrium, $F_e = F_s$ or



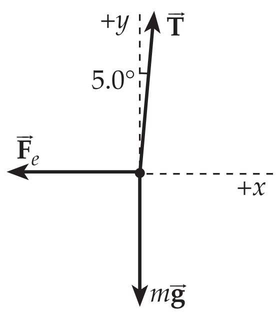
$$\frac{k_e Q^2}{L^2} = kx = k(L - L_i)$$

Thus,

$$Q = \sqrt{\frac{k(L - L_i)L^2}{k_e}} = \sqrt{\frac{(100\text{ N/m})(0.500\text{ m} - 0.400\text{ m})(0.500\text{ m})^2}{8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2}}$$

$$Q = 1.67\times 10^{-5}\text{ C} = \boxed{16.7\ \mu\text{C}}$$

15.15 Consider the free-body diagram of one of the spheres given at the right. Here, T is the tension in the string and F_e is the repulsive electrical force exerted by the other sphere.



$$\Sigma F_y = 0 \Rightarrow T\cos 5.0° = mg, \quad \text{or} \quad T = \frac{mg}{\cos 5.0°}$$

$$\Sigma F_x = 0 \Rightarrow F_e = T\sin 5.0° = mg\tan 5.0°$$

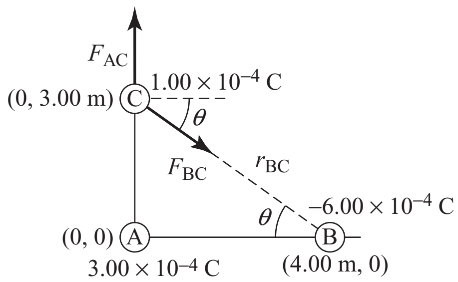
At equilibrium, the distance separating the two spheres is $r = 2L\sin 5.0°$.

Thus, $F_e = mg\tan 5.0°$ becomes $\dfrac{k_e q^2}{(2L\sin 5.0°)^2} = mg\tan 5.0°$ and yields

$$q = (2L\sin 5.0°)\sqrt{\frac{mg\tan 5.0°}{k_e}}$$

$$= [2(0.300\text{ m})\sin 5.0°]\sqrt{\frac{(0.20\times 10^{-3}\text{ kg})(9.80\text{ m/s}^2)\tan 5.0°}{8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2}} = \boxed{7.2\text{ nC}}$$

15.16 In the sketch at the right,



$$r_{BC} = \sqrt{(4.00\text{ m})^2 + (3.00\text{ m})^2} = 5.00\text{ m}$$

and $\theta = \tan^{-1}\left(\dfrac{3.00\text{ m}}{4.00\text{ m}}\right) = 36.9°$

(a) $(F_{AC})_x = \boxed{0}$

(b) $(F_{AC})_y = k_e\dfrac{|q_A||q_C|}{r_{AC}^2} = (8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)\dfrac{(3.00\times 10^{-4}\text{ C})(1.00\times 10^{-4}\text{ C})}{(3.00\text{ m})^2} = \boxed{30.0\text{ N}}$

continued on next page

(c) $\left|F_{BC}\right| = k_e \dfrac{\left|q_B\right|\left|q_C\right|}{r_{BC}^2} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\dfrac{\left(6.00\times10^{-4}\ \text{C}\right)\left(1.00\times10^{-4}\ \text{C}\right)}{(5.00\ \text{m})^2} = \boxed{21.6\ \text{N}}$

(d) $\left(F_{BC}\right)_x = \left|F_{BC}\right|\cos\theta = (21.6\ \text{N})\cos(36.9°) = \boxed{17.3\ \text{N}}$

(e) $\left(F_{BC}\right)_y = -\left|F_{BC}\right|\sin\theta = -(21.6\ \text{N})\sin(36.9°) = \boxed{-13.0\ \text{N}}$

(f) $\left(F_R\right)_x = \left(F_{AC}\right)_x + \left(F_{BC}\right)_x = 0 + 17.3\ \text{N} = \boxed{17.3\ \text{N}}$

(g) $\left(F_R\right)_y = \left(F_{AC}\right)_y + \left(F_{BC}\right)_y = 30.0 - 13.0\ \text{N} = \boxed{17.0\ \text{N}}$

(h) $F_R = \sqrt{\left(F_R\right)_x^2 + \left(F_R\right)_y^2} = \sqrt{(17.3\ \text{N})^2 + (17.0\ \text{N})^2} = \boxed{24.3\ \text{N}}$

and $\varphi = \tan^{-1}\left[\dfrac{\left(F_R\right)_y}{\left(F_R\right)_x}\right] = \tan^{-1}\left(\dfrac{17.0\ \text{N}}{17.3\ \text{N}}\right) = \boxed{44.5°}$

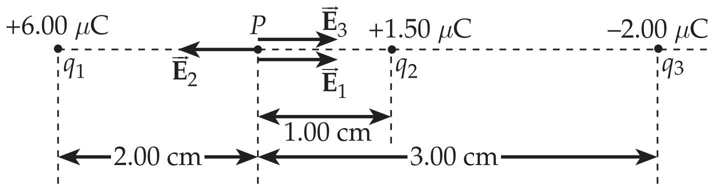
or $\boxed{\vec{\mathbf{F}}_R = 24.3\ \text{N}\ \ \text{at}\ \ 44.5°\ \text{above the}\ +x\text{-axis}}$

15.17 In order to suspend the object in the electric field, the electric force exerted on the object by the field must be directed upward and have a magnitude equal to the weight of the object. Thus, $F_e = qE = mg$, and the magnitude of the electric field must be

$$E = \frac{mg}{|q|} = \frac{\left(3.80\ \cancel{\text{g}}\right)\left(9.80\ \text{m/s}^2\right)}{18\times10^{-6}\ \text{C}}\left(\frac{1\ \text{kg}}{10^3\ \cancel{\text{g}}}\right) = \boxed{2.07\times10^3\ \text{N/C}}$$

The electric force on a negatively charged object is in the direction opposite to that of the electric field. Since the electric force must be directed upward, the electric field must be $\boxed{\text{directed downward.}}$

15.18 (a) Taking to the right as positive, the resultant electric field at point P is given by



$$E_R = E_1 + E_3 - E_2$$

$$= \frac{k_e\left|q_1\right|}{r_1^2} + \frac{k_e\left|q_3\right|}{r_3^2} - \frac{k_e\left|q_2\right|}{r_2^2}$$

$$= \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[\frac{6.00\times10^{-6}\ \text{C}}{(0.020\,0\ \text{m})^2} + \frac{2.00\times10^{-6}\ \text{C}}{(0.030\,0\ \text{m})^2} - \frac{1.50\times10^{-6}\ \text{C}}{(0.010\,0\ \text{m})^2}\right]$$

This gives $E_R = +2.00\times10^7\ \text{N/C}$

or $\vec{\mathbf{E}}_R = \boxed{2.00\times10^7\ \text{N/C to the right}}$

(b) $\vec{\mathbf{F}} = q\vec{\mathbf{E}}_R = \left(-2.00\times10^{-6}\ \text{C}\right)\left(2.00\times10^7\ \text{N/C}\right) = -40.0\ \text{N}$

or $\vec{\mathbf{F}} = \boxed{40.0\ \text{N to the left}}$

15.19 The force on a negative charge is opposite to the direction of the electric field and has magnitude $F = |q|E$. Thus,

$$F = \left|-6.00\times10^{-6}\ \text{C}\right|\left(5.25\times10^{5}\ \text{N/C}\right) = 3.15\ \text{N}$$

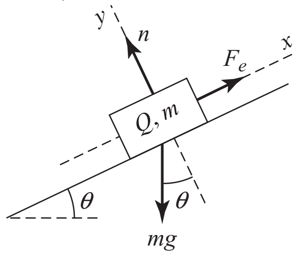
and $\vec{\mathbf{F}} = \boxed{3.15\ \text{N due north}}$

15.20 (a) The magnitude of the force on the electron is $F = |q|E = eE$, and the acceleration is

$$a = \frac{F}{m_e} = \frac{eE}{m_e} = \frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(300\ \text{N/C}\right)}{9.11\times10^{-31}\ \text{kg}} = \boxed{5.27\times10^{13}\ \text{m/s}^2}$$

(b) $v = v_0 + at = 0 + \left(5.27\times10^{13}\ \text{m/s}^2\right)\left(1.00\times10^{-8}\ \text{s}\right) = \boxed{5.27\times10^{5}\ \text{m/s}}$

15.21 (a) The electrical force must be directed up the incline and have a magnitude equal to the tangential component of the gravitational force.



$$\Sigma F_x = 0 \;\Rightarrow\; \text{F}_e - mg\sin\theta = 0$$

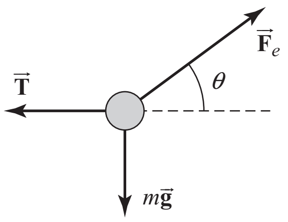
or $F_e = |Q|E = mg\sin\theta$ and $E = \boxed{mg\sin\theta/|Q|}$

(b) $$E = \frac{mg\sin\theta}{|Q|} = \frac{\left(5.40\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\sin 25.0^\circ}{7.00\times10^{-6}\ \text{C}} = 3.19\times10^{3}\ \text{N/C}$$

Since F_e must be directed up the incline and the electrical force on a negative charge is directed opposite to the field, it is necessary to have the electric field directed down the incline.

Thus, $\vec{\mathbf{E}} = \boxed{3.19\times10^{3}\ \text{N/C down the incline}}$

15.22 (a)



(b) $\Sigma F_y = 0 \quad\Rightarrow\quad F_e\sin\theta - mg = 0 \quad$ or $\quad F_e = \dfrac{mg}{\sin\theta}$

Since $F_e = qE$, this gives

$$E = \frac{mg}{q\sin\theta} = \frac{\left(5.8\times10^{-3}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)}{\left(68\times10^{-6}\ \text{C}\right)\sin 37^\circ} = \boxed{1.4\times10^{3}\ \text{N/C}}$$

(c) $\Sigma F_x = 0 \quad\Rightarrow\quad F_e\cos\theta - T = 0 \quad$ or $\quad T = F_e\cos\theta = \left(\dfrac{mg}{\sin\theta}\right)\cos\theta = \dfrac{mg}{\tan\theta}$

and $$T = \frac{\left(5.8\times10^{-3}\ \text{kg}\right)\left(9.8\ \text{m/s}^2\right)}{\tan 37^\circ} = \boxed{7.5\times10^{-2}\ \text{N}}$$

15.23 (a) $a = \dfrac{F}{m} = \dfrac{qE}{m_p} = \dfrac{\left(1.60\times10^{-19}\text{ C}\right)\left(640\text{ N/C}\right)}{1.673\times10^{-27}\text{ kg}} = \boxed{6.12\times10^{10}\text{ m/s}^2}$

(b) $t = \dfrac{\Delta v}{a} = \dfrac{1.20\times10^{6}\text{ m/s}}{6.12\times10^{10}\text{ m/s}^2} = 1.96\times10^{-5}\text{ s} = \boxed{19.6\ \mu\text{s}}$

(c) $\Delta x = \dfrac{v_f^2 - v_0^2}{2a} = \dfrac{\left(1.20\times10^{6}\text{ m/s}\right)^2 - 0}{2\left(6.12\times10^{10}\text{ m/s}^2\right)} = \boxed{11.8\text{ m}}$

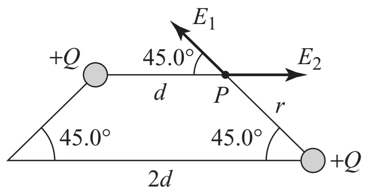
(d) $KE_f = \dfrac{1}{2}m_p v_f^2 = \dfrac{1}{2}\left(1.673\times10^{-27}\text{ kg}\right)\left(1.20\times10^{6}\text{ m/s}\right)^2 = \boxed{1.20\times10^{-15}\text{ J}}$

15.24 (a) Please refer to the solution of Problem 15.13 earlier in this chapter. There it is shown that the resultant electric force experienced by the 2.00 μC located at the origin is $\vec{\mathbf{F}} = 0.438$ N at 85.2° below the $+x$-axis. Since the electric field at a location is defined as the force per unit charge experienced by a test charge placed in that location, the electric field at the origin in the charge configuration of Figure P15.13 is

$$\vec{\mathbf{E}} = \frac{\vec{\mathbf{F}}}{q_0} = \frac{0.438\text{ N}}{2.00\times10^{-6}\text{ C}}\ \text{at } -85.2° = \boxed{2.19\times10^{5}\text{ N/C at 85.2° below the } +x\text{-axis}}$$

(b) The electric force experienced by the charge at the origin is directly proportional to the magnitude of that charge. Thus, doubling the magnitude of this charge would double the magnitude of the electric force. However, the electric field is the force per unit charge and the field would be unchanged if the charge was doubled. This is easily seen in the calculation of part (a) above. Doubling the magnitude of the charge at the origin would double both the numerator and the denominator of the ratio $\vec{\mathbf{F}}/q_0$, but the value of the ratio (i.e., the electric field) would be unchanged.

15.25 From the figure at the right, observe that $2r\cos 45.0° + d = 2d$. Thus,

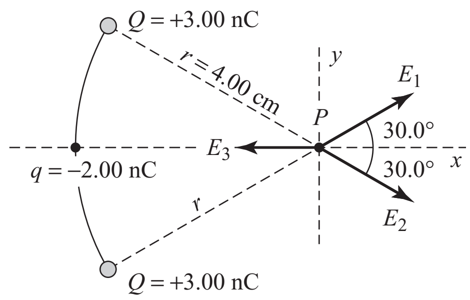


$$r = \frac{d}{2\cos 45.0°} = \frac{d}{2\left(\sqrt{2}/2\right)} = \frac{d}{\sqrt{2}}$$

$$E_x = E_2 - E_1\cos 45.0° = \frac{k_eQ}{d^2} - \frac{k_eQ}{r^2}\left(\frac{\sqrt{2}}{2}\right) = \frac{k_eQ}{d^2} - \frac{k_eQ}{\left(d^2/2\right)}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\left(1-\sqrt{2}\right)\frac{k_eQ}{d^2}}$$

$$E_y = E_1\sin 45.0° + 0 = \frac{k_eQ}{r^2}\left(\frac{\sqrt{2}}{2}\right) = \frac{k_eQ}{\left(d^2/2\right)}\left(\frac{\sqrt{2}}{2}\right) = \boxed{\sqrt{2}\,\frac{k_eQ}{d^2}}$$

15.26 (a) Observe the figure at the right:



$$E_1 = E_2 = \frac{k_eQ}{r^2} \quad \text{and} \quad E_3 = \frac{k_e|q|}{r^2}$$

$$E_y = E_1\sin 30.0° - E_2\sin 30.0° = 0$$

$$E_x = E_1\cos 30.0° + E_2\cos 30.0° - E_3$$

continued on next page

$$E_x = \frac{2k_eQ}{r^2}\cos 30.0° - \frac{k_e|q|}{r^2} = \frac{k_e}{r^2}\left(2Q\cos 30.0° - |q|\right)$$

$$= \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}{\left(4.00\times10^{-2}\ \text{m}\right)^2}\left[2\left(3.00\times10^{-9}\ \text{C}\right)\cos 30.0° - 2.00\times10^{-9}\ \text{C}\right]$$

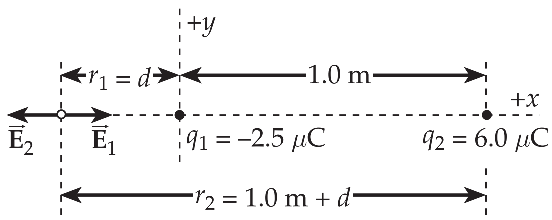
$$E_x = +1.80\times10^{4}\ \text{N/C}$$

Then, $E = \sqrt{E_x^2 + E_y^2} = 1.80\times10^{4}\ \text{N/C}$ and $\vec{\mathbf{E}} = \boxed{1.80\times10^{4}\ \text{N/C to the right}}$

(b) $F_e = q'E = \left(-5.00\times10^{-9}\ \text{C}\right)\left(1.80\times10^{4}\ \text{N/C}\right) = -9.00\times10^{-5}\ \text{N}$

or $\vec{\mathbf{F}}_e = \boxed{9.00\times10^{-5}\ \text{N to the left}}$

15.27 If the resultant field is zero, the contributions from the two charges must be in opposite directions and also have equal magnitudes. Choose the line connecting the charges as the x-axis, with the origin at the $-2.5\ \mu$C charge. Then, the two contributions will have opposite directions only in the regions $x < 0$ and $x > 1.0$ m. For the magnitudes to be equal, the point must be nearer the smaller charge. Thus, the point of zero resultant field is on the x-axis at $x < 0$.



Requiring equal magnitudes gives

$$\frac{k_e|q_1|}{r_1^2} = \frac{k_e|q_2|}{r_2^2} \quad \text{or} \quad \frac{2.5\ \mu\text{C}}{d^2} = \frac{6.0\ \mu\text{C}}{(1.0\ \text{m}+d)^2}$$

Thus, $(1.0\ \text{m}+d)\sqrt{\dfrac{2.5}{6.0}} = d$

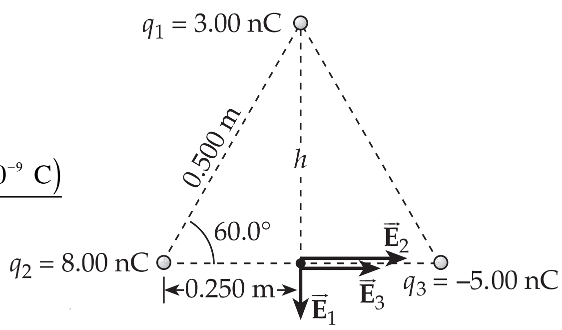
Solving for d yields

$d = 1.8$ m, or $\boxed{1.8\ \text{m to the left of the } -2.5\ \mu\text{C charge}}$

15.28 The altitude of the triangle is

$$h = (0.500\ \text{m})\sin 60.0° = 0.433\ \text{m}$$

and the magnitudes of the fields due to each of the charges are



$$E_1 = \frac{k_eq_1}{h^2} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.00\times10^{-9}\ \text{C}\right)}{(0.433\ \text{m})^2}$$

$$= 144\ \text{N/C}$$

$$E_2 = \frac{k_e|q_2|}{r_2^2} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(8.00\times10^{-9}\ \text{C}\right)}{(0.250\ \text{m})^2} = 1.15\times10^{3}\ \text{N/C}$$

continued on next page

and $E_3 = \frac{k_e|q_3|}{r_3^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(5.00\times10^{-9}\ \text{C})}{(0.250\ \text{m})^2} = 719\ \text{N/C}$

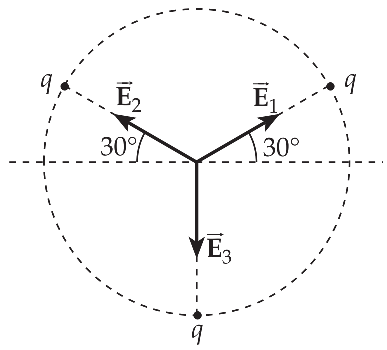
Thus, $\Sigma E_x = E_2 + E_3 = 1.87\times10^3\ \text{N/C}$ and $\Sigma E_y = -E_1 = -144\ \text{N/C}$
giving

$$E_R = \sqrt{(\Sigma E_x)^2 + (\Sigma E_y)^2} = 1.88\times10^3\ \text{N/C}$$

and $\theta = \tan^{-1}(\Sigma E_y/\Sigma E_x) = \tan^{-1}(-0.0769) = -4.40°$

Hence $\boxed{\vec{\mathbf{E}}_R = 1.88\times10^3\ \text{N/C at } 4.40° \text{ below the } +x\text{-axis}}$

15.29 From the symmetry of the charge distribution, students should recognize that the resultant electric field at the center is



$$\boxed{\vec{\mathbf{E}}_R = 0}$$

If one does not recognize this intuitively, consider:

$$\vec{\mathbf{E}}_R = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2 + \vec{\mathbf{E}}_3$$

so $E_x = E_{1x} - E_{2x} = \frac{k_e|q|}{r^2}\cos 30° - \frac{k_e|q|}{r^2}\cos 30° = 0$

and $E_y = E_{1y} + E_{2y} - E_3 = \frac{k_e|q|}{r^2}\sin 30° + \frac{k_e|q|}{r^2}\sin 30° - \frac{k_e|q|}{r^2} = 0$

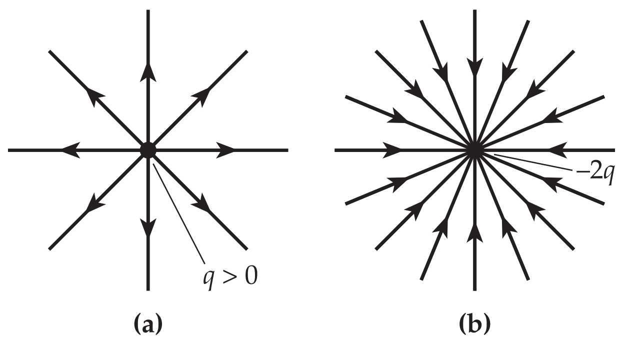
Thus, $E_R = \sqrt{E_x^2 + E_y^2} = \boxed{0}$

15.30 The magnitude of q_2 is three times the magnitude of q_1 because 3 times as many lines emerge from q_2 as enter q_1. $|q_2| = 3|q_1|$

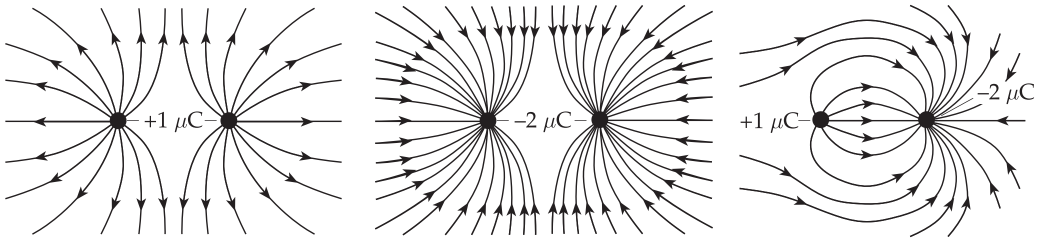
(a) Then, $\boxed{q_1/q_2 = -1/3}$

(b) $\boxed{q_2 > 0}$ because lines emerge from it, and $\boxed{q_1 < 0}$ because lines terminate on it.

15.31 Note in the sketches at the right that electric field lines originate on positive charges and terminate on negative charges. The density of lines is twice as great for the $-2q$ charge in (b) as it is for the $1q$ charge in (a).

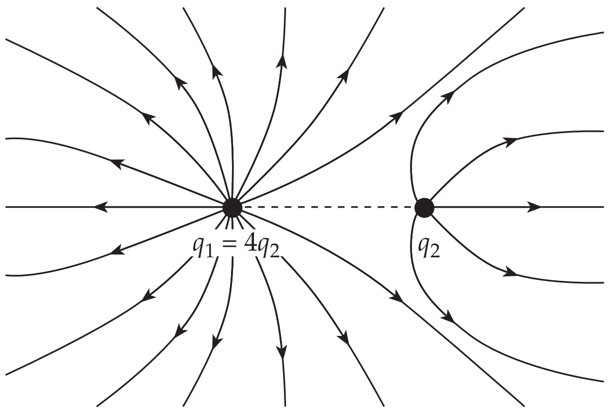


15.32 Rough sketches for these charge configurations are shown below.



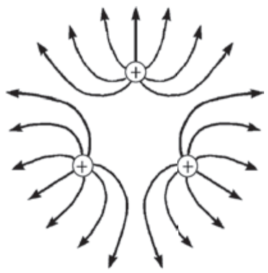
(a) **(b)** **(c)**

15.33 (a) The sketch for (a) is shown at the right. Note that four times as many lines should leave q_1 as emerge from q_2 although, for clarity, this is not shown in this sketch.

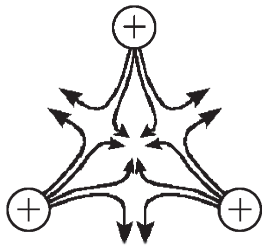


(b) The field pattern looks the same here as that shown for (a) with the exception that the arrows are reversed on the field lines.

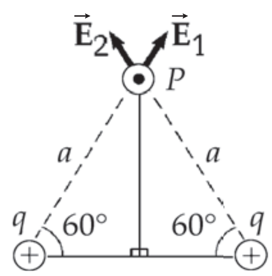
15.34 (a) The electric field in the plane of the charges has the general appearance shown at the right:



(b) It is zero $\boxed{\text{at the center of the triangle}}$, where (by symmetry) one can see that the three charges individually produce fields that cancel out. In addition to the center of the triangle, the electric field lines in the second figure to the right indicate three other points near the middle of each leg of the triangle where $E = 0$, but they are more difficult to find mathematically.



(c) Using the sketch at the right, observe that $E_1 = E_2 = k_e q/a^2$ and the resultant field at point P is $\vec{\mathbf{E}} = \vec{\mathbf{E}}_1 + \vec{\mathbf{E}}_2$. Thus,



$$E_x = E_1 \cos 60.0° - E_2 \cos 60.0° = 0$$

and $$E_y = E_1 \sin 60.0° + E_2 \sin 60.0° = \frac{2k_e q}{a^2} \sin 60.0°$$

The magnitude of the resultant field is then

$$E = \sqrt{E_x^2 + E_y^2} = \sqrt{0 + E_y^2} = E_y = \boxed{\frac{1.73 k_e q}{a^2}}$$

continued on next page

(d) Since $E_x = 0$ and $E_y > 0$, the resultant field at point P is directed vertically upward in the $\boxed{\text{positive } y\text{-direction.}}$

15.35 (a) $\boxed{\text{Zero}}$ net charge on each surface of the sphere.

(b) The negative charge lowered into the sphere repels $\boxed{-5\mu\text{C on the outside}}$ surface, and leaves $\boxed{+5\ \mu\text{C on the inside}}$ surface of the sphere.

(c) The negative charge lowered inside the sphere neutralizes the inner surface, leaving $\boxed{\text{zero charge on the inside}}$. This leaves $\boxed{-5\mu\text{C on the outside}}$ surface of the sphere.

(d) When the object is removed, the sphere is left with $\boxed{-5.00\ \mu\text{C on the outside}}$ surface and $\boxed{\text{zero charge on the inside}}$.

15.36 (a) The dome is a closed conducting surface. Therefore, the electric field is $\boxed{\text{zero}}$ everywhere inside it.

At the surface and outside of this spherically symmetric charge distribution, the field is as if all the charge were concentrated at the center of the sphere.

(b) At the surface,

$$E = \frac{k_e q}{R^2} = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.0\times10^{-4}\ \text{C}\right)}{(1.0\ \text{m})^2} = \boxed{1.8\times10^6\ \text{N/C}}$$

(c) Outside the spherical dome, $E = k_e q/r^2$. Thus, at $r = 4.0$ m,

$$E = \frac{\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(2.0\times10^{-4}\ \text{C}\right)}{(4.0\ \text{m})^2} = \boxed{1.1\times10^5\ \text{N/C}}$$

15.37 For a uniformly charged sphere, the field is strongest at the surface.

Thus, $E_{\text{max}} = \dfrac{k_e q_{\text{max}}}{R^2}$,

or $q_{\text{max}} = \dfrac{R^2 E_{\text{max}}}{k_e} = \dfrac{(2.0\ \text{m})^2\left(3.0\times10^6\ \text{N/C}\right)}{8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2} = \boxed{1.3\times10^{-3}\ \text{C}}$

15.38 If the weight of the drop is balanced by the electric force, then $mg = |q|E = eE$, so the mass of the drop must be

$$m = \frac{eE}{g} = \frac{\left(1.6\times10^{-19}\ \text{C}\right)\left(3\times10^4\ \text{N/C}\right)}{9.8\ \text{m/s}^2} \approx 5\times10^{-16}\ \text{kg}$$

But $m = \rho V = \rho\left(4\pi r^3/3\right)$ and the radius of the drop is $r = \left[3m/4\pi\rho\right]^{1/3}$, which gives

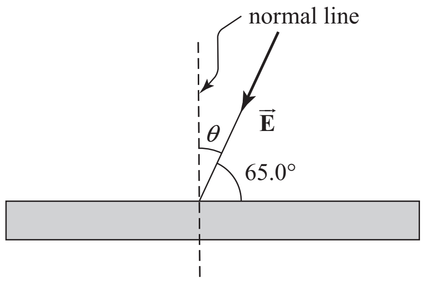
$$r = \left[\frac{3\left(5\times10^{-16}\ \text{kg}\right)}{4\pi\left(858\ \text{kg/m}^3\right)}\right]^{1/3} = 5.2\times10^{-7}\ \text{m} \qquad \text{or} \qquad r \sim \boxed{1\ \mu\text{m}}$$

15.39 (a) $F_e = ma = (1.67 \times 10^{-27}\ \text{kg})(1.52 \times 10^{12}\ \text{m/s}^2) = \boxed{2.54 \times 10^{-15}\ \text{N}}$ in the direction of the acceleration, or radially outward.

(b) The direction of the field is the direction of the force on a positive charge (such as the proton). Thus, the field is directed $\boxed{\text{radially outward}}$. The magnitude of the field is

$$E = \frac{F_e}{q} = \frac{2.54 \times 10^{-15}\ \text{N}}{1.60 \times 10^{-19}\ \text{C}} = \boxed{1.59 \times 10^{4}\ \text{N/C}}$$

15.40 When an electric field of magnitude E is incident on a surface of area A, the flux through the surface is



$$\Phi_E = EA\cos\theta$$

where θ is the angle between $\vec{\mathbf{E}}$ and the line normal to the surface. Thus, in this case,

$$\theta = 90.0° - 65.0° = 25.0°$$

and $\Phi_E = (435\ \text{N/C})(3.50\ \text{m}^2)\cos 25.0° = \boxed{1.38 \times 10^{3}\ \text{N} \cdot \text{m}^2/\text{C}}$

15.41 The area of the rectangular plane is $A = (0.350\ \text{m})(0.700\ \text{m}) = 0.245\ \text{m}^2$.

(a) When the plane is parallel to the yz-plane, its normal line is parallel to the x-axis and makes an angle $\theta = 0°$ with the direction of the field. The flux is then

$$\Phi_E = EA\cos\theta = (3.50 \times 10^{3}\ \text{N/C})(0.245\ \text{m}^2)\cos 0° = \boxed{858\ \text{N} \cdot \text{m}^2/\text{C}}$$

(b) When the plane is parallel to the x-axis, $\theta = 90°$ and $\Phi_E = \boxed{0}$.

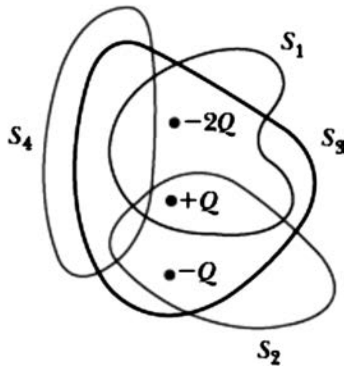
(c) $\Phi_E = EA\cos\theta = (3.50 \times 10^{3}\ \text{N/C})(0.245\ \text{m}^2)\cos 40.0° = \boxed{657\ \text{N} \cdot \text{m}^2/\text{C}}$

15.42 (a) Gauss's law states that the electric flux through any closed surface equals the net charge enclosed divided by ϵ_0. We choose to consider a closed surface in the form of a sphere, centered on the center of the charged sphere and having a radius infinitesimally larger than that of the charged sphere. The electric field then has a uniform magnitude and is perpendicular to our surface at all points on that surface. The flux through the chosen closed surface is therefore $\Phi_E = EA = E(4\pi r^2)$, and Gauss's law gives

$$\begin{aligned} Q_{\text{inside}} &= \epsilon_0\, \Phi_E = 4\pi\, \epsilon_0\, Er^2 \\ &= 4\pi(8.85 \times 10^{-12}\ \text{C}^2/\text{N} \cdot \text{m}^2)(575\ \text{N/C})(0.230\ \text{m})^2 = 3.38 \times 10^{-9}\ \text{C} = \boxed{3.38\ \text{nC}} \end{aligned}$$

(b) Since the electric field displays spherical symmetry, you can conclude that the charge distribution generating that field is $\boxed{\text{spherically symmetric}}$. Also, since the electric field lines are directed outward away from the sphere, the sphere must contain a $\boxed{\text{net positive charge}}$.

15.43 From Gauss's law, the electric flux through any closed surface is equal to the net charge enclosed divided by $\in_0$. Thus, the flux through each surface (with a positive flux coming outward from the enclosed interior and a negative flux going inward toward that interior) is



For S_1: $\Phi_E = Q_{\text{net}}/\in_0 = (+Q-2Q)/\in_0 = \boxed{-Q/\in_0}$

For S_2: $\Phi_E = Q_{\text{net}}/\in_0 = (+Q-Q)/\in_0 = \boxed{0}$

For S_3: $\Phi_E = Q_{\text{net}}/\in_0 = (-2Q+Q-Q)/\in_0 = \boxed{-2Q/\in_0}$

For S_4: $\Phi_E = Q_{\text{net}}/\in_0 = (0)/\in_0 = \boxed{0}$

15.44 (a) From Gauss's law, the total flux through this closed surface is

$$\Phi_E = \frac{Q_{\text{enclosed}}}{\in_0} = \frac{+5.80\times10^{-6}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{6.55\times10^5\ \text{N}\cdot\text{m}^2/\text{C}}$$

(b) When the charge is located in the center of this tetrahedron, we have total symmetry and the flux is the same through each of the four surfaces. Thus, the flux through any one face is

$$\Phi_{\text{1 face}} = \frac{\Phi_E}{4} = \frac{6.55\times10^5\ \text{N}\cdot\text{m}^2/\text{C}}{4} = \boxed{1.64\times10^5\ \text{N}\cdot\text{m}^2/\text{C}}$$

15.45 We choose a spherical Gaussian surface, concentric with the charged spherical shell and of radius r. Then, $\Sigma EA\cos\theta = E(4\pi r^2)\cos 0° = 4\pi r^2 E$.

(a) For $r > a$ (that is, outside the shell), the total charge enclosed by the Gaussian surface is $Q = +q - q = 0$. Thus, Gauss's law gives $4\pi r^2 E = 0$, or $\boxed{E = 0}$.

(b) Inside the shell, $r < a$, and the enclosed charge is $Q = +q$.

Therefore, from Gauss's law, $4\pi r^2 E = \dfrac{q}{\in_0}$, or $E = \dfrac{q}{4\pi \in_0 r^2} = \dfrac{k_e q}{r^2}$

The field for $r < a$ is $\boxed{\vec{\mathbf{E}} = k_e q/r^2 \text{ directed radially outward}}$.

15.46 (a) The surface of the cube is a closed surface which surrounds a total charge of $Q = 1.70\times10^2\ \mu\text{C}$. Thus, by Gauss's law, the electric flux through the full surface of the cube is

$$\Phi_E = \frac{Q_{\text{inside}}}{\in_0} = \frac{1.70\times10^{-4}\ \text{C}}{8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2} = \boxed{1.92\times10^7\ \text{N}\cdot\text{m}^2/\text{C}}$$

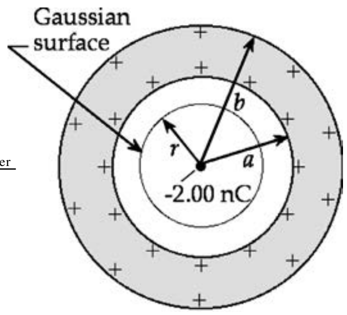
(b) Since the charge is located at the center of the cube, the six faces of the cube are symmetrically positioned around the location of the charge. Thus, one-sixth of the flux passes through each of the faces, or

$$\Phi_{\text{face}} = \frac{\Phi_E}{6} = \frac{1.92\times10^7\ \text{N}\cdot\text{m}^2/\text{C}}{6} = \boxed{3.20\times10^6\ \text{N}\cdot\text{m}^2/\text{C}}$$

(c) The answer to part (b) would change because the charge could now be at different distances from each face of the cube, but the answer to part (a) would be unchanged because the flux through the entire closed surface depends only on the total charge inside the surface.

15.47 Note that with the point charge -2.00 nC positioned at the center of the spherical shell, we have complete spherical symmetry in this situation. Thus, we can expect the distribution of charge on the shell, as well as the electric fields both inside and outside of the shell, to also be spherically symmetric.

(a) We choose a spherical Gaussian surface, centered on the center of the conducting shell, with radius $r = 1.50 \text{ m} < a$ as shown at the right. Gauss's law gives



$$\Phi_E = EA = E\left(4\pi r^2\right) = \frac{Q_{\text{inside}}}{\epsilon_0} \text{ or } E = \frac{Q_{\text{inside}}}{4\pi \epsilon_0 r^2} = \frac{k_e Q_{\text{center}}}{r^2}$$

so $$E = \frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(-2.00\times10^{-9} \text{ C}\right)}{(1.50 \text{ m})^2}$$

and $E = \boxed{-7.99 \text{ N/C}}$. The negative sign means that the field is $\boxed{\text{radial inward}}$.

(b) All points at $r = 2.20$ m are in the range $a < r < b$, and hence are located within the conducting material making up the shell. Under conditions of electrostatic equilibrium, the field is $\boxed{E = 0}$ at all points inside a conducting material.

(c) If the radius of our Gaussian surface is $r = 2.50 \text{ m} > b$, Gauss's law (with total spherical symmetry) leads to $E = \dfrac{Q_{\text{inside}}}{4\pi \epsilon_0 r^2} = \dfrac{k_e Q_{\text{inside}}}{r^2}$ just as in part (a). However, now $Q_{\text{inside}} = Q_{\text{shell}} + Q_{\text{center}} = +3.00 \text{ nC} - 2.00 \text{ nC} = +1.00 \text{ nC}$. Thus, we have

$$E = \frac{\left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(+1.00\times10^{-9} \text{ C}\right)}{(2.50 \text{ m})^2} = \boxed{+1.44 \text{ N/C}}$$

with the positive sign telling us that the field is $\boxed{\text{radial outward}}$ at this location.

(d) Under conditions of electrostatic equilibrium, all excess charge on a conductor resides entirely on its surface. Thus, the sum of the charge on the inner surface of the shell and that on the outer surface of the shell is $Q_{\text{shell}} = +3.00$ nC. To see how much of this is on the inner surface, consider our Gaussian surface to have a radius r that is infinitesimally larger than a. Then, all points on the Gaussian surface lie within the conducting material, meaning that $E = 0$ at all points and the total flux through the surface is $\Phi_E = 0$. Gauss's law then states that $Q_{\text{inside}} = Q_{\substack{\text{inner}\\ \text{surface}}} + Q_{\text{center}} = 0$, or

$$Q_{\substack{\text{inner}\\ \text{surface}}} = -Q_{\text{center}} = -(-2.00 \text{ nC}) = \boxed{+2.00 \text{ nC}}$$

The charge on the outer surface must be

$$Q_{\substack{\text{outer}\\ \text{surface}}} = Q_{\text{shell}} - Q_{\substack{\text{inner}\\ \text{surface}}} = 3.00 \text{ nC} - 2.00 \text{ nC} = \boxed{+1.00 \text{ nC}}$$

15.48 Please review Example 15.8 in your textbook. There it is shown that the electric field due to a nonconducting plane sheet of charge parallel to the xy-plane has a constant magnitude given by $E_z = |\sigma_{\text{sheet}}|/2\epsilon_0$, where σ_{sheet} is the uniform charge per unit area on the sheet. This field is everywhere perpendicular to the xy-plane, is directed away from the sheet if it has a positive charge density, and is directed toward the sheet if it has a negative charge density.

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In this problem, we have two plane sheets of charge, both parallel to the xy-plane and separated by a distance of 2.00 cm. The upper sheet has charge density $\sigma_{\text{sheet}} = -2\sigma$, while the lower sheet has $\sigma_{\text{sheet}} = +\sigma$. Taking upward as the positive z-direction, the fields due to each of the sheets in the three regions of interest are:

	Lower sheet (at $z = 0$)	**Upper sheet (at $z = 2.00$ cm)**
Region	Electric Field	Electric Field
$z < 0$	$E_z = -\dfrac{\lvert +\sigma \rvert}{2\in_0} = -\dfrac{\sigma}{2\in_0}$	$E_z = +\dfrac{\lvert -2\sigma \rvert}{2\in_0} = +\dfrac{\sigma}{\in_0}$
$0 < z < 2.00$ cm	$E_z = +\dfrac{\lvert +\sigma \rvert}{2\in_0} = +\dfrac{\sigma}{2\in_0}$	$E_z = +\dfrac{\lvert -2\sigma \rvert}{2\in_0} = +\dfrac{\sigma}{\in_0}$
$z > 2.00$ cm	$E_z = +\dfrac{\lvert +\sigma \rvert}{2\in_0} = +\dfrac{\sigma}{2\in_0}$	$E_z = -\dfrac{\lvert -2\sigma \rvert}{2\in_0} = -\dfrac{\sigma}{\in_0}$

When both plane sheets of charge are present, the resultant electric field in each region is the vector sum of the fields due to the individual sheets for that region.

(a) For $z < 0$: $$E_z = E_{z,\text{lower}} + E_{z,\text{upper}} = -\frac{\sigma}{2\in_0} + \frac{\sigma}{\in_0} = \boxed{+\frac{\sigma}{2\in_0}}$$

(b) For $0 < z < 2.00$ cm: $$E_z = E_{z,\text{lower}} + E_{z,\text{upper}} = +\frac{\sigma}{2\in_0} + \frac{\sigma}{\in_0} = \boxed{+\frac{3\sigma}{2\in_0}}$$

(c) For $z > 2.00$ cm: $$E_z = E_{z,\text{lower}} + E_{z,\text{upper}} = +\frac{\sigma}{2\in_0} - \frac{\sigma}{\in_0} = \boxed{-\frac{\sigma}{2\in_0}}$$

15.49 The radius of each sphere is small in comparison to the distance to the nearest neighboring charge (the other sphere). Thus, we shall assume that the charge is uniformly distributed over the surface of each sphere and, in its interaction with the other charge, treat it as though it were a point charge. In this model, we then have two identical point charges, of magnitude 35.0 mC, separated by a total distance of 310 m (the length of the cord plus the radius of each sphere). Each of these charges repels the other with a force of magnitude

$$F_e = k_e\frac{Q^2}{r^2} = \left(8.99\times10^9 \text{ N}\cdot\text{m}^2/\text{C}^2\right)\frac{\left(35.0\times10^{-3}\text{ C}\right)^2}{\left(3.10\times10^2\text{ m}\right)^2} = 115 \text{ N}$$

Thus, to counterbalance this repulsion and hold each sphere in equilibrium, the cord must have a tension of $\boxed{115 \text{ N}}$, so it will exert a 115 N on that sphere, directed toward the other sphere.

15.50 (a) As shown in Example 15.8 in the textbook, the electric field due to a nonconducting plane sheet of charge has a constant magnitude of $E = \sigma/2\in_0$, where σ is the uniform charge per unit area on the sheet. The direction of the field at all locations is perpendicular to the plane sheet and directed away from the sheet if σ is positive, and toward the sheet if σ is negative. Thus, if $\sigma = +5.20\ \mu\text{C/m}^2$, the magnitude of the electric field at all distances greater than zero from the plane (including the distance of 8.70 cm) is

$$E = \frac{\sigma}{2\in_0} = \frac{+5.20\times10^{-6}\text{ C/m}^2}{2\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)} = \boxed{2.94\times10^5\text{ N/C}}$$

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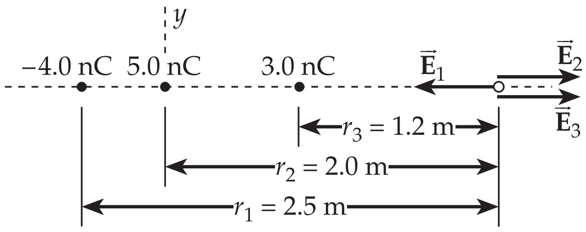
(b) The field does not vary with distance as long as the distance is small compared with the dimensions of the sheet.

15.51 The three contributions to the resultant electric field at the point of interest are shown in the sketch at the right.



The magnitude of the resultant field is

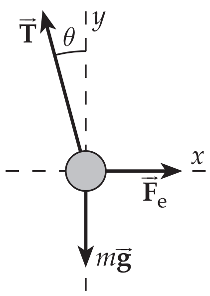
$$E_R = -E_1 + E_2 + E_3$$

$$E_R = -\frac{k_e|q_1|}{r_1^2} + \frac{k_e|q_2|}{r_2^2} + \frac{k_e|q_3|}{r_3^2} = k_e\left[-\frac{|q_1|}{r_1^2} + \frac{|q_2|}{r_2^2} + \frac{|q_3|}{r_3^2}\right]$$

$$E_R = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left[-\frac{4.0\times10^{-9}\ \text{C}}{(2.5\ \text{m})^2} + \frac{5.0\times10^{-9}\ \text{C}}{(2.0\ \text{m})^2} + \frac{3.0\times10^{-9}\ \text{C}}{(1.2\ \text{m})^2}\right]$$

$$E_R = +24\ \text{N/C} \text{ or } \vec{\mathbf{E}}_R = \boxed{24\ \text{N/C in the } +x\text{-direction}}$$

15.52 Consider the force diagram shown at the right.



$$\Sigma F_y = 0 \Rightarrow T\cos\theta = mg \quad \text{or} \quad T = \frac{mg}{\cos\theta}$$

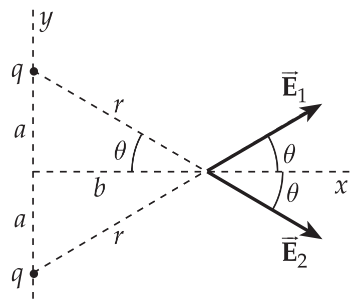
$$\Sigma F_x = 0 \Rightarrow F_e = T\sin\theta = \left(\frac{mg}{\cos\theta}\right)\sin\theta = mg\tan\theta$$

Since $F_e = qE$, we have

$$qE = mg\tan\theta, \text{ or } q = mg\tan\theta/E$$

$$q = \frac{(2.00\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)\tan 15.0°}{1.00\times10^3\ \text{N/C}} = +5.25\times10^{-6}\ \text{C} = \boxed{+5.25\ \mu\text{C}}$$

15.53 (a) At a point on the x-axis, the contributions by the two charges to the resultant field have equal magnitudes given by $E_1 = E_2 = k_e q/r^2$.



The components of the resultant field are

$$E_y = E_{1y} - E_{2y} = \left(\frac{k_e q}{r^2}\right)\sin\theta - \left(\frac{k_e q}{r^2}\right)\sin\theta = 0$$

and $$E_x = E_{1x} + E_{2x} = \left(\frac{k_e q}{r^2}\right)\cos\theta + \left(\frac{k_e q}{r^2}\right)\cos\theta = \left[\frac{k_e(2q)}{r^2}\right]\cos\theta$$

Since $\dfrac{\cos\theta}{r^2} = \dfrac{b/r}{r^2} = \dfrac{b}{r^3} = \dfrac{b}{(a^2+b^2)^{3/2}}$, the resultant field is

$$\vec{\mathbf{E}}_R = \boxed{\frac{k_e(2q)b}{(a^2+b^2)^{3/2}} \text{ in the } +x\text{-direction}}$$

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(b) Note that the result of part (a) may be written as $E_R = \dfrac{k_e(Q)b}{\left(a^2+b^2\right)^{3/2}}$, where $Q = 2q$ is the total charge in the charge distribution generating the field.

In the case of a uniformly charged circular ring, consider the ring to consist of a very large number of pairs of charges uniformly spaced around the ring. Each pair consists of two identical charges located diametrically opposite each other on the ring. The total charge of pair number i is Q_i. At a point on the axis of the ring, this pair of charges generates an electric field contribution that is parallel to the axis and has magnitude $E_i = k_e b Q_i \big/ \left(a^2+b^2\right)^{3/2}$.

The resultant electric field of the ring is the summation of the contributions by all pairs of charges, or

$$E_R = \Sigma E_i = \left[\frac{k_e b}{\left(a^2+b^2\right)^{3/2}}\right]\Sigma Q_i = \frac{k_e b Q}{\left(a^2+b^2\right)^{3/2}}$$

where $Q = \Sigma Q_i$ is the total charge on the ring.

$$\vec{\mathbf{E}}_R = \boxed{\frac{k_e Q b}{\left(a^2+b^2\right)^{3/2}} \text{ in the } +x\text{-direction}}$$

15.54 It is desired that the electric field exert a retarding force on the electrons, slowing them down and bringing them to rest. For the retarding force to have maximum effect, it should be anti-parallel to the direction of the electron's motion. Since the force an electric field exerts on negatively charged particles (such as electrons) is in the direction opposite to the field, $\boxed{\text{the electric field should be in the direction of the electron's motion.}}$

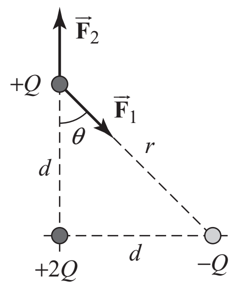
The work a retarding force of magnitude $F_e = |q|E = eE$ does on the electrons as they move distance d is $W = F_e d\cos 180° = -F_e d = -eEd$. The work-energy theorem $(W = \Delta KE)$ then gives

$$-eEd = KE_f - KE_i = 0 - K$$

and the magnitude of the electric field required to stop the electrons in distance d is

$$E = \frac{-K}{-ed} \quad \text{or} \quad \boxed{E = K/ed}$$

15.55 Consider the sketch at the right and observe:



$$\theta = \tan^{-1}\left(\frac{d}{d}\right) = 45.0°$$

and $r = \sqrt{d^2 + d^2} = d\sqrt{2}$

Thus,

$$F_x = F_{1x} + F_{2x} = +\frac{k_e|-Q|Q}{r^2}\sin 45.0° + 0 = +\frac{k_e Q^2}{2d^2}\left(\frac{\sqrt{2}}{2}\right) = +\frac{\sqrt{2}}{4}\left(\frac{k_e Q^2}{d^2}\right) = \boxed{+0.354\frac{k_e Q^2}{d^2}}$$

and $F_y = F_{1y} + F_{2y} = -\dfrac{k_e|-Q|Q}{r^2}\cos 45.0° + \dfrac{k_e(Q)(2Q)}{d^2} = -\dfrac{k_e Q^2}{2d^2}\left(\dfrac{\sqrt{2}}{2}\right) + \dfrac{2k_e Q^2}{d^2}$

or $F_y = \left(2 - \dfrac{\sqrt{2}}{4}\right)\dfrac{k_e Q^2}{d^2} = \boxed{+1.65\dfrac{k_e Q^2}{d^2}}$

15.56 (a) The downward electrical force acting on the ball is

$$F_e = qE = \left(2.00\times10^{-6}\ \text{C}\right)\left(1.00\times10^{5}\ \text{N/C}\right) = 0.200\ \text{N}$$

The total downward force acting on the ball is then

$$F = F_e + mg = 0.200\ \text{N} + \left(1.00\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) = 0.210\ \text{N}$$

Thus, the ball will behave as if it were in a modified gravitational field where the effective free-fall acceleration is

$$"g" = \frac{F}{m} = \frac{0.210\ \text{N}}{1.00\times10^{-3}\ \text{kg}} = 210\ \text{m/s}^2$$

The period of the pendulum will be

$$T = 2\pi\sqrt{\frac{L}{"g"}} = 2\pi\sqrt{\frac{0.500\ \text{m}}{210\ \text{m/s}^2}} = \boxed{0.307\ \text{s}}$$

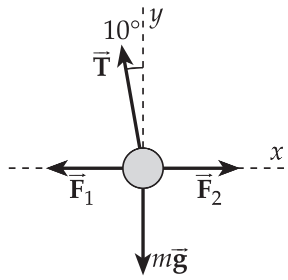
(b) $\boxed{\text{Yes}}$. The force of gravity is a significant portion of the total downward force acting on the ball. Without gravity, the effective acceleration would be

$$"g" = \frac{F_e}{m} = \frac{0.200\ \text{N}}{1.00\times10^{-3}\ \text{kg}} = 200\ \text{m/s}^2$$

giving $T = 2\pi\sqrt{\dfrac{0.500\ \text{m}}{200\ \text{m/s}^2}} = 0.314\ \text{s}$

a 2.28% difference from the correct value with gravity included.

15.57 The sketch at the right gives a force diagram of the positively charged sphere. Here, $F_1 = k_e|q|^2/r^2$ is the attractive force exerted by the negatively charged sphere, and $F_2 = qE$ is exerted by the electric field.



$$\Sigma F_y = 0 \Rightarrow T\cos10° = mg \quad \text{or} \quad T = \frac{mg}{\cos10°}$$

$$\Sigma F_x = 0 \Rightarrow F_2 = F_1 + T\sin10° \quad \text{or} \quad qE = \frac{k_e|q|^2}{r^2} + mg\tan10°$$

At equilibrium, the distance between the two spheres is $r = 2(L\sin10°)$. Thus,

$$E = \frac{k_e|q|}{4(L\sin10°)^2} + \frac{mg\tan10°}{q}$$

$$= \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(5.0\times10^{-8}\ \text{C}\right)}{4\left[(0.100\ \text{m})\sin10°\right]^2} + \frac{\left(2.0\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\tan10°}{\left(5.0\times10^{-8}\ \text{C}\right)}$$

or the needed electric field strength is $E = \boxed{4.4\times10^{5}\ \text{N/C}}$

15.58 (a) At any point on the x-axis in the range $0 < x < 1.00$ m, the contributions made to the resultant electric field by the two charges are both in the positive x-direction. Thus, it is not possible for these contributions to cancel each other and yield a zero field.

(b) Any point on the x-axis in the range $x < 0$ is located closer to the larger magnitude charge $(q = 5.00\ \mu\text{C})$ than the smaller magnitude charge $(|q| = 4.00\ \mu\text{C})$. Thus, the contribution to the resultant electric field by the larger charge will always have a greater magnitude than the contribution made by the smaller charge. It is not possible for these contributions to cancel to give a zero resultant field.

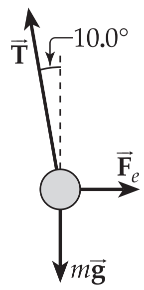
(c) If a point is on the x-axis in the region $x > 1.00$ m, the contributions made by the two charges are in opposite directions. Also, a point in this region is closer to the smaller magnitude charge than it is to the larger charge. Thus, there is a location in this region where the contributions of these charges to the total field will have equal magnitudes and cancel each other.

(d) When the contributions by the two charges cancel each other, their magnitudes must be equal. That is,

$$k_e\frac{(5.00\ \mu\text{C})}{x^2} = k_e\frac{(4.00\ \mu C)}{(x-1.00\text{ m})^2} \quad \text{or } x - 1.00\text{ m} = +x\sqrt{4/5}$$

Thus, the resultant field is zero at $\quad x = \dfrac{1.00\text{ m}}{1-\sqrt{4/5}} = \boxed{+9.47\text{ m}}$

15.59 The two spheres have charges q_1 and $q_2 = 2q_1$, so the repulsive force that one exerts on the other has magnitude $F_e = k_e\, q_1 q_2/r^2 = 2k_e q_1^2/r^2$.



From Figure P15.59 in the textbook, observe that the distance separating the two spheres is

$$r = 3.00\text{ cm} + 2\left[(5.00\text{ cm})\sin 10.0°\right] = 4.74\text{ cm} = 0.047\,4\text{ m}$$

From the force diagram of one sphere given on the right, observe that

$$\Sigma F_y = 0 \;\Rightarrow\; T\cos 10.0° = mg \;\text{ or }\; T = mg/\cos 10.0°$$

and $\quad \Sigma F_x = 0 \;\Rightarrow\; F_e = T\sin 10.0° = \left(\dfrac{mg}{\cos 10.0°}\right)\sin 10.0° = mg\tan 10.0°$

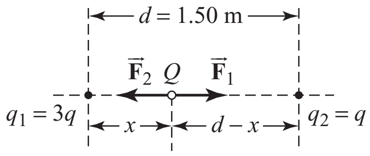
Thus, $\quad 2k_e\, q_1^2/r^2 = mg\tan 10.0°$

or $\quad q_1 = \sqrt{\dfrac{mgr^2\tan 10.0°}{2k_e}} = \sqrt{\dfrac{(0.0150\text{ kg})(9.80\text{ m/s}^2)(0.0474\text{ m})^2\tan 10.0°}{2(8.99\times 10^9\text{ N}\cdot\text{m}^2/\text{C}^2)}}$

giving $\quad q_1 = \boxed{5.69\times 10^{-8}\text{ C}}$ as the charge on one sphere,

and $\quad q_2 = 2q_1 = \boxed{1.14\times 10^{-7}\text{ C}}$ as the charge on the other sphere.

15.60 Consider the sketch at the right. The electrical forces acting on the third charge, Q, have magnitudes



$$F_1 = \frac{k_e q_1 Q}{x^2} = \frac{3k_e qQ}{x^2}$$

and $$F_2 = \frac{k_e q_2 Q}{(d-x)^2} = \frac{k_e qQ}{(d-x)^2}$$

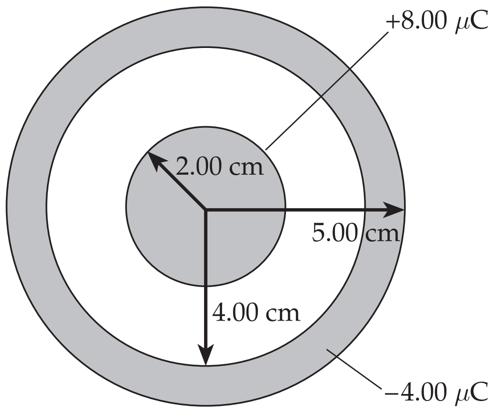
The bead with charge Q is in equilibrium when $F_1 = F_2$, or

$$\frac{3\,\cancel{k_e qQ}}{x^2} = \frac{\cancel{k_e qQ}}{(d-x)^2}$$

giving $\quad 3(d-x)^2 = x^2 \quad$ or $\quad \sqrt{3}(d-x) = x$

and $$x = \frac{\sqrt{3}d}{1+\sqrt{3}} = \frac{\sqrt{3}(1.50\text{ m})}{1+\sqrt{3}} = \boxed{0.951\text{ m}}$$

15.61 Because of the spherical symmetry of the charge distribution, any electric field present will be radial in direction. If a field does exist at distance R from the center, it is the same as if the net charge located within $r \le R$ were concentrated as a point charge at the center of the inner sphere. Charge located at $r > R$ does not contribute to the field at $r = R$.



(a) At $r = 1.00$ cm, $\boxed{E = 0}$ since static electric fields cannot exist within conducting materials.

(b) The net charge located at $r \le 3.00$ cm is $Q = +8.00\ \mu\text{C}$.

Thus, at $r = 3.00$ cm,

$$E = \frac{k_e Q}{r^2}$$

$$= \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(8.00\times10^{-6}\ \text{C})}{(3.00\times10^{-2}\ \text{m})^2} = \boxed{7.99\times10^7\ \text{N/C (outward)}}$$

(c) At $r = 4.50$ cm, $\boxed{E = 0}$, since this is located within conducting materials.

(d) The net charge located at $r \le 7.00$ cm is $Q = +4.00\ \mu\text{C}$.

Thus, at $r = 7.00$ cm,

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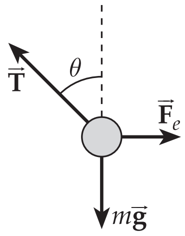
$$E = \frac{k_e Q}{r^2}$$

$$= \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(4.00\times10^{-6}\ \text{C}\right)}{\left(7.00\times10^{-2}\ \text{m}\right)^2} = \boxed{7.34\times10^{6}\ \text{N/C (outward)}}$$

15.62 Consider the free-body diagram of the rightmost charge given at the right.



$$\Sigma F_y = 0 \ \Rightarrow\ T\cos\theta = mg \qquad \text{or} \qquad T = mg/\cos\theta$$

and $\Sigma F_x = 0 \ \Rightarrow\ F_e = T\sin\theta = \left(mg/\cos\theta\right)\sin\theta = mg\tan\theta$

But, $F_e = \dfrac{k_e q^2}{r_1^2} + \dfrac{k_e q^2}{r_2^2} = \dfrac{k_e q^2}{\left(L\sin\theta\right)^2} + \dfrac{k_e q^2}{\left(2L\sin\theta\right)^2} = \dfrac{5k_e q^2}{4L^2\sin^2\theta}$

Thus, $\dfrac{5k_e q^2}{4L^2\sin^2\theta} = mg\tan\theta$ or $q = \sqrt{\dfrac{4L^2 mg\sin^2\theta\tan\theta}{5k_e}}$

If $\theta = 45.0°$, $m = 0.100$ kg, and $L = 0.300$ m, then

$$q = \sqrt{\frac{4(0.300\ \text{m})^2(0.100\ \text{kg})\left(9.80\ \text{m/s}^2\right)\sin^2(45.0°)\tan(45.0°)}{5\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)}}$$

or $q = 1.98\times10^{-6}\ \text{C} = \boxed{1.98\ \mu\text{C}}$

15.63 (a) When an electron (negative charge) moves distance Δx in the direction of an electric field, the work done on it is

$$W = F_e(\Delta x)\cos\theta = eE(\Delta x)\cos180° = -eE(\Delta x)$$

From the work-energy theorem $\left(W_{\text{net}} = KE_f - KE_i\right)$ with $KE_f = 0$, we have

$$-eE(\Delta x) = -KE_i, \quad \text{or} \quad E = \frac{KE_i}{e(\Delta x)} = \frac{1.60\times10^{-17}\ \text{J}}{\left(1.60\times10^{-19}\ \text{C}\right)(0.100\ \text{m})} = \boxed{1.00\times10^{3}\ \text{N/C}}$$

(b) The magnitude of the retarding force acting on the electron is $F_e = eE$, and Newton's second law gives the acceleration as $a = -F_e/m = -eE/m$. Thus, the time required to bring the electron to rest is

$$t = \frac{v - v_0}{a} = \frac{0 - \sqrt{2\left(KE_i\right)/m}}{-eE/m} = \frac{\sqrt{2m\left(KE_i\right)}}{eE}$$

or $t = \dfrac{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.60\times10^{-17}\ \text{J}\right)}}{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.00\times10^{3}\ \text{N/C}\right)} = 3.37\times10^{-8}\ \text{s} = \boxed{33.7\ \text{ns}}$

(c) After bringing the electron to rest, the electric force continues to act on it, causing the electron to $\boxed{\text{accelerate in the direction opposite to the field}}$ at a rate of

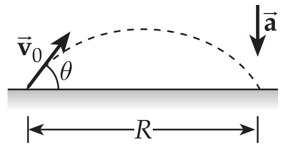
$$|a| = \frac{eE}{m} = \frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.00\times10^{3}\ \text{N/C}\right)}{9.11\times10^{-31}\ \text{kg}} = \boxed{1.76\times10^{14}\ \text{m/s}^2}$$

15.64 (a) The acceleration of the protons is downward (in the direction of the field) and

$$|a_y| = \frac{F_e}{m} = \frac{eE}{m} = \frac{(1.60\times10^{-19}\ \text{C})(720\ \text{N/C})}{1.67\times10^{-27}\ \text{kg}} = 6.90\times10^{10}\ \text{m/s}^2$$

The time of flight for the proton is twice the time required to reach the peak of the arc, or

$$t = 2t_{\text{peak}} = 2\left(\frac{v_{0y}}{|a_y|}\right) = \frac{2v_0\sin\theta}{|a_y|}$$



The horizontal distance traveled in this time is

$$R = v_{0x}t = (v_0\cos\theta)\left(\frac{2v_0\sin\theta}{|a_y|}\right) = \frac{v_0^2\sin2\theta}{|a_y|}$$

Thus, if $R = 1.27\times10^{-3}$ m, we must have

$$\sin2\theta = \frac{|a_y|R}{v_0^2} = \frac{(6.90\times10^{10}\ \text{m/s}^2)(1.27\times10^{-3}\ \text{m})}{(9\,550\ \text{m/s})^2} = 0.961$$

giving $2\theta = 73.9°$ or $2\theta = 180° - 73.9° = 106°$.

Hence, $\theta = \boxed{37.0° \text{ or } 53.0°}$

(b) The time of flight for each possible angle of projection is:

For $\theta = 37.0°$: $t = \dfrac{2v_0\sin\theta}{|a_y|} = \dfrac{2(9\,550\ \text{m/s})\sin37.0°}{6.90\times10^{10}\ \text{m/s}^2} = \boxed{1.67\times10^{-7}\ \text{s}}$

For $\theta = 53.0°$: $t = \dfrac{2v_0\sin\theta}{|a_y|} = \dfrac{2(9\,550\ \text{m/s})\sin53.0°}{6.90\times10^{10}\ \text{m/s}^2} = \boxed{2.21\times10^{-7}\ \text{s}}$

16

Electrical Energy and Capacitance

QUICK QUIZZES

1. Choice (b). The field exerts a force on the electron, causing it to accelerate in the direction opposite to that of the field. In this process, electrical potential energy is converted into kinetic energy of the electron. Note that the electron moves to a region of higher potential, but because the electron has negative charge this corresponds to a decrease in the potential energy of the electron.

2. Choice (a). The electron, a negatively charged particle, will move toward the region of higher electric potential. Because of the electron's negative charge, this corresponds to a decrease in electrical potential energy.

3. Choice (b). Charged particles always tend to move toward positions of lower potential energy. The electrical potential energy of a charged particle is $PE = qV$ and, for positively-charged particles, this decreases as V decreases. Thus, a positively-charged particle located at $x = A$ would move toward the left.

4. Choice (d). For a negatively-charged particle, the potential energy ($PE = qV$) decreases as V increases. A negatively charged particle would oscillate around $x = B$ which is a position of minimum potential energy for negative charges.

5. Choice (d). If the potential is zero at a point located a finite distance from charges, negative charges must be present in the region to make negative contributions to the potential and cancel positive contributions made by positive charges in the region.

6. Choice (c). Both the electric potential and the magnitude of the electric field decrease as the distance from the charged particle increases. However, the electric flux through the balloon does not change because it is proportional to the total charge enclosed by the balloon, which does not change as the balloon increases in size.

7. Choice (a). From the conservation of energy, the final kinetic energy of either particle will be given by

$$KE_f = KE_i + \left(PE_i - PE_f\right) = 0 + qV_i - qV_f = -q\left(V_f - V_i\right) = -q(\Delta V)$$

For the electron, $q = -e$ and $\Delta V = +1$ V giving $KE_f = -(-e)(+1\text{ V}) = +1$ eV.
For the proton, $q = +e$ and $\Delta V = -1$ V, so $KE_f = -(e)(-1\text{ V}) = +1$ eV, the same as that of the electron.

8. Choice (c). The battery moves negative charge from one plate and puts it on the other. The first plate is left with excess positive charge whose magnitude equals that of the negative charge moved to the other plate.

9. (a) C decreases. (b) Q stays the same. (c) E stays the same.
(d) ΔV increases. (e) The energy stored increases.

Because the capacitor is removed from the battery, charges on the plates have nowhere to go. Thus, the charge on the capacitor plates remains the same as the plates are pulled apart. Because $E = \sigma/\epsilon_0 = (Q/A)/\epsilon_0$, the electric field is constant as the plates are separated. Because $\Delta V = Ed$ and E does not change, ΔV increases as d increases. Because the same charge is stored at a higher potential difference, the capacitance $(C = Q/\Delta V)$ has decreased. Because energy stored $= Q^2/2C$ and Q stays the same while C decreases, the energy stored increases. The extra energy must have been transferred from somewhere, so work was done. This is consistent with the fact that the plates attract one another, and work must be done to pull them apart.

10. (a) C increases. (b) Q increases. (c) E stays the same.
(d) ΔV remains the same. (e) The energy stored increases.

The presence of a dielectric between the plates increases the capacitance by a factor equal to the dielectric constant. Since the battery holds the potential difference constant while the capacitance increases, the charge stored $(Q = C\Delta V)$ will increase. Because the potential difference and the distance between the plates are both constant, the electric field $(E = \Delta V/d)$ will stay the same. The battery maintains a constant potential difference. With ΔV constant while capacitance increases, the stored energy $[\text{energy stored} = \frac{1}{2}C(\Delta V)^2]$ will increase.

11. Choice (a). Increased random motions associated with an increase in temperature make it more difficult to maintain a high degree of polarization of the dielectric material. This has the effect of decreasing the dielectric constant of the material, and in turn, decreasing the capacitance of the capacitor.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The change in the potential energy of the proton is equal to the negative of the work done on it by the electric field. Thus,

$$\Delta PE = -W = -qE_x(\Delta x) = -\left(+1.6\times10^{-19}\text{ C}\right)\left(850\text{ N/C}\right)\left(2.5\text{ m} - 0\right) = -3.4\times10^{-16}\text{ J}$$

and (b) is the correct choice for this question.

2. Because electric forces are conservative, the kinetic energy gained is equal to the decrease in electrical potential energy, or

$$KE = -PE = -q(\Delta V) = -(-1\text{ e})\left(+1.00\times10^4\text{ V}\right) = +1.00\times10^4\text{ eV}$$

so the correct choice is (a).

3. In a uniform electric field, the change in electric potential is $\Delta V = -E_x(\Delta x)$, giving

$$E_x = -\frac{\Delta V}{\Delta x} = -\frac{\left(V_f - V_i\right)}{\left(x_f - x_i\right)} = -\frac{(190\text{ V} - 120\text{ V})}{(5.0\text{ m} - 3.0\text{ m})} = -35\text{ V/m} = -35\text{ N/C}$$

and it is seen that the correct choice is (d).

4. From conservation of energy, $KE_f + PE_f = KE_i + PE_i$, or $\frac{1}{2}mv_B^2 = \frac{1}{2}mv_A^2 + qV_A - qV_B$, the final speed of the nucleus is

$$v_B = \sqrt{v_A^2 + \frac{2q(V_A - V_B)}{m}}$$

$$= \sqrt{\left(6.20\times10^5 \text{ m/s}\right)^2 + \frac{2\left[2\left(1.60\times10^{-19} \text{ C}\right)(1.50-4.00)\times10^3 \text{ V}\right]}{6.63\times10^{-27} \text{ kg}}} = 3.78\times10^5 \text{ m/s}$$

Thus, the correct answer is choice (b).

5. In a series combination of capacitors, the equivalent capacitance is always less than any individual capacitance in the combination, meaning that choice (a) is false. Also, for a series combination of capacitors, the magnitude of the charge is the same on all plates of capacitors in the combination, making both choices (d) and (e) false. The potential difference across the capacitance C_i is $\Delta V_i = Q/C_i$, where Q is the common charge on each capacitor in the combination. Thus, the largest potential difference (voltage) appears across the capacitor with the *least* capacitance, making choice (b) the correct answer.

6. The total potential at a point due to a set of point charges q_i is

$$V = \sum_i kq_i/r_i$$

where r_i is the distance from the point of interest to the location of the charge q_i. Note that in this case, the point at the center of the circle is equidistant from the 4 point charges located on the rim of the circle. Note also that $q_2 + q_3 + q_4 = (+1.5 - 1.0 - 0.5)\ \mu\text{C} = 0$, so we have

$$V_{\text{center}} = \frac{k_e q_1}{r} + \frac{k_e q_2}{r} + \frac{k_e q_3}{r} + \frac{k_e q_4}{r} = \frac{k_e}{r}\left(q_1 + q_2 + q_3 + q_4\right) = \frac{k_e}{r}\left(q_1 + 0\right) = \frac{k_e q_1}{r} = V_1$$
$$= 4.5\times10^4 \text{ V}$$

or the total potential at the center of the circle is just that due to the first charge alone, and the correct answer is choice (b).

7. With the given specifications, the capacitance of this parallel-plate capacitor will be

$$C = \frac{\kappa \in_0 A}{d} = \frac{\left(1.00\times10^2\right)\left(8.85\times10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(1.00 \text{ cm}^2\right)}{1.00\times10^{-3} \text{ m}}\left(\frac{1 \text{ m}^2}{10^4 \text{ cm}^2}\right)$$

$$= 8.85\times10^{-11} \text{ F} = 88.5\times10^{-12} \text{ F} = 88.5 \text{ pF}$$

and the correct choice is (a).

8. Keeping the capacitor connected to the battery means that the potential difference between the plates is kept at a constant value equal to the voltage of the battery. Since the capacitance of a parallel-plate capacitor is $C = \kappa \in_0 A/d$, doubling the plate separation d, while holding other characteristics of the capacitor constant, means the capacitance will be decreased by a factor of 2. The energy stored in a capacitor may be expressed as $U = \frac{1}{2}C(\Delta V)^2$, so when the potential difference ΔV is held constant while the capacitance is decreased by a factor of 2, the stored energy decreases by a factor of 2, making (c) the correct choice for this question.

9. When the battery is disconnected, there is no longer a path for charges to use in moving onto or off of the plates of the capacitor. This means that the charge Q is constant. The capacitance of a parallel-plate capacitor is $C = \kappa \in_0 A/d$ and the dielectric constant is $\kappa \approx 1$ when the capacitor is air filled. When a dielectric with dielectric constant $\kappa = 2$ is inserted between the plates, the capacitance is doubled $\left(C_f = 2C_i\right)$. Thus, with Q constant, the potential difference between the plates, $\Delta V = Q/C$, is decreased by a factor of 2, meaning that choice (a) is a true statement. The electric field between the plates of a parallel-plate capacitor is $E = \Delta V/d$ and decreases when ΔV decreases, making choice (e) false and leaving (a) as the only correct choice for this question.

10. Once the capacitor is disconnected from the battery, there is no path for charges to move onto or off of the plates, so the charges on the plates are constant, and choice (e) can be eliminated. The capacitance of a parallel-plate capacitor is $C = \kappa \in_0 A/d$, so the capacitance decreases when the plate separation d is increased. With Q constant and C decreasing, the energy stored in the capacitor, $U = Q^2/2C$, increases, making choice (a) false and choice (b) true. The potential difference between the plates, $\Delta V = Q/C = Q \cdot d/\kappa \in_0 A$, increases and the electric field between the plates, $E = \Delta V/d = Q/\kappa \in_0 A$, is constant. This means that both choices (c) and (d) are false and leaves choice (b) as the only correct response.

11. Capacitances connected in parallel all have the same potential difference across them and the equivalent capacitance, $C_{\text{eq}} = C_1 + C_2 + C_3 + \ldots$, is larger than the capacitance of any one of the capacitors in the combination. Thus, choice (c) is a true statement. The charge on a capacitor is $Q = C(\Delta V)$, so with ΔV constant, but the capacitances different, the capacitors all store different charges that are proportional to the capacitances, making choices (a), (b), (d), and (e) all false. Therefore, (c) is the only correct answer.

12. For a series combination of capacitors, the magnitude of the charge is the same on all plates of capacitors in the combination. Also, the equivalent capacitance is always less than any individual capacitance in the combination. Therefore, choice (a) is true while choices (b) and (c) are both false. The potential difference across a capacitor is $\Delta V = Q/C$, so with Q constant, capacitors having different capacitances will have different potential differences across them, with the largest potential difference being across the capacitor with the smallest capacitance. This means that choices (d) and (e) are false, and choice (f) is true. Thus, both choices (a) and (f) are true statements.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The potential energy between a pair of point charges separated by distance R is $PE = k_e\, q_1 q_2/R$. Thus, the potential energy for each of the four systems is:

(a) $PE_a = k_e \dfrac{Q(2Q)}{r} = 2k_e \dfrac{Q^2}{r}$ (b) $PE_b = k_e \dfrac{(-Q)(-Q)}{r} = k_e \dfrac{Q^2}{r}$

(c) $PE_c = k_e \dfrac{Q(-Q)}{2r} = -\dfrac{1}{2} k_e \dfrac{Q^2}{r}$ (d) $PE_d = k_e \dfrac{(-Q)(-2Q)}{2r} = k_e \dfrac{Q^2}{r}$

Therefore, the correct ranking from largest to smallest is $(a) > (b) = (d) > (c)$.

4. To move like charges together from an infinite separation, at which the potential energy of the system of two charges is zero, requires *work* to be done on the system by an outside agent. Hence energy is stored, and potential energy is positive. As charges with opposite signs move together from an infinite separation, energy is released, and the potential energy of the set of charges becomes negative.

6. A sharp point on a charged conductor would produce a large electric field in the region near the point. An electric discharge could most easily take place at the point.

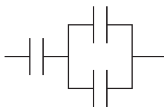
8. There are eight different combinations that use all three capacitors in the circuit. These combinations and their equivalent capacitances are:

All three capacitors in series: $C_{\text{eq}} = \left(\dfrac{1}{C_1} + \dfrac{1}{C_2} + \dfrac{1}{C_3}\right)^{-1}$

All three capacitors in parallel: $C_{\text{eq}} = C_1 + C_2 + C_3$

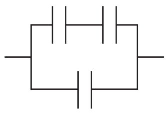
One capacitor in series with a parallel combination of the other two:

$$C_{\text{eq}} = \left(\frac{1}{C_1 + C_2} + \frac{1}{C_3}\right)^{-1},\ C_{\text{eq}} = \left(\frac{1}{C_3 + C_1} + \frac{1}{C_2}\right)^{-1},\ C_{\text{eq}} = \left(\frac{1}{C_2 + C_3} + \frac{1}{C_1}\right)^{-1}$$



One capacitor in parallel with a series combination of the other two:

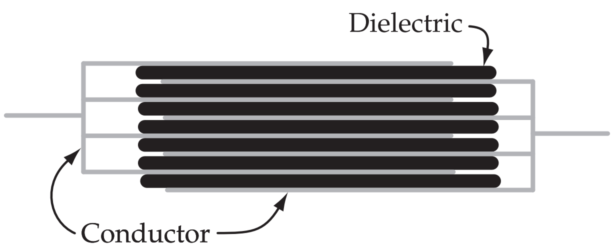
$$C_{\text{eq}} = \left(\frac{C_1 C_2}{C_1 + C_2}\right) + C_3,\ C_{\text{eq}} = \left(\frac{C_3 C_1}{C_3 + C_1}\right) + C_2,\ C_{\text{eq}} = \left(\frac{C_2 C_3}{C_2 + C_3}\right) + C_1$$



10. (a) If the wires are disconnected from the battery and not allowed to touch each other or another object, the charge on the plates is unchanged.

(b) If, after being disconnected from the battery, the wires are connected to each other, electrons will rapidly flow from the negatively charged plate to the positively charged plate to leave the capacitor uncharged with both plates neutral.

12. The primary choice would be the dielectric. You would want to choose a dielectric that has a large dielectric constant and dielectric strength, such as strontium titanate, where $\kappa \approx 233$ (Table 16.1). A convenient choice could be thick plastic or Mylar. Secondly, geometry would be a factor. To maximize capacitance, one would want the individual plates as close as possible, since the capacitance is proportional to the inverse of the plate separation—hence the need for a dielectric with a high dielectric strength. Also, one would want to build, instead of a single parallel-plate capacitor, several capacitors in parallel. This could be achieved through "stacking" the plates of the capacitor. For example, you can alternately lay down sheets of a conducting material, such as aluminum foil, sandwiched between your sheets of insulating dielectric. Making sure that none of the conducting sheets are in contact with their nearest neighbors, connect every other plate together as illustrated in the figure below.



This technique is often used when "home-brewing" signal capacitors for radio applications, as they can withstand huge potential differences without flashover (without either discharge between plates around the dielectric or dielectric breakdown). One variation on this technique is to sandwich together flexible materials such as aluminum roof flashing and thick plastic, so the whole product can be rolled up into a "capacitor burrito" and placed in an insulating tube, such as a PVC pipe, and then filled with motor oil (again to prevent flashover).

14. The material of the dielectric may be able to withstand a larger electric field than air can withstand before breaking down to pass a spark between the capacitor plates.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 6.16×10^{-17} N

(b) 3.69×10^{10} m/s^2 in the direction of the electric field

(c) 7.38 cm

4. 6.67×10^{11} electrons

6. **(a)** 1.10×10^{-2} N to the right **(b)** 1.98×10^{-3} J **(c)** -1.98×10^{-3} J

(d) -49.5 V

8. **(a)** -2.31 kV **(b)** Protons would require a greater potential difference.

(c) $\Delta V_{\mathrm{p}}/\Delta V_{\mathrm{e}} = -m_{\mathrm{p}}/m_{\mathrm{e}}$

10. 40.2 kV

12. **(a)** +5.39 kV **(b)** +10.8 kV

14. –9.08 J

16. **(a)** See Solution. **(b)** $3k_e q/a$ **(c)** See Solution.

18. **(a)** See Solution. **(b)** $V = (22.5\ \mathrm{V \cdot m})\left(\dfrac{1}{1.20\ \mathrm{m} - x} - \dfrac{2}{x}\right)$

(c) –37.5 V **(d)** $x = 0.800$ m

20. **(a)** Conservation of energy alone yields one equation with two unknowns.

(b) Conservation of linear momentum

(c) $v_p = 1.05 \times 10^7$ m/s, $v_\alpha = 2.64 \times 10^6$ m/s

22. 5.4×10^5 V

24. **(a)** $V = 4\sqrt{2}k_e Q/a$ **(b)** $W = 4\sqrt{2}k_e qQ/a$

26. **(a)** 3.00 μF **(b)** 36.0 μC

28. **(a)** 1.00 μF **(b)** 100 V

30. 31.0 Å

32. 1.23 kV

34. **(a)** 17.0 μF **(b)** 9.00 V

(c) 45.0 μC on C_1, 108 μC on C_2

36. 3.00 pF and 6.00 pF

38. **(a)** 6.00 μF **(b)** 12.0 μF **(c)** 432 μC

(d) $Q_4 = 144\ \mu\text{C},\ Q_2 = 72.0\ \mu\text{C},\ Q_{\substack{\text{rightmost}\\ \text{branch}}} = 216\ \mu\text{C}$ **(e)** $Q_{24} = Q_8 = 216\ \mu\text{C}$

(f) 9.00 V **(g)** 27.0 V

40. **(a)** $2C$ **(b)** $Q_1 > Q_3 > Q_2$ **(c)** $\Delta V_1 > \Delta V_2 = \Delta V_3$

(d) Q_1 and Q_3 increase, Q_2 decreases

42. **(a)** 6.04 μF **(b)** 83.6 μC

44. **(a)** 5.96 μF

(b) 89.4 μC on the 20.0 μF capacitor, 63.0 μC on the 6.00 μF capacitor, 26.3 μC on the 15.0 μF capacitor, and 26.3 μC on the 3.00 μF capacitor

46. **(a)** $C_{\text{eq}} = 12.0\ \mu\text{F},\ E_{\text{stored,total}} = 8.64 \times 10^{-4}\ \text{J}$

(b) $E_{\text{stored},1} = 5.76 \times 10^{-4}\ \text{J},\ E_{\text{stored},2} = 2.88 \times 10^{-4}\ \text{J}$

It will always be true that $E_{\text{stored},1} + E_{\text{stored},2} = E_{\text{stored,total}}$.

(c) 5.66 V; C_2, with the largest capacitance, stores the most energy.

48. 9.79 kg

50. **(a)** 13.3 nC **(b)** 272 nC

52. 1.04 m

54. 0.443 mm

56. **(a)** 13.5 mJ

(b) $E_{\text{stored},2} = 3.60\ \text{mJ},\ E_{\text{stored},3} = 5.40\ \text{mJ},\ E_{\text{stored},4} = 1.80\ \text{mJ},\ E_{\text{stored},6} = 2.70\ \text{mJ}$

(c) The energy stored in the equivalent capacitance equals the sum of the energies stored in the individual capacitors.

58. $C_1 = \frac{1}{2}C_p \pm \sqrt{\frac{1}{4}C_p^2 - C_pC_s}, \quad C_2 = \frac{1}{2}C_p \mp \sqrt{\frac{1}{4}C_p^2 - C_pC_s}$

60. **(a)** $1.8 \times 10^4\ \text{V}$ **(b)** $-3.6 \times 10^4\ \text{V}$ **(c)** $-1.8 \times 10^4\ \text{V}$

(d) $-5.4 \times 10^{-2}\ \text{J}$

62. **(a)** $C = \frac{ab}{k_e(b-a)}$ **(b)** See Solution.

64. $\kappa = 2.33$

66. **(a)** $\dfrac{2k_e q}{d\sqrt{5}}$ **(b)** $\dfrac{4k_e q^2}{d\sqrt{5}}$ **(c)** $\dfrac{4k_e q^2}{d\sqrt{5}}$

(d) $\sqrt{\dfrac{8k_e q^2}{md\sqrt{5}}}$

68. **(a)** 0.1 mm **(b)** 4.4 mm

PROBLEM SOLUTIONS

16.1 (a) Because the electron has a negative charge, it experiences a force in the direction opposite to the field and, when released from rest, will move in the negative x-direction. The work done on the electron by the field is

$$W = F_x(\Delta x) = (qE_x)\Delta x = (-1.60\times10^{-19}\text{ C})(375\text{ N/C})(-3.20\times10^{-2}\text{ m})$$
$$= \boxed{1.92\times10^{-18}\text{ J}}$$

(b) The change in the electric potential energy is the negative of the work done on the particle by the field. Thus,

$$\Delta PE = -W = \boxed{-1.92\times10^{-18}\text{ J}}$$

(c) Since the Coulomb force is a conservative force, conservation of energy gives $\Delta KE + \Delta PE = 0$, or $KE_f = \frac{1}{2}m_e v_f^2 = KE_i - \Delta PE = 0 - \Delta PE$, and

$$v_f = \sqrt{\frac{-2(\Delta PE)}{m_e}} = \sqrt{\frac{-2(-1.92\times10^{-18}\text{ J})}{9.11\times10^{-31}\text{ kg}}} = \boxed{2.05\times10^{6}\text{ m/s in the } -x\text{-direction}}$$

16.2 (a) $F = qE = (1.60\times10^{-19}\text{ C})(385\text{ N/C}) = \boxed{6.16\times10^{-17}\text{ N}}$

(b) $a = \dfrac{F}{m_p} = \dfrac{6.16\times10^{-17}\text{ N}}{1.67\times10^{-27}\text{ kg}} = \boxed{3.69\times10^{10}\text{ m/s}^2\text{ in the direction of the electric field}}$

(c) $\Delta x = v_0 t + \dfrac{1}{2}at^2 = 0 + \dfrac{1}{2}(3.69\times10^{10}\text{ m/s}^2)(2.00\times10^{-6}\text{ s})^2$

$= 7.38\times10^{-2}\text{ m} = \boxed{7.38\text{ cm}}$

16.3 The work done by the agent moving the charge out of the cell is

$$W_{\text{input}} = -W_{\text{field}} = -(-\Delta PE_e) = +q(\Delta V)$$
$$= (1.60\times10^{-19}\text{ C})(+90\times10^{-3}\text{ J/C}) = \boxed{1.4\times10^{-20}\text{ J}}$$

16.4 Assuming the sphere is isolated, the excess charge on it is uniformly distributed over its surface. Under this spherical symmetry, the electric field outside the sphere is the same as if all the excess charge on the sphere were concentrated as a point charge located at the center of the sphere.

continued on next page

Thus, at $r = 8.00\text{ cm} > R_{\text{sphere}} = 5.00\text{ cm}$, the electric field is $E = k_e Q/r^2$. The required charge then has magnitude $|Q| = Er^2/k_e$, and the number of electrons needed is

$$n = \frac{|Q|}{e} = \frac{Er^2}{k_e e} = \frac{(1.50\times10^5\ \text{N/C})(8.00\times10^{-2}\ \text{m})^2}{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})} = \boxed{6.67\times10^{11}\ \text{electrons}}$$

16.5 $E = \dfrac{|\Delta V|}{d} = \dfrac{25\times10^3\ \text{J/C}}{1.5\times10^{-2}\ \text{m}} = \boxed{1.7\times10^6\ \text{N/C}}$

16.6 (a) $\vec{\mathbf{F}} = q\vec{\mathbf{E}} = (+40.0\times10^{-6}\ \text{C})(+275\ \text{N/C}) = \boxed{1.10\times10^{-2}\ \text{N directed toward the right}}$

(b) $W_{AB} = F(\Delta x)\cos\theta = (1.10\times10^{-2}\ \text{N})(0.180\ \text{m})\cos 0° = \boxed{1.98\times10^{-3}\ \text{J}}$

(c) $\Delta PE = -W_{AB} = \boxed{-1.98\times10^{-3}\ \text{J}}$

(d) $\Delta V = V_B - V_A = \dfrac{\Delta PE}{q} = \dfrac{-1.98\times10^{-3}\ \text{J}}{+40.0\times10^{-6}\ \text{C}} = \boxed{-49.5\ \text{V}}$

16.7 (a) $E = \dfrac{|\Delta V|}{d} = \dfrac{600\ \text{J/C}}{5.33\times10^{-3}\ \text{m}} = \boxed{1.13\times10^5\ \text{N/C}}$

(b) $F = |q|E = \dfrac{|q||\Delta V|}{d} = \dfrac{(1.60\times10^{-19}\ \text{C})(600\ \text{J/C})}{5.33\times10^{-3}\ \text{m}} = \boxed{1.80\times10^{-14}\ \text{N}}$

(c) $W = F\cdot s\cos\theta$

$$= (1.80\times10^{-14}\ \text{N})\left[(5.33-2.90)\times10^{-3}\ \text{m}\right]\cos 0° = \boxed{4.37\times10^{-17}\ \text{J}}$$

16.8 (a) Using conservation of energy, $\Delta KE + \Delta PE = 0$, with $KE_f = 0$ since the particle is "stopped," we have

$$\Delta PE = -\Delta KE = -\left(0 - \frac{1}{2}m_e v_i^2\right) = +\frac{1}{2}(9.11\times10^{-31}\ \text{kg})(2.85\times10^7\ \text{m/s})^2$$
$$= +3.70\times10^{-16}\ \text{J}$$

The required stopping potential is then

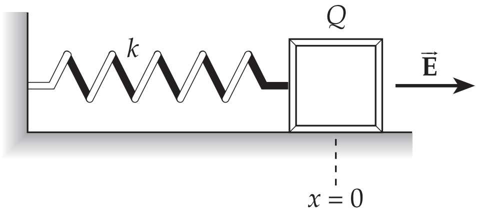
$$\Delta V = \frac{\Delta PE}{q} = \frac{+3.70\times10^{-16}\ \text{J}}{-1.60\times10^{-19}\ \text{C}} = -2.31\times10^3\ \text{V} = \boxed{-2.31\ \text{kV}}$$

(b) Being more massive than electrons, protons traveling at the same initial speed will have more initial kinetic energy and $\boxed{\text{require a greater magnitude stopping potential}}$.

(c) Since $\Delta V_{\text{stopping}} = \Delta PE/q = (-\Delta KE)/q = (-mv^2/2)/q$, the ratio of the stopping potential for a proton to that for an electron having the same initial speed is

$$\frac{\Delta V_p}{\Delta V_e} = \frac{-m_p v_i^2/2(+e)}{-m_e v_i^2/2(-e)} = \boxed{-m_p/m_e}$$

16.9 (a) We use conservation of energy, $\Delta(KE)+\Delta(PE_s)+\Delta(PE_e)=0$, recognizing that $\Delta(KE)=0$ since the block is at rest at both the beginning and end of the motion. The change in the elastic potential energy is given by $\Delta(PE_s)=\frac{1}{2}kx_{\max}^2-0$, where $x_{\max}$ is the maximum stretch of the spring. The change in the electrical potential energy is the negative of the work the electric field does, $\Delta(PE_e)=-W=-F_e(\Delta x)=-(QE)x_{\max}$. Thus, $0+\frac{1}{2}kx_{\max}^2-(QE)x_{\max}=0$, which yields



$$x_{\max}=\frac{2QE}{k}=\frac{2(35.0\times10^{-6}\text{ C})(4.86\times10^{4}\text{ V/m})}{78.0\text{ N/m}}=4.36\times10^{-2}\text{ m}=\boxed{4.36\text{ cm}}$$

(b) At equilibrium, $\Sigma F=F_s+F_e=0$, or $-kx_{\text{eq}}+QE=0$. Therefore,

$$x_{\text{eq}}=\frac{QE}{k}=\frac{1}{2}x_{\max}=\boxed{2.18\text{ cm}}$$

The amplitude is the distance from the equilibrium position to each of the turning points (at $x=0$ and $x=4.36$ cm), so $\boxed{A=2.18\text{ cm}=x_{\max}/2}$.

(c) From conservation of energy, $\Delta(KE)+\Delta(PE_s)+\Delta(PE_e)=0$, we have $0+\frac{1}{2}kx_{\max}^2+Q\Delta V=0$. Since $x_{\max}=2A$, this gives

$$\Delta V=-\frac{kx_{\max}^2}{2Q}=-\frac{k(2A)^2}{2Q}\qquad\text{or}\qquad\boxed{\Delta V=-\frac{2kA^2}{Q}}$$

16.10 Using $\Delta y=v_{0y}t+\frac{1}{2}a_yt^2$ for the full flight gives $0=v_{0y}t_f+\frac{1}{2}a_yt_f^2$, or $a_y=-2v_{0y}/t_f$, where t_f is the full time of the flight. Then, using $v_y^2=v_{0y}^2+2a_y(\Delta y)$ for the upward part of the flight gives

$$(\Delta y)_{\max}=\frac{0-v_{0y}^2}{2a_y}=\frac{-v_{0y}^2}{2(-2v_{0y}/t_f)}=\frac{v_{0y}t_f}{4}=\frac{(20.1\text{ m/s})(4.10\text{ s})}{4}=20.6\text{ m}$$

From Newton's second law,

$$a_y=\frac{\Sigma F_y}{m}=\frac{-mg-qE}{m}=-\left(g+\frac{qE}{m}\right)$$

Equating this to the earlier result gives $a_y=-(g+qE/m)=-2v_{0y}/t_f$, so the electric field strength is

$$E=\left(\frac{m}{q}\right)\left[\frac{2v_{0y}}{t_f}-g\right]=\left(\frac{2.00\text{ kg}}{5.00\times10^{-6}\text{ C}}\right)\left[\frac{2(20.1\text{ m/s})}{4.10\text{ s}}-9.80\text{ m/s}^2\right]=1.95\times10^{3}\text{ N/C}$$

Thus, $(\Delta V)_{\max}=(\Delta y_{\max})E=(20.6\text{ m})(1.95\times10^{3}\text{ N/C})=4.02\times10^{4}\text{ V}=\boxed{40.2\text{ kV}}$

16.11 (a) $$V_A=\frac{k_eq}{r_A}=\frac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(-1.60\times10^{-19}\text{ C})}{0.250\times10^{-2}\text{ m}}=\boxed{-5.75\times10^{-7}\text{ V}}$$

(b) $$V_B=\frac{k_eq}{r_B}=\frac{(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2)(-1.60\times10^{-19}\text{ C})}{0.750\times10^{-2}\text{ m}}=\boxed{-1.92\times10^{-7}\text{ V}}$$

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$$\Delta V = V_B - V_A = -1.92\times10^{-7}\ \text{V} - \left(-5.75\times10^{-7}\ \text{V}\right) = \boxed{+3.83\times10^{-7}\ \text{V}}$$

(c) $\boxed{\text{No}}$. The original electron will be repelled by the negatively charged particle which suddenly appears at point A. Unless the electron is fixed in place, it will move in the opposite direction, away from points A and B, thereby lowering the potential difference between these points.

16.12 (a) $V_A = \sum_i \dfrac{k_e q_i}{r_i} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\dfrac{-15.0\times10^{-9}\ \text{C}}{2.00\times10^{-2}\ \text{m}} + \dfrac{27.0\times10^{-9}\ \text{C}}{2.00\times10^{-2}\ \text{m}}\right) = \boxed{+5.39\ \text{kV}}$

(b) $V_B = \sum_i \dfrac{k_e q_i}{r_i} = \left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(\dfrac{-15.0\times10^{-9}\ \text{C}}{1.00\times10^{-2}\ \text{m}} + \dfrac{27.0\times10^{-9}\ \text{C}}{1.00\times10^{-2}\ \text{m}}\right) = \boxed{+10.8\ \text{kV}}$

16.13 (a) Calling the 2.00 μC charge q_3,

$$V = \sum_i \frac{k_e q_i}{r_i} = k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{\sqrt{r_1^2 + r_2^2}}\right)$$

$$= \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{8.00\times10^{-6}\ \text{C}}{0.060\ 0\ \text{m}} + \frac{4.00\times10^{-6}\ \text{C}}{0.030\ 0\ \text{m}} + \frac{2.00\times10^{-6}\ \text{C}}{\sqrt{(0.060\ 0)^2 + (0.030\ 0)^2}\ \text{m}}\right)$$

$$V = \boxed{2.67\times10^6\ \text{V}}$$

(b) Replacing 2.00×10^{-6} C by -2.00×10^{-6} C in part (a) yields

$$V = \boxed{2.13\times10^6\ \text{V}}$$

16.14 $W = q(\Delta V) = q\left(V_f - V_i\right)$, and $V_f = 0$ since the final location of the 8.00 μC is an infinite distance from other charges. The potential, due to the other charges, at the initial location of the 8.00 μC is $V_i = k_e\left(q_1/r_1 + q_2/r_2\right)$. Thus, the required energy for the move is

$$W = q\left[0 - k_e\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)\right]$$

$$= -\left(8.00\times10^{-6}\ \text{C}\right)\left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\frac{2.00\times10^{-6}\ \text{C}}{0.030\ 0\ \text{m}} + \frac{4.00\times10^{-6}\ \text{C}}{\sqrt{(0.030\ 0)^2 + (0.060\ 0)^2}\ \text{m}}\right)$$

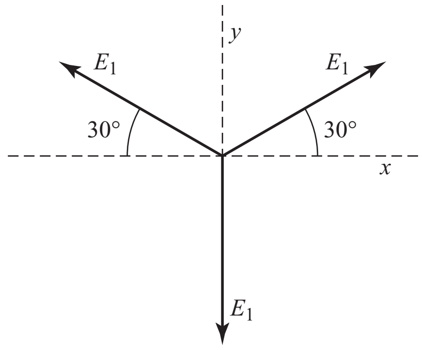
$$W = \boxed{-9.08\ \text{J}}$$

16.15 (a) $V = \sum_i \dfrac{k_e q_i}{r_i} = \left(8.99\times10^9\ \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(\dfrac{5.00\times10^{-9}\ \text{C}}{0.175\ \text{m}} - \dfrac{3.00\times10^{-9}\ \text{C}}{0.175\ \text{m}}\right) = \boxed{103\ \text{V}}$

(b) $PE = \dfrac{k_e q_1 q_2}{r_{12}} = \left(8.99\times10^9\ \dfrac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\dfrac{\left(5.00\times10^{-9}\ \text{C}\right)\left(-3.00\times10^{-9}\ \text{C}\right)}{0.350\ \text{m}} = \boxed{-3.85\times10^{-7}\ \text{J}}$

The negative sign means that $\boxed{\text{positive work must be done}}$ to separate the charges by an infinite distance (that is, bring them up to a state of zero potential energy).

16.16 (a) At the center of the triangle, each of the identical charges produce a field contribution of magnitude $E_1 = k_e q/a^2$. The three contributions are oriented as shown at the right and the components of the resultant field are:



$$E_x = \Sigma E_x = +E_1 \cos 30° - E_1 \cos 30° = 0$$

$$E_y = \Sigma E_y = +E_1 \sin 30° - E_1 + E_1 \sin 30° = 0$$

Thus, the resultant field has magnitude

$$E = \sqrt{E_x^2 + E_y^2} = \boxed{0}$$

(b) The total potential at the center of the triangle is

$$V = \Sigma V_i = \Sigma \frac{k_e q_i}{r_i} = \frac{k_e q}{a} + \frac{k_e q}{a} + \frac{k_e q}{a} = \boxed{\frac{3k_e q}{a}}$$

(c) Imagine a test charge placed at the center of the triangle. Since the field is zero at the center, the test charge will experience no electrical force at that point. The fact that the potential is not zero at the center means that work would have to be done by an external agent to move a test charge from infinity to the center.

16.17 The Pythagorean theorem gives the distance from the midpoint of the base to the charge at the apex of the triangle as

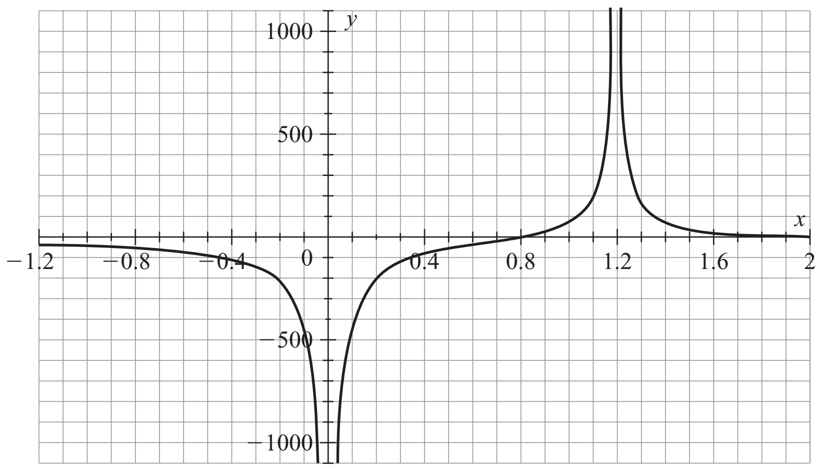
$$r_3 = \sqrt{(4.00 \text{ cm})^2 - (1.00 \text{ cm})^2} = \sqrt{15} \text{ cm} = \sqrt{15} \times 10^{-2} \text{ m}$$

Then, the potential at the midpoint of the base is $V = \sum_i k_e q_i / r_i$, or

$$V = \left(8.99 \times 10^9 \ \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}\right)\left(\frac{(-7.00 \times 10^{-9} \text{ C})}{0.010\,0 \text{ m}} + \frac{(-7.00 \times 10^{-9} \text{ C})}{0.010\,0 \text{ m}} + \frac{(+7.00 \times 10^{-9} \text{ C})}{\sqrt{15} \times 10^{-2} \text{ m}}\right)$$

$$= -1.10 \times 10^4 \text{ V} = \boxed{-11.0 \text{ kV}}$$

16.18 (a) See the sketch below:



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(b) At the point $(x, 0)$, where $0 < x < 1.20$ m, the potential is

$$V = \sum_i \frac{k_e q_i}{r_i} = \frac{k_e(-2q)}{x} + \frac{k_e q}{1.20\text{ m} - x} = k_e q\left(\frac{1}{1.20\text{ m} - x} - \frac{2}{x}\right)$$

or

$$V = \left(8.99\times10^{9}\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(2.50\times10^{-9}\text{ C}\right)\left(\frac{1}{1.20\text{ m} - x} - \frac{2}{x}\right) = \boxed{(22.5\text{ V}\cdot\text{m})\left(\frac{1}{1.20\text{ m} - x} - \frac{2}{x}\right)}$$

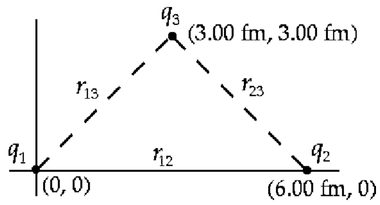
(c) At $x = +\,0.600$ m, the potential is

$$V = (22.5\text{ V}\cdot\text{m})\left(\frac{1}{1.20\text{ m} - 0.600\text{ m}} - \frac{2}{0.600\text{ m}}\right) = -\frac{22.5\text{ V}\cdot\text{m}}{0.600\text{ m}} = \boxed{-37.5\text{ V}}$$

(d) When $0 < x < 1.20$ m and $V = 0$, we have $1/(1.20\text{ m} - x) - 2/x = 0$, or $x = 2.40\text{ m} - 2x$.

This yields $x = 2.40\text{ m}/3 = \boxed{0.800\text{ m}}$.

16.19 (a) When the charge configuration consists of only the two protons (q_1 and q_2 in the sketch), the potential energy of the configuration is



$$PE_a = \frac{k_e q_1 q_2}{r_{12}} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{6.00\times10^{-15}\text{ m}}$$

or $PE_a = \boxed{3.84\times10^{-14}\text{ J}}$

(b) When the alpha particle (q_3 in the sketch) is added to the configuration, there are three distinct pairs of particles, each of which possesses potential energy. The total potential energy of the configuration is now

$$PE_b = \frac{k_e q_1 q_2}{r_{12}} + \frac{k_e q_1 q_3}{r_{13}} + \frac{k_e q_2 q_3}{r_{23}} = PE_a + 2\left(\frac{k_e\left(2e^2\right)}{r_{13}}\right)$$

where use has been made of the facts that $q_1 q_3 = q_2 q_3 = e(2e) = 2e^2$ and

$r_{13} = r_{23} = \sqrt{(3.00\text{ fm})^2 + (3.00\text{ fm})^2} = \sqrt{18.0}\text{ fm} = \sqrt{18.0}\times10^{-15}$ m. Also, note that the first term in this computation is just the potential energy computed in part (a). Thus,

$$PE_b = PE_a + \frac{4k_e e^2}{r_{13}}$$

$$= 3.84\times10^{-14}\text{ J} + \frac{4\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{\sqrt{18.0}\times10^{-15}\text{ m}} = \boxed{2.55\times10^{-13}\text{ J}}$$

(c) If we start with the three-particle system of part (b) and allow the alpha particle to escape to infinity [thereby returning us to the two-particle system of part (a)], the change in electric potential energy will be

$$\Delta PE = PE_a - PE_b = 3.84\times10^{-14}\text{ J} - 2.55\times10^{-13}\text{ J} = \boxed{-2.17\times10^{-13}\text{ J}}$$

(d) Conservation of energy, $\Delta KE + \Delta PE = 0$, gives the speed of the alpha particle at infinity in the situation of part (c) as $\frac{1}{2}m_\alpha v_\alpha^2 - 0 = -\Delta PE$, or

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$$v_\alpha = \sqrt{\frac{-2(\Delta PE)}{m_\alpha}} = \sqrt{\frac{-2\left(-2.17\times10^{-13}\ \text{J}\right)}{6.64\times10^{-27}\ \text{kg}}} = \boxed{8.08\times10^{6}\ \text{m/s}}$$

(e) When, starting with the three-particle system, the two protons are both allowed to escape to infinity, there will be no remaining pairs of particles and hence no remaining potential energy. Thus, $\Delta PE = 0 - PE_b = -PE_b$, and conservation of energy gives the change in kinetic energy as $\Delta KE = -\Delta PE = +PE_b$. Since the protons are identical particles, this increase in kinetic energy is split equally between them giving $KE_{\text{proton}} = \frac{1}{2}m_p v_p^2 = \frac{1}{2}\left(PE_b\right)$,

or $$v_p = \sqrt{\frac{PE_b}{m_p}} = \sqrt{\frac{2.55\times10^{-13}\ \text{J}}{1.67\times10^{-27}\ \text{kg}}} = \boxed{1.24\times10^{7}\ \text{m/s}}$$

16.20 (a) If a proton and an alpha particle, initially at rest 4.00 fm apart, are released and allowed to recede to infinity, the final speeds of the two particles will differ because of the difference in the masses of the particles. Thus, attempting to solve for the final speeds by use of conservation of energy alone leads to a situation of having $\boxed{\text{one equation with two unknowns}}$, and does not permit a solution.

(b) In the situation described in part (a) above, one can obtain a second equation with the two unknown final speeds by using $\boxed{\text{conservation of linear momentum.}}$ Then, one would have two equations which could be solved simultaneously for both unknowns.

(c) From conservation of energy: $\left[\left(\frac{1}{2}m_\alpha v_\alpha^2 + \frac{1}{2}m_p v_p^2\right) - 0\right] + \left[0 - k_e q_\alpha q_p / r_i\right] = 0$, or

$$m_\alpha v_\alpha^2 + m_p v_p^2 = \frac{2k_e q_\alpha q_p}{r_i} = \frac{2\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(3.20\times10^{-19}\ \text{C}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{4.00\times10^{-15}\ \text{m}}$$

yielding $m_\alpha v_\alpha^2 + m_p v_p^2 = 2.30\times10^{-13}\ \text{J}$ **[1]**

From conservation of linear momentum,

$$m_\alpha v_\alpha + m_p v_p = 0 \qquad \text{or} \qquad |v_\alpha| = \left(\frac{m_p}{m_\alpha}\right)v_p \qquad \textbf{[2]}$$

Substituting Equation [2] into Equation [1] gives

$$m_\alpha\left(\frac{m_p}{m_\alpha}\right)^2 v_p^2 + m_p v_p^2 = 2.30\times10^{-13}\ \text{J} \qquad \text{or} \qquad \left(\frac{m_p}{m_\alpha}+1\right)m_p v_p^2 = 2.30\times10^{-13}\ \text{J}$$

and

$$v_p = \sqrt{\frac{2.30\times10^{-13}\ \text{J}}{\left(m_p/m_\alpha+1\right)m_p}} = \sqrt{\frac{2.30\times10^{-13}\ \text{J}}{\left(1.67\times10^{-27}/6.64\times10^{-27}+1\right)\left(1.67\times10^{-27}\ \text{kg}\right)}} = \boxed{1.05\times10^{7}\ \text{m/s}}$$

Then, Equation [2] gives the final speed of the alpha particle as

$$|v_\alpha| = \left(\frac{m_p}{m_\alpha}\right)v_p = \left(\frac{1.67\times10^{-27}\ \text{kg}}{6.64\times10^{-27}\ \text{kg}}\right)\left(1.05\times10^{7}\ \text{m/s}\right) = \boxed{2.64\times10^{6}\ \text{m/s}}$$

16.21 (a) Conservation of energy gives

$$KE_f = KE_i + \left(PE_i - PE_f\right) = 0 + k_e q_1 q_2 \left(\frac{1}{r_i} - \frac{1}{r_f}\right)$$

With $q_1 = +8.50$ nC, $q_2 = -2.80$ nC, $r_i = 1.60\ \mu$m, and $r_f = 0.500\ \mu$m, this becomes

$$KE_f = \left(8.99\times10^9\ \frac{\text{N}\cdot\text{m}^2}{\text{C}^2}\right)\left(8.50\times10^{-9}\ \text{C}\right)\left(-2.80\times10^{-9}\ \text{C}\right)\left(\frac{1}{1.60\times10^{-6}\ \text{m}} - \frac{1}{0.500\times10^{-6}\ \text{m}}\right)$$

yielding $KE_f = \boxed{0.294\ \text{J}}$

(b) When $r = r_f = 0.500\ \mu$m and $KE = KE_f = 0.294$ J, the speed of the sphere having mass $m = 8.00\ \text{mg} = 8.00\times10^{-6}$ kg is

$$v_f = \sqrt{\frac{2\left(KE_f\right)}{m}} = \sqrt{\frac{2(0.294\ \text{J})}{8.00\times10^{-6}\ \text{kg}}} = \boxed{271\ \text{m/s}}$$

16.22 The excess charge on the metal sphere will be uniformly distributed over its surface. In this spherically symmetric situation, the electric field and the electric potential outside the sphere is the same as if all the excess charge were concentrated as a point charge at the center of the sphere. Thus, for points outside the sphere,

$$E = k_e \frac{Q}{r^2} \qquad \text{and} \qquad V = k_e \frac{Q}{r} = E\cdot r$$

If the sphere has a radius of $r = 18\ \text{cm} = 0.18$ m and the air breaks down when $E = 3.0\times10^6\ \text{V/m}$, the electric potential at the surface of the sphere when breakdown occurs is

$$V = \left(3.0\times10^6\ \text{V/m}\right)(0.18\ \text{m}) = \boxed{5.4\times10^5\ \text{V}}$$

16.23 From conservation of energy, $\left(KE + PE_e\right)_f = \left(KE + PE_e\right)_i$, which gives $0 + k_e Qq/r_f = \frac{1}{2} m_\alpha v_i^2 + 0$, or

$$r_f = \frac{2k_e Qq}{m_\alpha v_i^2} = \frac{2k_e(79e)(2e)}{m_\alpha v_i^2}$$

$$r_f = \frac{2\left(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)(158)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(6.64\times10^{-27}\ \text{kg}\right)\left(2.00\times10^7\ \text{m/s}\right)^2} = \boxed{2.74\times10^{-14}\ \text{m}}$$

16.24 (a) The distance from any one of the corners of the square to the point at the center is one half the length of the diagonal of the square, or

$$r = \frac{diagonal}{2} = \frac{\sqrt{a^2 + a^2}}{2} = \frac{a\sqrt{2}}{2} = \frac{a}{\sqrt{2}}$$

Since the charges have equal magnitudes and are all the same distance from the center of the square, they make equal contributions to the total potential. Thus,

$$V_{\text{total}} = 4V_{\substack{\text{single}\\\text{charge}}} = 4\frac{k_e Q}{r} = 4\frac{k_e Q}{a/\sqrt{2}} = \boxed{4\sqrt{2}k_e \frac{Q}{a}}$$

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(b) The work required to carry charge q from infinity to the point at the center of the square is equal to the increase in the electric potential energy of the charge, or

$$W = PE_{\text{center}} - PE_{\infty} = qV_{\text{total}} - 0 = q\left(4\sqrt{2}k_e \frac{Q}{a}\right) = \boxed{4\sqrt{2}k_e \frac{qQ}{a}}$$

16.25 (a) $C = \epsilon_0 \dfrac{A}{d} = \left(8.85\times10^{-12}\ \dfrac{\text{C}^2}{\text{N}\cdot\text{m}^2}\right)\dfrac{\left(1.0\times10^6\ \text{m}^2\right)}{(800\ \text{m})} = \boxed{1.1\times10^{-8}\ \text{F}}$

(b) $Q_{\max} = C(\Delta V)_{\max} = C\left(E_{\max}d\right) = \epsilon_0 \dfrac{A}{\cancel{d}}\left(E_{\max}\cancel{d}\right) = \epsilon_0\ AE_{\max}$

$$= \left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(1.0\times10^6\ \text{m}^2\right)\left(3.0\times10^6\ \text{N/C}\right) = \boxed{27\ \text{C}}$$

16.26 (a) $C = \dfrac{Q}{\Delta V} = \dfrac{27.0\ \mu\text{C}}{9.00\ \text{V}} = \boxed{3.00\ \mu\text{F}}$

(b) $Q = C(\Delta V) = (3.00\ \mu\text{F})(12.0\ \text{V}) = \boxed{36.0\ \mu\text{C}}$

16.27 (a) The capacitance of this air-filled (dielectric constant, $\kappa = 1.00$) parallel-plate capacitor is

$$C = \frac{k\,\epsilon_0\,A}{d} = \frac{(1.00)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(2.30\times10^{-4}\ \text{m}^2\right)}{1.50\times10^{-3}\ \text{m}} = 1.36\times10^{-12}\ \text{F} = \boxed{1.36\ \text{pF}}$$

(b) $Q = C(\Delta V) = \left(1.36\times10^{-12}\ \text{F}\right)(12.0\ \text{V}) = 1.63\times10^{-11}\ \text{C} = 16.3\times10^{-12}\ \text{C} = \boxed{16.3\ \text{pC}}$

(c) $E = \dfrac{\Delta V}{d} = \dfrac{12.0\ \text{V}}{1.50\times10^{-3}\ \text{m}} = \boxed{8.00\times10^3\ \text{V/m}} = \boxed{8.00\times10^3\ \text{N/C}}$

16.28 (a) $C = \dfrac{Q}{V} = \dfrac{10.0\ \mu\text{C}}{10.0\ \text{V}} = \boxed{1.00\ \mu\text{F}}$

(b) $V = \dfrac{Q}{C} = \dfrac{100\ \mu\text{C}}{1.00\ \mu\text{F}} = \boxed{100\ \text{V}}$

16.29 (a) $E = \dfrac{\Delta V}{d} = \dfrac{20.0\ \text{V}}{1.80\times10^{-3}\ \text{m}} = 1.11\times10^4\ \text{V/m} = \boxed{11.1\ \text{kV/m}}$ toward the negative plate

(b) $C = \dfrac{\epsilon_0\ A}{d} = \dfrac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(7.60\times10^{-4}\ \text{m}^2\right)}{1.80\times10^{-3}\ \text{m}} = 3.74\times10^{-12}\ \text{F} = \boxed{3.74\ \text{pF}}$

(c) $Q = C(\Delta V) = \left(3.74\times10^{-12}\ \text{F}\right)(20.0\ \text{V}) = 7.48\times10^{-11}\ \text{C} = \boxed{74.8\ \text{pC}}$ on one plate and $\boxed{-74.8\ \text{pC}}$ on the other plate.

16.30 $C = \epsilon_0\ A/d$, so

$$d = \frac{\epsilon_0\ A}{C} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(21.0\times10^{-12}\ \text{m}^2\right)}{60.0\times10^{-15}\ \text{F}} = 3.10\times10^{-9}\ \text{m}$$

$$d = \left(3.10\times10^{-9}\ \text{m}\right)\left(\frac{1\ \text{Å}}{10^{-10}\ \text{m}}\right) = \boxed{31.0\ \text{Å}}$$

16.31 (a) Assuming the capacitor is air-filled $(\kappa = 1)$, the capacitance is

$$C = \frac{\epsilon_0 A}{d} = \frac{\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(0.200\ \text{m}^2\right)}{3.00\times10^{-3}\ \text{m}} = \boxed{5.90\times10^{-10}\ \text{F}}$$

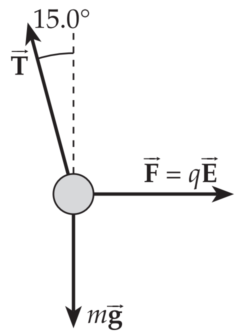
(b) $Q = C(\Delta V) = \left(5.90\times10^{-10}\ \text{F}\right)(6.00\ \text{V}) = \boxed{3.54\times10^{-9}\ \text{C}}$

(c) $E = \dfrac{\Delta V}{d} = \dfrac{6.00\ \text{V}}{3.00\times10^{-3}\ \text{m}} = \boxed{2.00\times10^{3}\ \text{V/m}} = \boxed{2.00\times10^{3}\ \text{N/C}}$

(d) $\sigma = \dfrac{Q}{A} = \dfrac{3.54\times10^{-9}\ \text{C}}{0.200\ \text{m}^2} = \boxed{1.77\times10^{-8}\ \text{C/m}^2}$

(e) Increasing the distance separating the plates decreases the capacitance, the charge stored, and the electric field strength between the plates. This means that all of the $\boxed{\text{previous answers will be decreased}}$.

16.32 $\Sigma F_y = 0 \ \Rightarrow\ T\cos 15.0° = mg \quad \text{or} \quad T = \dfrac{mg}{\cos 15.0°}$



$\Sigma F_x = 0 \ \Rightarrow\ qE = T\sin 15.0° = mg\tan 15.0°$

or $\quad E = \dfrac{mg\tan 15.0°}{q}$

$$\Delta V = Ed = \frac{mgd\tan 15.0°}{q}$$

$$\Delta V = \frac{\left(350\times10^{-6}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)\left(0.040\ 0\ \text{m}\right)\tan 15.0°}{30.0\times10^{-9}\ \text{C}} = 1.23\times10^{3}\ \text{V} = \boxed{1.23\ \text{kV}}$$

16.33 (a) Capacitors in a series combination store the same charge, $Q = C_{eq}(\Delta V)$, where C_{eq} is the equivalent capacitance and ΔV is the potential difference maintained across the series combination. The equivalent capacitance for the given series combination is $1/C_{eq} = 1/C_1 + 1/C_2$, or $C_{eq} = C_1C_2/(C_1 + C_2)$, giving

$$C_{eq} = \frac{(2.50\ \mu\text{F})(6.25\ \mu\text{F})}{2.50\ \mu\text{F} + 6.25\ \mu\text{F}} = 1.79\ \mu\text{F}$$

and the charge stored on each capacitor in the series combination is

$$Q = C_{eq}(\Delta V) = (1.79\ \mu\text{F})(6.00\ \text{V}) = \boxed{10.7\ \mu\text{C}}$$

(b) When connected in parallel, each capacitor has the same potential difference, $\Delta V = 6.00$ V, maintained across it. The charge stored on each capacitor is then

For $C_1 = 2.50\ \mu$F: $\quad Q_1 = C_1(\Delta V) = (2.50\ \mu\text{F})(6.00\ \text{V}) = \boxed{15.0\ \mu\text{C}}$

For $C_2 = 6.25\ \mu$F: $\quad Q_2 = C_2(\Delta V) = (6.25\ \mu\text{F})(6.00\ \text{V}) = \boxed{37.5\ \mu\text{C}}$

16.34 (a) $C_{eq} = C_1 + C_2 = 5.00\ \mu\text{F} + 12.0\ \mu\text{F} = \boxed{17.0\ \mu\text{F}}$

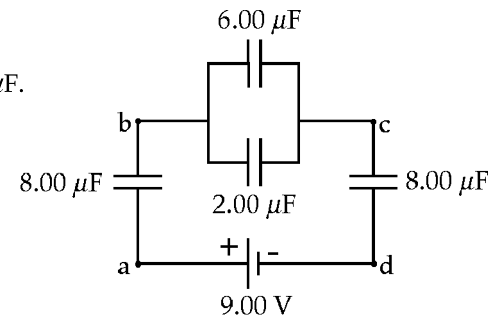
(b) In a parallel combination, the full potential difference maintained between the terminals of the battery exists across each capacitor. Thus,

$$\Delta V_1 = \Delta V_2 = \Delta V_{battery} = \boxed{9.00\ \text{V}}$$

(c) $Q_1 = C_1(\Delta V_1) = (5.00\ \mu\text{F})(9.00\ \text{V}) = \boxed{45.0\ \mu\text{C}}$

$Q_2 = C_2(\Delta V_2) = (12.0\ \mu\text{F})(9.00\ \text{V}) = \boxed{108\ \mu\text{C}}$

16.35 (a) First, we replace the parallel combination between points b and c by its equivalent capacitance, $C_{bc} = 2.00\ \mu\text{F} + 6.00\ \mu\text{F} = 8.00\ \mu\text{F}$. Then, we have three capacitors in series between points a and d. The equivalent capacitance for this circuit is therefore



$$\frac{1}{C_{eq}} = \frac{1}{C_{ab}} + \frac{1}{C_{bc}} + \frac{1}{C_{cd}} = \frac{3}{8.00\ \mu\text{F}}$$

giving $C_{eq} = \dfrac{8.00\ \mu\text{F}}{3} = \boxed{2.67\ \mu\text{F}}$

(b) The charge stored on each capacitor in the series combination is

$$Q_{ab} = Q_{bc} = Q_{cd} = C_{eq}(\Delta V_{ad}) = (2.67\ \mu\text{F})(9.00\ \text{V}) = 24.0\ \mu\text{C}$$

Then, note that $\Delta V_{bc} = Q_{bc}/C_{bc} = 24.0\ \mu\text{C}/8.00\ \mu\text{F} = 3.00\ \text{V}$. The charge on each capacitor in the original circuit is:

On the $8.00\ \mu\text{F}$ between a and b: $Q_8 = Q_{ab} = \boxed{24.0\ \mu\text{C}}$

On the $8.00\ \mu\text{F}$ between c and d: $Q_8 = Q_{cd} = \boxed{24.0\ \mu\text{C}}$

On the $2.00\ \mu\text{F}$ between b and c: $Q_2 = C_2(\Delta V_{bc}) = (2.00\ \mu\text{F})(3.00\ \text{V}) = \boxed{6.00\ \mu\text{C}}$

On the $6.00\ \mu\text{F}$ between b and c: $Q_6 = C_6(\Delta V_{bc}) = (6.00\ \mu\text{F})(3.00\ \text{V}) = \boxed{18.0\ \mu\text{C}}$

(c) Note that $\Delta V_{ab} = Q_{ab}/\text{C}_{ab} = 24.0\ \mu\text{C}/8.00\ \mu\text{F} = 3.00\ \text{V}$, and that $\Delta V_{cd} = Q_{cd}/\text{C}_{cd} = 24.0\ \mu\text{C}/8.00\ \mu\text{F} = 3.00\ \text{V}$. We earlier found that $\Delta V_{bc} = 3.00\ \text{V}$, so we conclude that the potential difference across each capacitor in the circuit is

$$\Delta V_8 = \Delta V_2 = \Delta V_6 = \Delta V_8 = \boxed{3.00\ \text{V}}$$

16.36 $C_{parallel} = C_1 + C_2 = 9.00\ \text{pF} \quad \Rightarrow \quad C_1 = 9.00\ \text{pF} - C_2$ **[1]**

$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2} \quad \Rightarrow \quad C_{series} = \frac{C_1 C_2}{C_1 + C_2} = 2.00\ \text{pF}$$

Thus, using Equation [1], $C_{series} = \dfrac{(9.00\ \text{pF} - C_2)C_2}{(9.00\ \text{pF} - C_2) + C_2} = 2.00\ \text{pF}$ which reduces to

continued on next page

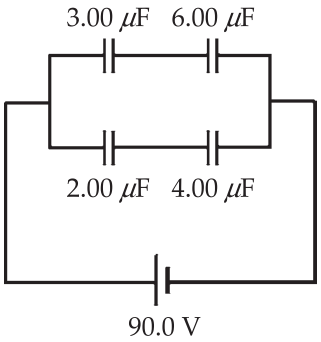
$$C_2^2 - (9.00\text{ pF})C_2 + 18.0\ (\text{pF})^2 = 0\text{, or }(C_2 - 6.00\text{ pF})(C_2 - 3.00\text{ pF}) = 0$$

Therefore, either $C_2 = 6.00$ pF and, from Equation [1], $C_1 = 3.00$ pF

or $C_2 = 3.00$ pF and $C_1 = 6.00$ pF.

We conclude that the two capacitances are $\boxed{3.00\text{ pF and }6.00\text{ pF}}$.

16.37 (a) The equivalent capacitance of the series combination in the upper branch is



$$\frac{1}{C_{\text{upper}}} = \frac{1}{3.00\ \mu\text{F}} + \frac{1}{6.00\ \mu\text{F}} = \frac{2+1}{6.00\ \mu\text{F}}$$

or $C_{\text{upper}} = 2.00\ \mu\text{F}$

Likewise, the equivalent capacitance of the series combination in the lower branch is

$$\frac{1}{C_{\text{lower}}} = \frac{1}{2.00\ \mu\text{F}} + \frac{1}{4.00\ \mu\text{F}} = \frac{2+1}{4.00\ \mu\text{F}} \quad\text{or}\quad C_{\text{lower}} = 1.33\ \mu\text{F}$$

These two equivalent capacitances are connected in parallel with each other, so the equivalent capacitance for the entire circuit is

$$C_{\text{eq}} = C_{\text{upper}} + C_{\text{lower}} = 2.00\ \mu\text{F} + 1.33\ \mu\text{F} = \boxed{3.33\ \mu\text{F}}$$

(b) Note that the same potential difference, equal to the potential difference of the battery, exists across both the upper and lower branches. The charge stored on each capacitor in the series combination in the upper branch is

$$Q_3 = Q_6 = Q_{\text{upper}} = C_{\text{upper}}(\Delta V) = (2.00\ \mu\text{F})(90.0\text{ V}) = \boxed{180\ \mu\text{C}}$$

and the charge stored on each capacitor in the series combination in the lower branch is

$$Q_2 = Q_4 = Q_{\text{lower}} = C_{\text{lower}}(\Delta V) = (1.33\ \mu\text{F})(90.0\text{ V}) = \boxed{120\ \mu\text{C}}$$

(c) The potential difference across each of the capacitors in the circuit is:

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{120\ \mu\text{C}}{2.00\ \mu\text{F}} = \boxed{60.0\text{ V}} \qquad \Delta V_4 = \frac{Q_4}{C_4} = \frac{120\ \mu\text{C}}{4.00\ \mu\text{F}} = \boxed{30.0\text{ V}}$$

$$\Delta V_3 = \frac{Q_3}{C_3} = \frac{180\ \mu\text{C}}{3.00\ \mu\text{F}} = \boxed{60.0\text{ V}} \qquad \Delta V_6 = \frac{Q_6}{C_6} = \frac{180\ \mu\text{C}}{6.00\ \mu\text{F}} = \boxed{30.0\text{ V}}$$

16.38 (a) The equivalent capacitance of the series combination in the rightmost branch of the circuit is

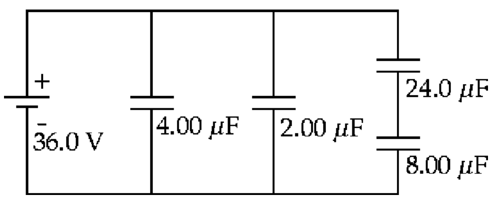


Figure P16.38

$$\frac{1}{C_{\text{right}}} = \frac{1}{24.0\ \mu\text{F}} + \frac{1}{8.00\ \mu\text{F}} = \frac{1+3}{24.0\ \mu\text{F}}$$

or $C_{\text{right}} = \boxed{6.00\ \mu\text{F}}$

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(b) The equivalent capacitance of the three capacitors now connected in parallel with each other and with the battery is

$$C_{eq} = 4.00\ \mu\text{F} + 2.00\ \mu\text{F} + 6.00\ \mu\text{F} = \boxed{12.0\ \mu\text{F}}$$

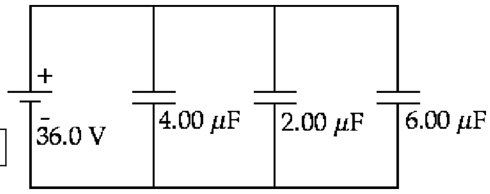

Diagram 1

(c) The total charge stored in this circuit is

$$Q_{total} = C_{eq}(\Delta V) = (12.0\ \mu\text{F})(36.0\ \text{V})$$

or $Q_{total} = \boxed{432\ \mu\text{C}}$

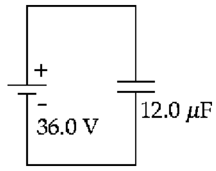

Diagram 2

(d) The charges on the three capacitors shown in Diagram 1 are:

$$Q_4 = C_4(\Delta V) = (4.00\ \mu\text{F})(36.0\ \text{V}) = \boxed{144\ \mu\text{C}}$$

$$Q_2 = C_2(\Delta V) = (2.00\ \mu\text{F})(36.0\ \text{V}) = \boxed{72\ \mu\text{C}}$$

$$Q_{right} = C_{right}(\Delta V) = (6.00\ \mu\text{F})(36.0\ \text{V}) = \boxed{216\ \mu\text{C}}$$

$\boxed{\text{Yes.}}$ $Q_4 + Q_2 + Q_{right} = Q_{total}$ as it should.

(e) The charge on each capacitor in the series combination in the rightmost branch of the original circuit (Figure P16.38) is

$$Q_{24} = Q_8 = Q_{right} = \boxed{216\ \mu\text{C}}$$

(f) $\Delta V_{24} = \dfrac{Q_{24}}{C_{24}} = \dfrac{216\ \mu\text{C}}{24.0\ \mu\text{F}} = \boxed{9.00\ \text{V}}$

(g) $\Delta V_8 = \dfrac{Q_8}{C_8} = \dfrac{216\ \mu\text{C}}{8.00\ \mu\text{F}} = \boxed{27.0\ \text{V}}$ Note that $\Delta V_8 + \Delta V_{24} = \Delta V = 36.0\ \text{V}$ as it should.

16.39

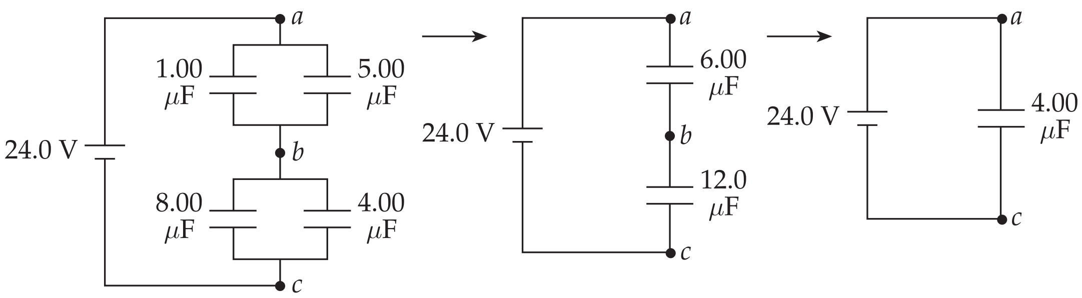

Figure 1 **Figure 2** **Figure 3**

The circuit may be reduced in steps as shown above.

Using Figure 3, $Q_{ac} = (4.00\ \mu\text{F})(24.0\ \text{V}) = 96.0\ \mu\text{C}$

Then, in Figure 2, $(\Delta V)_{ab} = \dfrac{Q_{ac}}{C_{ab}} = \dfrac{96.0\ \mu\text{C}}{6.00\ \mu\text{F}} = 16.0\ \text{V}$

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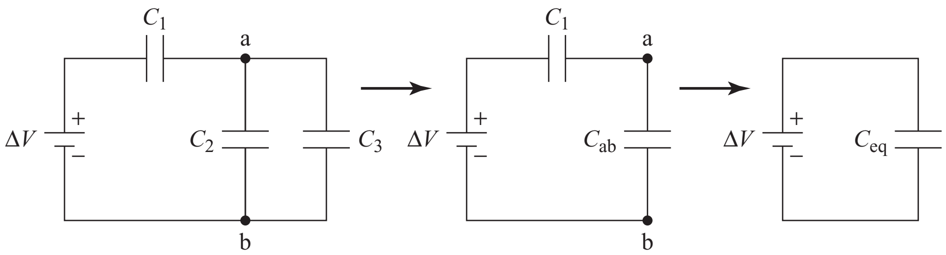
and $(\Delta V)_{bc} = (\Delta V)_{ac} - (\Delta V)_{ab} = 24.0\text{ V} - 16.0\text{ V} = 8.00\text{ V}$

Finally, using Figure 1,

$$Q_1 = C_1(\Delta V)_{ab} = (1.00\ \mu\text{F})(16.0\text{ V}) = \boxed{16.0\ \mu\text{C}}, \quad Q_5 = (5.00\ \mu\text{F})(\Delta V)_{ab} = \boxed{80.0\ \mu\text{C}}$$

$$Q_8 = (8.00\ \mu\text{F})(\Delta V)_{bc} = \boxed{64.0\ \mu\text{C}}, \quad \text{and} \quad Q_4 = (4.00\ \mu\text{F})(\Delta V)_{bc} = \boxed{32.0\ \mu\text{C}}$$

16.40 (a) Consider the simplification of the circuit as shown below:



Since C_2 and C_3 are connected in parallel, $C_{\text{ab}} = C_2 + C_3 = C + 5C = 6C$.

Now observe that C_1 and C_{ab} are connected in series, giving

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_{\text{ab}}} \quad \text{or} \quad C_{\text{eq}} = \frac{C_1 C_{\text{ab}}}{C_1 + C_{\text{ab}}} = \frac{(3C)(6C)}{3C + 6C} = \boxed{2C}$$

(b) Since capacitors C_1 and C_{ab} are connected in series,

$$Q_1 = Q_{\text{ab}} = Q_{\text{eq}} = C_{\text{eq}}(\Delta V) = 2C(\Delta V)$$

Then, $\Delta V_{\text{ab}} = \dfrac{Q_{\text{ab}}}{C_{\text{ab}}} = \dfrac{2C(\Delta V)}{6C} = \dfrac{\Delta V}{3}$, giving $Q_2 = C_2(\Delta V_{\text{ab}}) = \dfrac{C(\Delta V)}{3}$

Also, $Q_3 = C_3(\Delta V_{\text{ab}}) = \dfrac{5C(\Delta V)}{3}$. Therefore, $\boxed{Q_1 > Q_3 > Q_2}$

(c) Since capacitors C_1 and C_{ab} are in series with the battery,

$$\Delta V_1 = \Delta V - \Delta V_{\text{ab}} = \Delta V - \frac{\Delta V}{3} = \frac{2}{3}\Delta V$$

Also, with capacitors C_2 and C_3 in parallel between points a and b,

$$\Delta V_2 = \Delta V_3 = \Delta V_{\text{ab}} = \frac{\Delta V}{3}$$

Thus, $\boxed{\Delta V_1 > \Delta V_2 = \Delta V_3}$

(d) Consider the following steps:

(i) Increasing C_3 while C_1 and C_2 remain constant will increase $C_{\text{ab}} = C_2 + C_3$.

Therefore, the equivalent capacitance, $C_{\text{eq}} = C_1\left(\dfrac{C_{\text{ab}}}{C_1 + C_{\text{ab}}}\right)$, will increase.

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(ii) Since C_1 and C_{ab} are in series, $Q_1 = Q_{ab} = C_{eq}(\Delta V)$. Thus, $\boxed{Q_1 \text{ will increase}}$ as C_{eq} increases. Also, Q_{ab} experiences the same increase.

(iii) Because $\Delta V_1 = Q_1/C_1$, an increase in Q_1 causes ΔV_1 to increase and causes $\Delta V_{ab} = \Delta V - \Delta V_1$ to decrease. Thus, since $Q_2 = C_2(\Delta V_{ab})$, $\boxed{Q_2 \text{ will decrease}}$.

(iv) With capacitors C_2 and C_3 in parallel between points a and b, we have $Q_{ab} = Q_2 + Q_3$ or $Q_3 = Q_{ab} - Q_2$. Thus, with Q_{ab} increasing [see Step (ii)] while Q_2 is decreasing [see Step (iii)], we see that $\boxed{Q_3 \text{ will increase}}$.

16.41 (a) From $Q = C(\Delta V)$, $Q_{25} = (25.0\ \mu\text{F})(50.0\ \text{V}) = 1.25\times10^3\ \mu\text{C} = \boxed{1.25\ \text{mC}}$

and $Q_{40} = (40.0\ \mu\text{F})(50.0\ \text{V}) = 2.00\times10^3\ \mu\text{C} = \boxed{2.00\ \text{mC}}$

(b) Since the negative plate of one capacitor was connected to the positive plate of the other, the net charge stored in the new parallel combination is

$$Q = Q_{40} - Q_{25} = 2.00\times10^3\ \mu\text{C} - 1.25\times10^3\ \mu\text{C} = 750\ \mu\text{C}$$

The two capacitors, now in parallel, have a common potential difference ΔV across them. The new charges on each of the capacitors are $Q'_{25} = C_1(\Delta V)$ and $Q'_{40} = C_2(\Delta V)$. Thus,

$$Q'_{25} = \frac{C_1}{C_2}Q'_{40} = \left(\frac{25\ \mu\text{F}}{40\ \mu\text{F}}\right)Q'_{40} = \frac{5}{8}Q'_{40}$$

and the total change now stored in the combination may be written as

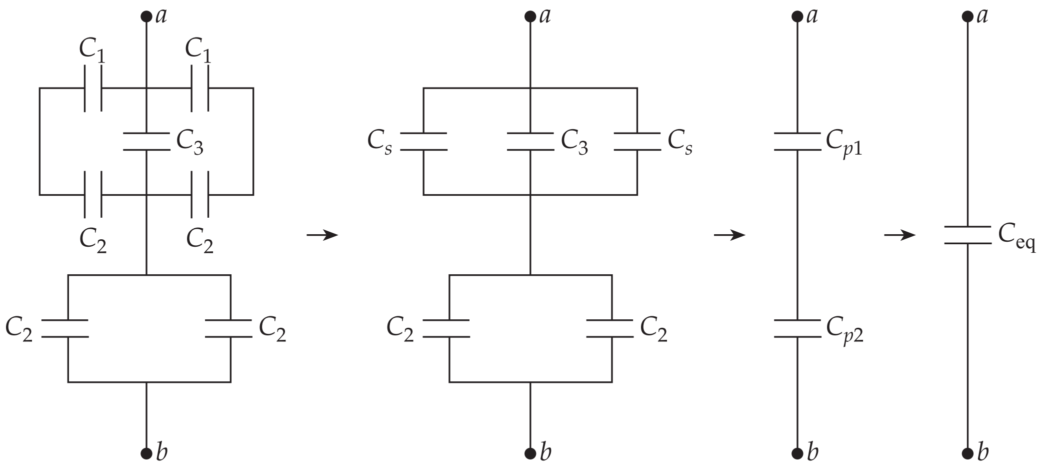
$$Q = Q'_{40} + Q'_{25} = Q'_{40} + \frac{5}{8}Q'_{40} = \frac{13}{8}Q'_{40} = 750\ \mu\text{C}$$

giving $Q'_{40} = \dfrac{8}{13}(750\ \mu\text{C}) = \boxed{462\ \mu\text{C}}$ and $Q'_{25} = Q - Q'_{40} = (750 - 462)\ \mu\text{C} = \boxed{288\ \mu\text{C}}$

(c) The potential difference across each capacitor in the new parallel combination is

$$\Delta V = \frac{Q}{C_{eq}} = \frac{Q}{C_1 + C_2} = \frac{750\ \mu\text{C}}{65.0\ \mu\text{F}} = \boxed{11.5\ \text{V}}$$

16.42 (a) The original circuit reduces to a single equivalent capacitor in the steps shown below.



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$$C_s = \left(\frac{1}{C_1} + \frac{1}{C_2}\right)^{-1} = \left(\frac{1}{5.00\ \mu\text{F}} + \frac{1}{10.0\ \mu\text{F}}\right)^{-1} = 3.33\ \mu\text{F}$$

$$C_{p1} = C_s + C_3 + C_s = 2(3.33\ \mu\text{F}) + 2.00\ \mu\text{F} = 8.66\ \mu\text{F}$$

$$C_{p2} = C_2 + C_2 = 2(10.0\ \mu\text{F}) = 20.0\ \mu\text{F}$$

$$C_{\text{eq}} = \left(\frac{1}{C_{p1}} + \frac{1}{C_{p2}}\right)^{-1} = \left(\frac{1}{8.66\ \mu\text{F}} + \frac{1}{20.0\ \mu\text{F}}\right)^{-1} = \boxed{6.04\ \mu\text{F}}$$

(b) The total charge stored between points a and b is

$$Q_{\text{total}} = C_{\text{eq}}(\Delta V)_{ab} = (6.04\ \mu\text{F})(60.0\ \text{V}) = 362\ \mu\text{C}$$

Then, looking at the third figure, observe that the charges of the series capacitors of that figure are $Q_{p1} = Q_{p2} = Q_{\text{total}} = 362\ \mu\text{C}$. Thus, the potential difference across the upper parallel combination shown in the second figure is

$$(\Delta V)_{p1} = \frac{Q_{p1}}{C_{p1}} = \frac{362\ \mu\text{C}}{8.66\ \mu\text{F}} = 41.8\ \text{V}$$

Finally, the charge on C_3 is

$$Q_3 = C_3(\Delta V)_{p1} = (2.00\ \mu\text{F})(41.8\ \text{V}) = \boxed{83.6\ \mu\text{C}}$$

16.43 From $Q = C(\Delta V)$, the initial charge of each capacitor is

$$Q_1 = (1.00\ \mu\text{F})(10.0\ \text{V}) = 10.0\ \mu\text{C} \quad \text{and} \quad Q_2 = (2.00\ \mu\text{F})(0) = 0$$

After the capacitors are connected in parallel, the potential difference across one is the same as that across the other. This gives

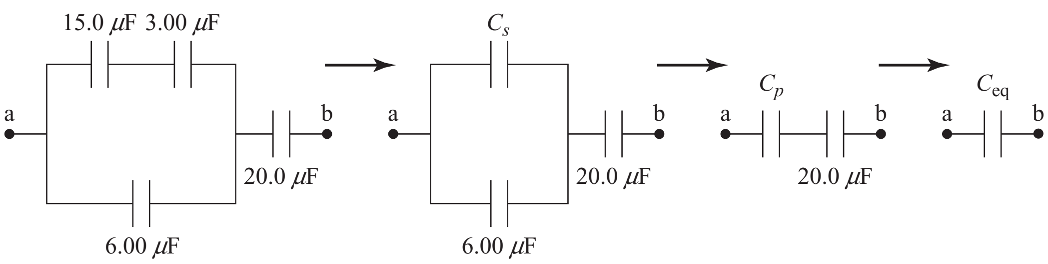
$$\Delta V = \frac{Q_1'}{1.00\ \mu\text{F}} = \frac{Q_2'}{2.00\ \mu\text{F}} \quad \text{or} \quad Q_2' = 2Q_1' \qquad \textbf{[1]}$$

From conservation of charge, $Q_1' + Q_2' = Q_1 + Q_2 = 10.0\ \mu\text{C}$. Then, substituting from Equation [1], this becomes

$Q_1' + 2Q_1' = 10.0\ \mu\text{C}$, giving $\qquad Q_1' = \boxed{10\ \mu\text{C}/3} = \boxed{3.33\ \mu\text{C}}$

Finally, from Equation [1], $\qquad Q_2' = \boxed{20\ \mu\text{C}/3} = \boxed{6.67\ \mu\text{C}}$

16.44 (a) We simplify the circuit in stages as shown below:



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$$\frac{1}{C_s}=\frac{1}{15.0\ \mu\text{F}}+\frac{1}{3.00\ \mu\text{F}} \quad \text{or} \quad C_s=\frac{(15.0\ \mu\text{F})(3.00\ \mu\text{F})}{15.0\ \mu\text{F}+3.00\ \mu\text{F}}=2.50\ \mu\text{F}$$

$$C_p=C_s+6.00\ \mu\text{F}=2.50\ \mu\text{F}+6.00\ \mu\text{F}=8.50\ \mu\text{F}$$

$$\frac{1}{C_{\text{eq}}}=\frac{1}{C_p}+\frac{1}{20.0\ \mu\text{F}} \quad \text{or} \quad C_{\text{eq}}=\frac{(8.50\ \mu\text{F})(20.0\ \mu\text{F})}{8.50\ \mu\text{F}+20.0\ \mu\text{F}}=\boxed{5.96\ \mu\text{F}}$$

(b) $Q_{20}=Q_{C_p}=Q_{\text{eq}}=C_{\text{eq}}(\Delta V_{\text{ab}})=(5.96\ \mu\text{F})(15.0\ \text{V})=\boxed{89.4\ \mu\text{C}}$

$$\Delta V_{C_p}=\frac{Q_{C_p}}{C_p}=\frac{89.4\ \mu\text{C}}{8.50\ \mu\text{F}}=10.5\ \text{V}$$

so $Q_6=C_6\left(\Delta V_{C_p}\right)=(6.00\ \mu\text{F})(10.5\ \text{V})=\boxed{63.0\ \mu\text{C}}$

and $Q_{15}=Q_3=Q_s=C_s\left(\Delta V_{C_p}\right)=(2.50\ \mu\text{F})(10.5\ \text{V})=\boxed{26.3\ \mu\text{C}}$

The charges are 89.4 μC on the 20 μF capacitor, 63.0 μC on the 6 μF capacitor, and 26.3 μC on both the 15 μF and 3 μF capacitors.

16.45 Energy stored $=\dfrac{Q^2}{2C}=\dfrac{1}{2}C(\Delta V)^2=\dfrac{1}{2}\left(4.50\times10^{-6}\ \text{F}\right)(12.0\ \text{V})^2=\boxed{3.24\times10^{-4}\ \text{J}}$

16.46 (a) The equivalent capacitance of a series combination of C_1 and C_2 is

$$\frac{1}{C_{\text{eq}}}=\frac{1}{18.0\ \mu\text{F}}+\frac{1}{36.0\ \mu\text{F}}=\frac{2+1}{36.0\ \mu\text{F}} \quad \text{or} \quad \boxed{C_{\text{eq}}=12.0\ \mu\text{F}}$$

When this series combination is connected to a 12.0-V battery, the total stored energy is

$$\text{Total energy stored}=\frac{1}{2}C_{\text{eq}}(\Delta V)^2=\frac{1}{2}\left(12.0\times10^{-6}\ \text{F}\right)(12.0\ \text{V})^2=\boxed{8.64\times10^{-4}\ \text{J}}$$

(b) The charge stored on each of the two capacitors in the series combination is

$$Q_1=Q_2=Q_{\text{total}}=C_{\text{eq}}(\Delta V)=(12.0\ \mu\text{F})(12.0\ \text{V})=144\ \mu\text{C}=1.44\times10^{-4}\ \text{C}$$

and the energy stored in each of the individual capacitors is

$$\text{Energy stored in } C_1=\frac{Q_1^2}{2C_1}=\frac{\left(1.44\times10^{-4}\ \text{C}\right)^2}{2\left(18.0\times10^{-6}\ \text{F}\right)}=\boxed{5.76\times10^{-4}\ \text{J}}$$

and $$\text{Energy stored in } C_2=\frac{Q_2^2}{2C_2}=\frac{\left(1.44\times10^{-4}\ \text{C}\right)^2}{2\left(36.0\times10^{-6}\ \text{F}\right)}=\boxed{2.88\times10^{-4}\ \text{J}}$$

Energy stored in C_1 + Energy stored in $C_2=5.76\times10^{-4}\ \text{J}+2.88\times10^{-4}\ \text{J}=8.64\times10^{-4}\ \text{J}$, which is the same as the total stored energy found in part (a). This must be true $\boxed{\text{if the computed equivalent capacitance is truly equivalent to the original combination.}}$

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(c) If C_1 and C_2 had been connected in parallel rather than in series, the equivalent capacitance would have been $C_{eq} = C_1 + C_2 = 18.0\ \mu\text{F} + 36.0\ \mu\text{F} = 54.0\ \mu\text{F}$. If the total energy stored $\left[\frac{1}{2}C_{eq}(\Delta V)^2\right]$ in this parallel combination is to be the same as was stored in the original series combination, it is necessary that

$$\Delta V = \sqrt{\frac{2(\text{Total energy stored})}{C_{eq}}} = \sqrt{\frac{2(8.64\times10^{-4}\ \text{J})}{54.0\times10^{-6}\ \text{F}}} = \boxed{5.66\ \text{V}}$$

Since the two capacitors in parallel have the same potential difference across them, the energy stored in the individual capacitors $\left[\frac{1}{2}C(\Delta V)^2\right]$ is directly proportional to their capacitances. $\boxed{\text{The larger capacitor, } C_2\text{, stores the most energy in this case.}}$

16.47 (a) The energy initially stored in the capacitor is

$$(\text{Energy stored})_1 = \frac{Q_i^2}{2C_i} = \frac{1}{2}C_i(\Delta V)_i^2 = \frac{1}{2}(3.00\ \mu\text{F})(6.00\ \text{V})^2 = \boxed{54.0\ \mu\text{J}}$$

(b) When the capacitor is disconnected from the battery, the stored charge becomes isolated with no way off the plates. Thus, the charge remains constant at the value Q_i as long as the capacitor remains disconnected. Since the capacitance of a parallel-plate capacitor is $C = \kappa \in_0 A/d$, when the distance d separating the plates is doubled, the capacitance is decreased by a factor of 2 $(C_f = C_i/2 = 1.50\ \mu\text{F})$. The stored energy (with Q unchanged) becomes

$$(\text{Energy stored})_2 = \frac{Q_i^2}{2C_f} = \frac{Q_i^2}{2(C_i/2)} = 2\left(\frac{Q_i^2}{2C_i}\right) = 2(\text{Energy stored})_1 = \boxed{108\ \mu\text{J}}$$

(c) When the capacitor is reconnected to the battery, the potential difference between the plates is reestablished at the original value of $\Delta V = (\Delta V)_i = 6.00$ V, while the capacitance remains at $C_f = C_i/2 = 1.50\ \mu\text{F}$. The energy stored under these conditions is

$$(\text{Energy stored})_3 = \frac{1}{2}C_f(\Delta V)_i^2 = \frac{1}{2}(1.50\ \mu\text{F})(6.00\ \text{V})^2 = \boxed{27.0\ \mu\text{J}}$$

16.48 The energy transferred to the water is

$$W = \frac{1}{100}\left[\frac{1}{2}Q(\Delta V)\right] = \frac{(50.0\ \text{C})(1.00\times10^8\ \text{V})}{200} = 2.50\times10^7\ \text{J}$$

Thus, if m is the mass of water boiled away, $W = m\left[c(\Delta T) + L_v\right]$ becomes

$$2.50\times10^7\ \text{J} = m\left[\left(4186\ \frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)(100^\circ\text{C} - 30.0^\circ\text{C}) + 2.26\times10^6\ \text{J/kg}\right]$$

giving $$m = \frac{2.50\times10^7\ \text{J}}{\left[2.93\times10^5\ \text{J/kg} + 2.26\times10^6\ \text{J/kg}\right]} = \boxed{9.79\ \text{kg}}$$

16.49 (a) Note that the charge on the plates remains constant at the original value, Q_0, as the dielectric is inserted. Thus, the change in the potential difference, $\Delta V = Q/C$, is due to a change in capacitance alone. The ratio of the final and initial capacitances is

$$\frac{C_f}{C_i} = \frac{\kappa \in_0 A/d}{\in_0 A/d} = \kappa \quad \text{and} \quad \frac{C_f}{C_i} = \frac{Q_0/(\Delta V)_f}{Q_0/(\Delta V)_i} = \frac{(\Delta V)_i}{(\Delta V)_f} = \frac{85.0\ \text{V}}{25.0\ \text{V}} = 3.40$$

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Thus, the dielectric constant of the inserted material is $\boxed{\kappa = 3.40}$, and the material is probably $\boxed{\text{nylon}}$ (see Table 16.1).

(b) If the dielectric only partially filled the space between the plates, leaving the remaining space air-filled, the equivalent dielectric constant would be somewhere between $\kappa = 1.00$ (air) and $\kappa = 3.40$. The resulting potential difference would then lie somewhere between $(\Delta V)_i = 85.0$ V and $(\Delta V)_f = 25.0$ V.

16.50 (a) If the maximum electric field that can exist between the plates before breakdown (i.e., the dielectric strength) is E_{max}, the maximum potential difference across the plates is $\Delta V_{max} = E_{max} \cdot d$, where d is the plate separation. The maximum change on either plate then has magnitude

$$Q_{max} = C(\Delta V_{max}) = C(E_{max} \cdot d)$$

Since the capacitance of a parallel-plate capacitor is $C = \kappa \in_0 A/d$, the maximum charge is

$$Q_{max} = \left(\frac{\kappa \in_0 A}{\cancel{d}}\right)(E_{max} \cdot \cancel{d}) = \kappa \in_0 AE_{max}$$

The area of each plate is $A = 5.00 \text{ cm}^2 = 5.00 \times 10^{-4} \text{ m}^2$, and when air is the dielectric, $\kappa = 1.00$ and $E_{max} = 3.00 \times 10^6$ V/m (see Table 16.1). Thus,

$$Q_{max} = (1.00)(8.85 \times 10^{-12} \text{ C/N} \cdot \text{m}^2)(5.00 \times 10^{-4} \text{ m}^2)(3.00 \times 10^6 \text{ V/m})$$

$$= 1.33 \times 10^{-8} \text{ C} = \boxed{13.3 \text{ nC}}$$

(b) If the dielectric is now polystyrene $(\kappa = 2.56 \text{ and } E_{max} = 24.0 \times 10^6 \text{ V/m})$, then

$$Q_{max} = (2.56)(8.85 \times 10^{-12} \text{ C/N} \cdot \text{m}^2)(5.00 \times 10^{-4} \text{ m}^2)(24.0 \times 10^6 \text{ V/m})$$

$$= 2.72 \times 10^{-7} \text{ C} = \boxed{272 \text{ nC}}$$

16.51 (a) The dielectric constant for Teflon® is $\kappa = 2.1$, so the capacitance is

$$C = \frac{\kappa \in_0 A}{d} = \frac{(2.1)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(175 \times 10^{-4} \text{ m}^2)}{0.040\,0 \times 10^{-3} \text{ m}}$$

$$C = 8.1 \times 10^{-9} \text{ F} = \boxed{8.1 \text{ nF}}$$

(b) For Teflon®, the dielectric strength is $E_{max} = 60 \times 10^6$ V/m, so the maximum voltage is

$$\Delta V_{max} = E_{max} d = (60 \times 10^6 \text{ V/m})(0.040\,0 \times 10^{-3} \text{ m})$$

$$\Delta V_{max} = 2.4 \times 10^3 \text{ V} = \boxed{2.4 \text{ kV}}$$

16.52 Before the capacitor is rolled, the capacitance of this parallel-plate capacitor is

$$C = \frac{\kappa \in_0 A}{d} = \frac{\kappa \in_0 (w \times L)}{d}$$

where A is the surface area of one side of a foil strip. Thus, the required length is

$$L = \frac{C \cdot d}{\kappa \in_0 w} = \frac{(9.50 \times 10^{-8} \text{ F})(0.025\,0 \times 10^{-3} \text{ m})}{(3.70)(8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2)(7.00 \times 10^{-2} \text{ m})} = \boxed{1.04 \text{ m}}$$

16.53 (a) $V = \dfrac{m}{\rho} = \dfrac{1.00\times10^{-12}\ \text{kg}}{1100\ \text{kg/m}^3} = \boxed{9.09\times10^{-16}\ \text{m}^3}$

Since $V = 4\pi r^3/3$, the radius is $r = \left[3V/4\pi\right]^{1/3}$, and the surface area is

$$A = 4\pi r^2 = 4\pi\left[\frac{3V}{4\pi}\right]^{2/3} = 4\pi\left[\frac{3\left(9.09\times10^{-16}\ \text{m}^3\right)}{4\pi}\right]^{2/3} = \boxed{4.54\times10^{-10}\ \text{m}^2}$$

(b) $C = \dfrac{\kappa \epsilon_0 A}{d} = \dfrac{(5.00)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)\left(4.54\times10^{-10}\ \text{m}^2\right)}{100\times10^{-9}\ \text{m}} = \boxed{2.01\times10^{-13}\ \text{F}}$

(c) $Q = C(\Delta V) = \left(2.01\times10^{-13}\ \text{F}\right)\left(100\times10^{-3}\ \text{V}\right) = \boxed{2.01\times10^{-14}\ \text{C}}$

and the number of electronic charges is

$$n = \frac{Q}{e} = \frac{2.01\times10^{-14}\ \text{C}}{1.60\times10^{-19}\ \text{C}} = \boxed{1.26\times10^5}$$

16.54 For a parallel-plate capacitor, $C = \kappa \epsilon_0 A/d$ and $Q = \sigma A = C(\Delta V)$. Thus, $\sigma \cancel{A} = \left(\kappa \epsilon_0 \cancel{A}/d\right)(\Delta V)$, and $d = \left(\kappa \epsilon_0/\sigma\right)(\Delta V)$. With air as the dielectric material $(\kappa = 1.00)$, the separation of the plates must be

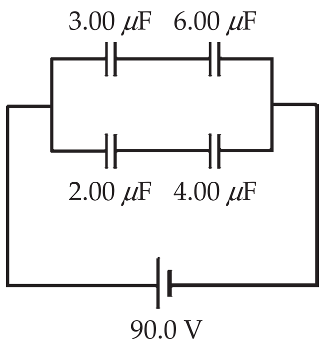
$$d = \frac{(1.00)\left(8.85\times10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}^2\right)(150\ \text{V})}{\left(3.00\times10^{-10}\ \text{C/cm}^2\right)\left(10^4\ \text{cm}^2/1\ \text{m}^2\right)} = 4.43\times10^{-4}\ \text{m} = \boxed{0.443\ \text{mm}}$$

16.55 Since the capacitors are in series, the equivalent capacitance is given by

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} = \frac{d_1}{\epsilon_0 A} + \frac{d_2}{\epsilon_0 A} + \frac{d_3}{\epsilon_0 A} = \frac{d_1 + d_2 + d_3}{\epsilon_0 A}$$

or $C_{eq} = \boxed{\dfrac{\epsilon_0 A}{d} \text{ where } d = d_1 + d_2 + d_3}$

16.56 (a) Please refer to the solution of Problem 16.37 where the following results were obtained:



$C_{eq} = 3.33\ \mu\text{F}$ $\quad Q_3 = Q_6 = 180\ \mu\text{C}$ $\quad Q_2 = Q_4 = 120\ \mu\text{C}$

The total energy stored in the full circuit is then

$$(\text{Energy stored})_{\text{total}} = \frac{1}{2}C_{eq}(\Delta V)^2 = \frac{1}{2}\left(3.33\times10^{-6}\ \text{F}\right)(90.0\ \text{V})^2$$
$$= 1.35\times10^{-2}\ \text{J} = 13.5\times10^{-3}\ \text{J} = \boxed{13.5\ \text{mJ}}$$

(b) The energy stored in each individual capacitor is

For $2.00\ \mu\text{F}$: $\quad (\text{Energy stored})_2 = \dfrac{Q_2^2}{2C_2} = \dfrac{\left(120\times10^{-6}\ \text{C}\right)^2}{2\left(2.00\times10^{-6}\ \text{F}\right)} = 3.60\times10^{-3}\ \text{J} = \boxed{3.60\ \text{mJ}}$

For $3.00\ \mu\text{F}$: $\quad (\text{Energy stored})_3 = \dfrac{Q_3^2}{2C_3} = \dfrac{\left(180\times10^{-6}\ \text{C}\right)^2}{2\left(3.00\times10^{-6}\ \text{F}\right)} = 5.40\times10^{-3}\ \text{J} = \boxed{5.40\ \text{mJ}}$

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For 4.00 μF: $\quad (\text{Energy stored})_4 = \dfrac{Q_4^2}{2C_4} = \dfrac{(120\times10^{-6}\text{ C})^2}{2(4.00\times10^{-6}\text{ F})} = 1.80\times10^{-3}\text{ J} = \boxed{1.80\text{ mJ}}$

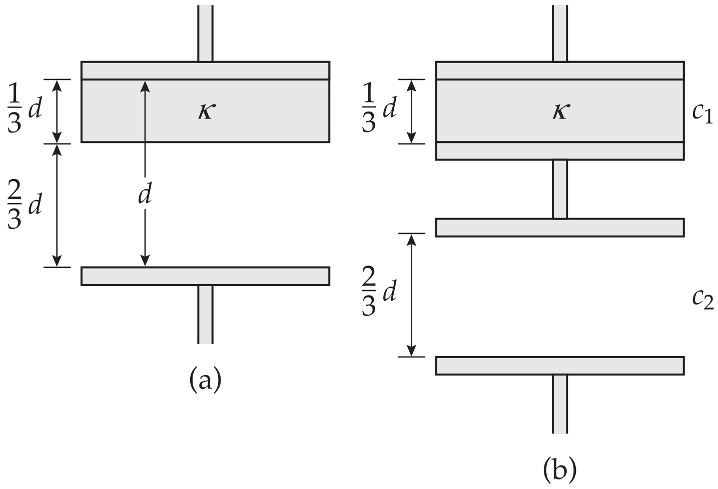
For 6.00 μF: $\quad (\text{Energy stored})_6 = \dfrac{Q_6^2}{2C_6} = \dfrac{(180\times10^{-6}\text{ C})^2}{2(6.00\times10^{-6}\text{ F})} = 2.70\times10^{-3}\text{ J} = \boxed{2.70\text{ mJ}}$

(c) The total energy stored in the individual capacitors is

$$\text{Energy stored} = (3.60+5.40+1.80+2.70)\text{ mJ} = \boxed{13.5\text{ mJ}} = (\text{Energy stored})_{\text{total}}$$

Thus, the sums of the energies stored in the individual capacitors equals the total energy stored by the system.

16.57 In the absence of a dielectric, the capacitance of the parallel-plate capacitor is $C_0 = \epsilon_0 A/d$.



With the dielectric inserted, it fills one-third of the gap between the plates as shown in sketch (a) at the right. We model this situation as consisting of a pair of capacitors, C_1 and C_2, connected in series as shown in sketch (b) at the right. In reality, the lower plate of C_1 and the upper plate of C_2 are one and the same, consisting of the lower surface of the dielectric shown in sketch (a). The capacitances in the model of sketch (b) are given by

$$C_1 = \frac{\kappa\epsilon_0 A}{d/3} = \frac{3\kappa\epsilon_0 A}{d} \quad \text{and} \quad C_2 = \frac{\epsilon_0 A}{2d/3} = \frac{3\epsilon_0 A}{2d}$$

The equivalent capacitance of the series combination is

$$\frac{1}{C_{\text{eq}}} = \frac{d}{3\kappa\epsilon_0 A} + \frac{2d}{3\epsilon_0 A} = \left(\frac{1}{\kappa}+2\right)\left(\frac{d}{3\epsilon_0 A}\right) = \left(\frac{2\kappa+1}{\kappa}\right)\frac{d}{3\epsilon_0 A} = \left(\frac{2\kappa+1}{3\kappa}\right)\frac{d}{\epsilon_0 A} = \left(\frac{2\kappa+1}{3\kappa}\right)\frac{1}{C_0}$$

and $\boxed{C_{\text{eq}} = [3\kappa/(2\kappa+1)]C_0}$.

16.58 For the parallel combination: $C_p = C_1 + C_2$ which gives $\quad C_2 = C_p - C_1$ **[1]**

For the series combination: $\dfrac{1}{C_s} = \dfrac{1}{C_1} + \dfrac{1}{C_2} \quad$ or $\quad \dfrac{1}{C_2} = \dfrac{1}{C_s} - \dfrac{1}{C_1} = \dfrac{C_1 - C_s}{C_s C_1}$

Thus, we have $C_2 = \dfrac{C_s C_1}{C_1 - C_s}$ and equating this to Equation [1] above gives

$$C_p - C_1 = \frac{C_s C_1}{C_1 - C_s} \quad \text{or} \quad C_p C_1 - C_p C_s - C_1^2 + \cancel{C_s C_1} = \cancel{C_s C_1}$$

We write this result as: $\quad C_1^2 - C_p C_1 + C_p C_s = 0$

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and use the quadratic formula to obtain $\boxed{C_1 = \frac{1}{2}C_p \pm \sqrt{\frac{1}{4}C_p^2 - C_pC_s}}$

Then, Equation [1] gives $\boxed{C_2 = \frac{1}{2}C_p \mp \sqrt{\frac{1}{4}C_p^2 - C_pC_s}}$

16.59 For a parallel-plate capacitor with plate separation d,

$$\Delta V_{\max} = E_{\max} \cdot d \qquad \text{or} \qquad d = \frac{\Delta V_{\max}}{E_{\max}}$$

The capacitance is then

$$C = \frac{\kappa \in_0 A}{d} = \kappa \in_0 A\left(\frac{E_{\max}}{\Delta V_{\max}}\right)$$

and the needed area of the plates is $A = C \cdot \Delta V_{\max}/\kappa \in_0 E_{\max}$, or

$$A = \frac{\left(0.250\times10^{-6}\text{ F}\right)\left(4.00\times10^{3}\text{ V}\right)}{(3.00)\left(8.85\times10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(2.00\times10^{8}\text{ V/m}\right)} = \boxed{0.188\text{ m}^2}$$

16.60 (a) The 1.0-μC is located 0.50 m from point P, so its contribution to the potential at P is

$$V_1 = k_e\frac{q_1}{r_1} = \left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{1.0\times10^{-6}\text{ C}}{0.50\text{ m}}\right) = \boxed{1.8\times10^{4}\text{ V}}$$

(b) The potential at P due to the –2.0-μC charge located 0.50 m away is

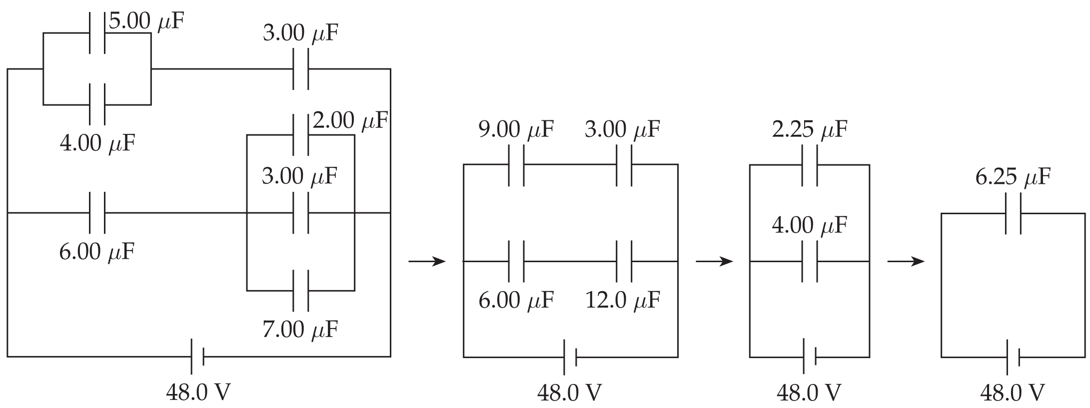
$$V_2 = k_e\frac{q_2}{r_2} = \left(8.99\times10^{9}\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(\frac{-2.0\times10^{-6}\text{ C}}{0.50\text{ m}}\right) = \boxed{-3.6\times10^{4}\text{ V}}$$

(c) The total potential at point P is $V_P = V_1 + V_2 = (+1.8-3.6)\times10^4\text{ V} = \boxed{-1.8\times10^{4}\text{ V}}$

(d) The work required to move a charge $q = 3.0\ \mu\text{C}$ to point P from infinity is

$$W = q\Delta V = q\left(V_P - V_\infty\right) = \left(3.0\times10^{-6}\text{ C}\right)\left(-1.8\times10^{4}\text{ V} - 0\right) = \boxed{-5.4\times10^{-2}\text{ J}}$$

16.61 The stages for the reduction of this circuit are shown below.



Thus, $C_{\text{eq}} = \boxed{6.25\ \mu\text{F}}$

16.62 (a) Due to spherical symmetry, the charge on each of the concentric spherical shells will be uniformly distributed over that shell. Inside a spherical surface having a uniform charge distribution, the electric field due to the charge on that surface is zero. Thus, in this region, the potential due to the charge on that surface is constant and equal to the potential at the surface. Outside a spherical surface having a uniform charge distribution, the potential due to the charge on that surface is given by $V = k_e q/r$, where r is the distance from the center of that surface and q is the charge on that surface.

In the region between a pair of concentric spherical shells, with the inner shell having charge $+Q$ and the outer shell having radius b and charge $-Q$, the total electric potential at distance r from the center is given by

$$V = V_{\substack{\text{due to}\\ \text{inner shell}}} + V_{\substack{\text{due to}\\ \text{outer shell}}} = \frac{k_e Q}{r} + \frac{k_e(-Q)}{b} = k_e Q\left(\frac{1}{r} - \frac{1}{b}\right)$$

The potential difference between the two shells is therefore

$$\Delta V = V|_{r=a} - V|_{r=b} = k_e Q\left(\frac{1}{a} - \frac{1}{b}\right) - k_e Q\left(\frac{1}{b} - \frac{1}{b}\right) = k_e Q\left(\frac{b-a}{ab}\right)$$

The capacitance of this device is given by

$$C = \frac{Q}{\Delta V} = \boxed{\frac{ab}{k_e(b-a)}}$$

(b) When $b >> a$, then $b - a \approx b$. Thus, in the limit as $b \to \infty$, the capacitance found above becomes

$$C \to \frac{ab}{k_e(b)} = \frac{a}{k_e} = \boxed{4\pi \in_0 a}$$

16.63 The energy stored in a charged capacitor is $E_{\text{stored}} = \frac{1}{2}C(\Delta V)^2$. Hence,

$$\Delta V = \sqrt{\frac{2E_{\text{stored}}}{C}} = \sqrt{\frac{2(300\text{ J})}{30.0\times 10^{-6}\text{ F}}} = 4.47\times 10^3\text{ V} = \boxed{4.47\text{ kV}}$$

16.64 From $Q = C(\Delta V)$, the capacitance of the capacitor with air between the plates is

$$C_0 = \frac{Q_0}{\Delta V} = \frac{150\ \mu\text{C}}{\Delta V}$$

After the dielectric is inserted, the potential difference is held to the original value, but the charge changes to $Q = Q_0 + 200\ \mu\text{C} = 350\ \mu\text{C}$. Thus, the capacitance with the dielectric slab in place is

$$C = \frac{Q}{\Delta V} = \frac{350\ \mu\text{C}}{\Delta V}$$

The dielectric constant of the dielectric slab is therefore

$$\kappa = \frac{C}{C_0} = \left(\frac{350\ \mu\text{C}}{\cancel{\Delta V}}\right)\left(\frac{\cancel{\Delta V}}{150\ \mu\text{C}}\right) = \frac{350}{150} = \boxed{2.33}$$

16.65 The charges initially stored on the capacitors are

$$Q_1 = C_1(\Delta V)_i = (6.0\ \mu\text{F})(250\ \text{V}) = 1.5\times10^3\ \mu\text{C}$$

and $$Q_2 = C_2(\Delta V)_i = (2.0\ \mu\text{F})(250\ \text{V}) = 5.0\times10^2\ \mu\text{C}$$

When the capacitors are connected in parallel, with the negative plate of one connected to the positive pl ate of the other, the net stored charge is

$$Q = Q_1 - Q_2 = 1.5\times10^3\ \mu\text{C} - 5.0\times10^2\ \mu\text{C} = 1.0\times10^3\ \mu\text{C}$$

The equivalent capacitance of the parallel combination is $C_{\text{eq}} = C_1 + C_2 = 8.0\ \mu\text{F}$. Thus, the final potential difference across each of the capacitors is

$$(\Delta V)' = \frac{Q}{C_{\text{eq}}} = \frac{1.0\times10^3\ \mu\text{C}}{8.0\ \mu\text{F}} = 125\ \text{V}$$

and the final charge on each capacitor is

$$Q_1' = C_1(\Delta V)' = (6.0\ \mu\text{F})(125\ \text{V}) = 750\ \mu\text{C} = \boxed{0.75\ \text{mC}}$$

and $$Q_2' = C_2(\Delta V)' = (2.0\ \mu\text{F})(125\ \text{V}) = 250\ \mu\text{C} = \boxed{0.25\ \text{mC}}$$

16.66 (a) The distance from the charge $2q$ to either of the charges on the y-axis is $r = \sqrt{d^2 + (2d)^2} = \sqrt{5}\,d$. Thus,

$$V = \sum_i \frac{k_e q_i}{r_i} = \frac{k_e q}{\sqrt{5}\,d} + \frac{k_e q}{\sqrt{5}\,d} = \boxed{\frac{2k_e q}{\sqrt{5}\,d}}$$

(b) $$PE_{2q} = \frac{k_e q_1 q_3}{r_1} + \frac{k_e q_2 q_3}{r_2} = \frac{k_e q(2q)}{\sqrt{5}\,d} + \frac{k_e q(2q)}{\sqrt{5}\,d} = \boxed{\frac{4k_e q^2}{\sqrt{5}\,d}}$$

(c) From conservation of energy with $PE = 0$ at $r = \infty$,

$$KE_f = KE_i + PE_i - PE_f = 0 + \frac{4k_e q^2}{\sqrt{5}\,d} - 0 = \boxed{\frac{4k_e q^2}{\sqrt{5}\,d}}$$

(d) $$v_f = \sqrt{\frac{2(KE_f)}{m}} = \sqrt{\frac{2}{m}\left(\frac{4k_e q^2}{\sqrt{5}\,d}\right)} = \boxed{\left(\frac{8k_e q^2}{\sqrt{5}\,md}\right)^{\frac{1}{2}}}$$

16.67 When excess charge resides on a spherical surface that is far removed from any other charge, this excess charge is uniformly distributed over the spherical surface, and the electric potential at the surface is the same as if all the excess charge were concentrated at the center of the spherical surface.

In the given situation, we have two charged spheres, initially isolated from each other, with charges and potentials of $Q_A = +6.00\ \mu\text{C}$ and $V_A = k_e Q_A / R_A$, where $R_A = 12.0$ cm, $Q_B = -4.00\ \mu\text{C}$, and $V_B = k_e Q_B / R_B$, with $R_B = 18.0$ cm.

When these spheres are then connected by a long conducting thread, the charges are redistributed (yielding charges of Q_A' and Q_B', respectively) until the two surfaces come to a common potential $\left(V_A' = kQ_A'/R_A = V_B' = kQ_B'/R_B\right)$. When equilibrium is established, we have:

From conservation of charge: $Q_A' + Q_B' = Q_A + Q_B \quad \Rightarrow \quad Q_A' + Q_B' = +2.00\ \mu\text{C}$ **[1]**

continued on next page

From equal potentials: $\dfrac{kQ_A'}{R_A}=\dfrac{kQ_B'}{R_B} \quad \Rightarrow \quad Q_B'=\left(\dfrac{R_B}{R_B}\right)Q_A' \quad \text{or} \quad Q_B'=1.50\,Q_A'$ **[2]**

Substituting Equation [2] into [1] gives $\quad Q_A'=\dfrac{+2.00\ \mu\text{C}}{2.50}=\boxed{0.800\ \mu\text{C}}$

Then, Equation [2] gives $\quad Q_B'=1.50\left(0.800\ \mu\text{C}\right)=\boxed{1.20\ \mu\text{C}}$

16.68 The electric field between the plates is directed downward with magnitude

$$\left|E_y\right|=\frac{\Delta V}{D}=\frac{100\ \text{V}}{2.0\times10^{-3}\ \text{m}}=5.0\times10^{4}\ \text{N/m}$$

Since the gravitational force experienced by the electron is negligible in comparison to the electrical force acting on it, the vertical acceleration is

$$a_y=\frac{F_y}{m_e}=\frac{qE_y}{m_e}=\frac{\left(-1.60\times10^{-19}\ \text{C}\right)\left(-5.0\times10^{4}\ \text{N/m}\right)}{9.11\times10^{-31}\ \text{kg}}=+8.8\times10^{15}\ \text{m/s}^2$$

(a) At the closest approach to the bottom plate, $v_y=0$. Thus, the vertical displacement from point O is found from $v_y^2=v_{0y}^2+2a_y\left(\Delta y\right)$ as

$$\Delta y=\frac{0-\left(v_0\sin\theta_0\right)^2}{2a_y}=\frac{-\left[-\left(5.6\times10^{6}\ \text{m/s}\right)\sin45^\circ\right]^2}{2\left(8.8\times10^{15}\ \text{m/s}^2\right)}=-8.9\times10^{-4}\ \text{m}=-0.89\ \text{mm}$$

The minimum distance above the bottom plate is then

$$d=\frac{D}{2}+\Delta y=1.0\ \text{mm}-0.89\ \text{mm}=\boxed{0.1\ \text{mm}}$$

(b) The time for the electron to go from point O to the upper plate ($\Delta y=+1.0$ mm) is found from $\Delta y=v_{0y}t+\frac{1}{2}a_yt^2$ as

$$+1.0\times10^{-3}\ \text{m}=\left[-\left(5.6\times10^{6}\ \frac{\text{m}}{\text{s}}\right)\sin45^\circ\right]t+\frac{1}{2}\left(8.8\times10^{15}\ \frac{\text{m}}{\text{s}^2}\right)t^2$$

Solving for t gives a positive solution of $t=1.1\times10^{-9}$ s. The horizontal displacement from point O at this time is

$$\Delta x=v_{0x}t=\left[\left(5.6\times10^{6}\ \text{m/s}\right)\cos45^\circ\right]\left(1.1\times10^{-9}\ \text{s}\right)=\boxed{4.4\ \text{mm}}$$

17

Current and Resistance

QUICK QUIZZES

1. Choice (d). Negative charges moving in one direction are equivalent to positive charges moving in the opposite direction. Thus, I_a, I_b, I_c, and I_d are equivalent to the movement of 5, 3, 4, and 2 charges respectively, giving $I_d < I_b < I_c < I_a$.

2. Choice (b). Under steady-state conditions, the current is the same in all parts of the wire. Thus, the drift velocity, given by $v_d = I/nqA$, is inversely proportional to the cross-sectional area.

3. Choices (c) and (d). Neither circuit (a) nor circuit (b) applies a difference in potential across the bulb. Circuit (a) has both lead wires connected to the same battery terminal. Circuit (b) has a low resistance path (a "short") between the two battery terminals as well as between the bulb terminals.

4. Choice (b). The slope of the line tangent to the curve at a point is the reciprocal of the resistance at that point. Note that as ΔV increases, the slope (and hence $1/R$) increases. Thus, the resistance decreases.

5. Choice (b). From Ohm's Law, $R = \Delta V/I = 120\text{ V}/6.00\text{ A} = 20.0\ \Omega$.

6. Choice (b). Consider the expression for resistance: $R = \rho\dfrac{L}{A} = \rho\dfrac{L}{\pi r^2}$. Doubling all linear dimensions increases the numerator of this expression by a factor of 2 but increases the denominator by a factor of 4. Thus, the net result is that the resistance will be reduced to one-half of its original value.

7. Choice (a). The resistance of the shorter wire is half that of the longer wire. The power dissipated, $P = (\Delta V)^2/R$, (and hence the rate of heating) will be greater for the shorter wire. Consideration of the expression $P = I^2R$ might initially lead one to think that the reverse would be true. However, one must realize that the currents will not be the same in the two wires.

8. Choice (b). $I_a = I_b > I_c = I_d > I_e = I_f$. Charges constituting the current I_a leave the positive terminal of the battery and then split to flow through the two bulbs; thus, $I_a = I_c + I_e$. Because the potential difference ΔV is the same across the two bulbs and because the power delivered to a device is $P = I(\Delta V)$, the 60-W bulb with the higher power rating must carry the greater current, meaning that $I_c > I_e$. Because charge does not accumulate in the bulbs, all the charge flowing into a bulb from the left has to flow out on the right; consequently $I_c = I_d$ and $I_e = I_f$. The two currents leaving the bulbs recombine to form the current back into the battery, $I_f + I_d = I_b$.

9. Choice (a). The power dissipated by a resistor may be expressed as $P = I^2R$, where I is the current carried by the resistor of resistance R. Since resistors connected in series carry the same current, the resistor having the largest resistance will dissipate the most power.

10. Choice (c). Increasing the diameter of a wire increases the cross-sectional area. Thus, the cross-sectional area of A is greater than that of B, and from $R = \rho L/A$, we see that $R_A < R_B$. Since the power dissipated in a resistance may be expressed as $P = (\Delta V)^2/R$, the wire having the smallest resistance dissipates the most power for a given potential difference.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The average current in a conductor is the charge passing a given point per unit time, or $I = \Delta Q/\Delta t = e(\Delta n)/\Delta t$, so the number of electrons passing this point per second is

$$\frac{\Delta n}{\Delta t} = \frac{I}{e} = \frac{1.6\ \text{C/s}}{1.6\times10^{-19}\ \text{C/electron}} = 1.0\times10^{19}\ \text{electron/s}$$

making (c) the correct choice.

2. The drift velocity of charge carriers in a conductor is given by $v_d = I/nqA$. Wires A and B carry the same current I. Also, they are made of the same material, so the density and charge of the charge carriers (n and q) are the same for the two wires. Since the cross-sectional area is $A = \pi r^2$, and $r_A = 2r_B$, wire A has a cross-section 4 times that of B. This makes $v_A = v_B/4$, and the correct choice is (e).

3. By cutting the wire of length L and cross-sectional area A into pieces and placing the pieces side by side, a new wire with length $L' = L/3$, and cross-sectional area $A' = 3A$ is created. The new resistance is

$$R' = \frac{\rho L'}{A'} = \frac{\rho L/3}{3A} = \frac{1}{9}\left(\frac{\rho L}{A}\right) = \frac{1}{9}R$$

so the correct choice is (a).

4. The resistance of a conductor having length L' and cross-sectional area A is $R = \rho L'/A$. Thus, the resistance of each wire is: $R_1 = \rho L/\pi r^2$, $R_2 = \rho L/\pi(2r)^2 = \rho L/2\pi r^2 = \frac{1}{2}R_1$, and $R_3 = \rho(2L)/\pi(3r)^2 = 4\rho L/9\pi r^2 = \frac{4}{9}R_1$. We see that wire 3 has the smallest resistance, and choice (c) is the correct answer.

5. The ampere is a unit of current (1 amp = 1 coulomb/second). Hence, the ampere-hour is a unit of charge

$$1\ \text{ampere-hour} = 1\ \text{ampere}\cdot 1\ \text{hour} = (1\ \text{C/s})(3\,600\ \text{s}) = 3\,600\ \text{C}$$

Thus, the ampere-hour rating is a measure of the charge the battery can supply, and choice (d) is the correct answer.

6. When the potential across the device is 2 V, the current is 2 A, so the resistance is $R = \Delta V/I = 2\ \text{V}/2\ \text{A} = 1\ \Omega$, and (a) is the correct choice.

7. The power consumption of the set is $P = (\Delta V)I = (120\ \text{V})(2.5\ \text{A}) = 3.0\times10^2\ \text{W} = 0.30\ \text{kW}$. Thus, the energy used in 8.0 h of operation is $E = P\cdot t = (0.30\ \text{kW})(8.0\ \text{h}) = 2.4\ \text{kWh}$, at a cost of $cost = (2.4\ \text{kWh})(8.0\ \text{cents/kWh}) = 19\ \text{cents}$. The correct choice is (c).

8. The temperature variation of resistance is given by $R = R_0\left[1+\alpha(T-T_0)\right]$, where R_0 is the resistance of the conductor at the reference temperature T_0, usually 20.0°C. Using the given resistance at $T = 90.0°\text{C}$, the temperature coefficient of resistivity for this conducting material is found to be

$$\alpha = \frac{1}{T-T_0}\left(\frac{R}{R_0}-1\right) = \frac{1}{90.0°\text{C}-20.0°\text{C}}\left(\frac{10.55\ \Omega}{10.00\ \Omega}-1\right) = 7.86\times10^{-4}\ °\text{C}^{-1}$$

The resistance of this conductor at $T = -20.0°C$, where $\Delta T = -20.0°C - (20.0°C) = -40.0°C$, will be

$$R = (10.00\ \Omega)\left[1 + \left(7.86\times10^{-4}\ °C^{-1}\right)(-40.0°C)\right] = 9.69\ \Omega$$

so (b) is the correct choice.

9. When the potential difference across the device is 3.0 V, the current is 2.5 A, so the resistance is $R = \Delta V/I = 3.0\ V/2.5\ A = 1.2\ \Omega$, and (b) is the correct choice.

10. Resistors in a parallel combination all have the same potential difference across them. Thus, from Ohm's law, $I = \Delta V/R$, the resistor with the smallest resistance carries the largest current. Choice (a) is the correct response.

11. The current through the resistor is $I = \Delta V/R = 1.0\ V/10.0\ \Omega = 0.10\ A$, and the charge passing through in a 20 s interval is $\Delta Q = I\cdot\Delta t = (0.10\ C/s)(20\ s) = 2.0\ C$. Thus, (c) is the correct choice.

12. Resistors in a series combination all carry the same current. Thus, from Ohm's law, $\Delta V = IR$, the resistor with the highest resistance has the greatest voltage drop across it. Choice (c) is the correct response.

13. $$\frac{P_A}{P_B} = \frac{(\Delta V)^2/R_A}{(\Delta V)^2/R_B} = \frac{R_B}{R_A} = \frac{\rho L_B/A_B}{\rho L_A/A_A} = \frac{\rho L_B/\left(\pi d_B^2/4\right)}{\rho L_A/\left(\pi d_A^2/4\right)} = \left(\frac{L_B}{L_A}\right)\left(\frac{d_A}{d_B}\right)^2 = \left(\frac{1}{2}\right)(2)^2 = 2$$

Thus, $P_A/P_B = 2$, and the correct choice is (c).

14. The resistance of a conductor may be expressed as $R = \rho L/A$, so the cross-sectional area is $A = \rho L/R$. Thus,

$$\frac{A_A}{A_B} = \frac{\rho L/R_A}{\rho L/R_B} = \frac{R_B}{R_A} = \frac{R_B}{3R_B} = \frac{1}{3}$$

and choice (e) is the correct answer.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The amplitude of atomic vibrations increases with temperature, thereby scattering electrons more efficiently and increasing the resistivity of the material.

4. (a) The number of cars would correspond to charge Q.

(b) The rate of flow of cars past a point would correspond to current.

6. (a) The 25 W bulb has the higher resistance. Because $R = (\Delta V)^2/P$, and both operate from 120 V, the bulb dissipating the least power has the higher resistance.

(b) When the voltage is constant, the current and power are directly proportional to each other, $P = (\Delta V)I = (\text{constant})I$. Thus, the higher power bulb (100 W) carries more current.

8. An electrical shock occurs when your body serves as a conductor between two points having a difference in potential. The concept behind the admonition is to avoid simultaneously touching points that are at different potentials.

10. The knob is connected to a variable resistor. As you increase the magnitude of the resistance in the circuit, the current is reduced, and the bulb dims.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 5.57×10^{-5} m/s **(b)** the drift speed is smaller

4. 3.4×10^{21} electrons

6. 159 mA

8. 0.130 mm/s

10. 8.89 Ω

12. **(a)** 1.8 m **(b)** 0.28 mm

14. $3.22\times10^{-8}\,\Omega\cdot\text{m}$

16. 1.3

18. **(a)** 2.8×10^{8} A **(b)** 1.8×10^{7} A

20. **(a)** 5.0×10^{5} V

(b) Rubber gloves and soles can increase resistance to current passing through the body.

22. 6.3 Ω

24. **(a)** 1.6×10^{2} °C

(b) The expansion of the cross-sectional area contributes slightly more than the expansion of the length of the wire, so the answer would be slightly reduced.

26. 2.3×10^{2} °C

28. $1.1\times10^{-3}\,(^\circ\text{C})^{-1}$

30. **(a)** Yes, the design goal can be met with $R_{0,\text{Carbon}} = 4.4\ \Omega$ and $R_{0,\text{Nichrome}} = 5.6\ \Omega$.

(b) $L_{\text{Carbon}} = 0.89$ m, $L_{\text{Nichrome}} = 26$ m

32. 63.3°C

34. **(a)** $0.29 **(b)** $2.6

36. **(a)** 50 MW **(b)** 0.071 or 7.1%

38. **(a)** 3.2×10^{5} J **(b)** 18 min

40. **(a)** 184 W **(b)** 461°C

42. **(a)** 0.66 kWh **(b)** $0.079 = 7.9 cents

44. **(a)** $1.61 **(b)** 0.582 cents **(c)** $0.416 = 41.6 cents

46. **(a)** 3.5×10^{2} W **(b)** 41 Ω **(c)** 4.0×10^{1} Ω

(d) $d=\sqrt{4\rho_0 L/\pi R_0}$ **(e)** 0.38 mm

48. **(a)** 2.7×10^{-3} Ω **(b)** 1.45×10^{3} K **(c)** 1.7×10^{-2} Ω

(d) 1.1 V **(e)** They radiate only a small portion of the energy consumed as visible light.

50. 15.0 h

52. 1.5×10^{2} °C

54. 90 μV

56. **(a)** 9.3 m **(b)** 0.93 mm

58. **(a)** 18 C **(b)** 3.6 A

60. **(a)** 9.1 Ω

(b) We assume the temperature coefficient of resistivity for Nichrome remains constant over this temperature range.

62. No, the fuse should limit the current to 3.87 A or less.

64. **(a)** See Solution. **(b)** 1.420 Ω versus 1.418 Ω

66. **(a)** 470 W **(b)** 1.60 mm or more **(c)** 2.93 mm or more

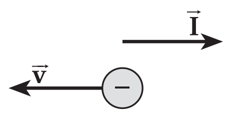
PROBLEM SOLUTIONS

17.1 (a) The charge that moves past the cross section is $\Delta Q = I(\Delta t)$, and the number of electrons is

$$n=\frac{\Delta Q}{|e|}=\frac{I(\Delta t)}{|e|}$$

$$=\frac{\left(80.0\times10^{-3}\ \text{C/s}\right)\left[(10.0\ \text{min})(60.0\ \text{s/min})\right]}{1.60\times10^{-19}\ \text{C}}=\boxed{3.00\times10^{20}\ \text{electrons}}$$

(b) The negatively charged electrons move in the direction opposite to the conventional current flow.



17.2 (a) From Example 17.2 in the textbook, the density of charge carriers (electrons) in a copper wire is $n = 8.46\times10^{28}$ electrons/m^3. With $A=\pi r^2$ and $|q|=e$, the drift speed of electrons in this wire is

$$v_d=\frac{I}{n|q|A}=\frac{3.70\ \text{C/s}}{\left(8.46\times10^{28}\ \text{m}^{-3}\right)\left(1.60\times10^{-19}\ \text{C}\right)\pi\left(1.25\times10^{-3}\ \text{m}\right)^2}=\boxed{5.57\times10^{-5}\ \text{m/s}}$$

continued on next page

(b) $\boxed{\text{The drift speed is smaller because more electrons are being conducted.}}$ With a greater number of charge carriers in motion, they do not have to move as fast to have a specified number of them passing a given point each second.

17.3 The period of the electron in its orbit is $T = 2\pi r/v$, and the current represented by the orbiting electron is $I = \Delta Q/\Delta t = |e|/T = v|e|/2\pi r$, or

$$I = \frac{\left(2.19\times10^{6}\ \text{m/s}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{2\pi\left(5.29\times10^{-11}\ \text{m}\right)} = 1.05\times10^{-3}\ \text{C/s} = \boxed{1.05\ \text{mA}}$$

17.4 Since $I = \Delta Q/\Delta t$, we have $\Delta Q = I(\Delta t)$, and the number of electrons passing through in time Δt is $N = \Delta Q/e = I(\Delta t)/e$. Thus,

$$N = \frac{(0.15\ \text{C/s})(1\ \text{h})(3\,600\ \text{s/1 h})}{1.60\times10^{-19}\ \text{C/electron}} = \boxed{3.4\times10^{21}\ \text{electrons}}$$

17.5 The resistance of a wire of length L and diameter d is $R = \rho L/A = \rho L\big/\left(\pi d^2/4\right)$, giving $d^2 = 4\rho L/\pi R$. Using Table 17.1, the needed diameter is found to be

$$d = \sqrt{\frac{4\rho L}{\pi R}} = \sqrt{\frac{4\left(2.82\times10^{-8}\ \Omega\cdot\text{m}\right)(32.0\ \text{m})}{\pi(2.50\ \Omega)}} = 6.78\times10^{-4}\ \text{m} = \boxed{0.678\ \text{mm}}$$

17.6 The mass of a single gold atom is $m_{\text{atom}} = M/N_A$, where M is the molecular weight of gold and N_A is Avogadro's number. Thus, the number of atoms deposited, and hence the number of ions moving to the negative electrode, is

$$n = \frac{m}{m_{\text{atom}}} = \frac{mN_A}{M} = \frac{\left(3.25\times10^{-3}\ \text{kg}\right)\left(6.02\times10^{23}\ \text{atoms/mol}\right)}{(197\ \text{g/mol})\left(10^{-3}\ \text{kg/g}\right)} = 9.93\times10^{21}$$

The current in the cell is then

$$I = \frac{\Delta Q}{\Delta t} = \frac{ne}{\Delta t} = \frac{\left(9.93\times10^{21}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{(2.78\ \text{h})(3\,600\ \text{s/1 h})} = 0.159\ \text{A} = \boxed{159\ \text{mA}}$$

17.7 The drift speed of electrons in the line is $v_d = I/nqA = I\big/n|e|\left(\pi d^2/4\right)$. The time to travel the length of the 200-km line is then $\Delta t = L/v_d = Ln|e|\left(\pi d^2\right)\big/4I$, or

$$\Delta t = \frac{\left(200\times10^{3}\ \text{m}\right)\left(8.5\times10^{28}/\text{m}^3\right)\left(1.6\times10^{-19}\ \text{C}\right)\pi(0.02\ \text{m})^2}{4(1\,000\ \text{A})\left(3.156\times10^{7}\ \text{s/yr}\right)} = \boxed{27\ \text{yr}}$$

17.8 The mass of a single aluminum atom is $m_{\text{atom}} = M/N_A$, where the molecular weight (mass per mole) of aluminum is $M = 27.0\ \text{g/mol}$, and Avogadro's number (number of atoms per mole) is $N_A = 6.02\times10^{23}/\text{mol}$. If each aluminum atom contributes one conduction electron, the number of conduction electrons per unit volume is

$$n = \frac{\text{mass per unit volume}}{\text{mass per atom}} = \frac{\text{density}}{m_{\text{atom}}} = \frac{\rho N_A}{M}$$

and the drift speed is $v_d = I/nqA = MI/\rho N_A qA$. Thus, for the given case, we find

continued on next page

$$v_d = \frac{(27.0\ \text{g/mol})(5.00\ \text{C/s})}{\left[\left(2.70\ \dfrac{\text{g}}{\text{cm}^3}\right)\left(\dfrac{10^6\ \text{cm}^3}{1\ \text{m}^3}\right)\right]\left(\dfrac{6.02\times10^{23}}{\text{mol}}\right)(1.60\times10^{-19}\ \text{C})(4.00\times10^{-6}\ \text{m}^2)}$$

or $v_d = 1.30\times10^{-4}\ \text{m/s} = \boxed{0.130\ \text{mm/s}}$.

17.9 (a) Using the periodic table on the inside back cover of the textbook, we find

$$M_{\text{Fe}} = 55.85\ \text{g/mol} = (55.85\ \text{g/mol})(1\ \text{kg}/10^3\ \text{g}) = \boxed{55.85\times10^{-3}\ \text{kg/mol}}$$

(b) From Table 9.1, the density of iron is $\rho_{\text{Fe}} = 7.86\times10^3\ \text{kg/m}^3$, so the molar density is

$$(\text{molar density})_{\text{Fe}} = \frac{\rho_{\text{Fe}}}{M_{\text{Fe}}} = \frac{7.86\times10^3\ \text{kg/m}^3}{55.85\times10^{-3}\ \text{kg/mol}} = \boxed{1.41\times10^5\ \text{mol/m}^3}$$

(c) The density of iron atoms is

$$\text{density of atoms} = N_A(\text{molar density})$$

$$= \left(6.02\times10^{23}\ \frac{\text{atoms}}{\text{mol}}\right)\left(1.41\times10^5\ \frac{\text{mol}}{\text{m}^3}\right) = \boxed{8.49\times10^{28}\ \frac{\text{atoms}}{\text{m}^3}}$$

(d) With two conduction electrons per iron atom, the density of charge carriers is

$$n = (\text{charge carriers/atom})(\text{density of atoms})$$

$$= \left(2\ \frac{\text{electrons}}{\text{atom}}\right)\left(8.49\times10^{28}\ \frac{\text{atoms}}{\text{m}^3}\right) = \boxed{1.70\times10^{29}\ \text{electrons/m}^3}$$

(e) With a current of $I = 30.0$ A and cross-sectional area $A = 5.00\times10^{-6}\ \text{m}^2$, the drift speed of the conduction electrons in this wire is

$$v_d = \frac{I}{nqA} = \frac{30.0\ \text{C/s}}{(1.70\times10^{29}\ \text{m}^{-3})(1.60\times10^{-19}\ \text{C})(5.00\times10^{-6}\ \text{m}^3)} = \boxed{2.21\times10^{-4}\ \text{m/s}}$$

17.10 From Ohm's law, $R = \dfrac{\Delta V}{I} = \dfrac{1.20\times10^2\ \text{V}}{13.5\ \text{A}} = \boxed{8.89\ \Omega}$

17.11 $(\Delta V)_{\max} = I_{\max}R = (80\times10^{-6}\ \text{A})R$

Thus, if $R = 4.0\times10^5\ \Omega$, $(\Delta V)_{\max} = \boxed{32\ \text{V}}$

and if $R = 2\,000\ \Omega$, $(\Delta V)_{\max} = \boxed{0.16\ \text{V}}$

17.12 The volume of the copper is

$$V = \frac{m}{density} = \frac{1.00\times10^{-3}\ \text{kg}}{8.92\times10^3\ \text{kg/m}^3} = 1.12\times10^{-7}\ \text{m}^3$$

Since $V = A\cdot L$, this gives $A\cdot L = 1.12\times10^{-7}\ \text{m}^3$. **[1]**

continued on next page

(a) From $R = \frac{\rho L}{A}$, where ρ is the resistivity of copper, we find that

$$A = \left(\frac{\rho}{R}\right)L = \left(\frac{1.7\times10^{-8}\ \Omega\cdot\text{m}}{0.500\ \Omega}\right)L = \left(3.4\times10^{-8}\ \text{m}\right)L$$

Inserting this expression for A into Equation [1] gives

$$\left(3.4\times10^{-8}\ \text{m}\right)L^2 = 1.12\times10^{-7}\ \text{m}^3,\ \text{which yields}\quad L = \boxed{1.8\ \text{m}}$$

(b) From Equation [1], $A = \frac{\pi d^2}{4} = \frac{1.12\times10^{-7}\ \text{m}^3}{L}$, or

$$d = \sqrt{\frac{4\left(1.12\times10^{-7}\ \text{m}^3\right)}{\pi L}} = \sqrt{\frac{4\left(1.12\times10^{-7}\ \text{m}^3\right)}{\pi(1.8\ \text{m})}}$$

$$= 2.8\times10^{-4}\ \text{m} = \boxed{0.28\ \text{mm}}$$

17.13 (a) From Ohm's law, $R = \frac{\Delta V}{I} = \frac{1.20\times10^2\ \text{V}}{9.25\ \text{A}} = \boxed{13.0\ \Omega}$

(b) Using $R = \rho L/A$ and data from Table 17.1, the required length is found to be

$$L = \frac{RA}{\rho} = \frac{R\left(\pi r^2\right)}{\rho} = \frac{(13.0\ \Omega)\pi\left(0.791\times10^{-3}\ \text{m}\right)^2}{150\times10^{-8}\ \Omega\cdot\text{m}} = \boxed{17.0\ \text{m}}$$

17.14 From $R = \rho L/A = \rho L/\left(\pi d^2/4\right)$, the resistivity is

$$\rho = \frac{\pi R d^2}{4L} = \frac{\pi(1.60\ \Omega)\left(0.800\times10^{-3}\ \text{m}\right)^2}{4(25.0\ \text{m})} = \boxed{3.22\times10^{-8}\ \Omega\cdot\text{m}}$$

17.15 (a) $R = \frac{\Delta V}{I} = \frac{12\ \text{V}}{0.40\ \text{A}} = \boxed{30\ \Omega}$

(b) From $R = \rho L/A$,

$$\rho = \frac{R\cdot A}{L} = \frac{(30\ \Omega)\left[\pi\left(0.40\times10^{-2}\ \text{m}\right)^2\right]}{3.2\ \text{m}} = \boxed{4.7\times10^{-4}\ \Omega\cdot\text{m}}$$

17.16 Using $R = \rho L/A$ and data from Table 17.1, we have $\rho_{\text{Cu}}L_{\text{Cu}}/\pi r_{\text{Cu}}^2 = \rho_{\text{Al}}L_{\text{Al}}/\pi r_{\text{Al}}^2$. This reduces to $r_{\text{Al}}^2/r_{\text{Cu}}^2 = \rho_{\text{Al}}/\rho_{\text{Cu}}$, and yields

$$\frac{r_{\text{Al}}}{r_{\text{Cu}}} = \sqrt{\frac{\rho_{\text{Al}}}{\rho_{\text{Cu}}}} = \sqrt{\frac{2.82\times10^{-8}\ \Omega\cdot\text{m}}{1.7\times10^{-8}\ \Omega\cdot\text{m}}} = \boxed{1.3}$$

17.17 From Ohm's law, $R = \Delta V/I$, and $R = \rho L/A = \rho L/\left(\pi d^2/4\right)$, the resistivity is

$$\rho = \frac{\pi R d^2}{4L} = \frac{\pi(\Delta V)d^2}{4IL} = \frac{\pi(9.11\ \text{V})\left(2.00\times10^{-3}\ \text{m}\right)^2}{4(36.0\ \text{A})(50.0\ \text{m})} = 1.59\times10^{-8}\ \Omega\cdot\text{m}$$

Then, from Table 17.1, we see that the wire is made of $\boxed{\text{silver}}$.

17.18 With different orientations of the block, three different values of the ratio L/A are possible. These are:

$$\left(\frac{L}{A}\right)_1 = \frac{10\text{ cm}}{(20\text{ cm})(40\text{ cm})} = \frac{1}{80\text{ cm}} = \frac{1}{0.80\text{ m}},$$

$$\left(\frac{L}{A}\right)_2 = \frac{20\text{ cm}}{(10\text{ cm})(40\text{ cm})} = \frac{1}{20\text{ cm}} = \frac{1}{0.20\text{ m}},$$

and $$\left(\frac{L}{A}\right)_3 = \frac{40\text{ cm}}{(10\text{ cm})(20\text{ cm})} = \frac{1}{5.0\text{ cm}} = \frac{1}{0.050\text{ m}}$$

(a) $$I_{\max} = \frac{\Delta V}{R_{\min}} = \frac{\Delta V}{\rho(L/A)_{\min}} = \frac{(6.0\text{ V})(0.80\text{ m})}{1.7\times 10^{-8}\ \Omega\cdot\text{m}} = \boxed{2.8\times 10^{8}\text{ A}}$$

(b) $$I_{\min} = \frac{\Delta V}{R_{\max}} = \frac{\Delta V}{\rho(L/A)_{\max}} = \frac{(6.0\text{ V})(0.050\text{ m})}{1.7\times 10^{-8}\ \Omega\cdot\text{m}} = \boxed{1.8\times 10^{7}\text{ A}}$$

17.19 The volume of material, $V = AL_0 = \left(\pi r_0^2\right)L_0$, in the wire is constant. Thus, as the wire is stretched to decrease its radius, the length increases such that $\left(\pi r_f^2\right)L_f = \left(\pi r_0^2\right)L_0$ giving $L_f = \left(r_0/r_f\right)^2 L_0 = \left(r_0/0.25r_0\right)^2 L_0 = (4.0)^2 L_0 = 16L_0$. The new resistance is then

$$R_f = \rho\frac{L_f}{A_f} = \rho\frac{L_f}{\pi r_f^2} = \rho\frac{16L_0}{\pi\left(0.25r_0\right)^2} = 256\left(\rho\frac{L_0}{\pi r_0^2}\right) = 256R_0 = 256(1.00\ \Omega) = \boxed{256\ \Omega}$$

17.20 (a) From Ohm's law, $\Delta V = IR = \left(500\times 10^{-3}\text{ A}\right)\left(1.0\times 10^{6}\ \Omega\right) = \boxed{5\times 10^{5}\text{ V}}$

(b) Rubber-soled shoes and rubber gloves can increase the resistance to current and help reduce the likelihood of a serious shock.

17.21 If a conductor of length L has a uniform electric field E maintained within it, the potential difference between the ends of the conductor is $\Delta V = EL$. But, from Ohm's law, the relation between the potential difference across a conductor and the current through it is $\Delta V = IR$, where $R = \rho L/A$. Combining these relations, we obtain

$$\Delta V = EL = IR = I\left(\rho L/A\right) \qquad \text{or} \qquad \boxed{E = \rho(I/A) = \rho J}$$

17.22 Using $R = R_0\left[1+\alpha\left(T - T_0\right)\right]$, with $R_0 = 6.00\ \Omega$ at $T_0 = 20.0°\text{C}$ and $\alpha_{\text{silver}} = 3.8\times 10^{-3}\ (°\text{C})^{-1}$ (from Table 17.1 in the textbook), the resistance at $T = 34.0°\text{C}$ is

$$R = (6.00\ \Omega)\left[1 + 3.8\times 10^{-3}\ (°\text{C})^{-1}(34.0°\text{C} - 20.0°\text{C})\right] = \boxed{6.3\ \Omega}$$

17.23 From Ohm's law, $\Delta V = I_i R_i = I_f R_f$, so the current in Antarctica is

$$I_f = I_i\left(\frac{R_i}{R_f}\right) = I_i\left(\frac{R_0\left[1+\alpha\left(T_i - T_0\right)\right]}{R_0\left[1+\alpha\left(T_f - T_0\right)\right]}\right) = (1.00\text{ A})\left(\frac{1+\left[3.9\times 10^{-3}\ (°\text{C})^{-1}\right](58.0°\text{C} - 20.0°\text{C})}{1+\left[3.9\times 10^{-3}\ (°\text{C})^{-1}\right](-88.0°\text{C} - 20.0°\text{C})}\right)$$

or $I_f = \boxed{2.0\text{ A}}$

17.24 (a) Given: Aluminum wire with $\alpha = 3.9\times10^{-3}\ (°\text{C})^{-1}$ (see Table 17.1 in textbook), and $R_0 = 30.0\ \Omega$ at $T_0 = 20.0°\text{C}$. If $R = 46.2\ \Omega$ at temperature T, solving $R = R_0\left[1+\alpha\left(T-T_0\right)\right]$ gives the final temperature as

$$T = T_0 + \frac{\left(R/R_0\right)-1}{\alpha} = 20.0°\text{C} + \frac{\left(46.2\ \Omega/30.0\ \Omega\right)-1}{3.9\times10^{-3}\ (°\text{C})^{-1}} = \boxed{1.6\times10^{2}\ °\text{C}}$$

(b) The expansion of the cross-sectional area contributes slightly more than the expansion of the length of the wire, so the answer would be slightly reduced.

17.25 The volume of the gold wire may be written as $V = A\cdot L = m/\rho_d$, where ρ_d is the density of gold. Thus, the cross-sectional area is $A = m/\rho_d L$. The resistance of the wire is $R = \rho_e L/A$, where ρ_e is the electrical resistivity. Therefore,

$$R = \frac{\rho_e L}{m/\rho_d L} = \frac{\rho_e \rho_d L^2}{m} = \frac{\left(2.44\times10^{-8}\ \Omega\cdot\text{m}\right)\left(19.3\times10^{3}\ \text{kg/m}^3\right)\left(2.40\times10^{3}\ \text{m}\right)^2}{1.00\times10^{-3}\ \text{kg}}$$

giving $R = 2.71\times10^{6}\ \Omega = \boxed{2.71\ \text{M}\Omega}$

17.26 For aluminum, the resistivity at room temperature is $\rho_{0,\text{Al}} = 2.82\times10^{-8}\ \Omega\cdot\text{m}$, and the temperature coefficient of resistivity is $\alpha_{\text{Al}} = 3.9\times10^{-3}\ (°\text{C})^{-1}$. Thus, if at some temperature, the aluminum has a resistivity of ρ_{Al}, solving $\rho_{\text{Al}} = \rho_{0,\text{Al}}\left[1+\alpha_{\text{Al}}\left(T-T_0\right)\right]$ for that temperature gives $T = T_0 + \left\{\left[\left(\rho_{\text{Al}}/\rho_{0,\text{Al}}\right)-1\right]/\alpha_{\text{Al}}\right\}$, where $T_0 = 20°\text{C}$.

When $\rho_{\text{Al}} = 3\rho_{0,\text{Cu}} = 3\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right) = 5.1\times10^{-8}\ \Omega\cdot\text{m}$, the temperature must be

$$T = 20°\text{C} + \frac{\left(\dfrac{5.1\times10^{-8}\ \Omega\cdot\text{m}}{2.82\times10^{-8}\ \Omega\cdot\text{m}}\right)-1}{3.9\times10^{-3}\ (°\text{C})^{-1}} = \boxed{2.3\times10^{2}\ °\text{C}}$$

17.27 At 80°C,

$$I = \frac{\Delta V}{R} = \frac{\Delta V}{R_0\left[1+\alpha\left(T-T_0\right)\right]} = \frac{5.0\ \text{V}}{\left(2.0\times10^{2}\ \Omega\right)\left[1+\left(-0.5\times10^{-3}\ °\text{C}^{-1}\right)\left(80°\text{C}-20°\text{C}\right)\right]}$$

or $I = 2.6\times10^{-2}\ \text{A} = \boxed{26\ \text{mA}}$

17.28 If $R = 41.0\ \Omega$ at $T = 20.0°\text{C}$ and $R = 41.4\ \Omega$ at $T = 29.0°\text{C}$, then $R = R_0\left[1+\alpha\left(T-T_0\right)\right]$ gives the temperature coefficient of resistivity of the material making up this wire as

$$\alpha = \frac{R-R_0}{R_0\left(T-T_0\right)} = \frac{41.4\ \Omega - 41.0\ \Omega}{(41.0\ \Omega)(29.0°\text{C}-20.0°\text{C})} = \boxed{1.1\times10^{-3}\ (°\text{C})^{-1}}$$

17.29 (a) The resistance at 20.0°C is

$$R_0 = \rho\frac{L}{A} = \frac{\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)(34.5\ \text{m})}{\pi\left(0.25\times10^{-3}\ \text{m}\right)^2} = 3.0\ \Omega$$

and the current will be $I = \dfrac{\Delta V}{R_0} = \dfrac{9.0\ \text{V}}{3.0\ \Omega} = \boxed{3.0\ \text{A}}$

continued on next page

(b) At 30.0°C,

$$R = R_0\left[1+\alpha\left(T-T_0\right)\right]$$

$$= (3.0\ \Omega)\left[1+\left(3.9\times10^{-3}\ (°\text{C})^{-1}\right)(30.0°\text{C}-20.0°\text{C})\right] = 3.1\ \Omega$$

Thus, the current is $I = \dfrac{\Delta V}{R} = \dfrac{9.0\text{ V}}{3.1\ \Omega} = \boxed{2.9\text{ A}}$.

17.30 We call the carbon resistor 1 and the Nichrome resistor 2. Then, connecting the resistors end to end, the total resistance is

$$R = R_1 + R_2 = R_{01}\left[1+\alpha_1\cdot\Delta T\right] + R_{02}\left[1+\alpha_2\cdot\Delta T\right] = R_{01}+R_{02}+\left(R_{01}\alpha_1 + R_{02}\alpha_2\right)\Delta T$$

If this is not to vary with temperature, it is necessary that $R_{01}\alpha_1 + R_{02}\alpha_2 = 0$, or

$$R_{01} = -\left(\frac{\alpha_2}{\alpha_1}\right)R_{02} = -\left(\frac{0.4\times10^{-3}\ (°\text{C})^{-1}}{-0.5\times10^{-3}\ (°\text{C})^{-1}}\right)R_{02} = 0.8R_{02}$$

If the constant total resistance is to be $R = 10.0\ \Omega$, it is necessary that

$$R_{01} + R_{02} = 0.8R_{02} + R_{02} = 10.0\ \Omega$$

or $R_{02} = \dfrac{10.0\ \Omega}{1.8} = 5.6\ \Omega$ and $R_{01} = 10.0\ \Omega - R_{02} = 4.4\ \Omega$

(a) $\boxed{\text{Yes}}$ it is possible for her to meet the design goal with this method.

(b) From $R = \rho L/A = \rho L/\pi r^2$, we have $L = \left(\pi r^2\right)\cdot R/\rho$. This yields

$$L_1 = \left(\pi r^2\right)\frac{R_{01}}{\rho_{01}} = \pi\left(1.50\times10^{-3}\text{ m}\right)^2\frac{4.4\ \Omega}{3.5\times10^{-5}\ \Omega\cdot\text{m}} = \boxed{0.89\text{ m}}\ \text{(carbon)}$$

and $$L_2 = \left(\pi r^2\right)\frac{R_{02}}{\rho_{02}} = \pi\left(1.50\times10^{-3}\text{ m}\right)^2\frac{5.6\ \Omega}{150\times10^{-8}\ \Omega\cdot\text{m}} = \boxed{26\text{ m}}\ \text{(Nichrome)}$$

17.31 (a) From $R = \rho L/A$, the initial resistance of the mercury is

$$R_i = \frac{\rho L_i}{A_i} = \frac{\left(9.4\times10^{-7}\ \Omega\cdot\text{m}\right)(1.000\ 0\text{ m})}{\pi\left(1.00\times10^{-3}\text{ m}\right)^2/4} = \boxed{1.2\ \Omega}$$

(b) Since the volume of mercury is constant, $V = A_f\cdot L_f = A_i\cdot L_i$ gives the final cross-sectional area as $A_f = A_i\cdot\left(L_i/L_f\right)$. Thus, the final resistance is given by $R_f = \rho L_f/A_f = \rho L_f^2/A_i\cdot L_i$. The fractional change in the resistance is then

$$\Delta = \frac{R_f - R_i}{R_i} = \frac{R_f}{R_i} - 1 = \frac{\rho L_f^2/\left(A_i\cdot L_i\right)}{\rho L_i/A_i} - 1 = \left(\frac{L_f}{L_i}\right)^2 - 1$$

$$\Delta = \left(\frac{100.04}{100.00}\right)^2 - 1 = \boxed{8.0\times10^{-4}}\ \text{or}\ \boxed{\text{a }0.080\%\text{ increase}}$$

17.32 The resistance at the reference temperature of 20.0°C is

$$R_0 = \frac{R}{1+\alpha(T-T_0)} = \frac{200.0\ \Omega}{1+\left[3.92\times10^{-3}\ (°\text{C})^{-1}\right](0°\text{C}-20.0°\text{C})} = 217\ \Omega$$

Solving $R = R_0\left[1+\alpha(T-T_0)\right]$ for T gives the temperature of the melting potassium as

$$T = T_0 + \frac{R-R_0}{\alpha R_0} = 20.0°\text{C} + \frac{253.8\ \Omega - 217\ \Omega}{\left[3.92\times10^{-3}\ (°\text{C})^{-1}\right](217\ \Omega)} = \boxed{63.3°\text{C}}$$

17.33 (a) The power consumed by the device is $P = I(\Delta V)$, so the current must be

$$I = \frac{P}{\Delta V} = \frac{1.00\times10^{3}\ \text{W}}{1.20\times10^{2}\ \text{V}} = \boxed{8.33\ \text{A}}$$

(b) From Ohm's law, the resistance is $R = \dfrac{\Delta V}{I} = \dfrac{1.20\times10^{2}\ \text{V}}{8.33\ \text{A}} = \boxed{14.4\ \Omega}$

17.34 (a) The energy used by a 100-W bulb in 24 h is

$$E = P\cdot\Delta t = (100\ \text{W})(24\ \text{h}) = (0.100\ \text{kW})(24\ \text{h}) = 2.4\ \text{kWh}$$

and the cost of this energy, at a rate of \$0.12 per kilowatt-hour is

$$cost = E\cdot rate = (2.4\ \text{kWh})(\$0.12/\text{kWh}) = \boxed{\$0.29}$$

(b) The energy used by the oven in 5.0 h is

$$E = P\cdot\Delta t = \left[I(\Delta V)\right]\cdot\Delta t = \left[(20.0\ \text{C/s})(220\ \text{J/C})\left(\frac{1\ \text{kW}}{10^{3}\ \text{J/s}}\right)\right](5.0\ \text{h}) = 22\ \text{kWh}$$

and the cost of this energy, at a rate of \$0.12 per kilowatt-hour is

$$cost = E\cdot rate = (22\ \text{kWh})(\$0.12/\text{kWh}) = \boxed{\$2.6}$$

17.35 (a) $R_{\text{Cu}} = \dfrac{\rho_{\text{Cu}}L}{A} = \dfrac{4\rho_{\text{Cu}}L}{\pi d^2} = \dfrac{4\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)(1.00\ \text{m})}{\pi\left(0.205\times10^{-2}\ \text{m}\right)^2} = 5.2\times10^{-3}\ \Omega$

and $P_{\text{Cu}} = I^2R_{\text{Cu}} = (20.0\ \text{A})^2\left(5.2\times10^{-3}\ \Omega\right) = \boxed{2.1\ \text{W}}$

(b) $R_{\text{Al}} = \dfrac{\rho_{\text{Al}}L}{A} = \dfrac{4\rho_{\text{Al}}L}{\pi d^2} = \dfrac{4\left(2.82\times10^{-8}\ \Omega\cdot\text{m}\right)(1.00\ \text{m})}{\pi\left(0.205\times10^{-2}\ \text{m}\right)^2} = 8.54\times10^{-3}\ \Omega$

and $P_{\text{Al}} = I^2R_{\text{Al}} = (20.0\ \text{A})^2\left(8.54\times10^{-3}\ \Omega\right) = \boxed{3.42\ \text{W}}$

(c) $\boxed{\text{No}}$, the aluminum wire would not be as safe. If surrounded by thermal insulation, it would get much hotter than the copper wire.

17.36 (a) The power loss in the line is $P_{\text{loss}} = I^2R = (1\,000\ \text{A})^2\left[(0.31\ \Omega/\text{km})(160\ \text{km})\right]$, or $P_{\text{loss}} = 5.0\times10^{7}\ \text{W} = \boxed{50\ \text{MW}}$.

(b) The total power transmitted is $P_{\text{input}} = (\Delta V)I = \left(700\times10^{3}\ \text{V}\right)(1\,000\ \text{A})$, or $P_{\text{input}} = 7.0\times10^{8}\ \text{W} = 700\ \text{MW}$.

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Thus, the fraction of the total transmitted power represented by the line losses is

$$fraction\ loss = \frac{P_{\text{loss}}}{P_{\text{input}}} = \frac{50\ \text{MW}}{700\ \text{MW}} = 0.071 \text{ or } \boxed{7.1\%}$$

17.37 The energy required to bring the water to the boiling point is

$$E = mc(\Delta T) = (0.500\ \text{kg})(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(100^\circ\text{C} - 23.0^\circ\text{C}) = 1.61\times10^5\ \text{J}$$

The power input by the heating element is

$$P_{\text{input}} = (\Delta V)I = (120\ \text{V})(2.00\ \text{A}) = 240\ \text{W} = 240\ \text{J/s}$$

Therefore, the time required is

$$t = \frac{E}{P_{\text{input}}} = \frac{1.61\times10^5\ \text{J}}{240\ \text{J/s}} = 671\ \text{s}\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{11.2\ \text{min}}$$

17.38 (a) $E = P\cdot t = (90\ \text{W})(1\ \text{h}) = (90\ \text{J/s})(3\,600\ \text{s}) = \boxed{3.2\times10^5\ \text{J}}$

(b) The power consumption of the color set is

$$P = (\Delta V)I = (120\ \text{V})(2.50\ \text{A}) = 300\ \text{W}$$

Therefore, the time required to consume the energy found in (a) is

$$t = \frac{E}{P} = \frac{3.2\times10^5\ \text{J}}{300\ \text{J/s}} = 1.1\times10^3\ \text{s}\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{18\ \text{min}}$$

17.39 (a) $R_{\text{A}} = \dfrac{(\Delta V)^2}{P_{\text{A}}} = \dfrac{(120\ \text{V})^2}{25.0\ \text{W}} = \boxed{576\ \Omega}$

and $R_{\text{B}} = \dfrac{(\Delta V)^2}{P_{\text{B}}} = \dfrac{(120\ \text{V})^2}{100\ \text{W}} = \boxed{144\ \Omega}$

(b) $\Delta t_{\text{A}} = \dfrac{Q}{I_{\text{A}}} = Q\left(\dfrac{R_{\text{A}}}{\Delta V}\right) = (1.00\ \text{C})\left(\dfrac{576\ \Omega}{120\ \text{V}}\right) = \boxed{4.80\ \text{s}}$

(c) The charge is the same. However, as it leaves the bulb, it is at a lower potential than when it entered the bulb.

(d) $P = \dfrac{W}{\Delta t}$, so $\Delta t_{\text{A}} = \dfrac{W}{P_{\text{A}}} = \dfrac{1.00\ \text{J}}{25.0\ \text{J/s}} = \boxed{0.040\,0\ \text{s}}$

(e) Energy enters the bulb by electrical transmission and leaves by heat and radiation.

(f) $E = P_{\text{A}}(\Delta t) = \left[(25.0\ \text{W})\left(\dfrac{1\ \text{kW}}{10^3\ \text{W}}\right)\right]\left[(30.0\ \text{d})\left(\dfrac{24.0\ \text{h}}{1\ \text{d}}\right)\right] = 18.0\ \text{kWh}$

and $cost = E\times rate = (18.0\ \text{kWh})(\$0.110/\text{kWh}) = \boxed{\$1.98}$

17.40 (a) At the operating temperature,

$$P = (\Delta V)I = (120\ \text{V})(1.53\ \text{A}) = \boxed{184\ \text{W}}$$

(b) From $R = R_0\left[1+\alpha\left(T-T_0\right)\right]$, the temperature T is given by $T = T_0 + (R-R_0)/\alpha R_0$. The resistances are given by Ohm's law as

$$R = \frac{(\Delta V)}{I} = \frac{120\ \text{V}}{1.53\ \text{A}} \quad \text{and} \quad R_0 = \frac{(\Delta V)_0}{I_0} = \frac{120\ \text{V}}{1.80\ \text{A}}$$

Therefore, the operating temperature is

$$T = 20.0°\text{C} + \frac{(120/1.53)-(120/1.80)}{\left(0.400\times10^{-3}\ (°\text{C})^{-1}\right)(120/1.80)} = \boxed{461°\text{C}}$$

17.41 The power loss per unit length of the cable is $P/L = (I^2R)/L = I^2(R/L)$. Thus, the resistance per unit length of the cable is

$$\frac{R}{L} = \frac{P/L}{I^2} = \frac{2.00\ \text{W/m}}{(300\ \text{A})^2} = 2.22\times10^{-5}\ \Omega/\text{m}$$

From $R = \rho L/A$, the resistance per unit length is also given by $R/L = \rho/A$. Hence, the cross-sectional area is $\pi r^2 = A = \rho/(R/L)$, and the required radius is

$$r = \sqrt{\frac{\rho}{\pi(R/L)}} = \sqrt{\frac{1.7\times10^{-8}\ \Omega\cdot\text{m}}{\pi\left(2.22\times10^{-5}\ \Omega/\text{m}\right)}} = 0.016\ \text{m} = \boxed{1.6\ \text{cm}}$$

17.42 (a) The rating of the 12-V battery is $I\cdot\Delta t = 55\ \text{A}\cdot\text{h}$. Thus, the stored energy is

$$\text{Energy stored} = P\cdot\Delta t = (\Delta V)I\cdot\Delta t = (12\ \text{V})(55\ \text{A}\cdot\text{h}) = 660\ \text{W}\cdot\text{h} = \boxed{0.66\ \text{kWh}}$$

(b) $\textit{value} = (0.66\ \text{kWh})(\$0.12/\text{kWh}) = \$0.079 = \boxed{7.9\ \text{cents}}$

17.43 $P = (\Delta V)I = \left(75.0\times10^{-3}\ \text{V}\right)\left(0.200\times10^{-3}\ \text{A}\right) = 1.50\times10^{-5}\ \text{W} = 15.0\times10^{-6}\ \text{W} = \boxed{15.0\ \mu\text{W}}$

17.44 (a) $E = P\cdot t = (40.0\ \text{W})(14.0\ \text{d})(24.0\ \text{h/d}) = 1.34\times10^4\ \text{Wh} = 13.4\ \text{kWh}$

$$\textit{cost} = E\cdot(\textit{rate}) = (13.4\ \text{kWh})(\$0.120/\text{kWh}) = \boxed{\$1.61}$$

(b) $E = P\cdot t = (0.970\ \text{kW})(3.00\ \text{min})(1\ \text{h}/60\ \text{min}) = 4.85\times10^{-2}\ \text{kWh}$

$$\textit{cost} = \left(4.85\times10^{-2}\ \text{kWh}\right)(\$0.120/\text{kWh}) = \$0.005\,82 = \boxed{0.582\ \text{cents}}$$

(c) $E = P\cdot t = (5.20\ \text{kW})(40.0\ \text{min})(1\ \text{h}/60\ \text{min}) = 3.47\ \text{kWh}$

$$\textit{cost} = E\cdot(\textit{rate}) = (3.47\ \text{kWh})(\$0.120/\text{kWh}) = \$0.416 = \boxed{41.6\ \text{cents}}$$

17.45 Total length of transmission lines: $L = 2(50.0\ \text{m}) = 100\ \text{m}$. Thus, the resistance of these lines is $R = (0.108\ \Omega/300\ \text{m})(100\ \text{m}) = 3.60\times10^{-2}\ \Omega$.

(a) The total potential drop along the transmission lines is $(\Delta V)_{\text{lines}} = IR$, giving

$$(\Delta V)_{\text{house}} = (\Delta V)_{\text{source}} - (\Delta V)_{\text{lines}} = 120\ \text{V} - (110\ \text{A})\left(3.60\times10^{-2}\ \Omega\right) = \boxed{116\ \text{V}}$$

(b) $P_{\text{delivered}} = I(\Delta V)_{\text{house}} = (110\ \text{A})(116\ \text{V}) = 1.28\times10^4\ \text{W} = \boxed{12.8\ \text{kW}}$

17.46 (a) The thermal energy needed to raise the water temperature by ΔT is $E = mc(\Delta T)$. If $\Delta T = 100°\text{C} - 20°\text{C} = 80°\text{C}$ and this energy is to be delivered in 4.00 min, the average power required is

$$P = \frac{E}{\Delta t} = \frac{mc(\Delta T)}{\Delta t} = \frac{(0.250\ \text{kg})(4\,186\ \text{J/kg}\cdot°\text{C})(100°\text{C} - 20°\text{C})}{(4.00\ \text{min})(60\ \text{s/1 min})} = \boxed{3.5\times 10^2\ \text{W}}$$

(b) The required resistance (at 100°C) of the heating element is then

$$R = \frac{(\Delta V)^2}{P} = \frac{(120\ \text{V})^2}{3.5\times 10^2\ \text{W}} = \boxed{41\ \Omega}$$

(c) The resistance at 20.0°C would then be

$$R_0 = \frac{R}{1+\alpha\left(T - T_0\right)} = \frac{41\ \Omega}{1+\left(0.4\times 10^{-3}\ °\text{C}^{-1}\right)(100°\text{C} - 20°\text{C})} = \boxed{4.0\times 10^1\ \Omega}$$

(d) We find the needed dimensions of a Nichrome wire for this heating element from $R_0 = \rho_0\, L/A = \rho_0\, L\big/\left(\pi d^2/4\right) = 4\rho_0\, L/\pi d^2$, where L is the length of the wire and d is its diameter. This gives the diameter as $\boxed{d = \sqrt{4\rho_0 L/\pi R_0}}$.

(e) If $L = 3.00$ m, the required diameter of the wire is

$$d = \left[\frac{4\rho_0 L}{\pi R_0}\right]^{\frac{1}{2}} = \left[\frac{4\left(150\times 10^{-8}\ \Omega\cdot\text{m}\right)(3.00\ \text{m})}{\pi\left(4.0\times 10^1\ \Omega\right)}\right]^{\frac{1}{2}} = 3.8\times 10^{-4}\ \text{m} = \boxed{0.38\ \text{mm}}$$

17.47 The power dissipated in a conductor is $P = (\Delta V)^2/R$, so the resistance may be written as $R = (\Delta V)^2/P$. Hence,

$$\frac{R_B}{R_A} = \frac{\cancel{(\Delta V)^2}}{P_B}\cdot\frac{P_A}{\cancel{(\Delta V)^2}} = \frac{P_A}{P_B} = 3 \qquad \text{or} \qquad R_B = 3R_A$$

Since $R = \rho L/A = \rho L\big/\left(\pi d^2/4\right)$, this result becomes

$$\frac{\cancel{4\rho L}}{\cancel{\pi}\, d_B^2} = 3\left(\frac{\cancel{4\rho L}}{\cancel{\pi}\, d_A^2}\right) \qquad \text{or} \qquad \frac{d_A^2}{d_B^2} = 3$$

and yields $\boxed{d_A/d_B = \sqrt{3}}$.

17.48 (a) For tungsten, Table 17.1 from the textbook gives the resistivity at $T_0 = 20.0°\text{C} = 293$ K as $\rho_0 = 5.6\times 10^{-8}\ \Omega\cdot\text{m}$ and the temperature coefficient of resistivity as $\alpha = 4.5\times 10^{-3}\ (°\text{C})^{-1} = 4.5\times 10^{-3}\ \text{K}^{-1}$. Thus, for a tungsten wire having a radius of 1.00 mm and a length of 15.0 cm, the resistance at $T_0 = 293$ K is

$$R_0 = \rho_0\frac{L}{A} = \rho_0\frac{L}{\left(\pi r^2\right)} = \left(5.6\times 10^{-8}\ \Omega\cdot\text{m}\right)\frac{\left(15.0\times 10^{-2}\ \text{m}\right)}{\pi\left(1.00\times 10^{-3}\ \text{m}\right)^2} = \boxed{2.7\times 10^{-3}\ \Omega}$$

(b) From Stefan's law (see Section 11.5 of the textbook), the radiated power is $P = \sigma AeT^4$, where A is the area of the radiating surface. Note that since we are computing the radiated power, not the net energy gained or lost as a result of radiation, the ambient temperature is not needed here. In the case of a wire, this is the cylindrical surface area $A = 2\pi rL$. The temperature of the wire when it is radiating a power of $P = 75.0$ W is given by $T = \left[P/\sigma Ae\right]^{1/4}$ as

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$$T=\left[\frac{75.0\text{ W}}{\left(5.669\times10^{-8}\text{ W/m}^2\cdot\text{K}^4\right)2\pi\left(1.00\times10^{-3}\text{ m}\right)(0.150\text{ m})(0.320)}\right]^{1/4}=\boxed{1.45\times10^3\text{ K}}$$

(c) Assuming a linear temperature variation of resistance, the resistance of the wire at this temperature is

$$R=R_0\left[1+\alpha\left(T-T_0\right)\right]=\left(2.7\times10^{-3}\ \Omega\right)\left[1+\left(4.5\times10^{-3}\text{ K}^{-1}\right)\left(1.45\times10^3\text{ K}-293\text{ K}\right)\right]$$

giving $R=\boxed{1.7\times10^{-2}\ \Omega}$

(d) The voltage drop across the wire when it is radiating 75.0 W and has the resistance found in part (c) above is given by $P=(\Delta V)^2/R$ as

$$\Delta V=\sqrt{R\cdot P}=\sqrt{\left(1.7\times10^{-2}\ \Omega\right)(75.0\text{ W})}=\boxed{1.1\text{ V}}$$

(e) $\boxed{\text{Tungsten bulbs radiate little of the energy they consume in the form of visible light}}$, making them inefficient sources of light.

17.49 The battery is rated to deliver the equivalent of 60.0 amperes of current (i.e., 60.0 C/s) for 1 hour. This is

$$Q=I\cdot\Delta t=(60.0\text{ A})(1\text{ h})=(60.0\text{ C/s})(3\,600\text{ s})=\boxed{2.16\times10^5\text{ C}}$$

17.50 The energy available in the battery is

$$\text{Energy stored}=P\cdot t=(\Delta V)I\cdot t=(12.0\text{ V})(90.0\text{ A}\cdot\text{h})=1.08\times10^3\text{ W}\cdot\text{h}$$

The two head lights together consume a total power of $P=2(36.0\text{ W})=72.0\text{ W}$, so the time required to completely discharge the battery is

$$\Delta t=\frac{\text{Energy stored}}{P}=\frac{1.08\times10^3\text{ W}\cdot\text{h}}{72.0\text{ W}}=\boxed{15.0\text{ h}}$$

17.51 (a) $$R=\frac{\rho L}{A}=\frac{\rho L}{\pi d^2/4}=\frac{4\rho L}{\pi d^2}=\frac{4\left(2.82\times10^{-8}\ \Omega\cdot\text{m}\right)(15.0\text{ m})}{\pi\left(0.600\times10^{-3}\text{ m}\right)^2}=\boxed{1.50\ \Omega}$$

(b) $$I=\frac{\Delta V}{R}=\frac{9.00\text{ V}}{1.50\ \Omega}=\boxed{6.00\text{ A}}$$

17.52 Using chemical symbols to denote the two different metals, the resistances are equal when

$$R_{0_{\text{Cu}}}\left[1+\alpha_{\text{Cu}}(\Delta T)\right]=R_{0_{\text{W}}}\left[1+\alpha_{\text{W}}(\Delta T)\right]$$

or $$\frac{R_{0_{\text{Cu}}}}{R_{0_{\text{W}}}}-1=\left[\alpha_{\text{W}}-\left(\frac{R_{0_{\text{Cu}}}}{R_{0_{\text{W}}}}\right)\alpha_{\text{Cu}}\right](\Delta T)$$

Thus, $$\Delta T=T-20.0\text{ °C}=\frac{\left(R_{0_{\text{Cu}}}/R_{0_{\text{W}}}\right)-1}{\alpha_{\text{W}}-\left(R_{0_{\text{Cu}}}/R_{0_{\text{W}}}\right)\alpha_{\text{Cu}}}$$

or $$T=20.0\text{ °C}+\frac{\left(R_{0_{\text{Cu}}}/R_{0_{\text{W}}}\right)-1}{\alpha_{\text{W}}-\left(R_{0_{\text{Cu}}}/R_{0_{\text{W}}}\right)\alpha_{\text{Cu}}}$$

continued on next page

$$T = 20.0\ ^\circ\text{C} + \frac{(5.00/4.75) - 1}{4.5\times10^{-3}\ (^\circ\text{C})^{-1} - (5.00/4.75)\left(3.9\times10^{-3}\ (^\circ\text{C})^{-1}\right)}$$

$$= 20.0\ ^\circ\text{C} + 1.3\times10^{2}\ ^\circ\text{C} = \boxed{1.5\times10^{2}\ ^\circ\text{C}}$$

17.53 From $P = (\Delta V)^2/R$, the total resistance needed is $R = (\Delta V)^2/P = (20\ \text{V})^2/48\ \text{W} = 8.3\ \Omega$.

Thus, from $R = \rho L/A$, the length of wire required is

$$L = \frac{R\cdot A}{\rho} = \frac{(8.3\ \Omega)\left(4.0\times10^{-6}\ \text{m}^2\right)}{3.0\times10^{-8}\ \Omega\cdot\text{m}} = 1.1\times10^{3}\ \text{m} = \boxed{1.1\ \text{km}}$$

17.54 The resistance of the 4.0 cm length of wire between the feet is

$$R = \frac{\rho L}{A} = \frac{\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)(0.040\ \text{m})}{\pi(0.022\ \text{m})^2/4} = 1.8\times10^{-6}\ \Omega$$

so the potential difference is

$$\Delta V = IR = (50\ \text{A})\left(1.8\times10^{-6}\ \Omega\right) = 9.0\times10^{-5}\ \text{V} = \boxed{90\ \mu\text{V}}$$

17.55 Ohm's law gives the resistance as $R = (\Delta V)/I$. From $R = \rho L/A$, the resistivity is given by $\rho = R\cdot(A/L)$. The results of these calculations for each of the three wires are summarized in the table below.

L (m)	R (Ω)	ρ ($\Omega\cdot$m)
0.540	10.4	1.41×10^{-6}
1.028	21.1	1.50×10^{-6}
1.543	31.8	1.50×10^{-6}

The average value found for the resistivity is

$$\rho_{av} = \frac{\Sigma\rho_i}{3} = \boxed{1.47\times10^{-6}\ \Omega\cdot\text{m}}$$

which differs from the value of $\rho = 150\times10^{-8}\ \Omega\cdot\text{m} = 1.50\times10^{-6}\ \Omega\cdot\text{m}$ given in Table 17.1 by $\boxed{2.0\%}$.

17.56 The volume of the material is

$$V = \frac{mass}{density} = \frac{50.0\ \text{g}}{7.86\ \text{g/cm}^3}\left(\frac{1\ \text{m}^3}{10^6\ \text{cm}^3}\right) = 6.36\times10^{-6}\ \text{m}^3$$

Since $V = A\cdot L$, the cross-sectional area of the wire is $A = V/L$.

(a) From $R = \rho L/A = \rho L/(V/L) = \rho L^2/V$, the length of the wire is given by

$$L = \sqrt{\frac{R\cdot V}{\rho}} = \sqrt{\frac{(1.5\ \Omega)\left(6.36\times10^{-6}\ \text{m}^3\right)}{11\times10^{-8}\ \Omega\cdot\text{m}}} = \boxed{9.3\ \text{m}}$$

continued on next page

(b) The cross-sectional area of the wire is $A = \pi d^2/4 = V/L$. Thus, the diameter is

$$d = \sqrt{\frac{4V}{\pi L}} = \sqrt{\frac{4\left(6.36\times10^{-6}\ \text{m}^3\right)}{\pi(9.3\ \text{m})}} = 9.3\times10^{-4}\ \text{m} = \boxed{0.93\ \text{mm}}$$

17.57 (a) The total power you now use while cooking breakfast is

$$P = (1\,200 + 500)\ \text{W} = 1.70\ \text{kW}$$

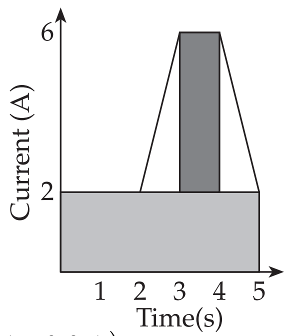
The cost to use this power for 0.500 h each day for 30.0 days is

$$cost = [P\times(\Delta t)]\times rate = \left[(1.70\ \text{kW})\left(0.500\ \frac{\text{h}}{\text{day}}\right)(30.0\ \text{days})\right](\$0.120/\text{kWh}) = \boxed{\$3.06}$$

(b) If you upgraded, the new power requirement would be $P = (2\,400 + 500)\ \text{W} = 2\,900\ \text{W}$, and the required current would be $I = P/\Delta V = 2\,900\ \text{W}/110\ \text{V} = 26.4\ \text{A} > 20\ \text{A}$.

$\boxed{\text{No}}$, your present circuit breaker cannot handle the upgrade.

17.58 (a) The charge passing through the conductor in the interval $0 \le t \le 5.0$ s is represented by the area under the I vs. t graph given in Figure P17.58. This area consists of two rectangles and two triangles. Thus,



$$\Delta Q = A_{\text{rectangle 1}} + A_{\text{rectangle 2}} + A_{\text{triangle 1}} + A_{\text{triangle 2}}$$

$$= (5.0\ \text{s} - 0)(2.0\ \text{A} - 0) + (4.0\ \text{s} - 3.0\ \text{s})(6.0\ \text{A} - 2.0\ \text{A})$$

$$+\frac{1}{2}(3.0\ \text{s} - 2.0\ \text{s})(6.0\ \text{A} - 2.0\ \text{A}) + \frac{1}{2}(5.0\ \text{s} - 4.0\ \text{s})(6.0\ \text{A} - 2.0\ \text{A})$$

$$\Delta Q = \boxed{18\ \text{C}}$$

(b) The constant current that would pass the same charge in 5.0 s is

$$I = \frac{\Delta Q}{\Delta t} = \frac{18\ \text{C}}{5.0\ \text{s}} = \boxed{3.6\ \text{A}}$$

17.59 (a) The power input to the motor is $P_{\text{input}} = (\Delta V)I = P_{\text{output}}/efficiency$, so the required current is

$$I = \frac{P_{\text{output}}}{(\Delta V)(efficiency)} = \frac{(2.50\ \text{hp})(746\ \text{W}/1\ \text{hp})}{(120\ \text{V})(0.900)} = \boxed{17.3\ \text{A}}$$

(b) $$E = P_{\text{input}}(\Delta t) = \frac{P_{\text{output}}}{efficiency}(\Delta t) = \frac{(2.50\ \text{hp})(0.746\ \text{kW}/1\ \text{hp})}{0.900}(3.00\ \text{h})$$

yielding $E = 6.22\ \text{kWh} = (6.22\ \text{kWh})\left(\frac{3.60\ \text{MJ}}{1\ \text{kWh}}\right) = \boxed{22.4\ \text{MJ}}$

(c) $cost = E\times rate = (6.22\ \text{kWh})(\$0.110/\text{kWh}) = \boxed{\$0.684}$

17.60 (a) $R = R_0\left[1+\alpha(\Delta T)\right] = (8.0\ \Omega)\left[1+\left(0.4\times10^{-3}\ {}^\circ\text{C}^{-1}\right)(350^\circ\text{C}-20^\circ\text{C})\right] = \boxed{9.1\ \Omega}$

(b) We assume that the temperature coefficient of resistivity for Nichrome remains constant over this temperature range.

17.61 The current in the wire is $I = \Delta V/R = 15.0\ \text{V}/0.100\ \Omega = 150\ \text{A}$. Then, from the expression for the drift velocity, $v_d = I/nqA$, the density of free electrons is

$$n = \frac{I}{v_d e\left(\pi r^2\right)} = \frac{150\ \text{A}}{\left(3.17\times10^{-4}\ \text{m/s}\right)\left(1.60\times10^{-19}\ \text{C}\right)\pi\left(5.00\times10^{-3}\ \text{m}\right)^2}$$

or $n = \boxed{3.77\times10^{28}/\text{m}^3}$

17.62 Each speaker has a resistance of $R = 4.00\ \Omega$ and can handle a maximum power of 60.0 W. From $P = I^2R$, the maximum safe current is

$$I_{\text{max}} = \sqrt{\frac{P_{\text{max}}}{R}} = \sqrt{\frac{60.0\ \text{W}}{4.00\ \Omega}} = 3.87\ \text{A}$$

Thus, the system $\boxed{\text{is not adequately protected}}$ by a 4.00 A fuse.

17.63 The cross-sectional area of the conducting material is $A = \pi\left(r_{\text{outer}}^2 - r_{\text{inner}}^2\right)$.

Thus,

$$R = \frac{\rho L}{A} = \frac{\left(3.5\times10^5\ \Omega\cdot\text{m}\right)\left(4.0\times10^{-2}\ \text{m}\right)}{\pi\left[\left(1.2\times10^{-2}\ \text{m}\right)^2 - \left(0.50\times10^{-2}\ \text{m}\right)^2\right]} = 3.7\times10^7\ \Omega = \boxed{37\ \text{M}\Omega}$$

17.64 (a) At temperature T, the resistance is $R = \dfrac{\rho L}{A}$, where $\rho = \rho_0\left[1+\alpha\left(T-T_0\right)\right]$,

$L = L_0\left[1+\alpha'\left(T-T_0\right)\right]$, and $A = A_0\left[1+2\alpha'\left(T-T_0\right)\right]$

Thus,

$$R = \left(\frac{\rho_0 L_0}{A_0}\right)\frac{\left[1+\alpha\left(T-T_0\right)\right]\cdot\left[1+\alpha'\left(T-T_0\right)\right]}{\left[1+2\alpha'\left(T-T_0\right)\right]} = \boxed{\frac{R_0\left[1+\alpha\left(T-T_0\right)\right]\cdot\left[1+\alpha'\left(T-T_0\right)\right]}{\left[1+2\alpha'\left(T-T_0\right)\right]}}$$

(b) $R_0 = \dfrac{\rho_0 L_0}{A_0} = \dfrac{\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)(2.00\ \text{m})}{\pi\left(0.100\times10^{-3}\right)^2} = 1.082\ \Omega$

Then $R = R_0\left[1+\alpha\left(T-T_0\right)\right]$ gives

$$R = (1.082\ \Omega)\left[1+\left(3.9\times10^{-3}\ {}^\circ\text{C}^{-1}\right)(80.0^\circ\text{C})\right] = \boxed{1.420\ \Omega}$$

The more complex formula gives

$$R = \frac{(1.420\ \Omega)\cdot\left[1+\left(17\times10^{-6}\ {}^\circ\text{C}^{-1}\right)(80.0^\circ\text{C})\right]}{\left[1+2\left(17\times10^{-6}\ {}^\circ\text{C}^{-1}\right)(80.0^\circ\text{C})\right]} = \boxed{1.418\ \Omega}$$

Note: Some rules for handing significant figures have been deliberately violated in this solution in order to illustrate the very small difference in the results obtained with these two expressions.

17.65 The power the beam delivers to the target is

$$P = (\Delta V)I = \left(4.0\times10^{6}\ \text{V}\right)\left(25\times10^{-3}\ \text{A}\right) = 1.0\times10^{5}\ \text{W}$$

The mass of cooling water that must flow through the tube each second if the rise in the water temperature is not to exceed 50°C is found from $P = (\Delta m/\Delta t)c(\Delta T)$ as

$$\frac{\Delta m}{\Delta t} = \frac{P}{c(\Delta T)} = \frac{1.0\times10^{5}\ \text{J/s}}{(4\,186\ \text{J/kg}\cdot{}^\circ\text{C})(50^\circ\text{C})} = \boxed{0.48\ \text{kg/s}}$$

17.66 **Note:** All potential differences in this solution have a value of $\Delta V = 120\ \text{V}$. First, we shall do a symbolic solution for many parts of the problem and then enter the specified numeric values for the cases of interest.

From the marked specifications on the cleaner, its internal resistance (assumed constant) is

$$R_{\text{vac}} = \frac{(\Delta V)^2}{P_1} \qquad \text{where } P_1 = 535\ \text{W} \tag{1}$$

If each of the two conductors in the extension cord has resistance R_c, the total resistance in the path of the current (outside of the power source) is

$$R_t = R_{\text{vac}} + 2R_c \tag{2}$$

so the current which will exist is $I = \Delta V/R_t$, and the power that is delivered to the cleaner is

$$P_{\text{delivered}} = I^2 R_{\text{vac}} = \left(\frac{\Delta V}{R_t}\right)^2 R_{\text{vac}} = \left(\frac{\Delta V}{R_t}\right)^2 \frac{(\Delta V)^2}{P_1} = \frac{(\Delta V)^4}{R_t^2 P_1} \tag{3}$$

The resistance of a copper conductor of length L and diameter d is

$$R_c = \rho_{\text{Cu}}\frac{L}{A} = \rho_{\text{Cu}}\frac{L}{\left(\pi d^2/4\right)} = \frac{4\rho_{\text{Cu}}L}{\pi d^2}$$

Thus, if $R_{c,\,\max}$ is the maximum allowed value of R_c, the minimum acceptable diameter of the conductor is

$$d_{\min} = \sqrt{\frac{4\rho_{\text{Cu}}L}{\pi R_{c,\,\max}}} \tag{4}$$

(a) If $R_c = 0.900\ \Omega$, then from Equations [2] and [1],

$$R_t = R_{\text{vac}} + 2(0.900\ \Omega) = \frac{(\Delta V)^2}{P_1} + 1.80\ \Omega = \frac{(120\ \text{V})^2}{535\ \text{W}} + 1.80\ \Omega$$

and, from Equation [3], the power delivered to the cleaner is

$$P_{\text{delivered}} = \frac{(120\ \text{V})^4}{\left[\dfrac{(120\ \text{V})^2}{535\ \text{W}} + 1.80\ \Omega\right]^2 (535\ \text{W})} = \boxed{470\ \text{W}}$$

continued on next page

If the minimum acceptable power delivered to the cleaner is P_{min}, then the maximum allowable total resistance is given by Equations [2] and [3] as

$$R_{t,\,\text{max}} = R_{\text{vac}} + 2R_{c,\,\text{max}} = \sqrt{\frac{(\Delta V)^4}{P_{\text{min}} P_1}} = \frac{(\Delta V)^2}{\sqrt{P_{\text{min}} P_1}}$$

so

$$R_{c,\,\text{max}} = \frac{1}{2}\left[\frac{(\Delta V)^2}{\sqrt{P_{\text{min}} P_1}} - R_{\text{vac}}\right] = \frac{1}{2}\left[\frac{(\Delta V)^2}{\sqrt{P_{\text{min}} P_1}} - \frac{(\Delta V)^2}{P_1}\right] = \frac{(\Delta V)^2}{2}\left[\frac{1}{\sqrt{P_{\text{min}} P_1}} - \frac{1}{P_1}\right]$$

(b) When $P_{\text{min}} = 525\ \text{W}$, then

$$R_{c,\,\text{max}} = \frac{(120\ \text{V})^2}{2}\left[\frac{1}{\sqrt{(525\ \text{W})(535\ \text{W})}} - \frac{1}{535\ \text{W}}\right] = 0.128\ \Omega$$

and, from Equation [4],

$$d_{\text{min}} = \sqrt{\frac{4\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)(15.0\ \text{m})}{\pi(0.128\ \Omega)}} = \boxed{1.60\ \text{mm}}$$

(c) When $P_{\text{min}} = 532\ \text{W}$, then

$$R_{c,\,\text{max}} = \frac{(120\ \text{V})^2}{2}\left[\frac{1}{\sqrt{(532\ \text{W})(535\ \text{W})}} - \frac{1}{535\ \text{W}}\right] = 0.037\,9\ \Omega$$

and $$d_{\text{min}} = \sqrt{\frac{4\left(1.7\times10^{-8}\ \Omega\cdot\text{m}\right)(15.0\ \text{m})}{\pi(0.037\,9\ \Omega)}} = \boxed{2.93\ \text{mm}}$$

18

Direct-Current Circuits

QUICK QUIZZES

1. True. When a battery is delivering a current, there is a voltage drop within the battery due to internal resistance, making the terminal voltage less than the emf.

2. Because of internal resistance, power is dissipated into the battery material, raising its temperature.

3. Choice (b). When the switch is opened, resistors R_1 and R_2 are in series, so that the total circuit resistance is larger than when the switch was closed. As a result, the current decreases.

4. Choice (a). When the switch is opened, resistors R_1 and R_2 are in series, so the total circuit resistance is larger than and the current through R_1 is less with the switch open than when it is closed. Since the power delivered to R_1 is $P = I^2 R_1$, $P_o < P_c$.

5. Choice (a). When the switch is closed, resistors R_1 and R_2 are in parallel, so that the total circuit resistance is smaller than when the switch was open. As a result, the current increases.

6. Choice (b). Observe that the potential difference across R_1 equals the terminal potential difference of the battery. If the battery has negligible internal resistance, the terminal potential difference is the same with the switch open or closed. Under these conditions, the power delivered to R_1, equal to $P = (\Delta V)^2/R_1$, is unchanged when the switch is closed.

7. The voltage drop across each bulb connected in parallel with each other and across the battery equals the terminal potential difference of the battery. As more bulbs are added, the current supplied by the battery increases. However, if the internal resistance is negligible, the terminal potential difference is constant and the current through each bulb is the same regardless of the number of bulbs connected. Under these conditions: (a) The brightness of a bulb, determined by the current flowing in the bulb, is unchanged as bulbs are added. (b) The individual currents in the bulbs, $I = \Delta V/R$, are constant as bulbs are added since ΔV does not change. (c) The total power delivered by the battery increases by an amount $(\Delta V)^2/R$ each time a bulb is added. (d) With the total delivered power increasing, energy is drawn from the battery at an increasing rate and the battery's lifetime decreases.

8. Adding bulbs in series with each other and the battery increases the total load resistance seen by the battery. This means that the current supplied by the battery decreases with each new bulb that is added. (a) The brightness of a bulb is determined by the power delivered to that bulb, $P_{\text{bulb}} = I^2 R$, which decreases as bulbs are added and the current decreases. (b) For a series connection, the individual currents in the bulbs are the same and equal to the total current supplied by the battery. This decreases as bulbs are added. (c) The total power delivered by the battery is given by $P_{\text{total}} = (\Delta V)I$, where ΔV is the terminal potential difference of the battery and I is the total current supplied by the battery. With negligible internal resistance, ΔV is constant. Thus, with I decreasing as bulbs are added, the total delivered power decreases. (d) With the delivered power decreasing, energy is drawn from the battery at a decreasing rate, which increases the lifetime of the battery.

9. Choice (c). After the capacitor is fully charged, current flows only around the outer loop of the circuit. This path has a total resistance of 3 Ω, so the 6-V battery will supply a current of 2 amperes.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The same potential difference exists across all elements connected in parallel with each other, while the current through each element is inversely proportional to the resistance of that element ($I = \Delta V/R$). Thus, both (b) and (c) are true statements while the other choices are false.

2. In a series connection, the same current exists in each element. The potential difference across a resistor in this series connection is directly proportional to the resistance of that resistor, $\Delta V = IR$, and independent of its location within the series connection. The only true statement among the listed choices is (c).

3. For these parallel resistors,

$$\frac{1}{R_{\text{eq}}} = \frac{1}{1.00\ \Omega} + \frac{1}{2.00\ \Omega} = \frac{2+1}{2.00\ \Omega} \text{ and } R_{\text{eq}} = \frac{2.00\ \Omega}{3} = 0.667\ \Omega$$

Choice (c) is the correct answer.

4. The total power dissipated is $P_{\text{total}} = P_1 + P_2 = 120\ \text{W} + 60.0\ \text{W} = 180\ \text{W}$, while the potential difference across this series combination is $\Delta V = 120\ \text{V}$. The current drawn through the series combination is then $I = \dfrac{P_{\text{total}}}{\Delta V} = \dfrac{180\ \text{W}}{120\ \text{V}} = 1.5\ \text{A}$, and (b) is the correct choice.

5. The equation of choice (b) is the result of a correct application of Kirchhoff's junction rule at either of the two junctions in the circuit. The equation of choice (c) results from a correct application of Kirchhoff's loop rule to the lower loop in the circuit, while the equation of choice (d) is obtained by correctly applying the loop rule to the loop forming the outer perimeter of the circuit. The equation of choice (a) is the result of an incorrect application (involving 2 sign errors) of the loop rule to the upper loop in the circuit. The correct answer is choice (a).

6. The equivalent resistance of the parallel combination consisting of the 4.0-Ω, 6.0-Ω, and 10-Ω resistors is $R_p = 1.9\ \Omega$. This resistance is in series with a 2.0-Ω resistor, making the total resistance of the circuit $R_{\text{total}} = 3.9\ \Omega$. The total current supplied by the battery is $I_{\text{total}} = \Delta V/R_{\text{total}} = 12\ \text{V}/3.9\ \Omega = 3.1\ \text{A}$. Thus, the potential difference across each resistor in the parallel combination is $\Delta V_p = R_p I_{\text{total}} = (1.9\ \Omega)(3.1\ \text{A}) = 5.9\ \text{V}$ and the current through the 10 Ω resistor is $I_{10} = \Delta V_p/10\ \Omega = 5.9\ \text{V}/10\ \Omega = 0.59\ \text{A}$. Choice (a) is the correct answer.

7. The potential difference across each element of this parallel combination is $\Delta V = 120\ \text{V}$, and the total power dissipated is $P_{\text{total}} = 1\,200\ \text{W} + 600\ \text{W} = 1\,800\ \text{W}$. The total current through the parallel combination, and hence, the current drawn from the external source is then $I = \dfrac{P_{\text{total}}}{\Delta V} = \dfrac{1\,800\ \text{W}}{120\ \text{V}} = 15\ \text{A}$. The correct choice is (e).

8. When the two identical resistors are in series, the current supplied by the battery is $I = \Delta V/2R$, and the total power delivered is $P_s = (\Delta V)I = (\Delta V)^2/2R$. With the resistors connected in parallel, the potential difference across each resistor is ΔV and the power delivered to each resistor is $P_1 = (\Delta V)^2/R$. Thus, the total power delivered in this case is

$$P_p = 2P_1 = 2\frac{(\Delta V)^2}{R} = 4\left[\frac{(\Delta V)^2}{2R}\right] = 4P_s = 4(8.0\ \text{W}) = 32\ \text{W}$$

and (b) is the correct choice.

9. The equivalent resistance for the series combination of five identical resistors is $R_{eq} = 5R$, and the equivalent capacitance of five identical capacitors in parallel is $C_{eq} = 5C$. The time constant for the circuit is therefore $\tau = R_{eq}C_{eq} = (5R)(5C) = 25RC$ and (d) is the correct choice.

10. When the switch is closed, the current has a large initial value but decreases exponentially in time. The bulb will glow brightly at first, but fade rapidly as the capacitor charges. After a time equal to many time constants of the circuit, the current is essentially zero and the bulb does not glow. The correct answer is choice (c).

11. The equivalent resistance of a group of resistors connected in parallel is the reciprocal of the inverses of the individual resistances and is always less than the smallest resistance in the group. Therefore, both (b) and (e) are true statements while all other choices are false.

12. When resistors are connected in series, Kirchhoff's junction rule requires that the current be the same in all resistors at each instant in time. Thus, the charge entering each resistor in a given time interval must be the same for all resistors, and both choices (b) and (d) are correct. The other choices are false since the potential difference, $\Delta V = IR$, and the power, $P = I^2R$, must differ when the resistances are different.

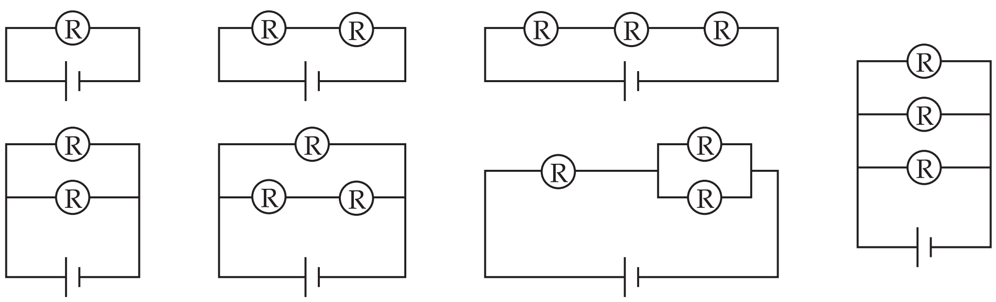
13. With the capacitor initially uncharged, the potential difference across the capacitor, $\Delta V = q/C$, starts at zero when the switch is first closed and rises exponentially toward the equilibrium value of $(\Delta V)_{max} = \Delta V_{battery} = 6.00$ V. The time constant of the circuit is the time required for the charge (and hence the potential difference) to increase from 0 to 63.2% of the maximum equilibrium value. Thus, after one time constant, the potential difference across the capacitor will be $\Delta V = 0.632(6.00\text{ V}) = 3.79$ V. The correct answer is choice (d).

14. The circuit breaker should be connected in series with the device so that an open circuit results when the circuit breaker trips, causing the current through the device to cease. Thus, choice (a) is the correct answer.

15. The equivalent resistance of a series combination of resistors is the algebraic sum of the individual resistances and is always greater than any individual resistance. Therefore, choices (a) and (d) are true statements and all others are false.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2.



4. A short circuit can develop when the last bit of insulation frays away between the two conductors in a lamp cord. Then the two conductors touch each other, creating a low resistance path in parallel with the lamp. The lamp will immediately go out, carrying no current and presenting no danger. A very large current will be produced in the power source, the house wiring, and the wire in the lamp cord up to and through the short. The circuit breaker will interrupt the circuit quickly but not before considerable heating and sparking is produced in the short-circuit path.

6. A wire or cable in a transmission line is thick and made of material with very low resistivity. Only when its length is very large does its resistance become significant. To transmit power over a long distance it is most efficient to use low current at high voltage. The power loss per unit length of the transmission line is $P_{\text{loss}}/L = I^2(R/L)$, where R/L is the resistance per unit length of the line. Thus, a low current is clearly desirable, but to transmit a significant amount of power $P = (\Delta V)I$ with low current, a high voltage must be used.

8. The bulbs of set A are wired in parallel. The bulbs of set B are wired in series, so removing one bulb produces an open circuit with infinite resistance and zero current.

10. (a) ii. The power delivered may be expressed as $P = I^2R$, and while resistors connected in series have the same current in each, they may have different values of resistance.

(b) ii. The power delivered may also be expressed as $P = (\Delta V)^2/R$, and while resistors connected in parallel have the same potential difference across them, they may have different values of resistance.

12. Compare two runs in series to two resistors connected in series. Compare three runs in parallel to three resistors connected in parallel. Compare one chairlift followed by two runs in parallel to a battery followed immediately by two resistors in parallel.

The junction rule for ski resorts says that the number of skiers coming into a junction must be equal to the number of skiers leaving. The loop rule would be stated as the total change in altitude must be zero for any skier completing a closed path.

14. Because water is a good conductor, if you should become part of a short circuit when fumbling with any electrical circuit while in a bathtub, the current would follow a pathway through you, the water, and to ground. Electrocution would be the obvious result.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** $27\ \Omega$ **(b)** 0.44 A **(c)** $3.0\ \Omega,\ 1.3$ A

4. **(a)** 1.13 A **(b)** $9.17\ \Omega$

6. **(a)** $35.0\ \Omega/3 = 11.7\ \Omega$

(b) 1.00 A in the 12.0-Ω and 8.00-Ω resistors, 2.00 A in the 6.00-Ω and 4.00-Ω resistors, and 3.00 A in the 5.00-Ω resistor

8. **(a)** $3.33\ \Omega$ **(b)** $7.33\ \Omega$ **(c)** $2.13\ \Omega$

(d) $4.13\ \Omega$ **(e)** 1.94 A **(f)** 3.88 V

(g) 4.12 V **(h)** 1.37 A

10. **(a)** $\Delta V_1 = \varepsilon/3,\ \Delta V_2 = 2\varepsilon/9,\ \Delta V_3 = 4\varepsilon/9,\ \Delta V_4 = 2\varepsilon/3$

(b) $I_1 = I,\ I_2 = I_3 = I/3,\ I_4 = 2I/3$

12. **(a)** $1.00\ \text{k}\Omega$ **(b)** $2.00\ \text{k}\Omega$ **(c)** $3.00\ \text{k}\Omega$

14. **(a)** Yes, $R_{eq} = 8.00\ \Omega$ **(b)** 2.25 A **(c)** 40.5 W

16. **(a)** $I_6 = 3.0\text{ A}, I_{12} = 2.0\text{ A}, I_{24} = 1.0\text{ A}$

(b) Answers are the same as for part (a).

18. **(a)** 0.909 A **(b)** $|\Delta V_{ab}| = 1.82$ V, with point b at the lower potential.

20. $I_2 = 2.0\text{ A}$ flowing from b toward a, $I_3 = 1.0\text{ A}$ in direction shown

22. **(a)** $60.0\ \Omega$ **(b)** 0.20 A **(c)** 2.4 W

(d) 0.50 W

24. **(a)** $4.59\ \Omega$ **(b)** 0.081 6 or 8.16%

26. **(a)** $1.7\times10^2\text{ A}$ **(b)** 0.20 A

28. **(a)** $18.0I_{12} + 13.0I_{18} = 30.0$ **(b)** $5.00I_{36} - 18.0I_{12} = 24.0$ **(c)** $I_{18} = I_{12} + I_{36}$

(d) $I_{36} = I_{18} - I_{12}$ **(e)** $5.00I_{18} - 23.0I_{12} = 24.0$

(f) $I_{12} = -0.416\text{ A}, I_{18} = 2.88\text{ A}$ **(g)** $I_{36} = 3.30\text{ A}$

(h) I_{12} flows opposite to the assumed direction and has magnitude 0.416 A.

30. See Solution.

32. **(a)** 2.00 ms **(b)** 180 μC **(c)** 114 μC

34. $1.3\times10^2\ \mu\text{C}$

36. 587 kΩ

38. **(a)** 8.0 A **(b)** 120 V **(c)** 0.80 A

(d) $5.8\times10^2\text{ W}$

40. **(a)** $I_{\substack{\text{coffee}\\\text{maker}}} = 10\text{ A}, I_{\text{toaster}} = 9.2\text{ A}, I_{\substack{\text{waffle}\\\text{maker}}} = 12\text{ A}$

(b) $I_{\text{total}} = 31\text{ A}$ **(c)** No, $I_{\text{total}} > 15\text{ A}$.

42. **(a)** $4.1\times10^{-11}\text{ J}$ **(b)** 0.56 μA

44. Sixteen distinct resistances are possible – See Solution for how these are produced.

46. **(a)** 2.0 kΩ **(b)** 15 V **(c)** 9.0 V

(d) Assumed negligible current through the voltmeter and negligible resistance in the ammeter.

48. **(a)** 40.0 W **(b)** $\Delta V_1 = 80.0\text{ V}, \Delta V_2 = \Delta V_3 = 40.0\text{ V}$

50. **(a)** 0.708 A **(b)** 2.51 W

(c) Only the circuit of Figure P18.50c. In the other circuits, the batteries can be combined into a single effective battery while the 5.00-Ω and 8.00-Ω resistors remain in parallel with each other.

(d) The power is lowest in Figure P18.50c. The circuits in Figures P18.50b and P18.50d have in effect 30-V batteries driving the current.

52. 14 Ω

54. 14 s

56. $R = 20\ \Omega$ or $R = 98\ \Omega$

58. See Solution.

60. $q_1 = (240\ \mu\text{C})\left(1 - e^{-1\,000\,t/6.0\text{ s}}\right),\ q_2 = (360\ \mu\text{C})\left(1 - e^{-1\,000\,t/6.0\text{ s}}\right)$

62. **(a)** $R_{eq} = 0.099\,9\ \Omega,\ I_{R_1} = 50.0\text{ A},\ I_{R_2} = I_{R_3} = I_{100} = 45.0\text{ mA}$

(b) $R_{eq} = 1.09\ \Omega,\ I_{R_1} = I_{R_2} = 4.55\text{ A},\ I_{R_3} = I_{100} = 45.5\text{ mA}$

(c) $R_{eq} = 9.99\ \Omega,\ I_{R_1} = I_{R_2} = I_{R_3} = 0.450\text{ A},\ I_{100} = 50.0\text{ mA}$

64. **(a)** In case 1, the two bulbs have the same current, power supplied, and brightness.

(b) In case 2, the two bulbs have the same current, power supplied, and brightness.

(c) The bulbs in case 2 are brighter, each having twice the current of a bulb in case 1.

(d) In case 1, both go out when one bulb fails. In case 2, the other bulb remains lit with unchanged brightness when one bulb fails.

66. **(a)** 61.6 mA **(b)** 0.235 μC **(c)** 1.96 A

PROBLEM SOLUTIONS

18.1 From $\Delta V = I(R + r)$, the internal resistance is

$$r = \frac{\Delta V}{I} - R = \frac{9.00\text{ V}}{0.117\text{ A}} - 72.0\ \Omega = \boxed{4.92\ \Omega}$$

18.2 (a) When the three resistors are in series, the equivalent resistance of the circuit is $R_{eq} = R_1 + R_2 + R_3 = 3(9.0\ \Omega) = \boxed{27\ \Omega}$.

(b) The terminal potential difference of the battery is applied across the series combination of the three 9.0-Ω resistors, so the current supplied by the battery and the current through each resistor in the series combination is

$$I = \frac{\Delta V}{R_{eq}} = \frac{12\text{ V}}{27\ \Omega} = \boxed{0.44\text{ A}}$$

continued on next page

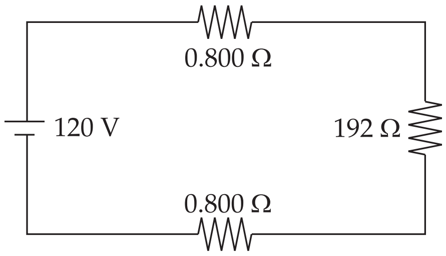
(c) If the three 9.0-Ω resistors are now connected in parallel with each other, the equivalent resistance is

$$\frac{1}{R_{eq}}=\frac{1}{9.0\ \Omega}+\frac{1}{9.0\ \Omega}+\frac{1}{9.0\ \Omega}=\frac{3}{9.0\ \Omega} \qquad \text{or} \qquad R_{eq}=\frac{9.0\ \Omega}{3}=\boxed{3.0\ \Omega}$$

When this parallel combination is connected to the battery, the potential difference across each resistor in the combination is $\Delta V = 12$ V, so the current through each of the resistors is

$$I=\frac{\Delta V}{R}=\frac{12\ \text{V}}{9.0\ \Omega}=\boxed{1.3\ \text{A}}$$

18.3 (a) The bulb acts as a 192-Ω resistor (see below), so the circuit diagram is:



(b) For the bulb in use as intended, $R_{bulb}=(\Delta V)^2/P=(120\ \text{V})^2/75.0\ \text{W}=192\ \Omega$.

Now, assuming the bulb resistance is unchanged, the current in the circuit shown is

$$I=\frac{\Delta V}{R_{eq}}=\frac{120\ \text{V}}{0.800\ \Omega+192\ \Omega+0.800\ \Omega}=0.620\ \text{A}$$

and the actual power dissipated in the bulb is

$$P=I^2R_{bulb}=(0.620\ \text{A})^2(192\ \Omega)=\boxed{73.8\ \text{W}}$$

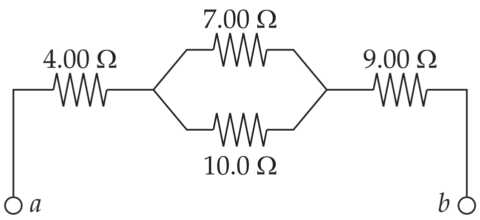
18.4 (a) When the 8.00-Ω resistor is connected across the 9.00-V terminal potential difference of the battery, the current through both the resistor and the battery is

$$I=\frac{\Delta V}{R}=\frac{9.00\ \text{V}}{8.00\ \Omega}=\boxed{1.13\ \text{A}}$$

(b) The relation between the emf and the terminal potential difference of a battery supplying current I is $\Delta V=\varepsilon-Ir$, where r is the internal resistance of the battery. Thus, if the battery has $r = 0.15\ \Omega$ and maintains a terminal potential difference of $\Delta V = 9.00$ V while supplying the current found above, the emf of this battery must be

$$\varepsilon=\Delta V+Ir=9.00\ \text{V}+(1.13\ \text{A})(0.15\ \Omega)=(9.00+0.17)\ \Omega=\boxed{9.17\ \Omega}$$

18.5 (a) The equivalent resistance of the two parallel resistors is



$$R_p=\left(\frac{1}{7.00\ \Omega}+\frac{1}{10.0\ \Omega}\right)^{-1}=4.12\ \Omega$$

Thus,

$$R_{ab}=R_4+R_p+R_9=(4.00+4.12+9.00)\ \Omega=\boxed{17.1\ \Omega}$$

continued on next page

(b) $I_{ab} = \dfrac{(\Delta V)_{ab}}{R_{ab}} = \dfrac{34.0\text{ V}}{17.1\ \Omega} = 1.99\text{ A, so } \boxed{I_4 = I_9 = 1.99\text{ A}}$

Also, $(\Delta V)_p = I_{ab}R_p = (1.99\text{ A})(4.12\ \Omega) = 8.20\text{ V}$

Then, $I_7 = \dfrac{(\Delta V)_p}{R_7} = \dfrac{8.20\text{ V}}{7.00\ \Omega} = \boxed{1.17\text{ A}}$

and $I_{10} = \dfrac{(\Delta V)_p}{R_{10}} = \dfrac{8.20\text{ V}}{10.0\ \Omega} = \boxed{0.820\text{ A}}$

18.6 (a) The parallel combination of the 6.00-Ω and 12.0-Ω resistors has an equivalent resistance of

$$\frac{1}{R_{p1}} = \frac{1}{6.00\ \Omega} + \frac{1}{12.0\ \Omega} = \frac{2+1}{12.0\ \Omega} \qquad \text{or} \qquad R_{p1} = \frac{12.0\ \Omega}{3} = 4.00\ \Omega$$

Similarly, the equivalent resistance of the 4.00-Ω and 8.00-Ω parallel combination is

$$\frac{1}{R_{p2}} = \frac{1}{4.00\ \Omega} + \frac{1}{8.00\ \Omega} = \frac{2+1}{8.00\ \Omega} \qquad \text{or} \qquad R_{p2} = \frac{8.00\ \Omega}{3}$$

The total resistance of the series combination between points *a* and *b* is then

$$R_{ab} = R_{p1} + 5.00\ \Omega + R_{p2} = 4.00\ \Omega + 5.00\ \Omega + \frac{8.00}{3}\ \Omega = \boxed{\frac{35.0}{3}\ \Omega}$$

(b) If $\Delta V_{ab} = 35.0\text{ V}$, the total current from *a* to *b* is

$$I_{ab} = \Delta V_{ab}/R_{ab} = 35.0\text{ V}/(35.0\ \Omega/3) = 3.00\text{ A}$$

and the potential differences across the two parallel combinations are

$$\Delta V_{p1} = I_{ab}R_{p1} = (3.00\text{ A})(4.00\ \Omega) = 12.0\text{ V, and}$$

$$\Delta V_{p2} = I_{ab}R_{p2} = (3.00\text{ A})\left(\frac{8.00}{3}\ \Omega\right) = 8.00\text{ V}$$

The individual currents through the various resistors are:

$$I_{12} = \Delta V_{p1}/12.0\ \Omega = \boxed{1.00\text{ A}}; \qquad I_6 = \Delta V_{p1}/6.00\ \Omega = \boxed{2.00\text{ A}};$$

$$I_5 = I_{ab} = \boxed{3.00\text{ A}}; \qquad I_8 = \Delta V_{p2}/8.00\ \Omega = \boxed{1.00\text{ A}};$$

and $I_4 = \Delta V_{p2}/4.00\ \Omega = \boxed{2.00\text{ A}}$

18.7 When connected in series, we have $R_1 + R_2 = 690\ \Omega$ **[1]**

which we may rewrite as $R_2 = 690\ \Omega - R_1$ **[1a]**

When in parallel, $\dfrac{1}{R_1} + \dfrac{1}{R_2} = \dfrac{1}{150\ \Omega}$ or $\dfrac{R_1R_2}{R_1+R_2} = 150\ \Omega$ **[2]**

continued on next page

Substitute Equations [1] and [1a] into Equation [2] to obtain:

$$\frac{R_1\left(690\ \Omega - R_1\right)}{690\ \Omega} = 150\ \Omega \qquad \text{or} \qquad R_1^2 - (690\ \Omega)R_1 + (690\ \Omega)(150\ \Omega) = 0 \qquad \textbf{[3]}$$

Using the quadratic formula to solve Equation [3] gives

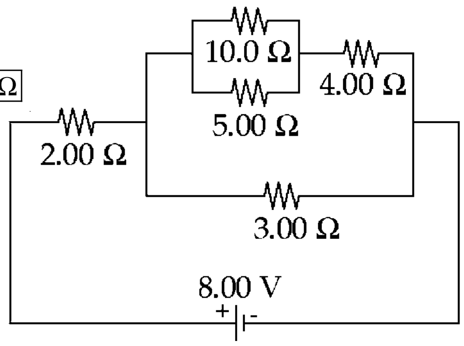
$$R_1 = \frac{690\ \Omega \pm \sqrt{(690\ \Omega)^2 - 4(690\ \Omega)(150\ \Omega)}}{2}$$

with two solutions of $R_1 = 470\ \Omega$ and $R_1 = 220\ \Omega$

Then Equation [1a] yields $R_2 = 220\ \Omega$ or $R_2 = 470\ \Omega$

Thus, the two resistors have resistances of $\boxed{220\ \Omega \text{ and } 470\ \Omega}$.

18.8 (a) The equivalent resistance of this first parallel combination is



$$\frac{1}{R_{p1}} = \frac{1}{10.0\ \Omega} + \frac{1}{5.00\ \Omega} \qquad \text{or} \qquad R_{p1} = \boxed{3.33\ \Omega}$$

(b) For this series combination,

$$R_{\text{upper}} = R_{p1} + 4.00\ \Omega = \boxed{7.33\ \Omega}$$

(c) For the second parallel combination,

$$\frac{1}{R_{p2}} = \frac{1}{R_{\text{upper}}} + \frac{1}{3.00\ \Omega} = \frac{1}{7.33\ \Omega} + \frac{1}{3.00\ \Omega} \qquad \text{or} \qquad R_{p2} = \boxed{2.13\ \Omega}$$

(d) For the second series combination (and hence the entire resistor network)

$$R_{\text{total}} = 2.00\ \Omega + R_{p2} = 2.00\ \Omega + 2.13\ \Omega = \boxed{4.13\ \Omega}$$

(e) The total current supplied by the battery is

$$I_{\text{total}} = \frac{\Delta V}{R_{\text{total}}} = \frac{8.00\ \text{V}}{4.13\ \Omega} = \boxed{1.94\ \text{A}}$$

(f) The potential drop across the $2.00\ \Omega$ resistor is

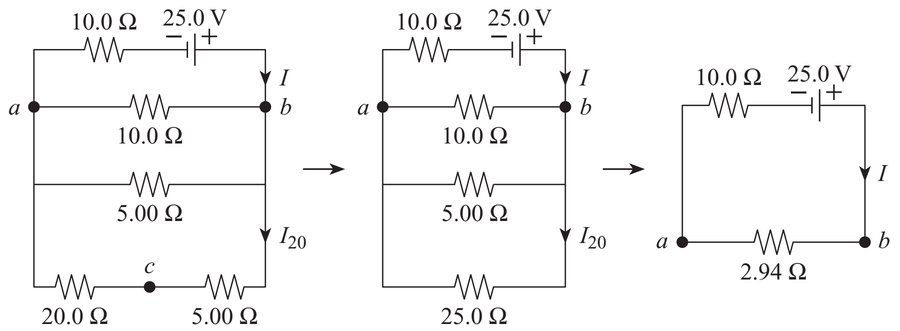
$$\Delta V_2 = R_2 I_{\text{total}} = (2.00\ \Omega)(1.94\ \text{A}) = \boxed{3.88\ \text{V}}$$

(g) The potential drop across the second parallel combination must be

$$\Delta V_{p2} = \Delta V - \Delta V_2 = 8.00\ \text{V} - 3.88\ \text{V} = \boxed{4.12\ \text{V}}$$

(h) So the current through the 3.00 Ω resistor is $I_{\text{total}} = \dfrac{\Delta V_{p2}}{R_3} = \dfrac{4.12\ \text{V}}{3.00\ \Omega} = \boxed{1.37\ \text{A}}$

18.9 (a) Using the rules for combining resistors in series and parallel, the circuit reduces as shown below:



Step 1 **Step 2** **Step 3**

From the figure of Step 3, observe that

$$I = \frac{25.0\text{ V}}{10.0\ \Omega + 2.94\ \Omega} = 1.93\text{ A} \quad \text{and} \quad \Delta V_{ab} = I(2.94\ \Omega) = (1.93\text{ A})(2.94\ \Omega) = \boxed{5.67\text{ V}}$$

(b) From the figure of Step 1, observe that $I_{20} = \dfrac{\Delta V_{ab}}{25.0\ \Omega} = \dfrac{5.67\text{ V}}{25.0\ \Omega} = \boxed{0.227\text{ A}}$

18.10 (a) The figures below show the simplification of the circuit in stages:

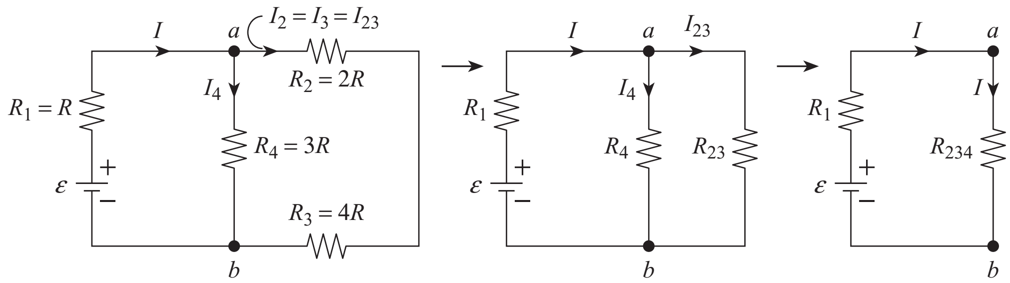


Figure 1 **Figure 2** **Figure 3**

Note that R_2 and R_3 are in series with equivalent resistance $R_{23} = R_2 + R_3 = 6R$. Then, R_4 and R_{23} are in parallel with equivalent resistance

$$R_{234} = \frac{R_4 R_{23}}{R_4 + R_{23}} = \frac{(3R)(6R)}{3R + 6R} = 2R$$

The total current supplied by the battery is then

$$I = \frac{\varepsilon}{R_1 + R_{234}} = \frac{\varepsilon}{R + 2R} = \frac{\varepsilon}{3R}$$

The potential difference across R_1 is $\Delta V_1 = IR_1 = \left(\dfrac{\varepsilon}{3R}\right)R = \boxed{\varepsilon/3}$

continued on next page

and that across R_4 is $\Delta V_4 = \Delta V_{ab} = IR_{234} = \left(\frac{\varepsilon}{3R}\right)(2R) = \boxed{2\varepsilon/3}$

The current through R_2 and R_3 is $I_{23} = \Delta V_{ab}/R_{23} = (2\varepsilon/3R)/6R = \varepsilon/9R$

so the potential difference across R_2 is $\Delta V_2 = I_{23}R_2 = \left(\frac{\varepsilon}{9R}\right)(2R) = \boxed{2\varepsilon/9}$

and that across R_3 is $\Delta V_3 = I_{23}R_3 = \left(\frac{\varepsilon}{9R}\right)(4R) = \boxed{4\varepsilon/9}$

(b) From above, we have $I_1 = \boxed{I}$ and $I_2 = I_3 = I_{23} = \varepsilon/9R = \boxed{I/3}$

The current through R_4 is $I_4 = \Delta V_4/R_4 = (2\varepsilon/3)/3R = 2\varepsilon/9R = \boxed{2I/3}$

18.11 The equivalent resistance is $R_{eq} = R + R_p$, where R_p is the total resistance of the three parallel branches:

$$R_p = \left(\frac{1}{120\ \Omega} + \frac{1}{40\ \Omega} + \frac{1}{R+5.0\ \Omega}\right)^{-1} = \left(\frac{1}{30\ \Omega} + \frac{1}{R+5.0\ \Omega}\right)^{-1} = \frac{(30\ \Omega)(R+5.0\ \Omega)}{R+35\ \Omega}$$

Thus, $$75\ \Omega = R + \frac{(30\ \Omega)(R+5.0\ \Omega)}{R+35\ \Omega} = \frac{R^2 + (65\ \Omega)R + 150\ \Omega^2}{R+35\ \Omega}$$

which reduces to $R^2 - (10\ \Omega)R - 2\,475\ \Omega^2 = 0$ or $(R - 55\ \Omega)(R + 45\ \Omega) = 0$.

Only the positive solution is physically acceptable, so $R = \boxed{55\ \Omega}$.

18.12 The sketch at the right shows the equivalent circuit when the switch is in the open position. For this simple series circuit,

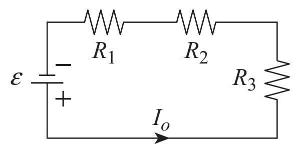


Figure 1

$$R_1 + R_2 + R_3 = \frac{\varepsilon}{I_o} = \frac{6.00\ \text{V}}{1.00 \times 10^{-3}\ \text{A}}$$

or $R_1 + R_2 + R_3 = 6.00\ \text{k}\Omega$ **[1]**

When the switch is closed in position a, the equivalent circuit is shown in Figure 2. The equivalent resistance of the two parallel resistors, R_2, is $R_p = R_2/2$ and the total resistance of the circuit is $R_a = R_1 + (R_2/2) + R_3$. Thus,

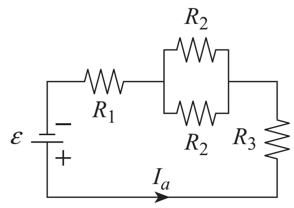


Figure 2

$$R_1 + \frac{R_2}{2} + R_3 = \frac{\varepsilon}{I_a} = \frac{6.00\ \text{V}}{1.20 \times 10^{-3}\ \text{A}}$$

or $R_1 + \frac{R_2}{2} + R_3 = 5.00\ \text{k}\Omega$ **[2]**

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When the switch is closed in position b, resistor R_3 is shorted out, leaving R_1 and R_2 in series with the battery as shown in Figure 3. This gives

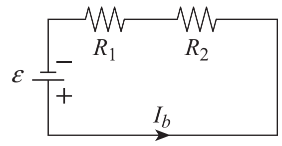


Figure 3

$$R_1 + R_2 = \frac{\varepsilon}{I_b} = \frac{6.00 \text{ V}}{2.00 \times 10^{-3} \text{ A}}$$

and $\quad R_1 + R_2 = 3.00 \text{ k}\Omega$ **[3]**

Substitute Equation [3] into Equation [1] to obtain

$$3.00 \text{ k}\Omega + R_3 = 6.00 \text{ k}\Omega \qquad \text{and} \qquad R_3 = 3.00 \text{ k}\Omega$$

Now, Equation [1] minus Equation [2] gives $R_2/2 = 1.00 \text{ k}\Omega$ or $R_2 = 2.00 \text{ k}\Omega$

Finally, Equation [3] tells that $R_1 + 2.00 \text{ k}\Omega = 3.00 \text{ k}\Omega$, or $R_1 = 1.00 \text{ k}\Omega$

In summary, we have

(a) $\boxed{R_1 = 1.00 \text{ k}\Omega}$, (b) $\boxed{R_2 = 2.00 \text{ k}\Omega}$, and (c) $\boxed{R_3 = 3.00 \text{ k}\Omega}$

18.13 The resistors in the circuit can be combined in the stages shown below to yield an equivalent resistance of $R_{ad} = (63/11)\ \Omega$.

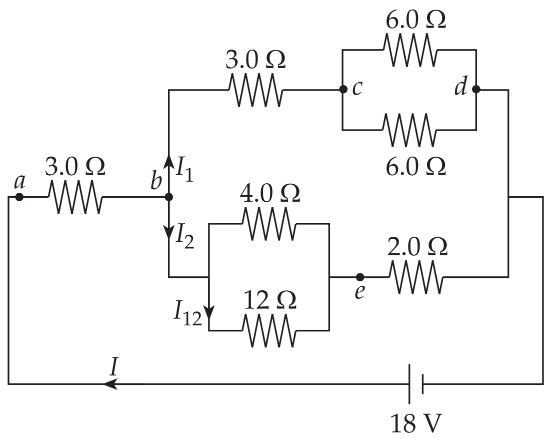


Figure 1

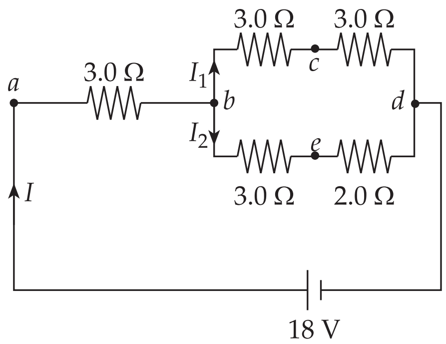


Figure 2

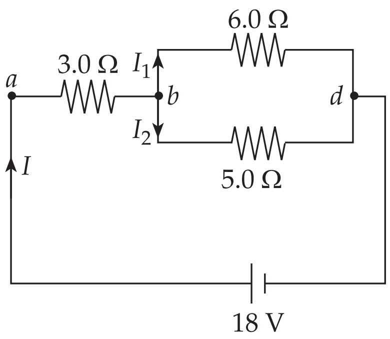


Figure 3

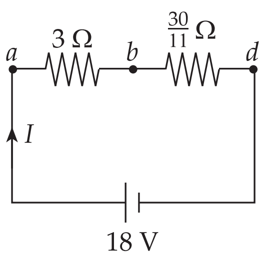


Figure 4

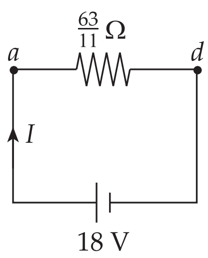


Figure 5

From Figure 5, $\quad I = \dfrac{(\Delta V)_{ad}}{R_{ad}} = \dfrac{18 \text{ V}}{(63/11)\ \Omega} = 3.1 \text{ A}$

continued on next page

Then, from Figure 4, $(\Delta V)_{bd} = I R_{bd} = (3.1\text{ A})(30/11\ \Omega) = 8.5\text{ V}$

Now, look at Figure 2 and observe that

$$I_2 = \frac{(\Delta V)_{bd}}{3.0\ \Omega + 2.0\ \Omega} = \frac{8.5\text{ V}}{5.0\ \Omega} = 1.7\text{ A}$$

so $(\Delta V)_{be} = I_2 R_{be} = (1.7\text{ A})(3.0\ \Omega) = 5.1\text{ V}$

Finally, from Figure 1, $I_{12} = \dfrac{(\Delta V)_{be}}{R_{12}} = \dfrac{5.1\text{ V}}{12\ \Omega} = \boxed{0.43\text{ A}}$

18.14 (a) The resistor network connected to the battery in Figure P18.14 can be reduced to a single equivalent resistance in the following steps. The equivalent resistance of the parallel combination of the 3.00 Ω and 6.00 Ω resistors is

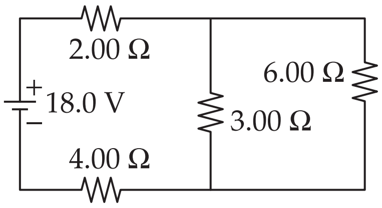


Figure P18.14

$$\frac{1}{R_p} = \frac{1}{3.00\ \Omega} + \frac{1}{6.00\ \Omega} = \frac{3}{6.00\ \Omega} \quad \text{or} \quad R_p = 2.00\ \Omega$$

This resistance is in series with the 4.00 Ω and the other 2.00 Ω resistor, giving a total equivalent resistance of $R_{\text{eq}} = 2.00\ \Omega + R_p + 4.00\ \Omega = \boxed{8.00\ \Omega}$.

(b) The current in the 2.00 Ω resistor is the total current supplied by the battery and is equal to

$$I_{\text{total}} = \frac{\Delta V}{R_{\text{eq}}} = \frac{18.0\text{ V}}{8.00\ \Omega} = \boxed{2.25\text{ A}}$$

(c) The power the battery delivers to the circuit is

$$P = (\Delta V) I_{\text{total}} = (18.0\text{ V})(2.25\text{ A}) = \boxed{40.5\text{ W}}$$

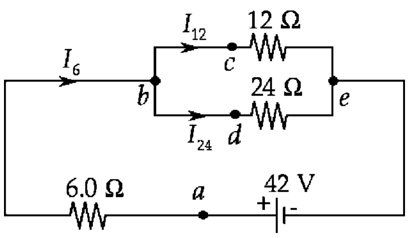
18.15 (a) Connect $\boxed{\text{two 50-}\Omega\text{ resistors in parallel}}$ to get 25 Ω. Then connect that parallel combination $\boxed{\text{in series with a 20-}\Omega\text{ resistor}}$ for a total resistance of 45 Ω.

(b) Connect $\boxed{\text{two 50-}\Omega\text{ resistors in parallel}}$ to get 25 Ω.

Also, $\boxed{\text{connect two 20-}\Omega\text{ resistors in parallel}}$ to get 10 Ω.

Then, $\boxed{\text{connect these two parallel combinations in series}}$ to obtain 35 Ω.

18.16 (a) The equivalent resistance of the parallel combination between points *b* and *e* is



$$\frac{1}{R_{be}} = \frac{1}{12\ \Omega} + \frac{1}{24\ \Omega} \quad \text{or} \quad R_{be} = 8.0\ \Omega$$

The total resistance between points *a* and *e* is then

$$R_{ae} = R_{ab} + R_{be} = 6.0\ \Omega + 8.0\ \Omega = 14\ \Omega$$

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The total current supplied by the battery (and also the current in the 6.0-Ω resistor) is

$$I_{\text{total}} = I_6 = \frac{\Delta V_{ae}}{R_{ae}} = \frac{42\text{ V}}{14\ \Omega} = \boxed{3.0\text{ A}}$$

The potential difference between points b and e is

$$\Delta V_{be} = R_{be} I_{\text{total}} = (8.0\ \Omega)(3.0\text{ A}) = 24\text{ V}$$

so $I_{12} = \frac{\Delta V_{be}}{R_{bce}} = \frac{24\text{ V}}{12\ \Omega} = \boxed{2.0\text{ A}}$ and $I_{24} = \frac{\Delta V_{be}}{R_{bde}} = \frac{24\text{ V}}{24\ \Omega} = \boxed{1.0\text{ A}}$

(b) Applying the junction rule at point b yields $\boxed{I_6 - I_{12} - I_{24} = 0}$ **[1]**

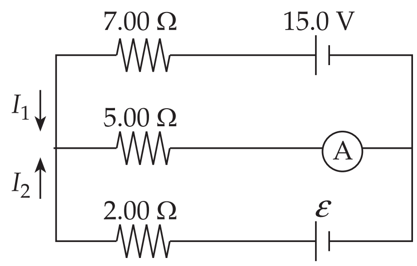
Using the loop rule on loop *abdea* gives $+42 - 6I_6 - 24I_{24} = 0$ or $\boxed{I_6 = 7.0 - 4I_{24}}$ **[2]**

and using the loop rule on loop *bcedb* gives $-12I_{12} + 24I_{24} = 0$ or $\boxed{I_{12} = 2I_{24}}$ **[3]**

Substituting Equations [2] and [3] into [1] yields $7I_{24} = 7.0$ or $\boxed{I_{24} = 1.0\text{ A}}$

Then, Equations [2] and [3] yield $\boxed{I_6 = 3.0\text{ A}}$ and $\boxed{I_{12} = 2.0\text{ A}}$

18.17 Going counterclockwise around the upper loop, applying Kirchhoff's loop rule, gives



$$+15.0\text{ V} - (7.00)I_1 - (5.00)(2.00\text{ A}) = 0$$

or $I_1 = \frac{15.0\text{ V} - 10.0\text{ V}}{7.00\ \Omega} = \boxed{0.714\text{ A}}$

From Kirchhoff's junction rule, $I_1 + I_2 - 2.00\text{ A} = 0$

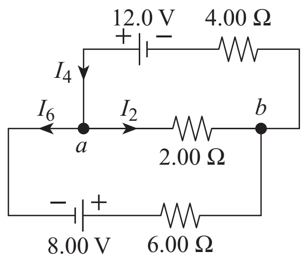
so $I_2 = 2.00\text{ A} - I_1 = 2.00\text{ A} - 0.714\text{ A} = \boxed{1.29\text{ A}}$

Going around the lower loop in a clockwise direction gives

$$+\varepsilon - (2.00)I_2 - (5.00)(2.00\text{ A}) = 0$$

or $\varepsilon = (2.00\ \Omega)(1.29\text{ A}) + (5.00\ \Omega)(2.00\text{ A}) = \boxed{12.6\text{ V}}$

18.18 (a) Applying Kirchhoff's junction rule at a gives



$$I_2 = I_4 - I_6 \qquad \textbf{[1]}$$

Going counterclockwise around the lower loop, and applying Kirchhoff's loop rule, we obtain

$$+8.00\text{ V} - (6.00\ \Omega)I_6 + (2.00\ \Omega)I_2 = 0$$

or $I_6 = \frac{4}{3} + \frac{1}{3}I_2$ **[2]**

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Applying the loop rule as we go counterclockwise around the upper loop:

$$-(2.00\ \Omega)I_2 - (4.00\ \Omega)I_4 + 12.0\text{ V} = 0 \qquad \text{or} \qquad I_4 = 3.00 - \frac{1}{2}I_2 \qquad \textbf{[3]}$$

Substituting Equations [2] and [3] into Equation [1] yields

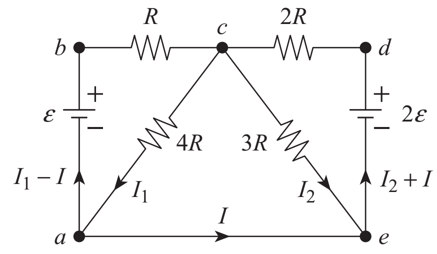
$$\left(1+\frac{1}{2}+\frac{1}{3}\right)I_2 = 3.00 - \frac{4}{3} \qquad \text{and} \qquad I_2 = \boxed{0.909\text{ A}}$$

(b) The potential difference between points a and b is

$$\Delta V_{ab} = -I_2(2.00\ \Omega) = -(0.909\text{ A})(2.00\ \Omega) = \boxed{-1.82\text{ V}}$$

or $|\Delta V_{ab}| = 1.82$ V with point b at the lower potential.

18.19 Consider the circuit diagram at the right, in which Kirchhoff's junction rule has already been applied at points a and e.



Applying the loop rule around loop *abca* gives

$$\varepsilon - R(I_1 - I) - 4RI_1 = 0$$

or $$I_1 = \frac{1}{5}\left(\frac{\varepsilon}{R}+I\right) \qquad \textbf{[1]}$$

Next, applying the loop rule around loop *cedc* gives

$$-3RI_2 + 2\varepsilon - 2R(I_2 + I) = 0 \qquad \text{or} \qquad I_2 = \frac{2}{5}\left(\frac{\varepsilon}{R} - I\right) \qquad \textbf{[2]}$$

Finally, applying the loop rule around loop *caec* gives

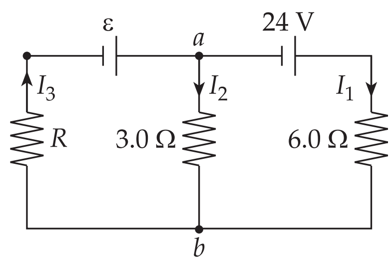
$$-4RI_1 + 3RI_2 = 0 \qquad \text{or} \qquad 4I_1 = 3I_2 \qquad \textbf{[3]}$$

Substituting Equations [1] and [2] into Equation [3] yields $I = \dfrac{\varepsilon}{5R}$

Thus, if $\varepsilon = 250$ V and $R = 1.00\text{ k}\Omega = 1.00\times10^3\ \Omega$, the current in the wire between a and e is

$$I = \frac{250\text{ V}}{5(1.00\times10^3\ \Omega)} = 50.0\times10^{-3}\text{ A} = \boxed{50.0\text{ mA flowing from } a \text{ toward } e.}$$

18.20 Following the path of I_1 from a to b and recording changes in potential gives



$$V_b - V_a = +24\text{ V} - (6.0\ \Omega)(3.0\text{ A}) = +6.0\text{ V}$$

Now, following the path of I_2 from a to b and recording changes in potential gives

$$V_b - V_a = -(3.0\ \Omega)I_2 = +6.0\text{ V, or } I_2 = \boxed{-2.0\text{ A}}$$

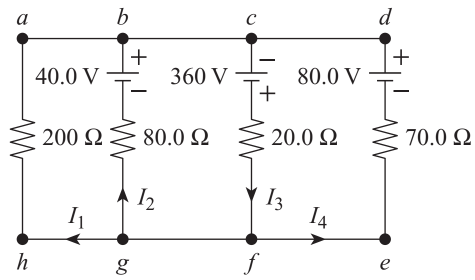
continued on next page

Thus, I_2 is directed $\boxed{\text{from } b \text{ toward } a}$ and has magnitude of 2.0 A.

Applying Kirchhoff's junction rule at point *a* gives

$$I_3 = I_1 + I_2 = 3.0\ \text{A} + (-2.0\ \text{A}) = \boxed{1.0\ \text{A}}$$

18.21 (a) The circuit diagram at the right shows the assumed directions of the current in each resistor. Note that the total current flowing out of the section of wire connecting points *g* and *f* must equal the current flowing into that section. Thus,



$$I_3 = I_1 + I_2 + I_4 \qquad \textbf{[1]}$$

Applying the loop rule around loop *abgha* gives

$$-200I_1 - 40.0 + 80.0I_2 = 0 \qquad \text{or} \qquad I_2 = \frac{1}{2}(5I_1 + 1.00) \qquad \textbf{[2]}$$

Next, applying the loop rule around loop *bcfgb* gives

$$+360 - 20.0I_3 - 80.0I_2 + 40.0 = 0 \qquad \text{or} \qquad I_3 = 20.0 - 4I_2 \qquad \textbf{[3]}$$

Finally, applying the loop rule around the outer loop *abcdefgha* yields

$$-80.0 + 70I_4 - 200I_1 = 0 \qquad \text{or} \qquad I_4 = \frac{1}{7}(20I_1 + 8.00) \qquad \textbf{[4]}$$

To solve this set of equations, we first substitute Equation [2] into Equation [3] to obtain

$$I_3 = 20.0 - 2(5I_1 + 1.00) \qquad \text{or} \qquad I_3 = 18.0 - 10I_1 \qquad \textbf{[3']}$$

Now, substitute Equations [2], [4], and [3′] into Equation [1] to find

$$\left(10 + \frac{5}{2} + 1 + \frac{20}{7}\right)I_1 = 18.0 - \frac{1.00}{2} - \frac{8.00}{7} \qquad \text{and} \qquad \boxed{I_1 = 1.00\ \text{A}}$$

Substituting this result back into Equations [2], [4], and [3′] gives

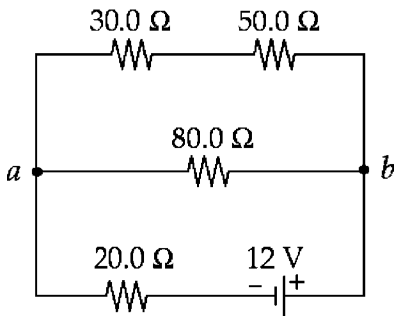
$$\boxed{I_2 = 3.00\ \text{A}} \qquad \boxed{I_4 = 4.00\ \text{A}} \qquad \text{and} \qquad \boxed{I_3 = 8.00\ \text{A}}$$

(b) The potential difference across the 200-Ω resistor is

$$\Delta V_{200} = I_1 R_{200} = (1.00\ \text{A})(200\ \Omega) = \boxed{200\ \text{V}}$$

with point *a* at a lower potential than point *h*.

18.22 (a) The 30.0-Ω and 50.0-Ω resistors in the upper branch are in series, and add to give a total resistance of $R_{\text{upper}} = 80.0\ \Omega$ for this path. This 80.0-Ω resistance is in parallel with the 80.0-Ω resistance of the middle branch, and the rule for combining resistors in parallel yields a total resistance of $R_{ab} = 40.0\ \Omega$ between points a and b. This resistance is in series with the 20.0-Ω resistor, so the total equivalent resistance of the circuit is



$$R_{\text{eq}} = 20.0\ \Omega + R_{ab} = 20.0\ \Omega + 40.0\ \Omega = \boxed{60.0\ \Omega}$$

(b) The current supplied to this circuit by the battery is $I_{\text{total}} = \dfrac{\Delta V}{R_{\text{eq}}} = \dfrac{12\ \text{V}}{60.0\ \Omega} = \boxed{0.20\ \text{A}}$.

(c) The power delivered by the battery is $P_{\text{total}} = R_{\text{eq}} I_{\text{total}}^2 = (60.0\ \Omega)(0.20\ \text{A})^2 = \boxed{2.4\ \text{W}}$.

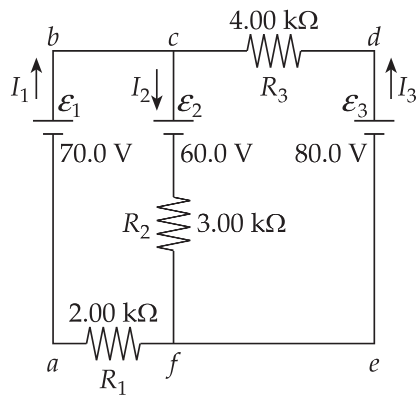
(d) The potential difference between points a and b is

$$\Delta V_{ab} = R_{ab} I_{\text{total}} = (40.0\ \Omega)(0.20\ \text{A}) = 8.0\ \text{V}$$

and the current in the upper branch is $I_{\text{upper}} = \dfrac{\Delta V_{ab}}{R_{\text{upper}}} = \dfrac{8.0\ \text{V}}{80.0\ \Omega} = 0.10\ \text{A}$, so the power delivered to the 50.0 Ω resistor is

$$P_{50} = R_{50} I_{\text{upper}}^2 = (50.0\ \Omega)(0.10\ \text{A})^2 = \boxed{0.50\ \text{W}}$$

18.23 (a) We name the currents I_1, I_2, and I_3 as shown.



Applying Kirchhoff's loop rule to loop *abcfa* gives $+\mathcal{E}_1 - \mathcal{E}_2 - R_2 I_2 - R_1 I_1 = 0$

or $\quad 3I_2 + 2I_1 = 10.0\ \text{mA}$

and $\quad I_1 = 5.00\ \text{mA} - 1.50 I_2$ **[1]**

Applying the loop rule to loop *edcfe* yields

$+\mathcal{E}_3 - R_3 I_3 - \mathcal{E}_2 - R_2 I_2 = 0 \quad$ or $\quad 3I_2 + 4I_3 = 20.0\ \text{mA}$

and $\quad I_3 = 5.00\ \text{mA} - 0.750 I_2$ **[2]**

Finally, applying Kirchhoff's junction rule at junction c gives

$$I_2 = I_1 + I_3 \qquad \textbf{[3]}$$

Substituting Equations [1] and [2] into [3] yields

$$I_2 = 5.00\ \text{mA} - 1.50 I_2 + 5.00\ \text{mA} - 0.750 I_2 \quad \text{or} \quad 3.25 I_2 = 10.0\ \text{mA}$$

This gives $I_2 = (10.0\ \text{mA})/3.25 = \boxed{3.08\ \text{mA}}$. Then, Equation [1] yields

$$I_1 = 5.00\ \text{mA} - 1.50\left(\frac{10.0\ \text{mA}}{3.25}\right) = \boxed{0.385\ \text{mA}}$$

continued on next page

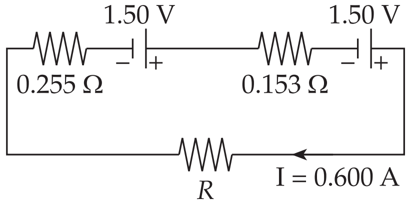
and, from Equation [2], $I_3 = 5.00\text{ mA} - 0.750\left(\frac{10.0\text{ mA}}{3.25}\right) = \boxed{2.69\text{ mA}}$

(b) Start at point c and go to point f, recording changes in potential to obtain

$$V_f - V_c = -\mathcal{E}_2 - R_2 I_2 = -60.0\text{ V} - \left(3.00\times10^3\ \Omega\right)\left(3.08\times10^{-3}\text{ A}\right) = -69.2\text{ V}$$

or $|\Delta V|_{cf} = \boxed{69.2\text{ V and point } c \text{ is at the higher potential}}$.

18.24 (a) Applying Kirchhoff's loop rule to the circuit gives



$$+3.00\text{ V} - (0.255\ \Omega + 0.153\ \Omega + R)(0.600\text{ A}) = 0$$

or $R = \frac{3.00\text{ V}}{0.600\text{ A}} - (0.255\ \Omega + 0.153\ \Omega) = \boxed{4.59\ \Omega}$

(b) The total power input to the circuit is

$$P_{\text{input}} = \left(\mathcal{E}_1 + \mathcal{E}_2\right) I = (1.50\text{ V} + 1.50\text{ V})(0.600\text{ A}) = 1.80\text{ W}$$

The power loss by heating within the batteries is

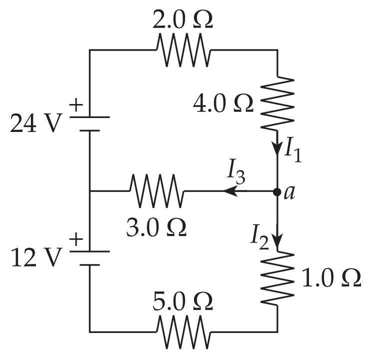
$$P_{\text{loss}} = I^2\left(r_1 + r_2\right) = (0.600\text{ A})^2(0.255\ \Omega + 0.153\ \Omega) = 0.147\text{ W}$$

Thus, the fraction of the power input that is dissipated internally is

$$\frac{P_{\text{loss}}}{P_{\text{input}}} = \frac{0.147\text{ W}}{1.80\text{ W}} = \boxed{0.0816 \text{ or } 8.16\%}$$

18.25 (a) $\boxed{\text{No.}}$ Some simplification could be made by recognizing that the 2.0-Ω and 4.0-Ω resistors are in series, adding to give a total of 6.0 Ω; and the 5.0-Ω and 1.0-Ω resistors form a series combination with a total resistance of 6.0 Ω. The circuit cannot be simplified any further, and Kirchhoff's rules must be used to analyze the circuit.

(b) Applying Kirchhoff's junction rule at junction a gives



$$I_1 = I_2 + I_3 \qquad \textbf{[1]}$$

Using Kirchhoff's loop rule on the upper loop yields

$$+24\text{ V} - (2.0 + 4.0)I_1 - (3.0)I_3 = 0$$

or $I_3 = 8.0\text{ A} - 2I_1$ **[2]**

For the lower loop: $+12\text{ V} + (3.0)I_3 - (1.0 + 5.0)I_2 = 0$

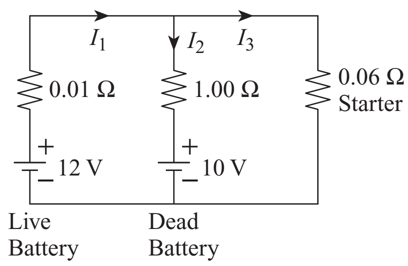
Using Equation [2], this reduces to

$$I_2 = \frac{12\text{ V} + 3.0\left(8.0\text{ A} - 2I_1\right)}{6.0} \quad \text{or} \quad I_2 = 6.0\text{ A} - I_1 \qquad \textbf{[3]}$$

Substituting Equations [2] and [3] into [1] gives $\boxed{I_1 = 3.5\text{ A}}$.

Then, Equation [3] gives $\boxed{I_2 = 2.5\text{ A}}$, and Equation [2] yields $\boxed{I_3 = 1.0\text{ A}}$.

18.26 Using Kirchhoff's loop rule on the outer perimeter of the circuit gives



$$+12\text{ V}-(0.01)I_1-(0.06)I_3=0$$

or $\quad I_1=1.2\times10^3\text{ A}-6.0I_3$ **[1]**

For the rightmost loop, the loop rule gives

$$+10\text{ V}+(1.00)I_2-(0.06)I_3=0$$

or $\quad I_2=0.06I_3-10\text{ A}$ **[2]**

Applying Kirchhoff's junction rule at either junction gives

$$I_1=I_2+I_3 \qquad \textbf{[3]}$$

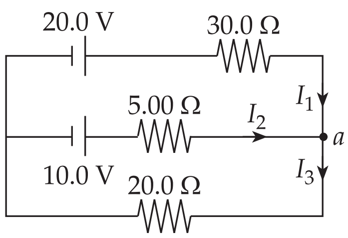
(a) Substituting Equations [1] and [2] into [3] yields

$$7.1I_3=1.2\times10^3\text{ A and }\boxed{I_3=1.7\times10^2\text{ A (in starter)}}$$

(b) Then, Equation [2] gives $\boxed{I_2=0.20\text{ A (in dead battery)}}$.

18.27 (a) $\boxed{\text{No.}}$ This multi-loop circuit does not contain any resistors in series (i.e., connected so all the current in one must pass through the other) nor in parallel (connected so the voltage drop across one is always the same as that across the other). Thus, this circuit cannot be simplified any further, and Kirchhoff's rules must be used to analyze it.

(b) Assume currents I_1, I_2, and I_3 in the directions shown. Then, using Kirchhoff's junction rule at junction a gives



$$I_3=I_1+I_2 \qquad \textbf{[1]}$$

Applying Kirchhoff's loop rule on the lower loop,

$$+10.0\text{ V}-(5.00)I_2-(20.0)I_3=0$$

or $\quad I_2=2.00\text{ A}-4I_3$ **[2]**

and for the loop around the perimeter of the circuit, $\quad 20.0\text{ V}-30.0I_1-20.0I_3=0$

or $\quad I_1=0.667\text{ A}-0.667I_3$ **[3]**

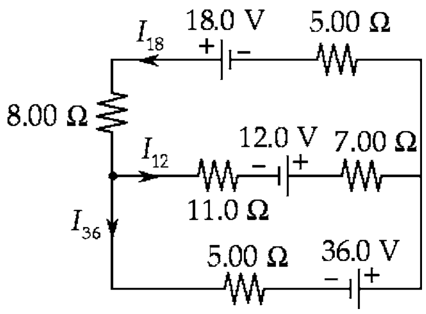
Substituting Equations [2] and [3] into [1]: $I_3=0.667\text{ A}-0.667I_3+2.00\text{ A}-4I_3$

which reduces to $5.67I_3=2.67\text{ A}$ and gives $\boxed{I_3=0.471\text{ A}}$.

Then, Equation [2] gives $\boxed{I_2=0.116\text{ A}}$, and from Equation [3], $\boxed{I_1=0.353\text{ A}}$.

All currents are in the directions indicated in the circuit diagram given above.

18.28 (a) Going counterclockwise around the upper loop, Kirchhoff's loop rule gives



$$-11.0I_{12}+12.0-7.00I_{12}-5.00I_{18}+18.0-8.00I_{18}=0$$

or $\boxed{18.0I_{12}+13.0I_{18}=30.0}$ **[1]**

(b) Going counterclockwise around the lower loop:

$$-5.00I_{36}+36.0+7.00I_{12}-12.0+11.0I_{12}=0$$

or $\boxed{5.00I_{36}-18.0I_{12}=24.0}$ **[2]**

(c) Applying the junction rule at the node in the left end of the circuit gives

$\boxed{I_{18}=I_{12}+I_{36}}$ **[3]**

(d) Solving Equation [3] for I_{36} yields $\boxed{I_{36}=I_{18}-I_{12}}$. **[4]**

(e) Substituting Equation [4] into [2] gives $5.00(I_{18}-I_{12})-18.0I_{12}=24.0$, or

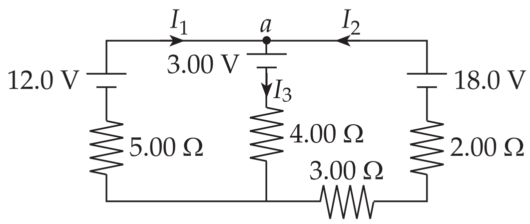
$\boxed{5.00I_{18}-23.0I_{12}=24.0}$ **[5]**

(f) Solving Equation [5] for I_{18} yields $I_{18}=(24.0+23.0I_{12})/5.00$. Substituting this into Equation [1] and simplifying gives $389I_{12}=-162$, and $\boxed{I_{12}=-0.416\text{ A}}$. Then, from Equation [1], $I_{18}=(30.0-18.0I_{12})/13.0$ which yields $\boxed{I_{18}=2.88\text{ A}}$.

(g) Equation [4] gives $I_{36}=2.88\text{ A}-(-0.416\text{ A})$, or $\boxed{I_{36}=3.30\text{ A}}$.

(h) The negative sign in the answer for I_{12} means that this current flows in the opposite direction to that shown in the circuit diagram and assumed during this solution. That is, the actual current in the middle branch of the circuit flows from right to left and has a magnitude of 0.416 A.

18.29 Applying Kirchhoff's junction rule at junction *a* gives



$$I_3=I_1+I_2 \qquad \textbf{[1]}$$

Using Kirchhoff's loop rule on the leftmost loop yields

$$-3.00\text{ V}-(4.00)I_3-(5.00)I_1+12.0\text{ V}=0$$

so $I_1=(9.00\text{ A}-4.00I_3)/5.00$ or $I_1=1.80\text{ A}-0.800I_3$ **[2]**

For the rightmost loop,

$$-3.00\text{ V}-(4.00)I_3-(3.00+2.00)I_2+18.0\text{ V}=0$$

and $I_2=(15.0\text{ A}-4.00I_3)/5.00$ or $I_2=3.00\text{ A}-0.800I_3$ **[3]**

continued on next page

Substituting Equations [2] and [3] into [1] and simplifying gives $2.60I_3 = 4.80$ and $I_3 = 1.846$ A. Then Equations [2] and [3] yield $I_1 = 0.323$ A and $I_2 = 1.523$ A.

Therefore, the potential differences across the resistors are

$$\Delta V_2 = I_2(2.00\ \Omega) = \boxed{3.05\ \text{V}},\ \Delta V_3 = I_2(3.00\ \Omega) = \boxed{4.57\ \text{V}}$$

$$\Delta V_4 = I_3(4.00\ \Omega) = \boxed{7.38\ \text{V}},\ \text{and}\ \Delta V_5 = I_1(5.00\ \Omega) = \boxed{1.62\ \text{V}}$$

18.30 The time constant is $\tau = RC$. Considering units, we find

$$RC \to (\text{Ohms})(\text{Farads}) = \left(\frac{\text{Volts}}{\text{Amperes}}\right)\left(\frac{\text{Coulombs}}{\text{Volts}}\right) = \left(\frac{\text{Coulombs}}{\text{Amperes}}\right)$$

$$= \left(\frac{\text{Coulombs}}{\text{Coulombs/Second}}\right) = \text{Second}$$

or $\tau = RC$ has units of time.

18.31 (a) The time constant is: $\tau = RC = (75.0 \times 10^3\ \Omega)(25.0 \times 10^{-6}\ \text{F}) = \boxed{1.88\ \text{s}}$.

(b) At $t = \tau$, $q = 0.632Q_{\text{max}} = 0.632(C\varepsilon) = 0.632(25.0 \times 10^{-6}\ \text{F})(12.0\ \text{V}) = \boxed{1.90 \times 10^{-4}\ \text{C}}$.

18.32 (a) $\tau = RC = (100\ \Omega)(20.0 \times 10^{-6}\ \text{F}) = 2.00 \times 10^{-3}\ \text{s} = \boxed{2.00\ \text{ms}}$

(b) $Q_{\text{max}} = C\varepsilon = (20.0 \times 10^{-6}\ \text{F})(9.00\ \text{V}) = 1.80 \times 10^{-4}\ \text{C} = \boxed{180\ \mu\text{C}}$

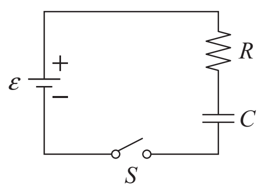
(c) $Q = Q_{\text{max}}\left(1 - e^{-t/\tau}\right) = Q_{\text{max}}\left(1 - e^{-\tau/\tau}\right) = Q_{\text{max}}\left(1 - \dfrac{1}{e}\right) = \boxed{114\ \mu\text{C}}$

18.33 (a) The time constant of an RC circuit is $\tau = RC$. Thus,

$$\tau = (1.00 \times 10^6\ \Omega)(5.00 \times 10^{-6}\ \text{F}) = \boxed{5.00\ \text{s}}$$

(b) $Q_{\text{max}} = C\varepsilon = (5.00\ \mu\text{F})(30.0\ \text{V}) = \boxed{150\ \mu\text{C}}$

(c) To obtain the current through the resistor at time t after the switch is closed, recall that the charge on the capacitor at that time is $q = C\varepsilon\left(1 - e^{-t/\tau}\right)$ and the potential difference across a capacitor is $V_c = q/C$. Thus,



$$V_c = \frac{C\varepsilon\left(1 - e^{-t/\tau}\right)}{C} = \varepsilon\left(1 - e^{-t/\tau}\right)$$

Then, considering switch S to have been closed at time $t = 0$, apply Kirchhoff's loop rule around the circuit shown above to obtain

$$+\varepsilon - iR - V_c = 0 \qquad \text{or} \qquad i = \frac{\varepsilon - \varepsilon\left(1 - e^{-t/\tau}\right)}{R}$$

The current in the circuit at time t after the switch is closed is then $i = (\varepsilon/R)e^{-t/\tau}$, so the current in the resistor at $t = 10.0$ s is

$$i = \left(\frac{30.0\ \text{V}}{1.00 \times 10^6\ \Omega}\right)e^{-\frac{10.0\ \text{s}}{5.00\ \text{s}}} = (30.0\ \mu\text{A})e^{-2.00} = \boxed{4.06\ \mu\text{A}}$$

18.34 Assuming the capacitor is initially uncharged and the switch is closed at $t = 0$, the charge on the capacitor at time $t > 0$ is $q = Q_{max}(1 - e^{-t/\tau})$, where $Q_{max} = C\varepsilon$ and τ is the time constant of the circuit. When $\varepsilon = 30$ V, $C = 5.0\ \mu$F, and $R = 1.0$ MΩ, the time constant is

$$\tau = RC = \left(1.0\times10^{6}\ \Omega\right)\left(5.0\times10^{-6}\ \text{F}\right) = 5.0\ \text{s}$$

and we find the charge on the capacitor at $t = 10$ s to be

$$q = \left(5.0\ \mu\text{F}\right)\left(30\ \text{V}\right)\left(1 - e^{-10\ \text{s}/5.0\ \text{s}}\right) = \boxed{1.3\times10^{2}\ \mu\text{C}}$$

18.35 (a) The charge remaining on the capacitor after time t is $q = Qe^{-t/\tau}$.

Thus, if $q = 0.750Q$, then $e^{-t/\tau} = 0.750$ and $-t/\tau = \ln(0.750)$,

or $\quad t = -\tau\ln(0.750) = -(1.50\ \text{s})\ln(0.750) = \boxed{0.432\ \text{s}}$

(b) $\tau = RC$, so $\quad C = \dfrac{\tau}{R} = \dfrac{1.50\ \text{s}}{250\times10^{3}\ \Omega} = 6.00\times10^{-6}\ \text{F} = \boxed{6.00\ \mu\text{F}}$

18.36 At time t after the switch is closed, the potential difference between the plates of the initially uncharged capacitor is

$$\Delta V = qC = CQ_{max}\left(1 - e^{-t/\tau}\right) = \varepsilon\left(1 - e^{-t/RC}\right)$$

Thus, $\quad e^{-t/RC} = 1 - \dfrac{\Delta V}{\varepsilon} \quad$ and $\quad -\dfrac{t}{RC} = \ln\left(1 - \dfrac{\Delta V}{\varepsilon}\right)$

giving $\quad R = \dfrac{-(t/C)}{\ln(1 - \Delta V/\varepsilon)}$

If $\varepsilon = 10.0$ V, $C = 10.0\ \mu$F, and $\Delta V = 4.00$ V at $t = 3.00$ s after closing the switch, the resistance must be

$$R = \frac{-\left(3.00\ \text{s}/10.0\times10^{-6}\ \text{F}\right)}{\ln\left(1 - 4.00\ \text{V}/10.0\ \text{V}\right)} = \frac{-3.00\times10^{5}\ \Omega}{\ln(0.600)} = 5.87\times10^{5}\ \Omega = \boxed{587\ \text{k}\Omega}$$

18.37 The current drawn by a single 75-W bulb connected to a 120-V source is $I_1 = P/\Delta V = 75\ \text{W}/120\ \text{V}$. Thus, the number of such bulbs that can be connected in parallel with this source before the total current drawn will equal 30.0 A is

$$n = \frac{30.0\ \text{A}}{I_1} = (30.0\ \text{A})\left(\frac{120\ \text{V}}{75\ \text{W}}\right) = \boxed{48}$$

18.38 (a) The equivalent resistance of the parallel combination is

$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = \left(\frac{1}{150\ \Omega} + \frac{1}{25\ \Omega} + \frac{1}{50\ \Omega}\right)^{-1} = 15\ \Omega$$

so the total current supplied to the circuit is

$$I_{total} = \frac{\Delta V}{R} = \frac{120\ \text{V}}{15\ \Omega} = \boxed{8.0\ \text{A}}$$

continued on next page

(b) Since the appliances are connected in parallel, the voltage across each one is $\Delta V = \boxed{120\text{ V}}$.

(c) $$I_{\text{lamp}} = \frac{\Delta V}{R_{\text{lamp}}} = \frac{120\text{ V}}{150\ \Omega} = \boxed{0.80\text{ A}}$$

(d) $$P_{\text{heater}} = \frac{(\Delta V)^2}{R_{\text{heater}}} = \frac{(120\text{ V})^2}{25\ \Omega} = \boxed{5.8\times 10^2\text{ W}}$$

18.39 From $P = (\Delta V)^2/R$, the resistance of the element is

$$R = \frac{(\Delta V)^2}{P} = \frac{(240\text{ V})^2}{3\,000\text{ W}} = 19.2\ \Omega$$

When the element is connected to a 120-V source, we find that

(a) $I = \dfrac{\Delta V}{R} = \dfrac{120\text{ V}}{19.2\ \Omega} = \boxed{6.25\text{ A}}$, and

(b) $$P = (\Delta V)I = (120\text{ V})(6.25\text{ A}) = \boxed{750\text{ W}}$$

18.40 (a) The current drawn by each appliance operating separately is

Coffee Maker: $$I = \frac{P}{\Delta V} = \frac{1\,200\text{ W}}{120\text{ V}} = \boxed{10\text{ A}}$$

Toaster: $$I = \frac{P}{\Delta V} = \frac{1100\text{ W}}{120\text{ V}} = \boxed{9.2\text{ A}}$$

Waffle Maker: $$I = \frac{P}{\Delta V} = \frac{1\,400\text{ W}}{120\text{ V}} = \boxed{12\text{ A}}$$

(b) If the three appliances are operated simultaneously, they will draw a total current of $I_{\text{total}} = (10 + 9.2 + 12)\text{ A} = \boxed{31\text{ A}}$.

(c) $\boxed{\text{No.}}$ The total current required exceeds the limit of the circuit breaker, so they cannot be operated simultaneously. In fact, with a 15 A limit, no two of these appliances could be operated at the same time without tripping the breaker.

18.41 (a) The area of each surface of this axon membrane is

$$A = L(2\pi r) = (0.10\text{ m})\left[2\pi\left(10\times 10^{-6}\text{ m}\right)\right] = 2\pi\times 10^{-6}\text{ m}^2$$

and the capacitance is

$$C = \kappa \epsilon_0 \frac{A}{d} = 3.0\left(8.85\times 10^{-12}\text{ C}^2/\text{N}\cdot\text{m}^2\right)\left(\frac{2\pi\times 10^{-6}\text{ m}^2}{1.0\times 10^{-8}\text{ m}}\right) = 1.67\times 10^{-8}\text{ F}$$

In the resting state, the charge on the outer surface of the membrane is

$$Q_i = C(\Delta V)_i = \left(1.67\times 10^{-8}\text{ F}\right)\left(70\times 10^{-3}\text{ V}\right) = 1.17\times 10^{-9}\text{ C} \rightarrow \boxed{1.2\times 10^{-9}\text{ C}}$$

The number of potassium ions required to produce this charge is

$$N_{\text{K}^+} = \frac{Q_i}{e} = \frac{1.17\times 10^{-9}\text{ C}}{1.6\times 10^{-19}\text{ C}} = \boxed{7.3\times 10^{9}\text{ K}^+\text{ ions}}$$

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and the charge per unit area on this surface is

$$\sigma = \frac{Q_i}{A} = \frac{1.17\times10^{-9}\ \text{C}}{2\pi\times10^{-6}\ \text{m}^2}\left(\frac{1\ e}{1.6\times10^{-19}\ \text{C}}\right)\left(\frac{10^{-20}\ \text{m}^2}{1\ \text{Å}^2}\right) = \frac{1\ e}{8.6\times10^{4}\ \text{Å}^2} = \boxed{\frac{1\ e}{(290\ \text{Å})^2}}$$

This corresponds to a low charge density of one electronic charge per square of side 290 Å, compared to a normal atomic spacing of one atom every few Å.

(b) In the resting state, the net charge on the inner surface of the membrane is $-Q_i = -1.17\times10^{-9}$ C, and the net positive charge on this surface in the excited state is

$$Q_f = C(\Delta V)_f = (1.67\times10^{-8}\ \text{F})(+30\times10^{-3}\ \text{V}) = +5.0\times10^{-10}\ \text{C}$$

The total positive charge which must pass through the membrane to produce the excited state is therefore

$$\Delta Q = Q_f - Q_i$$

$$= +5.0\times10^{-10}\ \text{C} - (-1.17\times10^{-9}\ \text{C}) = 1.67\times10^{-9}\ \text{C} \to \boxed{1.7\times10^{-9}\ \text{C}}$$

corresponding to

$$N_{\text{Na}^+} = \frac{\Delta Q}{e} = \frac{1.67\times10^{-9}\ \text{C}}{1.6\times10^{-19}\ \text{C/Na}^+\ \text{ion}} = \boxed{1.0\times10^{10}\ \text{Na}^+\ \text{ions}}$$

(c) If the sodium ions enter the axon in a time of $\Delta t = 2.0$ ms, the average current is

$$I = \frac{\Delta Q}{\Delta t} = \frac{1.67\times10^{-9}\ \text{C}}{2.0\times10^{-3}\ \text{s}} = 8.3\times10^{-7}\ \text{A} = \boxed{0.83\ \mu\text{A}}$$

(d) When the membrane becomes permeable to sodium ions, the initial influx of sodium ions neutralizes the capacitor with no required energy input. The energy input required to charge the now neutral capacitor to the potential difference of the excited state is

$$W = \frac{1}{2}C(\Delta V)_f^2 = \frac{1}{2}(1.67\times10^{-8}\ \text{F})(30\times10^{-3}\ \text{V})^2 = \boxed{7.5\times10^{-12}\ \text{J}}$$

18.42 The capacitance of the 10 cm length of axon was found to be $C = 1.67\times10^{-8}$ F in the solution of Problem 18.41.

(a) When the membrane becomes permeable to potassium ions, these ions flow out of the axon with no energy input required until the capacitor is neutralized. To maintain this outflow of potassium ions and charge the now neutral capacitor to the resting action potential requires an energy input of

$$W = \frac{1}{2}C(\Delta V)^2 = \frac{1}{2}(1.67\times10^{-8}\ \text{F})(70\times10^{-3}\ \text{V})^2 = \boxed{4.1\times10^{-11}\ \text{J}}$$

(b) As found in the solution of Problem 18.41, the charge on the inner surface of the membrane in the resting state is -1.17×10^{-9} C, and the charge on this surface in the excited state is $+5.0\times10^{-10}$ C. Thus, the positive charge which must flow out of the axon as it goes from the excited state to the resting state is

$$\Delta Q = 5.0\times10^{-10}\ \text{C} + 1.17\times10^{-9}\ \text{C} = 1.67\times10^{-9}\ \text{C}$$

continued on next page

and the average current during the 3.0 ms required to return to the resting state is

$$I = \frac{\Delta Q}{\Delta t} = \frac{1.67\times10^{-9}\text{ C}}{3.0\times10^{-3}\text{ s}} = 5.6\times10^{-7}\text{ A} = \boxed{0.56\ \mu\text{A}}$$

18.43 From Figure 18.28, the duration of an action potential pulse is 4.5 ms. From the solution of Problem 18.41, the energy input required to reach the excited state is $W_1 = 7.5\times10^{-12}$ J. The energy input required during the return to the resting state is found in Problem 18.42 to be $W_2 = 4.1\times10^{-11}$ J. Therefore, the average power input required during an action potential pulse is

$$P = \frac{W_{\text{total}}}{\Delta t} = \frac{W_1 + W_2}{\Delta t} = \frac{7.5\times10^{-12}\text{ J} + 4.1\times10^{-11}\text{ J}}{4.5\times10^{-3}\text{ s}} = 1.1\times10^{-8}\text{ W} = \boxed{11\text{ nW}}$$

18.44 <u>Using a single resistor $\rightarrow$ 3 distinct values: $R_1 = 2.0\ \Omega$, $R_2 = 4.0\ \Omega$, $R_3 = 6.0\ \Omega$</u>

<u>2 resistors in Series</u> $\rightarrow$ 2 additional distinct values: $R_4 = 2.0\ \Omega + 6.0\ \Omega = 8.0\ \Omega$, and $R_5 = 4.0\ \Omega + 6.0\ \Omega = 10\ \Omega$. Note: 2.0 Ω and 4.0 Ω in series duplicates R_3 above.

<u>2 resistors in Parallel</u> $\rightarrow$ 3 additional distinct values:

$R_6 = 2.0\ \Omega$ and $4.0\ \Omega$ in parallel $= 1.3\ \Omega$

$R_7 = 2.0\ \Omega$ and $6.0\ \Omega$ in parallel $= 1.5\ \Omega$

$R_8 = 4.0\ \Omega$ and $6.0\ \Omega$ in parallel $= 2.4\ \Omega$

<u>3 resistors in Series</u> $\rightarrow$ 1 additional distinct value:

$R_9 = 2.0\ \Omega + 4.0\ \Omega + 6.0\ \Omega = 12\ \Omega$

<u>3 resistors in Parallel</u> $\rightarrow$ 1 additional distinct value:

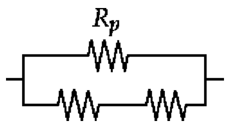
$R_{10} = 2.0\ \Omega$, $4.0\ \Omega$, and $6.0\ \Omega$ in parallel $= 1.1\ \Omega$

<u>1 resistor in Parallel with Series combination of the other 2:</u> $\rightarrow$ 3 additional values:

$R_{11} = \left(R_p = 2.0\ \Omega;\ 4.0\ \Omega \text{ and } 6.0\ \Omega \text{ in series}\right) = 1.7\ \Omega$

$R_{12} = \left(R_p = 4.0\ \Omega;\ 2.0\ \Omega \text{ and } 6.0\ \Omega \text{ in series}\right) = 2.7\ \Omega$

$R_{12} = \left(R_p = 6.0\ \Omega;\ 2.0\ \Omega \text{ and } 4.0\ \Omega \text{ in series}\right) = 3.0\ \Omega$

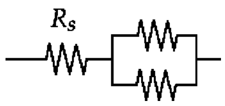


<u>1 resistor in Series with Parallel combination of the other 2:</u> $\rightarrow$ 3 additional values:

$R_{14} = \left(R_s = 2.0\ \Omega;\ 4.0\ \Omega \text{ and } 6.0\ \Omega \text{ in parallel}\right) = 4.4\ \Omega$

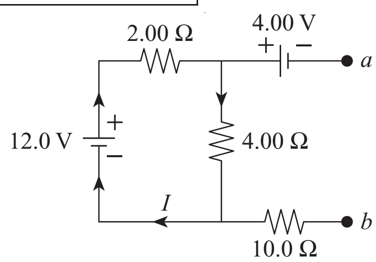
$R_{15} = \left(R_s = 4.0\ \Omega;\ 2.0\ \Omega \text{ and } 6.0\ \Omega \text{ in parallel}\right) = 5.5\ \Omega$

$R_{16} = \left(R_s = 6.0\ \Omega;\ 2.0\ \Omega \text{ and } 4.0\ \Omega \text{ in Parallel}\right) = 7.3\ \Omega$



$\boxed{\text{Thus, 16 distinct values of resistance are possible using these three resistors.}}$

18.45 Since the circuit is open at points *a* and *b*, no current flows through the 4.00-V battery or the 10.0-Ω resistor. A current *I* will flow around the closed path through the 2.00-Ω resistor, 4.00-Ω resistor, and the 12.0-V battery as shown in the sketch at the right. This current has magnitude



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$$I = \frac{\Delta V}{R_{\text{path}}} = \frac{12.0 \text{ V}}{2.00\ \Omega + 4.00\ \Omega} = 2.00 \text{ A}$$

Along the path from point a to point b, the change in potential that occurs is given by

$$\Delta V_{ab} = +\varepsilon_4 - IR_4 = +4.00 \text{ V} - (2.00 \text{ A})(4.00\ \Omega) = -4.00 \text{ V}$$

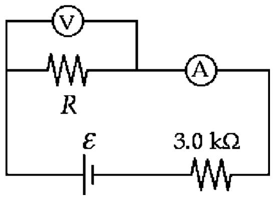
(a) The potential difference between points a and b has magnitude

$$\boxed{|\Delta V_{ab}| = 4.00 \text{ V}}$$

(b) Since the change in potential in going from a to b was negative, we conclude that $\boxed{\text{point } a \text{ is at the higher potential}}$.

18.46 (a) $R = \dfrac{\Delta V}{I} = \dfrac{6.0 \text{ V}}{3.0 \times 10^{-3} \text{ A}} = 2.0 \times 10^3\ \Omega = \boxed{2.0 \text{ k}\Omega}$

(b) The resistance in the circuit consists of a series combination with an equivalent resistance of $R_{eq} = 2.0 \text{ k}\Omega + 3.0 \text{ k}\Omega = 5.0 \text{ k}\Omega$. The emf of the battery is then



$$\varepsilon = IR_{eq} = \left(3.0 \times 10^{-3} \text{ A}\right)\left(5.0 \times 10^3\ \Omega\right) = \boxed{15 \text{ V}}$$

(c) $\Delta V_3 = IR_3 = \left(3.0 \times 10^{-3} \text{ A}\right)\left(3.0 \times 10^3\ \Omega\right) = \boxed{9.0 \text{ V}}$

(d) In this solution, $\boxed{\text{we have assumed that we have ideal devices in the circuit.}}$ In particular, we have assumed that the battery has negligible internal resistance, the voltmeter has an extremely large resistance and draws negligible current, and the ammeter has an extremely low resistance and a negligible voltage drop across it.

18.47 (a) The resistors combine to an equivalent resistance of $R_{eq} = \boxed{15\ \Omega}$ as shown.

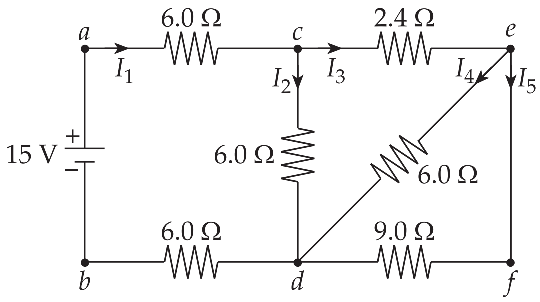


Figure 1

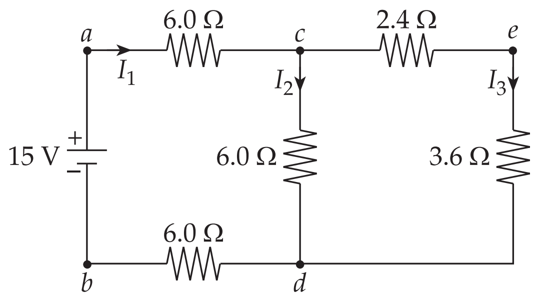


Figure 2

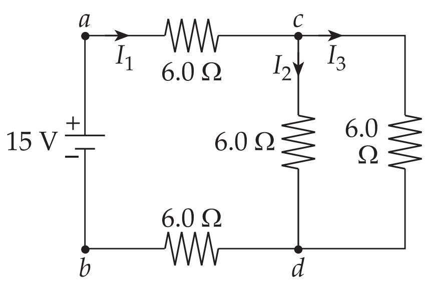


Figure 3

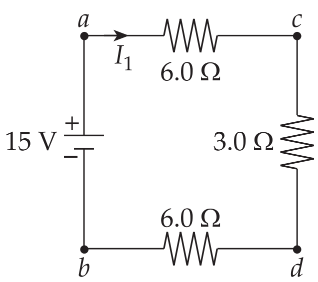


Figure 4

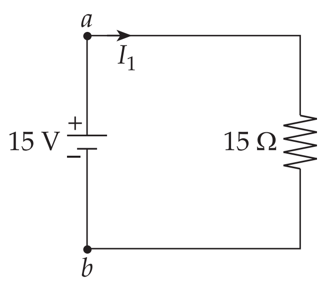


Figure 5

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(b) From Figure 5, $I_1 = \frac{\Delta V_{ab}}{R_{eq}} = \frac{15\text{ V}}{15\ \Omega} = \boxed{1.0\text{ A}}$

Then, from Figure 4,

$$\Delta V_{ac} = \Delta V_{db} = I_1(6.0\ \Omega) = 6.0\text{ V and }\Delta V_{cd} = I_1(3.0\ \Omega) = 3.0\text{ V}$$

From Figure 3, $I_2 = I_3 = \frac{\Delta V_{cd}}{6.0\ \Omega} = \frac{3.0\text{ V}}{6.0\ \Omega} = \boxed{0.50\text{ A}}$

From Figure 2, $\Delta V_{ed} = I_3(3.6\ \Omega) = 1.8\text{ V}$

Then, from Figure 1, $I_4 = \frac{\Delta V_{ed}}{6.0\ \Omega} = \frac{1.8\text{ V}}{6.0\ \Omega} = \boxed{0.30\text{ A}}$

and $I_5 = \frac{\Delta V_{fd}}{9.0\ \Omega} = \frac{\Delta V_{ed}}{9.0\ \Omega} = \frac{1.8\text{ V}}{9.0\ \Omega} = \boxed{0.20\text{ A}}$

(c) From Figure 2, $\Delta V_{ce} = I_3(2.4\ \Omega) = \boxed{1.2\text{ V}}$. All the other needed potential differences were calculated above in part (b). The results were

$$\Delta V_{ac} = \Delta V_{db} = \boxed{6.0\text{ V}};\ \Delta V_{cd} = \boxed{3.0\text{ V}};\text{ and }\Delta V_{fd} = \Delta V_{ed} = \boxed{1.8\text{ V}}$$

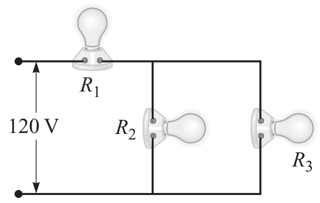
(d) The power dissipated in each resistor is found from $P = (\Delta V)^2/R$ with the following results:

$$P_{ac} = \frac{(\Delta V)^2_{ac}}{R_{ac}} = \frac{(6.0\text{ V})^2}{6.0\ \Omega} = \boxed{6.0\text{ W}} \qquad P_{ce} = \frac{(\Delta V)^2_{ce}}{R_{ce}} = \frac{(1.2\text{ V})^2}{2.4\ \Omega} = \boxed{0.60\text{ W}}$$

$$P_{ed} = \frac{(\Delta V)^2_{ed}}{R_{ed}} = \frac{(1.8\text{ V})^2}{6.0\ \Omega} = \boxed{0.54\text{ W}} \qquad P_{fd} = \frac{(\Delta V)^2_{fd}}{R_{fd}} = \frac{(1.8\text{ V})^2}{9.0\ \Omega} = \boxed{0.36\text{ W}}$$

$$P_{cd} = \frac{(\Delta V)^2_{cd}}{R_{cd}} = \frac{(3.0\text{ V})^2}{6.0\ \Omega} = \boxed{1.5\text{ W}} \qquad P_{db} = \frac{(\Delta V)^2_{db}}{R_{db}} = \frac{(6.0\text{ V})^2}{6.0\ \Omega} = \boxed{6.0\text{ W}}$$

18.48 (a) From $P = (\Delta V)^2/R$, the resistance of each of the three bulbs is given by



$$R = \frac{(\Delta V)^2}{P} = \frac{(120\text{ V})^2}{60.0\text{ W}} = 240\ \Omega$$

As connected, the parallel combination of R_2 and R_3 is in series with R_1. Thus, the equivalent resistance of the circuit is

$$R_{eq} = R_1 + \left(\frac{1}{R_2} + \frac{1}{R_3}\right)^{-1} = 240\ \Omega + \left(\frac{1}{240\ \Omega} + \frac{1}{240\ \Omega}\right)^{-1} = 360\ \Omega$$

The total power delivered to the circuit is

$$P = \frac{(\Delta V)^2}{R_{eq}} = \frac{(120\text{ V})^2}{360\ \Omega} = \boxed{40.0\text{ W}}$$

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(b) The current supplied by the source is $I = \frac{\Delta V}{R_{eq}} = \frac{120\text{ V}}{360\ \Omega} = \frac{1}{3}\text{ A}$. Thus, the potential difference across R_1 is

$$(\Delta V)_1 = I R_1 = \left(\frac{1}{3}\text{ A}\right)(240\ \Omega) = \boxed{80.0\text{ V}}$$

The potential difference across the parallel combination of R_2 and R_3 is then

$$(\Delta V)_2 = (\Delta V)_3 = (\Delta V)_{source} - (\Delta V)_1 = 120\text{ V} - 80.0\text{ V} = \boxed{40.0\text{ V}}$$

18.49 When the two resistors are connected in series, the equivalent resistance is $R_s = R_1 + R_2$ and the power delivered when a current $I = 5.00\text{ A}$ flows through the series combination is

$$P_s = I^2 R_s = (5.00\text{ A})^2 (R_1 + R_2) = 225\text{ W}$$

Thus, $R_1 + R_2 = \frac{225\text{ W}}{25.0\text{ A}^2}$ giving $R_1 + R_2 = 9.00\ \Omega$ **[1]**

When the resistors are connected in parallel, the equivalent resistance is $R_p = R_1 R_2 / (R_1 + R_2)$ and the power delivered by the same current $(I = 5.00\text{ A})$ is

$$P_p = I^2 R_p = (5.00\text{ A})^2 \left(\frac{R_1 R_2}{R_1 + R_2}\right) = 50.0\text{ W}$$

giving $R_p = \frac{50.0\text{ W}}{25.0\text{ A}^2}$ or $\frac{R_1 R_2}{R_1 + R_2} = 2.00\ \Omega$ **[2]**

Substituting Equation [1] into Equation [2] yields

$$\frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 (9.00\ \Omega - R_1)}{9.00\ \Omega} = 2.00\ \Omega$$

or $R_1^2 - (9.00\ \Omega) R_1 + 18.0\ \Omega^2 = 0$. This quadratic equation factors as

$$(R_1 - 3.00\ \Omega)(R_1 - 6.00\ \Omega) = 0$$

Thus, either $R_1 = 3.00\ \Omega$ or $R_1 = 6.00\ \Omega$, and from Equation [1], we find that either $R_2 = 6.00\ \Omega$ or $R_2 = 3.00\ \Omega$. Therefore, the pair contains one $\boxed{3.00\ \Omega}$ resistor and one $\boxed{6.00\ \Omega}$ resistor.

18.50 (a) Recognize that the 5.00-Ω and the 8.00-Ω resistors are connected in parallel and that the effective resistance of this parallel combination is

$$R_p = \frac{(5.00\ \Omega)(8.00\ \Omega)}{5.00\ \Omega + 8.00\ \Omega} = 3.08\ \Omega$$

This resistance is in series with the 10.0-Ω resistor, giving a total resistance for the circuit of $R_{eq} = 10.0\ \Omega + 3.08 = 13.1\ \Omega$. Thus, the current supplied by the battery is

$$I_{total} = \frac{\varepsilon}{R_{eq}} = \frac{15.0\text{ V}}{13.1\ \Omega} = 1.15\text{ A}$$

and the potential difference across the parallel combination is

$$\Delta V_p = I_{total} R_p = (1.15\text{ A})(3.08\ \Omega) = 3.54\text{ V}$$

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The current through the 5.00-Ω in this parallel combination is then

$$I_5 = \frac{\Delta V_p}{R_5} = \frac{3.54\ \text{V}}{5.00\ \Omega} = \boxed{0.708\ \text{A}}$$

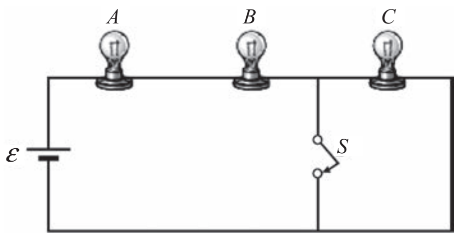
(b) The power delivered to the 5.00-Ω resistor is

$$P_5 = I_5^2 R_5 = (0.708\ \text{A})^2 (5.00\ \Omega) = \boxed{2.51\ \text{W}}$$

(c) $\boxed{\text{Only the circuit in Figure P18.50c}}$ requires the use of Kirchhoff's rules for solution. In the other circuits, the batteries can be combined into a single effective battery while the 5.00-Ω and 8.00-Ω resistors remain in parallel with each other.

(d) $\boxed{\text{The power delivered is lowest in Figure 18.50c.}}$ The circuits in Figures P18.50b and P18.50d have in effect 30.0-V batteries driving current through the 10.0-Ω resistor, thus delivering more power than the circuit in Figure 18.50a. In Figure 18.50c, the two 15.0-V batteries tend to oppose each other's efforts to drive current through the 10.0-Ω resistor, making them less effective than the single 15.0-V battery of Figure 18.50a.

18.51 (a) When switch S is open, all three bulbs are in series and the equivalent resistance is $R_{\text{eq}}^{\text{open}} = R + R + R = \boxed{3R}$.

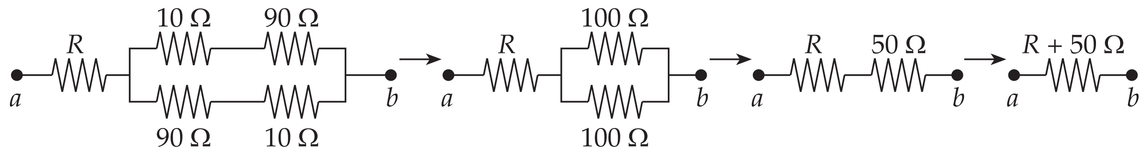


When the switch is closed, bulb C is shorted across and no current will flow through that bulb. This leaves bulbs A and B in series with an equivalent resistance of $R_{\text{eq}}^{\text{closed}} = R + R = \boxed{2R}$.

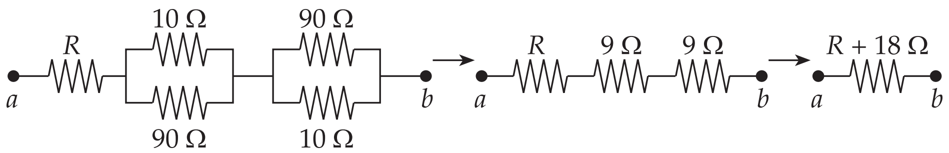
(b) With the switch open, the power delivered by the battery is $P_{\text{open}} = \dfrac{\mathcal{E}^2}{R_{\text{eq}}^{\text{open}}} = \boxed{\dfrac{\mathcal{E}^2}{3R}}$, and with the switch closed, $P_{\text{closed}} = \mathcal{E}^2 / R_{\text{eq}}^{\text{closed}} = \boxed{\mathcal{E}^2/2R}$.

(c) When the switch is open, the three bulbs have equal brightness. When S is closed, $\boxed{\text{bulb } C \text{ goes out, while } A \text{ and } B \text{ remain equal at a greater brightness}}$ than they had when the switch was open.

18.52 With the switch open, the circuit may be reduced as follows:



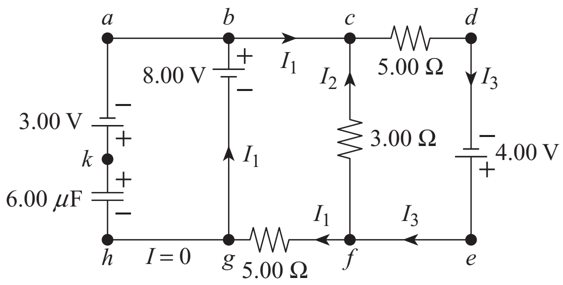
With the switch closed, the circuit reduces as shown below:



Since the equivalent resistance with the switch closed is one-half that when the switch is open, we have

$$R + 18\ \Omega = \frac{1}{2}(R + 50\ \Omega), \text{ which yields } R = \boxed{14\ \Omega}$$

18.53 (a) Note the assumed directions of the three distinct currents in the circuit diagram at the right. Applying the junction rule at point *c* gives



$$I_1 = I_3 - I_2 \qquad \textbf{[1]}$$

Applying Kirchhoff's loop rule to loop *gbcfg* gives

$$+8.00 \text{ V} + 3.00 I_2 - 5.00 I_1 = 0 \qquad \text{or} \qquad 5.00 I_1 - 3.00 I_2 = 8.00 \text{ V} \qquad \textbf{[2]}$$

Finally, applying Kirchhoff's loop rule to loop *fcdef* yields

$$-3.00 I_2 - 5.00 I_3 + 4.00 \text{ V} = 0 \qquad \text{or} \qquad 5.00 I_3 + 3.00 I_2 = 4.00 \text{ V} \qquad \textbf{[3]}$$

Substituting Equation [1] into Equation [2] gives

$$5.00 I_3 - 8.00 I_2 = 8.00 \text{ V}$$

and subtracting this result from Equation [3] yields $I_2 = -4.00 \text{ V}/11.0$

Equation [3] then gives the current in the 4.00-V battery as

$$I_3 = \frac{4.00 \text{ V} - 3.00(-4.00 \text{ V}/11.0)}{5.00} = +1.02 \text{ A} \quad \text{or} \quad I_3 = \boxed{1.02 \text{ A down}}$$

(b) From above, the current in the 3.00-Ω resistor is

$$I_2 = -\frac{4.00 \text{ V}}{11.0} = -0.364 \text{ A} \qquad \text{or} \qquad I_2 = \boxed{0.364 \text{ A down}}$$

(c) Equation [1] now gives the current in the 8.00-V battery as

$$I_1 = 1.02 \text{ A} - (-0.364 \text{ A}) = +1.38 \text{ A} \qquad \text{or} \qquad I_1 = \boxed{1.38 \text{ A up}}$$

(d) Once the capacitor is charged, the current in the 3.00-V battery is $I = \boxed{0}$ because of the open circuit between the plates of the capacitor.

(e) To obtain the potential difference between the plates of the capacitor, we start at the negative plate and go to the positive plate (noting the changes in potential) along path *hgbak*. The result is

$$\Delta V_{hk} = +8.00 \text{ V} + 3.00 \text{ V} = 11.0 \text{ V}$$

so the charge on the capacitor is

$$Q_{hk} = C_{hk}(\Delta V_{hk}) = (6.00 \ \mu\text{F})(11.0 \text{ V}) = \boxed{66.0 \ \mu\text{C}}$$

18.54 At time *t*, the charge on the capacitor will be $q = Q_{\max}\left(1 - e^{-t/\tau}\right)$, where

$$\tau = RC = \left(2.0 \times 10^6 \ \Omega\right)\left(3.0 \times 10^{-6} \text{ F}\right) = 6.0 \text{ s}$$

When $q = 0.90 Q_{\max}$, this gives $0.90 = 1 - e^{-t/\tau}$ or $e^{-t/\tau} = 0.10$. Thus, $-t/\tau = \ln(0.10)$, giving $t = -(6.0 \text{ s})\ln(0.10) = \boxed{14 \text{ s}}$.

18.55 (a) For the first measurement, the equivalent circuit is as shown in Figure 1. From this,

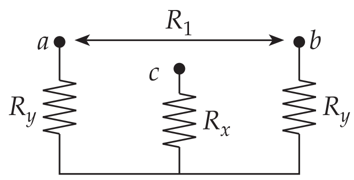


Figure 1

$$R_{ab} = R_1 = R_y + R_y = 2R_y$$

so $R_y = \frac{1}{2}R_1$ **[1]**

For the second measurement, the equivalent circuit is shown in Figure 2. This gives

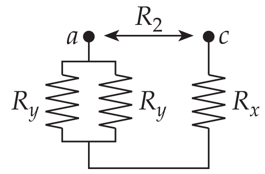


Figure 2

$$R_{ac} = R_2 = \frac{1}{2}R_y + R_x \quad \textbf{[2]}$$

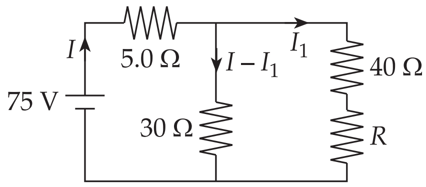
Substitute [1] into [2] to obtain

$$R_2 = \frac{1}{2}\left(\frac{1}{2}R_1\right) + R_x, \text{ or } \boxed{R_x = R_2 - \frac{1}{4}R_1}$$

(b) If $R_1 = 13\ \Omega$ and $R_2 = 6.0\ \Omega$, then $\boxed{R_x = 2.8\ \Omega}$.

Since this exceeds the limit of $2.0\ \Omega$, the antenna is $\boxed{\text{inadequately grounded}}$.

18.56 Assume a set of currents as shown in the circuit diagram at the right. Applying Kirchhoff's loop rule to the leftmost loop gives



$$+75 - (5.0)I - (30)(I - I_1) = 0$$

or $7I - 6I_1 = 15$ **[1]**

For the rightmost loop, the loop rule gives

$$-(40 + R)I_1 + (30)(I - I_1) = 0, \qquad \text{or} \qquad I = \left(\frac{7}{3} + \frac{R}{30}\right)I_1 \quad \textbf{[2]}$$

Substituting Equation [2] into [1] and simplifying gives

$$310I_1 + 7(I_1R) = 450 \quad \textbf{[3]}$$

Also, it is known that $P_R = I_1^2 R = 20\text{ W}$, so $\quad I_1R = \frac{20\text{ W}}{I_1}$ **[4]**

Substitution of Equation [4] into [3] yields

$$310I_1 + \frac{140}{I_1} = 450 \qquad \text{or} \qquad 310I_1^2 - 450I_1 + 140 = 0$$

Using the quadratic formula: $I_1 = \dfrac{-(-450) \pm \sqrt{(-450)^2 - 4(310)(140)}}{2(310)}$,

yielding $I_1 = 1.0\text{ A}$ and $I_1 = 0.452\text{ A}$. Then, from $R = \dfrac{20\text{ W}}{I_1^2}$, we find two possible values for the resistance R. These are: $\boxed{R = 20\ \Omega \quad \text{or} \quad R = 98\ \Omega}$

18.57 When connected in series, the equivalent resistance is $R_{eq} = R_1 + R_2 + \cdots + R_n = nR$. Thus, the current is $I_s = (\Delta V)/R_{eq} = (\Delta V)/nR$, and the power consumed by the series configuration is

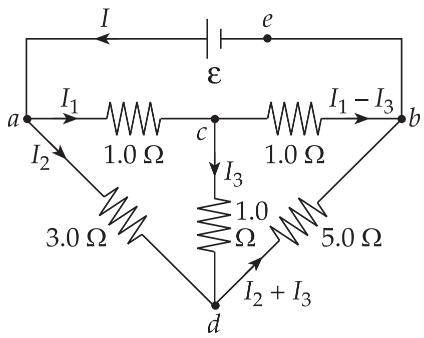
$$P_s = I_s^2 R_{eq} = \frac{(\Delta V)^2}{(nR)^2}(nR) = \frac{(\Delta V)^2}{nR}$$

For the parallel connection, the power consumed by each individual resistor is $P_1 = (\Delta V)^2/R$, and the total power consumption is

$$P_p = nP_1 = \frac{n(\Delta V)^2}{R}$$

Therefore, $\dfrac{P_s}{P_p} = \dfrac{(\Delta V)^2}{nR}\cdot\dfrac{R}{n(\Delta V)^2} = \dfrac{1}{n^2}$ or $\boxed{P_s = \dfrac{1}{n^2}P_p}$

18.58 Consider a battery of emf ε connected between points a and b as shown. Applying Kirchhoff's loop rule to loop *acbea* gives



$$-(1.0)I_1 - (1.0)(I_1 - I_3) + \varepsilon = 0$$

or $\quad I_3 = 2I_1 - \varepsilon$ **[1]**

Applying the loop rule to loop *adbea* gives

$$-(3.0)I_2 - (5.0)(I_2 + I_3) + \varepsilon = 0$$

or $\quad 8I_2 + 5I_3 = \varepsilon$ **[2]**

For loop *adca*, the loop rule yields

$$-(3.0)I_2 + (1.0)I_3 + (1.0)I_1 = 0 \text{ or } I_2 = \frac{I_1 + I_3}{3}$$ **[3]**

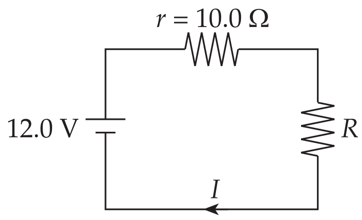
Substituting Equation [1] into [3] gives $\quad I_2 = I_1 - \varepsilon/3$ **[4]**

Now, substitute Equations [1] and [4] into [2] to obtain $18I_1 = \dfrac{26}{3}\varepsilon$, which reduces to $I_1 = \dfrac{13}{27}\varepsilon$.
Then, Equation [4] gives $I_2 = \left[\dfrac{13}{27} - \dfrac{9}{27}\right]\varepsilon = \dfrac{4}{27}\varepsilon$, and [1] yields $I_3 = -\dfrac{1}{27}\varepsilon$.

Then, applying Kirchhoff's junction rule at junction a gives

$$I = I_1 + I_2 = \frac{13}{27}\varepsilon + \frac{4}{27}\varepsilon = \frac{17}{27}\varepsilon. \text{ Therefore, } R_{ab} = \frac{\varepsilon}{I} = \frac{\varepsilon}{(17\varepsilon/27)} = \boxed{\frac{27}{17}\ \Omega}.$$

18.59 (a) and (b): With R the value of the load resistor, the current in a series circuit composed of a 12.0 V battery, an internal resistance of 10.0 Ω, and a load resistor is



$$I = \frac{12.0 \text{ V}}{R + 10.0\ \Omega}$$

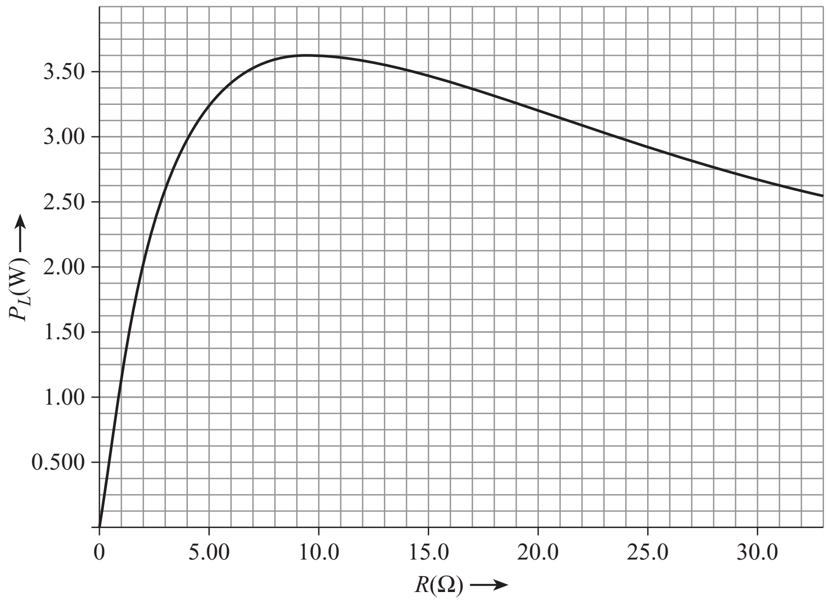
and the power delivered to the load resistor is

$$P_L = I^2 R = \boxed{\frac{(144 \text{ V}^2)R}{(R + 10.0\ \Omega)^2}}$$

continued on next page

Some typical data values for the graph are

R (Ω)	P_L (W)
1.00	1.19
5.00	3.20
10.0	3.60
15.0	3.46
20.0	3.20
25.0	2.94
30.0	2.70



The curve peaks at $P_L = 3.60$ W at a load resistance of $R = 10.0\ \Omega$.

18.60 The total resistance in the circuit is

$$R = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1} = \left(\frac{1}{2.0\ \text{k}\Omega} + \frac{1}{3.0\ \text{k}\Omega}\right)^{-1} = 1.2\ \text{k}\Omega$$

and the total capacitance is $C = C_1 + C_2 = 2.0\ \mu\text{F} + 3.0\ \mu\text{F} = 5.0\ \mu\text{F}$.

Thus, $Q_{\max} = C\varepsilon = (5.0\ \mu\text{F})(120\ \text{V}) = 600\ \mu\text{C}$

and $\tau = RC = (1.2 \times 10^3\ \Omega)(5.0 \times 10^{-6}\ \text{F}) = 6.0 \times 10^{-3}\ \text{s} = \dfrac{6.0\ \text{s}}{1\,000}$

The total stored charge at any time t is then

$$q = q_1 + q_2 = Q_{\max}\left(1 - e^{-t/\tau}\right) \quad \text{or} \quad q_1 + q_2 = (600\ \mu\text{C})\left(1 - e^{-1\,000\,t/6.0\ \text{s}}\right) \qquad \textbf{[1]}$$

Since the capacitors are in parallel with each other, the same potential difference exists across both at any time.

Therefore, $$(\Delta V)_C = \frac{q_1}{C_1} = \frac{q_2}{C_2}, \quad \text{or} \quad q_2 = \left(\frac{C_2}{C_1}\right)q_1 = 1.5\,q_1 \qquad \textbf{[2]}$$

Substituting Equation [2] into [1] gives

$$2.5q_1 = (600\ \mu\text{C})\left(1 - e^{-1\,000\,t/6.0\ \text{s}}\right) \quad \text{and} \quad q_1 = \boxed{(240\ \mu\text{C})\left(1 - e^{-1\,000\,t/6.0\ \text{s}}\right)}$$

Then, Equation [2] yields $q_2 = 1.5(240\ \mu\text{C})\left(1 - e^{-1\,000\,t/6.0\ \text{s}}\right) = \boxed{(360\ \mu\text{C})\left(1 - e^{-1\,000\,t/6.0\ \text{s}}\right)}$

18.61 (a) With 4.0×10^3 cells, each with an emf of 150 mV, connected in series, the total terminal potential difference is

$$\Delta V = (4.0 \times 10^3)(150 \times 10^{-3}\ \text{V}) = 6.0 \times 10^2\ \text{V}$$

continued on next page

When delivering a current of $I = 1.0$ A, the power output is

$$P = I(\Delta V) = (1.0\ \text{A})\left(6.0\times10^2\ \text{V}\right) = \boxed{6.0\times10^2\ \text{W}}$$

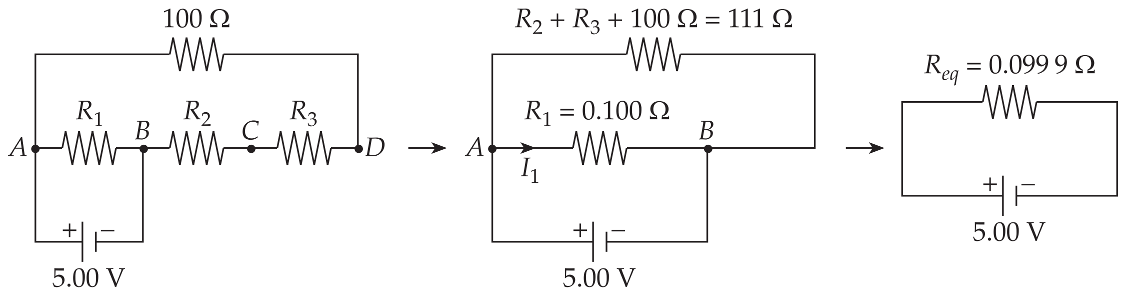
(b) The energy released in one shock is

$$E_1 = P(\Delta t)_1 = \left(6.0\times10^2\ \text{W}\right)\left(2.0\times10^{-3}\ \text{s}\right) = \boxed{1.2\ \text{J}}$$

(c) The energy released in 300 such shocks is $E_{\text{total}} = 300E_1 = 300(1.2\ \text{J}) = 3.6\times10^2$ J. For a 1.0-kg object to be given a gravitational potential energy of this magnitude, the height the object must be lifted above the reference level is

$$h = \frac{PE_g}{mg} = \frac{3.6\times10^2\ \text{J}}{(1.0\ \text{kg})\left(9.80\ \text{m/s}^2\right)} = \boxed{37\ \text{m}}$$

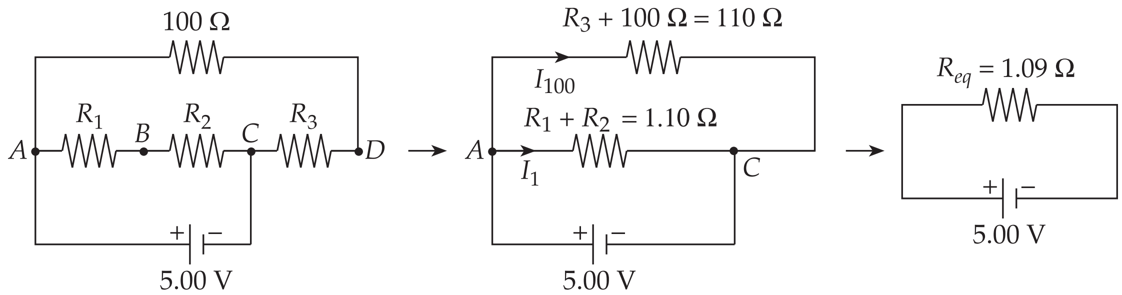
18.62 (a) When the power supply is connected to points A and B, the circuit reduces as shown below to an equivalent resistance of $R_{\text{eq}} = \boxed{0.099\,9\ \Omega}$.



From the center figure above, observe that $I_{R_1} = I_1 = \dfrac{5.00\ \text{V}}{0.100\ \Omega} = \boxed{50.0\ \text{A}}$

and $\quad I_{R_2} = I_{R_3} = I_{100} = \dfrac{5.00\ \text{V}}{111\ \Omega} = 0.045\,0\ \text{A} = \boxed{45.0\ \text{mA}}$

(b) When the power supply is connected to points A and C, the circuit reduces as shown below to an equivalent resistance of $R_{\text{eq}} = \boxed{1.09\ \Omega}$.



From the center figure above, observe that $I_{R_1} = I_{R_2} = I_1 = \dfrac{5.00\ \text{V}}{1.10\ \Omega} = \boxed{4.55\ \text{A}}$

and $\quad I_{R_3} = I_{100} = \dfrac{5.00\ \text{V}}{110\ \Omega} = 0.045\,5\ \text{A} = \boxed{45.5\ \text{mA}}$

(c) When the power supply is connected to points A and D, the circuit reduces as shown below to an equivalent resistance of $R_{\text{eq}} = \boxed{9.99\ \Omega}$.

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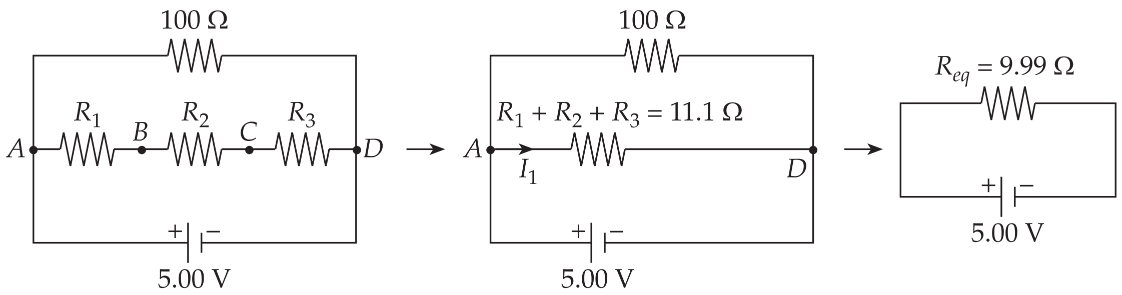
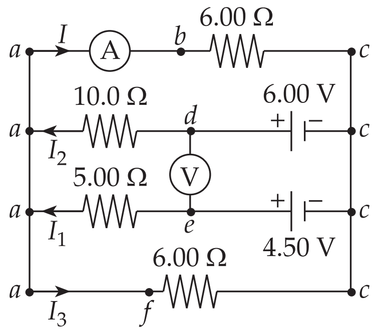


From the center figure above, observe that $I_{R_1} = I_{R_2} = I_{R_3} = I_1 = \dfrac{5.00\text{ V}}{11.1\ \Omega} = \boxed{0.450\text{ A}}$

and $\quad I_{100} = \dfrac{5.00\text{ V}}{100\ \Omega} = 0.050\,0\text{ A} = \boxed{50.0\text{ mA}}$

18.63 In the circuit diagram at the right, note that all points labeled *a* are at the same potential and equivalent to each other. Also, all points labeled *c* are equivalent.



To determine the voltmeter reading, go from point *e* to point *d* along the path *ecd*, keeping track of all changes in potential to find

$$\Delta V_{ed} = V_d - V_e = -4.50\text{ V} + 6.00\text{ V} = \boxed{+1.50\text{ V}}$$

Apply Kirchhoff's loop rule around loop *abcfa* to find

$$-(6.00\ \Omega)I + (6.00\ \Omega)I_3 = 0 \qquad \text{or} \qquad I_3 = I \qquad \textbf{[1]}$$

Apply Kirchhoff's loop rule around loop *abcda* to find

$$-(6.00\ \Omega)I + 6.00\text{ V} - (10.0\ \Omega)I_2 = 0 \qquad \text{or} \qquad I_2 = 0.600\text{ A} - 0.600I \qquad \textbf{[2]}$$

Apply Kirchhoff's loop rule around loop *abcea* to find

$$-(6.00\ \Omega)I + 4.50\text{ V} - (5.00\ \Omega)I_1 = 0 \qquad \text{or} \qquad I_1 = 0.900\text{ A} - 1.20I \qquad \textbf{[3]}$$

Finally, apply Kirchhoff's junction rule at either point *a* or point *c* to obtain

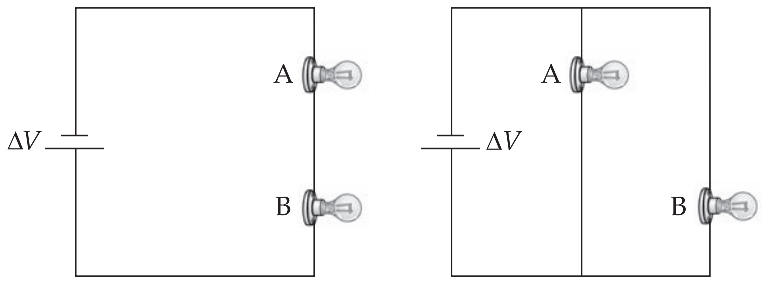
$$I + I_3 = I_1 + I_2 \qquad \textbf{[4]}$$

Substitute Equations [1], [2], and [3] into Equation [4] to obtain the current through the ammeter. This gives

$$I + I = 0.900\text{ A} - 1.20I + 0.600\text{ A} - 0.600I$$

or $\quad 3.80I = 1.50\text{ A} \qquad$ and $\qquad I = 1.50\text{ A}/3.80 = \boxed{0.395\text{ A}}$

18.64 In the figure given below, note that all bulbs have the same resistance, R.



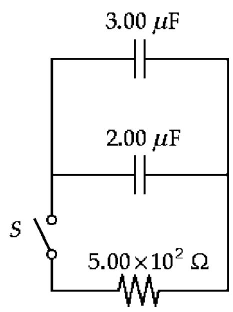
(a) In the series situation, Case 1, the same current I_1 flows through both bulbs. Thus, the same power, $P_1 = I_1^2 R$, is supplied to each bulb. Since the brightness of a bulb is proportional to the power supplied to it, they will have the same brightness. We conclude that the $\boxed{\text{bulbs have the same current, power supplied, and brightness}}$.

(b) In the parallel case, Case 2, the same potential difference ΔV is maintained across each of the bulbs. Thus, the $\boxed{\text{same current}}$ $I_2 = \Delta V/R$ will flow in each branch of this parallel circuit. This means that, again, the $\boxed{\text{same power}}$ $P_2 = I_2^2 R$ is supplied to each bulb, and the two bulbs will have $\boxed{\text{equal brightness}}$.

(c) The total resistance of the single branch of the series circuit (Case 1) is $2R$. Thus, the current in this case is $I_1 = \Delta V/2R$. Note that this is one half of the current I_2 that flows through each bulb in the parallel circuit (Case 2). Since the power supplied is proportional to the square of the current, the power supplied to each bulb in Case 2 is four times that supplied to each bulb in Case 1. Thus, the bulbs in $\boxed{\text{Case 2}}$ are much brighter than those in Case 1.

(d) If either bulb goes out in Case 1, the only conducting path of the circuit is broken and all current ceases. Thus, in the series case, $\boxed{\text{the other bulb must also go out}}$. If one bulb goes out in Case 2, there is still a continuous conducting path through the other bulb. Neglecting any internal resistance of the battery, the battery continues to maintain the same potential difference ΔV across this bulb as was present when both bulbs were lit. Thus, in the parallel case, $\boxed{\text{the second bulb remains lit}}$ with $\boxed{\text{unchanged current and brightness}}$ when one bulb fails.

18.65 (a) The equivalent capacitance of this parallel combination is



$$C_{eq} = C_1 + C_2 = 3.00\ \mu\text{F} + 2.00\ \mu\text{F} = 5.00\ \mu\text{F}$$

When fully charged by a 12.0-V battery, the total stored charge before the switch is closed is

$$Q_0 = C_{eq}(\Delta V) = (5.00\ \mu\text{F})(12.0\ \text{V}) = 60.0\ \mu\text{C}$$

Once the switch is closed, the time constant of the resulting RC circuit is

$$\tau = RC_{eq} = (5.00 \times 10^2\ \Omega)(5.00\ \mu\text{F}) = 2.50 \times 10^{-3}\ \text{s} = 2.50\ \text{ms}$$

Thus, at $t = 1.00$ ms after closing the switch, the remaining total stored charge is

$$q = Q_0 e^{-t/\tau} = (60.0\ \mu\text{C})e^{-1.00\ \text{ms}/2.50\ \text{ms}} = (60.0\ \mu\text{C})e^{-0.400} = 40.2\ \mu\text{C}$$

continued on next page

The potential difference across the parallel combination of capacitors is then

$$\Delta V = \frac{q}{C_{\text{eq}}} = \frac{40.2\ \mu\text{C}}{5.00\ \mu\text{F}} = 8.04\ \text{V}$$

and the charge remaining on the 3.00-μF capacitor will be

$$q_3 = C_3(\Delta V) = (3.00\ \mu\text{F})(8.04\ \text{V}) = \boxed{24.1\ \mu\text{C}}$$

(b) The charge remaining on the 2.00-μF capacitor at this time is

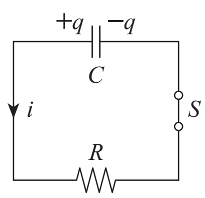
$$q_2 = q - q_3 = 40.2\ \mu\text{C} - 24.1\ \mu\text{C} = \boxed{16.1\ \mu\text{C}}$$

or alternately, $q_2 = C_2(\Delta V) = (2.00\ \mu\text{F})(8.04\ \text{V}) = \boxed{16.1\ \mu\text{C}}$

(c) Since the resistor is in parallel with the capacitors, it has the same potential difference across it as do the capacitors at all times. Thus, Ohm's law gives

$$I = \frac{\Delta V}{R} = \frac{8.04\ \text{V}}{5.00\times 10^{2}\ \Omega} = 1.61\times 10^{-2}\ \text{A} = \boxed{16.1\ \text{mA}}$$

18.66 (a) If the switch S in the circuit at the right is closed at $t = 0$, the charge remaining on the capacitor at time t is $q = Q_0 e^{-t/\tau}$, where $Q_0 = 5.10\ \mu\text{C}$ is the initial charge and



$$\tau = RC = (1.30\times 10^{3}\ \Omega)(2.00\times 10^{-9}\ \text{F}) = 2.60\ \mu\text{s}$$

is the time constant of the circuit.

The potential difference across the capacitor, Δv_C, at time t is

$$\Delta v_C = \frac{q}{C} = \frac{Q_0 e^{-t/\tau}}{C}$$

Applying Kirchhoff's loop rule to the above circuit gives $\Delta v_C - iR = 0$, or

$$i = \frac{\Delta v_C}{R} = \frac{Q_0 e^{-t/\tau}}{RC} = \left(\frac{Q_0}{\tau}\right)e^{-t/\tau}$$

The current through the resistor at time $t = 9.00\ \mu\text{s}$ is then

$$i = \left(\frac{Q_0}{\tau}\right)e^{-t/\tau} = \left(\frac{5.10\ \mu\text{C}}{2.60\ \mu\text{s}}\right)e^{-\frac{9.00\ \mu\text{s}}{2.60\ \mu\text{s}}} = 6.16\times 10^{-2}\ \text{C/s} = \boxed{61.6\ \text{mA}}$$

(b) The charge remaining on the capacitor at $t = 8.00\ \mu\text{s}$ is

$$q = Q_0 e^{-t/\tau} = (5.10\ \mu\text{C})e^{-\frac{8.00\ \mu\text{s}}{2.60\ \mu\text{s}}} = \boxed{0.235\ \mu\text{C}}$$

(c) The maximum current in the circuit occurs when the switch is first closed (at $t = 0$) and is given by

$$i_0 = \left(\frac{Q_0}{\tau}\right)e^{-0} = \frac{Q_0}{\tau} = \frac{5.10\ \cancel{\mu}\text{C}}{2.60\ \cancel{\mu}\text{s}} = \boxed{1.96\ \text{A}}$$

19

Magnetism

QUICK QUIZZES

1. Choice (b). The force that a magnetic field exerts on a charged particle moving through it is given by $F = qvB\sin\theta = qvB_{\perp}$, where $B_{\perp}$ is the component of the field perpendicular to the particle's velocity. Since the particle moves in a straight line, the magnetic force (and hence $B_{\perp}$, since $qv \neq 0$) must be zero.

2. Choice (c). The magnetic force exerted on a charge by a magnetic field is proportional to the charge's velocity relative to the field. If the charge is stationary, as in this situation, there is no magnetic force.

3. Choice (c). The torque that a planar current loop will experience when it is in a magnetic field is given by $\tau = BIA\sin\theta$. Note that this torque depends on the strength of the field, the current in the coil, the area enclosed by the coil, and the orientation of the plane of the coil relative to the direction of the field. However, it *does not depend on the shape* of the loop.

4. Choice (a). The magnetic force acting on the particle is always perpendicular to the velocity of the particle and hence to the displacement the particle is undergoing. Under these conditions, the force does no work on the particle, and the particle's kinetic energy remains constant.

5. Choices (a) and (c). The magnitude of the force per unit length between two parallel current carrying wires is $F/\ell = (\mu_0 I_1 I_2)/(2\pi d)$. The magnitude of this force can be doubled by doubling the magnitude of the current in either wire. It can also be doubled by decreasing the distance between them, d, by half. Thus, both choices (a) and (c) are correct.

6. Choice (b). The two forces are an action-reaction pair. They act on different wires and have equal magnitudes but opposite directions.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. If the magnitude of the magnetic force on the wire equals the weight of the wire, then $BI\ell\sin\theta = w$, or $B = w/I\ell\sin\theta$. The magnitude of the magnetic field is a minimum when $\theta = 90°$ and $\sin\theta = 1$. Thus,

$$B_{\min} = \frac{w}{I\ell} = \frac{1.0\times10^{-2}\ \text{N}}{(0.10\ \text{A})(0.50\ \text{m})} = 0.20\ \text{T}$$

and (a) is the correct answer for this question.

2. The electron moves in a horizontal plane in a direction of 35° N of E, which is the same as 55° E of N. Since the magnetic field at this location is horizontal and directed due north, the angle between the direction of the electron's velocity and the direction of the magnetic field is 55°. The magnitude of the magnetic force experienced by the electron is then

$$F = |q|vB\sin\theta = \left(1.6\times10^{-19}\ \text{C}\right)\left(2.5\times10^{6}\ \text{m/s}\right)\left(0.10\times10^{-4}\ \text{T}\right)\sin 55° = 3.3\times10^{-18}\ \text{N}$$

The right-hand rule number 1 predicts a force directed upward, away from the Earth's surface for a positively charged particle moving in the direction of the electron. However, the *negatively charged* electron will experience a force in the opposite direction, downward toward the Earth's surface. Thus, the correct choice is (d).

3. The z-axis is perpendicular to the plane of the loop, and the angle between the direction of this normal line and the direction of the magnetic field is $\theta = 30.0°$. Thus, the magnitude of the torque experienced by this coil containing $N = 10$ turns is

$$\tau = BIAN\sin\theta = (0.010\text{ T})(2.0\text{ A})\left[(0.20\text{ m})(0.30\text{ m})\right](10)\sin 30.0° = 6.0\times10^{-3}\text{ N}\cdot\text{m}$$

meaning that (c) is the correct choice.

4. The magnitude of the magnetic field at distance r from a long straight wire carrying current I is $B = \mu_0 I/2\pi r$. Thus, for the described situation,

$$B = \frac{\left(4\pi\times10^{-7}\text{ T·m/A}\right)(1\text{ A})}{2\pi(2\text{ m})} = 1\times10^{-7}\text{ T}$$

making (d) the correct response.

5. Since the proton follows a semicircular path, not a helical path, it entered perpendicularly to the field. A charged particle moving perpendicular to a magnetic field experiences a centripetal force of magnitude $F_c = mv^2/r = qvB$ and follows a circular path of radius $r = mv/qB$. The speed of this proton must be

$$v = \frac{qBr}{m} = \frac{\left(1.6\times10^{-19}\text{ C}\right)(0.050\text{ T})\left(1.0\times10^{-3}\text{ m}\right)}{1.67\times10^{-27}\text{ kg}} = 4.8\times10^{3}\text{ m/s}$$

and choice (e) is the correct answer.

6. The force per unit length between this pair of wires is

$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(3\text{ A})^2}{2\pi(2\text{ m})} = 9\times10^{-7}\text{ N} \sim 1\times10^{-6}\text{ N}$$

and (d) is the best choice for this question.

7. The magnitude of the magnetic field inside the specified solenoid is

$$B = \mu_0 nI = \mu_0\left(\frac{N}{\ell}\right)I = \left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)\left(\frac{120}{0.50\text{ m}}\right)(2.0\text{ A}) = 6.0\times10^{-4}\text{ T}$$

which is choice (e).

8. The force that a magnetic field exerts on a moving charge is always perpendicular to both the direction of the field and the direction of the particle's motion. Since the force is perpendicular to the direction of motion, it does no work on the particle and hence does not alter its speed. Because the speed is unchanged, both the kinetic energy and the magnitude of the linear momentum will be constant. Correct answers among the list of choices are (d) and (e). All other choices are false.

9. The magnitude of the magnetic force experienced by a charged particle in a magnetic field is given by $F = qvB\sin\theta$, where v is the speed of the particle and θ is the angle between the direction of the particle's velocity and the direction of the magnetic field. If either $v = 0$ [choice (e)] or $\sin\theta = 0$ [choice (c)], this force has zero magnitude. All other choices are false, so the correct answers are (c) and (e).

10. By the right-hand rule number 1, when the proton first enters the field, it experiences a force directed upward, toward the top of the page. This will deflect the proton upward, and as the proton's velocity changes direction, the force changes direction, always staying perpendicular to the velocity. The force, being perpendicular to the motion, causes the particle to follow a circular path, with no change in speed, as long as it is in the field. After completing a half circle, the proton will exit the field traveling toward the left. The correct answer is choice (d).

11. The contribution made to the magnetic field at point P by the lower wire is directed out of the page, while the contribution due to the upper wire is directed into the page. Since point P is equidistant from the two wires, and the wires carry the same magnitude currents, these two oppositely directed contributions to the magnetic field have equal magnitudes and cancel each other. Therefore, the total magnetic field at point P is zero, making (a) the correct answer for this question.

12. The magnetic field due to the current in the vertical wire is directed into the page on the right side of the wire and out of the page on the left side. The field due to the current in the horizontal wire is out of the page above this wire and into the page below the wire. Thus, the two contributions to the total magnetic field have the same directions at points B (both out of the page) and D (both contributions into the page), while the two contributions have opposite directions at points A and C. The magnitude of the total magnetic field will be greatest at points B and D where the two contributions are in the same direction, and smallest at points A and C where the two contributions are in opposite directions and tend to cancel. The correct choices for this question are (a) and (c).

13. The torque exerted on a single turn coil carrying current I by a magnetic field B is $\tau = BIA\sin\theta$. The line perpendicular to the plane of each coil is also perpendicular to the direction of the magnetic field (i.e., $\theta = 90°$). Since B and I are the same for all three coils, the torques exerted on them are proportional to the area A enclosed by each of the coils. Coil A is rectangular with area $A_A = (1 \text{ m})(2 \text{ m}) = 2 \text{ m}^2$. Coil B is elliptical with semi-major axis $a = 0.75$ m and semi-minor axis $b = 0.5$ m, giving an area $A_B = \pi ab$ or $A_B = \pi(0.75 \text{ m})(0.5 \text{ m}) = 1.2 \text{ m}^2$. Coil C is triangular with area $A_C = \frac{1}{2}(base)(height)$, or $A_C = \frac{1}{2}(1 \text{ m})(3 \text{ m}) = 1.5 \text{ m}^2$. Thus, $A_A > A_C > A_B$, meaning that $\tau_A > \tau_C > \tau_B$ and choice (b) is the correct answer.

14. Any point in region I is closer to the upper wire which carries the larger current. At all points in this region, the outward-directed field due the upper wire will have a greater magnitude than will the inward-directed field due to the lower wire. Thus, the resultant field in region I will be nonzero and out of the page, meaning that choice (d) is a true statement and choice (a) is false. In region II, the field due to each wire is directed into the page, so their magnitudes add and the resultant field cannot be zero at any point in this region. This means that choice (b) is false. In region III, the field due to the upper wire is directed into the page while that due to the lower wire is out of the page. Since points in this region are closer to the wire carrying the smaller current, there are points in this region where the magnitudes of the oppositely directed fields due to the two wires will possibly have equal magnitudes, canceling each other and producing a zero resultant field. Thus, choice (c) is true and choice (e) is false. The correct answers for this question are choices (c) and (d).

15. According to right-hand rule number 2, the magnetic field at point P due to the current in the wire is directed out of the page, meaning that choices (c) and (e) are false. The magnitude of this field is given by $B = \mu_0 I/2\pi r$, so choices (b) and (d) are false. Choice (a) is correct about both the magnitude and direction of the field and is the correct answer for the question.

16. The magnetic field inside a solenoid, carrying current I, with N turns and length L, is

$$B = \mu_0 nI = \mu_0 \left(\frac{N}{L}\right) I. \text{ Thus, } B_A = \frac{\mu_0 N_A I}{L_A},\ B_B = \frac{\mu_0 N_A I}{2L_A} = \frac{1}{2} B_A, \text{ and } B_C = \frac{\mu_0 (2N_A) I}{L_A/2} = 4B_A.$$

Therefore, we see that $B_C > B_A > B_B$, and choice (d) gives the correct rankings.

17. Using right-hand rule number 1, we see that the magnetic force exerted on the positively charged proton in the situation described is in the positive y-direction, making choice (c) the correct answer.

18. The correct answers to each of the parts of this question are as follows:

(a) Yes. (b) No. (c) Yes. (d) Yes, unless the object moves parallel to the field.
(e) No. (f) Yes, unless the current is parallel to the field. (g) Yes.
(h) Yes, unless the electrons move parallel to the field.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. It should point straight down toward the surface of the Earth.

4. No. The force that a constant magnetic field exerts on a charged particle is dependent on the velocity of that particle. If the particle has zero velocity, it will experience no magnetic force and cannot be set in motion by a constant magnetic field.

6. Yes. Regardless of which pole is used, the magnetic field of the magnet induces magnetic poles in the nail, with each end of the nail taking on a magnetic polarity opposite to that of the pole of the magnet nearest to it. These opposite magnetic poles then attract each other.

8. The magnet causes domain alignment in the iron, inducing magnetic poles such that the iron becomes magnetic and attracted to the original magnet. Now that the iron is magnetic, it can produce an identical effect in another piece of iron.

10. No. The magnetic field created by a single current loop resembles that of a bar magnet—strongest inside the loop, and decreasing in strength as you move away from the loop. Neither is the field uniform in direction—the magnetic field lines form closed paths that curve around the conductor forming the current loop and pass through the area enclosed by that current loop.

12. Near the poles the magnetic field of Earth points almost straight downward (or straight upward), in the direction (or opposite to the direction) the charges are moving. As a result, there is little or no magnetic force exerted on the charged particles at the pole to deflect them away from Earth.

14. The loop can be mounted on an axle so it is free to rotate. The current loop will tend to rotate in a manner that brings the plane of the loop perpendicular to the direction of the field. As the current in the loop is increased, the torque causing it to rotate will increase in magnitude.

16. (a) The blue magnet experiences an upward magnetic force equal to its weight. The yellow magnet is repelled by the red magnets with a force whose magnitude equals the weight of the yellow magnet plus the magnitude of the reaction force exerted on this magnet by the blue magnet.

(b) The rods prevent motion to the side and prevent the magnets from rotating under their mutual torques. Its constraint changes unstable equilibrium into stable.

(c) Most likely, the disks are magnetized perpendicular to their flat faces, making one face a north pole and the other a south pole. The yellow magnet has a pole on its lower face which is the same as the pole on the upper faces of the red magnets. The pole on the lower face of the blue magnet is the same as that on the upper face of the yellow magnet.

(d) If the upper magnet were inverted, the yellow and blue magnets would attract each other and stick firmly together. The yellow magnet would continue to be repelled by and float above the red magnets.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** a′) toward the left b′) into the page c′) out of the page
d′) toward top of page e′) into the page f′) out of the page

(b) The answer for each subpart is opposite to that given in part (a) above.

4. **(a)** toward top of page **(b)** out of page ($q < 0$) **(c)** zero force

(d) into the page

6. $\vec{\mathbf{B}} = 2.09 \times 10^{-2}$ T in the negative y-direction

8. **(a)** 7.90×10^{-12} N **(b)** 0

10. 807 N

12. **(a)** toward the left **(b)** into the page **(c)** out of the page

(d) toward top of page **(e)** into the page **(f)** out of the page

14. **(a)** 0.12 N

(b) Both the direction of the field and the direction of the current must be known before the direction of the force can be determined.

16. $\vec{\mathbf{B}}_{\min} = 0.245$ T eastward

18. **(a)** 9.0×10^{-3} N at 15° above the horizontal in the northward direction

(b) 2.3×10^{-3} N horizontal and due west

20. **(a)** 0.109 A **(b)** toward the right

22. $B = \mu_k mg/Id$

24. 5.8 N into the page

26. 4.33×10^{-3} N·m

28. 9.98 N·m, clockwise about the y-axis when viewed from above

30. 118 N·m

32. **(a)** $+x$-direction, zero torque about x-axis

(b) $-x$-direction, zero torque about x-axis

(c) No. The two forces are equal in magnitude and opposite in direction, canceling each other, and can have no effect on the motion of the loop.

(d) in the yz-plane at 130° counterclockwise from $+y$-direction. Torque is counterclockwise about x-axis.

(e) counterclockwise about x-axis

(f) $0.135\ \text{A}\cdot\text{m}^2$ **(g)** 130° **(h)** 0.155 N·m

34. **(a)** 1.25×10^{-13} N **(b)** $7.49\times10^{13}\ \text{m/s}^2$

36. 0.150 mm

38. 3.11 cm

40. **(a)** $v = 2(KE)/qBR$ **(b)** $m = q^2B^2R^2/2(KE)$

42. **(a)** $r_d = \sqrt{2}\cdot r_p$ **(b)** $r_\alpha = \sqrt{2}\cdot r_p$

44. **(a)** toward the left **(b)** out of the page **(c)** lower left to upper right

46. 675 A downward

48. **(a)** 40.0 μT into the page **(b)** 5.00 μT out of the page **(c)** 1.67 μT out of the page

50. **(a)** $\vec{\mathbf{B}}_{\text{net}} = 4.00\ \mu$T toward the bottom of the page

(b) $\vec{\mathbf{B}}_{\text{net}} = 6.67\ \mu$T upward at 77.0° to the left of vertical

52. **(a)** in the $-y$-direction **(b)** upward, in the positive z-direction

(c) The magnitude of the upward magnetic force must equal that of the downward gravitational force.

(d) $d = qv\mu_0 I/2\pi mg$ **(e)** 5.40 cm

54. **(a)** in the $+y$-direction **(b)** $B_P = \mu_0 Ix/\pi\left(x^2+d^2\right)$

(c) $B_P\big|_{x=0} = 0$. This is as expected since the two field contributions have equal magnitudes and opposite directions at the point midway between the wires.

56. **(a)** 8.0 A **(b)** opposite directions

(c) Reversing the direction of one current changes the force from repulsion to attraction. Doubling the magnitude of one current doubles the magnitude of the force.

58. 2.70×10^{-5} N toward the left

60. 4.8×10^{4} turns

62. 207 W

64. **(a)** $B = mg/NIw$ directed out of the page

(b) The forces on the two sides of the loop are equal in magnitude and opposite in direction, canceling each other, and do not affect the state of balance of the system.

(c) $B = 0.26\ \text{T}$

66. **(a)** 5.00 cm **(b)** $8.78 \times 10^{6}\ \text{m/s}$

68. $3.92 \times 10^{-2}\ \text{T}$

70. **(a)** $6.2\ \text{m/s}^2$ **(b)** 0.40 s

72. **(a)** opposite directions **(b)** 68 A

74. 0.59 T

76. **(a)** $1.00 \times 10^{-5}\ \text{T}$ **(b)** $8.00 \times 10^{-5}\ \text{N}$ directed toward wire 1

(c) $1.60 \times 10^{-5}\ \text{T}$ **(d)** $8.00 \times 10^{-5}\ \text{N}$ directed toward wire 2

PROBLEM SOLUTIONS

19.1 Remember that the direction of the magnetic force exerted on the negatively charged electron is opposite to the direction predicted by right-hand rule number 1. The magnetic field near the Earth's equator is horizontal and directed toward the north. The magnetic force experienced by a moving charged particle is always perpendicular to the plane formed by the vectors representing the magnetic field and the particle's velocity.

(a) When the velocity of a positively charged particle is downward, right-hand rule number 1 predicts a magnetic force toward the east. Hence, the force experienced by the negatively charged electron (and also the deflection of its velocity) is directed $\boxed{\text{toward the west}}$.

(b) When the particle moves northward, its velocity is parallel to the magnetic field, and it will experience $\boxed{\text{zero force and zero deflection}}$.

(c) The direction of the force on the negatively charged electron (and the deflection of its velocity) will be $\boxed{\text{vertically upward}}$.

(d) The direction of the force on the negatively charged electron (and the deflection of its velocity) will be $\boxed{\text{vertically downward}}$.

19.2 (a) For a positively charged particle, the direction of the force is that predicted by the right-hand rule number one. These are:

(a′) $\boxed{\text{in plane of page and to left}}$ (b′) $\boxed{\text{into the page}}$

(c′) $\boxed{\text{out of the page}}$ (d′) $\boxed{\text{in plane of page and toward the top}}$

(e′) $\boxed{\text{into the page}}$ (f′) $\boxed{\text{out of the page}}$

continued on next page

(b) For a negatively charged particle, the direction of the force is exactly opposite what the right-hand rule number 1 predicts for positive charges. Thus, the answers for part (b) are $\boxed{\text{reversed from those given in part (a)}}$.

19.3 Since the particle is positively charged, use the right-hand rule number 1. In this case, start with the fingers of the right hand in the direction of $\vec{\mathbf{v}}$ and the thumb pointing in the direction of $\vec{\mathbf{F}}$. As you start closing the hand, the fingers point in the direction of $\vec{\mathbf{B}}$ after they have moved 90°. The results are

(a) $\boxed{\text{into the page}}$ (b) $\boxed{\text{toward the right}}$ (c) $\boxed{\text{toward bottom of page}}$

19.4 Hold the right hand with the fingers in the direction of $\vec{\mathbf{v}}$ so that as you close your hand, the fingers move toward the direction of $\vec{\mathbf{B}}$. The thumb will point in the direction of the force (and hence the deflection) if the particle has a positive charge. The results are

(a) $\boxed{\text{toward top of page}}$ (b) $\boxed{\text{out of the page}}$, since the charge is negative

(c) $\theta = 180° \Rightarrow \boxed{\text{zero force}}$ (d) $\boxed{\text{into the page}}$

19.5 (a) The proton experiences maximum force when it moves perpendicular to the magnetic field, and the magnitude of this maximum force is

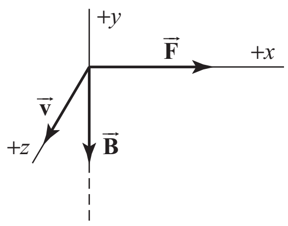
$$F_{\max} = qvB\sin 90° = \left(1.60\times10^{-19}\text{ C}\right)\left(6.00\times10^{6}\text{ m/s}\right)(1.50\text{ T})(1) = \boxed{1.44\times10^{-12}\text{ N}}$$

(b) $$a_{\max} = \frac{F_{\max}}{m_p} = \frac{1.44\times10^{-12}\text{ N}}{1.67\times10^{-27}\text{ kg}} = \boxed{8.62\times10^{14}\text{ m/s}^2}$$

(c) Since the magnitude of the charge of an electron is the same as that of a proton, the $\boxed{\text{force experienced by the electron would have the same magnitude}}$ but would be in the opposite direction due to the negative charge of the electron.

(d) The $\boxed{\text{acceleration of the electron would have a much greater magnitude}}$ than that of the proton because of the mass of the electron is much smaller.

19.6 Since the acceleration (and hence the magnetic force) is in the positive x-direction, the magnetic field must be in the negative y-direction (see sketch at the right) according to right-hand rule number 1.



The magnitude of the magnetic field is found from $F_m = qvB\sin\theta$ as

$$B = \frac{F_m}{qv\sin\theta} = \frac{ma}{qv\sin\theta}$$

or $$B = \frac{\left(1.67\times10^{-27}\text{ kg}\right)\left(2.00\times10^{13}\text{ m/s}^2\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(1.00\times10^{7}\text{ m/s}\right)\sin 90.0°} = 2.09\times10^{-2}\text{ T}$$

yielding $\boxed{\vec{\mathbf{B}} = 2.09\times10^{-2}\text{ T in the negative } y\text{-direction}}$.

19.7 The gravitational force is small enough to be ignored, so the magnetic force must supply the needed centripetal acceleration. Thus,

$$m\frac{v^2}{r} = qvB\sin 90°,\ \text{or}\ v = \frac{qBr}{m},\ \text{where}\ r = R_E + 1000\ \text{km} = 7.38\times 10^6\ \text{m}$$

$$v = \frac{(1.60\times 10^{-19}\ \text{C})(4.00\times 10^{-8}\ \text{T})(7.38\times 10^6\ \text{m})}{1.67\times 10^{-27}\ \text{kg}} = \boxed{2.83\times 10^7\ \text{m/s}}$$

If $\vec{\mathbf{v}}$ is $\boxed{\text{toward the west}}$ and $\vec{\mathbf{B}}$ is northward, $\vec{\mathbf{F}}$ will be directed downward as required.

19.8 The speed attained by the electron is found from $\frac{1}{2}mv^2 = |q|(\Delta V)$, or

$$v = \sqrt{\frac{2e(\Delta V)}{m}} = \sqrt{\frac{2(1.60\times 10^{-19}\ \text{C})(2\,400\ \text{V})}{9.11\times 10^{-31}\ \text{kg}}} = 2.90\times 10^7\ \text{m/s}$$

(a) Maximum force occurs when the electron moves perpendicular to the field.

$$F_{\max} = |q|vB\sin 90° = (1.60\times 10^{-19}\ \text{C})(2.90\times 10^7\ \text{m/s})(1.70\ \text{T}) = \boxed{7.90\times 10^{-12}\ \text{N}}$$

(b) Minimum force occurs when the electron moves parallel to the field.

$$F_{\min} = |q|vB\sin 0° = \boxed{0}$$

19.9 The magnetic force experienced by a moving charged particle has magnitude $F_m = qvB\sin\theta$, where θ is the angle between the directions of the particle's velocity and the magnetic field. Thus,

$$\sin\theta = \frac{F_m}{qvB} = \frac{8.20\times 10^{-13}\ \text{N}}{(1.60\times 10^{-19}\ \text{C})(4.00\times 10^6\ \text{m/s})(1.70\ \text{T})} = 0.754$$

and $\theta = \sin^{-1}(0.754) = \boxed{48.9°}$ or $\theta = 180° - 48.9° = \boxed{131°}$

19.10 The force on a single ion is

$$F_1 = qvB\sin\theta$$

$$= (1.60\times 10^{-19}\ \text{C})(0.851\ \text{m/s})(0.254\ \text{T})\sin(51.0°) = 2.69\times 10^{-20}\ \text{N}$$

The total number of ions present is

$$N = \left(3.00\times 10^{20}\ \frac{\text{ions}}{\text{cm}^3}\right)(100\ \text{cm}^3) = 3.00\times 10^{22}$$

Thus, assuming all ions move in the same direction through the field, the total force is

$$F = N\cdot F_1 = (3.00\times 10^{22})(2.69\times 10^{-20}\ \text{N}) = \boxed{807\ \text{N}}$$

19.11 Gravitational force:

$$F_g = mg = (9.11\times 10^{-31}\ \text{kg})(9.80\ \text{m/s}^2) = \boxed{8.93\times 10^{-30}\ \text{N downward}}$$

continued on next page

Electric force:

$$F_e = qE = \left(-1.60\times10^{-19}\ \text{C}\right)\left(-100\ \text{N/C}\right) = \boxed{1.60\times10^{-17}\ \text{N upward}}$$

Magnetic force:

$$F_m = qvB\sin\theta = \left(-1.60\times10^{-19}\ \text{C}\right)\left(6.00\times10^{6}\ \text{m/s}\right)\left(50.0\times10^{-6}\ \text{T}\right)\sin\left(90.0°\right)$$

$$= 4.80\times10^{-17}\ \text{N in direction opposite right hand rule prediction}$$

$$F_m = \boxed{4.80\times10^{-17}\ \text{N downward}}$$

19.12 Hold the right hand with the fingers in the direction of the current so, as you close the hand, the fingers move toward the direction of the magnetic field. The thumb then points in the direction of the force. The results are

(a) $\boxed{\text{to the left}}$ (b) $\boxed{\text{into the page}}$ (c) $\boxed{\text{out of the page}}$

(d) $\boxed{\text{toward top of page}}$ (e) $\boxed{\text{into the page}}$ (f) $\boxed{\text{out of the page}}$

19.13 From $F = BIL\sin\theta$, the magnetic field is

$$B = \frac{F/L}{I\sin\theta} = \frac{0.12\ \text{N/m}}{(15\ \text{A})\sin 90°} = \boxed{8.0\times10^{-3}\ \text{T}}$$

The direction of $\vec{\mathbf{B}}$ must be $\boxed{\text{the } +z\text{-direction}}$ to have $\vec{\mathbf{F}}$ in the $-y$-direction when $\vec{\mathbf{I}}$ is in the $+x$-direction.

19.14 (a) $F = BIL\sin\theta = (0.28\ \text{T})(3.0\ \text{A})(0.14\ \text{m})\sin 90° = \boxed{0.12\ \text{N}}$

(b) $\boxed{\text{Neither the direction of the magnetic field nor that of the current is given}}$. Both must be known before the direction of the force can be determined. In this problem, you can only say that the force is perpendicular to both the wire and the field.

19.15 Use the right-hand rule number 1, holding your right hand with the fingers in the direction of the current and the thumb pointing in the direction of the force. As you close your hand, the fingers will move toward the direction of the magnetic field. The results are

(a) $\boxed{\text{into the page}}$ (b) $\boxed{\text{toward the right}}$ (c) $\boxed{\text{toward the bottom of the page}}$

19.16 In order to just lift the wire, the magnetic force exerted on a unit length of the wire must be directed upward and have a magnitude equal to the weight per unit length. That is, the magnitude is

$$\frac{F}{\ell} = BI\sin\theta = \left(\frac{m}{\ell}\right)g \qquad \text{giving} \qquad B = \left(\frac{m}{\ell}\right)\frac{g}{I\sin\theta}$$

To find the minimum possible field, the magnetic field should be perpendicular to the current $(\theta = 90.0°)$. Then,

$$B_{\min} = \left(\frac{m}{\ell}\right)\frac{g}{I\sin 90.0°} = \left[0.500\ \frac{\text{g}}{\text{cm}}\left(\frac{1\ \text{kg}}{10^3\ \text{g}}\right)\left(\frac{10^2\ \text{cm}}{1\ \text{m}}\right)\right]\frac{9.80\ \text{m/s}^2}{(2.00\ \text{A})(1)} = \boxed{0.245\ \text{T}}$$

To find the direction of the field, hold the right hand with the thumb pointing upward (direction of the force) and the fingers pointing southward (direction of current). Then, as you close the hand, the fingers point eastward. The magnetic field should be directed $\boxed{\text{eastward}}$.

19.17 $F = BIL\sin\theta = (0.300\ \text{T})(10.0\ \text{A})(5.00\ \text{m})\sin(30.0°) = \boxed{7.50\ \text{N}}$

19.18 (a) The magnitude is

$$F = BIL\sin\theta = \left(0.60\times10^{-4}\ \text{T}\right)(15\ \text{A})(10.0\ \text{m})\sin(90°) = \boxed{9.0\times10^{-3}\ \text{N}}$$

$\vec{\mathbf{F}}$ is perpendicular to $\vec{\mathbf{B}}$. Using the right-hand rule number 1, the orientation of $\vec{\mathbf{F}}$ is found to be $\boxed{\text{15° above the horizontal in the northward direction}}$.

(b) $F = BIL\sin\theta = \left(0.60\times10^{-4}\ \text{T}\right)(15\ \text{A})(10.0\ \text{m})\sin(165°) = \boxed{2.3\times10^{-3}\ \text{N}}$

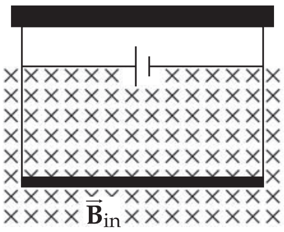
and, from the right-hand rule number 1, the direction is $\boxed{\text{horizontal and due west}}$.

19.19 For minimum field, $\vec{\mathbf{B}}$ should be perpendicular to the wire. If the force is to be northward, the field must be directed $\boxed{\text{downward}}$.

To keep the wire moving, the magnitude of the magnetic force must equal that of the kinetic friction force. Thus, $BIL\sin 90° = \mu_k(mg)$, or

$$B = \frac{\mu_k(m/L)g}{I\sin 90°} = \frac{(0.200)(1.00\ \text{g/cm})\left(9.80\ \text{m/s}^2\right)}{(1.50\ \text{A})(1.00)}\left(\frac{1\ \text{kg}}{10^3\ \text{g}}\right)\left(\frac{10^2\ \text{cm}}{1\ \text{m}}\right) = \boxed{0.131\ \text{T}}$$

19.20 (a) To have zero tension in the wires, the magnetic force per unit length must be directed upward and equal to the weight per unit length of the conductor. Thus,



$$\frac{\left|\vec{\mathbf{F}}_m\right|}{L} = BI = \frac{mg}{L},\ \text{or}$$

$$I = \frac{(m/L)g}{B} = \frac{(0.040\ \text{kg/m})\left(9.80\ \text{m/s}^2\right)}{3.60\ \text{T}} = \boxed{0.109\ \text{A}}$$

(b) From the right-hand rule number 1, the current must be $\boxed{\text{to the right}}$ if the force is to be upward when the magnetic field is into the page.

19.21 (a) $\boxed{\text{The magnetic force must be directed upward and its magnitude must equal } mg}$, the weight of the wire. Then, the net force acting on the wire will be zero, and it can move upward at constant speed.

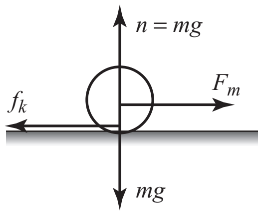
(b) The magnitude of the magnetic force must be $BIL\sin\theta = mg$ and for minimum field $\theta = 90°$. Thus,

$$B_{\text{min}} = \frac{mg}{I\,L} = \frac{(0.015\ \text{kg})\left(9.80\ \text{m/s}^2\right)}{(5.0\ \text{A})(0.15\ \text{m})} = \boxed{0.20\ \text{T}}$$

For the magnetic force to be directed upward when the current is toward the left, $\vec{\mathbf{B}}$ must be directed $\boxed{\text{out of the page}}$.

(c) If the field exceeds 0.20 T, the upward magnetic force exceeds the downward force of gravity, so $\boxed{\text{the wire accelerates upward}}$.

19.22 As shown in end view in the sketch at the right, the rod (and hence, the current) is horizontal, while the magnetic field is vertical. For the rod to move with constant velocity (zero acceleration), it is necessary that



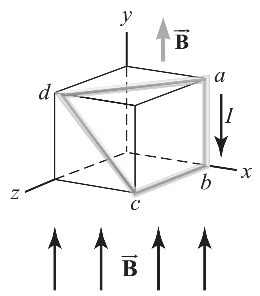
$$\Sigma F_x = F_m - f_k = BI\ell \sin 90.0° - \mu_k n = 0$$

or $$B = \frac{\mu_k n}{I\ell \sin 90.0°} = \frac{\mu_k (mg)}{Id(1.00)}$$

reducing to $B = \boxed{\mu_k mg/Id}$

19.23 For each segment, the magnitude of the force is given by $F = BIL\sin\theta$, and the direction is given by the right-hand rule number 1. The results of applying these to each of the four segments, with $B = 0.020\,0$ T and $I = 5.00$ A, are summarized below.

Segment	L (m)	θ	F (N)	Direction
ab	0.400	180°	0	no direction
bc	0.400	90.0°	0.040 0	negative *x*
cd	$0.400\sqrt{2}$	45.0°	0.040 0	negative *z*
da	$0.400\sqrt{2}$	90.0°	0.056 6	parallel to *xz*-plane at 45° to both +*x*- and +*z*-directions



19.24 The magnitude of the force is

$$F = BIL\sin\theta = (5.0\times10^{-5}\ \text{N})(2.2\times10^{3}\ \text{A})(58\ \text{m})\sin 65° = \boxed{5.8\ \text{N}}$$

and the right-hand rule number 1 shows its direction to be $\boxed{\text{into the page}}$.

19.25 The torque exerted on a single turn current loop in a magnetic field is given by $\tau = BIA\sin\theta$, where A is the area enclosed by the loop and θ is the angle between the line normal to the plane of the loop and the direction of the magnetic field B. For maximum torque, $\theta = 90.0°$, so we have

$$\tau_{\max} = BI\left(\frac{\pi d^2}{4}\right)\sin 90.0° = \frac{1}{4}(3.00\times10^{-3}\ \text{T})(5.00\ \text{A})\pi(0.100\ \text{m})^2(1.00)$$
$$= 1.18\times10^{-4}\ \text{N}\cdot\text{m} = 118\times10^{-6}\ \text{N}\cdot\text{m} = \boxed{118\ \mu\text{N}\cdot\text{m}}$$

19.26 The magnitude of the torque is $\tau = NBIA\sin\theta$, where θ is the angle between the field and the line perpendicular to the plane of the loop. The circumference of the loop is $2\pi r = 2.00$ m, so the radius is $r = \dfrac{1.00\ \text{m}}{\pi}$ and the area is $A = \pi r^2 = \dfrac{1}{\pi}\ \text{m}^2$.

Thus, $$\tau = (1)(0.800\ \text{T})(17.0\times10^{-3}\ \text{A})\left(\frac{1}{\pi}\ \text{m}^2\right)\sin 90.0° = \boxed{4.33\times10^{-3}\ \text{N}\cdot\text{m}}$$

19.27 The area of the loop is $A = \pi ab$, where $a = 0.200$ m and $b = 0.150$ m. Since the field is parallel to the plane of the loop, $\theta = 90.0°$ and the magnitude of the torque is

$$\tau = NBIA\sin\theta$$

$$= 8\left(2.00\times10^{-4}\text{ T}\right)(6.00\text{ A})\left[\pi(0.200\text{ m})(0.150\text{ m})\right]\sin 90.0° = \boxed{9.05\times10^{-4}\text{ N}\cdot\text{m}}$$

The torque is directed to make the left-hand side of the loop move toward you and the right-hand side move away.

19.28 Note that the angle between the field and the line perpendicular to the plane of the loop is $\theta = 90.0° - 30.0° = 60.0°$. Then, the magnitude of the torque is

$$\tau = NBIA\sin\theta = 100(0.800\text{ T})(1.20\text{ A})\left[(0.400\text{ m})(0.300\text{ m})\right]\sin 60.0° = \boxed{9.98\text{ N}\cdot\text{m}}$$

With current in the –y-direction, the outside edge of the loop will experience a force directed out of the page (+z-direction) according to the right-hand rule number 1. Thus, the loop will rotate $\boxed{\text{clockwise as viewed from above}}$.

19.29 (a) The torque exerted on a coil by a uniform magnetic field is $\tau = NBIA\sin\theta$, with maximum torque occurring when $\theta = 90°$. Thus, the current in the coil must be

$$I = \frac{\tau_{max}}{NBA} = \frac{0.15\text{ N}\cdot\text{m}}{(200)(0.90\text{ T})\left[\left(3.0\times10^{-2}\text{ m}\right)\left(5.0\times10^{-2}\text{ m}\right)\right]} = \boxed{0.56\text{ A}}$$

(b) If I has the value found above and θ is now 25°, the torque on the coil is

$$\tau = NBIA\sin\theta = (200)(0.90\text{ T})(0.56\text{ A})\left[(0.030\text{ m})(0.050\text{ m})\right]\sin 25° = \boxed{0.064\text{ N}\cdot\text{m}}$$

19.30 The resistance of the loop is

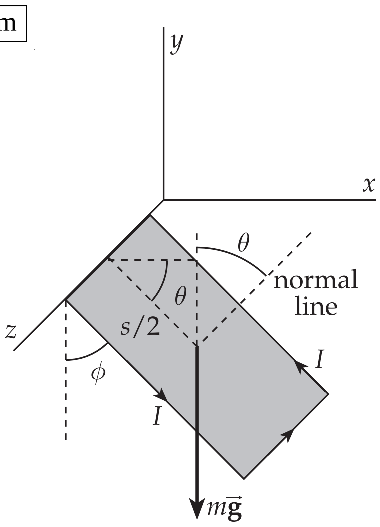
$$R = \frac{\rho L}{A} = \frac{\left(1.70\times10^{-8}\ \Omega\cdot\text{m}\right)(8.00\text{ m})}{1.00\times10^{-4}\text{ m}^2} = 1.36\times10^{-3}\ \Omega$$

and the current in the loop is $I = \dfrac{\Delta V}{R} = \dfrac{0.100\text{ V}}{1.36\times10^{-3}\ \Omega} = 73.5$ A.

The magnetic field exerts torque $\tau = NBIA\sin\theta$ on the loop, and this is a maximum when $\sin\theta = 1$. The wire forms a square loop with each side 2.00 m long. Thus,

$$\tau_{max} = NBIA = (1)(0.400\text{ T})(73.5\text{ A})(2.00\text{ m})^2 = \boxed{118\text{ N}\cdot\text{m}}$$

19.31 (a) Let θ be the angle the plane of the loop makes with the horizontal as shown in the sketch at the right. Then, the angle it makes with the vertical is $\phi = 90.0° - \theta$. The number of turns on the loop is



$$N = \frac{L}{circumference} = \frac{4.00\text{ m}}{4(0.100\text{ m})} = 10.0$$

The torque about the z-axis due to gravity is $\tau_g = mg\left(\dfrac{s}{2}\cos\theta\right)$, where $s = 0.100$ m is the length of one side of the loop. This torque tends to rotate the loop clockwise. The torque due to the magnetic force tends to rotate the loop counterclockwise about

continued on next page

the z-axis and has magnitude $\tau_m = NBIA\sin\theta$. At equilibrium, $\tau_m = \tau_g$, or $NBI(s^2)\sin\theta = mg(s\cos\theta)/2$. This reduces to

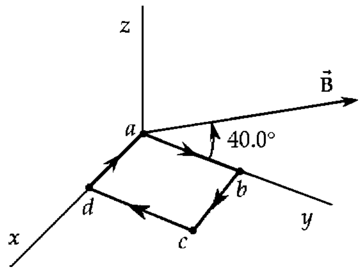
$$\tan\theta = \frac{mg}{2NBIs} = \frac{(0.100\text{ kg})(9.80\text{ m/s}^2)}{2(10.0)(0.010\,0\text{ T})(3.40\text{ A})(0.100\text{ m})} = 14.4$$

Since $\tan\theta = \tan(90.0° - \phi) = \cot\phi$, the angle the loop makes with the vertical at equilibrium is $\phi = \cot^{-1}(14.4) = \boxed{3.97°}$.

(b) At equilibrium,

$$\tau_m = NBI(s^2)\sin\theta$$
$$= (10.0)(0.010\,0\text{ T})(3.40\text{ A})(0.100\text{ m})^2\sin(90.0° - 3.97°) = \boxed{3.39\times10^{-3}\text{ N}\cdot\text{m}}$$

19.32 (a) The current in segment ab is in the $+y$-direction. Thus, by right-hand rule 1, the magnetic force on it is in the $\boxed{+x\text{-direction}}$. This force is parallel to the x-axis and therefore has zero torque about that axis.



(b) The current in segment cd is in the $-y$-direction, and the right-hand rule 1 gives the direction of the magnetic force as the $\boxed{-x\text{-direction}}$. This force is parallel to the x-axis and can exert no torque about that axis.

(c) $\boxed{\text{No}}$. These two forces are equal in magnitude and opposite in directions, so their sum is zero. Further, each force has zero torque about the axis the loop is hinged on. Since the two forces cancel each other and are both parallel to the rotation axis, they can have no effect on the motion of the loop.

(d) The magnetic force is perpendicular to both the direction of the current in bc (the x-axis) and the magnetic field. As given by right-hand rule 1, this places it $\boxed{\text{in the } yz\text{-plane at } 130° \text{ counterclockwise from the } +y\text{-axis}}$. The force acting on segment bc tends to rotate it counterclockwise about the x-axis, so the torque is in $\boxed{\text{the } +x \text{ direction}}$.

(e) The loop tends to rotate $\boxed{\text{counterclockwise about the } x\text{-axis}}$.

(f) $\mu = IAN = (0.900\text{ A})[(0.500\text{ m})(0.300\text{ m})](1) = \boxed{0.135\text{ A}\cdot\text{m}^2}$

(g) The magnetic moment vector is perpendicular to the plane of the loop (the xy-plane) and is therefore parallel to the z-axis. Because the current flows clockwise around the loop, the magnetic moment vector is directed downward, in the negative z-direction. This means that the angle between it and the direction of the magnetic field is $\theta = 90.0° + 40.0° = \boxed{130°}$.

(h) $\tau = \mu B\sin\theta = (0.135\text{ A}\cdot\text{m}^2)(1.50\text{ T})\sin(130°) = \boxed{0.155\text{ N}\cdot\text{m}}$

19.33 (a) From $KE = \frac{1}{2}m_e v^2$, the speed of the electron is

$$v = \sqrt{\frac{2(KE)}{m_e}} = \sqrt{\frac{2(3.30\times10^{-19}\text{ J})}{9.11\times10^{-31}\text{ kg}}} = \boxed{8.51\times10^5\text{ m}}$$

continued on next page

(b) The magnetic force acting on the electron must provide the necessary centripetal acceleration. Thus, $m_e v^2/r = qvB\sin\theta$, which gives

$$r = \frac{m_e v}{qB\sin\theta} = \frac{(9.11\times10^{-31}\ \text{kg})(8.51\times10^{5}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.235\ \text{T})\sin 90.0°}$$
$$= 2.06\times10^{-5}\ \text{m} = 20.6\times10^{-6}\ \text{m} = \boxed{20.6\ \mu\text{m}}$$

19.34 (a) $F = qvB\sin\theta = (1.60\times10^{-19}\ \text{C})(5.02\times10^{6}\ \text{m/s})(0.180\ \text{T})\sin(60.0°) = \boxed{1.25\times10^{-13}\ \text{N}}$

(b) $a = \dfrac{F}{m} = \dfrac{1.25\times10^{-13}\ \text{N}}{1.67\times10^{-27}\ \text{kg}} = \boxed{7.49\times10^{13}\ \text{m/s}^2}$

19.35 For the particle to pass through with no deflection, the net force acting on it must be zero. Thus, the magnetic force and the electric force must be in opposite directions and have equal magnitudes. This gives

$F_m = F_e$, or $qvB = qE$, which reduces to $\boxed{v = E/B}$

19.36 The speed of the particles emerging from the velocity selector is $v = E/B$ (see Problem 35). In the deflection chamber, the magnetic force supplies the centripetal acceleration, so

$$qvB = \frac{mv^2}{r} \qquad \text{or} \qquad r = \frac{mv}{qB} = \frac{m(E/B)}{qB} = \frac{mE}{qB^2}$$

Using the given data, the radius of the path is found to be

$$r = \frac{(2.18\times10^{-26}\ \text{kg})(950\ \text{V/m})}{(1.60\times10^{-19}\ \text{C})(0.930\ \text{T})^2} = 1.50\times10^{-4}\ \text{m} = \boxed{0.150\ \text{mm}}$$

19.37 From conservation of energy, $(KE+PE)_f = (KE+PE)_i$, we find that $\frac{1}{2}mv^2 + qV_f = 0 + qV_i$, or the speed of the particle is

$$v = \sqrt{\frac{2q(V_i - V_f)}{m}} = \sqrt{\frac{2q(\Delta V)}{m}} = \sqrt{\frac{2(1.60\times10^{-19}\ \text{C})(250\ \text{V})}{2.50\times10^{-26}\ \text{kg}}} = 5.66\times10^{4}\ \text{m/s}$$

The magnetic force supplies the centripetal acceleration giving $qvB = \dfrac{mv^2}{r}$

or $$r = \frac{mv}{qB} = \frac{(2.50\times10^{-26}\ \text{kg})(5.66\times10^{4}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.500\ \text{T})} = 1.77\times10^{-2}\ \text{m} = \boxed{1.77\ \text{cm}}$$

19.38 Since the centripetal acceleration is furnished by the magnetic force acting on the ions, $qvB = \dfrac{mv^2}{r}$, or the radius of the path is $r = \dfrac{mv}{qB}$. Thus, the distance between the impact points (that is, the difference in the diameters of the paths followed by the U_{238} and the U_{235} isotopes) is

$$\Delta d = 2(r_{238} - r_{235}) = \frac{2v}{qB}(m_{238} - m_{235})$$
$$= \frac{2(3.00\times10^{5}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(0.600\ \text{T})}\left[(238\ \text{u} - 235\ \text{u})\left(1.66\times10^{-27}\ \frac{\text{kg}}{\text{u}}\right)\right]$$

or $\Delta d = 3.11\times10^{-2}\ \text{m} = \boxed{3.11\ \text{cm}}$

19.39 In the perfectly elastic, head-on collision between the α-particle and the initially stationary proton, conservation of momentum requires that $m_p v_p + m_\alpha v_\alpha = m_\alpha v_0$ while conservation of kinetic energy also requires that $v_0 - 0 = -(v_\alpha - v_p)$ or $v_p = v_\alpha + v_0$. Using the fact that $m_\alpha = 4m_p$ and combining these equations gives

$$m_p\left(v_\alpha + v_0\right) + \left(4m_p\right)v_\alpha = \left(4m_p\right)v_0 \qquad \text{or} \qquad v_\alpha = 3v_0/5$$

and $\quad v_p = \left(3v_0/5\right) + v_0 = 8v_0/5$

Thus, $\quad v_\alpha = \dfrac{3}{5}v_0 = \dfrac{3}{5}\left(\dfrac{5}{8}v_p\right) = \dfrac{3}{8}v_p$

After the collision, each particle follows a circular path in the horizontal plane with the magnetic force supplying the centripetal acceleration. If the radius of the proton's trajectory is R, and that of the alpha particle is r, we have

$$q_p v_p B = m_p \frac{v_p^2}{R} \qquad \text{or} \qquad R = \frac{m_p v_p}{q_p B} = \frac{m_p v_p}{eB}$$

and $\quad q_\alpha v_\alpha B = m_\alpha \dfrac{v_\alpha^2}{r} \qquad \text{or} \qquad r = \dfrac{m_\alpha v_\alpha}{q_\alpha B} = \dfrac{\left(4m_p\right)\left(3v_p/8\right)}{(2e)B} = \dfrac{3}{4}\left(\dfrac{m_p v_p}{eB}\right) = \boxed{\dfrac{3}{4}R}$

19.40 (a) A charged particle follows a circular path when it moves perpendicular to the magnetic field. The magnetic force acting on the particle provides the required centripetal acceleration. Therefore, $F = qvB\sin 90° = m v^2/R$. Since the kinetic energy is $KE = mv^2/2$, we rewrite the force as $F = qvB\sin 90° = 2(KE)/R$, and solving for the speed v gives $\boxed{v = 2(KE)/qBR}$.

(b) From $KE = mv^2/2$, the mass of the particle is

$$m = \frac{2(KE)}{v^2} = \frac{2(KE)}{4(KE)^2/q^2B^2R^2} = \boxed{\frac{q^2B^2R^2}{2(KE)}}$$

19.41 (a) Within the velocity selector, the electric and magnetic fields exert forces in opposite directions on charged particles passing through. For particles having a particular speed, these forces have equal magnitudes, and the particles pass through without deflection. The selected speed is found from $F_e = qE = qvB = F_m$, giving $v = E/B$. In the deflection chamber, the selected particles follow a circular path having a diameter of $d = 2r = 2mv/qB$. Thus, the mass to charge ratio for these particles is

$$\frac{m}{q} = \frac{Bd}{2v} = \frac{Bd}{2(E/B)} = \frac{B^2 d}{2E} = \frac{(0.0931\text{ T})^2(0.396\text{ m})}{2(8\,250\text{ V/m})} = \boxed{2.08\times10^{-7}\text{ kg/C}}$$

(b) If the particle is doubly ionized (i.e., two electrons have been removed from the neutral atom), then $q = 2e$, and the mass of the ion is

$$m = (2e)\left(\frac{m}{q}\right) = 2\left(1.60\times10^{-19}\text{ C}\right)\left(2.08\times10^{-7}\text{ kg/C}\right) = \boxed{6.66\times10^{-26}\text{ kg}}$$

(c) Assuming this is an element, the mass of the ion should be approximately equal to the atomic weight multiplied by the atomic mass unit (see Table C.5 in Appendix C of the textbook). This would give the atomic weight as

$$\text{At. wt.} \approx \frac{m}{1\text{ u}} = \frac{6.66\times10^{-26}\text{ kg}}{1.66\times10^{-27}\text{ kg}} = 40.1\text{, suggesting that the element is } \boxed{\text{calcium}}.$$

19.42 Consider a particle of mass m and charge q accelerated from rest through a potential difference $\Delta V = V_i - V_f$, Applying conservation of energy gives

$$KE_f + PE_f = 0 + PE_i \qquad \text{or} \qquad KE_f = \frac{1}{2}mv^2 = PE_i - PE_f = qV_i - qV_f = q(\Delta V)$$

or the speed given the particle is $v = \sqrt{2q(\Delta V)/m}$.

If the particle now enters a magnetic field of strength B, moving perpendicular to the direction of the field, it will follow a circular path of radius

$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q(\Delta V)}{m}} \qquad \text{which reduces to} \qquad r = \sqrt{\frac{2m(\Delta V)}{qB^2}} \qquad \textbf{[1]}$$

For a proton (mass m_p and charge $q = e$), Equation [1] gives

$$r_p = \sqrt{\frac{2m_p(\Delta V)}{eB^2}}$$

(a) For the deuteron (mass $m_d = 2m_p$ and charge $q_d = e$), Equation [1] gives

$$r_d = \sqrt{\frac{2m_d(\Delta V)}{q_d B^2}} = \sqrt{\frac{2(2m_p)(\Delta V)}{eB^2}} = \sqrt{2}\left(\sqrt{\frac{2m_p(\Delta V)}{eB^2}}\right) = \boxed{\sqrt{2}\cdot r_p}$$

(b) For the alpha particle (mass $m_\alpha = 4m_p$ and charge $q_\alpha = 2e$), we find

$$r_\alpha = \sqrt{\frac{2m_\alpha(\Delta V)}{q_\alpha B^2}} = \sqrt{\frac{2(4m_p)(\Delta V)}{(2e)B^2}} = \sqrt{2}\left(\sqrt{\frac{2m_p(\Delta V)}{eB^2}}\right) = \boxed{\sqrt{2}\cdot r_p}$$

19.43 Treat the lightning bolt as a long, straight conductor. Then, the magnetic field is

$$B = \frac{\mu_0 I}{2\pi r} = \frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(1.00\times 10^4\ \text{A})}{2\pi(100\ \text{m})} = 2.00\times 10^{-5}\ \text{T} = \boxed{20.0\ \mu\text{T}}$$

19.44 Imagine grasping the conductor with the right hand so the fingers curl around the conductor in the direction of the magnetic field. The thumb then points along the conductor in the direction of the current. The results are

(a) $\boxed{\text{toward the left}}$ (b) $\boxed{\text{out of page}}$ (c) $\boxed{\text{lower left to upper right}}$

19.45 The magnetic field at distance r from a long conducting wire is $B = \mu_0 I/2\pi r$. Thus, if $B = 1.0\times 10^{-15}$ T at $r = 4.0$ cm, the current must be

$$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi(0.040\ \text{m})(1.0\times 10^{-15}\ \text{T})}{4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A}} = \boxed{2.0\times 10^{-10}\ \text{A}}$$

19.46 Model the tornado as a long, straight, vertical conductor and imagine grasping it with the right hand so the fingers point northward on the western side of the tornado (that is, at the observatory's location.) The thumb is directed downward, meaning that the $\boxed{\text{conventional current is downward or negative charge flows upward}}$.

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The magnitude of the current is found from $B = \mu_0 I/2\pi r$ as

$$I = \frac{2\pi r B}{\mu_0} = \frac{2\pi\left(9.00\times10^{3}\text{ m}\right)\left(1.50\times10^{-8}\text{ T}\right)}{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}} = \boxed{675\text{ A}}$$

19.47 From $B = \mu_0 I/2\pi r$, the required distance is

$$r = \frac{\mu_0 I}{2\pi B} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(20\text{ A})}{2\pi\left(1.7\times10^{-3}\text{ T}\right)} = 2.4\times10^{-3}\text{ m} = \boxed{2.4\text{ mm}}$$

19.48 Assume that the wire on the right is wire 1 and that on the left is wire 2. Also, choose the positive direction for the magnetic field to be out of the page and negative into the page.

(a) At the point half way between the two wires,

$$B_{\text{net}} = -B_1 - B_2 = -\left[\frac{\mu_0 I_1}{2\pi r_1} + \frac{\mu_0 I_2}{2\pi r_2}\right] = -\frac{\mu_0}{2\pi}\left[\frac{I}{d/2} + \frac{I}{d/2}\right] = -\frac{\mu_0}{2\pi}\left[\frac{4I}{d}\right]$$

$$= -\frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)}{2\pi}\left[\frac{4(5.00\text{ A})}{0.100\text{ m}}\right] = -4.00\times10^{-5}\text{ T}$$

or $B_{\text{net}} = \boxed{40.0\ \mu\text{T into the page}}$

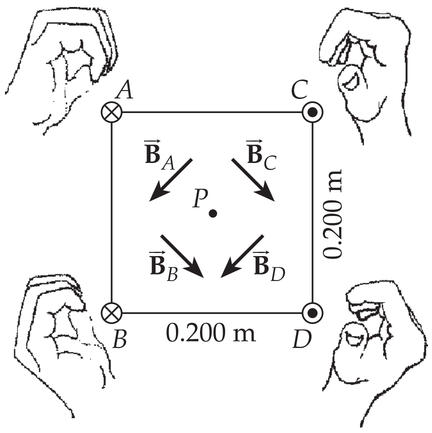
(b) At point P_1, $B_{\text{net}} = +B_1 - B_2 = \frac{\mu_0}{2\pi}\left[+\frac{I_1}{r_1} - \frac{I_2}{r_2}\right] = \frac{\mu_0}{2\pi}\left[+\frac{I}{d} - \frac{I}{2d}\right] = \frac{\mu_0}{2\pi}\left[+\frac{I}{2d}\right]$

$$B_{\text{net}} = \frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{2\pi}\left[\frac{5.00\text{ A}}{2(0.100\text{ m})}\right] = 5.00\times10^{-6}\text{ T} = \boxed{5.00\ \mu\text{T out of page}}$$

(c) At point P_2, $B_{\text{net}} = -B_1 + B_2 = \frac{\mu_0}{2\pi}\left[-\frac{I_1}{r_1} + \frac{I_2}{r_2}\right] = \frac{\mu_0}{2\pi}\left[-\frac{I}{3d} + \frac{I}{2d}\right] = \frac{\mu_0}{2\pi}\left[\frac{I}{6d}\right]$

$$B_{\text{net}} = \frac{4\pi\times10^{-7}\text{ T}\cdot\text{m/A}}{2\pi}\left[\frac{5.00\text{ A}}{6(0.100\text{ m})}\right] = 1.67\times10^{-6}\text{ T} = \boxed{1.67\ \mu\text{T out of page}}$$

19.49 The distance from each wire to point P is given by $r = \frac{1}{2}\sqrt{s^2 + s^2} = s/\sqrt{2}$, where $s = 0.200$ m.



At point P, the magnitude of the magnetic field produced by each of the wires is

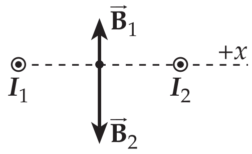
$$B = \frac{\mu_0 I}{2\pi r} = \frac{\left(4\pi\times10^{-7}\text{ T}\cdot\text{m/A}\right)(5.00\text{ A})\sqrt{2}}{2\pi(0.200\text{ m})} = 7.07\ \mu\text{T}$$

Carrying currents into the page, the field A produces at P is directed to the left and down at –135°, while B creates a field to the right and down at – 45°. Carrying currents toward you, C produces a field downward and to the right at – 45°, while D's contribution is down and to the left at –135°. The horizontal components of these equal magnitude contributions cancel in pairs, while the vertical components all add. The total field is then

$$B_{\text{net}} = 4(7.07\ \mu\text{T})\sin 45.0° = \boxed{20.0\ \mu\text{T toward the bottom of the page}}$$

19.50 Wire 1 has current $I_1 = 3.00$ A and wire 2 has $I_2 = 5.00$ A, with both currents directed out of the page. Choose the line running from wire 1 to wire 2 as the positive x-direction.

(a) At the point midway between the wires, the field due to each wire is parallel to the y-axis, and the net field is

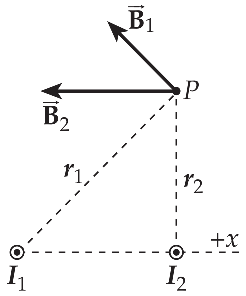


$$B_{net} = +B_{1y} - B_{2y} = \mu_0\left(I_1 - I_2\right)/\left(2\pi d/2\right)$$

Thus, $$B_{net} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)}{2\pi\left(0.100\ \text{m}\right)}\left(3.00\ \text{A} - 5.00\ \text{A}\right) = -4.00\times10^{-6}\ \text{T}$$

or $$\vec{\mathbf{B}}_{net} = \boxed{4.00\ \mu\text{T toward the bottom of the page}}$$

(b) At point P, B_1 is directed at $\theta_1 = +135°$.



The magnitude of B_1 is

$$B_1 = \frac{\mu_0 I_1}{2\pi r_1} = \frac{\mu_0 I_1}{2\pi\left(d\sqrt{2}\right)} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3.00\ \text{A}\right)}{2\pi\left(0.200\sqrt{2}\ \text{m}\right)} = 2.12\ \mu\text{T}$$

The contribution from wire 2 is in the $-x$-direction and has magnitude

$$B_2 = \frac{\mu_0 I_2}{2\pi r_2} = \frac{\mu_0 I_2}{2\pi d} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(5.00\ \text{A}\right)}{2\pi\left(0.200\ \text{m}\right)} = 5.00\ \mu\text{T}$$

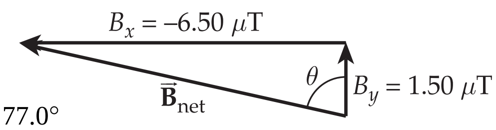
Therefore, the components of the net field at point P are

$$B_x = B_1\cos135° + B_2\cos180°$$

$$= \left(2.12\ \mu\text{T}\right)\cos135° + \left(5.00\ \mu\text{T}\right)\cos180° = -6.50\ \mu\text{T}$$

and $$B_y = B_1\sin135° + B_2\sin180° = \left(2.12\ \mu\text{T}\right)\sin135° + 0 = +1.50\ \mu\text{T}$$

Thus, $B_{net} = \sqrt{B_x^2 + B_y^2} = 6.67\ \mu\text{T}$ at



$$\theta = \tan^{-1}\left(\frac{|B_x|}{B_y}\right) = \tan^{-1}\left(\frac{6.50\ \mu\text{T}}{1.50\ \mu\text{T}}\right) = 77.0°$$

or $$\vec{\mathbf{B}}_{net} = \boxed{6.67\ \mu\text{T upward at 77.0° to the left of vertical}}$$

19.51 Call the wire along the x-axis wire 1 and the other wire 2. Also, choose the positive direction for the magnetic fields at point P to be out of the page.

At point P, $$B_{net} = +B_1 - B_2 = \frac{\mu_0 I_1}{2\pi r_1} - \frac{\mu_0 I_2}{2\pi r_2} = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right)$$

or $$B_{net} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)}{2\pi}\left(\frac{7.00\ \text{A}}{3.00\ \text{m}} - \frac{6.00\ \text{A}}{4.00\ \text{m}}\right) = +1.67\times10^{-7}\ \text{T}$$

$$\vec{\mathbf{B}}_{net} = \boxed{0.167\ \mu\text{T out of the page}}$$

19.52 (a) Imagine the horizontal *xy*-plane being perpendicular to the page, with the positive *x*-axis coming out of the page toward you and the positive *y*-axis toward the right edge of the page. Then, the vertically upward positive *z*-axis is directed toward the top of the page. With the current in the wire flowing in the positive *x*-direction, the right-hand rule 2 gives the direction of the magnetic field *above the wire* as being toward the left, or $\boxed{\text{in the } -y\text{-direction}}$.

(b) With the positively charged proton moving in the –*x*-direction (into the page), right-hand rule 1 gives the direction of the magnetic force on the proton as being directed toward the top of the page or $\boxed{\text{upward, in the positive } z\text{-direction}}$.

(c) Since the proton moves with constant velocity, a zero net force acts on it. Thus, the $\boxed{\text{magnitude of the magnetic force must equal that of the gravitational force}}$.

(d) $\Sigma F_z = ma_z = 0 \quad \Rightarrow \quad F_m = F_g$ or $qvB = mg$, where $B = \mu_0 I/2\pi d$. This gives $qv\mu_0 I/2\pi d = mg$, or the distance the proton is above the wire must be $\boxed{d = qv\mu_0 I/2\pi mg}$.

(e)
$$d = \frac{qv\mu_0 I}{2\pi mg} = \frac{\left(1.60\times10^{-19}\ \text{C}\right)\left(2.30\times10^{4}\ \text{m/s}\right)\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.20\times10^{-6}\ \text{A}\right)}{2\pi\left(1.67\times10^{-27}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right)}$$

$$d = 5.40\times10^{-2}\ \text{m} = \boxed{5.40\ \text{cm}}$$

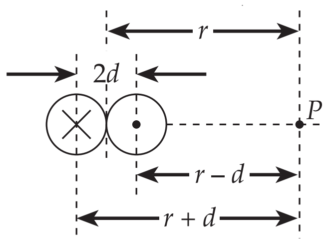
19.53 (a) From $B = \mu_0 I/2\pi r$, observe that the field is inversely proportional to the distance from the conductor. Thus, the field will have one-tenth its original value if the distance is increased by a factor of 10. The required distance is then $r' = 10r = 10(0.400\ \text{m}) = \boxed{4.00\ \text{m}}$.

(b) A point in the plane of the conductors and 40.0 cm from the center of the cord is located 39.85 cm from the nearer wire and 40.15 cm from the far wire. Since the currents are in opposite directions, so are their contributions to the net field. Therefore, $B_{\text{net}} = B_1 - B_2$, or

$$B_{\text{net}} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right) = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(2.00\ \text{A}\right)}{2\pi}\left(\frac{1}{0.398\,5\ \text{m}} - \frac{1}{0.401\,5\ \text{m}}\right)$$

$$= 7.50\times10^{-9}\ \text{T} = \boxed{7.50\ \text{nT}}$$

(c) Call *r* the distance from cord center to field point *P* and $2d = 3.00$ mm the distance between centers of the conductors.



$$B_{\text{net}} = \frac{\mu_0 I}{2\pi}\left(\frac{1}{r-d} - \frac{1}{r+d}\right) = \frac{\mu_0 I}{2\pi}\left(\frac{2d}{r^2 - d^2}\right)$$

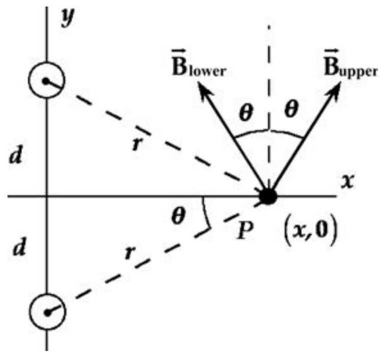
$$7.50\times10^{-10}\ \text{T} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(2.00\ \text{A}\right)}{2\pi}\left(\frac{3.00\times10^{-3}\ \text{m}}{r^2 - 2.25\times10^{-6}\ \text{m}^2}\right)$$

so $r = \boxed{1.26\ \text{m}}$

The field of the two-conductor cord is weak to start with and falls off rapidly with distance.

(d) The cable creates $\boxed{\text{zero}}$ field at exterior points, since a loop in Ampère’s law encloses zero total current.

19.54 (a) Point P is equidistant from the two wires which carry identical currents. Thus, the contributions of the two wires, $\vec{\mathbf{B}}_{\text{upper}}$ and $\vec{\mathbf{B}}_{\text{lower}}$, to the magnetic field at P will have equal magnitudes. The horizontal components of these contributions will cancel, while the vertical components add. The resultant field will be vertical $\boxed{\text{in the } +y\text{-direction}}$.



(b) The distance of each wire from point P is $r = \sqrt{x^2 + d^2}$, and the cosine of the angle that $\vec{\mathbf{B}}_{\text{upper}}$ and $\vec{\mathbf{B}}_{\text{lower}}$ make with the vertical is $\cos\theta = x/r$. The magnitude of either $\vec{\mathbf{B}}_{\text{upper}}$ or $\vec{\mathbf{B}}_{\text{lower}}$ is $B_{\text{wire}} = \mu_0 I/2\pi r$, and the vertical components of either of these contributions have values of

$$\left(B_{\text{wire}}\right)_y = \left(B_{\text{wire}}\right)\cos\theta = \left(\frac{\mu_0 I}{2\pi r}\right)\frac{x}{r} = \frac{\mu_0 I x}{2\pi r^2}$$

The magnitude of the resultant field at point P is then

$$B_P = 2\left(B_{\text{wire}}\right)_y = \frac{\mu_0 I x}{\pi r^2} = \boxed{\frac{\mu_0 I\, x}{\pi\left(x^2 + d^2\right)}}$$

(c) The point midway between the two wires is the origin (0, 0). From the above result for part (b), the resultant field at this midpoint is $B_P\big|_{x=0} = \boxed{0}$. This is as expected, because right-hand rule 2 shows that at the midpoint the field due to the upper wire is toward the right, while that due to the lower wire is toward the left. Thus, the two fields cancel, yielding a zero resultant field.

19.55 (a) The magnetic force per unit length on each of two parallel wires separated by the distance d and carrying currents I_1 and I_2 has the magnitude

$$\frac{F}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi d}$$

In this case, we have

$$\frac{F}{\ell} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(1.25\ \text{A})(3.50\ \text{A})}{2\pi\left(2.50 \times 10^{-2}\ \text{m}\right)} = \boxed{3.50 \times 10^{-5}\ \text{N/m}}$$

(b) The magnetic forces two parallel wires exert on each other are attractive if their currents are in the same direction and repulsive if the currents flow in opposite directions. In this case, the currents in the two wires are in opposite directions, so the forces are $\boxed{\text{repulsive}}$.

19.56 (a) The force per unit length that parallel conductors exert on each other is $F/\ell = \mu_0 I_1 I_2/2\pi d$. Thus, if $F/\ell = 2.0 \times 10^{-4}\ \text{N/m}$, $I_1 = 5.0\ \text{A}$, and $d = 4.0\ \text{cm}$, the current in the second wire must be

$$I_2 = \frac{2\pi d}{\mu_0 I_1}\left(\frac{F}{\ell}\right) = \frac{2\pi\left(4.0 \times 10^{-2}\ \text{m}\right)}{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(5.0\ \text{A})}\left(2.0 \times 10^{-4}\ \text{N/m}\right) = \boxed{8.0\ \text{A}}$$

(b) Since parallel conductors carrying currents in the same direction attract each other (see Section 19.8 in the textbook), the currents in these conductors which repel each other must be in $\boxed{\text{opposite directions.}}$

continued on next page

(c) The result of reversing the direction of either of the currents would be that the $\boxed{\text{force of interaction would change from a force of repulsion to an attractive force}}$. The expression for the force per unit length, $F/\ell = \mu_0 I_1 I_2 / 2\pi d$, shows that doubling either of the currents would $\boxed{\text{double the magnitude of the force of interaction}}$.

19.57 In order for the system to be in equilibrium, the repulsive magnetic force per unit length on the top wire must equal the weight per unit length of this wire.

Thus, $F/\ell = \mu_0 I_1 I_2 / 2\pi d = 0.080\ \text{N/m}$, and the distance between the wires will be

$$d = \frac{\mu_0 I_1 I_2}{2\pi(0.080\ \text{N/m})} = \frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(60.0\ \text{A})(30.0\ \text{A})}{2\pi(0.080\ \text{N/m})}$$

$$= 4.5\times 10^{-3}\ \text{m} = \boxed{4.5\ \text{mm}}$$

19.58 The magnetic forces exerted on the top and bottom segments of the rectangular loop are equal in magnitude and opposite in direction. Thus, these forces cancel, and we only need consider the sum of the forces exerted on the right and left sides of the loop. Choosing to the left (toward the long, straight wire) as the positive direction, the sum of these two forces is

$$F_{\text{net}} = +\frac{\mu_0 I_1 I_2 \ell}{2\pi c} - \frac{\mu_0 I_1 I_2 \ell}{2\pi(c+a)} = \frac{\mu_0 I_1 I_2 \ell}{2\pi}\left(\frac{1}{c} - \frac{1}{c+a}\right)$$

or $$F_{\text{net}} = \frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(5.00\ \text{A})(10.0\ \text{A})(0.450\ \text{m})}{2\pi}\left(\frac{1}{0.100\ \text{m}} - \frac{1}{0.250\ \text{m}}\right)$$

$$= +2.70\times 10^{-5}\ \text{N} = \boxed{2.70\times 10^{-5}\ \text{N to the left}}$$

19.59 The magnetic field inside a solenoid which carries current I is given by $B = \mu_0 n I$, where $n = N/\ell$ is the number of turns of wire per unit length. Thus, the current in the windings of this solenoid must be

$$I = \frac{B}{\mu_0 n} = \frac{B\ell}{\mu_0 N} = \frac{(1.00\times 10^{-4}\ \text{T})(0.400\ \text{m})}{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(10^3)} = 3.18\times 10^{-2}\ \text{A} = \boxed{31.8\ \text{mA}}$$

19.60 The magnetic field inside of a solenoid is $B = \mu_0 n I = \mu_0 (N/L) I$. Thus, the number of turns on this solenoid must be

$$N = \frac{BL}{\mu_0 I} = \frac{(9.0\ \text{T})(0.50\ \text{m})}{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(75\ \text{A})} = \boxed{4.8\times 10^4\ \text{turns}}$$

19.61 (a) From $R = \rho L/A$, the required length of wire to be used is

$$L = \frac{R\cdot A}{\rho} = \frac{(5.00\ \Omega)\left[\pi(0.500\times 10^{-3}\ \text{m})^2/4\right]}{1.7\times 10^{-8}\ \Omega\cdot\text{m}} = 58\ \text{m}$$

The total number of turns on the solenoid (that is, the number of times this length of wire will go around a 1.00-cm radius cylinder) is

$$N = \frac{L}{2\pi r} = \frac{58\ \text{m}}{2\pi(1.00\times 10^{-2}\ \text{m})} = 9.2\times 10^2 = \boxed{920}$$

continued on next page

(b) From $B = \mu_0 nI$, the number of turns per unit length on the solenoid is

$$n = \frac{B}{\mu_0 I} = \frac{4.00\times10^{-2}\ \text{T}}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(4.00\ \text{A}\right)} = 7.96\times10^{3}\ \text{turns/m}$$

Thus, the required length of the solenoid is

$$\ell = \frac{N}{n} = \frac{9.2\times10^{2}\ \text{turns}}{7.96\times10^{3}\ \text{turns/m}} = 0.12\ \text{m} = \boxed{12\ \text{cm}}$$

19.62 The wire used to wind the solenoid has diameter $d_{\text{wire}} = 0.100\ \text{cm} = 1.00\times10^{-3}\ \text{m}$. Thus, when wound in a single layer with adjacent turns touching each other, the number of turns per unit length on the solenoid is $n = 1.00\times10^{3}$ per meter. The total number of turns on the solenoid, $N = n\ell$, will then be

$$N = \left(1.00\times10^{3}/\text{m}\right)\left(0.750\ \text{m}\right) = 7.50\times10^{2}$$

If the magnetic field inside the solenoid is to be $B = 8.00$ mT, the required current in the windings is

$$I = \frac{B}{\mu_0 n} = \frac{8.00\times10^{-3}\ \text{T}}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(1.00\times10^{3}/\text{m}\right)} = 6.37\ \text{A}$$

The length of wire required for the solenoid is $L = N\cdot(\text{solenoid circumference})$, or

$$L = N\left(\pi d_{\text{solenoid}}\right) = \left(7.50\times10^{2}\right)\pi\left(0.100\ \text{m}\right) = 75.0\pi\ \text{meters}$$

and its resistance will be

$$R = \frac{\rho_{\text{Cu}} L}{A} = \frac{\rho_{\text{Cu}} L}{\pi\left(d_{\text{wire}}^{2}/4\right)} = \frac{4\left(1.70\times10^{-8}\ \Omega\cdot\text{m}\right)\left(75.0\pi\ \text{m}\right)}{\pi\left(1.00\times10^{-3}\ \text{m}\right)^{2}} = 5.10\ \Omega$$

The power that must be delivered to the solenoid is then

$$P = I^{2}R = (6.37\ \text{A})^{2}(5.10\ \Omega) = \boxed{207\ \text{W}}$$

19.63 (a) The magnetic force supplies the centripetal acceleration, so $qvB = mv^{2}/r$. The magnetic field inside the solenoid is then found to be

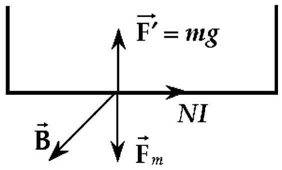
$$B = \frac{mv}{qr} = \frac{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.0\times10^{4}\ \text{m/s}\right)}{\left(1.60\times10^{-19}\ \text{C}\right)\left(2.0\times10^{-2}\ \text{m}\right)} = 2.8\times10^{-6}\ \text{T} = \boxed{2.8\ \mu\text{T}}$$

(b) From $B = \mu_0 nI$, the current is the solenoid is found to be

$$I = \frac{B}{\mu_0 n} = \frac{2.8\times10^{-6}\ \text{T}}{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left[\left(25\ \text{turns/cm}\right)\left(100\ \text{cm/1 m}\right)\right]}$$

$$= 8.9\times10^{-4}\ \text{A} = \boxed{0.89\ \text{mA}}$$

19.64 (a) When switch S is closed, a total current NI (current I in a total of N conductors) flows toward the right through the lower side of the coil. This results in a downward force of magnitude $F_m = B(NI)w$ being exerted on the coil by the magnetic field, with the requirement that the balance exert a upward force $F' = mg$ on the coil to bring the system back into balance.



In order for the magnetic force to be downward, the right-hand rule number 1 shows that the magnetic field must be directed $\boxed{\text{out of the page}}$ toward the reader. For the system to be restored to balance, it is necessary that

$$F_m = F' \qquad \text{or} \qquad B(NI)w = mg\text{, giving } B = \boxed{mg/NIw}$$

(b) The magnetic field exerts forces of equal magnitudes and opposite directions on the two sides of the coil. $\boxed{\text{These forces cancel each other and do not affect the balance}}$ of the coil. Hence the dimension of the sides is not needed.

(c) $$B = \frac{mg}{NIw} = \frac{(20.0\times10^{-3}\text{ kg})(9.80\text{ m/s}^2)}{(50)(0.30\text{ A})(5.0\times10^{-2}\text{ m})} = \boxed{0.26\text{ T}}$$

19.65 (a) The magnetic field at the center of a circular current loop of radius R and carrying current I is $B = \mu_0 I/2R$. The direction of the field at this center is given by right-hand rule number 2. Taking out of the page (toward the reader) as positive, the net magnetic field at the common center of these coplanar loops has magnitude

$$|B_{\text{net}}| = |B_2 - B_1| = \left|\frac{\mu_0 I_2}{2r_2} - \frac{\mu_0 I_1}{2r_1}\right| = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})}{2}\left|\frac{3.00\text{ A}}{9.00\times10^{-2}\text{ m}} - \frac{5.00\text{ A}}{12.0\times10^{-2}\text{ m}}\right|$$
$$= \left|-5.24\times10^{-6}\text{ T}\right| = \boxed{5.24\ \mu\text{T}}$$

(b) Since we chose out of the page as the positive direction, and now find that $B_{\text{net}} < 0$, we conclude the net magnetic field at the center is $\boxed{\text{into the page}}$.

(c) To have $B_{\text{net}} = 0$, it is necessary that $I_2/r_2 = I_1/r_1$, or

$$r_2 = \left(\frac{I_2}{I_1}\right)r_1 = \left(\frac{3.00\text{ A}}{5.00\text{ A}}\right)(12.0\text{ cm}) = \boxed{7.20\text{ cm}}$$

19.66 The angular momentum of a point mass moving in a circular path is

$$L = I\omega = (mr^2)\left(\frac{v}{r}\right) = mvr$$

where m is the mass of the particle, v is its speed, and r is the radius of its path.

(a) The magnetic force experienced by the moving electron supplies the needed centripetal acceleration, so

$$m\left(\frac{v^2}{r}\right) = qvB\sin 90.0° \qquad \text{or} \qquad mv = qBr$$

Thus, $L = mvr = (qBr)r = qBr^2$, and the radius of the path must be

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$$r = \sqrt{\frac{L}{qB}} = \sqrt{\frac{4.00\times10^{-25}\ \text{kg}\cdot\text{m}^2/\text{s}}{\left(1.60\times10^{-19}\ \text{C}\right)\left(1.00\times10^{-3}\ \text{T}\right)}} = 5.00\times10^{-2}\ \text{m} = \boxed{5.00\ \text{cm}}$$

(b) The speed of the electron may now be found from $L = mvr$ as

$$v = \frac{L}{mr} = \frac{4.00\times10^{-25}\ \text{kg}\cdot\text{m}^2/\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(5.00\times10^{-2}\ \text{m}\right)} = \boxed{8.78\times10^{6}\ \text{m/s}}$$

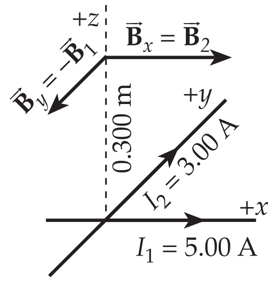
19.67 Assume wire 1 is along the x-axis and wire 2 is along the y-axis.

(a) Choosing out of the page as the positive field direction, the field at point P is

$$B = B_1 - B_2 = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right) = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)}{2\pi}\left(\frac{5.00\ \text{A}}{0.400\ \text{m}} - \frac{3.00\ \text{A}}{0.300\ \text{m}}\right)$$

$$= 5.00\times10^{-7}\ \text{T} = \boxed{0.500\ \mu\text{T out of the page}}$$

(b) At 30.0 cm above the intersection of the wires, the field components are as shown at the right, where



$$B_y = -B_1 = -\frac{\mu_0 I_1}{2\pi r}$$

$$= -\frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(5.00\ \text{A})}{2\pi(0.300\ \text{m})} = -3.33\times10^{-6}\ \text{T}$$

and $$B_x = B_2 = \frac{\mu_0 I_2}{2\pi r} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(3.00\ \text{A})}{2\pi(0.300\ \text{m})} = 2.00\times10^{-6}\ \text{T}$$

With $B_z = 0$, the resultant field is parallel to the xy-plane and

$$B = \sqrt{B_x^2 + B_y^2} = 3.88\times10^{-6}\ \text{T} \quad \text{at} \quad \theta = \tan^{-1}\left(\frac{B_y}{B_x}\right) = -59.0^\circ$$

or $\vec{\mathbf{B}} = \boxed{3.88\ \mu\text{T parallel to the } xy\text{-plane and } 59.0^\circ \text{ clockwise from the } +x\text{-direction}}$

19.68 For the rail to move at constant velocity, the net force acting on, it must be zero. Thus, the magnitude of the magnetic force must equal that of the friction force, giving $BIL = \mu_k(mg)$, or

$$B = \frac{\mu_k(mg)}{IL} = \frac{(0.100)(0.200\ \text{kg})\left(9.80\ \text{m/s}^2\right)}{(10.0\ \text{A})(0.500\ \text{m})} = \boxed{3.92\times10^{-2}\ \text{T}}$$

19.69 (a) Since the magnetic field is directed from N to S (that is, from left to right within the artery), positive ions with velocity in the direction of the blood flow experience a magnetic deflection toward electrode A. Negative ions will experience a force deflecting them toward electrode B. This separation of charges creates an electric field directed from A toward B. At equilibrium, the electric force caused by this field must balance the magnetic force, so

$$qvB = qE = q(\Delta V/d)$$

continued on next page

or $\quad v = \dfrac{\Delta V}{Bd} = \dfrac{160\times 10^{-6}\ \text{V}}{(0.040\,0\ \text{T})(3.00\times 10^{-3}\ \text{m})} = \boxed{1.33\ \text{m/s}}$

(b) The magnetic field is directed from N to S. If the charge carriers are negative moving in the direction of $\vec{\mathbf{v}}$, the magnetic force is directed toward point B. Negative charges build up at point B, making the potential at A higher than that at B. If the charge carriers are positive moving in the direction of $\vec{\mathbf{v}}$, the magnetic force is directed toward A, so positive charges build up at A. This also makes the potential at A higher than that at B. Therefore the sign of the potential difference $\boxed{\text{does not depend on the charge of the ions}}$.

19.70 (a) The magnetic force acting on the wire is directed upward and of magnitude $F_m = BIL\sin 90° = BI$.

Thus, $\quad a_y = \dfrac{\Sigma F_y}{m} = \dfrac{F_m - mg}{m} = \dfrac{BIL - mg}{m} = \dfrac{BI}{(m/L)} - g$, or

$$a_y = \frac{(4.0\times 10^{-3}\ \text{T})(2.0\ \text{A})}{5.0\times 10^{-4}\ \text{kg/m}} - 9.80\ \text{m/s}^2 = \boxed{6.2\ \text{m/s}^2}$$

(b) Using $\quad \Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, with $v_{0y} = 0$, gives

$$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(0.50\ \text{m})}{6.2\ \text{m/s}^2}} = \boxed{0.40\ \text{s}}$$

19.71 Label the wires 1, 2, and 3, as shown in Figure 1. Also, let B_1, B_2, and B_3, respectively, represent the magnitudes of the fields produced by the currents in those wires, and observe that $\theta = 45°$.

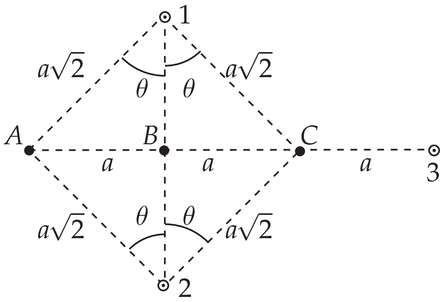


FIGURE 1

At point A, $B_1 = B_2 = \mu_0 I/2\pi\left(a\sqrt{2}\right)$, or

$$B_1 = B_2 = \frac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(2.0\ \text{A})}{2\pi(0.010\ \text{m})\sqrt{2}} = 28\ \mu\text{T}$$

and $\quad B_3 = \dfrac{\mu_0 I}{2\pi(3a)} = \dfrac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(2.0\ \text{A})}{2\pi(0.030\ \text{m})} = 13\ \mu\text{T}$

These field contributions are oriented as shown in Figure 2. Observe that the horizontal components of $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$ cancel while their vertical components add to $\vec{\mathbf{B}}_3$. The resultant field at point A is then

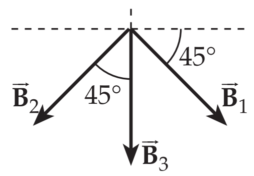


FIGURE 2

$$B_A = (B_1 + B_2)\cos 45° + B_3 = 53\ \mu\text{T, or}$$

$$\vec{\mathbf{B}}_A = \boxed{53\ \mu\text{T directed toward the bottom of the page}}$$

At point B, $\; B_1 = B_2 = \dfrac{\mu_0 I}{2\pi a} = \dfrac{(4\pi\times 10^{-7}\ \text{T}\cdot\text{m/A})(2.0\ \text{A})}{2\pi(0.010\ \text{m})} = 40\ \mu\text{T}$

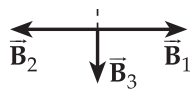


FIGURE 3

and $B_3 = \dfrac{\mu_0 I}{2\pi(2a)} = 20\ \mu\text{T}$. These contributions are oriented as shown in Figure 3. Thus, the resultant field at B is

continued on next page

$\vec{\mathbf{B}}_B = \vec{\mathbf{B}}_3 = \boxed{20\ \mu\text{T directed toward the bottom of the page}}$

At point C, $B_1 = B_2 = \mu_0 I/2\pi\left(a\sqrt{2}\right)$, while $B_3 = \mu_0 I/2\pi a$. These contributions are oriented as shown in Figure 4. Observe that the horizontal components of $\vec{\mathbf{B}}_1$ and $\vec{\mathbf{B}}_2$ cancel, while their vertical components add to oppose $\vec{\mathbf{B}}_3$. The magnitude of the resultant field at C is

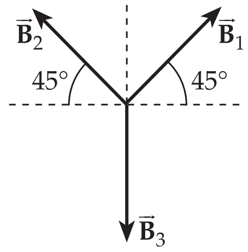
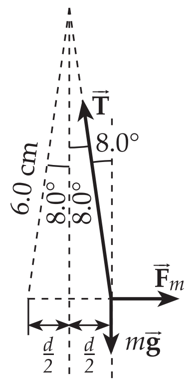


FIGURE 4

$$B_C = \left(B_1 + B_2\right)\sin 45° - B_3 = \frac{\mu_0 I}{2\pi a}\left(\frac{2\sin 45°}{\sqrt{2}} - 1\right) = \boxed{0}$$

19.72 (a) Since one wire repels the other, the currents must be in $\boxed{\text{opposite directions}}$.

(b) Consider a free-body diagram of one of the wires as shown at the right.



$$\Sigma F_y = 0 \;\Rightarrow\; T\cos 8.0° = mg$$

or $T = \dfrac{mg}{\cos 8.0°}$

$$\Sigma F_x = 0 \;\Rightarrow\; F_m = T\sin 8.0° = \left(\frac{mg}{\cos 8.0°}\right)\sin 8.0°$$

or $F_m = (mg)\tan 8.0°$. Thus, $\dfrac{\mu_0 I^2 L}{2\pi d} = (mg)\tan 8.0°$, which gives

$$I = \sqrt{\frac{d\left[(m/L)g\right]\tan 8.0°}{\mu_0/2\pi}}$$

Observe that the distance between the two wires is

$$d = 2\left[(6.0\text{ cm})\sin 8.0°\right] = 1.7\text{ cm, so}$$

$$I = \sqrt{\frac{\left(1.7\times10^{-2}\text{ m}\right)\left(0.040\text{ kg/m}\right)\left(9.80\text{ m/s}^2\right)\tan 8.0°}{2.0\times10^{-7}\text{ T}\cdot\text{m/A}}} = \boxed{68\text{ A}}$$

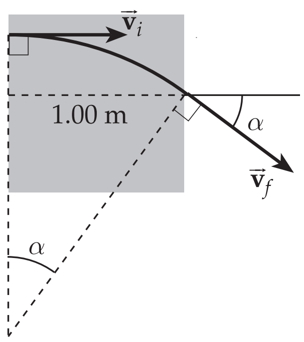
19.73 **Note:** We solve part (b) before part (a) for this problem.

(b) Since the magnetic force supplies the centripetal acceleration for this particle, $qvB = mv^2/r$, or the radius of the path is $r = mv/qB = p/qB$, where

$$p = mv = \sqrt{2m(KE)} = \sqrt{2\left(1.67\times10^{-27}\text{ kg}\right)\left(5.00\times10^{6}\text{ eV}\right)\left(1.60\times10^{-19}\text{ J/eV}\right)}$$

$$= 5.17\times10^{-20}\text{ kg}\cdot\text{m/s}$$

Consider the circular path shown at the right and observe that the desired angle is



$$\alpha = \sin^{-1}\left(\frac{1.00\text{ m}}{r}\right) = \sin^{-1}\left[\frac{(1.00\text{ m})qB}{p}\right]$$

or

$$\alpha = \sin^{-1}\left[\frac{(1.00\text{ m})\left(1.60\times10^{-19}\text{ C}\right)(0.050\,0\text{ T})}{5.17\times10^{-20}\text{ kg}\cdot\text{m/s}}\right] = \boxed{8.90°}$$

continued on next page

(a) The linear momentum of the particle has constant magnitude $p = mv$, and its vertical component as the particle leaves the field is $p_y = -p\sin\alpha$, or

$$p_y = -\left(5.17\times10^{-20}\ \text{kg}\cdot\text{m/s}\right)\sin(8.90°) = \boxed{-8.00\times10^{-21}\ \text{kg}\cdot\text{m/s}}$$

19.74 The force constant of the spring system is found from the elongation produced by the weight acting alone.

$$k = \frac{F}{x_1} = \frac{mg}{x_1} \qquad \text{where} \qquad x_1 = 0.50\ \text{cm}$$

The total force stretching the springs when the field is turned on is

$$\Sigma F_y = F_m + mg = kx_{\text{total}} \qquad \text{where} \qquad x_{\text{total}} = x_1 + 0.30\ \text{cm} = 0.80\ \text{cm}$$

Thus, the downward magnetic force acting on the wire is

$$F_m = kx_{\text{total}} - mg = \left(\frac{mg}{x_1}\right)x_{\text{total}} - mg = \left(\frac{x_{\text{total}}}{x_1} - 1\right)mg$$

$$= \left(\frac{0.80\ \text{cm}}{0.50\ \text{cm}} - 1\right)\left(10.0\times10^{-3}\ \text{kg}\right)\left(9.80\ \text{m/s}^2\right) = 5.9\times10^{-2}\ \text{N}$$

Since the magnetic force is given by $F_m = BIL\sin 90°$, the magnetic field is

$$B = \frac{F_m}{IL} = \frac{F_m}{(\Delta V/R)L} = \frac{(12\ \Omega)\left(5.9\times10^{-2}\ \text{N}\right)}{(24\ \text{V})\left(5.0\times10^{-2}\ \text{m}\right)} = \boxed{0.59\ \text{T}}$$

19.75 The magnetic force is very small in comparison to the weight of the ball, so we treat the motion as that of a freely falling body. Then, as the ball approaches the ground, it has velocity components with magnitudes of

$$v_x = v_{0x} = 20.0\ \text{m/s, and}$$

$$v_y = \sqrt{v_{0y}^2 + 2a_y(\Delta y)} = \sqrt{0 + 2\left(-9.80\ \text{m/s}^2\right)(-20.0\ \text{m})} = 19.8\ \text{m/s}$$

The velocity of the ball is perpendicular to the magnetic field and, just before it reaches the ground, has magnitude $v = \sqrt{v_x^2 + v_y^2} = 28.1\ \text{m/s}$. Thus, the magnitude of the magnetic force is

$$F_m = qvB\sin\theta$$

$$= \left(5.00\times10^{-6}\ \text{C}\right)(28.1\ \text{m/s})(0.010\,0\ \text{T})\sin 90.0° = \boxed{1.41\times10^{-6}\ \text{N}}$$

19.76 (a) $$B_1 = \frac{\mu_0 I_1}{2\pi d} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(5.00\ \text{A})}{2\pi(0.100\ \text{m})} = \boxed{1.00\times10^{-5}\ \text{T}}$$

(b) $$\frac{F_{21}}{\ell} = B_1 I_2 = \left(1.00\times10^{-5}\ \text{T}\right)(8.00\ \text{A}) = \boxed{8.00\times10^{-5}\ \text{N directed toward wire 1}}$$

(c) $$B_2 = \frac{\mu_0 I_2}{2\pi d} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(8.00\ \text{A})}{2\pi(0.100\ \text{m})} = \boxed{1.60\times10^{-5}\ \text{T}}$$

(d) $$\frac{F_{12}}{\ell} = B_2 I_1 = \left(1.60\times10^{-5}\ \text{T}\right)(5.00\ \text{A}) = \boxed{8.00\times10^{-5}\ \text{N directed toward wire 2}}$$

20

Induced Voltages and Inductance

QUICK QUIZZES

1. *b*, *c*, *a*. At each instant, the magnitude of the induced emf is proportional to the magnitude of the rate of change of the magnetic field (hence, proportional to the absolute value of the slope of the curve shown on the graph).

2. Choice (c). Taking downward as the normal direction, the north pole approaching from above produces an increasing positive flux through the area enclosed by the loop. To oppose this, the induced current must generate a negative flux through the interior of the loop, or the induced magnetic field must point upward through the enclosed area of the loop. Imagine gripping the wire of the loop with your right hand so the fingers curl upward through the area enclosed by the loop. You will find that your thumb, indicating the direction of the induced current, is directed counterclockwise around the loop as viewed from above.

3. Choice (b). If the positive z-direction is chosen as the normal direction, the increasing counterclockwise current in the left-hand loop produces an increasing positive flux through the area enclosed by this loop. The magnetic field lines due to this current curl around and pass through the area of the xy-plane outside the left-hand loop in the negative direction. Thus, the right-hand loop has an increasing negative flux through it. To counteract this effect, the induced current must produce positive flux, or generate a magnetic field in the positive z-direction, through the area enclosed by the right-hand loop. Imagine gripping the wire of the right-hand loop with the right hand so the fingers point in the positive z-direction as they penetrate the area enclosed by this loop. You should find that the thumb is directed counterclockwise around the loop as viewed from above the xy-plane.

4. Choice (a). All charged particles within the metal bar move straight downward with the bar. According to right-hand rule number 1, positive changes moving downward through a magnetic field that is directed northward will experience magnetic forces toward the east. This means that the free electrons (negative charges) within the metal will experience westward forces and will drift toward the west end of the bar, leaving the east end with a net positive charge.

5. Choice (b). According to Equation 20.3, when B and v are constant, the emf depends only on the length of the wire cutting across the magnetic field lines. Thus, you want the long dimension of the rectangular loop perpendicular to the velocity vector. This means that the short dimension is parallel to the velocity vector, and (b) is the correct choice. From a more conceptual point of view, you want the rate of change of area in the magnetic field to be the largest, which you do by thrusting the long dimension into the field.

6. Choice (b). When the iron rod is inserted into the solenoid, the inductance of the coil increases. As a result, more potential difference appears across the coil than before. Consequently, less potential difference appears across the bulb, and its brightness decreases.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The flux through a flat coil in a uniform magnetic field is a maximum when the plane of the coil is perpendicular to the direction of the field. Thus, with the field parallel to the y-axis, the flux is a maximum when the plane of the coil is parallel to the xz-plane, and (c) is the correct choice.

2. Choose the positive z-direction to be the reference direction ($\theta = 0°$) for the normal to the plane of the coil. Then, the change in flux through the coil is

$$\Delta\Phi_B = \left(B_f \cos\theta_f - B_i \cos\theta_i\right)A = \left[(3.0\text{ T})\cos 0° - (1.0\text{ T})\cos 180°\right](0.50\text{ m})^2 = 1.0\text{ Wb}$$

and the magnitude of the induced emf is

$$|\varepsilon| = N\frac{|\Delta\Phi_B|}{\Delta t} = (10)\left(\frac{1.0\text{ Wb}}{2.0\text{ s}}\right) = 5.0\text{ V}$$

The correct answer is choice (b).

3. The magnitude of the voltage drop across the inductor is

$$|\varepsilon_L| = L\left|\frac{\Delta I}{\Delta t}\right| = (5.00\text{ H})(2.00\text{ A/s}) = 10.0\text{ V}$$

and (d) is the correct choice.

4. The angular velocity of the rotating coil is $\omega = 10.0\text{ rev/s} = 20\pi\text{ rad/s}$ and the maximum emf induced in the coil is

$$\varepsilon_{\max} = NBA\omega = (100)(0.050\,0\text{ T})\left(0.100\text{ m}^2\right)(20\pi\text{ rad/s}) = 31.4\text{ V}$$

showing the correct choice to be (a).

5. The motional emf induced in a conductor of length ℓ moving at speed v through a magnetic field of magnitude B is $\Delta V = B_\perp \ell v = (B\sin\theta)\ell v$, where $B_\perp = B\sin\theta$ is the component of the field perpendicular to the velocity of the conductor. In the described situation,

$$\begin{aligned}\Delta V &= B_\perp \ell v = (B\sin\theta)\ell v = \left[\left(60.0\times10^{-6}\text{ T}\right)\sin 60.0°\right](12\text{ m})(60.0\text{ m/s})\\ &= 3.7\times10^{-2}\text{ V} = 37\text{ mV}\end{aligned}$$

and the correct choice is (c).

6. As the bar slides to the right along the rails, the magnetic flux through the conducting path formed by the bar, rails, and the resistive element at the left end is directed out of the page and increasing in magnitude. Thus, the induced current must generate a flux directed into the page through the area enclosed by the current path. This means the induced current must be in the clockwise direction and choice (b) is correct. Also, as the induced current flows, the rod will experience a magnetic force that tends to impede the motion of the rod. Therefore, an external force must be exerted on the bar to keep it moving at constant speed, and choice (d) is also correct.

7. The amplitude of the induced emf in the coil of a generator is directly proportional to the angular velocity of the coil $\left(\varepsilon_{\max} = NBA\omega\right)$. Therefore, when the rate of rotation is doubled, the amplitude of the induced emf is also doubled, and the correct choice is (b).

8. An emf is induced in the coil by any action which causes a change in the flux through the coil. The actions described in choices (a), (b), (d), and (e) all change the flux through the coil and induce an emf. However, moving the coil through the constant field without changing its orientation with respect to the field will not cause a change of flux. Thus, choice (c) is the correct answer.

9. With the current in the long wire flowing in the direction shown in Figure MCQ20.9, the magnetic flux through the rectangular loop is directed into the page. If this current is decreasing in time, the *change* in the flux is directed opposite to the flux itself (or out of the page). The induced current will then flow clockwise around the loop, producing a flux directed into the page through the loop and opposing the change in flux due to the decreasing current in the long wire. The correct choice for this question is (b).

10. A current flowing counterclockwise in the outer loop of Figure MCQ20.10 produces a magnetic flux through the inner loop that is directed out of the page. If this current is increasing in time, the *change* in the flux is in the same direction as the flux itself (or out of the page). The induced current in the inner loop will then flow clockwise around the loop, producing a flux through the loop directed into the page, opposing the change in flux due to the increasing current in the outer loop. The correct answer is choice (b).

11. <u>As the bar magnet approaches the loop from above</u>, with its south end downward as shown in Figure MCQ20.11, magnetic flux through the area enclosed by the loop is directed upward and increasing in magnitude. To oppose this increasing upward flux, the induced current in the loop will flow clockwise, as seen from above, producing a flux directed downward through the area enclosed by the loop. <u>After the bar magnet has passed through the plane of the loop</u> and is departing with its north end upward, a decreasing flux is directed upward through the loop. To oppose this decreasing upward flux, the induced current in the loop flows counterclockwise as seen from above, producing flux directed upward through the area enclosed by the loop. From this analysis, we see that (a) is the only true statement among the listed choices.

12. With the magnetic field perpendicular to the plane of the page in Figure MCQ20.12, the flux through the closed loop to the left of the bar is given by $\Phi_B = BA$, where B is the magnitude of the field and A is the area enclosed by the loop. Any action which produces a change in this product, BA, will induce a current in the loop and cause the bulb to light. Such actions include increasing or decreasing the magnitude of the field (B) and moving the bar to the right or left and changing the enclosed area A. Thus, the bulb will light during all of the actions in choices (a), (b), (c), and (d).

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. Consider the copper tube to be a large set of rings stacked one on top of the other. As the magnet falls toward or falls away from each ring, a current is induced in the ring. Thus, there is a current in the copper tube around its circumference.

4. (a) The flux is calculated as $\Phi_B = BA\cos\theta = B_\perp A$. The flux is therefore maximum when the magnetic field vector is perpendicular to the plane of the loop.

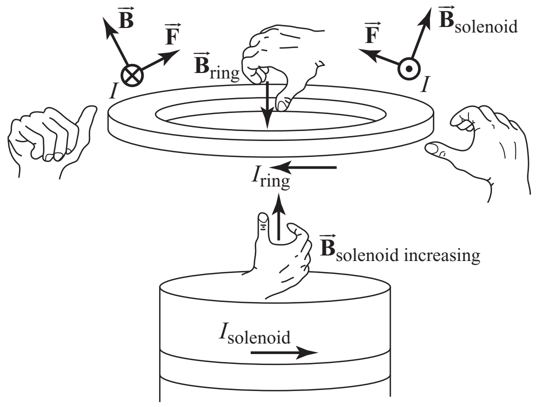
(b) The flux is zero when the magnetic field is parallel to the plane of the loop (and hence, $\theta = 90°$, making $B_\perp = B\cos\theta = 0$).

6. As water falls, it gains velocity and kinetic energy. It then pushes against the blades of a turbine, transferring this energy to the rotor or coil of a large generator. The rotor moves in a strong external magnetic field and a voltage is induced in the coil. This induced emf is the voltage source for the current in our electric power lines.

8. No. Once the bar is in motion and the charges are separated, no external force is necessary to maintain the motion. During the initial acceleration of the bar, an external applied force will be necessary to overcome both the inertia of the bar and a retarding magnetic force exerted on the bar.

10. Let us assume the north pole of the magnet faces the ring. As the bar magnet falls toward the conducting ring, a magnetic field is induced in the ring pointing upward. This upward directed field will oppose the motion of the magnet, preventing it from moving as a freely falling body. Try it for yourself to show that an upward force also acts on the falling magnet if the south end faces the ring.

12. The increasing counterclockwise current in the solenoid coil produces an upward magnetic field that increases rapidly. The increasing upward flux of this field through the ring induces an emf to produce a clockwise current in the ring. At each point on the ring, the field of the solenoid has a radially outward component as well as an upward component. This radial field component exerts an upward force on the current at each point in the ring. The resultant magnetic force on the ring is upward and exceeds the weight of the ring. Thus, the ring accelerates upward off of the solenoid.



14. As the magnet moves at high speed past the fixed coil, the magnetic flux through the coil changes very rapidly, increasing as the magnet approaches the coil and decreasing as the magnet moves away. The rapid change in flux through the coil induces a large emf, large enough to cause a spark across the gap in the spark plug.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** $1.00 \times 10^{-7}\ \text{T} \cdot \text{m}^2$ **(b)** $8.66 \times 10^{-8}\ \text{T} \cdot \text{m}^2$ **(c)** 0

4. zero

6. **(a)** 6.98 mT **(b)** $1.97 \times 10^{-5}\ \text{T} \cdot \text{m}^2$

8. 0.10 mV

10. 34 mV

12. 2.26 mV

14. into the page

16. **(a)** left to right **(b)** no induced current **(c)** right to left

18. **(a)** $1.88 \times 10^{-7}\ \text{T} \cdot \text{m}^2$ **(b)** $6.27 \times 10^{-8}\ \text{V}$

20. 8.8 A

22. **(a)** $|\varepsilon| = NB_0\pi r^2/t$ **(b)** clockwise **(c)** $I = B_0\pi r^2/tR$

24. 1.20 mV, west end is positive

26. **(a)** counterclockwise when viewed from the right end

(b) clockwise when viewed from the right end

(c) no induced current

28. **(a)** $6.0\ \mu\text{T}$

(b) The magnitude and direction of the Earth's field varies from one location to another, so the induced voltage in the wire will change.

30. $1.00\ \text{m/s}$

32. 13 mV

34. **(a)** 7.5 kV

(b) when the plane of the coil is parallel to the magnetic field

36. **(a)** 8.0 A **(b)** 3.2 A **(c)** 60 V

38. **(a)** 5.9 mH **(b)** $-24\ \text{V}$

40. See Solution.

42. $1.92\times10^{-5}\ \text{T}\cdot\text{m}^2$

44. **(a)** $1.00\ \text{k}\Omega$ **(b)** 3.00 ms

46. **(a)** 0 **(b)** 3.8 V

(c) 6.0 V **(d)** 2.2 V

48. **(a)** 2.00 ms **(b)** 0.176 A

(c) 1.50 A **(d)** 3.22 ms

50. **(a)** 4.44 mH **(b)** 0.555 mJ

52. **(a)** $1.3\ \Omega$ **(b)** 4.8×10^2 turns **(c)** 0.48 m

(d) 0.76 mH **(e)** 0.46 ms **(f)** 3.6 A

(g) 3.2 ms **(h)** 4.9 mJ

54. **(a)** increasing **(b)** $62.2\ \text{mT/s}$

56. **(a)** $0.73\ \text{m/s}$, counterclockwise **(b)** 0.65 mW

(c) Work is being done on the bar by an external force to maintain constant speed.

58. **(a)** 2.1×10^6 m/s **(b)** from side to side **(c)** 1.7×10^{10} V

(d) The very large induced emf would lead to powerful spontaneous electrical discharges. The strong electric and magnetic fields would disrupt the flow of ions in their bodies.

60. 0.158 mV

62. 0.120 A, clockwise

64. 1.60 A

66. **(a)** $\varepsilon = NB\ell v$ **(b)** $I = NB\ell v/R$ **(c)** $P = N^2B^2\ell^2v^2/R$

(d) $F = N^2B^2\ell^2v/R$ **(e)** clockwise **(f)** toward the left

PROBLEM SOLUTIONS

20.1 The angle between the direction of the constant field and the normal to the plane of the loop is $\theta = 0°$, so

$$\Phi_B = BA\cos\theta = (0.50\ \text{T})\left[(8.0\times10^{-2}\ \text{m})(12\times10^{-2}\ \text{m})\right]\cos 0° = \boxed{4.8\times10^{-3}\ \text{T}\cdot\text{m}^2}$$

20.2 The magnetic flux through the loop is given by $\Phi_B = BA\cos\theta$, where B is the magnitude of the magnetic field, A is the area enclosed by the loop, and θ is the angle the magnetic field makes with the normal to the plane of the loop. Thus,

$$\Phi_B = BA\cos\theta = (5.00\times10^{-5}\ \text{T})\left[20.0\ \text{cm}^2\left(\frac{10^{-2}\ \text{m}}{1\ \text{cm}}\right)^2\right]\cos\theta = (1.00\times10^{-7}\ \text{T}\cdot\text{m}^2)\cos\theta$$

(a) When $\vec{\mathbf{B}}$ is perpendicular to the plane of the loop, $\theta = 0°$ and $\Phi_B = \boxed{1.00\times10^{-7}\ \text{T}\cdot\text{m}^2}$.

(b) If $\theta = 30.0°$, then $\Phi_B = (1.00\times10^{-7}\ \text{T}\cdot\text{m}^2)\cos 30.0° = \boxed{8.66\times10^{-8}\ \text{T}\cdot\text{m}^2}$.

(c) If $\theta = 90.0°$, then $\Phi_B = (1.00\times10^{-7}\ \text{T}\cdot\text{m}^2)\cos 90.0° = \boxed{0}$.

20.3 $\Phi_B = BA\cos\theta = B(\pi r^2)\cos\theta$, where θ is the angle between the direction of the field and the normal to the plane of the loop.

(a) If the field is perpendicular to the plane of the loop, $\theta = 0°$, and

$$B = \frac{\Phi_B}{(\pi r^2)\cos\theta} = \frac{8.00\times10^{-3}\ \text{T}\cdot\text{m}^2}{\pi(0.12\ \text{m})^2\cos 0°} = \boxed{0.177\ \text{T}}$$

(b) If the field is directed parallel to the plane of the loop, $\theta = 90°$, and

$$\Phi_B = BA\cos\theta = BA\cos 90° = \boxed{0}$$

20.4 The magnetic field lines are everywhere parallel to the surface of the cylinder, so no magnetic field lines penetrate the cylindrical surface. The total flux through the cylinder is $\boxed{\text{zero}}$.

20.5 (a) Every field line that comes up through the area A on one side of the wire goes back down through area A on the other side of the wire. Thus, the net flux through the coil is $\boxed{\text{zero}}$.

(b) The magnetic field is parallel to the plane of the coil, so $\theta = 90.0°$. Therefore, $\Phi_B = BA\cos\theta = BA\cos 90.0° = \boxed{0}$.

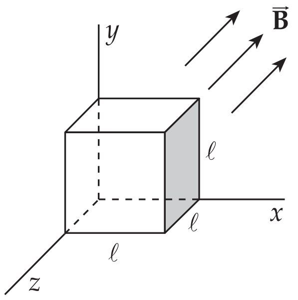
20.6 (a) The magnitude of the field inside the solenoid is

$$B = \mu_0 nI = \mu_0\left(\frac{N}{\ell}\right)I = \left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(\frac{400}{0.360\ \text{m}}\right)(5.00\ \text{A})$$
$$= 6.98\times10^{-3}\ \text{T} = \boxed{6.98\ \text{mT}}$$

(b) The field inside a solenoid is directed perpendicular to the cross-sectional area, so $\theta = 0°$ and the flux through a loop of the solenoid is

$$\Phi_B = BA\cos\theta = B\left(\pi r^2\right)\cos\theta$$
$$= \left(6.98\times10^{-3}\ \text{T}\right)\pi\left(3.00\times10^{-2}\ \text{m}\right)^2\cos 0° = \boxed{1.97\times10^{-5}\ \text{T}\cdot\text{m}^2}$$

20.7 (a) The magnetic flux through an area A may be written as



$$\Phi_B = (B\cos\theta)A$$
$$= (\text{component of } B \text{ perpendicular to } A)\cdot A$$

Thus, the flux through the shaded side of the cube is

$$\Phi_B = B_x\cdot A = (5.0\ \text{T})\cdot\left(2.5\times10^{-2}\ \text{m}\right)^2 = \boxed{3.1\times10^{-3}\ \text{T}\cdot\text{m}^2}$$

(b) Unlike electric field lines, magnetic field lines always form closed loops, without beginning or end. Therefore, no magnetic field lines originate or terminate within the cube and any line entering the cube at one point must emerge from the cube at some other point. The net flux through the cube, and indeed through any *closed* surface, is $\boxed{\text{zero}}$.

20.8 $$|\varepsilon| = \frac{|\Delta\Phi_B|}{\Delta t} = \frac{(\Delta B)A\cos\theta}{\Delta t} = \frac{(1.5\ \text{T}-0)\left[\pi\left(1.6\times10^{-3}\ \text{m}\right)^2\right]\cos 0°}{120\times10^{-3}\ \text{s}} = 1.0\times10^{-4}\ \text{V} = \boxed{0.10\ \text{mV}}$$

20.9 (a) As loop A moves parallel to the long straight wire, the magnetic flux through loop A does not change. Hence, there is $\boxed{\text{no induced current}}$ in this loop.

(b) As loop B moves to the left away from the straight wire, the magnetic flux through this loop is directed out of the page, and is decreasing in magnitude. To oppose this change in flux, the induced current flows $\boxed{\text{counterclockwise around loop B}}$ producing a magnetic flux directed out of the page through the area enclosed by loop B.

(c) As loop C moves to the right away from the straight wire, the magnetic flux through this loop is directed into the page and is decreasing in magnitude. In order to oppose this change in flux, the induced current flows $\boxed{\text{clockwise around loop C}}$ producing a magnetic flux directed into the page through the area enclosed by loop C.

20.10 $$|\varepsilon| = \frac{|\Delta\Phi_B|}{\Delta t} = \frac{B|\Delta A|\cos\theta}{\Delta t} = \frac{(0.15\ \text{T})\left[\pi(0.12\ \text{m})^2 - 0\right]\cos 0°}{0.20\ \text{s}} = 3.4\times10^{-2}\ \text{V} = \boxed{34\ \text{mV}}$$

20.11 The magnitude of the induced emf is

$$|\varepsilon| = \frac{|\Delta\Phi_B|}{\Delta t} = \frac{|\Delta(B\cos\theta)|A}{\Delta t}$$

If the normal to the plane of the loop is considered to point in the original direction of the magnetic field, then $\theta_i = 0°$ and $\theta_f = 180°$. Thus, we find

$$|\varepsilon| = \frac{|(0.20\text{ T})\cos 180° - (0.30\text{ T})\cos 0°|\,\pi(0.30\text{ m})^2}{1.5\text{ s}} = 9.4\times10^{-2}\text{ V} = \boxed{94\text{ mV}}$$

20.12 With the field directed perpendicular to the plane of the coil, the flux through the coil is $\Phi_B = BA\cos 0° = BA$. As the magnitude of the field decreases, the magnitude of the induced emf in the coil is

$$|\varepsilon| = \frac{|\Delta\Phi_B|}{\Delta t} = \left|\frac{\Delta B}{\Delta t}\right| A = (0.050\,0\text{ T/s})\left[\pi(0.120\text{ m})^2\right] = 2.26\times10^{-3}\text{ V} = \boxed{2.26\text{ mV}}$$

20.13 The magnetic field changes from zero to a magnitude of 2.5 T, directed at 45° to the plane of the band, in a time of $\Delta t = 0.18$ s. If the band has a diameter of 6.5 cm, the magnitude of the average induced emf in the metal band is

$$|\varepsilon| = \frac{\Delta\Phi_B}{\Delta t} = \frac{|\Delta B|}{\Delta t}A\cos\theta = \frac{(2.5\text{ T}-0)}{0.18\text{ s}}\pi\frac{\left(6.5\times10^{-2}\text{ m}\right)^2}{4}\cos 45° = 3.3\times10^{-2}\text{ V} = \boxed{33\text{ mV}}$$

20.14 When the switch is closed, the magnetic field due to the current from the battery will be directed toward the left along the axis of the cylinder. To oppose this increasing leftward flux, the induced current in the other loop must produce a field directed to the right through the area it encloses. Thus, the induced current is directed $\boxed{\text{out of the page}}$ through the resistor.

20.15 (a) When the magnet moves to the left, the flux through the interior of the coil is directed toward the right and is decreasing in magnitude. To oppose this change in flux, the magnetic field generated by the induced current should be directed to the right along the axis of the coil. The current must then be $\boxed{\text{left to right}}$ through the resistor.

(b) When the magnet moves to the right, the flux through the interior of the coil is directed toward the right and is increasing in magnitude. To oppose this increasing flux, the magnetic field generated by the induced current should be directed toward the left along the axis of the coil. The current must then be $\boxed{\text{right to left}}$ through the resistor.

20.16 When the switch is closed, the current from the battery produces a magnetic field directed toward the right along the axis of both coils.

(a) As the battery current is growing in magnitude, the induced current in the rightmost coil opposes the increasing rightward directed field by generating a field toward to the left along the axis. Thus, the induced current must be $\boxed{\text{left to right}}$ through the resistor.

(b) Once the battery current, and the field it produces, have stabilized, the flux through the rightmost coil is constant, and there is $\boxed{\text{no induced current}}$.

(c) As the switch is opened, the battery current and the field it produces rapidly decrease in magnitude. To oppose this decrease in the rightward directed field, the induced current must produce a field toward the right along the axis, so the induced current is $\boxed{\text{right to left}}$ through the resistor.

20.17 (a) The current is $\boxed{\text{zero}}$. The magnetic flux the current produces through the right side of the loop is directed into the page, and is equal in magnitude to the outward-directed flux the current produces through the left side of the loop. Thus, the net flux through the loop has a constant value of zero and does not induce a current.

(b) The flux through the loop due to the long wire is directed out of the page and is increasing in magnitude. To oppose this increasing outward flux, the induced current must generate a magnetic field that is directed into the page through the area enclosed by the loop. Thus, the induced current in the loop must be $\boxed{\text{clockwise}}$.

20.18 The initial magnetic field inside the solenoid is

$$B = \mu_0 nI = \mu_0\left(\frac{N}{\ell}\right)I = \left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(\frac{100}{0.200\ \text{m}}\right)(3.00\ \text{A}) = 1.88\times10^{-3}\ \text{T}$$

(a) $\Phi_B = BA_{\text{loop}}\cos\theta = \left(1.88\times10^{-3}\ \text{T}\right)\left(1.00\times10^{-2}\ \text{m}\right)^2\cos0° = \boxed{1.88\times10^{-7}\ \text{T}\cdot\text{m}^2}$

(b) When the current is zero, the flux through the loop is $\Phi_B = 0$, and the average induced emf has been

$$|\varepsilon| = \frac{|\Delta\Phi_B|}{\Delta t} = \frac{\left|0-1.88\times10^{-7}\ \text{T}\cdot\text{m}^2\right|}{3.00\ \text{s}} = \boxed{6.27\times10^{-8}\ \text{V}}$$

20.19 (a) The initial field inside the solenoid is

$$B_i = \mu_0 nI_i = \left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(\frac{300}{0.200\ \text{m}}\right)(2.00\ \text{A}) = \boxed{3.77\times10^{-3}\ \text{T}}$$

(b) The final field inside the solenoid is

$$B_f = \mu_0 nI_f = \left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(\frac{300}{0.200\ \text{m}}\right)(5.00\ \text{A}) = \boxed{9.42\times10^{-3}\ \text{T}}$$

(c) The 4-turn coil encloses an area $A = \pi r^2 = \pi\left(1.50\times10^{-2}\ \text{m}\right)^2 = \boxed{7.07\times10^{-4}\ \text{m}^2}$

(d) The change in flux through each turn of the 4-turn coil during the 0.900-s period is

$$\Delta\Phi_B = (\Delta B)A = \left(9.42\times10^{-3}\ \text{T} - 3.77\times10^{-3}\ \text{T}\right)\left(7.07\times10^{-4}\ \text{m}^2\right) = \boxed{3.99\times10^{-6}\ \text{Wb}}$$

(e) The average induced emf in the 4-turn coil is

$$\varepsilon = N_2\left(\frac{\Delta\Phi_B}{\Delta t}\right) = 4\left(\frac{3.99\times10^{-6}\ \text{Wb}}{0.900\ \text{s}}\right) = \boxed{1.77\times10^{-5}\ \text{V}}$$

Since the current increases at a constant rate during this time interval, the induced emf at any instant during the interval is the same as the average value given above.

(f) The induced emf is small, so the current in the 4-turn coil will also be very small. This means that the magnetic field generated by this current will be negligibly small in comparison to the field generated by the solenoid.

20.20 The magnitude of the average emf is

$$|\varepsilon| = \frac{N|\Delta\Phi_B|}{\Delta t} = \frac{NBA|\Delta(\cos\theta)|}{\Delta t}$$

$$= \frac{200(1.1\text{ T})(100\times10^{-4}\text{ m}^2)|\cos 180° - \cos 0°|}{0.10\text{ s}} = 44\text{ V}$$

Therefore, the average induced current is $I = \frac{|\varepsilon|}{R} = \frac{44\text{ V}}{5.0\ \Omega} = \boxed{8.8\text{ A}}$.

20.21 If the magnetic field makes an angle of 28.0° with the plane of the coil, the angle it makes with the normal to the plane of the coil is $\theta = 62.0°$. Thus,

$$|\varepsilon| = \frac{N(\Delta\Phi_B)}{\Delta t} = \frac{NB(\Delta A)\cos\theta}{\Delta t}$$

$$= \frac{200(50.0\times10^{-6}\text{ T})\left[(39.0\text{ cm}^2)(1\text{ m}^2/10^4\text{ cm}^2)\right]\cos 62.0°}{1.80\text{ s}}$$

$$= 1.02\times10^{-5}\text{ V} = \boxed{10.2\ \mu\text{V}}$$

20.22 With the magnetic field perpendicular to the plane of the coil, the flux through each turn of the coil is $\Phi_B = BA = B(\pi r^2)$. Since the area remains constant, the change in flux due to the changing magnitude of the magnetic field is $\Delta\Phi_B = (\Delta B)\pi r^2$.

(a) The induced emf is $|\varepsilon| = N\left|\frac{\Delta\Phi}{\Delta t}\right| = N\left[\frac{|B_0 - 0|\pi r^2}{t-0}\right] = \boxed{\frac{NB_0\pi r^2}{t}}$.

(b) When looking down on the coil from a location on the positive z-axis, the magnetic field (in the positive z-direction) is directed up toward you and increasing in magnitude. This means the change in the flux through the coil is directed upward. In order to oppose this change in flux, the induced current must produce a magnetic field directed downward through the area enclosed by the coil. Thus, the current must flow $\boxed{\text{clockwise}}$ as seen from your viewing location.

(c) Since the turns of the coil are connected in series, the total resistance of the coil is $R_{eq} = NR$. Thus, the magnitude of the induced current is

$$I = \frac{|\varepsilon|}{R_{eq}} = \frac{NB_0\pi r^2/t}{NR} = \boxed{\frac{B_0\pi r^2}{tR}}$$

20.23 The motional emf induced in a metallic object of length ℓ moving through a magnetic field at speed v is given by $\varepsilon = B_\perp v\ell$, where $B_\perp$ is the component of the magnetic field perpendicular to the velocity of the object. Thus,

$$\varepsilon = (35.0\times10^{-6}\text{ T})(25.0\text{ m/s})(15.0\text{ m}) = 1.31\times10^{-2}\text{ V} = \boxed{13.1\text{ mV}}$$

20.24 The vertical component of the Earth's magnetic field is perpendicular to the horizontal velocity of the wire. Thus, the magnitude of the motional emf induced in the wire is

$$\varepsilon = B_\perp \ell v = (40.0\times10^{-6}\text{ T})(2.00\text{ m})(15.0\text{ m/s}) = 1.20\times10^{-3}\text{ V} = \boxed{1.20\text{ mV}}$$

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Imagine holding your right hand horizontal with the fingers pointing north (the direction of the wire's velocity), such that when you close your hand the fingers curl downward (in the direction of $B_\perp$). Your thumb will then be pointing westward. By right-hand rule number 1, the magnetic force on charges in the wire would tend to move positive charges westward. Thus, $\boxed{\text{the west end of the wire will be positive relative to the east end}}$.

20.25 The vertical component of the Earth's magnetic field is perpendicular to the horizontal velocity of the metallic truck body. Thus, the motional emf induced across the width of the truck is

$$\varepsilon = B_\perp \ell v = \left(35\times10^{-6}\ \text{T}\right)\left[(79.8\ \text{in})\left(\frac{1\ \text{m}}{39.37\ \text{in}}\right)\right](37\ \text{m/s}) = 2.6\times10^{-3}\ \text{V} = \boxed{2.6\ \text{mV}}$$

20.26 (a) As the loop passes position A, the flux through the area enclosed by the loop is directed right to left and is increasing in magnitude. The induced current must flow $\boxed{\text{counterclockwise as seen from the right end}}$ in order to generate flux passing left to right through the loop, opposing the increase in flux due to the magnet.

(b) When the loop reaches position B, the flux through the enclosed area is again directed right to left but is now decreasing in magnitude. The induced current must flow $\boxed{\text{clockwise as seen from the right end}}$ to generate additional flux passing right to left through the loop, opposing the decrease in flux due to the magnet.

(c) At position C, the flux through the loop is not changing so there is no induced emf, and hence $\boxed{\text{no induced current}}$, in the loop.

20.27 (a) Observe that only the horizontal component, B_h, of Earth's magnetic field is effective in exerting a vertical force on charged particles in the antenna. For the magnetic force, $F_m = qvB_h \sin\theta$, on positive charges in the antenna to be directed upward and have maximum magnitude (when $\theta = 90°$), the car should move $\boxed{\text{eastward}}$ through the northward horizontal component of the magnetic field.

(b) $\varepsilon = B_h \ell v$, where B_h is the horizontal component of the magnetic field.

$$\varepsilon = \left[\left(50.0\times10^{-6}\ \text{T}\right)\cos 65.0°\right](1.20\ \text{m})\left[\left(65.0\ \frac{\text{km}}{\text{h}}\right)\left(\frac{0.278\ \text{m/s}}{1\ \text{km/h}}\right)\right]$$

$$= \boxed{4.58\times10^{-4}\ \text{V}}$$

20.28 (a) Since $\varepsilon = B_\perp \ell v$, the magnitude of the vertical component of the Earth's magnetic field at this location is

$$B_{\text{vertical}} = B_\perp = \frac{\varepsilon}{\ell v} = \frac{0.45\ \text{V}}{(25\ \text{m})\left(3.0\times10^3\ \text{m/s}\right)} = 6.0\times10^{-6}\ \text{T} = \boxed{6.0\ \mu\text{T}}$$

(b) $\boxed{\text{Yes.}}$ The magnitude and direction of the Earth's field varies from one location to the other, so the induced voltage in the wire changes. Further, the voltage will change if the tether cord changes its orientation relative to the Earth's field.

20.29 The metal bar of length ℓ and moving at speed v through the magnetic field experiences an induced emf of magnitude $\varepsilon = B_\perp \ell v$, where $B_\perp = B\cos\theta$ is the component of the magnetic field perpendicular to the velocity of the bar as shown in figure (a) below.

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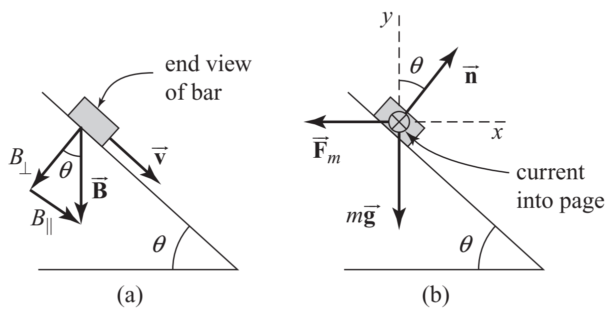

As the bar slides down the rails, the magnetic flux through the conducting path formed by the bar, the rails, and the resistor R is directed downward and is increasing in magnitude. Thus, the induced current must flow counterclockwise around the conducting path to generate an upward flux, opposing the increase in flux due to the field B. This current flows into the page as indicated in figure (b) and has magnitude $I = \varepsilon/R = B\ell v\cos\theta/R$.

According to right-hand rule number 1, the bar will experience a magnetic force $\vec{\mathbf{F}}_m$ directed horizontally toward the left as shown in figure (b). The magnitude of this force is

$$F_m = BI\ell = B\left(\frac{B\ell v\cos\theta}{R}\right)\ell = \frac{B^2\ell^2 v\cos\theta}{R}$$

Now, consider the free-body diagram of the bar in figure (b), where $\vec{\mathbf{n}}$ is the normal force exerted on the bar by the rails. If the bar is to move with constant velocity (i.e., be in equilibrium), it is necessary that

$$\Sigma F_y = 0 \quad \Rightarrow \quad n\cos\theta = mg \qquad \text{or} \qquad n = \frac{mg}{\cos\theta}$$

and $$\Sigma F_x = 0 \quad \Rightarrow \quad F_m = n\sin\theta \qquad \text{or} \qquad \frac{B^2\ell^2 v\cos\theta}{R} = \left(\frac{mg}{\cos\theta}\right)\sin\theta = mg\tan\theta$$

Thus, the equilibrium speed of the bar is

$$v = \frac{(mg\tan\theta)R}{B^2\ell^2\cos\theta} = \frac{(0.200\text{ kg})(9.80\text{ m/s}^2)\tan 25.0°(1.00\ \Omega)}{(0.500\text{ T})^2(1.20\text{ m})^2\cos 25.0°} = \boxed{2.80\text{ m/s}}$$

20.30 From $\varepsilon = B\ell v$, the required speed is

$$v = \frac{\varepsilon}{B\ell} = \frac{IR}{B\ell} = \frac{(0.500\text{ A})(6.00\ \Omega)}{(2.50\text{ T})(1.20\text{ m})} = \boxed{1.00\text{ m/s}}$$

20.31 (a) In the initial orientation of the coil, the magnitude of the flux passing through the loop is $|\Phi_B| = NBA$, where A is the area enclosed by the loop and N is the number of turns on the loop. After the loop has rotated 90°, the magnetic field is now parallel to the plane of the loop and the flux through the loop is zero. The average emf induced in the loop as it rotates is

$$\varepsilon = \frac{|\Delta\Phi_B|}{\Delta t} = \frac{NBA - 0}{\Delta t} = \frac{28(1.25\text{ T})(2.80\times10^{-2}\text{ m})^2}{0.335\text{ s}} = 8.19\times10^{-2}\text{ V} = \boxed{81.9\text{ mV}}$$

(b) The average induced current is $I = \dfrac{\varepsilon}{R} = \dfrac{81.9\text{ mV}}{0.780\ \Omega} = \boxed{105\text{ mA}}$.

20.32 Note that the vertical component of the magnetic field is always parallel to the plane of the coil, and can never contribute to the flux through the coil. The maximum induced emf in the coil is then

$$\varepsilon_{max} = NB_{horizontal}A\omega = 100\left(2.0\times10^{-5}\text{ T}\right)(0.20\text{ m})^2\left[\left(1500\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\right]$$

$$= 1.3\times10^{-2}\text{ V} = \boxed{13\text{ mV}}$$

20.33 Note the similarity between the situation in this problem and a generator. In a generator, one normally has a loop rotating in a constant magnetic field so the flux through the loop varies sinusoidally in time. In this problem, we have a stationary loop in an oscillating magnetic field, and the flux through the loop varies sinusoidally in time. In both cases, a sinusoidal emf $\varepsilon = \varepsilon_{max}\sin\omega t$, where $\varepsilon_{max} = NBA\omega$, is induced in the loop.

The loop in this case consists of a single band $(N = 1)$ around the perimeter of a red blood cell with diameter $d = 8.0\times10^{-6}$ m. The angular frequency of the oscillating flux through the area of this loop is $\omega = 2\pi f = 2\pi(60\text{ Hz}) = 120\pi$ rad/s. The maximum induced emf is then

$$\varepsilon_{max} = NBA\omega = B\left(\frac{\pi d^2}{4}\right)\omega = \frac{\left(1.0\times10^{-3}\text{ T}\right)\pi\left(8.0\times10^{-6}\text{ m}\right)^2\left(120\pi\text{ s}^{-1}\right)}{4} = \boxed{1.9\times10^{-11}\text{ V}}$$

20.34 (a) Using $\varepsilon_{max} = NBA\omega$, we find

$$\varepsilon_{max} = 1\,000(0.20\text{ T})\left(0.10\text{ m}^2\right)\left[\left(60\ \frac{\text{rev}}{\text{s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right)\right]$$

$$= 7.5\times10^3\text{ V} = \boxed{7.5\text{ kV}}$$

(b) The maximum induced emf occurs when the flux through the coil is changing the most rapidly. This is when the plane of the coil is $\boxed{\text{parallel to the magnetic field}}$.

20.35 The angular frequency is $\omega = \left(120\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60\text{ s}}\right)\left(\frac{2\pi\text{ rad}}{1\text{ rev}}\right) = 4\pi$ rad/s.

(a) $\varepsilon_{max} = NBA\omega = 500(0.60\text{ T})\left[(0.080\text{ m})(0.20\text{ m})\right](4\pi\text{ rad/s}) = \boxed{60\text{ V}}$

(b) Note that the calculator must be in radians mode for the next calculation.

$$\varepsilon = \varepsilon_{max}\sin(\omega t) = (60\text{ V})\sin\left[(4\pi\text{ rad/s})\left(\frac{\pi}{32}\text{ s}\right)\right] = \boxed{57\text{ V}}$$

(c) The emf is first maximum when $\omega t = \pi/2$ radians, or when

$$t = \frac{\pi/2\text{ rad}}{\omega} = \frac{\pi/2\text{ rad}}{4\pi\text{ rad/s}} = \boxed{0.13\text{ s}}$$

20.36 (a) Immediately after the switch is closed, the motor coils are still stationary and the back emf is zero. Thus,

$$I = \frac{\varepsilon}{R} = \frac{240\text{ V}}{30\ \Omega} = \boxed{8.0\text{ A}}$$

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(b) At maximum speed, $\varepsilon_{\text{back}} = 145\text{ V}$ and

$$I = \frac{\varepsilon - \varepsilon_{\text{back}}}{R} = \frac{240\text{ V} - 145\text{ V}}{30\ \Omega} = \boxed{3.2\text{ A}}$$

(c) $\varepsilon_{\text{back}} = \varepsilon - IR = 240\text{ V} - (6.0\text{ A})(30\ \Omega) = \boxed{60\text{ V}}$

20.37 (a) When a coil having N turns and enclosing area A rotates at angular frequency ω in a constant magnetic field, the emf induced in the coil is

$$\varepsilon = \varepsilon_{\max}\sin\omega t, \quad \text{where} \quad \varepsilon_{\max} = NB_{\perp}A\omega$$

Here, $B_{\perp}$ is the magnitude of the magnetic field perpendicular to the rotation axis of the coil. In the given case, $B_{\perp} = 55.0\ \mu\text{T}$; $A = \pi ab$, where $a = (10.0\text{ cm})/2$ and $b = (4.00\text{ cm})/2$; and

$$\omega = 2\pi f = 2\pi\left(100\ \frac{\text{rev}}{\text{min}}\right)\left(\frac{1\text{ min}}{60.0\text{ s}}\right) = 10.5\ \text{rad/s}$$

Thus, $\varepsilon_{\max} = (10.0)\left(55.0\times10^{-6}\text{ T}\right)\left[\frac{\pi}{4}(0.100\text{ m})(0.040\,0\text{ m})\right](10.5\ \text{rad/s})$

or $\varepsilon_{\max} = 1.81\times10^{-5}\text{ V} = \boxed{18.1\ \mu\text{V}}$

(b) When the rotation axis is parallel to the field, then $B_{\perp} = 0$, giving $\varepsilon_{\max} = \boxed{0}$. It is easily understood that the induced emf is always zero in this case if you recognize that the magnetic field lines are always parallel to the plane of the coil, and the flux through the coil has a constant value of zero.

20.38 (a) In terms of its cross-sectional area A, length ℓ, and number of turns N, the self inductance of a solenoid is given as $L = \mu_0 N^2 A/\ell$. Thus, for the given solenoid,

$$L = \frac{\mu_0 N^2\left(\pi d^2/4\right)}{\ell} = \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(580)^2\,\pi\left(8.0\times10^{-2}\text{ m}\right)^2}{4(0.36\text{ m})}$$
$$= 5.9\times10^{-3}\text{ H} = \boxed{5.9\text{ mH}}$$

(b) $\varepsilon = -L\left(\frac{\Delta I}{\Delta t}\right) = -\left(5.9\times10^{-3}\text{ H}\right)(+4.0\ \text{A/s}) = -2.4\times10^{-2}\text{ V} = \boxed{-24\text{ mV}}$

20.39 From $|\varepsilon| = L|\Delta I/\Delta t|$, we have

$$L = \frac{|\varepsilon|}{|\Delta I/\Delta t|} = \frac{|\varepsilon|(\Delta t)}{|\Delta I|} = \frac{\left(12\times10^{-3}\text{ V}\right)(0.50\text{ s})}{|2.0\text{ A} - 3.5\text{ A}|} = 4.0\times10^{-3}\text{ H} = \boxed{4.0\text{ mH}}$$

20.40 The units of $N\Phi_B/I$ are $\text{T}\cdot\text{m}^2/\text{A}$. From the force on a moving charged particle, $F = qvB$, the magnetic field is $B = F/qv$, and we find that

$$1\text{ T} = 1\ \frac{\text{N}}{\text{C}\cdot(\text{m/s})} = 1\ \frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}}$$

Thus, $\text{T}\cdot\text{m}^2 = \left(\frac{\text{N}\cdot\text{s}}{\text{C}\cdot\text{m}}\right)\cdot\text{m}^2 = \frac{(\text{N}\cdot\text{m})\cdot\text{s}}{\text{C}} = \left(\frac{\text{J}}{\text{C}}\right)\cdot\text{s} = \text{V}\cdot\text{s}$, and $\text{T}\cdot\text{m}^2/\text{A} = \text{V}\cdot\text{s}/\text{A}$.

The units of $\varepsilon/(\Delta I/\Delta t)$ are $\text{V}/(\text{A/s}) = \text{V}\cdot\text{s}/\text{A}$, the same as the units of $N\Phi_B/I$.

20.41 (a) $$L = \frac{\mu_0 N^2 A}{\ell} = \frac{\left(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A}\right)(400)^2\left[\pi\left(2.5\times 10^{-2}\ \text{m}\right)^2\right]}{0.20\ \text{m}} = 2.0\times 10^{-3}\ \text{H} = \boxed{2.0\ \text{mH}}$$

(b) From $|\varepsilon| = L(\Delta I/\Delta t)$, $\dfrac{\Delta I}{\Delta t} = \dfrac{|\varepsilon|}{L} = \dfrac{75\times 10^{-3}\ \text{V}}{2.0\times 10^{-3}\ \text{H}} = \boxed{38\ \text{A/s}}$

20.42 From $|\varepsilon| = L(\Delta I/\Delta t)$, the self-inductance is

$$L = \frac{|\varepsilon|}{\Delta I/\Delta t} = \frac{24.0\times 10^{-3}\ \text{V}}{10.0\ \text{A/s}} = 2.40\times 10^{-3}\ \text{H}$$

Then, from $L = N\Phi_B/I$, the magnetic flux through each turn is

$$\Phi_B = \frac{L\cdot I}{N} = \frac{\left(2.40\times 10^{-3}\ \text{H}\right)(4.00\ \text{A})}{500} = \boxed{1.92\times 10^{-5}\ \text{T}\cdot\text{m}^2}$$

20.43 (a) In the series circuit of Figure P20.43, maximum current occurs after the switch has been closed for a very long time, when current has stabilized and the back emf due to the inductance has decreased to zero. This maximum current is given by

$$I_{\text{max}} = \frac{\varepsilon}{R} = \frac{24.0\ \text{V}}{4.50\ \Omega} = \boxed{5.33\ \text{A}}$$

(b) The time constant of the *RL* circuit is

$$\tau = \frac{L}{R} = \frac{12.0\ \text{H}}{4.50\ \Omega} = \frac{12.0\ \Omega\cdot\text{s}}{4.50\ \Omega} = \boxed{2.67\ \text{s}}$$

(c) If the switch in the *RL* circuit is closed at time $t = 0$, the current as a function of time is given by $I = I_{\text{max}}\left(1 - e^{-t/\tau}\right)$.

Thus, $e^{-t/\tau} = 1 - I/I_{\text{max}}$, or $t = -\tau \ln\left(1 - I/I_{\text{max}}\right)$. With $\tau = 2.67$ s, the current in this circuit will be $I = 0.950 I_{\text{max}}$ at time

$$t = -(2.67\ \text{s})\ln(1 - 0.950) = \boxed{8.00\ \text{s}}$$

20.44 (a) The time constant of the *RL* circuit is $\tau = L/R$, and that of the *RC* circuit is $\tau = RC$. If the two time constants have the same value, then $RC = L/R$, or

$$R = \sqrt{\frac{L}{C}} = \sqrt{\frac{3.00\ \text{H}}{3.00\times 10^{-6}\ \text{F}}} = 1.00\times 10^{3}\ \Omega = \boxed{1.00\ \text{k}\Omega}$$

(b) The common value of the two time constants is

$$\tau = \frac{L}{R} = \frac{3.00\ \text{H}}{1.00\times 10^{3}\ \Omega} = 3.00\times 10^{-3}\ \text{s} = \boxed{3.00\ \text{ms}}$$

20.45 (a) $I_{\text{max}} = \varepsilon/R$, so $\varepsilon = I_{\text{max}}R = (8.0\ \text{A})(0.30\ \Omega) = \boxed{2.4\ \text{V}}$.

(b) The time constant is $\tau = L/R$, giving

$$L = \tau R = (0.25\ \text{s})(0.30\ \Omega) = 7.5\times 10^{-2}\ \text{H} = \boxed{75\ \text{mH}}$$

continued on next page

(c) The current as a function of time is $I = I_{max}\left(1 - e^{-t/\tau}\right)$, so at $t = \tau$,

$$I = I_{max}\left(1 - e^{-1}\right) = 0.632 I_{max} = 0.632(8.0\ \text{A}) = \boxed{5.1\ \text{A}}$$

(d) At $t = \tau$, $I = 5.1$ A, and the voltage drop across the resistor is

$$\Delta V_R = -IR = -(5.1\ \text{A})(0.30\ \Omega) = \boxed{-1.5\ \text{V}}$$

(e) Applying Kirchhoff's loop rule to the circuit shown in Figure P20.43 gives $\varepsilon + \Delta V_R + \Delta V_L = 0$. Thus, at $t = \tau$, we have

$$\Delta V_L = -(\varepsilon + \Delta V_R) = -(2.4\ \text{V} - 1.5\ \text{V}) = \boxed{-0.90\ \text{V}}$$

20.46 The current in the RL circuit at time t is $I = \dfrac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right)$. The potential difference across the resistor is $\Delta V_R = RI = \varepsilon\left(1 - e^{-t/\tau}\right)$, and from Kirchhoff's loop rule, the potential difference across the inductor is

$$\Delta V_L = \varepsilon - \Delta V_R = \varepsilon\left[1 - \left(1 - e^{-t/\tau}\right)\right] = \varepsilon e^{-t/\tau}$$

(a) At $t = 0$, $\Delta V_R = \varepsilon\left(1 - e^{-0}\right) = \varepsilon(1 - 1) = \boxed{0}$.

(b) At $t = \tau$, $\Delta V_R = \varepsilon\left(1 - e^{-1}\right) = (6.0\ \text{V})(1 - 0.368) = \boxed{3.8\ \text{V}}$.

(c) At $t = 0$, $\Delta V_L = \varepsilon e^{-0} = \varepsilon = \boxed{6.0\ \text{V}}$.

(d) At $t = \tau$, $\Delta V_L = \varepsilon e^{-1} = (6.0\ \text{V})(0.368) = \boxed{2.2\ \text{V}}$.

20.47 From $I = I_{max}\left(1 - e^{-t/\tau}\right)$, we obtain $e^{-t/\tau} = 1 - I/I_{max}$. If $I/I_{max} = 0.900$ at $t = 3.00$ s, then

$$e^{-(3.00\ \text{s})/\tau} = 0.100 \qquad \text{or} \qquad \tau = \frac{-3.00\ \text{s}}{\ln(0.100)} = 1.30\ \text{s}$$

Since the time constant of an RL circuit is $\tau = L/R$, the resistance is

$$R = \frac{L}{\tau} = \frac{2.50\ \text{H}}{1.30\ \text{s}} = \boxed{1.92\ \Omega}$$

20.48 (a) $\tau = \dfrac{L}{R} = \dfrac{8.00\ \text{mH}}{4.00\ \Omega} = \boxed{2.00\ \text{ms}}$

(b) $I = \dfrac{\varepsilon}{R}\left(1 - e^{-t/\tau}\right) = \left(\dfrac{6.00\ \text{V}}{4.00\ \Omega}\right)\left(1 - e^{-250\times10^{-6}\ \text{s}/2.00\times10^{-3}\ \text{s}}\right) = \boxed{0.176\ \text{A}}$

(c) $I_{max} = \dfrac{\varepsilon}{R} = \dfrac{6.00\ \text{V}}{4.00\ \Omega} = \boxed{1.50\ \text{A}}$

(d) $I = I_{max}\left(1 - e^{-t/\tau}\right)$ yields $e^{-t/\tau} = 1 - I/I_{max}$, and

$t = -\tau \ln\left(1 - I/I_{max}\right) = -(2.00\ \text{ms})\ln(1 - 0.800) = \boxed{3.22\ \text{ms}}$

20.49 (a) The energy stored by an inductor is $PE_L = \frac{1}{2}LI^2$, so the self inductance is

$$L = \frac{2(PE_L)}{I^2} = \frac{2(0.300\times10^{-3}\text{ J})}{(1.70\text{ A})^2} = 2.08\times10^{-4}\text{ H} = \boxed{0.208\text{ mH}}$$

(b) If $I = 3.0$ A, the stored energy will be

$$PE_L = \frac{1}{2}LI^2 = \frac{1}{2}(2.08\times10^{-4}\text{ H})(3.0\text{ A})^2 = 9.36\times10^{-4}\text{ J} = \boxed{0.936\text{ mJ}}$$

20.50 (a) The inductance of a solenoid is given by $L = \mu_0 N^2 A/\ell$, where N is the number of turns on the solenoid, A is its cross-sectional area, and ℓ is its length. For the given solenoid,

$$L = \frac{\mu_0 N^2(\pi r^2)}{\ell} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(300)^2\pi(5.00\times10^{-2}\text{ m})^2}{0.200\text{ m}}$$

$$= 4.44\times10^{-3}\text{ H} = \boxed{4.44\text{ mH}}$$

(b) When the solenoid described above carries a current of $I = 0.500$ A, the stored energy is

$$PE_L = \frac{1}{2}LI^2 = \frac{1}{2}(4.44\text{ mH})(0.500\text{ A})^2 = \boxed{0.555\text{ mJ}}$$

20.51 The current in the circuit at time t is $I = I_{\max}\left(1-e^{-t/\tau}\right)$, where $I_{\max} = \varepsilon/R$, and the energy stored in the inductor is $PE_L = \frac{1}{2}LI^2$.

(a) As $t\to\infty$, $I \to I_{\max} = \dfrac{\varepsilon}{R} = \dfrac{24\text{ V}}{8.0\ \Omega} = 3.0$ A, and

$$PE_L \to \frac{1}{2}LI_{\max}^2 = \frac{1}{2}(4.0\text{ H})(3.0\text{ A})^2 = \boxed{18\text{ J}}$$

(b) At $t = \tau$, $I = I_{\max}\left(1-e^{-1}\right) = (3.0\text{ A})(1-0.368) = 1.9$ A, and

$$PE_L = \frac{1}{2}(4.0\text{ H})(1.9\text{ A})^2 = \boxed{7.2\text{ J}}$$

20.52 (a) Use Table 17.1 to obtain the resistivity of the copper wire and find

$$R_{\text{wire}} = \frac{\rho_{\text{Cu}}L}{A_{\text{wire}}} = \frac{\rho_{\text{Cu}}L}{\pi r_{\text{wire}}^2} = \frac{(1.7\times10^{-8}\ \Omega\cdot\text{m})(60.0\text{ m})}{\pi(0.50\times10^{-3}\text{ m})^2} = \boxed{1.3\ \Omega}$$

(b) $$N = \frac{L}{\text{circumference of a loop}} = \frac{L}{2\pi r_{\text{solenoid}}} = \frac{60.0\text{ m}}{2\pi(2.0\times10^{-2}\text{ m})} = \boxed{4.8\times10^2\text{ turns}}$$

(c) The length of the solenoid is

$$\ell = N(\text{diameter of wire}) = N(2r_{\text{wire}}) = (480)2(0.50\times10^{-3}\text{ m}) = \boxed{0.48\text{ m}}$$

(d) $$L = \frac{\mu_0 N^2 A_{\text{solenoid}}}{\ell} = \frac{\mu_0 N^2 \pi r_{\text{solenoid}}^2}{\ell} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(480)^2\pi(2.0\times10^{-2}\text{ m})^2}{0.48\text{ m}}$$

giving $L = 7.6\times10^{-4}\text{ H} = \boxed{0.76\text{ mH}}$

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(e) $\tau = \dfrac{L}{R_{\text{total}}} = \dfrac{L}{R_{\text{wire}} + r_{\text{internal}}} = \dfrac{7.6 \times 10^{-4}\ \text{H}}{1.3\ \Omega + 0.350\ \Omega} = 4.6 \times 10^{-4}\ \text{s} = \boxed{0.46\ \text{ms}}$

(f) $I_{\max} = \dfrac{\varepsilon}{R_{\text{total}}} = \dfrac{6.0\ \text{V}}{1.3\ \Omega + 0.350\ \Omega} = \boxed{3.6\ \text{A}}$

(g) $I = I_{\max}\left(1 - e^{-t/\tau}\right)$, so when $I = 0.999 I_{\max}$, we have $e^{-t/\tau} = 1 - 0.999 = 0.001$. Thus, $-t/\tau = \ln(0.001)$, or $t = -\tau \cdot \ln(0.001) = -(0.46\ \text{ms}) \cdot \ln(0.001) = \boxed{3.2\ \text{ms}}$.

(h) $(PE_L)_{\max} = \dfrac{1}{2} L I_{\max}^2 = \dfrac{1}{2}\left(7.6 \times 10^{-4}\ \text{H}\right)(3.6\ \text{A})^2 = 4.9 \times 10^{-3}\ \text{J} = \boxed{4.9\ \text{mJ}}$

20.53 The flux due to the current in loop 1 passes from left to right through the area enclosed by loop 2. As loop 1 moves closer to loop 2, the magnitude of this flux through loop 2 is increasing. The induced current in loop 2 generates a magnetic field directed toward the left through the area it encloses in order to oppose the increasing flux from loop 1. This means that the induced current in loop 2 must flow $\boxed{\text{counterclockwise}}$ as viewed from the left end of the rod.

20.54 (a) The clockwise induced current in the loop produces a flux directed into the page through the area enclosed by the loop. Since this flux opposes the change in flux due to the external field, the outward-directed flux due to the external field must be increasing in magnitude. This means that the magnitude of the external field itself must be $\boxed{\text{increasing}}$ in time.

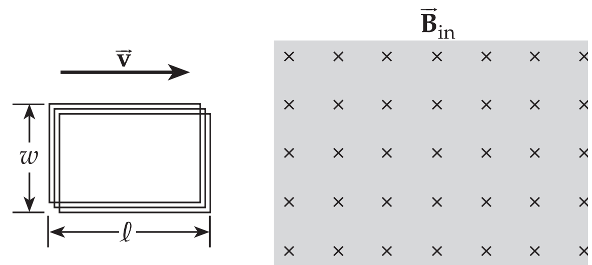
(b) The induced emf in the loop must be

$$\varepsilon = IR = (2.50\ \text{mA})(0.500\ \Omega) = 1.25\ \text{mV}$$

Since $\varepsilon = \dfrac{\Delta\Phi_B}{\Delta t} = \dfrac{\Delta(BA\cos 0°)}{\Delta t} = \left(\dfrac{\Delta B}{\Delta t}\right)A$, the rate of change of the field is

$$\frac{\Delta B}{\Delta t} = \frac{\varepsilon}{A} = \frac{\varepsilon}{\pi r^2} = \frac{1.25 \times 10^{-3}\ \text{V}}{\pi\left(8.00 \times 10^{-2}\ \text{m}\right)^2} = 6.22 \times 10^{-2}\ \text{T/s} = \boxed{62.2\ \text{mT/s}}$$

20.55 (a) After the right end of the coil has entered the field, but the left end has not, the flux through the area enclosed by the coil is directed into the page and is increasing in magnitude. This increasing flux induces an emf of magnitude



$$|\varepsilon| = \frac{\Delta\Phi_B}{\Delta t} = \frac{NB(\Delta A)}{\Delta t} = NBwv$$

in the loop. Note that in the above equation, $\Delta A = wv$ is the area enclosed by the coil that enters the field in time Δt. This emf produces a counterclockwise current in the loop to oppose the increasing inward flux. The magnitude of this current is $I = |\varepsilon|/R = NBwv/R$. The right end of the loop is now a conductor, of length Nw, carrying a current toward the top of the page through a field directed into the page. The field exerts a magnetic force of magnitude

$$F = BI(Nw) = B\left(\frac{NBwv}{R}\right)(Nw) = \boxed{\frac{N^2B^2w^2v}{R}} \quad \text{directed} \quad \boxed{\text{toward the left}}$$

on this conductor, and hence, on the loop.

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(b) When the loop is entirely within the magnetic field, the flux through the area enclosed by the loop is constant. Hence, there is no induced emf or current in the loop, and the field exerts $\boxed{\text{zero}}$ force on the loop.

(c) After the right end of the loop emerges from the field, and before the left end emerges, the flux through the loop is directed into the page and is decreasing. This decreasing flux induces an emf of magnitude $|\varepsilon| = NBwv$ in the loop, which produces an induced current directed clockwise around the loop so as to oppose the decreasing flux. The current has magnitude $I = |\varepsilon|/R = NBwv/R$. This current flowing upward, through conductors of total length Nw, in the left end of the loop, experiences a magnetic force given by

$$F = BI(Nw) = B\left(\frac{NBwv}{R}\right)(Nw) = \boxed{\frac{N^2B^2w^2v}{R}} \text{ directed } \boxed{\text{toward the left}}$$

20.56 (a) The motional emf induced in the bar must be $\varepsilon = IR$, where I is the current in this series circuit. Since $\varepsilon = B_\perp \ell v$, the speed of the moving bar must be

$$v = \frac{\varepsilon}{B_\perp \ell} = \frac{IR}{B_\perp \ell} = \frac{(8.5\times10^{-3}\text{ A})(9.0\ \Omega)}{(0.30\text{ T})(0.35\text{ m})} = \boxed{0.73\text{ m/s}}$$

The flux through the closed loop formed by the rails, the bar, and the resistor is directed into the page and is increasing in magnitude. To oppose this increasing inward flux, the induced current must generate a magnetic field directed out of the page through the area enclosed by the loop. This means the current will flow $\boxed{\text{countercloclockwise}}$.

(b) The rate at which energy is delivered to the resistor is

$$P = I^2R = (8.5\times10^{-3}\text{ A})^2(9.0\ \Omega) = 6.5\times10^{-4}\text{ W} = \boxed{0.65\text{ mW}}$$

(c) An external force directed to the right acts on the bar to balance the magnetic force to the left. Hence, $\boxed{\text{work is being done by the external force}}$, which is transformed into internal energy within the resistor.

20.57 (a) The current in the solenoid reaches $I_{sol} = 0.632 I_{max}$ in a time of $t = \tau = L/R$, where

$$L = \frac{\mu_0 N^2 A}{\ell} = \frac{(4\pi\times10^{-7}\text{ T}\cdot\text{m/A})(12\ 500)^2(1.00\times10^{-4}\text{ m}^2)}{7.00\times10^{-2}\text{ m}} = 0.280\text{ H}$$

Thus, $t = (0.280\text{ H})/(14.0\ \Omega) = 2.00\times10^{-2}\text{ s} = \boxed{20.0\text{ ms}}$.

(b) The change in the solenoid current during this time is

$$\Delta I_{sol} = 0.632 I_{max} - 0 = 0.632\left(\frac{\Delta V}{R}\right) = 0.632\left(\frac{60.0\text{ V}}{14.0\ \Omega}\right) = 2.71\text{ A}$$

so the average back emf is

$$|\varepsilon_{back}| = L\left(\frac{\Delta I_{sol}}{\Delta t}\right) = (0.280\text{ H})\left(\frac{2.71\text{ A}}{2.00\times10^{-2}\text{ s}}\right) = \boxed{37.9\text{ V}}$$

(c) The change in the magnitude of the magnetic field at the location of the coil is one-half the change in the magnitude of the field at the center of the solenoid. Thus, $\Delta B_{coil} = \frac{1}{2}\left[\mu_0 n_{sol}(\Delta I_{sol})\right]$, and the average rate of change of flux through each turn of the coil is

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$$\left(\frac{\Delta\Phi_B}{\Delta t}\right)_{\text{coil}} = \frac{(\Delta B)_{\text{coil}}\,A_{\text{coil}}}{\Delta t} = \frac{\frac{1}{2}\left[\mu_0 n_{\text{sol}}\left(\Delta I_{\text{sol}}\right)\right]A_{\text{coil}}}{\Delta t} = \frac{\mu_0 N_{\text{sol}}\left(\Delta I_{\text{sol}}\right)A_{\text{coil}}}{2\,\ell_{\text{sol}}\cdot(\Delta t)}$$

$$= \frac{\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(12\,500)(2.71\ \text{A})\left(1.00\times10^{-4}\ \text{m}^2\right)}{2\left(7.00\times10^{-2}\ \text{m}\right)\left(2.00\times10^{-2}\ \text{s}\right)} = \boxed{1.52\times10^{-3}\ \text{V}}$$

(d) $$I_{\text{coil}} = \frac{\left|\varepsilon_{\text{coil}}\right|}{R_{\text{coil}}} = \frac{N_{\text{coil}}\left(\Delta\Phi_B/\Delta t\right)_{\text{coil}}}{R_{\text{coil}}} = \frac{(820)\left(1.52\times10^{-3}\ \text{V}\right)}{24.0\ \Omega}$$

$$= 0.051\,9\ \text{A} = \boxed{51.9\ \text{mA}}$$

20.58 (a) The gravitational force exerted on the ship by the pulsar supplies the centripetal acceleration needed to hold the ship in orbit. Thus, $F_g = \dfrac{GM_{\text{pulsar}}m_{\text{ship}}}{r_{\text{orbit}}^2} = \dfrac{m_{\text{ship}}v^2}{r_{\text{orbit}}}$, giving

$$v = \sqrt{\frac{GM_{\text{pulsar}}}{r_{\text{orbit}}}} = \sqrt{\frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m/kg}^2\right)\left(2.0\times10^{30}\ \text{kg}\right)}{3.0\times10^7\ \text{m}}} = \boxed{2.1\times10^6\ \text{m/s}}$$

(b) The magnetic force acting on charged particles moving through a magnetic field is perpendicular to both the magnetic field and the velocity of the particles (and therefore perpendicular to the ship's length). Thus, the charged particles in the materials making up the spacecraft experience magnetic forces directed from one side of the ship to the other, meaning that the induced emf is directed $\boxed{\text{from side to side}}$ within the ship.

(c) $\varepsilon = B_\perp \ell v$, where $\ell = 2r_{\text{ship}} = 0.080\ \text{km} = 80\ \text{m}$ is the side-to-side dimension of the ship. This yields

$$\varepsilon = \left(1.0\times10^2\ \text{T}\right)(80\ \text{m})\left(2.1\times10^6\ \text{m/s}\right) = \boxed{1.7\times10^{10}\ \text{V}}$$

(d) The very large induced emf would lead to powerful spontaneous electric discharges. The strong electric and magnetic fields would disrupt the flow of ions in their bodies.

20.59 (a) To move the bar at uniform speed, the magnitude of the applied force must equal that of the magnetic force retarding the motion of the bar. Therefore, $F_{\text{app}} = BI\ell$. The magnitude of the induced current is

$$I = \frac{|\varepsilon|}{R} = \frac{\left(\Delta\Phi_B/\Delta t\right)}{R} = \frac{B\left(\Delta A/\Delta t\right)}{R} = \frac{B\,\ell v}{R}$$

so the field strength is $B = IR/\ell v$, giving $F_{\text{app}} = \left(IR/\ell v\right)I\ell = I^2R/v$, and the current is

$$I = \sqrt{\frac{F_{\text{app}}\cdot v}{R}} = \sqrt{\frac{(1.00\ \text{N})(2.00\ \text{m/s})}{8.00\ \Omega}} = \boxed{0.500\ \text{A}}$$

(b) $P_{\text{dissipated}} = I^2R = (0.500\ \text{A})^2(8.00\ \Omega) = \boxed{2.00\ \text{W}}$

(c) $P_{\text{input}} = F_{\text{app}}\cdot v = (1.00\ \text{N})(2.00\ \text{m/s}) = \boxed{2.00\ \text{W}}$

20.60 Since the magnetic field outside the solenoid is negligible in comparison to the field inside the solenoid, we shall assume that the flux through the single-turn square loop is the same as that through each turn of the solenoid. Then, the induced emf in the square loop is

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$$\varepsilon = \frac{\Delta\Phi_B}{\Delta t} = \frac{\Delta\left(B_{\substack{\text{inside}\\\text{solenoid}}} A_{\text{solenoid}}\right)}{\Delta t} = \frac{\Delta\left(\mu_0 n I_{\text{solenoid}}\right) A_{\text{solenoid}}}{\Delta t} = \mu_0 n \left(\frac{\Delta I_{\text{solenoid}}}{\Delta t}\right) A_{\text{solenoid}}$$

or $\varepsilon = \mu_0 n\left(\Delta I_{\text{solenoid}}/\Delta t\right)\pi r^2$, which gives

$$\varepsilon = \left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(3.50\times10^3/\text{m}\right)\left(28.5\ \text{A/s}\right)\pi\left(2.00\times10^{-2}\ \text{m}\right)^2$$
$$= 1.58\times10^{-4}\ \text{V} = 0.158\times10^{-3}\ \text{V} = \boxed{0.158\ \text{mV}}$$

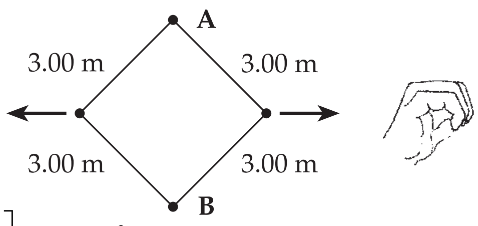
20.61 If d is the distance from the lightning bolt to the center of the coil, then

$$\left|\varepsilon_{\text{av}}\right| = \frac{N\left(\Delta\Phi_B\right)}{\Delta t} = \frac{N(\Delta B)A}{\Delta t} = \frac{N\left[\mu_0(\Delta I)/2\pi d\right]A}{\Delta t} = \frac{N\,\mu_0(\Delta I)A}{2\pi d(\Delta t)}$$

$$= \frac{100\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)\left(6.02\times10^6\ \text{A}-0\right)\left[\pi(0.800\ \text{m})^2\right]}{2\pi(200\ \text{m})\left(10.5\times10^{-6}\ \text{s}\right)}$$

$$= 1.15\times10^5\ \text{V} = \boxed{115\ \text{kV}}$$

20.62 When A and B are 3.00 m apart, the area enclosed by the loop consists of four triangular sections, each having hypotenuse of 3.00 m, altitude of 1.50 m, and base of $\sqrt{(3.00\ \text{m})^2-(1.50\ \text{m})^2} = 2.60$ m. The decrease in the enclosed area has been



$$\Delta A = A_i - A_f = (3.00\ \text{m})^2 - 4\left[\frac{1}{2}(1.50\ \text{m})(2.60\ \text{m})\right] = 1.20\ \text{m}^2$$

The average induced current has been

$$I_{\text{av}} = \frac{\left|\varepsilon_{\text{av}}\right|}{R} = \frac{\left(\Delta\Phi_B/\Delta t\right)}{R} = \frac{B(\Delta A/\Delta t)}{R} = \frac{(0.100\ \text{T})\left(1.20\ \text{m}^2/0.100\ \text{s}\right)}{10.0\ \Omega} = \boxed{0.120\ \text{A}}$$

As the enclosed area decreases, the flux due to the external field (directed into the page) through this area also decreases. Thus, the induced current will be directed $\boxed{\text{clockwise}}$ around the loop to create additional flux directed into the page through the enclosed area.

20.63 (a) $$\left|\varepsilon_{\text{av}}\right| = \frac{\left|\Delta\Phi_B\right|}{\Delta t} = \frac{B\left|\Delta A\right|}{\Delta t} = \frac{B\left[\left(\pi d^2/4\right)-0\right]}{\Delta t}$$

$$= \frac{(25.0\ \text{mT})\pi\left(2.00\times10^{-2}\ \text{m}\right)^2}{4\left(50.0\times10^{-3}\ \text{s}\right)} = \boxed{0.157\ \text{mV}}$$

As the inward-directed flux through the loop decreases, the induced current goes clockwise around the loop to create additional inward flux through the enclosed area. With positive charges accumulating at B, $\boxed{\text{point } B \text{ is positive relative to } A}$.

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(b) $$\left|\varepsilon_{\text{av}}\right| = \frac{\Delta\Phi_B}{\Delta t} = \frac{(\Delta B)A}{\Delta t} = \frac{\left[(100-25.0)\ \text{mT}\right]\pi\left(2.00\times10^{-2}\ \text{m}\right)^2}{4\left(4.00\times10^{-3}\ \text{s}\right)} = \boxed{5.89\ \text{mV}}$$

As the inward-directed flux through the enclosed area increases, the induced current goes counterclockwise around the loop in to create flux directed outward through the enclosed area. With positive charges now accumulating at A, $\boxed{\text{point } A \text{ is positive relative to } B}$.

20.64 The induced emf in the ring is

$$\left|\varepsilon_{\text{av}}\right| = \frac{\Delta\Phi_B}{\Delta t} = \frac{(\Delta B)A_{\text{solenoid}}}{\Delta t} = \frac{\left(\Delta B_{\text{solenoid}}/2\right)A_{\text{solenoid}}}{\Delta t} = \frac{1}{2}\left[\mu_0 n\left(\frac{\Delta I_{\text{solenoid}}}{\Delta t}\right)\right]A_{\text{solenoid}}$$

$$= \frac{1}{2}\left[\left(4\pi\times10^{-7}\ \text{T}\cdot\text{m/A}\right)(1\,000)(270\ \text{A/s})\left(\pi\left[3.00\times10^{-2}\ \text{m}\right]^2\right)\right] = 4.80\times10^{-4}\ \text{V}$$

Thus, the induced current in the ring is

$$I_{\text{ring}} = \frac{\left|\varepsilon_{\text{av}}\right|}{R} = \frac{4.80\times10^{-4}\ \text{V}}{3.00\times10^{-4}\ \Omega} = \boxed{1.60\ \text{A}}$$

20.65 (a) As the rolling axle (of length $\ell = 1.50$ m) moves perpendicularly to the uniform magnetic field, an induced emf of magnitude $|\varepsilon| = B\ell v$ will exist between its ends. The current produced in the closed-loop circuit by this induced emf has magnitude

$$I = \frac{\left|\varepsilon_{\text{av}}\right|}{R} = \frac{\left(\Delta\Phi_B/\Delta t\right)}{R} = \frac{B\left(\Delta A/\Delta t\right)}{R} = \frac{B\ell v}{R} = \frac{(0.800\ \text{T})(1.50\ \text{m})(3.00\ \text{m/s})}{0.400\ \Omega} = \boxed{9.00\ \text{A}}$$

(b) The induced current through the axle will cause the magnetic field to exert a retarding force of magnitude $F_r = BI\ell$ on the axle. This force is directed opposite to the velocity $\vec{\mathbf{v}}$ to oppose the motion of the axle. If the axle is to continue moving at constant speed, an applied force in the direction of $\vec{\mathbf{v}}$, and having magnitude $F_{\text{app}} = F_r$, must be exerted on the axle.

$$F_{\text{app}} = BI\ell = (0.800\ \text{T})(9.00\ \text{A})(1.50\ \text{m}) = \boxed{10.8\ \text{N}}$$

(c) Using right-hand rule number 1, observe that positive charges within the moving axle experience a magnetic force toward the rail containing point b, and negative charges experience a force directed toward the rail containing point a. Thus, the rail containing b is positive relative to the other rail, so $\boxed{b \text{ is at the higher potential}}$.

(d) $\boxed{\text{No}}$. Both the velocity $\vec{\mathbf{v}}$ of the rolling axle and the magnetic field $\vec{\mathbf{B}}$ are unchanged. Thus, the polarity of the induced emf in the moving axle is unchanged, and the current continues to be directed from b to a through the resistor R.

20.66 (a) The time required for the coil to move distance ℓ and exit the field is $t = \ell/v$, where v is the constant speed of the coil. Since the speed of the coil is constant, the flux through the area enclosed by the coil decreases at a constant rate. Thus, the instantaneous induced emf is the same as the average emf over the interval t seconds in duration, or

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{(0-BA)}{t-0} = N\frac{B\ell^2}{t} = \frac{NB\ell^2}{\ell/v} = \boxed{NB\ell v}$$

(b) The current induced in the coil is $I = \varepsilon/R = \boxed{NB\ell v/R}$.

continued on next page

(c) Since the coil moves with constant velocity, the power delivered to the coil must equal the power being dissipated within the coil. This is given by $P = I^2 R$, or

$$P = \left(\frac{N^2 B^2 \ell^2 v^2}{R^2}\right) R = \boxed{\frac{N^2 B^2 \ell^2 v^2}{R}}$$

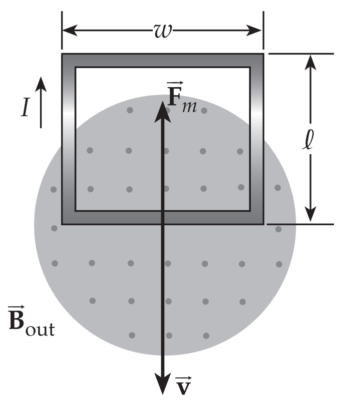
(d) The rate that the applied force does work must equal the power delivered to the coil, so $F_{\text{app}} \cdot v = P$, or

$$F_{\text{app}} = \frac{P}{v} = \frac{N^2 B^2 \ell^2 v^2 / R}{v} = \boxed{\frac{N^2 B^2 \ell^2 v}{R}}$$

(e) As the coil is emerging from the field, the flux through the area it encloses is directed into the page and is decreasing in magnitude. The induced current must flow $\boxed{\text{clockwise}}$ around the coil to generate a magnetic field directed into the page through the area enclosed by the coil, opposing the decrease in the inward flux.

(f) As the coil is emerging from the field, the left side of the coil carries an induced current directed toward the top of the page through a magnetic field that is directed into the page. Right-hand rule number 1 then shows that this side of the coil will experience $\boxed{\text{a magnetic force directed to the left}}$, opposing the motion of the coil.

20.67 (a) As the bottom conductor of the loop falls, it cuts across the magnetic field lines coming out of the page. This induces an emf of magnitude $|\varepsilon| = Bwv$ in this conductor, with the left end at the higher potential. As a result, an induced current of magnitude



$$I = \frac{|\varepsilon|}{R} = \frac{Bwv}{R}$$

flows clockwise around the loop. The field then exerts an upward force of magnitude

$$F_m = BIw = B\left(\frac{Bwv}{R}\right)w = \frac{B^2 w^2 v}{R}$$

on this current-carrying conductor forming the bottom of the loop. If the loop is falling at terminal speed, the magnitude of this force must equal the downward gravitational force acting on the loop. That is, when $v = v_t$, we must have

$$F_m = \frac{B^2 w^2 v_t}{R} = Mg \qquad \text{or} \qquad v_t = \boxed{\frac{MgR}{B^2 w^2}}$$

(b) A larger resistance would make the current smaller, so the loop must reach higher speed before the magnitude of the magnetic force will equal the gravitational force.

(c) The magnetic force is proportional to the product of the field and the current, while the current itself is proportional to the field. If B is cut in half, the speed must become four times larger to compensate and yield a magnetic force with magnitude equal to the that of the gravitational force.

21

Alternating Current Circuits and Electromagnetic Waves

QUICK QUIZZES

1. Choice (c). The average power is proportional to the rms current which is non-zero even though the average current is zero. (a) is only valid for an open circuit, for which $R \to \infty$. (b) and (d) can never be true because $i_{\text{av}} = 0$ for AC currents.

2. Choice (b). Choices (a) and (c) are incorrect because the unaligned sine curves in Figure 21.9 mean the voltages are out of phase, and so we cannot simply add the maximum (or rms) voltages across the elements. (In other words, $\Delta V \neq \Delta V_R + \Delta V_L + \Delta V_C$ even though $\Delta v = \Delta v_R + \Delta v_L + \Delta v_C$.)

3. Choice (b). Closing switch A replaces a single resistor with a parallel combination of two resistors. Since the equivalent resistance of a parallel combination is always less than the lowest resistance in the combination, the total resistance of the circuit decreases, which causes the impedance $Z = \sqrt{R_{\text{total}}^2 + \left(X_L - X_C\right)^2}$ to decrease.

4. Choice (a). Closing switch A replaces a single resistor with a parallel combination of two resistors. Since the equivalent resistance of a parallel combination is always less than the lowest resistance in the combination, the total resistance of the circuit decreases, which causes the phase angle, $\phi = \tan^{-1}\left[\left(X_L - X_C\right)/R\right]$, to increase.

5. Choice (a). Closing switch B replaces a single capacitor with a parallel combination of two capacitors. Since the equivalent capacitance of a parallel combination is greater than that of either of the individual capacitors, the total capacitance increases, which causes the capacitive reactance $X_C = 1/2\pi fC$ to decrease. Thus, the net reactance, $X_L - X_C$, increases causing the phase angle, $\phi = \tan^{-1}\left[\left(X_L - X_C\right)/R\right]$, to increase.

6. Choice (b). Inserting an iron core in the inductor increases both the self-inductance and the inductive reactance, $X_L = 2\pi fL$. This means the net reactance, $X_L - X_C$, and hence the impedance, $Z = \sqrt{R_{\text{total}}^2 + \left(X_L - X_C\right)^2}$, increases, causing the current (and therefore, the bulb's brightness) to decrease.

7. Choices (b) and (c). Since pressure is *force per unit area*, changing the size of the area without altering the intensity of the radiation striking that area will not cause a change in radiation pressure. In (b), the smaller disk absorbs less radiation, resulting in a smaller force. For the same reason, the momentum in (c) is reduced.

8. Choices (b) and (d). The frequency and wavelength of light waves are related by the equation $\lambda f = v$ or $f = v/\lambda$, where the speed of light v is a constant within a given medium. Thus, the frequency and wavelength are inversely proportional to each other; when one increases the other must decrease.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. In an electromagnetic wave, the electric field $\vec{\mathbf{E}}$, the magnetic field $\vec{\mathbf{B}}$, and the direction of propagation of the wave are always mutually perpendicular to each other. Thus, with $\vec{\mathbf{B}}$ (in the $-x$-direction) and the direction of propagation ($+y$-direction) both in the xy-plane, $\vec{\mathbf{E}}$ must be parallel to the z-axis, meaning that either (c) or (d) must be the correct answer. To choose between these possible answers, recall the right-hand rule for electromagnetic waves (see Section 21.11 in the textbook). Hold your right hand, with the fingers extended, so the thumb is in the direction of propagation ($+y$) and your palm is facing the direction of $\vec{\mathbf{B}}$ ($-x$-direction). Then the orientation of the extended fingers is the direction of the electric field $\vec{\mathbf{E}}$. You should find this to be the negative z-direction, so the correct choice is (d).

2. When the frequency doubles, the rms current $I_{\text{rms}} = \Delta V_{L,\text{rms}}/X_L = \Delta V_{L,\text{rms}}/2\pi fL$ is cut in half. Thus, the new current is $I_{\text{rms}} = 3.0\text{ A}/2 = 1.5\text{ A}$, and (e) is the correct answer.

3. At the resonance frequency, $X_L = X_C$ and the impedance is $Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = R$. Thus, the rms current is $I_{\text{rms}} = \Delta V_{\text{rms}}/Z = (120\text{ V})/(20\ \Omega) = 6.0\text{ A}$, and (b) is the correct choice.

4. $\Delta V_{L,\text{rms}} = I_{\text{rms}} X_L = I_{\text{rms}}(2\pi fL) = (2.0\text{ A})2\pi(60.0\text{ Hz})\left[(1.0/2\pi)\text{H}\right] = 120\text{ V}$, and the correct answer is choice (c).

5. $\Delta V_{C,\text{rms}} = I_{\text{rms}} X_C = \dfrac{I_{\text{rms}}}{2\pi fC} = \dfrac{1.00\times10^{-3}\text{ A}}{2\pi(60.0\text{ Hz})\left[(1.00/2\pi)\times10^{-6}\text{ F}\right]} = 16.7\text{ V}$, so choice (a) is correct.

6. The battery produces a constant current in the primary coil, which generates a constant flux through the secondary coil. With no change in flux through the secondary coil, there is no induced voltage across the secondary coil, and choice (e) is the correct answer.

7. The speed c, frequency f, and wavelength λ of an electromagnetic wave are related by $c = f\lambda$. The wavelength of the waves in the oven is then

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\text{ m/s}}{2.45\times10^{9}\text{ Hz}} = 0.122\text{ m} = 12.2\text{ cm}$$

and (b) is the correct choice.

8. When a power source, AC or DC, is first connected to a RL combination, the presence of the inductor impedes the buildup of a current in the circuit. The value of the current starts at zero and increases as the back emf induced across the inductor decreases somewhat in magnitude. Thus, the correct choice is (c).

9. The voltage across the capacitor is proportional to the stored charge. This charge, and hence the voltage Δv_C, is a maximum when the current has zero value and is in the process of reversing direction after having been flowing in one direction for a half cycle. Thus, the voltage across the capacitor lags behind the current by 90°, and (a) is the correct choice.

10. In an AC circuit, both an inductor and a capacitor store energy for one half of the cycle of the current and return that energy to the circuit during the other half of the cycle. On the other hand, a resistor converts electrical potential energy into thermal energy during all parts of the cycle of the current. Thus, only the resistor has a non-zero average power loss. The power delivered to a circuit by a generator is given by $P_{\text{av}} = I_{\text{rms}}\Delta V_{\text{rms}}\cos\phi$, where ϕ is the phase difference between the generator voltage and the current. Among the listed choices, the only *true* statement is choice (c).

11. In an *RLC* circuit, the instantaneous voltages Δv_R, Δv_L, and Δv_C (across the resistance, inductance, and capacitance respectively) are not in phase with each other. The instantaneous voltage Δv_R is in phase with the current, Δv_L leads the current by 90°, while Δv_C lags behind the current by 90°. The instantaneous values of these three voltages do add algebraically to give the instantaneous voltage across the *RLC* combination, but the rms voltages across these components do not add algebraically. The rms voltages across the three components must be added as vectors (or phasors) to obtain the correct rms voltage across the *RLC* combination. Among the listed choices, choice (e) is the *false* statement.

12. If the voltage is a maximum when the current is zero, the voltage is either leading or lagging the current by 90° (or a quarter cycle) in phase. Thus, the element could be *either* an inductor or a capacitor. It could not be a resistor since the voltage across a resistor is always in phase with the current. If the current and voltage were out of phase by 180°, one would be a maximum in one direction when the other was a maximum value in the opposite direction. The correct choice for this question is (d).

13. At resonance of the *RLC* series circuit, $X_L = X_C$, and the impedance becomes

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + 0} = R$$

so choice (c) is correct.

14. At a frequency of $f = 5.0 \times 10^2$ Hz, the inductive reactance, capacitive reactance, and impedance are $X_L = 2\pi f L$, $X_C = \dfrac{1}{2\pi f C}$, and $Z = \sqrt{R^2 + \left(X_L - X_C\right)^2}$, respectively. This yields

$$Z = \sqrt{(20.0\ \Omega)^2 + \left[2\pi\left(5.0 \times 10^2\ \text{Hz}\right)(0.120\ \text{H}) - \frac{1}{2\pi\left(5.0 \times 10^2\ \text{Hz}\right)\left(0.75 \times 10^{-6}\ \text{F}\right)}\right]^2} = 51\ \Omega,$$

and $I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{120\ \text{V}}{51\ \Omega} = 2.3\ \text{A}$. Choice (a) is the correct answer.

15. Choices (c) and (d) are the only true statements. Electromagnetic waves consist of oscillating electric and magnetic fields that are perpendicular to each other and to the direction of propagation. In a vacuum, all electromagnetic waves travel at the same speed, c. The electromagnetic spectrum consists of waves having broad ranges of frequencies and wavelengths, which are related by $\lambda f = c$.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The phase angle in an *RLC* series circuit is given by

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{2\pi f L - 1/2\pi f C}{R}\right)$$

which is clearly frequency dependent. As $f \to 0$, $\phi \to \tan^{-1}(-\infty) = -90°$ and as $f \to \infty$, $\phi \to \tan^{-1}(+\infty) = +90°$. At resonance, $X_L = X_C$, and $\phi = \tan^{-1}(0) = 0°$.

4. An antenna that is a conducting line responds to the electric field of the electromagnetic wave—the oscillating electric field causes an electric force on electrons in the wire along its length. The movement of electrons along the wire is detected as a current by the radio and is amplified. Thus, a line antenna must have the same orientation as the broadcast antenna. A loop antenna responds to the magnetic field in the radio wave. The varying magnetic field induces a varying current in the loop (by Faraday's law), and this signal is amplified. The loop should be in the vertical plane containing the line of sight to the broadcast antenna, so the magnetic field lines go through the area of the loop.

6. (a) In an electromagnetic wave, electric and magnetic fields oscillate at right angles to each other and perpendicular to the direction of propagation of the wave.

(b) Energy is the quantity transported by the oscillating electric and magnetic fields of the electromagnetic wave.

8. Consider a typical metal rod antenna for a car radio. Charges in the rod respond to the electric field portion of the carrier wave. Variations in the amplitude of the incoming radio wave cause the electrons in the rod to vibrate with amplitudes emulating those of the carrier wave. Likewise, for frequency modulation, the variations of the frequency of the carrier wave cause constant-amplitude vibrations of the electrons in the rod but at frequencies that imitate those of the carrier.

10. The brightest portion of your face shows where you radiate the most. Your nostrils and the openings of your ear canals are particularly bright. Brighter still are the pupils of your eyes.

12. The changing magnetic field of the solenoid induces eddy currents in the conducting core. This is accompanied by I^2R conversion of electrically-transmitted energy into internal energy in the conductor.

14. The voltages are not added in a scalar form, but in a vector form, as shown in the phasor diagrams throughout the chapter. Kirchhoff's loop rule is true at any instant, but the voltages across different circuit elements are not simultaneously at their maximum values. Do not forget that an inductor can induce an emf in itself and that the voltage across it is 90° in phase *ahead* of the current in the circuit.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 1.70×10^2 V **(b)** 2.40×10^2 Ω

(c) The 100-W bulb will have the lower resistance.

4. **(a)** $I_{1,\text{rms}} = I_{2,\text{rms}} = 1.25$ A, $I_{3,\text{rms}} = 0.833$ A

(b) $R_1 = R_2 = 96.0\ \Omega,\ R_3 = 144\ \Omega$

6. **(a)** 170 V **(b)** 30.0 Hz **(c)** 120 V

(d) 6.00 A **(e)** 8.49 A **(f)** 720 W

(g) 6.9 A **(h)** radians

8. **(a)** 141 mA **(b)** 235 mA

10. **(a)** 221 Ω **(b)** 0.163 A **(c)** 0.231 A

(d) No. The charge is a maximum when the voltage is a maximum, but the voltage across a capacitor is 90° out of phase with the current.

12. **(a)** 69.3 V **(b)** 40 Hz **(c)** 0.354 A

(d) 196 Ω **(e)** 20.3 μF

14. **(a)** $12.6\ \Omega$ **(b)** 6.19 A **(c)** 8.75 A

16. **(a)** 15.0 Hz **(b)** 84.9 V **(c)** $47.1\ \Omega$

(d) 1.80 A **(e)** 2.55 A **(f)** 0

(g) $i = (2.55\text{ A})\sin(30\pi t - \pi/2)$

(h) 20.9 ms

18. **(a)** $57.5\ \Omega$ **(b)** 1.39 A

20. **(a)** 194 V **(b)** The current leads the voltage by 49.9°.

22. **(a)** 138 V **(b)** 104 V **(c)** 729 V

(d) 641 V **(e)** See Solution.

24. **(a)** 0.11 A **(b)** $\Delta V_{R,\max} = 1.3\times10^2\text{ V}, \Delta V_{L,\max} = 1.2\times10^2\text{ V}$

(c) $\Delta v_R = 1.3\times10^2\text{ V}, \Delta v_L = 0, \Delta v_{\text{source}} = 1.3\times10^2\text{ V}$

(d) $\Delta v_R = 0, \Delta v_L = 1.2\times10^2\text{ V}, \Delta v_{\text{source}} = 1.2\times10^2\text{ V}$

26. **(a)** $88.4\ \Omega$ **(b)** $107\ \Omega$ **(c)** 1.12 A

(d) The voltage lags the current by 55.8°.

(e) It changes the impedance, and therefore the current, in the circuit.

28. 2.79 kHz

30. **(a)** 0.11 A **(b)** $\Delta V_{R,\max} = 1.3\times10^2\text{ V}, \Delta V_{C,\max} = 1.2\times10^2\text{ V}$

(c) $\Delta v_R = 0, \Delta v_C = 1.2\times10^2\text{ V}, \Delta v_{\text{source}} = 1.2\times10^2\text{ V}, q_C = 300\ \mu\text{C}$

(d) $\Delta v_R = 1.3\times10^2\text{ V}, \Delta v_C = 0, \Delta v_{\text{source}} = 1.3\times10^2\text{ V}, q_C = 0$

32. **(a)** $66.8\ \Omega$ **(b)** 0.953 A **(c)** 45.4 W

34. 88.0 W

36. **(a)** No, the algebraic sum of the rms voltage drops is 20.9 V.

(b) to the resistor **(c)** 3.2 W

38. 2.31 kHz

40. **(a)** $Z = R = 15\ \Omega$ **(b)** 41 Hz

(c) at resonance **(d)** 2.5 A

42. **(a)** 480 W **(b)** 0.192 W

(c) 30.7 mW **(d)** 0.192 W

(e) 30.7 mW; Maximum power is delivered at the resonance frequency.

44. **(a)** 9.23 V **(b)** 30.0 W

46. **(a)** fewer turns **(b)** 25 mA **(c)** 20 turns

48. **(a)** 29.0 kW **(b)** 0.577% **(c)** See Solution.

50. **(a)** 6.80×10^2 y **(b)** 8.31 min **(c)** 2.56 s

52. 2.998×10^8 m/s

54. 0.80 or 80%

56. **(a)** $I = mgc/2A$ **(b)** $1.46 \times 10^9 \text{ W/m}^2$

(c) Propulsion by light pressure in a significant gravity field is impractical because of the enormous power requirements. In addition, no material is perfectly reflecting, so the absorbed energy would melt the reflecting surface.

58. **(a)** $8.89 \times 10^{-8} \text{ W/m}^2$ **(b)** 11.4 MW

60. 11.0 m

62. The radio listeners hear the news 8.4 ms before the studio audience because radio waves travel so much faster than sound waves.

64. **(a)** 6.0036×10^{14} Hz **(b)** 3.6×10^{11} Hz

66. 1.1×10^7 m/s

68. $\sim 10^6$ J

70. $X_{C,i} = 3R$

72. **(a)** 0.63 pF **(b)** 8.4 mm **(c)** 25 Ω

74. **(a)** 6.0 Ω **(b)** 12 mH

76. **(a)** 6.2 mW/cm^2**,** 24% higher than the maximum allowed leakage from a microwave

(b) 0.024 mW/cm^2

PROBLEM SOLUTIONS

21.1 For an AC circuit containing only resistance (the filament of the lightbulb), the power dissipated is $P = I_{\text{rms}}^2 R = \left(\Delta V_{\text{rms}}/R\right)^2 R = \Delta V_{\text{rms}}^2/R = \left(\Delta V_{\text{max}}/\sqrt{2}\right)^2/R$.

(a) $$R = \frac{\Delta V_{\text{rms}}^2}{P} = \frac{\left(170 \text{ V}/\sqrt{2}\right)^2}{75.0 \text{ W}} = \boxed{193\ \Omega}$$

continued on next page

(b) $R = \dfrac{\Delta V_{\text{rms}}^2}{P} = \dfrac{\left(170\ \text{V}/\sqrt{2}\right)^2}{100.0\ \text{W}} = \boxed{145\ \Omega}$

21.2 (a) $\Delta V_{R,\max} = \sqrt{2}\left(\Delta V_{R,\text{rms}}\right) = \sqrt{2}\left(1.20\times10^2\ \text{V}\right) = \boxed{1.70\times10^2\ \text{V}}$

(b) $P_{\text{av}} = I_{\text{rms}}^2 R = \dfrac{\Delta V_{\text{rms}}^2}{R} \Rightarrow R = \dfrac{\Delta V_{\text{rms}}^2}{P_{\text{av}}} = \dfrac{\left(1.20\times10^2\ \text{V}\right)^2}{60.0\ \text{W}} = \boxed{2.40\times10^2\ \Omega}$

(c) Because $R = \dfrac{\Delta V_{\text{rms}}^2}{P_{\text{av}}}$ (see above), if the bulbs are designed to operate at the same voltage,

$\boxed{\text{the 100 W will have the lower resistance}}$.

21.3 For a simple resistance, $i(t) = v(t)/R = \left(\Delta V_{\max}\sin\omega t\right)/R = I_{\max}\sin\omega t$. Thus, if $i = 0.600 I_{\max}$ at $t = 7.00$ ms, we have

$$\omega t = 2\pi f t = \sin^{-1}\left(i/I_{\max}\right) \qquad \text{and} \qquad f = \frac{\sin^{-1}\left(i/I_{\max}\right)}{2\pi t}$$

giving $f = \dfrac{\sin^{-1}(0.600)}{2\pi t} = \dfrac{0.644\ \text{rad}}{(2\pi\ \text{rad})\left(7.00\times10^{-3}\ \text{s}\right)} = 14.6\ \text{s}^{-1} = \boxed{14.6\ \text{Hz}}$.

21.4 All lamps are connected in parallel with the voltage source, so $\Delta V_{\text{rms}} = 120$ V for each lamp. Also, for each bulb, the current is $I_{\text{rms}} = P_{\text{av}}/\Delta V_{\text{rms}}$ and the resistance is $R = \Delta V_{\text{rms}}/I_{\text{rms}}$.

(a) For bulbs 1 and 2: $I_{1,\text{rms}} = I_{2,\text{rms}} = \dfrac{150\ \text{W}}{120\ \text{V}} = \boxed{1.25\ \text{A}}$

For bulb 3: $I_{3,\text{rms}} = \dfrac{100\ \text{W}}{120\ \text{V}} = \boxed{0.833\ \text{A}}$

(b) $R_1 = R_2 = \dfrac{120\ \text{V}}{1.25\ \text{A}} = \boxed{96.0\ \Omega}$ and $R_3 = \dfrac{120\ \text{V}}{0.833\ \text{A}} = \boxed{144\ \Omega}$

21.5 The total resistance (series connection) is $R_{\text{eq}} = R_1 + R_2 = 8.20\ \Omega + 10.4\ \Omega = 18.6\ \Omega$, so the current in the circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{R_{\text{eq}}} = \frac{15.0\ \text{V}}{18.6\ \Omega} = 0.806\ \text{A}$$

The power to the speaker is then $P_{\text{av}} = I_{\text{rms}}^2 R_{\text{speaker}} = (0.806\ \text{A})^2(10.4\ \Omega) = \boxed{6.76\ \text{W}}$.

21.6 The general form of the generator voltage is $\Delta v = \left(\Delta V_{\max}\right)\sin(\omega t)$, so by inspection

(a) $\Delta V_{R,\max} = \boxed{170\ \text{V}}$ and (b) $f = \dfrac{\omega}{2\pi} = \dfrac{60\pi\ \text{rad/s}}{2\pi\ \text{rad}} = \boxed{30.0\ \text{Hz}}$

(c) $\Delta V_{R,\text{rms}} = \dfrac{\Delta V_{R,\max}}{\sqrt{2}} = \dfrac{170\ \text{V}}{\sqrt{2}} = \boxed{120\ \text{V}}$

(d) $I_{\text{rms}} = \dfrac{\Delta V_{R,\text{rms}}}{R} = \dfrac{120\ \text{V}}{20.0\ \Omega} = \boxed{6.00\ \text{A}}$

continued on next page

(e) $I_{max} = \sqrt{2} I_{rms} = \sqrt{2}(6.00 \text{ A}) = \boxed{8.49 \text{ A}}$

(f) $P_{av} = I_{rms}^2 R = (6.00 \text{ A})^2 (20.0\ \Omega) = \boxed{720 \text{ W}}$

(g) At $t = 0.005\,0$ s, the instantaneous current is

$$i = \frac{\Delta v}{R} = \frac{(170 \text{ V})}{20.0\ \Omega} \sin\left[(60\pi \text{ rad/s})(0.005\,0 \text{ s})\right] = \frac{(170 \text{ V})}{20.0\ \Omega} \sin(0.94 \text{ rad}) = \boxed{6.9 \text{ A}}$$

(h) The argument of the sine function has units of $[\omega t] = (\text{rad/s})(\text{s}) = \boxed{\text{radians}}$.

21.7 (a) The expression for capacitive reactance is $X_C = 1/2\pi fC$. Thus, if $X_C < 175\ \Omega$, it is necessary that

$$f = \frac{1}{2\pi C X_C} > \frac{1}{2\pi(22.0 \times 10^{-6} \text{ F})(175\ \Omega)} \qquad \text{or} \qquad \boxed{f > 41.3 \text{ Hz}}$$

(b) For C_1, the reactance is $X_{C,1} = 1/2\pi fC_1$, while for C_2, $X_{C,2} = 1/2\pi fC_2$. Thus, for the same frequencies, the ratio of the reactance for the two capacitors is

$$\frac{X_{C,2}}{X_{C,1}} = \left(\frac{1}{\cancel{2\pi f} C_2}\right)\left(\frac{\cancel{2\pi f} C_1}{1}\right) = \frac{C_1}{C_2} \qquad \text{or} \qquad X_{C,2} = \left(\frac{C_1}{C_2}\right) X_{C,1}$$

If $C_1 = 22.0\ \mu\text{F}$, $C_2 = 44.0\ \mu\text{F}$, and $X_{C,1} < 175\ \Omega$, we have

$$X_{C,2} < \left(\frac{22.0\ \mu\text{F}}{44.0\ \mu\text{F}}\right)(175\ \Omega) \qquad \text{or} \qquad \boxed{X_{C,2} < 87.5\ \Omega}$$

21.8 $I_{max} = \sqrt{2}\, I_{rms} = \dfrac{\sqrt{2}(\Delta V_{rms})}{X_C} = \sqrt{2}(\Delta V_{rms}) 2\pi fC$

(a) $I_{max} = \sqrt{2}(120 \text{ V}) 2\pi (60.0 \text{ Hz})(2.20 \times 10^{-6} \text{ C/V}) = 0.141 \text{ A} = \boxed{141 \text{ mA}}$

(b) $I_{max} = \sqrt{2}(240 \text{ V}) 2\pi (50.0 \text{ Hz})(2.20 \times 10^{-6} \text{ C/V}) = 0.235 \text{ A} = \boxed{235 \text{ mA}}$

21.9 $I_{rms} = \dfrac{\Delta V_{rms}}{X_C} = 2\pi fC(\Delta V_{rms})$, so

$$f = \frac{I_{rms}}{2\pi C(\Delta V_{rms})} = \frac{0.30 \text{ A}}{2\pi(4.0 \times 10^{-6} \text{ F})(30 \text{ V})} = \boxed{4.0 \times 10^2 \text{ Hz}}$$

21.10 (a) $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(60.0 \text{ Hz})(12.0 \times 10^{-6} \text{ F})} = \boxed{221\ \Omega}$

(b) $I_{rms} = \dfrac{\Delta V_{C,rms}}{X_C} = \dfrac{36.0 \text{ V}}{221\ \Omega} = \boxed{0.163 \text{ A}}$

(c) $I_{max} = \sqrt{2}\, I_{rms} = \sqrt{2}(0.163 \text{ A}) = \boxed{0.231 \text{ A}}$

(d) $\boxed{\text{No.}}$ The charge is a maximum when the voltage is a maximum, but the voltage across a capacitor is 90° out of phase with the current.

21.11 $I_{\max} = \dfrac{\Delta V_{\max}}{X_C} = 2\pi f C\left(\Delta V_{\max}\right) = 2\pi(90.0\text{ Hz})\left(3.70\times10^{-6}\text{ F}\right)(48.0\text{ V})$

or $I_{\max} = 1.00\times10^{-1}\text{ A} = \boxed{100\text{ mA}}$

21.12 (a) By inspection, $\Delta V_{C,\max} = 98.0\text{ V}$, so $\Delta V_{C,\text{rms}} = \dfrac{\Delta V_{C,\max}}{\sqrt{2}} = \dfrac{98.0\text{ V}}{\sqrt{2}} = \boxed{69.3\text{ V}}$

(b) Also by inspection, $\omega = 80\pi\text{ rad/s}$, so $f = \dfrac{\omega}{2\pi} = \dfrac{80\pi\text{ rad/s}}{2\pi\text{ rad}} = \boxed{40\text{ Hz}}$

(c) $I_{\text{rms}} = \dfrac{I_{\max}}{\sqrt{2}} = \dfrac{0.500\text{ A}}{\sqrt{2}} = \boxed{0.354\text{ A}}$

(d) $X_C = \dfrac{\Delta V_{C,\max}}{I_{\max}} = \dfrac{98.0\text{ V}}{0.500\text{ A}} = \boxed{196\ \Omega}$

(e) $X_C = \dfrac{1}{2\pi f C} = \dfrac{1}{\omega C}$, so $C = \dfrac{1}{\omega X_C} = \dfrac{1}{(80\pi\text{ rad/s})(196\ \Omega)} = 2.03\times10^{-5}\text{ F} = \boxed{20.3\ \mu\text{F}}$

21.13 $X_L = 2\pi(60.0\text{ Hz})L = 54.0\ \Omega \quad\Rightarrow\quad 2\pi L = \dfrac{54.0\ \Omega}{60.0\text{ s}^{-1}} = 0.900\ \Omega\cdot\text{s}$

Then, when $\Delta V_{\text{rms}} = 100\text{ V}$ and $f = 50.0\text{ Hz}$, the maximum current will be

$$I_{\max} = \frac{\Delta V_{\max}}{X_L} = \frac{\sqrt{2}\left(\Delta V_{\text{rms}}\right)}{(2\pi L)f} = \frac{\sqrt{2}(100\text{ V})}{(0.900\ \Omega\cdot\text{s})(50.0\text{ Hz})} = \boxed{3.14\text{ A}}$$

21.14 (a) $X_L = 2\pi f L = 2\pi(80.0\text{ Hz})\left(25.0\times10^{-3}\text{ H}\right) = \boxed{12.6\ \Omega}$

(b) $I_{\text{rms}} = \dfrac{\Delta V_{L,\text{rms}}}{X_L} = \dfrac{78.0\text{ V}}{12.6\ \Omega} = \boxed{6.19\text{ A}}$

(c) $I_{\max} = \sqrt{2}\,I_{\text{rms}} = \sqrt{2}(6.19\text{ A}) = \boxed{8.75\text{ A}}$

21.15 (a) $I_{\max} = \dfrac{\Delta V_{\max}}{X_L} = \dfrac{\Delta V_{\max}}{2\pi f L}$, so

$$L = \frac{\Delta V_{\max}}{2\pi f I_{\max}} = \frac{100\text{ V}}{2\pi(50.0\text{ Hz})(7.50\text{ A})} = 4.24\times10^{-2}\text{ H} = \boxed{42.4\text{ mH}}$$

(b) $I_{\max} = \Delta V_{\max}/X_L = \Delta V_{\max}/\omega L$, or $I_{\max}$ is inversely proportional to ω. Thus, $I_{\max,1}/I_{\max,2} = \omega_2/\omega_1$, or

$$\omega_2 = \left(\frac{I_{\max,1}}{I_{\max,2}}\right)\omega_1 = \left(\frac{7.50\text{ A}}{2.50\text{ A}}\right)\left[2\pi(50.0\text{ Hz})\right] = \boxed{942\text{ rad/s}}$$

21.16 Given: $v_L = \left(1.20\times10^2\text{ V}\right)\sin(30\pi t)$ and $L = 0.500\text{ H}$

(a) By inspection, $\omega = 30\pi\text{ rad/s}$, so $f = \dfrac{\omega}{2\pi} = \dfrac{30\pi\text{ rad/s}}{2\pi} = \boxed{15.0\text{ Hz}}$

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(b) Also by inspection, $\Delta V_{L,\max} = 1.20\times10^2$ V, so

$$\Delta V_{L,\text{rms}} = \frac{\Delta V_{L,\max}}{\sqrt{2}} = \frac{1.20\times10^2\ \text{V}}{\sqrt{2}} = \boxed{84.9\ \text{V}}$$

(c) $X_L = 2\pi fL = \omega L = (30\pi\ \text{rad/s})(0.500\ \text{H}) = \boxed{47.1\ \Omega}$

(d) $I_{\text{rms}} = \dfrac{\Delta V_{L,\text{rms}}}{X_L} = \dfrac{84.9\ \text{V}}{47.1\ \Omega} = \boxed{1.80\ \text{A}}$

(e) $I_{\max} = \sqrt{2}\,I_{\text{rms}} = \sqrt{2}(1.80\ \text{A}) = \boxed{2.55\ \text{A}}$

(f) The phase difference between the voltage across an inductor and the current through the inductor is $\phi_L = 90°$, so the average power delivered to the inductor is

$$P_{L,\text{av}} = I_{\text{rms}}\Delta V_{L,\text{rms}}\cos\phi_L = I_{\text{rms}}\Delta V_{L,\text{rms}}\cos(90°) = \boxed{0}$$

(g) When a sinusoidal voltage with a peak value $\Delta V_{L,\max}$ is applied to an inductor, the current through the inductor also varies sinusoidally in time, with the same frequency as the applied voltage, and has a maximum value of $I_{\max} = \Delta V_{L,\max}/X_L$. However, the current lags behind the voltage in phase by a quarter-cycle, or $\pi/2$ radians. Thus, if the voltage is given by $\Delta v_L = \Delta V_{L,\max}\sin(\omega t)$, the current as a function of time is $i = I_{\max}\sin(\omega t - \pi/2)$. In the case of the given inductor, the current through it will be $\boxed{i = (2.55\ \text{A})\sin(30\pi t - \pi/2)}$.

(h) When $i = +1.00$ A, we have $\sin(30\pi t - \pi/2) = (1.00\ \text{A}/2.55\ \text{A})$, or

$$30\pi t - \pi/2 = \sin^{-1}(1.00\ \text{A}/2.55\ \text{A}) = \sin^{-1}(0.392) = 0.403\ \text{rad}$$

and $$t = \frac{\pi/2\ \text{rad} + 0.403\ \text{rad}}{30\pi\ \text{rad/s}} = 2.09\times10^{-2}\ \text{s} = \boxed{20.9\ \text{ms}}$$

21.17 From $L = N\Phi_B/I$ (see Section 20.5 in the textbook), the total flux through the coil is $\Phi_{B,\text{total}} = N\Phi_B = L\cdot I$, where Φ_B is the flux through a single turn on the coil. Thus,

$$(\Phi_{B,\text{total}})_{\max} = L\cdot I_{\max} = L\cdot\left[\frac{\Delta V_{\max}}{X_L}\right]$$

$$= L\frac{\sqrt{2}(\Delta V_{\text{rms}})}{2\pi fL} = \frac{\sqrt{2}(120\ \text{V})}{2\pi(60.0\ \text{Hz})} = \boxed{0.450\ \text{T}\cdot\text{m}^2}$$

21.18 (a) The applied voltage is $\Delta v = \Delta V_{\max}\sin(\omega t) = (80.0\ \text{V})\sin(150t)$, so $\Delta V_{\max} = 80.0$ V and $\omega = 2\pi f = 150$ rad/s. The impedance for the circuit is

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + (2\pi fL - 1/2\pi fC)^2} = \sqrt{R^2 + (\omega L - 1/\omega C)^2}$$

or $$Z = \sqrt{(40.0\ \Omega)^2 + \left[(150\ \text{rad/s})(80.0\times10^{-3}\ \text{H}) - \frac{1}{(150\ \text{rad/s})(125\times10^{-6}\ \text{F})}\right]^2}$$

$$= \boxed{57.5\ \Omega}$$

(b) $I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{80.0\ \text{V}}{57.5\ \Omega} = \boxed{1.39\ \text{A}}$

21.19 $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(60.0\text{ Hz})(40.0\times10^{-6}\text{ F})} = 66.3\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50.0\ \Omega)^2 + (0 - 66.3\ \Omega)^2} = 83.0\ \Omega$

(a) $I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{30.0\text{ V}}{83.0\ \Omega} = \boxed{0.361\text{ A}}$

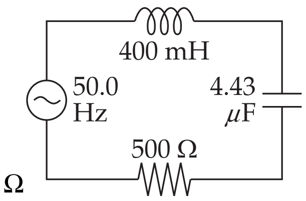
(b) $\Delta V_{R,\text{rms}} = I_{\text{rms}}R = (0.361\text{ A})(50.0\ \Omega) = \boxed{18.1\text{ V}}$

(c) $\Delta V_{C,\text{rms}} = I_{\text{rms}}X_C = (0.361\text{ A})(66.3\ \Omega) = \boxed{23.9\text{ V}}$

(d) $\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right) = \tan^{-1}\left(\dfrac{0 - 66.3\ \Omega}{50.0\ \Omega}\right) = -53.0°$

so $\boxed{\text{the voltage lags behind the current by }53°}$.

21.20 (a) $X_L = 2\pi fL = 2\pi(50.0\text{ Hz})(400\times10^{-3}\text{ H}) = 126\ \Omega$



$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(50.0\text{ Hz})(4.43\times10^{-6}\text{ F})} = 719\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(500\ \Omega)^2 + (126\ \Omega - 719\ \Omega)^2} = 776\ \Omega$

$\Delta V_{\max} = I_{\max}Z = (0.250\text{ A})(776\ \Omega) = \boxed{194\text{ V}}$

(b) $\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right) = \tan^{-1}\left(\dfrac{126\ \Omega - 719\ \Omega}{500\ \Omega}\right) = -49.9°$

Thus, $\boxed{\text{the current leads the voltage by }49.9°}$.

21.21 (a) $X_L = 2\pi fL = 2\pi(240\text{ Hz})(2.50\text{ H}) = 3.77\times10^3\ \Omega$

$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(240\text{ Hz})(0.250\times10^{-6}\text{ F})} = 2.65\times10^3\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(900\ \Omega)^2 + \left[(3.77 - 2.65)\times10^3\ \Omega\right]^2} = \boxed{1.44\text{ k}\Omega}$

(b) $I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{140\text{ V}}{1.44\times10^3\ \Omega} = \boxed{0.097\ 2\text{ A}}$

(c) $\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right) = \tan^{-1}\left[\dfrac{(3.77 - 2.65)\times10^3\ \Omega}{900\ \Omega}\right] = \boxed{51.2°}$

(d) $\phi > 0$, so $\boxed{\text{the voltage leads the current}}$

21.22 $X_L = 2\pi fL = 2\pi(60.0\text{ Hz})(0.100\text{ H}) = 37.7\ \Omega$

$X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(60.0\text{ Hz})(10.0\times10^{-6}\text{ F})} = 265\ \Omega$

$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(50.0\ \Omega)^2 + (37.7\ \Omega - 265\ \Omega)^2} = 233\ \Omega$

(a) $\Delta V_{R,\text{rms}} = I_{\text{rms}}R = (2.75\text{ A})(50.0\ \Omega) = \boxed{138\text{ V}}$

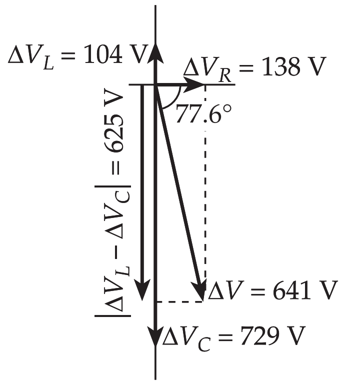
(b) $\Delta V_{L,\text{rms}} = I_{\text{rms}}X_L = (2.75\text{ A})(37.7\ \Omega) = \boxed{104\text{ V}}$

continued on next page

(c) $\Delta V_C = I_{\text{rms}} X_C = (2.75\text{ A})(265\ \Omega) = \boxed{729\text{ V}}$

(d) $\Delta V_{\text{rms}} = I_{\text{rms}} Z = (2.75\text{ A})(233\ \Omega) = \boxed{641\text{ V}}$

(e)



21.23 (a) $X_L = 2\pi f L = 2\pi(60.0\text{ Hz})(460\times10^{-3}\text{ H}) = 173\ \Omega$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(60.0\text{ Hz})(21.0\times10^{-6}\text{ F})} = 126\ \Omega$$

Thus, $\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right) = \tan^{-1}\left(\dfrac{173\ \Omega - 126\ \Omega}{150\ \Omega}\right) = \boxed{+17.4°}$

(b) $X_L > X_C$ and $\phi > 0$, so the voltage leads the current. Hence, $\boxed{\text{the voltage}}$ reaches its maximum first.

21.24 $X_L = 2\pi f L = 2\pi(60\text{ Hz})(2.8\text{ H}) = 1.1\times10^3\ \Omega$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(1.2\times10^3\ \Omega)^2 + (1.1\times10^3\ \Omega - 0)^2} = 1.6\times10^3\ \Omega$$

(a) $I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{170\text{ V}}{1.6\times10^3\ \Omega} = \boxed{0.11\text{ A}}$

(b) $\Delta V_{R,\max} = I_{\max} R = (0.11\text{ A})(1.2\times10^3\ \Omega) = \boxed{1.3\times10^2\text{ V}}$

$\Delta V_{L,\max} = I_{\max} X_L = (0.11\text{ A})(1.1\times10^3\ \Omega) = \boxed{1.2\times10^2\text{ V}}$

(c) When the instantaneous current is a maximum $(i = I_{\max})$, the instantaneous voltage across the resistor is $\Delta v_R = iR = I_{\max} R = \Delta V_{R,\max} = \boxed{1.3\times10^2\text{ V}}$. The instantaneous voltage across an inductor is always 90° or a quarter cycle out of phase with the instantaneous current. Thus, when $i = I_{\max}$, $\Delta v_L = \boxed{0}$.

Kirchhoff's loop rule always applies to the instantaneous voltages around a closed path. Thus, for this series circuit $\Delta v_{\text{source}} = \Delta v_R + \Delta v_L$, and at this instant when $i = I_{\max}$, we have $\Delta v_{\text{source}} = I_{\max} R + 0 = \boxed{1.3\times10^2\text{ V}}$.

(d) When the instantaneous current i is zero, the instantaneous voltage across the resistor is $\Delta v_R = iR = \boxed{0}$. Again, the instantaneous voltage across an inductor is a quarter cycle out of phase with the current. Thus, when $i = 0$, we must have $\Delta v_L = \Delta V_{L,\max} = \boxed{1.2\times10^2\text{ V}}$. Then, applying Kirchhoff's loop rule to the instantaneous voltages around the series circuit at the instant when $i = 0$ gives $\Delta v_{\text{source}} = \Delta v_R + \Delta v_L = 0 + \Delta V_{L,\max} = \boxed{1.2\times10^2\text{ V}}$.

21.25 $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(60.0\text{ Hz})(20.0\times10^{-12}\text{ F})} = 1.33\times10^{8}\ \Omega$

$Z_{RC} = \sqrt{R_{\text{body}}^2 + X_C^2} = \sqrt{(50.0\times10^{3}\ \Omega)^2 + (1.33\times10^{8}\ \Omega)^2} = 1.33\times10^{8}\ \Omega$

and $I_{\text{rms}} = \dfrac{(\Delta V_{\text{secondary}})_{\text{rms}}}{Z_{RC}} = \dfrac{5\ 000\text{ V}}{1.33\times10^{8}\ \Omega} = 3.76\times10^{-5}\text{ A}$

Therefore, $\Delta V_{\text{body, rms}} = I_{\text{rms}} R_{\text{body}} = (3.76\times10^{-5}\text{ A})(50.0\times10^{3}\ \Omega) = \boxed{1.88\text{ V}}$.

21.26 (a) $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(60.0\text{ Hz})(30.0\times10^{-6}\text{ F})} = \boxed{88.4\ \Omega}$

(b) $Z = \sqrt{R^2 + (0 - X_C)^2} = \sqrt{R^2 + X_C^2} = \sqrt{(60.0\ \Omega)^2 + (88.4\ \Omega)^2} = \boxed{107\ \Omega}$

(c) $I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{1.20\times10^{2}\text{ V}}{107\ \Omega} = \boxed{1.12\text{ A}}$

(d) The phase angle in this *RC* circuit is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{0 - 88.4\ \Omega}{60.0\ \Omega}\right) = -55.8^\circ$$

Since $\phi < 0$, $\boxed{\text{the voltage lags behind the current by } 55.8^\circ}$.

(e) $\boxed{\text{Adding an inductor will change the impedance and the current in the circuit.}}$ If the added inductive reactance is $X_L < 2X_C$, the impedance will be decreased and the current will increase. However, if $X_L > 2X_C$, the impedance will be increased and the current will decrease.

21.27 (a) $X_L = 2\pi fL = 2\pi(50.0\text{ Hz})(0.150\text{ H}) = \boxed{47.1\ \Omega}$

(b) $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(50.0\text{ Hz})(5.00\times10^{-6}\text{ F})} = \boxed{637\ \Omega}$

(c) $Z = \dfrac{\Delta V_{\max}}{I_{\max}} = \dfrac{240\text{ V}}{1.00\times10^{-1}\text{ A}} = 2.40\times10^{3}\ \Omega = \boxed{2.40\text{ k}\Omega}$

(d) $Z = \sqrt{R^2 + (X_L - X_C)^2}$ so $R = \sqrt{Z^2 - (X_L - X_C)^2}$

$R = \sqrt{(2.40\times10^{3}\ \Omega)^2 - (47.1\ \Omega - 637\ \Omega)^2} = 2.33\times10^{3}\ \Omega = \boxed{2.33\text{ k}\Omega}$

(e) $\phi = \tan^{-1}\left(\dfrac{X_L - X_C}{R}\right) = \tan^{-1}\left(\dfrac{47.1\ \Omega - 637\ \Omega}{2.33\times10^{3}\ \Omega}\right) = \boxed{-14.2^\circ}$

21.28 $X_L = \omega L$ and $X_C = 1/\omega C$, where $\omega = 2\pi f$. At resonance (i.e., when $X_L = X_C$), we have

$$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}} \qquad \text{or} \qquad f_0 = \frac{1}{2\pi\sqrt{LC}}$$

With $L = 57.0\ \mu\text{H}$ and $C = 57.0\ \mu\text{F}$, this resonance frequency is

$$f_0 = \frac{1}{2\pi\sqrt{(57.0\times10^{-6}\text{ H})(57.0\times10^{-6}\text{ F})}} = 2.79\times10^{3}\text{ Hz} = \boxed{2.79\text{ kHz}}$$

21.29 $X_L = 2\pi fL = 2\pi(50.0\text{ Hz})(0.185\text{ H}) = 58.1\ \Omega$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(50.0\text{ Hz})(65.0\times10^{-6}\text{ F})} = 49.0\ \Omega$$

$$Z_{ad} = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(40.0\ \Omega)^2 + (58.1\ \Omega - 49.0\ \Omega)^2} = 41.0\ \Omega$$

and $I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z_{ad}} = \dfrac{(\Delta V_{\max}/\sqrt{2})}{Z_{ad}} = \dfrac{150\text{ V}}{(41.0\ \Omega)\sqrt{2}} = 2.59\text{ A}$

(a) $Z_{ab} = R = 40.0\ \Omega$, so $(\Delta V_{\text{rms}})_{ab} = I_{\text{rms}}\,Z_{ab} = (2.59\text{ A})(40.0\ \Omega) = \boxed{104\text{ V}}$

(b) $Z_{bc} = X_L = 58.1\ \Omega$, and $(\Delta V_{\text{rms}})_{bc} = I_{\text{rms}}\,Z_{bc} = (2.59\text{ A})(58.1\ \Omega) = \boxed{150\text{ V}}$

(c) $Z_{cd} = X_C = 49.0\ \Omega$, and $(\Delta V_{\text{rms}})_{cd} = I_{\text{rms}}\,Z_{cd} = (2.59\text{ A})(49.0\ \Omega) = \boxed{127\text{ V}}$

(d) $Z_{bd} = |X_L - X_C| = 9.10\ \Omega$, so $(\Delta V_{\text{rms}})_{bd} = I_{\text{rms}}\,Z_{bd} = (2.59\text{ A})(9.10\ \Omega) = \boxed{23.6\text{ V}}$

21.30 $X_C = \dfrac{1}{2\pi fC} = \dfrac{1}{2\pi(60\text{ Hz})(2.5\times10^{-6}\text{ F})} = 1.1\times10^3\ \Omega$

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(1.2\times10^3\ \Omega)^2 + (0 - 1.1\times10^3\ \Omega)^2} = 1.6\times10^3\ \Omega$$

(a) $I_{\max} = \dfrac{\Delta V_{\max}}{Z} = \dfrac{170\text{ V}}{1.6\times10^3\ \Omega} = \boxed{0.11\text{ A}}$

(b) $\Delta V_{R,\max} = I_{\max}R = (0.11\text{ A})(1.2\times10^3\ \Omega) = \boxed{1.3\times10^2\text{ V}}$

$\Delta V_{C,\max} = I_{\max}X_C = (0.11\text{ A})(1.1\times10^3\ \Omega) = \boxed{1.2\times10^2\text{ V}}$

(c) When the instantaneous current i is zero, the instantaneous voltage across the resistor is $|\Delta v_R| = iR = \boxed{0}$. The instantaneous voltage across a capacitor is always 90°, or a quarter cycle, out of phase with the instantaneous current. Thus, when $i = 0$, $|\Delta v_C| = \Delta V_{C,\max} = \boxed{1.2\times10^2\text{ V}}$, and

$$q_C = C(\Delta v_C) = (2.5\times10^{-6}\text{ F})(1.2\times10^2\text{ V}) = 3.0\times10^{-4}\text{ C} = \boxed{300\ \mu\text{C}}$$

Kirchhoff's loop rule always applies to the instantaneous voltages around a closed path. Thus, for this series circuit, $\Delta v_{\text{source}} + \Delta v_R + \Delta v_C = 0$, and at this instant when $i = 0$, we have $|\Delta v_{\text{source}}| = 0 + |\Delta v_C| = \Delta V_{C,\max} = \boxed{1.2\times10^2\text{ V}}$.

(d) When the instantaneous current is a maximum $(i = I_{\max})$, the instantaneous voltage across the resistor is $|\Delta v_R| = |iR| = I_{\max}R = \Delta V_{R,\max} = \boxed{1.3\times10^2\text{ V}}$. Again, the instantaneous voltage across a capacitor is a quarter cycle out of phase with the current. Thus, when $i = I_{\max}$, we must have $|\Delta v_C| = \boxed{0}$ and $q_C = C|\Delta v_C| = \boxed{0}$. Then, applying Kirchhoff's loop rule to the instantaneous voltages around the series circuit at the instant when $i = I_{\max}$ and $|\Delta v_C| = 0$ gives

$$\Delta v_{\text{source}} + \Delta v_R + \Delta v_C = 0 \quad\Rightarrow\quad |\Delta v_{\text{source}}| = |\Delta v_R| = \Delta V_{R,\max} = \boxed{1.3\times10^2\text{ V}}$$

21.31 (a) $Z = \dfrac{\Delta V_{\text{rms}}}{I_{\text{rms}}} = \dfrac{104\text{ V}}{0.500\text{ A}} = \boxed{208\ \Omega}$

(b) $P_{\text{av}} = I_{\text{rms}}^2 R$ gives $R = \dfrac{P_{\text{av}}}{I_{\text{rms}}^2} = \dfrac{10.0\text{ W}}{(0.500\text{ A})^2} = \boxed{40.0\ \Omega}$

(c) $Z = \sqrt{R^2 + X_L^2}$, so $X_L = \sqrt{Z^2 - R^2} = \sqrt{(208\ \Omega)^2 - (40.0\ \Omega)^2} = 204\ \Omega$

and $L = \dfrac{X_L}{2\pi f} = \dfrac{204\ \Omega}{2\pi(60.0\text{ Hz})} = \boxed{0.541\text{ H}}$

21.32 Given $v = \Delta V_{\max}\sin(\omega t) = (90.0\text{ V})\sin(350t)$, observe that $\Delta V_{\max} = 90.0\text{ V}$ and $\omega = 350\text{ rad/s}$. Also, the net reactance is $X_L - X_C = 2\pi fL - 1/2\pi fC = \omega L - 1/\omega C$.

(a) $X_L - X_C = \omega L - \dfrac{1}{\omega C} = (350\text{ rad/s})(0.200\text{ H}) - \dfrac{1}{(350\text{ rad/s})(25.0\times10^{-6}\text{ F})} = -44.3\ \Omega$

so the impedance is $Z = \sqrt{R^2\left(X_L - X_C\right)^2} = \sqrt{(50.0\ \Omega)^2 + (-44.3\ \Omega)^2} = \boxed{66.8\ \Omega}$

(b) $I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{\Delta V_{\max}/\sqrt{2}}{Z} = \dfrac{90.0\text{ V}}{\sqrt{2}(66.8\ \Omega)} = \boxed{0.953\text{ A}}$

(c) The phase difference between the applied voltage and the current is

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) = \tan^{-1}\left(\frac{-44.3\ \Omega}{50.0\ \Omega}\right) = -41.5°$$

so the average power delivered to the circuit is

$$P_{\text{av}} = I_{\text{rms}}\Delta V_{\text{rms}}\cos\phi = I_{\text{rms}}\left(\frac{\Delta V_{\max}}{\sqrt{2}}\right)\cos\phi = (0.953\text{ A})\left(\frac{90.0\text{ V}}{\sqrt{2}}\right)\cos(-41.5°) = \boxed{45.4\text{ W}}$$

21.33 If $\Delta v = (100\text{ V})\sin\left[(1\,000\text{ rad/s})t\right]$, then $\Delta V_{\max} = 100\text{ V}$ and $\omega = 1\,000\text{ rad/s}$.

Thus, $X_L = \omega L = (1\,000\text{ rad/s})(0.500\text{ H}) = 500\ \Omega$

and $X_C = \dfrac{1}{\omega C} = \dfrac{1}{(1\,000\text{ rad/s})(5.00\times10^{-6}\text{ C})} = 200\ \Omega$

Therefore, $Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{(400\ \Omega)^2 + (300\ \Omega)^2} = 500\ \Omega$

and $I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{\Delta V_{\max}/\sqrt{2}}{Z} = \dfrac{100\text{ V}}{\sqrt{2}(500\ \Omega)} = \dfrac{0.200\text{ A}}{\sqrt{2}}$

The power delivered to the circuit equals the power dissipated in the resistor, or

$$P = I_{\text{rms}}^2 R = \left(\frac{0.200\text{ A}}{\sqrt{2}}\right)^2 (400\ \Omega) = \boxed{8.00\text{ W}}$$

21.34 The rms current in the circuit is

$$I_{\text{rms}} = \frac{\Delta V_{\text{rms}}}{Z} = \frac{160\text{ V}}{80.0\ \Omega} = 2.00\text{ A}$$

continued on next page

and the average power delivered to the circuit is

$$P_{av} = I_{rms}\left(\Delta V_{rms}\cos\phi\right) = I_{rms}\Delta V_{R,rms} = I_{rms}\left(I_{rms}R\right) = I_{rms}^2 R = (2.00\text{ A})^2(22.0\ \Omega) = \boxed{88.0\text{ W}}$$

21.35 (a) $P_{av} = I_{rms}^2 R = I_{rms}\left(I_{rms}R\right) = I_{rms}\left(\Delta V_{R,\,rms}\right)$, so $I_{rms} = \dfrac{P_{av}}{\Delta V_{R,\,rms}} = \dfrac{14\text{ W}}{50\text{ V}} = 0.28\text{ A}$

Thus, $R = \dfrac{\Delta V_{R,\,rms}}{I_{rms}} = \dfrac{50\text{ V}}{0.28\text{ A}} = \boxed{1.8\times 10^2\ \Omega}$

(b) $Z = \sqrt{R^2 + X_L^2}$, which yields

$$X_L = \sqrt{Z^2 - R^2} = \sqrt{\left(\frac{\Delta V_{rms}}{I_{rms}}\right)^2 - R^2} = \sqrt{\left(\frac{90\text{ V}}{0.28\text{ A}}\right)^2 - \left(1.8\times 10^2\ \Omega\right)^2} = 2.7\times 10^2\ \Omega$$

and $L = \dfrac{X_L}{2\pi f} = \dfrac{2.7\times 10^2\ \Omega}{2\pi(60\text{ Hz})} = \boxed{0.72\text{ H}}$

21.36 $X_L = 2\pi fL = 2\pi(600\text{ Hz})\left(6.0\times 10^{-3}\text{ H}\right) = 23\ \Omega$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(600\text{ Hz})\left(25\times 10^{-6}\text{ F}\right)} = 11\ \Omega$$

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{(25\ \Omega)^2 + (23\ \Omega - 11\ \Omega)^2} = 28\ \Omega$$

(a) $\Delta V_{R,\,rms} = I_{rms}R = \left(\dfrac{\Delta V_{rms}}{Z}\right)R = \left(\dfrac{10\text{ V}}{28\ \Omega}\right)(25\ \Omega) = 8.9\text{ V}$

$\Delta V_{L,\,rms} = I_{rms}X_L = \left(\dfrac{\Delta V_{rms}}{Z}\right)X_L = \left(\dfrac{10\text{ V}}{28\ \Omega}\right)(23\ \Omega) = 8.2\text{ V}$

$\Delta V_{C,\,rms} = I_{rms}X_C = \left(\dfrac{\Delta V_{rms}}{Z}\right)X_C = \left(\dfrac{10\text{ V}}{28\ \Omega}\right)(11\ \Omega) = 3.9\text{ V}$

$\boxed{\text{No}}$, $\Delta V_{R,\,rms} + \Delta V_{L,\,rms} + \Delta V_{C,\,rms} = 8.9\text{ V} + 8.2\text{ V} + 3.8\text{ V} = 20.9\text{ V} \neq 10\text{ V}$

(b) The $\boxed{\text{power delivered to the resistor}}$ is the greatest. No power losses occur in an ideal capacitor or inductor.

(c) $P_{av} = I_{rms}^2 R = \left(\dfrac{\Delta V_{rms}}{Z}\right)^2 R = \left(\dfrac{10\text{ V}}{28\ \Omega}\right)^2 (25\ \Omega) = \boxed{3.2\text{ W}}$

21.37 The resonance frequency of a series *RLC* circuit is $f_0 = 1\big/\left(2\pi\sqrt{LC}\right)$. Thus, if $L = 1.40\ \mu$H and the desired resonance frequency is $f_0 = 99.7$ MHz, the needed capacitance is

$$C = \frac{1}{4\pi^2 f_0^2 L} = \frac{1}{4\pi^2\left(99.7\times 10^6\text{ Hz}\right)^2\left(1.40\times 10^{-6}\text{ H}\right)} = 1.82\times 10^{-12}\text{ F} = \boxed{1.82\text{ pF}}$$

21.38 The resonance frequency of a series *RLC* circuit is $f_0 = 1\big/\left(2\pi\sqrt{LC}\right)$. Thus, the ratio of the resonance frequencies when the same inductance is used with two different capacitances in the circuit is

$$\frac{f_{0,2}}{f_{0,1}} = \left(\frac{1}{2\pi\sqrt{LC_2}}\right)\left(\frac{2\pi\sqrt{LC_1}}{1}\right) = \sqrt{\frac{C_1}{C_2}}$$

continued on next page

If $f_{0,1} = 2.84$ kHz when $C_1 = 6.50\ \mu$F, the resonance frequency when the capacitance is $C_2 = 9.80\ \mu\text{F}$ will be

$$f_{0,2} = f_{0,1}\sqrt{\frac{C_1}{C_2}} = (2.84\text{ kHz})\sqrt{\frac{6.50\ \mu\text{F}}{9.80\ \mu\text{F}}} = \boxed{2.31\text{ kHz}}$$

21.39 $f_0 = \dfrac{1}{2\pi\sqrt{LC}}$, so $C = \dfrac{1}{4\pi^2 f_0^2 L}$

For $f_0 = (f_0)_{\min} = 500\text{ kHz} = 5.00\times 10^5\text{ Hz}$

$$C = C_{\max} = \frac{1}{4\pi^2\left(5.00\times 10^5\text{ Hz}\right)^2\left(2.0\times 10^{-6}\text{ H}\right)} = 5.1\times 10^{-8}\text{ F} = \boxed{51\text{ nF}}$$

For $f_0 = (f_0)_{\max} = 1600\text{ kHz} = 1.60\times 10^6\text{ Hz}$

$$C = C_{\min} = \frac{1}{4\pi^2\left(1.60\times 10^6\text{ Hz}\right)^2\left(2.0\times 10^{-6}\text{ H}\right)} = 4.9\times 10^{-9}\text{ F} = \boxed{4.9\text{ nF}}$$

21.40 (a) At resonance, $X_L = X_C$, so the impedance will be

$$Z = \sqrt{R^2 + \left(X_L - X_C\right)^2} = \sqrt{R^2 + 0} = R = \boxed{15\ \Omega}$$

(b) When $X_L = X_C$, we have $2\pi fL = \dfrac{1}{2\pi fC}$, which yields

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.20\text{ H})\left(75\times 10^{-6}\text{ F}\right)}} = \boxed{41\text{ Hz}}$$

(c) The current is a maximum $\boxed{\text{at resonance}}$, where the impedance has its minimum value of $Z = R$.

(d) At $f = 60$ Hz, $X_L = 2\pi(60\text{ Hz})(0.20\text{ H}) = 75\ \Omega$, $X_C = \dfrac{1}{2\pi(60\text{ Hz})\left(75\times 10^{-6}\text{ F}\right)} = 35\ \Omega$,

and

$$Z = \sqrt{(15\ \Omega)^2 + (75\ \Omega - 35\ \Omega)^2} = 43\ \Omega$$

Thus, $I_{\text{rms}} = \dfrac{\Delta V_{\text{rms}}}{Z} = \dfrac{\left(\Delta V_{\max}/\sqrt{2}\right)}{Z} = \dfrac{150\text{ V}}{\sqrt{2}(43\ \Omega)} = \boxed{2.5\text{ A}}$.

21.41 The resonance frequency of an LC circuit is $f_0 = 1/\left(2\pi\sqrt{LC}\right)$. If the capacitance remains constant while the inductance increases by 1.000%, the new resonance frequency will be

$$f_0' = \frac{1}{2\pi\sqrt{(1.010L)C}} = \left(\frac{1}{\sqrt{1.010}}\right)\frac{1}{2\pi\sqrt{LC}} = \frac{f_0}{\sqrt{1.010}}$$

The beat frequency detected in this case will be $f_{\text{beat}} = f_0 - f_0'$, or

$$f_{\text{beat}} = \left(1 - \frac{1}{\sqrt{1.010}}\right)f_0 = \left(1 - \frac{1}{\sqrt{1.010}}\right)(725\text{ kHz}) = \boxed{3.60\text{ kHz}}$$

21.42 The resonance frequency is $\omega_0 = 2\pi f_0 = 1/\sqrt{LC}$. Also, $X_L = \omega L$ and $X_C = 1/\omega C$.

(a) At resonance, $X_C = X_L = \omega_0 L = \left(\dfrac{1}{\sqrt{LC}}\right)L = \sqrt{\dfrac{L}{C}} = \sqrt{\dfrac{3.00\text{ H}}{3.00\times10^{-6}\text{ F}}} = 1\,000\ \Omega.$

Thus, $Z = \sqrt{R^2 + 0^2} = R$, $I_{\text{rms}} = \Delta V_{\text{rms}}/Z = 120\text{ V}/30.0\ \Omega = 4.00\text{ A}$, and $P_{\text{av}} = I_{\text{rms}}^2 R = (4.00\text{ A})^2(30.0\ \Omega) = \boxed{480\text{ W}}$.

(b) At $\omega = \frac{1}{2}\omega_0$; $X_L = \frac{1}{2}\left(X_L\big|_{\omega_0}\right) = 500\ \Omega$, $X_C = 2\left(X_C\big|_{\omega_0}\right) = 2\,000\ \Omega$,

$$Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{(30.0\ \Omega)^2 + (500\ \Omega - 2\,000\ \Omega)^2} = 1\,500\ \Omega$$

and $I_{\text{rms}} = \dfrac{120\text{ V}}{1\,500\ \Omega} = 0.080\,0\text{ A}$. Thus, $P_{\text{av}} = I_{\text{rms}}^2 R = (0.080\,0\text{ A})^2(30.0\ \Omega) = \boxed{0.192\text{ W}}$.

(c) At $\omega = \frac{1}{4}\omega_0$; $X_L = \frac{1}{4}\left(X_L\big|_{\omega_0}\right) = 250\ \Omega$, $X_C = 4\left(X_C\big|_{\omega_0}\right) = 4\,000\ \Omega$, $Z = 3\,750\ \Omega$,

and $I_{\text{rms}} = \dfrac{120\text{ V}}{3\,750\ \Omega} = 0.032\,0\text{ A}$.

Therefore, $P_{\text{av}} = I_{\text{rms}}^2 R = (0.032\,0\text{ A})^2(30.0\ \Omega) = 3.07\times10^{-2}\text{ W} = \boxed{30.7\text{ mW}}$.

(d) At $\omega = 2\omega_0$; $X_L = 2\left(X_L\big|_{\omega_0}\right) = 2\,000\ \Omega$, $X_C = \frac{1}{2}\left(X_C\big|_{\omega_0}\right) = 500\ \Omega$, $Z = 1\,500\ \Omega$, and

$I_{\text{rms}} = \dfrac{120\text{ V}}{1\,500\ \Omega} = 0.080\,0\text{ A}$. Then, $P_{\text{av}} = I_{\text{rms}}^2 R = (0.080\,0\text{ A})^2(30.0\ \Omega) = \boxed{0.192\text{ W}}$.

(e) At $\omega = 4\omega_0$; $X_L = 4\left(X_L\big|_{\omega_0}\right) = 4\,000\ \Omega$, $X_C = \frac{1}{4}\left(X_C\big|_{\omega_0}\right) = 250\ \Omega$, $Z = 3\,750\ \Omega$,

and $I_{\text{rms}} = \dfrac{120\text{ V}}{3\,750\ \Omega} = 0.032\,0\text{ A}$. Hence,

$$P_{\text{av}} = I_{\text{rms}}^2 R = (0.032\,0\text{ A})^2(30.0\ \Omega) = 3.07\times10^{-2}\text{ W} = \boxed{30.7\text{ mW}}$$

The power delivered to the circuit is a maximum when the rms current is a maximum. This occurs when the frequency of the source is equal to the resonance frequency of the circuit.

21.43 The maximum output voltage $(\Delta V_{\text{max}})_2$ is related to the maximum input voltage $(\Delta V_{\text{max}})_1$ by the expression $(\Delta V_{\text{max}})_2 = \dfrac{N_2}{N_1}(\Delta V_{\text{max}})_1$, where N_1 and N_2 are the number of turns on the primary coil and the secondary coil, respectively. Thus, for the given transformer,

$$(\Delta V_{\text{max}})_2 = \frac{1\,500}{250}(170\text{ V}) = 1.02\times10^3\text{ V}$$

and the rms voltage across the secondary is $(\Delta V_{\text{rms}})_2 = \dfrac{(\Delta V_{\text{max}})_2}{\sqrt{2}} = \dfrac{1.02\times10^3\text{ V}}{\sqrt{2}} = \boxed{721\text{ V}}$.

21.44 (a) The output voltage of the transformer is

$$\Delta V_{2,\text{rms}} = \left(\frac{N_2}{N_1}\right)\Delta V_{1,\text{rms}} = \left(\frac{1}{13}\right)(120\text{ V}) = \boxed{9.23\text{ V}}$$

continued on next page

(b) Assuming an ideal transformer, $P_{output} = P_{input}$, and the power delivered to the CD player is

$$(P_{av})_2 = (P_{av})_1 = I_{1,\text{rms}}(\Delta V_{1,\text{rms}}) = (0.250 \text{ A})(120 \text{ V}) = \boxed{30.0 \text{ W}}$$

21.45 The power input to the transformer is

$$(P_{av})_{input} = (\Delta V_{1,\text{ rms}}) I_{1,\text{ rms}} = (3\,600 \text{ V})(50 \text{ A}) = 1.8 \times 10^5 \text{ W}$$

For an ideal transformer, $(P_{av})_{ouput} = (\Delta V_{2,\text{ rms}}) I_{2,\text{ rms}} = (P_{av})_{input}$, so the current in the long-distance power line is

$$I_{2,\text{ rms}} = \frac{(P_{av})_{input}}{(\Delta V_{2,\text{ rms}})} = \frac{1.8 \times 10^5 \text{ W}}{100\,000 \text{ V}} = 1.8 \text{ A}$$

The power dissipated as heat in the line is then

$$P_{lost} = I_{2,\text{ rms}}^2 R_{line} = (1.8 \text{ A})^2 (100\ \Omega) = 3.2 \times 10^2 \text{ W}$$

The percentage of the power delivered by the generator that is lost in the line is

$$\%\text{ Lost} = \frac{P_{lost}}{P_{input}} \times 100\% = \left(\frac{3.2 \times 10^2 \text{ W}}{1.8 \times 10^5 \text{ W}}\right) \times 100\% = \boxed{0.18\%}$$

21.46 (a) Since the transformer is to step the voltage *down* from 120 volts to 6.0 volts, the secondary must have $\boxed{\text{fewer turns}}$ than the primary.

(b) For an ideal transformer, $(P_{av})_{input} = (P_{av})_{ouput}$, or $(\Delta V_{1,\text{ rms}}) I_{1,\text{ rms}} = (\Delta V_{2,\text{ rms}}) I_{2,\text{ rms}}$, so the current in the primary will be

$$I_{1,\text{ rms}} = \frac{(\Delta V_{2,\text{ rms}}) I_{2,\text{ rms}}}{\Delta V_{1,\text{ rms}}} = \frac{(6.0 \text{ V})(500 \text{ mA})}{120 \text{ V}} = \boxed{25 \text{ mA}}$$

(c) The ratio of the secondary to primary voltages is the same as the ratio of the number of turns on the secondary and primary coils, $\Delta V_2/\Delta V_1 = N_2/N_1$. Thus, the number of turns needed on the secondary coil of this step down transformer is

$$N_2 = N_1\left(\frac{\Delta V_2}{\Delta V_1}\right) = (400)\left(\frac{6.0 \text{ V}}{120 \text{ V}}\right) = \boxed{20 \text{ turns}}$$

21.47 (a) At 90% efficiency, $(P_{av})_{output} = 0.90(P_{av})_{input}$. Thus, if $(P_{av})_{output} = 1\,000$ kW, the input power to the primary is

$$(P_{av})_{input} = \frac{(P_{av})_{output}}{0.90} = \frac{1\,000 \text{ kW}}{0.90} = \boxed{1.1 \times 10^3 \text{ kW}}$$

(b) $$I_{1,\text{ rms}} = \frac{(P_{av})_{input}}{\Delta V_{1,\text{ rms}}} = \frac{1.1 \times 10^3 \text{ kW}}{\Delta V_{1,\text{ rms}}} = \frac{1.1 \times 10^6 \text{ W}}{3\,600 \text{ V}} = \boxed{3.1 \times 10^2 \text{ A}}$$

(c) $$I_{2,\text{ rms}} = \frac{(P_{av})_{output}}{\Delta V_{2,\text{ rms}}} = \frac{1\,000 \text{ kW}}{\Delta V_{2,\text{ rms}}} = \frac{1.0 \times 10^6 \text{ W}}{120 \text{ V}} = \boxed{8.3 \times 10^3 \text{ A}}$$

21.48 $R_{\text{line}} = (4.50\times10^{-4}\ \Omega/\text{m})(6.44\times10^{5}\ \text{m}) = 290\ \Omega$

(a) The power transmitted is $(P_{\text{av}})_{\text{transmitted}} = (\Delta V_{\text{rms}})I_{\text{rms}}$, so

$$I_{\text{rms}} = \frac{(P_{\text{av}})_{\text{transmitted}}}{\Delta V_{\text{rms}}} = \frac{5.00\times10^{6}\ \text{W}}{500\times10^{3}\ \text{V}} = 10.0\ \text{A}$$

Thus, $(P_{\text{av}})_{\text{loss}} = I_{\text{rms}}^2 R_{\text{line}} = (10.0\ \text{A})^2(290\ \Omega) = 2.90\times10^{4}\ \text{W} = \boxed{29.0\ \text{kW}}$.

(b) The power input to the line is

$$(P_{\text{av}})_{\text{input}} = (P_{\text{av}})_{\text{transmitted}} + (P_{\text{av}})_{\text{loss}} = 5.00\times10^{6}\ \text{W} + 2.90\times10^{4}\ \text{W} = 5.03\times10^{6}\ \text{W}$$

and the fraction of input power lost during transmission is

$$\textit{fraction} = \frac{(P_{\text{av}})_{\text{loss}}}{(P_{\text{av}})_{\text{input}}} = \frac{2.90\times10^{4}\ \text{W}}{5.03\times10^{6}\ \text{W}} = \boxed{0.005\,77\ \text{ or }\ 0.577\%}$$

(c) It is impossible to deliver the needed power at the generator voltage of 4.50 kV. The maximum line current with an input voltage of 4.50 kV to the line occurs when the line is shorted out at the customer's end, and this current is

$$(I_{\text{rms}})_{\text{max}} = \frac{\Delta V_{\text{rms}}}{R_{\text{line}}} = \frac{4\,500\ \text{V}}{290\ \Omega} = 15.5\ \text{A}$$

The maximum input power is then

$$(P_{\text{input}})_{\text{max}} = (\Delta V_{\text{rms}})(I_{\text{rms}})_{\text{max}}$$

$$= (4.50\times10^{3}\ \text{V})(15.5\ \text{A}) = 6.98\times10^{4}\ \text{W} = 69.8\ \text{kW}$$

This is far short of meeting the customer's request, and all of this power is lost in the transmission line.

21.49 From $v = \lambda f$, the wavelength is

$$\lambda = \frac{v}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{75\ \text{Hz}} = 4.00\times10^{6}\ \text{m} = 4\,000\ \text{km}$$

The required length of the antenna is then

$L = \lambda/4 = \boxed{1\,000\ \text{km}}$, or about 621 miles. Not very practical at all.

21.50 (a) $t = \dfrac{d}{c} = \dfrac{6.44\times10^{18}\ \text{m}}{3.00\times10^{8}\ \text{m/s}}\left(\dfrac{1\ \text{y}}{3.156\times10^{7}\ \text{s}}\right) = \boxed{6.80\times10^{2}\ \text{y}}$

(b) From Table C.4 (in Appendix C of the textbook), the average Earth-Sun distance is $d = 1.496\times10^{11}$ m, giving the transit time as

$$t = \frac{d}{c} = \frac{1.496\times10^{11}\ \text{m}}{3.00\times10^{8}\ \text{m/s}}\left(\frac{1\ \text{min}}{60\ \text{s}}\right) = \boxed{8.31\ \text{min}}$$

continued on next page

(c) Also from Table C.4, the average Earth-Moon distance is $d = 3.84 \times 10^8$ m, giving the time for the round trip as

$$t = \frac{2d}{c} = \frac{2\left(3.84 \times 10^8 \text{ m}\right)}{3.00 \times 10^8 \text{ m/s}} = \boxed{2.56 \text{ s}}$$

21.51 (a) The solar energy incident each second on 1.00 m^2 of the surface of Earth's atmosphere is

$$U_{\text{total}} = 1\,370 \ \frac{\text{W}}{\text{m}^2} = 1370 \ \frac{\text{J/s}}{\text{m}^2} = 1370 \ \frac{\text{N} \cdot \text{m}}{\text{m}^2 \cdot \text{s}} = 1370 \ \frac{\text{N}}{\text{m} \cdot \text{s}}$$

Of this, 38.0% is reflected and 62.0% is absorbed. From Equation 21.29 and Equation 21.30 in the textbook, the radiation pressures P_1 due to the reflected radiation and P_2 due to the absorbed radiation are given by

$$P_1 = \frac{2\,U_{\text{reflected}}}{c} = \frac{2\left(0.380\,U_{\text{total}}\right)}{c} = \frac{0.760\,U_{\text{total}}}{c}$$

and $$P_2 = \frac{U_{\text{absorbed}}}{c} = \frac{0.620\,U_{\text{total}}}{c}$$

The total radiation pressure is then

$$P_{\text{rad}} = P_1 + P_2 = \frac{(0.760 + 0.620)\,U_{\text{total}}}{c}$$

or $$P_{\text{rad}} = \frac{1.38\left(1\,370 \text{ N/m} \cdot \text{s}\right)}{3.00 \times 10^8 \text{ m/s}} = 6.30 \times 10^{-6} \text{ N/m}^2 = \boxed{6.30 \times 10^{-6} \text{ Pa}}$$

(b) In comparison, atmospheric pressure at the surface of the Earth is

$$\frac{P_{\text{atm}}}{P_{\text{rad}}} = \frac{101 \times 10^3 \text{ Pa}}{6.30 \times 10^{-6} \text{ Pa}} = \boxed{1.60 \times 10^{10} \text{ times greater than the radiation pressure}}$$

21.52 $$c = \frac{1}{\sqrt{\mu_0 \in_0}} = \frac{1}{\sqrt{\left(4\pi \times 10^{-7} \text{ N} \cdot \text{s}^2/\text{C}^2\right)\left(8.854 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2\right)}}$$

or $c = \boxed{2.998 \times 10^8 \text{ m/s}}$

21.53 (a) The frequency of an electromagnetic wave is $f = c/\lambda$, where c is the speed of light and λ is the wavelength of the wave. The frequencies of the two light sources are then

Red: $$f_{\text{red}} = \frac{c}{\lambda_{\text{red}}} = \frac{3.00 \times 10^8 \text{ m/s}}{660 \times 10^{-9} \text{ m}} = \boxed{4.55 \times 10^{14} \text{ Hz}}$$

and

Infrared: $$f_{\text{IR}} = \frac{c}{\lambda_{\text{IR}}} = \frac{3.00 \times 10^8 \text{ m/s}}{940 \times 10^{-9} \text{ m}} = \boxed{3.19 \times 10^{14} \text{ Hz}}$$

(b) The intensity of an electromagnetic wave is proportional to the square of its amplitude. If 67% of the incident intensity of the red light is absorbed, then the intensity of the

continued on next page

emerging wave is $(100\% - 67\%) = 33\%$ of the incident intensity, or $I_f = 0.33 I_i$. Hence, we must have

$$\frac{E_{\max,f}}{E_{\max,i}} = \sqrt{\frac{I_f}{I_i}} = \sqrt{0.33} = \boxed{0.57}$$

21.54 If I_0 is the incident intensity of a light beam, and I is the intensity of the beam after passing through length L of a fluid having concentration C of absorbing molecules, the Beer-Lambert law states that $\log_{10}(I/I_0) = -\in CL$, where $\in$ is a constant.

For 660-nm light, the absorbing molecules are oxygenated hemoglobin. Thus, if 33% of this wavelength light is transmitted through blood, the concentration of oxygenated hemoglobin in the blood is

$$C_{\text{HBO2}} = \frac{-\log_{10}(0.33)}{\in L} \qquad \textbf{[1]}$$

The absorbing molecules for 940-nm light are deoxygenated hemoglobin, so if 76% of this light is transmitted through the blood, the concentration of these molecules in the blood is

$$C_{\text{HB}} = \frac{-\log_{10}(0.76)}{\in L} \qquad \textbf{[2]}$$

Dividing Equation [2] by Equation [1] gives the ratio of deoxygenated hemoglobin to oxygenated hemoglobin in the blood as

$$\frac{C_{\text{HB}}}{C_{\text{HBO2}}} = \frac{\log_{10}(0.76)}{\log_{10}(0.33)} = 0.25 \quad \text{or} \quad C_{\text{HB}} = 0.25 C_{\text{HBO2}}$$

Since all the hemoglobin in the blood is either oxygenated or deoxygenated, it is necessary that $C_{\text{HB}} + C_{\text{HBO2}} = 1.00$, and we now have $0.25 C_{\text{HBO2}} + C_{\text{HBO2}} = 1.0$. The fraction of hemoglobin that is oxygenated in this blood is then

$$C_{\text{HBO2}} = \frac{1.0}{1.0 + 0.25} = 0.80 \quad \text{or} \quad \boxed{80\%}$$

Someone with only 80% oxygenated hemoglobin in the blood is probably in serious trouble, needing supplemental oxygen immediately.

21.55 From $\mathit{Intensity} = \dfrac{E_{\max} B_{\max}}{2\mu_0}$ and $\dfrac{E_{\max}}{B_{\max}} = c$, we find $\mathit{Intensity} = \dfrac{c B_{\max}^2}{2\mu_0}$.

Thus,

$$B_{\max} = \sqrt{\frac{2\mu_0}{c}(\mathit{Intensity})} = \sqrt{\frac{2(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})}{3.00 \times 10^8 \text{ m/s}}(1\,370 \text{ W/m}^2)} = \boxed{3.39 \times 10^{-6} \text{ T}}$$

and $E_{\max} = B_{\max} c = (3.39 \times 10^{-6} \text{ T})(3.00 \times 10^8 \text{ m/s}) = \boxed{1.02 \times 10^3 \text{ V/m}}$

21.56 (a) To exert an upward force on the disk, the laser beam should be aimed vertically upward, striking the lower surface of the disk. To just levitate the disk, the upward force exerted on the disk by the beam should equal the weight of the disk.

continued on next page

The momentum that electromagnetic radiation of intensity I, incident normally on a perfectly reflecting surface of area A, delivers to that surface in time Δt is given by Equation 21.30 in the textbook as $\Delta p = 2U/c = 2(IA\Delta t)/c$. Thus, from the impulse-momentum theorem, the average force exerted on the reflecting surface is $F = \Delta p/\Delta t = 2IA/c$. To just levitate the surface, $F = 2IA/c = mg$, and the required intensity of the incident radiation is $\boxed{I = mgc/2A}$.

(b) $$I = \frac{mgc}{2A} = \frac{mgc}{2\pi r^2} = \frac{(5.00\times10^{-3}\ \text{kg})(9.80\ \text{m/s}^2)(3.00\times10^{8}\ \text{m/s})}{2\pi(4.00\times10^{-2}\ \text{m})^2} = \boxed{1.46\times10^{9}\ \text{W/m}^2}$$

(c) Propulsion by light pressure in a significant gravity field is impractical because of the enormous power requirements. In addition, no material is perfectly reflecting, so the absorbed energy would melt the reflecting surface.

21.57 The distance between adjacent antinodes in a standing wave is $\lambda/2$.

Thus, $\lambda = 2(6.00\ \text{cm}) = 12.0\ \text{cm} = 0.120\ \text{m}$, and

$$c = \lambda f = (0.120\ \text{m})(2.45\times10^{9}\ \text{Hz}) = \boxed{2.94\times10^{8}\ \text{m/s}}$$

21.58 (a) The intensity of the radiation at distance $r = 20.0$ ly from the star is

$$I = \frac{P}{4\pi r^2} = \frac{4.00\times10^{28}\ \text{W}}{4\pi\left[(20.0\ \text{ly})(9.461\times10^{15}\ \text{m/ly})\right]^2} = \boxed{8.89\times10^{-8}\ \text{W/m}^2}$$

(b) The power of the starlight intercepted by Earth, with cross-sectional area of $A_{cs} = \pi R_E^2$, is

$$P = IA_{cs} = I\cdot\pi R_E^2 = (8.89\times10^{-8}\ \text{W/m}^2)\pi(6.38\times10^{6}\ \text{m})^2$$

or $P = 1.14\times10^{7}\ \text{W} = \boxed{11.4\ \text{MW}}$

21.59 (a) $$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{5.00\times10^{19}\ \text{Hz}} = 6.00\times10^{-12}\ \text{m} = \boxed{6.00\ \text{pm}}$$

(b) $$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{4.00\times10^{9}\ \text{Hz}} = 7.50\times10^{-2}\ \text{m} = \boxed{7.50\ \text{cm}}$$

21.60 $$\lambda = \frac{c}{f} = \frac{3.00\times10^{8}\ \text{m/s}}{27.33\times10^{6}\ \text{Hz}} = \boxed{11.0\ \text{m}}$$

21.61 (a) For the AM band,

$$\lambda_{\min} = \frac{c}{f_{\max}} = \frac{3.00\times10^{8}\ \text{m/s}}{1\,600\times10^{3}\ \text{Hz}} = \boxed{188\ \text{m}}$$

$$\lambda_{\max} = \frac{c}{f_{\min}} = \frac{3.00\times10^{8}\ \text{m/s}}{540\times10^{3}\ \text{Hz}} = \boxed{556\ \text{m}}$$

(b) For the FM band,

$$\lambda_{\min} = \frac{c}{f_{\max}} = \frac{3.00\times10^{8}\ \text{m/s}}{108\times10^{6}\ \text{Hz}} = \boxed{2.78\ \text{m}}$$

$$\lambda_{\max} = \frac{c}{f_{\min}} = \frac{3.00\times10^{8}\ \text{m/s}}{88\times10^{6}\ \text{Hz}} = \boxed{3.4\ \text{m}}$$

21.62 The transit time for the radio wave is

$$t_R = \frac{d_R}{c} = \frac{100 \times 10^3 \text{ m}}{3.00 \times 10^8 \text{ m/s}} = 3.33 \times 10^{-4} \text{ s} = 0.333 \text{ ms}$$

and that for the sound wave is

$$t_s = \frac{d_s}{v_{\text{sound}}} = \frac{3.0 \text{ m}}{343 \text{ m/s}} = 8.7 \times 10^{-3} \text{ s} = 8.7 \text{ ms}$$

Thus, $\boxed{\text{the radio listeners hear the news 8.4 ms before the studio audience}}$ because radio waves travel so much faster than sound waves.

21.63 If an object of mass m is attached to a spring of spring constant k, the natural frequency of vibration of that system is $f = \sqrt{k/m}/2\pi$. Thus, the resonance frequency of the C=O double bond will be

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m_{\substack{\text{oxygen}\\ \text{atom}}}}} = \frac{1}{2\pi}\sqrt{\frac{2\,800 \text{ N/m}}{2.66 \times 10^{-26} \text{ kg}}} = \boxed{5.2 \times 10^{13} \text{ Hz}}$$

and the light with this frequency has wavelength

$$\lambda = \frac{c}{f} = \frac{3.00 \times 10^8 \text{ m/s}}{5.2 \times 10^{13} \text{ Hz}} = 5.8 \times 10^{-6} \text{ m} = \boxed{5.8 \ \mu\text{m}}$$

The infrared region of the electromagnetic spectrum ranges from $\lambda_{\text{max}} \approx 1$ mm down to $\lambda_{\text{min}} = 700$ nm $= 0.7 \ \mu$m. Thus, $\boxed{\text{the computed wavelength falls within the infrared region}}$.

21.64 (a) Since the space station and the ship are moving toward one another, the frequency after being Doppler shifted is $f_O = f_S(1 + u/c)$, where u is the relative speed between the observer and source. Thus,

$$f_O = \left(6.000\,0 \times 10^{14} \text{ Hz}\right)\left[1 + \frac{1.800\,0 \times 10^5 \text{ m/s}}{3.000\,0 \times 10^8 \text{ m/s}}\right] = \boxed{6.003\,6 \times 10^{14} \text{ Hz}}$$

(b) The change in frequency is

$$\Delta f = f_O - f_S = 6.003\,6 \times 10^{14} \text{ Hz} - 6.000\,0 \times 10^{14} = \boxed{3.6 \times 10^{11} \text{ Hz}}$$

21.65 Since you and the car ahead of you are moving away from each other (getting farther apart) at a rate of $u = 120 \text{ km/h} - 80 \text{ km/h} = 40 \text{ km/h}$, the Doppler-shifted frequency you will detect is $f_O = f_S(1 - u/c)$, and the change in frequency is

$$\Delta f = f_O - f_S = -f_S\left(\frac{u}{c}\right) = -\left(4.3 \times 10^{14} \text{ Hz}\right)\left(\frac{40 \text{ km/h}}{3.0 \times 10^8 \text{ m/s}}\right)\left(\frac{0.278 \text{ m/s}}{1 \text{ km/h}}\right) = \boxed{-1.6 \times 10^7 \text{ Hz}}$$

The frequency you will detect will be

$$f_O = f_S + \Delta f = 4.3 \times 10^{14} \text{ Hz} - 1.6 \times 10^7 \text{ Hz} = \boxed{4.299\,999\,84 \times 10^{14} \text{ Hz}}$$

21.66 The driver was driving toward the warning lights, so the correct form of the Doppler shift equation is $f_O = f_S(1 + u/c)$. The frequency emitted by the yellow warning light is

$$f_S = \frac{c}{\lambda_S} = \frac{3.00 \times 10^8 \text{ m/s}}{580 \times 10^{-9} \text{ m}} = 5.17 \times 10^{14} \text{ Hz}$$

continued on next page

and the frequency the driver claims that she observed is

$$f_o = \frac{c}{\lambda_o} = \frac{3.00\times10^8 \text{ m/s}}{560\times10^{-9} \text{ m}} = 5.36\times10^{14} \text{ Hz}$$

The speed with which she would have to approach the light for the Doppler effect to yield this claimed shift is

$$u = c\left(\frac{f_O}{f_S} - 1\right) = \left(3.00\times10^8 \text{ m/s}\right)\left(\frac{5.36\times10^{14} \text{ Hz}}{5.17\times10^{14} \text{ Hz}} - 1\right) = \boxed{1.1\times10^7 \text{ m/s}}$$

21.67 The energy incident on the mirror in time Δt is $U = P_{\text{laser}} \cdot \Delta t$, where P_{laser} is the power transmitted by the laser beam

$$P_{\text{laser}} = 25.0\times10^{-3} \text{ W} = 25.0\times10^{-3} \text{ J/s} = 25.0\times10^{-3} \text{ N}\cdot\text{m/s}$$

From Equation 21.30 in the textbook, the rate of change in the momentum of the mirror as the beam reflects from it is

$$\frac{\Delta p}{\Delta t} = \frac{2U/c}{\Delta t} = \frac{2P_{\text{laser}}}{c}$$

The impulse-momentum theorem then gives the force exerted on the mirror as $F = \Delta p/\Delta t = 2P_{\text{laser}}/c$, and the radiation pressure on the mirror is

$$P_{\text{rad}} = \frac{F}{A} = \frac{2P_{\text{laser}}/c}{A} = \frac{2P_{\text{laser}}}{cA}$$

where $A = \pi r^2 = \pi d^2/4$ is the area of the mirror illuminated (i.e., the cross-sectional area of the laser beam). Thus,

$$P_{\text{rad}} = \frac{2P_{\text{laser}}}{c\left(\pi d^2/4\right)} = \frac{8P_{\text{laser}}}{\pi c d^2} = \frac{8\left(25.0\times10^{-3} \text{ N}\cdot\text{m/s}\right)}{\pi\left(3.00\times10^8 \text{ m/s}\right)\left(2.00\times10^{-3} \text{ m}\right)^2}$$

or $\quad P_{\text{rad}} = 5.31\times10^{-5} \text{ N/m}^2 = \boxed{5.31\times10^{-5} \text{ Pa}}$

21.68 Suppose you cover a 1.7 m-by-0.3 m section of beach blanket. Suppose the elevation angle of the Sun is 60°. Then the target area you fill in the Sun's field of view is $(1.7 \text{ m})(0.3 \text{ m})\cos 30° = 0.4 \text{ m}^2$.

The intensity the radiation at Earth's surface is $I_{\text{surface}} = 0.6 I_{\text{incoming}}$, and only 50% of this is absorbed. Since $I = P_{\text{av}}/A = (\Delta E/\Delta t)/A$, the absorbed energy is

$$\Delta E = \left(0.5 I_{\text{surface}}\right)A(\Delta t) = \left[0.5\left(0.6 I_{\text{incoming}}\right)\right]A(\Delta t)$$

$$= (0.5)(0.6)\left(1\,370 \text{ W/m}^2\right)\left(0.4 \text{ m}^2\right)\left(3\,600 \text{ s}\right) = 6\times10^5 \text{ J} \quad \text{or} \quad \boxed{\sim10^6 \text{ J}}$$

21.69 $Z = \sqrt{R^2 + \left(X_C\right)^2} = \sqrt{R^2 + (2\pi f C)^{-2}}$

$$= \sqrt{(200\ \Omega)^2 + \left[2\pi(60 \text{ Hz})\left(5.0\times10^{-6} \text{ F}\right)\right]^{-2}} = 5.7\times10^2\ \Omega$$

Thus, $\quad P_{\text{av}} = I_{\text{rms}}^2 R = \left(\frac{\Delta V_{\text{rms}}}{Z}\right)^2 R = \left(\frac{120 \text{ V}}{5.7\times10^2\ \Omega}\right)^2 (200\ \Omega) = 8.9 \text{ W} = 8.9\times10^{-3} \text{ kW}$

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and $cost = \Delta E \cdot (rate) = P_{av} \cdot \Delta t \cdot (rate)$

$$= (8.9 \times 10^{-3}\ \text{kW})(24\ \text{h})(8.0\ \text{cents/kWh}) = \boxed{1.7\ \text{cents}}$$

21.70 For a parallel-plate capacitor, $C = \in A/d$: cutting the plate separation d in half will double the capacitance $(C_f = 2C_i)$. Since the capacitive reactance is $X_C = 1/(2\pi f C)$, doubling the capacitance reduces the capacitive reactance by a factor of 2, giving $X_{C,f} = X_{C,i}/2$.

The current in the circuit is $I_{rms} = \Delta V_{rms}/Z$, so the impedance must be cut in half $(Z_f = Z_i/2)$ if the current doubles while the applied voltage remains constant. With an inductive reactance equal to the resistance $(X_L = R)$, we then have

$$\sqrt{R^2 + (R - X_{C,f})^2} = \frac{1}{2}\sqrt{R^2 + (R - X_{C,i})^2} \qquad \text{or} \qquad 4\left[R^2 + (R - X_{C,i}/2)^2\right] = R^2 + (R - X_{C,i})^2$$

Simplifying gives $6R^2 = 2RX_{C,i}$, and $X_{C,i} = 6R^2/2R = \boxed{3R}$.

21.71 $R = \dfrac{(\Delta V)_{DC}}{I_{DC}} = \dfrac{12.0\ \text{V}}{0.630\ \text{A}} = 19.0\ \Omega$

$$Z = \sqrt{R^2 + (2\pi f L)^2} = \frac{\Delta V_{rms}}{I_{rms}} = \frac{24.0\ \text{V}}{0.570\ \text{A}} = 42.1\ \Omega$$

Thus, $L = \dfrac{\sqrt{Z^2 - R^2}}{2\pi f} = \dfrac{\sqrt{(42.1\ \Omega)^2 - (19.0\ \Omega)^2}}{2\pi(60.0\ \text{Hz})} = 9.97 \times 10^{-2}\ \text{H} = \boxed{99.7\ \text{mH}}$

21.72 (a) The frequency of a 3.0-cm radar wave, and hence the desired resonance frequency of the tuning circuit, $f_0 = 1/2\pi\sqrt{LC}$, is

$$f_0 = f = \frac{c}{\lambda} = \frac{3.00 \times 10^8\ \text{m/s}}{3.00 \times 10^{-2}\ \text{m}} = 1.0 \times 10^{10}\ \text{Hz}$$

Therefore, the required capacitance is

$$C = \frac{1}{(2\pi f_0)^2 L} = \frac{1}{(2\pi \times 10^{10}\ \text{Hz})^2 (400 \times 10^{-12}\ \text{H})} = 6.3 \times 10^{-13}\ \text{F} = \boxed{0.63\ \text{pF}}$$

(b) $C = \dfrac{\in_0 A}{d} = \dfrac{\in_0 \ell^2}{d}$, so

$$\ell = \sqrt{\frac{C \cdot d}{\in_0}} = \sqrt{\frac{(6.3 \times 10^{-13}\ \text{F})(1.0 \times 10^{-3}\ \text{m})}{8.85 \times 10^{-12}\ \text{C}^2/\text{N}\cdot\text{m}}} = 8.4 \times 10^{-3}\ \text{m} = \boxed{8.4\ \text{mm}}$$

(c) $X_C = X_L = (2\pi f_0)L = 2\pi(1.0 \times 10^{10}\ \text{Hz})(400 \times 10^{-12}\ \text{H}) = \boxed{25\ \Omega}$

21.73 (a) $\dfrac{E_{max}}{B_{max}} = c$, so $B_{max} = \dfrac{E_{max}}{c} = \dfrac{0.20 \times 10^{-6}\ \text{V/m}}{3.00 \times 10^8\ \text{m/s}} = \boxed{6.7 \times 10^{-16}\ \text{T}}$

(b) $Intensity = \dfrac{E_{max} B_{max}}{2\mu_0} = \dfrac{(0.20 \times 10^{-6}\ \text{V/m})(6.7 \times 10^{-16}\ \text{T})}{2(4\pi \times 10^{-7}\ \text{T}\cdot\text{m/A})} = \boxed{5.3 \times 10^{-17}\ \text{W/m}^2}$

(c) $P_{av} = (Intensity) \cdot A = (Intensity)\left[\dfrac{\pi d^2}{4}\right]$

$$= (5.3 \times 10^{-17}\ \text{W/m}^2)\left[\frac{\pi(20.0\ \text{m})^2}{4}\right] = \boxed{1.7 \times 10^{-14}\ \text{W}}$$

21.74 (a) $Z = \dfrac{\Delta V_{\text{rms}}}{I_{\text{rms}}} = \dfrac{12\text{ V}}{2.0\text{ A}} = \boxed{6.0\ \Omega}$

(b) $R = \dfrac{\Delta V_{\text{DC}}}{I_{\text{DC}}} = \dfrac{12\text{ V}}{3.0\text{ A}} = 4.0\ \Omega$

From $Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (2\pi f L)^2}$, we find

$$L = \frac{\sqrt{Z^2 - R^2}}{2\pi f} = \frac{\sqrt{(6.0\ \Omega)^2 - (4.0\ \Omega)^2}}{2\pi(60\text{ Hz})} = 1.2\times 10^{-2}\text{ H} = \boxed{12\text{ mH}}$$

21.75 (a) From Equation 21.30 in the textbook, the momentum imparted in time Δt to a perfectly reflecting sail of area A by normally incident radiation of intensity I is $\Delta p = 2U/c = 2(IA\Delta t)/c$.

From the impulse-momentum theorem, the average force exerted on the sail is then

$$F_{\text{av}} = \frac{\Delta p}{\Delta t} = \frac{2(IA\Delta t)/c}{\Delta t} = \frac{2IA}{c} = \frac{2(1\,340\text{ W/m}^2)(6.00\times 10^4\text{ m}^2)}{3.00\times 10^8\text{ m/s}} = \boxed{0.536\text{ N}}$$

(b) $a_{\text{av}} = \dfrac{F_{\text{av}}}{m} = \dfrac{0.536\text{ N}}{6\,000\text{ kg}} = \boxed{8.93\times 10^{-5}\text{ m/s}^2}$

(c) From $\Delta x = v_0 t + \dfrac{1}{2}at^2$, with $v_0 = 0$, the time is

$$t = \sqrt{\frac{2(\Delta x)}{a_{\text{av}}}} = \sqrt{\frac{2(3.84\times 10^8\text{ m})}{8.93\times 10^{-5}\text{ m/s}^2}} = (2.93\times 10^6\text{ s})\left(\frac{1\text{ d}}{8.64\times 10^4\text{ s}}\right) = \boxed{33.9\text{ d}}$$

21.76 (a) The intensity of radiation at distance r from a point source, which radiates total power P, is $I = P/A = P/4\pi r^2$. Thus, at distance $r = 2.0$ in from a cell phone radiating a total power of $P = 2.0\text{ W} = 2.0\times 10^3\text{ mW}$, the intensity is

$$I = \frac{2.0\times 10^3\text{ mW}}{4\pi\left[(2.0\text{ in})(2.54\text{ cm/1 in})\right]^2} = \boxed{6.2\text{ mW/cm}^2}$$

$\boxed{\text{This intensity is 24\% higher than the maximum allowed leakage from a microwave}}$ at this distance of 2.0 inches.

(b) If when using a Bluetooth headset (emitting 2.5 mW of power) in the ear at distance $r_h = 2.0\text{ in} = 5.1\text{ cm}$ from the brain, the cell phone (emitting 2.0 W of power) is located in the pocket at distance $r_p = 1.0\text{ m} = 1.0\times 10^2\text{ cm}$ from the brain, the total radiation intensity at the brain is

$$I_{\text{total}} = I_{\text{phone}} + I_{\text{headset}} = \frac{2.0\times 10^3\text{ mW}}{4\pi(1.0\times 10^2\text{ cm})^2} + \frac{2.5\text{ mW}}{4\pi(5.1\text{ cm})^2}$$

or $\quad I_{\text{total}} = 1.6\times 10^{-2}\ \dfrac{\text{mW}}{\text{cm}^2} + 7.6\times 10^{-3}\ \dfrac{\text{mW}}{\text{cm}^2} = 2.4\times 10^{-2}\ \dfrac{\text{mW}}{\text{cm}^2} = \boxed{0.024\text{ mW/cm}^2}$

22

Reflection and Refraction of Light

QUICK QUIZZES

1. Choice (a). In part (a), you can see clear reflections of the headlights and the lights on the top of the truck. The reflection is specular. In part (b), although bright areas appear on the roadway in front of the headlights, the reflection is not as clear, and no separate reflection of the lights from the top of the truck is visible. The reflection in part (b) is mostly diffuse.

2. Beams 2 and 4 are reflected; beams 3 and 5 are refracted.

3. Choice (b). When light goes from one material into one having a higher index of refraction, it refracts toward the normal line of the boundary between the two materials. If, as the light travels through the new material, the index of refraction continues to increase, the light ray will refract more and more toward the normal line.

4. Choice (c). Both the wave speed and the wavelength decrease as the index of refraction increases. The frequency is unchanged.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The index of refraction of a material in which light has speed v is $n = c/v = c/\lambda f$. Thus,

$$\frac{n_{\text{liquid}}}{n_{\text{air}}} = \frac{c/(\lambda_{\text{liquid}} f)}{c/(\lambda_{\text{air}} f)} = \frac{\lambda_{\text{air}}}{\lambda_{\text{liquid}}} \quad \text{and} \quad n_{\text{liquid}} = \left(\frac{\lambda_{\text{air}}}{\lambda_{\text{liquid}}}\right) n_{\text{air}} = \left(\frac{495 \text{ nm}}{434 \text{ nm}}\right)(1.00) = 1.14$$

so the correct choice is (c).

2. The critical angle as light passes from medium 1 into medium 2 is $\theta_c = \sin^{-1}(n_2/n_1)$, so as light is incident on crown glass ($n_2 = 1.52$) from carbon disulfide ($n_1 = 1.63$),

$$\theta_c = \sin^{-1}\left(\frac{1.52}{1.63}\right) = 68.8°$$

and choice (b) is the correct response.

3. The energy of a photon is $E = hf = hc/\lambda$. Thus, if n 800-nm photons have the same energy as four 200-nm photons, it is necessary that

$$n(hc/800 \text{ nm}) = 4(hc/200 \text{ nm}) \quad \text{or} \quad n = 4(800 \text{ nm}/200 \text{ nm}) = 16$$

Therefore, the correct answer is (e).

4. Observe that the angle of refraction is greater than the angle of incidence as the light ray passes from medium 1 into medium 2. Thus, the speed of light increases as the light crosses the boundary between these materials. Since $n = c/v$, the index of refraction of medium 2 is less than that of medium 1, or $n_1 > n_2$. The correct choice is (d).

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5. Total internal reflection will occur when light, in attempting to go from a medium with one index of refraction n_1 into a second medium where it travels faster than in the first medium (or where $n_2 < n_1$), strikes the surface at an angle of incidence greater than or equal to the critical angle. The correct choice is (b).

6. Water and air have different indices of refraction, with $n_{\text{water}} \approx 4n_{\text{air}}/3$. In passing from one of these media into the other, light will be refracted (deviated in direction) unless the angle of incidence is zero (in which case, the angle of refraction is also zero). Thus, rays B and D cannot be correct. In refraction, the incident ray and the refracted ray are never on the same side of the line normal to the surface at the point of contact, so ray A cannot be correct. Also in refraction, the ray makes a smaller angle with the normal in the medium having the highest index of refraction. Therefore, ray E cannot be correct, leaving only ray C as a likely path. Choice (c) is the correct answer.

7. When light is in water, the relationships between the values of its frequency, speed, and wavelength to the values of the same quantities in air are

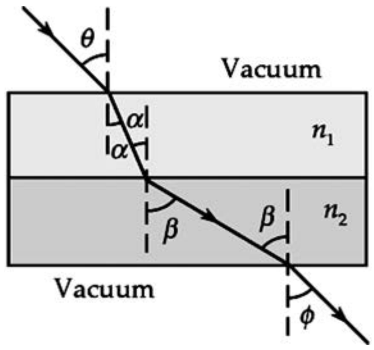
$$f_{\text{water}} = f_{\text{air}}, \quad \lambda_{\text{water}} = \left(\frac{n_{\text{air}}}{n_{\text{water}}}\right)\lambda_{\text{air}} \approx \frac{3}{4}\lambda_{\text{air}}, \quad \text{and} \quad v_{\text{water}} = \left(\frac{n_{\text{air}}}{n_{\text{water}}}\right)v_{\text{air}} = \left(\frac{3}{4}\right)c$$

Therefore, only choice (b) is a completely true statement.

8. In a dispersive medium, the index of refraction is largest for the shortest wavelength. Thus, the violet light will be refracted (or bent) the most as it passes through a surface of the crown glass, making (a) the correct choice.

9. For any medium, other than vacuum, the index of refraction for red light is slightly lower than that for blue light. This means that when light goes from vacuum (or air) into glass, the red light deviates from its original direction less than does the blue light. Also, as the light reemerges from the glass into vacuum (or air), the red light again deviates less than the blue light. If the two surfaces of the glass are parallel to each other, the red and blue rays will emerge traveling parallel to each other but displaced laterally from one another. The sketch that best illustrates this process is C, so choice (c) is the best answer.

10. Consider the sketch at the right and apply Snell's law to the refraction at each of the three surfaces. The resulting equations are



$$(1.00)\sin\theta = n_1 \sin\alpha \qquad (1^{\text{st}} \text{ surface})$$

$$n_1 \sin\alpha = n_2 \sin\beta \qquad (2^{\text{nd}} \text{ surface})$$

and $$n_2 \sin\beta = (1.00)\sin\phi \qquad (3^{\text{rd}} \text{ surface})$$

Combining these three equations yields $(1.00)\sin\phi = (1.00)\sin\theta$, and $\phi = \theta$. Hence, choice (c) is the correct answer.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. (a) From Snell's law, $\sin\theta_2 = (n_1/n_2)\sin\theta_1$. Thus, if $n_2 < n_1$, then $\sin\theta_2 > \sin\theta_1$ and $\theta_2 > \theta_1$, so the ray refracts away from the normal.

(b) The index of refraction is defined as $n = c/v = c/\lambda f$. The wavelength may then be written as $\lambda = c/nf = (c/f)/n = \lambda_0/n$, where λ_0 is the wavelength of the light in vacuum. Thus, as the ray moves into a medium of lower index of refraction, the wavelength will increase.

(c) The frequency at which wavefronts move away from a boundary equals the frequency with which they arrive at the boundary. That is, the frequency of the light stays the same as it moves between the two materials.

4. A mirage occurs when light changes direction as it moves between batches of air having different indices of refraction. The different indices of refraction occur because the air has different densities at different temperatures. Two images are seen: one from a direct path from the object to you, and the second arriving by rays originally heading toward Earth but refracted to your eye. On a hot day, the Sun makes the surface of asphalt hot, so the air is hot directly above it, becoming cooler as one moves higher into the sky. The "water" we see far in front of us is an image of the blue sky. Adding to the effect is the fact that the image shimmers as the air changes in temperature, giving the appearance of moving water.

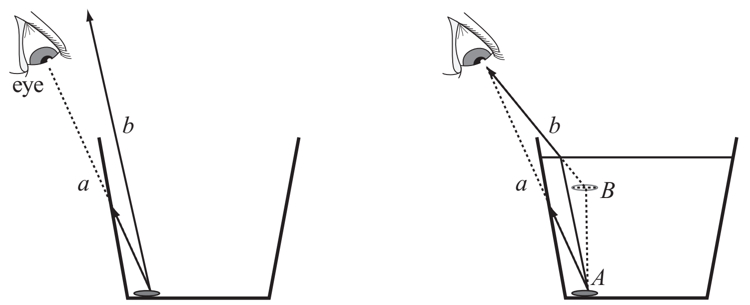
6. The upright image of the hill is formed by light that has followed a direct path from the hill to the eye of the observer. The second image is a result of refraction in the atmosphere. Some light is reflected from the hill toward the water. As this light passes through warmer layers of air directly above the water, it is refracted back up toward the eye of the observer, resulting in the observation of an inverted image of the hill directly below the upright image.

8. The index of refraction of water is 1.333, quite different from that of air, which has an index of refraction of about 1. The boundary between the air and water is therefore easy to detect, because of the differing refraction effects above and below the boundary. (Try looking at a glass half full of water.) The index of refraction of liquid helium, however, happens to be much closer to that of air. Consequently, the refractive differences above and below the helium-air boundary are harder to see.

10. Total internal reflection can only occur when light attempts to move from a medium of high index of refraction to a medium of lower index of refraction. Thus, light moving from air ($n = 1$) to water ($n = 1.333$) cannot undergo total internal reflection.

12. With no water in the cup, light rays from the coin do not reach the eye because they are blocked by the side of the cup. With water in the cup, light rays are bent away from the normal as they leave the water so that some reach the eye.

In figure (a) below, ray *a* is blocked by the side of the cup so it cannot enter the eye, and ray *b* misses the eye. In figure (b), ray *a* is still blocked by the side of the cup, but ray *b* refracts at the water's surface so that it reaches the eye. Ray *b* seems to come from position B, directly above the coin at position A.



(a) (b)

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 1.99×10^{-15} J **(b)** 12.4 keV

(c) decreased **(d)** increased

4. **(a)** 550 nm **(b)** 366 nm

(c) 2.26 eV **(d)** The energy does not change.

6. **(a)** $\lambda = hc/E$ **(b)** Higher energy photons have shorter wavelengths.

8. **(a)** 1.94 m **(b)** 50.0° above the horizontal

10. See Solution.

12. **(a)** The angle of refraction increases with wavelength, so the longest wavelength deviates the least from the original path.

(b) $\lambda = 400$ nm, $\theta_r = 16.0°$; $\lambda = 500$ nm, $\theta_r = 16.1°$; $\lambda = 650$ nm, $\theta_r = 16.3°$

14. 80.0°

16. **(a)** *B* **(b)** *A*, *B*, and *C*

18. **(a)** $\theta_{r,\text{top}} = 19.5°$ **(b)** $\theta_{i,\text{bottom}} = 19.5°$, $\theta_{r,\text{bottom}} = 30.0°$

(c) 0.386 cm **(d)** 2.00×10^{8} m/s **(e)** 1.06×10^{-10} s

(f) The angle of incidence affects the distance the ray travels in the glass and hence affects the travel time.

20. 1.22

22. 6.30 cm

24. **(a)** See Solution. **(b)** 42.0° **(c)** 63.1°

(d) 26.9° **(e)** 107 m

26. See Solution.

28. 0.39°

30. $\theta_{\text{red}} = 34.9°$, $\theta_{\text{violet}} = 34.5°$

32. 4.5°

34. **(a)** 43.3° **(b)** 42.2° **(c)** 40.4°

36. 48.5°

38. 67.3°

40. **(a)** 34.2° **(b)** 34.2°

(c) and **(d)** Neither thickness nor index of refraction affects the result.

42. **(a)** 24.42° **(b)** See Solution.

(c) 33.44° **(d)** Total internal reflection still occurs.

(e) rotate clockwise **(f)** 2.9°

44. 4.54 m

46. **(a)** 23.7° **(b)** $\theta_r \to \theta_i = 30.0°$ **(c)** $\theta_r > \theta_i = 30.0°$

48. **(a)** $\theta_{\text{incidence}} \le 90.0°$ **(b)** $\theta_{\text{incidence}} \le 29.9°$

(c) Total internal reflection is not possible since $n_{\text{polystyrene}} < n_{\text{carbon disulfide}}$.

50. 77.5°

52. **(a)** $R_{\min} = nd/(n-1)$

(b) $R_{\min} \to 0$ as $d \to 0$; $R_{\min}$ decreases as n increases; $R_{\min}$ increases as $n \to 1$. All are reasonable behaviors.

(c) $R_{\min} = 350\ \mu\text{m}$

54. $\phi = 8.0°$

56. 82 complete reflections

58. 1.33

60. 23.1°

PROBLEM SOLUTIONS

22.1 The total distance the light travels is

$$\Delta d = 2\left(D_{\substack{\text{center to}\\ \text{center}}} - R_{\text{Earth}} - R_{\text{Moon}}\right)$$

$$= 2\left(3.84\times10^{8} - 6.38\times10^{6} - 1.76\times10^{6}\right)\text{ m} = 7.52\times10^{8}\text{ m}$$

Therefore, $v = \dfrac{\Delta d}{\Delta t} = \dfrac{7.52\times10^{8}\text{ m}}{2.51\text{ s}} = \boxed{3.00\times10^{8}\text{ m/s}}$

22.2 (a) The energy of a photon is $E = hf = hc/\lambda$, where Planck's constant is $h = 6.63\times10^{-34}\text{ J}\cdot\text{s}$, and the speed of light in vacuum is $c = 3.00\times10^{8}\text{ m/s}$. If $\lambda = 1.00\times10^{-10}\text{ m}$,

$$E = \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{1.00\times10^{-10}\text{ m}} = \boxed{1.99\times10^{-15}\text{ J}}$$

continued on next page

(b) $E = \left(1.99\times10^{-15}\ \text{J}\right)\left(\dfrac{1\ \text{eV}}{1.602\times10^{-19}\ \text{J}}\right) = 1.24\times10^{4}\ \text{eV} = \boxed{12.4\ \text{keV}}$

(c) and (d) For the x-rays to be more penetrating, the photons should be more energetic. Since the energy of a photon is directly proportional to the frequency and inversely proportional to the wavelength, $\boxed{\text{the wavelength should decrease}}$, which is the same as saying $\boxed{\text{the frequency should increase}}$.

22.3 (a) $E = hf = \left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(5.00\times10^{17}\ \text{Hz}\right)\left(\dfrac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{2.07\times10^{3}\ \text{eV}} = \boxed{2.07\ \text{keV}}$

(b) $E = hf = \dfrac{hc}{\lambda} = \dfrac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{3.00\times10^{2}\ \text{nm}}\left(\dfrac{1\ \text{nm}}{10^{-9}\ \text{m}}\right) = 6.63\times10^{-19}\ \text{J}$

$$E = 6.63\times10^{-19}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{4.14\ \text{eV}}$$

22.4 (a) $\lambda_0 = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\ \text{m/s}}{5.45\times10^{14}\ \text{Hz}} = 5.50\times10^{-7}\ \text{m} = \boxed{550\ \text{nm}}$

(b) From Table 22.1, the index of refraction for benzene is $n = 1.501$. Thus, the wavelength in benzene is

$$\lambda_n = \frac{\lambda_0}{n} = \frac{550\ \text{nm}}{1.501} = \boxed{366\ \text{nm}}$$

(c) $E = hf = \left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(5.45\times10^{14}\ \text{Hz}\right)\left(\dfrac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{2.26\ \text{eV}}$

(d) The energy of the photon is proportional to the frequency, which does not change as the light goes from one medium to another. Thus, when the photon enters benzene, $\boxed{\text{the energy does not change}}$.

22.5 The speed of light in a medium with index of refraction n is $v = c/n$, where c is its speed in vacuum.

(a) For water, $n = 1.333$, and $v = \dfrac{3.00\times10^{8}\ \text{m/s}}{1.333} = \boxed{2.25\times10^{8}\ \text{m/s}}$.

(b) For crown glass, $n = 1.52$, and $v = \dfrac{3.00\times10^{8}\ \text{m/s}}{1.52} = \boxed{1.97\times10^{8}\ \text{m/s}}$.

(c) For diamond, $n = 2.419$, and $v = \dfrac{3.00\times10^{8}\ \text{m/s}}{2.419} = \boxed{1.24\times10^{8}\ \text{m/s}}$.

22.6 (a) The energy of a photon is

$$E = hf = h\left(\frac{c}{\lambda}\right) = \frac{hc}{\lambda}$$

Thus, $\boxed{\lambda = hc/E}$

(b) $\boxed{\text{Higher energy photons have shorter wavelengths.}}$

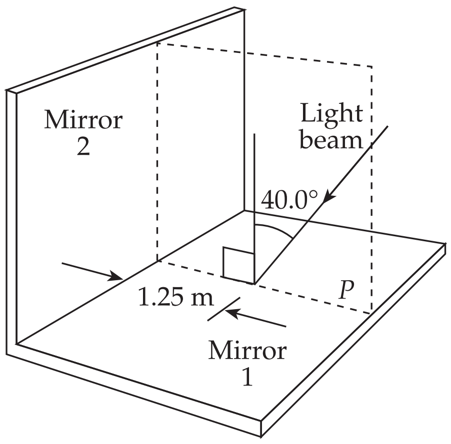
22.7 From Snell's law, $n_2 \sin\theta_2 = n_1 \sin\theta_1$. Thus, when $\theta_1 = 45.0°$ and the first medium is air ($n_1 = 1.00$), we have $\sin\theta_2 = (1.00)\sin 45.0°/n_2$.

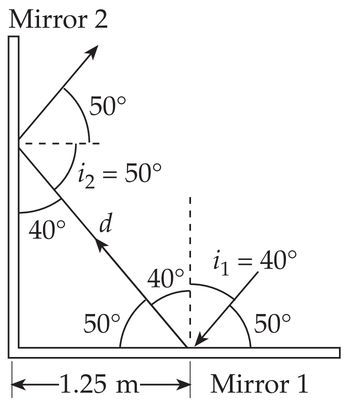
(a) For quartz, $n_2 = 1.458$, and $\theta_2 = \sin^{-1}\left(\dfrac{(1.00)\sin 45.0°}{1.458}\right) = \boxed{29.0°}$.

(b) For carbon disulfide, $n_2 = 1.628$, and $\theta_2 = \sin^{-1}\left(\dfrac{(1.00)\sin 45.0°}{1.628}\right) = \boxed{25.7°}$.

(c) For water, $n_2 = 1.333$, and $\theta_2 = \sin^{-1}\left(\dfrac{(1.00)\sin 45.0°}{1.333}\right) = \boxed{32.0°}$.

22.8





(a) From geometry, $1.25 \text{ m} = d\sin 40.0°$, so $d = \boxed{1.94 \text{ m}}$.

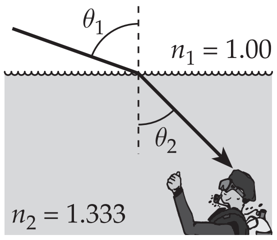
(b) $\boxed{50.0° \text{ above horizontal}}$, or parallel to the incident ray

22.9 $n_1 \sin\theta_1 = n_2 \sin\theta_2$

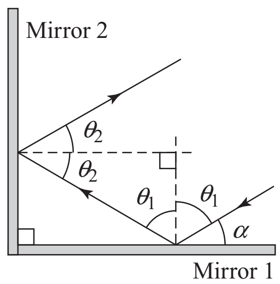
$\sin\theta_1 = 1.333 \sin 45.0°$

$\theta_1 = \sin^{-1}(1.333 \sin 45.0°) = 70.5°$

Thus, the sun appears to be $\boxed{19.5° \text{ above the horizontal}}$.



22.10 In the sketch at the right, observe that the law of reflection is obeyed as the ray reflects from each of the mirrors. Also, note that the normal lines to the two mirrors intersect at a right angle since the two mirrors are perpendicular to each other. Considering the right triangle formed by the two normal lines and the ray, and recalling that the sum of the interior angles of any triangle is 180°, we find that



$$\theta_1 + \theta_2 + 90.0° = 180° \qquad \text{or} \qquad \theta_2 = 90.0° - \theta_1$$

Looking at the point where the incident ray strikes mirror 1, we see that $\alpha = 90° - \theta_1$. Thus, both the incident ray and the final outgoing reflected ray are at angle $90° - \theta_1$ above the horizontal and hence, $\boxed{\text{parallel to each other}}$.

22.11 (a) From Snell's law, $n_2 = \dfrac{n_1 \sin\theta_1}{\sin\theta_2} = \dfrac{(1.00)\sin 30.0^\circ}{\sin 19.24^\circ} = \boxed{1.52}$.

(b) $\lambda_2 = \dfrac{\lambda_0}{n_2} = \dfrac{632.8 \text{ nm}}{1.52} = \boxed{416 \text{ nm}}$

(c) $f = \dfrac{c}{\lambda_0} = \dfrac{3.00\times10^8 \text{ m/s}}{632.8\times10^{-9} \text{ m}} = \boxed{4.74\times10^{14} \text{ Hz}}$ in the air and also in the solution

(d) $v_2 = \dfrac{c}{n_2} = \dfrac{3.00\times10^8 \text{ m/s}}{1.52} = \boxed{1.97\times10^8 \text{ m/s}}$

22.12 (a) When light refracts from air $(n_1 = 1.00)$ into the crown glass, Snell's law gives the angle of refraction as

$$\theta_2 = \sin^{-1}\left(\sin 25.0^\circ / n_{\text{crown glass}}\right)$$

For first quadrant angles, the sine of the angle increases as the angle increases. Thus, from the above equation, note that θ_2 will increase when the index of refraction of the crown glass decreases. From Figure 22.13, we see that the angle of refraction will increase with wavelength, so the longer wavelengths $\boxed{\text{deviate the least}}$ from the original path.

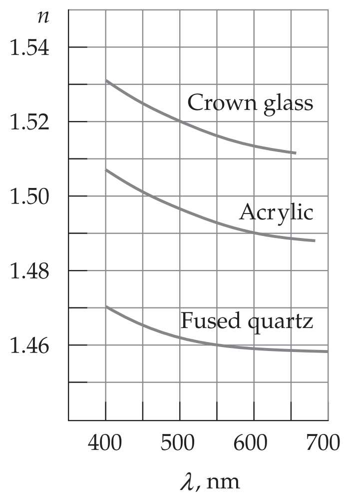


Figure 22.13

(b) From Figure 22.13, observe that the index of refraction of crown glass for the given wavelengths is

$$\lambda = 400 \text{ nm}, \; n_{\text{crown glass}} = 1.53; \qquad \lambda = 500 \text{ nm}, \; n_{\text{crown glass}} = 1.52;$$

and $\lambda = 650 \text{ nm}, \; n_{\text{crown glass}} = 1.51$

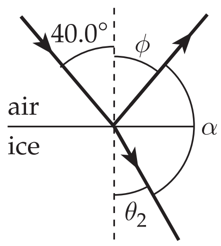
Thus, Snell's law gives

$$\lambda = 400 \text{ nm:} \quad \theta_2 = \sin^{-1}(\sin 25.0^\circ/1.53) = \boxed{16.0^\circ}$$

$$\lambda = 500 \text{ nm:} \quad \theta_2 = \sin^{-1}(\sin 25.0^\circ/1.52) = \boxed{16.1^\circ}$$

$$\lambda = 650 \text{ nm:} \quad \theta_2 = \sin^{-1}(\sin 25.0^\circ/1.51) = \boxed{16.3^\circ}$$

22.13 From Snell's law,



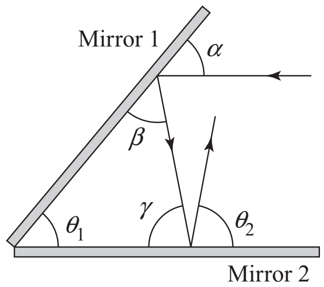
$$\theta_2 = \sin^{-1}\left[\frac{n_1 \sin\theta_1}{n_2}\right] = \sin^{-1}\left[\frac{(1.00)\sin 40.0^\circ}{1.309}\right] = 29.4^\circ$$

and from the law of reflection, $\phi = \theta_1 = 40.0^\circ$.

Hence, the angle between the reflected and refracted rays is

$$\alpha = 180^\circ - \theta_2 - \phi = 180.0^\circ - 29.4^\circ - 40.0^\circ = \boxed{110.6^\circ}$$

22.14 Consider the sketch at the right and note that the incident horizontal ray is parallel to the surface of mirror 2. Thus, the angle the incident ray makes with mirror 1 must be $\alpha = \theta_1 = 50.0°$. Since the ray must obey the law of reflection at mirror 1, the angle β must be $\beta = \alpha = 50.0°$. Recalling that the sum of the interior angles of a triangle is always 180.0°, we find that



$$\gamma = 180.0° - \theta_1 - \beta = 180.0° - 50.0° - 50.0° = 80.0°$$

Hence, in order to obey the law of reflection at mirror 2, the angle the outgoing reflected ray makes with the surface of mirror 2 must be $\theta_2 = \gamma = \boxed{80.0°}$.

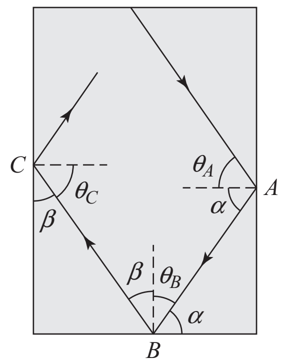
22.15 The index of refraction of zircon is $n = 1.923$.

(a) $v = \dfrac{c}{n} = \dfrac{3.00\times10^8\ \text{m/s}}{1.923} = \boxed{1.56\times10^8\ \text{m/s}}$

(b) The wavelength in the zircon is $\lambda_n = \dfrac{\lambda_0}{n} = \dfrac{632.8\ \text{nm}}{1.923} = \boxed{329.1\ \text{nm}}$.

(c) The frequency is $f = \dfrac{v}{\lambda_n} = \dfrac{c}{\lambda_0} = \dfrac{3.00\times10^8\ \text{m/s}}{632.8\times10^{-9}\ \text{m}} = \boxed{4.74\times10^{14}\ \text{Hz}}$.

22.16 The sketch at the right shows the path of the ray inside the glass slab. Considering the reflections at points A and B, the law of reflection tells us that $\alpha = \theta_A$ and $\beta = \theta_B$. Also, we observe that



$$\theta_B = 90° - \alpha = 90° - \theta_A$$

and $\theta_C = 90° - \beta = 90° - \theta_B = 90° - \left(90° - \theta_A\right) = \theta_A$

Thus, if $\theta_A = 55°$, then $\theta_B = 35°$ and $\theta_C = 55°$.

In order for part of the ray to leave the glass slab and enter the surrounding medium at a reflection point, the angle of incidence at that point must be less than the critical angle, $\theta_c = \sin^{-1}(n_2/n_{\text{glass}}) = \sin^{-1}(n_2/1.52)$.

(a) If the surrounding medium is air, then $n_2 = 1.00$ and $\theta_c = \sin^{-1}(1.00/1.52) = 41.1°$. Thus, we see that total internal reflection will occur at points A and C, but part of the ray can refract into the surrounding air at $\boxed{\text{point } B}$.

(b) When the surrounding medium is carbon disulfide, $n_2 = 1.628 > n_{\text{glass}}$. Thus, the critical angle does not exist, and total internal reflection will not occur when the ray attempts to go from the glass into the carbon disulfide. This means that part of the ray will enter the carbon disulfide at $\boxed{\text{points } A,\ B,\ \text{and } C}$.

22.17 The incident light reaches the left-hand mirror at distance

$$d/2 = (1.00\ \text{m})\tan 5.00° = 0.087\,5\ \text{m}$$

above its bottom edge. The reflected light first reaches the right-hand mirror at height

$$d = 2\left(0.087\,5\ \text{m}\right) = 0.175\ \text{m}$$

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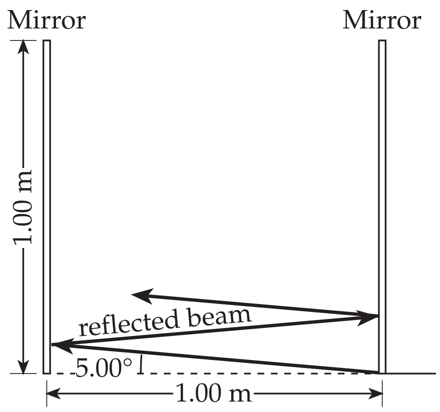
It bounces between the mirrors with distance d between points of contact with a given mirror.



Since the full 1.00 length of the right-hand mirror is available for reflections, the number of reflections from this mirror will be

$$N_{\text{right}} = \frac{1.00 \text{ m}}{0.175 \text{ m}} = 5.71 \quad \rightarrow \quad \boxed{5 \text{ full reflections}}$$

Since the first reflection from the left-hand mirror occurs at a height of $d/2 = 0.087\,5$ m, the total number of that can occur from this mirror is

$$N_{\text{left}} = 1 + \frac{1.00 \text{ m} - 0.087\,5 \text{ m}}{0.175 \text{ m}} = 6.21 \quad \rightarrow \quad \boxed{6 \text{ full reflections}}$$

22.18 (a) From Snell's law, the angle of refraction at the first surface is

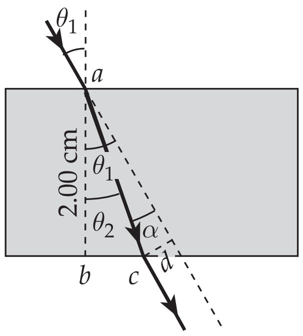
$$\theta_2 = \sin^{-1}\left[\frac{n_{\text{air}} \sin\theta_1}{n_{\text{glass}}}\right] = \sin^{-1}\left[\frac{(1.00)\sin 30.0°}{1.50}\right] = \boxed{19.5°}$$

(b) Since the upper and lower surfaces are parallel, the normal lines where the ray strikes these surfaces are parallel. Hence, the angle of incidence at the lower surface will be $\theta_2 = \boxed{19.5°}$. The angle of refraction at this surface is then

$$\theta_3 = \sin^{-1}\left[\frac{n_{\text{glass}} \sin\theta_{\text{glass}}}{n_{\text{air}}}\right] = \sin^{-1}\left[\frac{(1.50)\sin 19.5°}{1.00}\right] = \boxed{30.0°}$$

Thus, the light emerges traveling parallel to the incident beam.

(c) Consider the sketch at the right, and let h represent the distance from point a to c (that is, the hypotenuse of triangle abc). Then,



$$h = \frac{2.00 \text{ cm}}{\cos\theta_2} = \frac{2.00 \text{ cm}}{\cos 19.5°} = 2.12 \text{ cm}$$

Also, $\alpha = \theta_1 - \theta_2 = 30.0° - 19.5° = 10.5°$, so

$$d = h\sin\alpha = (2.12 \text{ cm})\sin 10.5° = \boxed{0.386 \text{ cm}}$$

(d) The speed of the light in the glass is

$$v = \frac{c}{n_{\text{glass}}} = \frac{3.00 \times 10^8 \text{ m/s}}{1.50} = \boxed{2.00 \times 10^8 \text{ m/s}}$$

(e) The time required for the light to travel through the glass is

$$t = \frac{h}{v} = \frac{2.12 \text{ cm}}{2.00 \times 10^8 \text{ m/s}}\left(\frac{1 \text{ m}}{10^2 \text{ cm}}\right) = \boxed{1.06 \times 10^{-10} \text{ s}}$$

(f) Changing the angle of incidence will change the angle of refraction and therefore the distance h the light travels in the glass. Thus, $\boxed{\text{the travel time will also change}}$.

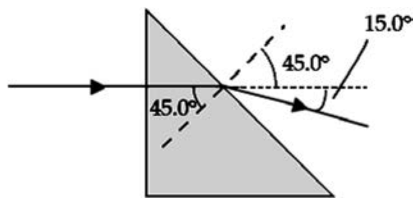
22.19 From Snell's law, the angle of incidence at the air-oil interface is

$$\theta = \sin^{-1}\left[\frac{n_{\text{oil}}\sin\theta_{\text{oil}}}{n_{\text{air}}}\right] = \sin^{-1}\left[\frac{(1.48)\sin 20.0^\circ}{1.00}\right] = \boxed{30.4^\circ}$$

and the angle of refraction as the light enters the water is

$$\theta' = \sin^{-1}\left[\frac{n_{\text{oil}}\sin\theta_{\text{oil}}}{n_{\text{water}}}\right] = \sin^{-1}\left[\frac{(1.48)\sin 20.0^\circ}{1.333}\right] = \boxed{22.3^\circ}$$

22.20 Since the light ray strikes the first surface at normal incidence, it passes into the prism without deviation. Thus, the angle of incidence at the second surface (hypotenuse of the triangular prism) is $\theta_1 = 45.0^\circ$ as shown in the sketch at the right. The angle of refraction is

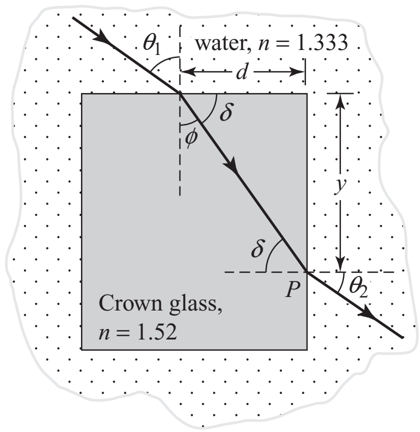


$$\theta_2 = 45.0^\circ + 15.0^\circ = 60.0^\circ$$

and Snell's law gives the index of refraction of the prism material as

$$n_1 = \frac{n_2\sin\theta_2}{\sin\theta_1} = \frac{(1.00)\sin(60.0^\circ)}{\sin(45.0^\circ)} = \boxed{1.22}$$

22.21 Applying Snell's law where the ray first enters the glass gives



$$\phi = \sin^{-1}\left(\frac{n_{\text{water}}\sin\theta_1}{n_{\text{glass}}}\right) = \sin^{-1}\left[\frac{(1.333)\sin 42.0^\circ}{1.52}\right] = 35.9^\circ$$

Thus, $\delta = 90.0^\circ - \phi = 90.0^\circ - 35.9^\circ = 54.1^\circ$.

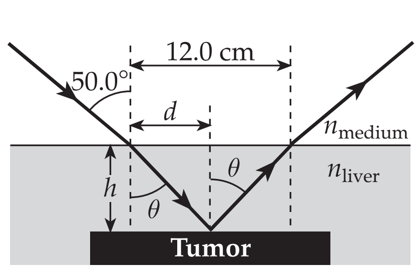
(a) The distance down to point P, where the ray emerges from the glass, is now seen to be

$$y = d\tan\delta = (3.50\text{ cm})\tan 54.1^\circ = \boxed{4.84\text{ cm}}$$

(b) The angle of refraction as the ray leaves the block is given by Snell's law as

$$\theta_2 = \sin^{-1}\left(\frac{n_{\text{glass}}\sin\delta}{n_{\text{water}}}\right) = \sin^{-1}\left[\frac{(1.52)\sin 54.1^\circ}{1.333}\right] = \boxed{67.5^\circ}$$

22.22 From Snell's law, $\sin\theta = \left(\dfrac{n_{\text{medium}}}{n_{\text{liver}}}\right)\sin 50.0^\circ$.



But $\dfrac{n_{\text{medium}}}{n_{\text{liver}}} = \dfrac{c/v_{\text{medium}}}{c/v_{\text{liver}}} = \dfrac{v_{\text{liver}}}{v_{\text{medium}}} = 0.900$

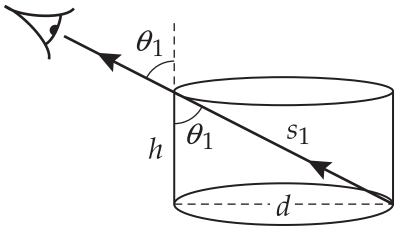
so $\theta = \sin^{-1}\left[(0.900)\sin 50.0^\circ\right] = 43.6^\circ$

continued on next page

From the law of reflection,

$$d = \frac{12.0\text{ cm}}{2} = 6.00\text{ cm} \qquad \text{and} \qquad h = \frac{d}{\tan\theta} = \frac{6.00\text{ cm}}{\tan(43.6°)} = \boxed{6.30\text{ cm}}$$

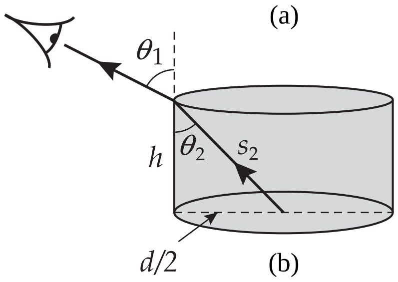
22.23 (a) Before the container is filled, the ray's path is as shown in figure (a) at the right. From this figure, observe that



(a)

$$\sin\theta_1 = \frac{d}{s_1} = \frac{d}{\sqrt{h^2+d^2}} = \frac{1}{\sqrt{(h/d)^2+1}}$$

After the container is filled, the ray's path is shown in figure (b). From this figure, we find that



(b)

$$\sin\theta_2 = \frac{d/2}{s_2} = \frac{d/2}{\sqrt{h^2+(d/2)^2}} = \frac{1}{\sqrt{4(h/d)^2+1}}$$

From Snell's law, $n_{\text{air}}\sin\theta_1 = n\sin\theta_2$, or

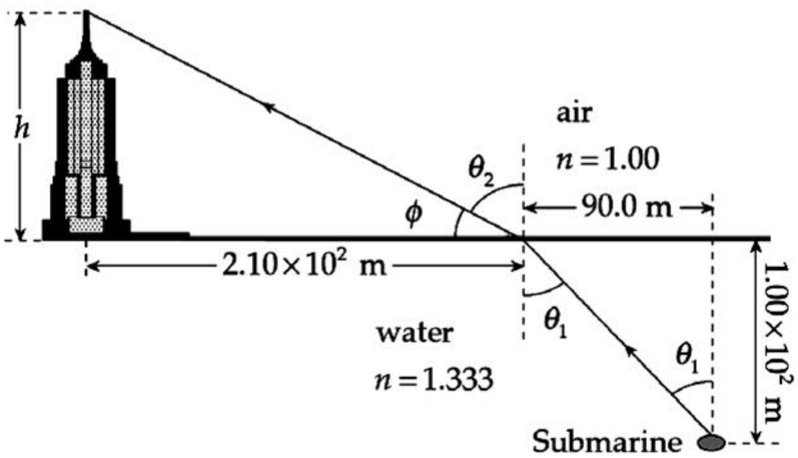
$$\frac{1.00}{\sqrt{(h/d)^2+1}} = \frac{n}{\sqrt{4(h/d)^2+1}} \qquad \text{and} \qquad 4(h/d)^2+1 = n^2(h/d)^2+n^2$$

Simplifying, this gives $(4-n^2)(h/d)^2 = n^2-1$ or $\boxed{\dfrac{h}{d} = \sqrt{\dfrac{n^2-1}{4-n^2}}}$

(b) If $d = 8.00$ cm and $n = n_{\text{water}} = 1.333$, then

$$h = (8.00\text{ cm})\sqrt{\frac{(1.333)^2-1}{4-(1.333)^2}} = \boxed{4.73\text{ cm}}$$

22.24 (a) A sketch illustrating the situation and the two triangles needed in the solution is given below:



(b) The angle of incidence at the water surface is

$$\theta_1 = \tan^{-1}\left(\frac{90.0\text{ m}}{1.00\times10^2\text{ m}}\right) = \boxed{42.0°}$$

continued on next page

(c) Snell's law gives the angle of refraction as

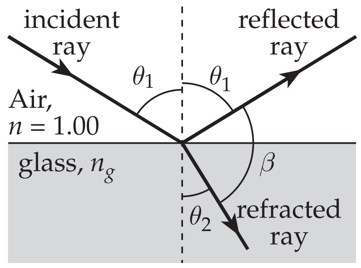
$$\theta_2 = \sin^{-1}\left(\frac{n_{\text{water}}\sin\theta_1}{n_{\text{air}}}\right) = \sin^{-1}\left(\frac{(1.333)\sin 42.0°}{1.00}\right) = \boxed{63.1°}$$

(d) The refracted beam makes angle $\phi = 90.0° - \theta_2 = \boxed{26.9°}$ with the horizontal.

(e) Since $\tan\phi = h/(2.10\times10^2\text{ m})$, the height of the target is

$$h = \left(2.10\times10^2\text{ m}\right)\tan(26.9°) = \boxed{107\text{ m}}$$

22.25 As shown at the right, $\theta_1 + \beta + \theta_2 = 180°$.



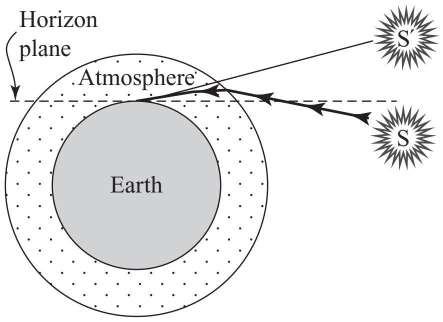
When $\beta = 90°$, this gives $\theta_2 = 90° - \theta_1$.

Then, from Snell's law

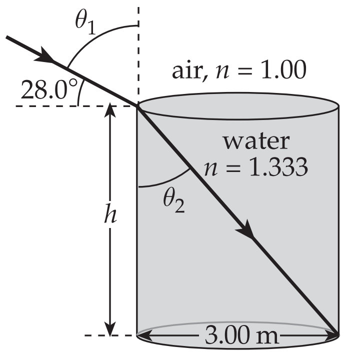
$$\sin\theta_1 = \frac{n_g \sin\theta_2}{n_{\text{air}}}$$
$$= n_g \sin(90° - \theta_1) = n_g \cos\theta_1$$

Thus, when $\beta = 90°$, $\dfrac{\sin\theta_1}{\cos\theta_1} = \tan\theta_1 = n_g$ or $\boxed{\theta_1 = \tan^{-1}(n_g)}$.

22.26 The index of refraction of the atmosphere decreases with increasing altitude because of the decrease in density of the atmosphere with increasing altitude. As indicated in the ray diagram at the right, the Sun located at S below the horizon appears to be located at S′.



22.27 When the Sun is 28.0° above the horizon, the angle of incidence for sunlight at the air-water boundary is



$$\theta_1 = 90.0° - 28.0° = 62.0°$$

Thus, the angle of refraction is

$$\theta_2 = \sin^{-1}\left[\frac{n_{\text{air}}\sin\theta_1}{n_{\text{water}}}\right]$$
$$= \sin^{-1}\left[\frac{(1.00)\sin 62.0°}{1.333}\right] = 41.5°$$

The depth of the tank is then $h = \dfrac{3.00\text{ m}}{\tan\theta_2} = \dfrac{3.00\text{ m}}{\tan(41.5°)} = \boxed{3.39\text{ m}}$.

22.28 The angles of refraction for the two wavelengths are

$$\theta_{red} = \sin^{-1}\left(\frac{n_{air}\sin\theta_1}{n_{red}}\right) = \sin^{-1}\left(\frac{1.000\sin 30.00°}{1.615}\right) = 18.03°$$

and $$\theta_{blue} = \sin^{-1}\left(\frac{n_{air}\sin\theta_1}{n_{blue}}\right) = \sin^{-1}\left(\frac{1.000\sin 30.00°}{1.650}\right) = 17.64°$$

Thus, the angle between the two refracted rays is

$$\Delta\theta = \theta_{red} - \theta_{blue} = 18.03° - 17.64° = \boxed{0.39°}$$

22.29 Using Snell's law gives

(a) $$\theta_{blue} = \sin^{-1}\left(\frac{n_{air}\sin\theta_i}{n_{blue}}\right) = \sin^{-1}\left(\frac{(1.000)\sin 83.00°}{1.340}\right) = \boxed{47.79°}$$

(b) $$\theta_{red} = \sin^{-1}\left(\frac{n_{air}\sin\theta_i}{n_{red}}\right) = \sin^{-1}\left(\frac{(1.000)\sin 83.00°}{1.331}\right) = \boxed{48.22°}$$

22.30 Using Snell's law gives

$$\theta_{red} = \sin^{-1}\left(\frac{n_{air}\sin\theta_i}{n_{red}}\right) = \sin^{-1}\left(\frac{(1.00)\sin 60.0°}{1.512}\right) = \boxed{34.9°}$$

and $$\theta_{violet} = \sin^{-1}\left(\frac{n_{air}\sin\theta_i}{n_{violet}}\right) = \sin^{-1}\left(\frac{(1.00)\sin 60.0°}{1.530}\right) = \boxed{34.5°}$$

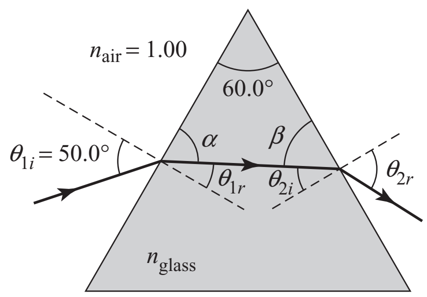
22.31 Using Snell's law gives

$$\theta_{red} = \sin^{-1}\left(\frac{n_{air}\sin\theta_i}{n_{red}}\right) = \sin^{-1}\left(\frac{(1.000)\sin 50.00°}{1.455}\right) = 31.77°$$

and $$\theta_{violet} = \sin^{-1}\left(\frac{n_{air}\sin\theta_i}{n_{violet}}\right) = \sin^{-1}\left(\frac{(1.000)\sin 50.00°}{1.468}\right) = 31.45°$$

Thus, the dispersion is $\theta_{red} - \theta_{violet} = 31.77° - 31.45° = \boxed{0.32°}$.

22.32 For the violet light, $n_{glass} = 1.66$, and



$$\theta_{1r} = \sin^{-1}\left(\frac{n_{air}\sin\theta_{1i}}{n_{glass}}\right)$$

$$= \sin^{-1}\left(\frac{1.00\sin 50.0°}{1.66}\right) = 27.5°$$

$\alpha = 90° - \theta_{1r} = 62.5°$, $\beta = 180.0° - 60.0° - \alpha = 57.5°$,

and $\theta_{2i} = 90.0° - \beta = 32.5°$. The final angle of refraction of the violet light is

$$\theta_{2r} = \sin^{-1}\left(\frac{n_{glass}\sin\theta_{2i}}{n_{air}}\right) = \sin^{-1}\left(\frac{1.66\sin 32.5°}{1.00}\right) = 63.1°$$

continued on next page

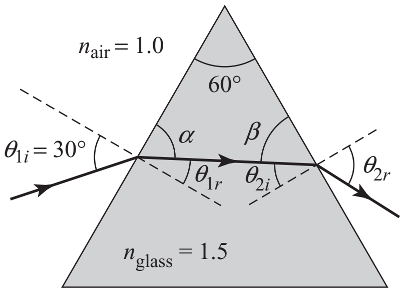
Following the same steps for the red light $(n_{\text{glass}} = 1.62)$ gives

$$\theta_{1r} = 28.2°,\ \alpha = 61.8°,\ \beta = 58.2°,\ \theta_{2i} = 31.8°, \text{ and } \theta_{2r} = 58.6°$$

Thus, the angular dispersion of the emerging light is

$$Dispersion = \theta_{2r}\big|_{\text{violet}} - \theta_{2r}\big|_{\text{red}} = 63.1° - 58.6° = \boxed{4.5°}$$

22.33 (a) The angle of incidence at the first surface is $\boxed{\theta_{1i} = 30°}$, and the angle of refraction is



$$\theta_{1r} = \sin^{-1}\left(\frac{n_{\text{air}} \sin\theta_{1i}}{n_{\text{glass}}}\right) = \sin^{-1}\left(\frac{1.0 \sin 30°}{1.5}\right) = \boxed{19°}$$

Also, $\alpha = 90° - \theta_{1r} = 71°$ and $\beta = 180° - 60° - \alpha = 49°$.

Therefore, the angle of incidence at the second surface is $\theta_{2i} = 90° - \beta = \boxed{41°}$. The angle of refraction at this surface is

$$\theta_{2r} = \sin^{-1}\left(\frac{n_{\text{glass}} \sin\theta_{2i}}{n_{\text{air}}}\right) = \sin^{-1}\left(\frac{1.5 \sin 41°}{1.0}\right) = \boxed{80°}$$

(b) The angle of reflection at each surface equals the angle of incidence at that surface. Thus,

$$(\theta_1)_{\text{reflection}} = \theta_{1i} = \boxed{30°}, \text{ and } (\theta_2)_{\text{reflection}} = \theta_{2i} = \boxed{41°}$$

22.34 As light goes from a medium having a refractive index n_1 to a medium with refractive index $n_2 < n_1$, the critical angle is given by the relation $\sin\theta_c = n_2/n_1$. Table 22.1 gives the refractive index for various substances at $\lambda_0 = 589$ nm.

(a) For fused quartz surrounded by air, $n_1 = 1.458$ and $n_2 = 1.00$, giving $\theta_c = \sin^{-1}(1.00/1.458) = \boxed{43.3°}$.

(b) In going from polystyrene ($n_1 = 1.49$) to air, $\theta_c = \sin^{-1}(1.00/1.49) = \boxed{42.2°}$.

(c) From sodium chloride ($n_1 = 1.544$) to air, $\theta_c = \sin^{-1}(1.00/1.544) = \boxed{40.4°}$.

22.35 When light is coming from a medium of refractive index n_1 into water ($n_2 = 1.333$), the critical angle is given by $\theta_c = \sin^{-1}(1.333/n_1)$.

(a) For fused quartz, $n_1 = 1.458$, giving $\theta_c = \sin^{-1}(1.333/1.458) = \boxed{66.1°}$.

(b) In going from polystyrene ($n_1 = 1.49$) to water, $\theta_c = \sin^{-1}(1.333/1.49) = \boxed{63.5°}$.

(c) From sodium chloride ($n_1 = 1.544$) to water, $\theta_c = \sin^{-1}(1.333/1.544) = \boxed{59.7°}$.

22.36 Using Snell's law, the index of refraction of the liquid is found to be

$$n_{\text{liquid}} = (n_{\text{air}} \sin\theta_i)/\sin\theta_r$$

continued on next page

Thus, the critical angle for light going from this liquid into air is

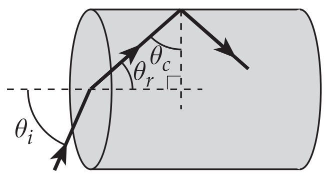
$$\theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{liquid}}}\right) = \sin^{-1}\left[\frac{\cancel{n_{\text{air}}}}{(\cancel{n_{\text{air}}}\sin\theta_i)/\sin\theta_r}\right] = \sin^{-1}\left[\frac{\sin 22.0^\circ}{\sin 30.0^\circ}\right] = \boxed{48.5^\circ}$$

22.37 When light attempts to cross a boundary from one medium of refractive index n_1 into a new medium of refractive index $n_2 < n_1$, total internal reflection will occur if the angle of incidence exceeds the critical angle given by $\theta_c = \sin^{-1}(n_2/n_1)$.

(a) If $n_1 = 1.53$ and $n_2 = n_{\text{air}} = 1.00$, then $\theta_c = \sin^{-1}\left(\dfrac{1.00}{1.53}\right) = \boxed{40.8^\circ}$.

(b) If $n_1 = 1.53$ and $n_2 = n_{\text{water}} = 1.333$, then $\theta_c = \sin^{-1}\left(\dfrac{1.333}{1.53}\right) = \boxed{60.6^\circ}$.

22.38 The critical angle for this material in air is

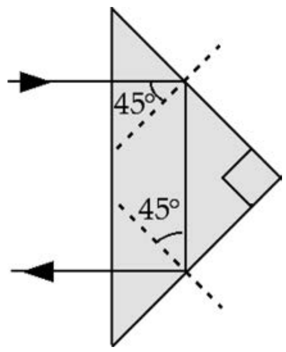


$$\theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{pipe}}}\right) = \sin^{-1}\left(\frac{1.00}{1.36}\right) = 47.3^\circ$$

Thus, $\theta_r = 90.0^\circ - \theta_c = 42.7^\circ$ and from Snell's law,

$$\theta_i = \sin^{-1}\left(\frac{n_{\text{pipe}}\sin\theta_r}{n_{\text{air}}}\right) = \sin^{-1}\left(\frac{(1.36)\sin 42.7^\circ}{1.00}\right) = \boxed{67.3^\circ}$$

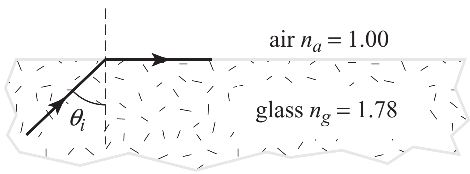
22.39 The angle of incidence at each of the shorter faces of the prism is 45°, as shown in the figure at the right. For total internal reflection to occur at these faces, it is necessary that the critical angle be less than 45°. With the prism surrounded by air, the critical angle is given by $\sin\theta_c = n_{\text{air}}/n_{\text{prism}} = 1.00/n_{\text{prism}}$, so it is necessary that



$$\sin\theta_c = \frac{1.00}{n_{\text{prism}}} < \sin 45^\circ$$

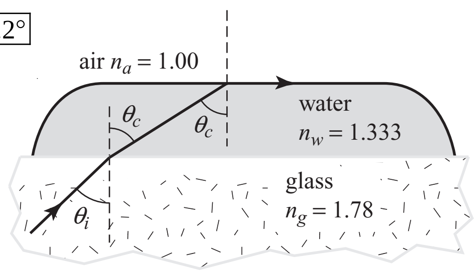
or $\quad n_{\text{prism}} > \dfrac{1.00}{\sin 45^\circ} = \dfrac{1.00}{\sqrt{2}/2} = \boxed{\sqrt{2}}$

22.40 (a) The minimum angle of incidence for which total internal reflection occurs is the critical angle. At the critical angle, the angle of refraction is 90°, as shown in the figure at the right. From Snell's law, $n_g \sin\theta_i = n_a \sin 90^\circ$, the critical angle for the glass-air interface is found to be



$$\theta_i = \theta_c = \sin^{-1}\left(\frac{n_a \sin 90^\circ}{n_g}\right) = \sin^{-1}\left(\frac{1.00}{1.78}\right) = \boxed{34.2^\circ}$$

(b) When the slab of glass has a layer of water on top, we want the angle of incidence at the water-air interface to equal the critical angle for that combination of media. At this angle, Snell's law gives



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$$n_w \sin\theta_c = n_a \sin 90° = 1.00$$

and $\sin\theta_c = 1.00/n_w$

Now, considering the refraction at the glass-water interface, Snell's law gives $n_g \sin\theta_i = n_g \sin\theta_c$. Combining this with the result for $\sin\theta_c$ from above, we find the required angle of incidence in the glass to be

$$\theta_i = \sin^{-1}\left(\frac{n_w \sin\theta_c}{n_g}\right) = \sin^{-1}\left(\frac{n_w\left(1.00/n_w\right)}{n_g}\right) = \sin^{-1}\left(\frac{1.00}{n_g}\right) = \sin^{-1}\left(\frac{1.00}{1.78}\right) = \boxed{34.2°}$$

(c) and (d) Observe in the calculation of part (b) that all the physical properties of the intervening layer (water in this case) canceled, and the result of part (b) is identical to that of part (a). This will always be true when the upper and lower surfaces of the intervening layer are parallel to each other. Neither the thickness nor the index of refraction of the intervening layer affects the result.

22.41 (a) Snell's law can be written as $\sin\theta_1/\sin\theta_2 = v_1/v_2$. At the critical angle of incidence $\left(\theta_1 = \theta_c\right)$, the angle of refraction is 90°, and Snell's law becomes $\sin\theta_c = v_1/v_2$. At the concrete-air boundary,

$$\theta_c = \sin^{-1}\left(\frac{v_1}{v_2}\right) = \sin^{-1}\left(\frac{343 \text{ m/s}}{1850 \text{ m/s}}\right) = \boxed{10.7°}$$

(b) Sound can be totally reflected only if it is initially traveling in the slower medium. Hence, at the concrete-air boundary, the sound must be traveling in $\boxed{\text{air}}$.

(c) $\boxed{\text{Sound in air falling on the wall from most directions is 100\% reflected}}$, so the wall is a good mirror.

22.42 (a) The index of refraction for diamond is $n_{\text{diamond}} = 2.419$, and the critical angle at a diamond-air boundary is

$$\theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{diamond}}}\right) = \sin^{-1}\left(\frac{1.000}{2.419}\right) = \boxed{24.42°}$$

(b) With the face of the diamond tilted up 35.0° from the horizontal, the normal line to this face at point *P* is tipped over 35.0° from the vertical. Thus, the angle of incidence at point *P* is $\theta_i = 35.0° > \theta_c = 24.42°$. Since this angle of incidence exceeds the critical angle, total internal reflection occurs at point *P*.

(c) If the diamond is immersed in water ($n_{\text{water}} = 1.333$), the critical angle is

$$\theta_c = \sin^{-1}\left(\frac{n_{\text{water}}}{n_{\text{diamond}}}\right) = \sin^{-1}\left(\frac{1.333}{2.419}\right) = \boxed{33.44°}$$

(d) The light continues to enter the top surface of the diamond at normal incidence, so the angle of incidence at point *P* continues to be $\theta_i = 35.0° > \theta_c = 33.44°$. Since this angle of incidence (barely) exceeds the critical angle for the diamond-water boundary, $\boxed{\text{total internal reflection still occurs at point } P}$.

(e) To have light exit the diamond at point *P*, we need to decrease the angle of incidence at this point. Thus, we should $\boxed{\text{rotate the diamond clockwise}}$, thereby bringing the normal line closer to the incident ray at point *P*.

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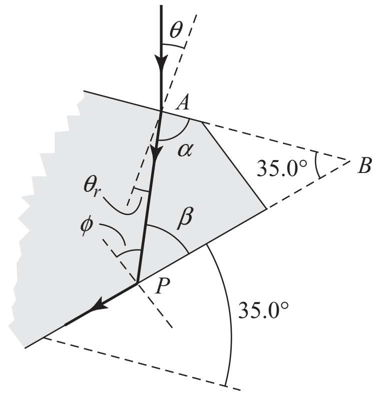
(f) Rotating the diamond clockwise by an angle θ changes the angle of refraction at point A where the ray enters the diamond. To find what this new angle of refraction will be, we extend the line of the top of the diamond and the line of the face containing point P until they intersect at point B as shown at the right. Summing the interior angles in triangle ABP gives



$$\alpha + \beta + 35.0° = 180°$$

or $$(90.0° - \theta_r) + (90.0° - \phi) + 35.0° = 180°$$

and $$\theta_r = 35.0° - \phi$$

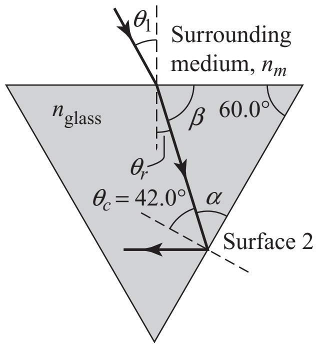
If the ray is to exit the diamond and enter the water at point P, the angle of incidence at this point must be less than or equal to the critical angle found in part (c). Thus, we require $\phi \le 33.44°$ and see that we must have

$$\theta_r \ge 35.0° - 33.44° = 1.6°$$

Applying Snell's law at point A gives the minimum required rotation as

$$\theta_{\min} = \sin^{-1}\left(\frac{n_{\text{diamond}}\sin\theta_r}{n_{\text{water}}}\right) = \sin^{-1}\left[\frac{2.419\sin(1.6°)}{1.333}\right] = \boxed{2.9°}$$

22.43 If $\theta_c = 42.0°$ at the boundary between the prism glass and the surrounding medium, then $\sin\theta_c = n_2/n_1$ gives



$$\frac{n_m}{n_{\text{glass}}} = \sin 42.0°$$

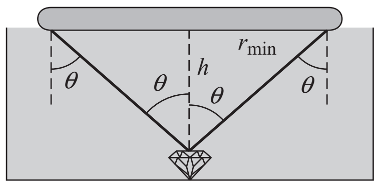
From the geometry shown in the figure at the right,

$$\alpha = 90.0° - 42.0° = 48.0°,\ \beta = 180° - 60.0° - \alpha = 72.0°$$

and $\theta_r = 90.0° - \beta = 18.0°$. Thus, applying Snell's law at the first surface gives

$$\theta_1 = \sin^{-1}\left(\frac{n_{\text{glass}}\sin\theta_r}{n_m}\right) = \sin^{-1}\left(\frac{\sin\theta_r}{n_m/n_{\text{glass}}}\right) = \sin^{-1}\left(\frac{\sin 18.0°}{\sin 42.0°}\right) = \boxed{27.5°}$$

22.44 The circular raft must cover the area of the surface through which light from the diamond could emerge. Thus, it must form the base of a cone (with apex at the diamond) whose half angle is θ, where θ is greater than or equal to the critical angle.



The critical angle at the water-air boundary is

$$\theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right) = \sin^{-1}\left(\frac{1.00}{1.333}\right) = 48.6°$$

Thus, the minimum diameter of the raft is

$$2r_{\min} = 2h\tan\theta_{\min} = 2h\tan\theta_c = 2(2.00\text{ m})\tan 48.6° = \boxed{4.54\text{ m}}$$

22.45 At the air-ice boundary, Snell's law gives the angle of refraction in the ice as

$$\sin\theta_{1r} = \frac{n_{\text{air}}\sin\theta_{1i}}{n_{\text{ice}}}$$

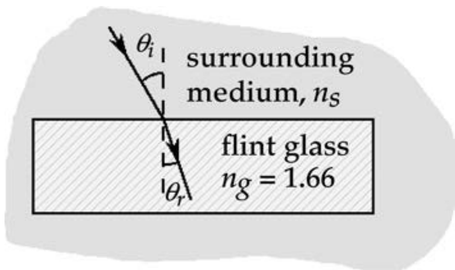
Since the sides of the ice layer are parallel, the angle of incidence at the ice-water boundary is $\theta_{2i} = \theta_{1r}$. Then, from Snell's law, the angle of refraction in the water is

$$\theta_{2r} = \sin^{-1}\left(\frac{n_{\text{ice}}\sin\theta_{2i}}{n_{\text{water}}}\right) = \sin^{-1}\left(\frac{n_{\text{ice}}\sin\theta_{1r}}{n_{\text{water}}}\right) = \sin^{-1}\left[\frac{\cancel{n_{\text{ice}}}}{n_{\text{water}}}\left(\frac{n_{\text{air}}\sin\theta_{1i}}{\cancel{n_{\text{ice}}}}\right)\right]$$

or $$\theta_{2r} = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_{1i}}{n_{\text{water}}}\right) = \sin^{-1}\left[\frac{(1.00)\sin 30.0^\circ}{1.333}\right] = \boxed{22.0^\circ}$$

Note that all of the properties of the ice canceled out in the above calculation, and the result is the same as if the ice had not been present. This will always be true when the intermediate medium has parallel sides.

22.46 When light coming from the surrounding medium is incident on the surface of the glass slab, Snell's law gives $n_g \sin\theta_r = n_s \sin\theta_i$, or



$$\sin\theta_r = (n_s/n_g)\sin\theta_i$$

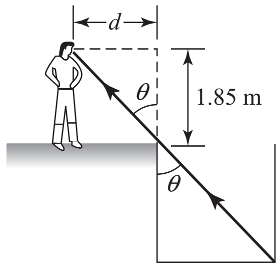
(a) If $\theta_i = 30.0^\circ$ and the surrounding medium is water ($n_s = 1.333$), the angle of refraction is

$$\theta_r = \sin^{-1}\left[\frac{1.333\sin(30.0^\circ)}{1.66}\right] = \boxed{23.7^\circ}$$

(b) From Snell's law given above, we see that as $n_s \to n_g$ we have $\sin\theta_r \to \sin\theta_i$, or the angle of refraction approaches the angle of incidence, $\boxed{\theta_r \to \theta_i = 30.0^\circ}$.

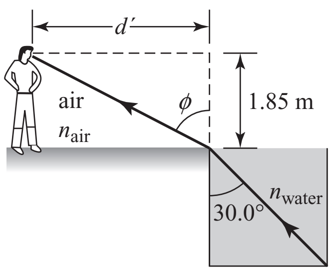
(c) If $n_s > n_g$, then $\sin\theta_r = (n_s/n_g)\sin\theta_i > \sin\theta_i$, or $\boxed{\theta_r > \theta_i}$.

22.47 (a) Given that the angle θ shown in the figure at the right is 30.0°, the maximum distance the observer can be from the pool and continue to see the lower edge on the opposite side of the pool is



$$d = (1.85\text{ m})\tan 30.0^\circ = \boxed{1.07\text{ m}}$$

(b) If the pool is now completely filled with water, the light ray coming to the observer's eye from the lower opposite edge of the pool will refract at the surface of the water as shown in the second figure. The angle of refraction is

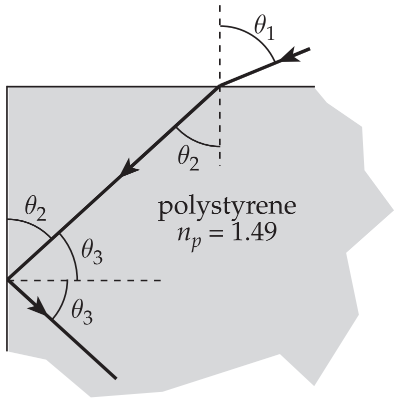


$$\phi = \sin^{-1}\left(\frac{n_{\text{water}}\sin 30.0^\circ}{n_{\text{air}}}\right) = \sin^{-1}\left[\frac{(1.333)\sin 30.0^\circ}{1.00}\right] = 41.8^\circ$$

The maximum distance the observer can now be from the pool and still see the same boundary is

$$d' = (1.85\text{ m})\tan\phi = (1.85\text{ m})\tan 41.8^\circ = \boxed{1.65\text{ m}}$$

22.48 (a) For polystyrene surrounded by air, total internal reflection at the left vertical face requires that



$$\theta_3 \ge \theta_c = \sin^{-1}\left(\frac{n_{\text{air}}}{n_p}\right) = \sin^{-1}\left(\frac{1.00}{1.49}\right) = 42.2°$$

From the geometry shown in the figure at the right,

$$\theta_2 = 90.0° - \theta_3 \le 90.0° - 42.2° = 47.8°$$

Thus, use of Snell's law at the upper surface gives

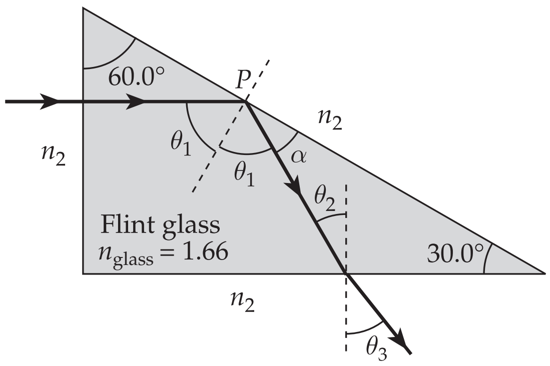
$$\sin\theta_1 = \frac{n_p \sin\theta_2}{n_{\text{air}}} \le \frac{(1.49)\sin 47.8°}{1.00} = 1.10$$

so it is seen that $\boxed{\text{any angle of incidence} \le 90°}$ at the upper surface will yield total internal reflection at the left vertical face.

(b) Repeating the steps of part (a) with the index of refraction of air replaced by that of water yields $\theta_3 \ge 63.5°$, $\theta_2 \le 26.5°$, $\sin\theta_1 \le 0.499$, and $\theta_1 \le \boxed{29.9°}$.

(c) Total internal reflection is $\boxed{\text{not possible since } n_{\text{polystyrene}} < n_{\text{carbon disulfide}}}$.

22.49 (a) From the geometry of the figure at the right, observe that $\theta_1 = 60.0°$. Therefore,



$$\alpha = 90.0° - \theta_1 = 30.0°$$

and $\quad (\theta_2 + 90.0°) + \alpha + 30.0° = 180.0°$

Thus, $\theta_2 = 180.0° - 120.0° - \alpha = 30.0°$

Since the prism is immersed in water, $n_2 = 1.333$ and Snell's law gives

$$\theta_3 = \sin^{-1}\left(\frac{n_{\text{glass}} \sin\theta_2}{n_2}\right) = \sin^{-1}\left(\frac{(1.66)\sin 30.0°}{1.333}\right) = \boxed{38.5°}$$

(b) For refraction to occur at point P, it is necessary that $\theta_c > \theta_1$.

Thus, $\quad \theta_c = \sin^{-1}\left(\frac{n_2}{n_{\text{glass}}}\right) > \theta_1$

which gives $n_2 > n_{\text{glass}} \sin\theta_1 = (1.66)\sin 60.0° = \boxed{1.44}$.

22.50 Applying Snell's law to this refraction, recognizing that $n_{\text{air}} = 1.00$, gives

$$n_{\text{glass}} \sin\theta_2 = n_{\text{air}} \sin\theta_1 = \sin\theta_1$$

If $\theta_1 = 2\theta_2$, this becomes

$$n_{\text{glass}} \sin\theta_2 = \sin(2\theta_2) = 2\sin\theta_2 \cos\theta_2$$

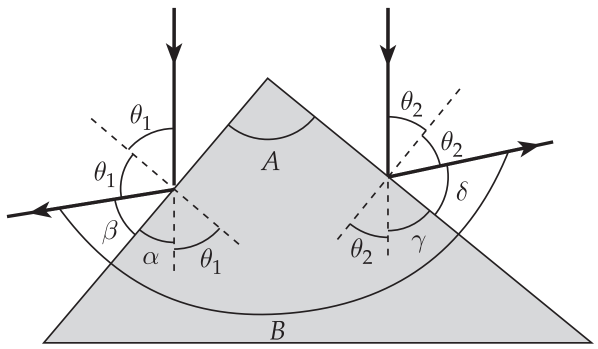
continued on next page

or $\cos\theta_2 = \dfrac{n_{\text{glass}}}{2}$ and $\theta_2 = \cos^{-1}\left(\dfrac{n_{\text{glass}}}{2}\right)$

Then, the desired angle of incidence is

$$\theta_1 = 2\theta_2 = 2\cos^{-1}\left(\frac{n_{\text{glass}}}{2}\right) = 2\cos^{-1}\left(\frac{1.56}{2}\right) = \boxed{77.5°}$$

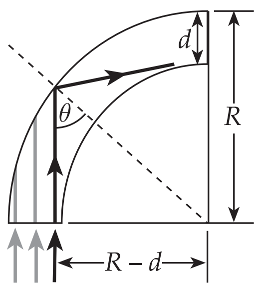
22.51 In the figure at the right, observe that $\beta = 90° - \theta_1$ and $\alpha = 90° - \theta_1$. Thus, $\beta = \alpha$.



Similarly, on the right side of the prism, $\delta = 90° - \theta_2$ and $\gamma = 90° - \theta_2$, giving $\delta = \gamma$.

Next, observe that the angle between the reflected rays is $B = (\alpha + \beta) + (\gamma + \delta)$, so $B = 2(\alpha + \gamma)$. Finally, observe that the left side of the prism is sloped at angle α from the vertical, and the right side is sloped at angle γ. Thus, the angle between the two sides is $A = \alpha + \gamma$, and we obtain the result $B = 2(\alpha + \gamma) = \boxed{2A}$.

22.52 (a) Observe in the sketch at the right that a ray originally traveling along the inner edge will have the smallest angle of incidence when it strikes the outer edge of the fiber in the curve. Thus, if this ray is totally internally reflected, all of the others are also totally reflected.



For this ray to be totally internally reflected it is necessary that

$$\theta \geq \theta_c \quad \text{or} \quad \sin\theta \geq \sin\theta_c = \frac{n_{\text{air}}}{n_{\text{pipe}}} = \frac{1}{n}$$

But $\sin\theta = \dfrac{R-d}{R}$, so we must have $\dfrac{R-d}{R} \geq \dfrac{1}{n}$

which simplifies to $R \geq nd/(n-1)$, or $\boxed{R_{\text{min}} = nd/(n-1)}$.

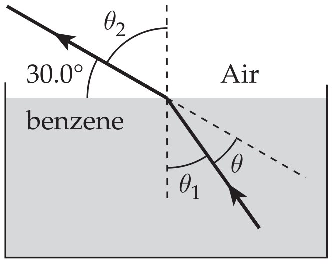
(b) As $d \to 0$, $R_{\text{min}} \to 0$. This is $\boxed{\text{reasonable}}$ behavior.

As n increases, the minimum acceptable radius of curvature decreases. This is $\boxed{\text{reasonable}}$ behavior. $\left(R_{\text{min}} = \dfrac{nd}{n-1} = \dfrac{d}{1-1/n}\right)$

As $n \to 1$, R_{min} increases. This is $\boxed{\text{reasonable}}$ behavior.

(c) $R_{\text{min}} = \dfrac{nd}{n-1} = \dfrac{(1.40)(100\ \mu\text{m})}{1.40-1} = \boxed{350\ \mu\text{m}}$

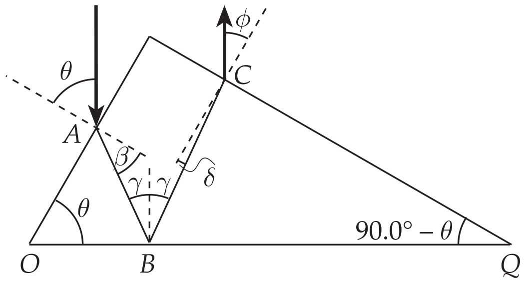
22.53 Consider light which leaves the lower end of the wire and travels parallel to the wire while in the benzene. If the wire appears straight to an observer looking along the dry portion of the wire, this ray from the lower end of the wire must enter the observer's eye as he sights along the wire. Thus, the ray must refract and travel parallel to the wire in air. The angle of refraction is then $\theta_2 = 90.0° - 30.0° = 60.0°$. From Snell's law, the angle of incidence was



$$\theta_1 = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_2}{n_{\text{benzene}}}\right) = \sin^{-1}\left(\frac{(1.00)\sin 60.0°}{1.50}\right) = 35.3°$$

and the wire is bent by angle $\theta = \theta_2 - \theta_1 = 60.0° - \theta_1 = 60.0° - 35.3° = \boxed{24.7°}$.

22.54 In the sketch at the right, the angle of incidence at A is the same as the prism angle at point O. This is true because tipping the line OA up angle θ from the horizontal necessarily tips its normal line over at angle θ from the vertical. Given that $\theta = 60.0°$, application of Snell's law at point A gives



$$1.50\sin\beta = (1.00)\sin 60.0° \quad \text{or} \quad \beta = 35.3°$$

From triangle AOB, we calculate the angle of incidence and reflection, γ, at point B:

$$\theta + (90.0° - \beta) + (90.0° - \gamma) = 180° \qquad \text{or} \qquad \gamma = \theta - \beta = 60.0° - 35.3° = 24.7°$$

Now, we find the angle of incidence at point C using triangle BCQ:

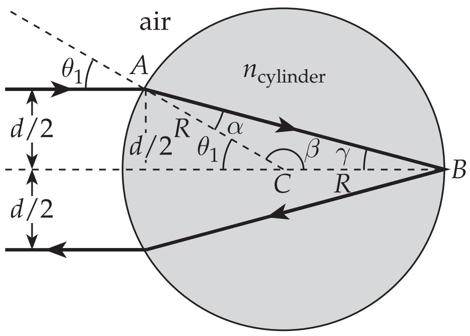
$$(90.0° - \gamma) + (90.0° - \delta) + (90.0° - \theta) = 180°$$

or $\quad \delta = 90.0° - (\theta + \gamma) = 90.0° - 84.7° = 5.3°$

Finally, application of Snell's law at point C gives $(1.00)\sin\phi = (1.50)\sin(5.3°)$,

or $\phi = \sin^{-1}(1.50\sin 5.3°) = \boxed{8.0°}$.

22.55 The path of a light ray during a reflection and/or refraction process is always reversible. Thus, if the emerging ray is parallel to the incident ray, the path which the light follows through this cylinder must be symmetric about the center line as shown at the right.

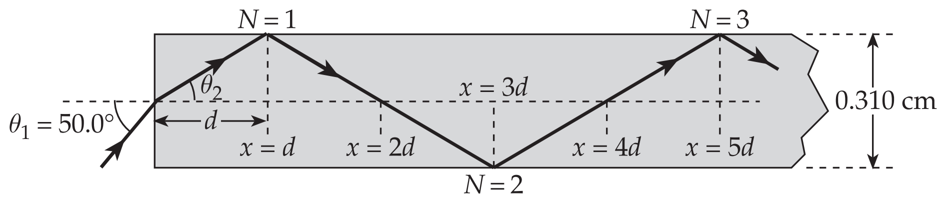


Thus, $\theta_1 = \sin^{-1}\left(\frac{d/2}{R}\right) = \sin^{-1}\left(\frac{1.00\text{ m}}{2.00\text{ m}}\right) = 30.0°$

Triangle ABC is isosceles, so $\gamma = \alpha$ and $\beta = 180° - \alpha - \gamma = 180° - 2\alpha$. Also, $\beta = 180° - \theta_1$, which gives $\alpha = \theta_1/2 = 15.0°$. Then, from applying Snell's law at point A,

$$n_{\text{cylinder}} = \frac{n_{\text{air}}\sin\theta_1}{\sin\alpha} = \frac{(1.00)\sin 30.0°}{\sin 15.0°} = \boxed{1.93}$$

22.56



The angle of refraction as the light enters the left end of the slab is

$$\theta_2 = \sin^{-1}\left(\frac{n_{\text{air}}\sin\theta_1}{n_{\text{slab}}}\right) = \sin^{-1}\left(\frac{(1.00)\sin 50.0°}{1.48}\right) = 31.2°$$

Observe from the figure that the first reflection occurs at $x = d$, the second reflection is at $x = 3d$, the third is at $x = 5d$, and so forth. In general, the Nth reflection occurs at $x = (2N-1)d$, where

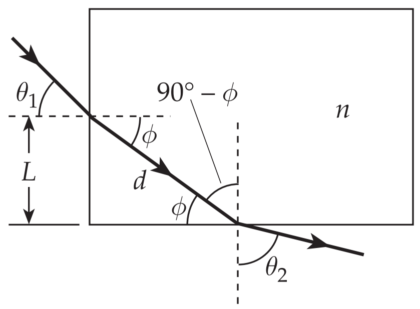
$$d = \frac{(0.310\text{ cm}/2)}{\tan\theta_2} = \frac{0.310\text{ cm}}{2\tan 31.2°} = 0.256\text{ cm}$$

Therefore, the number of reflections made before reaching the other end of the slab at $x = L = 42$ cm is found from $L = (2N-1)d$ to be

$$N = \frac{1}{2}\left(\frac{L}{d}+1\right) = \frac{1}{2}\left(\frac{42\text{ cm}}{0.256\text{ cm}}+1\right) = 82.5 \text{ or } \boxed{\text{82 complete reflections}}$$

22.57 (a) If $\theta_1 = 45.0°$, application of Snell's law at the point where the beam enters the plastic block gives

$$(1.00)\sin 45.0° = n\sin\phi \qquad \textbf{[1]}$$



Application of Snell's law at the point where the beam emerges from the plastic, with $\theta_2 = 76.0°$, gives

$$n\sin(90.0°-\phi) = (1.00)\sin 76.0° \quad \text{or} \quad (1.00)\sin 76.0° = n\cos\phi \qquad \textbf{[2]}$$

Dividing Equation [1] by Equation [2], we obtain

$$\tan\phi = \frac{\sin 45.0°}{\sin 76.0°} = 0.729 \quad \text{and} \quad \phi = 36.1°$$

Thus, from Equation [1], $n = \dfrac{\sin 45.0°}{\sin\phi} = \dfrac{\sin 45.0°}{\sin 36.1°} = \boxed{1.20}$

(b) Observe from the figure above that $\sin\phi = L/d$. Thus, the distance the light travels inside the plastic is $d = L/\sin\phi$, and if $L = 50.0$ cm $= 0.500$ m, the time required is

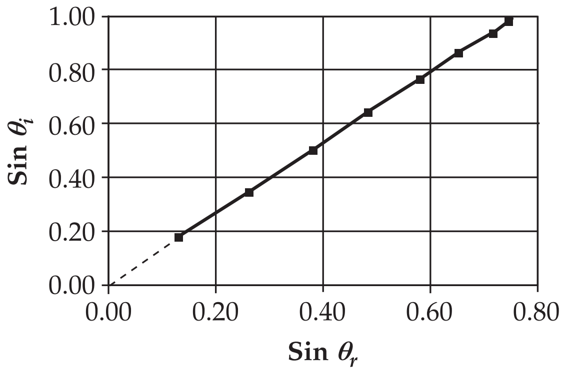
$$\Delta t = \frac{d}{v} = \frac{L/\sin\phi}{c/n} = \frac{nL}{c\sin\phi} = \frac{1.20(0.500\text{ m})}{(3.00\times10^8\text{ m/s})\sin 36.1°} = 3.39\times10^{-9}\text{ s} = \boxed{3.39\text{ ns}}$$

22.58 Snell's law would predict that $n_{\text{air}} \sin\theta_i = n_{\text{water}} \sin\theta_r$, or since $n_{\text{air}} = 1.00$,

$$\sin\theta_i = n_{\text{water}} \sin\theta_r$$

Comparing this equation to the equation of a straight line, $y = mx + b$, shows that if Snell's law is valid, a graph of $\sin\theta_i$ versus $\sin\theta_r$ should yield a straight line that would pass through the origin if extended and would have a slope equal to n_{water}.

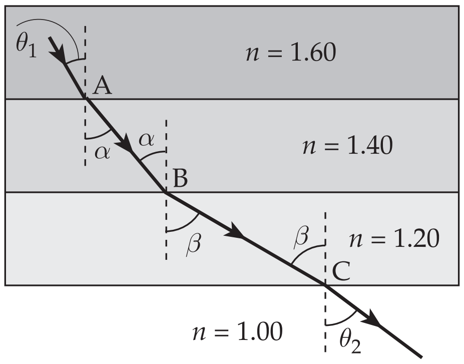
θ_i (deg)	θ_r (deg)	$\sin\theta_i$	$\sin\theta_r$
10.0	7.50	0.174	0.131
20.0	15.1	0.342	0.261
30.0	22.3	0.500	0.379
40.0	28.7	0.643	0.480
50.0	35.2	0.766	0.576
60.0	40.3	0.866	0.647
70.0	45.3	0.940	0.711
80.0	47.7	0.985	0.740



The straightness of the graph line and the fact that its extension passes through the origin demonstrates the validity of Snell's law. Using the end points of the graph line to calculate its slope gives the value of the index of refraction of water as

$$n_{\text{water}} = slope = \frac{0.985 - 0.174}{0.740 - 0.131} = \boxed{1.33}$$

22.59 Applying Snell's law at points A, B, and C gives



$$1.40 \sin\alpha = 1.60 \sin\theta_1 \qquad \textbf{[1]}$$

$$1.20 \sin\beta = 1.40 \sin\alpha \qquad \textbf{[2]}$$

and $$1.00 \sin\theta_2 = 1.20 \sin\beta \qquad \textbf{[3]}$$

Combining Equations [1], [2], and [3] yields

$$\sin\theta_2 = 1.60 \sin\theta_1 \qquad \textbf{[4]}$$

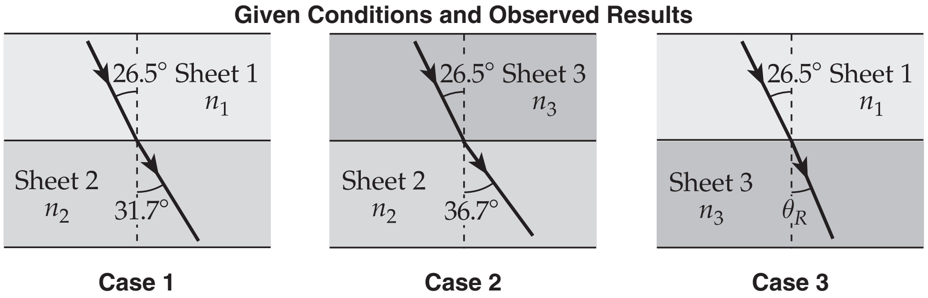
Note that Equation [4] is exactly what Snell's law would yield if the second and third layers of this "sandwich" were ignored. This will always be true if the surfaces of all the layers are parallel to each other.

(a) If $\theta_1 = 30.0°$, then Equation [4] gives $\theta_2 = \sin^{-1}(1.60 \sin 30.0°) = \boxed{53.1°}$.

(b) At the critical angle of incidence on the lowest surface, $\theta_2 = 90.0°$. Then, Equation [4] gives

$$\theta_1 = \sin^{-1}\left(\frac{\sin\theta_2}{1.60}\right) = \sin^{-1}\left(\frac{\sin 90.0°}{1.60}\right) = \boxed{38.7°}$$

22.60



For the first placement, Snell's law gives $n_2 = \dfrac{n_1 \sin 26.5°}{\sin 31.7°}$

In the second placement, application of Snell's law yields

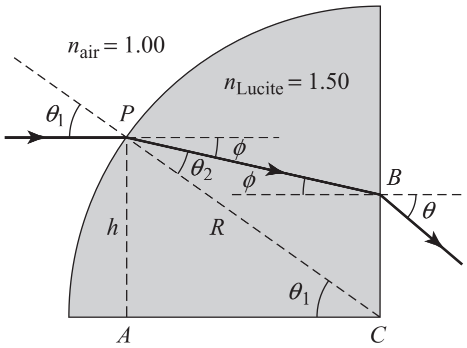
$$n_3 \sin 26.5° = n_2 \sin 36.7° = \left(\frac{n_1 \sin 26.5°}{\sin 31.7°}\right)\sin 36.7°, \text{ or } n_3 = \frac{n_1 \sin 36.7°}{\sin 31.7°}$$

Finally, using Snell's law in the third placement gives

$$\sin\theta_R = \frac{n_1 \sin 26.5°}{n_3} = (n_1 \sin 26.5°)\left(\frac{\sin 31.7°}{n_1 \sin 36.7°}\right) = 0.392$$

and $\theta_R = \boxed{23.1°}$

22.61 From the right triangle PAC in the figure at the right, observe that



$$\theta_1 = \sin^{-1}\left(\frac{h}{R}\right) = \sin^{-1}\left(\frac{6.00 \text{ cm}}{12.0 \text{ cm}}\right) = 30.0°$$

This is also the angle of incidence at point P where the light ray enters the Lucite. Snell's law then gives

$$\theta_2 = \sin^{-1}\left(\frac{n_{\text{air}} \sin\theta_1}{n_{\text{Lucite}}}\right) = \sin^{-1}\left[\frac{\sin 30.0°}{1.50}\right] = 19.5°$$

Now, by observing the vertical angles at point P, we find that $\theta_2 + \phi = \theta_1$, so the angle of incidence at point B where the ray exits from the Lucite is $\phi = \theta_1 - \theta_2 = 10.5°$. The angle of refraction at point B is then given by Snell's law as

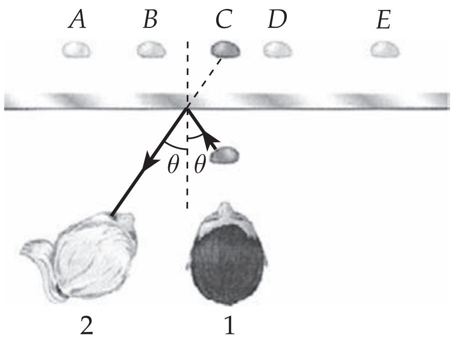
$$\theta = \sin^{-1}\left(\frac{n_{\text{Lucite}} \sin\phi}{n_{\text{air}}}\right) = \sin^{-1}\left[(1.50)\sin 10.5°\right] = \boxed{15.9°}$$

23

Mirrors and Lenses

QUICK QUIZZES

1. At C.



2. Choice (c). Since $n_{\text{water}} > n_{\text{air}}$, the virtual image of the fish formed by refraction at the flat water surface is closer to the surface than is the fish. See Equation 23.9 in the textbook.

3. (a) False. A concave mirror forms an inverted image when the object distance is greater than the focal length.

(b) False. The magnitude of the magnification produced by a concave mirror is greater than 1 if the object distance is less than the radius of curvature.

(c) True.

4. Choice (b). In this case, the index of refraction of the lens material is less than that of the surrounding medium. Under these conditions, a biconvex lens will be divergent.

5. Although a ray diagram only uses 2 or 3 rays (those whose direction is easily determined using only a straight edge), an infinite number of rays leaving the object will always pass through the lens.

6. (a) False. A virtual image is formed on the left side of the lens if $p < f$.

(b) True. An upright, virtual image is formed when $p < f$, while an inverted, real image is formed when $p > f$.

(c) False. A magnified, real image is formed if $2f > p > f$, and a magnified, virtual image is formed if $p < f$.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The image formed by a flat mirror of a real object is always an upright, virtual image that is the same size as the object and located as far behind the mirror as the object is in front of the mirror. Thus, statements (b), (c), and (e) are all true, while statements (a) and (d) are false.

2. Since the size of the image is 1.50 times the size of the object, the magnitude of the magnification is $|M| = 1.50$. Because the image is upright, $M > 0$. Thus, $M = -q/p = +1.50$, which gives $q = -1.50p$. Then from the mirror equation, we have

$$f = \frac{R}{2} = \frac{qp}{p+q} = \frac{(-1.50p)p}{p-1.50p} = +3.00p = +3.00(+30.0 \text{ cm}) = +90.0 \text{ cm}$$

and the correct choice is (d).

3. For a convergent lens, $f > 0$, and because the image is real, $q > 0$. The thin-lens equation, $1/p + 1/q = 1/f$, then gives

$$p = \frac{qf}{q-f} = \frac{(12.0 \text{ cm})(8.00 \text{ cm})}{12.0 \text{ cm} - 8.00 \text{ cm}} = +24.0 \text{ cm}$$

Since $p > 0$, the object is in front (in this case, to the left) of the lens, and the correct choice is (c).

4. For a converging lens, the focal length is positive, or $f > 0$. Since the object is virtual, we know that the object distance is negative, or $p < 0$ and $p = -|p|$. Thus, the thin-lens equation gives the image distance as

$$q = \frac{pf}{p-f} = \frac{-|p|f}{-|p|-f} = +\left(\frac{|p|}{|p|+f}\right)f$$

Since $|p|$ and f are positive quantities, we see that $q > 0$ and the image is real. Also, since $|p|/(|p|+f) < 1$, we see that $q < f$. Thus, we have shown that choices (a) and (d) are false statements, while choices (b), (c), and (e) are all true.

5. From the mirror equation, $1/p + 1/q = 2/R = 1/f$, with $f < 0$ since the mirror is convex, the image distance is found to be

$$q = \frac{pf}{p-f} = \frac{(16.0 \text{ cm})(-6.00 \text{ cm})}{16.0 \text{ cm} - (-6.00 \text{ cm})} = -4.36 \text{ cm}$$

Since $q < 0$, the image is virtual and located 4.36 cm <u>behind</u> the mirror. Choice (d) is the correct answer.

6. For a divergent lens, $f < 0$, and because the object is real, $p > 0$. The thin-lens equation, $1/p + 1/q = 1/f$, then gives

$$q = \frac{pf}{p-f} = \frac{(10.0 \text{ cm})(-16.0 \text{ cm})}{10.0 \text{ cm} - (-16.0 \text{ cm})} = -6.15 \text{ cm}$$

Since $q < 0$, the image is in front (in this case, to the left) of the lens, and the correct choice is (b).

7. A concave mirror forms inverted, real images of real objects located outside the focal point ($p > f$), and upright, magnified, virtual images of real objects located inside the focal point ($p < f$). Virtual images, located behind the mirror, have negative image distances by the sign convention of Table 23.1. Choices (d) and (e) are true statements, and all other choices are false.

8. A convergent lens forms inverted, real images of real objects located outside the focal point ($p > f$). When $p > 2f$, the real image is diminished in size, and the image is enlarged if $2f > p > f$. For real objects located inside the focal point ($p < f$) of the convergent lens, the image is upright, virtual, and enlarged. In the given case, $p > 2f$, so the image is real, inverted, and diminished in size. Choice (c) is the correct answer.

9. With a real object in front of a convex mirror, the image is always upright, virtual, diminished in size, and located between the mirror and the focal point. Thus, of the available choices, only choice (c) is a true statement.

10. For a real object ($p > 0$) and a diverging lens ($f < 0$), the image distance given by the thin-lens equation is

$$q = \frac{pf}{p-f} = \frac{|p|\left(-|f|\right)}{|p| - \left(-|f|\right)} = -\frac{|p||f|}{|p| + |f|} < 0$$

and the magnification is

$$M = -\frac{q}{p} = -\frac{-|q|}{|p|} > 0$$

Thus, the image is always virtual and upright, meaning that choice (b) is a true statement while all other choices are false.

11. Light rays reflecting from the fish, and passing through the flat refracting surface of the water ($n_{\text{water}} > n_{\text{air}}$) on the way to the fisherman's eye, form a virtual image located closer to the surface than the fish's actual depth (see Example 23.6 in the textbook). Thus, the fisherman should aim below the apparent location of the fish, and (b) is the correct choice.

12. The image that a plane reflecting surface forms of a real object is an upright, virtual image located as far behind the reflecting surface as the object is in front of it. Choices (a) and (d) must be eliminated because they portray inverted images. Also, the tip of the image arrow should be closer to the mirror than the base of the image arrow, just as is true of the ends of the object arrow. Thus, choice (c) must be eliminated, leaving (b) as best describing the image formed by the mirror.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The stream appears shallower than its true depth because refraction of light coming from pebbles on the bottom of the stream forms virtual images located closer to the surface than the actual objects.

4. Chromatic aberration is produced when light passes *through* a material, as it does when passing through the glass of a lens. A mirror, silvered on its front surface, never has light passing through it, so this aberration cannot occur. This is only one of many reasons why large telescopes use mirrors rather than lenses for their primary optical elements.

6. Light rays diverge from the position of a virtual image just as they do from an actual object. Thus, a virtual image can be as easily photographed as any object can. Of course, the camera would have to be placed near the axis of the lens or mirror in order to intercept the light rays.

8. Actually no physics is involved here. The design is chosen so your eyelashes will not brush against the glass as you blink. A reason involving a little physics is that with this design, when you direct your gaze near the outer circumference of the lens, you receive a ray that has passed through glass with more nearly parallel surfaces of entry and exit. Then the lens minimally distorts the direction to the object you are looking at.

10. Both words are inverted. However, OXIDE looks the same right-side-up and upside-down. LEAD does not.

12. (a) No. The screen is needed to reflect the light toward your eye.

(b) Yes. The light is traveling toward your eye and diverging away from the position of the image, the same as if the object were located at that position.

14. The correct answer is choice (d). The entire image would appear because any portion of the lens can form the image. The image would be dimmer because the card reduces the light intensity on the screen by 50%.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 1.0 m behind nearest mirror **(b)** the palm

(c) 5.0 m behind nearest mirror **(d)** back of the hand

(e) 7.0 m behind nearest mirror **(f)** the palm **(g)** all are virtual

4. 4.58 m

6. **(a)** $R = +2.22$ cm **(b)** $M = +10.0$

8. $R = -0.790$ cm

10. The mirror is concave with $R \approx 60$ cm and $f \approx 30$ cm.

12. **(a)** $p = +15.0$ cm **(b)** $|R| = 60.0$ cm

14. **(a)** 60.0 cm

(b) $q = 42.9$ cm, $M = -0.429$, so the image is real, inverted, and 42.9 cm in front of mirror.

(c) $q = -15.0$ cm, $M = 1.50$, so the image is virtual, upright, and 15.0 cm behind the mirror.

16. **(a)** a real object located 16 cm in front of the mirror

(b) upright and one-third the size of the object

18. **(a)** $p = 8.00$ cm **(b)** See Solution. **(c)** virtual

20. **(a)** As the ball moves from $p = 3.00$ m to $p = 0.500$ m, the image moves from $q = +0.600$ m to $q = +\infty$. As the ball moves from $p = 0.500$ m to $p = 0$, the image moves from $q = -\infty$ to $q = 0$.

(b) at $t = 0.639$ s when $p = 1.00$ m, and at $t = 0.782$ s when $p = 0$

22. in the water, 9.00 cm inside the wall of the bowl

24. **(a)** 1.50 m **(b)** 1.75 m

26. 4.8 cm

28. See Solution.

30. **(a)** at either 36.2 cm or 13.8 cm from the screen

(b) When $q = 36.2$ cm, $M = -2.62$. When $q = 13.8$ cm, $M = -0.381$.

32. **(a)** See Solution. **(b)** 40 cm behind the mirror **(c)** +2

(d) The mirror equation yields $q = -40.0$ cm and $M = +2.00$.

34. **(a)** See Solution. **(b)** See Solution.

(c) Graph (a) yields an upright, virtual image located 13.3 cm from the lens and one-third the size of the object. Graph (b) yields an upright, virtual image located 6.7 cm from the lens and two-thirds the size of the object. Both agree with the algebraic solutions.

36. $f = +5.68$ cm

38. **(a)** 12.3 cm to the left of the lens **(b)** $M = +0.615$ **(c)** See Solution.

40. **(a)** $q = pf/(p-f)$ **(b)** $q < 0$ for all values of $p > 0$ and $f < 0$

(c) $q > 0$ only if $p > f$ when $f > 0$

42. **(a)** $f_2 = -11.1$ cm **(b)** $M = +2.50$

(c) virtual and upright

44. **(a)** 13.3 cm **(b)** $M = -5.91$ **(c)** inverted

46. **(a)** $q_1 = +30.0$ cm **(b)** 20.0 cm beyond the second lens

(c) $p_2 = -20.0$ cm **(d)** $q_2 = +4.00$ cm **(e)** $M_1 = -2.00$

(f) $M_2 = +0.200$ **(g)** $M_{\text{total}} = -0.400$ **(h)** real, inverted

48. **(a)** $q = 5f/4$ **(b)** $M = -1/4$ **(c)** real, inverted, opposite side

50. **(a)** $f = -12.0$ cm **(b)** $q = f = -12.0$ cm **(c)** $q = -9.00$ cm

(d) $q = -6.00$ cm **(e)** $q = -4.00$ cm

52. **(a)** 25.3 cm to the right of the mirror

(b) virtual **(c)** upright **(d)** $M = +8.05$

54. $\Delta x = 21.3$ cm

56. See the Solution.

58. See the Solution.

60. **(a)** $p = 4f/3$ **(b)** $p = 3f/4$

(c) $M_a = -3,\ M_b = +4$

62. $f_{\text{mirror}} = 11.7$ cm

64. **(a)** 0.708 cm in front or the spherical ornament **(b)** See Solution.

66. **(a)** convex **(b)** at the 30.0-cm mark **(c)** $f = -20.0$ cm

PROBLEM SOLUTIONS

23.1 (a) Due to the finite value of the speed of light, the light arriving at your eye must have reflected from your face at a slightly earlier time. Thus, the image viewed in the mirror $\boxed{\text{shows you younger than your actual age}}$.

(b) If you stand 40 cm in front of the mirror, the time required for light scattered from your face to travel to the mirror and back to your eye is

$$\Delta t = \frac{2d}{c} = \frac{2(0.40 \text{ m})}{3.0\times10^{8} \text{ m/s}} = 2.7\times10^{-9} \text{ s} \qquad \text{or} \qquad \boxed{\sim 10^{-9} \text{ s}}$$

23.2 (a) With the palm located 1.0 m in front of the nearest mirror, that mirror forms an image, I_{P1}, located $\boxed{\text{1.0 m behind the nearest mirror}}$.

(b) The image I_{P1} is an image $\boxed{\text{of the palm}}$.

(c) The woman's left hand is located 2.0 m in front of the farthest mirror, which forms an image located 2.0 m behind this mirror (and hence, 5.0 m in front of the nearest mirror). This image, I_{B1}, serves as an object for the nearest mirror, which then forms an image, I_{B2}, located $\boxed{\text{5.0 m behind the nearest mirror}}$.

(d) The images I_{B1} and I_{B2} are both images of the $\boxed{\text{back of the hand}}$.

(e) The image I_{P1} [see part (a)] serves as an object located 4.0 m in front of the farthest mirror, which forms an image I_{P2}, located 4.0 m behind that mirror and 7.0 m in front of the nearest mirror. This image then serves as an object for the nearest mirror, which forms an image I_{P3}, located $\boxed{\text{7.0 m behind the nearest mirror}}$.

(f) The images I_{P2} and I_{P3} are both images $\boxed{\text{of the palm}}$.

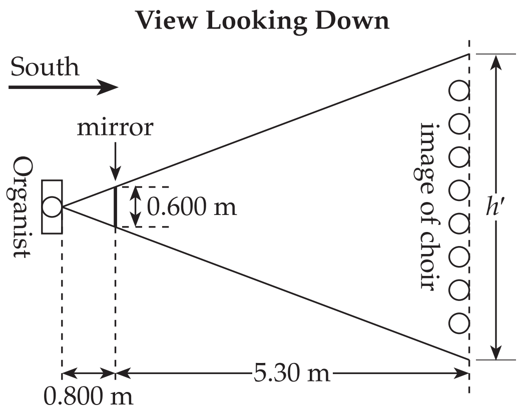
(g) Since all images are located behind the mirrors, $\boxed{\text{all are virtual images}}$.

23.3 (1) The first image in the left-hand mirror is 5.00 ft behind the mirror or $\boxed{\text{10.0 ft from the person}}$.

(2) The first image in the right-hand mirror serves as an object for the left-hand mirror. It is located 10.0 ft behind the right-hand mirror, which is 25.0 ft from the left-hand mirror. Thus, the second image in the left-hand mirror is 25.0 ft behind the mirror or $\boxed{\text{30.0 ft from the person}}$.

(3) The first image in the left-hand mirror serves as an object for the right-hand mirror. It is located 20.0 ft in front of the right-hand mirror and forms an image 20.0 ft behind that mirror. This image then serves as an object for the left-hand mirror. The distance from this object to the left-hand mirror is 35.0 ft. Thus, the third image in the left-hand mirror is 35.0 ft behind the mirror or $\boxed{\text{40.0 ft from the person}}$.

23.4 The virtual image is as far behind the mirror as the choir is in front of the mirror. Thus, the image is 5.30 m behind the mirror.



The image of the choir is

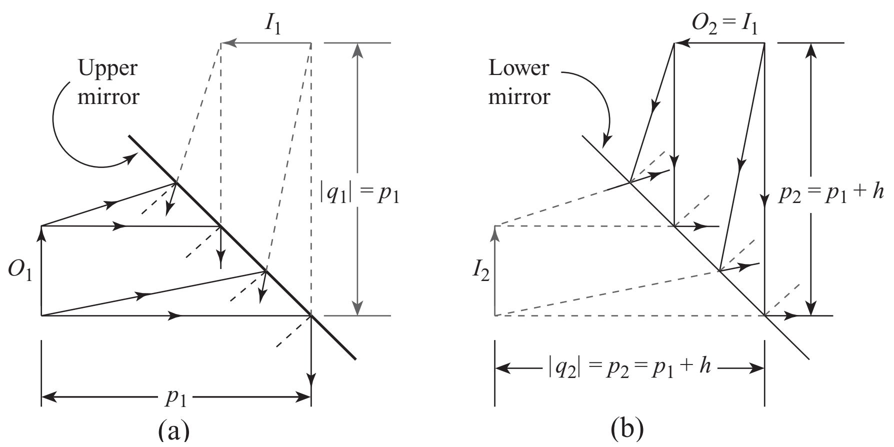
$$0.800 \text{ m} + 5.30 \text{ m} = 6.10 \text{ m}$$

from the organist. Using similar triangles gives

$$\frac{h'}{0.600 \text{ m}} = \frac{6.10 \text{ m}}{0.800 \text{ m}}$$

or $$h' = (0.600 \text{ m})\left(\frac{6.10 \text{ m}}{0.800 \text{ m}}\right) = \boxed{4.58 \text{ m}}$$

23.5 When an object is in front of a plane mirror, that mirror forms an upright, virtual image that is the same size as the object and as far behind the mirror as the object is in front of the mirror. This statement is true even if the mirror is rotated, as shown in the ray diagrams given below. In figure (a), a real object O_1 is distance p_1 in front of the upper mirror in the periscope. This mirror forms the virtual image I_1 at distance p_1 behind the mirror. As shown in figure (b), this image serves as the object for the lower mirror in the periscope, and is distance $p_2 = p_1 + h$ in front of the lower mirror. The lower mirror then forms the final image I_2, an upright, virtual image, located distance $p_2 = p_1 + h$ behind this mirror.



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(a) As shown in the above ray diagrams, the final image is $\boxed{\text{distance } |q_2| = p_1 + h}$ behind the lower mirror, where p_1 is the distance from the original object to the upper mirror and h is the vertical distance between the two mirrors in the periscope.

(b) The final image is behind the mirror and is $\boxed{\text{virtual}}$.

(c) As seen from the ray diagrams, the final image I_2 is oriented the same way as the original object, and is therefore $\boxed{\text{upright}}$.

(d) The overall magnification is

$$M = M_1 M_2 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right) = \left(-\frac{-p_1}{p_1}\right)\left(-\frac{-p_2}{p_2}\right) = (+1)(+1) = \boxed{+1}$$

The final image is therefore upright and the same size as the original object.

(e) $\boxed{\text{No}}$. The images formed by plane mirrors are upright in both directions. Just as plane mirrors do not reverse up and down, neither do they reverse left and right.

23.6 (a) Since the object is in front of the mirror, $p > 0$. With the image behind the mirror, $q < 0$. The mirror equation gives the radius of curvature as

$$\frac{2}{R} = \frac{1}{p} + \frac{1}{q} = \frac{1}{1.00 \text{ cm}} - \frac{1}{10.0 \text{ cm}} = \frac{10-1}{10.0 \text{ cm}}$$

or $R = 2\left(\frac{10.0 \text{ cm}}{9}\right) = \boxed{+2.22 \text{ cm}}$

(b) The magnification is $M = -\frac{q}{p} = -\frac{(-10.0 \text{ cm})}{1.00 \text{ cm}} = \boxed{+10.0}$.

23.7 (a) The center of curvature of a convex mirror is behind the mirror. Therefore, the radius of curvature, and hence the focal length $f = R/2$, is negative. With the image behind the mirror, the image is virtual and $q = -10.0$ cm. The mirror equation then gives

$$p = \frac{qf}{q-f} = \frac{(-10.0 \text{ cm})(-15.0 \text{ cm})}{-10.0 \text{ cm} - (-15.0 \text{ cm})} = +30.0 \text{ cm}$$

The object should be placed $\boxed{\text{30.0 cm in front of the mirror}}$.

(b) The magnification of the mirror is

$$M = -\frac{q}{p} = -\frac{(-10.0 \text{ cm})}{+30.0 \text{ cm}} = \boxed{+0.333}$$

Therefore, the image is upright and one-third the size of the object.

23.8 The lateral magnification is given by $M = -q/p$. Therefore, the image distance is

$$q = -Mp = -(0.013\,0)(30.0 \text{ cm}) = -0.390 \text{ cm}$$

The mirror equation, $\frac{2}{R} = \frac{1}{p} + \frac{1}{q}$, or $R = \frac{2pq}{p+q}$, gives

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$$R = \frac{2(30.0\text{ cm})(-0.390\text{ cm})}{30.0\text{ cm} - 0.390\text{ cm}} = \boxed{-0.790\text{ cm}}$$

The negative sign tells us that the surface is convex, as expected.

23.9 (a) The center of curvature of a concave mirror is in front of the mirror. Therefore, both the radius of curvature and the focal length, $f = R/2$, are positive. Since the image is virtual, the image distance is negative and $q = -20.0$ cm. With $R = +40.0$ cm and $f = +20.0$ cm, the mirror equation gives

$$p = \frac{qf}{q-f} = \frac{(-20.0\text{ cm})(+20.0\text{ cm})}{-20.0\text{ cm} - (+20.0\text{ cm})} = +10.0\text{ cm}$$

Thus, the object should be placed $\boxed{\text{10.0 cm in front of the mirror}}$.

(b) The magnification of the mirror is

$$M = -\frac{q}{p} = -\frac{(-20.0\text{ cm})}{+10.0\text{ cm}} = \boxed{+2.00}$$

Therefore, the image is upright and twice the size of the object.

23.10 The image was initially upright but became inverted when Dina was more than 30 cm from the mirror. From this information, we know that the mirror must be $\boxed{\text{concave}}$, because a convex mirror will form only upright, virtual images of real objects.

When the object is located at the focal point of a concave mirror, the rays leaving the mirror are parallel, and no image is formed. Since Dina observed that her image disappeared when she was about 30 cm from the mirror, we know that the focal length must be $\boxed{f \approx 30\text{ cm}}$. Also, for spherical mirrors, $R = 2f$. Thus, the radius of curvature of this concave mirror must be $\boxed{R \approx 60\text{ cm}}$.

23.11 The enlarged, virtual images formed by a concave mirror are upright, so $M > 0$.

Thus, $M = -\dfrac{q}{p} = \dfrac{h'}{h} = \dfrac{5.00\text{ cm}}{2.00\text{ cm}} = +2.50$, giving

$$q = -2.50\,p = -2.50(+3.00\text{ cm}) = -7.50\text{ cm}$$

The mirror equation then gives

$$f = \frac{pq}{p+q} = \frac{(3.00\text{ cm})(-7.50\text{ cm})}{3.00\text{ cm} - 7.50\text{ cm}} = \boxed{+5.00\text{ cm}}$$

23.12 Realize that the magnitude of the radius of curvature, $|R|$, is the same for both sides of the hubcap. For the convex side, $R = -|R|$; and for the concave side, $R = +|R|$. The object distance p is positive (real object) and has the same value in both cases. Also, we write the virtual image distance as $q = -|q|$ in each case. The mirror equation then gives:

For the convex side, $\dfrac{1}{-|q|} = \dfrac{2}{-|R|} - \dfrac{1}{p}$ or $|q| = \dfrac{|R|\,p}{|R| + 2p}$ **[1]**

For the concave side, $\dfrac{1}{-|q|} = \dfrac{2}{|R|} - \dfrac{1}{p}$ or $|q| = \dfrac{|R|\,p}{|R| - 2p}$ **[2]**

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Comparing Equations [1] and [2], we observe that the smaller magnitude image distance, $|q| = 10.0$ cm, occurs with the convex side of the mirror. Hence, we have

$$\frac{1}{-10.0\text{ cm}} = \frac{2}{-|R|} - \frac{1}{p} \qquad \textbf{[3]}$$

and for the concave side, $|q| = 30.0$ cm gives

$$\frac{1}{-30.0\text{ cm}} = \frac{2}{|R|} - \frac{1}{p} \qquad \textbf{[4]}$$

(a) Adding Equations [3] and [4] yields $\dfrac{2}{p} = \dfrac{3+1}{30.0\text{ cm}}$, or $p = \boxed{+15.0\text{ cm}}$.

(b) Subtracting [3] from [4] gives $\dfrac{4}{|R|} = \dfrac{3-1}{30.0\text{ cm}}$, or $|R| = \boxed{60.0\text{ cm}}$.

23.13 The image is upright, so $M > 0$, and we have

$$M = -\frac{q}{p} = +2.0, \text{ or } q = -2.0\,p = -2.0(25\text{ cm}) = -50\text{ cm}$$

The radius of curvature is then found to be

$$\frac{2}{R} = \frac{1}{p} + \frac{1}{q} = \frac{1}{25\text{ cm}} - \frac{1}{50\text{ cm}} = \frac{2-1}{50\text{ cm}}, \text{ or } R = 2\left(\frac{0.50\text{ m}}{+1}\right) = \boxed{1.0\text{ m}}$$

23.14 (a) For spherical mirrors, both concave and convex, the radius of curvature is related to the focal length by $R = 2f$. Hence, the radius of curvature of this concave mirror, having $f = +30.0$ cm, is $\boxed{R = +60.0\text{ cm}}$.

(b) When $p = +100$ cm (i.e., a real object located 100 cm in front of the mirror),

$$q = \frac{pf}{p-f} = \frac{(+100\text{ cm})(+30.0\text{ cm})}{100\text{ cm} - 30.0\text{ cm}} = +42.9\text{ cm}$$

and $$M = -\frac{q}{p} = -\frac{42.9\text{ cm}}{100\text{ cm}} = -0.429$$

Thus, the image is $\boxed{\text{real}}$ $(q > 0)$, $\boxed{\text{inverted}}$ $(M < 0)$, $\boxed{\text{diminished in size}}$ $(|M| < 1)$, and $\boxed{\text{located 42.9 cm in front of the mirror}}$.

(c) When $p = +10.0$ cm (i.e., a real object located 10.0 cm in front of the mirror),

$$q = \frac{pf}{p-f} = \frac{(+10.0\text{ cm})(+30.0\text{ cm})}{10.0\text{ cm} - 30.0\text{ cm}} = -15.0\text{ cm}$$

and $$M = -\frac{q}{p} = -\frac{(-15.0\text{ cm})}{10.0\text{ cm}} = +1.50$$

Thus, the image is $\boxed{\text{virtual}}$ $(q < 0)$, $\boxed{\text{upright}}$ $(M > 0)$, $\boxed{\text{enlarged}}$ $(|M| > 1)$, and $\boxed{\text{located 15.0 cm behind the mirror}}$.

23.15 The focal length of the mirror may be found from the given object and image distances as $1/f = 1/p + 1/q$, or

$$f = \frac{pq}{p+q} = \frac{(152\text{ cm})(18.0\text{ cm})}{152\text{ cm} + 18.0\text{ cm}} = +16.1\text{ cm}$$

For an upright image twice the size of the object, the magnification is

$$M = -q/p = +2.00\text{, giving } q = -2.00\,p$$

Then, using the mirror equation again, $1/p + 1/q = 1/f$ becomes

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{p} - \frac{1}{2.00\,p} = \frac{2-1}{2.00\,p} = \frac{1}{f}$$

or $\quad p = \dfrac{f}{2.00} = \dfrac{16.1\text{ cm}}{2.00} = \boxed{8.05\text{ cm}}$

23.16 (a) The mirror is convex, so $f < 0$, and we have $f = -|f| = -8.0$ cm. The image is virtual, so $q < 0$, or $q = -|q|$. Since we also know that $|q| = p/3$, the mirror equation gives

$$\frac{1}{p} + \frac{1}{q} = \frac{1}{p} - \frac{3}{p} = \frac{1}{f} \qquad \text{or} \qquad -\frac{2}{p} = \frac{1}{-8.0\text{ cm}} \qquad \text{and} \qquad p = +16\text{ cm}$$

This means that we have $\boxed{\text{a real object located 16 cm in front of the mirror}}$.

(b) The magnification is $M = -q/p = +|q|/p = \boxed{+1/3}$. Thus, the image is $\boxed{\text{upright}}$ and one-third the size of the object.

23.17 (a) Since the mirror is convex, $R < 0$. Thus, $R = -0.550$ m and $f = R/2 = -0.275$ m. The mirror equation then yields

$$q = \frac{pf}{p-f} = \frac{(+10.0\text{ m})(-0.275\text{ m})}{+10.0\text{ m} - (-0.275\text{ m})} = -0.268\text{ m} = -26.8\text{ cm}$$

The image is located $\boxed{\text{26.8 cm behind the mirror}}$.

(b) The magnification of the image is $M = -q/p$. Since $p > 0$ and $q < 0$ in this case, we see that $M > 0$. Therefore, the image is $\boxed{\text{upright}}$.

(c) $M = -\dfrac{q}{p} = -\dfrac{(-26.8\text{ cm})}{10.0\text{ m}} = -\dfrac{(-26.8\text{ cm})}{10.0 \times 10^{2}\text{ cm}} = +2.68 \times 10^{-2} = \boxed{+0.0268}$

23.18 (a) Since the mirror is concave, $R > 0$, giving $R = +24.0$ cm and $f = R/2 = +12.0$ cm. Because the image is upright ($M > 0$) and three times the size of the object ($|M| = 3.00$), we have

$$M = -\frac{q}{p} = +3.00 \qquad \text{and} \qquad q = -3p$$

The mirror equation then gives

$$\frac{1}{p} - \frac{1}{3.00\,p} = \frac{2.00}{3.00\,p} = \frac{1}{12.0\text{ cm}} \qquad \text{or} \qquad \boxed{p = +8.00\text{ cm}}$$

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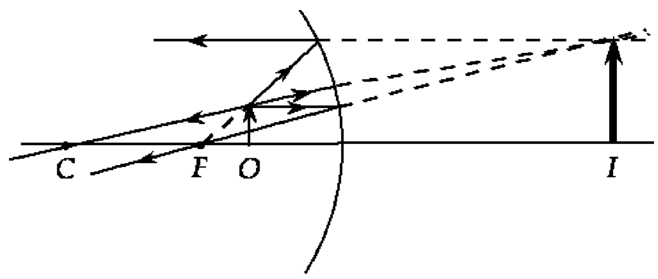
(b) The needed ray diagram, with the object 8.00 cm in front of the mirror, is shown below:



(c) From a carefully drawn scale drawing, you should find that the image is $\boxed{\text{upright, virtual, 24.0 cm behind the mirror, and three times the size of the object}}$.

23.19 (a) An image formed on a screen is a real image. Thus, the mirror must be $\boxed{\text{concave}}$ since, of mirrors, only concave mirrors can form real images of real objects.

(b) The magnified, real images formed by concave mirrors are inverted, so $M < 0$ and

$$M = -\frac{q}{p} = -5\text{, or } p = \frac{q}{5} = \frac{5.0\text{ m}}{5} = 1.0\text{ m}$$

The object should be $\boxed{\text{1.0 m in front of the mirror}}$.

(a—revisited) Now that we have both the object distance and image distance, we can use the mirror equation to be more specific about the required mirror. The needed focal length of the concave mirror is given by

$$f = \frac{pq}{p+q} = \frac{(1.0\text{ m})(5.0\text{ m})}{1.0\text{ m} + 5.0\text{ m}} = \boxed{0.83\text{ m}}$$

23.20 (a) From $\frac{1}{p} + \frac{1}{q} = \frac{2}{R}$, we find $q = \frac{Rp}{2p - R} = \frac{(1.00\text{ m})p}{2p - 1.00\text{ m}}$.

The table gives the image position at a few critical points in the motion. Between $p = 3.00$ m and $p = 0.500$ m, the real image moves from 0.600 m to positive infinity. From $p = 0.500$ cm to $p = 0$, the virtual image moves from negative infinity to 0.

Object Distance, *p*	**Image Distance, *q***
3.00 m	0.600 m
0.500 m	$\pm\infty$
0	0

Note the "jump" in the image position as the ball passes through the focal point of the mirror.

(b) The ball and its image coincide when $p = 0$ and when $1/p + 1/p = 2/p = 2/R$, or $p = R = 1.00$ m.

From $\Delta y = v_{0y}t + \frac{1}{2}a_y t^2$, with $v_{0y} = 0$, the times for the ball to fall from $y = +3.00$ m to these positions are found to be

$p = 1.00$ m position: $$t = \sqrt{\frac{2(\Delta y)}{a_y}} = \sqrt{\frac{2(-2.00\text{ m})}{-9.80\text{ m/s}^2}} = \boxed{0.639\text{ s}}\text{ and}$$

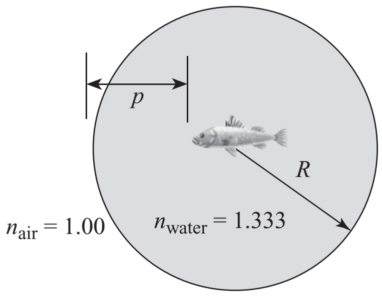
$p = 0$ position: $$t = \sqrt{\frac{2(-3.00\text{ m})}{-9.80\text{ m/s}^2}} = \boxed{0.782\text{ s}}$$

23.21 From $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$, with $R\to\infty$, the image position is found to be

$$q=-\frac{n_2}{n_1}p=-\left(\frac{1.00}{1.309}\right)(50.0\text{ cm})=-38.2\text{ cm}$$

or the virtual image is $\boxed{\text{38.2 cm below the upper surface of the ice}}$.

23.22 The location of the image formed by refraction at this spherical surface is described by Equation 23.7 from the textbook, which states $n_1/p+n_2/q=(n_2-n_1)/R$ and uses the sign convention of Table 23.2. As light crosses this surface, passing from the water into air, we have $n_1=n_{\text{water}}=1.333$, $n_2=n_{\text{air}}=1.00$, $p=+10.0$ cm (object is in front of the surface), and $R=-15.0$ cm (center of curvature is in front of the surface). Thus, the image distance is found from



$$\frac{n_2}{q}=\frac{n_2-n_1}{R}-\frac{n_1}{p}\quad\text{or}\quad\frac{1.00}{q}=\frac{1.00-1.333}{-15.0\text{ cm}}-\frac{1.333}{+10.0\text{ cm}}=\frac{+0.666-3.999}{30.0\text{ cm}}=\frac{-3.333}{30.0\text{ cm}}$$

This yields $q=30.0\text{ cm}/(-3.333)=-9.00$ cm. Thus, the goldfish appears to be in the water, $\boxed{\text{9.00 cm inside the wall of the bowl}}$.

23.23 Since the center of curvature of the surface is on the side the light comes from, $R<0$, giving $R=-4.0$ cm. Then, $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$ becomes

$$\frac{1.00}{q}=\frac{1.00-1.50}{-4.0\text{ cm}}-\frac{1.50}{4.0\text{ cm}}\text{, or } q=-4.0\text{ cm}$$

Thus, the magnification $M=\frac{h'}{h}=-\left(\frac{n_1}{n_2}\right)\frac{q}{p}$ gives

$$h'=-\left(\frac{n_1q}{n_2p}\right)h=-\frac{1.50(-4.0\text{ cm})}{1.00(4.0\text{ cm})}(2.5\text{ mm})=\boxed{3.8\text{ mm}}$$

23.24 For a plane refracting surface, $R\to\infty$, and $\frac{n_1}{p}+\frac{n_2}{q}=\frac{n_2-n_1}{R}$ becomes $q=-\frac{n_2}{n_1}p$.

(a) Considering the bottom of the pool as the object, $p=2.00$ m when the pool is full, and

$$q=-\left(\frac{1.00}{1.333}\right)(2.00\text{ m})=-1.50\text{ m}$$

or the pool appears to be $\boxed{1.50\text{ m}}$ deep.

(b) If the pool is half filled, then $p=1.00$ m, and $q=-0.750$ m. Thus, the bottom of the pool appears to be 0.75 m below the water surface or $\boxed{1.75\text{ m}}$ below ground level.

23.25 As parallel rays from the Sun (object distance, $p \to \infty$) enter the transparent sphere from air $(n_1 = 1.00)$, the center of curvature of the surface is on the side the light is going toward (back side). Thus, $R > 0$. It is observed that a real image is formed on the surface opposite the Sun, giving the image distance as $q = +2R$. Then,

$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \qquad \text{becomes} \qquad 0 + \frac{n}{2R} = \frac{n - 1.00}{R}$$

which reduces to $n = 2n - 2.00$, and gives $n = \boxed{2.00}$.

23.26 Light scattered from the bottom of the plate undergoes two refractions, once at the top of the plate and once at the top of the water. All surfaces are planes $(R \to \infty)$, so the image distance for each refraction is $q = -(n_2/n_1)\,p$. At the top of the plate,

$$q_{1\text{B}} = -\left(\frac{n_{\text{water}}}{n_{\text{glass}}}\right) p_{1\text{B}} = -\left(\frac{1.333}{1.66}\right)(8.00\ \text{cm}) = -6.42\ \text{cm}$$

or the first image is 6.42 cm below the top of the plate. This image serves as a real object for the refraction at the top of the water, so the final image of the bottom of the plate is formed at

$$q_{2\text{B}} = -\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right) p_{2\text{B}} = -\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right)\left(12.0\ \text{cm} + |q_{1\text{B}}|\right)$$

$$= -\left(\frac{1.00}{1.333}\right)(18.4\ \text{cm}) = -13.8\ \text{cm or 13.8 cm below the water surface.}$$

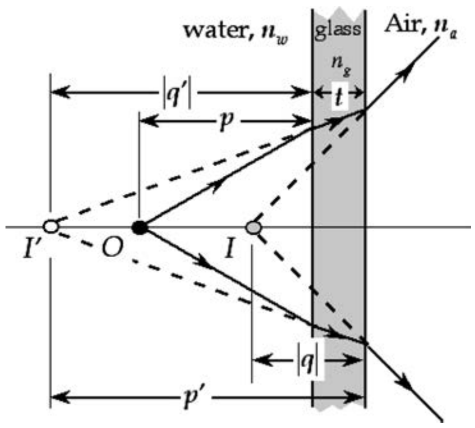
Now, consider light scattered from the top of the plate. It undergoes a single refraction, at the top of the water. This refraction forms an image of the top of the plate at

$$q_{\text{T}} = -\left(\frac{n_{\text{air}}}{n_{\text{water}}}\right) p_{\text{T}} = -\left(\frac{1.00}{1.333}\right)(12.0\ \text{cm}) = -9.00\ \text{cm}$$

or 9.00 cm below the water surface.

The apparent thickness of the plate is then $\Delta y = |q_{2\text{B}}| - |q_{\text{T}}| = 13.8\ \text{cm} - 9.00\ \text{cm} = \boxed{4.8\ \text{cm}}$.

23.27 In the drawing at the right, object O (the jellyfish) is located distance p in front of a plane water-glass interface. Refraction at that interface produces a virtual image I' at distance $|q'|$ in front it. This image serves as the object for refraction at the glass-air interface. This object is located distance $p' = |q'| + t$ in front of the second interface, where t is the thickness of the layer of glass. Refraction at the glass-air interface produces a final virtual image, I, located distance $|q|$ in front of this interface.



From $n_1/p + n_2/q = (n_2 - n_1)/R$ with $R \to \infty$ for a plane, the relation between the object and image distances for refraction at a flat surface is $q = -(n_2/n_1)\,p$. Thus, the image distance for the refraction at the water-glass interface is $q' = -(n_g/n_w)\,p$. This gives an object distance for the refraction at the glass-air interface of $p' = (n_g/n_w)p + t$ and a final image position (measured from the glass-air interface) of

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$$q=-\frac{n_a}{n_g}p'=-\frac{n_a}{n_g}\left[\left(\frac{n_g}{n_w}\right)p+t\right]=-\left[\left(\frac{n_a}{n_w}\right)p+\left(\frac{n_a}{n_g}\right)t\right]$$

(a) If the jellyfish is located 1.00 m (or 100 cm) in front of a 6.00-cm thick pane of glass, then $p=+100$ cm, $t=6.00$ cm, and the position of the final image relative to the glass-air interface is

$$q=-\left[\left(\frac{1.00}{1.333}\right)(100\text{ cm})+\left(\frac{1.00}{1.50}\right)(6.00\text{ cm})\right]=-79.0\text{ cm}$$

It appears to be $\boxed{\text{in the water, 79.0 cm back of the outer surface of the glass pane}}$.

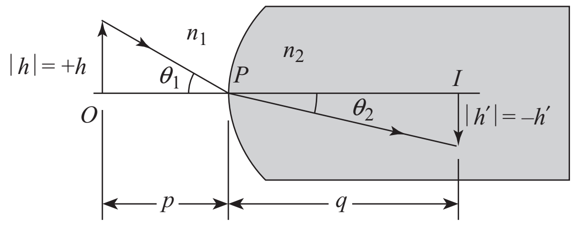
(b) If the thickness of the glass is negligible ($t\to 0$), the distance of the final image from the glass-air interface is

$$q=-\frac{n_a}{n_g}\left[\left(\frac{n_g}{n_w}\right)p+0\right]=-\left(\frac{n_a}{n_w}\right)p=-\left(\frac{1.00}{1.333}\right)(100\text{ cm})=-75.0\text{ cm}$$

Now, it appears to be $\boxed{\text{75.0 cm back of the outer surface of the glass pane}}$.

(c) Comparing the results of parts (a) and (b), we see that the 6.00-cm thickness of the glass in part (a) made a 4.00-cm difference in the apparent position of the jellyfish. We conclude that the thicker the glass, the greater the distance between the final image and the outer surface of the glass.

23.28 We assume the ray shown in the diagram at the right is a paraxial ray so θ_1 and θ_2 are both sufficiently small to allow us to write Snell's law as



$$n_1\tan\theta_1=n_2\tan\theta_2$$

Also, note that we are assuming transverse distances measured upward from the axis *OPI* are positive, while those measured downward from this axis are negative. Looking at the right triangle having distance $\overline{OP}$ as its base, we see that $\tan\theta_1=h/p$, and looking at the triangle having $\overline{PI}$ as a base, we have $\tan\theta_2=|h'|/q=-h'/q$. Thus, Snell's law becomes $n_1(h/p)=-n_2(h'/q)$, and the magnification becomes

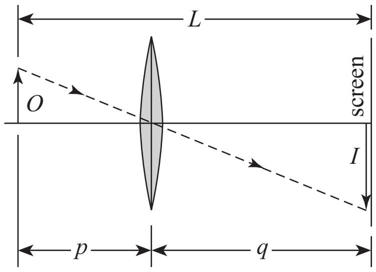
$$\boxed{M=\frac{h'}{h}=-\frac{n_1 q}{n_2 p}}$$

23.29 With $R_1=+2.00$ cm and $R_2=+2.50$ cm, the lens maker's equation gives the focal length as

$$\frac{1}{f}=(n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=(1.50-1)\left(\frac{1}{2.00\text{ cm}}-\frac{1}{2.50\text{ cm}}\right)=+0.050\,0\text{ cm}^{-1}$$

or $\quad f=\dfrac{1}{0.050\,0\text{ cm}^{-1}}=\boxed{+20.0\text{ cm}}$

23.30 Consider the figure at the right showing an object O located distance L in front of a screen. A convergent lens is positioned to focus an image I on the screen. Observe that the sum of the object and image distances is $p+q=L$, giving $p=L-q$. Also, from the thin-lens equation, we have



$$p=\frac{fq}{q-f} \qquad \text{giving} \qquad L-q=\frac{fq}{q-f}$$

Simplifying the last result gives $(L-q)(q-f)=fq$, which reduces to $q^2-Lq+Lf=0$. Solving by use of the quadratic formula, with $L=50.0$ cm and $f=10.0$ cm, gives

$$q=\frac{L\pm\sqrt{L^2-4Lf}}{2}=\frac{50.0\text{ cm}\pm\sqrt{2500\text{ cm}^2-2000\text{ cm}^2}}{2}=25.0\text{ cm}\pm 11.2\text{ cm}$$

(a) Thus, there are 2 locations, generally known as "conjugate positions," where the lens could be positioned to form a clear image on the screen. These positions are:

(i) $\boxed{36.2\text{ cm from the screen}}$ $\rightarrow$ $\left(q=36.2\text{ cm and } p=13.8\text{ cm}\right)$

(ii) $\boxed{13.8\text{ cm from the screen}}$ $\rightarrow$ $\left(q=13.8\text{ cm and } p=36.2\text{ cm}\right)$

(b) If $q=36.2$ cm, the magnification is $M=-\dfrac{q}{p}=-\dfrac{36.2\text{ cm}}{13.8\text{ cm}}=\boxed{-2.62}$.

If $q=13.8$ cm, the magnification is $M=-\dfrac{q}{p}=-\dfrac{13.8\text{ cm}}{36.2\text{ cm}}=\boxed{-0.381}$.

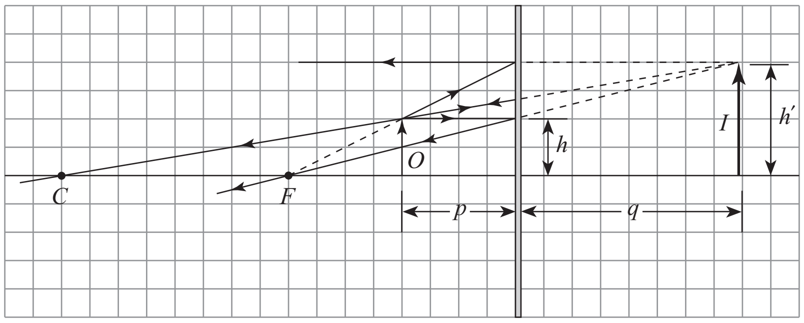
23.31 The focal length of a converging lens is positive, so $f=+10.0$ cm. The thin-lens equation then yields an image distance of $q=\dfrac{pf}{p-f}=\dfrac{p(10.0\text{ cm})}{p-10.0\text{ cm}}$.

(a) When $p=+20.0$ cm, $q=\dfrac{(20.0\text{ cm})(10.0\text{ cm})}{20.0\text{ cm}-10.0\text{ cm}}=+20.0$ cm, and $M=-\dfrac{q}{p}=-1.00$, so the image is located $\boxed{20.0\text{ cm beyond the lens}}$, is $\boxed{\text{real } (q>0)}$, is $\boxed{\text{inverted } (M<0)}$, and is $\boxed{\text{the same size as the object } \left(|M|=1.00\right)}$.

(b) When $p=f=+10.0$ cm, the object is at the focal point and $\boxed{\text{no image is formed}}$. Instead, $\boxed{\text{parallel rays emerge from the lens}}$.

(c) When $p=5.00$ cm, $q=\dfrac{(5.00\text{ cm})(10.0\text{ cm})}{5.00\text{ cm}-10.0\text{ cm}}=-10.0$ cm, and $M=-\dfrac{q}{p}=+2.00$, so the image is located $\boxed{10.0\text{ cm in front of the lens}}$, is $\boxed{\text{virtual } (q<0)}$, is $\boxed{\text{upright } (M>0)}$, and is $\boxed{\text{twice the size of the object } \left(|M|=2.00\right)}$.

23.32 (a) To approximate paraxial rays, the rays should be drawn so they reflect at the vertical plane that passes through the vertex of the mirror, rather than at the mirror's surface as is generally done in the textbook. For this reason, the concave surface of the mirror appears flat in the ray diagram given below.



(b) In the diagram above, each square of the grid represents 5 cm. Thus, we see that the image I is located $\boxed{\text{40 cm behind the mirror, or } q = -40 \text{ cm}}$.

(c) From the above ray diagram, the height h' of the upright image I is seen to be twice the height h of the object O. Therefore, the magnification is $\boxed{M = h'/h = +2}$.

(d) From the mirror equation, the image distance is $q = pf/(p-f)$, or

$$q = \frac{(+20.0 \text{ cm})(+40.0 \text{ cm})}{20.0 \text{ cm} - 40.0 \text{ cm}} = \boxed{-40.0 \text{ cm}}$$

and the magnification is $M = -\dfrac{q}{p} = -\dfrac{(-40.0 \text{ cm})}{+20.0 \text{ cm}} = \boxed{+2.00}$

23.33 For a divergent lens, the focal length is negative. Hence, $f = -20.0$ cm in this case. The thin-lens equation gives the image distance as $q = pf/(p-f)$, and the magnification is given by $M = -q/p$.

(i) (a) $q = \dfrac{(40.0 \text{ cm})(-20.0 \text{ cm})}{40.0 \text{ cm} - (-20.0 \text{ cm})} = -13.3 \text{ cm} \quad \Rightarrow \quad \boxed{\text{13.3 cm in front of the lens.}}$

(b) $q < 0 \quad \Rightarrow \quad \boxed{\text{virtual image}}$

(c) $M = -q/p > 0 \quad \Rightarrow \quad \boxed{\text{upright image}}$

(d) $M = -\dfrac{(-13.3 \text{ cm})}{+40.0 \text{ cm}} = \boxed{+0.333}$

(ii) (a) $q = \dfrac{(20.0 \text{ cm})(-20.0 \text{ cm})}{20.0 \text{ cm} - (-20.0 \text{ cm})} = -10.0 \text{ cm} \quad \Rightarrow \quad \boxed{\text{10.0 cm in front of the lens.}}$

(b) $q < 0 \quad \Rightarrow \quad \boxed{\text{virtual image}}$

(c) $M = -q/p > 0 \quad \Rightarrow \quad \boxed{\text{upright image}}$

(d) $M = -\dfrac{(-10.0 \text{ cm})}{+20.0 \text{ cm}} = \boxed{+0.500}$

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(iii) (a) $q = \dfrac{(10.0 \text{ cm})(-20.0 \text{ cm})}{10.0 \text{ cm} - (-20.0 \text{ cm})} = -6.67 \text{ cm} \quad \Rightarrow \quad \boxed{6.67 \text{ cm in front of the lens}}$

(b) $q < 0 \quad \Rightarrow \quad \boxed{\text{virtual image}}$

(c) $M = -q/p > 0 \quad \Rightarrow \quad \boxed{\text{upright image}}$

(d) $M = -\dfrac{(-6.67 \text{ cm})}{+10.0 \text{ cm}} = \boxed{+0.667}$

23.34 (a) and (b) Your scale drawings should look similar to those given below:

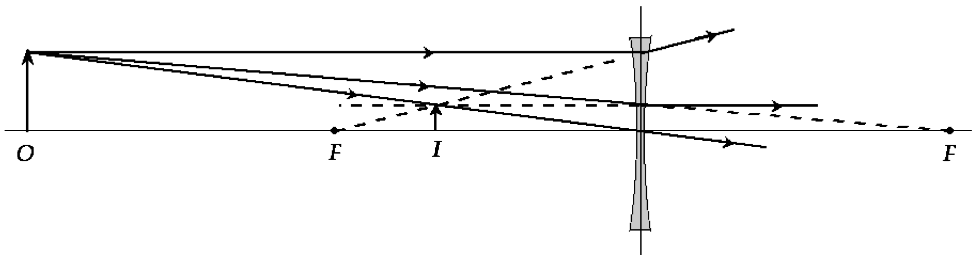


Figure (a)

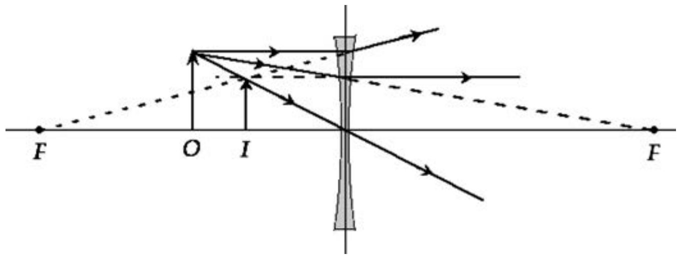


Figure (b)

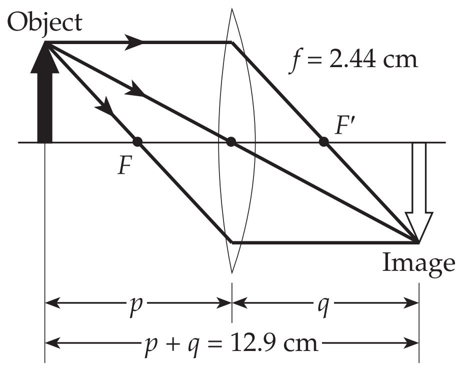
A carefully drawn-to-scale version of Figure (a) should yield an upright, virtual image located 13.3 cm in front of the lens and one-third the size of the object. Similarly, a carefully drawn-to-scale version of Figure (b) should yield an upright, virtual image located 6.7 cm in front of the lens and two-thirds the size of the object.

(c) The results of the graphical solution are consistent with the algebraic answers found in Problem 23.33, allowing for small deviances due to uncertainties in measurement. Graphical answers may vary, depending on the size of the graph paper and accuracy of the drawing.

23.35 (a) The real image case is shown in the ray diagram. Notice that $p + q = 12.9$ cm, or $q = 12.9 \text{ cm} - p$. The thin-lens equation, with $f = 2.44$ cm, then gives

$$\frac{1}{p} + \frac{1}{12.9 \text{ cm} - p} = \frac{1}{2.44 \text{ cm}}$$

or $\quad p^2 - (12.9 \text{ cm})p + 31.5 \text{ cm}^2 = 0$



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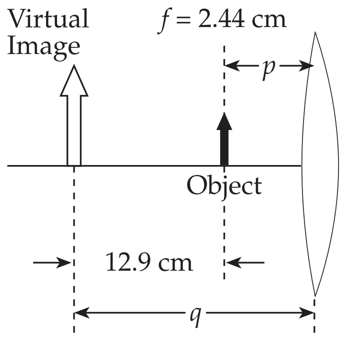
Using the quadratic formula to solve gives

$$\boxed{p = 9.63\text{ cm or } p = 3.27\text{ cm}}$$

Both are valid solutions for the real image case.



(b) The virtual image case is shown in the second diagram. Note that in this case, $q = -(12.9\text{ cm} + p)$, so the thin-lens equation gives

$$\frac{1}{p} - \frac{1}{12.9\text{ cm} + p} = \frac{1}{2.44\text{ cm}}$$

or $\quad p^2 + (12.9\text{ cm})p - 31.5\text{ cm}^2 = 0$

The quadratic formula then gives $p = 2.10\text{ cm}$ or $p = -15.0\text{ cm}$.

Since the object is real, the negative solution must be rejected, leaving $\boxed{p = 2.10\text{ cm}}$.

23.36 We must first realize that we are looking at an upright, enlarged, virtual image. Thus, we have a real object located between a converging lens and its front-side focal point, so $q < 0$, $p > 0$, and $f > 0$.

The magnification is $M = -\dfrac{q}{p} = +2$, giving $q = -2p$. Then, from the thin-lens equation, $\dfrac{1}{p} - \dfrac{1}{2p} = +\dfrac{1}{2p} = \dfrac{1}{f}$, or $f = 2p = 2(2.84\text{ cm}) = \boxed{5.68\text{ cm}}$.

23.37 It is desired to form a magnified, real image on the screen using a single thin lens. To do this, a converging lens must be used, and the image will be inverted. The magnification then gives

$$M = \frac{h'}{h} = \frac{-1.80\text{ m}}{24.0 \times 10^{-3}\text{ m}} = -\frac{q}{p}, \text{ or } q = 75.0\,p$$

Also, we know that $p + q = 3.00$ m. Therefore, $p + 75.0\,p = 3.00$ m, giving

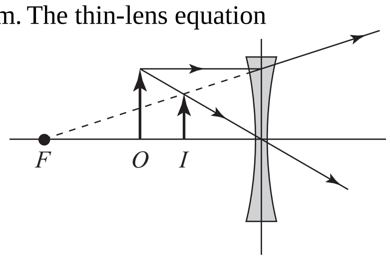
(b) $\quad p = \dfrac{3.00\text{ m}}{76.0} = 3.95 \times 10^{-2}\text{ m} = \boxed{39.5\text{ mm}}$

(a) The thin-lens equation then gives $\dfrac{1}{p} + \dfrac{1}{75.0\,p} = \dfrac{76.0}{75.0\,p} = \dfrac{1}{f}$, or

$$f = \left(\frac{75.0}{76.0}\right)p = \left(\frac{75.0}{76.0}\right)(39.5\text{ mm}) = \boxed{39.0\text{ mm}}$$

23.38 (a) This is a real object, so the object distance is $p = +20.0$ cm. The thin-lens equation gives the image distance as

$$q = \frac{pf}{p-f} = \frac{(20.0\text{ cm})(-32.0\text{ cm})}{20.0\text{ cm} - (-32.0\text{ cm})} = -12.3\text{ cm}$$



so the image is $\boxed{12.3\text{ cm to the left of the lens}}$.

(b) The magnification is $M = -\dfrac{q}{p} = -\dfrac{(-12.3\text{ cm})}{+20.0\text{ cm}} = \boxed{+0.615}$.

(c) The ray diagram for this arrangement is shown above.

23.39 Since the light rays incident to the first lens are parallel, $p_1 = \infty$, and the thin-lens equation gives $q_1 = f_1 = -10.0\text{ cm}$.

The virtual image formed by the first lens serves as the object for the second lens, so $p_2 = 30.0\text{ cm} + |q_1| = 40.0\text{ cm}$. If the light rays leaving the second lens are parallel, then $q_2 = \infty$, and the thin-lens equation gives $f_2 = p_2 = \boxed{40.0\text{ cm}}$.

23.40 (a) Solving the thin-lens equation for the image distance q gives

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{p-f}{pf} \qquad \text{or} \qquad \boxed{q = \frac{pf}{p-f}}$$

(b) For a real object, $p > 0$ and $p = |p|$. Also, for a diverging lens, $f < 0$ and $f = -|f|$. The result of part (a) then becomes

$$q = \frac{|p|(-|f|)}{|p| - (-|f|)} = -\frac{|p||f|}{|p| + |f|}$$

Thus, we see that $\boxed{q < 0 \text{ for all numeric values of } |p| \text{ and } |f|}$. Since negative image distances mean virtual images, we conclude that a diverging lens will always form virtual images of real objects.

(c) For a real object, $p > 0$ and $p = |p|$. Also, for a converging lens, $f > 0$ and $f = |f|$. The result of part (a) then becomes

$$q = \frac{|p||f|}{|p| - |f|} > 0 \quad \text{if} \quad |p| - |f| > 0$$

Since q must be positive for a real image, we see that a converging lens will form real images of real objects only when $|p| > |f|$ (or $\boxed{p > f}$ since both p and f are positive in this situation).

23.41 The thin-lens equation gives the image position for the first lens as

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(30.0\text{ cm})(15.0\text{ cm})}{30.0\text{ cm} - 15.0\text{ cm}} = +30.0\text{ cm}$$

and the magnification by this lens is $M_1 = -\dfrac{q_1}{p_1} = -\dfrac{30.0\text{ cm}}{30.0\text{ cm}} = -1.00$.

The real image formed by the first lens serves as the object for the second lens, so $p_2 = 40.0\text{ cm} - q_1 = +10.0\text{ cm}$. Then, the thin-lens equation gives

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(10.0\text{ cm})(15.0\text{ cm})}{10.0\text{ cm} - 15.0\text{ cm}} = -30.0\text{ cm}$$

and the magnification by the second lens is

$$M_2 = -\frac{q_2}{p_2} = -\frac{(-30.0\text{ cm})}{10.0\text{ cm}} = +3.00$$

Thus, the final, virtual image is located $\boxed{30.0\text{ cm in front of the second lens}}$,

and the overall magnification is $M = M_1 M_2 = (-1.00)(+3.00) = \boxed{-3.00}$.

23.42 (a) With $p_1 = +15.0$ cm, the thin-lens equation gives the position of the image formed by the first lens as

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(15.0\text{ cm})(10.0\text{ cm})}{15.0\text{ cm} - 10.0\text{ cm}} = +30.0\text{ cm}$$

This image serves as the object for the second lens, with an object distance of $p_2 = 10.0\text{ cm} - q_1 = 10.0\text{ cm} - 30.0\text{ cm} = -20.0\text{ cm}$ (a virtual object). If the image formed by this lens is at the position of O_1, the image distance is

$$q_2 = -(10.0\text{ cm} + p_1) = -(10.0\text{ cm} + 15.0\text{ cm}) = -25.0\text{ cm}$$

The thin-lens equation then gives the focal length of the second lens as

$$f_2 = \frac{p_2 q_2}{p_2 + q_2} = \frac{(-20.0\text{ cm})(-25.0\text{ cm})}{-20.0\text{ cm} - 25.0\text{ cm}} = \boxed{-11.1\text{ cm}}$$

(b) The overall magnification is

$$M = M_1 M_2 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right) = \left(-\frac{30.0\text{ cm}}{15.0\text{ cm}}\right)\left[-\frac{(-25.0\text{ cm})}{(-20.0\text{ cm})}\right] = \boxed{+2.50}$$

(c) Since $q_2 < 0$, the final image is $\boxed{\text{virtual}}$; and since $M > 0$, it is $\boxed{\text{upright}}$.

23.43 From the thin-lens equation, $q_1 = \dfrac{p_1 f_1}{p_1 - f_1} = \dfrac{(4.00\text{ cm})(8.00\text{ cm})}{4.00\text{ cm} - 8.00\text{ cm}} = -8.00\text{ cm}$.

The magnification by the first lens is $M_1 = -\dfrac{q_1}{p_1} = -\dfrac{(-8.00\text{ cm})}{4.00\text{ cm}} = +2.00$.

The virtual image formed by the first lens is the object for the second lens, so $p_2 = 6.00\text{ cm} + |q_1| = +14.0\text{ cm}$, and the thin-lens equation gives

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(14.0\text{ cm})(-16.0\text{ cm})}{14.0\text{ cm} - (-16.0\text{ cm})} = -7.47\text{ cm}$$

The magnification by the second lens is $M_2 = -\dfrac{q_2}{p_2} = -\dfrac{(-7.47\text{ cm})}{14.0\text{ cm}} = +0.533$, so the overall magnification is $M = M_1 M_2 = (+2.00)(+0.533) = +1.07$.

The position of the final image is $\boxed{\text{7.47 cm in front of the second lens}}$, and its height is $h' = M\,h = (+1.07)(1.00\text{ cm}) = \boxed{1.07\text{ cm}}$.

Since $M > 0$, the final image is $\boxed{\text{upright}}$; and since $q_2 < 0$, this image is $\boxed{\text{virtual}}$.

23.44 (a) We start with the final image and work backward. From Figure P23.44, observe that $q_2 = -(50.0\text{ cm} - 31.0\text{ cm}) = -19.0$ cm. The thin-lens equation then gives

$$p_2 = \frac{q_2 f_2}{q_2 - f_2} = \frac{(-19.0\text{ cm})(20.0\text{ cm})}{-19.0\text{ cm} - 20.0\text{ cm}} = +9.74\text{ cm}$$

continued on next page

The image formed by the first lens serves as the object for the second lens and is located 9.74 cm in front of the second lens.

Thus, the image distance for the first lens is $q_1 = 50.0\text{ cm} - 9.74\text{ cm} = 40.3\text{ cm}$, and the thin-lens equation gives

$$p_1 = \frac{q_1 f_1}{q_1 - f_1} = \frac{(40.3\text{ cm})(10.0\text{ cm})}{40.3\text{ cm} - 10.0\text{ cm}} = +13.3\text{ cm}$$

The original object should be located $\boxed{13.3\text{ cm}}$ in front of the first lens.

(b) The overall magnification is

$$M = M_1 M_2 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right) = \left(-\frac{40.3\text{ cm}}{13.3\text{ cm}}\right)\left(-\frac{(-19.0\text{ cm})}{9.74\text{ cm}}\right) = \boxed{-5.91}$$

(c) Since $M < 0$, the final image is $\boxed{\text{inverted}}$.

23.45 **Note:** Final answers to this problem are highly sensitive to round-off error. To avoid this, we retain extra digits in intermediate answers and round only the final answers to the correct number of significant figures.

Since the final image is to be real and in the film plane, $q_2 = +d$.

Then, the thin-lens equation gives $p_2 = \dfrac{q_2 f_2}{q_2 - f_2} = \dfrac{d(13.0\text{ cm})}{d - 13.0\text{ cm}}$.

The object of the second lens (L_2) is the image formed by the first lens (L_1), so

$$q_1 = (12.0\text{ cm} - d) - p_2 = 12.0\text{ cm} - d\left(1 + \frac{13.0\text{ cm}}{d - 13.0\text{ cm}}\right) = 12.0\text{ cm} - \frac{d^2}{d - 13.0\text{ cm}}$$

If $d = 5.00\text{ cm}$, then $q_1 = +15.125\text{ cm}$; and when $d = 10.0\text{ cm}$, $q_1 = +45.3\text{ cm}$.

From the thin-lens equation, $p_1 = \dfrac{q_1 f_1}{q_1 - f_1} = \dfrac{q_1(15.0\text{ cm})}{q_1 - 15.0\text{ cm}}$.

When $q_1 = +15.125\text{ cm}$ $(d = 5.00\text{ cm})$, then $p_1 = 1.82 \times 10^3\text{ cm} = 18.2\text{ m}$.

When $q_1 = +45.3\text{ cm}$ $(d = 10.0\text{ cm})$, then $p_1 = 22.4\text{ cm} = 0.224\text{ m}$.

Thus, the range of focal distances for this camera is $\boxed{0.224\text{ m to }18.2\text{ m}}$.

23.46 (a) From the thin-lens equation, the image distance for the first lens is

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(15.0\text{ cm})(10.0\text{ cm})}{15.0\text{ cm} - 10.0\text{ cm}} = \boxed{+30.0\text{ cm}}$$

(b) With $q_1 = +30.0\text{ cm}$, the image of the first lens is located 30.0 cm in back of that lens. Since the second lens is only 10.0 cm beyond the first lens, this means that the first lens is trying to form its image at a location $\boxed{\text{20.0 cm beyond the second lens}}$.

continued on next page

(c) The image the first lens forms (or would form if allowed to do so) serves as the object for the second lens. Considering the answer to part (b) above, we see that this will be a virtual object, with object distance $\boxed{p_2 = -20.0\text{ cm}}$.

(d) From the thin-lens equation, the image distance for the second lens is

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-20.0\text{ cm})(5.00\text{ cm})}{-20.0\text{ cm} - 5.00\text{ cm}} = \boxed{+4.00\text{ cm}}$$

(e) $M_1 = -\dfrac{q_1}{p_1} = -\dfrac{30.0\text{ cm}}{15.0\text{ cm}} = \boxed{-2.00}$

(f) $M_2 = -\dfrac{q_2}{p_2} = -\dfrac{4.00\text{ cm}}{(-20.0\text{ cm})} = \boxed{+0.200}$

(g) $M_{\text{total}} = M_1 M_2 = (-2.00)(+0.200) = \boxed{-0.400}$

(h) Since $q_2 > 0$, the final image is $\boxed{\text{real}}$, and since $M_{\text{total}} < 0$, that image is $\boxed{\text{inverted}}$ relative to the original object.

23.47 Since $q = +8.00$ cm when $p = +10.0$ cm, we find that

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{q} = \frac{1}{10.0\text{ cm}} + \frac{1}{8.00\text{ cm}} = \frac{18.0}{80.0\text{ cm}}$$

Then, when $p = 20.0$ cm,

$$\frac{1}{q} = \frac{1}{f} - \frac{1}{p} = \frac{18.0}{80.0\text{ cm}} - \frac{1}{20.0\text{ cm}} = \frac{18.0 - 4.00}{80.0\text{ cm}} = \frac{14.0}{80.0\text{ cm}}$$

or $\quad q = \dfrac{80.0\text{ cm}}{14.0} = +5.71\text{ cm}$

Thus, a $\boxed{\text{real}}$ image is formed $\boxed{\text{5.71 cm in front of the mirror}}$.

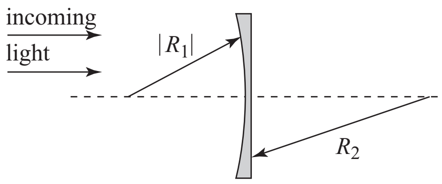
23.48 (a) We are given that $p = 5f$, with both p and f being positive. The thin-lens equation then gives

$$q = \frac{pf}{p - f} = \frac{(5f)f}{5f - f} = \boxed{\frac{5f}{4}}$$

(b) $M = -\dfrac{q}{p} = -\dfrac{(5f/4)}{5f} = \boxed{-\dfrac{1}{4}}$

(c) Since $q > 0$, the image is $\boxed{\text{real}}$. Because $M < 0$, the image is $\boxed{\text{inverted}}$. Since the object is real, it is located in front of the lens, and with $q > 0$, the image is located in back of the lens. Thus, the image is on the $\boxed{\text{opposite side}}$ of the lens from the object.

23.49 In the sketch at the right, the center of curvature of the left side of the biconcave lens is on the side of the incoming light. Thus, by the convention of Table 23.2, the radius of curvature of this side is negative while the radius of curvature of the right side is positive. If $R_1 = -32.5$ cm and $R_2 = 42.5$ cm, the lens maker's equation gives the focal length of the lens as



$$\frac{1}{f} = (n-1)\left(\frac{1}{-32.5 \text{ cm}} - \frac{1}{42.5 \text{ cm}}\right) = (1-n)\left(5.43\times10^{-2} \text{ cm}^{-1}\right)$$

(a) For a very distant object ($p \to \infty$), the thin-lens equation gives the image distance as $q = f$. Thus, if the index of refraction of the lens material is $n = 1.53$ for violet light,

$$q = f = \frac{1}{(1-1.53)\left(5.43\times10^{-2} \text{ cm}^{-1}\right)} = -34.7 \text{ cm}$$

and the image of violet light is formed $\boxed{\text{34.7 cm to the left of the lens}}$.

(b) If $n = 1.51$ for red light, the image distance for very distance source emitting red light is

$$q = f = \frac{1}{(1-1.51)\left(5.43\times10^{-2} \text{ cm}^{-1}\right)} = -36.1 \text{ cm}$$

The image of the very distant red light source is formed $\boxed{\text{36.1 cm to the left of the lens}}$.

23.50 (a) Using the sign convention from Table 23.2, the radii of curvature of the surfaces are $R_1 = -15.0$ cm and $R_2 = +10.0$ cm. The lens maker's equation then gives

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = (1.50-1)\left(\frac{1}{-15.0 \text{ cm}} - \frac{1}{10.0 \text{ cm}}\right) \text{ or } f = \boxed{-12.0 \text{ cm}}$$

(b) If $p \to \infty$, then $q = f = \boxed{-12.0 \text{ cm}}$.

The thin-lens equation gives $q = \dfrac{pf}{p-f} = \dfrac{p(-12.0 \text{ cm})}{p+12.0 \text{ cm}}$ and the following results:

(c) If $p = 3|f| = +36.0$ cm, $q = \boxed{-9.00 \text{ cm}}$.

(d) If $p = |f| = +12.0$ cm, $q = \boxed{-6.00 \text{ cm}}$.

(e) If $p = |f|/2 = +6.00$ cm, $q = \boxed{-4.00 \text{ cm}}$.

23.51 As light passes left to right through the lens, the image position is given by

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(100 \text{ cm})(80.0 \text{ cm})}{100 \text{ cm} - 80.0 \text{ cm}} = +400 \text{ cm}$$

This image serves as an object for the mirror with an object distance of $p_2 = 100 \text{ cm} - q_1 = -300$ cm (virtual object). From the mirror equation, the position of the image formed by the mirror is

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-300 \text{ cm})(-50.0 \text{ cm})}{-300 \text{ cm} - (-50.0 \text{ cm})} = -60.0 \text{ cm}$$

continued on next page

This image is the object for the lens as light now passes through it going right to left. The object distance for the lens is $p_3 = 100\text{ cm} - q_2 = 100\text{ cm} - (-60.0\text{ cm})$, or $p_3 = 160$ cm. From the thin- lens equation,

$$q_3 = \frac{p_3 f_3}{p_3 - f_3} = \frac{(160\text{ cm})(80.0\text{ cm})}{160\text{ cm} - 80.0\text{ cm}} = +160\text{ cm}$$

Thus, the final image is located $\boxed{\text{160 cm to the left of the lens}}$.

The overall magnification is $M = M_1 M_2 M_3 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right)\left(-\frac{q_3}{p_3}\right)$, or

$$M = \left(-\frac{400\text{ cm}}{100\text{ cm}}\right)\left[-\frac{(-60.0\text{ cm})}{(-300\text{ cm})}\right]\left(-\frac{160\text{ cm}}{160\text{ cm}}\right) = \boxed{-0.800}$$

Since $M < 0$, the final image is $\boxed{\text{inverted}}$.

23.52 (a) Since the object is midway between the lens and mirror, the object distance for the mirror is $p_1 = +12.5$ cm. The mirror equation gives the image position as

$$\frac{1}{q_1} = \frac{2}{R} - \frac{1}{p_1} = \frac{2}{+20.0\text{ cm}} - \frac{1}{12.5\text{ cm}} = \frac{5-4}{50.0\text{ cm}} = \frac{1}{50.0\text{ cm}} \quad \text{or} \quad q_1 = +50.0\text{ cm}$$

This image serves as the object for the lens, so $p_2 = 25.0\text{ cm} - q_1 = -25.0$ cm. Note that since $p_2 < 0$, this is a virtual object. The thin-lens equation gives the image position for the lens as

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-25.0\text{ cm})(-16.7\text{ cm})}{-25.0\text{ cm} - (-16.7\text{ cm})} = -50.3\text{ cm}$$

Since $q_2 < 0$, this image is located 50.3 cm in front of (to the right of) the lens or $\boxed{\text{25.3 cm behind (to the right of) the mirror}}$.

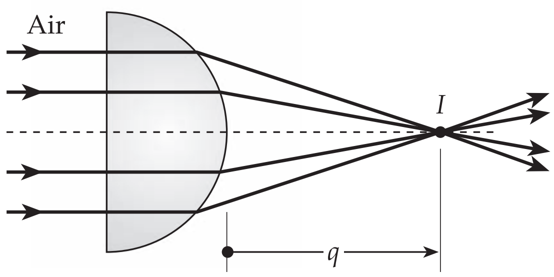
(b) With $q_2 < 0$, the final image is a $\boxed{\text{virtual}}$ image.

(c) and (d) The overall magnification is

$$M = M_1 M_2 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right) = \left(-\frac{50.0\text{ cm}}{12.5\text{ cm}}\right)\left[-\frac{(-50.3\text{ cm})}{(-25.0\text{ cm})}\right] = \boxed{+8.05}$$

Since $M > 0$, the final image is $\boxed{\text{upright}}$.

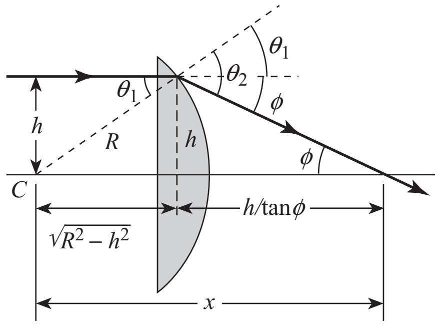
23.53 A hemisphere is too thick to be described as a thin lens. The light is undeviated on entry into the flat face. We next consider the light's exit from the curved surface, for which $R = -6.00$ cm.



The incident rays are parallel, so $p = \infty$.

Then, $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ becomes $0 + \frac{1.00}{q} = \frac{1.00 - 1.56}{-6.00\text{ cm}}$, from which $\boxed{q = 10.7\text{ cm}}$.

23.54 The diagram at the right shows a light ray traveling parallel to the principle axis of a plano-convex lens at distance h off the axis. This ray strikes the plane surface at normal incidence and passes into the glass undeviated. The angle of incidence at the spherical surface (of radius R) is θ_1, where $\sin\theta_1 = h/R$. If the index of refraction of the lens material is n, Snell's law gives the angle of refraction at the spherical surface as



$$\theta_2 = \sin^{-1}\left(\frac{n\sin\theta_1}{n_{\text{air}}}\right) = \sin^{-1}\left(\frac{nh}{R}\right)$$

The distance x from the center of curvature of the spherical surface to the point where the refracted ray crosses the principle axis of the lens is

$$x = \sqrt{R^2 - h^2} + \frac{h}{\tan\phi} \qquad \text{where} \qquad \phi = \theta_2 - \theta_1$$

If $R = 20.0$ cm, $n = 1.60$, and the first ray is distance $h_1 = 0.500$ cm off the axis, we find

$$\theta_1 = \sin^{-1}\left(\frac{0.500\text{ cm}}{20.0\text{ cm}}\right) = 1.43° \qquad \text{and} \qquad \theta_2 = \sin^{-1}\left[\frac{1.60(0.500\text{ cm})}{20.0\text{ cm}}\right] = 2.29°$$

so $$x_1 = \sqrt{(20.0\text{ cm})^2 - (0.500\text{ cm})^2} + \frac{0.500\text{ cm}}{\tan(2.29° - 1.43°)} = 53.3\text{ cm}$$

If the second ray is distance $h_2 = 12.0$ cm from the axis, then

$$\theta_1 = \sin^{-1}\left(\frac{12.0\text{ cm}}{20.0\text{ cm}}\right) = 36.9° \qquad \text{and} \qquad \theta_2 = \sin^{-1}\left[\frac{1.60(12.0\text{ cm})}{20.0\text{ cm}}\right] = 73.7°$$

giving $$x_2 = \sqrt{(20.0\text{ cm})^2 - (12.0\text{ cm})^2} + \frac{12.0\text{ cm}}{\tan(73.7° - 36.9°)} = 32.0\text{ cm}$$

The distance between the points where these two rays cross the principle axis is then

$$\Delta x = x_1 - x_2 = 53.3\text{ cm} - 32.0\text{ cm} = \boxed{21.3\text{ cm}}$$

23.55 (a) With light going through the piece of glass from left to right, the radius of the first surface is positive and that of the second surface is negative according to the sign convention of Table 23.2. Thus, $R_1 = +2.00$ cm, and $R_2 = -4.00$ cm.

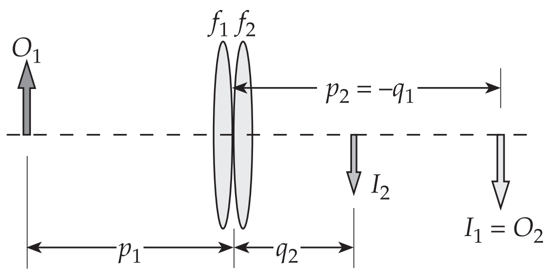
Applying $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ to the first surface gives $\frac{1.00}{1.00\text{ cm}} + \frac{1.50}{q_1} = \frac{1.50 - 1.00}{+2.00\text{ cm}}$, which yields $q_1 = -2.00$ cm. The first surface forms a virtual image 2.00 cm to the left of that surface, and 16.0 cm to the left of the second surface.

The image formed by the first surface is the object for the second surface, so $p_2 = +16.0$ cm, and $\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R}$ gives $\frac{1.50}{16.0\text{ cm}} + \frac{1.00}{q_2} = \frac{1.00 - 1.50}{-4.00\text{ cm}}$, or $q_2 = +32.0$ cm.

The final image is located $\boxed{\text{32.0 cm to the right of the second surface}}$.

(b) Since $q_2 > 0$, the final image formed by the piece of glass is $\boxed{\text{a real image}}$.

23.56 Consider an object O_1 at distance p_1 in front of the first lens. The thin-lens equation gives the image position for this lens as $\frac{1}{q_1} = \frac{1}{f_1} - \frac{1}{p_1}$.



The image, I_1, formed by the first lens serves as the object, O_2, for the second lens. With the lenses in contact, this will be a virtual object if I_1 is real and will be a real object if I_1 is virtual. In either case, if the thicknesses of the lenses may be ignored,

$$p_2 = -q_1 \qquad \text{and} \qquad \frac{1}{p_2} = -\frac{1}{q_1} = -\frac{1}{f_1} + \frac{1}{p_1}$$

Applying the thin-lens equation to the second lens, $\frac{1}{p_2} + \frac{1}{q_2} = \frac{1}{f_2}$ becomes

$$-\frac{1}{f_1} + \frac{1}{p_1} + \frac{1}{q_2} = \frac{1}{f_2} \qquad \text{or} \qquad \frac{1}{p_1} + \frac{1}{q_2} = \frac{1}{f_1} + \frac{1}{f_2}$$

Observe that this result is a thin-lens-type equation relating the position of the original object, O_1, and the position of the final image, I_2, formed by this two-lens combination. Thus, we see that we may treat two thin lenses in contact as a single lens having a focal length, f, given by

$\boxed{\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}}$.

23.57 From the thin-lens equation, the image distance for the first lens is

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(40.0\text{ cm})(30.0\text{ cm})}{40.0\text{ cm} - 30.0\text{ cm}} = +120\text{ cm}$$

and the magnification by this lens is $M_1 = -\frac{q_1}{p_1} = -\frac{120\text{ cm}}{40.0\text{ cm}} = -3.00$.

The real image formed by the first lens serves as the object for the second lens, with object distance of $p_2 = 110\text{ cm} - q_1 = -10.0\text{ cm}$ (a virtual object). The thin-lens equation gives the image distance for the second lens as

$$q_2 = \frac{p_2 f_2}{p_2 - f_2} = \frac{(-10.0\text{ cm}) f_2}{-10.0\text{ cm} - f_2}$$

(a) If $f_2 = -20.0$ cm, then $q_2 = +20.0$ cm and the magnification by the second lens is $M_2 = -q_2/p_2 = -(20.0\text{ cm})/(-10.0\text{ cm}) = +2.00$.

The final image is located $\boxed{\text{20.0 cm to the right of the second lens}}$, and the overall magnification is $M = M_1 M_2 = (-3.00)(+2.00) = \boxed{-6.00}$.

(b) Since $M < 0$, the final image is $\boxed{\text{inverted}}$.

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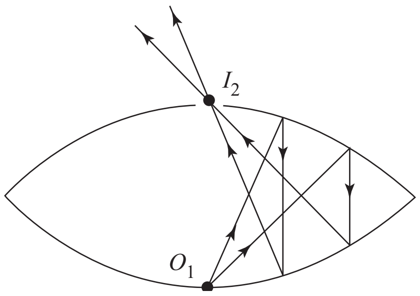
(c) If $f_2 = +20.0$ cm, then $q_2 = +6.67$ cm, and $M_2 = -\dfrac{q_2}{p_2} = -\dfrac{6.67\text{ cm}}{(-10.0\text{ cm})} = +0.667$.

The final image is $\boxed{\text{6.67 cm to the right of the second lens}}$, and the overall magnification is $M = M_1M_2 = (-3.00)(+0.667) = \boxed{-2.00}$.

Since $M < 0$, the final image is $\boxed{\text{inverted}}$.

23.58 The diagram at the right gives a ray diagram showing how this mirror system forms a final image I_2 located just above the opening in the upper mirror. The focal point of each mirror is at the center of the opposite mirror, so their focal length f is the vertical distance between the centers of the mirrors. The original object O_1 sits just above the surface at the center of the lower mirror. This places it on the principle axis and just inside the focal point of the upper mirror.



Therefore, the object distance for the upper mirror is $p_1 = f - \varepsilon$, where ε is a very small number approaching zero. The mirror equation then gives

$$q_1 = \frac{p_1 f}{p_1 - f} = \frac{(f-\varepsilon)f}{f-\varepsilon-f} = -\frac{f^2-\varepsilon f}{\varepsilon} = -\delta \qquad \text{and} \qquad \delta \to \infty \text{ when } \varepsilon \to 0$$

Thus, the upper mirror forms a virtual image, on the principle axis, far above this mirror. This image serves as the object for the lower mirror and is located far in front of that mirror. With $p_2 \approx |q_1| = \delta \gg f$, the image distance for the lower mirror is

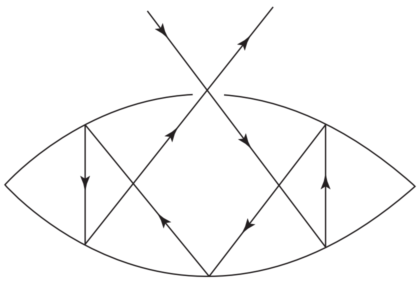
$$q_2 = \frac{p_2 f}{p_2 - f} = \frac{\delta f}{\delta - f} \approx +\frac{\delta f}{\delta} = +f$$

Since $q_2 > 0$, the final image I_2 is a $\boxed{\text{real image}}$ located just above the focal point of the lower mirror (at the opening in the upper mirror). The overall magnification for this mirror system is

$$M = M_1M_2 = \left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right) \approx \left(-\frac{-\delta}{f}\right)\left(-\frac{f}{\delta}\right) = +1$$

This means that the final image is $\boxed{\text{upright and the same size as the original object}}$.

When a flashlight beam is aimed at the final image, it passes through the opening in the upper mirror and reflects as shown in the ray diagram at the right. It emerges from the mirror as if it had reflected from that image, even obeying the law of reflection as it did so!



23.59 (a) The lens maker's equation, $\dfrac{1}{f}=(n-1)\left(\dfrac{1}{R_1}-\dfrac{1}{R_2}\right)$, gives

$$\frac{1}{5.00\text{ cm}}=(n-1)\left(\frac{1}{9.00\text{ cm}}-\frac{1}{-11.0\text{ cm}}\right)$$

which simplifies to $n=1+\dfrac{1}{5.00}\left(\dfrac{99.0}{11.0+9.00}\right)=\boxed{1.99}$.

(b) As light passes from left to right through the lens, the thin-lens equation gives the image distance as

$$q_1=\frac{p_1 f}{p_1-f}=\frac{(8.00\text{ cm})(5.00\text{ cm})}{8.00\text{ cm}-5.00\text{ cm}}=+13.3\text{ cm}$$

This image formed by the lens serves as an object for the mirror with object distance $p_2=20.0\text{ cm}-q_1=+6.67\text{ cm}$. The mirror equation then gives

$$q_2=\frac{p_2 R}{2p_2-R}=\frac{(6.67\text{ cm})(8.00\text{ cm})}{2(6.67\text{ cm})-8.00\text{ cm}}=+10.0\text{ cm}$$

This real image, formed 10.0 cm to the left of the mirror, serves as an object for the lens as light passes through it from right to left. The object distance is $p_3=20.0\text{ cm}-q_2=+10.0\text{ cm}$, and the thin-lens equation gives

$$q_3=\frac{p_3 f}{p_3-f}=\frac{(10.0\text{ cm})(5.00\text{ cm})}{10.0\text{ cm}-5.00\text{ cm}}=+10.0\text{ cm}$$

The final image is located $\boxed{\text{10.0 cm to the left of the lens}}$ and its overall magnification is

$$M=M_1M_2M_3=\left(-\frac{q_1}{p_1}\right)\left(-\frac{q_2}{p_2}\right)\left(-\frac{q_3}{p_3}\right)=\left(-\frac{13.3}{8.00}\right)\left(-\frac{10.0}{6.67}\right)\left(-\frac{10.0}{10.0}\right)=\boxed{-2.50}$$

(c) Since $M<0$, the final image is $\boxed{\text{inverted}}$.

23.60 From the thin-lens equation, the object distance is $p=\dfrac{qf}{q-f}$.

(a) If $q=+4f$, then $p=\dfrac{(4f)f}{4f-f}=\boxed{\dfrac{4f}{3}}$ or $\boxed{1.33f}$.

(b) When $q=-3f$, we find $p=\dfrac{(-3f)f}{-3f-f}=\boxed{3f/4}$, or $\boxed{0.750f}$.

(c) In case (a), $M=-\dfrac{q}{p}=-\dfrac{4f}{4f/3}=\boxed{-3}$, and in case (b), $M=-\dfrac{q}{p}=-\dfrac{-3f}{3f/4}=\boxed{+4}$.

23.61 If $R_1=-3.00$ m and $R_2=-6.00$ m, the focal length is given by

$$\frac{1}{f}=\left(\frac{n_1}{n_2}-1\right)\left(\frac{1}{-3.00\text{ m}}+\frac{1}{6.00\text{ m}}\right)=\left(\frac{n_1-n_2}{n_2}\right)\left(\frac{-1}{6.00\text{ m}}\right)$$

continued on next page

or $$f = \frac{(6.00\text{ m})n_2}{n_2 - n_1} \qquad \textbf{[1]}$$

(a) If $n_1 = 1.50$ and $n_2 = 1.00$, then $f = \dfrac{(6.00\text{ m})(1.00)}{1.00 - 1.50} = -12.0\text{ m}$.

The thin-lens equation gives $q = \dfrac{pf}{p-f} = \dfrac{(10.0\text{ m})(-12.0\text{ m})}{10.0\text{ m} + 12.0\text{ m}} = -5.45\text{ m}$.

A virtual image is formed $\boxed{\text{5.45 m to the left of the lens}}$.

(b) If $n_1 = 1.50$ and $n_2 = 1.33$, the focal length is $f = \dfrac{(6.00\text{ m})(1.33)}{1.33 - 1.50} = -46.9\text{ m}$, and

$$q = \frac{pf}{p-f} = \frac{(10.0\text{ m})(-46.9\text{ m})}{10.0\text{ m} + 46.9\text{ m}} = -8.24\text{ m}$$

The image is located $\boxed{\text{8.24 m to the left of the lens}}$.

(c) When $n_1 = 1.50$ and $n_2 = 2.00$, $f = \dfrac{(6.00\text{ m})(2.00)}{2.00 - 1.50} = +24.0\text{ m}$, and

$$q = \frac{pf}{p-f} = \frac{(10.0\text{ m})(24.0\text{ m})}{10.0\text{ m} - 24.0\text{ m}} = -17.1\text{ m}$$

The image is $\boxed{\text{17.1 m to the left of the lens}}$.

(d) Observe from Equation [1] that $f < 0$ if $n_1 > n_2$ and $f > 0$ when $n_1 < n_2$. Thus, a diverging lens can be changed to converging by surrounding it with a medium whose index of refraction exceeds that of the lens material.

23.62 The inverted image is formed by light that leaves the object and goes directly through the lens, never having reflected from the mirror. For the formation of this inverted image, we have

$$M = -\frac{q_1}{p_1} = -1.50 \qquad \text{giving} \qquad q_1 = +1.50\, p_1$$

The thin-lens equation then gives

$$\frac{1}{p_1} + \frac{1}{1.50\, p_1} = \frac{1}{10.0\text{ cm}} \qquad \text{or} \qquad p_1 = (10.0\text{ cm})\left(1 + \frac{1}{1.50}\right) = 16.7\text{ cm}$$

The upright image is formed by light that passes through the lens after reflecting from the mirror. The object for the lens in this upright image formation is the image formed by the mirror. In order for the lens to form the upright image at the same location as the inverted image, the image formed by the mirror must be located at the position of the original object (so the object distances, and hence image distances, are the same for both the inverted and upright images formed by the lens). Therefore, the object distance and the image distance for the mirror are equal, and their common value is

$$q_{\text{mirror}} = p_{\text{mirror}} = 40.0\text{ cm} - p_1 = 40.0\text{ cm} - 16.7\text{ cm} = +23.3\text{ cm}$$

continued on next page

The mirror equation, $\frac{1}{p_{\text{mirror}}}+\frac{1}{q_{\text{mirror}}}=\frac{2}{R}=\frac{1}{f_{\text{mirror}}}$, then gives

$$\frac{1}{f_{\text{mirror}}}=\frac{1}{23.3\text{ cm}}+\frac{1}{23.3\text{ cm}}=\frac{+2}{23.3\text{ cm}}\quad\text{or}\quad f_{\text{mirror}}=+\frac{23.3\text{ cm}}{2}=\boxed{+11.7\text{ cm}}$$

23.63 (a) The lens-maker's equation for a lens made of material with refractive index $n_1=1.55$ and immersed in a medium having refractive index n_2 is

$$\frac{1}{f}=\left(\frac{n_1}{n_2}-1\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)=\left(\frac{1.55-n_2}{n_2}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$$

Thus, when the lens is in air, we have $\frac{1}{f_{\text{air}}}=\left(\frac{1.55-1.00}{1.00}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ **[1]**

and when it is immersed in water, $\frac{1}{f_{\text{water}}}=\left(\frac{1.55-1.33}{1.33}\right)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$ **[2]**

Dividing Equation [1] by Equation [2] gives

$$\frac{f_{\text{water}}}{f_{\text{air}}}=\left(\frac{1.33}{1.00}\right)\left(\frac{1.55-1.00}{1.55-1.33}\right)=3.33$$

If $f_{\text{air}}=79.0$ cm, the focal length when immersed in water is

$$f_{\text{water}}=3.33(79.0\text{ cm})=\boxed{263\text{ cm}}$$

(b) The focal length for a mirror is determined by the law of reflection, which is independent of the material of which the mirror is made and of the surrounding medium. Thus, the focal length depends only on the radius of curvature and not on the material making up the mirror or the surrounding medium. This means that, for the mirror,

$$f_{\text{water}}=f_{\text{air}}=\boxed{79.0\text{ cm}}$$

23.64 (a) The spherical ornament serves as a convex mirror, forming an image that is three-fourths the size of the object. Since convex mirrors only form upright, virtual images for real objects, the magnification is positive. Thus,

$$M=\frac{h'}{h}=-\frac{q}{p}=+\frac{3}{4}\quad\text{and}\quad q=\frac{-3p}{4}$$

With $R<0$ (convex mirror) and a diameter of 8.50 cm, we have

$$R=-(8.50\text{ cm})/2=-\ 4.25\text{ cm}$$

The mirror equation, $1/p+1/q=2/R$, then gives

$$\frac{1}{p}-\frac{4}{3p}=\frac{2}{R}\quad\text{or}\quad p=-\frac{R}{6}=-\frac{(-4.25\text{ cm})}{6}=+0.708\text{ cm}$$

Hence, the object is $\boxed{\text{0.708 cm in front of the spherical ornament}}$.

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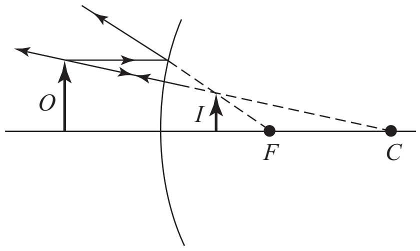
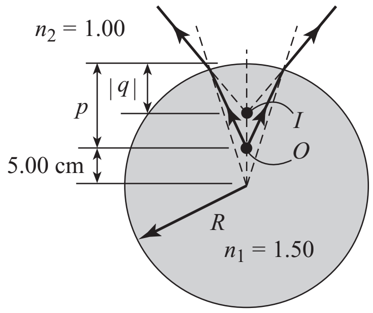
(b) The ray diagram at the right shows the formation of the upright, virtual image by this convex mirror.



23.65 The diagram at the right shows a bubble O, located 5.00 cm above the center of a glass sphere having radius $|R| = 15.0$ cm. This bubble is an actual distance of $p = 10.0$ cm below the refracting surface separating the glass and air. Refraction at this surface forms a virtual image I at distance $|q|$ below the surface as shown. The object distance p and image distance q are related by



$$\frac{n_1}{p} + \frac{n_2}{q} = \frac{n_2 - n_1}{R} \qquad \text{or} \qquad \frac{n_2}{q} = \frac{n_2 - n_1}{R} - \frac{n_1}{p}$$

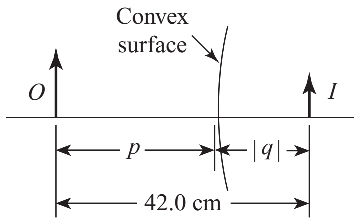
Thus, with $R < 0$ for the concave surface the light crosses going from glass into the air, we have $R = -15.0$ cm, and

$$\frac{1.00}{q} = \frac{1.00 - 1.50}{-15.0\text{ cm}} - \frac{1.50}{10.0\text{ cm}} = \frac{+1.00 - 4.50}{30.0\text{ cm}} = \frac{-3.50}{30.0\text{ cm}} \qquad \text{or} \qquad q = \frac{30.0\text{ cm}}{-3.50} = -8.57\text{ cm}$$

Therefore, the bubble has an apparent depth of $\boxed{8.57\text{ cm}}$ below the glass surface.

23.66 (a) The only upright images that spherical mirrors can form of real objects are virtual images. Therefore, we know that the image described is virtual and diminished in size (4.00 cm high in comparison to the 10.0-cm height of the object). Since the virtual images formed by concave mirrors are always enlarged, we must conclude that this image is formed by a $\boxed{\text{convex mirror}}$.

(b) The sketch at the right shows an upright, diminished, virtual image formed by a convex mirror. The distance between the object and this virtual image in this case is known to be $p + |q| = 42.0$ cm. The image distance for a virtual image is negative, so $q < 0$ and $|q| = -q$. Thus, we have



$$p - q = 42.0\text{ cm} \qquad \textbf{[1]}$$

We also know that the magnification is positive (upright image) and

$$M = \frac{h'}{h} = -\frac{q}{p} = \frac{4.00\text{ cm}}{10.0\text{ cm}} = 0.400 \qquad \text{or} \qquad q = -0.400p \qquad \textbf{[2]}$$

continued on next page

Substituting Equation [2] into Equation [1] gives

$$p-(-0.400)p = 42.0\text{ cm} \qquad \text{or} \qquad p = \frac{42.0\text{ cm}}{1.40} = 30.0\text{ cm}$$

With the object at the zero end of the meter stick, $\boxed{\text{the mirror is at the 30.0-cm mark}}$.

(c) The image distance is now seen to be $q = -0.400p = -0.400(30.0\text{ cm}) = -12.0\text{ cm}$. The mirror equation $(1/p + 1/q = 2/R = 1/f)$ then gives the focal length of this mirror as

$$f = \frac{pq}{p+q} = \frac{(30.0\text{ cm})(-12.0\text{ cm})}{30.0\text{ cm} - 12.0\text{ cm}} = \boxed{-20.0\text{ cm}}$$

24

Wave Optics

QUICK QUIZZES

1. Choice (c). The fringes on the screen are equally spaced only at small angles, where $\tan\theta \approx \sin\theta$ is a valid approximation.

2. Choice (c). The screen locations of the dark fringes of order m are given by $(y_{\text{dark}})_m = (\lambda L/d)(m+\frac{1}{2})$, with $m = 0$ corresponding to the first dark fringe on either side of the central maximum. The width of the central maximum is then $2(y_{\text{dark}})_0 = 2(\lambda L/d)(\frac{1}{2}) = \lambda L/d$. Thus, doubling the distance d between the slits will cut the width of the central maximum in half.

3. Choice (c). The screen locations of the bright fringes of order m are given by $(y_{\text{bright}})_m = (\lambda L/d)m$, and the distance between successive bright fringes for a given wavelength is

$$\Delta y_{\text{bright}} = \left(y_{\text{bright}}\right)_{m+1} - \left(y_{\text{bright}}\right)_m = \left(\lambda L/d\right)_{m+1} - \left(\lambda L/d\right)_m = \lambda L/d$$

Observe that this spacing is directly proportional to the wavelength. Thus, arranged from smallest to largest spacing between bright fringes, the order of the colors will be blue, green, red.

4. Choice (b). The space between successive bright fringes is proportional to the wavelength of the light. Since the wavelength in water is less than that in air, the bright fringes are closer together in the second experiment.

5. Choice (b). The outer edges of the central maximum occur where $\sin\theta = \pm\lambda/a$. Thus, as the width of the slit, a, becomes smaller, the width of the central maximum will increase.

6. The compact disc. The tracks of information on a compact disc are much closer together than on a phonograph record. As a result, the diffraction maxima from the compact disc will be farther apart than those from the record.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. With phase changes occurring in the reflections at both the air-oil boundary and the oil-water boundary, the condition for constructive interference in the reflected light is $2n_{\text{oil}}t = m\lambda$, where m is any integer. Thus, the minimum nonzero thickness of the oil which will strongly reflect the 530-nm light is $t_{\min} = \lambda/2n_{\text{oil}} = (530\text{ nm})/2(1.25) = 212\text{ nm}$, and (d) is the proper choice.

2. The bright fringe of order m occurs where $\delta = d\sin\theta = m\lambda$. For small angles, the sine of the angle is approximately equal to the angle expressed in radians. Thus, the angular position of the second order bright fringe in the case described is

$$\theta = 2\left(\frac{\lambda}{d}\right) = 2\left(\frac{5.0\times10^{-7}\text{ m}}{2.0\times10^{-5}\text{ m}}\right) = 0.050\text{ radians}$$

making (a) the correct choice.

3. In a single-slit diffraction pattern, formed on a screen at distance L from the slit, dark fringes occur where $\sin\theta_{dark} = m(\lambda/a) \approx \tan\theta_{dark} = y_{dark}/L$ and m is any nonzero integer. Thus, the width of the slit, a, in the described situation must be

$$a = \frac{(1)\lambda L}{\left(y_{dark}\right)_1} = \frac{\left(5.00\times10^{-7}\text{ m}\right)(1.00\text{ m})}{5.00\times10^{-3}\text{ m}} = 1.00\times10^{-4}\text{ m} = 0.100\text{ mm}$$

and the correct answer is seen to be choice (b).

4. From Malus's law, the intensity of the light transmitted through a polarizer having its transmission axis oriented at angle θ to the plane of polarization of the incident polarized light is $I = I_0\cos^2\theta$. Therefore, the intensity transmitted through the first polarizer having $\theta = 45° - 0 = 45°$ is $I_1 = I_0\cos^2(45°) = 0.50I_0$, and the intensity passing through the second polarizer having $\theta = 90° - 45° = 45°$ is $I_2 = (0.50I_0)\cos^2(45°) = 0.25I_0$. The fraction of the original intensity making it through both polarizers is then $I_2/I_0 = 0.25$, which is choice (b).

5. The spacing between successive bright fringes in a double-slit interference pattern is given by $\Delta y_{bright} = (\lambda L/d)(m+1) - (\lambda L/d)m = \lambda L/d$, where d is the slit separation. As d decreases, the spacing between the bright fringes will increase and choice (b) is the correct answer.

6. As discussed in question 5 above, the fringe spacing in a double-slit interference pattern is $\Delta y_{bright} = \lambda L/d$. Therefore, as the distance L between the screen and the plane of the slits is increased, the spacing between the bright fringes will increase, and (a) is the correct choice.

7. The fringe spacing in a double-slit interference pattern is $\Delta y_{bright} = \lambda L/d$. Note that this spacing is directly proportional to the wavelength of the light illuminating the slits. Thus, with $\lambda_2 > \lambda_1$, the spacing is greater in the second experiment, and choice (c) gives the correct answer.

8. The reflected light tends to be partially polarized, with the degree of polarization depending on the angle of incidence on the reflecting surface. Only if the angle of incidence equals the polarizing angle (or Brewster's angle) will the reflected light be completely polarized. The better answer for this question is choice (d).

9. The bright colored patterns are the result of interference between light reflected from the upper surface of the oil and light reflected from the lower surface of the oil film. Thus, the best answer is choice (e).

10. In a single-slit diffraction pattern, dark fringes occur where $y_{dark}/L \approx \sin\theta_{dark} = m(\lambda/a)$. The width of the central maximum is the distance between the locations of the first dark fringes on either side of the center (i.e., between the $m = \pm1$ dark fringes), giving

$$\text{width of central maximum} = (+1)\frac{\lambda}{a} - (-1)\frac{\lambda}{a} = \frac{2\lambda}{a}$$

Thus, if the width of the slit, a, is made half as large, the width of the central maximum will double and (d) is the correct choice.

11. The colors observed on the oil film are the result of constructive interference of waves reflected from the upper and lower surfaces of the film. For this to produce bright colored fringes, the thickness of the film must be on the same order of magnitude as the wavelength of the light. If the thickness of the oil film were smaller than half of the wavelengths of visible light, no colors would appear. If the thickness of the oil film were much larger, the constructive fringes for different colors would overlap and mix to white or gray. The best choice for this question is (b).

12. In principle, the ribs (or any set of parallel openings that waves pass through) can act as a diffraction grating. However, for any significant diffraction to occur as the waves pass through the openings, and also for the interference fringes to be separated sufficiently to be observed, the spacing between the slits must be on the same order of magnitude as the wavelength of the radiation. The spacing between the ribs is very large in comparison to the wavelength of x-rays, so the correct choice is (b).

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The wavelength of light is extremely small in comparison to the dimensions of your hand, so the diffraction of light around obstacles the size of your hand is totally negligible. However, sound waves have wavelengths that are comparable to the dimensions of the hand or even larger. Therefore, significant diffraction of sound waves occurs around hand-sized obstacles.

4. The spacing between double-slit interference fringes on a screen is proportional to the wavelength of the light ($\Delta y_{\text{bright}} = \lambda L/d$). Light having a wavelength λ_0 in vacuum will have wavelength $\lambda = \lambda_0/n$ in a medium with index of refraction n. Thus, the wavelength of the light is smaller in water than it is in air, meaning the interference fringes will be more closely spaced when the experiment is performed in water.

6. Every color produces its own interference pattern, and we see them superimposed. The central maximum is white. The first maximum is a full spectrum with violet on the inside and red on the outside. The second maximum is also a full spectrum, with red in it overlapping with violet in the third maximum. At larger angles, the light soon starts mixing to white again.

8. The skin on the tip of a finger has a series of closely spaced ridges and swirls on it. When the finger touches a smooth surface, the oils from the skin will be deposited on the surface in the pattern of the closely spaced ridges. The clear spaces between the lines of deposited oil can serve as the slits in a crude diffraction grating and produce a colored spectrum of the light passing through or reflecting from the glass surface.

10. Suppose the index of refraction of the coating is intermediate between vacuum and the glass. When the coating is very thin, light reflected from its top and bottom surfaces will interfere constructively, so you see the surface white and brighter. Once the thickness reaches one-quarter of the wavelength of violet light in the coating, destructive interference for violet light will make the surface look red. Then other colors in spectral order (blue, green, yellow, orange, and red) will interfere destructively, making the surface look red, magenta, and then blue. As the coating gets thicker, constructive interference is observed for violet light and then for other colors in spectral order. Even thicker coatings give constructive and destructive interference for several visible wavelengths, so the reflected light starts looking white again.

12. The reflected light is partially polarized, with the component parallel to the reflecting surface being the most intense. Therefore, the polarizing material should have its transmission axis oriented in the vertical direction in order to minimize the intensity of the reflected light from horizontal surfaces.

14. Due to gravity, the soap film tends to sag in its holder, being quite thin at the top and becoming thicker as one moves toward the bottom of the holding ring. Because light reflecting from the front surface of the film experiences a phase change, and light reflecting from the back surface of the film does not (see Figure 24.7 in the textbook), the film must be a minimum of a half wavelength thick before it can produce constructive interference in the reflected light. Thus, the light must be striking the film at some distance from the top of the ring before the thickness is sufficient to produce constructive interference for any wavelength in the visible portion of the spectrum.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 1.77 μm **(b)** 1.47 μm

4. 2.40 μm

6. **(a)** 1.52 cm **(b)** 2.13 cm

8. 2.9 μm

10. **(a)** 1.93 μm **(b)** $\delta = 3.00\lambda$ **(c)** a maximum

12. **(a)** 2.3°, 4.6°, and 6.9° **(b)** 1.1°, 3.4°, and 5.7°

(c) At small angles, $\theta \approx \sin\theta$. The approximation breaks down at larger angles.

14. **(a)** See Solution. **(b)** $\tan\theta = 2.51\times10^{-3}$

(c) 0.144°, 2.51×10^{-3}, $\sin\theta \approx \tan\theta$ only when θ is small

(d) 603 nm **(e)** 0.720° **(f)** 2.26 cm

16. **(a)** 640 nm

(b) make use of a higher order constructive interference (i.e., a larger value of m)

(c) 360 nm ($m = 1$), 600 nm ($m = 2$)

18. **(a)** 512 nm **(b)** $2.5m_1 = 2m_2 + 1$

20. 233 nm

22. **(a)** 541 nm

(b) 406 nm; The most strongly transmitted wavelengths are those which suffer destructive interference in reflection.

24. 8 dark fringes, including the one at the line of contact

26. 6.5×10^2 nm

28. 99.6 nm

30. **(a)** 97.8 nm **(b)** Yes, some of which are 293 nm, 489 nm, and 685 nm.

32. **(a)** 2.3 mm **(b)** 4.5 mm

34. 91.1 cm

36. 0.227 mm

38. 659 nm

40. **(a)** 2 complete orders **(b)** 10.9°

42. **(a)** Three maxima (for $m = -1$, $m = 0$, and $m = +1$) will be observed.

(b) for $m = -1$, $\theta = -45.2°$; for $m = 0$, $\theta = 0°$; and for $m = +1$, $\theta = +45.2°$

44. 7.35°

46. See Solution.

48. **(a)** 2.381×10^{-6} m **(b)** 29.65° **(c)** 1.138 m

(d) 26.69°, 1.140 m **(e)** 2.000×10^{-3} m

(f) 1.537×10^{-3} m; The two results agree to only 1 significant figure, showing that the calculation is sensitive to rounding of intermediate answers.

50. 78.1 nm and 469 nm

52. **(a)** 0.336 **(b)** 0.164

54. 36.9°

56. 60.5°

58. **(a)** 45.0° **(b)** 60.0° **(c)** 65.9°

60. **(a)** $I = I_0/2$ **(b)** 54.7°

62. 632.8 nm

64. **(a)** $6m_1 = 5m_2$ **(b)** $m_1 = 5$, $m_2 = 6$; 2.52 cm from the central maximum

66. maxima at $0°$, $\pm 29.1°$, and $\pm 76.4°$; minima at $\pm 14.1°$ and $\pm 46.8°$

68. 113 dark fringes (counting the $m = 0$ order at the line of contact)

70. **(a)** $I_f/I_i = 0$ **(b)** $I_f/I_i = 0.25$

72. See Solution.

74. $\alpha = 20.0 \times 10^{-6}\ (°\text{C})^{-1}$

PROBLEM SOLUTIONS

24.1 The location of the bright fringe of order m (measured from the position of the central maximum) is $(y_{\text{bright}})_m = (\lambda L/d)m$, $m = 0, \pm 1, \pm 2, \ldots$. Thus, the spacing between successive bright fringes is

$$\Delta y_{\text{bright}} = \left(y_{\text{bright}}\right)_{m+1} - \left(y_{\text{bright}}\right)_m = (\lambda L/d)(m+1) - (\lambda L/d)m = \lambda L/d$$

The wavelength of the laser light must be

$$\lambda = \frac{\left(\Delta y_{\text{bright}}\right)d}{L} = \frac{\left(1.58 \times 10^{-2}\text{ m}\right)\left(0.200 \times 10^{-3}\text{ m}\right)}{5.00\text{ m}} = 6.32 \times 10^{-7}\text{ m} = \boxed{632\text{ nm}}$$

24.2 (a) For a bright fringe of order m, the path difference is $\delta = m\lambda$, where $m = 0, 1, 2, \ldots$. At the location of the third order bright fringe, $m = 3$ and

$$\delta = 3\lambda = 3(589\text{ nm}) = 1.77\times10^{3}\text{ nm} = \boxed{1.77\ \mu\text{m}}$$

(b) For a dark fringe, the path difference is $\delta = \left(m + \dfrac{1}{2}\right)\lambda$, where $m = 0, 1, 2, \ldots$.

At the third dark fringe, $m = 2$ and

$$\delta = \left(2 + \frac{1}{2}\right)\lambda = \frac{5}{2}(589\text{ nm}) = 1.47\times10^{3}\text{ nm} = \boxed{1.47\ \mu\text{m}}$$

24.3 (a) The distance between the central maximum and the first order bright fringe is

$$\Delta y = y_{\text{bright}}\Big|_{m=1} - y_{\text{bright}}\Big|_{m=0} = \frac{\lambda L}{d}, \quad \text{or}$$

$$\Delta y = \frac{\lambda L}{d} = \frac{(546.1\times10^{-9}\text{ m})(1.20\text{ m})}{0.250\times10^{-3}\text{ m}} = 2.62\times10^{-3}\text{ m} = \boxed{2.62\text{ mm}}$$

(b) The distance between the first and second dark bands is

$$\Delta y = y_{\text{dark}}\Big|_{m=1} - y_{\text{dark}}\Big|_{m=0} = \frac{\lambda L}{d} = \boxed{2.62\text{ mm}}\ \text{as in (a) above.}$$

24.4 In the double-slit interference pattern, the bright fringe of order m is found where $d\sin\theta = m\lambda$ with $m = 0, \pm1, \pm2, \ldots$. Here, θ is the angle between the line to the central maximum and the line to the location of the bright fringe of interest. Thus, if $\theta = 15.0°$ for the $m = 1$ bright fringe of light having wavelength $\lambda = 620$ nm, the spacing between the slits must be

$$d = \frac{m\lambda}{\sin\theta} = \frac{(1)(620\times10^{-9}\text{ m})}{\sin 15.0°} = 2.40\times10^{-6}\text{ m} = \boxed{2.40\ \mu\text{m}}$$

24.5 (a) From $d\sin\theta = m\lambda$, the angle for the $m = 1$ maximum for the sound waves is

$$\theta = \sin^{-1}\left(\frac{m}{d}\lambda\right) = \sin^{-1}\left[\frac{m}{d}\left(\frac{v_{\text{sound}}}{f}\right)\right] = \sin^{-1}\left[\frac{1}{0.300\text{ m}}\left(\frac{354\text{ m/s}}{2\,000\text{ Hz}}\right)\right] = \boxed{36.2°}$$

(b) For 3.00-cm microwaves, the required slit spacing is

$$d = \frac{m\lambda}{\sin\theta} = \frac{(1)(3.00\text{ cm})}{\sin(36.2°)} = \boxed{5.08\text{ cm}}$$

(c) The wavelength of the light is $\lambda = d\sin\theta/m$, so the frequency is

$$f = \frac{c}{\lambda} = \frac{mc}{d\sin\theta} = \frac{(1)(3.00\times10^{8}\text{ m/s})}{(1.00\times10^{-6}\text{ m})\sin 36.2°} = \boxed{5.08\times10^{14}\text{ Hz}}$$

24.6 In a double-slit interference pattern, the screen position of the mth order maximum for wavelength λ is $y_m = (\lambda L/d)m$. The separation between the maxima of orders m_1 and m_2 is then $\Delta y = (\lambda L/d)(m_2 - m_1)$.

continued on next page

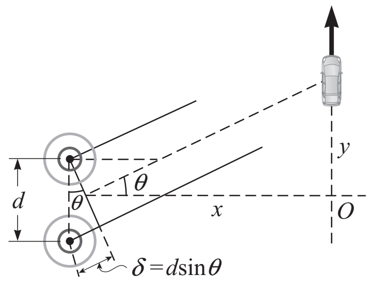
(a) If $\lambda = 588$ nm while $m_2 = 1$ and $m_1 = 0$, the separation is

$$\Delta y = \frac{\lambda L}{d}(1-0) = \frac{\lambda L}{d} = \frac{(588\times10^{-9}\ \text{m})(1.50\ \text{m})}{0.058\,0\times10^{-3}\ \text{m}} = 1.52\times10^{-2}\ \text{m} = \boxed{1.52\ \text{cm}}$$

(b) If $\lambda = 412$ nm, $m_2 = 4$, and $m_1 = 2$, the spacing between these maxima is

$$\Delta y = \frac{(412\times10^{-9}\ \text{m})(1.50\ \text{m})}{0.058\,0\times10^{-3}\ \text{m}}(4-2) = 2.13\times10^{-2}\ \text{m} = \boxed{2.13\ \text{cm}}$$

24.7 As indicated in the sketch at the right, the path difference between waves from the two antennas that travel toward the car is given by $\delta = d\sin\theta$. When $\delta = m\lambda$, where $m = 0, 1, 2, \ldots$, constructive interference produces the maximum of order m. Destructive interference produces the minimum of order m when $\delta = \left(m+\frac{1}{2}\right)\lambda$.



(a) At the $m = 2$ maximum, $\delta = d\sin\theta = 2\lambda$, or

$$\lambda = \frac{d\sin\theta}{m} = \frac{d}{2}\left(\frac{y}{\sqrt{x^2+y^2}}\right) = \frac{(300\,\text{m})}{2}\left[\frac{400\,\text{m}}{\sqrt{(1000\text{m})^2+(400\,\text{m})^2}}\right] = \boxed{55.7\,\text{m}}$$

(b) The next minimum encountered is the $m = 2$ minimum, and it occurs where $\delta = 5\lambda/2$, or

$$\theta = \sin^{-1}\left(\frac{5\lambda}{2d}\right) = \sin^{-1}\left[\frac{5(55.7\ \text{m})}{2(300\ \text{m})}\right] = 27.7°$$

At this point, $y = x\tan\theta = (1\,000\ \text{m})\tan 27.7° = 525$ m, so the car must travel an additional $\boxed{125\ \text{m}}$.

24.8 The angular position of the bright fringe of order m is given by $d\sin\theta = m\lambda$. Thus, if the $m = 1$ bright fringe is located at $\theta = 12°$ when $\lambda = 6.0\times10^2$ nm $= 6.0\times10^{-7}$ m, the slit spacing is

$$d = \frac{m\lambda}{\sin\theta} = \frac{(1)(6.0\times10^{-7}\ \text{m})}{\sin 12°} = 2.9\times10^{-6}\ \text{m} = \boxed{2.9\ \mu\text{m}}$$

24.9 The screen position of the mth order bright fringe of wavelength λ in a double-slit interference pattern is $y_m = (\lambda L/d)m$. Then, $\Delta y = (\lambda L/d)(m_2 - m_1)$ is the separation between the bright fringes of orders m_1 and m_2. If $\Delta y = 0.552$ mm for the first and second order bright fringes when $L = 1.75$ m, and the slit separation is $d = 0.552$ mm, the wavelength incident upon the set of slits is

$$\lambda = \frac{(\Delta y)d}{L(m_2-m_1)} = \frac{(0.552\times10^{-3}\ \text{m})(2.10\times10^{-3}\ \text{m})}{(1.75\ \text{m})(2-1)} = 6.62\times10^{-7}\ \text{m} = \boxed{662\ \text{nm}}$$

24.10 The angular deviation from the line of the central maximum is given by

$$\theta = \tan^{-1}\left(\frac{y}{L}\right) = \tan^{-1}\left(\frac{1.80\ \text{cm}}{140\ \text{cm}}\right) = 0.737°$$

(a) The path difference is then

$$\delta = d\sin\theta = (0.150\ \text{mm})\sin(0.737°) = 1.93\times10^{-3}\ \text{mm} = \boxed{1.93\ \mu\text{m}}$$

continued on next page

(b) $\delta = (1.93\times10^{-6}\text{ m})\left(\dfrac{\lambda}{643\times10^{-9}\text{ m}}\right) = \boxed{3.00\,\lambda}$

(c) Since the path difference for this position is a whole number of wavelengths (to three significant figures), the waves interfere constructively and produce a $\boxed{\text{maximum}}$ at this spot.

24.11 The distance between the central maximum (position of A) and the first minimum is

$$y = \frac{\lambda L}{d}\left(m+\frac{1}{2}\right)\bigg|_{m=0} = \frac{\lambda L}{2d}$$

Thus, $d = \dfrac{\lambda L}{2y} = \dfrac{(3.00\text{ m})(150\text{ m})}{2(20.0\text{ m})} = \boxed{11.3\text{ m}}$.

24.12 (a) The angular position of the bright fringe of order m is given by $d\sin\theta = m\lambda$, or $\theta_m = \sin^{-1}(m\lambda/d)$. If $d = 25\lambda$, the first three bright fringes are found at

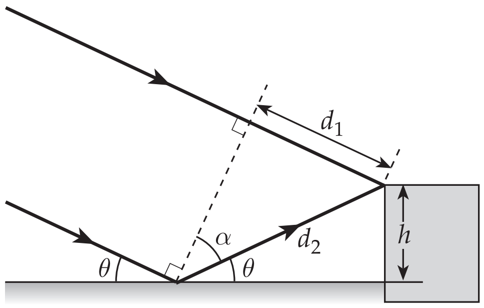
$$\theta_1 = \sin^{-1}\left(\frac{1}{25}\right) = \boxed{2.3°}, \quad \theta_2 = \sin^{-1}\left(\frac{2}{25}\right) = \boxed{4.6°}, \quad \text{and} \quad \theta_3 = \sin^{-1}\left(\frac{3}{25}\right) = \boxed{6.9°}$$

(b) The angular position of the dark fringe of order m is given by $d\sin\theta = (m+\frac{1}{2})\lambda$, or $\theta_m = \sin^{-1}\left[(m+\frac{1}{2})\lambda/d\right]$, $m = 0, \ \pm1, \ \pm2, \ldots$. If $d = 25\lambda$, the first three dark fringes are found at

$$\theta_0 = \sin^{-1}\left(\frac{1/2}{25}\right) = \boxed{1.1°}, \quad \theta_1 = \sin^{-1}\left(\frac{3/2}{25}\right) = \boxed{3.4°}, \quad \text{and} \quad \theta_2 = \sin^{-1}\left(\frac{5/2}{25}\right) = \boxed{5.7°}$$

(c) The answers are evenly spaced because $\boxed{\text{the angles are small and } \theta \approx \sin\theta}$. At larger angles, the approximation breaks down and the spacing isn't so regular.

24.13 As shown in the figure at the right, the path difference in the waves reaching the telescope is $\delta = d_2 - d_1 = d_2(1-\sin\alpha)$. If the first minimum $(\delta = \lambda/2)$ occurs when $\theta = 25.0°$, then

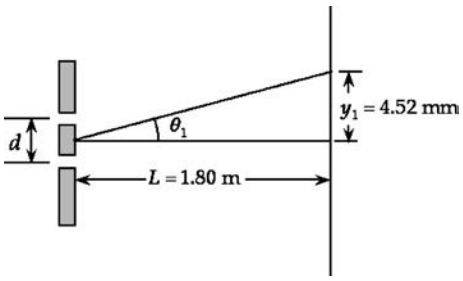


$$\alpha = 180° - (\theta + 90.0° + \theta) = 40.0°, \text{ and}$$

$$d_2 = \frac{\delta}{1-\sin\alpha} = \frac{\lambda/2}{1-\sin\alpha} = \frac{(250\text{ m}/2)}{1-\sin 40.0°} = 350\text{ m}$$

Thus, $h = d_2 \sin 25.0° = \boxed{148\text{ m}}$.

24.14 (a)



(b) $\tan\theta_1 = \dfrac{y_1}{L} = \dfrac{4.52\times10^{-3}\text{ m}}{1.80\text{ m}} = \boxed{2.51\times10^{-3}}$

continued on next page

(c) $\theta_1 = \tan^{-1}\left(2.51\times10^{-3}\right) = \boxed{0.144°}$ and $\sin\theta_1 = \sin(0.144°) = \boxed{2.51\times10^{-3}}$.
The sine and the tangent are very nearly the same but only because the angle is small.

(d) From $\delta = d\sin\theta = m\lambda$ for the order m bright fringe,

$$\lambda = \frac{d\sin\theta_1}{1} = \frac{\left(2.40\times10^{-4}\text{ m}\right)\sin(0.144°)}{1} = 6.03\times10^{-7}\text{ m} = \boxed{603\text{ nm}}$$

(e) $$\theta_5 = \sin^{-1}\left(\frac{5\lambda}{d}\right) = \sin^{-1}\left[\frac{5\left(6.03\times10^{-7}\text{ m}\right)}{2.40\times10^{-4}\text{ m}}\right] = \boxed{0.720°}$$

(f) $y_5 = L\tan\theta_5 = (1.80\text{ m})\tan(0.720°) = 2.26\times10^{-2}\text{ m} = \boxed{2.26\text{ cm}}$

24.15 The path difference in the two waves received at the home is $\delta = 2d$, where d is the distance from the home to the mountain. Neglecting any phase change upon reflection, the condition for destructive interference is

$$\delta = \left(m+\frac{1}{2}\right)\lambda \text{ with } m = 0, 1, 2, \ldots$$

so $$d_{\min} = \frac{\delta_{\min}}{2} = \left(0+\frac{1}{2}\right)\frac{\lambda}{2} = \frac{\lambda}{4} = \frac{300\text{ m}}{4} = \boxed{75.0\text{ m}}$$

24.16 (a) With a phase change in the reflection at the outer surface of the soap film and no change on reflection from the inner surface, the condition for constructive interference in the light reflected from the soap bubble is $2n_{\text{film}}t = (m+\frac{1}{2})\lambda$, where $m = 0,1,2,\ldots$. For the lowest order reflection, $m = 0$, so the wavelength is

$$\lambda = \frac{2n_{\text{water}}t}{(0+1/2)} = 4(1.333)(120\text{ nm}) = 6.4\times10^{2}\text{ nm} = \boxed{640\text{ nm}}$$

(b) To strongly reflect the same wavelength light, a thicker film will need to make use of a higher order constructive interference, i.e., $\boxed{\text{use a larger value of } m}$.

(c) The next greater thickness of soap film that can strongly reflect 640 nm light corresponds to $m = 1$, giving

$$t_1 = \frac{(1+1/2)\lambda}{2n_{\text{film}}} = \frac{3}{2}\left[\frac{640\text{ nm}}{2(1.333)}\right] = 3.6\times10^{2}\text{ nm} = \boxed{360\text{ nm}}$$

and the third such thickness (corresponding to $m = 2$) is

$$t_2 = \frac{(2+1/2)\lambda}{2n_{\text{film}}} = \frac{5}{2}\left[\frac{640\text{ nm}}{2(1.333)}\right] = 6.0\times10^{2}\text{ nm} = \boxed{600\text{ nm}}$$

24.17 Light reflecting from the first (glass-iodine) interface suffers a phase change, but light reflecting at the second (iodine-glass) interface does not have a phase change. Thus, the condition for constructive interference in the reflected light is $2n_{\text{film}}t = (m+\frac{1}{2})\lambda$, with $m = 0,1,2,\ldots$. The smallest film thickness capable of strongly reflecting the incident light is

$$t_{\min} = \frac{(m_{\min}+1/2)\lambda}{2n_{\text{film}}} = \frac{(0+1/2)\lambda}{2n_{\text{film}}} = \frac{6.00\times10^{2}\text{ nm}}{4(1.756)} = \boxed{85.4\text{ nm}}$$

24.18 (a) Phase changes are experienced by light reflecting at either surface of the oil film, a upper air-oil interface and a lower oil-water interface. Under these conditions, the requirement for constructive interference is $2n_{\text{film}}t = m_1\lambda$, with $m_1 = 0,1,2,\ldots$, and the requirement for destructive interference is $2n_{\text{film}}t = (m_2 + \frac{1}{2})\lambda$, with $m_2 = 0,1,2,\ldots$. To have the thinnest film that produces simultaneous constructive interference of $\lambda_1 = 640$ nm and destructive interference of $\lambda_2 = 512$ nm, it is necessary that

$$2n_{\text{film}}t = m_1(640\text{ nm}) = \left(m_2 + \frac{1}{2}\right)(512\text{ nm})$$

where both m_1 and m_2 are the smallest integers for which this is true. It is found that $m_1 = m_2 = 2$ are the smallest integer values that will satisfy this condition, giving the minimum acceptable film thickness as

$$t_{\min} = \frac{m_1\lambda_1}{2n_{\text{film}}} = \frac{\left(m_2 + \frac{1}{2}\right)\lambda_2}{2n_{\text{film}}} = \frac{2(640\text{ nm})}{2(1.25)} = \frac{(2.5)(512\text{ nm})}{2(1.25)} = \boxed{512\text{ nm}}$$

(b) From the discussion above, it is seen that in order to have a film thickness that produces simultaneous constructive interference of 640-nm light and destructive interference of 512-nm light, it is necessary that

$$m_1(640\text{ nm}) = \left(m_2 + \frac{1}{2}\right)(512\text{ nm}) \quad \text{or} \quad m_1\left(\frac{640\text{ nm}}{512\text{ nm}}\right) = m_2 + \frac{1}{2}$$

This gives $(1.25)m_1 = m_2 + \dfrac{1}{2}$, or $\boxed{2.5\,m_1 = 2m_2 + 1}$.

24.19 With $n_{\text{film}} = 1.52, > n_{\text{air}}$, light reflecting from the upper surface (air to glass transition) experiences a phase change, while light reflecting from the lower surface (a glass to air transition) does not experience such a change. Under these conditions, the requirement for constructive interference of normally incident waves reflecting from these two surfaces is $2n_{\text{film}}t = (m + \frac{1}{2})\lambda$, where m is any integer value. Thus, if the thickness of the film is $t = 0.420\ \mu\text{m} = 420$ nm, the wavelengths that will strongly reflect from the film are

$$\lambda = \frac{2n_{\text{film}}t}{m + 1/2} = \frac{4(1.52)(420\text{ nm})}{2m+1} \qquad m = 0, 1, 2, \ldots$$

yielding $m = 0 \Rightarrow \lambda = 2.55\times10^3$ nm (infrared); $m = 1 \Rightarrow \lambda = 851$ nm (infrared);

$m = 2 \Rightarrow \lambda = 511$ nm (visible); $m = 3 \Rightarrow \lambda = 365$ nm (ultraviolet),

with the wavelengths for all higher values of m being even shorter. Thus, the only visible light wavelength that can strongly reflect from this film is $\boxed{511\text{ nm}}$.

24.20 Since $n_{\text{air}} < n_{\text{oil}} < n_{\text{water}}$, light reflected from both top and bottom surfaces of the oil film experiences a phase change, resulting in zero net phase difference due to reflections. Therefore, the condition for constructive interference in reflected light is

$$2t = m\lambda_n = m\frac{\lambda}{n_{\text{film}}} \qquad \text{or} \qquad t = m\left(\frac{\lambda}{2n_{\text{film}}}\right) \text{ where } m = 0,\ 1,\ 2, \ldots$$

Assuming that $m = 1$, the thickness of the oil slick is $t = (1)\dfrac{\lambda}{2n_{\text{film}}} = \dfrac{600\text{ nm}}{2(1.29)} = \boxed{233\text{ nm}}$.

24.21 There will be a phase change of the radar waves reflecting from both surfaces of the polymer, giving zero net phase change due to reflections. The requirement for *destructive* interference in the reflected waves is then

$$2t = \left(m + \frac{1}{2}\right)\lambda_n \qquad \text{or} \qquad t = (2m+1)\frac{\lambda}{4n_{\text{film}}} \quad \text{where } m = 0, 1, 2, \ldots$$

If the film is as thin as possible, then $m = 0$, and the needed thickness is

$$t = \frac{\lambda}{4n_{\text{film}}} = \frac{3.00 \text{ cm}}{4(1.50)} = \boxed{0.500 \text{ cm}}$$

This anti-reflectance coating could be easily countered by changing the wavelength of the radar—to 1.50 cm—now creating maximum reflection!

24.22 (a) With $n_{\text{air}} < n_{\text{water}} < n_{\text{oil}}$, reflections at the air-oil interface experience a phase change, while reflections at the oil-water interface experience no phase change. With one phase change at the surfaces, the condition for constructive interference in the light reflected by the film is $2n_{\text{film}}t = (m + \frac{1}{2})\lambda$, where m is any positive integer. Thus,

$$\lambda_m = \frac{2n_{\text{film}}t}{m + \frac{1}{2}} = \frac{2(1.45)(280 \text{ nm})}{m + \frac{1}{2}} = \frac{812 \text{ nm}}{m + \frac{1}{2}} \qquad m = 0, 1, 2, 3, \ldots$$

The possible wavelengths are: $\lambda_0 = 1.62 \times 10^3$ nm, $\lambda_1 = 541$ nm, $\lambda_2 = 325$nm, ...

Of these, only $\boxed{\lambda_1 = 541 \text{ nm} \quad \text{(green)}}$ is in the visible portion of the spectrum.

(b) The wavelengths that will be most strongly transmitted are those that suffer destructive interference in the reflected light. With one phase change at the surfaces, the condition for *destructive* interference in the light reflected by the film is $2n_{\text{film}}t = m\lambda$, where m is any positive, nonzero integer. The possible wavelengths are

$$\lambda_m = \frac{2n_{\text{film}}t}{m} = \frac{2(1.45)(280 \text{ nm})}{m} = \frac{812 \text{ nm}}{m} \qquad m = 1, 2, 3, \ldots$$

or $\lambda_1 = 812$ nm, $\lambda_2 = 406$ nm, $\lambda_3 = 271$ nm, ...

of which only $\boxed{\lambda_2 = 406 \text{ nm} \quad \text{(violet)}}$ is in the visible spectrum.

24.23 (a) For maximum transmission, we want destructive interference in the light reflected from the front and back surfaces of the film.

If the surrounding glass has refractive index greater than 1.378, light reflected from the front surface suffers no phase change, and light reflected from the back does undergo a phase change. Under these conditions, the condition for destructive interference in the reflected light is $2d = m\lambda_n = m\lambda/n_{\text{film}}$, where m is a positive integer. Thus, for minimum nonzero film thickness, we require that $m = 1$ and find

$$d = \frac{\lambda}{2n_{\text{film}}} = \frac{656.3 \text{ nm}}{2(1.378)} = \boxed{238 \text{ nm}}$$

(b) The filter will expand. As d increases in $2n_{\text{film}}d = \lambda$, so does $\boxed{\lambda \text{ increase}}$.

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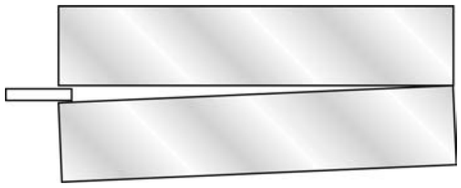
(c) Destructive interference of order 2 will occur for reflected light when $2d = 2\lambda/n_{\text{film}}$, or for the wavelength

$$\lambda = n_{\text{film}} d = (1.378)(238 \text{ nm}) = \boxed{328 \text{ nm}} \quad \text{(near ultraviolet)}$$

24.24 Light reflecting from the lower surface of the air layer experiences a phase change, but light reflecting from the upper surface of the layer does not. The requirement for a dark fringe (destructive interference) is then



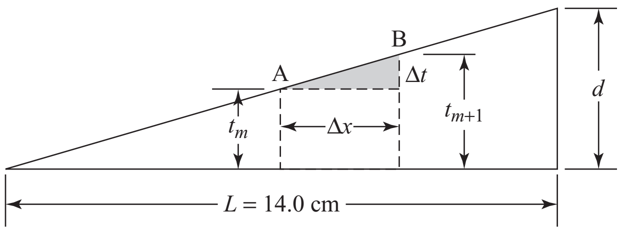
$$2n_{\text{film}} t = m\lambda, \quad \text{where } m = 0, 1, 2, \ldots$$

At the thickest part of the film $(t = 2.00\ \mu\text{m})$, the order number is

$$m = \frac{2n_{\text{film}} t}{\lambda} = \frac{2(1.00)(2.00\times10^{-6}\ \text{m})}{546.1\times10^{-9}\ \text{m}} = 7.32$$

Since m must be an integer, $m = 7$ is the order of the last dark fringe seen. Counting the $m = 0$ order along the edge of contact, a total of $\boxed{8 \text{ dark fringes}}$ will be seen.

24.25 When the hair is inserted between one end of the glass plates, which have length $L = 14.0$ cm, a wedge-shaped air film is created as shown at the right. The maximum thickness of this air wedge is equal to the diameter d of the hair. If the bright interference fringe of order m occurs at point A (where the air wedge has thickness t_m) and the adjacent bright fringe of order $m+1$ occurs at B (where the thickness is t_{m+1}), we may use the properties of similar triangles to write



$$\frac{d}{L} = \frac{\Delta t}{\Delta x} = \frac{t_{m+1} - t_m}{\Delta x} \qquad \text{or} \qquad d = \left(\frac{\Delta t}{\Delta x}\right)L \qquad \textbf{[1]}$$

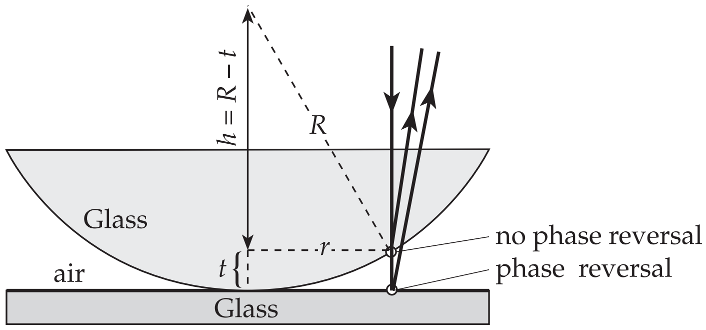
Since $n_{\text{air}} < n_{\text{glass}}$, light waves reflecting at the upper surface of the air film do not undergo a phase change, but those reflecting from the lower surface do experience such a change. Thus, the condition for constructive interference to produce a bright fringe is $2n_{\text{air}} t = (m+\frac{1}{2})\lambda$, or the thickness of the film at the location of the bright fringe of order m is $t_m = (m+\frac{1}{2})\lambda/2n_{\text{air}}$. The change in thickness between the locations of adjacent bright fringes is then

$$\Delta t = t_{m+1} - t_m = \frac{[(m+1)+1/2]\lambda}{2n_{\text{air}}} - \frac{[m+1/2]\lambda}{2n_{\text{air}}} = \frac{\lambda}{2n_{\text{air}}} = \frac{\lambda}{2}$$

If the observed spacing between adjacent bright fringes is $\Delta x = 0.580$ mm when using light of wavelength $\lambda = 650$ nm, Equation [1] gives the diameter of the hair as

$$d = \left(\frac{\lambda/2}{\Delta x}\right)L = \frac{\lambda L}{2(\Delta x)} = \frac{(650\times10^{-9}\ \text{m})(14.0\times10^{-2}\ \text{m})}{2(0.580\times10^{-3}\ \text{m})} = 7.84\times10^{-5}\ \text{m} = \boxed{78.4\ \mu\text{m}}$$

24.26



From the geometry shown in the figure, $R^2 = h^2 + r^2 = (R-t)^2 + r^2$, or

$$t = R - \sqrt{R^2 - r^2} = 3.0 \text{ m} - \sqrt{(3.0 \text{ m})^2 - (9.8\times10^{-3} \text{ m})^2} = 1.6\times10^{-5} \text{ m}$$

With a phase change upon reflection at the lower surface of the air layer but no change with reflection from the upper surface, the condition for a bright fringe is

$$2t = \left(m+\frac{1}{2}\right)\lambda_n = \left(m+\frac{1}{2}\right)\frac{\lambda}{n_{\text{air}}} = \left(m+\frac{1}{2}\right)\lambda \text{, where } m = 0, 1, 2, \ldots$$

At the 50th bright fringe, $m = 49$, and the wavelength is found to be

$$\lambda = \frac{2t}{m+1/2} = \frac{2(1.6\times10^{-5} \text{ m})}{49.5} = 6.5\times10^{-7} \text{ m} = \boxed{6.5\times10^{2} \text{ nm}}$$

24.27 There is a phase change due to reflection at the bottom of the air film but not at the top of the film. The requirement for a dark fringe is then

$$2n_{\text{air}}t = m\lambda \qquad \text{where} \qquad m = 0, 1, 2, \ldots$$

At the 19th dark ring (in addition to the dark center spot), the order number is $m = 19$, and the thickness of the film is

$$t = \frac{m\lambda}{2n_{\text{air}}} = \frac{19(500\times10^{-9} \text{ m})}{2(1.00)} = 4.75\times10^{-6} \text{ m} = \boxed{4.75 \ \mu\text{m}}$$

24.28 With a phase change due to reflection at each surface of the magnesium fluoride layer, there is zero net phase difference caused by reflections. The condition for destructive interference is then

$$2t = \left(m+\frac{1}{2}\right)\lambda_n = \left(m+\frac{1}{2}\right)\frac{\lambda}{n_{\text{film}}} \text{, where } m = 0, 1, 2, \ldots$$

For minimum thickness, $m = 0$, and the thickness is

$$t = (2m+1)\frac{\lambda}{4n_{\text{film}}} = (1)\frac{(550\times10^{-9} \text{ m})}{4(1.38)} = 9.96\times10^{-8} \text{ m} = \boxed{99.6 \text{ nm}}$$

24.29 Since the thin film has air on both sides of it, there is a phase change for light reflecting from the first surface but no change for light reflecting from the second surface. Under these conditions, the requirement to be met if waves reflecting from the two sides are to produce constructive interference and a strong reflection is

$$2n_{\text{film}}t = \left(m+\frac{1}{2}\right)\lambda \qquad \text{or} \qquad \lambda = \frac{4n_{\text{film}}t}{2m+1} \qquad m = 0, 1, 2, \ldots$$

continued on next page

With $n_{\text{film}} = 1.473$ and $t = 542$ nm, the wavelengths that produce strong reflections are given by $\lambda = 4(1.473)(524 \text{ nm})/(2m+1)$, which yields

$$m=0: \quad \lambda = 3.09\times10^{3} \text{ nm} \qquad m=1: \quad \lambda = 1.03\times10^{3} \text{ nm} \qquad m=2: \quad \lambda = 617 \text{ nm}$$

$$m=3: \quad \lambda = 441 \text{ nm} \qquad m=4: \quad \lambda = 343 \text{ nm} \qquad m=5: \quad \lambda = 281 \text{ nm}$$

and many other shorter wavelengths. Of these, the only ones in the range 300 nm to 700 nm are $\boxed{\text{617 nm, 441 nm, and 343 nm}}$.

24.30 With $n_{\text{air}} < n_{\text{film}} < n_{\text{glass}}$, light undergoes a phase change when it reflects from either of the two surfaces of the film. Therefore, the condition for destructive interference of normally incident light reflecting from these two surfaces is

$$2n_{\text{film}}t = \left(m+\frac{1}{2}\right)\lambda \qquad \text{or} \qquad t = (2m+1)\left(\frac{\lambda}{4n_{\text{film}}}\right) \qquad \text{where } m = 0,\ 1,\ 2,\ldots \qquad \textbf{[1]}$$

(a) The minimum thickness film that will minimize the light reflected from the lens is given by $m = 0$ in Equation [1], and has the value

$$t_{\min} = \frac{\lambda}{4n_{\text{film}}} = \frac{540 \text{ nm}}{4(1.38)} = \boxed{97.8 \text{ nm}}$$

(b) $\boxed{\text{Yes.}}$ From Equation [1], we see than any film thickness given by $t = (2m+1)t_{\min}$, where $t_{\min} = (\lambda/4n_{\text{film}}) = 97.8$ nm, will produce destructive interference and hence minimum reflected light for wavelength $\lambda = 540$ nm. For $m = 1,\ 2,\ 3,\ldots$, we obtain thicknesses of 293 nm, 489 nm, 685 nm, … .

24.31 In a single-slit diffraction pattern, with the slit having width a, the dark fringe of order m occurs at angle θ_m, where $\sin\theta_m = m(\lambda/a)$ and $m = \pm1,\ \pm2,\ \pm3,\ldots$. The location, on a screen located distance L from the slit, of the dark fringe of order m (measured from $y = 0$ at the center of the central maximum) is $(y_{\text{dark}})_m = L\tan\theta_m \approx L\sin\theta_m = m\lambda(L/a)$.

(a) The central maximum extends from the $m = -1$ dark fringe to the $m = +1$ dark fringe, so the width of this central maximum is

$$\text{Central max. width} = (y_{\text{dark}})_1 - (y_{\text{dark}})_{-1} = 1\left(\frac{\lambda L}{a}\right) - (-1)\left(\frac{\lambda L}{a}\right) = \frac{2\lambda L}{a}$$

$$= \frac{2(5.40\times10^{-7} \text{ m})(1.50 \text{ m})}{0.200\times10^{-3} \text{ m}} = 8.10\times10^{-3} \text{ m} = \boxed{8.10 \text{ mm}}$$

(b) The first order bright fringe extends from the $m = 1$ dark fringe to the $m = 2$ dark fringe, or

$$\left(\Delta y_{\text{bright}}\right)_1 = \left(y_{\text{dark}}\right)_2 - \left(y_{\text{dark}}\right)_1 = 2\left(\frac{\lambda L}{a}\right) - 1\left(\frac{\lambda L}{a}\right) = \frac{\lambda L}{a}$$

$$= \frac{(5.40\times10^{-7} \text{ m})(1.50 \text{ m})}{0.200\times10^{-3} \text{ m}} = 4.05\times10^{-3} \text{ m} = \boxed{4.05 \text{ mm}}$$

Note that the width of the first order bright fringe is exactly one-half the width of the central maximum.

24.32 (a) Dark bands occur where $\sin\theta = m(\lambda/a)$. At the first dark band, $m=1$, and the distance from the center of the central maximum is

$$y_1 = L\tan\theta \approx L\sin\theta = L\left(\frac{\lambda}{a}\right) = (1.5\text{ m})\left(\frac{600\times10^{-9}\text{ m}}{0.40\times10^{-3}\text{ m}}\right) = \boxed{2.3\text{ mm}}$$

(b) The central maximum extends from the $m=1$ dark band to the $m=-1$ dark band. Its width is $(y_{\text{dark}})_1 - (y_{\text{dark}})_{-1} = 1(\lambda L/a) - (-1)(\lambda L/a) = 2\lambda L/a$, or

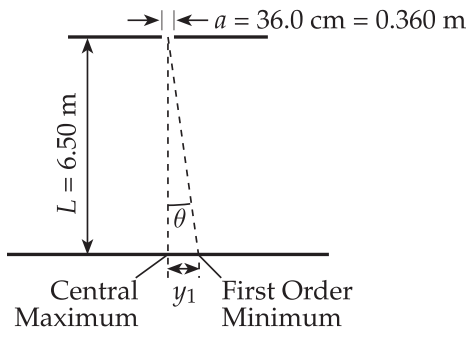
$$\text{width} = \frac{2(600\times10^{-9}\text{ m})(1.5\text{ m})}{0.40\times10^{-3}\text{ m}} = 4.5\times10^{-3}\text{ m} = \boxed{4.5\text{ mm}}$$

24.33 (a) Dark bands (minima) occur where $\sin\theta = m(\lambda/a)$. For the first minimum, $m=1$, and the distance from the center of the central maximum is $y_1 = L\tan\theta \approx L\sin\theta = L(\lambda/a)$. Thus, the needed distance to the screen is

$$L = y_1\left(\frac{a}{\lambda}\right) = (0.85\times10^{-3}\text{ m})\left(\frac{0.75\times10^{-3}\text{ m}}{587.5\times10^{-9}\text{ m}}\right) = \boxed{1.1\text{ m}}$$

(b) The width of the central maximum is $2y_1 = 2(0.85\text{ mm}) = \boxed{1.7\text{ mm}}$.

24.34 **Note:** The small angle approximation does not work well in this situation. Rather, you should proceed as follows.



At the first order minimum, $\sin\theta_1 = (1)\lambda/a$, or

$$\theta_1 = \sin^{-1}\left(\frac{\lambda}{a}\right) = \sin^{-1}\left(\frac{5.00\text{ cm}}{36.0\text{ cm}}\right) = 7.98^\circ$$

Then, $y_1 = L\tan\theta_1 = (6.50\text{ m})\tan 7.98^\circ$

$= 0.911\text{ m} = \boxed{91.1\text{ cm}}$

24.35 With the screen locations of the dark fringe of order m at

$$(y_{\text{dark}})_m = L\tan\theta_m \approx L\sin\theta_m = m(\lambda L/a) \quad \text{for} \quad m = \pm1, \pm2, \pm3, \ldots$$

the width of the central maximum is $\Delta y_{\substack{\text{central}\\\text{maximum}}} = (y_{\text{dark}})_{m=+1} - (y_{\text{dark}})_{m=-1} = 2(\lambda L/a)$, so

$$\lambda = \frac{a\left(\Delta y_{\substack{\text{central}\\\text{maximum}}}\right)}{2L} = \frac{(0.600\times10^{-3}\text{ m})(2.00\times10^{-3}\text{ m})}{2(1.30\text{ m})} = 4.62\times10^{-7}\text{ m} = \boxed{462\text{ nm}}$$

24.36 At the positions of the minima, $\sin\theta_m = m(\lambda/a)$ and $y_m = L\tan\theta_m \approx L\sin\theta_m = m[L(\lambda/a)]$.

Thus, $y_3 - y_1 = (3-1)[L(\lambda/a)] = 2[L(\lambda/a)]$, and

$$a = \frac{2L\lambda}{y_3 - y_1} = \frac{2(0.500\text{ m})(680\times10^{-9}\text{ m})}{3.00\times10^{-3}\text{ m}} = 2.27\times10^{-4}\text{ m} = \boxed{0.227\text{ mm}}$$

24.37 The locations of the dark fringes (minima) mark the edges of the maxima, and the widths of the maxima equals the spacing between successive minima.

At the locations of the minima, $\sin\theta_m = m(\lambda/a)$, and

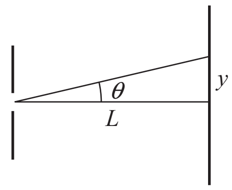
$$y_m = L\tan\theta_m \approx L\sin\theta_m = m\left[L(\lambda/a)\right] = m\left[(1.20\text{ m})\left(\frac{500\times10^{-9}\text{ m}}{0.500\times10^{-3}\text{ m}}\right)\right] = m(1.20\text{ mm})$$

Then, $\Delta y = \Delta m(1.20\text{ mm})$, and for successive minima, $\Delta m = 1$. Therefore, the width of each maximum, *other than the central maximum*, in this interference pattern is

$$\text{width} = \Delta y = (1)(1.20\text{ mm}) = \boxed{1.20\text{ mm}}$$

24.38 In a single-slit diffraction pattern, minima are found where

$$\sin\theta_{\text{dark}} = m\left(\frac{\lambda}{a}\right) \quad \text{where} \quad m = \pm1, \pm2, \ldots$$



or, using the small angle approximation $y = L\tan\theta \approx L\sin\theta$, at screen positions $y_m = m(\lambda L/a)$. Thus, if the $m = 2$ minimum is seen at $y = 1.40$ mm on a screen located distance $L = 85.0$ cm from a single slit of width $a = 0.800$ mm, the wavelength of the light passing through the slit must be

$$\lambda = \frac{y\,a}{mL} = \frac{(1.40\times10^{-3}\text{ m})(0.800\times10^{-3}\text{ m})}{2(85.0\times10^{-2}\text{ m})} = 6.59\times10^{-7}\text{ m} = \boxed{659\text{ nm}}$$

24.39 The grating spacing is $d = (1/3\,660)\text{cm} = (1/3.66\times10^5)$ m, and bright lines are found where $d\sin\theta = m\lambda$.

(a) The wavelength observed in the first-order spectrum is $\lambda = d\sin\theta_1$, or

$$\lambda = \left(\frac{1\text{ m}}{3.66\times10^5}\right)\left(\frac{10^9\text{ nm}}{1\text{ m}}\right)\sin\theta_1 = \left(\frac{10^4\text{ nm}}{3.66}\right)\sin\theta_1$$

This yields: at $\theta_1 = 10.1°$, $\lambda = \boxed{479\text{ nm}}$; at $\theta_1 = 13.7°$, $\lambda = \boxed{647\text{ nm}}$; and at $\theta_1 = 14.8°$, $\lambda = \boxed{698\text{ nm}}$.

(b) In the second order, $m = 2$. The second order images for the above wavelengths will be found at angles $\theta_2 = \sin^{-1}(2\lambda/d) = \sin^{-1}[2\sin\theta_1]$.

This yields: for $\lambda = 479$ nm, $\theta_2 = \boxed{20.5°}$; for $\lambda = 647$ nm, $\theta_2 = \boxed{28.3°}$; and for $\lambda = 698$ nm, $\theta_2 = \boxed{30.7°}$.

24.40 (a) The longest wavelength in the visible spectrum is 700 nm, and the grating spacing is $d = 1\text{ mm}/600 = 1.67\times10^{-3}\text{ mm} = 1.67\times10^{-6}\text{ m}$. The maximum viewing angle is $\theta_{\max} = 90.0°$, and maxima are found where $m\lambda = d\sin\theta$. Thus,

$$m_{\max} = \frac{d\sin 90.0°}{\lambda_{\text{red}}} = \frac{(1.67\times10^{-6}\text{ m})\sin 90.0°}{700\times10^{-9}\text{ m}} = 2.38$$

so $\boxed{\text{2 complete orders}}$ will be observed.

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(b) From $\lambda = d\sin\theta$, the angular separation of the red and violet edges in the first order will be

$$\Delta\theta = \sin^{-1}\left[\frac{\lambda_{\text{red}}}{d}\right] - \sin^{-1}\left[\frac{\lambda_{\text{violet}}}{d}\right] = \sin^{-1}\left[\frac{700\times10^{-9}\text{ m}}{1.67\times10^{-6}\text{ m}}\right] - \sin^{-1}\left[\frac{400\times10^{-9}\text{ m}}{1.67\times10^{-6}\text{ m}}\right]$$

or $\Delta\theta = \boxed{10.9^\circ}$

24.41 The grating spacing is $d = \dfrac{1.00\text{ cm}}{4\,500}\left(\dfrac{1.00\text{ m}}{10^2\text{ cm}}\right) = \dfrac{1.00\text{ m}}{4.50\times10^5}$.

From $d\sin\theta = m\lambda$, the angular separation between the given spectral lines will be $\Delta\theta = \sin^{-1}\left[m\lambda_{\text{red}}/d\right] - \sin^{-1}\left[m\lambda_{\text{violet}}/d\right]$, or

$$\Delta\theta = \sin^{-1}\left[\frac{m\left(656\times10^{-9}\text{ m}\right)\left(4.50\times10^5\right)}{1.00\text{ m}}\right] - \sin^{-1}\left[\frac{m\left(434\times10^{-9}\text{ m}\right)\left(4.50\times10^5\right)}{1.00\text{ m}}\right]$$

The results obtained are: for $m = 1$, $\Delta\theta = \boxed{5.91^\circ}$; for $m = 2$, $\Delta\theta = \boxed{13.2^\circ}$; and for $m = 3$, $\Delta\theta = \boxed{26.5^\circ}$. Complete orders for $m \geq 4$ are not visible.

24.42 The array of wires will act as a diffraction grating for the ultrasound waves, with maxima located at those angles given by the grating equation $d\sin\theta = m\lambda$, where $m = 0, \pm1, \pm2, \ldots$. The spacing between adjacent slits in this grating is $d = 1.30$ cm, and the wavelength of these ultrasound waves is

$$\lambda = \frac{v_{\text{sound}}}{f} = \frac{343\text{ m/s}}{37.2\times10^3\text{ Hz}} = 9.22\times10^{-3}\text{ m} = 9.22\text{ mm}$$

(a) The order number of the maximum found at angle θ is $m = (d/\lambda)\sin\theta$, so the maximum order that can be found will be the largest integer satisfying the relation

$$|m| \leq \left(\frac{d}{\lambda}\right)\sin|\theta_{\max}| = \left(\frac{d}{\lambda}\right)\sin 90.0^\circ = \frac{1.30\times10^{-2}\text{ m}}{9.22\times10^{-3}\text{ m}} = 1.41$$

Thus, $\boxed{\text{three maxima corresponding to } m=-1,\ m=0, \text{ and } m=+1 \text{ can be found}}$.

(b) We use $\theta = \sin^{-1}(m\lambda/d)$ to find the direction for each maximum to be:

for $m = -1$, $\theta = \boxed{-45.2^\circ}$; for $m = 0$, $\theta = \boxed{0^\circ}$; and for $m = +1$, $\theta = \boxed{+45.2^\circ}$.

24.43 For diffraction by a grating, the angle at which the maximum of order m occurs is given by $d\sin\theta_m = m\lambda$, where d is the spacing between adjacent slits on the grating. Thus,

$$d = \frac{m\lambda}{\sin\theta_m} = \frac{(1)\left(632.8\times10^{-9}\text{ m}\right)}{\sin 20.5^\circ} = 1.81\times10^{-6}\text{ m} = \boxed{1.81\ \mu\text{m}}$$

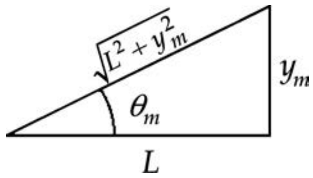
24.44 With 2 000 lines per centimeter, the grating spacing is

$$d = \frac{1}{2\,000}\text{ cm} = 5.00\times10^{-4}\text{ cm} = 5.00\times10^{-6}\text{ m}$$

Then, from $d\sin\theta = m\lambda$, the location of the first order for the red light is

$$\theta = \sin^{-1}\left(\frac{m\lambda}{d}\right) = \sin^{-1}\left(\frac{1\left(640\times10^{-9}\text{ m}\right)}{5.00\times10^{-6}\text{ m}}\right) = \boxed{7.35^\circ}$$

24.45 The spacing between adjacent slits on the grating is

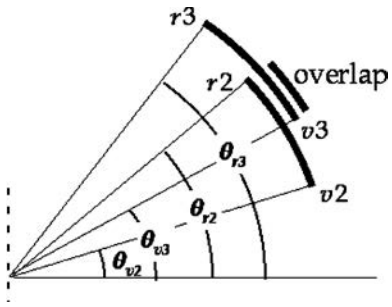


$$d=\frac{1\text{ cm}}{5\,310\text{ slits}}=\frac{10^{-2}\text{ m}}{5\,310}$$

The maximum of order m is located where $d\sin\theta_m=m\lambda$, so

$$\lambda=\frac{d}{m}\sin\theta_m=\frac{d}{m}\left(\frac{y_m}{\sqrt{L^2+y_m^2}}\right)=\frac{10^{-2}\text{ m}}{(1)(5\,310)}\left(\frac{0.488\text{ m}}{\sqrt{(1.72\text{ m})^2+(0.488\text{ m})^2}}\right)$$
$$=5.14\times10^{-7}\text{ m}=\boxed{514\text{ nm}}$$

24.46 From $d\sin\theta_m=m\lambda$, or $\sin\theta_m=m\lambda/d$, we see that



$$\sin\theta_{v2}=\frac{2\lambda_{\text{violet}}}{d}=\frac{800\text{ nm}}{d}\qquad \sin\theta_{v3}=\frac{3\lambda_{\text{violet}}}{d}=\frac{1\,200\text{ nm}}{d}$$

$$\sin\theta_{r2}=\frac{2\lambda_{\text{red}}}{d}=\frac{1\,400\text{ nm}}{d}\qquad \sin\theta_{r3}=\frac{3\lambda_{\text{red}}}{d}=\frac{2\,100\text{ nm}}{d}$$

Since, for $0°\le\theta\le90°$, θ increases as $\sin\theta$ increases, we have that

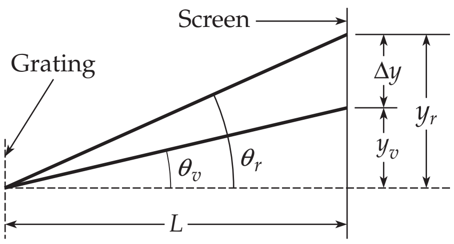
$$\theta_{v2}<\theta_{v3}<\theta_{r2}<\theta_{r3}$$

so $\boxed{\text{the second and third order spectra overlap in the range } \theta_{v3}\le\theta\le\theta_{r2}}$.

24.47 The grating spacing is $d=\dfrac{1\text{ cm}}{2\,750}=\dfrac{10^{-2}\text{ m}}{2.75\times10^{3}}=3.64\times10^{-6}\text{ m}$. From $d\sin\theta=m\lambda$, or $\theta=\sin^{-1}(m\lambda/d)$, the angular positions of the red and violet edges of the second-order spectrum are found to be

$$\theta_r=\sin^{-1}\left(\frac{2\lambda_{\text{red}}}{d}\right)=\sin^{-1}\left(\frac{2\left(700\times10^{-9}\text{ m}\right)}{3.64\times10^{-6}\text{ m}}\right)=22.6°$$

and $$\theta_v=\sin^{-1}\left(\frac{2\lambda_{\text{violet}}}{d}\right)=\sin^{-1}\left(\frac{2\left(400\times10^{-9}\text{ m}\right)}{3.64\times10^{-6}\text{ m}}\right)=12.7°$$



Note from the sketch at the right that $y_r=L\tan\theta_r$ and $y_v=L\tan\theta_v$, so the width of the spectrum on the screen is $\Delta y=L\left(\tan\theta_r-\tan\theta_v\right)$.

Since it is given that $\delta=(m+1/2)\lambda$, the distance from the grating to the screen must be

$$L=\frac{\Delta y}{\tan\theta_r-\tan\theta_v}=\frac{1.75\text{ cm}}{\tan(22.6°)-\tan(12.7°)}$$
$$=\boxed{9.17\text{ cm}}$$

24.48 (a) $$d=\frac{1\text{ cm}}{4.200\times10^{3}\text{ slits}}=2.381\times10^{-4}\text{ cm}=\boxed{2.381\times10^{-6}\text{ m}}$$

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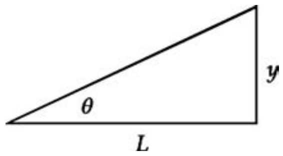
(b) $d\sin\theta_m = m\lambda \quad \Rightarrow \quad \theta_2 = \sin^{-1}\left(\dfrac{2\lambda}{d}\right) = \sin^{-1}\left[\dfrac{2\left(589.0\times10^{-9}\ \text{m}\right)}{2.381\times10^{-6}\ \text{m}}\right] = \boxed{29.65°}$

(c) $y_2 = L\tan\theta_2 = (2.000\ \text{m})\tan(29.65°) = \boxed{1.138\ \text{m}}$



(d) $\theta_2' = \sin^{-1}\left(\dfrac{2\lambda'}{d}\right) = \sin^{-1}\left[\dfrac{2\left(589.6\times10^{-9}\ \text{m}\right)}{2.381\times10^{-6}\ \text{m}}\right] = \boxed{29.69°}$

$y_2' = L\tan\theta_2' = (2.000\ \text{m})\tan(29.69°) = \boxed{1.140\ \text{m}}$

(e) $\Delta y = y_2' - y_2 = 1.140\ \text{m} - 1.138\ \text{m} = \boxed{2.000\times10^{-3}\ \text{m}}$

(f) $\Delta y = y_2' - y_2 = L\left[\tan\left(\sin^{-1}\left(\dfrac{2\lambda'}{d}\right)\right) - \tan\left(\sin^{-1}\left(\dfrac{2\lambda}{d}\right)\right)\right]$

$$= (2.000\ \text{m})\left[\tan\left(\sin^{-1}\left(\frac{2\left(589.6\times10^{-9}\ \text{m}\right)}{10^{-2}\ \text{m}/4.200\times10^{3}}\right)\right) - \tan\left(\sin^{-1}\left(\frac{2\left(589.0\times10^{-9}\ \text{m}\right)}{10^{-2}\ \text{m}/4.200\times10^{3}}\right)\right)\right]$$

$$= \boxed{1.537\times10^{-3}\ \text{m}}$$

The two answers agree to only one significant figure. The calculation is sensitive to rounding at intermediate steps.

24.49 (a) For a diffraction grating having distance d between adjacent slits, primary maxima for wavelength λ are produced at angles satisfying the grating equation, $d\sin\theta = m\lambda$. If the $m = 3$ maximum for $\lambda = 500$ nm is observed at $\theta = 32.0°$, the grating spacing is

$$d = \frac{m\lambda}{\sin\theta} = \frac{3\left(500\times10^{-9}\ \text{m}\right)}{\sin 32.0°} = 2.83\times10^{-6}\ \text{m} = 2.83\times10^{-4}\ \text{cm}$$

The number of rulings per centimeter on the grating is then

$$n = \frac{1\ \text{cm}}{d} = \frac{1\ \text{cm}}{2.83\times10^{-4}\ \text{cm}} = \boxed{3.53\times10^{3}\ \text{grooves/cm}}$$

(b) The order number of a maximum appearing at angle θ is $m = d\sin\theta/\lambda$. The magnitude of the largest order numbers visible is the largest integer satisfying the condition

$$|m| \le \left(\frac{d}{\lambda}\right)\sin\theta_{\text{max}} = \left(\frac{d}{\lambda}\right)\sin 90° = \frac{d}{\lambda} = \frac{2.83\times10^{-6}\ \text{m}}{500\times10^{-9}\ \text{m}} = 5.66$$

Thus, $\boxed{11\ \text{maxima}}$ for wavelength $\lambda = 500$ nm will be observable with this grating. These are the maxima corresponding to $m = 0,\ \pm1,\ \pm2,\ \pm3,\ \pm4,$ and ±5.

24.50 The grating spacing is $d = 1\ \text{cm}/1\,200 = 8.33\times10^{-4}\ \text{cm} = 8.33\times10^{-6}\ \text{m}$. Using $\sin\theta = m\lambda/d$ and the small angle approximation, the distance from the central maximum to the maximum of order m for wavelength λ is $y_m = L\tan\theta \approx L\sin\theta = (\lambda L/d)m$. Therefore, the spacing between successive maxima is $\Delta y = y_{m+1} - y_m = \lambda L/d$.

The longer wavelength in the light is found to be

$$\lambda_{\text{long}} = \frac{(\Delta y)d}{L} = \frac{\left(8.44\times10^{-3}\ \text{m}\right)\left(8.33\times10^{-6}\ \text{m}\right)}{0.150\ \text{m}} = 4.69\times10^{-7}\ \text{m} = \boxed{469\ \text{nm}}$$

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Since the third order maximum of the shorter wavelength falls halfway between the central maximum and the first order maximum of the longer wavelength, we have

$$\frac{3\lambda_{\text{short}}L}{d} = \left(\frac{0+1}{2}\right)\frac{\lambda_{\text{long}}L}{d} \qquad \text{or} \qquad \lambda_{\text{short}} = \left(\frac{1}{6}\right)(469\text{ nm}) = \boxed{78.1\text{ nm}}$$

24.51 (a) From Brewster's law, the index of refraction is

$$n_2 = \tan\theta_p = \tan(48.0°) = \boxed{1.11}$$

(b) From Snell's law, $n_2 \sin\theta_2 = n_1 \sin\theta_1$, the angle of refraction when $\theta_1 = \theta_p$ is

$$\theta_2 = \sin^{-1}\left(\frac{n_1 \sin\theta_p}{n_2}\right) = \sin^{-1}\left(\frac{(1.00)\sin 48.0°}{1.11}\right) = \boxed{42.0°}$$

Note that when $\theta_1 = \theta_p$, $\theta_2 = 90.0° - \theta_p$ as it should.

24.52 (a) From Malus's law, the fraction of the incident intensity of the unpolarized light that is transmitted by the polarizer is

$$I' = I_0\left(\cos^2\theta\right)_{\text{av}} = I_0\,(0.500)$$

The fraction of this intensity incident on the analyzer that will be transmitted is

$$I = I'\cos^2(35.0°) = \left[0.500 I_0\right]\cos^2(35.0°) = 0.336 I_0$$

Thus, the fraction of the incident unpolarized light transmitted is $I/I_0 = \boxed{0.336}$.

(b) The fraction of the original incident light absorbed by the analyzer is

$$\frac{I' - I}{I_0} = \frac{0.500 I_0 - 0.336 I_0}{I_0} = \boxed{0.164}$$

24.53 The more general expression for Brewster's angle is given in problem P24.57 as

$$\tan\theta_p = n_2/n_1$$

(a) When $n_1 = 1.00$ and $n_2 = 1.52$, $\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.52}{1.00}\right) = \boxed{56.7°}$.

(b) When $n_1 = 1.333$ and $n_2 = 1.52$, $\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.52}{1.333}\right) = \boxed{48.8°}$.

24.54 The polarizing angle for light in air striking a water surface is

$$\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.333}{1.00}\right) = 53.1°$$

This is the angle of incidence for the incoming sunlight (that is, the angle between the incident light and the normal to the surface). The altitude of the Sun is the angle between the incident light and the water surface. Thus, the altitude of the Sun is

$$\alpha = 90.0° - \theta_p = 90.0° - 53.1° = \boxed{36.9°}$$

24.55 (a) Brewster's angle (or the polarizing angle) is

$$\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{n_{\text{quartz}}}{n_{\text{air}}}\right) = \tan^{-1}\left(\frac{1.458}{1.000}\right) = \boxed{55.6°}$$

(b) When the angle of incidence is the polarizing angle, θ_p, the angle of refraction of the transmitted light is $\theta_2 = 90.0° - \theta_p$. Hence, $\theta_2 = 90.0° - 55.6° = \boxed{34.4°}$.

24.56 The critical angle for total internal reflection is $\theta_c = \sin^{-1}(n_2/n_1)$. Thus, if $\theta_c = 34.4°$ as light attempts to go from sapphire into air, the index of refraction of sapphire is

$$n_{\text{sapphire}} = n_1 = \frac{n_2}{\sin\theta_c} = \frac{1.00}{\sin 34.4°} = 1.77$$

Then, when light is incident on sapphire from air, the Brewster's angle is

$$\theta_p = \tan^{-1}\left(\frac{n_2}{n_1}\right) = \tan^{-1}\left(\frac{1.77}{1.00}\right) = \boxed{60.5°}$$

24.57 From Snell's law, the angles of incidence and refraction are related by $n_1 \sin\theta_1 = n_2 \sin\theta_2$.

If the angle of incidence is the polarizing angle (that is, $\theta_1 = \theta_p$), the refracted ray is perpendicular to the reflected ray (see Figure 24.28 in the textbook), and the angles of incidence and refraction are also related by $\theta_p + 90° + \theta_2 = 180°$, or $\theta_2 = 90° - \theta_p$.

Substitution into Snell's law then gives

$$n_1 \sin\theta_p = n_2 \sin(90° - \theta_p) = n_2 \cos\theta_p \qquad \text{or} \qquad \sin\theta_p/\cos\theta_p = \boxed{\tan\theta_p = n_2/n_1}$$

24.58 From Malus's law, the intensity of the light transmitted by the polarizer is $I = I_0 \cos^2\theta$, where I_0 is the intensity of the incident light, and θ is the angle between the direction of the plane of polarization of the incident light and the transmission axis of the polarizing disk. Thus, $\theta = \cos^{-1}\left(\sqrt{I/I_0}\right)$.

(a) $$\frac{I}{I_0} = \frac{1}{2.00} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\sqrt{\frac{1}{2.00}}\right) = \boxed{45.0°}$$

(b) $$\frac{I}{I_0} = \frac{1}{4.00} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\sqrt{\frac{1}{4.00}}\right) = \boxed{60.0°}$$

(c) $$\frac{I}{I_0} = \frac{1}{6.00} \quad \Rightarrow \quad \theta = \cos^{-1}\left(\sqrt{\frac{1}{6.00}}\right) = \boxed{65.9°}$$

24.59 From Malus's law, the intensity of the light transmitted by the first polarizer is $I_1 = I_i \cos^2\theta_1$. The plane of polarization of this light is parallel to the axis of the first plate and is incident on the second plate. Malus's law gives the intensity transmitted by the second plate as $I_2 = I_1 \cos^2(\theta_2 - \theta_1) = I_i \cos^2\theta_1 \cos^2(\theta_2 - \theta_1)$. This light is polarized parallel to the axis of the second plate and is incident upon the third plate. A final application of Malus's law gives the transmitted intensity as

$$I_f = I_2 \cos^2(\theta_3 - \theta_2) = I_i \cos^2\theta_1 \cos^2(\theta_2 - \theta_1)\cos^2(\theta_3 - \theta_2)$$

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With $\theta_1 = 20.0°$, $\theta_2 = 40.0°$, and $\theta_3 = 60.0°$, this result yields

$$I_f = (10.0 \text{ units})\cos^2(20.0°)\cos^2(20.0°)\cos^2(20.0°) = \boxed{6.89 \text{ units}}$$

24.60 (a) Using Malus's law, the intensity of the transmitted light is found to be $I = I_0 \cos^2(45°) = I_0\left(1/\sqrt{2}\right)^2$, or $\boxed{I = I_0/2}$.

(b) From Malus's law, $I/I_0 = \cos^2\theta$. Thus, if $I/I_0 = 1/3$, we obtain $\cos^2\theta = 1/3$, or

$$\theta = \cos^{-1}\left(1/\sqrt{3}\right) = \boxed{54.7°}$$

24.61 (a) If light has wavelength λ in vacuum, its wavelength in a medium of refractive index n is $\lambda_n = \lambda/n$. Thus, the wavelengths of the two components in the specimen are

$$\lambda_{n_1} = \frac{\lambda}{n_1} = \frac{546.1 \text{ nm}}{1.320} = \boxed{413.7 \text{ nm}}$$

and $$\lambda_{n_2} = \frac{\lambda}{n_2} = \frac{546.1 \text{ nm}}{1.333} = \boxed{409.7 \text{ nm}}$$

(b) The numbers of cycles of vibration each component completes while passing through the specimen of thickness $t = 1.000\ \mu$m are

$$N_1 = \frac{t}{\lambda_{n_1}} = \frac{1.000 \times 10^{-6} \text{ m}}{413.7 \times 10^{-9} \text{ m}} = 2.417$$

and $$N_2 = \frac{t}{\lambda_{n_2}} = \frac{1.000 \times 10^{-6} \text{ m}}{409.7 \times 10^{-9} \text{ m}} = 2.441$$

Thus, when they emerge, the two components are out of phase by $N_2 - N_1 = 0.024$ cycles. Since each cycle represents a phase angle of 360°, they emerge with a phase difference of

$$\Delta\phi = (0.024 \text{ cycles})(360°/\text{cycle}) = \boxed{8.6°}$$

24.62 In a single-slit diffraction pattern, the first dark fringe occurs where $\sin(\theta_{\text{dark}})_1 = (1)\lambda/a$. If no diffraction minima are to be observed, the maximum width the slit can have is that which would place the first dark fringe at the maximum viewable angle of 90.0°. That is, when $a = a_{\max}$, we will have $\sin(\theta_{\text{dark}})_1 = (1)\lambda/a_{\max} = \sin 90.0° = 1.00$, yielding

$$a_{\max} = \lambda = \boxed{632.8 \text{ nm}}$$

24.63 The light has passed through $\boxed{\text{a single slit}}$ since the central maximum is twice the width of other maxima (the space between the centers of successive dark fringes). In a double-slit pattern, the central maximum has the same width as all other maxima (compare Active Figures 24.1(b) and 24.16(b) in the textbook).

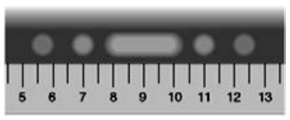


Figure P24.63

In single-slit diffraction the width of the central maximum on the screen is given by

$$\Delta y_{\substack{\text{central}\\\text{maximum}}} = 2L\tan\left(\theta_{\text{dark}}\right)_{m=1} \approx 2L\sin\left(\theta_{\text{dark}}\right)_{m=1} = 2L\left(\frac{(1)\lambda}{a}\right) = 2\left(\frac{\lambda L}{a}\right)$$

continued on next page

The width of the slit is then

$$a = \frac{2\lambda L}{\Delta y_{\substack{\text{central}\\\text{maximum}}}} = \frac{2\left(632.8\times10^{-9}\ \text{m}\right)(2.60\ \text{m})}{(10.3-7.6)\ \text{cm}}\left(\frac{10^{2}\ \text{cm}}{1\ \text{m}}\right) = 1.2\times10^{-4}\ \text{m} = \boxed{0.12\ \text{mm}}$$

24.64 (a) In a double-slit interference pattern, bright fringes on the screen occur where $(y_{\text{bright}})_m = m(\lambda L/d)$. Thus, if bright fringes of the wavelengths $\lambda_1 = 540$ nm and $\lambda_2 = 450$ nm are to coincide, it is necessary that

$$m_1\frac{(540\ \text{nm})\cancel{L}}{\cancel{d}} = m_2\frac{(450\ \text{nm})\cancel{L}}{\cancel{d}}$$

or, dividing both sides by 90 nm, $\boxed{6m_1 = 5m_2}$, where both m_1 and m_2 are integers.

(b) The condition found above may be written as $m_2 = (6/5)m_1$. Trial and error reveals that the smallest nonzero integer value of m_1 that will yield an integer value for m_2 is $\boxed{m_1 = 5}$, yielding $\boxed{m_2 = 6}$. Thus, the first overlap of bright fringes for the two given wavelengths occurs at the screen position (measured from the central maximum)

$$y_{\text{bright}} = \frac{5\left(540\times10^{-9}\ \text{m}\right)(1.40\ \text{m})}{0.150\times10^{-3}\ \text{m}} = \frac{6\left(450\times10^{-9}\ \text{m}\right)(1.40\ \text{m})}{0.150\times10^{-3}\ \text{m}}$$
$$= 2.52\times10^{-2}\ \text{m} = \boxed{2.52\ \text{cm}}$$

24.65 Dark fringes (destructive interference) occur where $d\sin\theta = (m+1/2)\lambda$ for $m = 0, 1, 2, \ldots$. Thus, if the second dark fringe $(m=1)$ occurs at $\theta = (18.0\ \text{min})(1.00°/60.0\ \text{min}) = 0.300°$, the slit spacing is

$$d = \left(m+\frac{1}{2}\right)\frac{\lambda}{\sin\theta} = \left(\frac{3}{2}\right)\frac{\left(546\times10^{-9}\ \text{m}\right)}{\sin(0.300°)} = 1.56\times10^{-4}\ \text{m} = \boxed{0.156\ \text{mm}}$$

24.66 The wavelength is $\lambda = v_{\text{sound}}/f = (340\ \text{m/s})/(2\,000\ \text{Hz}) = 0.170\ \text{m}$, and maxima occur where $d\sin\theta = m\lambda$, or $\theta = \sin^{-1}\left[m(\lambda/d)\right]$ for $m = 0, \pm1, \pm2, \ldots$. Since $d = 0.350$ m, $\lambda/d = 0.486$, which gives $\theta = \sin^{-1}\left[m(0.486)\right]$. For $m = 0, \pm1$, and ±2, this yields $\boxed{\text{maxima at } 0°,\ \pm29.1°, \text{ and } \pm76.4°}$. No solutions exist for $|m| \geq 3$, since that would imply $\sin\theta > 1$.

Minima occur where $d\sin\theta = (m+1/2)\lambda$, or $\theta = \sin^{-1}\left[(2m+1)\lambda/2d\right]$ for $m = 0, \pm1, \pm2, \ldots$. With $\lambda/d = 0.486$, this becomes $\theta = \sin^{-1}\left[(2m+1)(0.243)\right]$. For $m = 0$ and ±1, we find $\boxed{\text{minima at } \pm14.1° \text{ and } \pm46.8°}$. No solutions exist for $|m| \geq 2$, since that would imply $\sin\theta > 1$.

24.67 The source and its image, located 1.00 cm below the mirror, act as a pair of coherent sources. This situation may be treated as double-slit interference, with the slits separated by 2.00 cm, if it is remembered that the light undergoes a phase change upon reflection from the mirror. The existence of this phase change causes the conditions for constructive and destructive interference to be reversed. Therefore, dark bands (destructive interference) occur where $y = m(\lambda L/d)$ for $m = 0, 1, 2, \ldots$.

The $m = 0$ dark band occurs at $y = 0$ (that is, at mirror level). The first dark band above the mirror corresponds to $m = 1$, and is located at

$$y = (1)\left(\frac{\lambda L}{d}\right) = \frac{\left(500\times10^{-9}\ \text{m}\right)(100\ \text{m})}{2.00\times10^{-2}\ \text{m}} = 2.50\times10^{-3}\ \text{m} = \boxed{2.50\ \text{mm}}$$

24.68 Assuming the glass plates have refractive indices greater than that of both air and water, there will be a phase change at the reflection from the lower surface of the film but no change from reflection at the top of the film. Therefore, the condition for a dark fringe is

$$2t = m\lambda_n = m\left(\lambda/n_{\text{film}}\right) \text{ for } m = 0, 1, 2, \ldots$$

If the highest order dark band observed is $m = 84$ (a total of 85 dark bands counting the $m = 0$ order at the edge of contact), the maximum thickness of the air film is

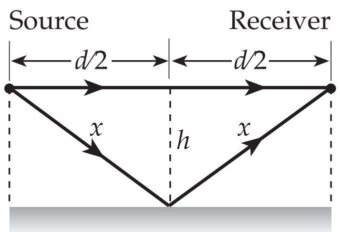
$$t_{\max} = \frac{m_{\max}}{2}\left(\frac{\lambda}{n_{\text{film}}}\right) = \frac{84}{2}\left(\frac{\lambda}{1.00}\right) = 42\,\lambda$$

When the film consists of water, the highest order dark fringe appearing will be

$$m_{\max} = 2t_{\max}\left(\frac{n_{\text{film}}}{\lambda}\right) = 2(42\,\lambda)\left(\frac{1.333}{\lambda}\right) = 112$$

Counting the $m = 0$ order, a total of $\boxed{113}$ dark fringes are now observed.

24.69 In the figure at the right, observe that the path difference between the direct and the indirect paths is



$$\delta = 2x - d = 2\sqrt{h^2 + (d/2)^2} - d$$

With a phase change (equivalent to a half-wavelength shift) occurring upon reflection at the ground, the condition for constructive interference is $\delta = (m + 1/2)\lambda$, and the condition for destructive interference is $\delta = m\lambda$. In both cases, the possible values of the order number are $m = 0, 1, 2, \ldots$.

(a) The wavelengths that will interfere constructively are $\lambda = \dfrac{\delta}{m + 1/2}$. The longest of these is for the $m = 0$ case and has a value of

$$\lambda = 2\delta = 4\sqrt{h^2 + (d/2)^2} - 2d = 4\sqrt{(50.0 \text{ m})^2 + (300 \text{ m})^2} - 2(600 \text{ m}) = \boxed{16.6 \text{ m}}$$

(b) The wavelengths that will interfere destructively are $\lambda = \delta/m$, and the largest finite one of these is for the $m = 1$ case. That wavelength is

$$\lambda = \delta = 2\sqrt{h^2 + (d/2)^2} - d = 2\sqrt{(50.0 \text{ m})^2 + (300 \text{ m})^2} - 600 \text{ m} = \boxed{8.28 \text{ m}}$$

24.70 From Malus's law, the intensity of the light transmitted by the first polarizer is $I_1 = I_i \cos^2\theta_1$. The plane of polarization of this light is parallel to the axis of the first plate and is incident on the second plate. Malus's law gives the intensity transmitted by the second plate as $I_2 = I_1\cos^2(\theta_2 - \theta_1) = I_i\cos^2\theta_1\cos^2(\theta_2 - \theta_1)$. This light is polarized parallel to the axis of the second plate and is incident upon the third plate. A final application of Malus's law gives the transmitted intensity as

$$I_f = I_2\cos^2(\theta_3 - \theta_2) = I_i\cos^2\theta_1\cos^2(\theta_2 - \theta_1)\cos^2(\theta_3 - \theta_2)$$

(a) If $\theta_1 = 45°$, $\theta_2 = 90°$, and $\theta_3 = 0°$, then

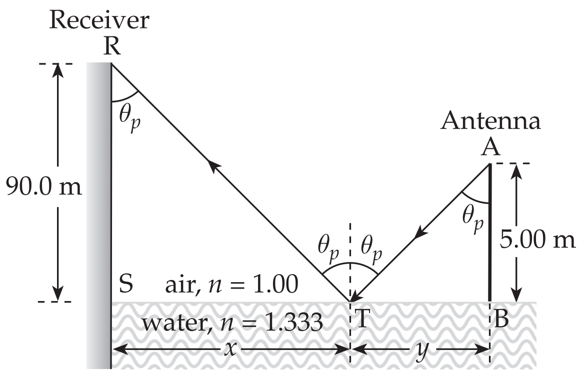
$$I_f/I_i = \cos^2 45°\cos^2(90° - 45°)\cos^2(0° - 90°) = \boxed{0}$$

continued on next page

(b) If $\theta_1 = 0°$, $\theta_2 = 45°$, and $\theta_3 = 90°$, then

$$I_f/I_i = \cos^2 0° \cos^2(45° - 0°)\cos^2(90° - 45°) = \boxed{0.25}$$

24.71 If the signal from the antenna to the receiver station is to be completely polarized by reflection from the water, the angle of incidence where it strikes the water must equal the polarizing angle from Brewster's law. This is given by



$$\theta_p = \tan^{-1}\left(\frac{n_{\text{water}}}{n_{\text{air}}}\right) = \tan^{-1}(1.333) = 53.1°$$

From the triangle RST in the sketch, the horizontal distance from the point of refection, T, to shore is given by

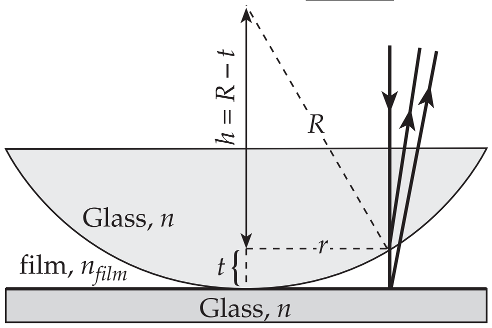
$$x = (90.0 \text{ m})\tan\theta_p = (90.0 \text{ m})(1.333) = 120 \text{ m}$$

and from triangle ABT, the horizontal distance from the antenna to point T is

$$y = (5.00 \text{ m})\tan\theta_p = (5.00 \text{ m})(1.333) = 6.67 \text{ m}$$

The total horizontal distance from ship to shore is then $x + y = 120 \text{ m} + 6.67 \text{ m} = \boxed{127 \text{ m}}$.

24.72 There will be a phase change associated with the reflection at one surface of the film but no change at the other surface of the film. Therefore, the condition for a dark fringe (destructive interference) is



$$2t = m\lambda_n = m\left(\frac{\lambda}{n_{\text{film}}}\right) \quad \text{where} \quad m = 0, 1, 2, \ldots$$

From the figure, note that $R^2 = r^2 + (R - t)^2 = r^2 + R^2 - 2Rt + t^2$, which reduces to $r^2 = 2Rt - t^2$. Since t will be very small in comparison to either r or R, we may neglect the term t^2, leaving $r \approx \sqrt{2Rt}$.

For a dark fringe, $t = \dfrac{m\lambda}{2n_{\text{film}}}$, so the radii of the dark rings will be

$$r \approx \sqrt{2R\left(\frac{m\lambda}{2n_{\text{film}}}\right)} = \boxed{\sqrt{\frac{m\lambda R}{n_{\text{film}}}}} \quad \text{for } m = 0, 1, 2, \ldots$$

24.73 In the single-slit diffraction pattern, destructive interference (or minima) occur where $\sin\theta = m(\lambda/a)$ for $m = 0, \pm1, \pm2, \ldots$. The screen locations, measured from the center of the central maximum, of these minima are at

$$y_m = L\tan\theta_m \approx L\sin\theta_m = m(\lambda L/a)$$

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If we assume the first order maximum is halfway between the first and second order minima, then its location is

$$y = \frac{y_1 + y_2}{2} = \frac{(1+2)(\lambda L/a)}{2} = \frac{3\lambda L}{2a}$$

and the slit width is

$$a = \frac{3\lambda L}{2y} = \frac{3(500\times10^{-9}\text{ m})(1.40\text{ m})}{2(3.00\times10^{-3}\text{ m})} = 3.50\times10^{-4}\text{ m} = \boxed{0.350\text{ mm}}$$

24.74 As light emerging from the glass reflects from the top of the air layer, there is no phase change produced. However, the light reflecting from the end of the metal rod at the bottom of the air layer does experience a phase change. Thus, the condition for constructive interference in the reflected light is $2t = \left(m+\frac{1}{2}\right)\lambda/n_{air} = \left(m+\frac{1}{2}\right)\lambda$.

As the metal rod expands, the thickness of the air layer decreases. The increase in the length of the rod is given by

$$\Delta L = |\Delta t| = \left(m_i + \tfrac{1}{2}\right)\frac{\lambda}{2} - \left(m_f + \tfrac{1}{2}\right)\frac{\lambda}{2} = |\Delta m|\frac{\lambda}{2}$$

The order number changes by one each time the film changes from bright to dark and back to bright. Thus, during the expansion, $\Delta m = 200$, and the measured change in the length of the rod is

$$\Delta L = (200)\frac{\lambda}{2} = (200)\frac{(500\times10^{-9}\text{ m})}{2} = 5.00\times10^{-5}\text{ m}$$

From $\Delta L = L_0\alpha(\Delta T)$, the coefficient of linear expansion of the rod is

$$\alpha = \frac{\Delta L}{L_0(\Delta T)} = \frac{5.00\times10^{-5}\text{ m}}{(0.100\text{ m})(25.0\,^\circ\text{C})} = \boxed{20.0\times10^{-6}\ (^\circ\text{C})^{-1}}$$

25

Optical Instruments

QUICK QUIZZES

1. Choice (c). The corrective lens for a farsighted eye is a converging lens, while that for a nearsighted eye is a diverging lens. Since a converging lens is required to form a real image of the Sun on the paper to start a fire, the campers should use the glasses of the farsighted person.

2. Choice (a). We would like to reduce the minimum angular separation for two objects below the angle subtended by the two stars in the binary system. We can do that by reducing the wavelength of the light—this in essence makes the aperture larger, relative to the light wavelength, increasing the resolving power. Thus, we would choose a blue filter.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The amount of light focused on the film by a camera is proportional to the area of the aperture through which the light enters the camera. Since the area of a circular opening varies as the square of the diameter of the opening, the light reaching the film is proportional to the square of the diameter of the aperture. Thus, increasing this diameter by a factor of 3 increases the amount of light by a factor of 9, and (c) is the correct choice.

2. The power of a lens in diopters equals the reciprocal of the focal length when that focal length is expressed in meters. Hence, the power of a lens having a focal length of 25 cm is

$$P = \frac{1}{f} = \frac{1}{0.25 \text{ m}} = 4.0 \text{ diopters}$$

and choice (b) is the correct answer.

3. Diffraction of light as it passes through, or reflects from, the objective element of a telescope can cause the images of two sources having a small angular separation to overlap and fail to be seen as separate images. The minimum angular separation two sources must have in order to be seen as separate sources is inversely proportional to the diameter of the objective element. Thus, using a large diameter objective element in a telescope increases its resolution, making (c) the correct choice.

4. When the eye is longer than normal, the lens-cornea system tends to form images of distant objects in front of the retina. Rays from near objects are more divergent and the lens-cornea system brings them into focus farther from the lens, on the retina. This means that the eye can see near objects clearly but is unable to focus on distant objects. Such an eye is nearsighted (myopia) and needs a diverging corrective lens to make the rays from distant objects more divergent before they enter the eye. Choice (b) is the correct answer.

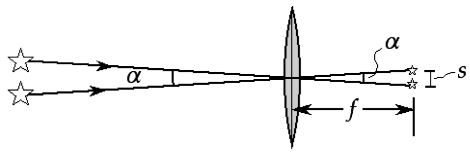
5. When the eye is shorter than normal, the lens-cornea system fails to bring light from near objects into focus by the time it reaches the retina, resulting in a blurry image. Light rays entering the pupil from distant objects are less divergent than those from near objects, and the lens-cornea system can focus them on the retina. Such an eye is farsighted, or has hyperopia, and needs a converging corrective lens to help bring rays from near objects to focus sooner. The correct choice is (c).

6. The corrective lens must form an upright, virtual image located 55 cm in front of the lens ($q = -55$ cm) when the object is 25 cm in front of the lens ($p = +25$ cm). The thin-lens equation then gives the required focal length as

$$f = \frac{pq}{p+q} = \frac{(25\text{ cm})(-55\text{ cm})}{25\text{ cm} - 55\text{ cm}} = +46\text{ cm}$$

and the correct choice is (c).

7. Stars are very distant, and the reciprocal of the object distance p in the thin-lens or mirror equation



$$1/p + 1/q = 1/f$$

is essentially zero. This means that $q = f$, or the images are formed at a distance equal to the focal length from the objective element. Thus, the angular separation of the images, and hence the stars, is

$$\alpha = \frac{s}{f} = \frac{20.0\times 10^{-3}\text{ m}}{2.00\text{ m}} = 1.00\times 10^{-2}\text{ rad}\left(\frac{360°}{2\pi\text{ rad}}\right) = 0.573°$$

and we see that choice (d) is the correct answer.

8. When a compound microscope is adjusted for most relaxed viewing (i.e., the final image formed by the eyepiece is at infinity), the approximate overall magnification produced by the microscope is given by the expression

$$m = -\frac{L}{f_o}\left(\frac{25\text{ cm}}{f_e}\right)$$

where L is the length of the microscope, f_o is the focal length of the objective lens, and f_e is the focal length of the eyepiece. With the microscope described, the approximate magnification is

$$m = -\frac{15\text{ cm}}{0.80\text{ cm}}\left(\frac{25\text{ cm}}{4.0\text{ cm}}\right) = -1.2\times 10^{2} \qquad \text{or} \qquad |m| \approx 120$$

and the correct answer is choice (d).

9. When using light of wavelength λ, the resolving power needed to distinguish two closely spaced spectral lines having a difference in wavelength of $\Delta\lambda$ is $R = \lambda/\Delta\lambda$. Thus, if two lines in the visible spectrum differ in wavelength by $\Delta\lambda = 0.1$ nm, the minimum resolving power of a diffraction grating that might be used to separate them is

$$R_{\min} = \frac{\lambda_{\min}}{\Delta\lambda} = \frac{400\text{ nm}}{0.1\text{ nm}} = 4\times 10^{3} = 4\ 000$$

and the correct choice is (b).

10. The angular separation of the two stars is $\theta = s/r = (10^{-5}\text{ ly})/(200\text{ ly}) = 5\times 10^{-8}\text{ radians}$. The limiting angle of resolution for a circular aperture is $\theta_{\min} = 1.22(\lambda/D)$. Requiring that $\theta_{\min} = \theta$, and assuming a wavelength at the center of the visible spectrum (550 nm), the required diameter of the aperture is found to be

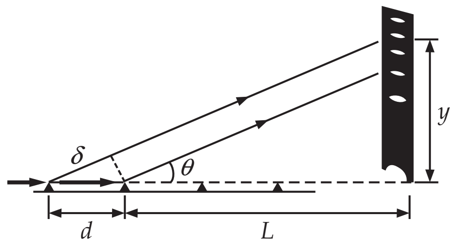
$$D = \frac{1.22\lambda}{\theta} = \frac{1.22\left(550\times 10^{-9}\text{ m}\right)}{5\times 10^{-8}\text{ rad}} = 1\times 10^{1}\text{ m} = 10\text{ m}$$

so (c) is the best choice of the listed possible answers.

11. Diffraction of the light as it passes through the opening of the detector limits the ability of that detector to resolve two closely spaced light sources. If the diameter of the opening of the detector does not change, the intensity of the light is not a factor in the resolution of the sources. Thus, if we neglect any contraction of the pupils of the eyes, your ability to resolve the two headlights will not change when they are switched to high beam, and choice (c) is the best response to the question. In reality, the natural response of the eyes to the increased light intensity is to contract the pupils to some degree, thereby somewhat decreasing their ability to resolve the two sources.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. Light nearly parallel to the horizontal ruler will scatter from rule marks, distance d apart, to produce a diffraction pattern on a vertical wall a distance L away. At height y on the wall, where the scattering angle (in radians) is $\theta = y/L$, light scattered from adjacent rule marks interferes constructively to produce a bright spot if the path difference is $\delta = m\lambda$, where $m = 1, 2, 3, \ldots$. The path difference is given by $\delta = d\cos\theta \approx d(1-\theta^2/2)$, where we have made use of a series expansion for $\cos\theta$, valid for very small angles. Combining these equations gives $m\lambda = d(1 - y_m^2/2L^2)$. Thus, the wavelength of the light may be calculated by measurement of the heights y_m of bright spots.



4. There will be an effect on the interference pattern—it will be distorted. The high temperature of the flame will change the index of refraction of air for the arm of the interferometer in which the match is held. As the index of refraction varies randomly, the wavelength of the light in that region will also vary randomly. As a result, the effective difference in length between the two arms will fluctuate, resulting in a wildly varying interference pattern.

6. The aperture of a camera is a close approximation to the iris of the eye. The retina of the eye corresponds to the film of the camera, and a close approximation to the cornea of the eye is the lens of the camera.

8. Large lenses are difficult to manufacture and machine with accuracy. Also, their large weight leads to sagging, which produces a distorted image. In reflecting telescopes, light does not pass through glass; hence, problems associated with chromatic aberrations are eliminated. Large-diameter reflecting telescopes are also technically easier to construct. Some designs use a rotating pool of mercury as the reflecting surface.

10. Under low ambient light conditions, a photoflash unit is used to ensure that light entering the camera lens will deliver sufficient energy for a proper exposure to each area of the film. Thus, the most important criterion is the additional energy per unit area (product of intensity and the duration of the flash, assuming this duration is less than the shutter speed) provided by the flash unit.

12. The angular magnification produced by a simple magnifier is $m = (25\text{ cm})/f$. Note that this is proportional to the optical power of a lens, $P = 1/f$, where the focal length f is expressed in meters. Thus, if the power of the lens is doubled, the angular magnification will also double.

14. In a nearsighted person the image of a distant object focuses in front of the retina. The cornea needs to be flattened so that its focal length is increased.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. 2.5 mm to 46 mm

4. The image is 19 mm across and easily fits on the 35-mm slide.

6. **(a)** See Solution. **(b)** $1/100\ \text{s}$

8. 1.05 m to 6.30 m

10. **(a)** farsighted **(b)** 18.0 cm **(c)** 38.0 cm

(d) virtual image, negative **(e)** 34.2 cm **(f)** +2.92 diopters

(g) $p = 20.0$ cm, $q = -40.0$ cm, $f = +40.0$ cm, $P = +2.50$ diopters

12. **(a)** $+50.8$ diopters $\leq P \leq +60.0$ diopters

(b) –0.800 diopters, diverging

14. **(a)** Yes, by using a bifocal or progressive lens.

(b) +1.78 diopters **(c)** –1.18 diopters

16. **(a)** -0.67 diopters **(b)** +0.67 diopters

18. **(a)** –25.0 cm **(b)** nearsighted **(c)** –3.70 diopters

20. **(a)** 4.17 cm in front of the lens **(b)** $m = 6.00$

22. **(a)** +5.0 **(b)** +6.0 **(c)** +4.2 cm

24. **(a)** $m_{\text{max}} = 2.39$ **(b)** $m = 1.39$

26. $m = -588$

28. $0.809\ \mu\text{m}$

30. **(a)** $L = f_o\left[(m+1)/m\right]$ **(b)** 2.00 cm toward the objective lens

32. 1.6×10^2 mi

34. **(a)** $m = 7.50$ **(b)** 0.944 m

36. **(a)** virtual image

(b) The final image is an infinite distance in front of the telescope.

(c) $f_e = -5.00$ cm, $f_o = 15.0$ cm

38. 1.7 m

40. **(a)** 2.29×10^{-4} rad **(b)** 43.7 m

42. 2.2×10^{11} m

44. 38 cm

46. **(a)** 3.6×10^{3} lines **(b)** 1.8×10^{3} lines

48. **(a)** 449 nm **(b)** smaller, because the wavelength is longer

50. 39.6 μm

52. 1.000 5

54. $\theta_{\min}\approx 2.0\times10^{-3}$ radians

56. **(a)** -4.3 diopters **(b)** -4.0 diopters, 44 cm

58. **(a)** 1.96 cm **(b)** 3.27 **(c)** 9.80

60. 5.07 mm

62. $m = 10.7$

PROBLEM SOLUTIONS

25.1 The *f*-number (or focal ratio) of a lens is defined to be the ratio of focal length of the lens to its diameter. Therefore, the *f*-number of the given lens is

$$f\text{-number} = \frac{f}{D} = \frac{28\text{ cm}}{4.0\text{ cm}} = \boxed{7.0}$$

25.2 If a camera has a lens with focal length of 55 mm and can operate at f-numbers that range from $f/1.2$ to $f/22$, the aperture diameters for the camera must range from

$$D_{\min} = \frac{f}{(f\text{-number})_{\max}} = \frac{55\text{ mm}}{22} = \boxed{2.5\text{ mm}}$$

to

$$D_{\max} = \frac{f}{(f\text{-number})_{\min}} = \frac{55\text{ mm}}{1.2} = \boxed{46\text{ mm}}$$

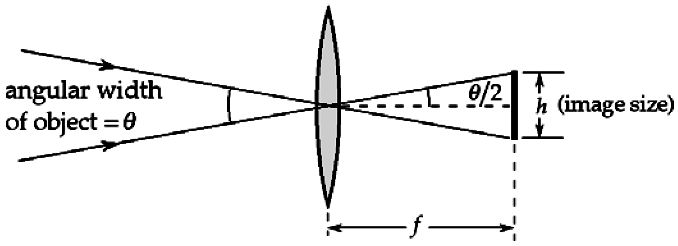
25.3 The thin-lens equation, $\frac{1}{p}+\frac{1}{q}=\frac{1}{f}$, gives the image distance as

$$q = \frac{pf}{p-f} = \frac{(100\text{ m})(52.0\text{ mm})}{100\text{ m} - 52.0\times10^{-3}\text{ m}} = 52.0\text{ mm}$$

From the magnitude of the lateral magnification, $|M| = h'/h = |-q/p|$, where the height of the image is $h' = 0.092\,0\text{ m} = 92.0\text{ mm}$, the height of the object (the building) must be

$$h = h'\left|-\frac{p}{q}\right| = (92.0\text{ mm})\left|-\frac{100\text{ m}}{52.0\text{ mm}}\right| = \boxed{177\text{ m}}$$

25.4 Consider rays coming from opposite edges of the object and passing undeviated through the center of the lens as shown at the right. For a very distant object, the image distance equals the focal length of the lens. If the angular width of the object is θ, the full image width on the film is



$$h = 2\left[f\tan(\theta/2)\right] = 2(55.0\text{ mm})\tan\left(\frac{20°}{2}\right) = \boxed{19\text{ mm}}$$

so the image easily fits within a 23.5 mm by 35.0 mm area.

25.5 The exposure time is being reduced by a factor of

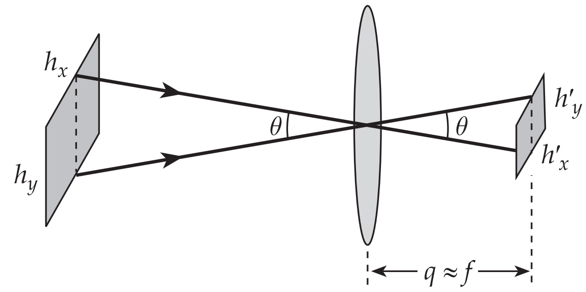
$$\frac{t_2}{t_1} = \frac{1/125\text{ s}}{1/15\text{ s}} = \frac{15}{125} = \frac{3}{25}$$

Thus, to maintain correct exposure, the intensity of the light reaching the film should be increased by a factor of 25/3. This is done by increasing the area of the aperture by a factor of 25/3, so in terms of the diameter, $\pi D_2^2/4 = (25/3)(\pi D_1^2/4)$, or $D_2 = D_1\sqrt{25/3}$.

The new *f*-number will be

$$(f\text{-number})_2 = \frac{f}{D_2} = \frac{f}{D_1\sqrt{25/3}} = (f\text{-number})_1\sqrt{\frac{3}{25}} = \frac{4.0\sqrt{3}}{5} = 1.4 \quad \text{or} \quad \boxed{f/1.4}$$

25.6 (a) The intensity is a measure of the rate at which energy is received by the film *per unit area of the image*, or $I \propto 1/A_{\text{image}}$. Consider an object with horizontal and vertical dimensions h_x and h_y as shown at the right. If the vertical dimension intercepts angle θ, the vertical dimension of the image is $h'_y = q\theta$, or $h'_y \propto q$.



Similarly for the horizontal dimension, $h'_x \propto q$, and the area of the image is $A_{\text{image}} = h'_x h'_y \propto q^2$. Assuming a very distant object, $q \approx f$, so $A_{\text{image}} \propto f^2$, and we conclude that $I \propto 1/f^2$.

The intensity of the light reaching the film is also proportional to the cross-sectional area of the lens and hence to the square of the diameter of that lens, or $I \propto D^2$. Combining this with our earlier conclusion gives

$$I \propto \frac{D^2}{f^2} = \frac{1}{(f/D)^2} \quad \text{or} \quad \boxed{I \propto \frac{1}{(f\text{-number})^2}}$$

(b) The total light energy delivered to the film is proportional to the product of intensity and exposure time, *It*. Thus, to maintain correct exposure, this product must be kept constant, or $I_2t_2 = I_1t_1$, giving

$$t_2 = \left(\frac{I_1}{I_2}\right)t_1 = \left[\frac{(f_2\text{-number})^2}{(f_1\text{-number})^2}\right]t_1 = \left(\frac{4.0}{1.8}\right)^2\left(\frac{1}{500}\text{ s}\right) \approx \boxed{1/100\text{ s}}$$

25.7 Since the exposure time is unchanged, the intensity of the light reaching the film must be doubled if the energy delivered is to be doubled. Using the result of Problem 25.6 (part a), we obtain

$$(f_2\text{-number})^2 = \left(\frac{I_1}{I_2}\right)(f_1\text{-number})^2 = \left(\frac{1}{2}\right)(11)^2 = 61, \quad \text{or} \quad f_2\text{-number} = \sqrt{61} = 7.8$$

Thus, you should use the $\boxed{f/8.0}$ setting on the camera.

25.8 The image must always be focused on the film, so the image distance is the distance between the lens and the film. From the thin-lens equation, $1/p + 1/q = 1/f$, the object distance is $p = qf/(q - f)$, and the range of object distances this camera can work with is from

$$p_{\min} = \frac{q_{\max} f}{q_{\max} - f} = \frac{(210 \text{ mm})(175 \text{ mm})}{210 \text{ mm} - 175 \text{ mm}} = 1.05 \times 10^3 \text{ mm} = \boxed{1.05 \text{ m}}$$

to

$$p_{\max} = \frac{q_{\min} f}{q_{\min} - f} = \frac{(180 \text{ mm})(175 \text{ mm})}{180 \text{ mm} - 175 \text{ mm}} = 6.30 \times 10^3 \text{ mm} = \boxed{6.30 \text{ m}}$$

25.9 The corrective lens must form an upright, virtual image at the near point of the eye (i.e., $q = -60.0$ cm in this case) for objects located 25.0 cm in front of the eye ($p = +25.0$ cm). From the thin-lens equation, $1/p + 1/q = 1/f$, the required focal length of the corrective lens is

$$f = \frac{pq}{p+q} = \frac{(25.0 \text{ cm})(-60.0 \text{ cm})}{25.0 \text{ cm} - 60.0 \text{ cm}} = \boxed{+42.9 \text{ cm}}$$

and the power (in diopters) of this lens will be

$$P = \frac{1}{f_{\text{in meters}}} = \frac{1}{+0.429 \text{ m}} = \boxed{+2.33 \text{ diopters}}$$

25.10 (a) The person is $\boxed{\text{farsighted}}$, able to see distant objects but unable to focus on objects at the normal near point for a human eye.

(b) With the corrective lens 2.00 cm in front of the eye, the object distance for an object 20.0 cm in front of the eye is $p = 20.0 \text{ cm} - 2.00 \text{ cm} = \boxed{18.0 \text{ cm}}$.

(c) The upright, virtual image formed by the corrective lens will serve as the object for the eye, and this object must be 40.0 cm in front of the eye. With the lens 2.00 cm in front of the eye, the magnitude of the image distance for the lens will be $|q| = 40.0 \text{ cm} - 2.00 \text{ cm} = \boxed{38.0 \text{ cm}}$.

(d) The image must be located in front of the corrective lens, so it is a $\boxed{\text{virtual image}}$, and the image distance is $\boxed{\text{negative}}$. Thus, $q = -38.0$ cm.

(e) From the thin-lens equation, $1/p + 1/q = 1/f$, the required focal length of the corrective lens is

$$f = \frac{pq}{p+q} = \frac{(18.0 \text{ cm})(-38.0 \text{ cm})}{18.0 \text{ cm} - 38.0 \text{ cm}} = \boxed{+34.2 \text{ cm}}$$

(f) The power of the corrective lens is then

$$P = \frac{1}{f_{\text{in meters}}} = \frac{1}{+0.342 \text{ m}} = \boxed{+2.92 \text{ diopters}}$$

continued on next page

(g) With a contact lens, the lens-to-eye distance would be zero, so we would have $\boxed{p = 20.0 \text{ cm}}$ and $\boxed{q = -40.0 \text{ cm}}$, giving a required focal length of

$$f = \frac{pq}{p+q} = \frac{(20.0 \text{ cm})(-40.0 \text{ cm})}{20.0 \text{ cm} - 40.0 \text{ cm}} = \boxed{+40.0 \text{ cm}}$$

and a power in diopters of

$$P = \frac{1}{f_{\text{in meters}}} = \frac{1}{+0.400 \text{ m}} = \boxed{+2.50 \text{ diopters}}$$

25.11 His lens must form an upright, virtual image of a very distant object ($p \approx \infty$) at his far point, 80.0 cm in front of the eye. Therefore, the focal length is $f = q = -80.0$ cm.

If this lens is to form a virtual image at his near point ($q = -18.0$ cm), the object distance must be

$$p = \frac{qf}{q-f} = \frac{(-18.0 \text{ cm})(-80.0 \text{ cm})}{-18.0 \text{ cm} - (-80.0 \text{ cm})} = \boxed{23.2 \text{ cm}}$$

25.12 (a) When the child clearly sees objects at her far point $(p_{\text{max}} = 125 \text{ cm})$, the lens-cornea combination has assumed a focal length suitable for forming the image on the retina $(q = 2.00 \text{ cm})$. The thin-lens equation gives the optical power under these conditions as

$$P_{\text{far}} = \frac{1}{f_{\text{in meters}}} = \frac{1}{p} + \frac{1}{q} = \frac{1}{1.25 \text{ m}} + \frac{1}{0.020\,0 \text{ m}} = +50.8 \text{ diopters}$$

When the eye is focused $(q = 2.00 \text{ cm})$ on objects at her near point $(p_{\text{min}} = 10.0 \text{ cm})$, the optical power of the lens-cornea combination is

$$P_{\text{near}} = \frac{1}{f_{\text{in meters}}} = \frac{1}{p} + \frac{1}{q} = \frac{1}{0.100 \text{ m}} + \frac{1}{0.020\,0 \text{ m}} = +60.0 \text{ diopters}$$

Therefore, the range of the power of the lens-cornea combination is

$$\boxed{+50.8 \text{ diopters} \le P \le +60.0 \text{ diopters}}$$

(b) If the child is to see very distant objects $(p \to \infty)$ clearly, her eyeglass lens must form an erect virtual image at the far point of her eye $(q = -125 \text{ cm})$. The optical power of the required lens is

$$P = \frac{1}{f_{\text{in meters}}} = \frac{1}{p} + \frac{1}{q} = 0 + \frac{1}{-1.25 \text{ m}} = \boxed{-0.800 \text{ diopters}}$$

Since the power, and hence the focal length, of this lens is negative, it is $\boxed{\text{diverging}}$.

25.13 (a) The lens should form an upright, virtual image at the far point $(q = -50.0 \text{ cm})$ for very distant objects $(p \approx \infty)$. Therefore, $f = q = -50.0$ cm, and the required power is

$$P = \frac{1}{f} = \frac{1}{-0.500 \text{ m}} = \boxed{-2.00 \text{ diopters}}$$

continued on next page

(b) If this lens is to form an upright, virtual image at the near point of the unaided eye $(q = -13.0 \text{ cm})$, the object distance should be

$$p = \frac{qf}{q-f} = \frac{(-13.0 \text{ cm})(-50.0 \text{ cm})}{-13.0 \text{ cm} - (-50.0 \text{ cm})} = \boxed{17.6 \text{ cm}}$$

25.14 (a) $\boxed{\text{Yes, a single lens can correct the patient's vision}}$. The patient needs corrective action in both the near vision (to allow clear viewing of objects between 45.0 cm and the normal near point of 25 cm) and the distant vision (to allow clear viewing of objects more than 85.0 cm away). A single lens solution is for the patient to wear a bifocal or progressive lens. Alternately, the patient must purchase two pairs of glasses, one for reading, and one for distant vision.

(b) To correct the near vision, the lens must form an upright, virtual image at the patient's near point ($q = -45.0$ cm) when a real object is at the normal near point ($p = +25.0$ cm). The thin-lens equation gives the needed focal length as

$$f = \frac{pq}{p+q} = \frac{(25.0 \text{ cm})(-45.0 \text{ cm})}{25.0 \text{ cm} - 45.0 \text{ cm}} = +56.3 \text{ cm}$$

so the required power in diopters is

$$P = \frac{1}{f_{\text{in meters}}} = \frac{1}{0.563 \text{ m}} = \boxed{+1.78 \text{ diopters}}$$

(c) To correct the distant vision, the lens must form an upright, virtual image at the patient's far point ($q = -85.0$ cm) for the most distant objects ($p \to \infty$). The thin-lens equation gives the needed focal length as $f = q = -85.0$ cm, so the needed power is

$$P = \frac{1}{f_{\text{in meters}}} = \frac{1}{-0.850 \text{ m}} = \boxed{-1.18 \text{ diopters}}$$

25.15 Considering the image formed by the cornea as a virtual object for the implanted lens, the object distance for this lens is $p = -5.33$ cm, and the image distance is $q = +2.80$ cm. The thin-lens equation then gives the focal length of the implanted lens as

$$f = \frac{pq}{p+q} = \frac{(-5.33 \text{ cm})(2.80 \text{ cm})}{-5.33 \text{ cm} + 2.80 \text{ cm}} = +5.90 \text{ cm}$$

so the power is $P = \dfrac{1}{f} = \dfrac{1}{+0.059\,0 \text{ m}} = \boxed{+17.0 \text{ diopters}}$.

25.16 (a) The upper portion of the lens should form an upright, virtual image of very distant objects $(p \approx \infty)$ at the far point of the eye $(q = -1.5 \text{ m})$. The thin-lens equation then gives $f = q = -1.5$ m, so the needed power is

$$P = \frac{1}{f} = \frac{1}{-1.5 \text{ m}} = \boxed{-0.67 \text{ diopters}}$$

continued on next page

(b) The lower part of the lens should form an upright, virtual image at the near point of the eye $(q = -30 \text{ cm})$ when the object distance is $p = 25$ cm. From the thin-lens equation,

$$f = \frac{pq}{p+q} = \frac{(25 \text{ cm})(-30 \text{ cm})}{25 \text{ cm} - 30 \text{ cm}} = +1.5 \times 10^2 \text{ cm} = +1.5 \text{ m}$$

Therefore, the power is $P = \dfrac{1}{f} = \dfrac{1}{+1.5 \text{ m}} = \boxed{+0.67 \text{ diopters}}$.

25.17 The corrective lens should form an upright, virtual image at the woman's far point $(q = -40.0 \text{ cm})$ for a very distant object $(p \to \infty)$. The thin-lens equation gives the required focal length as $f = q = -40.0 \text{ cm} = -0.400 \text{ m}$. Since $f < 0$, it is a $\boxed{\text{diverging lens}}$, and the required power is

$$P = \frac{1}{f_{\text{in meters}}} = \frac{1}{-0.400 \text{ m}} = \boxed{-2.50 \text{ diopters}}$$

25.18 (a) $f = \dfrac{1}{P} = \dfrac{1}{-4.00 \text{ diopters}} = -0.250 \text{ m} = \boxed{-25.0 \text{ cm}}$

(b) The corrective lens forms virtual images of very distant objects $(p \to \infty)$ at $q = f = -25.0$ cm. Thus, the person must be very $\boxed{\text{nearsighted}}$, unable to see objects clearly when they are more than $(25.0 + 2.00)$ cm $= 27.0$ cm from the eye.

(c) If contact lenses are to be worn, the far point of the eye will be 27.0 cm in front of the lens, so the needed focal length will be $f = q = -27.0$ cm, and the power is

$$P = \frac{1}{f_{\text{in meters}}} = \frac{1}{-0.270 \text{ m}} = \boxed{-3.70 \text{ diopters}}$$

25.19 (a) The simple magnifier (a converging lens) is to form an upright, virtual image located 25 cm in front of the lens $(q = -25 \text{ cm})$. The thin-lens equation then gives

$$p = \frac{qf}{q-f} = \frac{(-25 \text{ cm})(7.5 \text{ cm})}{-25 \text{ cm} - 7.5 \text{ cm}} = +5.8 \text{ cm}$$

so the stamp should be placed $\boxed{\text{5.8 cm in front of the lens}}$.

(b) When the image is at the near point of the eye, the angular magnification produced by the simple magnifier is

$$m = m_{\text{max}} = 1 + \frac{25 \text{ cm}}{f} = 1 + \frac{25 \text{ cm}}{7.5 \text{ cm}} = \boxed{4.3}$$

25.20 (a) A simple magnifier produces maximum magnification when it forms an upright, virtual image at the near point of the eye (25.0 cm for a normal eye). If the focal length of the magnifier is $f = +5.00$ cm, the required object distance for maximum magnification with a normal eye is

$$p = \frac{q\,f}{q-f} = \frac{(-25.0 \text{ cm})(+5.00 \text{ cm})}{-25.0 \text{ cm} - 5.00 \text{ cm}} = +4.17 \text{ cm}$$

or the object should be placed $\boxed{\text{4.17 cm in front of the lens}}$.

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(b) Under the conditions described in part (a), the angular magnification produced is

$$m = m_{\text{max}} = 1 + \frac{25.0 \text{ cm}}{f} = 1 + \frac{25.0 \text{ cm}}{5.00 \text{ cm}} = \boxed{6.00}$$

25.21 (a) From the thin-lens equation,

$$f = \frac{pq}{p+q} = \frac{(3.50 \text{ cm})(-25.0 \text{ cm})}{3.50 \text{ cm} - 25.0 \text{ cm}} = \boxed{+4.07 \text{ cm}}$$

(b) With the image at the normal near point, the angular magnification is

$$m = m_{\text{max}} = 1 + \frac{25.0 \text{ cm}}{f} = 1 + \frac{25.0 \text{ cm}}{4.07 \text{ cm}} = \boxed{+7.14}$$

25.22 (a) When the object is at the focal point of the magnifying lens, a virtual image is formed at infinity and parallel rays emerge from the lens. Under these conditions, the eye is most relaxed and the magnification produced is

$$m = \frac{25 \text{ cm}}{f} = \frac{25 \text{ cm}}{5.0 \text{ cm}} = \boxed{+5.0}$$

(b) When the object is positioned so the magnifier forms a virtual image at the near point of the eye ($q = -25$ cm), maximum magnification is produced and this is

$$m_{\text{max}} = 1 + \frac{25 \text{ cm}}{f} = 1 + \frac{25 \text{ cm}}{5.0 \text{ cm}} = \boxed{+6.0}$$

(c) From the thin-lens equation, the object distance needed to yield the maximum magnification computed in part (b) above is

$$p = \frac{qf}{q-f} = \frac{(-25 \text{ cm})(5.0 \text{ cm})}{-25 \text{ cm} - 5.0 \text{ cm}} = \boxed{+4.2 \text{ cm}}$$

25.23 (a) From the thin-lens equation, a real inverted image is formed at an image distance of

$$q = \frac{pf}{p-f} = \frac{(71.0 \text{ cm})(39.0 \text{ cm})}{71.0 \text{ cm} - 39.0 \text{ cm}} = +86.5 \text{ cm}$$

so the lateral magnification produced by the lens is

$$M = \frac{h'}{h} = -\frac{q}{p} = -\frac{86.5 \text{ cm}}{71.0 \text{ cm}} = -1.22$$

and the magnitude is $|M| = \boxed{1.22}$.

(b) If $|h|$ is the actual length of the leaf, the small-angle approximation gives the angular width of the leaf when viewed by the unaided eye from a distance of $d = 126 \text{ cm} + 71.0 \text{ cm} = 197 \text{ cm}$ as

$$\theta \approx \frac{|h|}{d} = \frac{|h|}{197 \text{ cm}}$$

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The length of the image formed by the lens is $|h'| = |M\,h| = 1.22|h|$, and its angular width when viewed from a distance of $d' = 126\text{ cm} - q = 39.5\text{ cm}$ is

$$\theta' \approx \frac{|h'|}{d'} = \frac{1.22|h|}{39.5\text{ cm}}$$

The angular magnification achieved by viewing the image instead of viewing the leaf directly is

$$\frac{\theta'}{\theta} \approx \frac{1.22|h|/39.5\text{ cm}}{|h|/197\text{ cm}} = \frac{1.22(197\text{ cm})}{39.5\text{ cm}} = \boxed{6.08}$$

25.24 (a) When a converging lens is used as a simple magnifier, maximum magnification is obtained when the upright, virtual image is formed at the near point of the eye ($|q| = 25.0$ cm for a normal eye). For the given lens, this maximum magnification is

$$m_{\text{max}} = 1 + \frac{25.0\text{ cm}}{f} = 1 + \frac{25.0\text{ cm}}{18.0\text{ cm}} = \boxed{2.39}$$

(b) When this simple magnifier is positioned for relaxed viewing (virtual image formed at infinity), the magnification produced is

$$m_{\text{relaxed}} = \frac{25.0\text{ cm}}{f} = \frac{25.0\text{ cm}}{18.0\text{ cm}} = \boxed{1.39}$$

25.25 The overall magnification is $m = M_1 m_e = M_1\left(25\text{ cm}/f_e\right)$, where M_1 is the lateral magnification produced by the objective lens. Therefore, the required focal length for the eyepiece is

$$f_e = \frac{M_1(25\text{ cm})}{m} = \frac{(-12)(25\text{ cm})}{-140} = \boxed{2.1\text{ cm}}$$

25.26 The approximate overall magnification of a compound microscope is given by $m = -(L/f_o)(25.0\text{ cm}/f_e)$, where L is the distance between the objective and eyepiece lenses, while f_o and f_e are the focal lengths of the objective and eyepiece lenses, respectively. Thus, the described microscope should have an approximate overall magnification of

$$m = -\frac{L}{f_o}\left(\frac{25.0\text{ cm}}{f_e}\right) = -\left(\frac{20.0\text{ cm}}{0.500\text{ cm}}\right)\left(\frac{25.0\text{ cm}}{1.70\text{ cm}}\right) = \boxed{-588}$$

25.27 The angular magnification produced by a telescope is $m = f_o/f_e$, where f_o is the focal length of the objective element, and f_e is that of the eyepiece lens. Also, the optical power of a lens is $P = 1/f_{\text{meters}}$, where f_{meters} is the focal length of that lens, expressed in meters. Thus, $f_o = 1/P_o$ and $f_e = 1/P_e$, so the angular magnification of this telescope is

$$m = \frac{f_o}{f_e} = \frac{1/P_o}{1/P_e} = \frac{P_e}{P_o} = \frac{35.0\text{ diopters}}{2.75\text{ diopters}} = \boxed{12.7}$$

25.28 It is specified that the final image the microscope forms of the blood cell is 29.0 cm in front of the eye and that the diameter of this image intercepts an angle of $\theta = 1.43$ mrad. The diameter of this final image must then be

$$h_e = r\theta = \left(29.0\times10^{-2}\text{ m}\right)\left(1.43\times10^{-3}\text{ rad}\right) = 4.15\times10^{-4}\text{ m}$$

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At this point, it is tempting to use Equation 25.7 from the textbook for the overall magnification of a compound microscope and compute $h = h_e/m$ as the size of the blood cell serving as the object for the microscope. However, the derivation of that equation is based on several assumptions, one of which is that the eye is relaxed and viewing a final image located an infinite distance in front of the eyepiece. This is clearly not true in this case, and the use of Equation 25.7 would introduce considerable error. Instead, we shall return to basics and use the thin-lens equation to find the size of the original object.

The image formed by the objective lens is the object for the eyepiece, and we label the size of this image as h'. The lateral magnification of the objective lens is $M_1 = h'/h = -q_1/p_1$ and that of the eyepiece is $M_e = h_e/h' = -q_e/p_e$. The overall magnification produced by the microscope is

$$M_{\text{total}} = \frac{h_e}{h} = \left(\frac{h'}{h}\right)\left(\frac{h_e}{h'}\right)$$

which gives the size of the original object as $h = h_e/|M_{\text{total}}|$.

From the thin-lens equation, the required object distance for the eyepiece is

$$p_e = \frac{q_e f_e}{q_e - f_e} = \frac{(-29.0\text{ cm})(0.950\text{ cm})}{-29.0\text{ cm} - 0.950\text{ cm}} = 0.920\text{ cm}$$

and the magnification produced by the eyepiece is

$$M_e = -\frac{q_e}{p_e} = -\frac{(-29.0\text{ cm})}{0.920\text{ cm}} = +31.5$$

The image distance for the objective lens is then

$$q_1 = L - p_e = 29.0\text{ cm} - 0.920\text{ cm} = 28.1\text{ cm}$$

and the object distance for this lens is

$$p_1 = \frac{q_1 f_o}{q_1 - f_o} = \frac{(28.1\text{ cm})(1.622\text{ cm})}{28.1\text{ cm} - 1.622\text{ cm}} = 1.72\text{ cm}$$

The magnification by the objective lens is

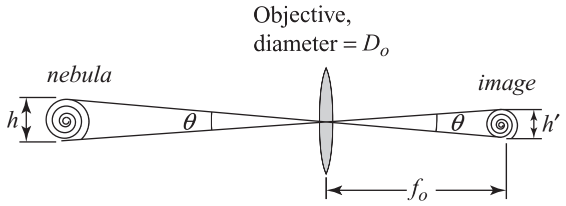
$$M_1 = -\frac{q_1}{p_1} = -\frac{(28.1\text{ cm})}{1.72\text{ cm}} = -16.3$$

and the overall lateral magnification is $M_{\text{total}} = M_1 M_e = (-16.3)(+31.5) = -513$.

The actual diameter of the red blood cell serving as the original object is found to be

$$h = \frac{h_e}{|M_{\text{total}}|} = \frac{4.15\times10^{-4}\text{ m}}{513} = 8.09\times10^{-7}\text{ m} = \boxed{0.809\ \mu\text{m}}$$

25.29 The sketch at the right shows an image of the nebula formed by an objective lens of diameter D_o and focal length f_o. The diameter of the image of the nebula formed on the film is given by



$$h' = f_o \cdot \theta$$

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where θ is the angular width of the nebula and its image as shown. If the intensity of the light arriving at the objective lens from the nebula is I, the energy entering the lens during an exposure time Δt is

$$E = IA_{\text{objective}}(\Delta t) = I\left(\frac{\pi D_o^2}{4}\right)\Delta t$$

The energy per unit area deposited on the film during the exposure time is then

$$\frac{E}{A_{\text{image}}} = \frac{E}{\pi(h')^2/4} = \frac{4E}{\pi\left(f_o\cdot\theta\right)^2} = \frac{4}{\pi\theta^2}\left(\frac{\pi}{4}\frac{ID_o^2\Delta t}{f_o^2}\right) = \left(\frac{I}{\theta^2}\right)\left(\frac{D_o}{f_o}\right)^2\Delta t$$

Telescope 1 has an objective diameter $D_{O,1} = 200$ mm, focal length $f_{O,1} = 2\,000$ mm, and uses an exposure time $(\Delta t)_1 = 1.50$ min. If telescope 2, with $D_{O,2} = 60.0$ mm, and $f_{O,2} = 900$ mm, is to deposit the same energy per unit area on the film as does telescope 1, it is necessary that

$$\frac{E}{A_{\text{image},2}} = \left(\frac{I}{\theta^2}\right)\left(\frac{D_{O,2}}{f_{O,2}}\right)^2(\Delta t)_2 = \left(\frac{I}{\theta^2}\right)\left(\frac{D_{O,1}}{f_{O,1}}\right)^2(\Delta t)_1 = \frac{E}{A_{\text{image},1}}$$

The required exposure time for the second telescope is therefore

$$(\Delta t)_2 = \frac{\cancel{(I/\theta^2)}(D_{O,1}/f_{O,1})^2}{\cancel{(I/\theta^2)}(D_{O,2}/f_{O,2})^2}(\Delta t)_1 = \left(\frac{D_{O,1}}{f_{O,1}}\right)^2\left(\frac{f_{O,2}}{D_{O,2}}\right)^2(\Delta t)_1$$

$$= \left(\frac{200\text{ mm}}{2\,000\text{ mm}}\right)^2\left(\frac{900\text{ mm}}{60.0\text{ mm}}\right)^2(1.50\text{ min}) = \boxed{3.38\text{ min}}$$

25.30 (a) For a refracting telescope, the overall length is $L = f_o + f_e$, and the magnification produced is $m = f_o/f_e$, where f_o and f_e are the focal lengths of the objective element and the eyepiece, respectively. Thus, we may write $f_e = f_o/m$ to obtain

$$L = f_o + \frac{f_o}{m} = f_o\left(1+\frac{1}{m}\right) = \boxed{f_o\left(\frac{m+1}{m}\right)}$$

(b) Using the result of part (a), the required change in the length of the telescope will be

$$\Delta L = f_o\left(\frac{m'+1}{m'} - \frac{m+1}{m}\right) = (2.00\text{ m})\left(\frac{101}{100} - \frac{51.0}{50.0}\right) = -2.00\times10^{-2}\text{ m} = -2.00\text{ cm}$$

or the telescope must be shortened by moving the eyepiece $\boxed{\text{2.00 cm forward}}$ toward the objective lens.

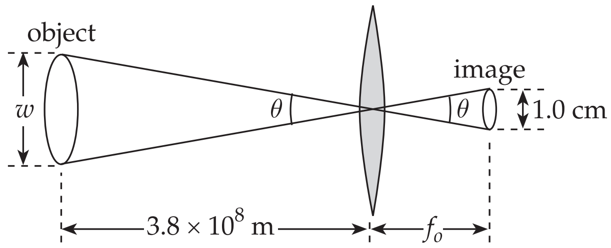
25.31 (a) The magnification of a telescope is $m = f_o/f_e$, where f_o and f_e are the focal lengths of the objective and eyepiece lenses, respectively. Thus, if $m = 34.0$, while $f_o = 86.0$ cm, the focal length of the eyepiece must be

$$f_e = \frac{f_o}{m} = \frac{86.0\text{ cm}}{34.0} = \boxed{2.53\text{ cm}}$$

(b) When the telescope is adjusted for relaxed-eye viewing, the distance between the objective and eyepiece lenses is

$$L = f_o + f_e = 86.0\text{ cm} + 2.53\text{ cm} = \boxed{88.5\text{ cm}}$$

25.32 The Moon may be considered an infinitely distant object $(p \to \infty)$ when viewed with this lens, so the image distance will be $q = f_o = 1\,500$ cm.



Considering the rays that pass undeviated through the center of this lens as shown in the sketch, observe that the angular widths of the image and the object are equal. Thus, if w is the linear width of an object forming a 1.00-cm-wide image, then

$$\theta = \frac{w}{3.8 \times 10^8 \text{ m}} = \frac{1.0 \text{ cm}}{f_o} = \frac{1.0 \text{ cm}}{1\,500 \text{ cm}}$$

or $$w = \left(3.8 \times 10^8 \text{ m}\right)\left(\frac{1.0 \text{ cm}}{1\,500 \text{ cm}}\right)\left(\frac{1 \text{ mi}}{1\,609 \text{ m}}\right) = \boxed{1.6 \times 10^2 \text{ mi}}$$

25.33 (a) From the thin-lens equation, $q = pf/(p-f)$, so the lateral magnification by the objective lens is $M = h'/h = -q/p = -f/(p-f)$. Therefore, the image size will be

$$h' = M\,h = -\frac{f\,h}{p-f} = \boxed{\frac{f\,h}{f-p}}$$

(b) If $p >> f$, then $f - p \approx -p$ and $h' \approx \boxed{-\frac{f\,h}{p}}$.

(c) Suppose the telescope observes the space station at the zenith. Then,

$$h' \approx -\frac{f\,h}{p} = -\frac{(4.00 \text{ m})(108.6 \text{ m})}{407 \times 10^3 \text{ m}} = -1.07 \times 10^{-3} \text{ m} = \boxed{-1.07 \text{ mm}}$$

25.34 Use the larger focal length (lowest power) lens as the objective element and the shorter focal length (largest power) lens for the eyepiece. The focal lengths are

$$f_o = \frac{1}{+1.20 \text{ diopters}} = +0.833 \text{ m, and } f_e = \frac{1}{+9.00 \text{ diopters}} = +0.111 \text{ m}$$

(a) The angular magnification (or magnifying power) of the telescope is then

$$m = \frac{f_o}{f_e} = \frac{+0.833 \text{ m}}{+0.111 \text{ m}} = \boxed{7.50}$$

(b) The length of the telescope is

$$L = f_o + f_e = 0.833 \text{ m} + 0.111 \text{ m} = \boxed{0.944 \text{ m}}$$

25.35 The lens for the left eye forms an upright, virtual image at $q_L = -50.0$ cm when the object distance is $p_L = 25.0$ cm, so the thin-lens equation gives its focal length as

$$f_L = \frac{p_L q_L}{p_L + q_L} = \frac{(25.0 \text{ cm})(-50.0 \text{ cm})}{25.0 \text{ cm} - 50.0 \text{ cm}} = 50.0 \text{ cm}$$

Similarly for the other lens, $q_R = -100$ cm when $p_R = 25.0$ cm, and $f_R = 33.3$ cm.

continued on next page

(a) Using the lens for the left eye as the objective,

$$m = \frac{f_o}{f_e} = \frac{f_L}{f_R} = \frac{50.0 \text{ cm}}{33.3 \text{ cm}} = \boxed{1.50}$$

(b) Using the lens for the right eye as the eyepiece and, for maximum magnification, requiring that the final image be formed at the normal near point $(q_e = -25.0 \text{ cm})$ gives the object distance for the eyepiece as

$$p_e = \frac{q_e f_e}{q_e - f_e} = \frac{(-25.0 \text{ cm})(33.3 \text{ cm})}{-25.0 \text{ cm} - 33.3 \text{ cm}} = +14.3 \text{ cm}$$

The maximum magnification by the eyepiece is then

$$m_e = 1 + \frac{25.0 \text{ cm}}{f_e} = 1 + \frac{25.0 \text{ cm}}{33.3 \text{ cm}} = +1.75$$

and the image distance for the objective is

$$q_1 = L - p_e = 10.0 \text{ cm} - 14.3 \text{ cm} = -4.3 \text{ cm}$$

The thin-lens equation then gives the object distance for the objective as

$$p_1 = \frac{q_1 f_1}{q_1 - f_1} = \frac{(-4.3 \text{ cm})(50.0 \text{ cm})}{-4.3 \text{ cm} - 50.0 \text{ cm}} = +4.0 \text{ cm}$$

The magnification by the objective is then

$$M_1 = -\frac{q_1}{p_1} = -\frac{(-4.3 \text{ cm})}{4.0 \text{ cm}} = +1.1$$

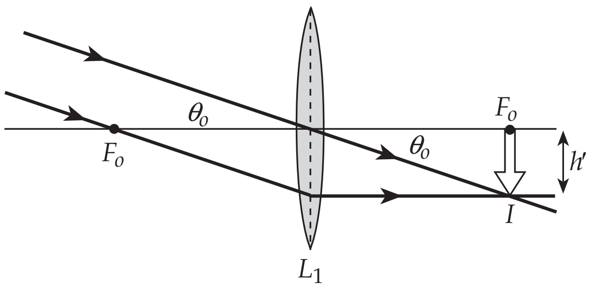
and the overall magnification is $m = M_1 m_e = (+1.1)(+1.75) = \boxed{1.9}$.

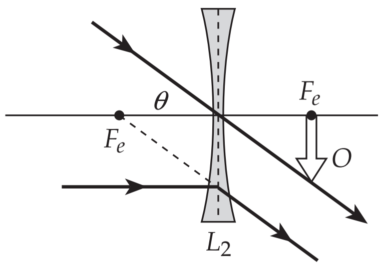
25.36 **Note**: We solve part (b) before answering part (a) in this problem.

(b) The objective forms a real, inverted image, diminished in size, of a very distant object at $q_1 = f_o$. This image is a virtual object for the eyepiece at $p_e = -|f_e|$, giving

$$\frac{1}{q_e} = \frac{1}{p_e} - \frac{1}{f_e} = \frac{1}{-|f_e|} + \frac{1}{|f_e|} = 0$$

and $\boxed{q_e \to \infty}$





(a) Parallel rays emerge from the eyepiece, so the eye observes a $\boxed{\text{virtual image}}$.

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(c) The angular magnification is $m = \dfrac{f_o}{|f_e|} = 3.00$, giving $f_o = 3.00|f_e|$. Also, the length of the telescope is $L = f_o + f_e = 3.00|f_e| - |f_e| = 10.0$ cm, giving

$$f_e = -|f_e| = -\frac{10.0 \text{ cm}}{2.00} = \boxed{-5.00 \text{ cm}} \text{ and } f_o = 3.00|f_e| = \boxed{15.0 \text{ cm}}$$

25.37 The angular separation of the lights is $\theta = d/h$, where $d = 1.00$ m is their linear separation, and h is the altitude of the satellite. If the lights are just resolved according to the Rayleigh criterion, then $\theta = \theta_{\min} = 1.22(\lambda/D)$, where λ is the wavelength of the light, and D is the diameter of the lens. Thus, the altitude of the satellite must be

$$h = \frac{d}{\theta} = \frac{d}{1.22(\lambda/D)} = \frac{d \cdot D}{1.22\lambda} = \frac{(1.00 \text{ m})(0.300 \text{ m})}{1.22\left(500 \times 10^{-9} \text{ m}\right)} = 4.92 \times 10^5 \text{ m} = \boxed{492 \text{ km}}$$

25.38 The angular separation of two objects seen on the ground is $\theta = d/h$, where d is their linear separation and h is your altitude. If the objects are just resolved according to the Rayleigh criterion, then $\theta = \theta_{\min} = 1.22(\lambda/D)$, where λ is the wavelength of the light, and D is the diameter of the aperture (your pupil). Thus, the minimum separation of objects you can distinguish is

$$d = h \cdot \theta_{\min} = \frac{1.22 \cdot h \cdot \lambda}{D} = \frac{1.22\left(9.50 \times 10^3 \text{ m}\right)\left(575 \times 10^{-9} \text{ m}\right)}{4.0 \times 10^{-3} \text{ m}} = \boxed{1.7 \text{ m}}$$

25.39 The limit of resolution in air is $\theta_{\min}\big|_{\text{air}} = 1.22\dfrac{\lambda}{D} = 0.60\ \mu\text{rad}$.

In oil, the limiting angle of resolution will be

$$\theta_{\min}\big|_{\text{oil}} = 1.22\frac{\lambda_{\text{oil}}}{D} = 1.22\frac{\left(\lambda/n_{\text{oil}}\right)}{D} = \left(1.22\frac{\lambda}{D}\right)\frac{1}{n_{\text{oil}}}$$

or $$\theta_{\min}\big|_{\text{oil}} = \frac{\theta_{\min}\big|_{\text{air}}}{n_{\text{oil}}} = \frac{0.60\ \mu\text{rad}}{1.5} = \boxed{0.40\ \mu\text{rad}}$$

25.40 (a) The wavelength of the light within the eye is $\lambda_n = \lambda/n$. Thus, the limiting angle of resolution for light passing through the pupil (a circular aperture with diameter $D = 2.00$ mm) is

$$\theta_{\min} = 1.22\frac{\lambda_n}{D} = 1.22\frac{\lambda}{nD} = 1.22\frac{\left(500 \times 10^{-9} \text{ m}\right)}{(1.33)\left(2.00 \times 10^{-3} \text{ m}\right)} = \boxed{2.29 \times 10^{-4} \text{ rad}}$$

(b) From $s = r\theta$, the distance from the eye that two points separated by a distance $s = 1.00$ cm will intercept this minimum angle of resolution is

$$r = \frac{s}{\theta_{\min}} = \frac{1.00 \text{ cm}}{2.29 \times 10^{-4} \text{ rad}} = 4.37 \times 10^3 \text{ cm} = \boxed{43.7 \text{ m}}$$

25.41 The angular separation of the headlights when viewed from a distance of $r = 10.0$ km is

$$\theta = \frac{s}{r} = \frac{2.00 \text{ m}}{10.0 \times 10^3 \text{ m}} = 2.00 \times 10^{-4} \text{ rad}$$

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If the headlights are to be just resolved, this separation must equal the limiting angle of resolution for the circular aperture, $\theta_{min} = 1.22\lambda/D$, so the diameter of the aperture is

$$D = \frac{1.22\lambda}{\theta_{min}} = \frac{1.22\lambda}{\theta} = \frac{1.22\left(885\times10^{-9}\text{ m}\right)}{2.00\times10^{-4}\text{ rad}} = 5.40\times10^{-3}\text{ m} = \boxed{5.40\text{ mm}}$$

25.42 The angular separation of the two stars is $\theta = d/r$, where d is their linear separation and r is their distance from Earth. If the stars are just resolved according to the Rayleigh criterion, then $\theta = \theta_{min} = 1.22(\lambda/D)$, where λ is the wavelength of the light, and D is the diameter of the aperture (telescope objective). Thus, the linear separation of the stars must be

$$d = r\cdot\theta_{min} = \frac{r\cdot1.22\lambda}{D} = \frac{\left[(23\text{ ly})\left(9.461\times10^{15}\text{ m/ly}\right)\right]1.22\left(575\times10^{-9}\text{ m}\right)}{0.68\text{ m}} = \boxed{2.2\times10^{11}\text{ m}}$$

25.43 If just resolved, the angular separation of the objects is $\theta = \theta_{min} = 1.22\dfrac{\lambda}{D}$

and $s = r\theta = \left(8.0\times10^{7}\text{ km}\right)\left[1.22\left(\dfrac{500\times10^{-9}\text{ m}}{5.00\text{ m}}\right)\right] = \boxed{9.8\text{ km}}$

25.44 If just resolved, the angular separation of the objects is $\theta = \theta_{min} = 1.22\dfrac{\lambda}{D}$

and $s = r\theta = \left(200\times10^{3}\text{ m}\right)\left[1.22\left(\dfrac{550\times10^{-9}\text{ m}}{0.35\text{ m}}\right)\right] = 0.38\text{ m} = \boxed{38\text{ cm}}$

25.45 The grating spacing is $d = 1\text{ cm}/6\,000 = 1.67\times10^{-4}\text{ cm} = 1.67\times10^{-6}\text{ m}$, and the highest order of 600 nm light that can be observed is

$$m_{max} = \frac{d\sin 90^\circ}{\lambda} = \frac{\left(1.67\times10^{-6}\text{ m}\right)(1)}{600\times10^{-9}\text{ m}} = 2.78 \;\rightarrow\; 2\text{ orders}$$

The total number of slits is $N = (15.0\text{ cm})(6\,000\text{ slits/cm}) = 9.00\times10^{4}$, and the resolving power of the grating in the second order is

$$R_{available} = Nm = \left(9.00\times10^{4}\right)2 = \boxed{1.80\times10^{5}}$$

The resolving power required to separate the given spectral lines is

$$R_{needed} = \frac{\lambda}{\Delta\lambda} = \frac{600.000\text{ nm}}{0.003\text{ nm}} = \boxed{2.0\times10^{5}}$$

These lines $\boxed{\text{cannot be separated}}$ with this grating.

25.46 The resolving power of a diffraction grating is $R = \lambda/\Delta\lambda = Nm$.

(a) The number of lines the grating must have to resolve the H_α line in the first order is

$$N = \frac{R}{m} = \frac{\lambda/\Delta\lambda}{(1)} = \frac{656.2\text{ nm}}{0.18\text{ nm}} = \boxed{3.6\times10^{3}\text{ lines}}$$

(b) In the second order $(m = 2)$, $N = \dfrac{R}{2} = \dfrac{656.2\text{ nm}}{2(0.18\text{ nm})} = \boxed{1.8\times10^{3}\text{ lines}}$.

25.47 A fringe shift occurs when the mirror moves distance $\lambda/4$. Thus, if the mirror moves distance $\Delta L = 0.180$ mm, the number of fringe shifts observed is

$$N_{\text{shifts}} = \frac{\Delta L}{\lambda/4} = \frac{4(\Delta L)}{\lambda} = \frac{4\left(0.180\times10^{-3}\text{ m}\right)}{550\times10^{-9}\text{ m}} = \boxed{1.31\times10^{3}}$$

25.48 (a) When the central spot in the interferometer pattern goes through a full cycle from bright to dark and back to bright, two fringe shifts have occurred, and the movable mirror has moved a distance of $2(\lambda/4) = \lambda/2$. Thus, if $N_{\text{cycles}} = 1\,700$ such cycles are observed as the mirror moves distance $d = 0.382$ mm, it must be true that $d = N_{\text{cycles}}\left(\lambda/2\right)$, or $\lambda = 2d/N_{\text{cycles}}$. The wavelength of the light illuminating the interferometer is therefore

$$\lambda = \frac{2\left(0.382\times10^{-3}\text{ m}\right)}{1\,700} = 4.49\times10^{-7}\text{ m} = \boxed{449\text{ nm}}$$

which is in the $\boxed{\text{blue}}$ region of the visible spectrum.

(b) Red light has a longer wavelength than blue light, so fewer wavelengths would cover the given displacement. Hence, $\boxed{N_{\text{cycles}}\text{ would be smaller}}$.

25.49 A fringe shift occurs when the mirror moves distance $\lambda/4$. Thus, the distance moved (length of the bacterium) as 310 shifts occur is

$$\Delta L = N_{\text{shifts}}\left(\frac{\lambda}{4}\right) = 310\left(\frac{650\times10^{-9}\text{ m}}{4}\right) = 5.04\times10^{-5}\text{ m} = \boxed{50.4\ \mu\text{m}}$$

25.50 A fringe shift occurs when the mirror moves distance $\lambda/4$. Thus, the distance the mirror moves as 250 fringe shifts are counted is

$$\Delta L = N_{\text{shifts}}\left(\frac{\lambda}{4}\right) = 250\left(\frac{632.8\times10^{-9}\text{ m}}{4}\right) = 3.96\times10^{-5}\text{ m} = \boxed{39.6\ \mu\text{m}}$$

25.51 When the optical path in one arm of a Michelson's interferometer increases by one wavelength, four fringe shifts will occur (one shift for every quarter-wavelength change in path length).

The number of wavelengths (in a vacuum) that fit in a distance equal to a thickness t is $N_{\text{vac}} = t/\lambda$. The number of wavelengths that fit in this thickness while traveling through the transparent material is $N_n = t/\lambda_n = t/\left(\lambda/n\right) = nt/\lambda$. Thus, the change in the number of wavelengths that fit in the path down this arm of the interferometer is

$$\Delta N = N_n - N_{\text{vac}} = (n-1)\frac{t}{\lambda}$$

and the number of fringe shifts that will occur as the thin sheet is inserted will be

$$\#\ \text{fringe shifts} = 4(\Delta N) = 4(n-1)\frac{t}{\lambda} = 4(1.40-1)\left(\frac{15.0\times10^{-6}\text{ m}}{600\times10^{-9}\text{ m}}\right) = \boxed{40}$$

25.52 The wavelength of light within the tube decreases from λ to $\lambda_n = \lambda/n_{gas}$ as the tube fills with gas. Thus, the number of wavelengths that will fit in the length L of the tube increases from L/λ to $n_{gas}L/\lambda$. Four fringe shifts occur for each additional wavelength that fits within the tube, so the number of fringes shifts to be seen as the tube fills with gas is

$$N_{\text{shifts}} = 4\left[\frac{n_{gas}L}{\lambda} - \frac{L}{\lambda}\right] = \frac{4L}{\lambda}\left(n_{gas} - 1\right)$$

Hence, $$n_{gas} = 1 + \left(\frac{\lambda}{4L}\right)N_{\text{shifts}} = 1 + \left[\frac{600\times10^{-9}\text{ m}}{4\left(5.00\times10^{-2}\text{ m}\right)}\right](160) = \boxed{1.000\ 5}$$

25.53 (a) For a refracting telescope, the magnification is $m = f_o/f_e$, where f_o and f_e are the focal lengths of the objective lens and the eyepiece, respectively. Thus, when the Yerkes telescope uses an eyepiece with $f_e = 2.50$ cm, the magnification is

$$m = \frac{f_o}{f_e} = \frac{20.0\text{ m}}{2.50\times10^{-2}\text{ m}} = 8.00\times10^{2} = \boxed{800}$$

(b) Standard astronomical telescopes form inverted images. Thus, the observer Martian polar caps are $\boxed{\text{upside down}}$.

25.54 When viewed from a distance of 50 meters, the angular length of a mouse (assumed to have an actual length of ≈ 10 cm) is

$$\theta = \frac{s}{r} = \frac{0.10\text{ m}}{50\text{ m}} = 2.0\times10^{-3}\text{ radians}$$

Thus, the limiting angle of resolution of the eye of the hawk must be

$$\theta_{\min} \approx \boxed{2.0\times10^{-3}\text{ radians}}$$

25.55 With 485 lines equally spaced in a height ℓ, the distance separating adjacent lines is $d = \ell/485$. When the screen is viewed from a distance L, the angular separation between adjacent lines is $\theta = d/L$. If the individual lines are not to be seen (i.e., the lines are to be unresolved), this angular separation must be less than the minimum angle of resolution, $\theta_{\min} = 1.22(\lambda/D)$ by the Rayleigh criterion. That is, we must have

$$\theta = \frac{d}{L} = \frac{\ell/485}{L} < \theta_{\min} = \frac{1.22\cdot\lambda}{D}$$

or $$\frac{L}{\ell} > \frac{D}{485(1.22\cdot\lambda)} = \frac{5.00\times10^{-3}\text{ m}}{485(1.22)\left(550\times10^{-9}\text{ m}\right)} = \boxed{15.4}$$

25.56 (a) Since this eye can already focus on objects located at the near point of a normal eye (25 cm), no correction is needed for near objects. To correct the distant vision, a corrective lens (located 2.0 cm from the eye) should form virtual images of very distant objects at 23 cm in front of the lens (or at the far point of the eye). Thus, we must require that $q = -23$ cm when $p \to \infty$. This gives

$$P = \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = 0 + \frac{1}{-0.23\text{ m}} = \boxed{-4.3\text{ diopters}}$$

continued on next page

(b) A corrective lens in contact with the cornea should form virtual images of very distant objects at the far point of the eye. Therefore, we require that $q = -25$ cm when $p \to \infty$, giving

$$P = \frac{1}{f} = \frac{1}{p} + \frac{1}{q} = 0 + \frac{1}{-0.25\text{ m}} = \boxed{-4.0\text{ diopters}}$$

When the contact lens $\left(f = \frac{1}{P} = -25\text{ cm} \right)$ is in place, the object distance which yields a virtual image at the near point of the eye (that is, $q = -16$ cm) is given by

$$p = \frac{qf}{q-f} = \frac{(-16\text{ cm})(-25\text{ cm})}{-16\text{ cm} - (-25\text{ cm})} = \boxed{44\text{ cm}}$$

25.57 (a) The lens should form an upright, virtual image at the near point of the eye, $q = -75.0$ cm, when the object distance is $p = 25.0$ cm. The thin-lens equation then gives

$$f = \frac{pq}{p+q} = \frac{(25.0\text{ cm})(-75.0\text{ cm})}{25.0\text{ cm} - 75.0\text{ cm}} = 37.5\text{ cm} = 0.375\text{ m}$$

so the needed power is $P = \frac{1}{f} = \frac{1}{0.375\text{ m}} = \boxed{+2.67\text{ diopters}}$.

(b) If the object distance must be $p = 26.0$ cm to position the image at $q = -75.0$ cm, the actual focal length is

$$f = \frac{pq}{p+q} = \frac{(26.0\text{ cm})(-75.0\text{ cm})}{26.0\text{ cm} - 75.0\text{ cm}} = 0.398\text{ m}$$

and $P = \frac{1}{f} = \frac{1}{0.398\text{ m}} = +2.51$ diopters

The error in the power is

$$\Delta P = (2.67 - 2.51)\text{ diopters} = \boxed{0.16\text{ diopters too low}}$$

25.58 (a) The image must be formed on the back of the eye (retina), so we must have $q = 2.00$ cm when $p = 1.00\text{ m} = 100$ cm. The thin-lens equation gives the required focal length as

$$f = \frac{pq}{p+q} = \frac{(100\text{ cm})(2.00\text{ cm})}{100\text{ cm} + 2.00\text{ cm}} = \boxed{1.96\text{ cm}}$$

(b) The *f*-number of a lens aperture is the focal length of the lens divided by the diameter of the aperture. Thus, the smallest *f*-number occurs with the largest diameter of the aperture. For the typical eyeball focused on objects 1.00 m away, this is

$$(f\text{-number})_{\min} = \frac{f}{D_{\max}} = \frac{1.96\text{ cm}}{0.600\text{ cm}} = \boxed{3.27}$$

(c) The largest *f*-number of the typical eyeball focused on a 1.00-m-distance object is

$$(f\text{-number})_{\max} = \frac{f}{D_{\min}} = \frac{1.96\text{ cm}}{0.200\text{ cm}} = \boxed{9.80}$$

25.59 (a) The implanted lens should give an image distance of $q = 22.4$ mm for distant $(p \to \infty)$ objects. The thin-lens equation then gives the focal length as $f = q = 22.4$ mm, so the power of the implanted lens should be

$$P_{\text{implant}} = \frac{1}{f} = \frac{1}{22.4 \times 10^{-3} \text{ m}} = \boxed{+44.6 \text{ diopters}}$$

(b) When the object distance is $p = 33.0$ cm, the corrective lens should produce parallel rays $(q \to \infty)$. Then the implanted lens will focus the final image on the retina. From the thin-lens equation, the required focal length is $f = p = 33.0$ cm, and the power of this lens should be

$$P_{\text{corrective}} = \frac{1}{f} = \frac{1}{0.330 \text{ m}} = \boxed{+3.03 \text{ diopters}}$$

25.60 We use $\dfrac{n_1}{p} + \dfrac{n_2}{q} = \dfrac{n_2 - n_1}{R}$, with $p \to \infty$ and q equal to the cornea to retina distance.

Then, $R = q\left(\dfrac{n_2 - n_1}{n_2}\right) = (2.00 \text{ cm})\left(\dfrac{1.34 - 1.00}{1.34}\right) = 0.507 \text{ cm} = \boxed{5.07 \text{ mm}}$.

25.61 When a converging lens forms a real image of a very distant object, the image distance equals the focal length of the lens. Thus, if the scout started a fire by focusing sunlight on kindling 5.00 cm from the lens, $f = q = 5.00$ cm.

(a) When the lens is used as a simple magnifier, maximum magnification is produced when the upright, virtual image is formed at the near point of the eye ($q = -15$ cm in this case). The object distance required to form an image at this location is

$$p = \frac{qf}{q - f} = \frac{(-15 \text{ cm})(5.0 \text{ cm})}{-15 \text{ cm} - 5.0 \text{ cm}} = \frac{15 \text{ cm}}{4.0}$$

and the lateral magnification produced is $M = -\dfrac{q}{p} = -\dfrac{-15 \text{ cm}}{15 \text{ cm}/4.0} = \boxed{+4.0}$. Note that adapting Equation 25.5 for use with this "abnormal" eye would give an angular magnification of $m_{\max} = 1 + |q|/f = 1 + 15 \text{ cm}/5.0 \text{ cm} = \boxed{+4.0}$.

(b) When the object is viewed directly while positioned at the near point of the eye, its angular size is $\theta_0 = h/15 \text{ cm}$. When the object is viewed by the relaxed eye while using the lens as a simple magnifier (with the object at the focal point so parallel rays enter the eye), the angular size of the upright, virtual image is $\theta = h/f$. Thus, the angular magnification gained by using the lens in this manner is

$$m = \frac{\theta}{\theta_0} = \frac{h/f}{h/15 \text{ cm}} = \frac{15 \text{ cm}}{f} = \frac{15 \text{ cm}}{5.0 \text{ cm}} = \boxed{+3.0}$$

25.62 The angular magnification is $m = \theta/\theta_0$, where θ is the angle subtended by the final image, and θ_0 is the angle subtended by the object as shown in the figure below. When the telescope is adjusted for minimum eyestrain, the rays entering the eye are parallel. Thus, the objective lens must form its image at the focal point of the eyepiece.

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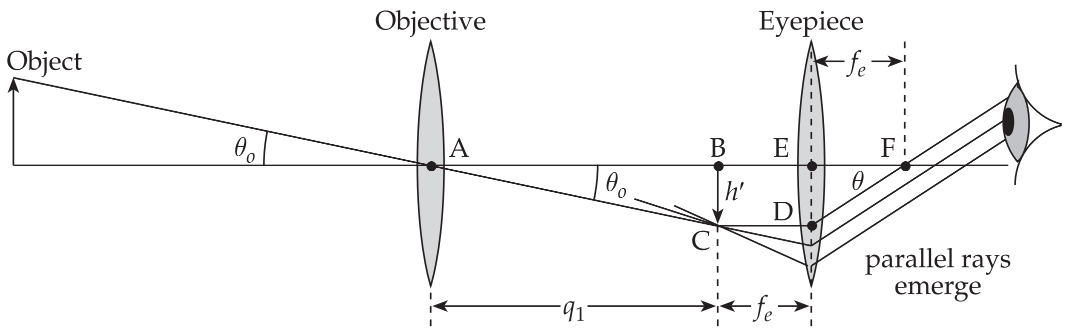

From triangle ABC, $\theta_0 \approx \tan\theta_0 = h'/q_1$, and from triangle DEF, $\theta \approx \tan\theta = h'/f_e$. The angular magnification is then $m = \dfrac{\theta}{\theta_0} = \dfrac{h'/f_e}{h'/q_1} = \dfrac{q_1}{f_e}$.

From the thin-lens equation, the image distance of the objective lens in this case is

$$q_1 = \frac{p_1 f_1}{p_1 - f_1} = \frac{(300 \text{ cm})(20.0 \text{ cm})}{300 \text{ cm} - 20.0 \text{ cm}} = 21.4 \text{ cm}$$

With an eyepiece of focal length $f_e = 2.00$ cm, the angular magnification for this telescope is

$$m = \frac{q_1}{f_e} = \frac{21.4 \text{ cm}}{2.00 \text{ cm}} = \boxed{10.7}$$

26

Relativity

QUICK QUIZZES

1. False. One of Einstein's two basic postulates in his special theory of relativity is the constancy of the speed of light. This postulate states that the speed of light in a vacuum has the same value in all inertial reference frames, regardless of the velocity of the observer or the velocity of the source emitting the light.

2. Choice (a). Less time will have passed for you in your frame of reference than for your employer back on Earth. Thus, to maximize your paycheck, you should choose to have your pay calculated according to the elapsed time on a clock on Earth.

3. False. Clocks, including biological clocks, in the observer's own reference frame appear to run at normal speeds. It is only clocks in reference frames moving relative to the observer that the observer will judge to be running slower than normal.

4. No. From your perspective you are at rest with respect to the cabin, so you will measure yourself as having your normal length and will require a normal-sized cabin.

5. **(i)** Choices (a) and (e). The outgoing rocket will appear to have a *shorter* length and a *slower* clock. **(ii)** Choices (a) and (e). The answers for the incoming rocket are the same as for the outgoing rocket. Length contraction and time dilation depend only on the magnitude of the relative velocity, not on the direction.

6. False. According to the mass-energy equivalence equation, the particle's total energy is $E = mc^2\big/\sqrt{1-v^2/c^2}$, and it is seen that as $v \to c$, the total energy becomes infinitely large. As the total energy becomes infinitely large, so will the kinetic energy given by $KE = E - mc^2$ and the relativistic momentum given by $p = \sqrt{E^2 - (mc^2)^2}\big/c$.

7. (a) False (b) False (c) True (d) False

A reflected photon does exert a force on the surface. Although a photon has zero mass, a photon does carry momentum. When it reflects from a surface, there is a change in the momentum, just like the change in momentum of a ball bouncing off a wall. According to the momentum interpretation of Newton's second law, a change in momentum results in a force on the surface. This concept is used in theoretical studies of space sailing. These studies propose building non-powered spacecraft with huge reflective sails oriented perpendicularly to the rays from the Sun. The large number of photons from the Sun reflecting from the surface of the sail will exert a force which, although small, will provide a continuous acceleration. This would allow the spacecraft to travel to other planets without fuel.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. Einstein based his special theory of relativity on the two postulates included in choices (d) and (e). The textbook refers to the postulate summarized in choice (d) as the principle of relativity and to the postulate in choice (e) as the constancy of the speed of light.

2. The length of the ship is parallel to the relative velocity between the ship and the observer. Thus, the observer at rest on Earth will measure the moving length of the ship to be shorter than its length when it was at rest. Choice (b) is the correct answer.

3. According to the second postulate of special relativity (the constancy of the speed of light), both observers will measure the light speed to be c. Therefore, both choices (b) and (c) are true statements, and the other choices are false.

4. The astronaut is moving with constant velocity and is therefore in an inertial reference frame. According to the principle of relativity, all the laws of physics are the same in her reference frame as in any other inertial reference frame. Thus, she should experience no effects due to her motion through space, and choice (e) is the correct answer.

5. The astronaut in the spaceship is at rest relative to the pendulum and measures the proper time required for it to complete one oscillation (its period). The observer on Earth is in motion relative to the pendulum and measures the time required for a full oscillation to be longer than does the astronaut (i.e., $T > P$), and the correct choice is (c).

6. If L_p is the proper length of each edge of the cube, the volume measured by the observer at rest relative to it is $V = L_p^3$. The observer moving relative to the cube sees the two dimensions that are perpendicular to his velocity as unchanged, with each having value L_p. The dimension parallel to his velocity is measured to have the contracted length $L = L_p/\gamma$. Thus, the observer moving relative to the cube measures its volume to be $V' = L_p \cdot L_p \cdot L = L_p^2 \cdot L_p/\gamma = V/5$, so the correct answer is (c).

7. The relativistic momentum of a particle is $p = \sqrt{E^2 - E_R^2}\big/c$, where E is the total energy of the particle and E_R is its rest energy. In this case, each particle has the same total energy, E. Thus, the particle with the smallest rest energy has the greatest momentum, while the particle with the greatest rest energy has the smallest momentum. This means that the correct ordering of the particles, from smallest to greatest momentum, is proton, electron, photon, and (e) is the correct choice.

8. The time interval between events appears to be dilated (or lengthened) to observers who are in motion relative to those events. Thus, the intervals between successive "ticks" of the clock in orbit will seem longer to observers on Earth than they do to an astronaut in orbit with the clock and at rest relative to it. This means the Earth-based observers detect the orbiting clock to run slower than the identical clock that was left at rest on Earth. The correct answer to this question is choice (c).

9. When the ground observer is moving at speed $v = 0.5c$ relative to the oscillating system, the time he will measure for the period is $T' = \gamma T$, where

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-(0.5)^2}} = 1.15$$

and the correct choice is (d).

10. Choice (d) is the correct response. Each observer measures the intervals between "ticks" of the clock in motion relative to him to be lengthened by the *same* factor $\gamma = 1\big/\sqrt{1-v^2/c^2}$, where v is the relative velocity between the two observers.

11. According to the second postulate of special relativity, the speed of light is the same in all inertial reference frames, regardless of the velocity of the observer or the velocity of the source emitting the light. Thus, the speed of light from the rapidly moving quasar should be measured to be c, the same as the speed of light emitted by a stationary source. This means that (b) is the correct answer.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The two observers will agree on the speed of light and on the *speed* at which they move relative to one another.

4. Special relativity describes inertial reference frames—that is, reference frames that are not accelerating. General relativity describes all reference frames.

6. You would see the same thing that you see when looking at a mirror when at rest. The theory of relativity tells us that all experiments will give the same results in all inertial frames of reference.

8. (a) Attempts to apply special relativity to this situation would lead one to say that each observer (one on Earth and one in orbit) would find the other's clock to run slow relative to their own clock. However, the observer in orbit is not in an inertial reference frame, and special relativity does not apply here. According to general relativity, the clock in orbit and undergoing an acceleration (the centripetal acceleration) will run slower.

(b) When the moving clock returns to Earth, they are again in inertial reference frames and subject to the same laws of physics. Thus, the identical clocks should tick at the same rate once again. However, they will not be synchronized. The clock that underwent the accelerations (i.e., the orbiting clock) will have permanently lost time (or will have "aged less") and will be behind the clock that remained on Earth.

10. The 8 light-years represents the proper length of a rod from Earth to Sirius, measured by an observer seeing both the rod and Sirius nearly at rest. The astronaut sees Sirius coming toward her at $0.8c$ but also sees the distance contracted to

$$d = (8\text{ ly})\sqrt{1-(v/c)^2} = (8\text{ ly})\sqrt{1-(0.8)^2} = 5\text{ ly}$$

So the travel time measured on her clock is $t = \dfrac{d}{v} = \dfrac{5\text{ ly}}{0.8c} = \dfrac{(5\text{ yr})c}{0.8c} = 6\,\text{yr}$.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 0.436 m **(b)** less than 0.436 m by an undetectable amount

4. **(a)** $70\ \text{beats/min}$ **(b)** $30\ \text{beats/min}$

6. **(a)** $65.0\ \text{beats/min}$ **(b)** $10.6\ \text{beats/min}$

8. 5.0 s

10. $26.0\ \text{min} + 1.06\times10^{-11}\ \text{s}$

12. **(a)** rectangular box **(b)** 1.2 m, 2.0 m, 2.0 m

14. **(a)** $4.96\times10^{-19}\ \text{kg}\cdot\text{m/s}$ **(b)** $3.52\times10^{-18}\ \text{kg}\cdot\text{m/s}$

(c) No. Neglecting relativity in this case introduces an 86% error in the result.

16. $0.94c$ toward the left

18. $+0.696c$

20. 0.161 Hz

22. **(a)** 939 MeV **(b)** 3.01 GeV **(c)** 2.07 GeV

24. **(a)** $KE \approx \gamma E_R = \gamma m_p c^2$ **(b)** $0.999\,99c$

26. **(a)** 0.582 MeV **(b)** 2.45 MeV

28. 2.51×10^{-28} kg with speed $0.987c$, and 8.84×10^{-28} kg with speed $0.868c$

30. **(a)** $KE_i + m_i c^2 = KE_f + m_f c^2$ **(b)** 236.052 588 u **(c)** 235.861 612 u

(d) 0.190 976 u **(e)** 177.893 MeV

32. **(a)** $0.023\,6c = 7.08 \times 10^3$ km/s **(b)** $(6.17 \times 10^{-4})c = 185$ km/s

34. **(a)** Yes. As the spring is compressed, positive work is done on it or energy is added to it. Since mass and energy are equivalent, mass has been added to the spring.

(b) $\Delta m = kx^2/2c^2$ **(c)** 2.5×10^{-17} kg

36. **(a)** 2.50 MeV/c **(b)** 4.60 GeV/c

38. 1.2×10^{13} m

40. **(a)** $t = \dfrac{2d}{c+v}$ **(b)** $t' = \dfrac{2d}{c}\sqrt{\dfrac{c-v}{c+v}}$

42. **(a)** $\sim 10^8$ km **(b)** $\sim 10^2$ s

44. **(a)** See Solution. **(b)** $0.943c$

46. $R_g = 1.47$ km for the Sun

48. **(a)** 21.0 yr **(b)** 14.7 ly

(c) 10.5 ly **(d)** 35.7 yr

50. **(a)** $0.946c$ **(b)** 0.160 ly

(c) 0.114 yr **(d)** 7.51×10^{22} J

PROBLEM SOLUTIONS

26.1 (a) Observers on Earth measure the time for the astronauts to reach Alpha Centauri as $\Delta t_E = 4.42$ yr. But these observers are moving relative to the astronaut's internal biological clock and hence experience a dilated version of the proper time interval Δt_p measured on that clock. From $\Delta t_E = \gamma \Delta t_p$, we find

$$\Delta t_p = \Delta t_E/\gamma = \Delta t_E\sqrt{1-(v/c)^2} = (4.42\text{ yr})\sqrt{1-(0.950)^2} = \boxed{1.38\text{ yr}}$$

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(b) The astronauts are moving relative to the span of space separating Earth and Alpha Centauri. Hence, they measure a length-contracted version of the proper distance, $L_p = 4.20$ ly. The distance measured by the astronauts is

$$L = L_p/\gamma = L_p\sqrt{1-(v/c)^2} = (4.20\text{ ly})\sqrt{1-(0.950)^2} = \boxed{1.31\text{ ly}}$$

26.2 (a) The length of the meterstick measured by the observer moving at speed $v = 0.900c$ relative to the meterstick is

$$L = L_p/\gamma = L_p\sqrt{1-(v/c)^2} = (1.00\text{ m})\sqrt{1-(0.900)^2} = \boxed{0.436\text{ m}}$$

(b) If the observer moves relative to Earth in the direction toward the meterstick, the velocity of the observer relative to the meterstick is greater than that in part (a). The measured length of the meterstick will be $\boxed{\text{less than } 0.436\text{ m}}$ under these conditions, but at the speed the observer could run, the effect would be too small to detect.

26.3 The contracted length of the ship, $L = L_p - \Delta L$, as measured by the Earth-based observer is $L = L_p - \Delta L = L_p\sqrt{1-(v/c)^2}$. This yields $\sqrt{1-(v/c)^2} = 1-(\Delta L/L_p)$, and solving for the speed v of the ship, we find $v = c\sqrt{1-(1-\Delta L/L_p)^2}$. Thus, if the proper length of the ship is $L_p = 28.0$ m, and the observed contraction is $\Delta L = 0.150$ m, the speed of the ship must be

$$v = c\sqrt{1-\left(1-\frac{0.150\text{ m}}{28.0\text{ m}}\right)^2} = \boxed{0.103c}$$

26.4 (a) The time for 70 beats, as measured by the astronaut and any observer at rest with respect to the astronaut, is $\Delta t_p = 1.0$ min. The observer in the ship then measures a rate of $\boxed{70\text{ beats/min}}$.

(b) The observer on Earth moves at $v = 0.90c$ relative to the astronaut and measures the time for 70 beats as

$$\Delta t = \gamma\Delta t_p = \frac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \frac{1.0\text{ min}}{\sqrt{1-(0.90)^2}} = 2.3\text{ min}$$

This observer then measures a beat rate of 70 beats/2.3 min = $\boxed{30\text{ beats/min}}$.

26.5 (a) $\Delta t = \gamma\Delta t_p = \dfrac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \dfrac{2.6\times10^{-8}\text{ s}}{\sqrt{1-(0.98)^2}} = \boxed{1.3\times10^{-7}\text{ s}}$

(b) $d = v(\Delta t) = \left[0.98\left(3.0\times10^8\text{ m/s}\right)\right]\left(1.3\times10^{-7}\text{ s}\right) = \boxed{38\text{ m}}$

(c) $d' = v(\Delta t_p) = \left[0.98\left(3.0\times10^8\text{ m/s}\right)\right]\left(2.6\times10^{-8}\text{ s}\right) = \boxed{7.6\text{ m}}$

26.6 (a) As measured by observers in the ship (that is, at rest relative to the astronaut), the time required for 75.0 beats is $\Delta t_p = 1.00$ min.

The time interval required for 75.0 beats as measured by the Earth observer is $\Delta t = \gamma\Delta t_p = (1.00\text{ min})\big/\sqrt{1-(0.500)^2}$, so the Earth observer measures a pulse rate of

$$rate = \frac{75.0}{\Delta t} = \frac{75.0\sqrt{1-(0.500)^2}}{1.00\text{ min}} = \boxed{65.0/\text{min}}$$

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(b) If $v = 0.990c$, then $\Delta t = \gamma \Delta t_p = \dfrac{1.00 \text{ min}}{\sqrt{1-(0.990)^2}}$

and the pulse rate observed on Earth is

$$rate = \frac{75.0}{\Delta t} = \frac{75.0\sqrt{1-(0.990)^2}}{1.00 \text{ min}} = \boxed{10.6/\text{min}}$$

That is, the life span of the astronaut (reckoned by the total number of his heartbeats) is much longer as measured by an Earth clock than by a clock aboard the space vehicle.

26.7 (a) To the observer on Earth, the muon appears to have a lifetime of

$$\Delta t = \frac{d}{v} = \frac{4.60 \times 10^3 \text{ m}}{0.990(3.00 \times 10^8 \text{ m/s})} = \boxed{1.55 \times 10^{-5} \text{ s}}$$

(b) $\gamma = \dfrac{1}{\sqrt{1-(v/c)^2}} = \dfrac{1}{\sqrt{1-(0.990)^2}} = \boxed{7.09}$

(c) To an observer at rest with respect to the muon, its proper lifetime is

$$\Delta t_p = \frac{\Delta t}{\gamma} = \frac{1.55 \times 10^{-5} \text{ s}}{7.09} = \boxed{2.19 \times 10^{-6} \text{ s}}$$

(d) The muon is at rest relative to the observer traveling with the muon. Thus, the muon travels zero distance as measured by this observer. However, during the observed lifetime of the muon, this observer sees Earth move toward the muon a distance of

$$d' = v(\Delta t_p) = \left[0.990(3.00 \times 10^8 \text{ m/s})\right](2.19 \times 10^{-6} \text{ s}) = 6.50 \times 10^2 \text{ m} = \boxed{650 \text{ m}}$$

(e) As the third observer travels toward the incoming muon, his speed relative to the muon is greater than that of the observer at rest on Earth. Thus, his observed gamma factor ($\Delta t = \gamma \Delta t_p$) is higher, and $\boxed{\text{he measures the muon's lifetime as longer}}$ than that measured by the observer at rest with respect to Earth.

26.8 $\Delta t = \gamma \Delta t_p = \dfrac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \dfrac{3.0 \text{ s}}{\sqrt{1-(0.80)^2}} = \boxed{5.0 \text{ s}}$

26.9 The proper length of the faster ship is three times that of the slower ship $(L_{pf} = 3L_{ps})$, yet they both appear to have the same contracted length, L. Thus,

$$L = L_{ps}\sqrt{1-(v_s/c)^2} = (3L_{ps})\sqrt{1-(v_f/c)^2}, \text{ or } 1-(v_s/c)^2 = 9 - 9(v_f/c)^2$$

This gives

$$v_f = \frac{c\sqrt{8+(v_s/c)^2}}{3} = \frac{\sqrt{8+(0.350)^2}}{3}c = \boxed{0.950c}$$

26.10 The driver is the observer at rest with respect to the clock measuring the 26.0-min time interval. Thus, this observer measures the proper time Δt_p, and the Earth-based observer measures the dilated time $\Delta t = \gamma \Delta t_p = [1-(v/c)^2]^{-\frac{1}{2}} \cdot \Delta t_p$. In this case, $v = 35.0$ m/s $\ll c$, so we use the

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binominal expansion $[1-x]^{-\frac{1}{2}} \approx 1+\frac{1}{2}x$ when $x \ll 1$. This gives the time measured by the observer fixed on Earth as

$$\Delta t = (26.0\text{ min})\left[1-(v/c)^2\right]^{-\frac{1}{2}} \approx (26.0\text{ min})\left(1+\frac{1}{2}(v/c)^2\right)$$

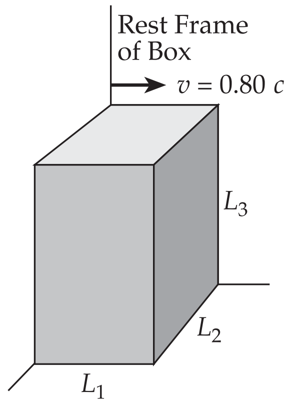
or $$\Delta t \approx 26.0\text{ min} + \frac{(26.0\ \cancel{\text{min}})}{2}\left(\frac{35.0\ \text{m/s}}{3.00\times10^{8}\ \text{m/s}}\right)^2\left(\frac{60.0\ \text{s}}{1\ \cancel{\text{min}}}\right) = \boxed{26.0\text{ min} + 1.06\times10^{-11}\ \text{s}}.$$

26.11 The trackside observer sees the supertrain length-contracted as

$$L = L_p\sqrt{1-(v/c)^2} = (100\text{ m})\sqrt{1-(0.95)^2} = 31\text{ m}$$

The supertrain $\boxed{\text{appears to fit in the tunnel}}$ with 50 m − 31 m = $\boxed{\text{19 m to spare}}$.

26.12 Length contraction occurs only in the dimension parallel to the motion.



(a) The sides labeled L_2 and L_3 in the figure at the right are unaffected, but the side labeled L_1 will appear contracted, giving the box a $\boxed{\text{rectangular}}$ shape, or more formally, the shape of a rectangular parallelepiped.

(b) The dimensions of the box, as measured by the observer moving at $v = 0.80c$ relative to it, are

$$L_2 = L_{2p} = \boxed{2.0\text{ m}},\quad L_3 = L_{3p} = \boxed{2.0\text{ m}},\ \text{and}$$
$$L_1 = L_{1p}\sqrt{1-(v/c)^2} = (2.0\text{ m})\sqrt{1-(0.80)^2} = \boxed{1.2\text{ m}}$$

26.13 (a) The classical expression for linear momentum is $p_{\text{classical}} = mv$, while the relativistic expression is $p = \gamma mv$, where $\gamma = 1\big/\sqrt{1-(v/c)^2}$. Thus, if $p = 3p_{\text{classical}}$, it is necessary that the gamma factor have a value $\gamma = 3$. Solving for the speed of the particle gives

$$v = c\sqrt{1-\frac{1}{\gamma^2}} = c\sqrt{1-\frac{1}{9}} = \frac{c\sqrt{8}}{3} = \boxed{\frac{2c\sqrt{2}}{3} = 0.943c}$$

(b) Observe that the calculation above did not depend on the mass of the particle involved. Thus, $\boxed{\text{the result is the same}}$ for a proton or any other particle.

26.14 (a) Classically,

$$p = mv = m(0.990c) = (1.67\times10^{-27}\ \text{kg})(0.990)(3.00\times10^{8}\ \text{m/s}) = \boxed{4.96\times10^{-19}\ \text{kg}\cdot\text{m/s}}$$

(b) By relativistic calculations,

$$p = \frac{mv}{\sqrt{1-(v/c)^2}} = \frac{m(0.990c)}{\sqrt{1-(0.990)^2}} = \frac{(1.67\times10^{-27}\ \text{kg})(0.990)(3.00\times10^{8}\ \text{m/s})}{\sqrt{1-(0.990)^2}}$$
$$= \boxed{3.52\times10^{-18}\ \text{kg}\cdot\text{m/s}}$$

(c) $\boxed{\text{No}}$, neglecting relativistic effects at such speeds would introduce an approximate 86% error in the result.

26.15 Momentum must be conserved, so the momenta of the two fragments must add to zero. Thus, their magnitudes must be equal, or $\gamma_2 m_2 v_2 = \gamma_1 m_1 v_1$. This gives

$$\frac{\left(1.67\times10^{-27}\ \text{kg}\right)v}{\sqrt{1-\left(v/c\right)^2}}=\frac{\left(2.50\times10^{-28}\ \text{kg}\right)\left(0.893c\right)}{\sqrt{1-\left(0.893\right)^2}}$$

and reduces to $\left[\dfrac{\left(1.67\times10^{-27}\ \text{kg}\right)\sqrt{1-\left(0.893\right)^2}}{\left(2.50\times10^{-28}\ \text{kg}\right)\left(0.893\right)}\right]\left(\dfrac{v}{c}\right)=\sqrt{1-\left(\dfrac{v}{c}\right)^2}$, or

$12.3\left(v/c\right)^2=1$, and yields $\boxed{v=0.285c}$.

26.16 We take to the right as the positive direction. Then, the velocities of the two ships relative to Earth are $v_{\text{RE}}=+0.70c$ and $v_{\text{LE}}=-0.70c$. The velocity of ship *L* relative to ship *R* is given by the relativistic relative velocity relation (Equation 26.7 in the textbook) as

$$v_{\text{LR}}=\frac{v_{\text{LE}}-v_{\text{RE}}}{1-\dfrac{v_{\text{LE}}v_{\text{RE}}}{c^2}}=\frac{(-0.70c)-0.70c}{1-\dfrac{(-0.70c)(0.70c)}{c^2}}=\frac{-1.40c}{1+0.49}=-0.94c=\boxed{0.94c\text{ toward the left}}$$

26.17 Taking to the right as the positive direction, the velocity of the electron relative to the laboratory is $v_{\text{EL}}=+0.90c$, and the velocity of the proton relative to the electron is $v_{\text{PE}}=-0.70c$. Thus, the relativistic addition of velocities (Equation 26.8 in the textbook) gives the velocity of the proton relative to the laboratory as

$$v_{\text{PL}}=\frac{v_{\text{PE}}+v_{\text{EL}}}{1+\dfrac{v_{\text{PE}}v_{\text{EL}}}{c^2}}=\frac{(-0.70c)+0.90c}{1+\dfrac{(-0.70c)(0.90c)}{c^2}}=\frac{+0.20c}{1-0.63}=+0.54c=\boxed{0.54c\text{ toward the right}}$$

26.18 We choose the direction of the spaceship's motion relative to Earth as the positive direction. Then, the spaceship's velocity relative to Earth is $v_{\text{SE}}=+0.750c$. It is desired to have the velocity of the rocket relative to Earth be $v_{\text{RE}}=+0.950c$. The relativistic relative velocity relation (Equation 26.7 in the textbook) then gives the required velocity of the rocket relative to the ship as

$$v_{\text{RS}}=\frac{v_{\text{RE}}-v_{\text{SE}}}{1-\dfrac{v_{\text{RE}}v_{\text{SE}}}{c^2}}=\frac{0.950c-0.750c}{1-\dfrac{(0.950c)(0.750c)}{c^2}}=\boxed{+0.696c}$$

26.19 Taking away from Earth as the positive direction, the velocity of ship *A* relative to Earth is $v_{\text{AE}}=+0.800c$, and the velocity of ship *B* relative to Earth is $v_{\text{BE}}=+0.900c$. The relativistic relative velocity relation (Equation 26.7 in the textbook) gives the velocity of ship *B* relative to ship *A* (and hence, the speed with which *B* is overtaking *A*) as

$$v_{\text{BA}}=\frac{v_{\text{BE}}-v_{\text{AE}}}{1-\dfrac{v_{\text{BE}}v_{\text{AE}}}{c^2}}=\frac{0.900c-0.800c}{1-\dfrac{(0.900c)(0.800c)}{c^2}}=\boxed{+0.357c}$$

26.20 We first determine the velocity of the pulsar relative to the rocket. Taking toward the Earth as the positive direction, the velocity of the pulsar relative to Earth is $v_{\text{PE}}=+0.950c$, and the velocity of the rocket relative to Earth is $v_{\text{RE}}=-0.995c$. The relativistic relative velocity relation (Equation 26.7 in the textbook) gives the velocity of the pulsar relative to the rocket as

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$$v_{\text{PR}} = \frac{v_{\text{PE}} - v_{\text{RE}}}{1 - \frac{v_{\text{PE}} v_{\text{RE}}}{c^2}} = \frac{0.950c - (-0.995c)}{1 - \frac{(0.950c)(-0.995c)}{c^2}} = +0.999\,87c$$

The time interval between successive pulses in the pulsar's own reference frame (i.e., on a clock at rest with respect to the event being timed) is $\Delta t_p = 1/10.0 \text{ Hz} = 0.100 \text{ s}$. The duration of this interval in the rocket's frame of reference is given by the time dilation relation as

$$\Delta t = \gamma \Delta t_p = \frac{\Delta t_p}{\sqrt{1 - (v_{PR}/c)^2}} = \frac{0.100 \text{ s}}{\sqrt{1 - (0.999\,87)^2}} = 6.20 \text{ s}$$

and the frequency of the pulses as observed in the rocket's reference frame is

$$f = \frac{1}{\Delta t} = \frac{1}{6.20 \text{ s}} = \boxed{0.161 \text{ Hz}}$$

26.21 Taking to the right as positive, it is given that the velocity of the rocket relative to observer A is $v_{\text{RA}} = +0.92c$. If observer B observes the rocket to have a velocity $v_{\text{RB}} = -0.95c$, the velocity of observer B relative to the rocket is $v_{\text{BR}} = +0.95c$. The relativistic velocity addition relation then gives the velocity of B relative to the stationary observer A as

$$v_{\text{BA}} = \frac{v_{\text{BR}} + v_{\text{RA}}}{1 + \frac{v_{\text{BR}} v_{\text{RA}}}{c^2}} = \frac{+0.95c + 0.92c}{1 + \frac{(0.95c)(0.92c)}{c^2}} = +0.998c \quad \text{or} \quad \boxed{0.998c \text{ toward the right}}$$

26.22 (a) $E_R = mc^2 = (1.67 \times 10^{-27} \text{ kg})(3.00 \times 10^8 \text{ m/s})^2 \left(\frac{1 \text{ MeV}}{1.60 \times 10^{-13} \text{ J}}\right) = \boxed{939 \text{ MeV}}$

(b) $E = \gamma mc^2 = \gamma E_R = \frac{E_R}{\sqrt{1 - (v/c)^2}}$

$$= \frac{939 \text{ MeV}}{\sqrt{1 - (0.950)^2}} = 3.01 \times 10^3 \text{ MeV} = \boxed{3.01 \text{ GeV}}$$

(c) $KE = E - E_R = 3.01 \times 10^3 \text{ MeV} - 939 \text{ MeV} = 2.07 \times 10^3 \text{ MeV} = \boxed{2.07 \text{ GeV}}$

26.23 (a) The total energy is 400 times the rest energy, or $E = \gamma E_R = 400 E_R$, so it is necessary that $\gamma = 400$. But $\gamma = 1\big/\sqrt{1 - (v/c)^2}$, and solving for the speed gives

$$v = c\sqrt{1 - \frac{1}{\gamma^2}} = c\sqrt{1 - \frac{1}{(400)^2}} = \boxed{0.999\,997c}$$

(b) The kinetic energy is $KE = E - E_R = 400E_R - E_R = 399E_R$. For a proton, $E_R = 938.3$ MeV. Thus,

$$KE = 399(938.3 \text{ MeV}) = \boxed{3.74 \times 10^5 \text{ MeV}}$$

26.24 (a) The rest energy of a proton is $E_R = 938.3$ MeV. If the kinetic energy is $KE = 175 \text{ GeV} \gg E_R$, we have $KE = (\gamma - 1)E_R \gg E_R$, or $\gamma - 1 \gg 1$, and $\gamma - 1 \approx \gamma$. Thus, we may write the approximate relation $KE = (\gamma - 1)E_R \approx \gamma E_E = \boxed{\gamma m_p c^2}$.

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(b) The gamma factor is $\gamma = 1\big/\sqrt{1-(v/c)^2}$, so solving for the speed gives $v = c\sqrt{1-\gamma^{-2}}$. Therefore, when $KE \gg E_R$ and we may use the approximation $KE \approx \gamma E_R$, we have $\gamma \approx E_R/KE$. Then, for the proton having $KE = 175$ GeV,

$$v = c\sqrt{1-\gamma^{-2}} \approx c\sqrt{1-\left(\frac{E_R}{KE}\right)^2} = c\sqrt{1-\left(\frac{938.3\text{ MeV}}{175\times10^3\text{ MeV}}\right)^2} = \boxed{0.999\,99c}$$

26.25 The nonrelativistic expression for kinetic energy is $KE = \frac{1}{2}mv^2$, while the relativistic expression is $KE = E - E_R = (\gamma-1)E_R = (\gamma-1)mc^2$, where $\gamma = 1\big/\sqrt{1-(v/c)^2}$. Thus, when the relativistic kinetic energy is twice the predicted nonrelativistic value, we have

$$\left(\frac{1}{\sqrt{1-(v/c)^2}}-1\right)mc^2 = 2\left(\frac{1}{2}mv^2\right) \qquad \text{or} \qquad 1 = \left[1+(v/c)^2\right]\sqrt{1-(v/c)^2}$$

Squaring both sides of the last result and simplifying gives

$$(v/c)^2\left[(v/c)^4+(v/c)^2-1\right] = 0$$

Ignoring the trivial solution $v/c = 0$, we must have $(v/c)^4+(v/c)^2-1 = 0$. This is a quadratic equation of the form $x^2+x-1=0$, with $x = (v/c)^2$. Applying the quadratic formula gives $x = \left(-1\pm\sqrt{5}\right)/2$. Since $x = (v/c)^2$, we ignore the negative solution and find

$$x = \left(\frac{v}{c}\right)^2 = \frac{-1+\sqrt{5}}{2} = 0.618$$

which yields $v = c\sqrt{0.618} = \boxed{0.786\,c}$.

26.26 The energy input to the electron will be $W = E_f - E_i = (\gamma_f - \gamma_i)E_R$, or

$$W = \left(\frac{1}{\sqrt{1-(v_f/c)^2}} - \frac{1}{\sqrt{1-(v_i/c)^2}}\right)E_R \qquad \text{where} \qquad E_R = 0.511\text{ MeV}$$

(a) If $v_f = 0.900c$ and $v_i = 0.500c$, then

$$W = \left(\frac{1}{\sqrt{1-(0.900)^2}} - \frac{1}{\sqrt{1-(0.500)^2}}\right)(0.511\text{ Mev}) = \boxed{0.582\text{ MeV}}$$

(b) When $v_f = 0.990c$ and $v_i = 0.900c$, we have

$$W = \left(\frac{1}{\sqrt{1-(0.990)^2}} - \frac{1}{\sqrt{1-(0.900)^2}}\right)(0.511\text{ Mev}) = \boxed{2.45\text{ MeV}}$$

26.27 (a) From the work-energy theorem, $W_{\text{net}} = \Delta KE$. The ship starts from rest, so $KE_i = 0$, and $\Delta KE = KE_f = (\gamma-1)E_R = (\gamma-1)mc^2$. Thus,

$$W_{\text{net}} = \left(\frac{1}{\sqrt{1-(0.700)^2}}-1\right)\left(2.40\times10^6\text{ kg}\right)\left(3.00\times10^8\text{ m/s}\right)^2 = \boxed{8.65\times10^{22}\text{ J}}$$

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(b) If all the increase in the kinetic energy of the ship comes from the rest energy of the fuel (i.e., $\Delta KE = E_{R,\text{fuel}} = m_{\text{fuel}}c^2$), the mass of the fuel required is

$$m_{\text{fuel}} = \frac{\Delta KE}{c^2} = \frac{8.65\times10^{22}\text{ J}}{\left(3.00\times10^{8}\text{ m/s}\right)^2} = \boxed{9.61\times10^{5}\text{ kg}}$$

26.28 Let m_1 be the mass of the fragment moving at $v_1 = 0.987c$, and m_2 be the mass having a speed of $v_2 = 0.868c$.

From conservation of mass-energy, $E_1 + E_2 = E_{R,i}$, or $\gamma_1 m_1 \cancel{c^2} + \gamma_2 m_2 \cancel{c^2} = m_i \cancel{c^2}$, giving

$$\frac{m_1}{\sqrt{1-(0.987)^2}} + \frac{m_2}{\sqrt{1-(0.868)^2}} = m_i \qquad \text{and} \qquad 6.22m_1 + 2.01m_2 = 3.34\times10^{-27}\text{ kg} \qquad \textbf{[1]}$$

Since the original particle was at rest, the momenta of the two fragments after decay must add to zero. Thus, the magnitudes must be equal, giving $p_2 = p_1$, or $\gamma_2 m_2 v_2 = \gamma_1 m_1 v_1$, and yielding

$$2.01m_2(0.868c) = 6.22m_1(0.987c) \qquad \text{or} \qquad m_2 = 3.52m_1 \qquad \textbf{[2]}$$

Substituting Equation [2] into [1] gives

$$(6.22+7.08)m_1 = 3.34\times10^{-27}\text{ kg, or } m_1 = \boxed{2.51\times10^{-28}\text{ kg}}$$

Equation [2] then yields $m_2 = 3.52\left(2.51\times10^{-28}\text{ kg}\right) = \boxed{8.84\times10^{-28}\text{ kg}}$.

26.29 The relativistic total energy is $E = \gamma E_R = \gamma mc^2$, and the momentum is $p = \gamma mv$, where $\gamma = 1\big/\sqrt{1-v^2/c^2}$. Thus, $E^2/c^2 = \gamma^2 m^2 c^2$, and $p^2 = \gamma^2 m^2 v^2$, so subtracting yields

$$\frac{E^2}{c^2} - p^2 = \gamma^2 m^2\left(c^2 - v^2\right) = \gamma^2 m^2 c^2\left(1 - v^2/c^2\right) = \left(\frac{1}{\cancel{1-v^2/c^2}}\right) m^2c^2\left(\cancel{1-v^2/c^2}\right) = m^2c^2$$

Rearranging, this becomes

$$\frac{E^2}{c^2} = p^2 + m^2c^2 \qquad \text{or} \qquad \boxed{E^2 = p^2c^2 + m^2c^4}$$

26.30 (a) Since the total relativistic energy of a particle is $E = KE + E_R = KE + mc^2$, requiring that total energy be conserved (i.e., total energy before reaction = total energy after reaction) gives $\boxed{KE_i + m_ic^2 = KE_f + m_fc^2}$, where m_i is the total mass of the particles present before the reaction, KE_i is the total kinetic energy before the reaction, m_f is the total mass of the particles present after the reaction, and KE_f is the final total kinetic energy.

(b) Using atomic masses from the tables in Appendix B of the textbook, the total mass of the initial particles is found to be

$$m_i = m_{^{235}_{92}\text{U}} + m_{^{1}_{0}\text{n}} = 235.043\,923\text{ u} + 1.008\,665\text{ u} = \boxed{236.052\,588\text{ u}}$$

(c) The total mass of the particles present after the reaction is

$$m_f = m_{^{148}_{57}\text{La}} + m_{^{87}_{35}\text{Br}} + m_{^{1}_{0}\text{n}}$$
$$= 147.932\,236\text{ u} + 86.920\,711\,19\text{ u} + 1.008\,665\text{ u} = \boxed{235.861\,612\text{ u}}$$

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(d) The mass that must have been converted to energy in the reaction is

$$\Delta m = m_i - m_f = (236.052\,588 - 235.861\,612)\ \text{u} = \boxed{0.190\,976\ \text{u}}$$

(e) Assuming that $KE_i = 0$, the total kinetic energy of the product particles is then

$$KE_f = \Delta mc^2 + KE_i = (0.190\,976\ \text{u})c^2\left(\frac{931.494\ \text{MeV}/c^2}{1\ \text{u}}\right) + 0 = \boxed{177.893\ \text{MeV}}$$

26.31 (a) $\gamma = \dfrac{E}{E_R} = \dfrac{20.0\ \text{GeV}}{0.511\ \text{MeV}}\left(\dfrac{10^3\ \text{MeV}}{1\ \text{GeV}}\right) = \boxed{3.91\times10^4}$

(b) $L = \dfrac{L_p}{\gamma} = \dfrac{3.00\times10^3\ \text{m}}{3.91\times10^4} = 7.67\times10^{-2}\ \text{m} = \boxed{7.67\ \text{cm}}$

26.32 (a) For an electron moving at $v_e = 0.750c$, the gamma factor is

$$\gamma_e = 1\Big/\sqrt{1-(v_e/c)^2} = 1\Big/\sqrt{1-(0.750)^2} = 1.51$$

The kinetic energy of a particle is $KE = E - E_R = (\gamma - 1)E_R$, so if the kinetic energy of a proton ($E_{R,p} = 938.3$ MeV) equals that of the electron ($E_{R,e} = 0.511$ MeV), we must have $(\gamma_p - 1)E_{R,p} = (\gamma_e - 1)E_{R,e}$, or

$$\gamma_p = 1 + (\gamma_e - 1)\frac{E_{R,e}}{E_{R,p}} = 1 + (1.51 - 1)\left(\frac{0.511\ \text{MeV}}{938.3\ \text{MeV}}\right) = 1 + 2.78\times10^{-4}$$

But $\gamma_p = 1\Big/\sqrt{1-(v_p/c)^2}$, so $(v_p/c)^2 = 1 - 1\big/\gamma_p^2$ and the speed of the proton must be

$$v_p = c\sqrt{1-\frac{1}{\gamma_p^2}} = c\sqrt{1-\frac{1}{\left(1+2.78\times10^{-4}\right)^2}} = \boxed{0.023\,6c = 7.08\times10^3\ \text{km/s}}$$

(b) As above, $\gamma_e = 1.51$ for an electron having speed $v_e = 0.750c$. The momentum of a particle is

$$p = \gamma mv = \frac{\gamma mc^2(v/c)}{c} = \frac{\gamma E_R(v/c)}{c} \qquad \text{or} \qquad pc = \gamma E_R(v/c)$$

If the momentum of a proton equals that of the electron, then $p_p c = p_e c$, or

$$\gamma_p E_{R,p}(v_p/c) = \gamma_e E_{R,e}(v_e/c)$$

and $$\frac{(v_p/c)}{\sqrt{1-(v_p/c)^2}} = \gamma_e\frac{E_{R,e}}{E_{R,p}}\left(\frac{v_e}{c}\right) = (1.51)\left(\frac{0.511\ \text{MeV}}{938.3\ \text{MeV}}\right)(0.750) = 6.17\times10^{-4}$$

Thus, $(v_p/c) = 6.17\times10^{-4}\sqrt{1-(v_p/c)^2}$, and

$(v_p/c)^2 = \left(6.17\times10^{-4}\right)^2 - \left(6.17\times10^{-4}\right)^2(v_p/c)^2$, which yields

$$(v_p/c)^2 = \frac{\left(6.17\times10^{-4}\right)^2}{1+\left(6.17\times10^{-4}\right)^2} = 3.81\times10^{-7}$$

and the speed of the proton must be

$$v_p = c\sqrt{3.81\times10^{-7}} = \left(3.00\times10^8\ \text{m/s}\right)\left(6.17\times10^{-4}\right) = 1.85\times10^5\ \text{m/s} = \boxed{185\ \text{km/s}}$$

26.33 $KE = E - E_R = (\gamma - 1)E_R$

so $\gamma = 1 + \dfrac{KE}{E_R} = \dfrac{1}{\sqrt{1-(v/c)^2}}$ giving $v = c\sqrt{1 - \dfrac{1}{(1+KE/E_R)^2}}$

(a) The speed of an electron having $KE = 2.00$ MeV will be

$$v_e = c\sqrt{1 - \frac{1}{(1+KE/E_{R,e})^2}} = c\sqrt{1 - \frac{1}{(1+2.00/0.511)^2}} = \boxed{0.979c}$$

(b) For a proton with $KE = 2.00$ MeV, the speed is

$$v_p = c\sqrt{1 - \frac{1}{(1+KE/E_{R,p})^2}} = c\sqrt{1 - \frac{1}{(1+2.00/938)^2}} = \boxed{0.065\,2c}$$

(c) $v_e - v_p = 0.979c - 0.065\,2c = \boxed{0.914c}$

26.34 (a) $\boxed{\text{Yes.}}$ As the spring is compressed, positive work is done on it or energy is added to it. Since mass and energy are equivalent, mass has been added to the spring.

(b) $\Delta m = \dfrac{\Delta E_R}{c^2} = \dfrac{\Delta PE_s}{c^2} = \dfrac{\frac{1}{2}kx^2}{c^2} = \boxed{\dfrac{kx^2}{2c^2}}$

(c) $\Delta m = \dfrac{(2.0\times10^2 \text{ N/m})(0.15 \text{ m})^2}{2(3.00\times10^8 \text{ m/s})^2} = \boxed{2.5\times10^{-17} \text{ kg}}$

26.35 An observer who moves at speed v relative to an object (or span of space) having proper length L_p sees a contracted length given by $L = L_p/\gamma = L_p\sqrt{1-(v/c)^2}$. Thus, if the proper distance to the star is $L_p = 5.00$ ly, and this length is to have a contracted value of $L = 2.00$ ly in the reference frame of the spacecraft, the speed of the spacecraft relative to the star must be

$$v = c\sqrt{1 - \left(\frac{L}{L_p}\right)^2} = c\sqrt{1 - \left(\frac{2.00 \text{ ly}}{5.00 \text{ ly}}\right)^2} = \boxed{0.917c}$$

26.36 From $E^2 = (pc)^2 + E_R^2$, with $E = 5E_R$, we find that $p = E_R\sqrt{24}/c$.

(a) For an electron, $p = \dfrac{(0.511 \text{ MeV})\sqrt{24}}{c} = \boxed{2.50 \text{ MeV}/c}$.

(b) For a proton, $p = \dfrac{(938.3 \text{ MeV})\sqrt{24}}{c} = 4.60\times10^3 \dfrac{\text{MeV}}{c} = \boxed{4.60 \text{ GeV}/c}$.

26.37 (a) Observers on Earth measure the distance to Andromeda to be $d = 2.00\times10^6 \text{ ly} = (2.00\times10^6 \text{ yr})c$. The time for the trip, in Earth's frame of reference, is $\Delta t = \gamma(\Delta t_p) = 30.0 \text{ yr}/\sqrt{1-(v/c)^2}$. The required speed is then

$$v = \frac{d}{\Delta t} = \frac{(2.00\times10^6 \text{ yr})c}{30.0 \text{ yr}/\sqrt{1-(v/c)^2}}$$

which gives $(1.50\times10^{-5})(v/c) = \sqrt{1-(v/c)^2}$. Squaring both sides of this equation and solving for v/c yields $v/c = 1/\sqrt{1+2.25\times10^{-10}}$. Then, the approximation $1/\sqrt{1+x} = 1 - x/2$ gives

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$$\frac{v}{c} \approx 1 - \frac{2.25 \times 10^{-10}}{2} = \boxed{1 - 1.13 \times 10^{-10}}$$

(b) $KE = (\gamma - 1)mc^2$, and

$$\gamma = \frac{1}{\sqrt{1-(v/c)^2}} = \frac{1}{\sqrt{1-\left(1-1.13\times 10^{-10}\right)^2}} = \frac{1}{\sqrt{2.26\times 10^{-10}}}$$

Thus,

$$KE = \left(\frac{1}{\sqrt{2.26\times 10^{-10}}} - 1\right)\left(1.00\times 10^6 \text{ kg}\right)\left(3.00\times 10^8 \text{ m/s}\right)^2 = \boxed{5.99\times 10^{27} \text{ J}}$$

(c) $cost = KE \times rate$

$$= \left[\left(5.99\times 10^{27} \text{ J}\right)\left(\frac{1 \text{ kWh}}{3.60\times 10^6 \text{ J}}\right)\right](\$0.13/\text{kWh}) = \boxed{\$2.16\times 10^{20}}$$

26.38 The clock, at rest in the ship's frame of reference, will measure a proper time of $\Delta t_p = 10$ h before sounding. Observers on Earth move at $v = 0.75c$ relative to the clock and measure an elapsed time of

$$\Delta t = \gamma\left(\Delta t_p\right) = \frac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \frac{10 \text{ h}}{\sqrt{1-(0.75)^2}} = 15 \text{ h}$$

The observers on Earth see the clock moving away at $0.75c$ and compute the distance traveled before the alarm sounds as

$$d = v(\Delta t) = \left[0.75\left(3.0\times 10^8 \text{ m/s}\right)\right](15 \text{ h})\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = \boxed{1.2\times 10^{13} \text{ m}}$$

26.39 (a) Since Dina and Owen are at rest in the same frame of reference (S′), they both see the ball traveling in the negative x' direction with speed $v'_{\text{ball}} = \boxed{0.800c}$. Note that the velocity of the ball relative to Dina is $v_{\text{BD}} = -0.800c$.

(b) The distance between Dina and Owen, measured in their own rest frame, is $L_p = 1.80\times 10^{12}$ m. Therefore, the time required for the ball to reach Dina, measured on her own clock, is

$$\Delta t_p = \frac{L_p}{v'_{\text{ball}}} = \frac{1.80\times 10^{12} \text{ m}}{0.800c} = \frac{2.25\times 10^{12} \text{ m}}{3.00\times 10^8 \text{ m/s}} = \boxed{7.50\times 10^3 \text{ s}}$$

(c) Ed sees a contracted length for the distance separating Dina and Owen. According to him, they are separated by a distance $L = L_p/\gamma = L_p\sqrt{1-(v/c)^2}$, where $v = 0.600c$ is the speed of the S′ frame relative to Ed's reference frame, S. Thus, according to Ed, the ball must travel a distance

$$L = \left(1.80\times 10^{12} \text{ m}\right)\sqrt{1-(0.600)^2} = \boxed{1.44\times 10^{12} \text{ m}}$$

(d) The ball has a velocity of $v_{\text{BD}} = -0.800c$ relative to Dina, and Dina moves at velocity $v_{\text{DE}} = +0.600c$ relative to Ed. The relativistic velocity addition relation (Equation 26.8 from the textbook) then gives the velocity of the ball relative to Ed as

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$$v_{\mathrm{BE}} = \frac{v_{\mathrm{BD}} + v_{\mathrm{DE}}}{1 + \frac{v_{\mathrm{BD}} v_{\mathrm{DE}}}{c^2}} = \frac{-0.800c + 0.600c}{1 + (-0.800)(0.600)} = \frac{-0.200c}{0.520} = -0.385c$$

Thus, the speed of the ball according to Ed is $v_{\mathrm{ball}} = |v_{\mathrm{BE}}| = \boxed{0.385c}$.

26.40 (a) As seen by an observer at rest relative to the mirror in frame S, the light must travel distance d before it strikes the mirror and then a distance $d - d_1$ back to the ship after reflection. Here, distance $d_1 = vt$ is the distance the ship moves toward the mirror in the time t between when the pulse was emitted from the ship and when the reflected pulse was received by the ship. Since all observers agree that light travels at speed c, the total travel time for the light is

$$t = \frac{d + (d - d_1)}{c} = \frac{2d - d_1}{c} = \frac{2d}{c} - \left(\frac{v}{c}\right)t \qquad \text{or} \qquad \left(1 + \frac{v}{c}\right)t = \frac{2d}{c}$$

Solving for the travel time t of the light gives $t = \boxed{2d/(c+v)}$.

(b) From the viewpoint of observers in the spacecraft, the spacecraft is at rest, and the mirror moves toward it at speed v. At the time the pulse starts toward the mirror, these observers see a contracted initial distance $d' = d\sqrt{1-(v/c)^2}$ to the mirror. If the total time of travel for the light is t', during the time $t'/2$ while the light is traveling toward the mirror, the mirror moves a distance $\Delta x = vt'/2$ closer to the ship. Thus, the light must travel a distance $d' - \Delta x$ before reflection and the same distance $d' - \Delta x$ back to the stationary ship after reflection. The total distance traveled by the light is then $D = 2(d' - \Delta x)$, and the time required (traveling at speed c) is

$$t' = \frac{D}{c} = \frac{2d' - 2v(t'/2)}{c} = \frac{2d'}{c} - \left(\frac{v}{c}\right)t' \qquad \text{or} \qquad \left(1 + \frac{v}{c}\right)t' = \frac{2d'}{c} = \frac{2d\sqrt{1-(v/c)^2}}{c}$$

Solving for the travel time t' measured by observers in the spacecraft then gives

$$(c+v)t' = 2d\sqrt{\frac{c^2 - v^2}{c^2}} \qquad \text{and} \qquad t' = \frac{2d}{c}\sqrt{\frac{(c+v)(c-v)}{(c+v)^2}} = \boxed{\frac{2d}{c}\sqrt{\frac{c-v}{c+v}}}$$

26.41 The length of the space ship, as measured by observers on Earth, is $L = L_p\sqrt{1-(v/c)^2}$. In Earth's frame of reference, the time required for the ship to pass overhead is

$$\Delta t = \frac{L}{v} = \frac{L_p\sqrt{1-(v/c)^2}}{v} = L_p\sqrt{\frac{1}{v^2} - \frac{1}{c^2}}$$

Thus, $$\frac{1}{v^2} = \frac{1}{c^2} + \left(\frac{\Delta t}{L_p}\right)^2 = \frac{1}{(3.00\times10^8\ \mathrm{m/s})^2} + \left(\frac{0.75\times10^{-6}\ \mathrm{s}}{300\ \mathrm{m}}\right)^2 = 1.74\times10^{-17}\ \frac{\mathrm{s}^2}{\mathrm{m}^2}$$

or $$v = \frac{1}{\sqrt{1.74\times10^{-17}\ \frac{\mathrm{s}^2}{\mathrm{m}^2}}} = \left(2.4\times10^8\ \frac{\mathrm{m}}{\mathrm{s}}\right)\left(\frac{c}{3.00\times10^8\ \mathrm{m/s}}\right) = \boxed{0.80c}$$

26.42 From $KE = (\gamma - 1)E_R$, we find

$$\gamma = 1 + \frac{KE}{E_R} = 1 + \frac{10^{13}\ \mathrm{MeV}}{938\ \mathrm{MeV}} = 1.07\times10^{10}$$

continued on next page

or $\gamma \sim 10^{10}$. Thus, the speed of the proton (and hence, the speed of the galaxy as seen by the proton) is $v = c\sqrt{1-1/\gamma^2} \sim c\sqrt{1-10^{-20}} \approx c$.

(a) The diameter of the galaxy, as seen in the proton's frame of reference, is

$$L = L_p\sqrt{1-(v/c)^2} = \frac{L_p}{\gamma} \sim \frac{10^5 \text{ ly}}{10^{10}} = 10^{-5} \text{ ly}$$

or $$L \approx 10^{-5} \text{ ly}\left(\frac{9.461\times10^{15} \text{ m}}{1 \text{ ly}}\right)\left(\frac{1 \text{ km}}{10^3 \text{ m}}\right) = 9.461\times10^7 \text{ km} \qquad \boxed{\sim 10^8 \text{ km}}$$

(b) The proton sees the galaxy rushing by at $v \approx c$. The time, in the proton's frame of reference, for the galaxy to pass is

$$\Delta t = \frac{L}{v} \sim \frac{10^8 \text{ km}}{c} = \frac{10^{11} \text{ m}}{3.00\times10^8 \text{ m/s}} = 333 \text{ s} \qquad \boxed{\sim 10^2 \text{ s}}$$

26.43 The difference between the relativistic momentum, $p = \gamma mv$, and the classical momentum, mv, is $\Delta p = \gamma mv - mv = (\gamma - 1)mv$.

(a) The error is 1.00% when $\Delta p/p = 0.010\,0$, or $(\gamma-1)mv = 0.010\,0\,\gamma mv$. This gives $\gamma = 1/0.990$, or $1-(v/c)^2 = (0.990)^2$, and yields $v = \boxed{0.141c}$.

(b) When the error is 10.0%, we have $\gamma = 1/0.900$, and $1-(v/c)^2 = (0.900)^2$. In this case, the speed of the particle is $v = c\sqrt{1-(0.900)^2} = \boxed{0.436c}$.

26.44 The kinetic energy gained by the electron will equal the loss of potential energy, so

$$KE = q(\Delta V) = e(1.02 \text{ MV}) = 1.02 \text{ MeV}$$

(a) If Newtonian mechanics remained valid, then $KE = \frac{1}{2}mv^2$, and the speed attained would be

$$v = \sqrt{\frac{2\,KE}{m}} = \sqrt{\frac{2(1.02 \text{ MeV})(1.60\times10^{-13} \text{ J/MeV})}{9.11\times10^{-31} \text{ kg}}} = 5.99\times10^8 \text{ m/s} \approx \boxed{2c}$$

(b) $KE = (\gamma-1)E_R$, so $\gamma = 1 + \dfrac{KE}{E_R} = 1 + \dfrac{1.02 \text{ MeV}}{0.511 \text{ MeV}} = 3.00$

The actual speed attained is

$$v = c\sqrt{1-1/\gamma^2} = c\sqrt{1-1/(3.00)^2} = \boxed{0.943c}$$

26.45 (a) When at rest, muons have a mean lifetime of $\Delta t_p = 2.2\ \mu$s. In a frame of reference where they move at $v = 0.95c$, the dilated mean lifetime of the muons will be

$$\tau = \gamma(\Delta t_p) = \frac{\Delta t_p}{\sqrt{1-(v/c)^2}} = \frac{2.2\ \mu\text{s}}{\sqrt{1-(0.95)^2}} = \boxed{7.0\ \mu\text{s}}$$

(b) In a frame of reference where the muons travel at $v = 0.95c$, the time required to travel 3.0 km is

$$t = \frac{d}{v} = \frac{3.0\times10^3 \text{ m}}{0.95(3.00\times10^8 \text{ m/s})} = 1.05\times10^{-5} \text{ s} = 10.5\ \mu\text{s}$$

continued on next page

If $N_0 = 5.0\times10^4$ muons started the 3.0 km trip, the number remaining at the end is

$$N = N_0 e^{-t/\tau} = \left(5.0\times10^4\right)e^{-10.5\ \mu\text{s}/7.0\ \mu\text{s}} = \boxed{1.1\times10^4}$$

26.46 The work required equals the increase in the gravitational potential energy, or $W = GM_{\text{Sun}}m/R_g$. If this is to equal the rest energy of the mass removed, then

$$mc^2 = \frac{GM_{\text{Sun}}m}{R_g} \qquad \text{or} \qquad R_g = \frac{GM_{\text{Sun}}}{c^2}$$

$$R_g = \frac{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.99\times10^{30}\ \text{kg}\right)}{\left(3.00\times10^8\ \text{m/s}\right)^2} = 1.47\times10^3\ \text{m} = \boxed{1.47\ \text{km}}$$

26.47 According to Earth-based observers, the times required for the two trips are:

For Speedo: $\Delta t_S = \dfrac{L_p}{v_S} = \dfrac{20.0\ \text{ly}}{0.950\,c} = \dfrac{20.0\ \text{yr}}{0.950} = 21.1\ \text{yr}$

For Goslo: $\Delta t_G = \dfrac{L_p}{v_G} = \dfrac{20.0\ \text{ly}}{0.750\,c} = \dfrac{20.0\ \text{yr}}{0.750} = 26.7\ \text{yr}$

Thus, after Speedo lands, he must wait and age at the same rate as planet-based observers, for an additional $\Delta T_{\text{age},S} = \Delta t_G - \Delta t_S = (26.7 - 21.1)\ \text{yr} = 5.6\ \text{yr}$ before Goslo arrives.

The time required for the trip according to Speedo's internal biological clock (which measures the proper time for his aging process during the trip) is

$$T_{\text{age},S} = \Delta t_{p,S} = \frac{\Delta t_S}{\gamma_S} = \Delta t_S\sqrt{1-\left(v_S/c\right)^2} = \left(21.1\ \text{yr}\right)\sqrt{1-(0.950)^2} = 6.59\ \text{yr}$$

When Goslo arrives, Speedo has aged a total of

$$T_S = T_{\text{age},S} + \Delta T_{\text{age},S} = 6.59\ \text{yr} + 5.6 = 12.2\ \text{yr}$$

The time required for the trip according to Goslo's internal biological clock (and hence the amount he ages) is

$$T_G = \Delta t_{p,G} = \frac{\Delta t_G}{\gamma_G} = \Delta t_G\sqrt{1-\left(v_G/c\right)^2} = \left(26.7\ \text{yr}\right)\sqrt{1-(0.750)^2} = 17.7\ \text{yr}$$

Thus, we see that when he arrives, $\boxed{\text{Goslo is older}}$ than Speedo, having aged an additional

$$T_G - T_S = 17.7\ \text{yr} - 12.2\ \text{yr} = \boxed{5.5\ \text{yr}}$$

26.48 (a) The proper lifetime is measured in the ship's reference frame, and Earth-based observers measure a dilated lifetime of

$$\Delta t = \gamma\Delta t_p = \frac{\Delta t_p}{\sqrt{1-\left(v/c\right)^2}} = \frac{15.0\ \text{yr}}{\sqrt{1-(0.700)^2}} = \boxed{21.0\ \text{yr}}$$

(b) $d = v(\Delta t) = \left[0.700\,c\right]\left(21.0\ \text{yr}\right) = \left[(0.700)\left(1.00\ \text{ly/yr}\right)\right]\left(21.0\ \text{yr}\right) = \boxed{14.7\ \text{ly}}$

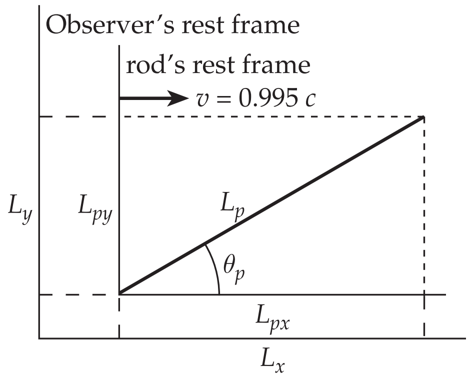
continued on next page

(c) Looking out the rear window, the astronauts see Earth recede at a rate of $v = 0.700c$. The distance it has receded, as measured by the astronauts, when the batteries fail is

$$d' = v(\Delta t_p) = (0.700c)(15.0\text{ yr}) = [(0.700)(1.00\text{ ly/yr})](15.0\text{ yr}) = \boxed{10.5\text{ ly}}$$

(d) Mission control gets signals for 21.0 yr while the battery is operating and then for 14.7 yr after the battery stops powering the transmitter, 14.7 ly away. The total time that signals are received is $21.0\text{ yr} + 14.7\text{ yr} = \boxed{35.7\text{ yr}}$.

26.49 **Note:** Excess digits are retained in some steps given below to more clearly illustrate the method of solution.



We are given that $L = 2.00$ m and $\theta = 30.0°$ (both measured in the observer's rest frame). The components of the rod's length as measured in the observer's rest frame are

$$L_x = L\cos\theta = (2.00\text{ m})\cos 30.0° = 1.732\text{ m}$$

and $L_y = L\sin\theta = (2.00\text{ m})\sin 30.0° = 1.00\text{ m}$

The component of length parallel to the motion has been contracted, but the component perpendicular to the motion is unaltered. Thus, $L_{py} = L_y = 1.00$ m and

$$L_{px} = \frac{L_x}{\sqrt{1-(v/c)^2}} = \frac{1.732\text{ m}}{\sqrt{1-(0.995)^2}} = 17.34\text{ m}$$

(a) The proper length of the rod is then

$$L_p = \sqrt{L_{px}^2 + L_{py}^2} = \sqrt{(17.34\text{ m})^2 + (1.00\text{ m})^2} = \boxed{17.4\text{ m}}$$

(b) The orientation angle in the rod's rest frame is

$$\theta_p = \tan^{-1}\left(\frac{L_{py}}{L_{px}}\right) = \tan^{-1}\left(\frac{1.00\text{ m}}{17.34\text{ m}}\right) = \boxed{3.30°}$$

26.50 (a) Taking toward Earth as the positive direction, the velocity of the ship relative to Earth is $v_{\text{SE}} = +0.600c$, and the velocity of the lander relative to the ship is $v_{\text{LS}} = +0.800c$. The relativistic velocity addition relation (Equation 26.8 in the textbook) then gives the velocity of the lander relative to Earth as

$$v_{\text{LE}} = \frac{v_{\text{LS}} + v_{\text{SE}}}{1 + \dfrac{v_{\text{LS}}v_{\text{SE}}}{c^2}} = \frac{0.800c + 0.600c}{1 + (0.800)(0.600)} = \frac{1.40c}{1.48} = \boxed{+0.946c}$$

(b) Observers at rest on Earth measure the proper length between the ship and Earth as $L_p = 0.200$ ly. The contracted distance measured by observers at rest relative to the spaceship is

$$L = L_p\sqrt{1-(v_{\text{SE}}/c)^2} = (0.200\text{ ly})\sqrt{1-(0.600)^2} = \boxed{0.160\text{ ly}}$$

continued on next page

(c) Observers on the ship see the lander start toward Earth from an initial distance of $L = 0.160$ ly. They see this distance diminish as the lander nibbles into it from one end at $v_{LS} = 0.800c$, and the Earth (as it appears to approach them) reducing it from the other end at $v_{SE} = 0.600c$. The time they compute it will take the lander and Earth to meet is

$$t = \frac{L}{v_{LS} + v_{SE}} = \frac{0.160 \text{ ly}}{1.40c} = \frac{0.160 \text{ yr}}{1.40} = \boxed{0.114 \text{ yr}}$$

(d) The kinetic energy of the lander as observed in the Earth reference frame is $KE = E - E_R = (\gamma - 1)mc^2$, where $\gamma = 1\big/\sqrt{1-(v_{LE}/c)^2}$. This gives

$$KE = \left(\frac{1}{\sqrt{1-(0.946)^2}} - 1\right)\left(4.00\times10^5 \text{ kg}\right)\left(3.00\times10^8 \text{ m/s}\right)^2 = \boxed{7.51\times10^{22} \text{ J}}$$

27

Quantum Physics

QUICK QUIZZES

1. True. When a photon scatters off an electron that was initially at rest, the electron must recoil to conserve momentum. The recoiling electron possesses kinetic energy that was gained from the photon in the scattering process. Thus, the scattered photon must have less energy than the incident photon.

2. Choice (b). Some energy is transferred to the electron in the scattering process. Therefore, the scattered photon must have less energy (and hence, lower frequency) than the incident photon.

3. Choice (c). Conservation of energy requires the kinetic energy given to the electron be equal to the difference between the energy of the incident photon and that of the scattered photon.

4. False. The de Broglie wavelength of a particle of mass m and speed v is $\lambda = h/p = h/mv$. Thus, the wavelength decreases when the momentum increases.

5. Choice (c). Two particles with the same de Broglie wavelength will have the same momentum $p = mv = h/\lambda$. If the electron and proton have the same momentum, they cannot have the same speed because of the difference in their masses. For the same reason, remembering that $KE = p^2/2m$, they cannot have the same kinetic energy. Because the kinetic energy is the only type of energy an isolated particle can have, and we have argued that the particles have different energies, the equation $f = E/h$ tells us that the particles do not have the same frequency.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. According to Einstein's photoelectric effect equation, $e\,\Delta V_s = KE_{\text{max}} = E_{\text{photon}} - \phi$, and the work function of the metal is

$$\phi = E_{\text{photon}} - e\,\Delta V_s = 3.56 \text{ eV} - e(1.10 \text{ V}) = 2.46 \text{ eV}$$

so the correct answer is choice (b).

2. From Wien's displacement law,

$$T = \frac{0.289\,8\times 10^{-2} \text{ m}\cdot\text{K}}{\lambda_{\text{max}}} = \frac{0.289\,8\times 10^{-2} \text{ m}\cdot\text{K}}{475\times 10^{-9} \text{ m}} = 6.10\times 10^{3} \text{ K} = 6\,100 \text{ K}$$

and choice (a) is correct.

3. An electron accelerated from rest through a potential difference of 50.0 V will have a kinetic energy of $KE = 50.0 \text{ eV} \ll E_{R,e} = 0.511$ MeV. Thus, it is classical, and its momentum may be expressed as $p = \sqrt{2m_e(KE)}$. The de Broglie wavelength is then

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e(KE)}} = \frac{h}{\sqrt{2m_e(e\,\Delta V)}} = \frac{6.63\times 10^{-34} \text{ J}\cdot\text{s}}{\sqrt{2\left(9.11\times 10^{-31} \text{ kg}\right)\left(1.60\times 10^{-19} \text{ C}\right)(50.0 \text{ V})}} = 0.174 \text{ nm}$$

and the correct choice is (b).

4. The maximum energy, or minimum wavelength, photon is produced when the electron loses all of its kinetic energy in a single collision. Therefore, $E_{\max} = hc/\lambda_{\min} = KE_i = e\,\Delta V$, or

$$\lambda_{\min} = \frac{hc}{e\,\Delta V} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\left(1.60\times10^{-19}\ \text{C}\right)\left(3.00\ \text{J/C}\right)} = 4.14\times10^{-7}\ \text{m} = 414\ \text{nm}$$

and we see that (c) is the correct choice.

5. One form of Heisenberg's uncertainty relation is $\Delta x \Delta p_x \geq h/4\pi$, which says that one cannot determine both the position and momentum of a particle with arbitrary accuracy. Another form of this relation is $\Delta E \Delta t \geq h/4\pi$, which sets a limit on how accurately the energy can be determined in a finite time interval. Thus, both (a) and (c) are true statements, and the other listed choices are false.

6. From the Compton shift formula, the change in the photon's wavelength is

$$\Delta\lambda = \lambda - \lambda_0 = \frac{h}{m_e c}(1-\cos\theta) = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}(1-\cos 180°)$$

$$= 4.85\times10^{-12}\ \text{m} = 4.85\times10^{-3}\ \text{nm}$$

and choice (d) is the correct answer.

7. The comparative masses of the particles of interest are $m_p \approx 1\,840\,m_e$ and $m_{\text{He}} \approx 4m_p$. Assuming the particles are all classical, their momenta are $p_p = m_p v$, $p_{\text{He}} = m_{\text{He}} v = 4m_p v$, and $p_e = m_e v_e = m_p v/1\,840$. Their de Broglie wavelengths are $\lambda_p = h/p_p = h/m_p v$, $\lambda_{\text{He}} = h/p_{\text{He}} = h/4m_p v = \lambda_p/4$, and $\lambda_e = h/p_e = h/(m_p v/1\,840) = 1\,840\lambda_p$. The ranking, from longest to shortest, of these wavelengths is $\lambda_e > \lambda_p > \lambda_{\text{He}}$, and choice (e) gives the correct order.

8. From Bragg's law, the angle θ at which constructive interference of order m will be observed when x-rays reflect from crystalline planes separated by distance d is given by $2d\sin\theta = m\lambda$. Thus, the separation distance of the planes in the crystal is $d = m\lambda/2\sin\theta$. With m and λ the same for the two crystals, we see that d is inversely proportional to $\sin\theta$. Since it is observed that $\theta_A > \theta_B$ (and hence, $\sin\theta_A > \sin\theta_B$), it must be true that $d_A < d_B$, and the correct answer is (c).

9. Diffraction, polarization, interference, and refraction are all processes associated with waves. However, to understand the photoelectric effect, we must think of the energy transmitted as light coming in discrete packets, or quanta, called photons. Thus, the photoelectric effect most clearly demonstrates the particle nature of light, and the correct choice is (b).

10. Electron diffraction by crystals, first detected by the Davisson-Germer experiment in 1927, confirmed de Broglie's hypothesis and, of the listed choices, most clearly demonstrates the wave nature of electrons. The correct answer is (e).

11. During the scattering process, the photon will transfer some of its energy to the originally stationary electron, and the electron will recoil following this process. Since the energy of a photon is $E_{\text{photon}} = hf$, where h is constant, the frequency f of the photon must decrease when the photon gives up energy. Thus, the correct choice is (a).

12. The magnitude of the charge is $|q| = e$ for both the electron and proton, and since the two particles are accelerated through the same potential difference, they are given identical kinetic energies $KE_e = KE_p = e\Delta V$. The momentum of a non-relativistic particle having kinetic energy $KE = e\Delta V$ and mass m, is $p = \sqrt{2mKE} = \sqrt{2m(e\Delta V)}$. The de Broglie wavelength, $\lambda = h/p$, is then seen to be inversely proportional to $\sqrt{m}$. Since the mass of an electron is smaller than that of a proton, we see that $\lambda_e > \lambda_p$ when the electron and proton have the same kinetic energy. Thus, the correct choice is (a).

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. A microscope can see details no smaller than the wavelength of the waves it uses to produce images. Electrons with kinetic energies of several electron volts have wavelengths of less than a nanometer, which is much smaller than the wavelength of visible light (having wavelengths ranging from about 400 to 700 nm). Therefore, an electron microscope can resolve details of much smaller size as compared to an optical microscope.

4. Measuring the position of a particle implies having photons reflect from it. However, collisions between photons and the particle will alter the velocity of the particle. Thus, the more accurately you try to measure one quantity, the more uncertainty you create in your knowledge of the other.

6. Light has both wave and particle characteristics. In Young's double-slit experiment, light behaves as a wave. In the photoelectric effect, it behaves like a particle. Light can be characterized as an electromagnetic wave with a particular wavelength or frequency, yet at the same time, light can be characterized as a stream of photons, each carrying a discrete energy, hf.

8. Ultraviolet light has a shorter wavelength and higher photon energy, $E_{\text{photon}} = hf = hc/\lambda$, than visible light.

10. Increasing the temperature of the substance increases the average kinetic energy of the electrons inside the material. This makes it slightly easier for an electron to escape from the material when it absorbs a photon.

12. Most stars radiate nearly as blackbodies. Of the two stars, Vega has the higher surface temperature and, in agreement with Wien's displacement law, radiates more intensely at shorter wavelengths and has a bluish appearance, while Arcturus has a more reddish appearance.

14. No, the crystal cannot produce diffracted beams of visible light. The angles where one could expect to observe diffraction maxima are given by Bragg's law, $2d\sin\theta = m\lambda$, where m is a nonzero integer. Because $\sin\theta \le 1$, this equation gives $m = (2d/\lambda)\sin\theta \le 2d/\lambda$. Since the wavelengths of visible light ($\lambda \sim 10^2$ nm) are much larger than the distances between atomic planes in crystals ($d \sim 1$ nm), there are no nonzero integer values for m that can satisfy Bragg's law for visible light.

16. Any object of macroscopic size—including a grain of dust—has an undetectably small wavelength, so any diffraction effects it might exhibit are very small, effectively undetectable. Recall historically how the diffraction of sound waves was at one time well known, but the diffraction of light was not.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** ~ 100 nm, ultraviolet **(b)** $\sim 10^{-1}$ nm, gamma rays

4. **(a)** 5.78×10^3 K **(b)** 501 nm

6. **(a)** 2.90×10^{-19} J/photon **(b)** 4.23×10^3 K **(c)** 1.65×10^{26} W
(d) 5.69×10^{44} photon/s

8. 5.7×10^3 photons/s

10. **(a)** 288 nm **(b)** 1.04×10^{15} Hz **(c)** 1.19 eV

12. **(a)** only lithium **(b)** 0.81 eV

14. **(a)** 1.90 eV **(b)** 0.216 V

16. **(a)** 8.29×10^{-11} m **(b)** 1.24×10^{-11} m **(c)** $\lambda_{\min}$ decreases

18. 0.455 nm

20. $m=2,\ \theta=25.8°;\ m=3,\ \theta=40.8°;\ m=4,\ \theta=60.8°$

Higher orders cannot be found since $\sin\theta$ cannot exceed a value of 1.00.

22. 1.03×10^{-3} nm

24. 4.85×10^{-3} nm

26. **(a)** 0.101 nm **(b)** 80.9°

28. **(a)** 1.99×10^{-11} m **(b)** 1.98×10^{-14} m

30. **(a)** 0.709 nm **(b)** 414 nm

32. **(a)** $p=\sqrt{2mq\Delta V}$ **(b)** $\lambda=h/\sqrt{2mq\Delta V}$

(c) The proton, with the larger mass, will have the shorter wavelength.

34. 23 m/s

36. **(a)** 0.250 m/s **(b)** 2.25 m

38. **(a)** See Solution. **(b)** 5.3 MeV

40. $v=c\sqrt{2}/2$

42. **(a)** 2.49×10^{-11} m **(b)** 0.29 nm

44. **(a)** $\lambda=2.82\times10^{-37}$ m **(b)** $\Delta E\geq1.06\times10^{-32}$ J **(c)** 2.88×10^{-35} %

46. 0.14 keV

48. 0.785 eV

50. **(a)** 148 days

(b) This result is totally contrary to observations of the photoelectric effect.

PROBLEM SOLUTIONS

27.1 From Wien's displacement law,

(a) $$T=\frac{0.289\,8\times10^{-2}\text{ m}\cdot\text{K}}{\lambda_{\max}}=\frac{0.289\,8\times10^{-2}\text{ m}\cdot\text{K}}{970\times10^{-9}\text{ m}}=2.99\times10^{3}\text{ K, or }\boxed{\approx 3\,000\text{ K}}$$

(b) $$T=\frac{0.289\,8\times10^{-2}\text{ m}\cdot\text{K}}{\lambda_{\max}}=\frac{0.289\,8\times10^{-2}\text{ m}\cdot\text{K}}{145\times10^{-9}\text{ m}}=2.00\times10^{4}\text{ K, or }\boxed{\approx 20\,000\text{ K}}$$

27.2 Using Wien's displacement law,

(a) $$\lambda_{max} = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{10^4\ \text{K}} = 2.898\times10^{-7}\ \text{m}\ \boxed{\sim 100\ \text{nm}}\ \boxed{\text{ultraviolet}}$$

(b) $$\lambda_{max} = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{10^7\ \text{K}} = 2.898\times10^{-10}\ \text{m}\ \boxed{\sim 10^{-1}\ \text{nm}}\ \boxed{\gamma\text{-rays}}$$

27.3 From Wien's displacement law,

$$\lambda_{max} = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{T} = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{306\ \text{K}} = 9.47\times10^{-6}\ \text{m} = \boxed{9.47\ \mu\text{m (infrared)}}$$

27.4 (a) The power radiated by an object with surface area A and absolute temperature T is given by Stefan's law (see Chapter 11 of the textbook) as $P = \sigma AeT^4$, where σ is the Stefan-Boltzmann constant equal to $5.669\,6\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4$. For an object that approximates a blackbody, the emissivity is $e = 1$. Thus, the temperature of our Sun's surface should be

$$T = \left[\frac{P}{\sigma Ae}\right]^{\frac{1}{4}} = \left[\frac{P}{\sigma\left(4\pi R_{Sun}^2\right)e}\right]^{\frac{1}{4}} = \left[\frac{3.85\times10^{26}\ \text{W}}{4\pi\left(5.669\,6\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(6.96\times10^8\ \text{m}\right)^2(1)}\right]^{\frac{1}{4}}$$

$$= \boxed{5.78\times10^3\ \text{K}}$$

(b) Wien's displacement law gives the peak wavelength of the radiation from the Sun as

$$\lambda_{max} = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{T} = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{5.78\times10^3\ \text{K}} = 5.01\times10^{-7}\ \text{m} = \boxed{501\ \text{nm}}$$

27.5 The energy of a photon is given by $E = hf = hc/\lambda$, where $h = 6.63\times10^{-34}\ \text{J}\cdot\text{s}$ is Planck's constant.

(a) $$E = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^8\ \text{m/s}\right)}{5.00\times10^{-2}\ \text{m}}\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{2.49\times10^{-5}\ \text{eV}}$$

(b) $$E = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^8\ \text{m/s}\right)}{500\times10^{-9}\ \text{m}}\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{2.49\ \text{eV}}$$

(c) $$E = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^8\ \text{m/s}\right)}{5.00\times10^{-9}\ \text{m}}\left(\frac{1.00\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{249\ \text{eV}}$$

27.6 (a) $$E_{peak} = \frac{hc}{\lambda_{max}} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^8\ \text{m/s}\right)}{685\times10^{-9}\ \text{m}} = \boxed{2.90\times10^{-19}\ \text{J/photon}}$$

(b) $$T = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{\lambda_{max}} = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{685\times10^{-9}\ \text{m}} = \boxed{4.23\times10^3\ \text{K}}$$

(c) The surface area of the spherical star is $A = 4\pi r^2$, and (considering it to be a blackbody) its emissivity is $e = 1$. Stefan's law (see Chapter 11 of the textbook) gives the radiated power as

$$P = \sigma AeT^4 = \sigma\left(4\pi r^2\right)eT^4$$

$$= 4\pi\left(5.669\,6\times10^{-8}\ \text{W/m}^2\cdot\text{K}^4\right)\left(8.50\times10^8\ \text{m}\right)^2(1)\left(4.23\times10^3\ \text{K}\right)^4 = \boxed{1.65\times10^{26}\ \text{W}}$$

continued on next page

(d) Assuming the average energy of an emitted photon equals the energy of photons at the peak of the radiation distribution, the number of photons emitted by the star each second is

$$\frac{\Delta N}{\Delta t}=\frac{P}{E_{\text{peak}}}=\frac{1.65\times10^{26}\ \text{J/s}}{2.90\times10^{-19}\ \text{J/photon}}=\boxed{5.69\times10^{44}\ \text{photon/s}}$$

27.7 The energy of a photon having frequency f is given by $E_{\text{photon}}=hf$, where Planck's constant has a value of $h=6.63\times10^{-34}\ \text{J}\cdot\text{s}$. This energy may be converted to units of electron volts by use of the conversion factor $1\ \text{eV}=1.60\times10^{-19}\ \text{J}$.

(a) $$E_{\text{photon}}=hf=\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(620\times10^{12}\ \text{Hz}\right)\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)=\boxed{2.57\ \text{eV}}$$

(b) $$E_{\text{photon}}=hf=\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.10\times10^{9}\ \text{Hz}\right)\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)=\boxed{1.28\times10^{-5}\ \text{eV}}$$

(c) $$E_{\text{photon}}=hf=\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(46.0\times10^{6}\ \text{Hz}\right)\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)=\boxed{1.91\times10^{-7}\ \text{eV}}$$

27.8 The energy entering the eye each second is $P=I\cdot A$, where A is the area of the pupil. If the light has wavelength λ, the energy of a single photon is $E_{\text{photon}}=hc/\lambda$. Hence, the number of photons entering the eye in time Δt is given by $N=\Delta E/E_{\text{photon}}=P\cdot(\Delta t)/E_{\text{photon}}$, or $N=IA(\Delta t)/(hc/\lambda)=IA(\Delta t)\lambda/hc$. With $I=4.0\times10^{-11}\ \text{W/m}^2$, $\lambda=500$ nm, $\Delta t=1.0$ s, and the opening of the pupil having an area of $A=\pi d^2/4$, where d is the pupil diameter, we find

$$\frac{N}{\Delta t}=\frac{\left(4.0\times10^{-11}\ \text{J/s}\cdot\text{m}^2\right)\left[\pi\left(8.5\times10^{-3}\ \text{m}\right)^2/4\right]\left(500\times10^{-9}\ \text{m}\right)}{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}=\boxed{5.7\times10^{3}\ \text{photons/s}}$$

27.9 (a) From the photoelectric effect equation, the work function is $\phi=hc/\lambda-KE_{\text{max}}$, or

$$\phi=\frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{350\times10^{-9}\ \text{m}}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)-1.31\ \text{eV}=\boxed{2.24\ \text{eV}}$$

(b) $$\lambda_c=\frac{hc}{\phi}=\frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{2.24\ \text{eV}}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right)=5.55\times10^{-7}\ \text{m}=\boxed{555\ \text{nm}}$$

(c) $$f_c=\frac{c}{\lambda_c}=\frac{3.00\times10^{8}\ \text{m/s}}{555\times10^{-9}\ \text{m}}=\boxed{5.41\times10^{14}\ \text{Hz}}$$

27.10 (a) At the cutoff wavelength, $KE_{\text{max}}=0$, so the photoelectric effect equation ($KE_{\text{max}}=hc/\lambda-\phi$) gives the cutoff wavelength as

$$\lambda_c=\frac{hc}{\phi}=\frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{4.31\ \text{eV}}\left(\frac{1\ \text{ev}}{1.60\times10^{-19}\ \text{J}}\right)=2.88\times10^{-7}\ \text{m}=\boxed{288\ \text{nm}}$$

(b) The lowest frequency of light that will free electrons from the material is $f_c=c/\lambda_c$, where λ_c is the cutoff wavelength (calculated above). Thus,

$$f_c=\frac{c}{\lambda_c}=\frac{3.00\times10^{8}\ \text{m/s}}{2.88\times10^{-7}\ \text{m}}=\boxed{1.04\times10^{15}\ \text{Hz}}$$

continued on next page

(c) The photoelectric effect equation may be written as $KE_{\max} = E_{\text{photon}} - \phi$. Therefore, if the photons incident on this surface have energy $E_{\text{photon}} = 5.50$ eV, the maximum kinetic energy of the ejected electrons is

$$KE_{\max} = E_{\text{photon}} - \phi = 5.50 \text{ eV} - 4.31 \text{ eV} = \boxed{1.19 \text{ eV}}$$

27.11 (a) $\phi = 6.35 \text{ eV}\left(\dfrac{1.60\times10^{-19} \text{ J}}{1 \text{ eV}}\right) = \boxed{1.02\times10^{-18} \text{ J}}$

(b) The energy of a photon having the cutoff frequency or cutoff wavelength equals the work function of the surface, or $E_{\text{photon}} = hf_c = hc/\lambda_c = \phi$. Thus, the cutoff frequency of a surface having a work function of $\phi = 6.35$ eV is

$$f_c = \frac{\phi}{h} = \frac{6.35 \text{ eV}}{6.63\times10^{-34} \text{ J}\cdot\text{s}}\left(\frac{1.60\times10^{-19} \text{ J}}{1 \text{ eV}}\right) = \boxed{1.53\times10^{15} \text{ Hz}}$$

(c) The cutoff wavelength is

$$\lambda_c = \frac{hc}{\phi} = \frac{c}{f_c} = \frac{3.00\times10^{8} \text{ m/s}}{1.53\times10^{15} \text{ Hz}} = 1.96\times10^{-7} \text{ m} = \boxed{196 \text{ nm}}$$

(d) $KE_{\max} = E_{\text{photon}} - \phi = 8.50 \text{ eV} - 6.35 \text{ eV} = \boxed{2.15 \text{ eV}}$

(e) $eV_s = KE_{\max}$, so the stopping potential is $V_s = KE_{\max}/e = 2.15 \text{ eV}/e = \boxed{2.15 \text{ V}}$.

27.12 (a) The energy of the incident photons is

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34} \text{ J}\cdot\text{s}\right)\left(3.00\times10^{8} \text{ m/s}\right)}{400\times10^{-9} \text{ m}}\left(\frac{1 \text{ eV}}{1.60\times10^{-19} \text{ J}}\right) = 3.11 \text{ eV}$$

For photoelectric emission to occur, it is necessary that $E_{\text{photon}} \geq \phi$. Thus, of the three metals given, $\boxed{\text{only lithium}}$ will exhibit the photoelectric effect.

(b) For lithium, $KE_{\max} = E_{\text{photon}} - \phi = 3.11 \text{ eV} - 2.30 \text{ eV} = \boxed{0.81 \text{ eV}}$.

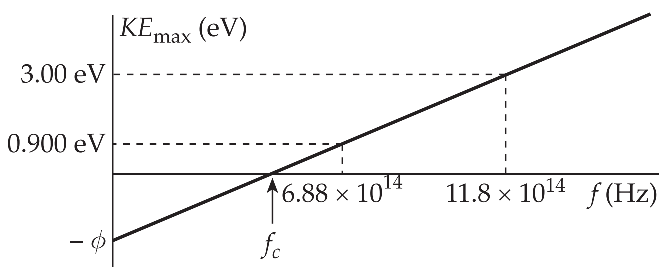
27.13 The two frequencies of the light allowed to strike the surface are

$$f_1 = \frac{c}{\lambda_1} = \frac{3.00\times10^{8} \text{ m/s}}{254\times10^{-9} \text{ m}} = 1.18\times10^{15} \text{ Hz} = 11.8\times10^{14} \text{ Hz}$$

and $$f_2 = \frac{c}{\lambda_2} = \frac{3.00\times10^{8} \text{ m/s}}{436\times10^{-9} \text{ m}} = 6.88\times10^{14} \text{ Hz}$$

The graph you draw should look somewhat like that given at the right.

The desired quantities, read from the axes intercepts of the graph line, should agree within their uncertainties with



$f_c = \boxed{4.8\times10^{14} \text{ Hz}}$ and $\phi = \boxed{2.0 \text{ eV}}$

27.14 (a) The maximum kinetic energy of the ejected electrons is related to the stopping potential by the expression $KE_{max} = e(\Delta V_s)$. Thus, if the stopping potential is $V_s = 0.376$ V when the incident light has wavelength $\lambda = 546.1$ nm, the photoelectric effect equation gives the work function of this metal as

$$\phi = \frac{hc}{\lambda} - KE_{max} = \frac{hc}{\lambda} - e(\Delta V_s)$$

$$= \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{546.1\times10^{-9}\ \text{m}}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) - 0.376\ \text{eV} = \boxed{1.90\ \text{eV}}$$

(b) If light of wavelength $\lambda = 587.5$ nm is incident on this metal, the maximum kinetic energy of the ejected electrons is

$$e(\Delta V_s) = KE_{max} = \frac{hc}{\lambda} - \phi$$

$$= \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{587.5\times10^{-9}\ \text{m}}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) - 1.90\ \text{eV} = 0.216\ \text{eV}$$

Thus, the stopping potential will be

$$\Delta V_s = \frac{KE_{max}}{e} = \frac{0.216\ \text{eV}}{e} = \boxed{0.216\ \text{V}}$$

27.15 Assuming the electron produces a single photon as it comes to rest, the energy of that photon is $E_{photon} = (KE)_i = e(\Delta V)$. The accelerating voltage is then

$$\Delta V = \frac{E_{photon}}{e} = \frac{hc}{e\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})\lambda} = \frac{1.24\times10^{-6}\ \text{V}\cdot\text{m}}{\lambda}$$

For $\lambda = 1.0\times10^{-8}$ m, $V = \dfrac{1.24\times10^{-6}\ \text{V}\cdot\text{m}}{1.0\times10^{-8}\ \text{m}} = \boxed{1.2\times10^{2}\ \text{V}}$

and for $\lambda = 1.0\times10^{-13}$ m, $V = \dfrac{1.24\times10^{-6}\ \text{V}\cdot\text{m}}{1.0\times10^{-13}\ \text{m}} = \boxed{1.2\times10^{7}\ \text{V}}$

27.16 A photon of maximum energy and minimum wavelength is produced when the electron gives up all its kinetic energy in a single collision, or $\lambda_{min} = hc/KE = hc/e\,\Delta V$.

(a) If $\Delta V = 15.0$ kV, $\lambda_{min} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(15.0\times10^{3}\ \text{V})} = \boxed{8.29\times10^{-11}\ \text{m}}$.

(b) If $\Delta V = 100$ kV, $\lambda_{min} = \dfrac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(100\times10^{3}\ \text{V})} = \boxed{1.24\times10^{-11}\ \text{m}}$.

(c) As seen above, $\boxed{\lambda_{min}\ \text{decreases}}$ when the potential difference, ΔV, increases.

27.17 A photon of maximum energy or minimum wavelength is produced when the electron gives up all of its kinetic energy in a single collision within the target. Thus, $E_{max} = hc/\lambda_{min} = KE_e = e\,\Delta V$. If $\lambda_{min} = 70.0\ \text{pm} = 70.0\times10^{-12}$ m, the required accelerating voltage is

$$\Delta V = \frac{hc}{e\lambda_{min}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(1.60\times10^{-19}\ \text{C})(70.0\times10^{-12}\ \text{m})} = 1.78\times10^{4}\ \text{V} = \boxed{17.8\ \text{kV}}$$

27.18 From the Bragg equation, $2d\sin\theta = m\lambda$, the interplanar spacing when 0.129-nm x-rays produce a first order diffraction maximum at $\theta = 8.15°$ is

$$d = \frac{m\lambda}{2\sin\theta} = \frac{(1)(0.129\text{ nm})}{2\sin 8.15°} = \boxed{0.455\text{ nm}}$$

27.19 Using Bragg's law, the wavelength is found to be

$$\lambda = \frac{2d\sin\theta}{m} = \frac{2(0.296\text{ nm})\sin 7.6°}{1} = \boxed{0.078\text{ nm}}$$

27.20 From the Bragg equation, $2d\sin\theta = m\lambda$, the angle at which the diffraction maximum of order m will be found when the first order maximum is at $\theta_1 = 12.6°$, is given by

$$\sin\theta_m = m(\lambda/2d) = m\sin\theta_1 = m\sin(12.6°)$$

Thus, additional orders will be found at

$m = 2:$	$\sin\theta_2 = 2\sin 12.6° = 0.436$	and	$\theta_2 = \sin^{-1}(0.436) = \boxed{25.8°}$
$m = 3:$	$\sin\theta_3 = 3\sin 12.6° = 0.654$	and	$\theta_3 = \sin^{-1}(0.654) = \boxed{40.8°}$
$m = 4:$	$\sin\theta_4 = 4\sin 12.6° = 0.873$	and	$\theta_4 = \sin^{-1}(0.873) = \boxed{60.8°}$
$m = 5:$	$\sin\theta_5 = 5\sin 12.6° = 1.09$	Impossible, since $\sin\theta \le 1$ for all θ.	

$\boxed{\text{No orders higher than } m = 4 \text{ will be seen}}$ because it is mathematically impossible for the $\sin\theta$ to be greater than one.

27.21 The interplanar spacing in the crystal is given by Bragg's law as

$$d = \frac{m\lambda}{2\sin\theta} = \frac{(1)(0.140\text{ nm})}{2\sin 14.4°} = \boxed{0.281\text{ nm}}$$

27.22 From the Compton shift equation, the wavelength shift of the scattered x-rays is

$$\Delta\lambda = \frac{h}{m_e c}(1-\cos\theta) = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{(9.11\times10^{-31}\text{ kg})(3.00\times10^{8}\text{ m/s})}(1-\cos 55.0°)$$

$$= 1.03\times10^{-12}\text{ m}\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right) = \boxed{1.03\times10^{-3}\text{ nm}}$$

27.23 If the scattered photon has energy equal to the kinetic energy of the recoiling electron, the energy of the incident photon is divided equally between them. Thus,

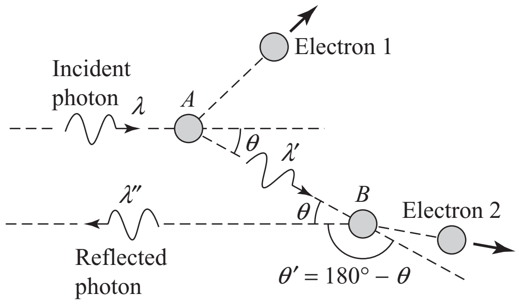
$$E_{\text{photon}} = \frac{(E_0)_{\text{photon}}}{2} \Rightarrow \frac{hc}{\lambda} = \frac{hc}{2\lambda_0},\text{ so } \lambda = 2\lambda_0 \text{ and } \Delta\lambda = 2\lambda_0 - \lambda_0 = \lambda_0 = 0.001\,60\text{ nm}$$

The Compton scattering formula, $\Delta\lambda = \lambda_C(1-\cos\theta)$, then gives the scattering angle as

$$\theta = \cos^{-1}\left(1 - \frac{\Delta\lambda}{\lambda_C}\right) = \cos^{-1}\left(1 - \frac{0.001\,60\text{ nm}}{0.002\,43\text{ nm}}\right) = \boxed{70.0°}$$

27.24 At point A, the incident photon scatters at angle θ from the first electron. The shift in wavelength that occurs in this scattering process is given by the Compton equation as

$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos\theta)$$



At point B, this intermediate photon scatters from the second electron at angle $\theta' = 180° - \theta$. Since $\cos(180° - \theta) = \cos 180° \cos\theta + \sin 180° \sin\theta = -\cos\theta$, the Compton equation gives the wavelength shift that occurs in the scattering at B as

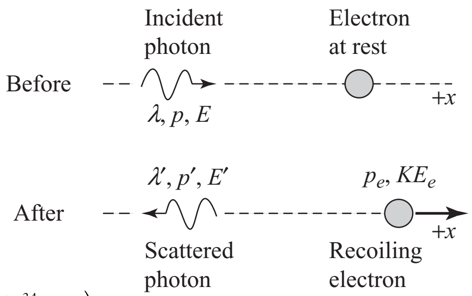
$$\lambda'' - \lambda' = \frac{h}{m_e c}(1 + \cos\theta)$$

Hence, the total wavelength shift that occurs in the two scattering processes is given by

$$\lambda'' - \lambda = (\lambda'' - \lambda') + (\lambda' - \lambda) = \frac{h}{m_e c}(1 + \cos\theta) + \frac{h}{m_e c}(1 - \cos\theta) = \frac{2h}{m_e c}$$

or $$\lambda'' - \lambda = \frac{2(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})}{(9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^{8}\ \text{m/s})} = 4.85 \times 10^{-12}\ \text{m} = \boxed{4.85 \times 10^{-3}\ \text{nm}}$$

27.25 The figure at the right shows the situation before and after the scattering process. Note that the scattering angle is $\theta = 180°$, so the Compton equation gives



$$\lambda' - \lambda = \frac{h}{m_e c}(1 - \cos 180°) = \frac{2h}{m_e c}$$

$$\lambda' = \lambda + \frac{2h}{m_e c} = 0.110 \times 10^{-9}\ \text{m} + \frac{2(6.63 \times 10^{-34}\ \text{J}\cdot\text{s})}{(9.11 \times 10^{-31}\ \text{kg})(3.00 \times 10^{8}\ \text{m/s})} = 0.115 \times 10^{-9}\ \text{m}$$

(a) The momentum of the incident photon is $p = h/\lambda$, while that of the scattered photon is $p' = -h/\lambda'$ (the negative sign is included since momentum is a vector quantity and the scattered photon travels in the negative x-direction). Thus, conservation of momentum gives $p_e - h/\lambda' = h/\lambda + 0$, or the momentum of the recoiling electron is

$$p_e = \frac{h}{\lambda} + \frac{h}{\lambda'} = (6.63 \times 10^{-34}\ \text{J}\cdot\text{s})\left(\frac{1}{0.110 \times 10^{-9}\ \text{m}} + \frac{1}{0.115 \times 10^{-9}\ \text{m}}\right)$$

$$= \boxed{1.18 \times 10^{-23}\ \text{kg}\cdot\text{m/s}}$$

(b) Assuming the recoiling electron is nonrelativistic, its kinetic energy is given by

$$KE_e = \frac{p_e^2}{2m_e} = \frac{(1.18 \times 10^{-23}\ \text{kg}\cdot\text{m/s})^2}{2(9.11 \times 10^{-31}\ \text{kg})} = 7.64 \times 10^{-17}\ \text{J}\left(\frac{1\ \text{eV}}{1.60 \times 10^{-19}\ \text{J}}\right) = \boxed{478\ \text{eV}}$$

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Note that this energy is very small in comparison to the rest energy of an electron. Thus, our assumption that the recoiling electron would be non-relativistic is seen to be valid.

27.26 First, observe that $v = 2.18 \times 10^6 \text{ m/s} << c$. Thus, the recoiling electron is nonrelativistic, and its kinetic energy is $KE_e = \frac{1}{2}m_e v^2$, while its momentum is given by $p_e = m_e v$.

(a) The Compton equation gives the wavelength of the scattered photon in terms of that of the incident photon as

$$\lambda = \lambda_0 + \frac{h}{m_e c}(1-\cos\theta) \qquad \textbf{[1]}$$

Conservation of energy gives the kinetic energy of the recoiling electron as $\frac{1}{2}m_e v^2 = hc/\lambda_0 - hc/\lambda = hc(\lambda - \lambda_0)/\lambda_0\lambda$. Substituting Equation [1] into this yields

$$\frac{1}{2}m_e v^2 = \frac{hc\left[(h/m_e c)(1-\cos\theta)\right]}{\lambda_0\left[\lambda_0 + (h/m_e c)(1-\cos\theta)\right]}$$

or $$\lambda_0\left[\lambda_0 + (h/m_e c)(1-\cos\theta)\right] = 2\left(\frac{h}{m_e v}\right)^2(1-\cos\theta)$$

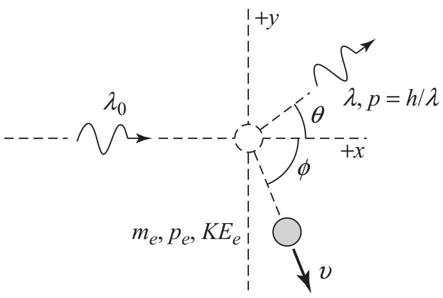
Simplifying, this equation is seen to be of the form $a\lambda_0^2 + b\lambda_0 + c = 0$, where $a = 1$, and the constants b and c are as computed below:

$$b = \frac{h}{m_e c}(1-\cos\theta) = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(1-\cos 17.4°)}{(9.11\times10^{-31}\text{ kg})(3.00\times10^8\text{ m/s})} = 1.11\times10^{-13}\text{ m}$$

$$c = -2\left(\frac{h}{m_e v}\right)^2(1-\cos\theta) = \frac{-2(6.63\times10^{-34}\text{ J}\cdot\text{s})^2(1-\cos 17.4°)}{(9.11\times10^{-31}\text{ kg})^2(2.18\times10^6\text{ m/s})^2} = -1.02\times10^{-20}\text{ m}^2$$

From the quadratic formula, $\lambda_0 = \left[-b \pm \sqrt{b^2 - 4ac}\right]/2a$, using only the upper sign since λ_0 must be positive, we find $\lambda_0 = 1.01\times10^{-10}\text{ m} = \boxed{0.101\text{ nm}}$.

(b) Choosing the x-axis to be the direction of the incident photon's motion, the initial momentum in the y-direction is zero. Thus, conserving momentum in the y-direction gives



$$p_e \sin\phi = p\sin\theta$$

where $\theta = 17.4°$, $p_e = m_e v$, and $p = h/\lambda$. From Equation [1] and the result of part (a), the wavelength of the scattered photon is

$$\lambda = \lambda_0 + \frac{h}{m_e c}(1-\cos\theta) = 1.01\times10^{-10}\text{ m} + \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(1-\cos 17.4°)}{(9.11\times10^{-31}\text{ kg})(3.00\times10^8\text{ m/s})}$$

or $\lambda = 1.011\times10^{-10}$ m. Thus, the scattering angle for the electron is $\phi = \sin^{-1}(p\sin\theta/p_e) = \sin^{-1}(h\sin\theta/\lambda m_e v)$, which gives

$$\phi = \sin^{-1}\left[\frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})\sin 17.4°}{(1.011\times10^{-10}\text{ m})(9.11\times10^{-31}\text{ kg})(2.18\times10^6\text{ m/s})}\right] = \boxed{80.9°}$$

27.27 (a) From $\lambda = h/p = h/mv$, the speed is

$$v = \frac{h}{m_e\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(5.00\times10^{-7}\ \text{m}\right)} = 1.46\times10^{3}\ \text{m/s} = \boxed{1.46\ \text{km/s}}$$

(b) $$\lambda = \frac{h}{m_e v} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.00\times10^{7}\ \text{m/s}\right)} = \boxed{7.28\times10^{-11}\ \text{m}}$$

27.28 The de Broglie wavelength of a particle of mass m is $\lambda = h/p$, where the momentum is given by $p = \gamma mv = mv\big/\sqrt{1-(v/c)^2}$. Note that when the particle is not relativistic, then $\gamma \approx 1$, and this relativistic expression for momentum reverts back to the classical expression.

(a) For a proton moving at speed $v = 2.00\times10^{4}\ \text{m/s}$, $v << c$ and $\gamma \approx 1$, so

$$\lambda = \frac{h}{m_p v} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(1.67\times10^{-27}\ \text{kg}\right)\left(2.00\times10^{4}\ \text{m/s}\right)} = \boxed{1.99\times10^{-11}\ \text{m}}$$

(b) For a proton moving at speed $v = 2.00\times10^{7}\ \text{m/s}$,

$$\lambda = \frac{h}{\lambda m_p v} = \frac{h}{m_p v}\sqrt{1-(v/c)^2}$$

$$= \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)}{\left(1.67\times10^{-27}\ \text{kg}\right)\left(2.00\times10^{7}\ \text{m/s}\right)}\sqrt{1-\left(\frac{2.00\times10^{7}\ \text{m/s}}{3.00\times10^{8}\ \text{m/s}}\right)^2} = \boxed{1.98\times10^{-14}\ \text{m}}$$

27.29 For relativistic particles, $p = \dfrac{\sqrt{E^2 - E_R^2}}{c}$ and $\lambda = \dfrac{h}{p} = \dfrac{hc}{\sqrt{E^2 - E_R^2}}$.

For 3.00 MeV electrons, $E = KE + E_R = 3.00\ \text{MeV} + 0.511\ \text{MeV} = 3.51\ \text{MeV}$, so

$$\lambda = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{\sqrt{(3.51\ \text{MeV})^2 - (0.511\ \text{MeV})^2}}\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right) = \boxed{3.58\times10^{-13}\ \text{m}}$$

27.30 (a) A 3.00 eV electron is nonrelativistic and its momentum is $p = \sqrt{2m_e KE}$, so its wavelength is

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2m_e KE}} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\sqrt{2\left(9.11\times10^{-31}\ \text{kg}\right)(3.00\ \text{eV})\left(1.60\times10^{-19}\ \text{J/1 eV}\right)}}$$

$$= 7.09\times10^{-10}\ \text{m} = \boxed{0.709\ \text{nm}}$$

(b) A photon has energy of $E_{\text{photon}} = hc/\lambda$, so its wavelength is $\lambda = hc/E_{\text{photon}}$. Because a photon has zero rest energy, all of its energy is kinetic energy. In this case, $E_{\text{photon}} = 3.00$ eV, and the wavelength is

$$\lambda = \frac{hc}{E_{\text{photon}}} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{(3.00\ \text{eV})\left(1.60\times10^{-19}\ \text{J/1 eV}\right)} = 4.14\times10^{-7}\ \text{m} = \boxed{414\ \text{nm}}$$

27.31 (a) The required electron momentum is

$$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{1.0\times10^{-11}\ \text{m}}\left(\frac{1\ \text{keV}}{1.60\times10^{-16}\ \text{J}}\right) = 4.1\times10^{-7}\ \frac{\text{keV}\cdot\text{s}}{\text{m}}$$

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and the total energy is

$$E = \sqrt{p^2c^2 + E_R^2}$$

$$= \sqrt{\left(4.1\times10^{-7}\ \frac{\text{keV}\cdot\text{s}}{\text{m}}\right)^2 \left(3.00\times10^8\ \text{m/s}\right)^2 + \left(511\ \text{keV}\right)^2} = 526\ \text{keV}$$

The kinetic energy is then

$$KE = E - E_R = 526\ \text{keV} - 511\ \text{keV} = \boxed{15\ \text{keV}}$$

(b) $$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^8\ \text{m/s}\right)}{1.0\times10^{-11}\ \text{m}}\left(\frac{1\ \text{keV}}{1.60\times10^{-16}\ \text{J}}\right) = \boxed{1.2\times10^2\ \text{keV}}$$

27.32 (a) From conservation of energy, the increase in kinetic energy must equal the decrease in potential energy, or $KE - 0 = q\,\Delta V$. Since the particle is nonrelativistic, its kinetic energy and momentum are related by the expression $KE = p^2/2m$. Thus, the momentum is $p^2 = 2m(KE) = 2mq\,\Delta V$, and $\boxed{p = \sqrt{2mq\,\Delta V}}$.

(b) The de Broglie wavelength is $\lambda = h/p$. Therefore, using the result of part (a) gives $\boxed{\lambda = h/\sqrt{2mq\,\Delta V}}$.

(c) The electron and proton have the same magnitude charge. Thus, when both are accelerated through the same magnitude potential difference, the resulting de Broglie wavelengths are inversely proportional to the square root of the masses of the particles. $\boxed{\text{The proton (with the larger mass) will have the shorter wavelength}}$.

27.33 From the uncertainty principle, the minimum uncertainty in the momentum of the electron is

$$\Delta p_x = \frac{h}{4\pi(\Delta x)} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(0.10\times10^{-9}\ \text{m}\right)} = 5.3\times10^{-25}\ \text{kg}\cdot\text{m/s}$$

so the uncertainty in the speed of the electron is

$$\Delta v_x = \frac{\Delta p_x}{m_e} = \frac{5.3\times10^{-25}\ \text{kg}\cdot\text{m/s}}{9.11\times10^{-31}\ \text{kg}} = 5.8\times10^5\ \text{m/s} \quad\text{or}\quad \sim 10^6\ \text{m/s}$$

If the speed is on the order of the uncertainty in the speed, then $\boxed{v \sim 10^6\ \text{m/s}}$.

27.34 Assuming the electron is nonrelativistic, the uncertainty in its momentum is $\Delta p = m_e\,\Delta v$. From the uncertainty principle, $\Delta x\,\Delta p_x \geq h/4\pi$, so if there is an uncertainty of $\Delta x = 2.5\ \mu\text{m} = 2.5\times10^{-6}\ \text{m}$ in the position of the electron, the minimum uncertainty in its speed is

$$\left(\Delta v_x\right)_{\min} = \frac{\left(\Delta p_x\right)_{\min}}{m_e} = \frac{h}{4\pi m_e\,(\Delta x)_{\max}} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(9.11\times10^{-31}\ \text{kg}\right)\left(2.5\times10^{-6}\ \text{m}\right)} = \boxed{23\ \text{m/s}}$$

27.35 The uncertainty in the magnitude of the velocity of each particle is

$$\Delta v_x = v_x\cdot(0.010\,0\%) = (500\ \text{m/s})\left(0.010\,0\times10^{-2}\right) = 5.00\times10^{-2}\ \text{m/s}$$

If the mass is known precisely, the uncertainty in momentum is $\Delta p_x = m(\Delta v_x)$, and the minimum uncertainty in position is $\Delta x_{\min} = h/4\pi(\Delta p_x) = h/4\pi m(\Delta v_x)$.

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For the electron:

$$\Delta x_{\min} = \frac{h}{4\pi m_e(\Delta v_x)} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(9.11\times10^{-31}\ \text{kg}\right)\left(5.00\times10^{-2}\ \text{m/s}\right)}$$

$$= 1.16\times10^{-3}\ \text{m} = \boxed{1.16\ \text{mm}}$$

For the bullet:

$$\Delta x_{\min} = \frac{h}{4\pi m(\Delta v_x)} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(0.020\,0\ \text{kg}\right)\left(5.00\times10^{-2}\ \text{m/s}\right)} = \boxed{5.28\times10^{-32}\ \text{m}}$$

27.36 (a) With uncertainty Δx in position, the minimum uncertainty in the speed is

$$\left(\Delta v_x\right)_{\min} = \frac{\left(\Delta p_x\right)_{\min}}{m} = \frac{h}{4\pi m(\Delta x)} = \frac{2\pi\ \text{J}\cdot\text{s}}{4\pi\left(2.00\ \text{kg}\right)\left(1.00\ \text{m}\right)} = \boxed{0.250\ \text{m/s}}$$

(b) If we knew Fuzzy's initial position and velocity exactly, his final position after an elapsed time t would be given by $x_f = x_i + v_x t$. Since we have uncertainty in both the initial position and the speed, the uncertainty in the final position is

$$\Delta x_f = \Delta x_i + \left(\Delta v_x\right)t = 1.00\ \text{m} + \left(0.250\ \text{m/s}\right)\left(5.00\ \text{s}\right) = \boxed{2.25\ \text{m}}$$

27.37 The maximum time one can use in measuring the energy of the particle is equal to the lifetime of the particle, or $\Delta t_{\max} \approx 2\ \mu\text{s}$. One form of the uncertainty principle is $\Delta E\,\Delta t \geq h/4\pi$. Thus, the minimum uncertainty one can have in the measurement of a muon's energy is

$$\Delta E_{\min} = \frac{h}{4\pi\,\Delta t_{\max}} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(2\times10^{-6}\ \text{s}\right)} = \boxed{3\times10^{-29}\ \text{J}}$$

27.38 (a) For a nonrelativistic particle, $KE = \frac{1}{2}mv^2 = (mv)^2/2m = \boxed{p^2/2m}$.

(b) From the uncertainty principle,

$$\Delta p_x \geq \frac{h}{4\pi(\Delta x)} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(1.0\times10^{-15}\ \text{m}\right)} = 5.3\times10^{-20}\ \text{kg}\cdot\text{m/s}$$

Since the momentum must be at least as large as its own uncertainty, the minimum kinetic energy is

$$KE_{\min} = \frac{p_{\min}^2}{2m} = \frac{\left(5.3\times10^{-20}\ \text{kg}\cdot\text{m/s}\right)^2}{2\left(1.67\times10^{-27}\ \text{kg}\right)}\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right) = \boxed{5.3\ \text{MeV}}$$

27.39 (a) $$p = \frac{h}{\lambda} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{3.00\times10^{-2}\ \text{m}} = \boxed{2.21\times10^{-32}\ \text{kg}\cdot\text{m/s}}$$

(b) $$f = \frac{c}{\lambda} = \frac{3.00\times10^{8}\ \text{m/s}}{3.00\times10^{-2}\ \text{m}} = \boxed{1.00\times10^{10}\ \text{Hz}}$$

(c) $$E = hf = \left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(1.00\times10^{10}\ \text{Hz}\right)$$

$$= 6.63\times10^{-24}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{4.14\times10^{-5}\ \text{eV}}$$

27.40 The de Broglie wavelength is $\lambda = h/p$, and the Compton wavelength is $\lambda_c = h/m_e c$. Thus, if $\lambda = \lambda_c$, it is necessary that $p = m_e c$. The relativistic expression for the momentum of an electron is $p = \gamma m_e v$, so if $p = m_e c$, we must have $\gamma \cancel{m_e} v = \cancel{m_e} c$, or $v/c = 1/\gamma = \sqrt{1-(v/c)^2}$.

Squaring both sides of this result gives $(v/c)^2 = 1-(v/c)^2$, or $2(v/c)^2 = 1$, and

$$v = \frac{c}{\sqrt{2}} = \boxed{\frac{c\sqrt{2}}{2}}$$

27.41 The total mechanical energy of this object is

$$E_{\text{mech}} = KE_f = PE_i = mgy_i = (2.0\ \text{kg})(9.80\ \text{m/s}^2)(5.0\ \text{m}) = 98\ \text{J}$$

A photon of light having wavelength $\lambda = 5.0\times10^{-7}$ m has an energy of

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{5.0\times10^{-7}\ \text{m}} = 4.0\times10^{-19}\ \text{J}$$

Thus, if all the initial gravitational potential energy of the object were converted to light with $\lambda = 5.0\times10^{-7}$ m, the number of photons that would be produced is

$$n = \frac{E_{\text{mech}}}{E_{\text{photon}}} = \frac{98\ \text{J}}{4.0\times10^{-19}\ \text{J/photon}} = \boxed{2.5\times10^{20}\ \text{photons}}$$

27.42 (a) Minimum wavelength photons are produced when an electron gives up all its kinetic energy in a single collision. Then, $E_{\text{photon}} = 50\,000$ eV and

$$\lambda_{\min} = \frac{hc}{E_{\text{photon}}} = \frac{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})}{(5.00\times10^{4}\ \text{eV})(1.60\times10^{-19}\ \text{J/eV})} = \boxed{2.49\times10^{-11}\ \text{m}}$$

(b) From Bragg's law, the interplanar spacing is

$$d = \frac{m\lambda}{2\sin\theta} = \frac{(1)(2.49\times10^{-11}\ \text{m})}{2\sin(2.5^\circ)} = 2.9\times10^{-10}\ \text{m} = \boxed{0.29\ \text{nm}}$$

27.43 (a) The peak radiation occurs at approximately 560 nm wavelength. From Wien's displacement law,

$$T = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{\lambda_{\max}} = \frac{0.289\,8\times10^{-2}\ \text{m}\cdot\text{K}}{560\times10^{-9}\ \text{m}} \approx \boxed{5\,200\ \text{K}}$$

(b) $\boxed{\text{Clearly, a firefly is not at this temperature, so this is not blackbody radiation}}$.

27.44 (a) From $v^2 = v_0^2 + 2a_y(\Delta y)$, Johnny's speed just before impact is

$$v = \sqrt{2g|\Delta y|} = \sqrt{2(9.80\ \text{m/s})(50.0\ \text{m})} = 31.3\ \text{m/s}$$

and his de Broglie wavelength is

$$\lambda = \frac{h}{mv} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{(75.0\ \text{kg})(31.3\ \text{m/s})} = \boxed{2.82\times10^{-37}\ \text{m}}$$

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(b) The energy uncertainty is

$$\Delta E \geq \frac{h}{4\pi(\Delta t)} = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{4\pi\left(5.00\times10^{-3}\ \text{s}\right)} = \boxed{1.06\times10^{-32}\ \text{J}}$$

(c) $$\%\ \text{error} = \frac{\Delta E}{mg|\Delta y|}(100\%) \geq \frac{\left(1.06\times10^{-32}\ \text{J}\right)(100\%)}{(75.0\ \text{kg})\left(9.80\ \text{m/s}^2\right)(50.0\ \text{m})} = \boxed{2.88\times10^{-35}\ \%}$$

27.45 The magnetic force supplies the centripetal acceleration for the electrons, so $m(v^2/r) = qvB$, or $p = mv = qrB$. The maximum kinetic energy is then $KE_{\text{max}} = p^2/2m = q^2r^2B^2/2m$, or

$$KE_{\text{max}} = \frac{\left(1.60\times10^{-19}\ \text{J}\right)^2(0.200\ \text{m})^2\left(2.00\times10^{-5}\ \text{T}\right)^2}{2\left(9.11\times10^{-31}\ \text{kg}\right)} = 2.25\times10^{-19}\ \text{J}$$

The work function of the surface is given by $\phi = E_{\text{photon}} - KE_{\text{max}} = hc/\lambda - KE_{\text{max}}$, or

$$\phi = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{450\times10^{-9}\ \text{m}} - 2.25\times10^{-19}\ \text{J}$$

$$= 2.17\times10^{-19}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{1.36\ \text{eV}}$$

27.46 The energy of the incident photon is $E_0 = 6.20$ keV, and its wavelength is $\lambda_0 = hc/E_0$.

In a head-on collision with an electron, the photon is back scattered, or the scattering angle is $\theta = 180°$. The Compton equation then gives the wavelength of the scattered photon as $\lambda = \lambda_0 + \lambda_c(1-\cos 180°) = \lambda_0 + 2\lambda_c$, where $\lambda_c = 2.43\times10^{-12}$ m is the Compton wavelength. Thus,

$$\lambda = \frac{hc}{E_0} + 2\lambda_c = \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{(6.20\ \text{keV})\left(1.60\times10^{-16}\ \text{J/1 keV}\right)} + 2\left(2.43\times10^{-12}\ \text{m}\right) = 2.05\times10^{-10}\ \text{m}$$

The energy of the scattered photon is $E = hc/\lambda$, and from conservation of energy, the kinetic energy of the recoiling electron is

$$KE_e = E_0 - E = 6.20\ \text{keV} - \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(3.00\times10^{8}\ \text{m/s}\right)}{2.05\times10^{-10}\ \text{m}}\left(\frac{1\ \text{keV}}{1.60\times10^{-16}\ \text{J}}\right)$$

or $KE_e = 6.20\ \text{keV} - 6.06\ \text{keV} = \boxed{0.14\ \text{keV}}$

27.47 From the photoelectric effect equation, $KE_{\text{max}} = E_{\text{photon}} - \phi = \dfrac{hc}{\lambda} - \phi.$

For $\lambda = \lambda_0$, $KE_{\text{max}} = 1.00$ eV, so $1.00\ \text{eV} = \dfrac{hc}{\lambda_0} - \phi$ **[1]**

For $\lambda = \dfrac{\lambda_0}{2}$, $KE_{\text{max}} = 4.00$ eV, giving $4.00\ \text{eV} = \dfrac{2hc}{\lambda_0} - \phi$ **[2]**

Multiplying Equation [1] by a factor of 2 and subtracting the result from Equation [2] gives the work function as $\phi = \boxed{2.00\ \text{eV}}$.

27.48 From the photoelectric effect equation, $KE_{\max} = E_{\text{photon}} - \phi = hc/\lambda - \phi$.

For $\lambda = 670$ nm, $KE_{\max} = E_1$, so $$E_1 = \frac{hc}{670\text{ nm}} - \phi \qquad [1]$$

For $\lambda = 520$ nm, $KE_{\max} = 1.50\,E_1$, giving $$1.50\,E_1 = \frac{hc}{520\text{ nm}} - \phi \qquad [2]$$

Multiplying Equation [1] by a factor of −1.50 and adding the result to Equation [2] gives

$$0 = hc\left(\frac{-1.50}{670\text{ nm}} + \frac{1.00}{520\text{ nm}}\right) + (1.50 - 1.00)\phi$$

The work function for the material is then

$$\phi = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{0.50}\left(\frac{1.50}{670\text{ nm}} - \frac{1.00}{520\text{ nm}}\right)\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right)\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right)$$

or $\phi = \boxed{0.785\text{ eV}}$.

27.49 (a) If the single photon produced has wavelength $\lambda = 1.00\times10^{-8}$ m, the kinetic energy of the electron was

$$KE_e = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{1.00\times10^{-8}\text{ m}} = 1.99\times10^{-17}\text{ J} = 124\text{ eV}$$

This electron is nonrelativistic and its speed is given by

$$v = \sqrt{\frac{2(KE_e)}{m_e}} = \sqrt{\frac{2(1.99\times10^{-17}\text{ J})}{9.11\times10^{-31}\text{ kg}}}\left(\frac{c}{3.00\times10^{8}\text{ m/s}}\right) = \boxed{0.022\,0\,c}$$

(b) When the single photon produced has wavelength $\lambda = 1.00\times10^{-13}$ m,

$$KE_e = \frac{hc}{\lambda} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{1.00\times10^{-13}\text{ m}}\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) = 12.4\text{ MeV}$$

the electron is highly relativistic and $KE = (\gamma - 1)E_R$, giving

$$\gamma = 1 + \frac{KE}{E_R} = 1 + \frac{12.4\text{ MeV}}{0.511\text{ MeV}} = 25.3$$

Then, $v = c\sqrt{1 - 1/\gamma^2} = c\sqrt{1 - 1/(25.3)^2} = \boxed{0.999\,2\,c}$.

27.50 (a) If light were a classical wave, the time required for a surface of area $A = \pi r^2$ to absorb energy E when illuminated with radiation of intensity I would be given by $E = P\cdot(\Delta t) = IA\cdot(\Delta t) = I(\pi r^2)(\Delta t)$. Using the given data values, we find the time to absorb energy $E = 1.00$ eV to be

$$\Delta t = \frac{E}{I(\pi r^2)} = \frac{1.00\text{ eV}(1.60\times10^{-19}\text{ J/1 eV})}{(500\text{ J/s})\pi(2.82\times10^{-15}\text{ m})^2}\left(\frac{1\text{ day}}{8.64\times10^{4}\text{ s}}\right) = \boxed{148\text{ days}}$$

(b) $\boxed{\text{This result is totally contrary to observations of the photoelectric effect.}}$ There, when a surface is illuminated with light above the threshold frequency of the surface, photoelectrons are seen to be ejected with a delay of less than 10^{-9} s.

28

Atomic Physics

QUICK QUIZZES

1. Choice (b). The allowed energy levels in a one-electron atom may be expressed as $E_n = -Z^2(13.6 \text{ eV})/n^2$, where Z is the atomic number. Thus, the ground state $(n = 1 \text{ level})$ in helium, with $Z = 2$, is lower than the ground state in hydrogen, with $Z = 1$.

2. (a) For $n = 5$, there are 5 allowed values of ℓ, namely $\ell = 0$, 1, 2, 3, and 4.

(b) Since m_ℓ ranges from $-\ell$ to $+\ell$ in integer steps, the largest allowed value of ℓ ($\ell = 4$ in this case) permits the greatest range of values for m_ℓ. For $n = 5$, there are 9 possible values for m_ℓ: –4, –3, –2, –1, 0, +1, +2, +3, and +4.

(c) For each value of ℓ, there are $2\ell + 1$ possible values of m_ℓ. Thus, there is 1 distinct pair with $\ell = 0$, 3 distinct pairs with $\ell = 1$, 5 distinct pairs with $\ell = 2$, 7 distinct pairs with $\ell = 3$, and 9 distinct pairs with $\ell = 4$. This yields a total of 25 distinct pairs of ℓ and m_ℓ that are possible when $n = 5$.

3. Choice (d). Krypton has a closed configuration consisting of filled $n = 1$, $n = 2$, and $n = 3$ shells as well as filled $4s$ and $4p$ subshells. The filled $n = 3$ shell (the next to outer shell in krypton) has a total of 18 electrons, 2 in the $3s$ subshell, 6 in the $3p$ subshell and 10 in the $3d$ subshell.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The energy levels in a single electron atom having atomic number Z are $E_n = -Z^2(13.6 \text{ eV})/n^2$. For beryllium, $Z = 4$, and for the ground state, $n = 1$. Thus,

$$E_1 = \frac{-4^2(13.6 \text{ eV})}{1^2} = -218 \text{ eV}$$

and the correct answer is choice (b).

2. Wavelengths of the hydrogen spectrum are given by $1/\lambda = R_H(1/n_f^2 - 1/n_i^2)$, where the Rydberg constant is $R_H = 1.097\,373\,2 \times 10^7 \text{ m}^{-1}$. Thus, with $n_f = 3$ and $n_i = 5$,

$$\frac{1}{\lambda} = 1.097\,373\,2 \times 10^7 \text{ m}^{-1}\left(\frac{1}{3^2} - \frac{1}{5^2}\right) = 7.80 \times 10^5 \text{ m}^{-1}$$

and $\lambda = 1/(7.80 \times 10^5 \text{ m}^{-1}) = 1.28 \times 10^{-6} \text{ m}$, so (a) is seen to be the correct choice.

3. There are 6 distinct possible downward transitions with 4 energy levels. These transitions are: $4 \to 1$, $4 \to 2$, $4 \to 3$, $3 \to 1$, $3 \to 2$, and $2 \to 1$. Thus, assuming that each transition has a unique photon energy, $E_{\text{photon}} = |\Delta E| = E_i - E_f$, associated with it, there are 6 different wavelengths $\lambda = hc/E_{\text{photon}}$ the atom could emit, and (e) is the correct choice.

4. With a principal quantum number of $n = 3$, there are 3 possible values of the orbital quantum number, $\ell = 0, 1, 2$. There are a total of $2(2\ell + 1)$ possible quantum states for each value of ℓ; $2\ell + 1$ possible values of the orbital magnetic quantum number m_ℓ, and 2 possible spin orientations $(m_s = \pm\frac{1}{2})$ for each value of m_ℓ. Thus, there are 10 $3d$ states (having $n = 3$, $\ell = 2$), 6 $3p$ states (with $n = 3$, $\ell = 1$), and 2 $3s$ states (with $n = 3$, $\ell = 0$), giving a grand total of $10 + 6 + 2 = 18$ $n = 3$ states, and the correct choice is (e).

5. The structure of the periodic table is the result of the Pauli exclusion principle, which states that no two electrons in an atom can ever have the same set of values for the set of quantum numbers n, ℓ, m_ℓ, and m_s. This principle is best summarized by choice (c).

6. Of the electron configurations listed, (b) and (e) are not allowed. Choice (b) is not possible because the Pauli exclusion principle limits the number of electrons in any p subshell to a maximum of 6. Choice (e) is impossible because the selection rules of quantum mechanics limit the maximum value of ℓ to $n - 1$. Thus, a $2d$ state ($n = 2$, $\ell = 2$) cannot exist.

7. All states associated with $\ell = 2$ are referred to as d states. Thus, all 10 possible quantum states having $n = 3$, $\ell = 2$ are called $3d$ states (see Question 4 above), and the correct answer is choice (c).

8. If it were possible for the spin quantum number to take on the four values $m_s = \pm\frac{3}{2}$ and $\pm\frac{1}{2}$, the first closed shell would occur for beryllium with 4 electrons in states of $(1, 0, 0, \frac{3}{2})$, $(1, 0, 0, \frac{1}{2})$, $(1, 0, 0, -\frac{1}{2})$, and $(1, 0, 0, -\frac{3}{2})$. The correct answer is choice (c).

9. Since the electron is in some bound quantum state of the atom, the atom is not ionized, and choice (a) is false. The fact that the electron is in a d state means that its orbital quantum number is $\ell = 2$, so choice (b) is false. Also, since the maximum value of ℓ is $n - 1$, choice (e) is false. Finally, the ground state of hydrogen is a $1s$ state, so choice (d) is false, leaving (c) as the only true statement in the list of choices.

10. According to de Broglie's interpretation of Bohr's quantization postulate, the circumference of the $n = 3$ orbit would be exactly 3 wavelengths of an electron in that orbit, and the circumference of the $n = 1$ orbit would equal the wavelength of an electron in this orbit. However, the de Broglie wavelength of the electron is given by

$$\lambda_n = \frac{h}{p_n} = \frac{h}{m_e v_n} = \frac{h}{m_e v_n r_n / r_n} = \frac{h r_n}{L_n} = \frac{h r_n}{n(h/2\pi)} = \frac{2\pi r_n}{n} = \frac{2\pi}{n}\left(\frac{n^2 \hbar^2}{m_e k_e e^2}\right) = n\left(\frac{2\pi\hbar^2}{m_e k_e e^2}\right)$$

Thus, the wavelength of an electron in the $n = 3$ orbit is 3 times longer than the wavelength of the electron when in the $n = 1$ orbit, and the circumference of the $n = 3$ orbit must be $C_3 = 3\lambda_3 = 3(3\lambda_1) = 9\lambda_1 = 9C_1$. Choice (c) is the correct answer for this question.

11. An electron in the ground state of hydrogen must be given 10.2 eV of energy to be excited to an $n = 2$ state in hydrogen. However, it would have to be given 13.6 eV of energy to remove it from the atom. Thus, incident electrons having 10.5 eV of kinetic energy have sufficient energy to raise the atom to an excited state but do not possess enough energy to ionize the atom. Choice (a) is the correct answer.

12. Choices (a), (b), (d), and (e) are basic postulates of the Bohr model of one-electron atoms. However, Bohr made no assumptions regarding quantization of charge. Thus, the correct answer for this question is choice (c).

13. A photon emitted by an atom carries energy away from the atom. Thus, the atom emitting the photon must move to a lower allowed energy state, and choice (c) is the correct answer.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. Neon signs do not emit a continuous spectrum, as can be determined by observing the light from the sign through a spectrometer. The discrete wavelengths a neon gas can produce are determined by the differences in the energies of the allowed states within the neon atom. The specific wavelengths and intensities account for the color of the sign.

4. In a neutral helium atom, one electron can be modeled as moving in an electric field created by the nucleus and the other electron. According to Gauss's law, if the electron is above the ground state it moves in the electric field of a net charge of $+2e-1e=+1e$. We say the nuclear charge is *screened* by the inner electron. The electron in a He^+ ion moves in the field of the unscreened nuclear charge of 2 protons. Then, the potential energy function for the electron is about double that of one electron in the neutral atom.

6. Classically, the electron can occupy any energy state. That is, all energies would be allowed. Therefore, if the electron obeyed classical mechanics, its spectrum, which originates from transitions between states, would be continuous rather than discrete.

8. Fundamentally, three quantum numbers describe an orbital wave function because we live in three-dimensional space. They arise mathematically from boundary conditions on the wave function, expressed as a product of a function of r, a function of θ, and a function of ϕ.

10. In both cases the answer is yes. Recall that the ionization energy of hydrogen is 13.6 eV. The electron can absorb a photon of energy less than 13.6 eV by making a transition to some intermediate state such as one with $n=2$. It can also absorb a photon of energy greater than 13.6 eV, but in doing so, the electron would be separated from the proton and have some residual kinetic energy.

12. It replaced the simple circular orbits in the Bohr theory with electron clouds. More important, quantum mechanics is consistent with Heisenberg's uncertainty principle, which tells us about the limits of accuracy in making measurements. In quantum mechanics, we talk about the probabilistic nature of the outcome of a measurement on a system, a concept which is incompatible with the Bohr theory. Finally, the Bohr theory of the atom contains only one quantum number n, while quantum mechanics provides the basis for additional quantum numbers to explain the finer details of atomic structure.

14. Each of the given atoms has a single electron in an $\ell=0$ $(\text{or } s)$ state outside a fully closed-shell core, shielded from all but one unit of the nuclear charge. Since they reside in very similar environments, one would expect these outer electrons to have nearly the same electrical potential energies and hence nearly the same ionization energies. This is in agreement with the given data values. Also, since the distance of the outer electron from the nuclear charge should tend to increase with Z (to allow for greater numbers of electrons in the core), one would expect the ionization energy to decrease somewhat as atomic number increases. This is also in agreement with the given data.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 1 875 nm, 1 281 nm, 1 094 nm

(b) All are in the infrared region of the spectrum.

4. 1.94 μm

6. 45 fm

8. **(a)** 2.19×10^{6} m/s **(b)** 13.6 eV **(c)** -27.2 eV

10. **(a)** 1 281 nm **(b)** 2.34×10^{14} Hz **(c)** 0.970 eV

12. **(a)** Transition II **(b)** Transition I **(c)** Transitions II and III

14. **(a)** 12.1 eV **(b)** 12.1 eV, 1.89 eV, 10.2 eV

16. See Solution.

18. **(a)** 6 distinct wavelengths **(b)** 1.88×10^{3} nm **(c)** Paschen series

20. **(a)** $2.89\times10^{34}\ \mathrm{kg\cdot m^2/s}$ **(b)** 2.75×10^{68} **(c)** 7.27×10^{-69}

22. **(a)** $KE_n = -\frac{1}{2}PE_n$ **(b)** $\Delta PE = 2E$ **(c)** $\Delta KE = -E$

24. **(a)** $E_n = -(122\text{ eV})/n^2$ **(b)** -7.63 eV **(c)** -30.5 eV

(d) $22.9\text{ eV} = 3.66\times10^{-18}$ J **(e)** 5.52×10^{15} Hz, 54.3 nm **(f)** deep ultraviolet region

26. **(a)** See Solution. **(b)** See Solution.

28. **(a)** 4: $\ell = 0$, 1, 2, and 3 **(b)** 7: $m_\ell = -3,\ -2,\ -1,\ 0, +1, +2,$ and $+3$.

30. **(a)** $1s^2 2s^2 2p^3$

(b) ($n=1,\ \ell=0,\ m_\ell=0,\ m_s=\pm\frac{1}{2}$); ($n=2,\ \ell=0,\ m_\ell=0,\ m_s=\pm\frac{1}{2}$);

($n=2,\ \ell=1,\ m_\ell=-1,\ m_s=\pm\frac{1}{2}$); ($n=2,\ \ell=1,\ m_\ell=0,\ m_s=\pm\frac{1}{2}$);

($n=2,\ \ell=1,\ m_\ell=1,\ m_s=\pm\frac{1}{2}$)

32. **(a)** 30 allowed states – see Solution for tables

(b) 36 possible combinations – six possible states for each electron independently

34. 0.031 1 nm

36. germanium

38. 137

40. **(a)** 4.20 mm **(b)** 1.05×10^{19} photons

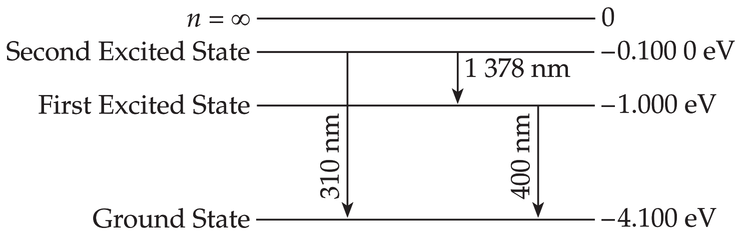
(c) 8.84×10^{16} photons/mm^3

42. **(a)** See Solution. **(b)** 2.54×10^{74} **(c)** 1.18×10^{-63} m

(d) This number is *much smaller* than the radius of an atomic nucleus ($\sim10^{-15}$ m), so the distance between quantized orbits of the Earth is too small to observe.

44. **(a)** 135 eV **(b)** ≈ 10 times the magnitude of the ground state energy

46.



PROBLEM SOLUTIONS

28.1 (a) The wavelengths in the Lyman series of hydrogen are given by $1/\lambda = R_H(1 - 1/n^2)$, where $n = 2, 3, 4, \ldots$, and the Rydberg constant is $R_H = 1.097\,373\,2 \times 10^7\ \text{m}^{-1}$. This can also be written as $\lambda = (1/R_H)\left[n^2/\left(n^2 - 1\right)\right]$, so the first three wavelengths in this series are

$$\lambda_1 = \frac{1}{1.097\,373\,2 \times 10^7\ \text{m}^{-1}}\left(\frac{2^2}{2^2 - 1}\right) = 1.215 \times 10^{-7}\ \text{m} = \boxed{121.5\ \text{nm}}$$

$$\lambda_2 = \frac{1}{1.097\,373\,2 \times 10^7\ \text{m}^{-1}}\left(\frac{3^2}{3^2 - 1}\right) = 1.025 \times 10^{-7}\ \text{m} = \boxed{102.5\ \text{nm}}$$

$$\lambda_3 = \frac{1}{1.097\,373\,2 \times 10^7\ \text{m}^{-1}}\left(\frac{4^2}{4^2 - 1}\right) = 9.720 \times 10^{-8}\ \text{m} = \boxed{97.20\ \text{nm}}$$

(b) These wavelengths are all in the $\boxed{\text{far ultraviolet region}}$ of the spectrum.

28.2 (a) The wavelengths in the Paschen series of hydrogen are given by $1/\lambda = R_H(1/3^2 - 1/n^2)$, where $n = 4, 5, 6, \ldots$, and the Rydberg constant is $R_H = 1.097\,373\,2 \times 10^7\ \text{m}^{-1}$. This can also be written as $\lambda = (1/R_H)[9n^2/(n^2 - 9)]$, so the first three wavelengths in this series are

$$\lambda_1 = \frac{1}{1.097\,373\,2 \times 10^7\ \text{m}^{-1}}\left[\frac{9(4)^2}{4^2 - 9}\right] = 1.875 \times 10^{-6}\ \text{m} = \boxed{1\,875\ \text{nm}}$$

$$\lambda_2 = \frac{1}{1.097\,373\,2 \times 10^7\ \text{m}^{-1}}\left[\frac{9(5)^2}{5^2 - 9}\right] = 1.281 \times 10^{-6}\ \text{m} = \boxed{1\,281\ \text{nm}}$$

$$\lambda_1 = \frac{1}{1.097\,373\,2 \times 10^7\ \text{m}^{-1}}\left[\frac{9(6)^2}{6^2 - 9}\right] = 1.094 \times 10^{-6}\ \text{m} = \boxed{1\,094\ \text{nm}}$$

(b) These wavelengths are all in the $\boxed{\text{infrared region}}$ of the spectrum.

28.3 (a) From Coulomb's law,

$$|F| = \frac{k_e|q_1 q_2|}{r^2} = \frac{\left(8.99 \times 10^9\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60 \times 10^{-19}\ \text{C}\right)^2}{\left(1.0 \times 10^{-10}\ \text{m}\right)^2}$$

$$= \boxed{2.3 \times 10^{-8}\ \text{N}}$$

continued on next page

(b) The electrical potential energy is

$$PE = \frac{k_e q_1 q_2}{r} = \frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(-1.60\times10^{-19}\ \text{C}\right)\left(1.60\times10^{-19}\ \text{C}\right)}{1.0\times10^{-10}\ \text{m}}$$

$$= -2.3\times10^{-18}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{-14\ \text{eV}}$$

28.4 If E_2, E_5, and E_6, are the energies of an electron in the second, fifth, and sixth excited states within this atom, respectively, the energies of the photons produced are: $\left(E_{\text{photon}}\right)_1 = hc/\lambda_1 = E_2 - E_5$, $\left(E_{\text{photon}}\right)_2 = hc/\lambda_2 = E_2 - E_6$, and $\left(E_{\text{photon}}\right)_3 = hc/\lambda_3 = E_5 - E_6$. Thus, we have $hc/\lambda_3 = E_5 - E_6 = \left(E_2 - E_6\right) - \left(E_2 - E_5\right)$, or $hc/\lambda_3 = hc/\lambda_2 - hc/\lambda_1$. This reduces to

$$\lambda_3 = \frac{\lambda_1\lambda_2}{\lambda_1 - \lambda_2} = \frac{(520\ \text{nm})(410\ \text{nm})}{520\ \text{nm} - 410\ \text{nm}} = 1.94\times10^{3}\ \text{nm} = \boxed{1.94\ \mu\text{m}}$$

28.5 (a) The electrical force supplies the centripetal acceleration of the electron, so $m_e v^2/r = k_e e^2/r^2$ or $v = \sqrt{k_e e^2/m_e r}$.

$$v = \sqrt{\frac{\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.0\times10^{-10}\ \text{m}\right)}} = \boxed{1.6\times10^{6}\ \text{m/s}}$$

(b) $\boxed{\text{No.}}$ $\dfrac{v}{c} = \dfrac{1.6\times10^{6}\ \text{m/s}}{3.00\times10^{8}\ \text{m/s}} = 5.3\times10^{-3} << 1$, so the electron is not relativistic.

(c) The de Broglie wavelength for the electron is $\lambda = h/p = h/m_e v$, or

$$\lambda = \frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{\left(9.11\times10^{-31}\ \text{kg}\right)\left(1.6\times10^{6}\ \text{m/s}\right)} = 4.5\times10^{-10}\ \text{m} = \boxed{0.45\ \text{nm}}$$

(d) $\boxed{\text{Yes.}}$ The wavelength and the atom are roughly the same size.

28.6 Assuming a head-on collision, the α-particle comes to rest momentarily at the point of closest approach. From conservation of energy,

$$KE_f + PE_f = KE_i + PE_i$$

or

$$0 + \frac{k_e(2e)(79e)}{r_f} = KE_i + \frac{k_e(2e)(79e)}{r_i}$$

With $r_i \to \infty$, this gives the distance of closest approach as

$$r_f = \frac{158 k_e e^2}{KE_i} = \frac{158\left(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\ \text{C}\right)^2}{5.0\ \text{MeV}\left(1.60\times10^{-13}\ \text{J/MeV}\right)}$$

$$= 4.5\times10^{-14}\ \text{m} = \boxed{45\ \text{fm}}$$

28.7 (a) $r_n = n^2 a_0$ yields $r_2 = 4\left(0.052\,9\ \text{nm}\right) = \boxed{0.212\ \text{nm}}$

(b) With the electrical force supplying the centripetal acceleration, $m_e v_n^2/r_n = k_e e^2/r_n^2$, giving $v_n = \sqrt{k_e e^2/m_e r_n}$, and $p_n = m_e v_n = \sqrt{m_e k_e e^2/r_n}$.

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Thus,

$$p_2 = \sqrt{\frac{m_e k_e e^2}{r_2}} = \sqrt{\frac{(9.11\times10^{-31}\ \text{kg})(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.6\times10^{-19}\ \text{C})^2}{0.212\times10^{-9}\ \text{m}}}$$

$$= \boxed{9.95\times10^{-25}\ \text{kg}\cdot\text{m/s}}$$

(c) $L_n = n\left(\frac{h}{2\pi}\right) \rightarrow L_2 = 2\left(\frac{6.63\times10^{-34}\ \text{J}\cdot\text{s}}{2\pi}\right) = \boxed{2.11\times10^{-34}\ \text{J}\cdot\text{s}}$

(d) $KE_2 = \frac{1}{2}mv_2^2 = \frac{p_2^2}{2m_e} = \frac{(9.95\times10^{-25}\ \text{kg}\cdot\text{m/s})^2}{2(9.11\times10^{-31}\ \text{kg})} = 5.44\times10^{-19}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{3.40\ \text{eV}}$

(e) $PE_2 = \frac{k_e(-e)e}{r_2} = -\frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(0.212\times10^{-9}\ \text{m})}$

$$= -1.09\times10^{-18}\ \text{J} = \boxed{-6.80\ \text{eV}}$$

(f) $E_2 = KE_2 + PE_2 = 3.40\ \text{eV} - 6.80\ \text{eV} = \boxed{-3.40\ \text{eV}}$

28.8 (a) With the electrical force supplying the centripetal acceleration, $m_e v_n^2/r_n = k_e e^2/r_n^2$, giving $v_n = \sqrt{k_e e^2/m_e r_n}$, where $r_n = n^2 a_0 = n^2(0.052\,9\ \text{nm})$.

Thus,

$$v_1 = \sqrt{\frac{k_e e^2}{m_e r_1}} = \sqrt{\frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(9.11\times10^{-31}\ \text{kg})(0.052\,9\times10^{-9}\ \text{m})}} = \boxed{2.19\times10^{6}\ \text{m/s}}$$

(b) $KE_1 = \frac{1}{2}m_e v_1^2 = \frac{1}{2}(9.11\times10^{-31}\ \text{kg})(2.19\times10^{6}\ \text{m/s})^2$

$$= 2.18\times10^{-18}\ \text{J}\left(\frac{1\ \text{eV}}{1.60\times10^{-19}\ \text{J}}\right) = \boxed{13.6\ \text{eV}}$$

(c) $PE_1 = \frac{k_e(-e)e}{r_1} = -\frac{(8.99\times10^{9}\ \text{N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\ \text{C})^2}{(0.052\,9\times10^{-9}\ \text{m})}$

$$= -4.35\times10^{-18}\ \text{J} = \boxed{-27.2\ \text{eV}}$$

28.9 Since the electrical force supplies the centripetal acceleration,

$$\frac{m_e v_n^2}{r_n} = \frac{k_e e^2}{r_n^2} \qquad \text{or} \qquad v_n^2 = \frac{k_2 e^2}{m_e r_n}$$

From $L_n = m_e r_n v_n = n\hbar$, we have $r_n = n\hbar/m_e v_n$, so

$$v_n^2 = \frac{k_2 e^2}{m_e}\left(\frac{m_e v_n}{n\hbar}\right)$$

which reduces to $\boxed{v_n = k_e e^2/n\hbar}$.

28.10 (a) The Rydberg equation is $1/\lambda = R_{\mathrm{H}}\left(1/n_f^2 - 1/n_i^2\right)$, or

$$\lambda = \frac{1}{R_{\mathrm{H}}}\left(\frac{n_i^2 n_f^2}{n_i^2 - n_f^2}\right)$$

With $n_i = 5$ and $n_f = 3$,

$$\lambda = \frac{1}{1.097\,373\,2\times 10^{7}\ \mathrm{m}^{-1}}\left[\frac{(25)(9)}{25-9}\right] = 1.281\times 10^{-6}\ \mathrm{m} = \boxed{1\,281\ \mathrm{nm}}$$

(b) $f = \dfrac{c}{\lambda} = \dfrac{3.00\times 10^{8}\ \mathrm{m/s}}{1\,281\times 10^{-9}\ \mathrm{m}} = \boxed{2.34\times 10^{14}\ \mathrm{Hz}}$

(c) $E_{\text{photon}} = \dfrac{hc}{\lambda} = \dfrac{\left(6.63\times 10^{-34}\ \mathrm{J\cdot s}\right)\left(3.00\times 10^{8}\ \mathrm{m/s}\right)}{1\,281\times 10^{-9}\ \mathrm{m}}\left(\dfrac{1\ \mathrm{eV}}{1.60\times 10^{-19}\ \mathrm{J}}\right) = \boxed{0.970\ \mathrm{eV}}$

28.11 The energy of the emitted photon is

$$E_{\text{photon}} = \frac{hc}{\lambda} = \frac{\left(6.626\times 10^{-34}\ \mathrm{J\cdot s}\right)\left(2.998\times 10^{8}\ \mathrm{m/s}\right)}{656\times 10^{-9}\ \mathrm{m}}\left(\frac{1\ \mathrm{eV}}{1.60\times 10^{-19}\ \mathrm{J}}\right) = 1.89\ \mathrm{eV}$$

This photon energy is also the difference in the electron's energy in its initial and final orbits. The energies of the electron in the various allowed orbits within the hydrogen atom are

$$E_n = -\frac{13.6\ \mathrm{eV}}{n^2} \qquad \text{where} \qquad n = 1,\ 2,\ 3,\ldots$$

giving $E_1 = -13.6\ \mathrm{eV}$, $E_2 = -3.40\ \mathrm{eV}$, $E_3 = -1.51\ \mathrm{eV}$, $E_4 = -0.850\ \mathrm{eV}, \ldots$.

Observe that $E_{\text{photon}} = E_3 - E_2$, so the transition was from $\boxed{\text{the } n = 3 \text{ orbit to the } n = 2 \text{ orbit}}$.

28.12 The change in the energy of the atom is

$$\Delta E = E_f - E_i = 13.6\ \mathrm{eV}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

Transition I: $\Delta E = 13.6\ \mathrm{eV}\left(\dfrac{1}{4} - \dfrac{1}{25}\right) = 2.86\ \mathrm{eV}$ (absorption)

Transition II: $\Delta E = 13.6\ \mathrm{eV}\left(\dfrac{1}{25} - \dfrac{1}{9}\right) = -0.967\ \mathrm{eV}$ (emission)

Transition III: $\Delta E = 13.6\ \mathrm{eV}\left(\dfrac{1}{49} - \dfrac{1}{16}\right) = -0.572\ \mathrm{eV}$ (emission)

Transition IV: $\Delta E = 13.6\ \mathrm{eV}\left(\dfrac{1}{16} - \dfrac{1}{49}\right) = 0.572\ \mathrm{eV}$ (absorption)

(a) Since $\lambda = \dfrac{hc}{E_{\text{photon}}} = \dfrac{hc}{-\Delta E}$, $\boxed{\text{transition II}}$ emits the shortest wavelength photon.

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(b) The atom gains the most energy in $\boxed{\text{transition I}}$.

(c) The atom loses energy in $\boxed{\text{transitions II and III}}$.

28.13 The energy absorbed by the atom is

$$E_{\text{photon}} = E_f - E_i = \frac{-13.6\text{ eV}}{n_f^2} - \frac{(-13.6\text{ eV})}{n_i^2} = +13.6\text{ eV}\left(\frac{1}{n_i^2} - \frac{1}{n_f^2}\right)$$

(a) $E_{\text{photon}} = 13.6\text{ eV}\left(\frac{1}{2^2} - \frac{1}{5^2}\right) = \boxed{2.86\text{ eV}}$

(b) $E_{\text{photon}} = 13.6\text{ eV}\left(\frac{1}{4^2} - \frac{1}{6^2}\right) = \boxed{0.472\text{ eV}}$

28.14 (a) The energy absorbed is

$$\Delta E = E_f - E_i = \frac{-13.6\text{ eV}}{n_f^2} - \frac{(-13.6\text{ eV})}{n_i^2} = 13.6\text{ eV}\left(\frac{1}{1} - \frac{1}{9}\right) = \boxed{12.1\text{ eV}}$$

(b) Three transitions are possible as the electron returns to the ground state. These transitions and the emitted photon energies are

$$n_i = 3 \to n_f = 1: \qquad |\Delta E| = 13.6\text{ eV}\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = \boxed{12.1\text{ eV}}$$

$$n_i = 3 \to n_f = 2: \qquad |\Delta E| = 13.6\text{ eV}\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \boxed{1.89\text{ eV}}$$

$$n_i = 2 \to n_f = 1: \qquad |\Delta E| = 13.6\text{ eV}\left(\frac{1}{1^2} - \frac{1}{2^2}\right) = \boxed{10.2\text{ eV}}$$

28.15 (a) The photon of longest wavelength is produced in the transition for which the atom gives up the smallest amount of energy. From Figure P28.15, this is seen to be the $n = 3$ to $n = 2$ transition, for which

$$E_{\text{photon}} = E_3 - E_2 = -1.512\text{ eV} - (-3.401\text{ eV}) = \boxed{1.889\text{ eV}}$$

(b) $$\lambda = \frac{hc}{E_{\text{photon}}} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{1.889\text{ eV}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right)$$
$$= 6.58\times10^{-7}\text{ m} = \boxed{658\text{ nm}}$$

(c) The photon of shortest wavelength is produced in the transition for which the atom gives up the greatest amount of energy. From Figure P28.15, this is seen to be the $n = 6$ to $n = 2$ transition, for which

$$E_{\text{photon}} = E_6 - E_2 = -0.378\text{ eV} - (-3.401\text{ eV}) = \boxed{3.02\text{ eV}}$$

(d) $$\lambda = \frac{hc}{E_{\text{photon}}} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{3.02\text{ eV}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right)$$
$$= 4.12\times10^{-7}\text{ m} = \boxed{412\text{ nm}}$$

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(e) The shortest wavelength is produced for the $n = \infty$ to $n = 2$ transition. The photon energy for this transition is $E_{\text{photon}} = 0 - (-3.401\text{ eV}) = 3.401\text{ eV}$, and its wavelength is

$$\lambda = \frac{hc}{E_{\text{photon}}} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{3.401\text{ eV}}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right)$$
$$= 3.66\times10^{-7}\text{ m} = \boxed{366\text{ nm}}$$

28.16 The magnetic force supplies the centripetal acceleration, so $mv^2/r = qvB$, or $r = mv/qB$. If angular momentum is quantized according to

$$L_n = mv_n r_n = 2n\hbar,\text{ then } mv_n = \frac{2n\hbar}{r_n}$$

and the allowed radii of the path are given by

$$r_n = \frac{1}{qB}\left(\frac{2n\hbar}{r_n}\right) \qquad \text{or} \qquad \boxed{r_n = \sqrt{\frac{2n\hbar}{qB}}}$$

28.17 (a) The energy of the emitted photon is

$$E_{\text{photon}} = E_4 - E_2 = -13.6\text{ eV}\left(\frac{1}{4^2} - \frac{1}{2^2}\right) = 2.55\text{ eV}$$

This photon has a wavelength of

$$\lambda = \frac{hc}{E_{\text{photon}}} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{(2.55\text{ eV})(1.60\times10^{-19}\text{ J/eV})} = 4.88\times10^{-7}\text{ m} = \boxed{488\text{ nm}}$$

(b) Since momentum must be conserved, the photon and the atom go in opposite directions with equal magnitude momenta. Thus, $p = m_{\text{atom}}v = h/\lambda$, or

$$v = \frac{h}{m_{\text{atom}}\lambda} = \frac{6.63\times10^{-34}\text{ J}\cdot\text{s}}{(1.67\times10^{-27}\text{ kg})(4.88\times10^{-7}\text{ m})} = \boxed{0.814\text{ m/s}}$$

28.18 (a) Starting from the $n = 4$ state, there are 6 possible transitions as the electron returns to the ground ($n = 1$) state. These transitions are: $n = 4 \to n = 1$, $n = 4 \to n = 2$, $n = 4 \to n = 3$, $n = 3 \to n = 1$, $n = 3 \to n = 2$, and $n = 2 \to n = 1$. Since there is a different change in energy associated with each of these transitions, there will be $\boxed{\text{6 distinct wavelengths}}$ observed in the emission spectrum of these atoms.

(b) The longest observed wavelength is produced by the transition involving the smallest change in energy. This is the $n = 4 \to n = 3$ transition, and the wavelength is

$$\lambda_{\max} = \frac{hc}{E_4 - E_3} = \frac{(6.63\times10^{-34}\text{ J}\cdot\text{s})(3.00\times10^{8}\text{ m/s})}{-13.6\text{ eV}\left(1/4^2 - 1/3^2\right)}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right)\left(\frac{1\text{ nm}}{10^{-9}\text{ m}}\right)$$

or $\lambda_{\max} = \boxed{1.88\times10^{3}\text{ nm}}$.

(c) Since the transition producing this wavelength terminates on the $n = 3$ level, this is part of the $\boxed{\text{Paschen series}}$.

28.19 (a) For the absorption of the photon to ionize the hydrogen atom, the electron in this atom must be in an excited state having an ionization energy less than or equal to the photon energy. That is, we must have $E_{\text{ionization}} = -E_n \le E_{\text{photon}} = 2.28 \text{ eV}$, or $E_n \ge -2.28$ eV. The state with the smallest value of n meeting this requirement is the $\boxed{n = 3}$ state, with $E_3 = -1.51$ eV.

(b) After the electron spends 1.51 eV of energy to escape from the atom, it will retain the remaining absorbed photon energy (2.28 eV − 1.51 eV = 0.77 eV) as kinetic energy. Its speed will then be

$$v = \sqrt{\frac{2KE}{m_e}} = \sqrt{\frac{2(0.77 \text{ eV})}{9.11\times10^{-31} \text{ kg}}\left(\frac{1.60\times10^{-19} \text{ J}}{1 \text{ eV}}\right)} = 5.2\times10^5 \text{ m/s} = \boxed{520 \text{ km/s}}$$

28.20 (a) $$L = M_M v r = M_M\left(\frac{2\pi r}{T}\right)r = \frac{\left(7.36\times10^{22} \text{ kg}\right)2\pi\left(3.84\times10^8 \text{ m}\right)^2}{2.36\times10^6 \text{ s}} = \boxed{2.89\times10^{34} \text{ kg}\cdot\text{m}^2/\text{s}}$$

(b) $$n = \frac{L}{\hbar} = \frac{2\pi\left(2.89\times10^{34} \text{ kg}\cdot\text{m}^2/\text{s}\right)}{1.05\times10^{-34} \text{ J}\cdot\text{s}} = \boxed{2.75\times10^{68}}$$

(c) The gravitational force supplies the centripetal acceleration, so $M_M v^2/r = GM_E M_M/r^2$, or $rv^2 = GM_E$. Then, from $L_n = M_M v_n r_n = n\hbar$, or $v_n = n\hbar/M_M r_n$, we have $r_n\left(n\hbar/M_M r_n\right)^2 = GM_E$, which gives $r_n = n^2\left(\hbar^2/GM_E M_M^2\right) = n^2 r_1$.

Therefore, when n increases by 1, the fractional change in the radius is

$$\frac{\Delta r}{r} = \frac{r_{n+1} - r_n}{r_n} = \frac{(n+1)^2 r_1 - n^2 r_1}{n^2 r_1} = \frac{2n+1}{n^2} \approx \frac{2}{n} \quad \text{for } n >> 1$$

$$\frac{\Delta r}{r} \approx \frac{2}{2.75\times10^{68}} = \boxed{7.27\times10^{-69}}$$

28.21 (a) $r_n = n^2 a_0 = n^2(0.052\,9 \text{ nm}) \Rightarrow r_3 = 3^2(0.052\,9 \text{ nm}) = \boxed{0.476 \text{ nm}}$

(b) In the Bohr model, the circumference of an allowed orbits must be an integral multiple of the de Broglie wavelength for the electron in that orbit, or $2\pi r_n = n\lambda$. Thus, the wavelength of the electron when in the $n = 3$ orbit in hydrogen is

$$\lambda = \frac{2\pi r_3}{3} = \frac{2\pi(0.476 \text{ nm})}{3} = \boxed{0.997 \text{ nm}}$$

28.22 (a) The Coulomb force supplies the necessary centripetal force to hold the electron in orbit, so $m_e v_n^2/r_n = k_e e^2/r_n^2$ or $m_e v_n^2 = k_e e^2/r_n$. But $m_e v_n^2 = 2KE_n$, and $k_e e^2/r_n = -PE_n$, where PE_n is the electrical potential energy of the electron-proton system when the electron is in an orbit of radius r_n. We then have $2KE_n = -PE_n$, or $\boxed{KE_n = -\frac{1}{2}PE_n}$.

(b) When the atom absorbs energy E, and the electron moves to a higher level, both the kinetic and potential energies will change. Conservation of energy requires that $E = \Delta KE + \Delta PE$. However, from the result of part (a), $\Delta KE = -\frac{1}{2}\Delta PE$, and we have

$$E = -\frac{1}{2}\Delta PE + \Delta PE = +\frac{1}{2}\Delta PE \qquad \text{or} \qquad \boxed{\Delta PE = 2E}$$

(c) $$\Delta KE = -\frac{1}{2}\Delta PE = -\frac{1}{2}(2E) \qquad \text{or} \qquad \boxed{\Delta KE = -E}$$

28.23 The radii for atomic number Z are $r_n = n^2\left(\hbar^2/m_e k_e e^2\right)/Z = n^2 a_0/Z$, so $r_1 = a_0/Z$, where $a_0 = 0.052\,9$ nm is the radius of the first Bohr orbit in hydrogen.

(a) For He^+, $Z = 2$ and $r_1 = 0.052\,9 \text{ nm}/2 = \boxed{0.026\,5 \text{ nm}}$.

(b) For Li^{2+}, $Z = 3$ and $r_1 = 0.052\,9 \text{ nm}/3 = \boxed{0.017\,6 \text{ nm}}$.

(c) For Be^{3+}, $Z = 4$ and $r_1 = 0.052\,9 \text{ nm}/4 = \boxed{0.013\,2 \text{ nm}}$.

28.24 (a) The energy levels in a single electron atom with nuclear charge $+Ze$ are $E_n = -Z^2(13.6 \text{ eV})/n^2$. For doubly ionized lithium, $Z = 3$, giving $\boxed{E_n = -(122 \text{ eV})/n^2}$.

(b) $$E_4 = \frac{-122 \text{ eV}}{4^2} = \boxed{-7.63 \text{ eV}}$$

(c) $$E_2 = \frac{-122 \text{ eV}}{2^2} = \boxed{-30.5 \text{ eV}}$$

(d) $$E_{\text{photon}} = E_i - E_f = -7.63 \text{ eV} - (-30.5 \text{ eV}) = \boxed{22.9 \text{ eV}}$$

$$E_{\text{photon}} = (22.9 \text{ eV})\left(\frac{1.60\times10^{-19} \text{ J}}{1 \text{ eV}}\right) = \boxed{3.66\times10^{-18} \text{ J}}$$

(e) $$f = \frac{E_{\text{photon}}}{h} = \frac{3.66\times10^{-18} \text{ J}}{6.63\times10^{-34} \text{ J}\cdot\text{s}} = \boxed{5.52\times10^{15} \text{ Hz}}$$

$$\lambda = \frac{c}{f} = \frac{3.00\times10^{8} \text{ m/s}}{5.52\times10^{15} \text{ Hz}} = 5.43\times10^{-8} \text{ m} = \boxed{54.3 \text{ nm}}$$

(f) This wavelength is in the $\boxed{\text{deep ultraviolet region}}$ of the spectrum.

28.25 The charge of the electron is $q_1 = -e$, and that of the nucleus is $q_2 = Ze$. The energy levels for the one-electron atoms and ions can then be written as $E_n = -k\mu Z^2/n^2$, where $k = k_e^2 e^4/2\hbar^2 =$ constant. The energy of the photon emitted in the $n = 3$ to $n = 2$ transition becomes $E_{\text{photon}} = E_3 - E_2 = k\mu Z^2\left(1/2^2 - 1/3^2\right) = \mu Z^2(5k/36)$, and the wavelength is

$$\lambda = \frac{hc}{E_{\text{photon}}} = \frac{1}{\mu Z^2}\left(\frac{36hc}{5k}\right)$$

For hydrogen, $Z_H = 1$ and the reduced mass is $\mu_H = m_p m_e/(m_p + m_e)$. Since the proton mass is very large in comparison to the electron mass, $m_p + m_e \approx m_p$ and $\mu_H \approx m_e$.

(a) For positronium, $Z_p = 1$ and the reduced mass is $\mu_p = m_e m_e/(m_e + m_e) = m_e/2$. Thus, the ratio of the wavelength for the $n = 3$ to $n = 2$ transition in positronium to that for the same transition in hydrogen is

$$\frac{\lambda_p}{\lambda_H} = \frac{\left(1/\mu_p Z_p^2\right)\cancel{(36hc/5k)}}{\left(1/\mu_H Z_H^2\right)\cancel{(36hc/5k)}} = \frac{\mu_H Z_H^2}{\mu_p Z_p^2} = \frac{m_e\cdot1^2}{(m_e/2)\cdot1^2} = 2$$

giving $\lambda_p = 2\lambda_H = 2(656.3 \text{ nm}) = \boxed{1313 \text{ nm} = 1.313\ \mu\text{m (infrared region)}}$.

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(b) For singly ionized helium, $Z_{\text{He}} = 2$, and the reduced mass is $\mu_{\text{He}} \approx 2m_p m_e/(2m_p + m_e)$. Again, $m_p \gg m_e$ and we find that $\mu_{\text{He}} \approx m_e$. The ratio of the wavelength for the $n = 3$ to $n = 2$ transition in singly ionized helium to that for the same transition in hydrogen is then

$$\frac{\lambda_{\text{He}}}{\lambda_{\text{H}}} = \frac{\mu_{\text{H}} Z_{\text{H}}^2}{\mu_{\text{He}} Z_{\text{He}}^2} = \frac{m_e \cdot 1^2}{m_e \cdot 2^2} = \frac{1}{4}$$

giving $\lambda_{\text{He}} = \dfrac{\lambda_{\text{H}}}{4} = \dfrac{656.3 \text{ nm}}{4} = \boxed{164.1 \text{ nm (ultraviolet region)}}$.

28.26 (a) For standing waves in a string fixed at both ends, $L = n\lambda/2$, or $\lambda = 2L/n$. According to the de Broglie hypothesis, $p = h/\lambda$. Combining these expressions gives $\boxed{p = mv = nh/2L}$.

(b) Using $E = \frac{1}{2}mv^2 = p^2/2m$, with p as found in (a) above:

$$E_n = \frac{n^2 h^2}{4L^2(2m)} \quad \text{or} \quad \boxed{E_n = n^2 E_0 \quad \text{where } E_0 = \frac{h^2}{8mL^2}}$$

28.27 In the 3d subshell, $n = 3$ and $\ell = 2$. The 10 possible quantum states are

$n = 3$	$\ell = 2$	$m_\ell = +2$	$m_s = +\frac{1}{2}$
$n = 3$	$\ell = 2$	$m_\ell = +2$	$m_s = -\frac{1}{2}$
$n = 3$	$\ell = 2$	$m_\ell = +1$	$m_s = +\frac{1}{2}$
$n = 3$	$\ell = 2$	$m_\ell = +1$	$m_s = -\frac{1}{2}$
$n = 3$	$\ell = 2$	$m_\ell = +0$	$m_s = +\frac{1}{2}$
$n = 3$	$\ell = 2$	$m_\ell = +0$	$m_s = -\frac{1}{2}$
$n = 3$	$\ell = 2$	$m_\ell = -1$	$m_s = +\frac{1}{2}$
$n = 3$	$\ell = 2$	$m_\ell = -1$	$m_s = -\frac{1}{2}$
$n = 3$	$\ell = 2$	$m_\ell = -2$	$m_s = +\frac{1}{2}$
$n = 3$	$\ell = 2$	$m_\ell = -2$	$m_s = -\frac{1}{2}$

28.28 (a) For a given value of the principal quantum number n, the orbital quantum number ℓ varies from 0 to $n-1$ in integer steps. Thus, if $n = 4$, there are $\boxed{4}$ possible values of ℓ: $\ell = 0$, 1, 2, and 3.

(b) For each possible value of the orbital quantum number ℓ, the orbital magnetic quantum number m_ℓ ranges from $-\ell$ to $+\ell$ in integer steps. When the principal quantum number is $n = 4$, and the largest allowed value of the orbital quantum number is $\ell = 3$, and there are $\boxed{7}$ distinct possible values for m_ℓ. These values are: $m_\ell = -3, -2, -1, 0, +1, +2$, and $+3$.

28.29 The 3d subshell has $n = 3$ and $\ell = 2$. For ρ-mesons, we also have $s = 1$. Thus, there are 15 possible quantum states, as summarized in the table below.

n	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
ℓ	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
m_ℓ	+2	+2	+2	+1	+1	+1	0	0	0	–1	–1	–1	–2	–2	–2
m_s	+1	0	–1	+1	0	–1	+1	0	–1	+1	0	–1	+1	0	–1

28.30 (a) The electronic configuration for nitrogen $(Z = 7)$ is $\boxed{1s^2 2s^2 2p^3}$.

(b) The quantum numbers for the 7 electrons can be:

$1s$ states	$n = 1$	$\ell = 0$	$m_\ell = 0$	$m_s = +\frac{1}{2}$
				$m_s = -\frac{1}{2}$
$2s$ states	$n = 2$	$\ell = 0$	$m_\ell = 0$	$m_s = +\frac{1}{2}$
				$m_s = -\frac{1}{2}$
$2p$ states	$n = 2$	$\ell = 1$	$m_\ell = -1$	$m_s = +\frac{1}{2}$
				$m_s = -\frac{1}{2}$
			$m_\ell = 0$	$m_s = +\frac{1}{2}$
				$m_s = -\frac{1}{2}$
			$m_\ell = 1$	$m_s = +\frac{1}{2}$
				$m_s = -\frac{1}{2}$

28.31 In the table of electronic configurations (Table 28.4), or the periodic table on the inside back cover of the text, look for the element whose last electron is in a $3p$ state and which has three electrons outside a closed shell. Its electron configuration will then end in $3s^2 3p^1$. You should find that the element is $\boxed{\text{aluminum}}$.

28.32 (a) For Electron #1 and also for Electron #2, $n = 3$ and $\ell = 1$. The other quantum numbers for each of the 30 allowed states are listed in the tables below.

	m_ℓ	m_s	m_ℓ	m_s	m_ℓ	m_s	m_ℓ	m_s	m_ℓ	m_s	m_ℓ	m_s
Electron #1	+1	$+\frac{1}{2}$	+1	$+\frac{1}{2}$	+1	$+\frac{1}{2}$	+1	$-\frac{1}{2}$	+1	$-\frac{1}{2}$	+1	$-\frac{1}{2}$
Electron #2	+1	$-\frac{1}{2}$	0	$\pm\frac{1}{2}$	−1	$\pm\frac{1}{2}$	+1	$+\frac{1}{2}$	0	$\pm\frac{1}{2}$	−1	$\pm\frac{1}{2}$

	m_ℓ	m_s	m_ℓ	m_s	m_ℓ	m_s	m_ℓ	m_s	m_ℓ	m_s	m_ℓ	m_s
Electron #1	0	$+\frac{1}{2}$	0	$+\frac{1}{2}$	0	$+\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$	0	$-\frac{1}{2}$
Electron #2	+1	$\pm\frac{1}{2}$	0	$-\frac{1}{2}$	−1	$\pm\frac{1}{2}$	+1	$\pm\frac{1}{2}$	0	$+\frac{1}{2}$	−1	$\pm\frac{1}{2}$

	m_ℓ	m_s	m_ℓ	m_s	m_ℓ	m_s	m_ℓ	m_s	m_ℓ	m_s	m_ℓ	m_s
Electron #1	−1	$+\frac{1}{2}$	−1	$+\frac{1}{2}$	−1	$+\frac{1}{2}$	−1	$-\frac{1}{2}$	−1	$-\frac{1}{2}$	−1	$-\frac{1}{2}$
Electron #2	+1	$\pm\frac{1}{2}$	0	$\pm\frac{1}{2}$	−1	$-\frac{1}{2}$	+1	$\pm\frac{1}{2}$	0	$\pm\frac{1}{2}$	−1	$+\frac{1}{2}$

There are $\boxed{\text{30 allowed states}}$, since Electron #1 can have any of three possible values of m_ℓ for both spin up and spin down, totaling six possible states. For each of these states, Electron #2 can be in either of the remaining five states.

(b) Were it not for the exclusion principle, there would be $\boxed{36}$ possible states, six for each electron independently.

28.33 (a) Observe the electron configurations given in the periodic table on the inside back cover of the textbook. Zirconium, with 40 electrons, has 4 electrons outside a closed krypton core. The krypton core, with 36 electrons, has all states up through the $4p$ subshell filled. Normally, one would expect the next 4 electrons to go into the $4d$ subshell. However, an exception to the rule occurs at this point, and the $5s$ subshell fills (with 2 electrons) before the $4d$ subshell starts filling. The two remaining electrons in zirconium are in an incomplete $4d$ subshell. Thus, $\boxed{n = 4 \text{ and } \ell = 2}$ for each of these electrons.

(b) For electrons in the $4d$ subshell, with $\ell = 2$, the possible values of m_ℓ are $\boxed{m_\ell = 0, \pm 1, \pm 2}$, and those for m_s are $\boxed{m_s = \pm 1/2}$.

(c) We have 40 electrons, so the electron configuration is:

$$\boxed{1s^2\, 2s^2\, 2p^6\, 3s^2\, 3p^6\, 3d^{10}\, 4s^2\, 4p^6\, 4d^2\, 5s^2 = [\text{Kr}]4d^2\, 5s^2}$$

28.34 For electrons accelerated through a potential difference of 40.0 kV, starting from rest, the kinetic energy is $KE = e(\Delta V) = e(40.0 \text{ kV}) = 40.0 \text{ keV}$. When these electrons strike the tungsten target, the shortest wavelength possible is produced when the electron loses all its kinetic energy and is brought to rest in a single collision, producing a single photon. The wavelength of the photon produced is $\lambda_{\min} = hc\big/\left(E_{\text{photon}}\right)_{\max} = hc/KE = hc/e(\Delta V)$. This gives

$$\lambda_{\min} = \frac{\left(6.63\times10^{-34} \text{ J}\cdot\text{s}\right)\left(3.00\times10^{8} \text{ m/s}\right)}{\left(1.60\times10^{-19} \text{ C}\right)\left(40.0\times10^{3} \text{ V}\right)} = 3.11\times10^{-11} \text{ m} = \boxed{0.0311 \text{ nm}}$$

28.35 (a) Note that $Z = 83$ for bismuth. With a vacancy in the L-shell, an electron in the M-shell is shielded from the nuclear charge by a total of 9 electrons, 2 electrons in the filled K-shell and 7 electrons in the partially filled L-shell. Thus, our estimate for the energy of this electron while it is in the M-shell is

$$E_{\text{M}} \approx \frac{-(Z-9)^2(13.6 \text{ eV})}{3^2} = \frac{-(74)^2(13.6 \text{ eV})}{9} = -8.27\times10^{3} \text{ eV} = -8.27 \text{ keV}$$

When this electron drops down to fill the vacancy in the L-shell, it will continue to be shielded from the nuclear charge by the 2 electrons in the filled K-shell. Our estimate for the energy of this electron in the L-shell is then

$$E_{\text{L}} \approx \frac{-(Z-2)^2(13.6 \text{ eV})}{2^2} = \frac{-(81)^2(13.6 \text{ eV})}{4} = -2.23\times10^{4} \text{ eV} = -22.3 \text{ keV}$$

Therefore, the estimate for the transitional energy, and hence the energy of the photon produced, in a M- to L-shell transition in bismuth is

$$E_{\text{photon}} = E_{\text{M}} - E_{\text{L}} \approx -8.27 \text{ keV} - (-22.3 \text{ keV}) = \boxed{14 \text{ keV}}$$

(b) The wavelength of the photon produced in the M- to L-shell transition should be approximately

$$\lambda = \frac{hc}{E_{\text{photon}}} \approx \frac{\left(6.63\times10^{-34} \text{ J}\cdot\text{s}\right)\left(3.00\times10^{8} \text{ m/s}\right)}{14 \text{ keV}}\left(\frac{1 \text{ keV}}{1.60\times10^{-16} \text{ J}}\right) = \boxed{8.9\times10^{-11} \text{ m}}$$

28.36 The energies in the K- and M-shells are

$$E_{\text{K}} \approx -\frac{(Z-1)^2(13.6 \text{ eV})}{(1)^2} \quad\text{and}\quad E_{\text{M}} \approx -\frac{(Z-9)^2(13.6 \text{ eV})}{(3)^2}$$

continued on next page

Thus,

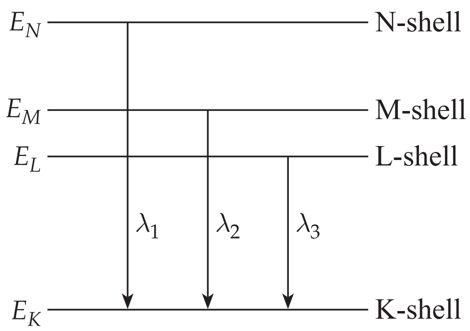
$$E_{\text{photon}} = E_{\text{M}} - E_{\text{K}} \approx (13.6\text{ eV})\left[-(Z-9)^2/9 + (Z-1)^2\right] = (13.6\text{ eV})\left(8Z^2/9 - 8\right)$$

and $E_{\text{photon}} = hc/\lambda$ gives $Z^2 = 9\left[8 + hc/(13.6\text{ eV})\lambda\right]/8$, or

$$Z \approx \sqrt{9 + \frac{9\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{8(13.6\text{ eV})\left(0.101\times10^{-9}\text{ m}\right)}\left(\frac{1\text{ eV}}{1.60\times10^{-19}\text{ J}}\right)} = 32.0$$

The element is $\boxed{\text{germanium}}$.

28.37 The transitions that produce the three longest wavelengths in the K series are shown at the right. The energy of the K-shell is $E_{\text{K}} = -69.5$ keV.



Thus, the energy of the L-shell is

$$E_{\text{L}} = E_{\text{K}} + \frac{hc}{\lambda_3}$$

or $$E_{\text{L}} = -69.5\text{ keV} + \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{0.021\,5\times10^{-9}\text{ m}}\left(\frac{1\text{ keV}}{1.60\times10^{-16}\text{ J}}\right)$$

$$= -69.5\text{ keV} + 57.8\text{ keV} = -11.7\text{ keV}$$

Similarly, the energies of the M- and N-shells are

$$E_{\text{M}} = E_{\text{K}} + \frac{hc}{\lambda_2} = -69.5\text{ keV} + \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(0.020\,9\times10^{-9}\text{ m}\right)\left(1.60\times10^{-16}\text{ J/keV}\right)} = -10.0\text{ keV}$$

and $$E_{\text{N}} = E_{\text{K}} + \frac{hc}{\lambda_1} = -69.5\text{ keV} + \frac{\left(6.63\times10^{-34}\text{ J}\cdot\text{s}\right)\left(3.00\times10^{8}\text{ m/s}\right)}{\left(0.018\,5\times10^{-9}\text{ m}\right)\left(1.60\times10^{-16}\text{ J/keV}\right)} = -2.30\text{ keV}$$

The ionization energies of the L-, M-, and N-shells are

$\boxed{\text{11.7 keV, 10.0 keV, and 2.30 keV, respectively}}$

28.38 According to the Bohr model, the radii of the electron orbits in hydrogen are given by

$$r_n = n^2 a_0,\text{ with } a_0 = 0.052\,9\text{ nm} = 5.29\times10^{-11}\text{ m}$$

Then, if $r_n \approx 1.00\ \mu\text{m} = 1.00\times10^{-6}$ m, the quantum number is

$$n = \sqrt{\frac{r_n}{a_0}} = \sqrt{\frac{1.00\times10^{-6}\text{ m}}{5.29\times10^{-11}\text{ m}}} \approx \boxed{137}$$

28.39 (a) $\Delta E = E_2 - E_1 = -13.6\text{ eV}/(2)^2 - \left(-13.6\text{ eV}/(1)^2\right) = \boxed{10.2\text{ eV}}$

(b) The average kinetic energy of the atoms must equal or exceed the needed excitation energy, or $\frac{3}{2}k_{\text{B}}T \geq \Delta E$, which gives

$$T \geq \frac{2(\Delta E)}{3k_{\text{B}}} = \frac{2(10.2\text{ eV})\left(1.60\times10^{-19}\text{ J/eV}\right)}{3\left(1.38\times10^{-23}\text{ J/K}\right)} = \boxed{7.88\times10^{4}\text{ K}}$$

28.40 (a) $L = c(\Delta t) = (3.00\times10^{8}\ \text{m/s})(14.0\times10^{-12}\ \text{s}) = 4.20\times10^{-3}\ \text{m} = \boxed{4.20\ \text{mm}}$

(b) $$N = \frac{E_{\text{pulse}}}{E_{\text{photon}}} = \frac{E_{\text{pulse}}}{hc/\lambda} = \frac{E_{\text{pulse}}\lambda}{hc}$$

$$= \frac{(3.00\ \text{J})(694.3\times10^{-9}\ \text{m})}{(6.63\times10^{-34}\ \text{J}\cdot\text{s})(3.00\times10^{8}\ \text{m/s})} = \boxed{1.05\times10^{19}\ \text{photons}}$$

(c) $$n = \frac{N}{V} = \frac{N}{L(\pi d^2/4)} = \frac{4N}{L(\pi d^2)}$$

$$= \frac{4(1.05\times10^{19}\ \text{photons})}{(4.20\ \text{mm})\pi(6.00\ \text{mm})^2} = \boxed{8.84\times10^{16}\ \text{photons/mm}^3}$$

28.41 (a) With one vacancy in the K-shell, an electron in the L-shell has one electron shielding it from the nuclear charge, so $Z_{\text{eff}} = Z - 1 = 24 - 1 = 23$. The estimated energy the atom gives up during a transition from the L-shell to the K-shell is then

$$\Delta E \approx E_i - E_f = -\frac{Z_{\text{eff}}^2(13.6\ \text{eV})}{n_i^2} - \left[-\frac{Z_{\text{eff}}^2(13.6\ \text{eV})}{n_f^2}\right] = Z_{\text{eff}}^2(13.6\ \text{eV})\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

or

$$\Delta E \approx (23)^2(13.6\ \text{eV})\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = 5.40\times10^{3}\ \text{eV} = \boxed{5.40\ \text{keV}}$$

(b) With a vacancy in the K-shell, we assume that $Z - 2 = 24 - 2 = 22$ electrons shield the outermost electron (in a 4s state) from the nuclear charge. Thus, for this outer electron, $Z_{\text{eff}} = 24 - 22 = 2$, and the estimated energy required to remove this electron from the atom is

$$E_{\text{ionization}} = E_f - E_i = 0 - E_i \approx -\left[-\frac{Z_{\text{eff}}^2(13.6\ \text{eV})}{n_i^2}\right] = \frac{2^2(13.6\ \text{eV})}{4^2} = \boxed{3.40\ \text{eV}}$$

(c) $KE = \Delta E - E_{\text{ionization}} = 5.40\ \text{keV} - 3.40\ \text{eV} \approx \boxed{5.40\ \text{keV}}$

28.42 (a) Requiring the angular momentum associated with the orbital motion of Earth to satisfy Bohr's postulate gives $M_E vr = n\hbar$, or the speed of the Earth in the nth allowed orbit would be $v_n = n\hbar/M_E r_n$. The gravitational force between Earth and the Sun supplies the needed centripetal acceleration as Earth moves in its orbit. Thus, $M_E v^2/r = GM_S M_E/r^2$, or $v^2 = GM_S/r$. Substituting for the speed in the nth orbit from above gives $(n\hbar/M_E r_n)^2 = GM_S/r_n$, which reduces to $\boxed{r_n = n^2\hbar^2/GM_S M_E^2}$.

(b) From Table 7.3 and the back flysheet of the textbook, $M_S = 1.991\times10^{30}\ \text{kg}$, $M_E = 5.98\times10^{24}\ \text{kg}$, $r = 1.496\times10^{11}\ \text{m}$, $G = 6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2$, and $\hbar = 1.05\times10^{-34}\ \text{J}\cdot\text{s}$. We then find the quantum number for Earth's orbit to be

$$n = \frac{M_E\sqrt{r_n GM_S}}{\hbar}$$

$$= \frac{(5.98\times10^{24}\ \text{kg})\sqrt{(1.496\times10^{11}\ \text{m})(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2)(1.991\times10^{30}\ \text{kg})}}{1.05\times10^{-34}\ \text{J}\cdot\text{s}}$$

or $n = \boxed{2.54\times10^{74}}$

continued on next page

(c) The difference in the radii of the nth and the $(n+1)$th allowed orbits of the Earth is

$$\Delta r = r_{n+1} - r_n = \frac{\left[(n+1)^2 - n^2\right]\hbar^2}{GM_S M_E^2} = \frac{[2n+1]\hbar^2}{GM_S M_E^2} \approx \frac{2n\hbar^2}{GM_S M_E^2}$$

Using the values from part (b) above gives

$$\Delta r \approx \frac{2n\hbar^2}{GM_S M_E^2} = \frac{2\left(2.54\times10^{74}\right)\left(1.05\times10^{-34}\ \text{J}\cdot\text{s}\right)^2}{\left(6.67\times10^{-11}\ \text{N}\cdot\text{m}^2/\text{kg}^2\right)\left(1.991\times10^{30}\ \text{kg}\right)\left(5.98\times10^{24}\ \text{kg}\right)^2}$$

$$= \boxed{1.18\times10^{-63}\ \text{m}}$$

(d) The result computed in part (d) above is *much smaller* than the radius of an atomic nucleus ($\sim 10^{-15}$ m), so the distance between quantized orbits of the Earth is too small to observe.

28.43 (a) $I = \dfrac{P}{A} = \dfrac{(\Delta E/\Delta t)}{\pi d^2/4} = \dfrac{4\left(3.00\times10^{-3}\ \text{J}/1.00\times10^{-9}\ \text{s}\right)}{\pi\left(30.0\times10^{-6}\ \text{m}\right)^2} = \boxed{4.24\times10^{15}\ \text{W/m}^2}$

(b) $E = IA(\Delta t) = \left(4.24\times10^{15}\ \dfrac{\text{W}}{\text{m}^2}\right)\left[\dfrac{\pi}{4}\left(0.600\times10^{-9}\ \text{m}\right)^2\right]\left(1.00\times10^{-9}\ \text{s}\right) = \boxed{1.20\times10^{-12}\ \text{J}}$

28.44 (a) Given that the de Broglie wavelength of the electron is $\lambda = 2a_0$, its momentum is $p = h/\lambda = h/2a_0$. The kinetic energy of this nonrelativistic electron is

$$KE = \frac{p^2}{2m_e} = \frac{h^2}{8m_e a_0^2}$$

$$= \frac{\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)^2\left(1\ \text{eV}/1.60\times10^{-19}\ \text{J}\right)}{8\left(9.11\times10^{-31}\ \text{kg}\right)\left(0.052\,9\times10^{-9}\ \text{m}\right)^2} = \boxed{135\ \text{eV}}$$

(b) The kinetic energy of this electron is $\boxed{\approx 10\ \text{times}}$ the magnitude of the ground state energy of the hydrogen atom, which is –13.6 eV.

28.45 In the Bohr model,

$$f = \frac{E_n - E_{n-1}}{h} = \frac{1}{h}\left[\frac{-m_e k_e^2 e^4}{2\hbar^2}\left(\frac{1}{n^2} - \frac{1}{(n-1)^2}\right)\right] = \frac{4\pi^2 m_e k_e^2 e^4}{2h^3}\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right]$$

which reduces to $\boxed{f = \dfrac{2\pi^2 m_e k_e^2 e^4}{h^3}\left(\dfrac{2n-1}{(n-1)^2 n^2}\right)}$.

28.46 $E_{\text{photon}} = \dfrac{hc}{\lambda} = \dfrac{\left(6.626\times10^{-34}\ \text{J}\cdot\text{s}\right)\left(2.998\times10^8\ \text{m/s}\right)}{\lambda\left(1.602\times10^{-19}\ \text{J/eV}\right)\left(10^{-9}\ \text{m/nm}\right)} = \dfrac{1\,240\ \text{eV}\cdot\text{nm}}{\lambda} = \Delta E$

For $\lambda = 310.0$ nm, $\Delta E = 4.000$ eV; $\lambda = 400.0$ nm, $\Delta E = 3.100$ eV; and $\lambda = 1\,378$ nm, $\Delta E = 0.900\,0$ eV.

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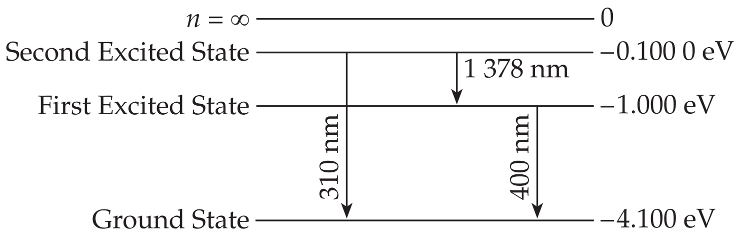
The ionization energy is 4.100 eV. The energy level diagram having the smallest number of levels, and consistent with these energy differences, is shown below.



29

Nuclear Physics

QUICK QUIZZES

1. False. In a sample containing a very large number of identical radioactive atoms, 50% of the original atoms remain after 1 half-life has elapsed. During the second half-life, half of the remaining 50% of the atoms decay, leaving 25% of the original atoms still present. Thus, after 2 half-lives have elapsed, only 75% of the original radioactive atoms have decayed. An individual radioactive atom may exist an indefinite time before decaying.

2. Choice (c). At the end of the first half-life interval, half of the original sample has decayed and half remains. During the second half-life interval, half of the remaining portion of the sample decays, leaving one-quarter of the original sample. During the third half-life, half of that quarter will decay. The total fraction of the sample that has decayed during the three half-lives is

$$\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2}\right)+\frac{1}{2}\left(\frac{1}{4}\right)=\frac{7}{8}$$

3. Choice (c). The half-life of a radioactive material is $T_{1/2}=\ln 2/\lambda$, where λ is the decay constant for that material. Thus, if $\lambda_A=2\lambda_B$, we have

$$\left(T_{1/2}\right)_A=\frac{\ln 2}{\lambda_A}=\frac{\ln 2}{2\lambda_B}=\frac{\left(T_{1/2}\right)_B}{2}=\frac{4\text{ h}}{2}=2\text{ h}$$

4. Choices (a) and (b). Reactions (a) and (b) both conserve total charge and total mass number as required. Reaction (c) violates conservation of mass number with the sum of the mass numbers being 240 before the reaction and being only 223 after the reaction.

5. Choice (b). The reactant nuclei in this endothermic reaction must supply $|Q|=2.17$ MeV of energy to be converted into mass during the reaction. However, the reactant nuclei must also supply the emerging particles with sufficient kinetic energy to allow momentum to be conserved during the reaction. Thus, the threshold kinetic energy for the reaction (minimum total kinetic energy of the particles going into the reaction) must exceed the value $|Q|=2.17$ MeV.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. Nuclei are approximately spherical with average radii of $r=r_0A^{1/3}$, where $r_0=$ constant. Thus, the ratio of the volume of a ^{20}Ne nucleus to that of a ^{4}He is

$$\frac{V_{\text{Ne}}}{V_{\text{He}}}=\frac{\frac{4}{3}\pi r_{\text{Ne}}^3}{\frac{4}{3}\pi r_{\text{He}}^3}=\frac{\left(r_0A_{\text{Ne}}^{1/3}\right)^3}{\left(r_0A_{\text{He}}^{1/3}\right)^3}=\frac{A_{\text{Ne}}}{A_{\text{He}}}=\frac{20}{4}=5$$

and (d) is the correct choice.

2. $m=(15.999\text{ u})\left(\dfrac{931.5\text{ MeV}/c^2}{1\text{ u}}\right)=1.49\times10^4\text{ MeV}/c^2$, and the correct answer is choice (e).

3. $m = (15.999 \text{ u})\left(\dfrac{1.66\times10^{-27}\text{ kg}}{1\text{ u}}\right) = 2.66\times10^{-26}$ kg, so choice (e) is the correct answer.

4. From the binding energy curve shown in Figure 29.4 of the textbook, the approximate binding energies per nucleon in the isotopes ^{35}Cl, ^{62}Ni, and ^{197}Au are seen to be 8.4 MeV, 8.8 MeV, and 7.9 MeV, respectively. Therefore, the correct ranking, from smallest to largest, is gold, chlorine, nickel, which is choice (a).

5. The nucleus $^{40}_{18}\text{X}$ contains $A = 40$ total nucleons, of which $Z = 18$ are protons. The remaining $A - Z = 40 - 18 = 22$ are neutrons, and choice (c) is correct.

6. In a large sample, one-half of the radioactive nuclei initially present remain in the sample after one half-life has elapsed. Hence, the fraction of the original number of radioactive nuclei remaining after n half-lives have elapsed is $(1/2)^n = 1/2^n$. In this case, the number of half-lives that have elapsed is $\Delta t/T_{1/2} = 14\text{ d}/3.6\text{ d} \approx 4$. Therefore, the approximate fraction of the original sample that remains undecayed is $1/2^4 = 1/16$, and the correct answer is choice (d).

7. The half-life of a substance is related to its decay constant by $T_{1/2} = \ln 2/\lambda$. The desired ratio is then

$$\frac{\left(T_{1/2}\right)_A}{\left(T_{1/2}\right)_B} = \frac{\ln 2/\lambda_A}{\ln 2/\lambda_B} = \frac{\lambda_B}{\lambda_A} = \frac{1}{3}$$

so (a) is the correct choice.

8. In the beta decay of $^{95}_{36}\text{Kr}$, the emitted particles are an electron, $^{0}_{-1}\text{e}$, and an antineutrino, $\bar{\nu}_e$. The emitted particles contain a total charge of $-e$ and zero nucleons. Thus, to conserve both charge and nucleon number, the daughter nucleus must be $^{95}_{37}\text{Rb}$, which contains $Z = 37$ protons and $A - Z = 95 - 37 = 58$ neutrons, making (a) the correct choice.

9. $^{32}_{15}\text{P}$ decays to $^{32}_{16}\text{S}$ by means of beta decay, with the decay equation being

$$^{32}_{15}\text{P} \rightarrow {}^{32}_{16}\text{S} + {}^{0}_{-1}\text{e} + \bar{\nu}_e$$

As will be discussed in Chapter 30, the antineutrino must be emitted along with the electron in order to conserve electron-lepton number. The correct choices are (c) and (e).

10. To conserve the total number of nucleons, it is necessary that $A + 4 = 234$, or $A = 230$. Conservation of charge demands that $Z + 2 = 90$, or $Z = 88$. We then see that the correct answer is choice (c).

11. In gamma decay, an unstable nucleus gives off excess energy by emitting a high-energy photon. The daughter nucleus is the same as the parent, with both the charge and nucleon number unchanged, simply in a lower energy state. Thus, the correct choice is (b).

12. The Q-value for the reaction $^{9}_{4}\text{Be} + {}^{4}_{2}\text{He} \rightarrow {}^{12}_{6}\text{C} + {}^{1}_{0}\text{n}$ is

$$Q = (\Delta m)c^2 = \left(m_{^9_4\text{Be}} + m_{^4_2\text{He}} - m_{^{12}_6\text{C}} - m_n\right)c^2$$

$$= [9.012\,182\text{ u} + 4.002\,603\text{ u} - 12.000\,000\text{ u} - 1.008\,665\text{ u}](931.5\text{ Mev/u})$$

$$= 5.70\text{ MeV}$$

so (d) is the correct choice.

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The nucleus was at rest before decay, so the total linear momentum will be zero both before and after decay. Thus, the alpha particle and the daughter nucleus carry equal amounts of momentum in opposite directions. Since kinetic energy can be written as $KE = p^2/2m$, the small-mass alpha particle has much more of the decay energy than the recoiling daughter nucleus.

4. Extra neutrons are required to overcome the increasing electrostatic repulsion of the protons. The neutrons participate in the net attractive effect of the nuclear force but feel no Coulomb repulsion.

6. An alpha particle is a doubly positive charged helium nucleus, is very massive, and does not penetrate very well. A beta particle is a singly negative charged electron, is very low mass, and only slightly more difficult to shield from. A gamma ray is a high-energy photon or high frequency electromagnetic wave and has high penetrating ability.

8. After one half-life, one-half the radioactive atoms have decayed. After the second half-life, one-half of the remaining atoms have decayed. Therefore, $\frac{1}{2}+\frac{1}{2}\left(\frac{1}{2}\right)=\frac{3}{4}$ of the original radioactive atoms have decayed after two half-lives.

10. With a very small mass in comparison to alpha particles, beta particles have greater penetrating ability than do alpha particles.

12. The alpha particle, with a mass about 7 000 times that of the beta particle, is much more difficult to deflect. Even under the influence of an electrical force whose magnitude is double that acting on the beta particle, the alpha particle is deviated from its original path by a much smaller amount.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. **(a)** 1.9 fm **(b)** 7.4 fm

4. **(a)** 4.8 fm **(b)** $4.6\times10^{-43}\ \text{m}^3$ **(c)** $2.3\times10^{17}\ \text{kg/m}^3$

6. **(a)** $r_{^{12}\text{C}} = 7.89\ \text{cm}$, $r_{^{13}\text{C}} = 8.21\ \text{cm}$

(b) $\sqrt{\dfrac{m_{^{12}\text{C}}}{m_{^{13}\text{C}}}} = \sqrt{\dfrac{12\ \text{u}}{13\ \text{u}}} = 0.961$ and $\dfrac{r_{^{12}\text{C}}}{r_{^{13}\text{C}}} = \dfrac{7.89\ \text{cm}}{8.21\ \text{cm}} = 0.961$

8. 16 km

10. **(a)** 1.11 MeV/nucleon **(b)** 7.07 MeV/nucleon **(c)** 8.79 MeV/nucleon

(d) 7.57 MeV/nucleon

12. For ${}^{55}_{25}\text{Mn}$, $E_b/A = 8.765$ MeV. For ${}^{56}_{26}\text{Fe}$, $E_b/A = 8.790$ MeV. For ${}^{59}_{27}\text{Co}$, $E_b/A = 8.768$ MeV. This gives us finer detail than is shown in Figure 29.4.

14. 7.93 MeV

16. 8.7×10^{3} Bq

18. **(a)** 0.755 **(b)** 0.570 **(c)** 9.77×10^{-4}

(d) No. The decay model depends on large numbers of nuclei. After some long but finite time, only one undecayed nucleus will remain. It is likely that the decay of this final nucleus will occur before infinite time.

20. 1.72×10^4 yr

22. 1.7×10^3 yr

24. **(a)** ${}^{12}_{6}\text{C}$ **(b)** ${}^{4}_{2}\text{He}$ **(c)** ${}^{14}_{6}\text{C}$

26. 4.28 MeV

28. **(a)** ${}^{66}_{28}\text{Ni} \rightarrow {}^{66}_{29}\text{Cu} + {}^{0}_{-1}\text{e} + \bar{\nu}_e$ **(b)** 186 keV

30. 2.81×10^4 yr

32. 1.00 MeV

34. **(a)** 8.023 829 u **(b)** 8.025 594 u **(c)** −1.64 MeV

(d) $m_p v = (m_{\text{Be}} + m_n)V$ **(e)** $\frac{1}{2}m_p v^2 = \frac{1}{2}(m_{\text{Be}} + m_n)V^2 - Q$ **(f)** 1.88 MeV

36. **(a)** 6.258 MeV, exothermic; 5.494 MeV, exothermic

(b) the first reaction releases more energy

(c) 1.88×10^{-15} m

38. **(a)** ${}^{4}_{2}\text{He} + {}^{14}_{7}\text{N} \rightarrow {}^{1}_{1}\text{H} + {}^{17}_{8}\text{O}$ **(b)** ${}^{7}_{3}\text{Li} + {}^{1}_{1}\text{H} \rightarrow {}^{4}_{2}\text{He} + {}^{4}_{2}\text{He}$

40. The equivalent dose is 5.0 rad of heavy ions.

42. **(a)** 2.00 J/kg **(b)** 4.78×10^{-4} °C

44. **(a)** 2.5×10^{-3} rem/x-ray **(b)** ≈ 38 times background levels

46. **(a)** 10 h **(b)** 3.2 m

48. 22.0 MeV

50. 156 keV

52. 12 mg

54. **(a)** 60.6 Bq/L **(b)** 40.6 days

56. **(a)** $N_0 = 2.5\times10^{24}$ **(b)** $R_0 = 2.3\times10^{12}$ Bq **(c)** 1.1×10^6 yr

PROBLEM SOLUTIONS

29.1 The average nuclear radii are $r = r_0 A^{1/3}$, where $r_0 = 1.2\times10^{-15}\ \text{m} = 1.2\ \text{fm}$ and A is the mass number.

(a) For ${}^{2}_{1}\text{H}$, $r = (1.2\ \text{fm})(2)^{1/3} = \boxed{1.5\ \text{fm}}$

(b) For ${}^{60}_{27}\text{Co}$, $r = (1.2\ \text{fm})(60)^{1/3} = \boxed{4.7\ \text{fm}}$

(c) For ${}^{197}_{79}\text{Au}$, $r = (1.2\ \text{fm})(197)^{1/3} = \boxed{7.0\ \text{fm}}$

(d) For ${}^{239}_{94}\text{Pu}$, $r = (1.2\ \text{fm})(239)^{1/3} = \boxed{7.4\ \text{fm}}$

29.2 (a) For ${}^{4}_{2}\text{He}$, $r = r_0 A^{1/3} = (1.2\times10^{-15}\ \text{m})(4)^{1/3} = 1.9\times10^{-15}\ \text{m} = \boxed{1.9\ \text{fm}}$

(b) For ${}^{238}_{92}\text{U}$, $r = r_0 A^{1/3} = (1.2\times10^{-15}\ \text{m})(238)^{1/3} = 7.4\times10^{-15}\ \text{m} = \boxed{7.4\ \text{fm}}$

29.3 From $M_E = \rho_{\text{nuclear}} V = \rho_{\text{nuclear}}(4\pi r^3/3)$, we find

$$r = \left(\frac{3M_E}{4\pi\rho_{\text{nuclear}}}\right)^{1/3} = \left[\frac{3(5.98\times10^{24}\ \text{kg})}{4\pi(2.3\times10^{17}\ \text{kg/m}^3)}\right]^{1/3} = \boxed{1.8\times10^{2}\ \text{m}}$$

29.4 (a) $r \approx r_0 A^{1/3} = (1.2\ \text{fm})(65)^{1/3} = \boxed{4.8\ \text{fm}}$

(b) $V = \frac{4}{3}\pi r^3 \approx \frac{4}{3}\pi(4.8\times10^{-15}\ \text{m})^3 = \boxed{4.6\times10^{-43}\ \text{m}^3}$

(c) $\rho = \frac{m}{V} \approx \frac{65\ \text{u}}{V} = \frac{65(1.66\times10^{-27}\ \text{kg})}{4.6\times10^{-43}\ \text{m}^3} = \boxed{2.3\times10^{17}\ \text{kg/m}^3}$

29.5 (a) $$F_{\max} = \frac{k_e q_1 q_2}{r_{\min}^2} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left[(2)(6)(1.60\times10^{-19}\ \text{C})^2\right]}{(1.00\times10^{-14}\ \text{m})^2} = \boxed{27.6\ \text{N}}$$

(b) $$a_{\max} = \frac{F_{\max}}{m_\alpha} = \frac{27.6\ \text{N}}{6.64\times10^{-27}\ \text{kg}} = \boxed{4.16\times10^{27}\ \text{m/s}^2}$$

(c) $$PE_{\max} = \frac{k_e q_1 q_2}{r_{\min}} = \frac{(8.99\times10^9\ \text{N}\cdot\text{m}^2/\text{C}^2)\left[(2)(6)(1.60\times10^{-19}\ \text{C})^2\right]}{1.00\times10^{-14}\ \text{m}}\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right),$$

yielding $PE_{\max} = \boxed{1.73\ \text{MeV}}$.

29.6 (a) From conservation of energy, $\Delta KE = -\Delta PE$, or $\frac{1}{2}mv^2 = q(\Delta V)$. Also, the centripetal acceleration is supplied by the magnetic force, so $mv^2/r = qvB$, or $v = qBr/m$. The energy equation then yields $r = \sqrt{2m(\Delta V)/qB^2}$. Applying this to the two isotopes of carbon in this case gives

$$r_{^{12}\text{C}} = \sqrt{\frac{2\left[12(1.66\times10^{-27}\ \text{kg})\right](1\,000\ \text{V})}{(1.60\times10^{-19}\ \text{C})(0.200\ \text{T})^2}} = 7.89\times10^{-2}\ \text{m} = \boxed{7.89\ \text{cm}}$$

continued on next page

and

$$r_{^{13}\text{C}} = \sqrt{\frac{2\left[13\left(1.66\times10^{-27}\text{ kg}\right)\right]\left(1\,000\text{ V}\right)}{\left(1.60\times10^{-19}\text{ C}\right)\left(0.200\text{ T}\right)^2}} = 8.21\times10^{-2}\text{ m} = \boxed{8.21\text{ cm}}$$

(b) From above, we see that the radii of the paths followed by singly ionized atoms of the two isotopes should be $r_{^{12}\text{C}} = \sqrt{2m_{^{12}\text{C}}\Delta V/eB^2}$ and $r_{^{13}\text{C}} = \sqrt{2m_{^{13}\text{C}}\Delta V/eB^2}$. Thus, the ratio of these radii will be

$$\frac{r_{^{12}\text{C}}}{r_{^{13}\text{C}}} = \frac{\sqrt{2m_{^{12}\text{C}}\Delta V/eB^2}}{\sqrt{2m_{^{13}\text{C}}(\Delta V)/eB^2}} = \sqrt{\frac{m_{^{12}\text{C}}}{m_{^{13}\text{C}}}} = \sqrt{\frac{12\text{ u}}{13\text{ u}}} = \boxed{0.961}$$

and the ratio of our computed radii is $\boxed{r_{^{12}\text{C}}/r_{^{13}\text{C}} = 7.89\text{ cm}/8.21\text{ cm} = 0.961}$, so they do agree.

29.7 (a) At the point of closest approach, $PE_f = KE_i$, so $k_e(2e)(79e)/r_{\min} = m_\alpha v_i^2/2$, or

$$v_i = \sqrt{\frac{2k_e(2e)(79e)}{m_\alpha r_{\min}}}$$

$$= \sqrt{\frac{316\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left(1.60\times10^{-19}\text{ C}\right)^2}{\left(6.64\times10^{-27}\text{ kg}\right)\left(3.2\times10^{-14}\text{ m}\right)}} = \sqrt{3.42}\times10^7\text{ m/s} = \boxed{1.9\times10^7\text{ m/s}}$$

(b) $$KE_i = PE_f = \frac{k_e q_1 q_2}{r_{\min}} = \frac{\left(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2\right)\left[(2)(79)\left(1.60\times10^{-19}\text{ C}\right)^2\right]}{3.2\times10^{-14}\text{ m}}$$

$$= 1.14\times10^{-12}\text{ J}\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) = \boxed{7.1\text{ MeV}}$$

29.8 If a star with a mass of two solar masses collapsed into a gigantic nucleus by converting all of its mass into neutrons, the total number of nucleons (all neutrons), and hence the atomic number, would be

$$A = \frac{m}{m_n} = \frac{2m_{\text{Sun}}}{m_n} = \frac{2\left(1.99\times10^{30}\text{ kg}\right)}{1.67\times10^{-27}\text{ kg}} = 2.38\times10^{57}$$

and its approximate radius would be

$$r \approx r_0 A^{1/3} = \left(1.2\times10^{-15}\text{ m}\right)\left(2.38\times10^{57}\right)^{1/3} = 1.6\times10^4\text{ m} = \boxed{16\text{ km}}$$

29.9 (a) The total binding energy for ${}^{24}_{12}\text{Mg}$ is $E_b = (\Delta m)c^2 = \left(12m_{^1\text{H}} + 12m_n - m_{^{24}\text{Mg}}\right)c^2$, and the average binding energy per nucleon is

$$\frac{E_b}{A} = \frac{\left[12\left(1.007\,825\text{ u}\right) + 12\left(1.008\,665\text{ u}\right) - 23.985\,042\text{ u}\right]\left(931.5\text{ MeV/u}\right)}{24}$$

$$= \boxed{8.26\text{ MeV/nucleon}}$$

(b) For ${}^{85}_{37}\text{Rb}$, $E_b = (\Delta m)c^2 = \left(37m_{^1\text{H}} + 48m_n - m_{^{85}\text{Rb}}\right)c^2$, yielding

$$\frac{E_b}{A} = \frac{\left[37\left(1.007\,825\text{ u}\right) + 48\left(1.008\,665\text{ u}\right) - 84.911\,789\text{ u}\right]\left(931.5\text{ MeV/u}\right)}{85}$$

$$= \boxed{8.70\text{ MeV/nucleon}}$$

29.10 (a) For ${}_1^2\text{H}$, $\Delta m = 1(1.007\ 825\,\text{u}) + 1(1.008\ 665\,\text{u}) - (2.014\ 102\,\text{u}) = 0.002\ 388\,\text{u}$, and

$$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.002\ 388\,\text{u})(931.5\ \text{MeV/u})}{2} = \boxed{1.11\ \text{MeV/nucleon}}$$

(b) For ${}_2^4\text{He}$, $\Delta m = 2(1.007\ 825\,\text{u}) + 2(1.008\ 665\,\text{u}) - (4.002\ 603\,\text{u}) = 0.030\ 377\,\text{u}$, and

$$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.030\ 377\,\text{u})(931.5\ \text{MeV/u})}{4} = \boxed{7.07\ \text{MeV/nucleon}}$$

(c) For ${}_{26}^{56}\text{Fe}$, $\Delta m = 26(1.007\ 825\,\text{u}) + 30(1.008\ 665\,\text{u}) - (55.934\ 942) = 0.528\ 458\,\text{u}$, and

$$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(0.528\ 458\,\text{u})(931.5\ \text{MeV/u})}{56} = \boxed{8.79\ \text{MeV/nucleon}}$$

(d) For ${}_{92}^{238}\text{U}$, $\Delta m = 92(1.007\ 825\,\text{u}) + 146(1.008\ 665\,\text{u}) - (238.050\ 783) = 1.934\ 207\,\text{u}$, and

$$\frac{E_b}{A} = \frac{(\Delta m)c^2}{A} = \frac{(1.934\ 207\,\text{u})(931.5\ \text{MeV/u})}{238} = \boxed{7.57\ \text{MeV/nucleon}}$$

29.11 For ${}_8^{15}\text{O}$, $\Delta m = 8(1.007\ 825\,\text{u}) + 7(1.008\ 665\,\text{u}) - (15.003\ 065) = 0.120\ 190$ u, and

$$E_b\big|_{^{15}\text{O}} = (\Delta m)c^2 = (0.120\,190\,\text{u})(931.5\ \text{MeV/u}) = 112.0\ \text{MeV}$$

For ${}_7^{15}\text{N}$, $\Delta m = 7(1.007\,825\,\text{u}) + 8(1.008\,665\,\text{u}) - (15.000\,109) = 0.123\,986$ u, and

$$E_b\big|_{^{15}\text{N}} = (\Delta m)c^2 = (0.123\,986\,\text{u})(931.5\ \text{MeV/u}) = 115.5\ \text{MeV}$$

Therefore, $\Delta E_b = E_b\big|_{^{15}\text{N}} - E_b\big|_{^{15}\text{O}} = \boxed{3.5\ \text{MeV}}$

29.12 $\Delta m = Z\,m_{^1_1\text{H}} + (A-Z)m_n - m$ and $E_b/A = \Delta m(931.5\ \text{MeV/u})/A$

Nucleus	Z	$(A-Z)$	m (in u)	Δm (in u)	E_b/A (in MeV)
${}_{25}^{55}\text{Mn}$	25	30	54.938 050	0.517 525	8.765
${}_{26}^{56}\text{Fe}$	26	30	55.934 942	0.528 458	8.790
${}_{27}^{59}\text{Co}$	27	32	58.933 200	0.555 355	8.768

Therefore, ${}_{26}^{56}\text{Fe}$ has a greater binding energy per nucleon than its neighbors. This gives us finer detail than is shown in Figure 29.4.

29.13 (a) For ${}_{11}^{23}\text{Na}$, $\Delta m = 11(1.007\ 825\,\text{u}) + 12(1.008\ 665\,\text{u}) - (22.989\ 770\,\text{u}) = 0.200\ 285\,\text{u}$, and for ${}_{12}^{23}\text{Mg}$, $\Delta m = 12(1.007\ 825\,\text{u}) + 11(1.008\ 665\,\text{u}) - (22.994\ 127\,\text{u}) = 0.195\ 088\,\text{u}$. The difference in the binding energy per nucleon for these two isobars is then

$$\frac{\Delta E_b}{A} = \frac{\left[(\Delta m)_{^{23}_{11}\text{Na}} - (\Delta m)_{^{23}_{12}\text{Mg}}\right]c^2}{A} = \frac{[0.200\,285\ \text{u} - 0.195\,088\ \text{u}](931.5\ \text{MeV/u})}{23}$$

$$= \boxed{0.210\ \text{MeV/nucleon}}$$

(b) The binding energy per nucleon is greater by 0.210 MeV/nucleon in ${}_{11}^{23}\text{Na}$. This is attributable to $\boxed{\text{less proton repulsion in } {}_{11}^{23}\text{Na than in } {}_{12}^{23}\text{Mg}}$.

29.14 The sum of the mass of ${}^{42}_{20}\text{Ca}$ plus the mass of a neutron exceeds the mass of ${}^{43}_{20}\text{Ca}$. This difference in mass must represent the mass equivalence of the energy spent removing the last neutron from ${}^{43}_{20}\text{Ca}$ to produce ${}^{42}_{20}\text{Ca}$ plus a free neutron. Thus,

$$E=\left(m_{{}^{42}_{20}\text{Ca}}+m_n-m_{{}^{43}_{20}\text{Ca}}\right)c^2=\left(41.958\,622\text{ u}+1.008\,665\text{ u}-42.958\,770\text{ u}\right)c^2$$

or

$$E=\left(0.008\,517\text{ u}\right)\left(931.5\text{ MeV/u}\right)=\boxed{7.93\text{ MeV}}$$

29.15 The mass of radon present at time t is equal to $m=m_{\text{atom}}N=m_{\text{atom}}N_0e^{-\lambda t}=m_0e^{-\lambda t}$, where m_{atom} is the mass of a single radon atom, N is the number or radon nuclei (and hence, atoms) present, and N_0 is the number present at time $t=0$, making m_0 the mass of radon present at $t=0$. The decay constant for radon is $\lambda=\ln 2/T_{1/2}=\ln 2/(3.83\text{ d})$, yielding

$$m=m_0e^{-\lambda t}=m_0e^{-\left(t/T_{1/2}\right)\ln 2}=\left(3.00\text{ g}\right)e^{-(1.50\text{ d}/3.83\text{ d})\ln 2}=\boxed{2.29\text{ g}}$$

29.16 The activity is $R=\lambda N=\lambda N_0e^{-\lambda t}=R_0e^{-\lambda t}$, where R_0 is the activity at time $t=0$, and the decay constant is $\lambda=\ln 2/T_{1/2}$. Thus,

$$R=R_0e^{-\left(t/T_{1/2}\right)\ln 2}=\left(1.1\times10^4\text{ Bq}\right)e^{-(2.0\text{ h}/6.05\text{ h})\ln 2}=\boxed{8.7\times10^3\text{ Bq}}$$

29.17 (a) $T_{1/2}=8.04\text{ d}\left(\dfrac{8.64\times10^4\text{ s}}{1\text{ d}}\right)=\boxed{6.95\times10^5\text{ s}}$

(b) $\lambda=\dfrac{\ln 2}{T_{1/2}}=\dfrac{\ln 2}{6.95\times10^5\text{ s}}=\boxed{9.97\times10^{-7}\text{ s}^{-1}}$

(c) $R=0.500\ \mu\text{Ci}=\left(0.500\times10^{-6}\text{ Ci}\right)\left(\dfrac{3.7\times10^{10}\text{ Bq}}{1\text{ Ci}}\right)=\boxed{1.9\times10^4\text{ Bq}}$

(d) From $R=\lambda N$, the number of radioactive nuclei in a 0.500 μCi of ${}^{131}\text{I}$ is

$$N=\frac{R}{\lambda}=\frac{1.9\times10^4\text{ s}^{-1}}{9.97\times10^{-7}\text{ s}^{-1}}=\boxed{1.9\times10^{10}\text{ nuclei}}$$

(e) The number of half-lives that have elapsed is $n=t/T_{1/2}=40.2\text{ d}/8.04\text{ d}=\boxed{5.00}$, so the remaining activity of the sample is

$$R=\frac{R_0}{2^n}=\frac{R_0}{2^{5.00}}=\frac{6.40\text{ mCi}}{32.0}=\boxed{0.200\text{ mCi}}$$

29.18 (a) From Equation (29.4a) in the textbook, the fraction remaining at $t=5.00$ yr will be

$$\frac{N}{N_0}==e^{-\lambda t}=e^{-\left(t/T_{1/2}\right)\ln 2}=e^{-(5.00\text{ yr}/12.33\text{ yr})\ln 2}=\boxed{0.755}$$

(b) At $t=10.0$ yr, $N/N_0=e^{-\lambda t}=e^{-\left(t/T_{1/2}\right)\ln 2}=e^{-(10.0\text{ yr}/12.33\text{ yr})\ln 2}=\boxed{0.570}$.

(c) At $t=123.3$ yr, $N/N_0=e^{-\lambda t}=e^{-\left(t/T_{1/2}\right)\ln 2}=e^{-(123.3\text{ yr}/12.33\text{ yr})\ln 2}=e^{-(10.0)\ln 2}=\boxed{9.77\times10^{-4}}$.

(d) No. The decay model depends on large numbers of nuclei. After some long but finite time, only one undecayed nucleus will remain. It is likely that the decay of this final nucleus will occur before infinite time.

29.19 From $R = \lambda N = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$, with $R = (0.842)R_0$, we find $e^{-\lambda t} = R/R_0$, and $\lambda t = -\ln(R/R_0)$. Since the half-life may be expressed as $T_{1/2} = \ln 2/\lambda$, this yields

$$T_{1/2} = -\frac{t\ln 2}{\ln(R/R_0)} = -\frac{(2.00\ \text{d})\ln 2}{\ln(0.842)} = \boxed{8.06\ \text{d}}$$

29.20 Using $R = \lambda N = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$, with $R/R_0 = 0.125$, gives $\lambda t = -\ln(R/R_0)$, or

$$t = -\frac{\ln(R/R_0)}{\lambda} = -T_{1/2}\left[\frac{\ln(R/R_0)}{\ln 2}\right] = -(5\,730\ \text{yr})\left[\frac{\ln(0.125)}{\ln 2}\right] = \boxed{1.72\times10^4\ \text{yr}}$$

29.21 (a) The initial activity is $R_0 = 10.0$ mCi, and at $t = 4.00$ h, $R = 8.00$ mCi. Then, from $R = \lambda N = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$, the decay constant is

$$\lambda = -\frac{\ln(R/R_0)}{t} = -\frac{\ln(0.800)}{4.00\ \text{h}} = \boxed{5.58\times10^{-2}\ \text{h}^{-1}}$$

and the half-life is $T_{1/2} = \dfrac{\ln 2}{\lambda} = \dfrac{\ln 2}{5.58\times10^{-2}\ \text{h}^{-1}} = \boxed{12.4\ \text{h}}$

(b) $$N_0 = \frac{R_0}{\lambda} = \frac{(10.0\times10^{-3}\ \text{Ci})(3.70\times10^{10}\ \text{s}^{-1}/1\ \text{Ci})}{(5.58\times10^{-2}\ \text{h}^{-1})(1\ \text{h}/3\,600\ \text{s})} = \boxed{2.39\times10^{13}\ \text{nuclei}}$$

(c) $$R = R_0\, e^{-\lambda t} = (10.0\ \text{mCi})e^{-(5.58\times10^{-2}\ \text{h}^{-1})(30\ \text{h})} = \boxed{1.9\ \text{mCi}}$$

29.22 The number of ${}^{90}_{38}\text{Sr}$ nuclei initially present is

$$N_0 = \frac{\text{total mass}}{\text{mass per nucleus}} = \frac{5.0\ \text{kg}}{(89.907\,7\ \text{u})(1.66\times10^{-27}\ \text{kg/u})} = 3.4\times10^{25}$$

The half-life of ${}^{90}_{38}\text{Sr}$ is $T_{1/2} = 29.1$ yr (Appendix B), so the initial activity is

$$R_0 = \lambda N_0 = \frac{N_0 \ln 2}{T_{1/2}} = \frac{(3.4\times10^{25})\ln 2}{(29.1\ \text{yr})(3.156\times10^7\ \text{s/yr})} = 2.6\times10^{16}\ \text{counts/s}$$

From $R = R_0 e^{-\lambda t}$, the time when the activity will be $R = 10.0$ counts/min is

$$t = -\frac{\ln(R/R_0)}{\lambda} = -(T_{1/2})\frac{\ln(R/R_0)}{\ln 2}$$

$$= -(29.1\ \text{yr})\frac{\ln\left[\dfrac{10.0\ \text{min}^{-1}}{2.6\times10^{16}\ \text{s}^{-1}}\left(\dfrac{1\ \text{min}}{60\ \text{s}}\right)\right]}{\ln 2} = \boxed{1.7\times10^3\ \text{yr}}$$

29.23 We identify the missing nuclide in each case by requiring that both the total mass number and the total charge number be the same on the two sides of the decay equation.

(a) ${}^{212}_{83}\text{Bi} \rightarrow \boxed{{}^{208}_{81}\text{Tl}} + {}^{4}_{2}\text{He}$ (b) ${}^{95}_{36}\text{Kr} \rightarrow \boxed{{}^{95}_{37}\text{Rb}} + {}^{0}_{-1}\text{e} + \bar{\nu}$ (c) $\boxed{{}^{144}_{60}\text{Nd}} \rightarrow {}^{4}_{2}\text{He} + {}^{140}_{58}\text{Ce}$

29.24 We complete the decay formula in each case by requiring that both the total mass number and the total charge number be the same on the two sides of the equation.

(a) ${}^{12}_{5}\text{B} \rightarrow \boxed{{}^{12}_{6}\text{C}} + {}^{0}_{-1}\text{e} + \bar{\nu}$ (b) ${}^{234}_{90}\text{Th} \rightarrow {}^{230}_{88}\text{Ra} + \boxed{{}^{4}_{2}\text{He}}$ (c) $\boxed{{}^{14}_{6}\text{C}} \rightarrow {}^{14}_{7}\text{N} + {}^{0}_{-1}\text{e} + \bar{\nu}$

29.25 The more massive ${}^{56}_{27}\mathrm{Co}$ decays into the less massive ${}^{56}_{26}\mathrm{Fe}$. To conserve charge, the charge of the emitted particle must be +1e. Since the parent and the daughter have the same mass number, the emitted particle must have essentially zero mass. Thus, the decay must be positron emission or $\boxed{\mathrm{e}^{+}\ \text{decay}}$. The decay equation is $\boxed{{}^{56}_{27}\mathrm{Co} \rightarrow {}^{56}_{26}\mathrm{Fe} + {}^{0}_{+1}\mathrm{e} + \nu_{\mathrm{e}}}$.

29.26 The energy released in the decay ${}^{238}_{92}\mathrm{U} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{234}_{90}\mathrm{Th}$ is

$$Q = (\Delta m)c^2 = \left[m_{^{238}\mathrm{U}} - \left(m_{^{4}\mathrm{He}} + m_{^{234}\mathrm{Th}}\right)\right]c^2$$

$$= \left[238.050\,783\ \mathrm{u} - \left(4.002\,603\ \mathrm{u} + 234.043\,583\ \mathrm{u}\right)\right](931.5\ \mathrm{MeV/u})$$

$$= \boxed{4.28\ \mathrm{MeV}}$$

29.27 The Q-value of a decay, $Q = (\Delta m)c^2$, is the amount of energy released in the decay. Here, Δm is the difference between the mass of the original nucleus and the total mass of the decay products. If $Q > 0$, the decay may occur spontaneously.

(a) For the decay ${}^{40}_{20}\mathrm{Ca} \rightarrow \mathrm{e}^{+} + {}^{40}_{19}\mathrm{K}$, the masses of the electrons do not automatically cancel. Thus, we add 20 electrons to each side of the decay to yield neutral atoms and obtain

$$\left({}^{40}_{20}\mathrm{Ca} + 20\mathrm{e}^{-}\right) \rightarrow \mathrm{e}^{+} + \left({}^{40}_{19}\mathrm{K} + 19\mathrm{e}^{-}\right) + \mathrm{e}^{-} \qquad \text{or} \qquad {}^{40}\mathrm{Ca}_{\text{atom}} \rightarrow \mathrm{e}^{+} + {}^{40}\mathrm{K}_{\text{atom}} + \mathrm{e}^{-}$$

Then, $Q = \left(m_{^{40}_{20}\mathrm{Ca}_{\text{atom}}} - m_{^{40}_{19}\mathrm{K}_{\text{atom}}} - 2m_e\right)c^2 = \left[39.962\,591\ \mathrm{u} - 39.963\,999\ \mathrm{u} - 2(0.000\,549\ \mathrm{u})\right]c^2$

or $\quad Q = (-0.002\,506\ \mathrm{u})c^2 < 0 \quad$ so the decay $\boxed{\text{cannot occur spontaneously}}$.

(b) In the decay ${}^{144}_{60}\mathrm{Nd} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{140}_{58}\mathrm{Ce}$, we may add 60 electrons to each side, forming all neutral atoms, and use masses from Appendix B to find

$$Q = \left(m_{^{144}_{60}\mathrm{Nd}} - m_{^{4}_{2}\mathrm{He}} - m_{^{140}_{58}\mathrm{Ce}}\right)c^2 = (143.910\,083\ \mathrm{u} - 4.002\,603\ \mathrm{u} - 139.905\,434\ \mathrm{u})c^2$$

or $\quad Q = (+0.002\,046\ \mathrm{u})c^2 > 0 \quad$ so the decay $\boxed{\text{can occur spontaneously}}$.

29.28 (a) $\boxed{{}^{66}_{28}\mathrm{Ni} \rightarrow {}^{66}_{29}\mathrm{Cu} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}_{\mathrm{e}}}$

(b) Because of the mass differences, neglect the kinetic energy of the recoiling daughter nucleus in comparison to that of the other decay products. Then, the maximum kinetic energy of the beta particle occurs when the antineutrino is given zero energy. That maximum is

$$KE_{\max} = \left(m_{^{66}\mathrm{Ni}} - m_{^{66}\mathrm{Cu}}\right)c^2 = (65.929\,1\ \mathrm{u} - 65.928\,9\ \mathrm{u})(931.5\ \mathrm{MeV/u})$$

$$= 0.186\ \mathrm{MeV} = \boxed{186\ \mathrm{keV}}$$

29.29 In the decay ${}^{3}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + {}^{0}_{-1}\mathrm{e} + \bar{\nu}_{\mathrm{e}}$, the antineutrino is massless. Adding 1 electron to each side of the decay gives $\left({}^{3}_{1}\mathrm{H} + \mathrm{e}^{-}\right) \rightarrow \left({}^{3}_{2}\mathrm{He} + 2\,\mathrm{e}^{-}\right) + \bar{\nu}_{\mathrm{e}}$, or ${}^{3}_{1}\mathrm{H}_{\text{atom}} \rightarrow {}^{3}_{2}\mathrm{He}_{\text{atom}} + \bar{\nu}_{\mathrm{e}}$. Therefore, using neutral atomic masses from Appendix B, the energy released is

$$E = (\Delta m)c^2 = \left(m_{^{3}\mathrm{H}} - m_{^{3}\mathrm{He}}\right)c^2 = (3.016\,049\ \mathrm{u} - 3.016\,029\ \mathrm{u})(931.5\ \mathrm{MeV/u})$$

$$= 0.018\,6\ \mathrm{MeV} = \boxed{18.6\ \mathrm{keV}}$$

29.30 The initial activity of the 1.00-kg carbon sample would have been

$$R_0 = \left(1.00\times10^3 \text{ g}\right)\left(\frac{15.0 \text{ counts/min}}{1.00 \text{ g}}\right) = 1.50\times10^4 \text{ min}^{-1}$$

From $R = \lambda N = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$, and $T_{1/2} = 5\,730$ yr for ^{14}C (Appendix B), the age of the sample is

$$t = -\frac{\ln\left(R/R_0\right)}{\lambda} = -T_{1/2}\frac{\ln\left(R/R_0\right)}{\ln 2}$$

$$= -\left(5\,730 \text{ yr}\right)\frac{\ln\left[\left(5.00\times10^2 \text{ min}^{-1}\right)/\left(1.50\times10^4 \text{ min}^{-1}\right)\right]}{\ln 2} = \boxed{2.81\times10^4 \text{ yr}}$$

29.31 From $R = \lambda N = \lambda N_0 e^{-\lambda t} = R_0 e^{-\lambda t}$, and $T_{1/2} = 5\,730$ yr for ^{14}C (Appendix B), the age of the sample is

$$t = -\frac{\ln\left(R/R_0\right)}{\lambda} = -T_{1/2}\frac{\ln\left(R/R_0\right)}{\ln 2} = -\left(5\,730 \text{ yr}\right)\frac{\ln(0.600)}{\ln 2} = \boxed{4.22\times10^3 \text{ yr}}$$

29.32 The energy released in the reaction is given by

$$Q = (\Delta m)c^2 = \left(m_{{}^1_1\text{H}} + m_{{}^{27}_{13}\text{Al}} - m_{{}^{27}_{14}\text{Si}} - m_n\right)c^2$$

$$= \left[1.007\,825 \text{ u} + 26.981\,539 \text{ u} - 26.986\,721 \text{ u} - 1.008\,665 \text{ u}\right](931.5 \text{ MeV/u})$$

$$= -5.61 \text{ MeV}$$

If we require total energy to be conserved and ignore the kinetic energy of the recoiling product nucleus, the kinetic energy of the emerging neutron must be

$$KE_f = KE_i + Q = 6.61 \text{ MeV} - 5.61 \text{ MeV} = \boxed{1.00 \text{ MeV}}$$

29.33 We identify the missing particles by requiring that both the total mass number and the total charge number be the same on the two sides of the equation and by remembering that some form of neutrino always accompanies the emission of a beta particle or positron.

(a) $\boxed{{}^{21}_{10}\text{Ne}} + {}^4_2\text{He} \rightarrow {}^{24}_{12}\text{Mg} + {}^1_0\text{n}$

(b) ${}^{235}_{92}\text{U} + {}^1_0\text{n} \rightarrow {}^{90}_{38}\text{Sr} + \boxed{{}^{144}_{54}\text{Xe}} + 2\,{}^1_0\text{n}$

(c) $2\,{}^1_1\text{H} \rightarrow {}^2_1\text{H} + \boxed{{}^0_{+1}\text{e}} + \boxed{\nu_e}$

29.34 (a) $m_i = m_{{}^1_1\text{H}} + m_{{}^7_3\text{Li}} = 1.007\,825 \text{ u} + 7.016\,004 \text{ u} = \boxed{8.023\,829 \text{ u}}$

(b) $m_f = m_{{}^7_4\text{Be}} + m_n = 7.016\,929 \text{ u} + 1.008\,665 \text{ u} = \boxed{8.025\,594 \text{ u}}$

(c) $Q = \left(m_i - m_f\right)c^2 = \left[8.023\,829 \text{ u} - 8.025\,594 \text{ u}\right](931.5 \text{ MeV/u}) = \boxed{-1.64 \text{ MeV}}$

(d) $\boxed{m_p v = \left(m_{\text{Be}} + m_n\right)V}$

(e) From conservation of energy, the total kinetic energy before the reaction, plus the energy released during the reaction, must equal the total kinetic energy after the reaction. That is, $KE_p + Q = KE_{\text{Be}} + KE_n$, or $\boxed{\frac{1}{2}m_p v^2 = \frac{1}{2}\left(m_{\text{Be}} + m_n\right)V^2 - Q}$.

continued on next page

(f) $$KE_{\min} = \left(1+\frac{m}{M}\right)|Q| = \left(1+\frac{m_p}{m_{^7_3\text{Li}}}\right)|Q| = \left(1+\frac{1.007\,825\text{ u}}{7.016\,004\text{ u}}\right)|-1.64\text{ MeV}| = \boxed{1.88\text{ MeV}}$$

29.35 We determine the product nucleus by requiring that both the total mass number and the total charge number be the same on the two sides of the reaction equation. The completed reaction equations are given below:

(a) $^{10}_{5}\text{B} + ^{4}_{2}\text{He} \rightarrow ^{1}_{1}\text{H} + \boxed{^{13}_{6}\text{C}}$ (b) $^{13}_{6}\text{C} + ^{1}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + \boxed{^{10}_{5}\text{B}}$

29.36 (a) For the first reaction: $\text{n} + ^{2}_{1}\text{H} \rightarrow ^{3}_{1}\text{H}$

$$Q_1 = \left(m_n + m_{^2_1\text{H}} - m_{^3_1\text{H}}\right)c^2 = [1.008\,665\text{ u} + 2.014\,102\text{ u} - 3.016\,049\text{ u}](931.5\text{ MeV/u})$$
$$= \boxed{6.258\text{ Mev}} > 0 \quad \Rightarrow \quad \boxed{\text{exothermic}}$$

For the second reaction: $^{1}_{1}\text{H} + ^{2}_{1}\text{H} \rightarrow ^{3}_{2}\text{He}$

$$Q_2 = \left(m_{^1_1\text{H}} + m_{^2_1\text{H}} - m_{^3_2\text{He}}\right)c^2 = [1.007\,825\text{ u} + 2.014\,102\text{ u} - 3.016\,029\text{ u}](931.5\text{ MeV/u})$$
$$= \boxed{5.494\text{ Mev}} > 0 \quad \Rightarrow \quad \boxed{\text{exothermic}}$$

(b) Since $Q_1 > Q_1$, $\boxed{\text{the first reaction released more energy}}$. One reason for this is that the product nucleus in the second reaction contains 2 protons. Some energy had to be left stored as electrical potential energy of this system, leaving less energy to be released as kinetic energy of the product nucleus.

(c) Assuming the electrical potential energy of the 2 protons in $^{3}_{2}\text{He}$ fully accounts for the difference in the Q-values of the two reactions, we have $\Delta Q = PE_e = k_e e^2/r$, where r is the distance separating the 2 protons. Thus,

$$r = \frac{k_e e^2}{\Delta Q} = \frac{(8.99\times10^9\text{ N}\cdot\text{m}^2/\text{C}^2)(1.60\times10^{-19}\text{ C})^2}{(6.258-5.494)\text{ MeV}}\left(\frac{1\text{ MeV}}{1.60\times10^{-13}\text{ J}}\right) = \boxed{1.88\times10^{-15}\text{ m}}$$

29.37 (a) Requiring that both charge and the number of nucleons (atomic mass number) be conserved, the reaction is found to be $\boxed{^{197}_{79}\text{Au} + ^{1}_{0}\text{n} \rightarrow ^{198}_{80}\text{Hg} + ^{0}_{-1}\text{e} + \bar{\nu}_e}$. Note that the antineutrino has been included to conserve electron-lepton number, which will be discussed in the next chapter.

(b) We add 79 electrons to both sides of the reaction equation given above to produce neutral atoms so we may use mass values from Appendix B. This gives $^{197}_{79}\text{Au}_{\text{atom}} + ^{1}_{0}\text{n} \rightarrow ^{198}_{80}\text{Hg}_{\text{atom}} + \bar{\nu}_e$, and, remembering that the antineutrino is massless, the Q-value is found to be

$$Q = (\Delta m)c^2 = \left(m_{^{197}_{79}\text{Au}} + m_n - m_{^{198}_{80}\text{Hg}}\right)c^2$$
$$= (196.966\,552\text{ u} + 1.008\,665\text{ u} - 197.966\,750\text{ u})(931.5\text{ MeV/u}) = 7.89\text{ MeV}$$

The kinetic energy carried away by the daughter nucleus is negligible. Thus, the energy released may be split in any manner between the electron and antineutrino, with the maximum kinetic energy of the electron being $\boxed{7.89\text{ MeV}}$.

29.38 We complete the reaction equation in each case by requiring that both the total mass number and the total charge number be the same on the two sides of the equation.

(a) $\boxed{^{4}_{2}\text{He}} + ^{14}_{7}\text{N} \rightarrow ^{1}_{1}\text{H} + ^{17}_{8}\text{O}$ (b) $^{7}_{3}\text{Li} + ^{1}_{1}\text{H} \rightarrow ^{4}_{2}\text{He} + \boxed{^{4}_{2}\text{He}}$

29.39 (a) Determine the product of the reaction by requiring that both the total mass number and the total charge number be the same on the two sides of the equation. The completed reaction equation is ${}^{7}_{3}\text{Li} + {}^{4}_{2}\text{He} \to \boxed{{}^{10}_{5}\text{B}} + {}^{1}_{0}\text{n}$.

(b) $$Q = \Delta mc^2 = \left[\left(m_{{}^{7}_{3}\text{Li}} + m_{{}^{4}_{2}\text{He}}\right) - \left(m_{{}^{10}_{5}\text{B}} + m_{{}^{1}_{0}\text{n}}\right)\right]c^2$$

$$= \left[(7.016\,004 \text{ u} + 4.002\,603 \text{ u}) - (10.012\,937 \text{ u} + 1.008\,665 \text{ u})\right](931.5 \text{ MeV/u})$$

$$= \boxed{-2.79 \text{ MeV}}$$

29.40 For equal amounts of biological damage, the two doses (in rem units) must be equal, or

$$(\text{heavy ion dose in rad}) \times \text{RBE}_{\text{heavy ions}} = (\text{x-ray dose in rad}) \times \text{RBE}_{\text{x-rays}}$$

or

$$\text{ion dose in rad} = \frac{(\text{x-ray dose in rad}) \times \text{RBE}_{\text{x-rays}}}{\text{RBE}_{\text{heavy ions}}} = \frac{(100 \text{ rad})(1.0)}{20} = \boxed{5.0 \text{ rad}}$$

29.41 For each rad of radiation, 10^{-2} J of energy is delivered to each kilogram of absorbing material. Thus, the total energy delivered in this whole body dose to a 75.0-kg person is

$$E = (25.0 \text{ rad})\left(10^{-2} \ \frac{\text{J/kg}}{\text{rad}}\right)(75.0 \text{ kg}) = \boxed{18.8 \text{ J}}$$

29.42 (a) Each rad of radiation delivers 10^{-2} J of energy to each kilogram of absorbing material. Thus, the energy delivered per unit mass with this dose is

$$\frac{E}{m} = (200 \text{ rad})\left(10^{-2} \ \frac{\text{J/kg}}{\text{rad}}\right) = \boxed{2.00 \text{ J/kg}}$$

(b) From $E = Q = mc(\Delta T)$, the expected temperature rise with this dosage is

$$\Delta T = \frac{E/m}{c} = \frac{2.00 \text{ J/kg}}{4\,186 \text{ J/kg}\cdot{}^\circ\text{C}} = \boxed{4.78 \times 10^{-4} \ {}^\circ\text{C}}$$

29.43 The rate of delivering energy to each kilogram of absorbing material is

$$\left(\frac{E/m}{\Delta t}\right) = (10 \text{ rad/s})\left(10^{-2} \ \frac{\text{J/kg}}{\text{rad}}\right) = 0.10 \ \frac{\text{J/kg}}{\text{s}}$$

The total energy needed per unit mass is

$$E/m = c(\Delta T) = \left(4\,186 \ \frac{\text{J}}{\text{kg}\cdot{}^\circ\text{C}}\right)(50^\circ\text{C}) = 2.1 \times 10^5 \text{ J/kg}$$

so the required time will be

$$\Delta t = \frac{\text{energy needed}}{\text{delivery rate}} = \frac{2.1 \times 10^5 \text{ J/kg}}{0.10 \text{ J/kg}\cdot\text{s}} = 2.1 \times 10^6 \text{ s}\left(\frac{1 \text{ d}}{8.64 \times 10^4 \text{ s}}\right) = \boxed{24 \text{ d}}$$

29.44 (a) The number of x-rays taken per year is

$$production = (8\ \text{x-ray/d})(5\ \text{d/week})(50\ \text{weeks/yr}) = 2.0\times10^{3}\ \text{x-ray/yr}$$

so the exposure per x-ray taken is

$$exposure\ rate = \frac{exposure}{production} = \frac{5.0\ \text{rem/yr}}{2.0\times10^{3}\ \text{x-ray/yr}} = \boxed{2.5\times10^{-3}\ \text{rem/x-ray}}$$

(b) The exposure due to background radiation is $0.13\ \text{rem/yr}$. Thus, the work-related exposure of $5.0\ \text{rem/yr}$ is

$$\frac{5.0\ \text{rem/yr}}{0.13\ \text{rem/yr}} \approx \boxed{38\ \text{times background levels}}$$

29.45 (a) From $N = R/\lambda = R_0 e^{-\lambda t}/\lambda = \left(T_{1/2} R_0/\ln 2\right)e^{-t\ln 2/T_{1/2}}$, the number of decays occurring during the 10-day period is

$$\Delta N = N_0 - N = \left(\frac{T_{1/2} R_0}{\ln 2}\right)\left(1 - e^{-t\ln 2/T_{1/2}}\right)$$

$$= \left[\frac{(14.3\ \text{d})(1.31\times10^{6}\ \text{decay/s})}{\ln 2}\left(\frac{8.64\times10^{4}\ \text{s}}{1\ \text{d}}\right)\left(1 - e^{-(10.0\ \text{d})\ln 2/14.3\ \text{d}}\right)\right]$$

$= \boxed{8.97\times10^{11}\ \text{decays}}$, and one electron is emitted per decay

(b) The total energy deposited is found to be

$$E = \left(700\ \frac{\text{keV}}{\text{decay}}\right)\left(8.97\times10^{11}\ \text{decays}\right)\left(\frac{1.60\times10^{-16}\ \text{J}}{1\ \text{keV}}\right) = \boxed{0.100\ \text{J}}$$

(c) The total absorbed dose (measured in rad) is given by

$$dose = \frac{\text{energy deposited per unit mass}}{\text{energy deposition per rad}} = \frac{(0.100\ \text{J}/0.100\ \text{kg})}{\left(10^{-2}\ \dfrac{\text{J/kg}}{\text{rad}}\right)} = \boxed{100\ \text{rad}}$$

29.46 (a) The dose (in rem) received in time Δt is given by

$$dose = (dose\ in\ rad)\times RBE = \left[\left(100\times10^{-3}\ \frac{\text{rad}}{\text{h}}\right)\Delta t\right]\times(1.00) = \left(0.100\ \frac{\text{rem}}{\text{h}}\right)\Delta t$$

If this dose is to be 1.0 rem, the required time is

$$\Delta t = \frac{1.0\ \text{rem}}{0.100\ \text{rem/h}} = \boxed{10\ \text{h}}$$

(b) Assuming the radiation is emitted uniformly in all directions, the intensity of the radiation is given by $I = I_0/4\pi r^2$. Therefore,

$$\frac{I_r}{I_1} = \frac{I_0/4\pi r^2}{I_0/4\pi(1.0\ \text{m})^2} = \frac{(1.0\ \text{m})^2}{r^2}$$

continued on next page

and $r = (1.0\text{ m})\sqrt{\dfrac{I_1}{I_r}} = (1.0\text{ m})\sqrt{\dfrac{100\text{ mrad/h}}{10\text{ mrad/h}}} = \boxed{3.2\text{ m}}$

29.47 (a) The mass of a ${}^{11}_{6}\text{C}$ atom is

$$m_{\text{atom}} = (11.011\text{ u})(1.661\times10^{-27}\text{ kg/u}) = 1.829\times10^{-26}\text{ kg} = 1.829\times10^{-23}\text{ g}$$

and the mass of 1 mole of ${}^{11}_{6}\text{C}$ is

$$M = m_{\text{atom}}N_A = \left(1.829\times10^{-23}\text{ g/atom}\right)\left(6.022\times10^{23}\text{ atoms/mol}\right) = 11.01\text{ g/mol}$$

The number of moles in a 3.50 μg sample is then

$$n = \frac{m_{\text{sample}}}{M} = \frac{3.50\times10^{-6}\text{ g}}{11.01\text{ g/mol}} = \boxed{3.18\times10^{-7}\text{ mol}}$$

(b) The number of nuclei in the original sample is

$$N_0 = nN_A = \left(3.18\times10^{-7}\text{ mol}\right)\left(6.022\times10^{23}\text{ atoms/mol}\right) = \boxed{1.91\times10^{17}\text{ atoms}}$$

or alternatively, $N_0 = \dfrac{m_{\text{sample}}}{m_{\text{atom}}} = \dfrac{3.50\times10^{-6}\text{ g}}{1.829\times10^{-23}\text{ g/atom}} = \boxed{1.91\times10^{17}\text{ atoms}}$.

(c) The initial activity is

$$R_0 = \lambda N_0 = \left(\frac{\ln 2}{T_{1/2}}\right)N_0 = \frac{\ln 2}{(20.4\text{ min})(60.0\text{ s/min})}\left(1.91\times10^{17}\right) = \boxed{1.08\times10^{14}\text{ Bq}}$$

(d) The activity after an elapsed time of $t = 8.00\text{ h} = 480\text{ min}$ will be

$$R = R_0e^{-\lambda t} = R_0e^{-t\ln 2/T_{1/2}} = \left(1.08\times10^{14}\text{ Bq}\right)e^{-(480\text{ min})\ln 2/(20.4\text{ min})} = \boxed{8.92\times10^{6}\text{ Bq}}$$

29.48 We must first calculate the Q value, given by $Q = (\Delta m)c^2 = \left[\left(m_{{}^1_0\text{n}} + m_{{}^4_2\text{He}}\right) - \left(m_{{}^2_1\text{H}} + m_{{}^3_1\text{H}}\right)\right]c^2$.

$$\begin{aligned}Q &= \left[(1.008\,665\text{ u} + 4.002\,603\text{ u}) - (2.014\,102\text{ u} + 3.016\,049\text{ u})\right](931.5\text{ MeV/u})\\ &= -17.6\text{ MeV}\end{aligned}$$

The threshold kinetic energy for the incident neutron is then

$$KE_{\text{min}} = \left(1 + \frac{m_{{}^1_0\text{n}}}{M_{{}^4_2\text{He}}}\right)|Q| = \left(1 + \frac{1.008\,665\text{ u}}{4.002\,603\text{ u}}\right)(17.6\text{ MeV}) = \boxed{22.0\text{ MeV}}$$

29.49 From $R = R_0e^{-\lambda t}$, the elapsed time is

$$t = -\frac{\ln(R/R_0)}{\lambda} = -T_{1/2}\frac{\ln(R/R_0)}{\ln 2} = -(14.0\text{ d})\frac{\ln(20.0\text{ mCi}/200\text{ mCi})}{\ln 2} = \boxed{46.5\text{ d}}$$

29.50 To compute the Q value for the reaction ${}^{14}_{6}\text{C} \to {}^{14}_{7}\text{N} + \text{e}^- + \bar{\nu}$, we add 6 electrons to each side of the equation to obtain $({}^{14}_{6}\text{C} + 6\text{e}^-) \to ({}^{14}_{7}\text{N} + 7\text{e}^-) + \bar{\nu}$, or ${}^{14}_{6}\text{C}_{\text{atom}} \to {}^{14}_{7}\text{N}_{\text{atom}} + \bar{\nu}$. Now, we use the atomic masses from Appendix B of the textbook. Since the neutrino is a massless particle, $Q = (m_{{}^{14}_{6}\text{C}_{\text{atom}}} - m_{{}^{14}_{7}\text{N}_{\text{atom}}})c^2$, giving

$$Q = (14.003\,242\text{ u} - 14.003\,074\text{ u})(931.5\text{ MeV/u}) = 0.156\text{ MeV} = \boxed{156\text{ keV}}$$

29.51 (a) If we assume all the ^{87}Sr nuclei in a gram of material came from the decay of ^{87}Rb nuclei, the original number of ^{87}Rb nuclei was $N_0 = 1.82\times10^{10} + 1.07\times10^{9} = 1.93\times10^{10}$. Then, from $N = N_0 e^{-\lambda t}$, the elapsed time is

$$t = -\frac{\ln(N/N_0)}{\lambda} = -\frac{T_{1/2}\ln(N/N_0)}{\ln 2} = -\frac{\left(4.8\times10^{10}\text{ yr}\right)\ln\left(\dfrac{1.82\times10^{10}}{1.93\times10^{10}}\right)}{\ln 2} = \boxed{4.1\times10^{9}\text{ yr}}$$

(b) $\boxed{\text{It could be no older. It could be younger if some } ^{87}\text{Sr were initially present.}}$

(c) $\boxed{\text{We have assumed that all the } ^{87}\text{Sr present came from the decay of } ^{87}\text{Rb.}}$

29.52 From $R = \lambda N = \lambda N_0 e^{-\lambda t}$, if we have an activity of R after time t has passed, the number of unstable nuclei that must have been present initially is given by $N_0 = Re^{\lambda t}/\lambda$. With $R = 10\text{ Ci} = 10(3.7\times10^{10}\text{ Bq}) = 3.7\times10^{11}\text{ decay/s}$, $\lambda = \ln 2/T_{1/2} = \ln 2/5.2\text{ yr}$, and $t = 30\text{ months} = 2.5\text{ yr}$, this yields

$$N_0 = \left(3.7\times10^{11}\text{ s}^{-1}\right)\left[\frac{(5.2\text{ yr})\left(3.156\times10^{7}\text{ s/1 yr}\right)}{\ln 2}\right]e^{(\ln 2/5.2\text{ yr})(2.5\text{ yr})} = 1.2\times10^{20}$$

Thus, the initial mass of ^{60}C required is found to be (using Appendix B from the text)

$$m = N_0 m_{\text{atom}} = \left(1.2\times10^{20}\right)(59.933\,822\text{ u}) = 7.2\times10^{21}\text{ u}$$

or $$m = 7.2\times10^{21}\text{ u}\left(\frac{1.66\times10^{-27}\text{ kg}}{1\text{ u}}\right)\left(\frac{10^{6}\text{ mg}}{1\text{ kg}}\right) = \boxed{12\text{ mg}}$$

29.53 The total activity of the working solution at $t = 0$ was

$$\left(R_0\right)_{\text{total}} = (2.5\text{ mCi/mL})(10\text{ mL}) = 25\text{ mCi}$$

Therefore, the initial activity of the 5.0-mL sample, drawn from the 250-mL working solution, was

$$\left(R_0\right)_{\text{sample}} = \left(R_0\right)_{\text{total}}\left(\frac{5.0\text{ mL}}{250\text{ mL}}\right) = (25\text{ mCi})\left(\frac{5.0\text{ mL}}{250\text{ mL}}\right) = 0.50\text{ mCi} = 5.0\times10^{-4}\text{ Ci}$$

With a half-life of 14.96 h for ^{24}Na (Appendix B), the activity of the sample after 48 h is

$$R = R_0 e^{-\lambda t} = R_0 e^{-t\ln 2/T_{1/2}} = \left(5.0\times10^{-4}\text{ Ci}\right)e^{-(48\text{ h})\ln 2/(14.96\text{ h})}$$

$$= 5.4\times10^{-5}\text{ Ci} = \boxed{54\ \mu\text{Ci}}$$

29.54 (a) The mass of a single ^{40}K atom is

$$m = (39.964\text{ u})\left(1.66\times10^{-27}\text{ kg/u}\right) = 6.63\times10^{-26}\text{ kg} = 6.63\times10^{-23}\text{ g}$$

Therefore, the number of ^{40}K nuclei in a liter of milk is

$$N = \frac{\text{total mass of } ^{40}\text{K present}}{\text{mass per atom}} = \frac{(2.00\text{ g/L})(0.011\,7/100)}{6.63\times10^{-23}\text{ g}} = 3.53\times10^{18}/\text{L}$$

continued on next page

and the activity due to potassium is

$$R = \lambda N = \frac{N \ln 2}{T_{1/2}} = \frac{\left(3.53 \times 10^{18}/\mathrm{L}\right)\ln 2}{\left(1.28 \times 10^{9}\ \mathrm{yr}\right)\left(3.156 \times 10^{7}\ \mathrm{s/yr}\right)} = \boxed{60.6\ \mathrm{Bq/L}}$$

(b) Using $R = R_0 e^{-\lambda t}$, the time required for the ^{131}I activity to decrease to the level of the potassium is given by

$$t = -\frac{\ln\left(R/R_0\right)}{\lambda} = -\frac{T_{1/2}\ln\left(R/R_0\right)}{\ln 2} = -\frac{(8.04\ \mathrm{d})\ln(60.6/2\,000)}{\ln 2} = \boxed{40.6\ \mathrm{d}}$$

29.55 The decay constant for ^{235}U is $\lambda_{235} = \ln 2/T_{1/2} = \ln 2/0.70 \times 10^{9}\ \mathrm{yr} = 9.9 \times 10^{-10}\ \mathrm{yr}^{-1}$, while that for ^{238}U is $\lambda_{238} = \ln 2/4.47 \times 10^{9}\ \mathrm{yr} = 1.55 \times 10^{-10}\ \mathrm{yr}^{-1}$. Assuming there were N_0 nuclei of each isotope present initially, the number of each type still present should be $N_{235} = N_0 e^{-\lambda_{235} t}$ and $N_{238} = N_0 e^{-\lambda_{238} t}$. With the currently observed ratio of ^{235}U to ^{238}U being 0.007, we have $N_{235}/N_{238} = e^{-(\lambda_{235}-\lambda_{238})t} = 0.007$, or our estimate of the elapsed time since the release of the elements forming our Earth is

$$t = -\frac{\ln(0.007)}{\lambda_{235} - \lambda_{236}} = -\frac{\ln(0.007)}{9.9 \times 10^{-10}\ \mathrm{yr}^{-1} - 1.55 \times 10^{-10}\ \mathrm{yr}^{-1}} = \boxed{5.9 \times 10^{9}\ \mathrm{yr}}$$

29.56 (a) Obtaining the mass of a single $^{239}_{94}$Pu atom from the table of Appendix B in the text, we find

$$N_0 = \frac{m_{\text{sample}}}{m_{\text{atom}}} = \frac{1.0\ \mathrm{kg}}{(239.052\,156\ \mathrm{u})\left(1.66 \times 10^{-27}\ \mathrm{kg/u}\right)} = \boxed{2.5 \times 10^{24}}$$

(b) The initial activity of this sample is

$$R_0 = \lambda N_0 = \left(\frac{\ln 2}{T_{1/2}}\right) N_0 = \left[\frac{\ln 2}{(24\,000\ \mathrm{yr})\left(3.156 \times 10^{7}\ \mathrm{s/yr}\right)}\right]\left(2.5 \times 10^{24}\right) = \boxed{2.3 \times 10^{12}\ \mathrm{Bq}}$$

(c) From $R = R_0 e^{-\lambda t}$, the time that must elapse before this sample will have a "safe" activity level of 0.10 Bq is

$$t = -\frac{\ln\left(R/R_0\right)}{\lambda} = -\frac{T_{1/2}\ln\left(R/R_0\right)}{\ln 2} = -\frac{(24\,000\ \mathrm{yr})\ln\left(0.10/2.3 \times 10^{12}\right)}{\ln 2} = \boxed{1.1 \times 10^{6}\ \mathrm{yr}}$$

30

Nuclear Physics and Elementary Particles

QUICK QUIZZES

1. Choice (a). This reaction fails to conserve charge and also fails to conserve baryon number. For each of these reasons, it cannot occur.

2. Choice (b). This reaction fails to conserve charge and cannot occur.

ANSWERS TO MULTIPLE CHOICE QUESTIONS

1. The total energy released was $E = (17\times10^3 \text{ ton})\left(4.0\times10^9 \text{ J}/1 \text{ ton}\right) = 6.8\times10^{13}$ J, and according to the mass-energy equivalence ($E = mc^2$), the mass converted was

$$m = \frac{E}{c^2} = \frac{6.8\times10^{13} \text{ J}}{\left(3.00\times10^8 \text{ m/s}\right)^2} = 7.6\times10^{-4} \text{ kg} = 0.76 \text{ g}$$

or $m \sim 1$ g, and the correct choice is seen to be (d).

2. The energy released in the decay $\text{n} \to \text{p} + \text{e}^- + \bar{\nu}_e$ is $Q = (\Delta m)c^2 = (m_n - m_p - m_e)c^2$, or combining the proton and electron to form a neutral hydrogen atom, $Q = (m_n - m_{^1_1\text{H}_{\text{atom}}})c^2$. We may then use the atomic masses from Appendix B in the textbook to obtain

$$Q = \left(1.008\,665 \text{ u} - 1.007\,825 \text{ u}\right)\left(931.5 \text{ MeV/u}\right) = 0.782 \text{ MeV}$$

Alternately, we may use the particle masses (in energy units) from Table 30.2 in the textbook to obtain

$$Q = (m_n - m_p - m_e)c^2 = (939.6 \text{ MeV}/c^2 - 938.3 \text{ MeV}/c^2 - 0.511 \text{ MeV}/c^2)c^2 = 0.789 \text{ MeV}$$

From either approach, we see that the best choice is (a).

3. Both the charge and mass of a particle are independent of its spin, so both choices (c) and (d) are false. A spin-$\frac{1}{2}$ particle could be among the decay products, provided it is possible for the spins of all the decay products to couple to $\frac{3}{2}$ and conserve angular momentum. Also, in a magnetic field, a spin-$\frac{3}{2}$ particle could have spin states of

$$m_s = 3/2,\ 1/2,\ -1/2, \text{ and } -3/2$$

so choices (a) and (e) are false, while choice (b) is true.

4. The decay $\text{p} \to {}^{0}_{+1}\text{e} + \nu_e$ would conserve charge $(+1 \to +1 + 0)$, electron lepton number $(0 \to -1 + 1)$, and strangeness $(0 \to 0 + 0)$, and can conserve energy if the total kinetic energy of the decay products equals the energy equivalent of the mass loss. However, it does not conserve baryon number $(+1 \to 0+0)$, and the decay cannot occur. The correct choice is then (b).

5. Positively charged particles, such as protons and alpha particles, have difficulty approaching the target nuclei because of Coulomb repulsion. Fast-moving particles may not stay in close proximity with a uranium nucleus long enough to have a good probability of producing a reaction. The best particles to trigger a fission reaction of the uranium nuclei are slow-moving neutrons, so choice (d) is the correct answer.

6. Slow neutrons have a much higher probability of causing fission in a collision with a nucleus in the fuel elements than do fast or high energy neutrons. The purpose of the moderator is to slow the neutrons down and without it the chain reaction would quickly die out. The correct choice is (c).

7. The annihilation ${}^{0}_{-1}\mathrm{e} + {}^{0}_{+1}\mathrm{e} \to \gamma$ can conserve energy $[2(0.511\text{ MeV}) = 1.02\text{ MeV}]$, does conserve charge $[-1+1=0]$, conserves baryon number $[0+0=0]$, and conserves lepton number $[+1-1=0]$. However, the total momentum is zero before annihilation, and the momentum of the single photon afterward is $p = 1.02\text{ MeV}/c \neq 0$. Thus, it cannot occur, and the correct choice is (b).

8. In the fission reaction ${}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \to {}^{137}_{53}\mathrm{I} + {}^{96}_{39}\mathrm{Y} + n({}^{1}_{0}\mathrm{n})$, where n is some unknown number of neutrons, we see that charge is conserved $(92+0=53+39+0)$ regardless of the value of n. The reaction must also conserve baryon number, so it is necessary that

$$235+1=137+96+n \qquad \text{or} \qquad n=3$$

and (c) is seen to be the correct choice.

9. The reaction of choice (c) fails to conserve charge $[0+1 \neq 0+0]$, while the reaction of choice (d) fails to conserve baryon number $[+1 \neq 2(+1)+0+0]$, so neither of these reactions can occur. The reactions of choices (a), (b), and (e) satisfy all conservation laws and may occur. The correct answers for this question are choices (c) and (d).

10. The reaction of choice (a) fails to conserve baryon number $[1+1 \neq 1+1-1]$, while the reaction of choice (e) fails to conserve tau-lepton number $[+1 \neq 0-1+0]$, so neither of these reactions can occur. The reactions of choices (b), (c), and (d) satisfy all conservation laws and may occur. The correct answers for this question are choices (a) and (e).

11. In a fusion reactor, the plasma must have a very high temperature so the nuclei have sufficient energy to overcome Coulomb repulsion and collide. Also, the plasma must be sufficiently dense to yield a good probability of collisions between nuclei and must be contained long enough to allow a large number of collisions to occur, so the reactions can be self-sustained. However, there are no requirements that the nuclei in the plasma be radioactive or that it consist only of hydrogen. Fusion of elements other than hydrogen does occur inside stars, but much higher temperatures are required for this to occur. The correct choices for this question are (a), (c), and (d).

12. When a particle of mass m and charge q enters a magnetic field perpendicular to the direction of the field, it is deviated into a circular path of radius $r = mv/qB = \sqrt{2m(KE)}\big/qB$. In this case, the kinetic energy, KE, and the magnetic field strength, B, are the same for the electron and the alpha particle. However, the ratio $\sqrt{m}\big/q$ is much smaller for the electron than for the alpha particle, so the path of the electron has a smaller radius. This means the electron is deflected more than the alpha particle, and the correct choice is (b).

ANSWERS TO EVEN NUMBERED CONCEPTUAL QUESTIONS

2. The two factors presenting the most technical difficulties are the requirements of a high plasma density and a high plasma temperature. These two conditions must be met simultaneously and make containment of the plasma very difficult.

4. Notice in the fusion reactions discussed in the text that the most commonly formed by-product of the reactions is helium, which is inert and not radioactive.

6. They are hadrons. Such particles decay into other strongly interacting particles such as p, n, and π with very short lifetimes. In fact, they decay so quickly that they cannot be detected directly. Decays which occur via the weak force have lifetimes of 10^{-13} s or longer; particles that decay via the electromagnetic force have lifetimes in the range of 10^{-16} s to 10^{-19} s.

8. Each flavor of quark can have three colors, designated as red, green, and blue. Antiquarks are colored antired, antigreen, and antiblue. Baryons consist of three quarks, each having a different color. Mesons consist of a quark of one color and an antiquark with a corresponding anticolor. Thus, baryons and mesons are colorless or white.

10. The decays of the neutral pion, eta, and neutral sigma occur by the electromagnetic interaction. These are the three shortest lifetimes in the table. All produce photons, which are the quanta of the electromagnetic force, and all conserve strangeness.

12. A neutron inside a nucleus is stable because it is in a lower energy state than a free neutron and lower in energy than it would be if it decayed into a proton (plus electron and antineutrino). The nuclear force gives it this lower energy by binding it inside the nucleus and by favoring pairing between neutrons and protons.

ANSWERS TO EVEN NUMBERED PROBLEMS

2. 192 MeV

4. 9.4×10^{2} g

6. 2.63 kg

8. **(a)** 3.1×10^{10} g **(b)** 1.3×10^{8} mol; 7.8×10^{31} atoms

(c) 2.6×10^{21} J **(d)** 5.5 yr

(e) Fission alone cannot meet the world's energy needs.

10. **(a)** 4×10^{15} g **(b)** 5×10^{3} yr

(c) The uranium comes from dissolving rock and minerals. Rivers carry such solutes into the oceans, but the Earth's supply of uranium is not renewable. However, if breeder reactors are used, the current ocean supply can last about a half-million years.

12. 5.49 MeV

14. **(a)** ${}^{13}_{7}\mathrm{N}$ **(b)** ${}^{13}_{6}\mathrm{C}$ **(c)** ${}^{14}_{7}\mathrm{N}$

(d) $^{15}_{8}\text{O}$ **(e)** $^{15}_{7}\text{N}$ **(f)** $^{12}_{6}\text{C}$

16. **(a)** 8.68 MeV

(b) The proton and the boron nucleus must each have enough kinetic energy to overcome the repulsive electrical force one exerts on the other.

18. 118 MeV

20. **(a)** may occur via the weak interaction

(b) violates conservation of baryon number – cannot occur by any interaction

22. **(a)** cannot occur, violates baryon number

(b) can occur, no laws violated

(c) can occur, no laws violated

(d) can occur, no laws violated

(e) cannot occur, violates baryon number and muon-lepton number

24. **(a)** See Solution.

(b) The second reaction violates conservation of strangeness and cannot occur via the strong or electromagnetic interactions.

(c) If a neutral kaon, K^0, was produced in addition to those shown, the second reaction could occur without violating any conservation laws. Because of the greater total mass of the product particles, the total kinetic energy of the incident particles in the this reaction would have to be greater than in the first reaction.

26. See Solution.

28. See Solution.

30. **(a)** $Q = -e,$ antiproton **(b)** $Q = 0,$ antineutron

32. **(a)** conservation of charge

(b) conservation of electron-lepton number and conservation of muon-lepton number

(c) conservation of baryon number

34. $\bar{\nu}_\mu$ and ν_e

36. a neutron

38. **(a)** 7.85×10^{13} J **(b)** $\$1.74 \times 10^{6}$ **(c)** $\$1.50 \times 10^{3}$

(d) At a 10% profit margin, the profit is $\$1.74 \times 10^{5}$ on each kilogram of deuterium.

40. **(a)** 3.61×10^{30} J **(b)** 1.63×10^{8} yr

42. The first reaction has a net 1u and 2d quarks both before and after the reaction. The second reaction has a net 1u and 2d quarks before the reaction and 1u, 3d, and 1$\overline{\text{s}}$ quark present afterwards.

44. $0.827\,9c$

PROBLEM SOLUTIONS

30.1 The energy consumed by a 100-W lightbulb in a 1.0-h time period is

$$E = P\cdot\Delta t = (100\ \text{J/s})(1.0\ \text{h})\left(\frac{3\,600\ \text{s}}{1\ \text{h}}\right) = 3.6\times10^{5}\ \text{J}$$

The number of fission events, yielding an average of 208 MeV each, required to produce this quantity of energy is

$$n = \frac{E}{208\ \text{MeV}} = \frac{3.6\times10^{5}\ \text{J}}{208\ \text{MeV}}\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \text{J}}\right) = \boxed{1.1\times10^{16}}$$

30.2 The energy released in the reaction ${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{98}_{40}\text{Zr} + {}^{135}_{52}\text{Te} + 3\,{}^{1}_{0}\text{n}$ is

$$Q = (\Delta m)c^2 = \left[m_{{}^{235}_{92}\text{U}} - 2m_n - m_{{}^{98}_{40}\text{Zr}} - m_{{}^{135}_{52}\text{Te}}\right]c^2$$

$$= \left[235.043\,923\ \text{u} - 2(1.008\,665\ \text{u}) - 97.912\,0\ \text{u} - 134.908\,7\ \text{u}\right](931.5\ \text{MeV/u})$$

$$= \boxed{192\ \text{MeV}}$$

30.3 The energy released in the reaction ${}^{1}_{0}\text{n} + {}^{235}_{92}\text{U} \rightarrow {}^{88}_{38}\text{Sr} + {}^{136}_{54}\text{Xe} + 12\,{}^{1}_{0}\text{n}$ is

$$Q = (\Delta m)c^2 = \left[m_{{}^{235}_{92}\text{U}} - 11m_n - m_{{}^{88}_{38}\text{Sr}} - m_{{}^{136}_{54}\text{Xe}}\right]c^2$$

$$= \left[235.043\,923\ \text{u} - 11(1.008\,665\ \text{u}) - 87.905\,614\ \text{u} - 135.907\,220\ \text{u}\right](931.5\ \text{MeV/u})$$

$$= \boxed{126\ \text{MeV}}$$

30.4 The total energy released was

$$E = \left(20\times10^{3}\ \cancel{\text{ton TNT}}\right)\left(\frac{4.0\times10^{9}\ \cancel{\text{J}}}{1\ \cancel{\text{ton TNT}}}\right)\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \cancel{\text{J}}}\right) = 5.0\times10^{26}\ \text{MeV}$$

The number of ^{235}U nuclei that must have undergone fission to yield this energy is

$$n = \frac{E}{208\ \text{MeV/nucleus}} = \frac{5.0\times10^{26}\ \text{MeV}}{208\ \text{MeV/nucleus}} = 2.4\times10^{24}\ \text{nuclei}$$

The mass of ^{235}U which will contain this number of atoms is

$$m = \left(\frac{n}{N_A}\right)M_{\text{mol}} = \left(\frac{2.4\times10^{24}\ \text{atoms}}{6.02\times10^{23}\ \text{atoms/mol}}\right)\left(235\ \frac{\text{g}}{\text{mol}}\right) = \boxed{9.4\times10^{2}\ \text{g}}$$

30.5 (a) With a specific gravity of 4.00, the density of soil is $\rho = 4.00\times10^3\ \text{kg/m}^3$. Thus, the mass of the top 1.00 m of soil is

$$m = \rho V = \left(4.00\times10^3\ \frac{\text{kg}}{\text{m}^3}\right)\left[(1.00\ \text{m})\left(43\,560\ \text{ft}^2\right)\left(\frac{1\ \text{m}}{3.281\ \text{ft}}\right)^2\right] = 1.62\times10^7\ \text{kg}$$

At a rate of 1 part per million, the mass of uranium in this soil is then

$$m_\text{U} = \frac{m}{10^6} = \frac{1.62\times10^7\ \text{kg}}{10^6} = \boxed{16.2\ \text{kg}}$$

(b) Since 0.720% of naturally occurring uranium is ${}^{235}_{92}\text{U}$, the mass of ${}^{235}_{92}\text{U}$ in the soil of part (a) is

$$m_{{}^{235}_{92}\text{U}} = \left(7.20\times10^{-3}\right)m_\text{U} = \left(7.20\times10^{-3}\right)(16.2\ \text{kg}) = 0.117\ \text{kg} = \boxed{117\ \text{g}}$$

30.6 With a power output of $P_\text{out} = 1.00\times10^9\ \text{W}$ and efficiency of $e = 0.400$, the total energy input required each day is

$$E_\text{input} = \frac{P_\text{out}\cdot\Delta t}{e} = \frac{\left(1.00\times10^9\ \cancel{\text{J}}/\text{s}\right)\left(1.00\ \cancel{\text{d}}\right)}{0.400}\left(\frac{1\ \text{MeV}}{1.60\times10^{-13}\ \cancel{\text{J}}}\right)\left(\frac{8.64\times10^4\ \text{s}}{1\ \cancel{\text{d}}}\right) = 1.35\times10^{27}\ \text{MeV}$$

At 200 MeV per fission event, the number of ${}^{235}\text{U}$ atoms consumed each day is

$$n = \frac{E_\text{input}}{200\ \text{MeV/atom}} = \frac{1.35\times10^{27}\ \text{MeV}}{200\ \text{MeV/atom}} = 6.75\times10^{24}\ \text{atoms}$$

The mass of ${}^{235}\text{U}$ which will contain this number of atoms is

$$m = \left(\frac{n}{N_A}\right)M_\text{mol} = \left(\frac{6.75\times10^{24}\ \text{atoms}}{6.02\times10^{23}\ \text{atoms/mol}}\right)\left(235\ \frac{\text{g}}{\text{mol}}\right) = 2.63\times10^3\ \text{g} = \boxed{2.63\ \text{kg}}$$

30.7 (a) For a sphere of radius a, the surface area is $A = 4\pi a^2$, and the volume is $V = 4\pi a^3/3$. Thus, the surface-to-volume ratio is

$$ratio_\text{sphere} = \frac{A}{V} = \frac{4\pi a^2}{4\pi a^3/3} = \boxed{3/a}$$

(b) For a cube of side s, $A = 6\cdot A_\text{face} = 6s^2$ and $V = s\cdot s\cdot s = s^3$. The surface-to-volume ratio for a cube is then

$$ratio_\text{cube} = \frac{A}{V} = \frac{6s^2}{s^3} = \frac{6}{s}$$

If the cube has the same volume as the sphere, then $s^3 = 4\pi a^3/3$, or $s = (4\pi/3)^{\frac{1}{3}}\,a$, and the surface-to-volume ratio for the cube becomes

$$ratio_\text{cube} = \frac{A}{V} = \frac{6s^2}{s^3} = \frac{6}{(4\pi/3)^{\frac{1}{3}}\,a} = \boxed{3.72/a}$$

(c) $\boxed{\text{The sphere has the better shape for minimum leakage}}$ since it has the smaller surface-to-volume ratio.

30.8 (a) The mass of ^{235}U in the reserve is

$$m_{^{235}\text{U}} = \left(\frac{0.70}{100}\right)\left(4.4\times10^{6}\text{ metric ton}\right)\left(10^{3}\,\frac{\text{kg}}{\text{ton}}\right)\left(10^{3}\,\frac{\text{g}}{\text{kg}}\right) = \boxed{3.1\times10^{10}\text{ g}}$$

(b) The number of moles is $n = m/M_{\text{mol}} = \left(3.1\times10^{10}\text{ g}\right)/\left(235\,\text{g/mol}\right) = \boxed{1.3\times10^{8}\text{ mol}}$, and the number of ^{235}U atoms in this reserve is

$$N = nN_A = \left(1.3\times10^{8}\,\text{mol}\right)\left(6.02\times10^{23}\text{ atoms/mol}\right) = \boxed{7.8\times10^{31}\,\text{atoms}}$$

(c) Assuming all atoms undergo fission and all released energy captured, the total energy available is

$$E = N\left(208\,\frac{\text{MeV}}{\text{atom}}\right) = \left(7.8\times10^{31}\text{ atoms}\right)\left(208\,\frac{\text{MeV}}{\text{atom}}\right)\left(\frac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right) = \boxed{2.6\times10^{21}\text{ J}}$$

(d) At a consumption rate of 1.5×10^{13} J/s, the maximum time this energy supply could last is

$$t = \frac{E}{P} = \frac{2.6\times10^{21}\text{ J}}{1.5\times10^{13}\text{ J/s}}\left(\frac{1\text{ yr}}{3.156\times10^{7}\text{ s}}\right) = \boxed{5.5\text{ yr}}$$

(e) $\boxed{\text{Fission alone cannot meet the world's energy needs}}$ at a price of \$130 or less per kilogram of uranium.

30.9 The total energy required for one year is

$$E = \left(2\,000\text{ kWh/month}\right)\left(3.60\times10^{6}\text{ J/kWh}\right)\left(12.0\text{ months}\right) = 8.64\times10^{10}\text{ J}$$

The number of fission events needed will be

$$N = \frac{E}{E_{\text{event}}} = \frac{8.64\times10^{10}\text{ J}}{\left(208\text{ MeV}\right)\left(1.60\times10^{-13}\text{ J/MeV}\right)} = 2.60\times10^{21}$$

and the mass of this number of ^{235}U atoms is

$$m = \left(\frac{N}{N_A}\right)M_{\text{mol}} = \left(\frac{2.60\times10^{21}\text{ atoms}}{6.02\times10^{23}\text{ atoms/mol}}\right)\left(235\text{ g/mol}\right) = \boxed{1.01\text{ g}}$$

30.10 (a) At a concentration of $c = 3\text{ mg/m}^3 = 3\times10^{-3}\text{ g/m}^3$, the mass of uranium dissolved in the oceans covering two-thirds of Earth's surface to an average depth of $h_{\text{av}} = 4$ km is $m_{\text{U}} = cV = c(\tfrac{2}{3}A)\cdot h_{\text{av}} = c[\tfrac{2}{3}(4\pi R_E^2)]\cdot h_{\text{av}}$, or

$$m_{\text{U}} = \left(3\times10^{-3}\,\frac{\text{g}}{\text{m}^3}\right)\left(\frac{2}{3}\right)4\pi\left(6.38\times10^{6}\text{ m}\right)^2\left(4\times10^{3}\text{ m}\right) = \boxed{4\times10^{15}\text{ g}}$$

(b) Fissionable ^{235}U makes up 0.7% of all uranium, so $m_{^{235}\text{U}} = 0.70m_{\text{U}}/100$. If we assume all of the ^{235}U is collected and caused to undergo fission, with the release of about 200 MeV per event, the potential energy supply is

$$E = \left(\text{number of }^{235}\text{U atoms}\right)\left(200\,\frac{\text{MeV}}{\text{atom}}\right) = \left[\left(\frac{0.7m_{\text{U}}/100}{M_{\text{mol}}}\right)N_A\right]\left(200\,\frac{\text{MeV}}{\text{atom}}\right)$$

continued on next page

and at a consumption rate of $P = 1.5\times10^{13}$ J/s, the time this could supply the world's energy needs is $t = E/P$, or

$$t = \left[\frac{0.7}{100}\left(\frac{m_{\mathrm{U}}}{M_{\mathrm{mol}}}\right)\right]\frac{(200\ \mathrm{MeV/atom})}{P}$$

$$= \left[\frac{0.7}{100}\left(\frac{4\times10^{15}\ \mathrm{g}}{235\ \mathrm{g/mol}}\right)\left(6.02\times10^{23}\ \frac{\mathrm{atoms}}{\mathrm{mol}}\right)\right]\frac{200\ \mathrm{MeV/atom}}{1.5\times10^{13}\ \mathrm{J/s}}\left(\frac{1.6\times10^{-13}\ \mathrm{J}}{1\ \mathrm{MeV}}\right)\left(\frac{1\ \mathrm{yr}}{3.156\times10^{7}\ \mathrm{s}}\right)$$

$$= \boxed{5\times10^{3}\ \mathrm{yr}}$$

(c) The uranium comes from dissolving rock and minerals. Rivers carry such solutes into the oceans, but the Earth's supply of uranium is not renewable. However, if breeder reactors are used, the current ocean supply can last about a half-million years.

30.11 (a) ${}^{4}_{2}\mathrm{He} + {}^{4}_{2}\mathrm{He} \rightarrow \boxed{{}^{8}_{4}\mathrm{Be}} + \gamma$

(b) ${}^{8}_{4}\mathrm{Be} + {}^{4}_{2}\mathrm{He} \rightarrow \boxed{{}^{12}_{6}\mathrm{C}} + \gamma$

(c) The total energy released in this pair of fusion reactions is

$$Q = (\Delta m)c^2 = \left[3m_{{}^{4}\mathrm{He}} - m_{{}^{12}\mathrm{C}}\right]c^2$$

$$= \left[3(4.002\ 603\ \mathrm{u}) - 12.000\ 000\ \mathrm{u}\right](931.5\ \ \mathrm{MeV/u}) = \boxed{7.27\ \mathrm{MeV}}$$

30.12 The energy released in the reaction ${}^{1}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{2}\mathrm{He} + \gamma$ is

$$Q = (\Delta m)c^2 = \left[m_{{}^{1}_{1}\mathrm{H}} + m_{{}^{2}_{1}\mathrm{H}} - m_{{}^{3}_{2}\mathrm{He}}\right]c^2$$

$$= \left[1.007\ 825\ \mathrm{u} + 2.014\ 102\ \mathrm{u} - 3.016\ 029\ \mathrm{u}\right](931.5\ \mathrm{MeV/u}) = \boxed{5.49\ \mathrm{MeV}}$$

30.13 The energy released in the reaction ${}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow {}^{3}_{1}\mathrm{H} + {}^{1}_{1}\mathrm{H}$ is $Q = \left[2m_{{}^{2}_{1}\mathrm{H}} - m_{{}^{3}_{1}\mathrm{H}} - m_{{}^{1}_{1}\mathrm{H}}\right]c^2$, or

$$Q = \left[2(2.014\ 102\ \mathrm{u}) - 3.016\ 049\ \mathrm{u} - 1.007\ 825\ \mathrm{u}\right](931.5\ \mathrm{MeV/u}) = \boxed{4.03\ \mathrm{MeV}}$$

30.14 (a) ${}^{1}_{1}\mathrm{H} + {}^{12}_{6}\mathrm{C} \rightarrow \boxed{{}^{13}_{7}\mathrm{N}} + \gamma$ (b) ${}^{13}_{7}\mathrm{N} \rightarrow \boxed{{}^{13}_{6}\mathrm{C}} + {}^{0}_{+1}\mathrm{e} + \nu$

(c) ${}^{13}_{6}\mathrm{C} + {}^{1}_{1}\mathrm{H} \rightarrow \boxed{{}^{14}_{7}\mathrm{N}} + \gamma$ (d) ${}^{14}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow \boxed{{}^{15}_{8}\mathrm{O}} + \gamma$

(e) ${}^{15}_{8}\mathrm{O} \rightarrow \boxed{{}^{15}_{7}\mathrm{N}} + {}^{0}_{+1}\mathrm{e} + \nu$ (f) ${}^{15}_{7}\mathrm{N} + {}^{1}_{1}\mathrm{H} \rightarrow \boxed{{}^{12}_{6}\mathrm{C}} + {}^{4}_{2}\mathrm{He}$

30.15 With the deuteron and triton at rest, the total momentum before the reaction is zero. To conserve momentum, the neutron and the alpha particle must move in opposite directions with equal magnitude momenta after the reaction, or $p_\alpha = p_n$. Neglecting relativistic effects, we use the classical relationship between momentum and kinetic energy, $KE = p^2/2m$, and write $\sqrt{2m_\alpha KE_\alpha} = \sqrt{2m_n KE_n}$, or $KE_\alpha = (m_n/m_\alpha)KE_n$.

To conserve energy, it is necessary that the kinetic energies of the reaction products satisfy the relation $KE_n + KE_\alpha = Q = 17.6$ MeV. Then, using the result from above, we have $KE_n + (m_n/m_\alpha)KE_n = 17.6$ MeV, or the kinetic energy of the emerging neutron must be

$$KE_n = \frac{17.6\ \mathrm{MeV}}{1 + (1.008\ 665\ \mathrm{u})/(4.002\ 603\ \mathrm{u})} = \boxed{14.1\ \mathrm{MeV}}$$

30.16 (a) The energy released in the reaction ${}^1_1\text{H} + {}^{11}_5\text{B} \rightarrow 3({}^4_2\text{He})$ is

$$Q = (\Delta m)c^2 = \left[m_{{}^1_1\text{H}} + m_{{}^{11}_5\text{B}} - 3m_{{}^4_2\text{He}}\right]c^2$$

$$= \left[1.007\,825\text{ u} + 11.009\,306\text{ u} - 3(4.002\,603\text{ u})\right](931.5\text{ MeV/u})$$

$$= \boxed{8.68\text{ MeV}}$$

(b) The proton and the boron nucleus both have positive charges but must come very close to one another in order for fusion to occur. Thus, they must have sufficient kinetic energy to overcome the repulsive Coulomb force one exerts on the other.

30.17 Note that pair production cannot occur in a vacuum. It must occur in the presence of a massive particle which can absorb at least some of the momentum of the photon and allow total linear momentum to be conserved.

When a particle-antiparticle pair is produced by a photon having the minimum possible frequency, and hence minimum possible energy, the nearby massive particle absorbs all the momentum of the photon, allowing both components of the particle-antiparticle pair to be left at rest. In such an event, the total kinetic energy afterwards is essentially zero, and the photon need only supply the total rest energy of the pair produced.

The minimum photon energy required to produce a proton-antiproton pair is $E_{\text{photon}} = 2(E_R)_{\text{proton}}$. Thus, the minimum photon frequency is

$$f = \frac{E_{\text{photon}}}{h} = \frac{2(E_R)_{\text{proton}}}{h} = \frac{2(938.3\text{ MeV})(1.60\times10^{-13}\text{ J/MeV})}{6.63\times10^{-34}\text{ J}\cdot\text{s}} = \boxed{4.53\times10^{23}\text{ Hz}}$$

and $\lambda = \dfrac{c}{f} = \dfrac{3.00\times10^{8}\text{ m/s}}{4.53\times10^{23}\text{ Hz}} = 6.62\times10^{-16}\text{ m} = \boxed{0.662\text{ fm}}$

30.18 The total kinetic energy after the pair production is

$$KE_{\text{total}} = E_{\text{photon}} - 2(E_R)_{\text{proton}} = 2.09\times10^{3}\text{ MeV} - 2(938.3\text{ MeV}) = 213\text{ MeV}$$

The kinetic energy of the antiproton is then

$$KE_{\bar{p}} = KE_{\text{total}} - KE_p = 213\text{ MeV} - 95.0\text{ MeV} = \boxed{118\text{ MeV}}$$

30.19 The total rest energy of the π^0 is converted into energy of the photons. Since the total momentum was zero before the decay, the two photons must go in opposite directions with equal magnitude momenta (and hence equal energies). Thus, the rest energy of the π^0 is split equally between the two photons, giving for each photon

$$E_{\text{photon}} = \frac{E_{R,\pi^0}}{2} = \frac{135\text{ MeV}}{2} = \boxed{67.5\text{ Mev}}$$

$$p_{\text{photon}} = \frac{E_{\text{photon}}}{c} = \boxed{67.5\text{ MeV}/c}$$

and $f = \dfrac{E_{\text{photon}}}{h} = \dfrac{67.5\text{ MeV}}{6.63\times10^{-34}\text{ J}\cdot\text{s}}\left(\dfrac{1.60\times10^{-13}\text{ J}}{1\text{ MeV}}\right) = \boxed{1.63\times10^{22}\text{ Hz}}$

30.20 Observe that the given reactions involve only mesons and baryons. With no leptons before or after the reactions, we do not have to consider the conservation laws concerning the various lepton numbers. All interactions always conserve both charge and baryon numbers. The strong interaction also conserves strangeness. Conservation of strangeness may be violated by the weak interaction but never by more than one unit. With these facts in mind consider the given interactions:

$K^0 \to \pi^+ + \pi^-$

	K^0	total before	π^+	π^-	total after
Charge	0	0	+1	−1	0
Baryon number	0	0	0	0	0
Strangeness	+1	+1	0	0	0

This reaction conserves both charge and baryon number but does violate strangeness by one unit. Thus, it $\boxed{\text{can occur via the weak interaction}}$ but not other interactions.

$\Lambda^0 \to \pi^+ + \pi^-$

	Λ^0	total before	π^+	π^-	total after
Charge	0	0	+1	−1	0
Baryon number	+1	+1	0	0	0
Strangeness	−1	−1	0	0	0

This reaction $\boxed{\text{fails to conserve baryon number and cannot occur}}$ via any interaction.

30.21

	Reaction	**Conservation Law Violated**
(a)	$p+\bar{p} \to \mu^+ + e^-$	$L_e: (0+0 \to 0+1)$; and $L_\mu: (0+0 \to -1+0)$
(b)	$\pi^- + p \to p + \pi^+$	Charge, Q: $(-1+1 \to +1+1)$
(c)	$p+p \to p+\pi^+$	Baryon Number, B: $(1+1 \to 1+0)$
(d)	$p+p \to p+p+n$	Baryon Number, B: $(1+1 \to 1+1+1)$
(e)	$\gamma + p \to n + \pi^0$	Charge, Q: $(0+1 \to 0+0)$

30.22

	Reaction	**Conservation Law Violated**
(a)	$p \to \pi^+ + \pi^0$	Baryon Number, B: $(1 \to 0+0)$
(b)	$p+p \to p+p+\pi^0$	No violations – The reaction can occur.
(c)	$\pi^+ \to \mu^+ + \nu_\mu$	No violations – The reaction can occur.
(d)	$n \to p + e^- + \bar{\nu}_e$	No violations – The reaction can occur.
(e)	$\pi^+ \to \mu^+ + n$	Baryon Number, B: $(0 \to 0+1)$ Muon-Lepton Number, L_μ: $(0 \to -1+0)$

30.23

	Reaction	Conservation Law	Conclusion
(a)	$\pi^- + \mathrm{p} \to 2\eta^0$	Baryon Number - violated: $(0+1 \to 0)$	Cannot Occur
(b)	$\mathrm{K}^- + \mathrm{n} \to \Lambda^0 + \pi^-$	All conservation laws observed.	May occur via the Strong Interaction
(c)	$\mathrm{K}^- \to \pi^- + \pi^0$	Strangeness violated by 1 unit $(-1 \to 0+0)$. All other conservation laws observed.	Can occur via Weak Interaction, but not by Electromagnetic or Strong Interactions.
(d)	$\Omega^- \to \Xi^- + \pi^0$	Strangeness violated by 1 unit $(-3 \to -2+0)$. All other conservation laws observed.	Can occur via Weak Interaction, but not by Electromagnetic or Strong Interactions.
(e)	$\eta^0 \to 2\gamma$	All conservation laws observed.	Allowed via all interactions, but photons are the mediator of the electromagnetic interaction and the lifetime of the η^0 is consistent with decay by that interaction.

30.24 (a) $\pi^+ + \mathrm{p} \to \mathrm{K}^+ + \Sigma^+$

Baryon number, B: $0+1 \to 0+1$ $\boxed{\Delta B = 0}$

Charge, Q: $1+1 \to 1+1$ $\boxed{\Delta Q = 0}$

$\pi^+ + \mathrm{p} \to \pi^+ + \Sigma^+$

Baryon number, B: $0+1 \to 0+1$ $\boxed{\Delta B = 0}$

Charge, Q: $1+1 \to 1+1$ $\boxed{\Delta Q = 0}$

(b) Strangeness is conserved in the first reaction:

Strangeness, S: $0+0 \to 1-1$ $\Delta S = 0$

The second reaction does not conserve strangeness:

Strangeness, S: $0+0 \to 0-1$ $\Delta S = -1$

The second reaction $\boxed{\text{cannot occur via the strong or electromagnetic interactions}}$.

(c) If $\boxed{\text{one of the neutral kaons}}$ were also produced in the second reaction, giving $\pi^+ + \mathrm{p} \to \pi^+ + \Sigma^+ + \mathrm{K}^0$, then strangeness would no longer be violated:

Strangeness, S: $0+0 \to 0-1+1$ $\Delta S = 0$

Because the total mass of the product particles in this reaction would be greater than that in the first reaction [see part (a)], the total incident energy of the reacting particles $\boxed{\text{would have to be greater for this reaction than for the first reaction}}$.

30.25 (a) $\Lambda^0 \to \mathrm{p} + \pi^-$

Strangeness, S: $-1 \to 0+0$ $\Rightarrow \Delta S \neq 0$ $\boxed{\text{Not Conserved}}$

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(b) $\pi^- + p \rightarrow \Lambda^0 + K^0$

Strangeness, S: $0+0 \rightarrow -1+1$ $\Rightarrow \Delta S = 0$ $\boxed{\text{Conserved}}$

(c) $\overline{p} + p \rightarrow \overline{\Lambda}^0 + \Lambda^0$

Strangeness, S: $0+0 \rightarrow +1-1$ $\Rightarrow \Delta S = 0$ $\boxed{\text{Conserved}}$

(d) $\pi^- + p \rightarrow \pi^- + \Sigma^+$

Strangeness, S: $0+0 \rightarrow 0-1$ $\Rightarrow \Delta S \neq 0$ $\boxed{\text{Not Conserved}}$

(e) $\Xi^- \rightarrow \Lambda^0 + \pi^-$

Strangeness, S: $-2 \rightarrow -1+0$ $\Rightarrow \Delta S \neq 0$ $\boxed{\text{Not Conserved}}$

(f) $\Xi^0 \rightarrow p + \pi^-$

Strangeness, S: $-2 \rightarrow 0+0$ $\Rightarrow \Delta S \neq 0$ $\boxed{\text{Not Conserved}}$

30.26

	proton	**u**	**u**	**d**	**total**
strangeness	0	0	0	0	0
baryon number	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1
charge	e	$2e/3$	$2e/3$	$-e/3$	e

	neutron	**u**	**d**	**d**	**total**
strangeness	0	0	0	0	0
baryon number	1	$\frac{1}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	1
charge	0	$2e/3$	$-e/3$	$-e/3$	0

30.27 The number of water molecules in one liter (mass = 1 000 g) of water is

$$N = \left(\frac{1\,000\text{ g}}{18.0\text{ g/mol}}\right)\left(6.02\times10^{23}\text{ molecules/mol}\right) = 3.34\times10^{25}\text{ molecules}$$

Each molecule contains 10 protons, 10 electrons, and 8 neutrons. Thus, there are

$$N_e = 10N = \boxed{3.34\times10^{26}\text{ electrons}},\ N_p = 10N = 3.344\times10^{26}\text{ protons},$$

and $N_n = 8N = 2.68\times10^{26}\text{ neutrons}$

Each proton contains 2 up quarks and 1 down quark, while each neutron has 1 up quark and 2 down quarks. Therefore, there are

$$N_u = 2N_p + N_n = \boxed{9.36\times10^{26}\text{ up quarks}},\text{ and } N_d = N_p + 2N_n = \boxed{8.70\times10^{26}\text{ down quarks}}.$$

30.28

K^0 Particle				
	K^0	d	$\bar{s}$	total
strangeness	1	0	1	1
baryon number	0	1/3	−1/3	0
charge	0	$-e/3$	$e/3$	0

Λ^0 Particle					
	Λ^0	u	d	s	total
strangeness	−1	0	0	−1	−1
baryon number	1	1/3	1/3	1/3	1
charge	0	$2e/3$	$-e/3$	$-e/3$	0

30.29 Compare the given quark states to the entries in Table 30.4:

(a) suu $= \boxed{\Sigma^+}$ (b) $\bar{\text{u}}$d $= \boxed{\pi^-}$

(c) $\bar{\text{s}}$d $= \boxed{\text{K}^0}$ (d) ssd $= \boxed{\Xi^-}$

30.30 (a) $\bar{\text{u}}\bar{\text{u}}\bar{\text{d}}$: charge $= \left(-\frac{2}{3}e\right) + \left(-\frac{2}{3}e\right) + \left(+\frac{1}{3}e\right) = \boxed{-e}$. This is the $\boxed{\text{antiproton}}$.

(b) $\bar{\text{u}}\bar{\text{d}}\bar{\text{d}}$: charge $= \left(-\frac{2}{3}e\right) + \left(+\frac{1}{3}e\right) + \left(+\frac{1}{3}e\right) = \boxed{0}$. This is the $\boxed{\text{antineutron}}$.

30.31 The reaction is $\Sigma^0 + \text{p} \to \Sigma^+ + \gamma + X$, or on the quark level, uds + uud → uus + 0 + x.

The left side has a net 3u, 2d, and 1s. The right side has 2u, 0d, and 1s, plus the quark composition of the unknown particle. To conserve the total number of each type of quark, the composition of the unknown particle must be $x = \boxed{\text{udd}}$. Therefore, the unknown particle must be a $\boxed{\text{neutron}}$.

30.32 (a) $\pi^- + \text{p} \to \Sigma^+ + \pi^0$ is forbidden by $\boxed{\text{conservation of charge}}$.

(b) $\mu^- \to \pi^- + \nu_e$ is forbidden by both $\boxed{\text{conservation of electron-lepton number}}$, and $\boxed{\text{conservation of muon-lepton number}}$.

(c) $\text{p} \to \pi^+ + \pi^+ + \pi^-$ is forbidden by $\boxed{\text{conservation of baryon number}}$.

30.33 To the reaction for nuclei, ${}^1_1\text{H} + {}^3_2\text{He} \to {}^4_2\text{He} + {}^{0}_{+1}\text{e} + \nu_e$, we add three electrons to both sides to obtain ${}^1_1\text{H}_{\text{atom}} + {}^3_2\text{He}_{\text{atom}} \to {}^4_2\text{He}_{\text{atom}} + {}^{0}_{-1}\text{e} + {}^{0}_{+1}\text{e} + \nu_e$. Then we use the masses of the neutral atoms from Appendix B of the textbook to compute

$$Q = (\Delta m)c^2 = \left[m_{{}^1_1\text{H}} + m_{{}^3_2\text{He}} - m_{{}^4_2\text{He}} - 2m_e\right]c^2$$

$$= \left[1.007\,825\ \text{u} + 3.016\,029\ \text{u} - 4.002\,603\ \text{u} - 2(0.000\,549\ \text{u})\right](931.5\ \text{MeV/u})$$

$$= \boxed{18.8\ \text{MeV}}$$

30.34 For the particle reaction, $\mu^+ + e^- \rightarrow 2\nu$, the lepton numbers before the event are $L_\mu = -1$ and $L_e = +1$. These values must be conserved by the reaction so one of the emerging neutrinos must have $L_\mu = -1$ while the other has $L_e = +1$. The emerging particles are $\boxed{\bar{\nu}_\mu}$ and $\boxed{\nu_e}$.

30.35 (a) $K^+ + p \rightarrow X + p$

Since this occurs via the strong interaction, all conservation rules must be observed.

Baryon Number:	$0+1 \rightarrow B+1$	$\Delta B = 0 \;\Rightarrow\; B = 0$	so X is not a baryon.
Strangeness:	$+1+0 \rightarrow S+0$	$\Delta S = 0 \Rightarrow S = +1$	or X has $S = +1$
Charge:	$+e+e \rightarrow Q+e$	$\Delta Q = 0 \Rightarrow Q = +e$	or X has $Q = +e$
Lepton Numbers:	$0+0 \rightarrow L+0$	$\Delta L = 0 \Rightarrow L = 0$	or X has $L_e = L_\mu = L_\tau = 0$

Of the particles in Table 30.2, the only non-baryon with $S = +1$ and $Q = +e$ is the positive kaon, $\boxed{K^+}$. Thus, this is an elastic scattering process.

The weak interaction observes all conservation rules except strangeness, and $\Delta S = \pm 1$.

(b) $\Omega^- \rightarrow X + \pi^-$

Baryon Number:	$1 \rightarrow B+0$	$\Delta B = 0 \;\Rightarrow\; B = +1$	or X has $B = +1$
Strangeness:	$-3 \rightarrow S+0$	$\Delta S = +1 \Rightarrow S = -2$	or X has $S = -2$
Charge:	$-e \rightarrow Q-e$	$\Delta Q = 0 \Rightarrow Q = 0$	or X has $Q = 0$
Lepton Numbers:	$0 \rightarrow L+0$	$\Delta L = 0 \Rightarrow L = 0$	or X has $L_e = L_\mu = L_\tau = 0$

The particle must be a neutral baryon with strangeness $S = -2$. Thus, it is the $\boxed{\Xi^0}$.

(c) $K^+ \rightarrow X + \mu^+ + \nu_\mu$

Baryon Number:	$0 \rightarrow B+0+0$	$\Delta B = 0 \;\Rightarrow\; B = 0$	so X is not a baryon.
Strangeness:	$+1 \rightarrow S+0+0$	$\Delta S = \pm 1 \Rightarrow S = 0,\ +2$	or X has $S = 0$ or $+2$
Charge:	$+e \rightarrow Q+e+0$	$\Delta Q = 0 \Rightarrow Q = 0$	or X has $Q = 0$
Lepton Numbers:	$0 \rightarrow L_e + 0 + 0$	$\Delta L_e = 0 \Rightarrow L_e = 0$	or X has $L_e = 0$
	$0 \rightarrow L_\mu - 1 + 1$	$\Delta L_\mu = 0 \Rightarrow L_\mu = +0$	or X has $L_\mu = 0$
	$0 \rightarrow L_\tau + 0 + 0$	$\Delta L_\tau = 0 \Rightarrow L_\tau = +0$	or X has $L_\tau = 0$

The particle must be a neutral non-baryon with strangeness $S = 0$ or $S = +2$, and $L_e = L_\mu = L_\tau = 0$. This is the neutral pion, $\boxed{\pi^0}$.

30.36 Assuming a head-on collision, the total momentum is zero both before and after the reaction $p + p \rightarrow p + \pi^+ + X$. Therefore, since the proton and the pion are at rest after the reaction, particle X must also be left at rest.

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Particle X must be a neutral baryon in order to conserve charge and baryon number in the reaction. Conservation of energy requires that the rest energy this particle be

$$E_{R,X} = 2\left(E_{R,p} + 70.4 \text{ MeV}\right) - E_{R,p} - E_{R,\pi^+} = E_{R,p} - E_{R,\pi^+} + 140.8 \text{ MeV}$$

or $E_{R,X} = 938.3 \text{ MeV} - 139.6 \text{ MeV} + 140.8 \text{ MeV} = 939.5 \text{ MeV}$

Particle X is $\boxed{\text{a neutron}}$.

30.37 If a neutron starts with kinetic energy $KE_i = 2.0$ MeV and loses one-half of its kinetic energy in each collision with a moderator atom, its kinetic energy after n collisions will be $KE_f = KE_i/2^n$.

The average kinetic energy associated with particles in a gas at temperature $T = 20.0°\text{C} = 293$ K (see Chapter 10 of the textbook) is

$$\overline{KE}_f = \frac{3}{2}k_BT = \frac{3}{2}\left(1.38\times10^{-23} \text{ J/K}\right)(293 \text{ K})\left(\frac{1 \text{ eV}}{1.60\times10^{-19} \text{ J}}\right) = 0.0379 \text{ eV}$$

Thus, the number of collisions the neutron must make before it reaches the energy associated with a room temperature gas is $n\ln 2 = \ln(KE_i/KE_f)$, or

$$n = \left(\frac{1}{\ln 2}\right)\ln\left(\frac{2.0\times10^6 \text{ eV}}{0.0379 \text{ eV}}\right) = \boxed{26}$$

30.38 (a) The number of deuterons in one kilogram of deuterium is

$$N = \frac{m}{m_{\text{atom}}} = \frac{1 \text{ kg}}{(2.01 \text{ u})\left(1.66\times10^{-27} \text{ kg/u}\right)} = 3.00\times10^{26}$$

Each occurrence of the reaction ${}^2_1\text{D} + {}^2_1\text{D} \to {}^3_2\text{He} + {}^1_0\text{n}$ consumes two deuterons and releases $Q = 3.27$ MeV of energy. The total energy available from the one kilogram of deuterium is then

$$E = \left(\frac{N}{2}\right)Q = \left(\frac{3.00\times10^{26}}{2}\right)(3.27 \text{ Mev})\left(\frac{1.60\times10^{-13} \text{ J}}{1 \text{ MeV}}\right) = \boxed{7.85\times10^{13} \text{ J}}$$

(b) At a rate of eight cents per kilowatt-hour, the value of this energy is

$$value = E\times rate = \left(7.85\times10^{13} \text{ J}\right)\left(\frac{\$0.08}{\text{kWh}}\right)\left(\frac{1 \text{ kWh}}{3.60\times10^6 \text{ J}}\right) = \$1.74\times10^6 = \boxed{\$1\,740\,000}$$

(c) Deuterium makes up four-twentieths or one-fifth of the mass of a heavy water molecule. Thus, five kilograms of heavy water are necessary to obtain one kilogram of deuterium. The cost for this water (5 kg)(\$300/kg) = $\boxed{\$1\,500}$.

(d) Whether it would be cost-effective depends on how much it cost to fuse the deuterium and how much net energy was produced. If the cost is nine-tenths of the value of the energy produced, each kilogram of deuterium would still yield a profit of \$174 000.

30.39 (a) The number of moles in 1.0 gal of water is

$$n = \frac{m}{M} = \frac{\rho V}{M} = \frac{\left(1.0 \dfrac{\text{g}}{\text{cm}^3}\right)(1.0 \text{ gal})\left(\dfrac{3.786 \text{ L}}{1 \text{ gal}}\right)\left(\dfrac{10^3 \text{ cm}^3}{1 \text{ L}}\right)}{18 \text{ g/mol}} = 2.1\times10^2 \text{ mol}$$

continued on next page

so the number of hydrogen atoms (2 per water molecule) will be

$$N_{\rm H} = 2(nN_A) = 2(2.1\times10^{2}\ \text{mol})(6.02\times10^{23}\ \text{mol}^{-1}) = 2.5\times10^{26}$$

and one of every 6 500 of these contains a deuteron. Thus, the number of deuterons contained in 1.0 gal of water is

$$N_{\rm D} = N_{\rm H}/6\,500 = 2.5\times10^{26}/6\,500 = 3.8\times10^{22}\ \text{deuterons}$$

and the available energy is

$$E = (3.8\times10^{22}\ \text{deuterons})(1.64\ \text{MeV/deuteron})(1.6\times10^{-13}\ \text{J/MeV}) = \boxed{1.0\times10^{10}\ \text{J}}$$

(b) At a consumption rate of $P = 1.0\times10^{4}\ \text{J/s}$, the time that this could supply a person's energy needs is

$$\Delta t = \frac{E}{P} = \frac{1.0\times10^{10}\ \text{J}}{1.0\times10^{4}\ \text{J/s}}\left(\frac{1\ \text{d}}{86\,400\ \text{s}}\right) = \boxed{12\ \text{d}}$$

30.40 (a) The number of water molecules in the oceans is

$$N_{\rm H_2O} = \left(\frac{m_{\substack{\text{ocean}\\\text{water}}}}{M_{\text{mol}}}\right)N_A = \left(\frac{1.32\times10^{21}\ \text{kg}}{18.0\times10^{-3}\ \text{kg/mol}}\right)(6.02\times10^{23}\ \text{molecules/mol})$$

$$= 4.41\times10^{46}\ \text{molecules}$$

Since there are 2 hydrogen atoms per water molecule, and the fraction of all hydrogen atoms which contain deuterons is 1.56×10^{-4}, the number of deuterons in the oceans is

$$N_D = (2N_{\rm H_2O})\times1.56\times10^{-4} = 2(4.41\times10^{46})(1.56\times10^{-4}) = 1.38\times10^{43}$$

Each fusion event consumes 2 deuterons, so the number of fusion events possible is $N_{\text{events}} = \frac{1}{2}N_D$. The reaction ${}^2_1\text{D} + {}^2_1\text{D} \to {}^3_2\text{He} + {}^1_0\text{n}$ releases 3.27 MeV per event, so the total energy that could be released is

$$E = (3.27\ \text{MeV})N_{\text{events}} = (3.27\ \text{MeV})\left(\tfrac{1}{2}N_D\right)$$

$$= (3.27\ \text{MeV})\left(\frac{1.38\times10^{43}}{2}\right)\left(\frac{1.60\times10^{-13}\ \text{J}}{1\ \text{MeV}}\right) = \boxed{3.61\times10^{30}\ \text{J}}$$

(b) One hundred times the current world electric power consumption is

$$P = 100(7.00\times10^{12}\ \text{W}) = 7.00\times10^{14}\ \text{J/s}$$

At this rate, the time the energy computed in part (a) would last is

$$t = \frac{E}{P} = \frac{3.61\times10^{30}\ \text{J}}{7.00\times10^{14}\ \text{J/s}}\left(\frac{1\ \text{yr}}{3.156\times10^{7}\ \text{s}}\right) = \boxed{1.63\times10^{8}\ \text{yr}}$$

30.41 Conserving energy in the decay $\pi^- \to \mu^- + \bar{\nu}_\mu$, with the π^- initially at rest gives $E_\mu + E_\nu = E_{R,\pi^-}$, or

$$E_\mu + E_\nu = 139.6\ \text{MeV} \qquad \textbf{[1]}$$

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Since the total momentum was zero before the decay, conservation of momentum requires the muon and antineutrino to recoil in opposite directions with equal magnitude momenta, or $p_\mu = p_\nu$. The relativistic relation between total energy and momentum of a particle gives for the antineutrino: $E_\nu = p_\nu c$, or $p_\nu = E_\nu / c$. The same relation applied to the muon gives $E_\mu^2 = \left(p_\mu c\right)^2 + E_{R,\mu}^2$. Since we must have $p_\mu = p_\nu$, this may be rewritten as $E_\mu^2 = \left(p_\nu c\right)^2 + E_{R,\mu}^2$, and using the fact that
$p_\nu c = E_\nu$, we have $E_\mu^2 = E_\nu{}^2 + E_{R,\mu}^2$. Rearranging and factoring then gives

$$E_\mu^2 - E_\nu^2 = \left(E_\mu + E_\nu\right)\left(E_\mu - E_\nu\right) = E_{R,\mu}^2 = (105.7 \text{ MeV})^2$$

and

$$E_\mu - E_\nu = \frac{(105.7 \text{ MeV})^2}{E_\mu + E_\nu} \qquad \textbf{[2]}$$

Substituting Equation [1] into [2] gives $E_\mu - E_\nu = (105.7 \text{ MeV})^2/139.6 \text{ MeV}$, or

$$E_\mu - E_\nu = 80.0 \text{ MeV} \qquad \textbf{[3]}$$

Subtracting Equation [3] from Equation [1] yields $2E_\nu = 59.6$ MeV, and

$$E_\nu = \boxed{29.8 \text{ MeV}}$$

30.42 The reaction $\pi^- + \text{p} \to \text{K}^0 + \Lambda^0$ is $\overline{\text{u}}\text{d} + \text{uud} \to \text{d}\overline{\text{s}} + \text{uds}$ at the quark level. There is $\boxed{\text{a net 1 u and 2 d quarks both before and after the reaction}}$. This reaction conserves the net number of each type quark.

For the reaction $\pi^- + \text{p} \to \text{K}^0 + \text{n}$, or $\overline{\text{u}}\text{d} + \text{uud} \to \text{d}\overline{\text{s}} + \text{udd}$, there is a net $\boxed{\text{1 u and 2 d before the reaction and 1 u, 3 d, and 1 }\overline{\text{s}}\text{ quark afterwards}}$. This reaction does not conserve the net number of each type quark.

30.43 To compute the energy released in each occurrence of the reaction

$$4\text{p} + 2\text{e}^- \;\to\; \alpha + 2\nu_\text{e} + 6\gamma$$

we add two electrons to each side to produce neutral atoms and obtain
$4({}^1_1\text{H}_\text{atom}) \;\to\; {}^4_2\text{He}_\text{atom} + 2\nu_\text{e} + 6\gamma$. Then, recalling that the neutrino and the photon both have zero rest mass, and using the neutral atomic masses from Appendix B in the textbook gives

$$Q = (\Delta m)c^2 = \left[4m_{{}^1_1\text{H}_\text{atom}} - m_{{}^4_2\text{He}_\text{atom}}\right]c^2$$

$$= \left[4(1.007\,825 \text{ u}) - 4.002\,603 \text{ u}\right](931.5 \text{ MeV/u})$$

$$= 26.7 \text{ Mev}$$

Each occurrence of this reaction consumes four protons. Thus, the energy released per proton consumed is $E_1 = 26.4 \text{ MeV}/4 \text{ protons} = 6.68 \text{ MeV/proton}$.

Therefore, the rate at which the Sun must be fusing protons to provide its power output is

$$rate = \frac{P}{E_1} = \frac{3.85\times10^{26} \text{ J/s}}{6.68 \text{ MeV/proton}}\left(\frac{1 \text{ MeV}}{1.60\times10^{-13} \text{ J}}\right) = \boxed{3.60\times10^{38} \text{ proton/s}}$$

30.44 With the kaon initially at rest, the total momentum was zero before the decay. Thus, to conserve momentum, the two pions must go in opposite directions and have equal magnitudes of momentum after the decay. Since the pions have equal mass, this means they must have equal speeds and hence, equal energies. The rest energy of the kaon is then split equally between the two pions, and the energy of each is

$$E_\pi = E_{R,\mathrm{K}^0}/2 = 497.7\ \text{Mev}/2 = 248.9\ \text{MeV}$$

The total energy of one of the pions is related to its rest energy by $E_\pi = \gamma E_{R,\pi}$, where $\gamma = 1\big/\sqrt{1-(v/c)^2}$. Therefore, the speed of each of the pions after the decay will be

$$v = c\sqrt{1-\left(\frac{E_{R,\pi}}{E_\pi}\right)^2} = c\sqrt{1-\left(\frac{139.6\ \text{MeV}}{248.9\ \text{MeV}}\right)^2} = \boxed{0.827\,9c}$$