TEST OF BELL AND MERMIN INEQUALITIS ON QUANTUM COMPUTER

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ABSTRACT

The tests of Bell and Mermin's inequalities are carried out on the 2-qubit and 3-qubit IBM quantum computer respectively. With the indications of clear violations of the two inequalities, the experimental results validate postulates in quantum mechanics and disprove the local realism argument in classical hidden variable theories proposed by Eistein, Podolsky, and Rosen.

1 Introduction

Quantum computation has developed more rapidly recently. In some certain problems, the computational speed of quantum machine far exceeds that of classical machines. As the mechanism of quantum computation is based on quantum mechanics, by demonstrating whether certain properties in quantum mechanics are concordant with the operations of quantum computation, we can investigate the current maturity of quantum computing. IBM released a 5-qubit universal quantum computer prototype accessible on the cloud in 2016 based on superconducting qubits: the IBM Quantum Experience. [1] It gives us a tool to conduct experiments and evaluate quantum computation.

In 1935, Albert Einstein, Boris Podolsky, and Nathan Rosen (EPR), argued that the description of physical reality provided by quantum mechanics was incomplete while they are trying to explain the phenomenon of quantum entanglement; alternatively, they proposed that there exists a pre-determined value in particles before measurements (realism) and that actions on one particle must travel through space-time to affect the other (locality). In response to the EPR's conclusion, J.S.Bell reexamined the assumption made in the hidden variable theory and derived the Bell's inequality accordingly in 1964. However, it turns out that the inequality violates physical observations that are proved by quantum mechanic. Thereby, Bell showed that the phenomenon of quantum entanglement is non-local and quantum mechanics is still valid. For further extension of Bell's inequality, Mermin's inequality was proposed for n-particle pure quantum system($n > 2$) and found to be violated for particles in GHZ states. This demonstrates a further proof of non-locality theorem in quantum mechanics and rules out the local realism argument proposed by EPR.

In our project, we code 2-bit and 3-bit entangled states on IBMQ experience to verify whether Bell's and Mermin's inequalities accord with the experimental results we obtained. Thereby, we can show whether laws of quantum mechanics apply to quantum computation. We structure this paper by discussing Bell's inequality and Mermin's inequality separately; within each discussion we will first reexamine the mathematical derivation of both inequalities, and then describe the quantum circuits we designed accordingly; finally, by analyzing the data we obtained through the circuits, we evaluate whether the two inequalities are valid in quantum computing.

2 Bell's Inequality

Local realism can be examined using Bell's inequality. To evaluate Bell's inequality in quantum computing, its mathematical derivation indicates its physical meaning, and gives us a direction for circuit design.

Theorem 1 (Bell's inequality)

$$C_h(a,c) - C_h(b,a) - C_h(b,c) \leq 1 \tag{1}$$

Where a, b and c refer to three arbitrary settings of the two analysers. The $C_h(a,c)$ is the correlation of the a and c settings, similarly, $C_h(b,a)$ and $C_h(b,c)$.

The Bell's inequality is only satisfied in classical mechanics, but not in quantum mechanics. It means the principle of locality presented by Einstein is not consistent with the predictions in quantum mechanics.

Theorem 2 (Principle of locality) *An object is directly influenced only by its immediate surrounding.*

In other words, this principle indicates that there doesn't exist the phenomenon of "action at a distance".

Definition 1 (Action at a distance) *An object can be moved, changed, or otherwise affected without being physically touched by another object.*

Proof 1 (Bell inequality) *[3] For simplicity, we derive Bell's inequality within the frame of the model that incorporates the essential features of various alternative theories, which agrees that it is possible to determine S_x and S_z simultaneously. Without loss of generality. We can choose some certain particles such that they have the following properties:*

If S_z is measured, we obtain a plus sign with certainty.

If S_x is measured, we obtain a minus sign with certainty.

We can denote them as $(\widehat{z}_+, \widehat{x}_-)$. However, it is worth noting that we are not asserting we can simultaneously measure S_z and S_x.

To satisfy conservation of angular momentum, if particle 1 is $(\widehat{z}_+, \widehat{x}_-)$, the particle 2 is $(\widehat{z}_-, \widehat{x}_+)$, such that the total angular momentum is zero.

Hence, if the observer A decides to measure S_z of particle 1, it can get $+$, in spite of whether B decides to measure S_z or S_x. In other words, the result of measurement on particle 1 is independent of the direction on which particle 2 is measured. It shows that the principle of locality is incorporated in this model.

So far, the principle of locality is consistent with quantum mechanical predictions. We now consider more complicated situation that has three unit vectors $\widehat{a}$, $\widehat{b}$, and $\widehat{c}$, which, in general, are not mutually orthogonal.

In the same way, we define notations to describe the results of states one obtains after measurement. For example, we denote as $(\widehat{a}_+, \widehat{b}_+, \widehat{c}_-)$, if observer A measures the particle1's $\widehat{a}$ to be plus, $\widehat{b}$ being plus, and $\widehat{c}$ being minus; therefore, the corresponding particle2 is $(\widehat{a}_-, \widehat{b}_-, \widehat{c}_+)$. The same logic applies the other 7 types.

In any given event, the particle pair in question must be a member of one of the 8 types shown in Table 1, these 8 possibilities are exclusive and adjoint with each other. The population of each type is indicated in the first column.

Now, we calculate probability of obtaining both $\widehat{a}$ and $\widehat{b}$ as plus. Apparently,

$$P(\widehat{a}+, \widehat{b}+) = \frac{N_3 + N_4}{\sum_i^8 N_i}, P(\widehat{a}+, \widehat{c}+) = \frac{N_2 + N_4}{\sum_i^8 N_i}, P(\widehat{c}+, \widehat{b}+) = \frac{N_3 + N_7}{\sum_i^8 N_i} \tag{2}$$

Because

$$N_3 + N_4 \leq (N_2 + N_4) + (N_3 + N_7), \tag{3}$$

Population	Particle 1	Particle 2
N_1	$(\widehat{a}+, \widehat{b}+, \widehat{c}+)$	$(\widehat{a}-, \widehat{b}-, \widehat{c}-)$
N_2	$(\widehat{a}+, \widehat{b}+, \widehat{c}-)$	$(\widehat{a}-, \widehat{b}-, \widehat{c}+)$
N_3	$(\widehat{a}+, \widehat{b}-, \widehat{c}+)$	$(\widehat{a}-, \widehat{b}+, \widehat{c}-)$
N_4	$(\widehat{a}+, \widehat{b}-, \widehat{c}-)$	$(\widehat{a}-, \widehat{b}+, \widehat{c}+)$
N_5	$(\widehat{a}-, \widehat{b}+, \widehat{c}+)$	$(\widehat{a}+, \widehat{b}-, \widehat{c}-)$
N_6	$(\widehat{a}-, \widehat{b}+, \widehat{c}-)$	$(\widehat{a}+, \widehat{b}-, \widehat{c}+)$
N_7	$(\widehat{a}-, \widehat{b}-, \widehat{c}+)$	$(\widehat{a}+, \widehat{b}+, \widehat{c}-)$
N_8	$(\widehat{a}-, \widehat{b}-, \widehat{c}-)$	$(\widehat{a}+, \widehat{b}+, \widehat{c}+)$

Table 1: Spin=component matching in the alternative theories

We obtain

$$P(\widehat{a}+, \widehat{b}+) \leq P(\widehat{a}+, \widehat{c}+) + P(\widehat{c}+, \widehat{b}+) \tag{4}$$

Now the Bell's inequality is proved.

2.1 The incompatibility of Bell's inequality and quantum mechanics

Now, we consider the incompatibility of Bell's inequality in quantum mechanics.

In quantum mechanics, we can't know how many particles are in a certain state as the particles' states are probabilistic before measurements. Hence,we can't use expression $\frac{N_i}{N_{all}}$ to describe the probability of particle in state i. Instead, we characterize all the particles' states by ket in Dirac's notation.

First, we evaluate $P(\widehat{a}+, \widehat{b}+)$.

Suppose particle 1's $\widehat{a}$ is $+$. Obviously, the particle 2's $\widehat{b}$ is $-$ by considering the conservation of angular momentum. However, in order to calculate the probability that $\widehat{a}$ and $\widehat{b}$ are both positive, we must find a new vector $\widehat{b}$ that is different from $\widehat{a}$ and forms an angle andθ_{ab} with $\widehat{a}$.

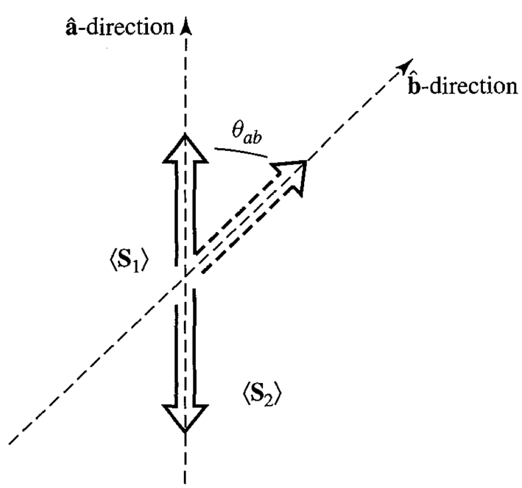


Figure 1: Evaluation of $P(\widehat{a}+, \widehat{b}+)$

The probability of $P(\widehat{a}+, \widehat{b}+)$ is given by: [3]

$$\frac{1}{2}cos^2[\frac{\pi - \theta_{ab}}{2}] = \frac{1}{2}sin^2(\frac{\theta_{ab}}{2}) \tag{5}$$

In the same way, we can obtain $P(\widehat{a}+, \widehat{c}+)$ and $P(\widehat{c}+, \widehat{b}+)$ respectively, where $\widehat{c}$ is a new vector distinct from $\widehat{a}$ and $\widehat{b}$. We can rewrite the Bell's inequality under principles of quantum mechanics:

$$sin^2(\frac{\theta_{ab}}{2}) \leq sin^2(\frac{\theta_{ac}}{2}) + sin^2(\frac{\theta_{ca}}{2}) \tag{6}$$

where θac is an angle between direction $\widehat{a}$ and direction $\widehat{c}$, and similarly θcb is the angle between $\widehat{c}$ and $\widehat{b}$.

Theorem 3 (The incompatibility of Bell's inequality and quantum mechanics) *Bell's inequality is NOT always satisfied in quantum mechanics.*

We can show it with a simple example.

Example 1 (Incompatibility) *It's simple to choose $\widehat{a}$, $\widehat{b}$, and $\widehat{c}$ to lie in a plane, and $\widehat{c}$ bisects the two directions defined by $\widehat{a}$ and $\widehat{b}$:*

$$\theta ab = 2\theta, \theta_{ac} = \theta_{cb} = \theta. \tag{7}$$

The inequality becomes

$$sin^2\theta \leq 2sin^2(\frac{\theta}{2}). \tag{8}$$

Then, using Mathematica, we plot the two functions in (8).

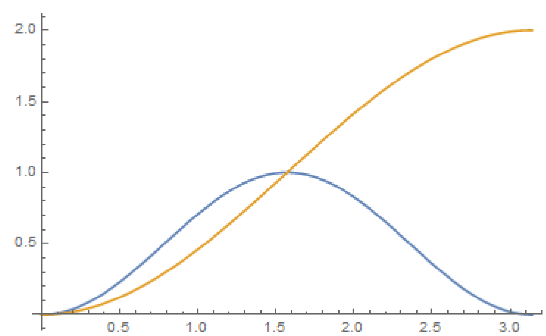


Figure 2: $sin^2\theta(blue), 2sin^2\frac{\theta}{2}(yellow) - \theta$

Therefore, we can show that the inequality is violated when

$$0 < \theta < \frac{\pi}{2} \tag{9}$$

2.2 Circuit implementation

In order to test whether Bell's inequality is valid in quantum computers, we construct the following circuits shown in Fig.3.

In the test of the inequality, a pair of Bell's states is needed to be created.

$$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{10}$$

This is done by applying a Hadamard gate and a CNOT gate, which creates entangled Qubits.

$$C_{12}H_1 |00\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) \tag{11}$$

Then, $|01\rangle$ and $|10\rangle$ states are created by applying NOT gate on the second Qubit.

$$X_2\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle) = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) \tag{12}$$

Afterwards, by applying the Z gate, the plus sign is flipped to be minus.

$$Z_2 \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle) = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) \tag{13}$$

Finally, to test the prediction from quantum mechanics that the relationship between the resultant probability of a state and the angle at which it is rotated:

$$P(\widehat{a}+, \widehat{b}+) = \frac{1}{2}\sin^2(\frac{\theta_{ab}}{2}) \tag{14}$$

we can verify this by applying a rotation gate around the Y axis:

$$Ry_2 \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle) = \frac{1}{\sqrt{2}}(\cos\frac{\theta}{2}|01\rangle + \sin\frac{\theta}{2}|00\rangle + \sin\frac{\theta}{2}|11\rangle - cos\frac{\theta}{2}|10\rangle) \tag{15}$$

By measuring the probability of the states $|00\rangle$ or $|11\rangle$, one can test the quantum mechanical relationship in (14)

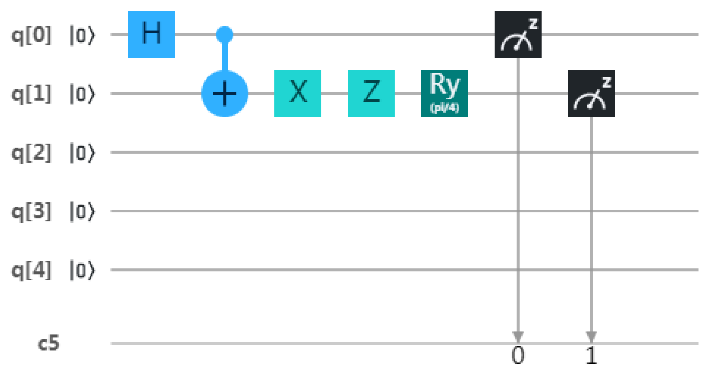


Figure 3: Circuit Diagram for Bell's inequality

2.3 Result

We implement our circuit through the IBM Quantum Experience website in order to perform our experiment. We run the circuit using various angles from 0 to π, and record the probability of getting the $|00\rangle$ state and the $|11\rangle$ state for each rotation. In order to obtain the probabilities, we run the circuit 8192 times for each angle.

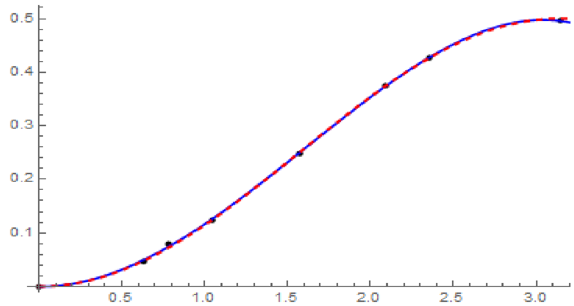
The experimental result for the state $|00\rangle$ is shown in Fig.4a. From the figure, the red dashed line stands for the function $\frac{1}{2}sin^2(\frac{\theta}{2})$, the blue dots are the data points and the blue line is the fitted curve.

The experimental result for the state $|11\rangle$ is shown in Fig.4b. The same notation as Fig.4a is used here.

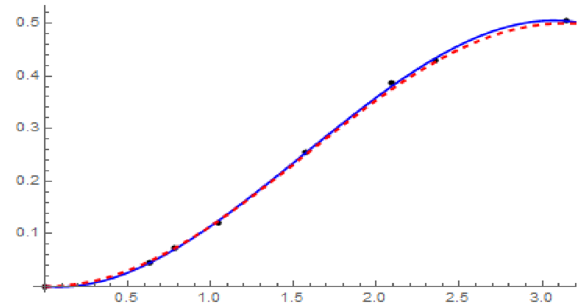
From the fitted figures we can see that the result is a pretty good fit to the quantum mechanical prediction. Using this result, we can show that equation(4) indeed implies equation(6) under quantum mechanics.

3 GHZ experiment

Another way of examining local realism is through the GHZ experiment. Greenberger-Horne-Zeilinger State, also known as GHZ state, is an entangled quantum state which includes at least three subsystem. GHZ state was mainly used to contradict the predictions from local hidden variable. Instead of using only two entangled particles like the test of Bell's inequality, GHZ experiment utilizes 3 entangled particles to clearly show the absolute contradiction between the predictions of local hidden variable theory, while the test of Bell's inequality shows only contradiction of statistical nature. With a better model compared to Bell's inequality, GHZ experiment also demonstrates the non-locality of quantum entanglement phenomenon.



(a) θ vs. $\frac{1}{2}sin^2(\frac{\theta}{2})$ for state $|00\rangle$



(b) θ vs. $\frac{1}{2}sin^2(\frac{\theta}{2})$ for state $|11\rangle$

Figure 4: Fitted Result

3.1 Mathematical derivation

The situation described below is a very similar version of the Bell's theorem. Instead of the 2-Qbit state that the bell's theorem uses, the GHZ experiment uses a 3-Qbit state to illustrate a similar result. This 3-Qbit version of the bell's theorem was first discovered by Daniel Greenberger, Michael Horne, and Anton Zeilinger in the late 1980s. [2]

Consider an entangled state consisting of three Qbits. Numbering the Qbits from left to right 2, 1, and 0.

$$\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle) \tag{16}$$

The entangled state described above can be obtained by doing the following operations on a standard state $|0\rangle_3$.

$$C_{21}C_{20}H_2X_2\,|000\rangle = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle) \tag{17}$$

Now we can obtain a special 3-Qbit state that will be used in the argument.

$$|\Psi\rangle = C_{21}H_2X_2\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle) \tag{18}$$

$$= \frac{1}{2}(|000\rangle - |110\rangle - |011\rangle - |101\rangle) \tag{19}$$

The state $|\Psi\rangle$ is special in the sense that it is invariant under permutations of the three Qbits. This means that other forms of $|\Psi\rangle$ can be obtain by permuting the subscripts 0, 1, and 2 of the operations performed in equation (18). In particular

$$|\Psi\rangle = C_{12}H_1X_1\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle) \tag{20}$$

It follows from equation (20) that

$$\begin{aligned} H_2H_1\,|\Psi\rangle &= H_2H_1C_{12}H_1X_1\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle) \\ &= (H_2H_1C_{12}H_1H_2)H_2X_1\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle) \\ &= C_{21}H_2X_1\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle) \end{aligned} \tag{21}$$

This is true because sandwiching a cNOT gate between four Hadamard gates is equivalent to a cNOT gate in reverse. Combining with the fact that $|\Psi\rangle$ is invariant under permutation of the Qbits, comparing equations (18) and (21) reveals that

$$H_2H_1\,|\Psi\rangle = Z_2X_1\,|\Psi\rangle \tag{22}$$
$$H_2H_0\,|\Psi\rangle = Z_2X_0\,|\Psi\rangle \tag{23}$$
$$H_1H_0\,|\Psi\rangle = Z_1X_0\,|\Psi\rangle \tag{24}$$

Now suppose we have three Qbits prepared in the state $|\Psi\rangle$. Without applying any operations to the state, we measure the state of each individual Qbit. Just by inspecting equation (19), we can see that we will get either none or two out of three of the Qbits in the state $|1\rangle$. Since the three Qbits are related in this way, there is a simple relation that demonstrates this result.

$$x_2 \oplus x_1 \oplus x_0 = 0 \tag{25}$$

Suppose instead of measuring the Qbits right after the initial preparation, we measure the Qbits after we apply the Hadamard gates to Qbits 1 and 2. According to equation (22), this operation has the same effect as flipping Qbit 1 in each term of the superposition in equation (19). We will now get a state with either one or all three of the Qbits in the state $|1\rangle$. Similarly, there is another relation that demonstrates the result.

$$x_2^H \oplus x_1^H \oplus x_0 = 1 \tag{26}$$

But there is nothing special about the Qbit 0 that we decided to isolate, we can repeat this process on Qbits 1 and 2 to obtain two additional equations.

$$x_2^H \oplus x_1 \oplus x_0^H = 1 \tag{27}$$

$$x_2 \oplus x_1^H \oplus x_0^H = 1 \tag{28}$$

Now an interesting argument can be made from the equations shown above. If one believes in any type of local hidden variable theories that suggests predetermined measurement outcome, one would agree that the measurement outcome of any one specific Qbit is independent of operation applied (before or after the measurement) on the other two Qbits. In other words, if predetermined measurement outcome holds, x_0 from equations (25) and (26) are identical. Same for x_1 from equations (25) and (27), and x_2 from equations (25) and (28). Following the same line of thought, we can show that the x_0^H, x_1^H, and x_2^H are the same in those equations.

We can then add up the left hand side of equations (25) - (28)

$$2 * (x_2 \oplus x_1 \oplus x_0 \oplus x_2^H \oplus x_1^H \oplus x_0^H) \tag{29}$$

Since each variable appeared exactly twice in the equation, the result of addition in modulo 2 will be 0. But on the contrary, when we add up the right hand side of equations (25) - (28), the result is $0 \oplus 1 \oplus 1 \oplus 1 = 1$. We can see that local hidden variable theories completely disagree with Quantum mechanical result. This showed theories that support predetermined measurement outcomes cannot be correct.

3.2 Circuit implementation

In order to show the non-locality property through the GHZ experiment, the following are the steps of how we use the quantum circuit to prove it.

Step 1: We created an entangled state consisting 3 Qbits by applying a Hadamard gate to the 0 Qbit and two Control Not gates which are C_{12} and C_{23}, respectively–$C_{23}C_{12}H_2\,|000\rangle$. And added a barrier behind them in order to make this entangled state as a whole when adding the other gates in the following steps.

By doing this, the probability becomes

$$\frac{1}{\sqrt{2}}(|000\rangle + |111\rangle) \tag{30}$$

Step 2: Next, since we are considering using the 3-Qbit state

$$|\Psi\rangle = \frac{1}{2}(|000\rangle - |110\rangle - |011\rangle - |101\rangle) \tag{31}$$

Therefore, we firstly applied a Phase Flip gate Z_2. The equation became

$$\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle) \tag{32}$$

Because the form of $|\Psi\rangle$is invariant under any permutation of the three Qbits. Consequently, after having the $|111\rangle$ flipped upside down, we applied the Not gate X_2 on Qbit 2, a Hadamard gate H_2 on Qbit 2, and a cNot gate C_{12} in series. After the cNot gate, we added a barrier in order to avoid the latter steps disturbing the previously-made circuit. Therefore, we have

$$|\Psi\rangle = C_{21}H_2X_2\frac{1}{\sqrt{2}}(|000\rangle - |111\rangle) \tag{33}$$

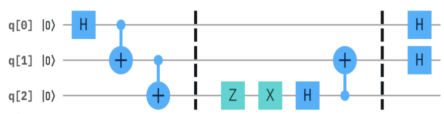
And after the cNot gate, we added a barrier in order to avoid the latter steps disturb the previously made state.

Step 3: To the point, we have finished the steps of preparing the testing state. If measure the prepared three Qbits, which is a superposition of computational-basis states, we will get that each state having none or two of the Qbits in state$|1\rangle$. Therefore, the following equation can be satisfied

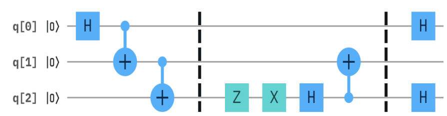
$$x_2 \oplus x_1 \oplus x_0 = 0 \tag{34}$$

(where $\oplus$ stands for addition by modulo 2)

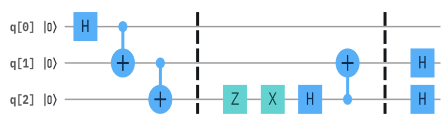
Then according to the mathematical Derivation in the previous sections, if we apply the Hadamard gate to two of the three Qbits at the same time before measuring all three Qbits, we would get a change of the previous state into a superposition of computational-basis states which having either one or three Qbits in state $|1\rangle$. (Equations 26 - 28)



(a) GHZ Circuit H_0H_1



(b) GHZ Circuit H_0H_2



(c) GHZ Circuit H_1H_2

Figure 5

Step 4: Eventually, we do addition by modulo 2 with the state vector on each occasions and the initial state which are

$$2x_0 \oplus 2x_1 \oplus 2x_2 \oplus 2x_0^H \oplus 2x_1^H \oplus 2x_2^H \tag{35}$$

3.3 Result

From circuit implementation and mathematical derivation of GHZ experiment, we are going to obtain 4 sets of probablity–circuit without Hadamard, Hadamard on Qbit 0 and 1, Hadamard on Qbit 0 and 2, Hadamard on Qbit 1 and 2.

Firstly, we ran the circuit without the Hadamard at the last part of the circuit. By looking at the state vectors, we would only obtain 4 states which are $|000\rangle$, $|011\rangle$, $|101\rangle$, $|110\rangle$. The probability diagram of this circuit is shown below

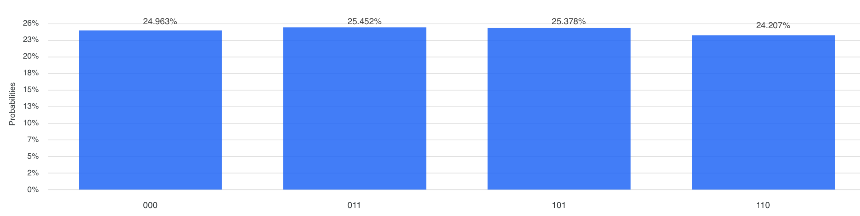


Figure 6: Probability result with no Hadamard

As previously mentioned in the circuit implementation, we did addition in modulo 2 for each state. The equation goes

$$x_2 \oplus x_1 \oplus x_0 = 0 \tag{36}$$

Since there are either 2 or 0 states in $|1\rangle$ for all four states, the result of doing addition in modulo 2 will always be 0.

Secondly, we applied Hadamard gate on Qbit 0 and 1. Seeing from the state vector, we would get four states which are $|001\rangle$, $|010\rangle$, $|100\rangle$, and $|111\rangle$. And the actual result run on the quantum computer is below.

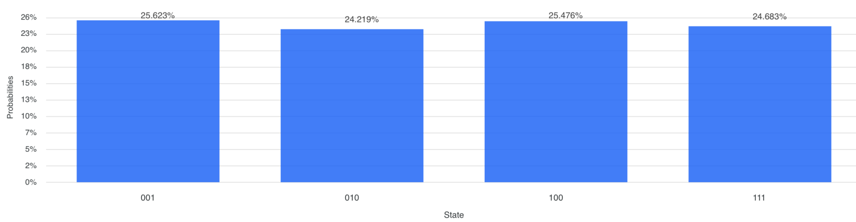


Figure 7: Probability result with H_0H_1

Similarly, we did addition by modulo 2 for each state. Since there are either 3 or 1 states in $|1\rangle$, the result would be 1 for all four states. The equation is shown below

$$x_2 \oplus x_1^H \oplus x_0^H = 1 \tag{37}$$

Thirdly, we applied Hadamard gate on Qbit 0 and 2. Theoretically, we are also going to get four states which are $|001\rangle$, $|010\rangle$, $|100\rangle$, and $|111\rangle$. And the probability diagram is below.

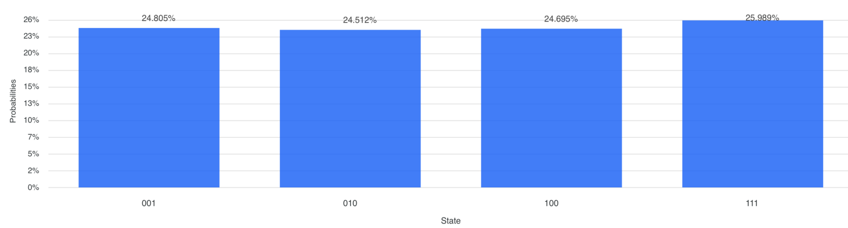


Figure 8: Probability result with H_0H_2

After doing addition in modulo 2 for each state, we still got 1 for the result of that addition because 1 or 3 states in $|1\rangle$. The equation is shown below

$$x_2^H \oplus x_1 \oplus x_0^H = 1 \tag{38}$$

Fourthly, we applied Hadamard gate on Qbit 1 and 2. Theoretically, we are also going to get four states which are $|001\rangle$, $|010\rangle$, $|100\rangle$, $|111\rangle$. And the probability diagram is below.

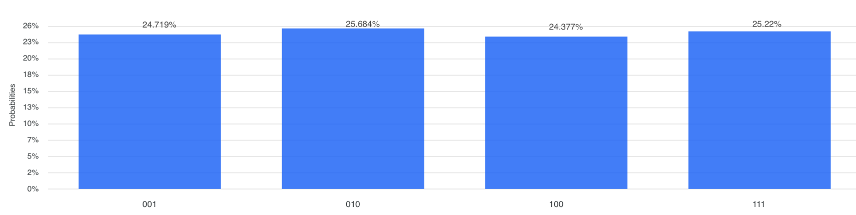


Figure 9: Probability result with H_1H_2

After doing addition in modulo 2 for each state, we are going to get 1 as the result since the number of states in $|1\rangle$ is either 1 or 3. And the equation is below

$$x_2^H \oplus x_1^H \oplus x_0 = 1 \tag{39}$$

So far, we got four set of results shown below

$$x_2 \oplus x_1 \oplus x_0 = 0 \tag{40}$$

$$x_2 \oplus x_1^H \oplus x_0^H = 1 \tag{41}$$

$$x_2^H \oplus x_1 \oplus x_0^H = 1 \tag{42}$$

$$x_2^H \oplus x_1^H \oplus x_0 = 1 \tag{43}$$

As mentioned in the circuit implementation, we are going to do addition in modulo 2 for both left hand side and right hand side of the equations above separately. And these two results will determine whether quantum mechanics is right, or wrong. If the result of LHS(abbreviation for "Left hand side") equals to that of RHS(abbreviation for "Right hand side"), then it proves the local hidden variable and prove that quantum mechanics is wrong. Oppositely, if they do not equal to each other, then quantum mechanics is right.

For LHS

$$2 * (x_2 \oplus x_1 \oplus x_0 \oplus x_2^H \oplus x_1^H \oplus x_0^H) = 0 \tag{44}$$

And for RHS

$$0 \oplus 1 \oplus 1 \oplus 1 = 1 \tag{45}$$

Apparently, $0 \neq 1$; LHS$\neq$RHS. Finally, the result proved that quantum mechanics is right while hidden variable theory is wrong.

4 Conclusion and discussion

In this paper, we have investigated the EPR paradox through two experiments that proves the validity of bell's inequality. From the two-Qbit experiment, we can see that the probability of getting a certain state of the entangled two-particle system can be predicted using quantum mechanics. Combing with Bell's inequality, we can show that the quantum mechanical prediction violates the inequality that the local hidden variable theories have to obey. This implies that quantum physics is incompatible with local hidden variable theories.

In the three-Qbit experiment, we used the GHZ's three-Qbit state to show a similar result. We see a totally uncorrelated result from local predetermined theories and quantum physics. And with so much experimental result supporting quantum mechanics, we can be safe to say that local hidden variable theories are incorrect.

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