DEFT: Smart Fee Calculation Approaches in Uniform Liquidity Constant Product Market Makers

Abstract

Deft is a decentralized exchange (DEX) designed to address the significant issue of impermanent loss in uniform liquidity constant product automated market makers (AMMs), a prevalent concern for liquidity providers (LPs) in traditional DEXs. As a fork of Uniswap V2, Deft introduces innovative features such as uniform liquidity and an advanced dynamic fee calculation mechanism to mitigate impermanent loss, creating a more secure and profitable environment for LPs. Additionally, Deft ensures a fair and efficient trading experience for liquidity takers (LTs), balancing the needs of all participants. By addressing these critical challenges and leveraging cutting-edge technology, Deft sets a new standard in the decentralized finance (DeFi) space, fostering greater confidence and participation among users.

Introduction

Decentralized exchanges (DEXs) are crucial for many blockchain applications, providing easy access to various services and crypto assets without the need for customer identity verification. Compared to centralized exchanges, DEXs are more accessible and can integrate seamlessly with other decentralized financial (DeFi) services at minimal cost [1]. DEXs handle a substantial amount of capital. The four largest DEXs on the Ethereum blockchain, Uniswap [2, 3], Curve Finance [4], SushiSwap [5], and Balancer [6], collectively achieve a trading volume of several billion dollars per day.

Automated Market Makers (AMMs) are a fundamental innovation in the decentralized finance (DeFi) ecosystem, enabling decentralized exchanges (DEXs) to function without the need for traditional order books. Instead of matching buy and sell orders, AMMs use mathematical formulas to price assets and facilitate trades through liquidity pools. These pools are funded by liquidity providers (LPs) who earn transaction fees in return for their contributions. [7]

Automated Market Makers (AMMs) use simple and sometimes complex mathematical formulas or algorithms to determine the quantities and prices of assets in liquidity pools. These mechanisms ensure that the pools never run out of any assets by adjusting prices based on the supply and demand within the pool. This constant adjustment helps maintaining the liquidity and enables seamless trading without the need for a traditional order book. Due to these automated adjustments natures of AMMs, the total value of assets in a liquidity pool can be lower than if the liquidity providers had kept the assets in their wallets or simply HODL[1]ed them. This difference is known as impermanent loss (IL) and is detailed in the liquidity providers' profit function [8, 9]. Many

[1] A slang term in the cryptocurrency community that stands for "Hold On for Dear Life."

believe impermanent loss is a significant issue, prompting DEXs like Bancor to introduce Impermanent Loss Protection. However, the growth and market share of Uniswap, the largest DEX which hasn't directly addressed this issue, suggest it may not be the most critical concern for liquidity providers. A recent research however showed that the impermanent loss is less severe than what was believed before, but still impacting the LP assets [10].

In this paper, we implement the PNL strategy introduced by in [11] into Uniswap V2 in a more practical manner. The main aim here is keeping the Liquidity Takers (LTs) incentivized as well as protecting the Liquidity Providers (LPs) from harsh price changes which will lead to impermanent loss. Considering the previous articles, the goal of this research is to reduce and optimize the impermanent loss which is a feature of Constant Product Market Makers (CPMM), rather than completely omitting the impermanent loss or making profit of it.

Literature Review

The initial researches focused on impermanent loss returns back to 2021. Angeris et al. [12, 13] analyzed UniswapV2 with assuming zero fees and demonstrate that liquidity providers' profits decline with changes in market prices. Additionally, they show that Automated Market Makers (AMMs) closely follow the reference market price. In another research, they computed the profit functions of liquidity providers in liquidity pools of CPMM. Evans [14] illustrates that similar results also hold for more generalized mean geometric models.

Park [15] has conducted comprehensive research focusing on conceptual flaws of DEXs, especially Automated market makers. This research assesses the current pricing functions of an AMM and corresponding pros and cons. This research states that linear price rules have problems such as increasing the cost of trading and threaten the long-term viability of the DeFi eco-system, except for uniform pricing for which sandwich attack profits are limited and smaller, but which invites excessive order splitting.

Bergault et al. [16] made research on bringing external oracle on the price discovery process in an AMM rather than solely rely on liquidity takers. They also adopted a mean variance analysis of profit and loss of liquidity providers and compared to those agents holding assets outside the DEX. Their research revealed that traditional CFMMs (including CPMMs) with different levels of transaction fees perform poorly relative to the theoretical efficient frontier and very often exhibit negative excess PnL. They also showed that allowing an AMM to get information about the current market exchange rate (through an oracle) can significantly improve performance. Hägele [1] has made a systematic review of the current centralized and decentralized exchange markets in DeFi ecosystem.

Bayraktar et al. [17] investigate the behavior of liquidity providers by modeling a decentralized cryptocurrency exchange (DEX) based on Uniswap v3. Their research revealed liquidity providers with different traits choose the best positions to provide liquidity, despite not knowing the size of incoming trades or the future asset prices. They compete with each other, and this competition shapes the exchange rates and arbitrage opportunities in the pool. We used Uniswap data to understand these LP traits and found that their strategies lead to exchange rate changes and liquidity behavior similar to what we see in real Uniswap pools. Pourpouneh et al. [18] present an execution framework for setting up Automated Market Makers (AMMs) on different blockchain platforms and within sharded blockchains. This plan includes economic incentives to encourage participation by ensuring fixed prices across different liquidity pools.

With using Monte Carlo simulations to examine impermanent loss in a dynamic context, Hafner et al. [10] revealed that price changes alone don't necessarily cause losses; trader and arbitrageur fees are crucial. They also conclude that creating an arbitrage-friendly environment can benefit liquidity providers, and recommend AMM developers encourage arbitrage rather than prevent it. Using minute-level ETH spot prices, Alexander et al. [19] examined the price discovery and efficiency of Uniswap-v2 and Uniswap-v3, comparing them to Coinbase and Bitstamp. Their findings showed that Uniswap-v3 significantly improved over Uniswap-v2 and nearly matched Bitstamp in price discovery. However, Uniswap-v3 lacks market efficiency, leading to numerous arbitrage opportunities that benefit professional traders over novices. Additionally, changes in long-memory patterns in ETH spot returns from Uniswap-v2 to Uniswap-v3 indicate that further technical enhancements are necessary for DEXs to attract more everyday investors.

Miori et al. [20] devised a method to analyze and classify Uniswap v3 traders based on their activity across various liquidity pools. They concentrated on a subset of 34 pools to ensure computational feasibility and to enhance interconnectedness within the ecosystem. By constructing transaction graphs for each trader and employing a modified graph2vec algorithm, they identified seven distinct clusters of traders exhibiting similar behaviors, including preferences for specific assets, trading frequency, and fee tolerance.

Loesch et al. [21] evaluated impermanent loss within the Uniswap-v3 ecosystem. They investigated the performance of different classes of liquidity providers (LPs) concerning impermanent loss (IL) and fees. Their hypothesis that active users who frequently adjust their positions would outperform others found no significant evidence. They also analyzed the duration of LP positions and discovered that longer-term LPs (over one month) experienced lower IL compared to short-term LPs or flash-LPs (up to a day). They suggested further research to identify medium-term LP strategies that might consistently outperform simply holding assets (HODLing).

Khakhar et al. [11] propose a new metric to delta-hedging liquidity positions via derivatives while introducing a new metric, Liquidity Position PNL, which directly measures the change in the net value of a Liquidity Position as a function of price movement in the underlying assets. Delta-hedging is a strategy which aims to reduce the directional risk associated with price movements of underlying financial instruments within a portfolio [22]. In other words, a portfolio is delta-hedged with respect to some assets when the change in price of these assets causes a negligible (or non-existent) change in portfolio value. Many recent works have investigated delta-hedging strategies with desirable properties, such as a delta-hedging strategy via reinforcement learning [23]. We use these researches to protect the liquidity providers as well as providing fair and reasonable arbitrage opportunities.

Model Formulation

In this section, we present the governing and fundamental equations of impermanent loss and profit and loss (PNL). We delve into the mathematical formulations that describe how impermanent loss occurs due to changes in asset prices within an automated market maker (AMM) environment. Additionally, we explore the equations that calculate the PNL for liquidity providers, taking into account fees earned and losses incurred due to price fluctuations. These equations are crucial for understanding the financial dynamics and risks involved in liquidity provision on decentralized exchanges (DEXs).

Constant Product Market Maker

DEXs are two-sided platforms which is shown in Fig. 1. Liquidity providers (LPs) interact with the platform by supplying assets to the liquidity pool in exchange for a fee. The liquidity pool depicted in Figure 1 consists of two assets denoted A and B, for example, USDT and WETH. Liquidity takers (LTs) are the other side of the DEX. We can categorize the LTs as traders and arbitrageurs. Traders can exchange one of the two assets for the other by paying a fee. Arbitrageurs also interact with the platform whenever they perceive price differences between the exchange rate of A and B at the DEX and that of other markets. If, for example, asset B is more valuable (relative to Asset A) at another exchange, arbitrageurs will buy B from the DEX by sending asset A and a fee to this DEX in exchange for asset B. At the same time, these arbitrageurs will sell B for A at another exchange to earn a risk-free profit.

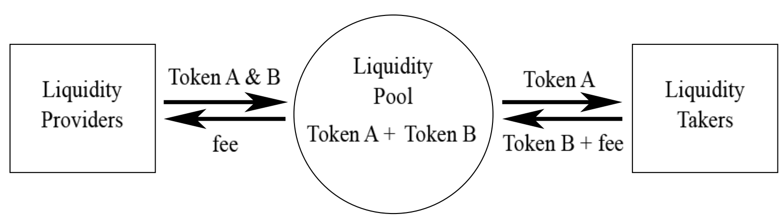

Figure 1.

A standard Uniswap liquidity pool allows the exchange of two assets via the constant product market maker mechanism which is called "*swap*". The aforementioned formula is presented in Eq.1:

$$xy = k = (x - \Delta x)(y + \Delta y) \quad \text{Equation 1}$$

If the corresponding fees of an exchange is enabled in a pair, the formula changes to the Eq.2:

$$(x - \Delta x)(y + (1 - \gamma)\Delta y) = k \quad \text{Equation 2}$$

where $x, y \in \mathbb{Q}^+$ are the current pool reserves of tokens X, Y respectively. Similarly, Δx (resp. Δy) denotes the quantity of token X (resp. Y) exchanged in an individual swap transaction. The signs in Eq.1 imply that a LT sells an amount Δy of token Y to the pool, while receiving Δx of token X back. The instantaneous exchange rate or spot price P between the two digital assets is given by the proportion of respective reserves in the pool, i.e.

$$P = \frac{dy}{dx} = -\frac{k}{x^2} = -\frac{y}{x} \quad \text{Equation 3}$$

Having the same spot prices, the average pricing mechanisms of different CPMM exchanges differ. As an example, the Uniswap v3 utilizes the time-weighted average price (TWAP) method, while the Uniswap v2 uses price accumulators instead.

Impermanent Loss

Per definition, the impermanent loss is defined as the difference between the final value of pool assets and the HODLed assets:

$$Impermanent\ Loss\ (IL) = \frac{\text{Final Value of Pool Assets}}{\text{Value If Assets Were Held}} - 1 \quad \text{Equation 4}$$

If we want to calculate the impermanent loss in Uniswap v2, considering the geometric mean of the reserve quantities which is also called as 'Liquidity', first we should calculate the value of the reserves held in the pool as a function of price in terms of L and P (V):

$$\begin{aligned} xy &= k \\ xy &= L^2 \\ y &= L\sqrt{P} \\ x &= \frac{L}{\sqrt{P}} \\ V &= xP + y \end{aligned} \quad \text{Equation 5}$$

Thus, the impermanent loss relation becomes as the preceding formula:

$$IL = \frac{2\sqrt{k}}{k + 1} - 1 \quad \text{Equation 6}$$

The impermanent loss formula can be related to the price changes relations. Let δ represent the price change from $P^{i}_{A \to B}$ (initial price of swapping tokens A to B) to $P^{f}_{A \to B}$ (final price of swapping tokens A to B):

$$\delta = \frac{P^{f}_{A \to B}}{P^{i}_{A \to B}} - 1 \qquad \text{Equation 7}$$

The impermanent loss formula can be presented as [13]:

$$IL = \frac{\mathrm{W}_1 \ * \ \delta_{\mathrm{LP}}}{\mathrm{W}_1 \ * \ \delta_{Ref}} - 1 = \frac{2\sqrt{\delta + 1}}{\delta + 2} - 1 \qquad \text{Equation 8}$$

with δ_{Ref} being the relative gain of the reference portfolio and W_1 the initial wealth of the liquidity provider.

Liquidity Position PNL

To hedge the difference between the final value of the liquidity pool assets and the initial value of the liquidity pool assets, thus the liquidity position PNL becomes as:

$$Liquidity\ Position\ PNL = \frac{\text{Final Value of Pool}}{\text{Initial Value of LP Investment}} - 1 \qquad \text{Equation 9}$$

For the AMMs with uniform liquidity (Uniswap v2), the aforementioned formula becomes as:

$$Liquidity\ Position\ PNL = \sqrt{\delta + 1} \qquad \text{Equation 10}$$

The algorithm

The core concept of the algorithm is finding the new coordination of the pool for a possible swap request. After calculating the delta and finding the new coordinate, the system now autonomously decides whether to alter swap fee to protect the LPs reaching for an impermanent loss state. The delta intervals for fee calculations are defined as the impermanent loss plot which is provided in Khakhar et al.'s research [11]. These intervals are demonstrated in Fig.2. Fee calculation relations are determined using the slope and mathematical nature of the impermanent loss graph.

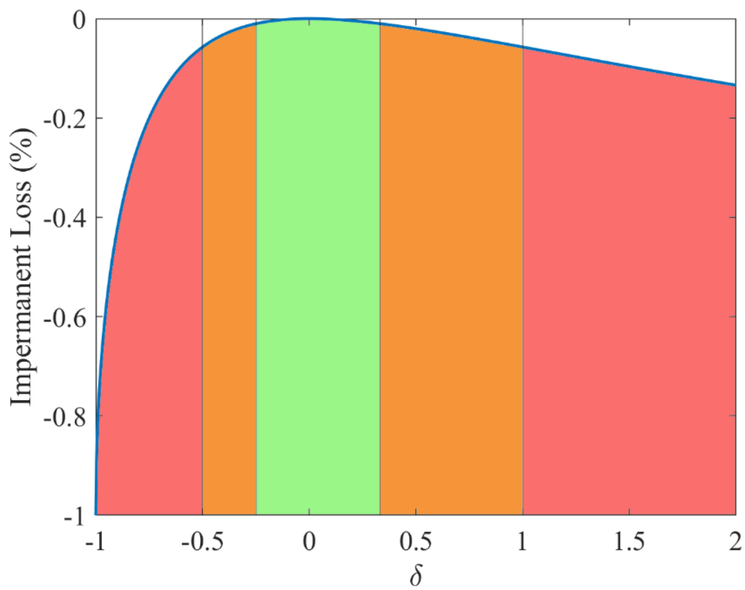

Figure 2. Smart fee calculation intervals

As it is provided in the Fig.2, the central interval $(-0.25 < \delta < 0.33)$ which is colored green, is safe interval. The price fluctuations are low and we have almost zero impermanent loss. In this period, the default fee of 0.3% is considered. The second interval that its color is orange $(-0.5 < \delta < -0.25 \,\&\, 0.33 < \delta < 1)$ is an interval which price fluctuations become significant. The impermanent loss starts to increase and reach to a high value. However, as the impermanent loss slope and amounts are not so high compared to the next interval, a linear regression algorithm is adopted for this region. The next interval is red and danger interval. In this region, which is mainly caused by arbitrageurs, the price and as a consequence, the reserves are changing drastically. Arbitrageurs make significant benefit from such swaps while the LPs make huge losses. For the purpose of stopping this loss, an exponential fee calculation mechanism is considered for this region where the slope is considerably high.

After defining the new coordinate for a swap, we can calculate the corresponding fee and charge the arbitrageurs. The entire algorithm is illustrated in Fig. 3.

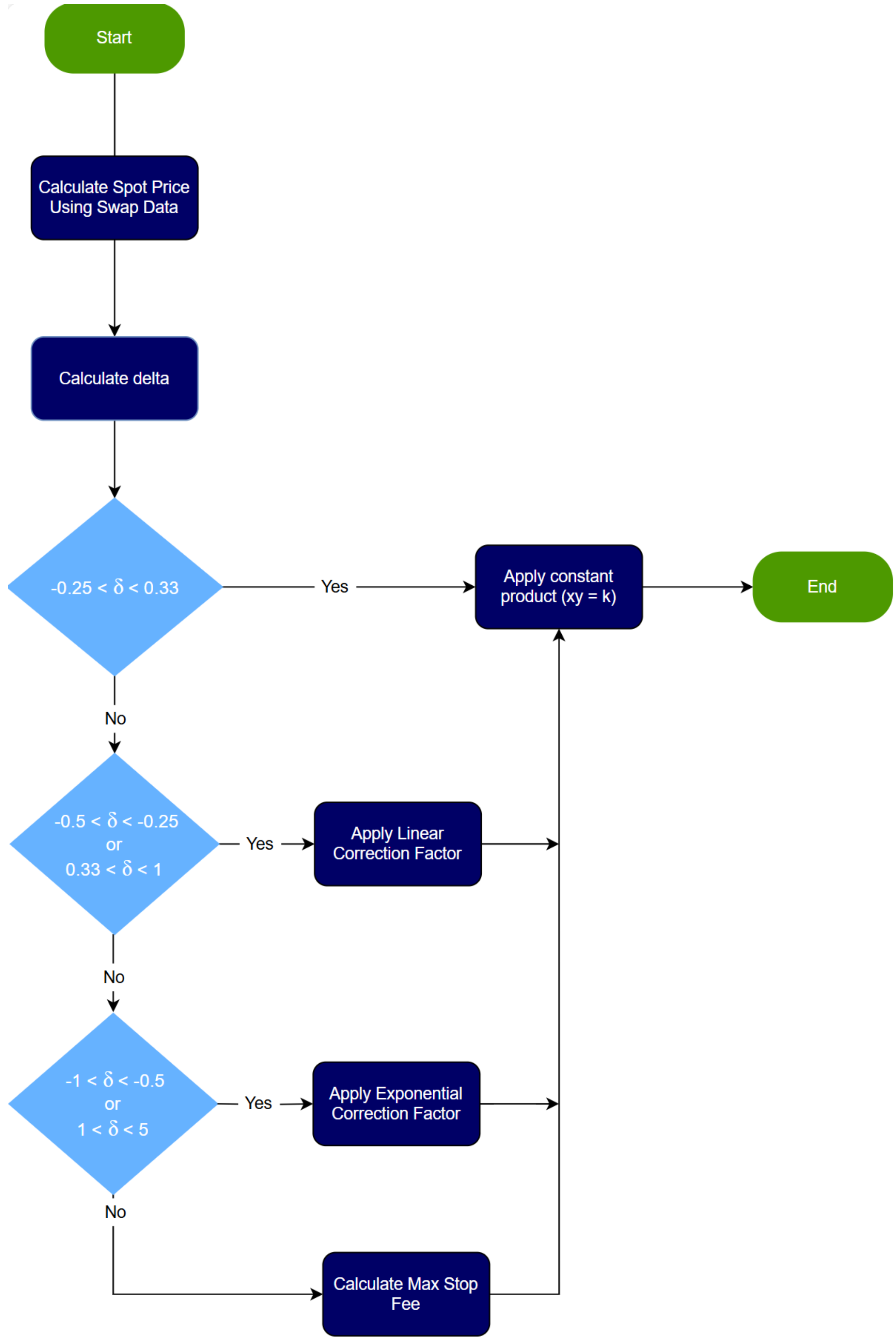

Figure 3. The algorithm for smart fee calculation

Relations for Smart Fee Calculations

As it is described in the previous section, the relations for fee computations are provided in this section. All the fees here are presented in the basis point of 1000.

Safe Zone: (Green Zone)

$$fee = 3$$

Alert Zone: (Orange Zone)

$$-0.5 < \delta < -0.25 \qquad fee = -68 \times \delta - 14$$

$$0.33 < \delta < 1 \qquad fee = 25.37 \times \delta - 5.37$$

Danger Zone: (Red Zone)

$$-1 < \delta < -0.5 \qquad fee = 8\exp(-1.8325814637483102 \times \delta)$$

$$1 < \delta < 5 \qquad fee = 15.905414575341013\exp(0.22907268296853878 \times \delta)$$

For the cases where the delta is bigger than 5, the fee amount is capped at 55.

To summarize this section, the fee amount calculation is defined based on the delta parameter intervals and the fee amount ranges from 0.003 to 0.055 in the worst case. Doing so, the LPs can benefit from high swap amounts and compared to the AMMs with static LP fees, Deft provides a great opportunity for LPs.

Conclusion

In conclusion, Deft is a groundbreaking decentralized exchange (DEX) that effectively addresses the critical issue of impermanent loss in uniform liquidity constant product AMMs. This issue has long been a significant concern for liquidity providers (LPs) in traditional DEXs, often deterring participation due to the potential for substantial losses. By being a fork of Uniswap V2, Deft leverages the proven foundation of a leading DEX while introducing innovative enhancements to improve the overall user experience.

Deft incorporates uniform liquidity and an advanced dynamic fee calculation mechanism to mitigate impermanent loss. The uniform liquidity model ensures that all trades draw from a single, deep liquidity pool, reducing slippage and enhancing capital efficiency. This results in a more stable and predictable environment for LPs, as their assets are consistently utilized in a more efficient manner. The advanced dynamic fee calculation mechanism adjusts transaction fees corresponding to each swap parameters, ensuring that fees remain fair and reflective of current volatility. This dynamic approach not only balances supply and demand but also incentivizes trading behaviors that promote market stability, further protecting LPs from adverse price movements.

Moreover, Deft ensures that these improvements do not come at the expense of liquidity takers (LTs). By maintaining fair and efficient trading conditions, Deft provides a seamless and user-friendly experience for all participants. This balanced approach fosters a thriving ecosystem where both LPs and LTs can benefit, encouraging greater participation and liquidity provision.

In essence, Deft sets a new standard in the DeFi space by addressing the persistent challenge of impermanent loss while maintaining a fair and efficient trading environment. By fostering a secure and profitable environment for LPs and ensuring a seamless experience for LTs, Deft paves the way for greater confidence and participation in the decentralized finance ecosystem.

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