

ZAMA

Recent advances in homomorphic compilation

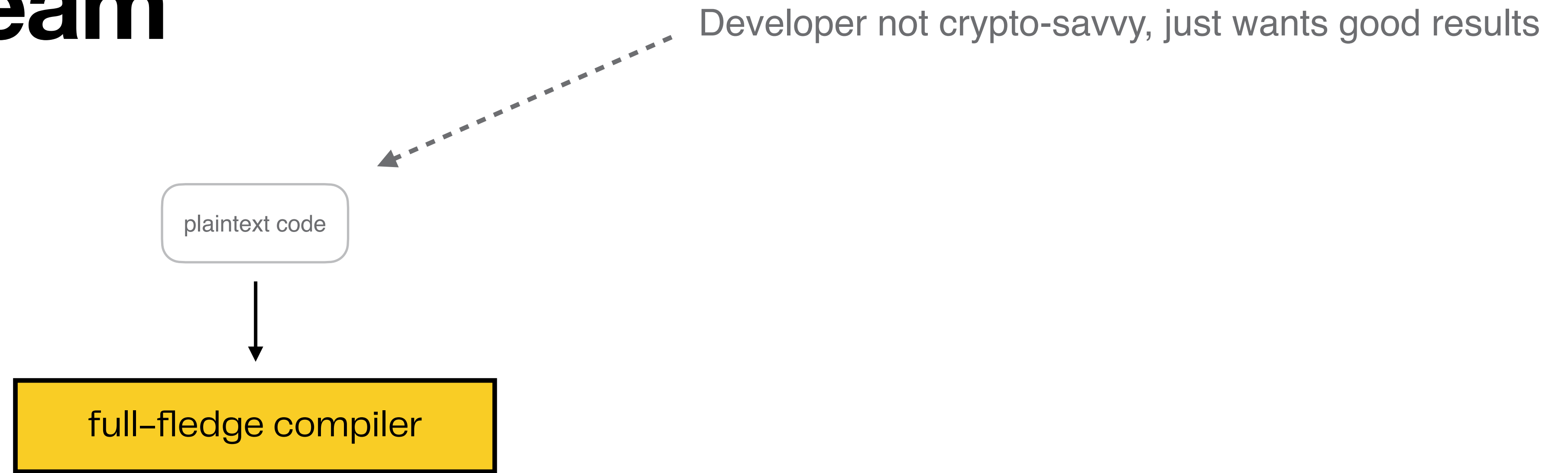
Pascal Paillier, Zama

[FHE.org](https://fhe.org) conference 2023

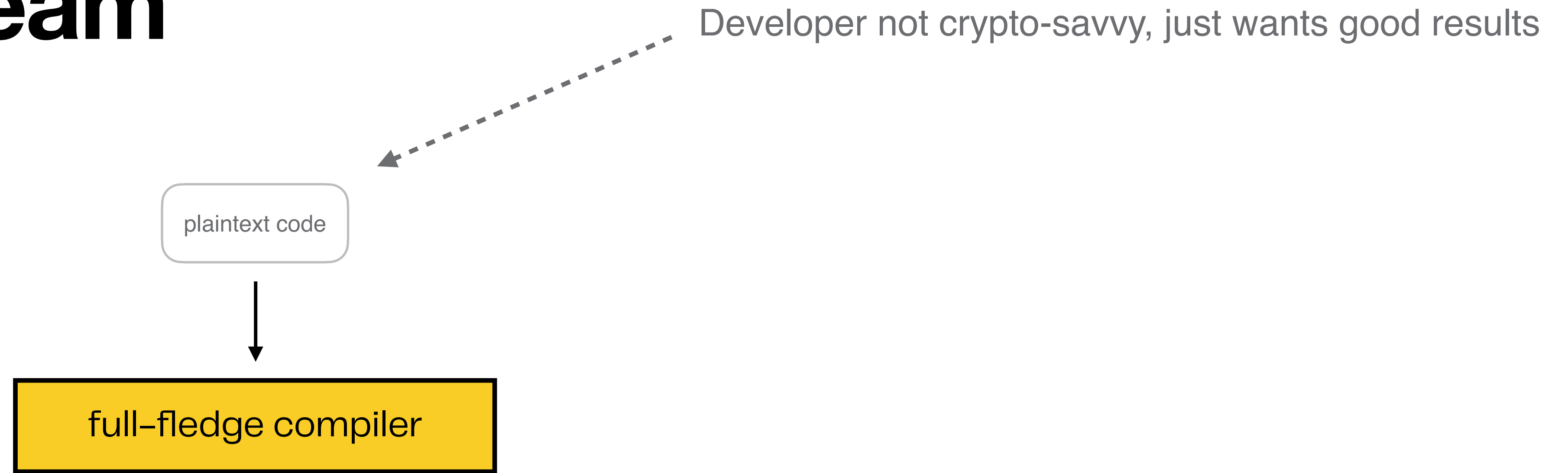


**What is homomorphic
compilation?**

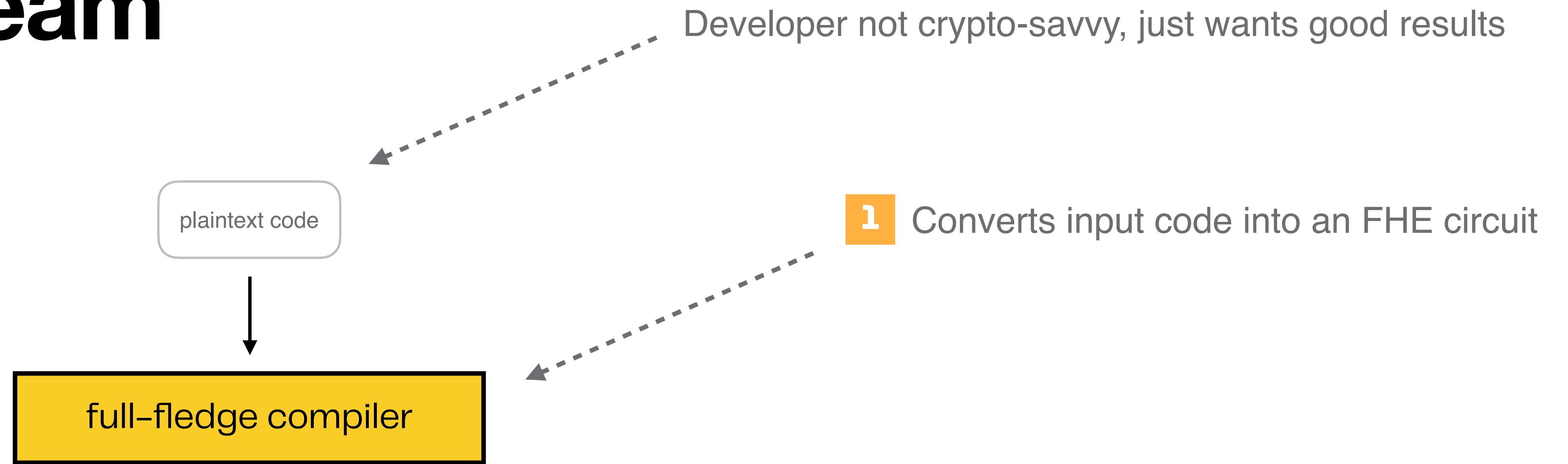
The big dream



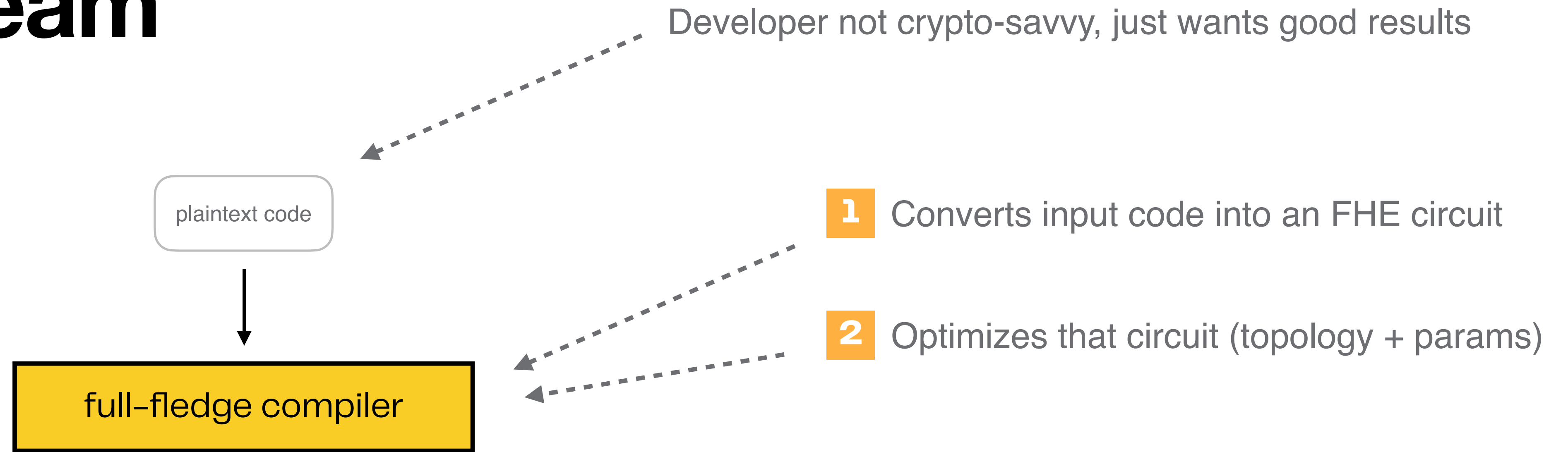
The big dream



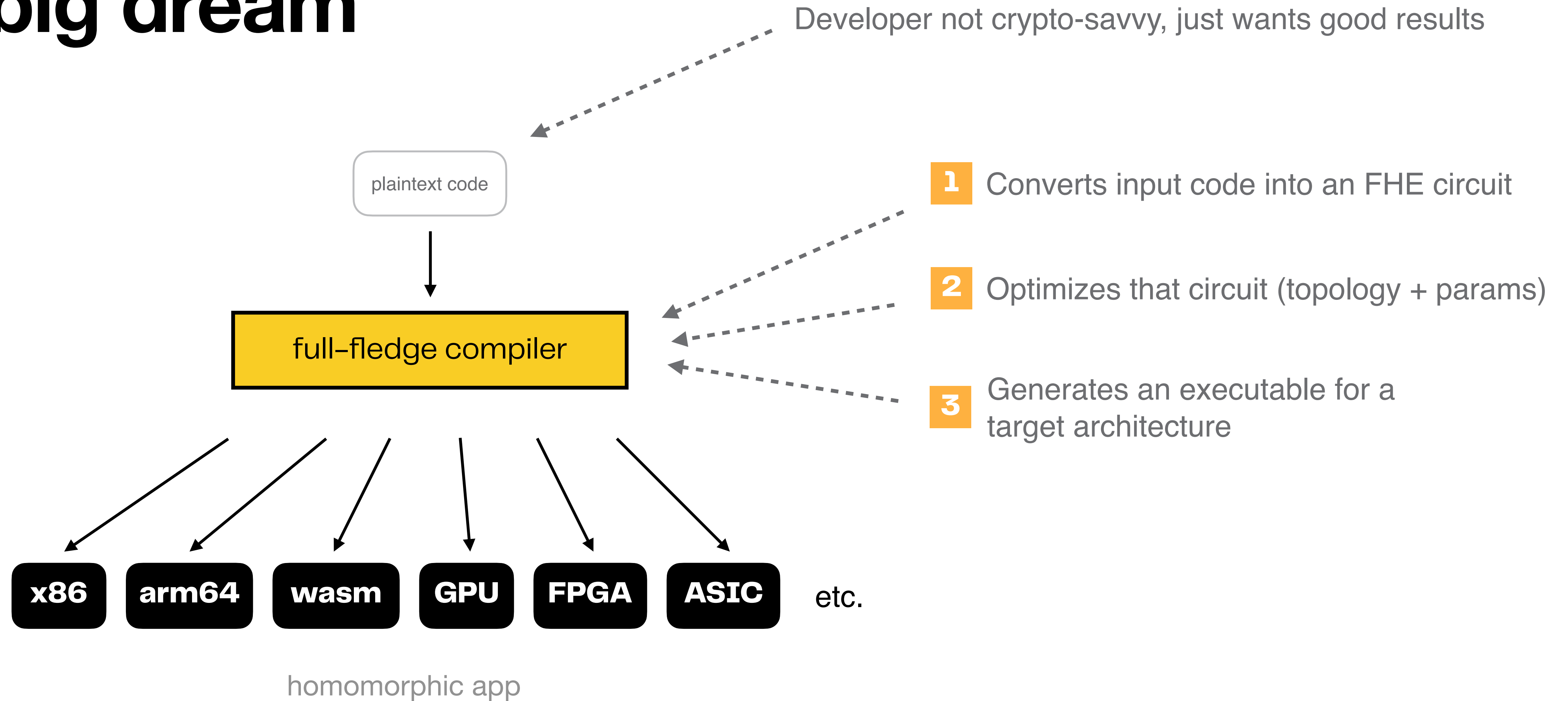
The big dream



The big dream

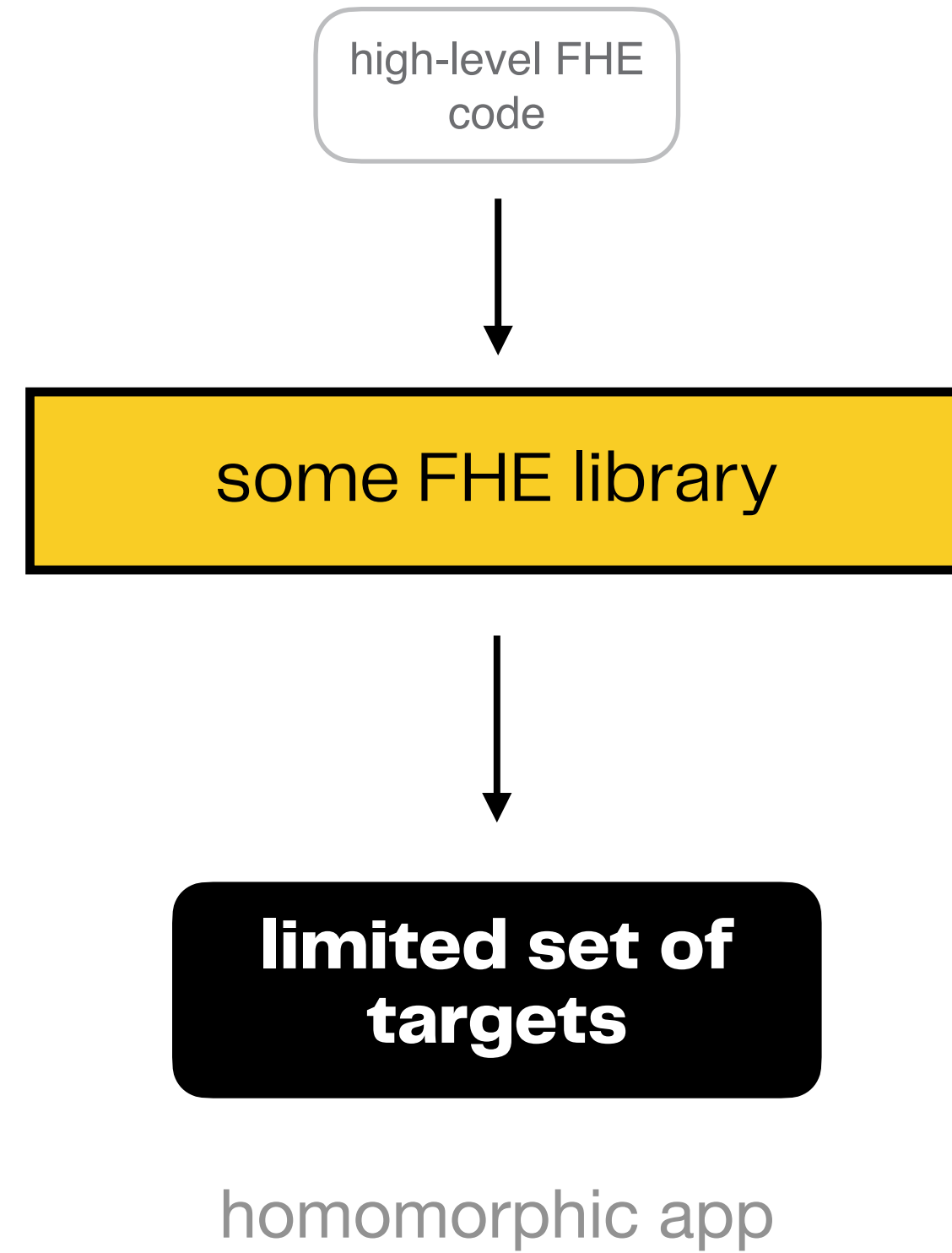


The big dream

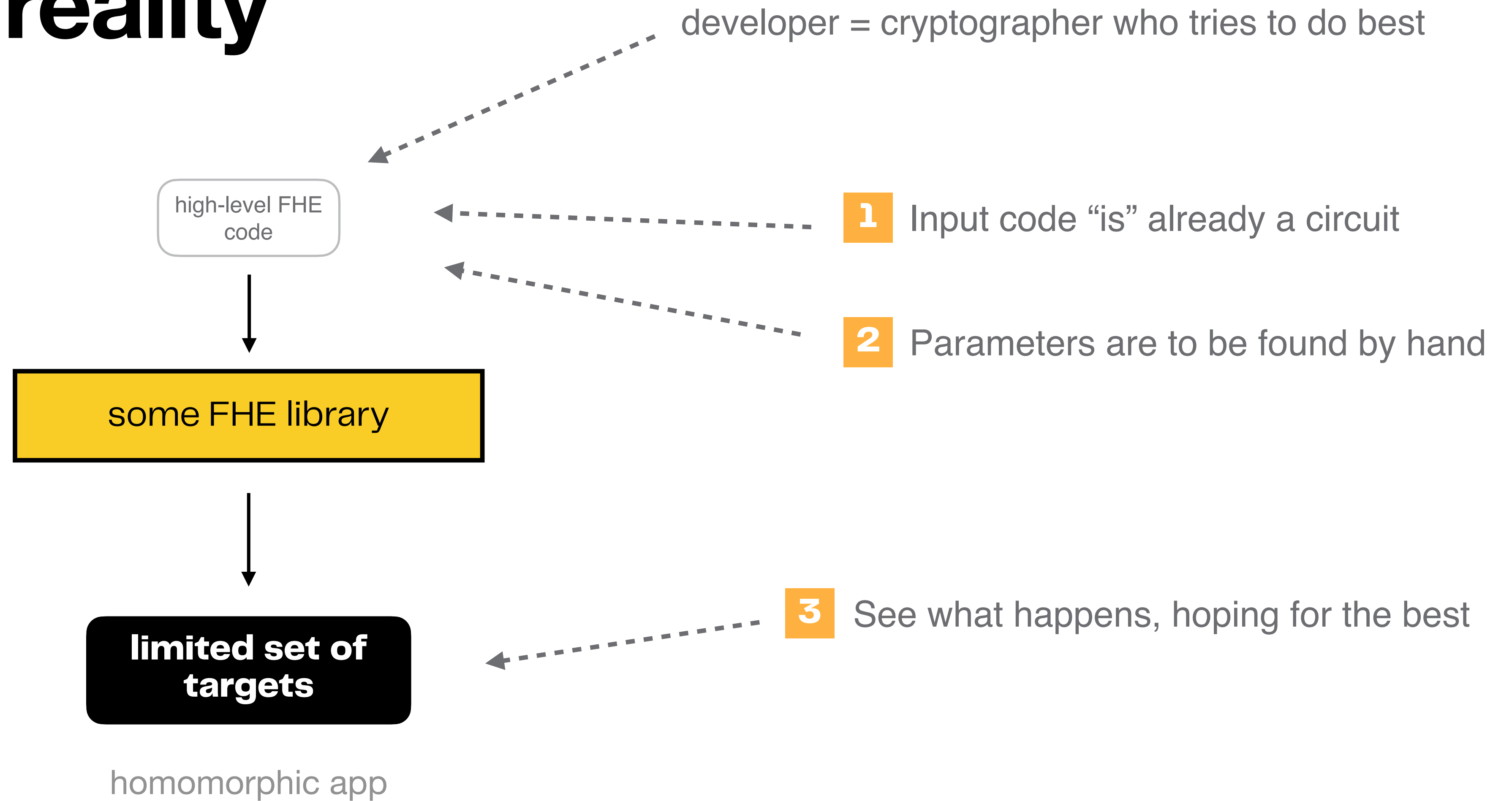


The (poor) reality

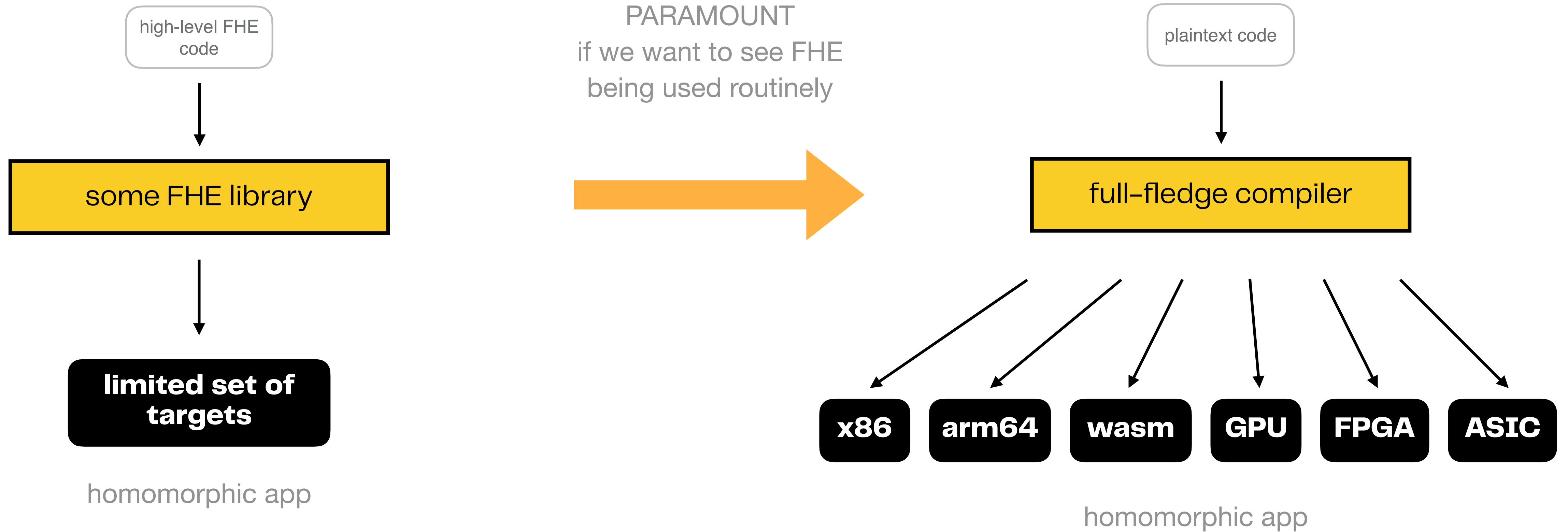
developer = cryptographer who tries to do best



The (poor) reality



What is blocking this transition?

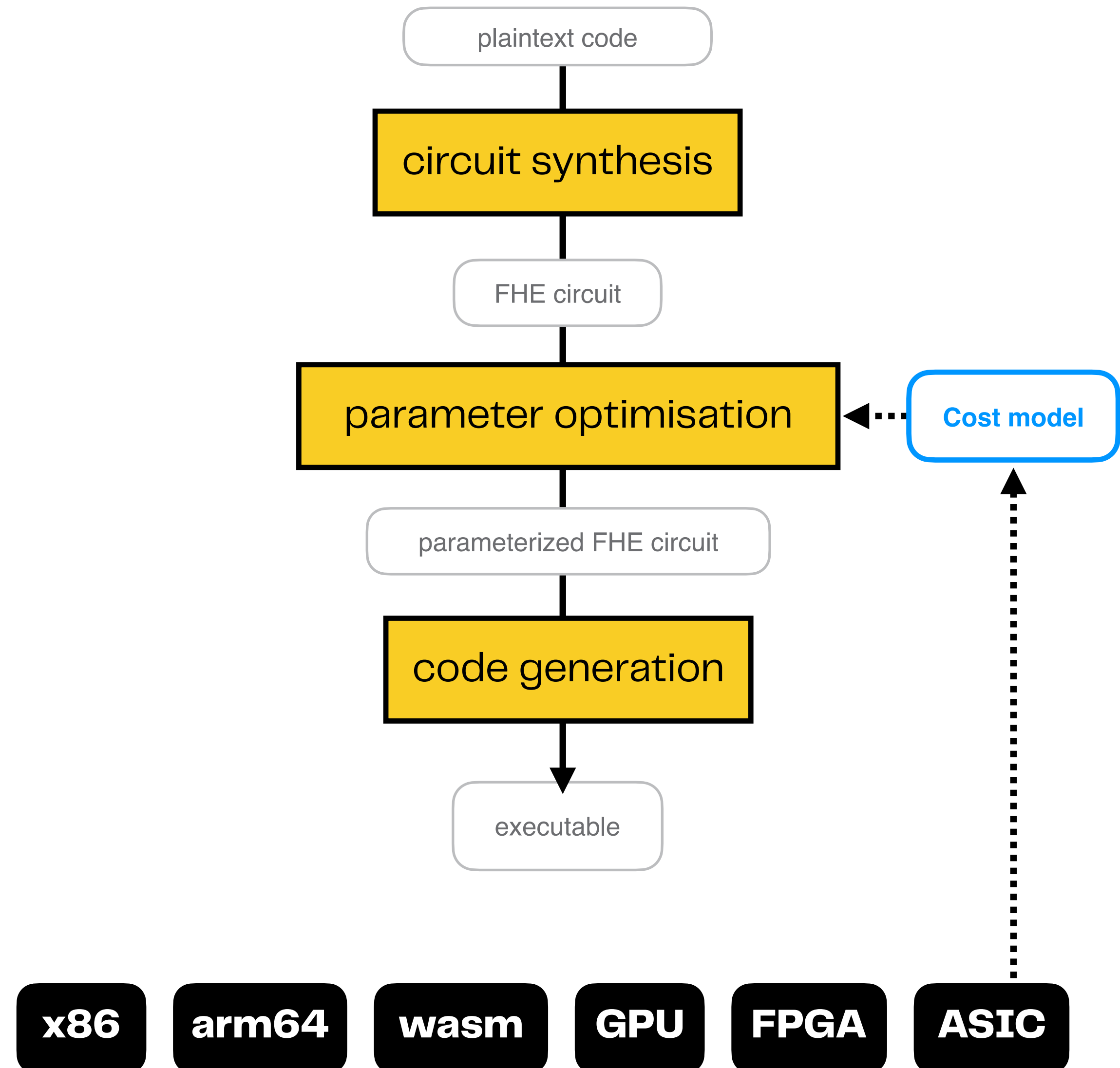


The inner ingredients of hom. compilers

1 Converts input code into an FHE circuit

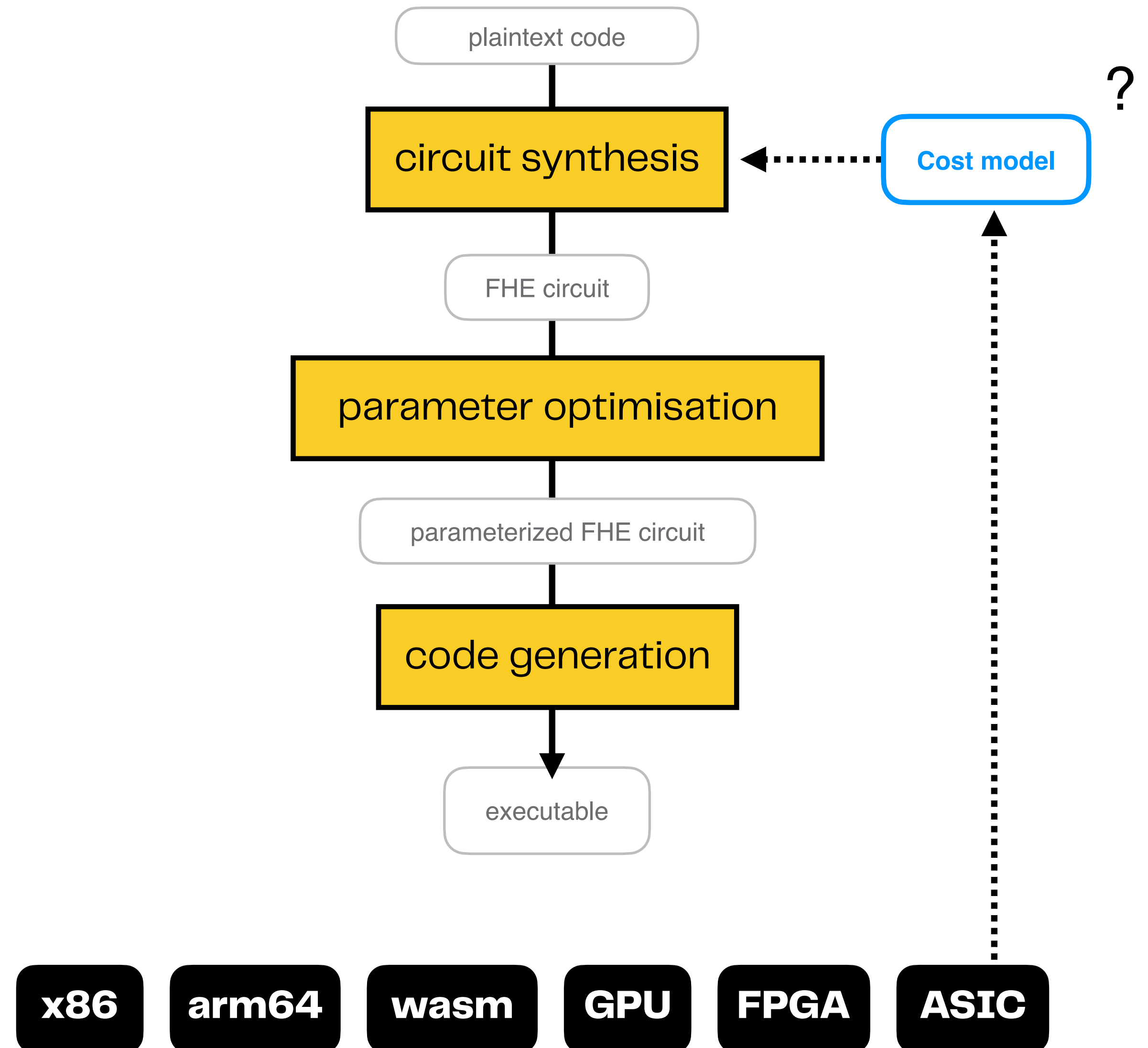
2 Optimizes that circuit (topology + params)

3 Generates an executable for a target architecture



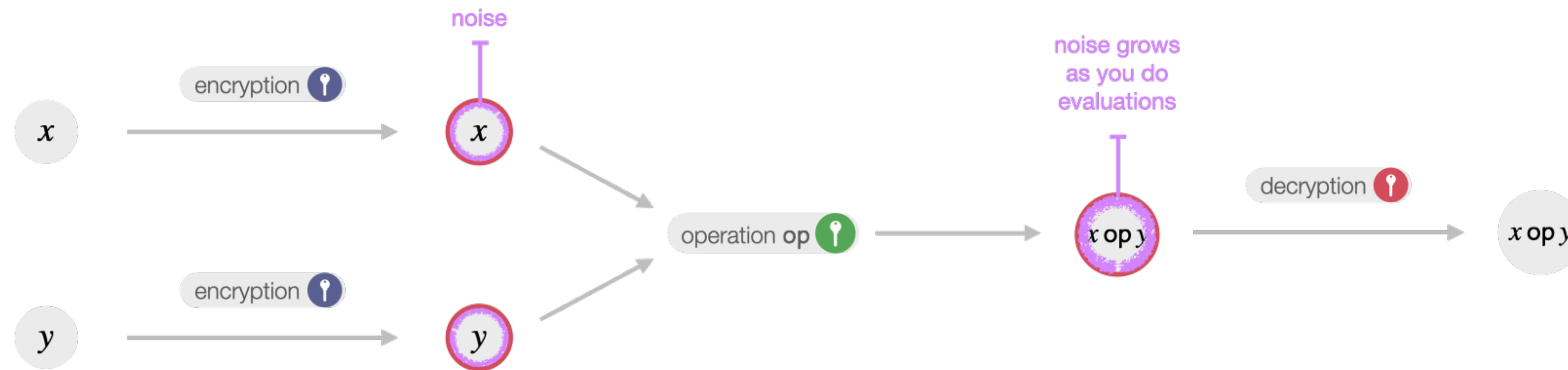
The inner ingredients of hom. compilers

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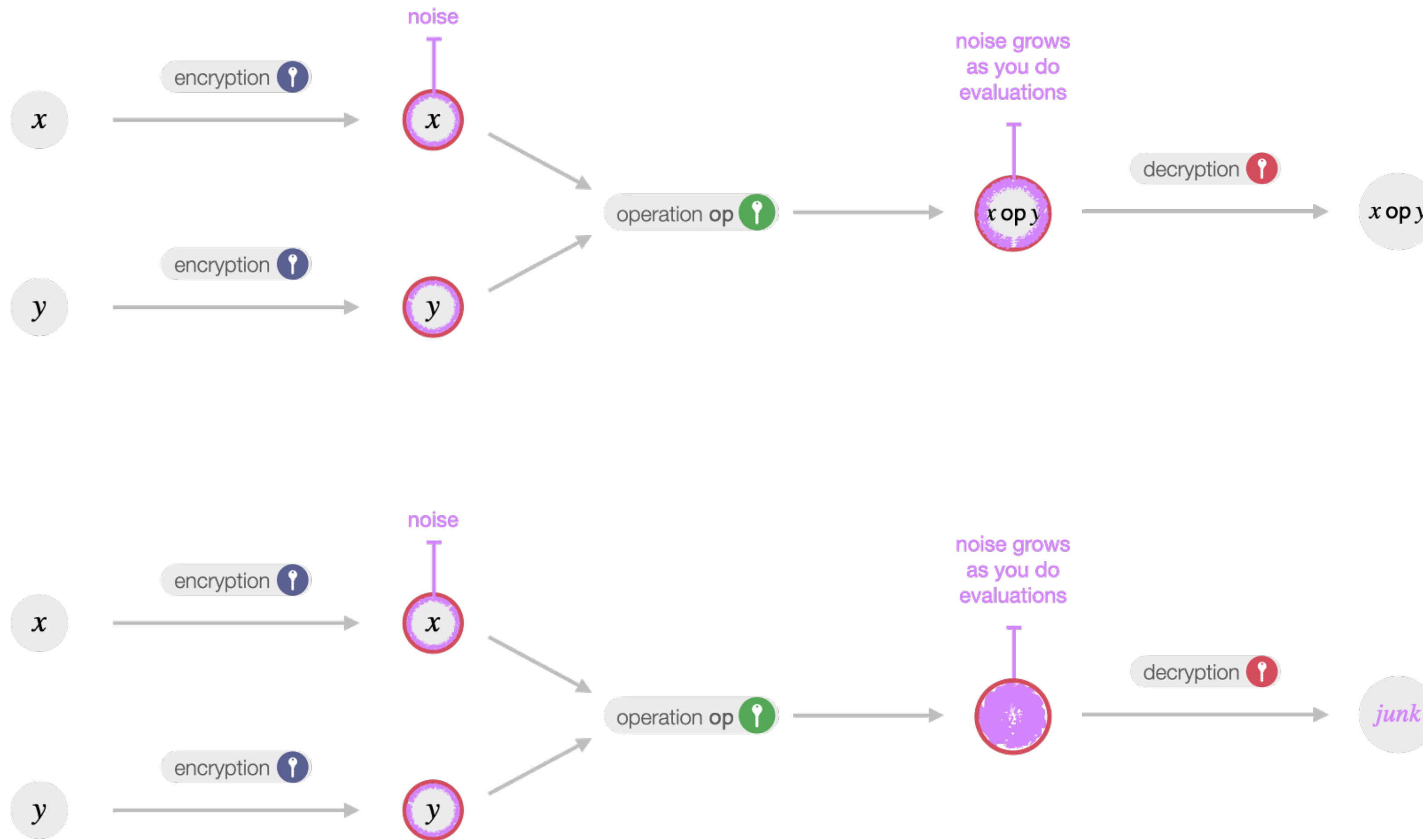


**What we are
building at Zama**

Homomorphic Encryption: Basic Concepts

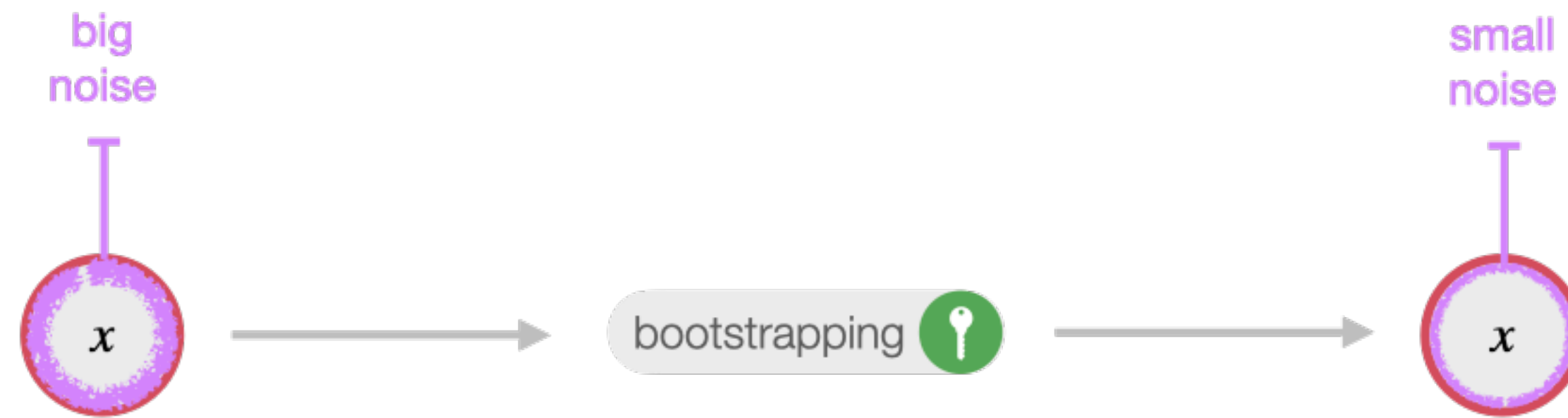


Homomorphic Encryption: Basic Concepts



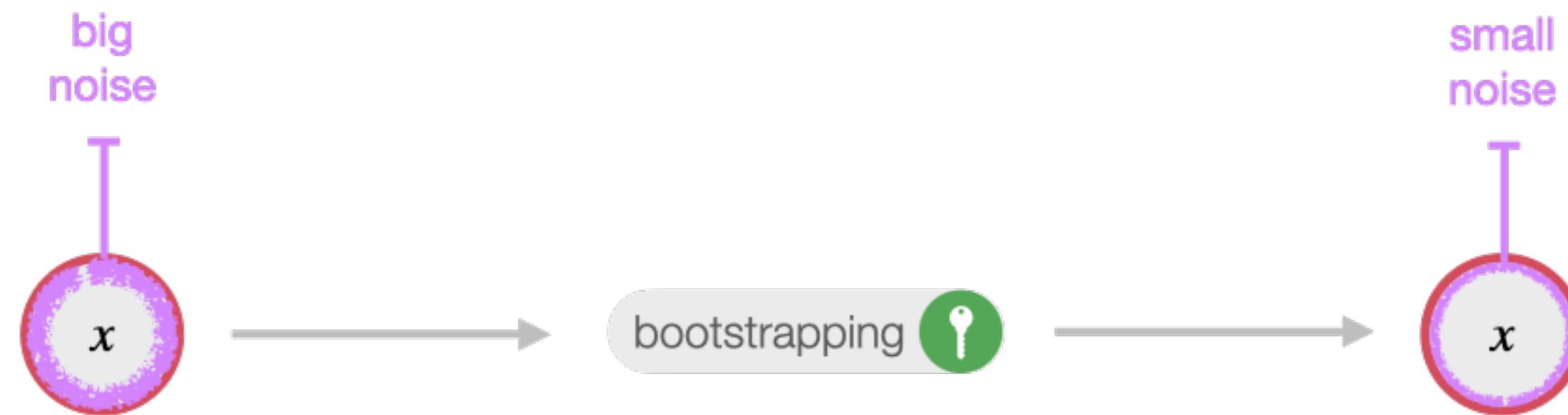
Homomorphic Encryption: Basic Concepts

Bootstrapping

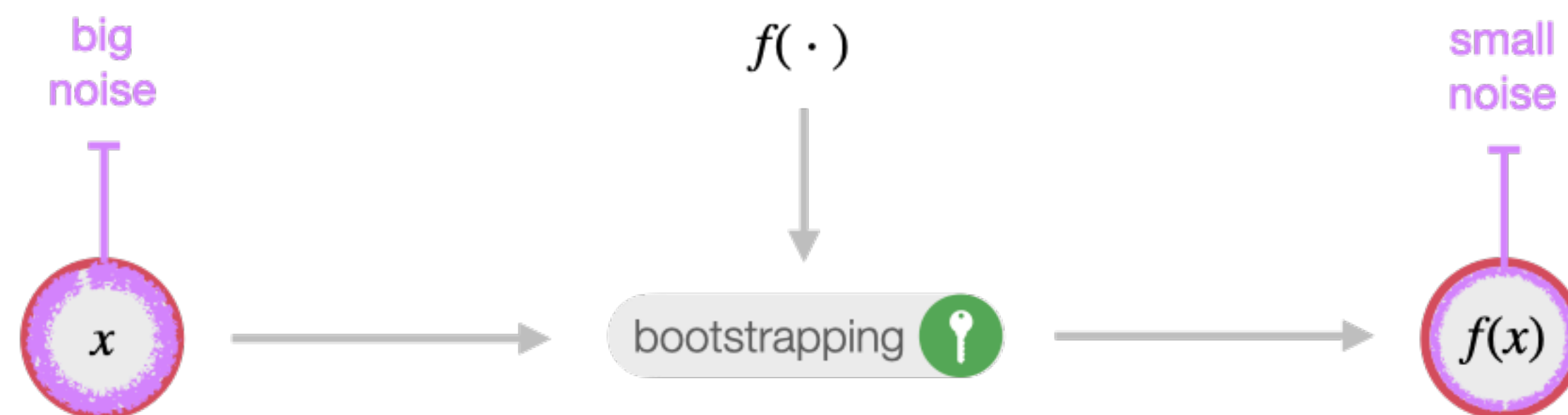


Homomorphic Encryption: Basic Concepts

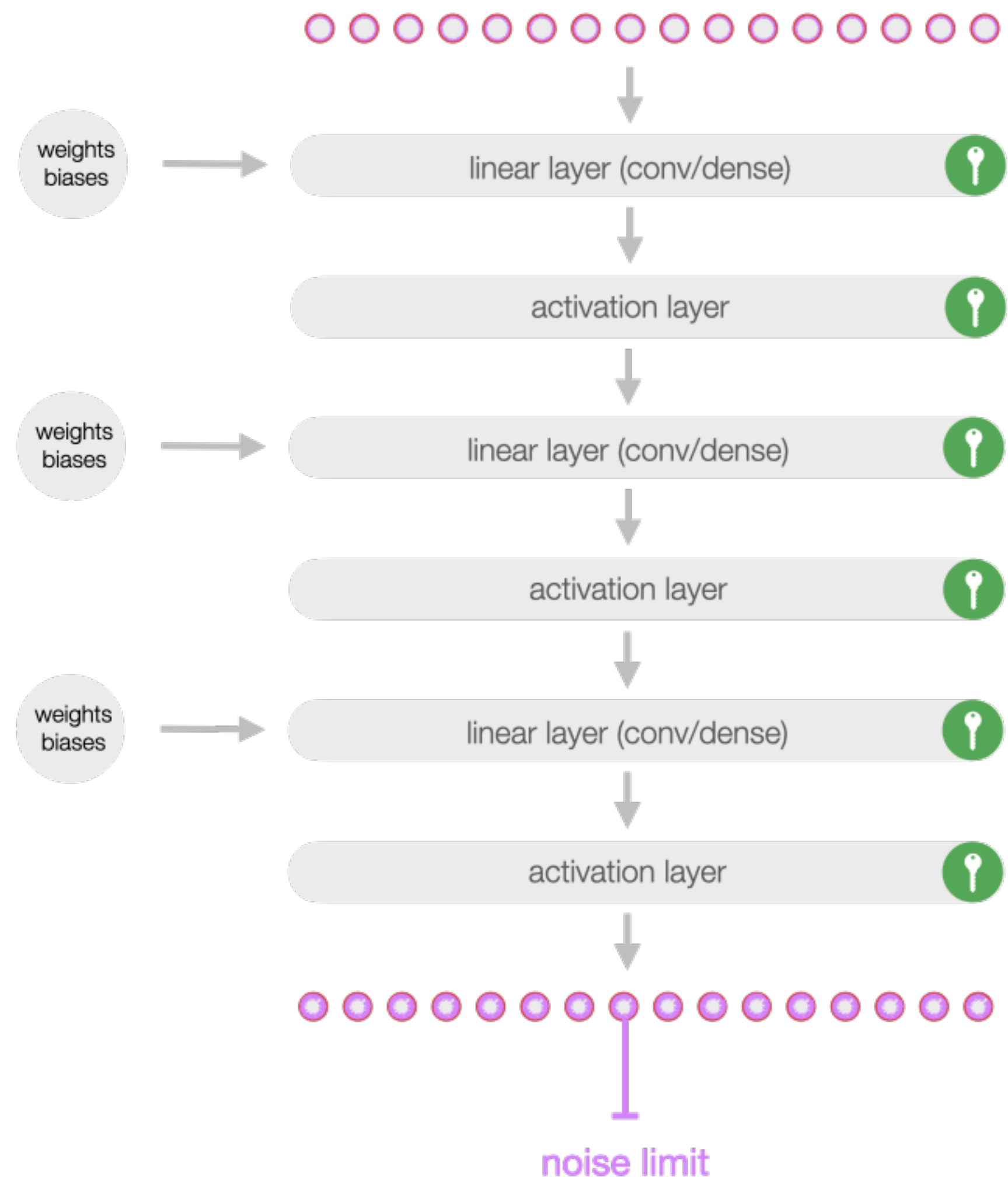
Bootstrapping



Programmable Bootstrapping
(PBS)



Homomorphic Inference: Leveled HE

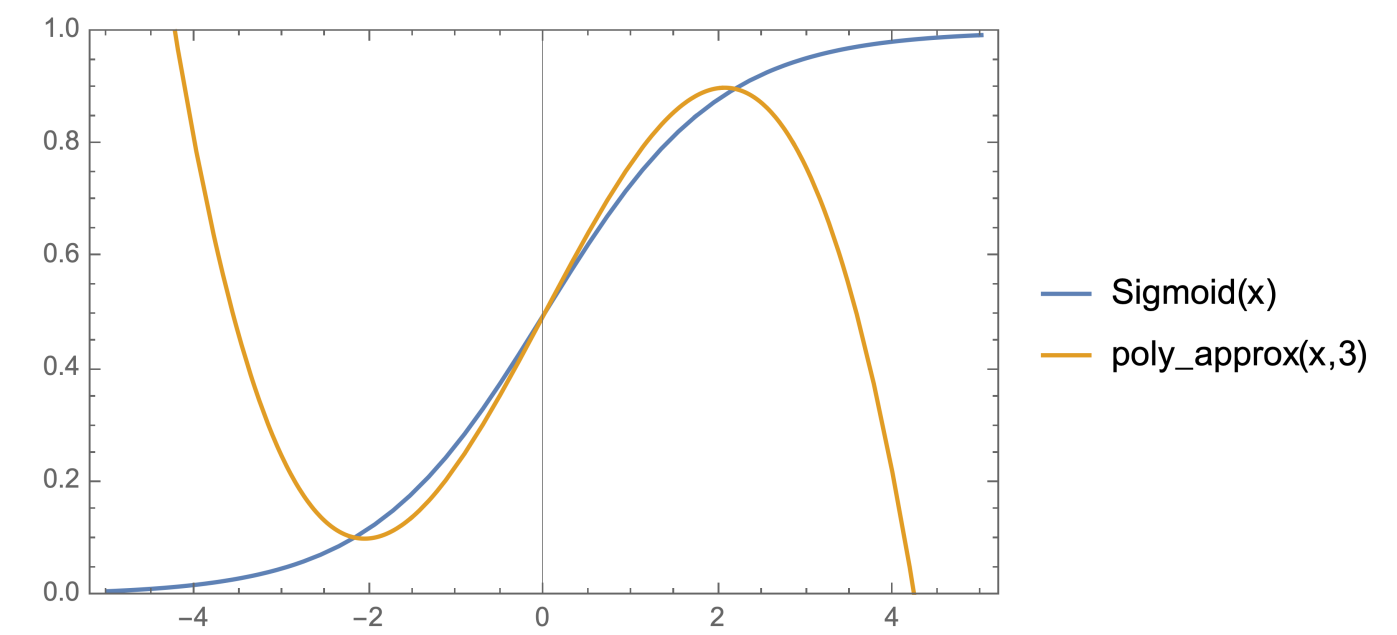


1

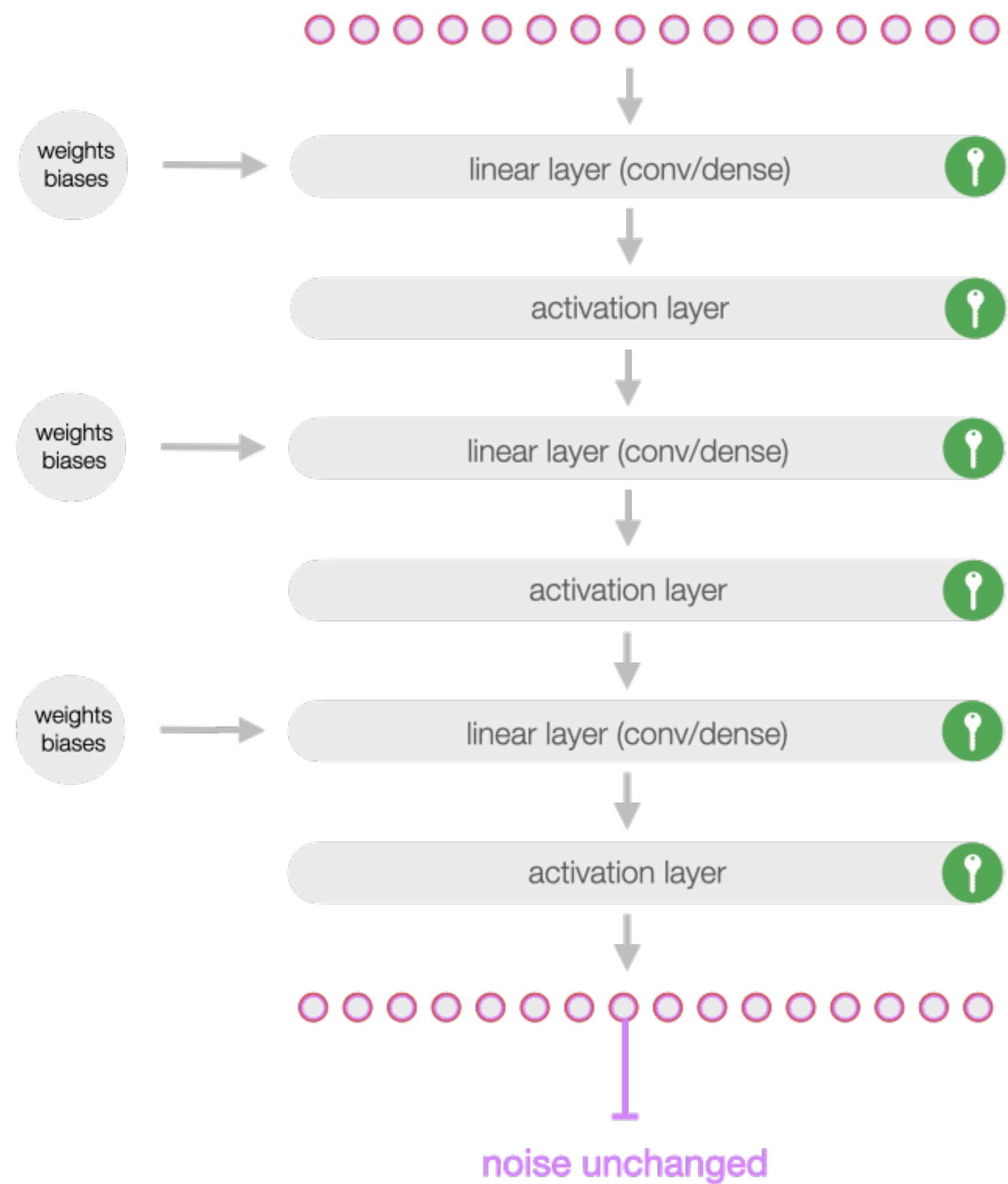
No bootstrapping: way too slow!

2

Activation layers: polynomial approximation



Homomorphic Inference: Fully HE

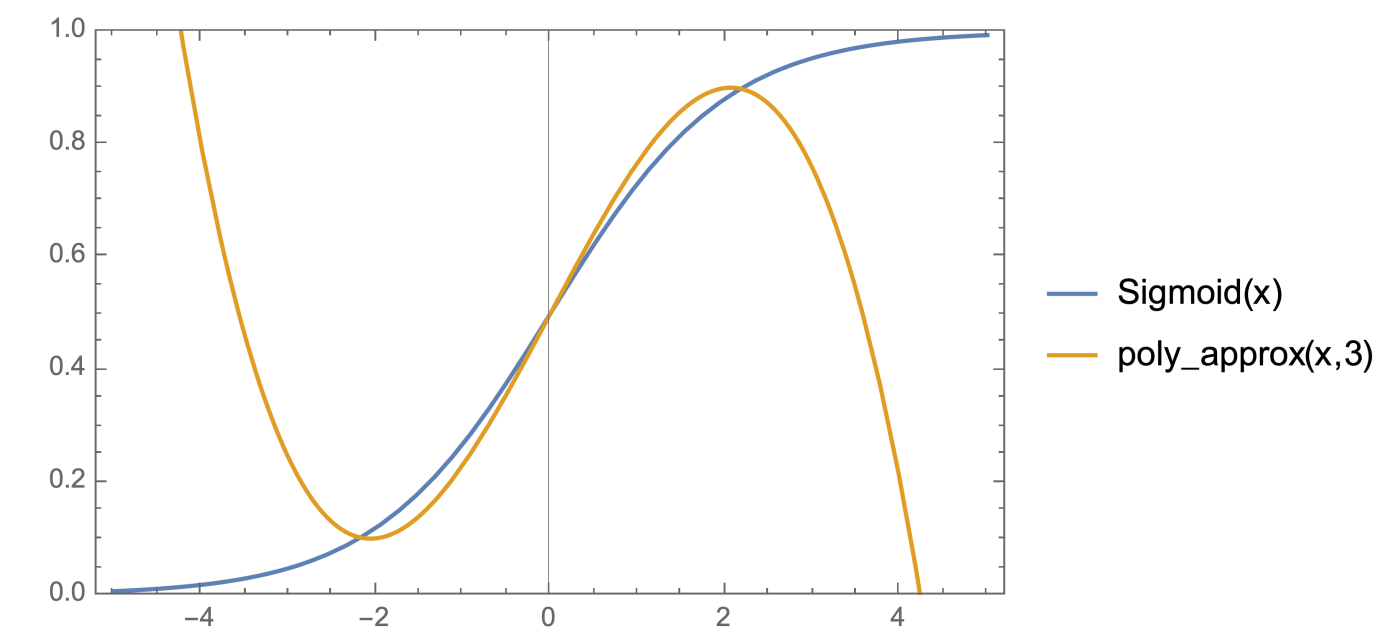


1

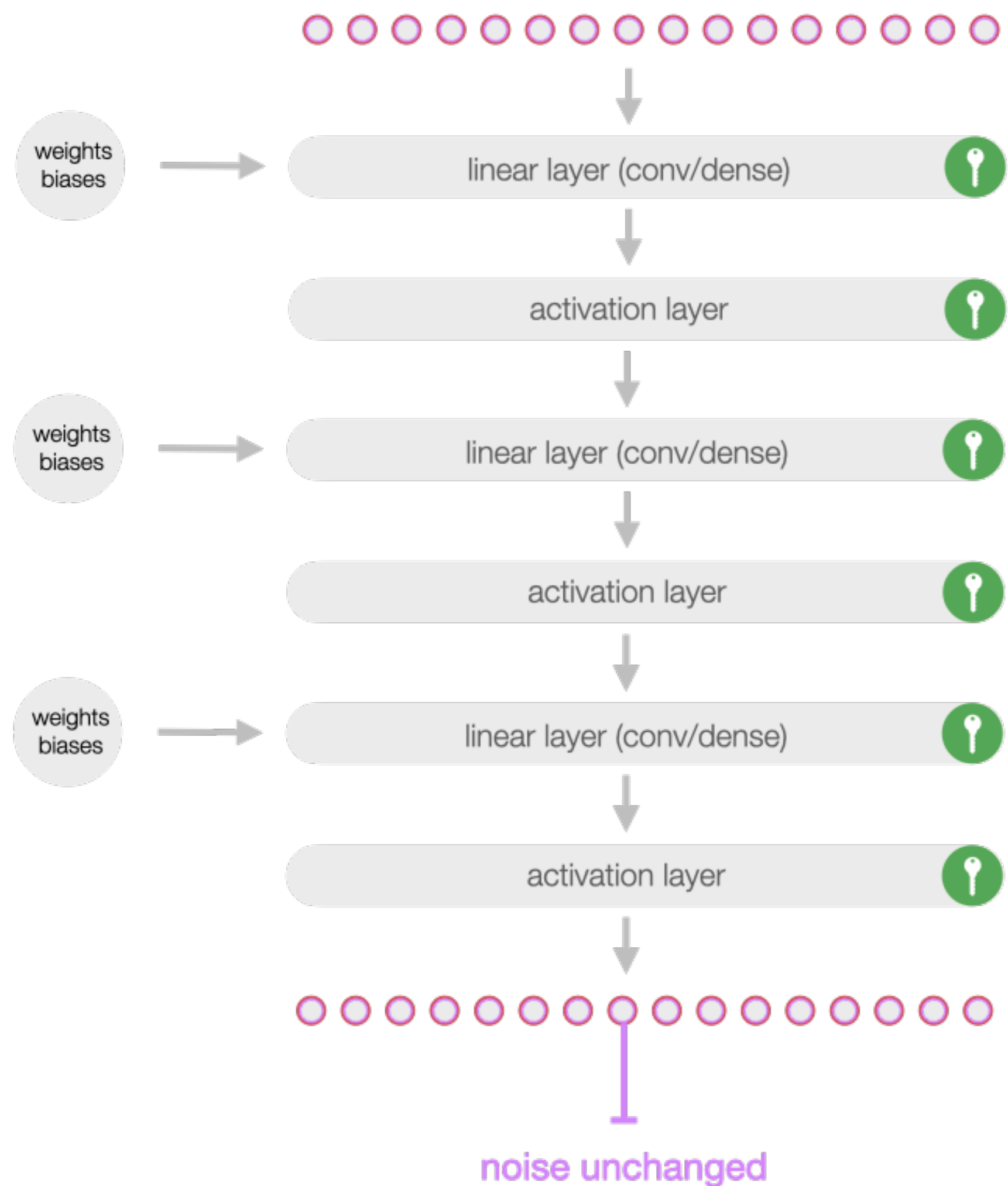
Bootstrapping (but super slow)

2

Activation layers: polynomial approximation

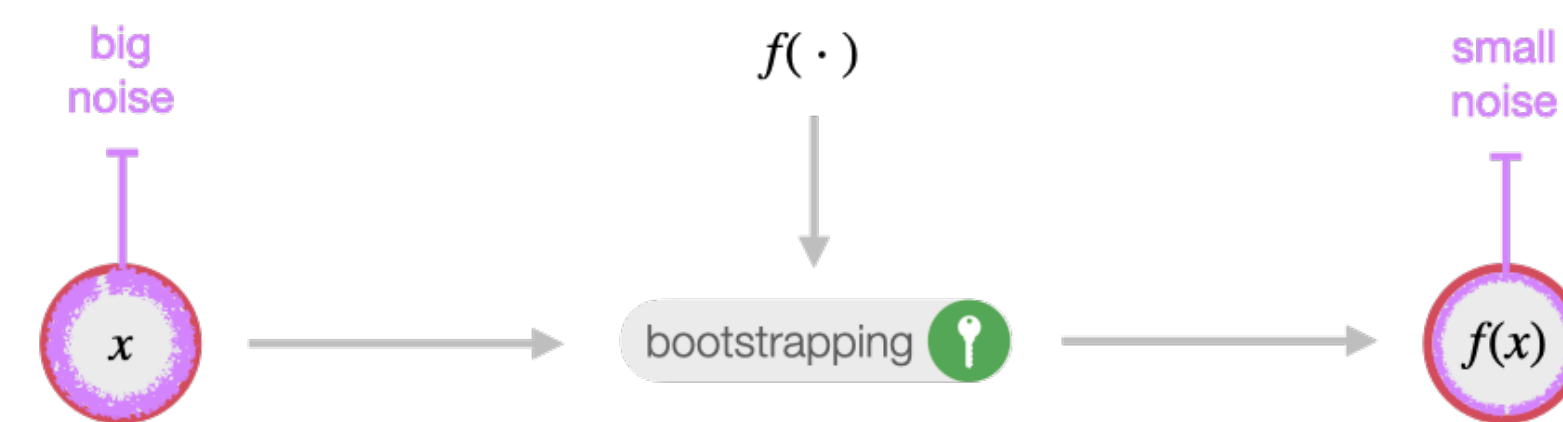


Homomorphic Inference: The “Zama way”



1+2

Activation layers: programmable bootstrapping



PBS is super fast: ~20 ms

A new computational paradigm



Kolmogorov Superposition
Theorem (KST)

1957

$$f(x_1, x_2, \dots, x_n) = \sum_{i=1}^{2n+1} g_i \left(\sum_{j=1}^n f_{ij}(x_j) \right)$$

univariate

A new computational paradigm



Kolmogorov Superposition
Theorem (KST) 1957

Ridge decomposition
or approximation

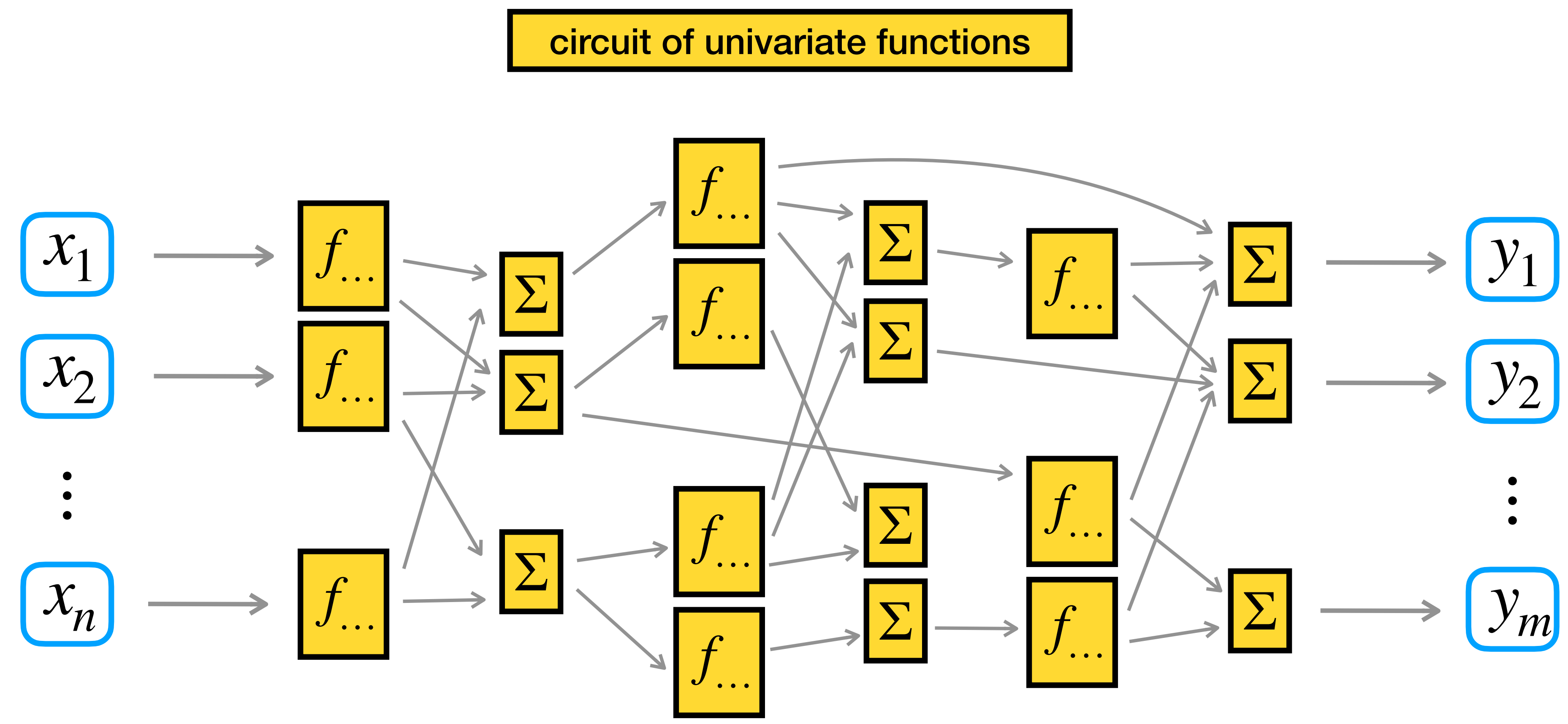
$$f(x_1, x_2, \dots, x_n) = \sum_{i=1}^{2n+1} g_i \left(\sum_{j=1}^n f_{ij}(x_j) \right)$$

↑ ↑
univariate univariate

$$f(x_1, \dots, x_n) \approx \sum_{i=1}^r g_i \left(\sum_{j=1}^n a_{ij} \cdot x_j \right)$$

↑ ↑
univariate $a_{ij} \in \mathbb{Z}$

A new computational paradigm



= graph mixing univariate functions and linear combinations

The Concrete Compiler

(and a glance at the whole Concrete stack)

Exact vs. approximate computing with TFHE

exact
paradigm

approximate
paradigm

plaintexts

$$m \in \mathbb{Z}_p$$

- p odd
- p even

continuous torus
encoding

$$\mu \in \mathbb{R}/\mathbb{Z}$$

$$\mu = \frac{m}{p} \pmod{1}$$

discretized torus
encoding

$$\mu \in \mathbb{Z}_{2^{32}}$$

$$\mu = \left\lfloor \frac{2^{32}}{p} m \right\rfloor \pmod{2^{32}}$$

noisy discretized
torus

$$\mu + \varepsilon \in \mathbb{Z}_{2^{32}}$$

$$\text{pdf} \left(\frac{\varepsilon}{2^{32}} \right) \approx \mathcal{N}(0, \sigma)$$

encrypted domain

$$\text{LWE}_{sk}(\mu + \varepsilon)$$

$$\in \mathbb{Z}_{2^{32}}^{n+1}$$

$$x \in [x^-, x^+] \subset \mathbb{R}$$

$$\text{pdf}(x) \approx \mathcal{N}(x, \sigma_x)$$

$$\Pr [x \notin [x^-, x^+]] = 0$$

$$\mu \in \mathbb{R}/\mathbb{Z}$$

$$\mu = \frac{x - x^-}{x^+ - x^-} \pmod{1}$$

$$\mu \in \mathbb{Z}_{2^{32}}$$

$$\mu = \left\lfloor \frac{x - x^-}{x^+ - x^-} \cdot 2^{32} \right\rfloor \pmod{2^{32}}$$

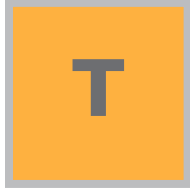
Modular TFHE circuits

exact paradigm

$$m \in \mathbb{Z}_p$$



= multisum



= lookup table



= grab

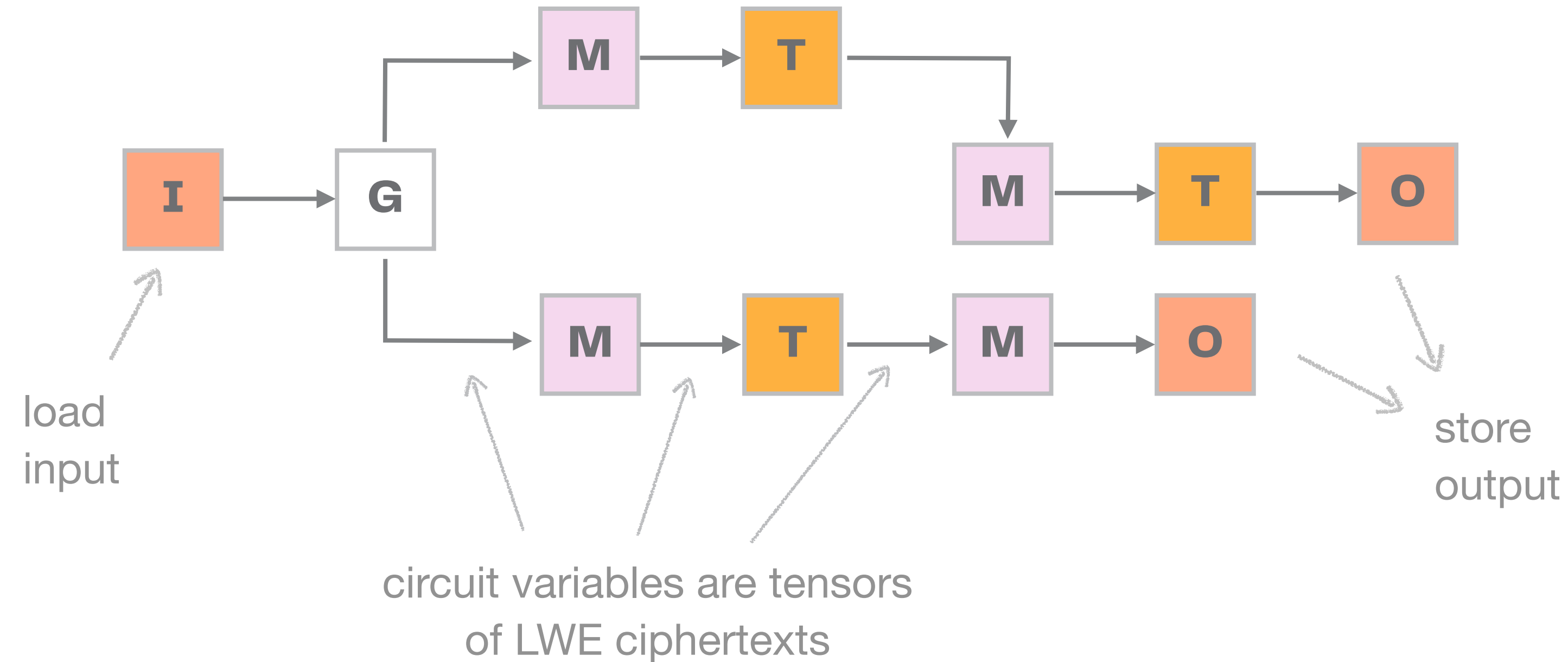
- concatenation
- splits
- reshape



= input

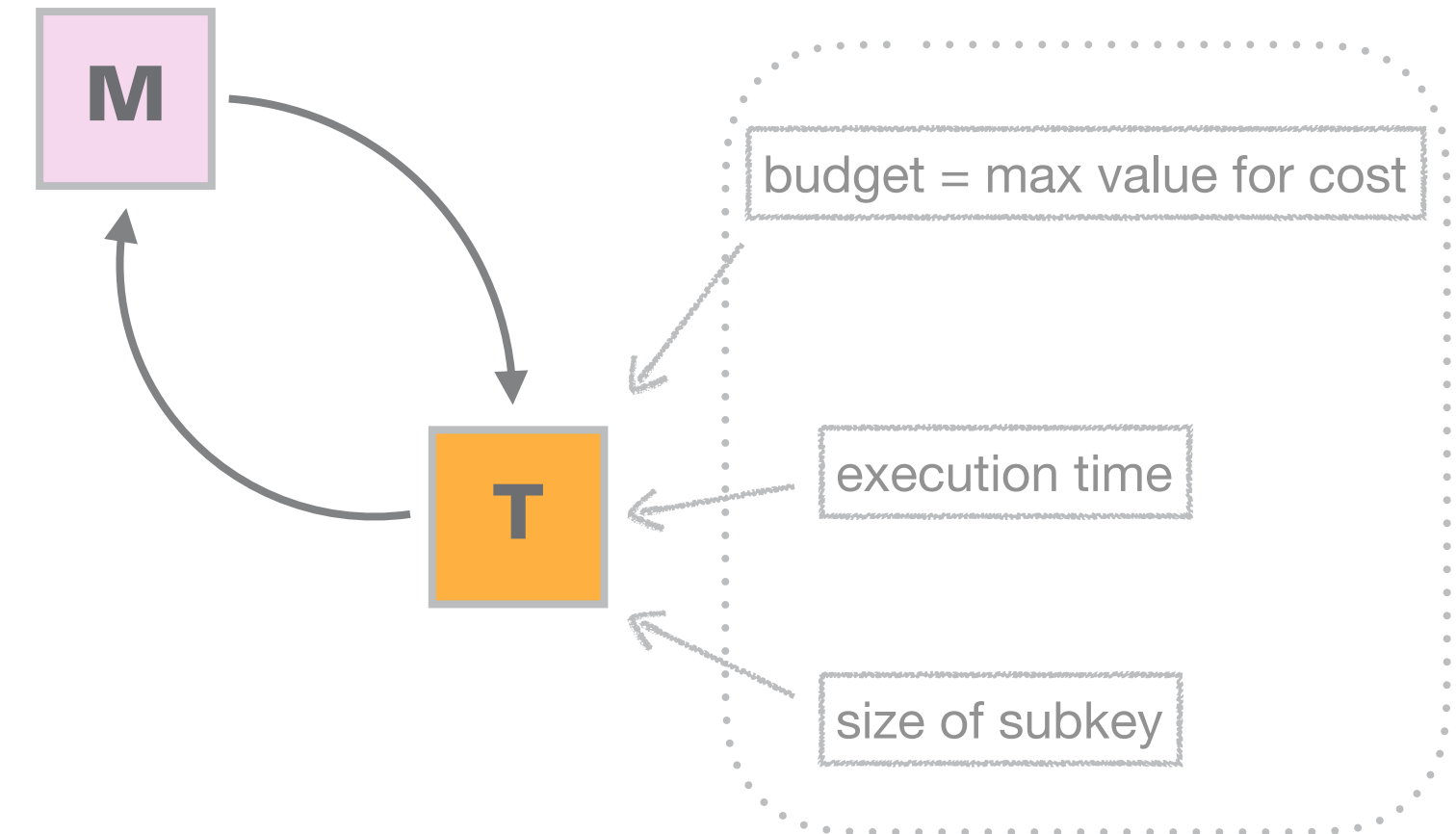


= output



$$\text{cost} = \sum_i |a_i|_p^2$$

$|a|_p = \min(a, p - a)$



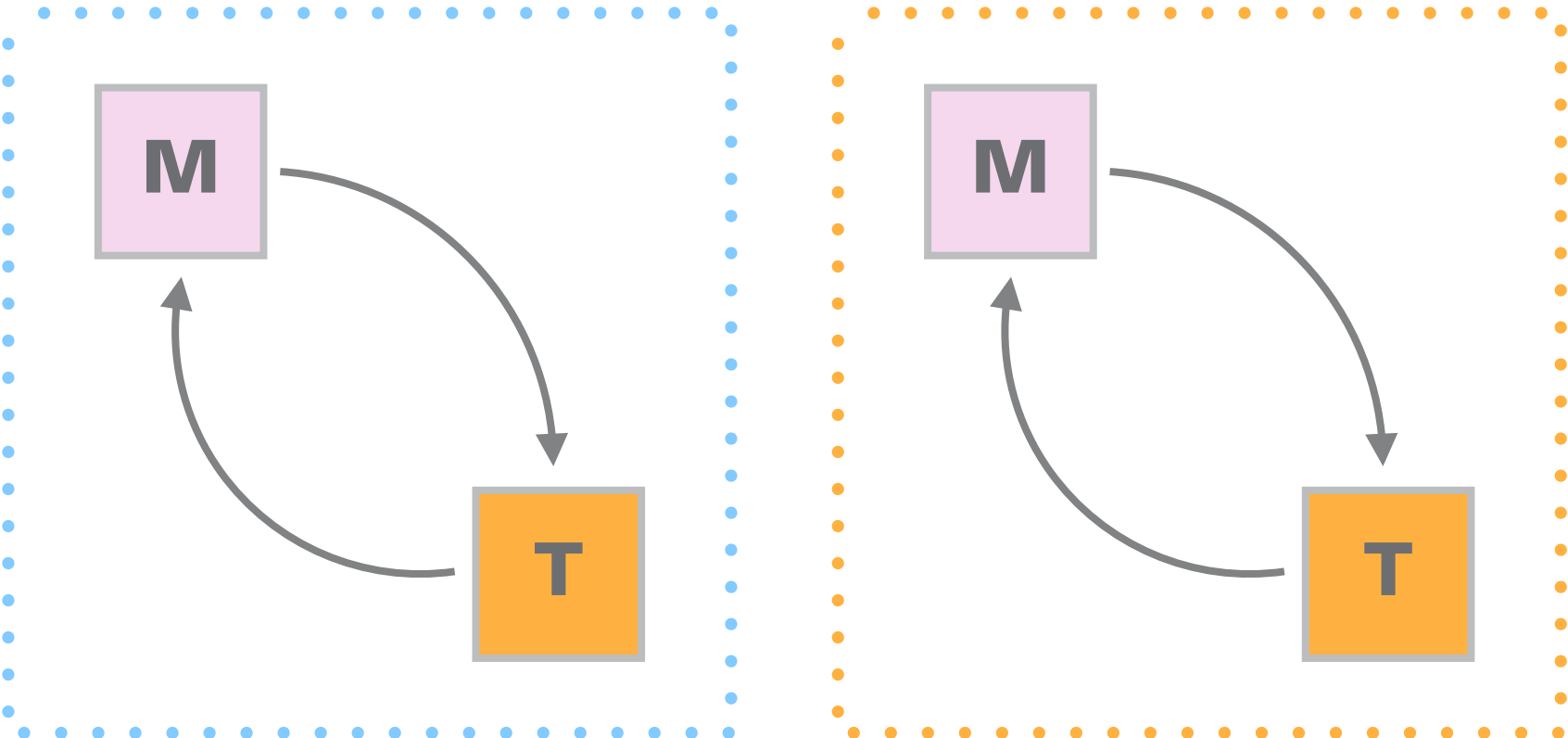
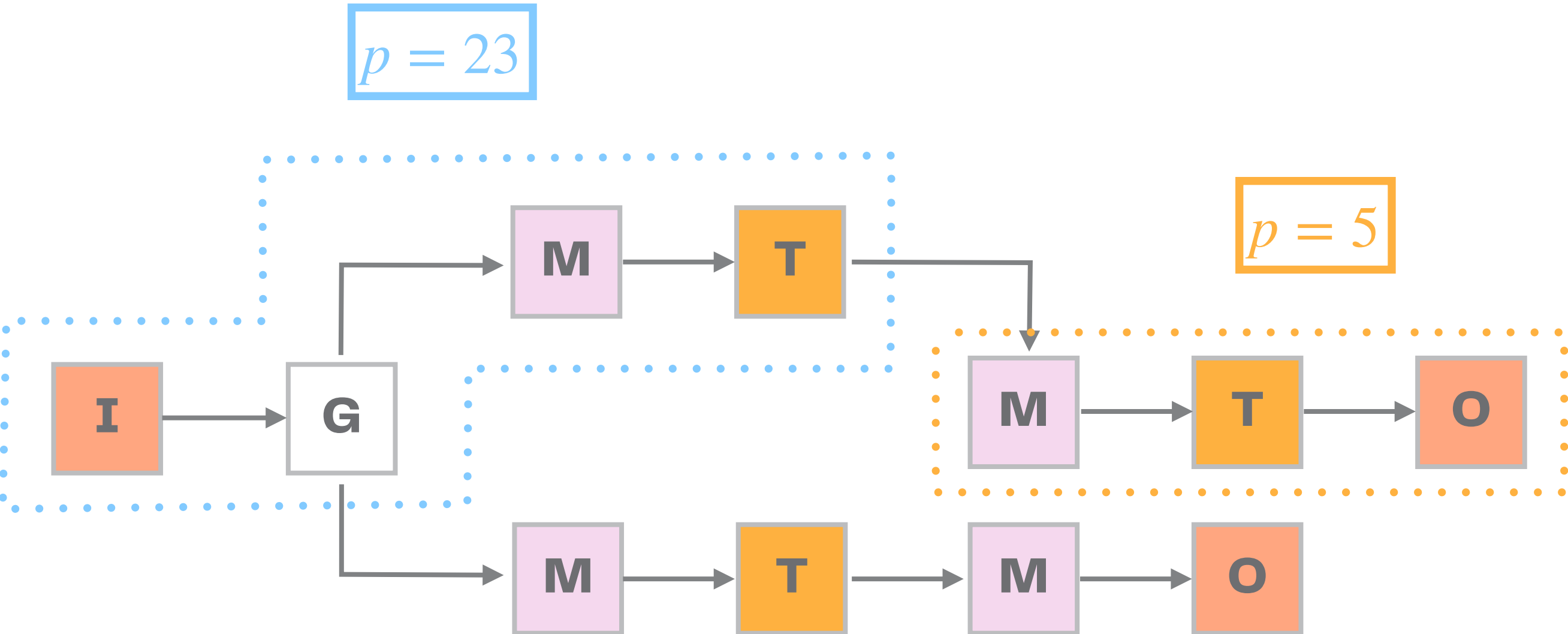
complex parametrization

Multi-modular TFHE circuits

exact paradigm

$$m \in \mathbb{Z}_p$$

Circuits typically switch back and forth between several moduli p_1, \dots, p_k



$p = 23$
 budget
 running time
 subkey size

\neq

$p = 5$
 budget
 running time
 subkey size

automated generation of optimal parameters from circuit topology

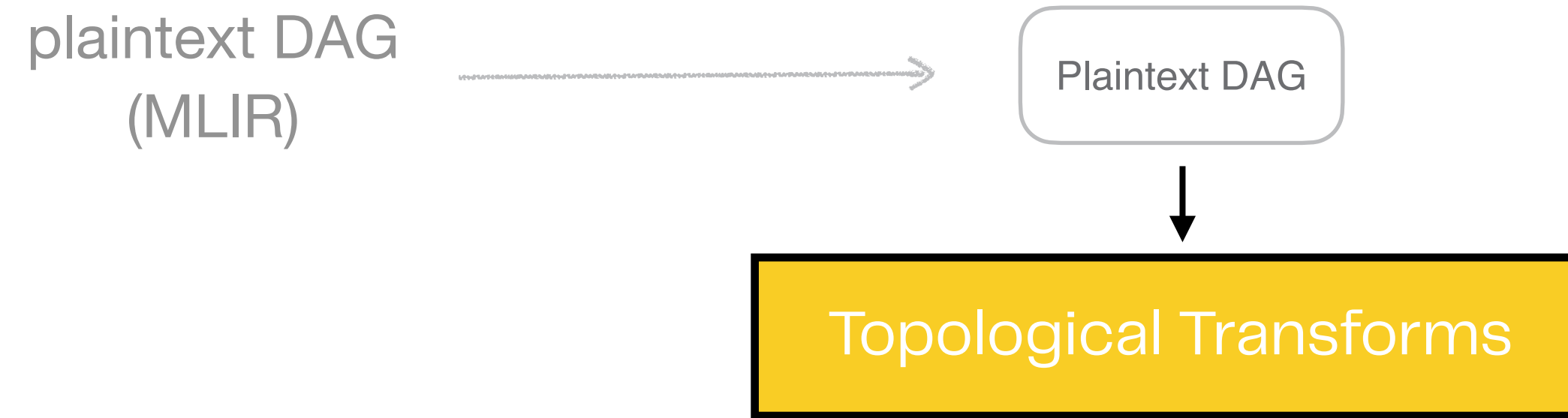
The Concrete Compiler (CC)

Plaintext DAG

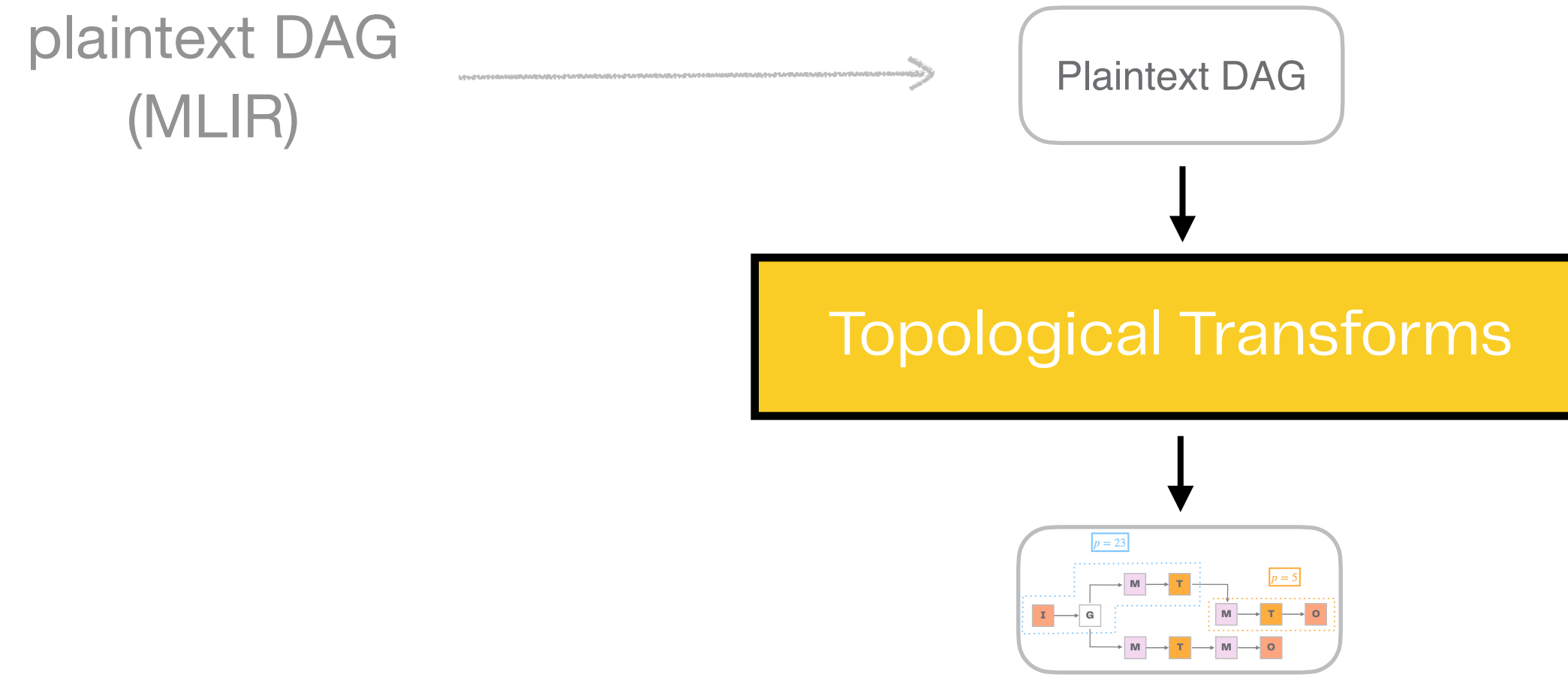
The Concrete Compiler (CC)



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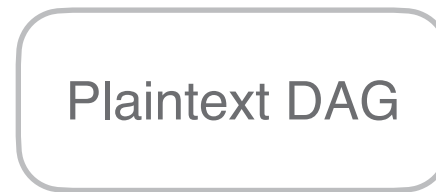


The Concrete Compiler (CC)

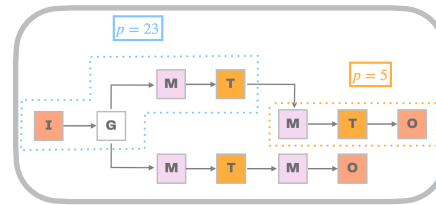


The Concrete Compiler (CC)

plaintext DAG
(MLIR)



TFHE DAG
(MLIR)



The Concrete Compiler (CC)

plaintext DAG
(MLIR)



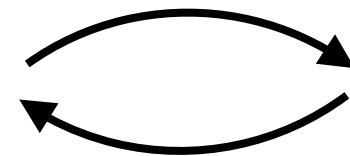
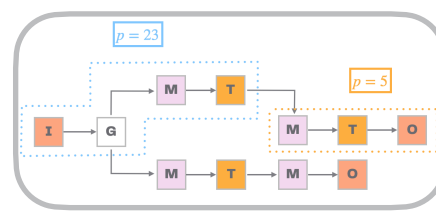
Plaintext DAG



Topological Transforms

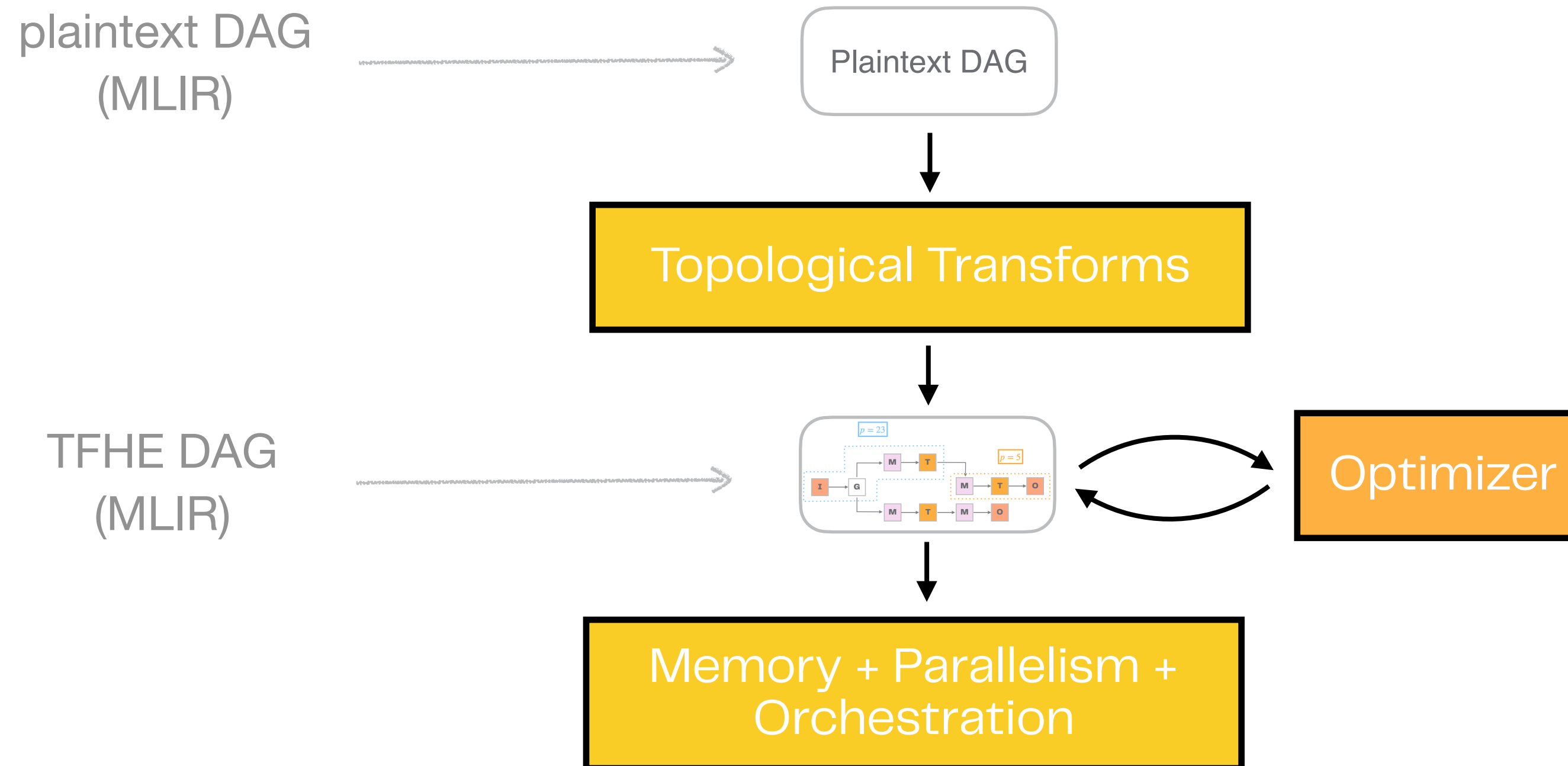


TFHE DAG
(MLIR)

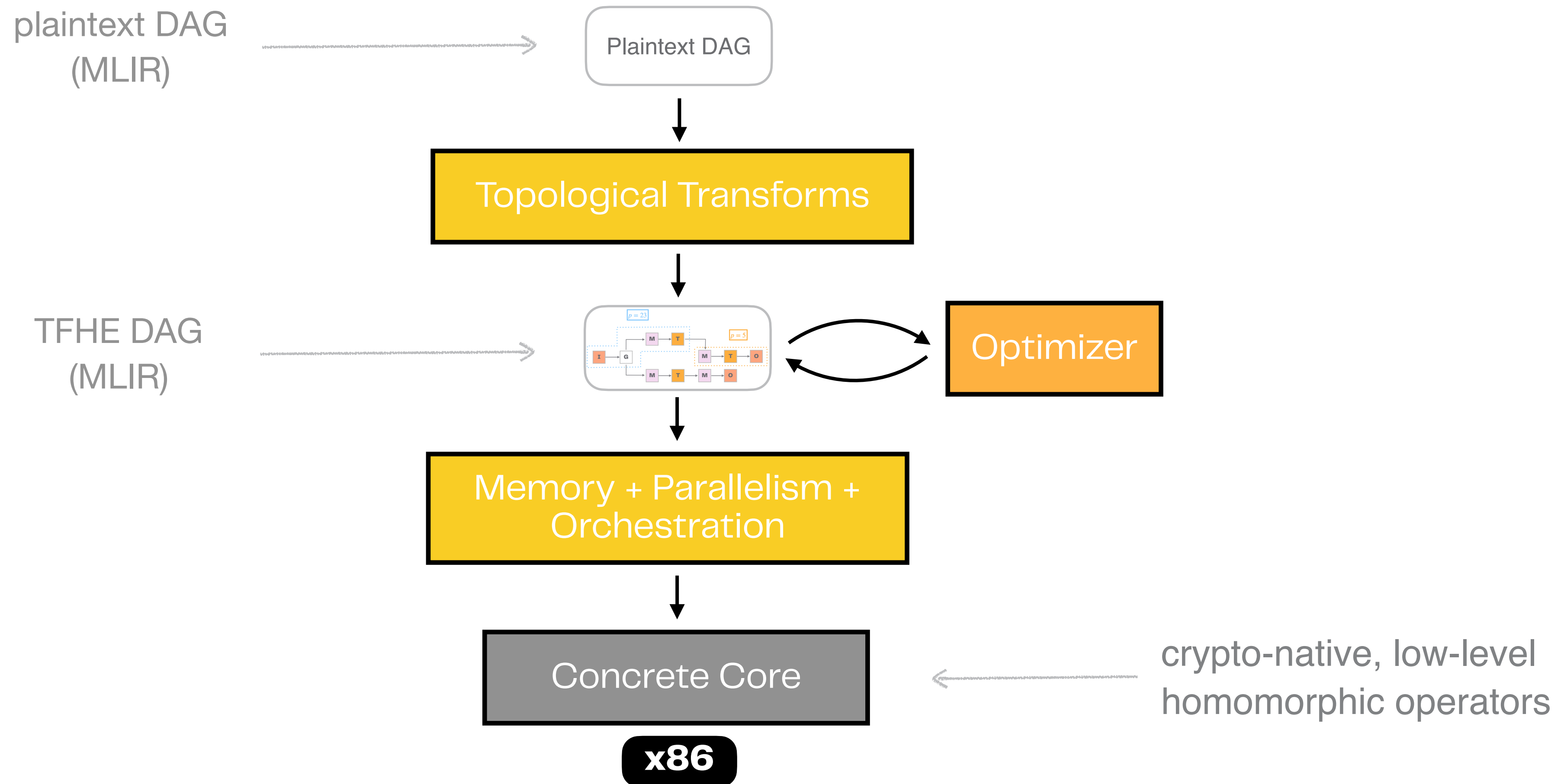


Optimizer

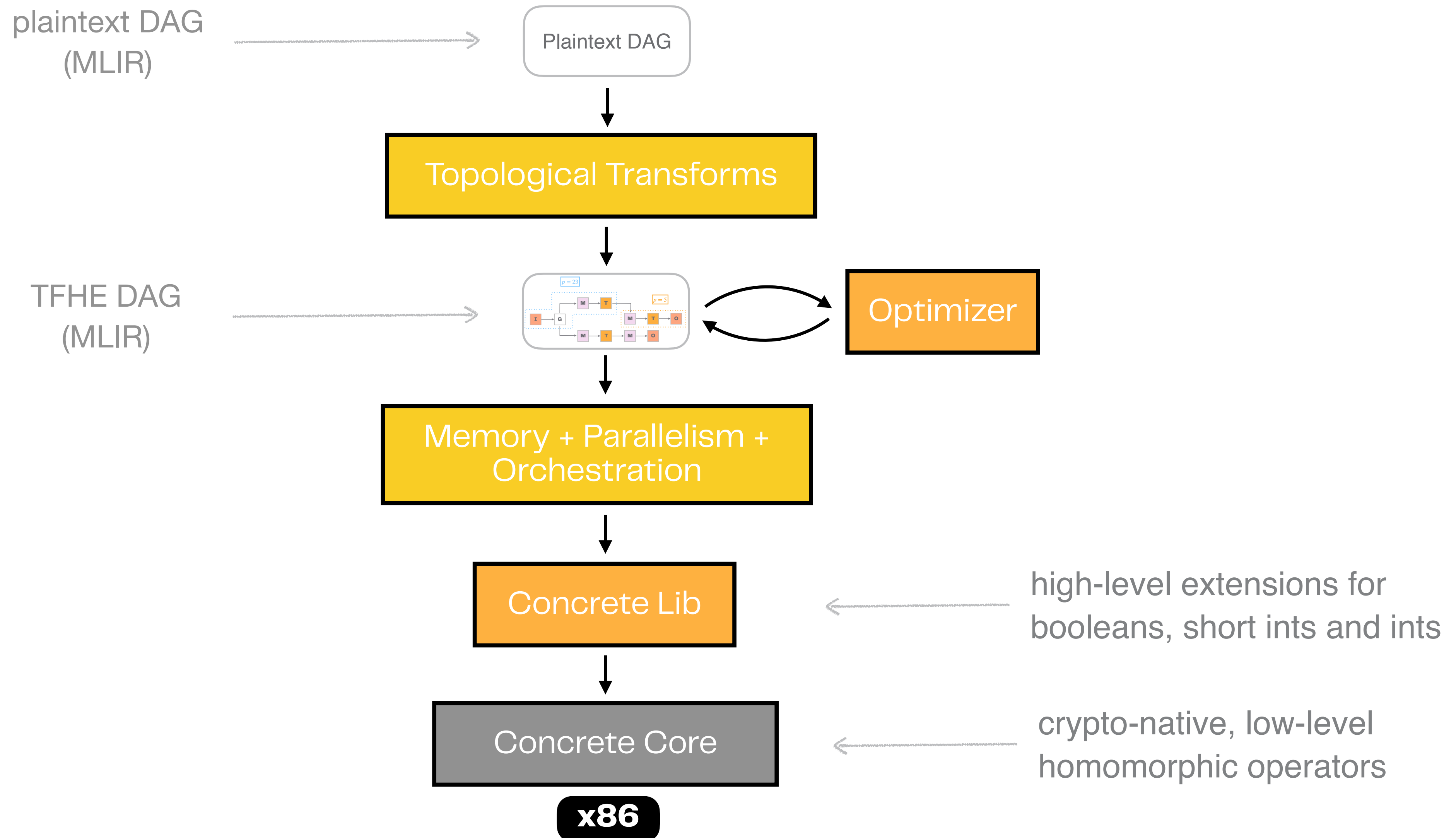
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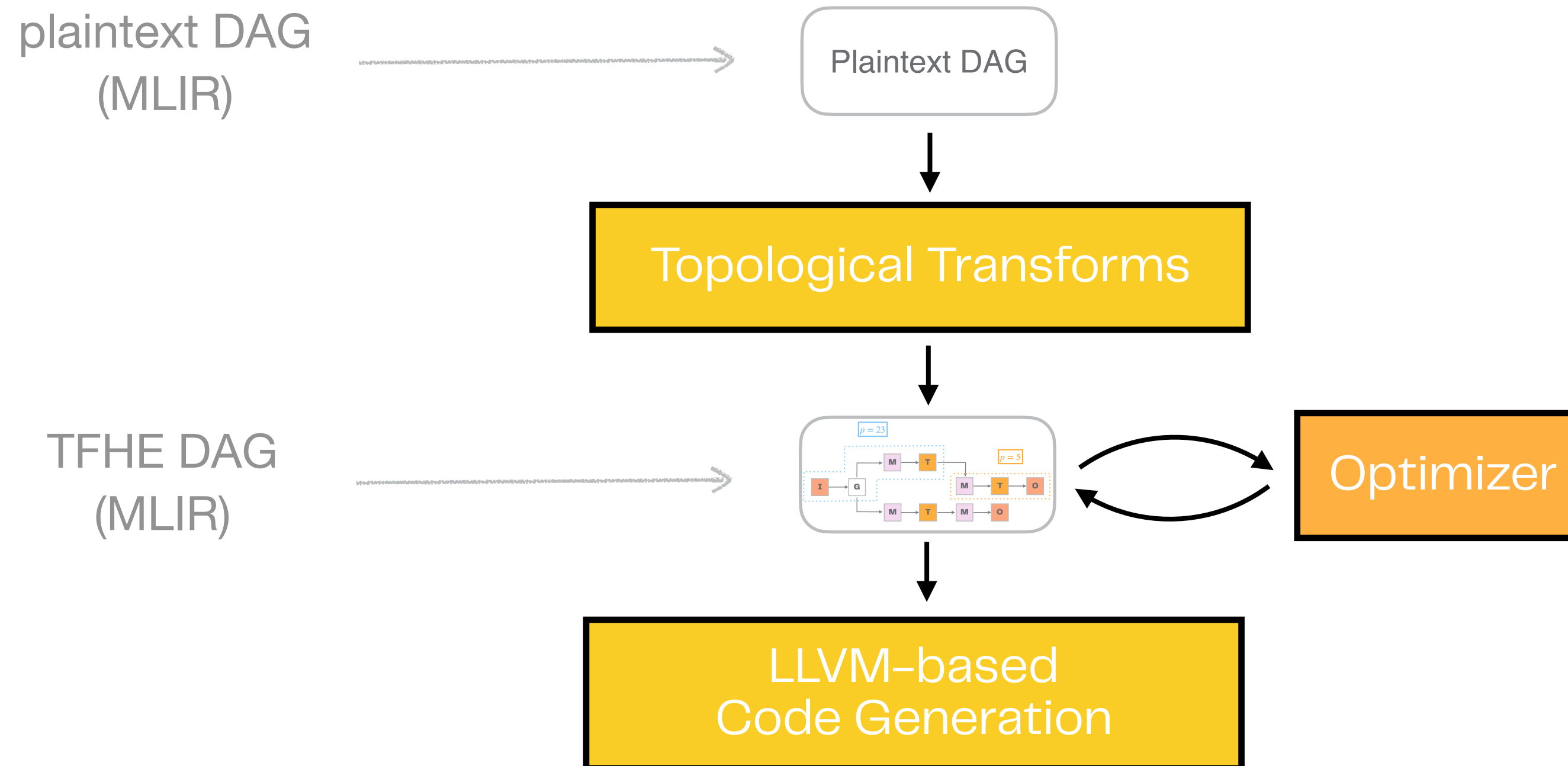
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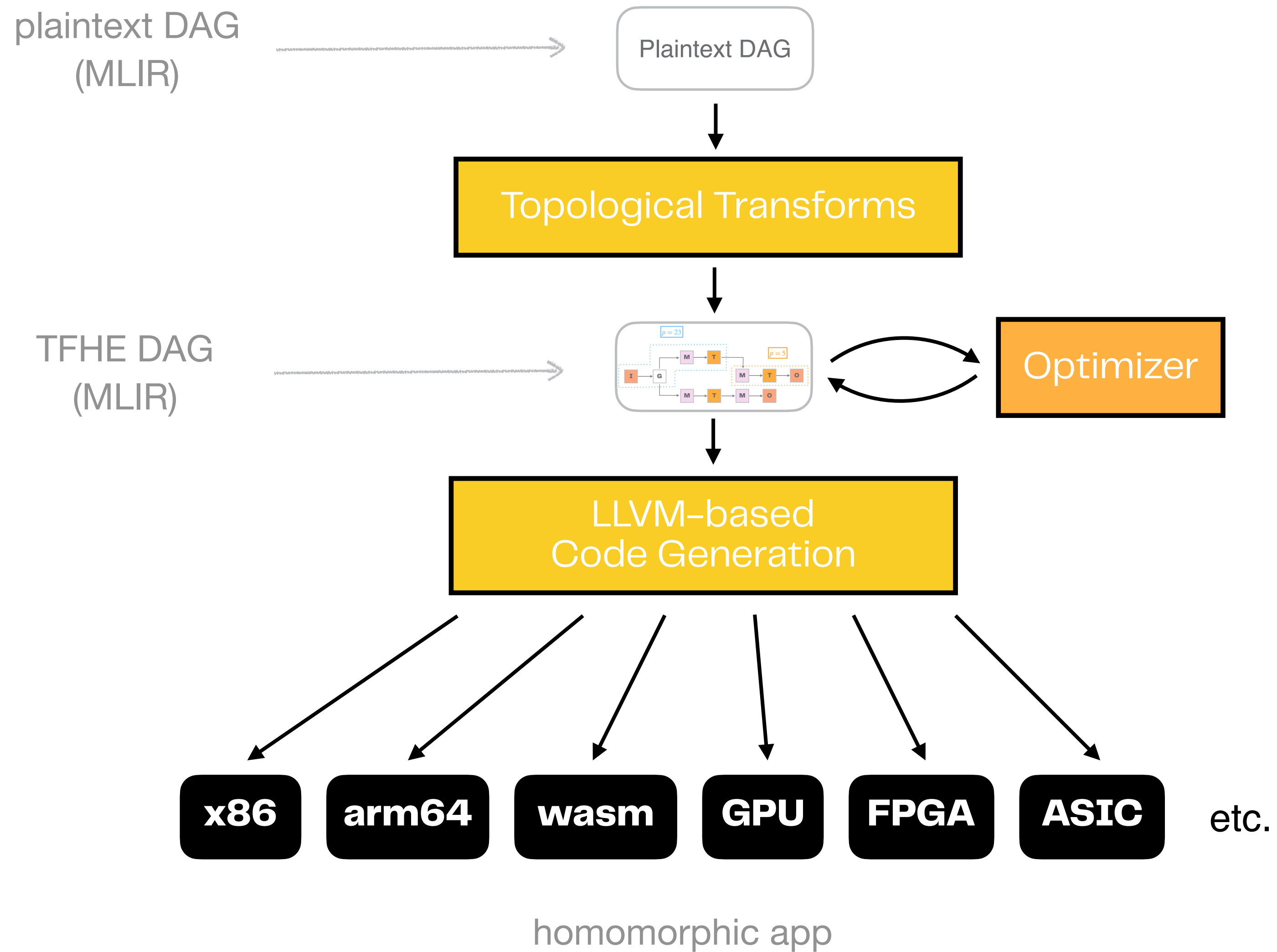
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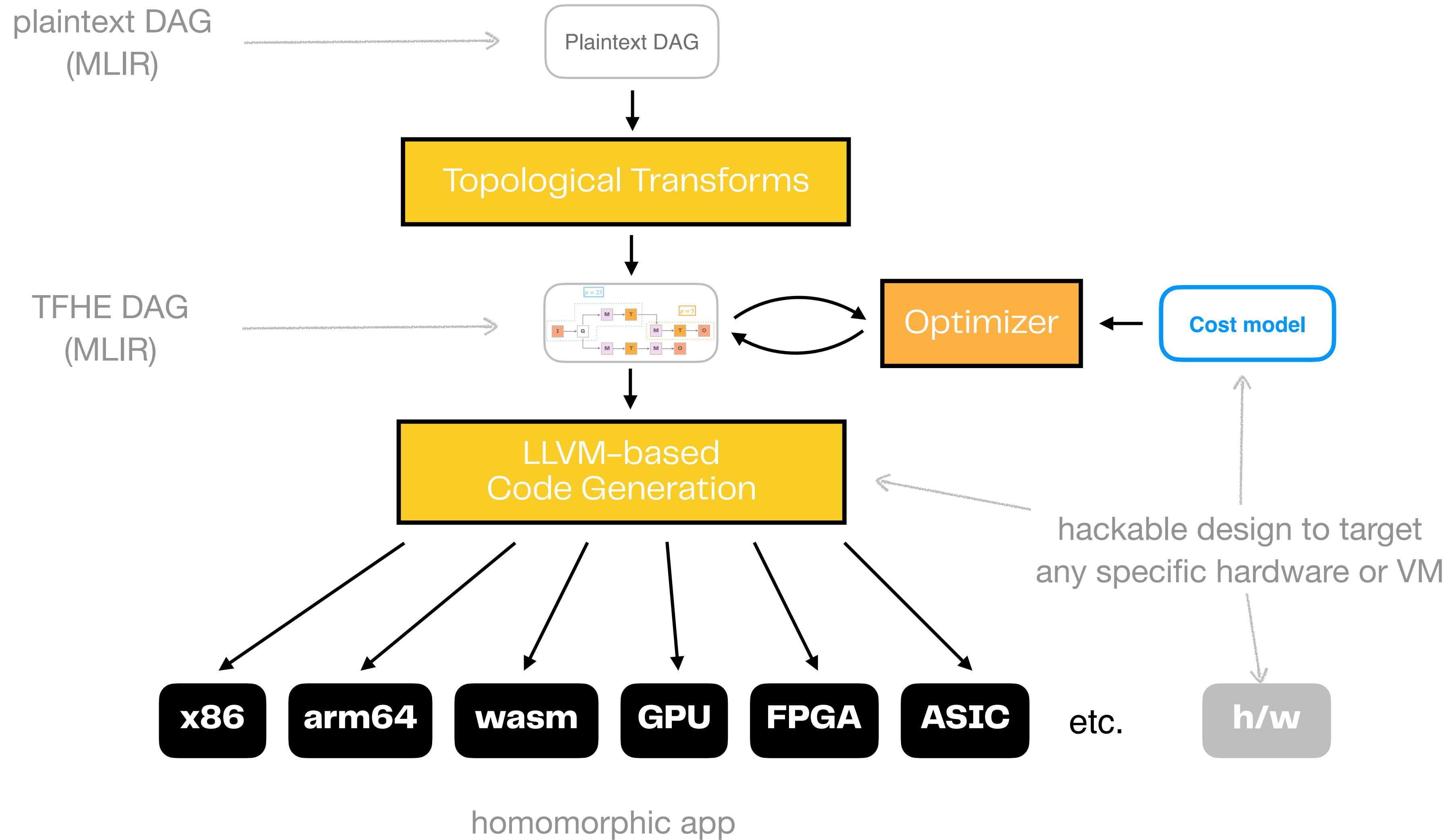
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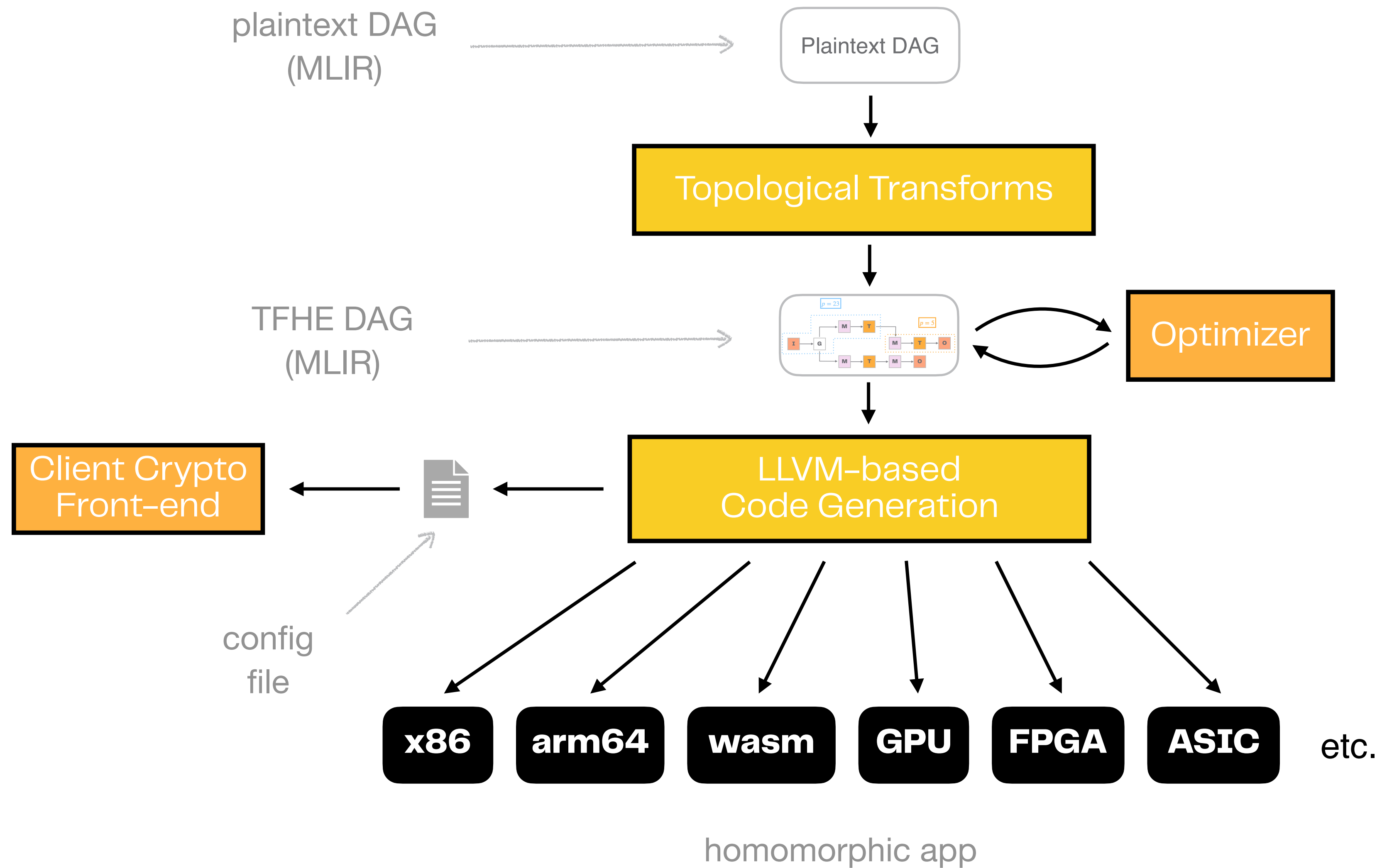
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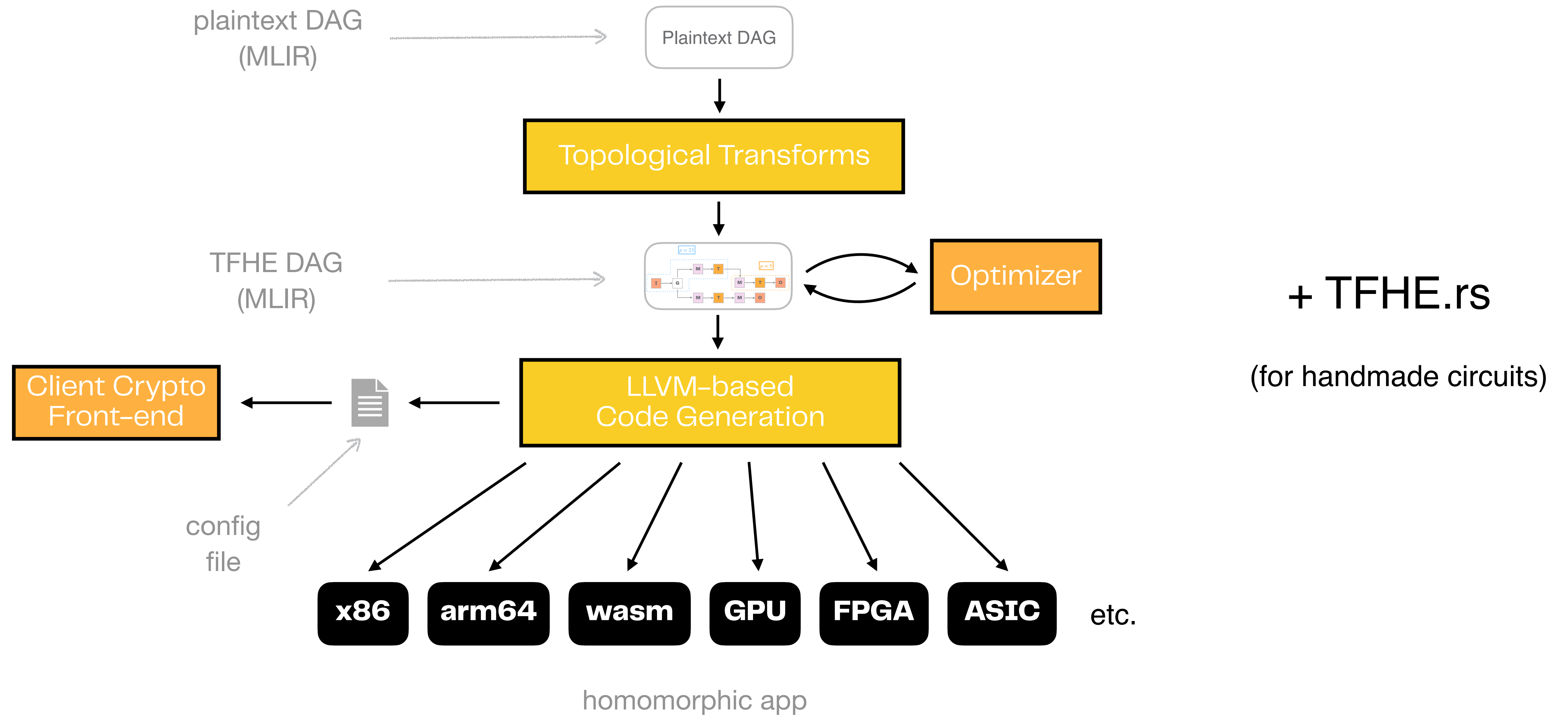
The Concrete Compiler (CC)



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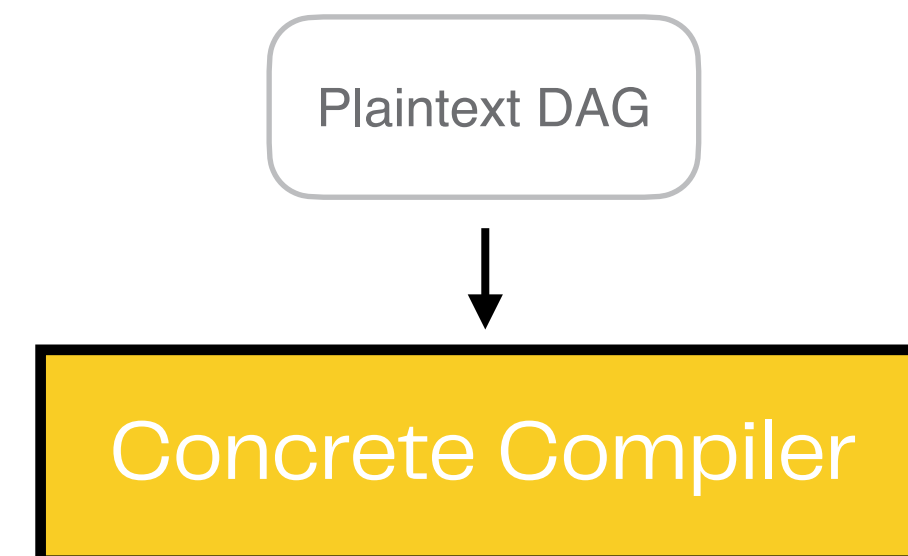
The Concrete Compiler (CC)



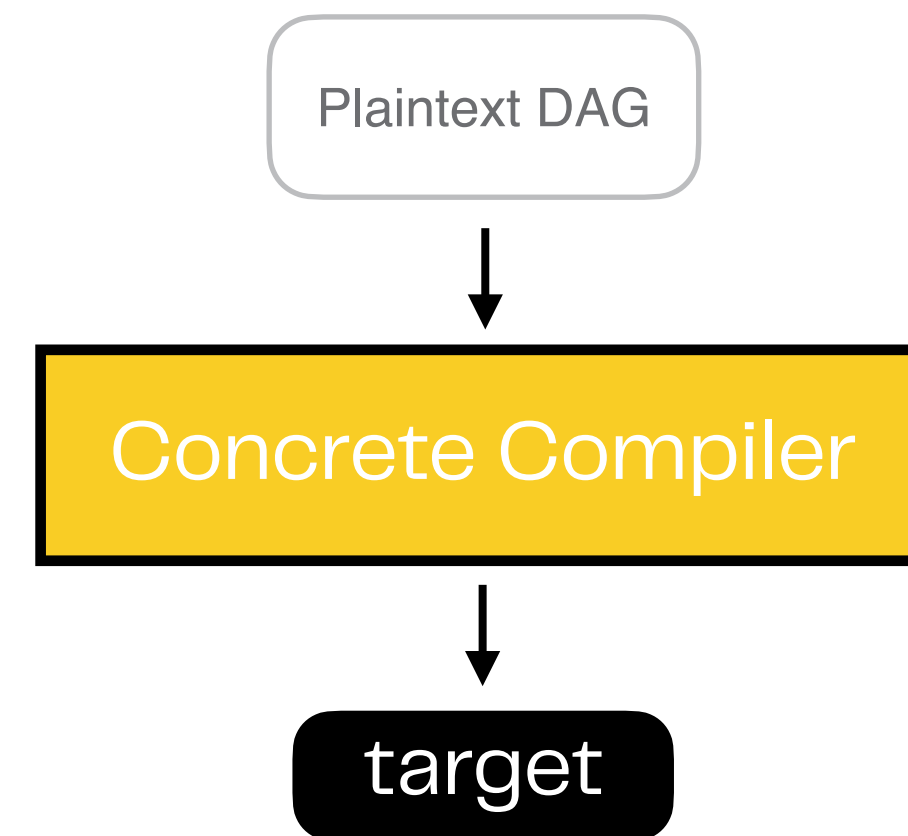
The Concrete stack is a versatile framework

Concrete Compiler

The Concrete stack is a versatile framework

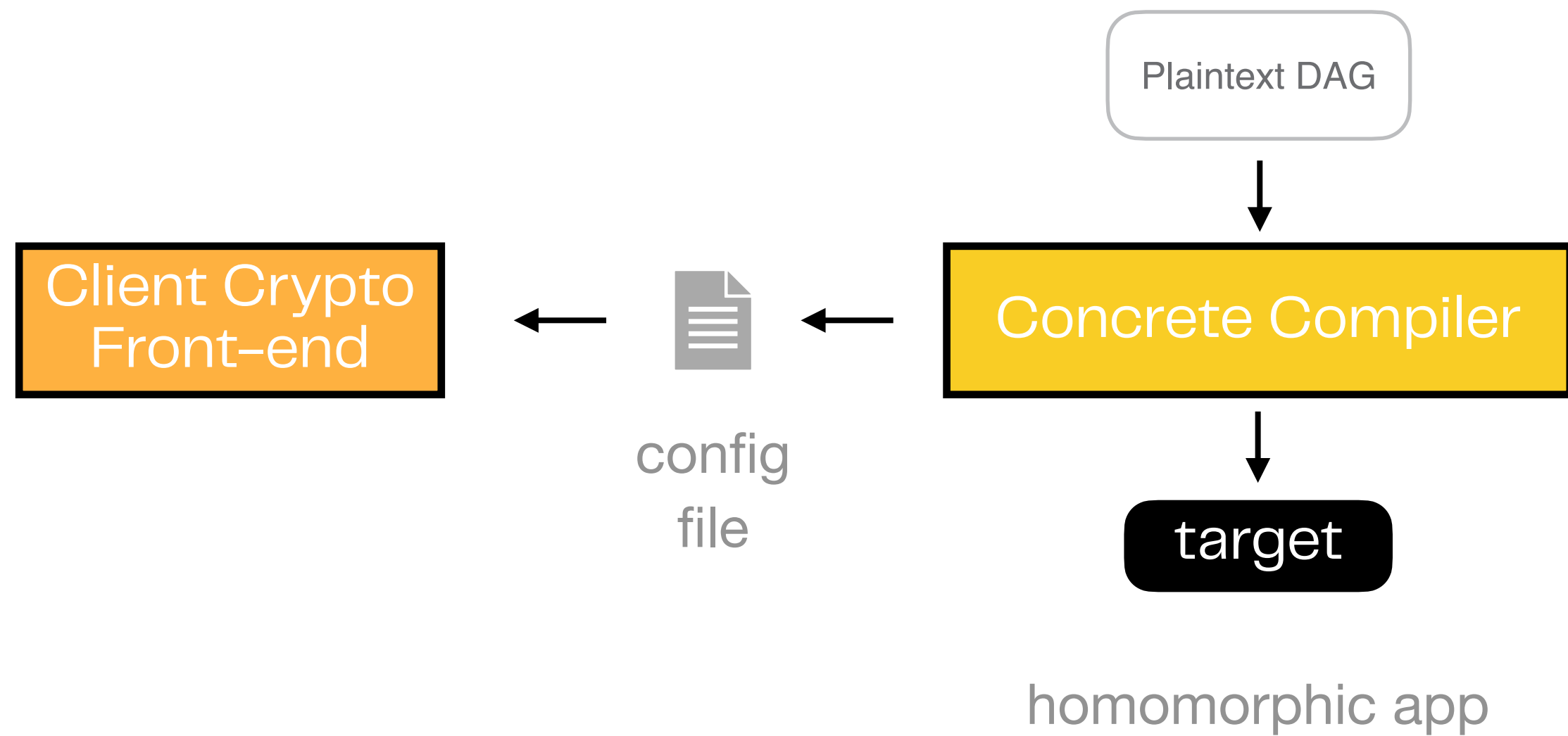


The Concrete stack is a versatile framework

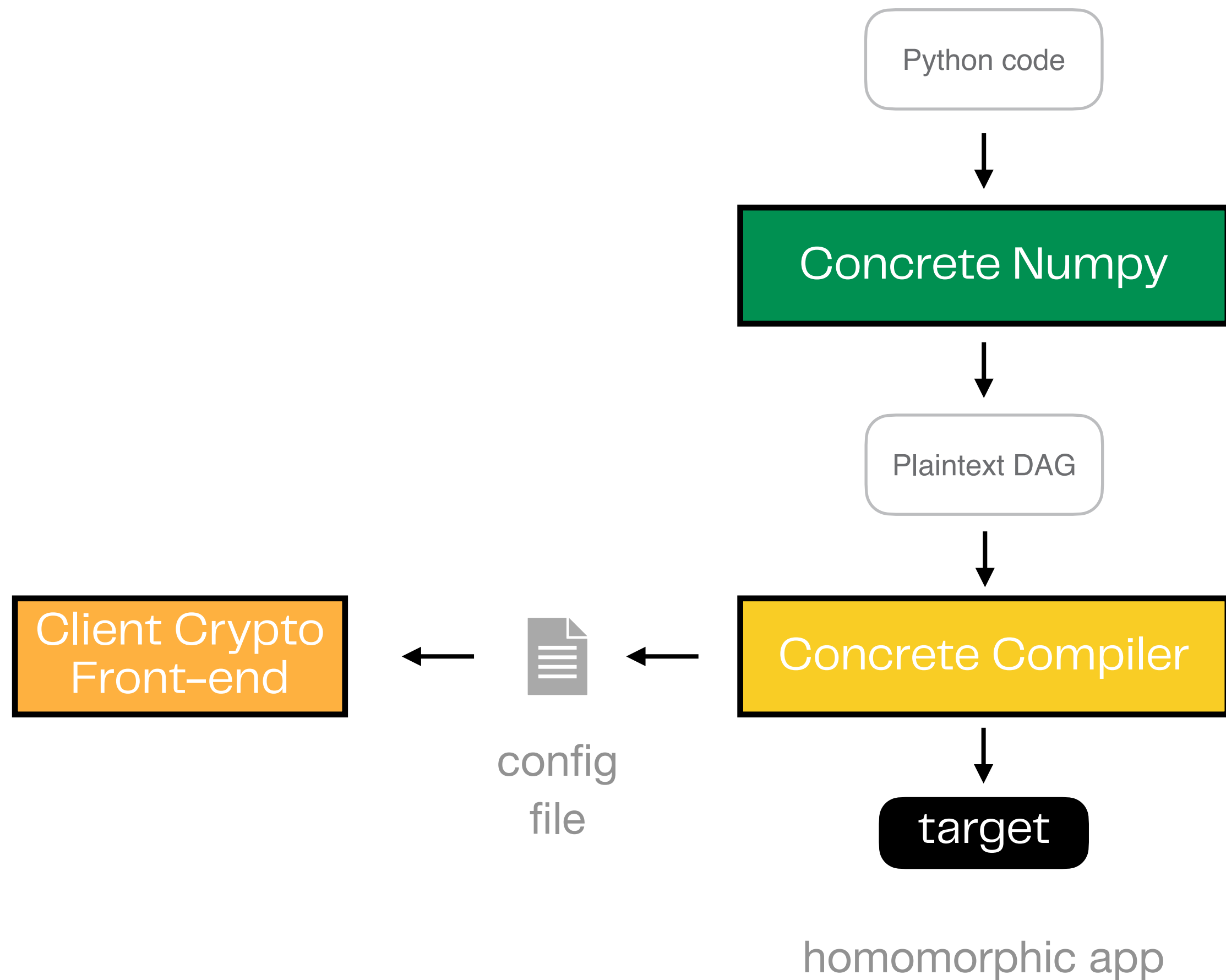


homomorphic app

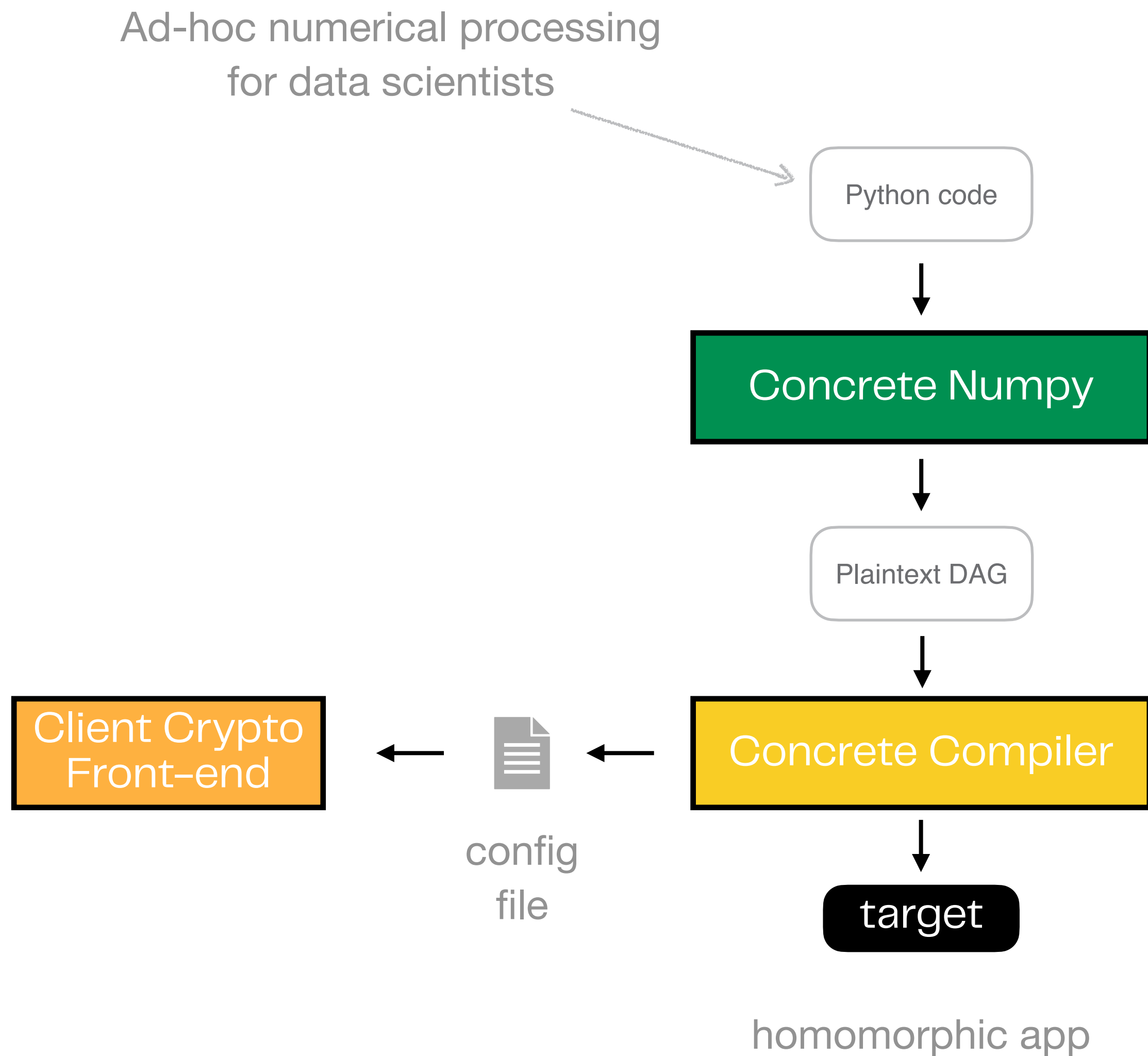
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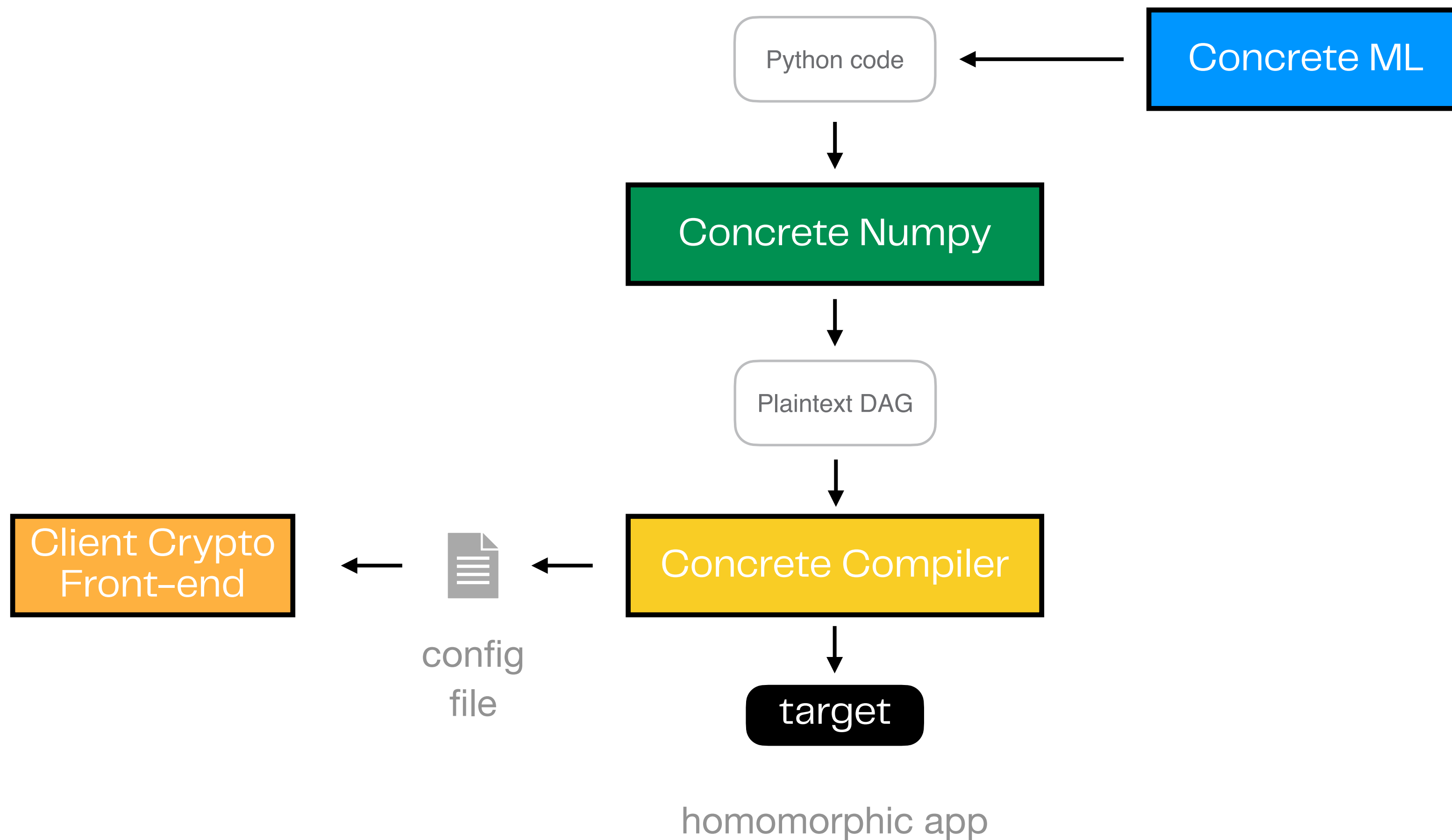
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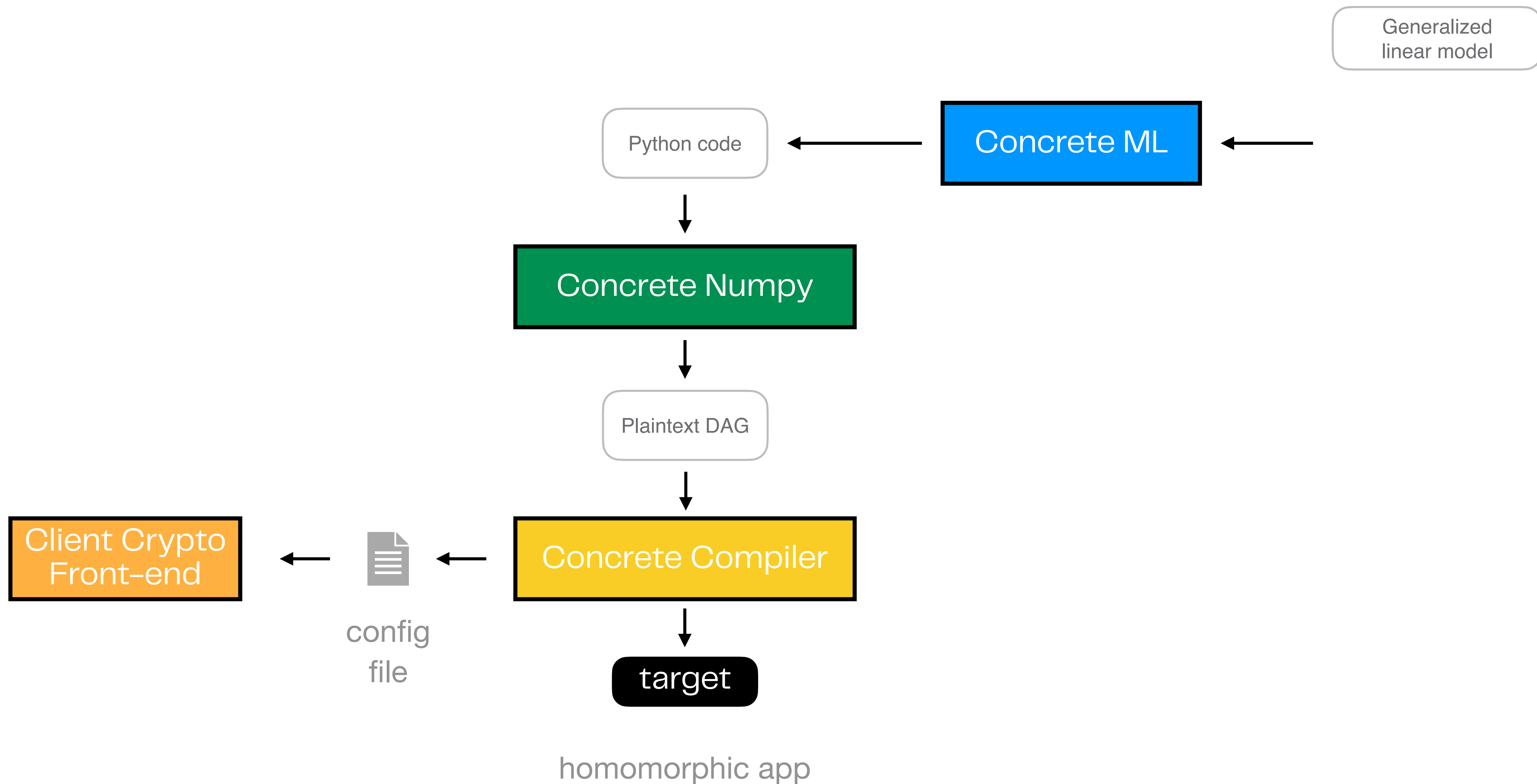
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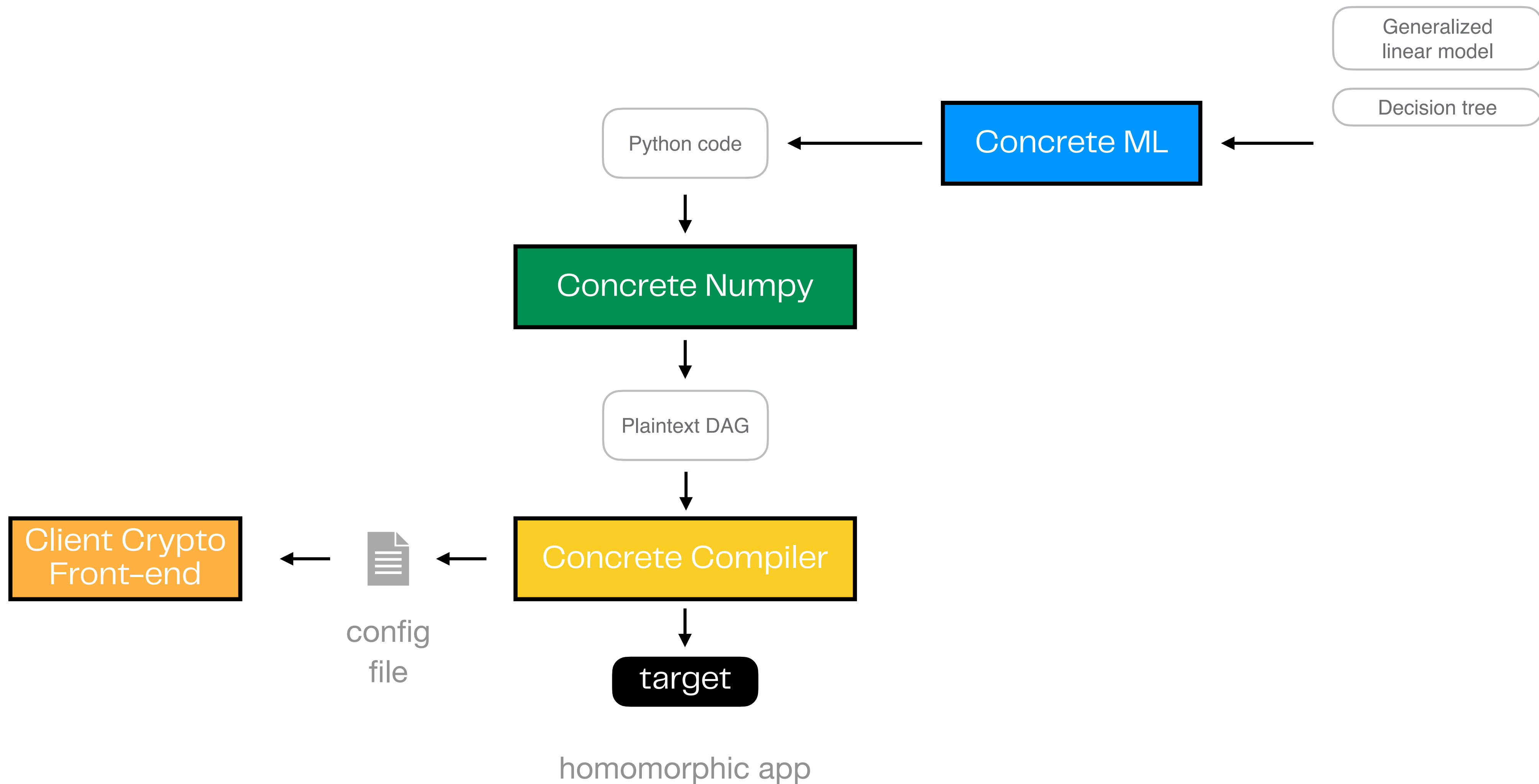
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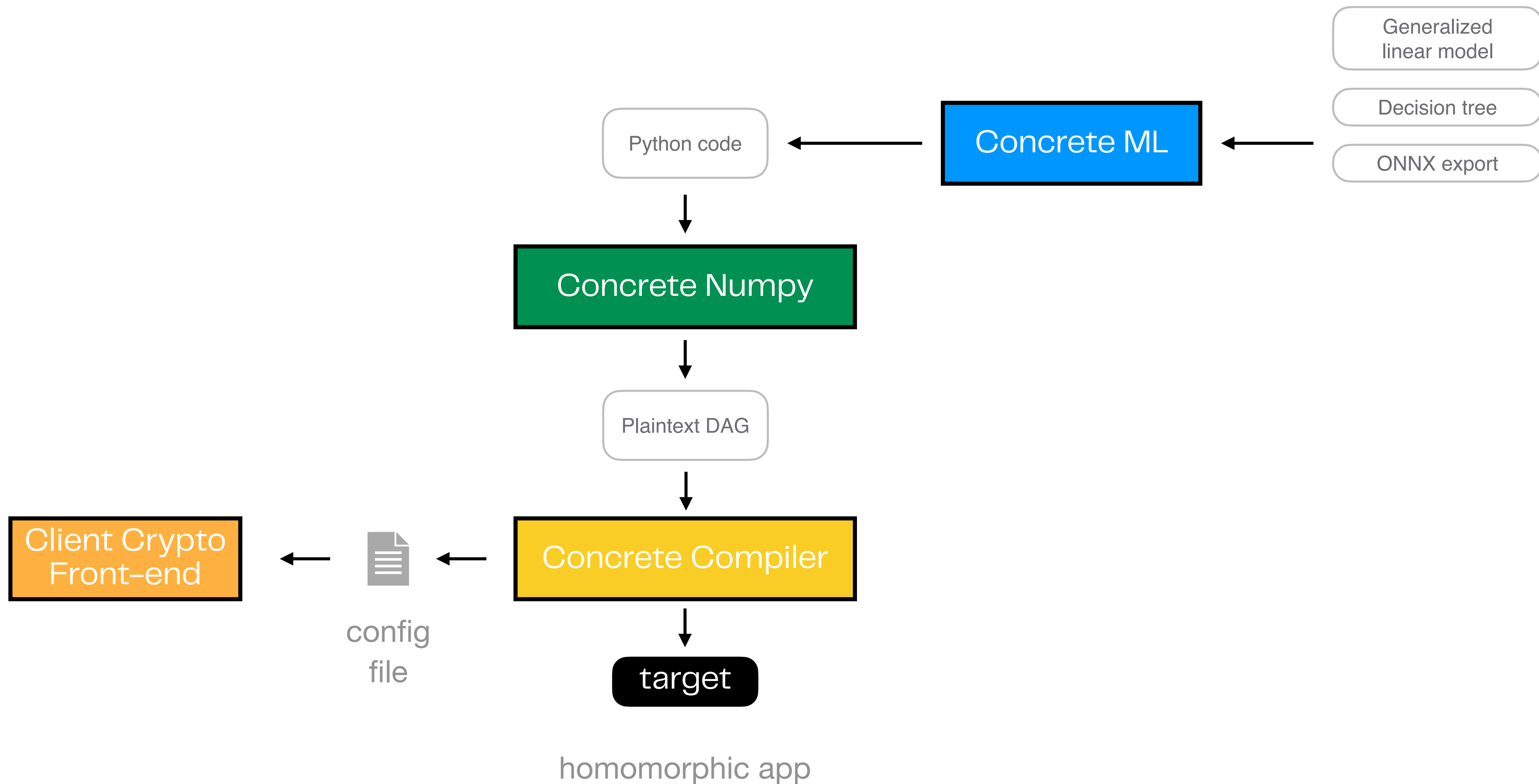
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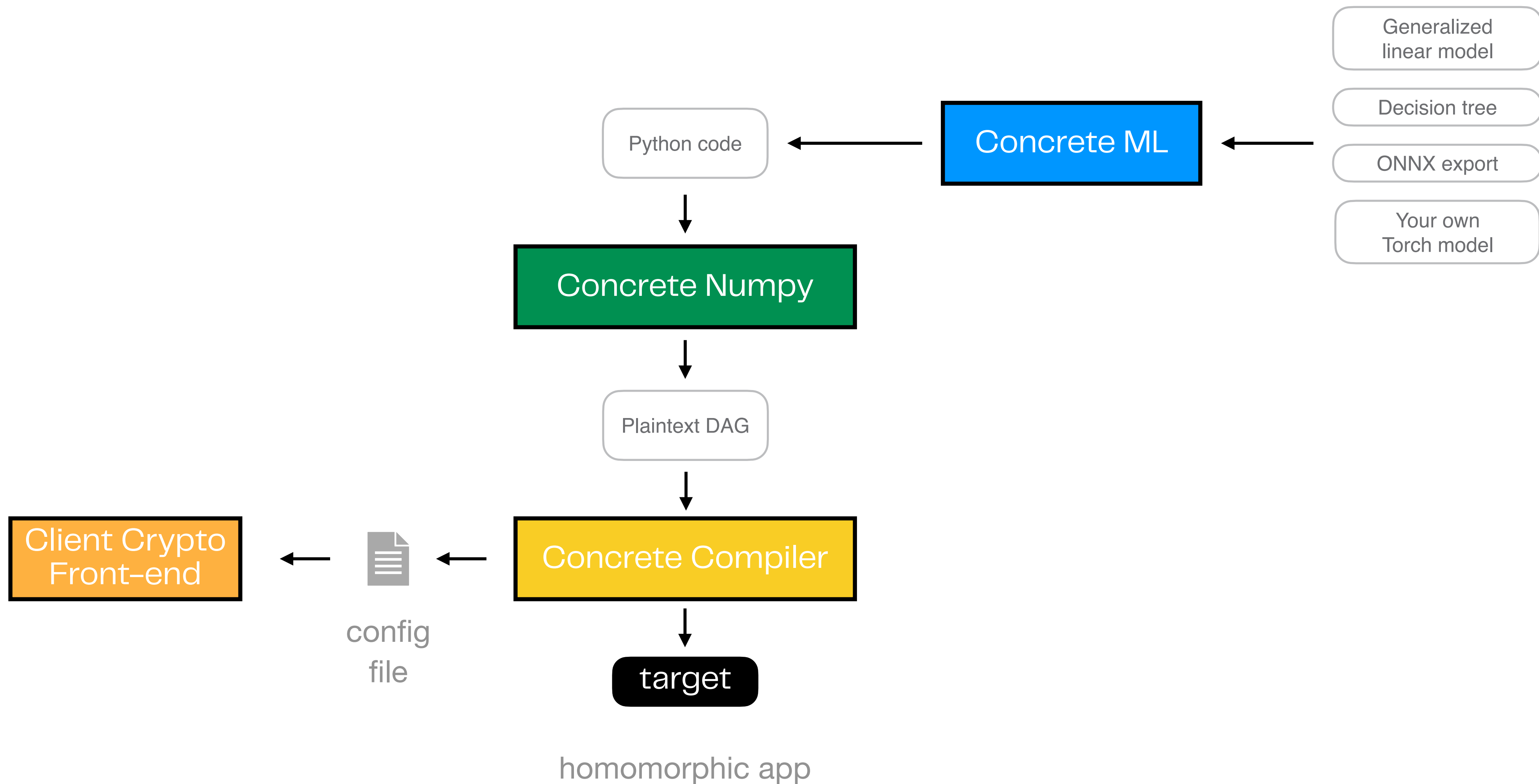
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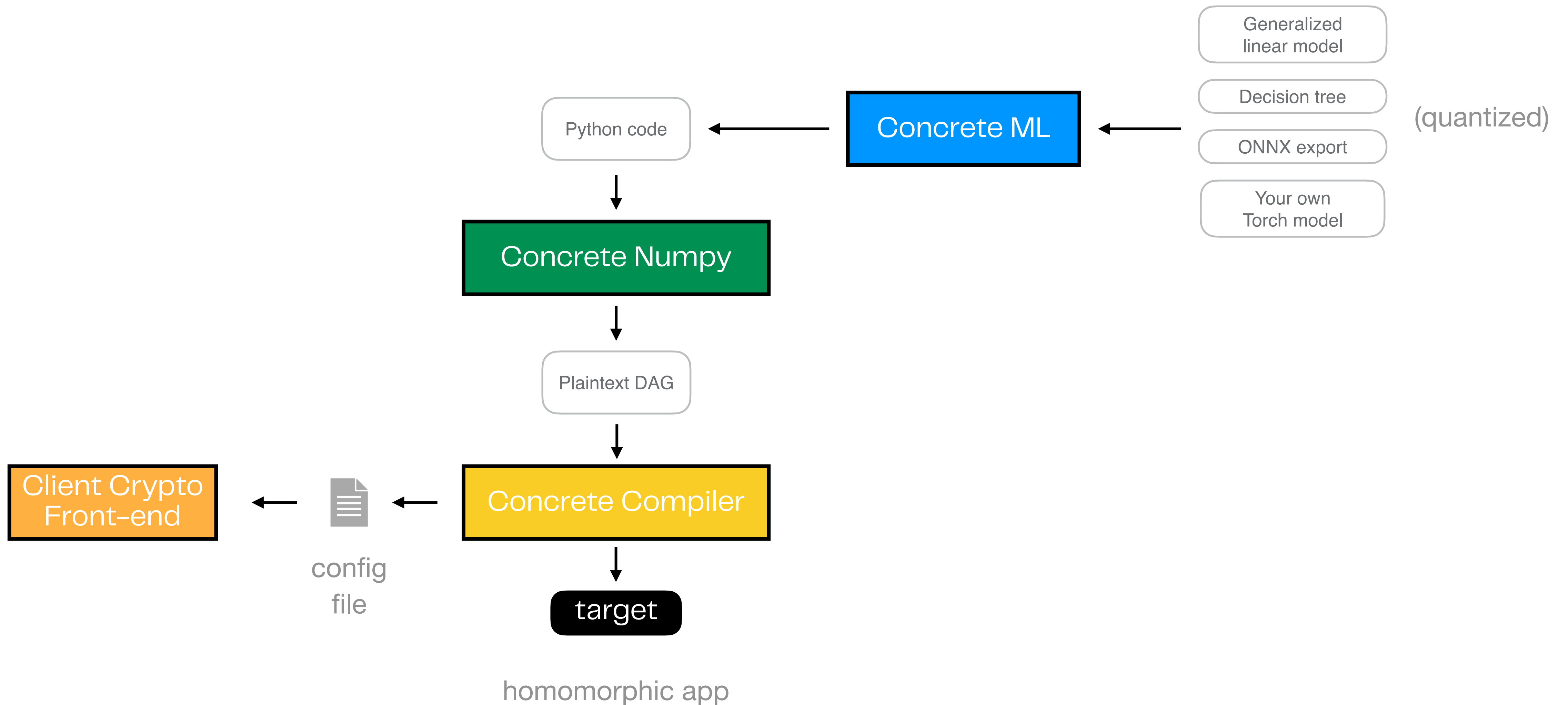
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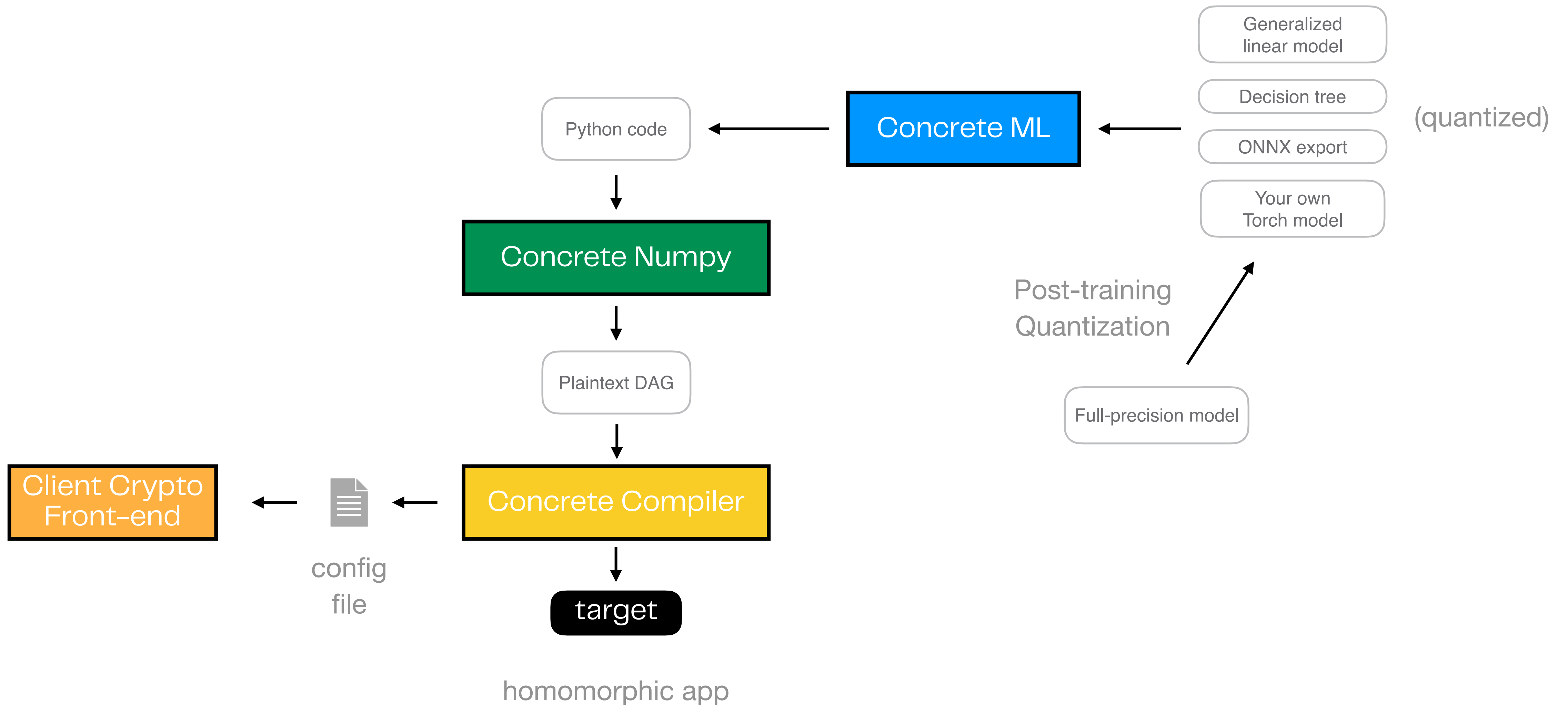
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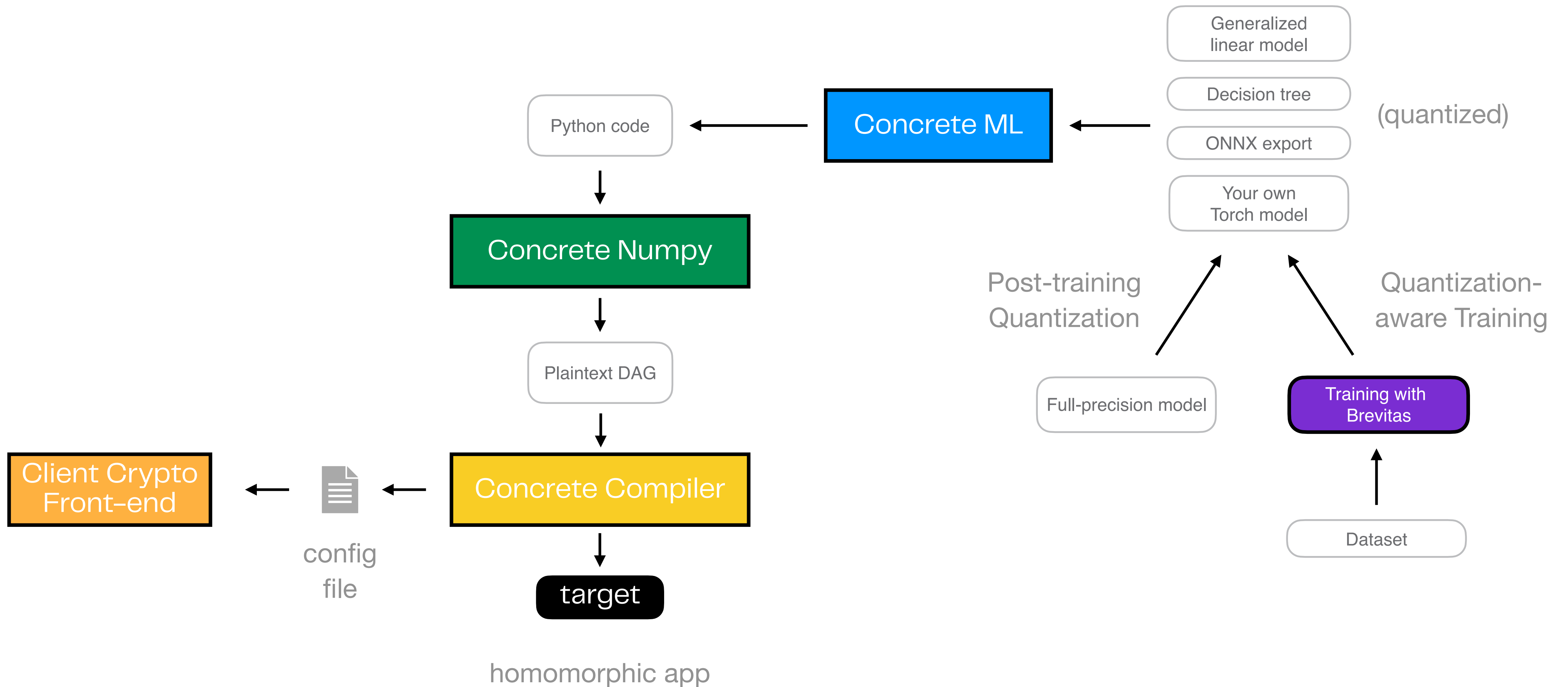
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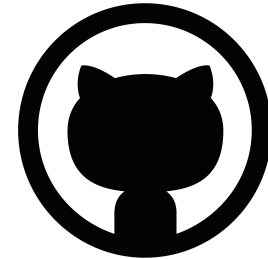


The Concrete stack is a versatile framework



Check it out!

Homomorphic
Everything



Clone from <https://github.com/zama-ai> and get support on <https://discord.fhe.org/> (#concrete channel)

