

Canonical Document Header (Informative / Non-Normative Metadata)

- Document ID: SC-AXIOM-000001
- Canonical Name: THE COHERENCE AXIOMS SPECIFICATION
- Document Class: SPEC
- Version: 1.0.0
- Issuance ID: v11l
- Release Status: ISSUED
- Effective Date: 2026-03-04
- Last Updated: 2026-03-19
- Issuing Authority: Coherence Research
- DOI / Persistent ID: 10.5281/zenodo.18043770
- Supersedes: SC-AXIOM-000001 (Issuance ID: v11k)
- Superseded By:
- Header Schema: SC-HDR-000001 (Issuance ID: v11)
- Integrity:
 - SHA-256: 05369abb763240e0a5689cd387706784c55fce98c0b9bc29338172b86ed891cc
 - Release Tag: 2026-03-19__v11l

Document Class Extension: SPEC (Informative / Non-Normative Metadata)

SPEC::Header-Schema-Version: 0.4

SPEC::Standard: Structural Coherence

SPEC::Spec-Subclass: axioms-spec

SPEC::Revisability: Versioned Only

SPEC::Canonical-Dependencies: SC-HDR-000001 (v11), SC-CORE-000001 (v39d)

Typed Structural Abbreviations (Meta-Structural; No New Primitives)

Expansion discipline (binding use). Any abbreviation introduced in this section (e.g., carrier $|X|$, restriction $X|_S$, outcome sets, quotient notation, preorders) **MUST NOT** acquire constitutive force by abbreviation alone. If an abbreviation is used inside any binding clause of the suite, it **MUST** be explicitly expandable into Core-admitted primitives / Kernel Terms, or else it **MUST** be treated as non-normative commentary.

Non-normative note: This section introduces abbreviations only. It adds no new primitives and no new binding constraints.

Predicate Expansion Register (Meta-Structural · Binding)

Purpose (Binding) This register makes every formal predicate used in canonical axiom statements explicitly expandable into Core-defined terms and admitted apparatus, preventing semantic import.

Predicate definitions (expandable abbreviations)

Adj

- Classification: abbrev
- Expansion: $\text{Adj}(X) := \text{Adjacency relation Adj}(X)$ as given per configuration (Core D1.1).
- Anchors: D1.1 Adjacency, D1.0 Configuration

Availability

- Classification: abbrev
- Expansion: Any binding use of “availability” / “available” in this Axioms document is shorthand for Availability as defined in the Core (D0.25): existence of a Core-admissible carrier witnessing the required continuation/redistribution/emergence condition under declared scope, with satisfaction/enforceability governed by D0.16/D0.17.
- Anchors: D0.25 Availability, D0.16 Constraint Satisfaction, D0.17 Enforceability, VE6 Admissible Pathway Existence, AR5 Pathway Admissibility, F6 Redistribution Support, F9 Degrees of Freedom for Redistribution

Persistence

- Classification: abbrev
- Expansion: Any binding use of “persistence” / “persists” / “coherence persistence” denotes Core-internal persistence under admissible recursion: there exists a Core-admissible recursion \mathcal{R} (D1.10) such that \mathcal{R} is enforceable (D0.17) and for every pathway $P \in \mathcal{R}$ and every intermediate configuration Z encountered along P , Z is Core-admissible (D0.1) and Z satisfies bounded coherence enforcement (AR4).
- Anchors: D1.10 Recursion, D0.17 Enforceability, D0.1 Core Admissibility, AR4 Bounded Coherence Enforcement

Out

- Classification: abbrev (meta-structural)
- Expansion: $\text{Out}(X)$ abbreviates the outcome set of Core-admissible redistribution results reachable from X : $\text{Out}(X) := \{ Y \mid \exists R: X \rightarrow Y \text{ such that } R \text{ is a redistribution (D1.5) that is enforceable (D0.17) and admissible under the redistribution admissibility constraints (AR1–AR3), with all intermediate configurations Core-admissible (D0.1) as required by those constraints } \}$.
- Anchors: D1.5 Redistribution, D0.17 Enforceability, D0.1 Core Admissibility, AR1 Redistribution Locality, AR2 Redistribution Preservation, AR3 Redistribution Boundedness

Driver-Resolution Preorder

- Classification: abbrev (meta-structural)
- Expansion: The preorder symbols “ \preceq_D ” and “ \prec_D ” are abbreviations used only for driver-resolution comparison via admissible redistribution outcomes: $D(A) \preceq_D D(B) :<=> \text{Out}(A) \neq \emptyset \wedge \text{Out}(B) \neq \emptyset \wedge (\forall B' \in \text{Out}(B) \exists A' \in \text{Out}(A) \text{ such that } \mathfrak{C}(A') \leq \mathfrak{C}(B'))$, where $\mathfrak{C}(\cdot)$ is expressive capacity (D1.3) and “ \leq ” is used only as the ordering interface defined there. Strictness is defined by $D(A) \prec_D D(B) :<=> D(A) \preceq_D D(B) \wedge \neg(D(B) \preceq_D D(A))$.
- Anchors: Out (this register), D1.3 Expressive Capacity, D1.5 Redistribution, AR1–AR3 Redistribution Admissibility

Gradedness

- Classification: governance (abbrev)
- Expansion: Any binding use of “graded” / “gradation” / “more or less coherent” MUST be expressed as an induced structural characterization using only Core-admitted forms:
 - (1) Ordering-only comparison: an explicit preorder/partial-order comparison over Core-typed objects (e.g., Expressive Capacity ordering (Core D1.3) or the Non-Divergence Preorder (Core D0.11)) as applicable; and/or
 - (2) Nesting / contraction: a regime-indexed admissible-set contraction statement using Regime Indexing and Nesting (Core D0.13) and Admissible-Set Contraction Under Nesting (Core D0.14); and/or
 - (3) Availability witnesses: existence/plurality of Core-admissible continuations/pathways/redistribution as witnessed by applicable availability constraints (e.g., Redistribution Support (Core F6) and Degrees of Freedom for Redistribution (Core F9)).

This expansion binds gradedness language to induced order, nesting contraction, and availability witness structure already governed by the Core.

- Anchors: Core D1.3 Expressive Capacity, Core D0.11 Non-Divergence Preorder, Core D0.13 Regime Indexing and Nesting, Core D0.14 Admissible-Set Contraction Under Nesting, Core F6 Redistribution Support, Core F9 Degrees of Freedom for Redistribution

Boundary

- Classification: abbrev
- Expansion: Boundary distinction (L vs N) in binding use is governed by Distinction Scope Closure (F14); any coupled-scope reliance must be representable via an interface carrier I satisfying Interface Representation (D0.29).
- Anchors: F14 Distinction Scope Closure, D0.29 Interface Representation

Coherent

- Classification: abbrev
- Expansion: Coherent(X) := X is Core-admissible (D0.1) AND (when invoked) satisfies bounded coherence enforcement (AR4) / coherence preservation under emergence (VE8) where scope applies.
- Anchors: D0.1 Core Admissibility Characterization, AR4 Bounded Coherence Enforcement, VE8 Coherence Preservation Under Emergence

Distinguish

- Classification: abbrev
- Expansion: Distinguish(L,N,E) is a distinction used with constraining force exactly when it triggers F14’s scope-posture requirement; threshold governance may be referenced via Differentiation Threshold (D0.22) if needed.
- Anchors: F14 Distinction Scope Closure, D0.22 Differentiation Threshold

MetaAdmissible

- Classification: abbrev
- Expansion: MetaAdmissible(S) := S is Canonical under the Core’s canonical material criterion (D0.7) and conforms under AR11 (Core conformance characterization).
- Anchors: D0.7 Canonical Material Criterion, AR11 Core Conformance Characterization

Minimal

- Classification: abbrev
- Expansion: $\text{Minimal}(T) := T$ satisfies the Term Schema minimality discipline (T5) and/or the global Minimal Sufficiency condition (F13) when ranging over kernel elements.
- Anchors: T5 Minimality / Non-Redundancy, F13 Minimal Sufficiency

Nameable

- Classification: meta-linguistic
- Expansion: Nameable is meta-linguistic; object-level nameability is already guaranteed by primitives ('formally nameable structural object') in D1.0/D1.7/D1.9. Avoid using Nameable as a binding predicate.
- Anchors: D1.0 Configuration, D1.7 Transformation, D1.9 Pathway

Null

- Classification: abbrev
- Expansion: $\text{Nullified}(C,E) :=$ 'C has loss of enforceability at E' per Enforceability (D0.17) and/or 'is nullified' per Nullification (D0.9) where appropriate.
- Note: "Null" (without arguments) is reserved as a meta-level designation (Axiom 0); $\text{Nullified}(C,E)$ is the configuration-level abbreviation.
- Anchors: D0.17 Enforceability (Canonical), D0.9 Nullification (Constraint Defeat)

Persistent

- Classification: meta-linguistic
- Expansion: Persistent is treated as non-constitutive prose unless explicitly expanded. When needed, expand to Conservation Through Transformation (F8) + pathway/recursion identity (D0.3) claims.
- Anchors: F8 Conservation Through Transformation, D0.3 Pathway Identity and Distinctness

Refer

- Classification: meta-linguistic
- Expansion: Refer is meta-linguistic and must not appear as an object-level predicate in binding math. Use explicit dependency references (T3/T3b) instead.
- Anchors: T3 Dependencies (Definitional), T3b Audit / Reflective References (Non-definitional)

Resolve

- Classification: abbrev
- Expansion: $\text{Resolve}(D(X),R)$ is used only as shorthand for the redistribution primitive: R is a redistribution that resolves at least one admissible driver (Core D1.5) for configuration X.
- Anchors: D1.5 Redistribution, D1.4 Driver (Structural Differential)

Terminal

- Classification: abbrev

- Expansion: $\text{Terminal}(X)$ is not a primitive; use Nonterminality rules instead. Replace $\text{Terminal}(X)$ with the negation-free witness form: ‘if X has an unresolved admissible driver then there exists an admissible redistribution to a non-equivalent configuration’ (AR9/AR10).
- Anchors: AR9 Nonterminality Under an Admissible Driver, AR10 Bounded Nonterminal Emergence

Interaction

- Classification: abbrev
- Expansion: Any binding use of $I : A \leftrightarrow C$ abbreviates the existence of an admissible redistribution $R : \Psi \rightarrow \Psi'$ (D1.5) for some configuration Ψ such that $A = \Psi|_S$ and $C = \Psi|_T$ for some $S, T \subseteq |\Psi|$, and there exists a boundary set $B \subseteq |\Psi|$ satisfying III.6 with cross-boundary propagation constrained to adjacency-compatible pathways (F11); the bidirectionality denotes that the redistribution spans the boundary distinction (F14) as a single coupled redistribution event rather than two independent transformation events.
- Anchors: D1.5 Redistribution, D1.7 Transformation, D1.9 Pathway, D1.1 Adjacency, F11 Locality-Compatible Propagation, F14 Distinction Scope Closure, III.6 Definition of Boundary

SelectionOrdering

- Classification: abbrev
- Expansion: $\text{SelectionOrdering}(\Psi_a, \Psi_b)$ iff there exists a Core-admissible recursion \mathcal{R} (D1.10) satisfying AR1–AR8 such that $(R_k \circ \dots \circ R_1)(\Psi_b) \sim \Psi_a$ for some $k \geq 1$ under F8 and D0.3, and no such $k \geq 1$ exists for Ψ_a under the same \mathcal{R} .
- Note: The infix notation “ $\Psi_a \prec \Psi_b$ ” is treated as a purely typographical shorthand for $\text{SelectionOrdering}(\Psi_a, \Psi_b)$ and carries no independent primitive status.
- Anchors: D1.10 Recursion, D1.6 Structural Equivalence, AR1 Continuity Enforcement, AR2 Adjacency Preservation, AR3 Locality Enforcement, AR4 Bounded Coherence Enforcement, AR5 Admissible Pathway Enforcement, AR6 Emergence Non-Equivalence Enforcement, AR7 Expressive Capacity Monotonicity, AR8 Primitive Closure Enforcement, F8 Conservation Through Transformation, D0.3 Pathway Identity and Distinctness

Projection

- Classification: abbrev
- Expansion: $\pi(\Psi)$ denotes an admissible transformation $T : \Psi \rightarrow \Psi|_B$ (D1.7) where $B \subseteq |\Psi|$ is a boundary set satisfying III.6, $\Psi|_B$ is a configuration under D1.0 via the restriction abbreviation, T satisfies AR1–AR3 and $\text{Coherent}(\Psi|_B)$ (D0.1, AR4), and π is projection-stable iff $\pi(\Psi) \sim \pi(R(\Psi))$ under D1.6 for all admissible redistributions R satisfying AR1–AR8.
- Anchors: D1.7 Transformation, D1.5 Redistribution, D1.0 Configuration, D1.1 Adjacency, D1.6 Structural Equivalence, D0.1 Core Admissibility Characterization, AR1 Continuity Enforcement, AR2 Adjacency Preservation, AR3 Locality Enforcement, AR4 Bounded Coherence Enforcement, AR1–AR8 Admissibility Rules, III.6 Definition of Boundary

Carrier. For any configuration X , let $|X|$ denote the carrier set induced by $\text{Adj}(X)$:

$$|X| := \{u \mid (\exists v) \text{Adj}_X(u, v) \vee \text{Adj}_X(v, u)\}.$$

Restriction (induced subconfiguration). For any $S \subseteq |X|$, define $X|_S$ as the configuration whose carrier is S and whose adjacency is Adj_X restricted to $S \times S$.

Neighborhood. For any $x \in |X|$, define $\text{Nbr}_X(x) := \{y \in |X| \mid \text{Adj}_X(x, y) \vee \text{Adj}_X(y, x)\}$.

Local driver (abbreviation). For any $x \in |X|$, define $D_{\text{loc}}(X, x) := D(X|_{\text{Nbr}_X(x)})$.

These abbreviations make element/subset talk well-typed without treating configurations themselves as sets.

Distinction, Locality, and Adjacency (Governed by the Core Specification)

Non-normative note: Distinction-handling discipline is governed by the Core Specification. In particular, **F11 (Locality-Compatible Propagation)** constrains any influence/propagation semantics to adjacency-mediated neighborhoods, and **F14 (Distinction Scope Closure)** requires that any binding use of a distinction $L \mid N$ explicitly adopt either a **Closed-Scope** posture (no cross-cut import) or a **Coupled-Scope** posture with an explicit enforceable interface representation. This Axioms Specification introduces no additional distinction-locality-adjacency constraints beyond those Core conditions.

THE COHERENCE AXIOMS SPECIFICATION

0.1 Scope

Axiom 0 establishes the minimal structural precondition for any Core-admissible system. It does not assert ontology, existence, substance, or worldhood.

It establishes only what must be admissible for coherence, recursion, and derivation to be possible.

0.2 Meta-Level Designation: Null

Null is a meta-level designation for absolute non-coherence.

Null denotes a hypothetical condition with:

- no adjacency
- no distinguishability
- no continuity
- no recursion compatibility
- no expressive capacity

Null is not a domain element, state, object, or configuration.

0.3 Referential Admissibility Constraint (Meta-Theoretic)

Any act of *formal designation* of a token as denoting a configuration (or other object of the coherence object-language) presupposes minimal coherence of the denoted target.

Accordingly, whenever a binding clause introduces a symbol X as ranging over configurations (or as denoting a configuration), X is admissible only if $\text{Coherent}(X)$ holds under Core admissibility (D0.1, AR11).

This constraint is meta-theoretic: it restricts what may be introduced into a coherence-preserving object-language, without introducing any new object-level predicate beyond Core-admitted terms.

0.4 Exclusion of Null

Null is not admissible as an object-language term.

Accordingly, no object-language token may be introduced with Null as its denotation; any such introduction is inadmissible for a coherence-preserving formalism.

Null cannot appear as:

- a referent
- a configuration
- a ground condition
- an element of any coherent domain

Null is structurally inadmissible.

0.5 Admissibility of Minimal Coherent Configurability

All specification statements are articulated within a coherence-permitting context. Accordingly, this specification is written under a **structural non-emptiness presupposition**: at least one coherent configuration is admissible in the purely structural sense (i.e., the admissible class is non-empty). This is a suite-level well-formedness presupposition, not a derived Core-level existence claim.

Define:

$$P_0 := \text{a minimally admissible coherent configuration}$$

where P_0 :

- is non-null
- supports adjacency in the minimal admissible sense
- is recursion-compatible
- carries minimal expressive capacity
- introduces no further structure beyond minimal coherence

No claim of uniqueness, substance, or ontology is made.

Canonical Statement (Axiom 0)

Null is structurally inadmissible. Therefore the discourse presupposes a minimally coherent configuration P_0 as a non-vacuity witness.

Appendix 0 — Technical Lemmas and Corollaries (Derived)

This appendix lists structural consequences of Axiom 0. These are not part of the axiom; they follow from it under the binding constraints.

Appendix 0A — Lemmas

Lemma 0A.1 — Meta-Theoretic Coherence Condition for Reference

Any lemma about “reference” is interpreted meta-theoretically as a restriction on admissible *formal designation* within the coherence-preserving object-language.

In particular: if a binding development treats a symbol X as denoting a configuration, then X must satisfy Core admissibility (D0.1, AR11), i.e., $\text{Coherent}(X)$ must hold.

Reference is admissible only when the target is representable within a coherence-preserving formalism.

Lemma 0A.2 — Null Cannot Occur in Any Coherent Domain

Let D be any coherent domain (object-level). Then:

$$\text{Coherent}(D) \Rightarrow \text{Null} \notin D$$

Interpretation: since Null is not a domain term (Section 0.2), no coherent domain can contain an element corresponding to Null .

Lemma 0A.3 — Intelligibility Presupposes Coherence (Meta-Theoretic)

Let $\text{Nameable}(X)$ mean: “there exists a well-formed term in the current representational layer that denotes X .” Then:

$$\text{Nameable}(X) \Rightarrow \text{Coherent}(X)$$

This is a meta-level constraint about representability, not an object-level predicate on configurations.

Lemma 0A.4 — Pre-Intelligible Coherence Is Admissible (Meta-Theoretic)

There may exist coherent configurations not nameable in the current representational layer:

$$\text{Coherent}(X) \wedge \neg \text{Nameable}(X) \Rightarrow \text{MetaAdmissible}(X)$$

Interpretation: lack of nameability is a limitation of the language layer, not a limitation on coherence.

Appendix 0B — Corollaries**Corollary 0B.1 — No Coherent Theory Can Use Null as a Denoting Term**

Any object-language intended to represent coherent configurations must exclude a denoting constant for Null; introducing such a term violates Lemma 0A.1–0A.2.

Corollary 0B.2 — Distinction Requires Coherence

If $\text{Distinguish}(X, Y)$ denotes any admissible act of distinguishing X from Y , then:

$$\text{Distinguish}(X, Y) \Rightarrow \text{Coherent}(X) \wedge \text{Coherent}(Y)$$

Distinction is admissible only over coherent targets (meta-theoretic constraint).

Corollary 0B.3 — A Coherence-Permitting Context Implies Admissibility of P_0

Within a coherence-permitting context:

$$\exists P_0 \text{ Coherent}(P_0)$$

and P_0 may be chosen minimal under the Core Specification's minimality ordering (F13) and expressive-capacity ordering (VE3).

Axiom I — Minimal Emergent Coherent Excitation

I.1 Scope

Section I derives the first nontrivial coherent configuration Ψ_0 emergent from P_0 (Section 0). All statements are structural and admissibility-based. No metaphysical, physical, or teleological assumptions are introduced.

I.2 Structural Premises

From Section 0 and the Coherence Core Specification:

1. **Minimal coherence of P_0**

$$\text{Coherent}(P_0) \wedge \text{Minimal}(P_0)$$

2. **Admissible pathway availability** (VE6; AR5):

$$\exists R : P_0 \rightarrow X$$

3. **Admissible internal driver** (D1.4; F4; AR4):

$$D(P_0) \text{ may be nonzero}$$

4. **Preservation constraints** (AR1–AR3; F1, F2, F5, F11): Any admissible emergence preserves continuity, non-negation, and adjacency.
5. **No external introduction / primitives** (VE5; S-3).
6. **Nonterminality under admissible drivers** (AR9): If a configuration admits a nonzero admissible internal driver, then some nontrivial redistribution must occur.

I.3 Minimal Generative Morphism

By AR9 applied to an admissible $D(P_0)$, there exists a nontrivial, adjacency-preserving, recursion-compatible morphism:

$$R_0 : P_0 \rightarrow \Psi_0$$

such that:

- R_0 depends only on the internal structure of P_0 (AR3)
- R_0 is driven by an admissible internal driver $D(P_0)$ (AR4, AR9)
- R_0 preserves continuity (AR1), non-negation (AR2), and adjacency (F1, F5, F11)
- R_0 introduces no external primitives (VE5)
- R_0 is minimally sufficient (F13)

I.4 Definition of the Minimal Excitation Ψ_0

Define:

$$\Psi_0 := R_0(P_0)$$

Ψ_0 satisfies:

- non-nullity (Section 0; AR1–AR3)
- coherence (F1–F5)
- admissible pathway availability (VE6; AR5)
- adjacency preservation (F1, F5, F11)
- axiomatic emergence constraint (VE3):

$$\mathcal{E}(P_0) < \mathcal{E}(\Psi_0)$$

By Corollary 0B.2, Ψ_0 is the first admissible configuration enabling nontrivial distinction.

I.5 Minimality and Structural Uniqueness

Let \sim denote adjacency-preserving structural isomorphism.

If multiple admissible morphisms exist producing candidates Ψ_0 and Ψ'_0 with the same intended minimality posture, then determinacy is interpreted conditionally as in the Core Specification: if a least candidate exists under the applicable leastness criterion (e.g., non-divergence preorder with leastness up to \sim), then any two least candidates are equivalent up to \sim .

In particular, the Axiom I use of ‘minimal’ is a selection posture (least-by-order, when a least exists), not a claim of uniqueness under a partial order in general.

I.6 Minimal Coherent Development Domain

Define:

$$\mathcal{D}_0 := \{\Psi_0, R_1(\Psi_0), R_2(R_1(\Psi_0)), \dots\}$$

Each element of \mathcal{D}_0 satisfies AR1–AR4, VE3, and AR9.

I.7 Canonical Statement of Axiom I

$\exists R_0 : P_0 \rightarrow \Psi_0$ such that Ψ_0 is the minimal admissible coherent excitation above P_0

Ψ_0 is determinate up to adjacency-preserving structural equivalence when a least candidate exists under the applicable leastness posture and initiates nonterminal coherent recursion

Appendix I — Structural Lemmas and Corollaries

Appendix I.A — Lemmas

Lemma I.A.1 — Uniqueness Up to Structural Equivalence

$$R_0(P_0) \sim R'_0(P_0)$$

Lemma I.A.2 — Irreducibility of Ψ_0

$$\mathcal{E}(X) < \mathcal{E}(\Psi_0) \Rightarrow X = P_0 \text{ or incoherent}$$

Lemma I.A.3 — First Adjacency Expansion (Scoped; Well-Typed)

Define the meta-structural predicate $\text{AdjExt}(P_0, \Psi_0)$ (“ Ψ_0 is an adjacency-extension emergence of P_0 ”) by:

$$\text{AdjExt}(P_0, \Psi_0) \stackrel{\text{def}}{\iff} \exists S \subset |\Psi_0| \left(P_0 \sim (\Psi_0|_S) \wedge \exists u \in |\Psi_0| \setminus S \exists v \in |\Psi_0| : \text{Adj}(\Psi_0)(u, v) \vee \text{Adj}(\Psi_0)(v, u) \right).$$

Then:

$$\text{AdjExt}(P_0, \Psi_0) \Rightarrow \exists S \subset |\Psi_0| : \text{Adj}(\Psi_0|_S) \subset \text{Adj}(\Psi_0).$$

Lemma I.A.4 — Seed of Recursive Development

$$\Psi_0 \Rightarrow \exists R_1 : \Psi_0 \rightarrow \Psi_1$$

(by VE6, AR5, and AR9)

Lemma I.A.5 — Conditional Driver Capability

If the development domain \mathcal{D}_0 is nontrivial beyond Ψ_0 , then:

$$D(\Psi_0) \neq 0$$

This condition is not required for Axiom I itself, only for further emergence.

Appendix I.B — Corollaries

Corollary I.B.1 — Every Coherent System on the P_0 Emergence Branch Contains a Ψ_0 -Equivalent State

$$\forall M, (\text{Coherent}(M) \wedge \exists Y \in M : (P_0 \rightarrow Y \rightarrow \dots)) \Rightarrow \exists \Psi'_0 \in M \text{ with } \Psi'_0 \sim \Psi_0$$

Corollary I.B.2 — No Coherent Structure Precedes P_0 or Bypasses Ψ_0

$$\neg \exists X \prec P_0$$

$$\neg \exists Y : (P_0 \rightarrow Y \rightarrow \dots), Y \approx \Psi_0$$

Corollary I.B.3 — Ψ_0 Enables First Admissible Distinction

Ψ_0 enables the first admissible object-level distinction structure

Axiom II — Coherent Differentiation

II.1 Scope

Section II derives the **first nontrivial coherent differentiation** of the minimal emergent excitation Ψ_0 identified in Section I.

All results are structural consequences of: - Foundational Conditions (F1–F14) - Valid Emergence Conditions (VE1–VE8) - Admissibility Rules (AR1–AR9) - In particular, **AR9 (Non-terminality Under an Admissible Driver)**

No physical, metaphysical, or teleological assumptions are introduced.

II.2 Structural Premises

From Section I and the Coherence Core Specification:

1. **Coherence of Ψ_0**

$$\text{Coherent}(\Psi_0)$$

2. **Expressive-capacity increase over P_0**

$$\mathcal{E}(P_0) < \mathcal{E}(\Psi_0)$$

3. **Admissible pathway availability (VE6; AR5):**

$$\exists R : \Psi_0 \rightarrow X$$

4. **Admissible internal driver (D1.4; F4; AR4):**

$$D(\Psi_0) \text{ may be nonzero}$$

5. **Nonterminality condition (AR9):** If $D(\Psi_0) \neq 0$ and admissible redistribution pathways exist, then Ψ_0 cannot remain structurally terminal.

II.3 Forced Differentiation Under AR9

Assume:

$$\text{Coherent}(\Psi_0) \wedge D(\Psi_0) \neq 0$$

By AR9, there must exist a redistribution:

$$R_1 : \Psi_0 \rightarrow \Psi_0^{(1)}$$

such that:

$$\Psi_0^{(1)} \approx \Psi_0$$

where: - R_1 preserves continuity (AR1) - preserves non-negation (AR2) - preserves adjacency (F1, F5, F11) - introduces no external primitives (VE5) - resolves at least one admissible internal driver (AR4)

Thus, **nontrivial differentiation is forced** whenever unresolved admissible drivers exist.

II.4 Definition of First Differentiated Configuration

Define:

$$\Psi_0^{(1)} := R_1(\Psi_0)$$

Define:

$$\Psi_1 := \Psi_0^{(1)}$$

$\Psi_0^{(1)}$ satisfies:

- coherence (F1–F5)
- expressive-capacity increase (VE3):

$$\mathcal{E}(\Psi_0) < \mathcal{E}(\Psi_0^{(1)})$$

- adjacency-preserving structural change
- non-equivalence:

$$\Psi_0^{(1)} \not\approx \Psi_0$$

This constitutes the **first admissible coherent differentiation**.

II.5 Minimality of First Differentiation

Among all admissible redistributions satisfying AR1–AR9, $\Psi_0^{(1)}$ is **minimal** with respect to expressive capacity:

$$\forall X \in \text{Out}(\Psi_0), \quad \mathcal{E}(\Psi_0) < \mathcal{E}(X) < \mathcal{E}(\Psi_0^{(1)}) \Rightarrow X \text{ violates VE3 or F13}$$

Thus, $\Psi_0^{(1)}$ is the **minimal nontrivial differentiation** of Ψ_0 .

II.6 Structural Uniqueness Up to Equivalence

If multiple admissible redistributions exist:

$$R_1, R'_1, \dots$$

producing:

$$\Psi_0^{(1)}, \Psi_0^{(1)'}, \dots$$

then:

$$\Psi_0^{(1)} \sim \Psi_0^{(1)'}$$

where “ \sim ” denotes adjacency-preserving structural isomorphism.

Thus, the first differentiation is **unique up to structural equivalence**.

II.7 Canonical Statement of Axiom II

If $\text{Coherent}(\Psi_0) \wedge D(\Psi_0) \neq 0$, then there exists a minimal coherent differentiation $\Psi_0^{(1)} \approx \Psi_0$

$\Psi_0^{(1)}$ is determinate up to adjacency-preserving structural equivalence when a least candidate exists under the applicable leastness posture

Appendix II — Structural Lemmas and Corollaries

Appendix II.A — Lemmas

Lemma II.A.1 — Non-Equivalence Requirement

$$\Psi_0^{(1)} \not\approx \Psi_0$$

Otherwise AR9 would be violated.

Lemma II.A.2 — Expressive-Capacity Increase

$$\mathcal{E}(\Psi_0^{(1)}) > \mathcal{E}(\Psi_0)$$

(by VE3)

Lemma II.A.3 — Nonterminality with Unresolved Drivers

If $D(\Psi_0) \neq 0$, then there exists an admissible redistribution $R \in \mathcal{R}$ such that

$$R(\Psi_0) \approx \Psi_0.$$

This is the negation-free witness form of nonterminality required by AR9/AR10 (i.e., unresolved admissible drivers force a nontrivial admissible redistribution).

Lemma II.A.4 — Driver Resolution Is Structural, Not Teleological

Redistribution resolves admissible drivers without: - optimization - preference - goal-directedness

Resolution is structural necessity under AR9.

Appendix II.B — Corollaries

Corollary II.B.1 — Differentiation Is Not Optional

Whenever admissible drivers exist, differentiation must occur.

Corollary II.B.2 — No Purely Static Coherence with Drivers

A coherent configuration with unresolved admissible drivers cannot persist unchanged.

Corollary II.B.3 — Foundation for Higher-Order Structure

All subsequent structure (boundaries, interaction, memory, selection, etc.) presupposes the existence of at least one coherent differentiation.

Completion Statement (Section II)

Section II establishes that:

- coherent differentiation is **forced** under admissible internal drivers
- the first differentiation is **minimal and unique up to equivalence**
- structural change is mandated by nonterminality, not assumed dynamics

This completes the derivation of **first-order differentiation** and provides the necessary foundation for higher-order canonical structures.

Axiom III — Constraint Stabilization and Boundary Formation

III.1 Scope

Section III derives the **necessary emergence of stabilized constraints and boundaries** from coherent differentiation.

Given: - minimal emergence (Ψ_0) (Section I), - necessary differentiation (Ψ_1) (Section II), this section formalizes how **coherence persists under recursion** by stabilizing gradients through constraint formation.

All results are structural and admissibility-derived. No geometric, physical, spatial, or teleological assumptions are introduced.

III.2 Structural Premises

From Sections I–II and the Coherence Core Specification:

1. **Coherent differentiated configuration exists**

$$\text{Coherent}(\Psi_1)$$

2. **Unresolved admissible internal driver persists** (D1.4; F4; AR4):

$$D(\Psi_1) \neq 0$$

3. **Admissible pathway availability** (VE6; AR5):

$$\exists R : \Psi_1 \rightarrow X$$

4. **Nonterminality under admissible drivers** (AR9): Admissible gradients cannot remain unresolved indefinitely.
5. **Preservation constraints** (AR1–AR3; F1, F2, F5, F11): All redistributions preserve continuity, non-negation, and adjacency.
6. **Cross-scale compatibility** (VE5).

Derived Abbreviation — Driver Resolution

Driver resolution is **not** a Core Specification primitive.

It is a **derived structural abbreviation** defined entirely in terms of existing Core primitives and admissibility rules.

For a configuration X admitting an internal driver $D(X)$, define:

$$\text{Resolve}(D(X))$$

as shorthand for the following condition:

There exists an admissible redistribution

$$R : X \rightarrow Y$$

such that: - R preserves adjacency, continuity, and non-negation (AR1–AR3), - R is admissible under bounded coherence (F4; AR4), - the unresolved internal driver is reduced under redistribution, in the sense that:

$$D(Y) \prec_D D(X)$$

where “ \prec_D ” denotes the Axioms-derived driver-resolution preorder defined via admissible redistribution outcomes.

This abbreviation introduces **no new primitives, no new operations, and no additional enforcement rules**.

It is used solely for notational economy when reasoning about comparative driver reduction across admissible redistributions.

III.2A Driver-Resolution Preorder (Structural)

Define an Axioms-derived preorder on admissible drivers (not a Core Specification primitive). This preorder is used only to compare driver-reduction pressure via admissible redistribution outcomes, and it uses only the ordering interface of expressive capacity $E(\cdot)$.

Outcome set (redistribution outcomes). For any configuration X , define the outcome set:

$$\text{Out}(X) := \{Y \mid \exists R : X \rightarrow Y \text{ such that } R \text{ is an admissible redistribution preserving AR1–AR3}\}.$$

This introduces no new operation beyond set-comprehension over admissible redistributions.

Preorder on drivers (simulation-style; reflexive and transitive). For drivers $D(A)$ and $D(B)$ associated with configurations A and B , define:

$$D(A) \preceq_D D(B) \quad :\Leftrightarrow \quad \text{Out}(A) \neq \emptyset \wedge \text{Out}(B) \neq \emptyset \wedge \forall B' \in \text{Out}(B) \exists A' \in \text{Out}(A) : \mathcal{E}(A') \leq \mathcal{E}(B').$$

Define strictness as usual:

$$D(A) \prec_D D(B) \quad :\Leftrightarrow \quad D(A) \preceq_D D(B) \wedge \neg(D(B) \preceq_D D(A)).$$

This definition is intentionally not total; incomparability is allowed under a partial order on $E(\cdot)$.

Domain note. If $\text{Out}(X)$ is empty in a context where AR9 applies (i.e., an admissible nonzero driver is present), then X is inadmissible by AR9. For notational hygiene, comparisons involving empty outcome sets are treated as undefined in this document.

III.2B Derived Operators (Canonical Abbreviations)

The following abbreviations introduce no new primitives.

Admissible redistribution class. Let \mathcal{R} denote the class of redistributions R satisfying AR1–AR8.

AR9 forcing hygiene. Whenever AR9 is invoked, forcing is conditional on non-emptiness of the admissible redistribution class: if $D(\Psi) \neq 0$ and $\exists R \in \mathcal{R}$ with $R(\Psi) \approx \Psi$, then a nontrivial redistribution occurs under AR9. **Perturbation (structural).** A *perturbation* is any $R \in \mathcal{R}$.

Persistence (recursion-closure).

$$\text{Persistent}(\Psi) := \exists k \geq 1, \exists R_1, \dots, R_k \in \mathcal{R} \text{ such that } (R_k \circ \dots \circ R_1)(\Psi) \sim \Psi.$$

Terminality. No terminality predicate is introduced in binding material; terminality claims are expressed only via the AR9/AR10 witness form (existence of a nontrivial admissible redistribution when drivers are unresolved) and/or by the driver count condition $D(\Psi) = 0$.

These abbreviations are used only as shorthands for recursion and equivalence statements. ##
III.3 Necessity of Constraint Stabilization

Unregulated recursion acting on differentiated gradients would lead to:

- unbounded gradient divergence, or
- rupture of adjacency (violating F1, F5, F11).

Both outcomes are inadmissible.

Therefore, coherence under recursion **requires** redistribution modes that:

- locally reduce gradient divergence, and
- preserve adjacency and continuity.

This necessitates **stabilized constraint formation**.

III.4 Boundary Formation Morphism

By AR4 and AR9, there exists a redistribution morphism:

$$R_2 : \Psi_1 \rightarrow \Psi_2$$

such that:

- R_2 redistributes gradients to locally bounded configurations
- R_2 preserves continuity (AR1) and non-negation (AR2)
- R_2 preserves adjacency (F1, F5, F11)
- R_2 introduces no external primitives (VE5)
- R_2 is minimally sufficient (F13)

III.5 Definition of Stabilized Configuration Ψ_2

Define:

$$\Psi_2 := R_2(\Psi_1)$$

Ψ_2 satisfies:

- coherence (F1–F5)
- admissible pathway availability (VE6; AR5)
- bounded internal gradients:

$$D(\Psi_2) \prec_D D(\Psi_1)$$

- stable adjacency partitioning

III.6 Definition of Boundary

A boundary is defined as an adjacency-local maximal driver-concentration region, stated in well-typed form.

Let $|X|$ denote the carrier induced by $\text{Adj}(X)$ (see Typed Structural Abbreviations). Let $\text{Nbr}_X(x)$ denote the adjacency neighborhood of x in X . Let $D_{\text{loc}}(X, x)$ denote the local driver abbreviation $D(X|_{\text{Nbr}_X(x)})$.

The driver-resolution preorder \preceq_D and its strict form \prec_D are as defined in III.2A.

Definition (boundary set). A boundary is any subset $B \subseteq |\Psi_2|$ such that for each $b \in B$, the local driver at b is \preceq_D -maximal within its adjacency neighborhood:

$$\forall b \in B, \nexists b' \in \text{Nbr}_{\Psi_2}(b) \text{ with } D_{\text{loc}}(\Psi_2, b) \prec_D D_{\text{loc}}(\Psi_2, b').$$

Boundaries are structural stabilizers (regions characterized by locally maximal unresolved driver concentration) rather than objects or surfaces.

III.7 Minimality and Structural Uniqueness

If two admissible stabilizations exist:

$$R_2(\Psi_1) = \Psi_2, \quad R'_2(\Psi_1) = \Psi'_2$$

then:

$$\Psi_2 \sim \Psi'_2$$

where \sim denotes adjacency-preserving structural equivalence.

Any configuration with weaker stabilization violates AR9 or F1–F5.

III.8 Canonical Statement of Axiom III

$\exists R_2 : \Psi_1 \rightarrow \Psi_2$ such that Ψ_2 stabilizes differentiated gradients via admissible constraints

Ψ_2 contains the first admissible boundary structures preserving coherence under recursion

Appendix III — Structural Lemmas and Corollaries

Appendix III.A — Lemmas

Lemma III.A.1 — Unbounded Gradients Are Inadmissible

$D(\Psi_1)$ is unbounded under $\prec_D \Rightarrow$ violation of F1 or F11

Lemma III.A.2 — Boundaries Are Structurally Necessary

$\text{Coherent}(\Psi_1) \wedge D(\Psi_1) \neq 0 \Rightarrow \exists B$

Lemma III.A.3 — Boundaries Preserve Adjacency

B partitions neighborhoods without rupture

Lemma III.A.4 — Boundary Formation Enables Persistent Structure

$\Psi_2 \Rightarrow \exists$ persistent coherent regions

Appendix III.B — Corollaries

Corollary III.B.1 — All Persistent Coherence Requires Boundaries

$\text{Persistent}(\Psi) \Rightarrow \exists B \subset \Psi$

Corollary III.B.2 — No Coherent Differentiation Persists Without Constraint

$\neg \exists \Psi_1$: coherent and unconstrained under recursion

Corollary III.B.3 — Boundary Formation Is Structurally Forced

Constraint stabilization follows necessarily from differentiation

Axiom IV — Interaction, Coupling, and Feedback Regulation

IV.1 Scope

Section IV derives the **necessary emergence of interaction and feedback regulation** from stabilized boundary-bearing coherent configurations.

Given: - minimal emergence (Ψ_0) (Section I), - necessary differentiation (Ψ_1) (Section II), - stabilized boundaries (Ψ_2) (Section III),

this section formalizes how **coherence remains dynamically stable under recursion** through interaction across boundaries and feedback-mediated regulation.

All results are structural, admissibility-derived, and Core-constrained.

IV.2 Structural Premises

From Sections I–III and the Coherence Core Specification:

1. **Boundary-bearing coherent configuration exists**

$$\text{Coherent}(\Psi_2)$$

2. **Boundaries partition adjacency neighborhoods** (from Section III)
3. **Admissible pathway availability** (VE6; AR5):

$$\exists R : \Psi_2 \rightarrow X$$

4. **Unresolved admissible internal driver persists** (D1.4; F4; AR4):

$$D(\Psi_2) \neq 0$$

5. **Nonterminality under admissible drivers** (AR9): Unresolved gradients cannot remain static.
6. **Preservation constraints** (AR1–AR3; F1, F2, F5, F11): All transformations preserve continuity, non-negation, and adjacency.

IV.3 Necessity of Interaction Across Boundaries

Boundaries create **adjacent but differentiated regions**.

Under recursion: - gradients propagate, - adjacency is preserved, - nonterminality forbids isolation.

Therefore, boundary-adjacent regions must **interact**.

Non-interaction would imply: - blocked adjacency (violating F5/F11), or - terminal gradient stasis (violating AR9).

Both are inadmissible.

Thus, **interaction is structurally forced**.

IV.4 Definition of Interaction

An **interaction** is defined as:

An adjacency-preserving exchange of gradient redistribution across a boundary.

Formally, for boundary B separating regions A and C :

$$\exists I : A \leftrightarrow C$$

such that: - gradients redistribute across B , - adjacency is preserved, - no external primitives are introduced (VE5).

IV.5 Coupling Morphism

By AR4 and AR9, there exists a coupling morphism:

$$R_3 : \Psi_2 \rightarrow \Psi_3$$

such that:

- R_3 mediates interactions across boundaries
- R_3 preserves continuity (AR1) and non-negation (AR2)
- R_3 preserves adjacency (F1, F5, F11)
- R_3 is internally driven (AR4)
- R_3 is minimally sufficient (F13)

IV.6 Definition of Feedback

A **feedback** is defined as:

An interaction whose effect counteracts unbounded gradient divergence and restores bounded coherence under recursion.

Formally:

$$F : \Psi_3 \rightarrow \Psi_2$$

such that:

$$D(F(\Psi_3)) \leq_D D(\Psi_2)$$

Feedback is **structural regulation**, not control or intention.

IV.7 Stabilized Interactive Configuration Ψ_3

Define:

$$\Psi_3 := R_3(\Psi_2)$$

Ψ_3 satisfies:

- coherence (F1–F5)
- admissible pathway availability (VE6; AR5)
- boundary-mediated interaction
- feedback-regulated gradient bounds
- persistence under recursive propagation

IV.8 Minimality and Structural Uniqueness

If two admissible coupling morphisms exist:

$$R_3(\Psi_2) = \Psi_3, \quad R'_3(\Psi_2) = \Psi'_3$$

then:

$$\Psi_3 \sim \Psi'_3$$

where \sim denotes adjacency-preserving structural equivalence.

Any weaker configuration violates AR9 or allows unbounded divergence.

IV.9 Canonical Statement of Axiom IV

$\exists R_3 : \Psi_2 \rightarrow \Psi_3$ such that Ψ_3 enables interaction and feedback-regulated coherence

Interaction and feedback are structurally necessary for coherence persistence under recursion

Appendix IV — Structural Lemmas and Corollaries

Appendix IV.A — Lemmas

Lemma IV.A.1 — Boundaries Necessarily Induce Interaction

$$\text{Boundary}(B) \Rightarrow \exists I \text{ across } B$$

Lemma IV.A.2 — Interaction Without Feedback Is Unstable

$$I \wedge \neg F \Rightarrow \text{gradient divergence}$$

Lemma IV.A.3 — Feedback Preserves Coherence

$$F(\Psi_3) \Rightarrow \text{Coherent}(\Psi_2)$$

Lemma IV.A.4 — Feedback Is Structurally Forced

$$\text{Coherent}(\Psi_2) \wedge D(\Psi_2) \neq 0 \Rightarrow \exists F$$

Appendix IV.B — Corollaries

Corollary IV.B.1 — All Persistent Coherent Systems Are Interactive

$$\text{Persistent}(\Psi) \Rightarrow \exists I$$

Corollary IV.B.2 — All Persistent Coherence Is Feedback-Regulated

$$\text{Persistent}(\Psi) \Rightarrow \exists F$$

Corollary IV.B.3 — Persistent Coherent Structures with Unresolved Drivers Are Not Isolated

$$\neg \exists \Psi : \text{Coherent}(\Psi) \wedge \text{Persistent}(\Psi) \wedge D(\Psi) \neq 0 \wedge \text{non-interactive}$$

Informal reading: terminal (driver-free) coherent configurations are admissible; isolation is excluded only for persistent coherence sustained in the presence of unresolved admissible drivers.

Axiom V — Memory, Retention, and Path Dependence

V.1 Scope

Section V derives the **necessary emergence of memory and path dependence** from feedback-regulated coherent interaction.

Given: - emergence (Ψ_0) (Section I), - differentiation (Ψ_1) (Section II), - boundary stabilization (Ψ_2) (Section III), - interaction and feedback regulation (Ψ_3) (Section IV),

this section formalizes how **coherent systems retain structural effects of prior states** under recursive propagation.

All results are structural, admissibility-based, and Core-constrained.

V.2 Structural Premises

From Sections I-IV and the Coherence Core Specification:

1. **Feedback-regulated coherent configuration exists**

$$\text{Coherent}(\Psi_3)$$

2. **Admissible pathway availability** (VE6; AR5):

$$\exists R : \Psi_3 \rightarrow X$$

3. **Feedback modifies subsequent configurations** (Section IV):

$$F(\Psi_3) \neq \Psi_3$$

4. **Non-negation holds** (F2): No relation simultaneously enforces and nullifies the same adjacency-compatible constraint.

5. **Nonterminality under admissible drivers** (AR9): Resolved gradients cannot return to a pre-interaction null configuration.

V.3 Necessity of Retention

Under recursion: - feedback alters configurations, - non-negation prevents constraint self-cancellation, - configuration structure conservation (F8, Core Specification) prevents silent deletion of configuration structure up to structural equivalence, - continuity preserves adjacency-compatible relational structure.

Therefore, any feedback-induced difference cannot be treated as if it never occurred while remaining coherent: it must remain representable as structural difference (possibly redistributed) in subsequent configurations.

This does not assert a psychological or semantic notion of memory; it asserts only structural retention in the sense of non-erasure of configuration structure and non-nullification of applied constraints.

V.4 Definition of Memory

A **memory** is defined as:

A retained structural modification resulting from prior interaction or feedback, preserved under recursive propagation.

Formally, for successive configurations:

$$\Psi_{n+1} = R(\Psi_n)$$

memory exists iff:

$$\Psi_{n+1} \neq \Psi_n \quad \text{and} \quad \Psi_{n+1} \text{ preserves modified adjacency}$$

Memory is not representation or storage; it is **structural persistence**.

V.5 Path Dependence

Because memory persists and non-negation holds:

$$\Psi_{n+k} \text{ depends on the sequence } \{\Psi_n, \Psi_{n+1}, \dots\}$$

not merely on the current state.

Thus, coherent evolution is **path dependent**.

Any claim of path-independence would imply: - erasure of prior structure (violating F2), or - terminal behavior (violating AR9).

Both are inadmissible.

V.6 Retentive Morphism

By AR4 and AR9, there exists a retentive morphism:

$$R_4 : \Psi_3 \rightarrow \Psi_4$$

such that:

- prior feedback effects persist,
- adjacency is preserved (F1, F5, F11),
- continuity and non-negation hold (AR1–AR2),
- expressive capacity does not decrease (VE3).

V.7 Memory-Bearing Configuration Ψ_4

Define:

$$\Psi_4 := R_4(\Psi_3)$$

Ψ_4 satisfies:

- coherence (F1–F5),

- admissible pathway availability (VE6; AR5),
- feedback-modified adjacency,
- retained structural differences,
- path-dependent evolution.

V.8 Minimality and Structural Uniqueness

If two admissible retentive morphisms exist:

$$R_4(\Psi_3) = \Psi_4, \quad R'_4(\Psi_3) = \Psi'_4$$

then:

$$\Psi_4 \sim \Psi'_4$$

under adjacency-preserving structural equivalence.

Any weaker configuration collapses to interaction-without-retention, which violates Section IV and AR9.

V.9 Canonical Statement of Axiom V

$\exists R_4 : \Psi_3 \rightarrow \Psi_4$ such that Ψ_4 retains structural effects of prior feedback

Memory and path dependence are structurally necessary for coherent persistence under recursion

Appendix V — Structural Lemmas and Corollaries

Appendix V.A — Lemmas

Lemma V.A.1 — Feedback Necessarily Produces Retained Structure

$$F(\Psi_3) \neq \Psi_3 \Rightarrow \exists \text{ retained modification}$$

Lemma V.A.2 — Retention Implies Path Dependence

$$\text{Retention} \Rightarrow \text{Path dependence}$$

Lemma V.A.3 — Erasure Violates Non-Negation

$$\text{No retention} \Rightarrow \text{structure erasure} \Rightarrow \neg \text{Coherent}$$

Lemma V.A.4 — Memory Is Structurally Forced

$$\text{Coherent}(\Psi_3) \wedge F \wedge R \Rightarrow \exists \text{Memory}$$

Appendix V.B — Corollaries

Corollary V.B.1 — All Persistent Coherent Systems Have Memory

$$\text{Persistent}(\Psi) \Rightarrow \exists \text{Memory}$$

Corollary V.B.2 — No Feedback-Regulated System Is Markovian

$$\neg \exists \Psi : \text{Coherent}(\Psi) \wedge \text{Memoryless}$$

Corollary V.B.3 — History Matters Structurally

$$\Psi_{n+k} \neq f(\Psi_n) \text{ alone}$$

Evolution depends on prior trajectory.

Axiom VI — Selection, Stabilization, and Attractor Formation

VI.1 Scope

Section VI derives the **necessary emergence of selection and attractor dynamics** from memory-bearing, feedback-regulated coherent systems.

Given: - emergence (Ψ_0) (Section I), - differentiation (Ψ_1) (Section II), - boundary stabilization (Ψ_2) (Section III), - interaction and feedback (Ψ_3) (Section IV), - memory and path dependence (Ψ_4) (Section V),

this section formalizes how **certain coherent configurations become preferentially stabilized** under recursive propagation.

No optimization, intention, or external criteria are introduced. Selection arises strictly from structural constraints.

VI.2 Structural Premises

From Sections I–V and the Coherence Core Specification:

1. **Memory-bearing coherent configuration exists**

$$\text{Coherent}(\Psi_4)$$

2. **Admissible pathway availability** (VE6; AR5):

$$\exists R : \Psi_4 \rightarrow X$$

3. **Path dependence holds** (Section V):

$$\Psi_{n+k} \text{ depends on prior trajectory}$$

4. **Feedback modulates gradient resolution** (Section IV):

$$F(\Psi_4) \neq \text{identity}$$

5. **Non-negation and continuity hold** (F1, F2).
6. **Nonterminality under admissible drivers** (AR9).

VI.3 Emergence of Selection

Under repeated recursion with memory:

- some redistributions reduce gradient tension more effectively,
- others amplify or destabilize gradients,
- feedback differentially reinforces certain structural pathways.

If no differential persistence occurred: - memory would have no effect, - recursion would collapse to equivalence classes, - path dependence would be inert.

Thus, **differential persistence is structurally forced**.

This constitutes **selection**.

VI.4 Definition of Selection

Selection is defined as:

The differential persistence of coherent configurations under recursive propagation due to feedback-mediated gradient resolution.

Formally, for two configurations Ψ_a, Ψ_b :

$\Psi_a \prec \Psi_b$ iff Ψ_b has a closure witness under some \mathcal{R} satisfying AR1–AR8 while Ψ_a has none under the same \mathcal{R} .

Selection is not choice or optimization; it is **structural survivability**.

VI.5 Attractor Formation

Because memory preserves prior effects and selection reinforces certain paths, there exist coherence-preserving subsets of the development domain that are recurrently re-entered under admissible perturbations.

These subsets define **attractors**.

Attractors are not necessarily fixed points; they may be: - steady configurations, - cycles, - bounded regimes of variation.

VI.6 Definition of an Attractor

An **attractor** is defined as:

A coherence-preserving subset of configurations that is invariant under admissible recursion and recurrently re-entered under admissible perturbations.

Formally:

$$\exists \mathcal{A} \subset \mathcal{D} \text{ such that } R(\Psi) \in \mathcal{A} \forall \Psi \in \mathcal{A}$$

VI.7 Stabilized Configuration Ψ_5

By AR4 and AR9, there exists a morphism:

$$R_5 : \Psi_4 \rightarrow \Psi_5$$

such that:

- feedback-reinforced structures persist,
- gradient fluctuations are bounded,
- expressive capacity does not decrease (VE3),
- adjacency and continuity are preserved (F1, F5, F11).

Define:

$$\Psi_5 := R_5(\Psi_4)$$

Ψ_5 is a **stabilized, selected configuration**.

VI.8 Minimality and Structural Uniqueness

If two stabilized configurations exist:

$$\Psi_5, \Psi'_5$$

both satisfying all constraints, then:

$$\Psi_5 \sim \Psi'_5$$

under adjacency-preserving structural equivalence.

Any weaker configuration fails to retain stability under perturbation, violating AR9 or Section V.

VI.9 Canonical Statement of Axiom VI

$\exists R_5 : \Psi_4 \rightarrow \Psi_5$ such that Ψ_5 is a selected, stabilized coherent configuration

Selection and attractor formation are structurally necessary outcomes of memory-bearing recursion

Appendix VI — Structural Lemmas and Corollaries

Appendix VI.A — Lemmas

Lemma VI.A.1 — Memory Necessitates Differential Persistence

Memory $\Rightarrow \exists$ differential persistence

Lemma VI.A.2 — Differential Persistence Defines Selection

Differential persistence \Rightarrow Selection

Lemma VI.A.3 — Selection Implies Attractor Formation

Selection $\Rightarrow \exists$ Attractor

Lemma VI.A.4 — Attractors Stabilize Coherent Evolution

$\Psi \in \mathcal{A} \Rightarrow R(\Psi) \in \mathcal{A}$

Appendix VI.B — Corollaries

Corollary VI.B.1 — No Persistent Coherent System Is Selection-Free

$\neg \exists \Psi : \text{Persistent}(\Psi) \wedge \neg \text{Selected}$

Corollary VI.B.2 — Stability Is Emergent, Not Imposed

Stability \Rightarrow Emergent from feedback and memory

Corollary VI.B.3 — Attractors Encode Structural History

\mathcal{A} reflects accumulated path dependence

Axiom VII — Hierarchy, Modularity, and Compositional Coherence

VII.1 Scope

Section VII derives the **necessary emergence of hierarchical and modular structure** from systems that already exhibit:

- selection and attractors (Section VI),
- memory and path dependence (Section V),
- feedback-regulated interaction (Section IV),
- nonterminal recursive evolution (AR9).

This section explains why coherent systems do not remain flat, and why **composition across scales is unavoidable**.

No organizational principle is imposed. Hierarchy emerges as a structural consequence of stabilized attractors interacting.

VII.2 Structural Premises

From Sections I–VI and the Coherence Core Specification:

1. **Stabilized attractors exist** (Section VI):

$$\exists \mathcal{A}_i \subset \mathcal{D}$$

2. **Attractors are persistent under perturbation:**

$$R(\Psi) \in \mathcal{A}_i \quad \forall \Psi \in \mathcal{A}_i$$

3. **Multiple attractors may coexist:**

$$\mathcal{A}_i \neq \mathcal{A}_j$$

4. **Attractors interact through adjacency-preserving propagation** (F5, F11).
5. **Nonterminality holds** (AR9): Recursive evolution cannot halt at a single stabilized level.

VII.3 Necessity of Modularization

If multiple attractors interact without constraint:

- interference destabilizes internal coherence,
- memory and feedback loops cross-couple destructively,
- selection loses effectiveness.

Thus, for attractors to persist under interaction, **internal coherence must be partially insulated**.

This insulation produces **modules**.

VII.4 Definition of a Module

A **module** is defined as a coherence-preserving subset $\mathcal{M} \subseteq \Psi$ that is **recursively invariant** under an admissible class of redistributions.

Let $\mathcal{R}(\mathcal{M})$ denote the set of admissible redistributions R (AR1–AR8) such that R maps \mathcal{M} into itself up to structural equivalence:

$$R \in \mathcal{R}(\mathcal{M}) \iff \forall \Psi \in \mathcal{M}, \exists \Psi' \in \mathcal{M} \text{ with } R(\Psi) \sim \Psi'.$$

\mathcal{M} is a module iff $\mathcal{R}(\mathcal{M})$ is nonempty and maximal (under set inclusion) among subsets satisfying this invariance property, subject to adjacency compatibility with the ambient configuration (F5, F11).

Modules are not isolated; they are **selectively permeable** through boundary-mediated interaction (Section IV).

VII.5 Emergence of Hierarchy

Because modules:

- stabilize internal structure,
- reduce destructive cross-coupling,
- allow recursion to proceed at higher expressive capacity,

modules themselves become **effective units of interaction**.

Thus, relations form **between modules**, not only within them.

This induces a **hierarchical ordering**:

$$\mathcal{M}_1 \prec \mathcal{M}_2 \prec \dots$$

where ordering reflects increasing expressive capacity and compositional depth.

Hierarchy is therefore **emergent composition**, not imposed stratification.

VII.6 Compositional Closure

At the modular level:

- internal details may be abstracted,
- only boundary-compatible interfaces propagate upward,
- recursion acts on composed structures.

Thus, higher-level coherence depends on **lower-level stability**, but is not reducible to it in description.

This yields **compositional closure**.

VII.7 Definition of Hierarchical Composition

A **hierarchical composition** is defined as:

A coherence-preserving mapping from interacting modules to a higher-order coherent structure.

Formally:

$$R_h : \{\mathcal{M}_i\} \rightarrow \Psi_h$$

such that:

- adjacency is preserved (F5, F11),
- expressive capacity increases (VE3),
- internal module coherence is not violated (F1–F3).

VII.8 Stabilized Hierarchical Configuration Ψ_6

By AR9, there exists a morphism:

$$R_6 : \Psi_5 \rightarrow \Psi_6$$

where:

- modules persist as identifiable subsystems,
- interactions occur primarily at module boundaries,
- higher-order attractors form over module interactions.

Define:

$$\Psi_6 := R_6(\Psi_5)$$

Ψ_6 is a **hierarchically organized coherent configuration**.

VII.9 Minimality and Structural Uniqueness

If two hierarchical organizations exist:

$$\Psi_6, \Psi'_6$$

both satisfying all binding constraints, then:

$$\Psi_6 \sim \Psi'_6$$

under adjacency-preserving structural equivalence.

Any configuration lacking modular separation fails to remain stable under continued interaction, violating AR9.

VII.10 Canonical Statement of Axiom VII

$\exists R_6 : \Psi_5 \rightarrow \Psi_6$ such that Ψ_6 is hierarchically and modularly organized

Hierarchy and modularity are structurally necessary outcomes of interacting stabilized attractors

Appendix VII — Structural Lemmas and Corollaries

Appendix VII.A — Lemmas

Lemma VII.A.1 — Interaction Necessitates Modular Insulation

Interacting attractors \Rightarrow Modularity

Lemma VII.A.2 — Modularity Enables Persistence Under Interaction

Modularity \Rightarrow Stability under recursion

Lemma VII.A.3 — Modules Induce Hierarchy

Persistent modules \Rightarrow Hierarchical composition

Lemma VII.A.4 — Hierarchy Increases Expressive Capacity

$$\mathcal{E}(\Psi_6) > \mathcal{E}(\Psi_5)$$

Appendix VII.B — Corollaries

Corollary VII.B.1 — No Persistent Coherent System Is Flat

$$\neg \exists \Psi : \text{Persistent}(\Psi) \wedge \neg \text{Hierarchical}$$

Corollary VII.B.2 — Reduction Without Composition Is Inadmissible

Pure reduction \Rightarrow Loss of coherence

Corollary VII.B.3 — Compositional Description Becomes Necessary

Hierarchy \Rightarrow Multi-level representation

Axiom VIII — Information, Constraint Encoding, and Symbolic Compression

VIII.1 Scope

Section VIII derives **information** as a necessary structural consequence of:

- hierarchical modular organization (Section VII),
- stabilized attractors (Section VI),
- memory and path dependence (Section V),
- feedback-regulated recursion (Section IV),
- nonterminal emergence (AR9).

Information is not assumed as a primitive. It emerges as **compressed constraint structure** required for scalable coherence.

VIII.2 Structural Premises

From Sections I–VII and the Coherence Core Specification:

1. **Hierarchical modular systems exist** (Section VII):

$$\exists \Psi_6$$

2. **Modules interact primarily through boundary interfaces.**
3. **Internal microstructure of modules exceeds boundary relevance:**

$$\mathcal{E}(\text{interior}) > \mathcal{E}(\text{boundary})$$

4. **Nonterminal recursion persists** (AR9).
5. **Adjacency-preserving propagation constraints apply** (F5, F11).

VIII.3 Necessity of Constraint Encoding

If all internal details of modules propagated upward:

- expressive capacity would explode,
- adjacency constraints would be violated,
- recursion would destabilize.

Thus, higher-order coherence requires **selective transmission** of constraints, not full internal state.

This selective transmission is **information**.

VIII.4 Definition of Information (Structural)

Information is defined as a boundary-level **constraint quotient** sufficient for adjacency-compatible interaction.

Let $\partial\mathcal{M}$ denote the boundary interface of a module \mathcal{M} (as a boundary-bearing subset under Section III). Define an interaction-outcome equivalence \sim_I on boundary-interface configurations by:

$$x \sim_I y \iff \forall I \in \mathcal{J}, I(x) \sim I(y),$$

where \mathcal{J} is the class of admissible interactions (Section IV) and \sim is adjacency-preserving structural equivalence.

Then the information carried by \mathcal{M} is the quotient:

$$I(\mathcal{M}) := \partial\mathcal{M} / \sim_I.$$

This definition introduces no representational primitive: it is an invariance class induced by admissible interaction.

VIII.5 Symbol Formation

A **symbol** is defined as a stable representative (up to \sim) of an information class that recurs under admissible replication and interaction (Sections IX and IV).

Formally, s is a symbol for an information class $[x]_{\sim_I}$ if $s \sim x$ and s is re-instantiated (up to \sim) across admissible lineages (Section IX).

The term *encoding* is used only as shorthand for a chosen representative (up to \sim) of an information class in $I(\mathcal{M})$; it introduces no additional primitive.

VIII.6 Emergence of Symbolic Layers

Repeated use of the same encodings yields:

- abstraction of internal variability,
- independence from specific microstates,
- robustness under perturbation.

Thus, a **symbolic layer** emerges:

$$\Psi_7 := R_7(\Psi_6)$$

where interaction increasingly occurs through symbolic encodings.

VIII.7 Minimality and Structural Uniqueness

Any encoding that:

- transmits more detail than necessary violates VE3,
- transmits less violates coherence constraints.

Thus, admissible encodings are minimal and unique up to equivalence.

VIII.8 Canonical Statement of Axiom VIII

$\exists R_7 : \Psi_6 \rightarrow \Psi_7$ such that Ψ_7 encodes constraints via compressed symbolic representations

Information emerges as the minimal compression required for coherent hierarchical interaction

Appendix VIII — Structural Lemmas and Corollaries

Appendix VIII.A — Lemmas

Lemma VIII.A.1 — Constraint Explosion Without Compression

No compression \Rightarrow Loss of coherence

Lemma VIII.A.2 — Boundary Sufficiency

Coherent interaction \Rightarrow Boundary-level encoding

Lemma VIII.A.3 — Symbol Stability

Reusable encodings \Rightarrow Symbol formation

Lemma VIII.A.4 — Information Is Structural, Not Semantic

Symbol \neq Meaning

Meaning may arise later but is not required for information.

Appendix VIII.B — Corollaries

Corollary VIII.B.1 — Information Is Scale-Relative

$I(\mathcal{M})$ depends on interaction scale

Corollary VIII.B.2 — Symbolic Layers Enable Further Hierarchy

$\Psi_7 \Rightarrow \exists \Psi_8$

Corollary VIII.B.3 — Description Becomes Possible

Symbolic compression \Rightarrow Representability

Axiom IX — Replication, Inheritance, and Lineages

IX.1 Scope

Section IX derives **replication and inheritance** as necessary structural consequences of:

- symbolic compression and information encoding (Section VIII),
- hierarchical modular organization (Section VII),
- stabilized attractors and selection (Section VI),
- memory and feedback-regulated recursion (Sections IV–V),
- nonterminal emergence (AR9).

Replication is not assumed as biological or intentional. It emerges as a **coherence-preserving strategy** under persistent interaction and variation.

IX.2 Structural Premises

From Sections I–VIII and the Coherence Core Specification:

1. **Symbolic encodings exist** (Section VIII):

$$\exists \Psi_7$$

2. **Symbols are reusable and boundary-stable.**
3. **Configurations undergo perturbation and selection** (Section VI).
4. **Memory preserves prior encodings** (Section V).
5. **Nonterminal recursion holds** (AR9).

IX.3 Necessity of Replication

If symbolic configurations were instantiated only once:

- perturbations would irreversibly destroy successful encodings,
- selection could not accumulate,
- coherence would stagnate or collapse.

Thus, persistence under nonterminal recursion requires **reproduction of successful encodings**.

Replication is therefore structurally necessary.

IX.4 Definition of Replication (Structural)

Replication is defined as:

The generation of a new coherent configuration that preserves a symbolic encoding under admissible transformations.

Formally, replication satisfies:

$$R_{\text{rep}} : \Psi \rightarrow \Psi'$$

such that:

$$I(\Psi') \sim I(\Psi)$$

where “ \sim ” denotes symbolic equivalence under constraint encoding.

IX.5 Inheritance

Because replication occurs under:

- imperfect boundary conditions,
- perturbations,
- internal variability,

replication is **never exact**.

Thus, replicated configurations inherit:

- core constraint encodings,
- with allowable variation.

Inheritance is defined as:

The persistence of symbolic encodings across replication events, modulo admissible variation.

IX.6 Lineages

Repeated replication with inheritance yields **lineages**:

$$\Psi^{(0)} \rightarrow \Psi^{(1)} \rightarrow \Psi^{(2)} \rightarrow \dots$$

where each element preserves symbolic equivalence class membership while permitting divergence.

Lineages enable:

- accumulation of selection,
- refinement of constraint encodings,
- long-term coherence.

IX.7 Stabilized Replicative Configuration Ψ_8

By AR9, there exists a morphism:

$$R_8 : \Psi_7 \rightarrow \Psi_8$$

where:

- symbolic encodings replicate,
- inheritance operates,
- selection acts across generations.

Define:

$$\Psi_8 := R_8(\Psi_7)$$

Ψ_8 is a **replicative coherent regime**.

IX.8 Minimality and Structural Uniqueness

Any system satisfying Sections I–VIII and AR9 must admit replication, or else lose accumulated coherence under perturbation.

Thus, replication is:

- minimal,
- structurally forced,
- unique up to equivalence of encoding and inheritance mechanisms.

IX.9 Canonical Statement of Axiom IX

$\exists R_8 : \Psi_7 \rightarrow \Psi_8$ such that symbolic encodings replicate with inheritance

Replication and lineage formation are necessary for nonterminal coherent evolution

Appendix IX — Structural Lemmas and Corollaries

Appendix IX.A — Lemmas

Lemma IX.A.1 — Selection Requires Replication

Selection \Rightarrow Replication

Lemma IX.A.2 — Replication Implies Inheritance

Replication \Rightarrow Constraint persistence with variation

Lemma IX.A.3 — Lineages Enable Accumulation

No lineages \Rightarrow No cumulative selection

Lemma IX.A.4 — Replication Is Not Teleological

Replication \neq Purpose

It is a structural consequence of persistence under variation.

Appendix IX.B — Corollaries

Corollary IX.B.1 — Any Persistent Coherent System Exhibits Lineage Structure

Persistent(Ψ) $\Rightarrow \exists$ lineage

Corollary IX.B.2 — Symbolic Evolution Is Lineage-Based

Symbolic adaptation \Rightarrow Replicative inheritance

Corollary IX.B.3 — Long-Term Coherence Requires Replication

\neg Replication $\Rightarrow \neg$ Nonterminal coherence

Axiom X — Observation, Measurement, and Projection Consistency

X.1 Scope

Section X derives **observation and measurement** as necessary structural consequences of:

- symbolic compression and information (Section VIII),
- replication and lineages (Section IX),
- hierarchical modular organization (Section VII),
- feedback, memory, and selection (Sections IV–VI),
- nonterminal coherent evolution (AR9).

Observation is not assumed as subjective, intentional, or epistemic. It emerges as a **consistency requirement across projections** in interacting coherent systems.

X.2 Structural Premises

From Sections I–IX and the Coherence Core Specification:

1. **Symbolic encodings exist and propagate** (Sections VIII–IX).
2. **Multiple coherent subsystems interact via boundary interfaces.**
3. **Each subsystem possesses internal state inaccessible in full to others.**
4. **Interaction occurs through projections of constraint encodings.**
5. **Nonterminal recursion persists** (AR9).

X.3 Necessity of Projection

If subsystems attempted to interact via full internal state exchange:

- expressive capacity would diverge,
- adjacency constraints would be violated,
- coherence would fail.

Thus interaction must proceed through **partial projections**.

A **projection** $\pi(\Psi)$ is an admissible transformation $T : \Psi \rightarrow \Psi|_B$ where B is a boundary set satisfying III.6, governed by AR1–AR3 and $\text{Coherent}(\Psi|_B)$, and projection-stable under admissible perturbation up to structural equivalence (D1.6).

X.4 Definition of Observation (Structural)

Observation is defined as:

The stabilization of a projected constraint encoding under repeated interaction between coherent systems.

Formally, an observation satisfies:

$$\pi(\Psi) \xrightarrow{\text{interaction}} \pi(\Psi)$$

as an invariant under admissible perturbations.

Observation is thus **structural agreement**, not perception.

X.5 Measurement

Measurement is defined as:

An observation that selects one among multiple admissible projected encodings via interaction with a measurement configuration.

Measurement requires:

- symbolic representations (Section VIII),
- replication and memory (Section IX),
- selection under perturbation (Section VI).

X.6 Projection Consistency Requirement

For coherent evolution to persist across interacting subsystems, their projections must be **mutually consistent**.

Define **projection consistency** as:

Compatibility of projected encodings such that no interaction induces contradiction or loss of coherence.

Formally:

$$\pi_A(\Psi) \sim \pi_B(\Psi)$$

under adjacency-preserving equivalence.

X.7 Emergence of Observer Configurations Ψ_9

By AR9, there exists a morphism:

$$R_9 : \Psi_8 \rightarrow \Psi_9$$

where:

- subsystems stabilize consistent projection interfaces,
- observations persist across interactions,
- measurements are reproducible up to adjacency-preserving projection equivalence.

Define:

$$\Psi_9 := R_9(\Psi_8)$$

Ψ_9 is a **coherent observational regime**.

X.8 Minimality and Structural Uniqueness

Any coherent system supporting:

- symbolic interaction,
- replication,
- selection,
- nonterminal evolution,

must satisfy projection consistency, or else collapse under contradiction.

Thus observation and measurement are:

- minimal,
- structurally necessary,
- unique up to equivalence of projection mappings.

X.9 Canonical Statement of Axiom X

$\exists R_9 : \Psi_8 \rightarrow \Psi_9$ such that coherent observation and measurement emerge via projection consistency

Observation is the stabilization of projected constraints under interaction, not an external epistemic act

Appendix X — Structural Lemmas and Corollaries

Appendix X.A — Lemmas

Lemma X.A.1 — Interaction Requires Projection

Direct full-state interaction \Rightarrow Incoherence

Lemma X.A.2 — Observation Is Not Subjective

Observation \neq Intentional perception

Lemma X.A.3 — Measurement Is Selection Over Projections

Measurement \Rightarrow Projection + selection

Lemma X.A.4 — Projection Inconsistency Destroys Coherence

\neg Projection consistency \Rightarrow \neg Persistent interaction

Appendix X.B — Corollaries

Corollary X.B.1 — Observers Are Coherent Subsystems

Observer := Projection-stabilizing subsystem

Corollary X.B.2 — Measurement Outcomes Are Relational

Outcome \Rightarrow Interaction-dependent

Corollary X.B.3 — Projection Consistency Grounds Objectivity

Objectivity := Cross-system projection agreement