# 5. 双目立体视觉系统

- 平行视图
- 图像校正
- 对应点搜索

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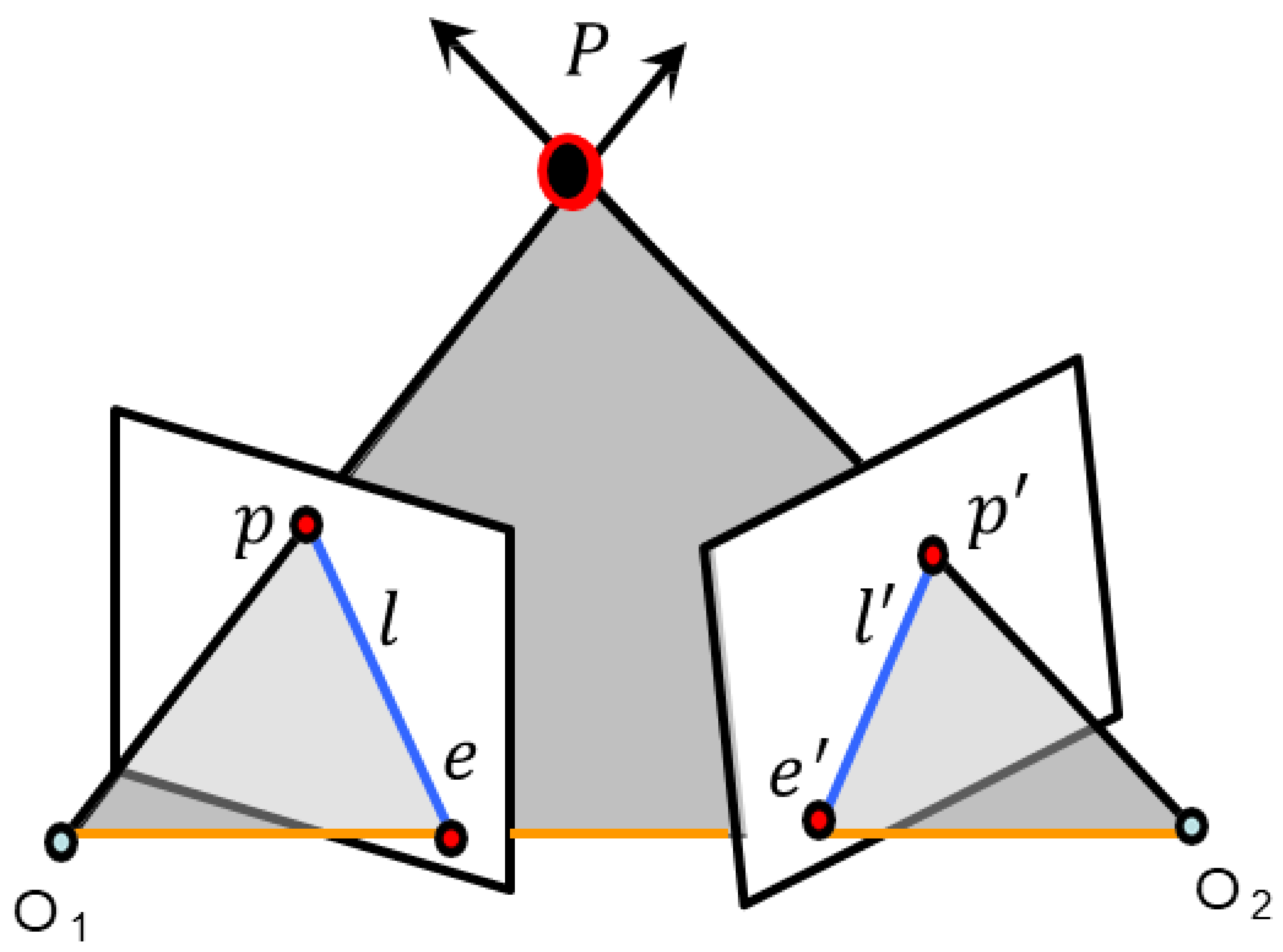
# 极几何

$E$ = 本质矩阵

| | |
|---|---|
| $K = K' = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$<br>$p$ 点像素坐标为 $(u,v)$<br>$p'$ 点像素坐标为 $(u',v')$ | $p'^T E p = 0$<br>$E = T \times R = [T_\times]R$ |

$F$ = 基础矩阵

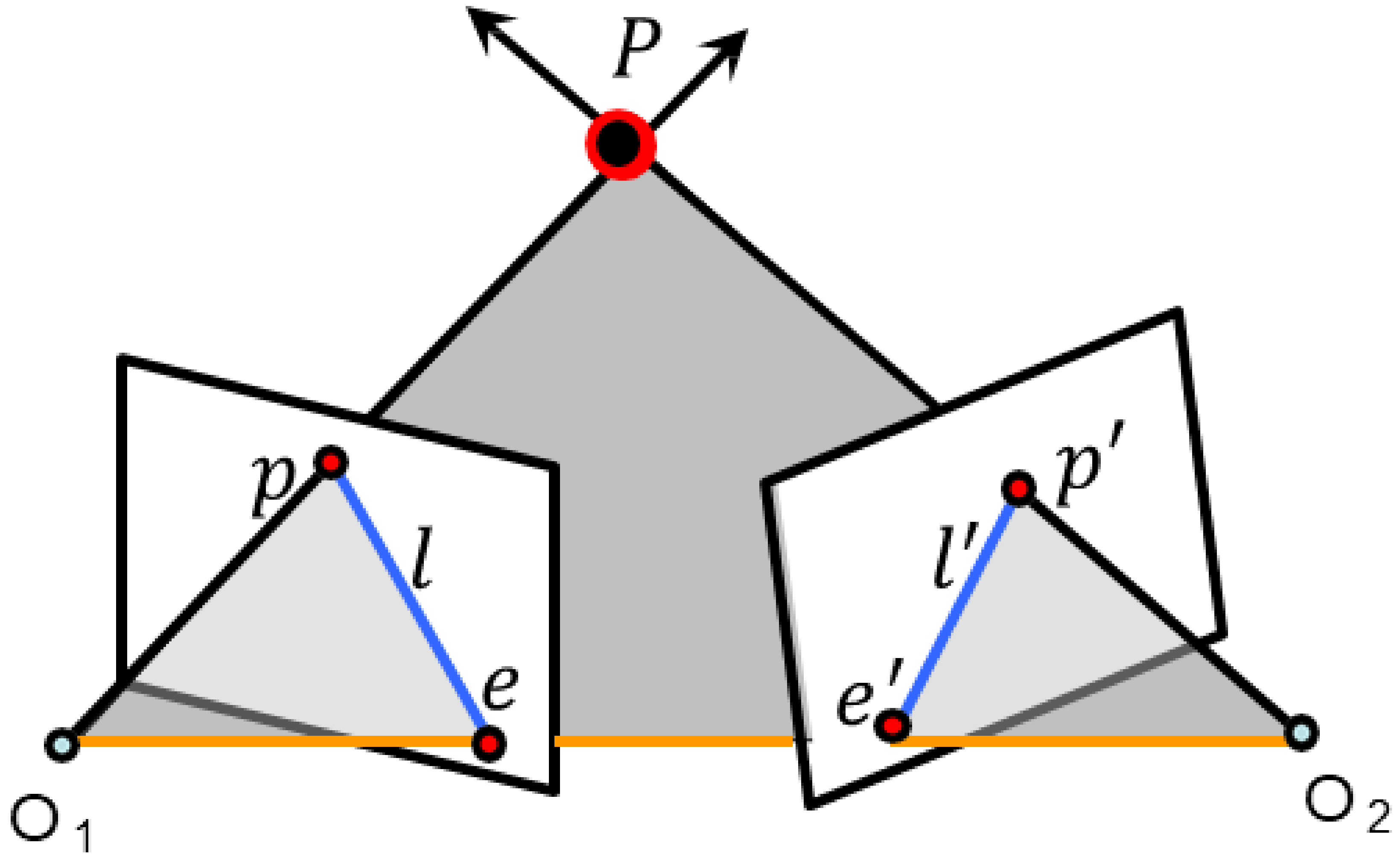
| | |
|---|---|
| 内参数分别为 $K$ 和 $K'$<br>$p$ 点像素坐标为 $(u,v)$<br>$p'$ 点像素坐标为 $(u',v')$ | $p'^T F p = 0$<br>$F = K'^{-T}[T_\times]RK^{-1}$<br>$= K'^{-T} E K^{-1}$ |



# 极几何

基础矩阵$F$

$$\boxed{F = K'^{-T}[T_\times]RK^{-1}}$$

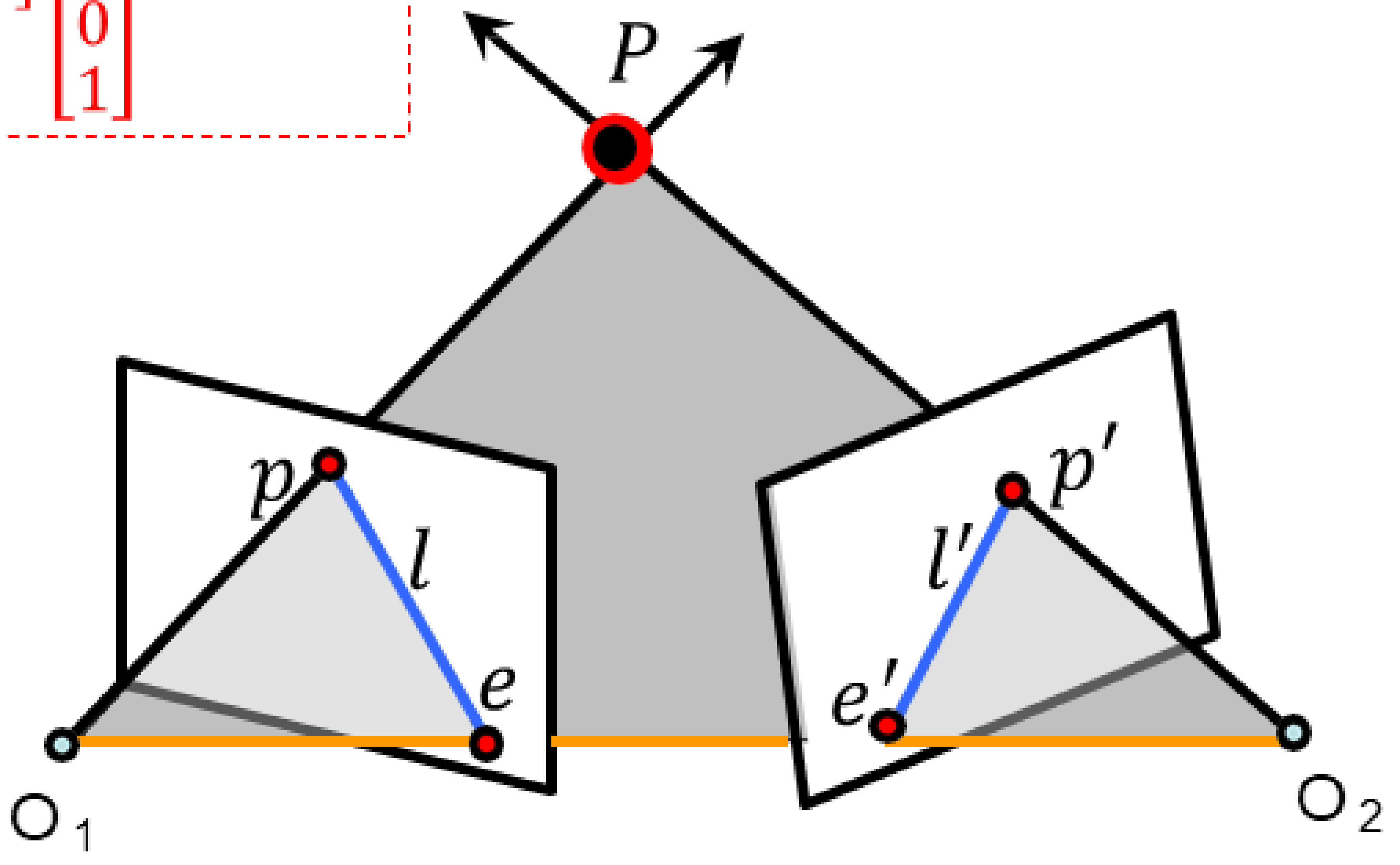


# 极几何

基础矩阵$F$

$$\boxed{F = K'^{-T}[T_{\times}]RK^{-1}}$$

$$e' = K'[R\ T]\begin{bmatrix}0\\0\\0\\1\end{bmatrix} = K'T$$



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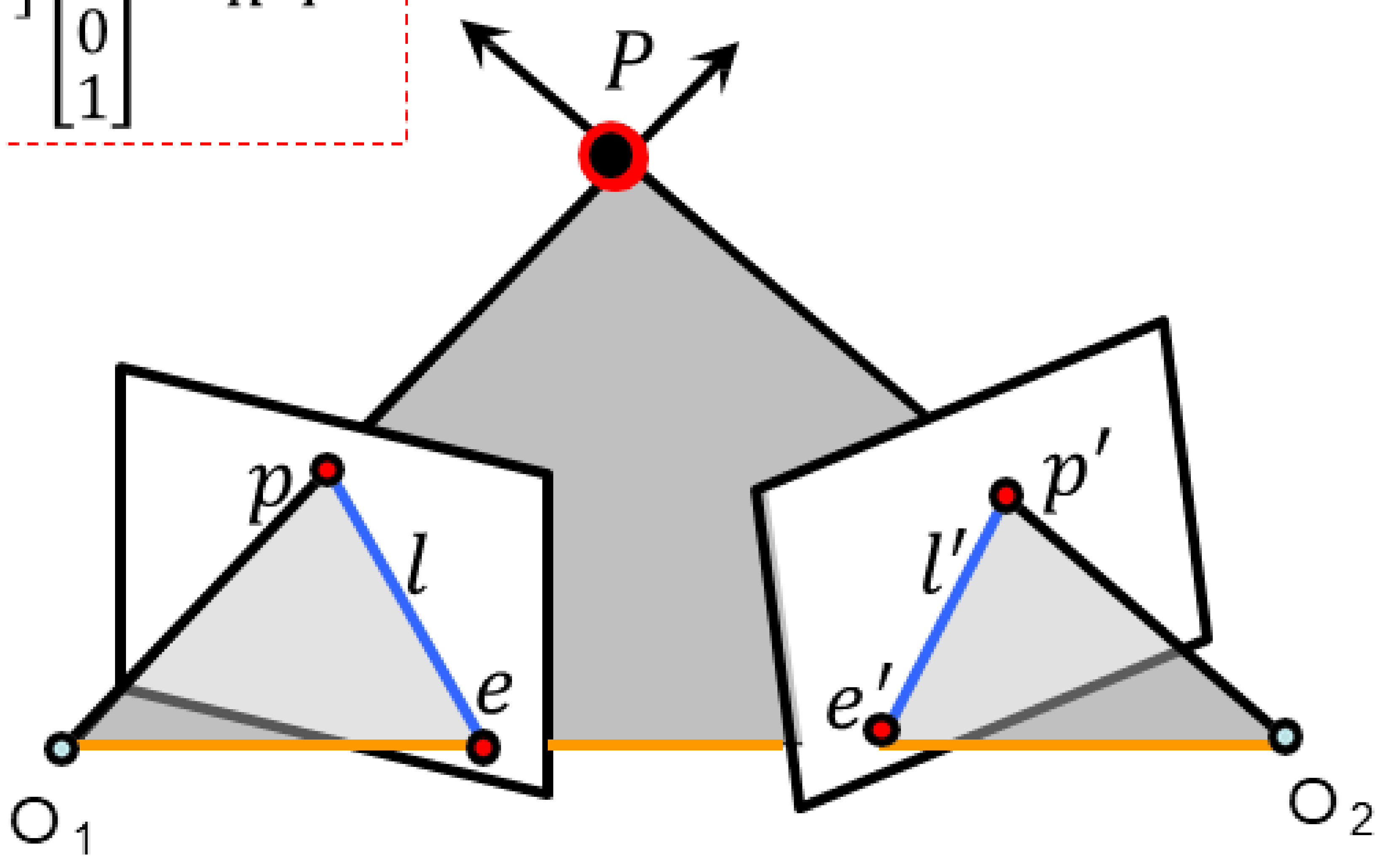
叉乘性质：对于任何向量$t$，如果M可逆，相差一个尺度情况下：

$$[t_{\times}]M = M^{-T}[(M^{-1}t)_{\times}]$$



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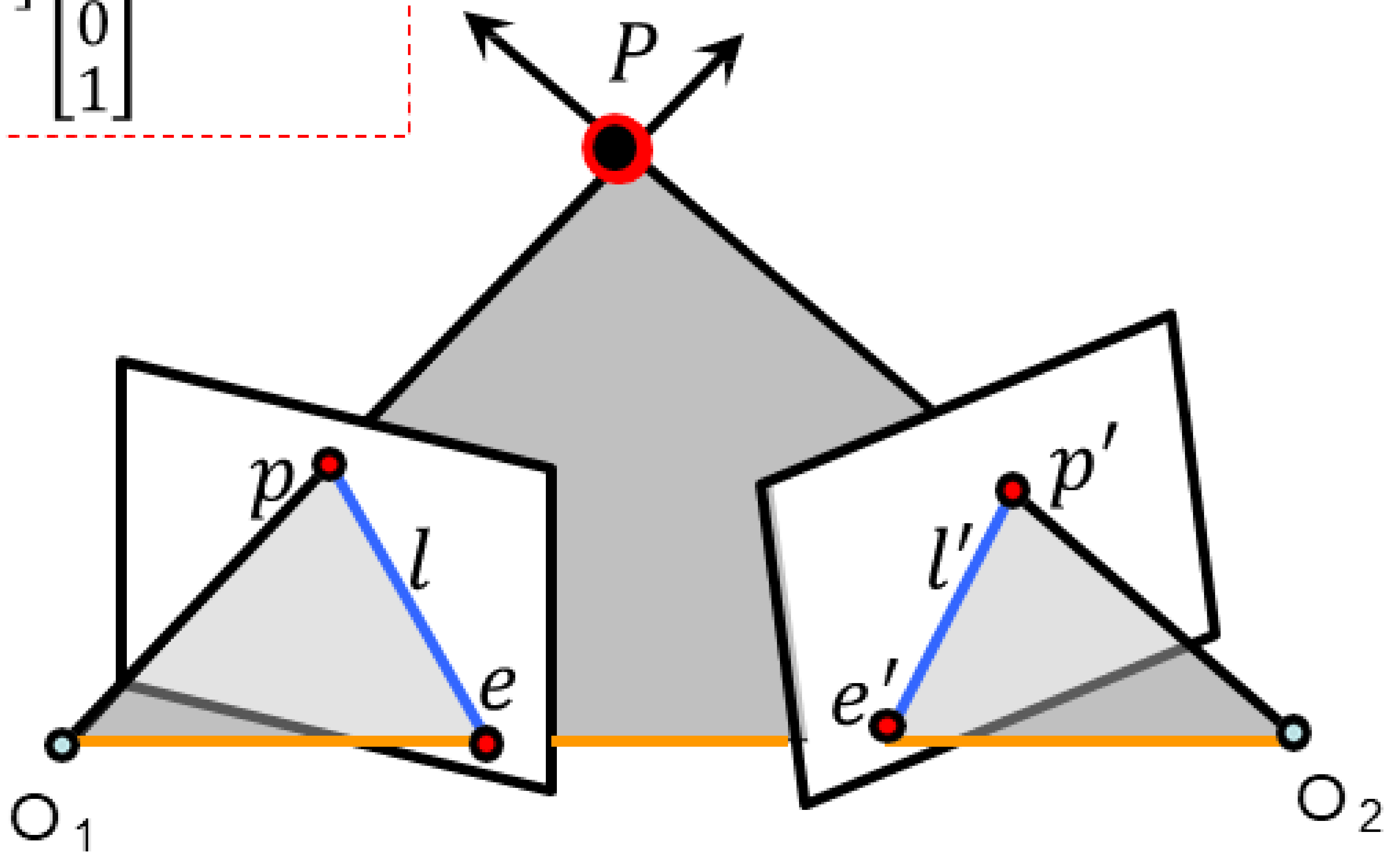


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$$[t_\times]M = M^{-T}[(M^{-1}t)_\times]$$

令 $t = T, M = K'^{-1}$

$$[T_\times]K'^{-1} = K'^{T}[(K'T)_\times]$$

$$[T_\times] = K'^{T}[(K'T)_\times]K'$$

# 极几何

## 基础矩阵$F$

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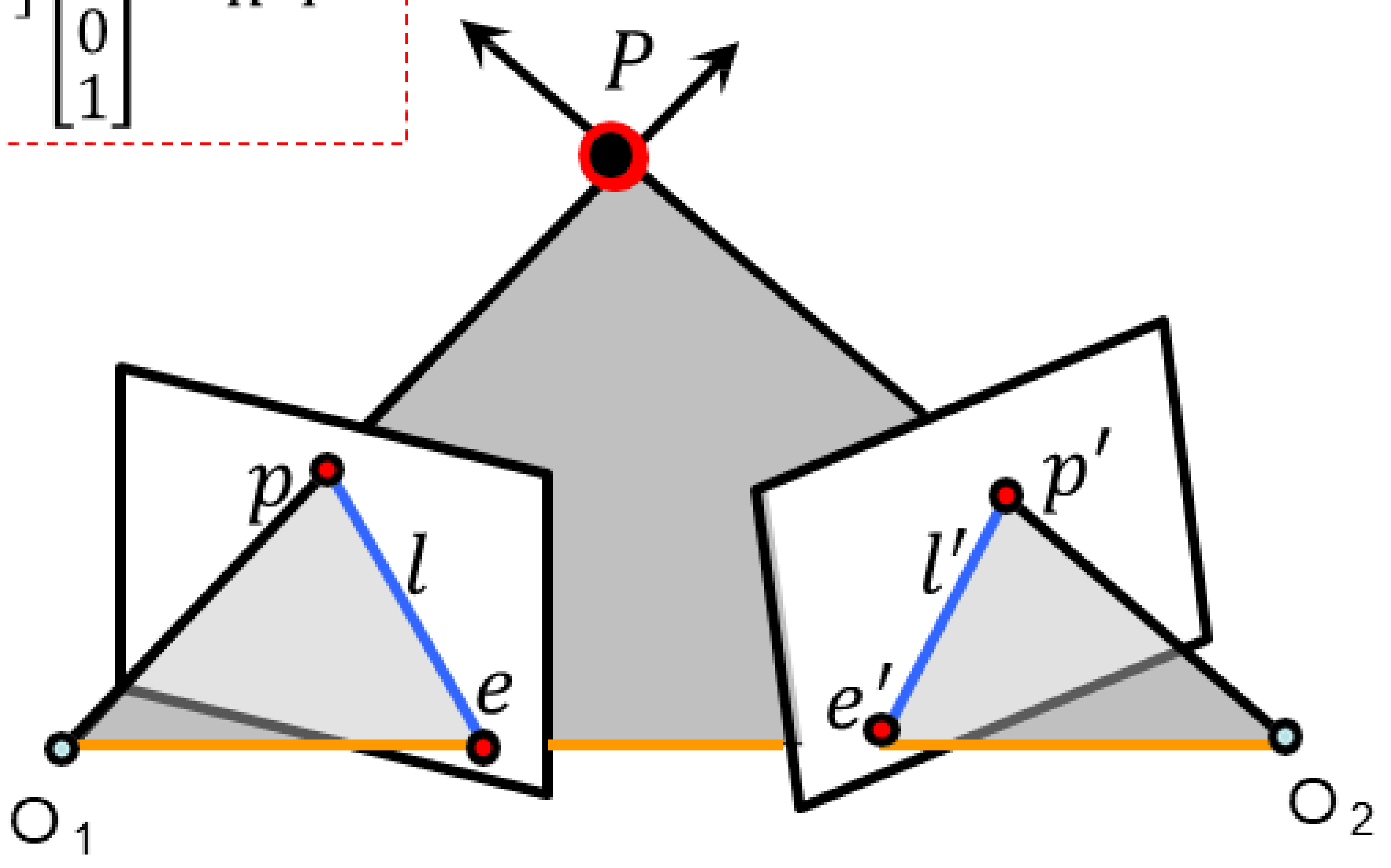
$$F = K'^{-T}[T_\times]RK^{-1} = K'^{-T}K'^{T}[(K'T)_\times]K'RK^{-1} = [(K'T)_\times]K'RK^{-1} = [e'_\times]K'RK^{-1}$$



# 极几何

**基础矩阵$F$**

$$\boxed{F = K'^{-T}[T_\times]RK^{-1}}$$

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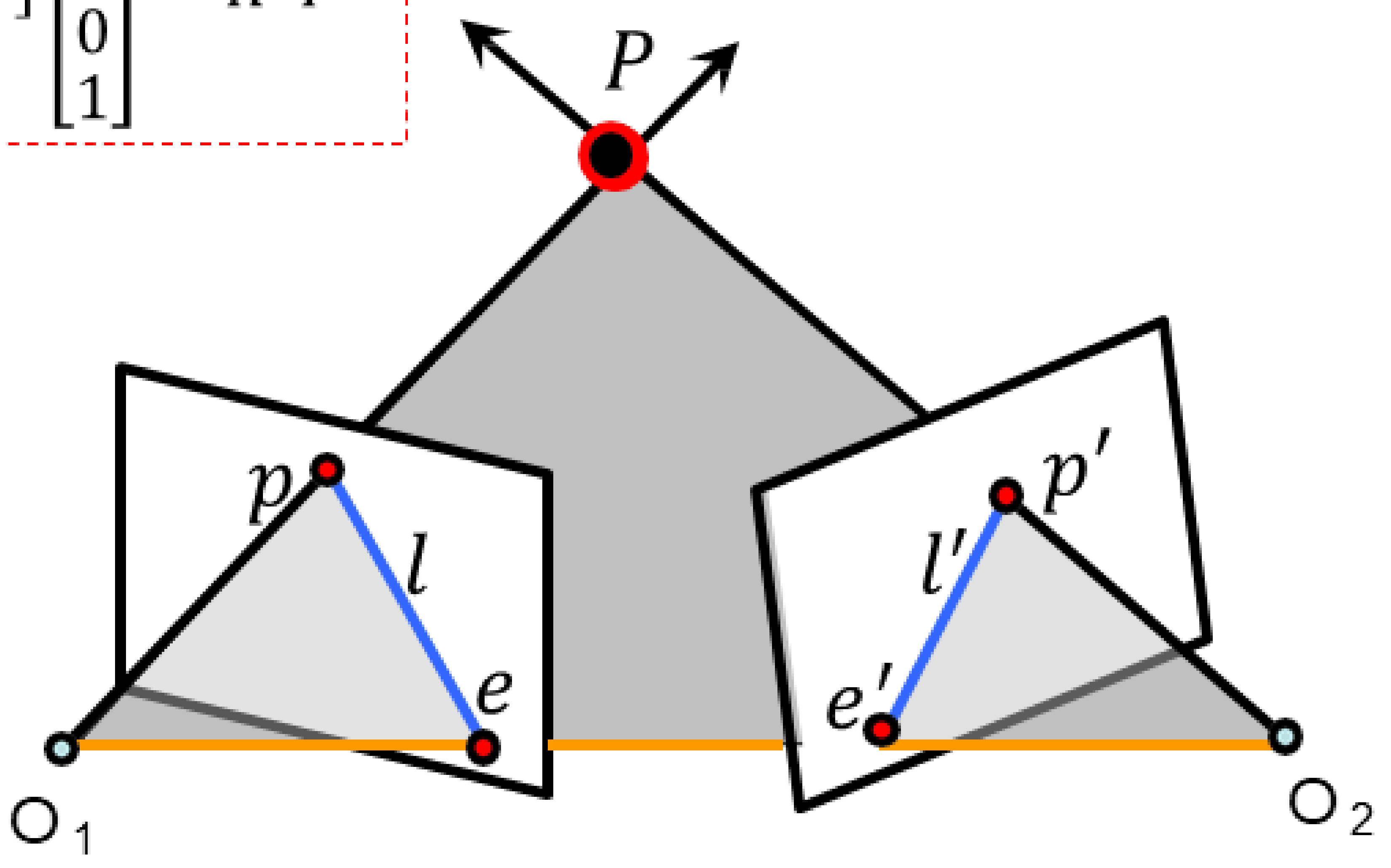


叉乘性质：对于任何向量$t$，如果M可逆，相差一个尺度情况下：

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$$[T_\times]K'^{-1} = K'^{T}[(K'T)_\times]$$

$$[T_\times] = K'^{T}[(K'T)_\times]K'$$

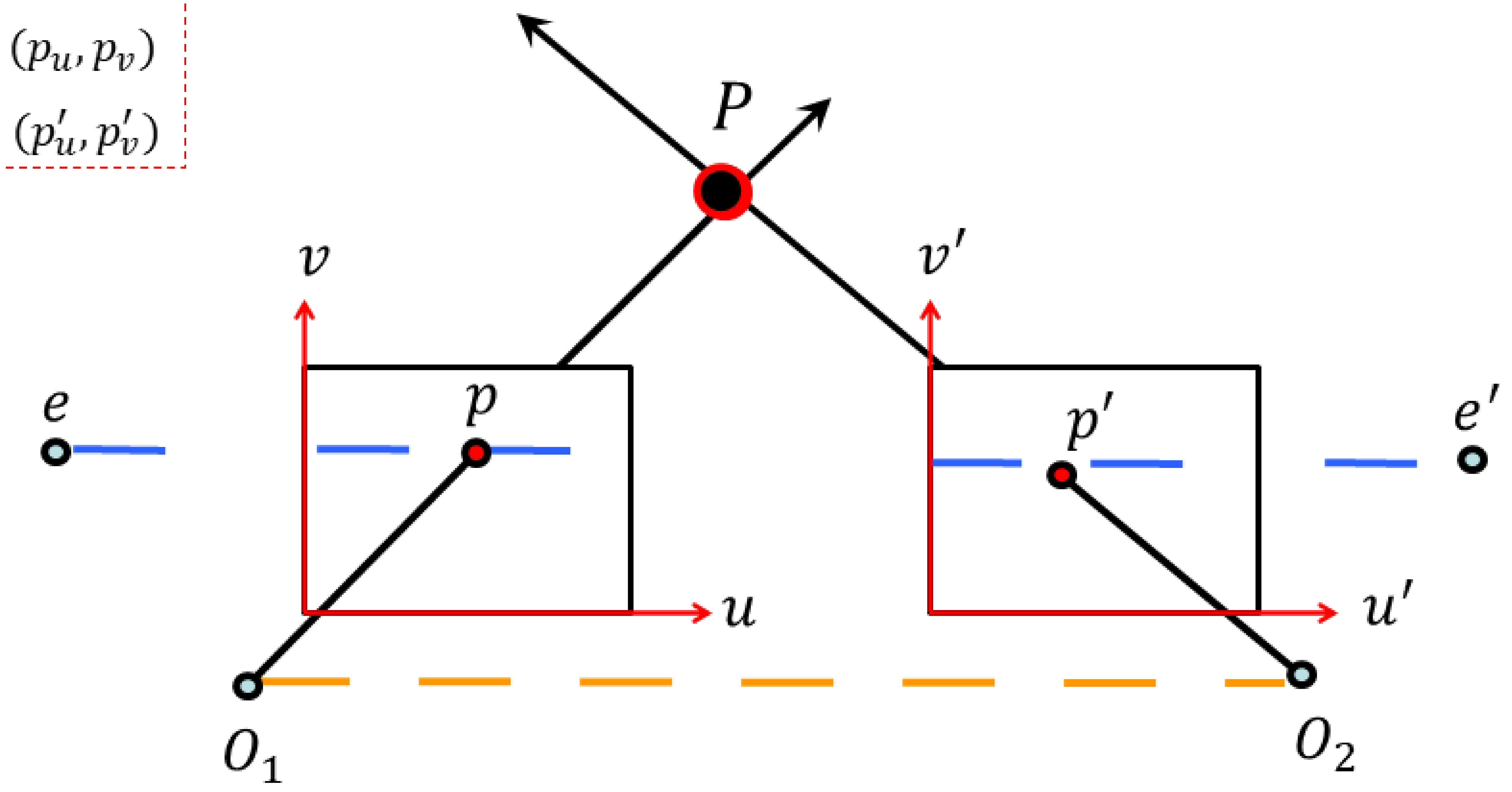
$$F = K'^{-T}[T_\times]RK^{-1} = K'^{-T}K'^{T}[(K'T)_\times]K'RK^{-1} = [(K'T)_\times]K'RK^{-1} = [e'_\times]K'RK^{-1}$$

$$\boxed{F = [e'_\times]K'RK^{-1}}$$

# 极几何特例：平行视图

$p$ 点像素坐标 $(p_u, p_v)$

$p'$ 点像素坐标 $(p'_u, p'_v)$




- 两个图像平面平行；
- 基线平行于图像平面，极点$e$和$e'$位于无穷远处

$\boxed{F = [e'_{\times}]K'RK^{-1}}$

# 平行视图的基础矩阵

$$K = K' \qquad R = I \qquad T = \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix} \qquad e' = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$\boxed{F = [e'_\times]K'RK^{-1}}$

# 平行视图的基础矩阵

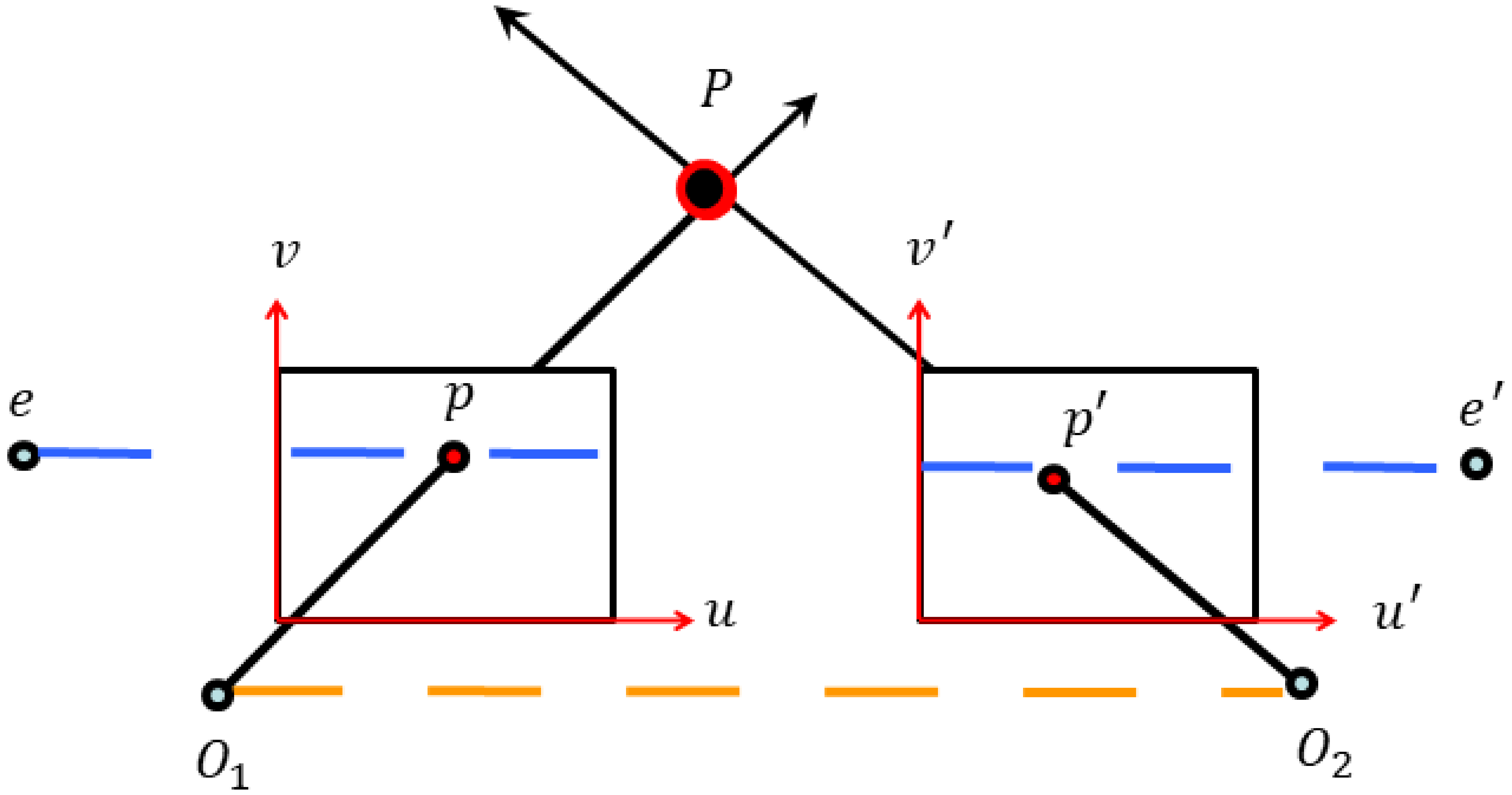
$$K = K' \qquad R = I \qquad T = \begin{bmatrix} T \\ 0 \\ 0 \end{bmatrix} \qquad e' = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$F = [e'_\times]K'RK^{-1} = [e'_\times] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

# 平行视图的极几何

$p$ 点像素坐标 $(p_u, p_v)$

$p'$ 点像素坐标 $(p'_u, p'_v)$




极线为?

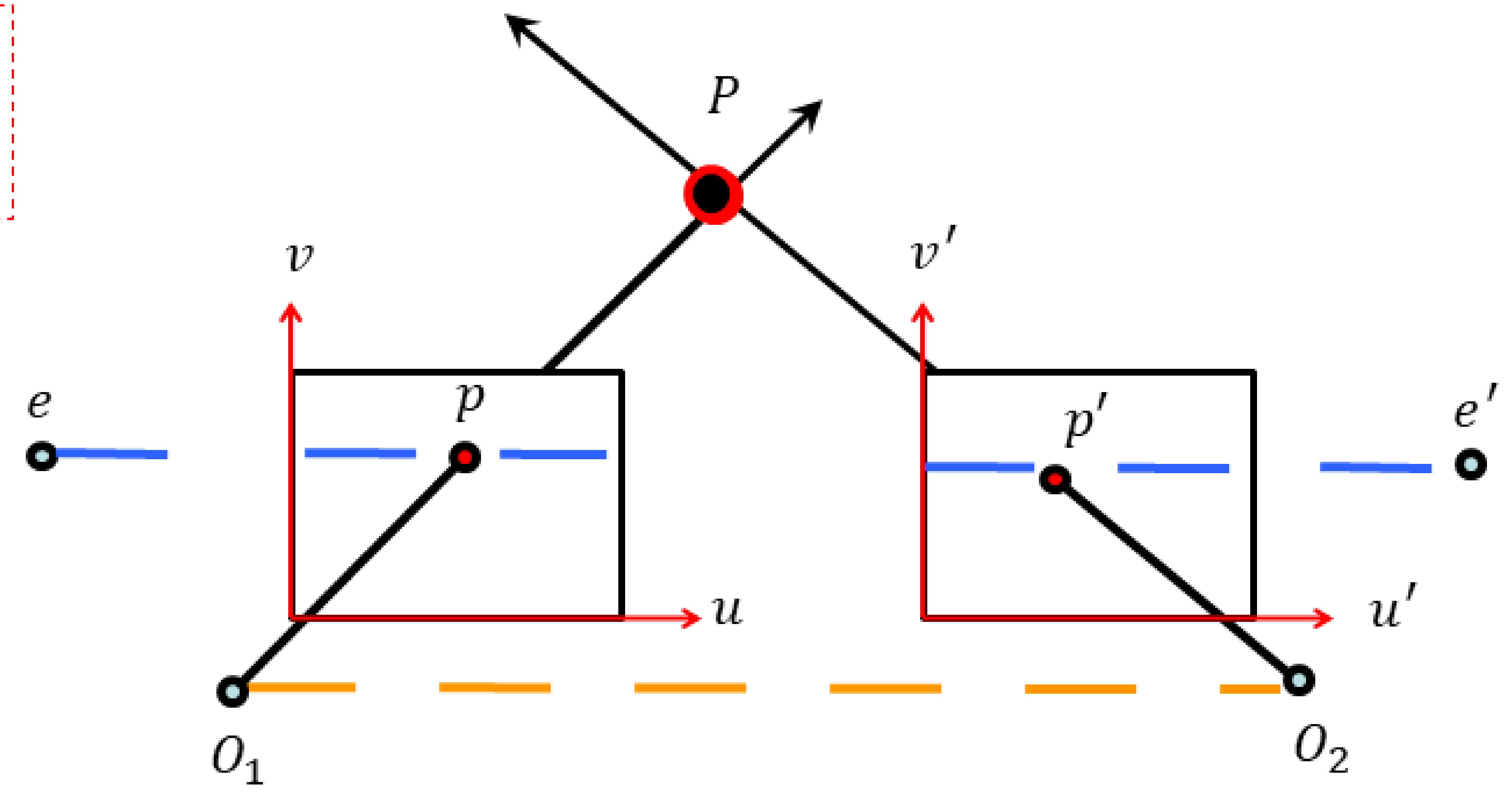
$$l = F^T p' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} p'_u \\ p'_v \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ -p'_v \end{bmatrix}$$

极线是水平的，平行于$u$轴！

# 平行视图的极几何

$p$ 点像素坐标 $(p_u, p_v)$

$p'$ 点像素坐标 $(p'_u, p'_v)$



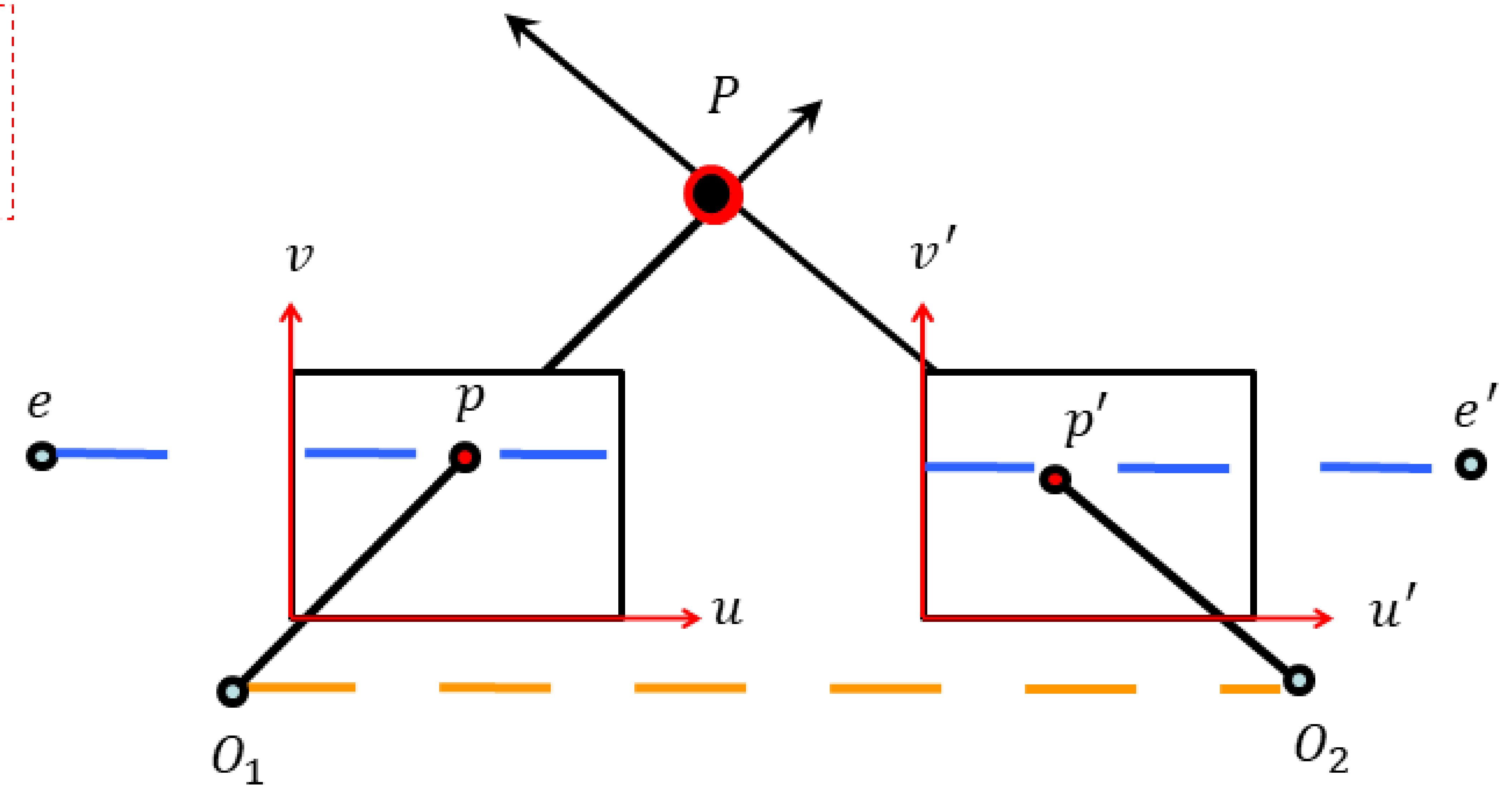

$p$和$p'$有何关系？ $p'^T F p = 0 \Longrightarrow (p'_u \quad p'_v \quad 1)\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}\begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = 0$

$\Downarrow$

$p$和$p'$的$v$坐标一样！ $\boxed{p_v = p'_v} \Longleftarrow p_v = p'_v$

# 平行视图的极几何

$p$ 点像素坐标 $(p_u, p_v)$
$p'$ 点像素坐标 $(p'_u, p'_v)$




$p$和$p'$有何关系？

极线是水平的，平行于$u$轴！

$p$和$p'$的$v$坐标一样！

$p'$点沿着扫描线寻找即可！！！

# 三角测量

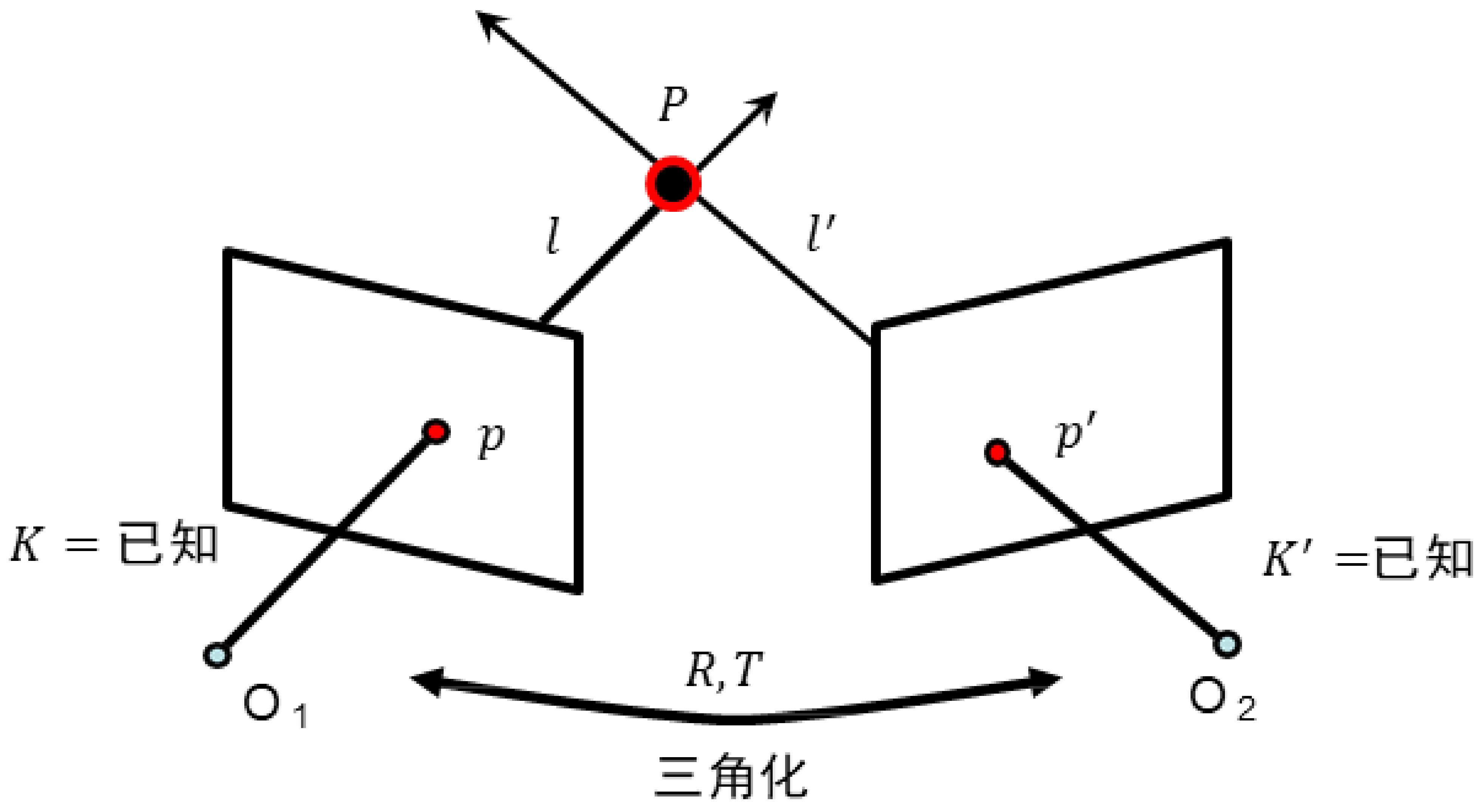
噪声的存在，两条直线通常不相交！

$$P = \cancel{l} \times l'$$

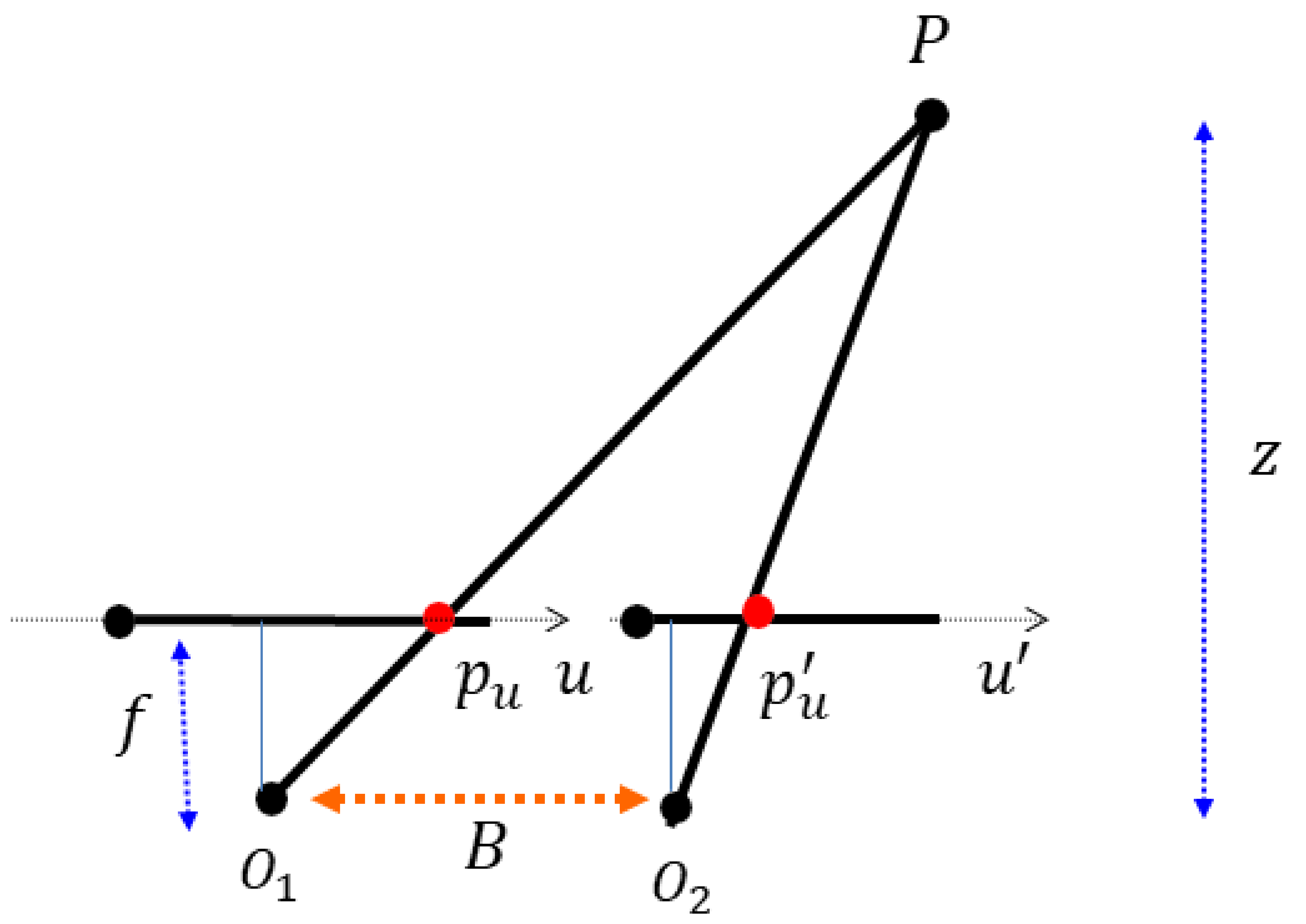
问题：已知$p$和$p'$，$K$和$K'$以及$R,T$

求解：$P$点的三维坐标？

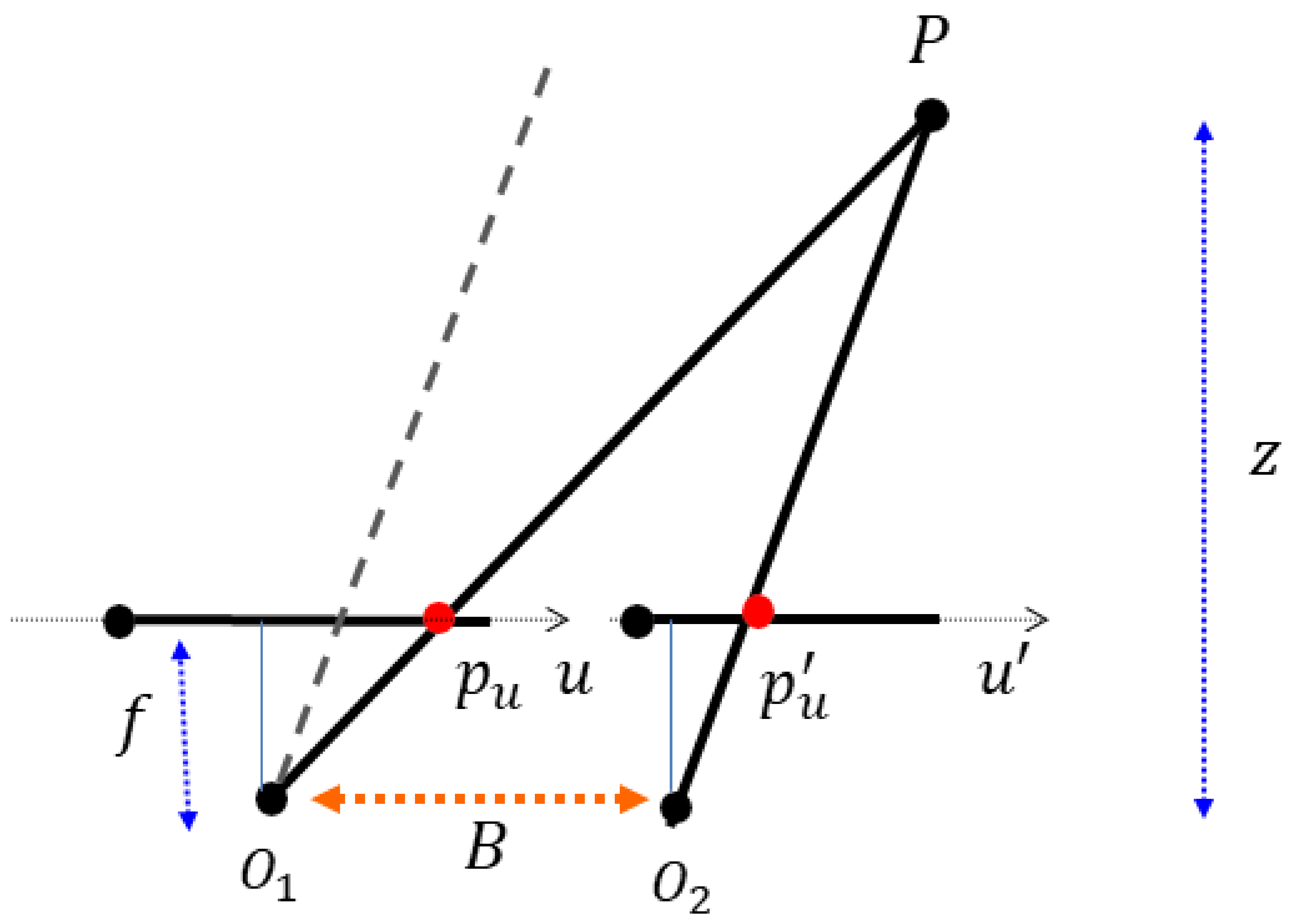
- ➢线性解法
- ➢非线性解法



# 平行视图的三角测量

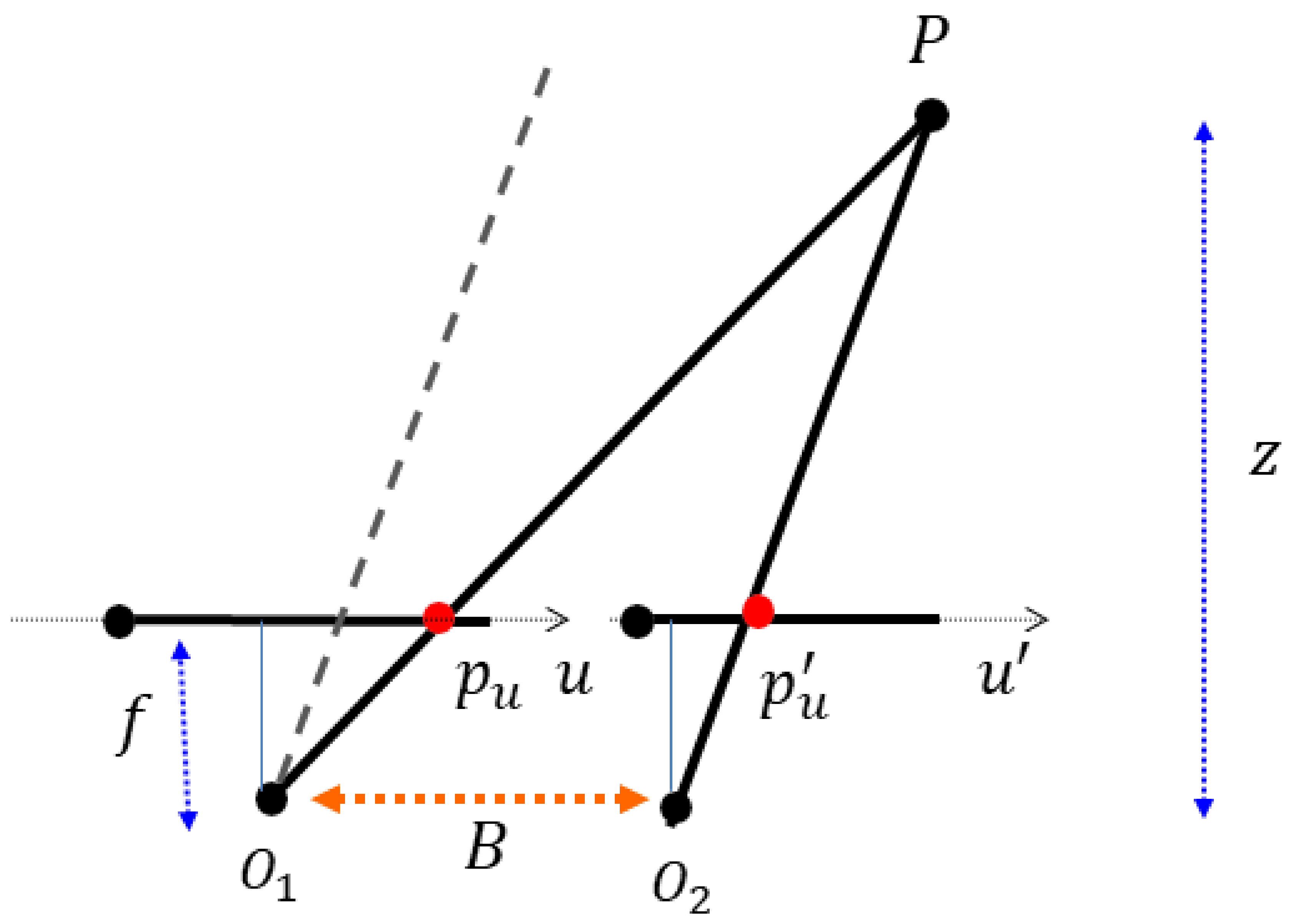


# 平行视图的三角测量




$$p_u - p'_u = \frac{B \cdot f}{z}$$

# 平行视图的三角测量



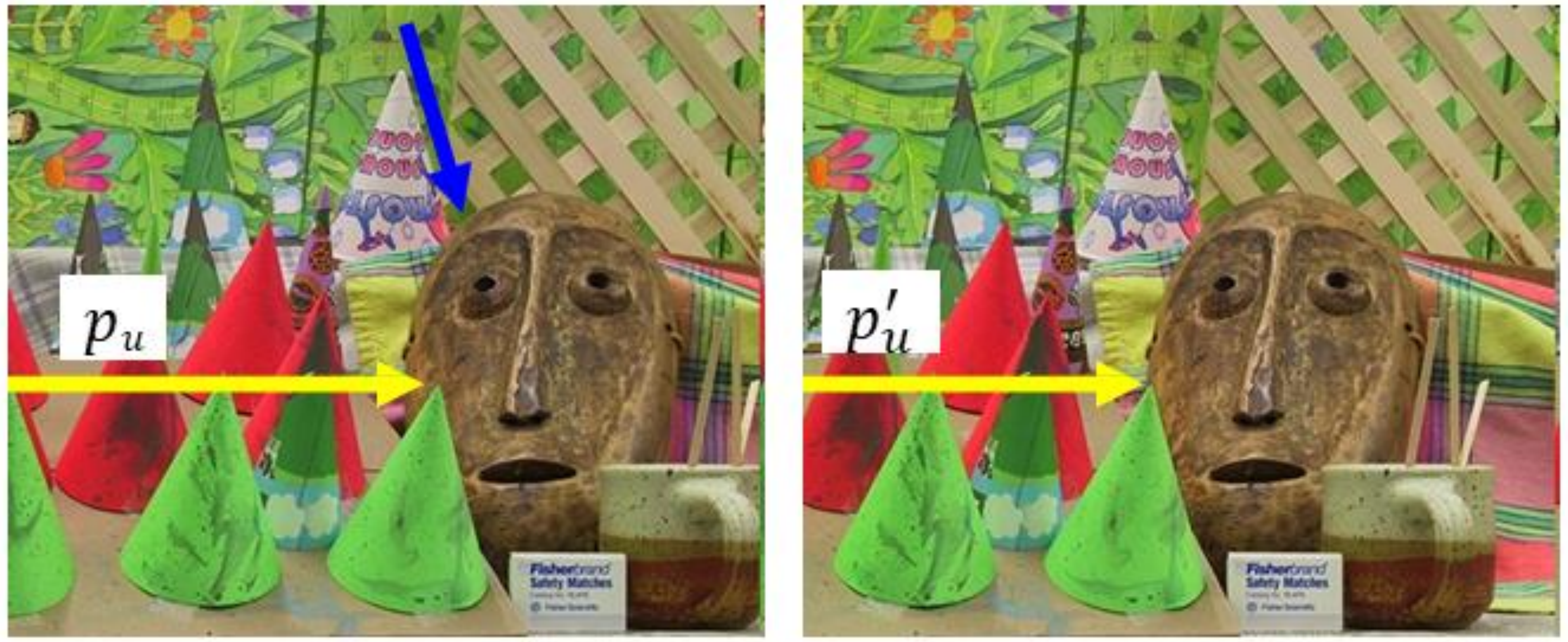

视差

$$\boxed{p_u - p'_u} = \frac{B \cdot f}{z}$$

视差与深度z成反比！

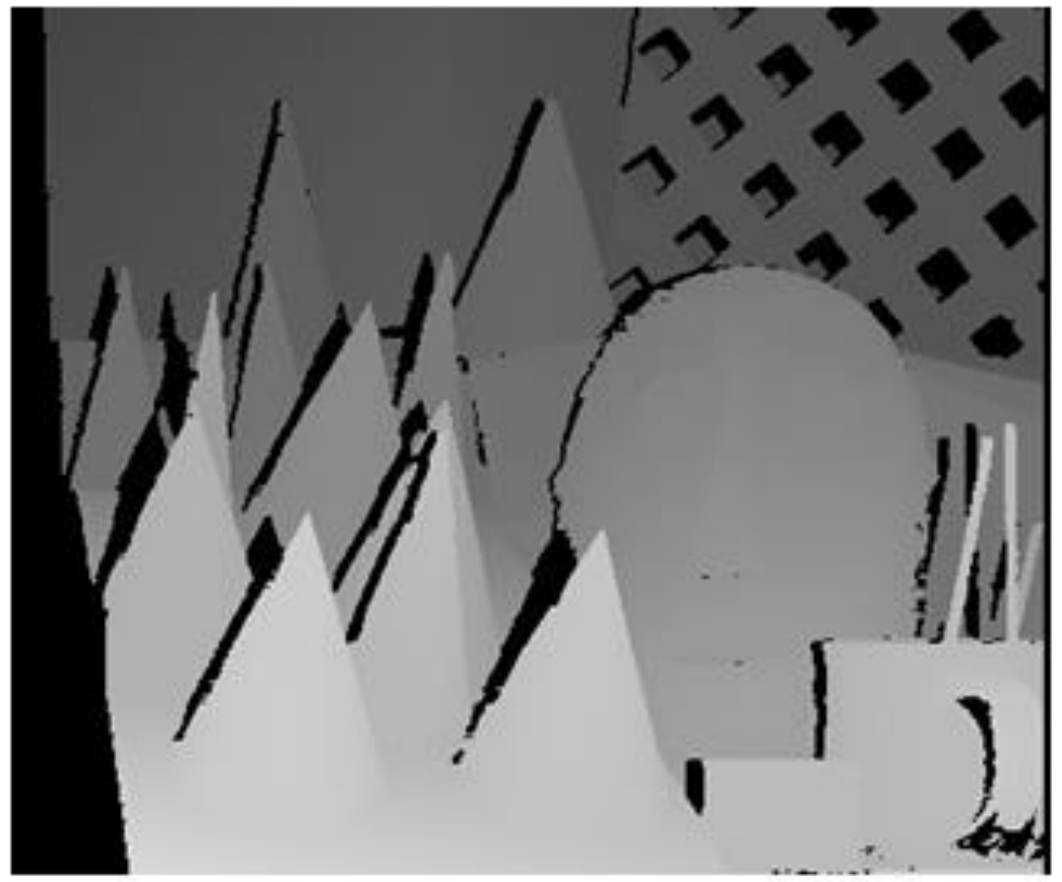
# 视差图

http://vision.middlebury.edu/stereo/




$$p_u - p'_u = \frac{B \cdot f}{z}$$

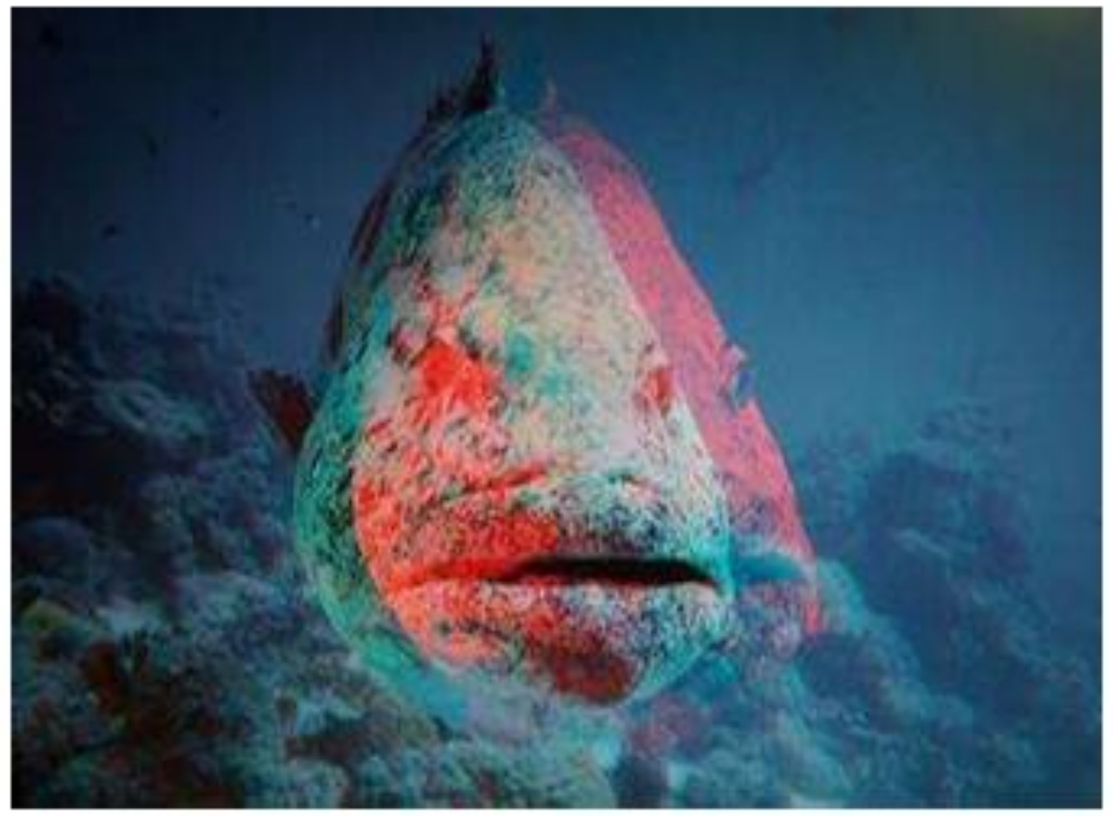
立体图像像对



视差图 / 深 度 图

# 视差原理应用

3D电影！



$$p_u - p'_u = \frac{B \cdot f}{z}$$

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- 对应点搜索