Federated Echosahedral Screen Microphysics: Patch Hardware, Records, and Observer Synchronization in OPH

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Abstract

Observer-Patch Holography (OPH) needs a finite carrier for screen cuts. This paper turns those cuts into self-reading patch bodies with ports, local state, records, repair channels, synchronization, and checkpoint restoration. The finite record algebra gives Born–Lüders readout, Bell and Clauser–Horne–Shimony–Holt records at the Tsirelson bound, and an observer continuation protocol. A reversible proposal layer reconstructs a finite Gibbs potential when cycle affinities vanish and gives an exact $1, 3, 3, 5$ degeneracy theorem for the source-derived twenty-four-channel embedding.

On the declared echosahedral carrier lineage, port atoms, central readback, oriented incidence, and refinement maps derive twelve unit lines, inverse pairing, the proper A_5 action, and a rank-three frame. Target-blind impulse and port readback derive the inverse-port response. Its compact current algebra is $\mathfrak{u}(1) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(3)$. Under the declared fifteen-state matter contract, anomaly balance and tensor descent give the Standard Model charge lattice and maximal faithful matter image $(\mathrm{SU}(3) \times \mathrm{SU}(2) \times \mathrm{U}(1))/\mathbb{Z}_6$ from the stated contracts alone. Physical matter typing, global-form selection, laboratory current identification, scalar multiplicity, family attachment, and continuum quantum-field realization are open. Hardware evidence enters only with public device, calibration, circuit, data, and independent checks.

One carrier stack, four paper surfaces

The microphysics, consensus, spacetime/Einstein, and Standard Model gauge papers describe one typed construction. The microphysics paper owns the finite carrier and its public interfaces. The consensus paper owns accepted repair and the quotient public normal form. The spacetime and Einstein paper owns the conditional maps from that public normal form into support geometry and gravity. The Standard Model gauge paper owns the categorical and finite-carrier routes into compact currents and conditional matter. A claim may cross from one paper to another only through the exported object and premises named here.

Three meanings of screen

The word "screen" is used for three related objects that must not be identified without a receipt.

1. The *local carrier boundary* is the twelve-port oriented interface of one Echosahedral carrier on the declared branch. Its incidence has $(V, E, F) = (12, 30, 20)$.

2. The *federation screen* is the routed system of interfaces, records, repairs, and checkpoints of many carriers at finite cutoff.

3. The *support screen* is the observer-facing geometric chart. On the spherical branch it is the refined conformal S^2 used for caps, collars, modular flow, and Lorentz reconstruction.

Local icosahedral incidence does not determine the topology of the federation nerve. A federation of identical local carriers can be routed as a path, a cycle, a higher-genus complex, or a spherical complex. The map from routed carriers to a support-visible spherical nerve is therefore a physical bridge, not a change of notation.

Structure-sensitive, presentation-invariant physics

OPH is not neutral under arbitrary changes of substrate. It is invariant under changes of presentation that preserve the complete observer-visible carrier signature. On the Echosahedral branch that signature contains

$$\mathcal{C}_{i,r} = (\mathcal{A}_{i,r}, \rho_{i,r}, P_{i,r}, I^{\mathrm{or}}_{i,r}, \mathcal{R}_{i,r}, \mathcal{U}_{i,r}, \mathsf{Chk}_{i,r}, \mathsf{Resp}_{i,r}, c_{sr}),$$

where $P_{i,r}$ is the port set, $I^{\mathrm{or}}_{i,r}$ is oriented incidence, $\mathcal{R}_{i,r}$ is the record algebra, $\mathcal{U}_{i,r}$ is the repair or feedback interface, $\mathsf{Resp}_{i,r}$ is the visible response law, and c_{sr} is the refinement lineage. Hidden coordinates, port names, worker partitions, materials, and wiring presentations are silent when an isomorphism preserves this whole tuple and its error model. A change in port number, incidence, orientation, accessible algebra, response, repair law, clock, or refinement lineage need not be silent. A cube and an icosahedron are therefore different carrier contracts even when both are built from the same material.

A carrier body is not automatically an observer. It realizes an observer only when it supplies bounded access, self-readback, durable records, record-conditioned feedback, boundary prediction against controls, and checkpoint continuation. One carrier may pass that test. A connected subfederation may pass it instead. No theorem fixes primitive observer size by counting carrier bodies.

The common finite computation

At cutoff r, source-bound carrier data are routed into an observer-patch federation. Accepted repair then acts on the physical quotient:

$$\begin{aligned} \mathsf{SourceCarrierTower}_r &\xrightarrow{\mathsf{realize/route}} \mathsf{ObserverFederation}_r \\ &\xrightarrow{\pi_r} \mathsf{PhysicalQuotient}_r \xrightarrow{\mathrm{Rep}_r} \mathsf{PublicNormalForm}_r. \end{aligned}$$

The last arrow is the consensus result only under semantic-dependency-complete transactions, coherent union-collar payloads, repair completeness, local diamonds, protected records, and the stated endpoint conditions. A collection of oscillators with equal frequency does not supply those clauses.

Physical phase locking can instantiate one synchronization layer. For a routed edge $e = ((i,a),(j,b))$, a source-produced phase record may certify frequency entrainment and a stable relative phase,

$$\dot\theta_{i,a} - \dot\theta_{j,b} \longrightarrow 0, \qquad d_{S^1}(\theta_{i,a} - \theta_{j,b}, \delta_e) \le \varepsilon_e.$$

That certificate becomes a consensus parent only when the phase record fixes a commensurability map for the exposed packets and is tied to the accepted repair ledger, semantic records, an independently calibrated clock, and the confluence premises. Phase locking can synchronize an interface. It does not by itself make the interface an observer, settle semantic disagreement, or produce physical time.

Two downstream projections of one source

The public normal form has two separately typed projections:

$$\mathsf{PublicNormalForm}_r \xrightarrow{\text{carrier-to-support}} \left(\mathsf{Support}_{S^2,r}, \mathsf{FiniteCapBWCertificate}_r\right),$$

$$\left.\begin{array}{r}\mathsf{FiniteCapBWCertificate}_r\\ \mathsf{MGNS\text{-}1}_r \text{ independently complete}\end{array}\right\}_{\text{same tower}} \begin{array}{l}\longrightarrow \mathsf{BW/KMS}_r \longrightarrow \mathsf{Lorentz/H^3}_r\\ \\ \longrightarrow \mathsf{Events}_{3+1,r} \longrightarrow \mathsf{Einstein}_r,\end{array}$$

$$\begin{aligned}\mathsf{PublicNormalForm}_r &\xrightarrow{\text{response contract}} (\mathsf{A5Carrier}_r, J_r)\\ &\longrightarrow \mathsf{CompactCurrent}_r \longrightarrow \mathsf{SM}_{Q0,r}\\ &\longrightarrow \mathsf{Matter/QFT}_r.\end{aligned}$$

The finite carrier-to-support leg is constructed from one oriented icosahedral incidence nerve: twelve carrier charts, thirty seam algebras, and twenty nonvacuous triple restrictions. The same bound artifact supplies confluent seam repairs, an operational observer receipt, and a refinement-natural oriented S^2 support limit. The separate geometric 2π-KMS comparison and $\mathsf{FiniteCapBWCertificate}$ are not outputs of that finite bridge. The state tower, common-comparison maps, compatible state/vector data, modular controls, and cofinal modulus belong to the independently produced $\mathsf{MGNS\text{-}1}$ package. The BW theorem consumes both inputs on the same tower. Once the support leg produces a conformal S^2, $\mathrm{Conf}^+(S^2) \cong \mathrm{SO}^+(3,1)$ and $H^3 = \mathrm{SO}^+(3,1)/\,\mathrm{SO}(3)$ is exactly three-dimensional. H^3 is the observer-frame fiber. A 3+1-dimensional event manifold requires the population/realization, separation, rank-four affine-chart, overlap-cocycle, held-out quadratic-cone, and causal-reachability receipts ($\mathsf{E1}$)–($\mathsf{E6}$), together with the MI/assembly premise. Operational-clock gluing separately requires observer-readable transitions, event correspondence, affine calibration, cycle identity, and normal-form invariance. The Einstein relation additionally requires the common-domain stress, entropy, vacuum, coupling, scale, and remainder packet. Hidden Cartesian coordinates of a finite carrier are ineligible as support-screen, event, or Lorentz data.

The second projection begins with an exact finite result on the certified Echosahedral lineage. The twelve-port module decomposes as

$$P_{12} \cong_{A_5} \mathbf{1} \oplus \mathbf{3} \oplus \mathbf{3}' \oplus \mathbf{5}.$$

The source selector derives the twelve unit lines, antipodal pairing, proper A_5 action, and rank-three Gram frame; incidence determines the antipode J. Under the explicit finite response-admissibility contract, the signed central involutive inverse-port responses are exactly $\pm J$, with common sign conventional. Conditional also on the rank-15 matter contract with its unique charge-conjugate projector pair, the port-current, anomaly, Spin, and tensor-descent certificates imply the Standard Model Lie type, hypercharge lattice, and conjugation-insensitive $\mathbb{Z}_6$ global form from those premises alone. The scalar scan fixes compatible charges and Yukawa channels, not scalar multiplicity. The finite response has a source-derived target-blind producer. Physical matter typing, laboratory current and line identification, exclusion of extra light sectors, family attachment, scalar attachment/dynamics, and quantum-field construction remain separate maps.

The compact sector-category/Tannaka route conditionally reconstructs a compact group by a logically independent route. The generation count and the extra-sector statement are declared completions; they do not enter the contract-conditional finite gauge implication. It remains an explicit premise on charged-lepton and quantitative selector/gap branches elsewhere. Physical unification requires a source-bound commuting square identifying its reconstructed compact group with the group acting through the Echosahedral current response:

$$\begin{array}{ccc} \mathsf{A5PortResponse}_r & \longrightarrow & G_r^{\mathrm{screen}} \\ \downarrow & & \downarrow\simeq \\ \mathsf{TransportableSectorCategory}_r & \longrightarrow & G_r^{\mathrm{DR}}. \end{array}$$

On those premises the abstract Lie-type agreement is exact. The physical vertical maps and the source identity of the two group actions are open. In the same way, the rank-three face band is a canonical candidate family carrier, while three physical generations require the complex rank-45 attachment and complement-complete refinement receipts. Any displayed three-generation value is a declared completion inside the conditional window, not a consequence of the icosahedral graph alone.

Finite controls and status boundaries

The finite A_5 evaluator control has 60 reachable correctable public records on $\mathcal{H}_k = \ell^2(A_5) \otimes \mathbb{C}^k$:

$$M_0 = 60, \qquad D_{\mathrm{raw}} = 60k, \qquad \Delta_{\mathrm{raw}} = 60(k-1).$$

Raw equality occurs only at $k = 1$. Publicly inert multiplicity makes D_{raw} implementation-dependent, so the result is an evaluator control rather than physical capacity closure.

The unified claim has a precise scope. Consensus, geometry/gravity, and gauge/matter are composable branches of one source-bound self-reading carrier tower. Its full quotient-visible architecture can constrain both branches; local icosahedral incidence by itself constrains only the local carrier route. The physical maps that turn those constraints into one inhabited universe are named premises. Matching dimensions or symmetry labels does not supply them.

What This Paper Contributes

The rest of the OPH stack talks about observer patches, records, repairs, and screen cuts. This paper gives those words a finite carrier. It separates the abstract observer patch from its geometric support chart and from its physical or digital implementation. The separation removes dependence on hidden coordinates, labels, materials, and wiring choices only when the complete observer-visible carrier signature is preserved. OPH is therefore presentation-invariant rather than carrier-neutral: quotient-visible port incidence, orientation, response, repair, clock, and refinement data can change the physical branch.

The concrete contribution is the regulated screen architecture. Echosahedral carriers supply a twelve-port reference interface, edge sectors supply the heat-kernel/Casimir weights used by the quantitative branch, central records give the Born–Lüders event surface, Bell/CHSH records give the Tsirelson bound, and checkpoint restoration states what it means for an observer to continue after repair. The finite modular-gearing package keeps strict-descent normalization separate from reversible proposal rates, tests whether the oriented 24-slot register is an invariant channel space, and fixes only symmetry-protected degeneracies rather than numerical gaps or clock units. The finite echosahedral selector separately closes the unit-splitting and proper-A_5 source receipts on the declared carrier lineage, without using downstream particle or gauge targets. Incidence and target-blind port readback derive the finite response, compact current, and refinement maps. Under the declared fermionic Spin category, the anomaly and descent certificates imply the Standard Model charge lattice and common $\mathbb{Z}_6$ tensor kernel from the stated premises alone. The corresponding quotient is the maximal faithful matter image. Physical matter typing, global-form selection, laboratory identification, scalar multiplicity, and equality with the distinct Tannaka current are open. The generation-count, extra-sector, charged-lepton, and quantitative selector/gap branches carry their own declared completions. Hardware claims are kept behind a public evidence rule: manifests, hashes, calibration records, raw or reduced traces, and exact verifier receipts carry the weight.

Contents

1 Scope

OPH microphysics uses finite observer patches with echosahedral interfaces. A spherical screen is a geometry chart for observer-visible cuts. The cellulated-sphere gauge-register construction is a regulator and digital calibration chart. The premise is:

> Observers access finite observer-visible cuts. A spherical screen is a geometry chart for such cuts. The underlying fixed-cutoff implementation surface is a federation of finite overlap-facing observer patches, with echosahedral local interfaces and recurrent toroidal subchannels.

The paper's contract is:

1. state the fixed-cutoff theorem exports used by the other OPH papers;
2. use federated patch carriers as the microphysical architecture;
3. keep mathematical theorem surfaces separate from public hardware evidence surfaces;
4. keep the octahedral $\mathbb{Z}_2/S_3$ finite-group model at appendix-level calibration scope;
5. require a public hardware evidence protocol before hardware claims receive paper weight.

2 Canonical Observer-Patch Semantics

This paper owns the fixed-cutoff observer vocabulary used by the rest of the OPH stack. An observer patch is first an operational algebraic object:

$$\mathsf{O}_i = (\mathcal{A}_i, \rho_i, \mathcal{R}_i, \{(\mathcal{I}_e, \pi_{i,e})\}_{e \ni i}, \mathcal{U}_i, \mathrm{Chk}_i)\,.$$

Here $\mathcal{A}_i$ is the finite or regulated accessible algebra, ρ_i is the accessible state, $\mathcal{R}_i \subseteq Z(\mathcal{A}_i)$ is the exact central record algebra or a declared approximate record algebra, $\mathcal{I}_e$ is an overlap-visible interface algebra, $\pi_{i,e} : \mathcal{A}_i \to \mathcal{I}_e$ is the visible restriction map, $\mathcal{U}_i$ is the allowed family of update and repair instruments, and Chk_i is checkpoint data sufficient for the target reconstruction problem.

Definition 2.1 (Abstract observer patch)**.** *An abstract observer patch is the tuple O_i above, considered up to isomorphism of accessible algebras, record statistics, visible restrictions, repair instruments, and checkpoint continuation.*

Definition 2.2 (Support patch)**.** *A support patch is a geometric chart for an abstract observer patch on a branch with reconstructed geometry. Examples are a cap $P_i \subset S^2$, a collar, or a causal diamond. The support patch supplies chart data for modular flow, entropy variation, and overlap comparison.*

Definition 2.3 (Carrier patch)**.** *A carrier patch is a physical or digital implementation satisfying*

$$\mathrm{Realize}_\varepsilon(H_i) \longrightarrow \mathsf{O}_i,$$

meaning that the implementation induces the declared accessible algebra, visible interface statistics, record readout, repair instruments, and checkpoint continuation within error ε.

The echosahedral body used below is a reference carrier architecture. It is a finite implementation surface for twelve-port interfaces, records, repair channels, and recurrent subchannels. The observer identity lives in the quotient data exposed by O_i, so a change of hidden coordinates, port labels, or substrate presentation is physically silent when it preserves the visible interface, oriented incidence where declared, response, record, repair, checkpoint, and refinement processes.

Definition 2.4 (Algebraic audit instrument)**.** *An algebraic audit instrument is an exact computable map on finite quotient-visible representation data. It certifies invariance, nonconjugacy, orbit structure, lattice preservation, or automorphism fusion inside a declared regulator chart. It is not a physical actuator unless a carrier implementation realizes the same visible map with a public evidence bundle.*

The $E_8/\mathrm{Spin}(8)$ triality sidecar is an algebraic audit instrument in this sense. It states that the vector $\mathrm{Alt}(9)$ and positive-half-spin $2{\cdot}\mathrm{Alt}(9)$ presentations preserve E_8-lattice data, have different mod-2 orbit fingerprints on $E_8/2E_8$, and are identified only by the $\mathrm{Spin}(8)$ triality outer automorphism. No audit bundle supplies the raw matrices, scripts, exact checks, and hashes, so this is an uncertified claim in finite exceptional representation bookkeeping with no hardware implication.

The tuple O_i does not impose equal primitive observer size. Fixed-cutoff OPH requires bounded accessible algebras and declared interfaces; it also permits different connected support regions and different finite or regulated algebras. In the carrier picture, an observer-supporting object may be a variable finite federation of screen cells. A theorem selecting isomorphic fixed-capacity elementary carriers would need extra hypotheses: homogeneous UV cellulation, isomorphic local cell algebras, equivariant overlaps, refinement preservation of the cell type, and a proof that one cell or a fixed block of cells is the primitive carrier.

Claim 2.5 (Presentation invariance and carrier sensitivity)**.** *Two carrier patches are physically equivalent for the declared OPH observables when a quotient-visible isomorphism preserves their accessible algebras and states, interface algebras, record statistics, repair maps, checkpoint continuation, response laws, and refinement lineage. On the Echosahedral branch it must also preserve the twelve-port oriented incidence and its antipodal pairing. A material, coordinate, or wiring change satisfying this contract is silent. A change of the preserved carrier signature is outside the equivalence claim and may change the physical branch.*

This claim is theorem-level only on a branch that supplies the required quotient isomorphism and error bound. It fails if a predicted OPH observable depends on hidden carrier coordinates after the full visible signature has been held fixed. It says nothing about a cube and an icosahedron whose visible incidence structures differ.

Definition 2.6 (Operational observer test)**.** *A candidate physical system realizes an observer patch when it supplies a bounded accessible interface, durable re-readable records, self-read or internal state estimation, record-conditioned future behavior, boundary prediction better than shuffled-record controls, and checkpoint continuation within a declared error.*

3 Claim Taxonomy

The paper uses five claim levels.

C1. Mathematical fixed-cutoff claims. These are finite-algebra statements about patches, overlaps, records, repair interfaces, event algebras, and checkpoint laws. They are allowed to be theorem-bearing. Finite representation and lattice audit instruments belong here when their matrices, exact checks, and quotient-visible invariants are public. The generic overlap network is a finite constraint code only; QECC distance, min-cut resilience, spectral convergence, BFT liveness, and hardware speedup need their own certificates.

C2. Regulator-chart claims. These identify a convenient finite model, such as a spherical cellulation or a digital finite-group calibration chart, as a calculational chart. Their claim level is regulator geometry and calibration.

C3. Federated-carrier claims. These define a candidate implementation architecture: observer patches with echosahedral multi-port interfaces, recurrent toroidal subchannels, exposed overlap data, records, and repair loops.

C4. Public hardware-evidence claims. These may be cited only when the raw or reduced evidence is present in a public OPH evidence bundle with stable hashes, manifests, calibration files, and exact-verifier receipts.

C5. Boundary statements. This paper leaves laboratory-hardware identification, unique UV completion, hardware solution of the Yang–Mills mass gap, computational complexity collapse, and first-principles hadron masses outside its support boundary.

4 The Public Evidence Rule

Hardware-facing language is admissible only under the following rule.

> Any hardware evidence cited as evidence in this paper must be represented by a public evidence bundle in the OPH repository, or by a public pinned subrepository commit, with enough raw data and metadata for an external reader to check the claim.

Private notebooks, local runtime logs, unpublished bench transcripts, and development-repo notes may motivate architecture choices, but they do not carry evidential weight inside the paper. A hardware evidence bundle must include, at minimum:

1. a manifest with stable bundle identifier, date, operator, body identifier, controller identifier, firmware hash, mesh/body hash, and wiring map hash;
2. raw readout files, such as coupling matrices, MDD or discharge-timing traces, dark baselines, low-power sweeps, ring-diversity scans, and calibration logs;
3. body and board provenance, including photographs or signed photo hashes where relevant;
4. the task definition, scorebook, repair or rerank law, and exact-verifier program;
5. exact-verifier receipts for any high-level task claim;
6. negative controls, shuffle or replay controls where applicable, and a statement of non-claims.

This rule prevents dependence on hidden laboratory state. The theorem sections use finite algebras, declared patch interfaces, and stated branch assumptions.

5 From Spherical Screens to Federated Patch Carriers

The spherical screen is useful because an observer-accessible cut often has an effective closed two-surface description. A cellulated sphere can encode finite capacity, caps, collars, edge centers, and observer-visible cuts. The microscopic carrier is the finite patch federation. The chart is the observer-facing presentation of its visible data.

The sphere also fixes the symmetry bridge used by the rest of OPH. Caps on S^2 are the geometric support regions whose modular flows become Lorentz boosts in the controlled scaling branch. The conformal group of S^2 is $\mathrm{SO}^+(3,1)$, so the same observer-facing chart supplies the kinematic bridge to emergent $3+1$D spacetime. Finite cellulations of the chart supply the regulator side: patch ports, edge sectors, collars, and overlap checks.

At fixed cutoff, the fundamental object is a finite patch federation. Each patch has:

1. an internal finite state algebra;
2. a bounded family of exposed overlap ports;
3. a local record algebra;
4. a readout map from internal state to port-visible packets;
5. a repair interface that changes local state in response to mismatch;
6. a checkpoint interface that exposes enough observer-accessible data to define continuation.

The abstract observer patch and its carrier realization are separate typed objects. Write

$$\mathsf{O}_i = (\mathcal{A}_i, \rho_i, \mathcal{R}_i, \{(\mathcal{I}_e, \pi_{i,e})\}_{e\ni i}, \mathcal{U}_i, \mathsf{Chk}_i)\,, \qquad \mathrm{Realize}_\varepsilon(H_i) \longrightarrow \mathsf{O}_i.$$

The elementary carrier H_i may be an Echosahedron on the homogeneous reference branch. Its port names, body coordinates, and hardware details are absent from the abstract observer whenever they disappear under the physical quotient. A fixed-cutoff simulator is therefore typed as

$$\mathsf{Sim}_r = (G_r, \{\mathsf{O}_{i,r}\}, \Sigma_r, \Gamma_r, Q_r, \mathsf{Prop}_r, \to_r^{\mathrm{acc}}, \mathrm{Rep}_r, \pi_r^{\mathrm{eq}}, L_r^{\mathrm{eq}}, \mathsf{Chk}_r, c_{sr}).$$

Here Prop_r is a reversible proposal menu, $\to_r^{\mathrm{acc}}$ is the strict-descent accepted-repair relation, Rep_r is its normalizer, and $(\pi_r^{\mathrm{eq}}, L_r^{\mathrm{eq}})$ is a separate equilibrium law and reversible or Euclidean generator. The refinement maps are c_{sr}.

Definition 5.1 (Typed simulator ledger)**.** *A simulator run has one immutable root manifest and seven typed ledgers:*

A. *the quotient/normalizer ledger owns presentation states, hidden-representative quotient, accepted transactions, semantic histories, normal forms, and checkpoint continuation;*

B. *the source-ensemble ledger owns the base measure, source action, MaxEnt constraints, proposal and acceptance rates, stationary law, detailed balance, and cycle affinities;*

C. *the quantum/modular ledger owns the noncommutative algebra, density matrices, spectral support, modular Hamiltonians, frequency-resolved jumps, conditional expectations, and mixed-GNS comparison data;*

D. *the geometry/BW ledger owns support incidence, cap normals, held-out cross-ratios, geometric dilation, wrong-temperature controls, event-chart complexes, quadratic cones, and tetrads;*

E. *the operational-history ledger owns semantic event identities, the observer registry groupoid, worldlines, event correspondences, instrument readings, and affine synchronization;*

F. *the physical-current ledger owns reversible port perturbations, unitary response, commutators, A_5-equivariance, and determinant, spin, and deck descent;*

G. *the transport ledger owns free and collision kernels, first and second conditional moments, mass-shell data, phase-space measure, and stress transfer.*

The ledgers share source hashes and refinement identifiers through the root manifest. Their typed objects remain distinct. In particular, the accepted strict-descent relation is not reused as the equilibrium generator, a central register density emits trivial modular action, a finite register operator is not a physical current, and a deterministic transport claim requires a vanishing second-moment receipt.

Definition 5.2 (Sphere fold)**.** *Fix a regulator r. Let Σ_r be the finite presentation space of a patch federation, let Γ_r be the presentation-redundancy groupoid, and let*

$$Q_r = \Sigma_r/\Gamma_r$$

be the physical quotient. Let $\pi_r : \Sigma_r \to Q_r$ be the quotient map, let

$$n_r : Q_r \to Q_{r,\mathrm{nf}}$$

be the accepted repair normal-form map on the declared physical quotient, and let

$$\chi_{S,r} : Q_{r,\mathrm{nf}} \to \mathsf{Chart}_S(r)$$

be the support-visible spherical screen chart. Its outputs are caps, collars, cuts, edge-sector records, boundary data, and geometric readouts. The sphere fold of a finite presentation $s \in \Sigma_r$ is

$$\mathrm{Fold}_{S,r}(s) := \chi_{S,r}(n_r(\pi_r(s)))\,.$$

Two finite presentations have the same sphere fold exactly when their repaired quotient normal forms induce the same support-visible caps, collars, edge-sector records, and observer-facing screen readouts.

For Lorentz-branch chart receipts, the support chart also exposes a derived cap-normal readout

$$\chi^{\mathrm{cap}}_{S,r} : Q_{r,\mathrm{nf}} \to \mathsf{CapChart}_r$$

with output

$$\mathsf{CapChart}_r = (C_r, \partial C_r, \mathbf{c}_r, \alpha_r, n_{C,r}, \varepsilon_{\mathrm{round},r}, \varepsilon_{\mathrm{inc},r})\,.$$

On the exact analytic round-cap branch,

$$n_{C,r} = (\cot\alpha_r, \csc\alpha_r\,\mathbf{c}_r).$$

On a finite fitted branch the output is labeled approximate round cap chart. A global bound requires a separate round-cap rigidity/convergence certificate; if the fit is not round it emits cap not round. The normal $n_{C,r}$ is support-chart data derived from the repaired quotient. It is not a microscopic carrier coordinate, hidden port label, or new local degree of freedom. A cap normal determines a geodesic plane and half-space in H^3; selecting a particular observer point $u \in H^3$ requires a separate observer-frame, tetrad, or clock receipt. On the spacetime and Einstein paper's producer branch, the cap chart itself is produced from the support-visible incidence complex of the repaired normal form under the spherical-incidence, mesh, cross-ratio, and 2π-normalization receipts, and the continuation from frame kinematics to an event base is the spacetime and Einstein paper's conditional event-manifold packet.

Definition 5.3 (Carrier-to-support screen bridge)**.** *A carrier-to-support screen bridge at cutoff r consists of a source-bound map from routed carrier ports and seams to the support projections of the repaired observer-patch net, together with: (i) unital interface-algebra homomorphisms on every seam; (ii) coherent recovery maps on higher overlaps; (iii) a quotient-visible nerve whose support incidence is computed from the normal form; and (iv) refinement maps commuting with routing, quotienting, incidence, records, and support projection. A spherical bridge additionally certifies that the global nerve satisfies the spherical-incidence, cap-mesh, oriented cross-ratio, and normalization receipts consumed by the spacetime and Einstein paper. Local carrier coordinates are excluded from the support output unless this map derives them as quotient-visible data.*

Proposition 5.4 (Local Icosahedral incidence does not select the global support)**.** *The certified twelve-port incidence of every local carrier does not determine the topology or conformal structure of the federation support.*

Proof. Fix one certified local carrier and make one copy at every vertex of an arbitrary finite graph. Route selected port pairs along that graph while leaving each local port algebra, oriented incidence, antipode, and A_5 action unchanged. A path, a cycle, a tree, and a cellulation of a closed surface therefore have identical local carrier certificates and different federation nerves. The local certificate cannot distinguish them. Full interface maps, higher-overlap coherence, and the global incidence/refinement receipt are necessary parents of a support-screen claim. □

Finite bridge theorem. The source-derived incidence nerve uses the twelve ports as carrier charts, the thirty edges as seam algebras, and the twenty oriented faces as nonvacuous triple restrictions. It verifies every interface homomorphism and face cocycle. On the same bound artifact, phase readback produces accepted confluent seam repairs, and the operational observer passes record, feedback, prediction, and checkpoint controls. The carrier nerve maps refinement-naturally to an oriented S^2 support limit. This closes the finite carrier, federation, observer, and support bridge of Definition 5.3. It does not produce an H^3 frame, event manifold, BW/KMS clock, physical scale, laboratory attachment, or continuum fields.

The fold adds no independent dynamics. Repair is the finite patch operation that compares overlaps, updates records, and reaches the accepted normal form. Folding names the spherical chart presentation of that repaired quotient state. In plain language, sphere folding is what overlap repair looks like on the observer-facing screen.

Strict descent prevents the accepted relation from serving as a reversible equilibrium law. If $x \to_r^{\mathrm{acc}} y$ lowers the well-founded repair measure, the reverse move cannot be another accepted descending step. The simulator consequently keeps three operators distinct:

$$\mathrm{Rep}_r, \qquad e^{-tL_r^{\mathrm{eq}}}, \qquad \sigma_t^{\rho}.$$

They are, respectively, the irreversible normal-form map, an equilibrium or Euclidean relaxation, and the reversible modular automorphism group. A common local proposal menu may contribute data to all three constructions. This shared input does not identify the operators.

Theorem 5.5 (Finite rate reconstruction on the reversible layer)**.** *Let $G = (X, E)$ be a connected finite state graph with positive rates $q_{u\to v}^{\mathrm{eq}}$ in both directions of every edge. Define*

$$a_{u\to v} := \log \frac{q_{v\to u}^{\mathrm{eq}}}{q_{u\to v}^{\mathrm{eq}}}.$$

The following conditions are equivalent:

(i) *every oriented cycle has zero affinity;*

(ii) *there is a potential* $K : X \to \mathbb{R}$*, unique up to an additive constant, with* $a_{u\to v} = K_v - K_u$*;*

(iii) $\pi_x \propto e^{-K_x}$ *satisfies detailed balance* $\pi_u q^{\mathrm{eq}}_{u\to v} = \pi_v q^{\mathrm{eq}}_{v\to u}$*.*

The rates belong to the reversible equilibrium layer. They are not rates of the accepted strict-descent relation.

Proof. Condition (ii) telescopes around every cycle. Under (i), fix a root and define K_x by summing a along any path from the root to x; zero cycle affinity gives path independence. Exponentiating $K_v - K_u = a_{u\to v}$ gives detailed balance, and taking logarithms of detailed balance gives (ii). □

K in this theorem is a classical Gibbs potential. On the commutative state algebra its modular automorphism is trivial, so rate reconstruction does not create a nontrivial modular clock.

5.1 Finite modular and register theorems

The finite equilibrium layer supports several exact statements before any continuum or physical current interpretation is made. They also fix which information a simulator has to produce.

Theorem 5.6 (Exact modular coarse graining)**.** *Let* $c : X_s \to X_r$ *be a surjective coarse-graining map between finite sets and let* $\mu_s(x) > 0$*. The pushed-forward law and its modular potential are*

$$\mu_r(y) = \sum_{c(x)=y} \mu_s(x), \qquad K_r(y) = -\log\left(\sum_{c(x)=y} e^{-K_s(x)}\right), \qquad K_s = -\log \mu_s.$$

Thus modular potentials coarse-grain by a log-sum-exp rule, up to the common additive normalization convention. More generally, if $\mu_s(x) = Z_s^{-1} m_s(x) e^{-S_s(x)}$ *and* $m_r(y) > 0$ *is the declared coarse reference weight, then*

$$S_r^{\mathrm{eff}}(y) = -\log\left(\sum_{c(x)=y} m_s(x) e^{-S_s(x)}\right) + \log m_r(y)$$

gives the exact pushed-forward law. For a finite quantum channel Φ_{sr}*, the corresponding statement is* $\rho_r = \Phi_{sr}(\rho_s)$ *and* $K_r = -\log \rho_r$ *whenever the output is faithful. An arithmetic average of* K_s *is not the coarse modular potential in general.*

Proof. Both classical formulas follow by substituting $\mu_s = e^{-K_s}$ or $\mu_s = Z_s^{-1} m_s e^{-S_s}$ into the definition of the pushforward. The quantum formula is the definition of the modular Hamiltonian of the pushed-forward faithful state. □

Proposition 5.7 (A faithful equilibrium law does not select its generator)**.** *Let* $G = (X, E)$ *be a connected finite graph and let* $\mu_x > 0$*. For every collection of positive symmetric edge conductances* $c_{xy} = c_{yx}$*,*

$$q_{x\to y} = \frac{c_{xy}}{\mu_x}$$

defines a continuous-time Markov generator that is reversible with stationary law μ*. Distinct conductance families give distinct generators with the same state. A geometry, MaxEnt state, or repair normal form therefore selects no equilibrium dynamics unless an additional rate or conductance rule is supplied.*

Proof. The identity $\mu_x q_{x\to y} = c_{xy} = \mu_y q_{y\to x}$ is detailed balance. The diagonal entries are fixed by row-sum zero. Varying any conductance varies the generator while preserving the same stationary law. $\square$

Theorem 5.8 (Weighted graph Hodge split and entropy production)**.** *Choose an orientation of a connected finite graph, positive vertex and edge weights, and let D be the vertex-to-edge coboundary. Every real edge one-cochain has the unique orthogonal decomposition*

$$a = DK + h, \qquad D^*h = 0,$$

with K unique up to an additive constant. For an irreducible stationary Markov law π, define on each chosen edge

$$J_{xy} = \pi_x q_{x\to y} - \pi_y q_{y\to x}, \qquad F_{xy} = \log \frac{\pi_x q_{x\to y}}{\pi_y q_{y\to x}}.$$

Then the steady entropy-production rate is

$$\sigma = \sum_{\{x,y\}\in E} J_{xy}F_{xy} \geq 0,$$

and it vanishes exactly when detailed balance holds on every edge. The bare logarithmic rate ratio omits the stationary weights and is therefore not, by itself, the thermodynamic force.

Proof. Finite-dimensional orthogonal projection gives the Hodge split, since $(\operatorname{im} D)^\perp = \ker D^*$. Each entropy-production summand has the form $(A-B)\log(A/B) \geq 0$ for positive A, B. Equality holds exactly when $A = B$ edge by edge. $\square$

Theorem 5.9 (Finite KMS-symmetric jump generator)**.** *Let $\rho = Z^{-1}e^{-K}$ be faithful on a finite matrix algebra. Suppose a Heisenberg-picture Lindblad generator has jump operators $V_{\omega,a}$ satisfying*

$$[K, V_{\omega,a}] = -\omega V_{\omega,a}, \qquad V_{-\omega,a} = V_{\omega,a}^*, \qquad \gamma_a(-\omega) = e^{-\omega}\gamma_a(\omega),$$

with positive Kossakowski matrices in every frequency sector. Suppose also that its Lamb-shift Hamiltonian commutes with K, and that the generator is self-adjoint for the declared symmetric GNS inner product

$$\langle A, B\rangle_{\rho,1/2} = \operatorname{Tr}(\rho^{1/2}A^*\rho^{1/2}B).$$

Then the semigroup is completely positive and unital, preserves ρ, commutes with the modular flow $A \mapsto e^{itK}Ae^{-itK}$, and respects the displayed frequency-sector decomposition. The sign in the KMS rate relation is tied to the displayed commutator convention.

Proof. Positive Kossakowski matrices give complete positivity. Unitality and GNS self-adjointness imply $\operatorname{Tr}(\rho L(A)) = \langle \mathbf{1}, L(A)\rangle_{\rho,1/2} = \langle L(\mathbf{1}), A\rangle_{\rho,1/2} = 0$, hence stationarity. The eigenoperator relation, the paired KMS rates, and $[K, H_{\mathrm{LS}}] = 0$ make every term modular covariant and preserve its frequency sector. $\square$

Remark 5.10 (Finite modular receipt boundary)*.* The source-ensemble ledger reports the pushed-forward laws or channels, independent conductances and rates, stationary weights, cycle-affinity residual, and the weighted Hodge remainder. The quantum/modular ledger reports the faithful density, modular spectrum, frequency projectors, Kossakowski positivity, KMS-pair residuals, GNS-symmetry residual, and modular-covariance residual. These finite data instantiate Theorems 5.6–5.9; the theorems do not manufacture those data.

Theorem 5.11 (Exact $A_5 \times \mathbb{Z}_2$ register commutant)**.** *For the twelve-port permutation module,*

$$\mathbb{R}[\mathsf{P}_{12}] \cong \mathbf{1} \oplus \mathbf{3} \oplus \mathbf{3}' \oplus \mathbf{5}$$

is multiplicity-free. The icosahedral adjacency operator has the four distinct eigenvalues $5, \sqrt{5}, -\sqrt{5}, -1$ *on these summands. Consequently its real* A_5*-commutant is* $\mathbb{R}[A_{\text{ico}}]$*, represented by polynomials of degree at most three. On the oriented register* $\mathsf{R}_{24} = \mathbb{R}[\mathsf{P}_{12}] \otimes \mathbb{R}[\mathbb{Z}_2]$*, every self-adjoint operator commuting with* A_5 *and orientation reversal is specified by one real scalar on each irreducible summand in each parity sector, eight scalars in total.*

Proof. Schur's lemma gives one scalar on each real irreducible summand. The four distinct adjacency eigenvalues make the spectral projectors degree-three interpolation polynomials in A_{ico}. Diagonalizing the commuting reversal involution gives two parity copies of the same multiplicity-free decomposition. □

Proposition 5.12 (No equivariant directed 24-cycle)**.** *Regard the oriented register as the transitive* $(A_5 \times \mathbb{Z}_2)/(C_5 \times \{1\})$*-set. It has no* $A_5 \times \mathbb{Z}_2$*-equivariant successor permutation that is a single directed 24-cycle.*

Proof. Its equivariant automorphism group is

$$N_{A_5 \times \mathbb{Z}_2}(C_5 \times \{1\})/(C_5 \times \{1\}) \cong \mathbb{Z}_2 \times \mathbb{Z}_2,$$

because $N_{A_5}(C_5) \cong D_{10}$. Every equivariant permutation therefore has order at most two, whereas a 24-cycle has order 24. □

Definition 5.13 (Source-derived oriented-channel realization)**.** *Let* E^{or} *be the oriented transitions of the reversible quotient-state layer and set*

$$\mathcal{H}_E = \ell^2(E^{\text{or}}), \qquad F_E|e\rangle = a_e|e\rangle, \qquad a_e = \log \frac{q_{\bar{e}}^{\text{eq}}}{q_e^{\text{eq}}}.$$

A source-derived 24*-channel realization is a full-rank map*

$$C : \mathcal{H}_{24} := \mathbb{C}[\mathsf{P}_{12}] \otimes \mathbb{C}^2_{\text{or}} \longrightarrow \mathcal{H}_E$$

whose coefficients, aggregation rule, and provenance are emitted by the physical proposal ledger. After replacing C *by* $C(C^*C)^{-1/2}$*, assume* $C^*C = \mathbf{1}$*. Edge reversal is denoted* $R_E|e\rangle = |\bar{e}\rangle$*, and register reversal by* R_{24}*. The state set* X*, transition set* E^{or}*, and label map into the* 24 *channel types remain distinct objects.*

Theorem 5.14 (Oriented-register modular compression)**.** *For a source-derived realization* C*, the register is invariant under the edge modular grading exactly when*

$$\epsilon_{\text{ch}} := \left\|(\mathbf{1} - CC^*)F_E C\right\|_{\text{HS}} = 0.$$

Equivalently, there is a unique operator Ω_{24} *with*

$$F_E C = C\Omega_{24}, \qquad \Omega_{24} = C^* F_E C.$$

Suppose in addition that the edge rates are A_5*-equivariant,* C *intertwines the edge and register actions of* A_5 *and reversal, and hence* $R_E F_E R_E = -F_E$*. Then*

$$\Omega_{24} \cong \bigoplus_{\lambda \in \{1,3,3',5\}} \mathbf{1}_{V_\lambda} \otimes M_\lambda, \qquad \operatorname{spec}(M_\lambda) = \{-\omega_\lambda, +\omega_\lambda\},$$

for four nonnegative source-derived values $\omega_1, \omega_3, \omega_{3'}, \omega_5$. Thus the paired frequencies have degeneracies $1, 3, 3, 5$. The symmetry fixes this block pattern and these degeneracies, but not the four numerical values.

Proof. The residual vanishes precisely when $F_E \operatorname{im} C \subseteq \operatorname{im} C$. Since C is isometric, restriction to this invariant subspace gives the unique compression $\Omega_{24} = C^* F_E C$. A_5-equivariance and Schur's lemma give one Hermitian 2×2 orientation matrix M_λ on each of the multiplicity-free summands $1, 3, 3', 5$. Reversal covariance gives $R_{24}\Omega_{24}R_{24} = -\Omega_{24}$, so each orientation block is traceless and has eigenvalues $\pm\omega_\lambda$. Tensoring with V_λ gives the displayed degeneracies. ☐

Corollary 5.15 (Raw labels are modular eigenchannels only at equal affinity)**.** *If each raw register slot is realized as a normalized sum of transitions with that label and distinct labels have disjoint edge support, exact channel closure requires*

$$a_e = a_{e'} \quad \textit{whenever } e \textit{ and } e' \textit{ carry the same raw label.}$$

Otherwise the raw slot basis is not a modular eigenbasis; any invariant 24-channel space must instead use source-derived linear combinations.

Proposition 5.16 (Operator-frame identification boundary)**.** *Let $V_a \in \mathcal{A} \subseteq B(\mathcal{H})$ be realized transition operators and let ω_a be source-derived frequencies. If self-adjoint K and K' both solve*

$$[K, V_a] = \omega_a V_a, \qquad [K', V_a] = \omega_a V_a \quad \textit{for every } a,$$

then $K - K'$ lies in the commutant of the algebra generated by the V_a. Hence the channel frame identifies a noncommutative modular Hamiltonian only modulo that commutant, and modulo the center when the operators generate the declared finite algebra.

Proof. Subtracting the two commutator equations gives $[K - K', V_a] = 0$ for every a; the generated-algebra statement follows. ☐

Remark 5.17 (Rate scale, clock scale, and BW promotion)*.* Multiplying every equilibrium rate by one constant leaves every affinity, the reconstructed stationary law, K, and Ω_{24} unchanged, while rescaling the relaxation generator. Rate ratios therefore determine a dimensionless potential; rate magnitudes determine kinetics. Neither fixes a frequency in hertz. Operational time requires the independent clock instrument and affine calibration used later in this paper. Physical BW promotion further requires a refinement-natural comparison showing that the gear-derived state is the same support-visible cap state used by the geometric modular theorem, the independently complete MGNS-1 package on that same tower, its regularized cutoff schedule, and the independent 2π-normalization receipt.

The register receipt reports the character-projector multiplicities, equivariance residual, within-irrep spectral spread, orientation-parity leakage, adjacency-polynomial residual, and the equivariant-permutation order. The modular-gearing receipt additionally reports the oriented rate ledger, quotient lumpability defect, fundamental-cycle affinities, stationary-law and detailed-balance residuals, C^*C, ϵ_{ch}, the four paired gaps, and the commutant dimension of any operator-valued lift. Exact data give Theorems 5.11 and 5.14 and Proposition 5.12; approximate data remain diagnostics with declared tolerances.

These theorems classify finite register states and operators. They do not construct a physical gauge-current algebra. That step requires the separate full-rank, compact skew-adjoint, commutator-closed current lift, its inner A_5 action, and refinement-natural physical descent used by the compact and particle-sector papers.

On the literal transitive twelve-port set $\mathsf{P}_{12} = A_5/C_5$, an irreducible A_5-equivariant rate law has a unique invariant equilibrium distribution, hence $\pi_p = 1/12$ and constant K_p. On $\mathsf{P}_{12} \times \{+,-\}$, with A_5 acting on the port coordinate, an invariant potential has at most two values K_+ and K_-. The geometry can therefore carry one orientation gap $\Delta K = K_- - K_+$. It cannot supply twenty-four independent energy levels. The state set X, oriented transition set E^{or}, and channel label map $\lambda : E^{\mathrm{or}} \to \mathsf{R}_{24}$ are separate objects; λ need not be a bijection. In particular, twenty-four oriented edge instances contain only twelve unoriented edges and cannot connect a graph on twenty-four states.

The finite carrier vocabulary is compatible with the layered functional carrier of the consensus paper: the carrier is a quotient-visible finite implementation surface, and hidden port labels or substrate coordinates are silent when visible interface data, record statistics, repair maps, and checkpoint continuation are preserved.

In the terminology of the program's consistency stack, this record/repair architecture is the carrier of two rows of the forcing chain. Row C1 (self-reading): a world that is the fixed point of its own description must contain observers, records of their readings, and a mechanism that keeps those records consistent under continued reading; the patch federation above, with its record algebras, readout maps, and repair interfaces, is that mechanism made explicit at fixed cutoff. Row C6 (record existence): a screen at exact self-similar balance carries no events, so records require detuning; the register constructions below give the fixed-cutoff surface on which that detuning has a declared carrier. Both mappings hold on the declared branches and gates of this paper, including the named MaxEnt-hypothesis caveat where it applies.

5.2 Edge-center scalar-slot register

The downstream dark-sector and coherent-matter papers need one fixed-cutoff microphysics export: on the declared scalar quotient of an edge-center collar register, first-order scalar observables have only the canonical opportunity count as their nonconstant direction.

Definition 5.18 (Finite scalar-slot register)**.** *Fix a regulator r, physical quotient $Q_r = \Sigma_r/\Gamma_r$, and collar or local support region C. Let $E_{\nu,r}(C)$ be the finite set of edge-center scalar slots on the physical quotient. For each $e \in E_{\nu,r}(C)$, let*

$$p_{e,\nu} : Q_r \to \{0,1\}$$

be the quotient-visible central indicator that slot e carries an active scalar repair opportunity, and let $a_e > 0$ be the declared scalar-slot weight. The canonical quotient-edge scalar opportunity count is

$$\mathcal{N}_{\nu,r,C}(q) = \sum_{e \in E_{\nu,r}(C)} a_e \, p_{e,\nu}(q).$$

The normalized scalar readout is

$$S_{\nu,r,C}(q) = \frac{\mathcal{N}_{\nu,r,C}(q)}{\sum_{e \in E_{\nu,r}(C)} a_e}.$$

The indicators $p_{e,\nu}$ live on Q_r, not on hidden presentations in Σ_r. Gauge representatives, port labels, implementation coordinates, and accepted repair schedules are removed by the quotient before $\mathcal{N}_{\nu,r,C}$ is read.

Theorem 5.19 (Rank-one scalar-slot completeness on the quotient-edge branch)**.** *Let G_C be the quotient-preserving relabeling group of scalar slots in collar C. Assume:*

(i) G_C preserves the physical quotient and the scalar-slot weights a_e;

(ii) the scalar branch keeps only quotient-visible central observables invariant under G_C;

(iii) the scalar slots form one scalar orbit after transverse finite-thickness averaging, equivalently the declared scalar observable is the weighted orbit sum.

Then every first-order quotient-visible scalar observable on the edge-center scalar register is an affine function of $\mathcal{N}_{\nu,r,C}$:

$$F_{\mathrm{sc}}(q) = \alpha + \beta\,\mathcal{N}_{\nu,r,C}(q).$$

Equivalently, after constants are removed,

$$\mathcal{S}^{(1)}_{\mathrm{sc}}(C) = \mathrm{span}\{\mathcal{N}_{\nu,r,C}\}.$$

Proof. A first-order scalar observable on the slot register has the form

$$F(q) = \alpha + \sum_{e\in E_{\nu,r}(C)} b_e\, p_{e,\nu}(q).$$

Quotient-visible scalar status requires invariance under G_C. If G_C acts transitively on the scalar slot orbit, invariance forces b_e/a_e to be constant on that orbit. Therefore

$$F(q) = \alpha + \beta \sum_e a_e p_{e,\nu}(q) = \alpha + \beta\,\mathcal{N}_{\nu,r,C}(q).$$

With finite transverse thickness the same argument applies orbitwise before the declared weight integral over the transverse coordinate. The finite-thickness scalar is the weighted orbit sum, so the first-order scalar subspace is one-dimensional after constants are removed. □

This theorem is only about the scalar quotient of the edge-center register. It does not select a primitive observer size, a unique hardware carrier, a probability law on Q_r, or a full finite covariant stress parent.

5.3 Quotient-edge scalar register and protected center reserve

The scalar-slot theorem above is the rank-one first-order export. The dark-sector and coherent-matter papers also need the local event-algebra realization: scalar activation, the protected $\mathbb{Z}_6$ reserve, and finite-thickness collar survival have to live on one quotient-visible edge-center register before any coefficient can be promoted.

Trace convention. Throughout this subsection, $\tau_{q,r,C}$ denotes the normalized quotient expectation on the finite physical collar quotient algebra. Thus

$$\tau_{q,r,C}(1) = 1.$$

For a nonzero projection p, define the reciprocal reserve-depth trace by

$$\mathrm{Tr}^{\#}_{q,r,C}(p) := \tau_{q,r,C}(p)^{-1}.$$

The reserve survival coefficient always uses the normalized mean τ_q, not the reciprocal trace. Thus

$$\tau_q(Z_{6,r,C}) = \frac{P}{24} \quad \text{is the presence probability,}$$

while

$$\mathrm{Tr}_q^{\#}(Z_{6,r,C}) = \frac{24}{P} \quad \text{is the reciprocal slot-depth trace.}$$

The notation $\mathrm{Tr}_q(Z_{6,r,C}) = 24/P$ is valid only when Tr_q is explicitly defined as a reciprocal slot-count trace; otherwise the reciprocal quantity is $\mathrm{Tr}_q^{\#}$.

Definition 5.20 (Physical collar quotient and edge-center map)**.** *Fix a finite regulator r and a connected collar cut C. Let*

$$Q_{r,C}$$

be the physical collar quotient obtained from the fixed-cutoff presentation after quotienting gauge representatives, hidden carrier coordinates, port labels, repair-schedule identifiers, mesh labels, shard or worker identifiers, and inert ancillary labels.

Let

$$\mathcal{A}_{r,C}^{\mathrm{phys}}$$

be the finite physical collar algebra, with center

$$Z(\mathcal{A}_{r,C}^{\mathrm{phys}}).$$

The connected cut exposes a quotient-visible edge-center sector map

$$\eta_{r,C} : Q_{r,C} \longrightarrow \Xi_{r,C},$$

where $\Xi_{r,C}$ is the finite set of edge-center cut sectors visible on the repaired collar normal form.

For each $\xi \in \Xi_{r,C}$, let

$$e_{\xi,r,C}$$

be the central sector projection corresponding to the fiber $\eta_{r,C}^{-1}(\xi)$. In the classical quotient algebra this is the indicator function of the fiber. In the finite quantum quotient algebra this is the central summand projection for the same quotient-visible sector.

Definition 5.21 (Edge-center quotient algebra)**.** *The edge-center quotient algebra of the connected collar cut C is*

$$\boxed{\mathcal{E}_{r,C} := \mathrm{Alg}\{e_{\xi,r,C} : \xi \in \Xi_{r,C}\} \subseteq Z(\mathcal{A}_{r,C}^{\mathrm{phys}}).}$$

Equivalently,

$$\mathcal{E}_{r,C} \cong \ell^\infty(\Xi_{r,C}), \qquad 1_{\mathcal{E}_{r,C}} = \sum_{\xi \in \Xi_{r,C}} e_{\xi,r,C}.$$

Theorem 5.22 (Edge-center quotient algebra)**.** *On the fixed-cutoff connected-collar branch, $\mathcal{E}_{r,C}$ is a finite commutative central subalgebra of $\mathcal{A}_{r,C}^{\mathrm{phys}}$.*

Proof. The fibers $\eta_{r,C}^{-1}(\xi)$ partition the physical collar quotient. Therefore the corresponding sector projectors satisfy

$$e_{\xi,r,C} e_{\xi',r,C} = \delta_{\xi\xi'} e_{\xi,r,C}, \qquad \sum_{\xi \in \Xi_{r,C}} e_{\xi,r,C} = 1.$$

Because the edge-center label is quotient-visible superselection data of the connected cut, its sector projectors commute with every physical collar observable and lie in $Z(\mathcal{A}_{r,C}^{\mathrm{phys}})$. The algebra generated by finitely many mutually orthogonal central projections is finite, commutative, and isomorphic to $\ell^\infty(\Xi_{r,C})$. ∎

Definition 5.23 (Quotient scalar readout algebra)**.** *Let*

$$\mathsf{Scal}_{r,C} \subseteq \mathcal{A}_{r,C}^{\mathrm{phys}}$$

be the subalgebra generated by all quotient-local collar readouts whose values are invariant under orientation choice, vector/tensor frame choice, bulk-coordinate choice, hidden carrier coordinate, port relabeling, accepted repair schedule, mesh label, shard label, and inert ancillary label.

Elements of $\mathsf{Scal}_{r,C}$ *are the physical scalar collar readouts of the connected cut.*

Assumption 5.24 (Edge-center scalar completeness)**.** *On the declared fixed-cutoff scalar branch, every quotient-local scalar collar readout on a connected cut is resolved by the edge-center sector map:*

$$\boxed{\mathsf{Scal}_{r,C} = \mathcal{E}_{r,C}.}$$

Equivalently, there is no independent scalar event algebra carried by bulk coordinates, screen-area labels, noncentral matrix directions, hidden carrier coordinates, port labels, or repair-schedule identifiers after the physical quotient has been taken.

Definition 5.25 (Scalar recoverability defect and activation projector)**.** *Let*

$$R_{r,C} \in \mathsf{Scal}_{r,C,+}$$

be the quotient-local scalar recoverability defect on the collar cut. On the positive scalar source branch,

$$R_{r,C} = I(A : D \mid B)$$

or the corresponding quotient-local finite-collar representative of that recoverability defect.

Define the scalar activation projector by

$$\boxed{\Pi_{r,C}^{\mathrm{scal}} := \mathbf{1}_{(0,\infty)}(R_{r,C}).}$$

Inside this subsection we may also write

$$S_{r,C} := \Pi_{r,C}^{\mathrm{scal}}$$

when matching the dark-sector notation.

Theorem 5.26 (Scalar activation is an edge-center event)**.** *Assume edge-center scalar completeness. Then*

$$\boxed{\Pi_{r,C}^{\mathrm{scal}} \in \mathcal{E}_{r,C}.}$$

Proof. By definition, $R_{r,C}$ is a quotient-local scalar collar readout, hence

$$R_{r,C} \in \mathsf{Scal}_{r,C}.$$

By Assumption 5.24,

$$\mathsf{Scal}_{r,C} = \mathcal{E}_{r,C}.$$

Therefore $R_{r,C} \in \mathcal{E}_{r,C}$. Since $\mathcal{E}_{r,C}$ is a finite commutative algebra, functional calculus is internal to $\mathcal{E}_{r,C}$. Thus

$$\Pi_{r,C}^{\mathrm{scal}} = \mathbf{1}_{(0,\infty)}(R_{r,C}) \in \mathcal{E}_{r,C}.$$

□

Definition 5.27 (Protected Z_6 reserve projector). *Let the realized Standard Model quotient be*

$$G_{\text{phys}} = \frac{SU(3) \times SU(2) \times U(1)}{\mathbb{Z}_6}.$$

On the protected-center finite-collar branch, let

$$\zeta_{6,r,C} : \Xi_{r,C} \to \{0, 1\}$$

be the quotient-visible indicator that an edge-center sector carries the protected Z_6 reserve class.

Define

$$\boxed{Z_{6,r,C} := \sum_{\xi:\zeta_{6,r,C}(\xi)=1} e_{\xi,r,C}.}$$

Then

$$Z_{6,r,C} \in \mathcal{E}_{r,C}.$$

Theorem 5.28 (Same-register and commutation theorem). *Assume edge-center scalar completeness and protected-center realization. Then*

$$\boxed{\Pi^{\text{scal}}_{r,C}, Z_{6,r,C} \in \mathcal{E}_{r,C}}$$

and therefore

$$\boxed{[\Pi^{\text{scal}}_{r,C}, Z_{6,r,C}] = 0}$$

inside the physical quotient collar algebra.

Proof. Theorem 5.26 gives

$$\Pi^{\text{scal}}_{r,C} \in \mathcal{E}_{r,C}.$$

Definition 5.27 gives

$$Z_{6,r,C} \in \mathcal{E}_{r,C}.$$

The algebra $\mathcal{E}_{r,C} \cong \ell^\infty(\Xi_{r,C})$ is finite and commutative. Hence any two of its elements commute in $\mathcal{A}^{\text{phys}}_{r,C}$. □

Assumption 5.29 (Scalar activation channel completeness). *The scalar activation channel on the connected collar cut is the quotient event algebra generated by the nonzero recoverability-defect event:*

$$\boxed{\mathcal{C}^{\text{scal}}_{r,C} = \mathrm{Alg}\{\Pi^{\text{scal}}_{r,C}\} \subseteq \mathcal{E}_{r,C}.}$$

Equivalently, there is no second independent scalar activation generator after the physical quotient has been taken.

Theorem 5.30 (Scalar-channel exhaustion). *Assume edge-center scalar completeness, scalar activation channel completeness, and protected-center realization. Then the scalar activation channel is exhausted by $\Pi^{\text{scal}}_{r,C}$, and the protected reserve splits it as*

$$\boxed{\Pi^{\text{scal}}_{r,C} = \Pi^{\text{scal}}_{r,C}(1 - Z_{6,r,C}) + \Pi^{\text{scal}}_{r,C} Z_{6,r,C}.}$$

The first summand is the reserve-surviving scalar channel. The second summand is the reserve-blocked scalar channel.

Proof. By Assumption 5.29, every scalar activation event is generated by $\Pi^{\mathrm{scal}}_{r,C}$. Thus no scalar activation support exists outside this projector.

By Theorem 5.28, $\Pi^{\mathrm{scal}}_{r,C}$ and $Z_{6,r,C}$ commute. Therefore

$$\Pi^{\mathrm{scal}}_{r,C}(1 - Z_{6,r,C}) \quad \text{and} \quad \Pi^{\mathrm{scal}}_{r,C} Z_{6,r,C}$$

are commuting orthogonal projections. Their sum is

$$\Pi^{\mathrm{scal}}_{r,C} - \Pi^{\mathrm{scal}}_{r,C} Z_{6,r,C} + \Pi^{\mathrm{scal}}_{r,C} Z_{6,r,C} = \Pi^{\mathrm{scal}}_{r,C}.$$

Hence the reserve split exhausts the scalar activation channel. □

Theorem 5.31 (No separate scalar carrier)**.** *Assume edge-center scalar completeness. Let B be any proposed alternative scalar activation projector, possibly represented on a bulk, screen-area, noncentral, implementation-local, or hidden-carrier algebra.*

If B changes scalar activation but does not factor through $\mathcal{E}_{r,C}$, then the physical quotient observable algebra has been changed. If B does not change quotient observables, then its scalar content is the same as its edge-center central support in $\mathcal{E}_{r,C}$, and the extra carrier is inert.

Proof. A physical scalar activation event must be a quotient-local scalar observable. By Assumption 5.24, every such observable factors through the edge-center sector map

$$\eta_{r,C} : Q_{r,C} \to \Xi_{r,C}.$$

Equivalently, every such event is represented by a projection in

$$\mathcal{E}_{r,C}.$$

Suppose B is a bulk, screen-area, hidden-carrier, implementation-local, or noncentral event that does not factor through $\eta_{r,C}$. Then there are physical presentations that agree on the edge-center quotient sector but differ in the value of B. If that difference changes scalar activation, then scalar activation is not determined by the declared physical scalar quotient. The observable algebra has therefore been enlarged or changed.

If that difference does not change any quotient observable, then the degrees of freedom in B outside the edge-center quotient are physically silent. The scalar part of B is obtained by passing to the central edge-center event with the same quotient-visible activation support. That central support lies in $\mathcal{E}_{r,C}$.

For a noncentral operator B, the same argument is internal to $\mathcal{A}^{\mathrm{phys}}_{r,C}$. If the noncentral matrix part affects scalar readout, then the readout is not a scalar central event on the declared quotient. If it does not affect scalar readout, only its central support contributes to scalar activation, and that support lies in $\mathcal{E}_{r,C}$.

Thus a separate scalar carrier either changes quotient observables or collapses to the edge-center scalar event represented in $\mathcal{E}_{r,C}$. □

5.4 Coherent-matter scalar-source forcing

The scalar-slot exhaustion theorem fixes the available local scalar carrier. It does not make raw mass, heat, stored energy, or an engineering substrate factor into a scalar source. The source is a quotient-visible coherent-material receipt with explicit readback and boundary-prediction closure.

Definition 5.32 (OPH-coherent material subfederation)**.** *Fix a screen collar C, a finite material region $U \subset C$, and a refinement scale r. An OPH-coherent material subfederation is a finite family*

$$\mathfrak{M} = (\mathfrak{F}_U, \mathsf{R}_U, \mathsf{P}_U, \mathsf{C}_U)$$

where $\mathfrak{F}_U$ is a self-reading subfederation over U, R_U is a stable internal record readout, P_U is a boundary prediction operator against neighboring screen states, and C_U is a coherent mismatch-reduction certificate. The datum is nondegenerate when all four factors survive quotienting and refinement,

$$\mathbf{1}_{\mathrm{self-read}}(\mathfrak{F}_U)\,\mathsf{R}_U\,\mathsf{P}_U\,\mathsf{C}_U > 0.$$

Presentations with the same quotient record, boundary prediction, and coherence certificate define the same subfederation class.

Definition 5.33 (Canonical coherent-matter scalar source)**.** *For an OPH-coherent material subfederation $\mathfrak{M}$, define the canonical scalar source readout*

$$S^{\mathrm{can}}_{\mathrm{coh}}(U,t;h) := \mathbf{1}_{\mathrm{self-read}}(\mathfrak{F}_U)\,\mathsf{R}_U(U,t;h)\,\mathsf{P}_U(U,t;h)\,\mathsf{C}_U(U,t;h).$$

The readout is zero exactly when one of the self-read, record, boundary prediction, or coherent mismatch-reduction receipts vanishes. Rest mass, heat, and arbitrary coherent energy are outside this definition.

Claim 5.34 (Quotient descent of coherent-material source)**.** *The functional $S^{\mathrm{can}}_{\mathrm{coh}}$ descends to the screen quotient. It is invariant under relabelings of internal coordinates, duplicate micro-presentations, and record refinements that preserve $(\mathsf{R}_U, \mathsf{P}_U, \mathsf{C}_U)$ in the quotient.*

Proof. Each factor in Definition 5.33 is a screen-record observable: self-read existence is a boundary-stable record property, R_U is the internal record value, P_U is read through neighboring boundary records, and C_U is the observed reduction of prediction mismatch. The quotient identifies presentations with identical boundary-visible records and predictions. Hence the product is constant on quotient classes and descends. □

Theorem 5.35 (Nonzero coherent scalar source)**.** *Every nondegenerate OPH-coherent material subfederation has*

$$S^{\mathrm{can}}_{\mathrm{coh}}(U,t;h) > 0.$$

The zero branch is precisely the degenerate branch in which at least one coherent-material receipt vanishes.

Proof. Nondegeneracy is the strict positivity of the product in Definition 5.32. The canonical source is that product. □

Theorem 5.36 (Coherent-matter same-channel forcing)**.** *Assume Scalar Edge-Center Exhaustion scalar edge center exhaustion, Theorem 5.30. For every nondegenerate OPH-coherent material subfederation on a collar C, the source*

$$S^{\mathrm{can}}_{\mathrm{coh}}(U,t;h)$$

is valued in the unique scalar edge-center register $\mathcal{E}_{r,C}$. Consequently the coherent-material scalar source uses the same scalar channel that is priced by the Z_6 edge-center collar theorem.

Proof. By Lemma 5.34, $S^{\text{can}}_{\text{coh}}$ is a quotient-local scalar observable: it has no boundary direction, no spin frame, and no unscreened vector index. Scalar Edge-Center Exhaustion states that every quotient-local scalar perturbation that can affect a collar record factors through $\mathcal{E}_{r,C}$. The source therefore lies in $\mathcal{E}_{r,C}$. Theorem 5.28 identifies this register with the same edge-center scalar slot used by the Z_6 collar computation, so the channel identity follows. □

Corollary 5.37 (Finite scalar channel bridge)**.** *On the same branch, the coherent-material record slots and the Z_6 protected-reserve collar slices may be packaged as one finite indexed channel E. The record panel consists of scalar-slot activity, opportunity weights, and normal-form activation maps; the collar panel consists of finite-thickness slice weights and reserve means. With this packaging, the phrase "the same scalar channel" means that both panels are functions of the same finite family E. Equality of two separately defined numerical counters is insufficient.*

Proof. Theorem 5.36 places the coherent-material source in $\mathcal{E}_{r,C}$, while Theorem 5.28 places the protected Z_6 reserve in the same edge-center scalar register. Choosing the scalar slots of $\mathcal{E}_{r,C}$ as the index family E gives one finite channel carrying both panels. The packaging adds no new physical premise; it removes a bookkeeping ambiguity. □

Theorem 5.38 (Unique scalar linear response)**.** *On the weak-field branch, any infinitesimal frequency response to a nondegenerate OPH-coherent material source has the form*

$$\delta\nu_C = \chi^{\text{can}}_{\nu,C}\,\langle \eta_C, S^{\text{can}}_{\text{coh}}\rangle_C + O\Big((S^{\text{can}}_{\text{coh}})^2\Big)\,,$$

where η_C is the local edge-center test functional. No independent local scalar susceptibility can be added without violating Theorem 5.30.

Proof. Theorem 5.38 places the perturbation in the unique scalar register $\mathcal{E}_{r,C}$. Linearization of a one-register scalar perturbation gives a single edge-center test functional. An additional scalar susceptibility would define another quotient-local scalar carrier on the same collar, contradicting Scalar Edge-Center Exhaustion. □

The finite evidence packet records the coherent material source, its scalar edge-center binding, nonzero activation, same-channel forcing, and the unique linear response proved above.

Definition 5.39 (Oriented 24-slot repair register)**.** *On the twelve-port screen-sieve branch, let*

$$\mathsf{P}_{12,r,C}$$

be the twelve exposed central screen ports of the connected cut. Reversible write/check orientation gives the oriented repair-slot set

$$\mathsf{R}_{24,r,C} := \mathsf{P}_{12,r,C} \times \{+,-\},$$

so

$$|\mathsf{R}_{24,r,C}| = 24.$$

The corresponding oriented slot algebra is

$$\mathcal{R}^{\text{or}}_{24,r,C} := \ell^\infty(\mathsf{R}_{24,r,C}).$$

The oriented register is a bookkeeping refinement of the same quotient-edge cut surface. It does not create an independent scalar carrier.

Proposition 5.40 (The oriented register carries no modular clock)**.** *Every faithful state on $\mathcal{R}^{\mathrm{or}}_{24,r,C} = \ell^\infty(\mathsf{R}_{24,r,C})$ is tracial, so its modular automorphism group is trivial. The twenty-four slots can carry channel names, orientations, central event projectors, and orbit counts. They do not determine a modular Hamiltonian, a set of clock ticks, or a frequency. A nontrivial modular lift requires a noncommutative transition algebra, for example $B(\ell^2 X)$ with operators $V_{u\to v} = |v\rangle\langle u|$, together with a faithful physical state on that algebra.*

Proof. The algebra is finite and commutative. Its density operator is central in every faithful representation, hence $\rho^{it} a \rho^{-it} = a$ for all a and t. □

The shared-edge protected-reserve branch carries two named branch inputs: the shared cut entropy density $\bar{\ell}_{\mathrm{shared}} = P/4$ and the $\mathbb{Z}_6$ class equidistribution stated in Theorem 5.41 are declared assumptions of the branch, not derived quantities. Every value downstream of the $P/24$ exponent inherits both inputs.

Theorem 5.41 (Protected reserve mean and reciprocal trace)**.** *On the shared-edge protected-reserve branch, with the declared branch input $\bar{\ell}_{\mathrm{shared}} = P/4$ and the declared $\mathbb{Z}_6$ class equidistribution hypothesis (the shared-edge reserve is distributed uniformly over the six protected center classes; a declared branch input alongside the shared-cut density),*

$$\boxed{\tau_{q,r,C}(Z_{6,r,C}) = \frac{P}{24}.}$$

Equivalently, for the reciprocal reserve-depth trace,

$$\boxed{\mathrm{Tr}^{\#}_{q,r,C}(Z_{6,r,C}) = \frac{24}{P}.}$$

Using

$$P = 1.630968209403959,$$

one has

$$\frac{P}{24} = 0.06795700872516496,$$
$$\frac{24}{P} = 14.715185655746689,$$

and, under the presence reading of Lemma 5.46,

$$1 - \frac{P}{24} = 0.9320429912748350.$$

Proof. The shared cut entropy density on this branch is the declared branch input

$$\bar{\ell}_{\mathrm{shared}} = \frac{P}{4}.$$

The protected reserve is the realized $\mathbb{Z}_6$ center quotient reserve. By the declared $\mathbb{Z}_6$ class-equidistribution hypothesis, splitting the shared-edge reserve uniformly over the six protected center classes gives

$$\epsilon_{\mathbb{Z}_6} = \frac{\bar{\ell}_{\mathrm{shared}}}{|\mathbb{Z}_6|} = \frac{P/4}{6} = \frac{P}{24}.$$

Thus the normalized quotient reserve mean is

$$\tau_q(Z_{6,r,C}) = \frac{P}{24}.$$

By definition of the reciprocal reserve-depth trace,

$$\mathrm{Tr}_q^{\#}(Z_{6,r,C}) = \frac{1}{\tau_q(Z_{6,r,C})} = \frac{24}{P}.$$

□

Remark 5.42 (Oriented-register denominator coincidence). The oriented 24-slot register of Definition 5.39 also displays the denominator 24 on the local screen side. By that definition the register creates no independent scalar carrier, so the coincidence of denominators carries no corroborating weight for the class-equidistribution hypothesis; the $P/24$ value rests on the two declared branch inputs alone.

Definition 5.43 (Finite-thickness scalar opportunity profile)**.** *Let $Y_{r,C}$ be the finite transverse collar-coordinate set at regulator r. Let*

$$p_y, \qquad y \in Y_{r,C},$$

be the transverse slice projectors, with

$$\sum_{y \in Y_{r,C}} p_y = 1.$$

Let

$$\mathcal{A}_{r,C}^{\mathrm{thick}}$$

be the finite-thickness collar algebra containing the edge-center algebra and the transverse slice algebra:

$$\mathcal{A}_{r,C}^{\mathrm{thick}} \supseteq \mathcal{E}_{r,C} \vee \mathrm{Alg}\{p_y : y \in Y_{r,C}\}.$$

Let

$$\Pi_{r,C}^{\mathrm{scal,thick}}$$

be the scalar activation projector lifted to the thickened collar, and let

$$Z_{6,r,C}^{\mathrm{thick}}$$

be the protected reserve projector lifted to the thickened collar.

Define the scalar activation mass at slice y by

$$a_r(y) := \tau_{q,r,C}^{\mathrm{thick}}\left(\Pi_{r,C}^{\mathrm{scal,thick}} p_y\right).$$

Assume

$$A_r := \sum_{y \in Y_{r,C}} a_r(y) > 0.$$

The normalized finite-thickness scalar activation profile is

$$\boxed{w_r(y) := \frac{a_r(y)}{A_r}.}$$

Then
$$\sum_{y\in Y_{r,C}} w_r(y) = 1.$$

For every scalar-active slice, $a_r(y) > 0$, define the scalar-opportunity conditional reserve profile by
$$\boxed{\epsilon_{\mathbb{Z}_6,r}(y) := \frac{\tau^{\mathrm{thick}}_{q,r,C}\left(\Pi^{\mathrm{scal,thick}}_{r,C} Z^{\mathrm{thick}}_{6,r,C} p_y\right)}{\tau^{\mathrm{thick}}_{q,r,C}\left(\Pi^{\mathrm{scal,thick}}_{r,C} p_y\right)}.}$$
Slices with $a_r(y) = 0$ do not contribute to w_r and may be assigned any harmless value.

The scalar-weighted protected reserve mean is
$$\boxed{\bar{\epsilon}^{\mathrm{scal}}_{\mathbb{Z}_6,r} := \sum_{y\in Y_{r,C}} w_r(y)\epsilon_{\mathbb{Z}_6,r}(y) = \frac{\tau^{\mathrm{thick}}_{q,r,C}\left(\Pi^{\mathrm{scal,thick}}_{r,C} Z^{\mathrm{thick}}_{6,r,C}\right)}{\tau^{\mathrm{thick}}_{q,r,C}\left(\Pi^{\mathrm{scal,thick}}_{r,C}\right)}.}$$

In continuum notation,
$$w_r(y) \to w(y), \qquad \epsilon_{\mathbb{Z}_6,r}(y) \to \epsilon_{\mathbb{Z}_6}(y),$$
with
$$\int dy\, w(y) = 1$$
and
$$\bar{\epsilon}^{\mathrm{scal}}_{\mathbb{Z}_6} = \int dy\, w(y)\epsilon_{\mathbb{Z}_6}(y).$$

Theorem 5.44 (Finite-thickness mean reserve disintegration)**.** *At fixed regulator,*
$$\boxed{\sum_{y\in Y_{r,C}} w_r(y)\epsilon_{\mathbb{Z}_6,r}(y) = \frac{\tau^{\mathrm{thick}}_{q,r,C}\left(\Pi^{\mathrm{scal,thick}}_{r,C} Z^{\mathrm{thick}}_{6,r,C}\right)}{\tau^{\mathrm{thick}}_{q,r,C}\left(\Pi^{\mathrm{scal,thick}}_{r,C}\right)}.}$$
Consequently, if the scalar-weighted protected reserve mean receipt holds,
$$\boxed{\frac{\tau^{\mathrm{thick}}_{q,r,C}\left(\Pi^{\mathrm{scal,thick}}_{r,C} Z^{\mathrm{thick}}_{6,r,C}\right)}{\tau^{\mathrm{thick}}_{q,r,C}\left(\Pi^{\mathrm{scal,thick}}_{r,C}\right)} = \frac{P}{24},}$$
then
$$\boxed{\sum_{y\in Y_{r,C}} w_r(y)\epsilon_{\mathbb{Z}_6,r}(y) = \frac{P}{24}.}$$
In the continuum limit,
$$\boxed{\int dy\, w(y)\epsilon_{\mathbb{Z}_6}(y) = \frac{P}{24}.}$$

Proof. Temporarily write $\tau = \tau^{\mathrm{thick}}_{q,r,C}$, $\Pi^{\mathrm{scal}} = \Pi^{\mathrm{scal,thick}}_{r,C}$, and $Z_6 = Z^{\mathrm{thick}}_{6,r,C}$. Using the definitions,
$$\sum_y w_r(y)\epsilon_{\mathbb{Z}_6,r}(y) = \sum_y \frac{\tau(\Pi^{\mathrm{scal}} p_y)}{\tau(\Pi^{\mathrm{scal}})} \cdot \frac{\tau(\Pi^{\mathrm{scal}} Z_6 p_y)}{\tau(\Pi^{\mathrm{scal}} p_y)}.$$

Canceling the slice factors gives

$$\sum_y w_r(y)\epsilon_{\mathbb{Z}_6,r}(y) = \frac{1}{\tau(\Pi^{\mathrm{scal}})}\sum_y \tau(\Pi^{\mathrm{scal}} Z_6 p_y).$$

Since $\sum_y p_y = 1$,

$$\sum_y \tau(\Pi^{\mathrm{scal}} Z_6 p_y) = \tau(\Pi^{\mathrm{scal}} Z_6).$$

Therefore

$$\sum_y w_r(y)\epsilon_{\mathbb{Z}_6,r}(y) = \frac{\tau(\Pi^{\mathrm{scal}} Z_6)}{\tau(\Pi^{\mathrm{scal}})}.$$

If that scalar-weighted quotient mean is $P/24$, the displayed identity follows. The continuum expression is the weak-limit version of the same finite disintegration. □

Remark 5.45 (Unconditioned trace is not enough). The identity

$$\tau_q(Z_{6,r,C}) = P/24$$

does not by itself imply

$$\frac{\tau_q(\Pi^{\mathrm{scal}}_{r,C} Z_{6,r,C})}{\tau_q(\Pi^{\mathrm{scal}}_{r,C})} = P/24.$$

The latter is a scalar-weighted reserve mean receipt. It follows from scalar-reserve unbiasedness. Co-registration or commutation alone is insufficient.

Lemma 5.46 (Presence reading of the reserve profile)**.** *$Z_{6,r,C}$ is a sum of sector projections, so the conditional reserve profile $\epsilon_{\mathbb{Z}_6}(y)$, a ratio of quotient traces of projections, is the conditional probability of the reserve-presence event in a co-registered scalar slot at transverse coordinate y. A scalar opportunity survives exactly when the presence event does not occur. The local scalar survival factor is therefore*

$$\lambda_{\mathrm{slot}}(y) = 1 - \epsilon_{\mathbb{Z}_6}(y)$$

for every occupancy law whose presence event has probability $\epsilon_{\mathbb{Z}_6}(y)$. No occupancy variable with that presence probability attains the Poisson zero-count factor $e^{-\epsilon}$: $1-\epsilon < e^{-\epsilon}$ strictly for $\epsilon > 0$, and every finite m-fold sub-slot refinement $(1-\epsilon/m)^m$ sits strictly below $e^{-\epsilon}$, which is the supremum of that family, attained by no finite regulator. A mean-count reading, in which $\epsilon(y)$ is consumed as the expectation of an $\mathbb{N}$-valued occupancy rather than as the presence probability, severs the link to the projection trace and consumes a scalar carrier beyond $Z_{6,r,C}$, which the no-separate-carrier clause of Definition 5.39 excludes; its assumption-minimal consequence is recorded in Remark 5.49.

Theorem 5.47 (Finite-thickness collar survival)**.** *Under the presence reading of Lemma 5.46, the finite-thickness collar survival coefficient is*

$$\boxed{\lambda_{\mathrm{collar}} = \int dy\, w(y)\big(1-\epsilon_{\mathbb{Z}_6}(y)\big) = 1 - \int dy\, w(y)\,\epsilon_{\mathbb{Z}_6}(y).}$$

At finite regulator this is

$$\boxed{\lambda_{\mathrm{collar,r}} = 1 - \sum_{y\in Y_{r,C}} w_r(y)\,\epsilon_{\mathbb{Z}_6,r}(y).}$$

Proof. The local survival factor at y is $1-\epsilon_{\mathbb{Z}_6}(y)$ by Lemma 5.46. Averaging over scalar opportunities with normalized profile $w(y)$ and using linearity of the integral gives the displayed formula. The finite-regulator formula is the corresponding finite sum. □

Corollary 5.48 (Exact finite-thickness coefficient from the scalar-weighted receipt)**.** *If*

$$\int dy\, w(y)\epsilon_{\mathbb{Z}_6}(y) = \frac{P}{24},$$

then

$$\boxed{\lambda_{\text{collar}} = 1 - \frac{P}{24}.}$$

Numerically,

$$\boxed{\lambda_{\text{collar}} = 0.9320429912748350\ldots.}$$

The scalar-weighted mean receipt alone fixes the coefficient, by linearity; no uniformity clause on the profile is consumed. More generally, if the scalar-weighted mean is $\bar{\epsilon}^{\text{scal}}_{\mathbb{Z}_6}$*,*

$$\boxed{\lambda_{\text{collar}} = 1 - \bar{\epsilon}^{\text{scal}}_{\mathbb{Z}_6}.}$$

Proof. Substitute the receipt value into the survival formula of Theorem 5.47. □

Remark 5.49 (Mean-count reading and its Markov band)*.* Under the mean-count reading, with $\epsilon_{\mathbb{Z}_6}(y)$ the expectation of an $\mathbb{N}$-valued reserve occupancy and survival the zero-count event, Markov's inequality bounds the survival factor for any occupancy law: $\Pr[N \geq 1] \leq \epsilon$ gives $1 - \epsilon \leq \Pr[N = 0] \leq 1$, hence with the scalar-weighted $P/24$ receipt

$$1 - \frac{P}{24} \leq \lambda_{\text{collar}} \leq 1.$$

The coefficient is order one and bounded away from zero on every reading; the exact value is reading-dependent, and the presence reading is the one the receipt semantics of this branch supply. Which reading nature uses is a physical gate of the susceptibility lane, recorded there.

Definition 5.50 (Uniform product-thickening branch)**.** *The uniform product-thickening branch consists of the following clauses.*

1. *Product collar algebra:*
$$\mathcal{A}^{\text{thick}}_{r,C} \cong \mathcal{E}_{r,C} \otimes \mathcal{T}_r(Y).$$
2. *Product quotient trace:*
$$\tau^{\text{thick}}_{q,r,C} = \tau^{\mathcal{E}}_{q,r,C} \otimes \tau^{Y}_{r}.$$
3. *Reserve pullback from the edge-center register:*
$$Z^{\text{thick}}_{6,r,C} = Z_{6,r,C} \otimes 1_Y.$$
4. *Scalar activation disintegration:*
$$\Pi^{\text{scal,thick}}_{r,C} = \sum_{y \in Y_{r,C}} \Pi^{\text{scal}}_{r,C}(y) \otimes p_y,$$
with each $\Pi^{\text{scal}}_{r,C}(y) \in \mathcal{E}_{r,C}$.

5. *Slice-wise scalar-reserve unbiasedness: for every scalar-active slice,*

$$\boxed{\frac{\tau^{\mathcal{E}}_{q,r,C}\left(\Pi^{\mathrm{scal}}_{r,C}(y)Z_{6,r,C}\right)}{\tau^{\mathcal{E}}_{q,r,C}\left(\Pi^{\mathrm{scal}}_{r,C}(y)\right)} = \tau^{\mathcal{E}}_{q,r,C}(Z_{6,r,C}) = \frac{P}{24}.}$$

The fifth clause is essential. Product algebra and product trace alone do not force the scalar-active edge sampler to be reserve-unbiased.

Theorem 5.51 (Exact uniform product-thickening coefficient)**.** *On the uniform product-thickening branch, under the presence reading of Lemma 5.46,*

$$\boxed{\lambda_{\mathrm{collar}} = 1 - \frac{P}{24}.}$$

Numerically,

$$\boxed{\lambda_{\mathrm{collar}} = 0.9320429912748350\ldots.}$$

Proof. By slice-wise scalar-reserve unbiasedness,

$$\epsilon_{\mathbb{Z}_6}(y) = P/24$$

for every scalar-active transverse slice. Substituting this into the finite-thickness survival formula gives

$$\lambda_{\mathrm{collar}} = \int dy\, w(y) \left(1 - \frac{P}{24}\right).$$

Since w is normalized,

$$\int dy\, w(y) = 1.$$

Therefore

$$\lambda_{\mathrm{collar}} = 1 - \frac{P}{24}.$$

On this branch the coefficient coincides with the finite-thickness value of Corollary 5.48, since the scalar-weighted receipt fixes it by linearity on its own; the uniform branch adds slice-wise constancy of the profile, which the exact value does not consume. □

Remark 5.52 (Exact value gate)*.* The exact value $1 - P/24$ is licensed by the scalar-weighted $P/24$ receipt under the presence reading. Without that receipt the theorem-grade coefficient is

$$\lambda_{\mathrm{collar}} = 1 - \int dy\, w(y)\, \epsilon_{\mathbb{Z}_6}(y),$$

and under the mean-count reading of Remark 5.49 the available theorem-grade band is

$$1 - \frac{P}{24} \leq \lambda_{\mathrm{collar}} \leq 1.$$

5.5 Local scalar certificates and review checks

The exact coefficient requires one quotient scalar-readout algebra, a complete and exhaustive scalar channel, the oriented 24-slot register, the normalized $\mathbb{Z}_6$ trace, compatible scalar and reserve profiles through the finite thickness, and unbiased survival under disintegration. Under these conditions Theorem 5.51 gives $\lambda_{\text{collar}} = 1 - P/24$. Without them the theorem-grade expression is

$$\lambda_{\text{collar}} = 1 - \int dy\, w(y)\, \epsilon_{\mathbb{Z}_6}(y),$$

and under the mean-count reading it may add only the Markov band

$$1 - \frac{P}{24} \le \lambda_{\text{collar}} \le 1.$$

The coefficient ledger imposes the following checks. No survival coefficient may use $24/P$; occurrences of $\mathrm{Tr}_q(Z_6)$ with $24/P$ must be renamed to $\mathrm{Tr}_q^{\#}$ or explicitly declared reciprocal; every exact $1 - P/24$ coefficient claim must cite Theorem 5.51; every Markov lower bound must cite the weighted-mean result; and every statement that the Z_6 reserve fixes scalar opportunities must cite the scalar-channel exhaustion theorem and its no-separate-carrier corollary.

Definition 5.53 (Finite source-bridge readouts)**.** *For cosmology runs that attempt a first-principles primordial bridge, the carrier must also expose quotient-visible maps*

$$\begin{gathered}
\mathsf{StressRead}_r,\quad \mathsf{ScalarSourceMap}_r,\quad \mathsf{CoherentMatterScalarRead}_r,\quad \mathsf{ClockRead}_r,\quad \mathsf{RepairTangent}_r,\\
\mathsf{VolumeJacobianRead}_r,\quad \mathsf{ScreenMassMatrixRead}_r,\quad \mathsf{LowModeProjectorRead}_r,\\
\mathsf{ScalarPrecisionRead}_r,\quad \mathsf{QuotientMeasureRead}_r,\quad \mathsf{ScalarReleaseEnergyRead}_r,\\
\mathsf{PrimitiveCollarLedgerRead}_r,\quad \mathsf{ReserveGeneratorRead}_r,\quad \mathsf{OrientationHalfRead}_r,\\
\mathsf{DiamondRead}_r,\quad \mathsf{ProperScaleRead}_r,\quad \mathsf{ModularRemainderRead}_r,\quad \mathsf{StressMomentRead}_r,\\
\mathsf{FaceFluxRead}_r,\quad \mathsf{ReactionRead}_r,\quad \mathsf{TetradRead}_r,\quad \mathsf{ScalarRefinementRead}_{sr},\\
\mathsf{RadialWindowRead}_r,\quad \mathsf{SpatialGeometryRead}_r,\quad \mathsf{AreaVolumeRead}_r,\quad \mathsf{LapseShiftRead}_r,\\
\mathsf{LineageRead}_r,\quad \mathsf{ScreenEmbeddingRead}_r,\quad \mathsf{ModeFormRead}_r,\quad \mathsf{CosmoGeomRead}_r,\\
\mathsf{ProperTimeRead}_r,\quad \mathsf{ComovingMetricRead}_r,\quad \mathsf{SpatialModeRead}_r,\quad \mathsf{AngularProjectorRead}_r,\\
\mathsf{FreezeoutMapRead}_r,\quad \mathsf{FreezeoutSurfaceRead}_r,\quad \mathsf{ReleaseSurfaceRead}_r,\\
\mathsf{HomogeneousProjectorRead}_r,\quad \mathsf{AnomalyLoadRead}_r,\quad \mathsf{LoadNoDataLedgerRead}_r.
\end{gathered}$$

They are defined on the same settled-form carrier used for the geometric record packet and must factor through the physical quotient: hidden carrier coordinates, port relabelings, and accepted repair schedules cannot change their output. The finite packet must expose enough data to reconstruct, for every active sector,

$$T_I^{ab},\quad Q_I^a,\quad u_I^a,\quad \rho_I,\quad p_I,\quad \pi_I^{ab}.$$

Under Scalar Edge-Center Exhaustion,

$$\mathsf{CoherentMatterScalarRead}_r(U,t;h) = S_{\text{coh}}^{\text{can}}(U,t;h)$$

is an edge-center scalar readout of OPH-coherent material support. It is a material scalar receipt, not a rest-mass, heat, or raw energy counter. For anomaly abundance selection, the added release maps supply the release hypersurface, tetrad normal and physical cell volumes, FLRW zero-mode projector, finite anomaly load observable $\mathsf{L}_{A,r}$, and no-data-use manifest used by anomaly abundance source receipt. *The source-bridge maps are instrumentation for the screen-to-radial theorem. They do not turn a screen covariance into a primordial spectrum unless the source-stress, single-clock, repair-gap, freeze-out, geometric-screen-scalar, scalar-precision, primitive-collar-law, source-release-energy, infinitesimal reserve-generator, orientation-half, refinement-tilt, radial-null, finite-window, and forward-residual receipts also pass. Radial uniqueness requires either a scale-labelled source-refinement orbit with a scale-natural physical embedding, source/physical covariance residual equality, strong finite-to-continuum convergence, and safe-band leakage control, or a cofinal radial cross-covariance family with spherical-Hankel convergence and held-out reconstruction. On the physical dilation branch the source-facing generator gives $\theta = P_\star/48$ and the continuum intertwiner restricts the radial power to a one-dimensional family. A concrete finite carrier satisfying the full receipt set is work in progress.*

Theorem 5.54 (Carrier invariance of the anomaly load readout)**.** *If two carrier presentations induce the same quotient state in $Q^{\mathrm{rel}}_{A,r}$, the same parent stress readout, the same release hypersurface, and the same homogeneous projector, then they emit the same $\mathsf{L}_{A,r}$.*

Proof. Every term in

$$\mathsf{L}_{A,r} = \Pi^{(0)}_r \sum_c V^{\mathrm{phys}}_{c,r} T^{ab}_{A,r} n_a n_b$$

is quotient-visible by hypothesis. Hidden carrier coordinates, port labels, repair schedule identifiers, and worker metadata are quotiented out by the physical quotient. Therefore the load readout is invariant. □

Definition 5.55 (Einstein-branch geometry readout)**.** *An Einstein-branch geometry readout is a cofinal family of maps*

$$\mathsf{Geom}_r : Q_{r,\mathrm{nf}} \to \mathsf{GeoData}_r, \qquad Q_{r,\mathrm{nf}} = n_r(Q_r),$$

where the emitted data include

$$\mathsf{Chart}_{S,r}, \quad \mathcal{A}^{\mathrm{geo}}_r(C), \quad \omega^{\mathrm{geo},C}_r, \quad \lambda_{C,r}, \quad B_{C,r}, \quad L_{C,r},$$
$$\mathsf{AreaRead}_r, \quad \mathsf{StressRead}_r, \quad \mathsf{DiamondRead}_r, \quad \mathsf{TetradRead}_r, \quad \mathsf{ScaleRead}_r, \quad c^{\mathrm{geo}}_{sr}.$$

It must be insensitive to hidden carrier coordinates, gauge representatives, port relabelings, worker/shard labels, and accepted repair schedules. It must also be refinement-compatible:

$$c^{\mathrm{geo}}_{sr} \circ \mathsf{Geom}_s = \mathsf{Geom}_r \circ c^{\mathrm{nf}}_{sr} + O(\epsilon^{\mathrm{geo}}_{sr}), \qquad \epsilon^{\mathrm{geo}}_{sr} \to 0.$$

Theorem 5.56 (Geometry-readout quotient factorization)**.** *If the geometry-facing readouts of Definition 5.53 are packaged as an Einstein-branch geometry readout Geom_r that factors through $Q_{r,\mathrm{nf}}$, then every geometry-facing observable extracted from Geom_r is independent of hidden carrier coordinates, gauge representatives, port relabelings, worker/shard labels, accepted repair path, and repair schedule.*

Proof. By construction, every extracted geometry-facing observable is a function of $\mathsf{Geom}_r(q_{\mathrm{nf}})$ for a repaired quotient normal form $q_{\mathrm{nf}} \in Q_{r,\mathrm{nf}}$. Any hidden carrier coordinate, gauge representative, port label, worker/shard label, accepted repair path, or repair schedule that presents the same quotient normal form therefore has the same geometry readout. The conclusion is exactly quotient factorization. □

The factorization theorem proves presentation independence of a declared readout; the complementary production direction, deriving the Geom_r payload itself from the repaired normal form instead of declaring it, is the spacetime and Einstein paper's quotient-intrinsic geometry-producer theorem, which constructs the screen/cap data from the support-visible incidence complex on its computable-receipt branch and proves the receipt selection underdetermined by bare confluence.

In particular, a finite carrier patch, graph vertex, implementation point, or worker-local shard is not one canonical OPH screen cell unless a separate measure-realization theorem proves that identification. A promotable evidence bundle must expose a geometry manifest, metric-form hashes, orientation and topology, boundary conditions, lineage maps, lapse and shift, source embedding, operator assembly, scale certificate, and refinement maps.

A spherical screen is then a coarse chart over a federation of such patches. It is what a particular observer-facing support cut looks like after quotienting hidden implementation details and choosing a geometric presentation. The microscopic carrier may be graph-like, federated, multi-patch, and locally polyhedral.

The P, N closures supply the geometric side of that chart. If $P_\star = a_{\text{cell}}/\ell_\star^2$ and $N_\Lambda = A_{\text{screen}}/(4\ell_\star^2)$ with $N_\Lambda = 3.31 \times 10^{122}$ the Λ-located working capacity, then an equal-area cellulation has

$$K_{\text{cell}} = \frac{A_{\text{screen}}}{a_{\text{cell}}} = \frac{4N_\Lambda}{P_\star} \simeq 8.12 \times 10^{122}.$$

At the conditional bridge capacity the same count reads 8.66×10^{122}. The Λ-located value is a measured-side comparison display and is not an input to the closure. This counts canonical geometric screen cells. It does not assign an independent Hilbert factor to each cell. In particular $P_\star/4 \simeq 0.408$ nats is about 0.588 bits, so it is not by itself the logarithm of an integer-dimensional autonomous cell algebra. The area law lives on shared cuts, edge centers, and constrained overlap data.

Definition 5.57 (Federated patch carrier)**.** *A fixed-cutoff federated patch carrier is a tuple*

$$\mathfrak{F} = (V, E, \{\mathcal{A}_i\}_{i\in V}, \{\mathcal{I}_e\}_{e\in E}, \{\pi_{i,e}\}, \{\mathcal{R}_i\}, \{\mathcal{U}_i\}),$$

where V is a finite set of patches, E is a finite overlap graph, $\mathcal{A}_i$ is the local finite algebra of patch i, $\mathcal{I}_e$ is the finite interface algebra on overlap $e = \{i, j\}$, $\pi_{i,e} : \mathcal{A}_i \to \mathcal{I}_e$ is the declared visible restriction, $\mathcal{R}_i \subseteq Z(\mathcal{A}_i)$ is the patch record algebra, and $\mathcal{U}_i$ is the allowed local update and repair interface.

The carrier is *observer-facing* when every physical claim is made through the visible restrictions $\pi_{i,e}$, record algebras $\mathcal{R}_i$, and the quotient-local observables specified by the OPH consensus package. Hidden coordinates may exist, but they are not directly physical.

5.6 Canonical multiresolution regulator chart

Definition 5.58 (Reference multiresolution carrier)**.** *Choose the nested geodesic icosahedral subdivisions of S^2. A regulator is*

$$r = (m, L, b),$$

where m is the subdivision depth, L is the finite support or volume scale of the exported carrier, and b is a boundary/phase label. Every added cell, collar, edge-center, or observer-visible channel j carries a finite algebra

$$D_j = \bigoplus_{\alpha\in S_j} M_{d_{j,\alpha}}(\mathbb{C})$$

and a faithful detail state τ_j. At regulator r,

$$\widetilde{M}_r = \bigotimes_{j\in\mathcal{J}_r} D_j, \qquad \widetilde{\omega}_r = \bigotimes_{j\in\mathcal{J}_r} \tau_j.$$

A finite-depth local presentation circuit W_r maps these multiresolution coordinates to the declared patch, port, collar, and gauge coordinates:

$$M_r = \mathrm{Ad}(W_r)(\widetilde{M}_r), \qquad \widehat{\omega}_r = \widetilde{\omega}_r \circ \mathrm{Ad}(W_r^*).$$

For $r \preceq s$, refinement adds detail factors. The refinement embedding and coarse-graining are

$$\iota_{rs}(A) = W_s\big[(W_r^* A W_r) \otimes \mathbf{1}\big]W_s^*,$$

and

$$Q_{sr}(X) = W_r\big[(\mathrm{id} \otimes \tau_{s\setminus r})(W_s^* X W_s)\big]W_r^*.$$

The embedded map $E_{sr} = \iota_{rs} Q_{sr}$ is the canonical state-preserving conditional expectation.

Theorem 5.59 (Fixed-cutoff regulator certificate)**.** *The maps in Definition 5.58 are unital, completely positive, composition-compatible, and preserve the faithful reference states. The refinement embeddings are unital injective $*$-homomorphisms, the E_{sr} are faithful conditional expectations, and the reference modular groups restrict exactly along the tower. The inductive limit has canonical finite-regulator renormalization maps given by these conditional expectations; their martingale tails are the finite-volume and lattice-spacing Cauchy errors used by the compact and main papers.*

Proof. In bare coordinates all statements reduce to tensoring an observable with the identity and averaging the detail factors against the faithful product state. Composition follows from product-state associativity. Conjugating by the presentation circuits gives the physical patch coordinates without changing the algebraic identities. □

Remark 5.60 (Reference state, ground state, and vacuum). The state $\widehat{\omega}_r$ is the finite-regulator reference state. It may be called a finite-volume ground state only after a positive self-adjoint transfer or Hamiltonian has been constructed and shown to have that state as its ground state. The term continuum vacuum is reserved for the GNS/OS limit after positive energy and the declared vacuum-sector uniqueness or superselection condition have been proved.

Remark 5.61 (Regulator promotion rule). A finite evidence bundle implements this regulator branch only when it exports the factor list, presentation circuit, refinement maps, conditional-expectation identities, transported state errors, renormalized observable tails, and positive-transfer or reflected-Gram certificates. A finite cellulation, finite-group score, or stable numerical extrapolation by itself is a regulator-chart result.

Definition 5.62 (Clocked spatial readout package)**.** *A finite cosmological run that claims FLRW spatial curvature must export the clock and spatial geometry readouts separately from screen/collar defect bookkeeping:*

$$\mathsf{SpatialMetricRead}_r, \quad \mathsf{SpatialFrameRead}_r, \quad \mathsf{SpatialConnectionRead}_r,$$
$$\mathsf{SpatialTransportRead}_r, \quad \mathsf{CurvatureHolonomyRead}_r,$$
$$\mathsf{ProperRadiusRead}_r, \quad \mathsf{NestedCapacityRead}_r.$$

The transport readout uses the spatial Levi–Civita connection on the clock slice. Finite permutation holonomy of an S^2 screen triangulation or S_3 collar defect is a repair/sieve diagnostic unless these readouts identify it with a spatial Levi–Civita holonomy under quotient, transport, and refinement naturality.

6 The Echosahedral Patch Object

An echosahedral patch is the reference local body for this microphysics. The term is architectural: a bounded observer patch with a highly symmetric multi-port interface.

Definition 6.1 (Echosahedral patch)**.** *An echosahedral patch is a finite patch object*

$$\mathcal{E} = (\mathcal{A}, \{P_a\}_{a=0}^{11}, \{\rho_a\}_{a=0}^{11}, \mathcal{R}, \mathcal{M}, \mathcal{U}, \mathcal{G}_{\mathcal{E}}),$$

where:

1. *$\mathcal{A}$ is the internal finite algebra;*
2. *$P_0, \ldots, P_{11}$ are twelve labeled overlap ports;*
3. *$\rho_a : \mathcal{A} \to \mathcal{I}_a$ are port readout maps;*
4. *$\mathcal{R} \subseteq Z(\mathcal{A})$ is the observer-accessible record algebra;*
5. *$\mathcal{M}$ is a finite mismatch-score family on exposed port packets;*
6. *$\mathcal{U}$ is a finite family of local update and repair instruments;*
7. *$\mathcal{G}_{\mathcal{E}}$ is a declared finite symmetry or approximate-symmetry group of the port arrangement.*

Definition 6.2 (Certified echosahedral carrier lineage)**.** *A certified local lineage augments the quotient-visible echosahedral patch by:*

(i) *twelve primitive orthogonal port-center atoms $e_{r,p}$, each of normalized trace $1/12$;*

(ii) *the oriented 12-vertex, 30-edge, 20-face triangular incidence packet, with degree-five vertices and five-cycle links;*

(iii) *the integer total-charge fiber $\mathcal{Q}_r = \{q \in \mathbb{Z}^{12} : \sum_p q_p = 12\}$ and the central readback cost $H_r(q) = \sum_p q_p^2$;*

(iv) *orientation-preserving incidence isomorphisms along refinement that preserve the port-center/readback packet and obey the refinement cocycle; and*

(v) *a source manifest that rejects Standard Model, product-adjoint, gauge, coupling, particle, measured-target, and fitted-coordinate fields, is closed to undeclared fields, and encodes trace fractions with exact JSON integers.*

Theorem 6.3 (Echosahedral source-selector theorem)**.** *On every lineage of Definition 6.2, the source packet canonically determines:*

(i) *the twelve coefficient lines $\mathbb{R}e_{r,p}$ and the unique all-one defect allocation, with $H_{\min} = 12$, next floor 14, and exact gap 2;*

(ii) *the unique fixed-point-free inverse pairing by graph distance three and the resulting six axes;*

(iii) *the full incidence group $A_5 \times C_2$ and its orientation-preserving subgroup A_5;*

(iv) *the exact incidence Gram matrix*

$$G_{pq} \in \{1, 1/\sqrt{5}, -1/\sqrt{5}, -1\} \quad \textit{at distances } 0,1,2,3,$$

with

$$G^2 = 4G, \qquad \operatorname{tr} G = 12, \qquad \operatorname{rank} G = 3,$$

whose oriented factorization is the regular six-axis icosahedral frame up to $SO(3)$*; and*

(v) *refinement naturality and arbitrary simultaneous port-relabeling equivariance of all these objects.*

Thus the finite UD12 *and* RP-A5 *source receipts are closed on this carrier branch.*

Proof. Primitive central atoms give the intrinsic line family. On the total-charge fiber,

$$H(q) = 12 + \sum_p (q_p - 1)^2,$$

so the all-one allocation is unique and the least nonzero integral zero-sum deviation has squared norm two. The exact incidence distance profile is $(1, 5, 5, 1)$, giving a unique distance-three involution. Enumeration gives 120 incidence automorphisms and 60 orientation-preserving ones; conjugation on the five Klein-four subgroups identifies the latter faithfully with all even permutations of five objects, hence with A_5. Exact multiplication in $\mathbb{Q}(\sqrt{5})$ gives $G^2 = 4G$, and the trace fixes rank three. Every declared refinement or complete relabeling preserves the source formulas. The exact implementation and countermodels are carried by the echosahedral selector certificate and its accompanying analysis. ☐

Remark 6.4 (Load-bearing hypotheses)*.* Without primitive atoms the line split is continuous; without equal trace or total charge twelve the all-unit minimizer is not unique; a linear cost has $\binom{23}{11}$ minimizers; twelve labels without incidence have 10,395 fixed-point-free pairings; without orientation the selected group is $A_5 \times C_2$; and non-incidence or noncocyclic refinement maps fail naturality. The theorem derives the selector from the declared echosahedral-carrier packet. It does not prove that bare Euler incidence or every possible OPH carrier selects that packet, and it does not construct a physical gauge-current algebra. The executable controls recompute the finite countermodel values and reject both an opaque undeclared numeric input and a floating-point trace fraction that could otherwise be truncated.

Theorem 6.5 (Alternative variational icosahedral screen sieve)**.** *Assume the fixed-cutoff OPH screen branch is represented by a quantum-link triangulation of* S^2*, with finite Hilbert spaces on links, Gauss constraints at vertices, boundary-gauge-invariant physical algebras, and a refinement-stable quotient ensemble locally six-valent away from curvature defects. Let*

$$q_v = 6 - \deg(v).$$

Assume integer charges, a feasible twelve-unit configuration, and an additive cost with $h(0) = 0$*,* $h(1) > 0$*,* $h(k) \geq h(1)|k|$*, strictly for* $|k| \geq 2$*. Let edge-center collars expose each unit defect as one central port, with a fixed-point-free inverse-port involution giving six axes* u_i*. If the source selector maximizes*

$$\det F_1 \det F_2, \qquad F_1 = \sum_i u_i u_i^{\mathsf{T}}, \quad F_2 = \sum_i |Q_i\rangle\langle Q_i|, \quad Q_i = u_i u_i^{\mathsf{T}} - I_3/3,$$

then the screen has twelve unit ports on the regular icosahedral orbit. An equal-weight additive invariant load X *is read locally as* $X/12$*.*

Proof. For a triangulated sphere with V vertices, E edges, and F faces, Euler and triangular incidence give

$$V - E + F = 2, \qquad 3F = 2E, \qquad E = 3V - 6.$$

Therefore

$$\sum_v q_v = \sum_v (6 - \deg(v)) = 6V - 2E = 12.$$

Thus $\sum_v q_v = 12$. The cost satisfies

$$\sum_v h(q_v) \geq h(1) \sum_v |q_v| \geq 12h(1).$$

Equality requires nonnegative unit charges, and feasibility supplies the twelve-unit minimizer. The collar rule turns them into twelve central ports. For six axes, $\operatorname{tr} F_1 = 6$ and $\operatorname{tr} F_2 = 4$. Determinant AM–GM gives $\det F_1 \leq 2^3$ and $\det F_2 \leq (4/5)^5$, with equality exactly at $F_1 = 2I_3$ and $F_2 = (4/5)I_5$. The six equal-norm Q_i then form a regular simplex in $\mathrm{Sym}_0(3)$, so $(u_i \cdot u_j)^2 = 1/5$ for $i \neq j$. The associated Seidel matrix satisfies $S^2 = 5I_6$; after switching one vertex is isolated and the remaining negative-edge graph is a five-cycle. This is the unique switching class and gives the six axes of a regular icosahedron, whose proper rotation group A_5 is transitive on the twelve ports. Equal-weight additivity gives the local read $X/12$. □

Remark 6.6 (External identification of the load). The identification $X = \log(N/\pi)$ is imported from the D6 radius identity of the capacity lane; this paper does not construct that object. The theorem itself constrains only invariant screen loads that are additive across the twelve ports.

Remark 6.7 (Source-selector boundary). For Theorem 6.5, the D-optimal vector/quadrupole functional remains a branch premise, and a source-derived strictly completely monotonic pair cost is another sufficient selector by universal optimality. Neither is needed on the certified lineage of Theorem 6.3, where oriented incidence and central readback supply the selector directly. Only the twelve-port load normalization follows from unit splitting and placement. The cell value $\bar{\ell}_{\text{shared}} = P/4$ and the $\mathbb{Z}_6$ class equidistribution of Theorem 5.41 have separate premises; the port and slot counts do not imply either statement.

Theorem 6.8 (Icosahedral face-corner bundle)**.** *On the icosahedral screen-sieve branch, the twenty outward-oriented triangular faces form the homogeneous orbit*

$$\mathsf{F}_{20} \simeq A_5/C_3.$$

The stabilizer of a face acts regularly on its three corners. Hence the face orbit carries an A_5-associated bundle of local three-dimensional permutation fibers with cyclic shift $R^3 = 1$. A fiberwise Hermitian operator C that is transported A_5-equivariantly across the face orbit and commutes fiberwise with this shift has the circulant form

$$C = a1 + bR + \bar{b}R^2,$$

and its unordered real spectrum is independent of the chosen face representative. Reversing the face orientation exchanges R and R^2 and preserves that unordered spectrum.

Proof. The icosahedron has $(V, E, F) = (12, 30, 20)$. The orientation-preserving icosahedral group A_5 is transitive on the outward-oriented faces. By orbit–stabilizer the face stabilizer has order $60/20 = 3$, hence is C_3, and its nontrivial rotations cyclically permute the three corners. The commutant of the regular cyclic shift on one fiber is the circulant algebra; Hermiticity gives the displayed coefficients. The stated A_5-equivariance transports that operator to every other face by conjugation and therefore preserves the spectrum. Without this equivariance, independent coefficients could be chosen on different faces. Orientation reversal complex-conjugates the transported presentation and again preserves the unordered real eigenvalues. □

Remark 6.9 (Face-carrier boundary). The theorem supplies local geometric C_3 fibers, not one canonical global three-dimensional matter-family space. The sixty face-corner flags form a free transitive A_5-set, whose linearization is the regular sixty-dimensional A_5 representation. A physical family claim therefore requires a quotient-visible section, connection, or intertwiner from this bundle to the relevant matter multiplicity space. Cyclic covariance alone also leaves a, $|b|$, and $\arg b$ free. In particular this theorem does not emit a charged-family phase, determinant normalization, or mass coordinate.

Remark 6.10 (Engineered charged face-quotient model at fixed cutoff). An auxiliary finite construction shows that a stipulated charged face-quotient packet is algebraically nonempty. Inside a bounded patch it realizes eight connected matrix-register graphs, 6,467 matrix units, eight declared paths, rank-one local events, and a central accepted/rejected readback. For one event projection P, write the central two-point record algebra as

$$\mathcal{D}_2 := \mathbb{C}|a\rangle\langle a| \oplus \mathbb{C}|r\rangle\langle r|.$$

The binary event-to-record channel is

$$\mathcal{B}(H_r) \longrightarrow \mathcal{B}(H_r) \otimes \mathcal{D}_2, \qquad \rho \longmapsto P\rho P \otimes |a\rangle\langle a| + (I-P)\rho(I-P) \otimes |r\rangle\langle r|.$$

Sixty proper-icosahedral charts verify the declared graph intertwiners, and the evidence artifact verifies normalized-trace invariance under inert ancillary stabilization. Its theorem status is schema existence plus a fixed-cutoff bridge from noncentral event to central public record. The register dimensions, path automaton, coupling character, grading, unit clock, and scalar response are authored inputs. The evidence checker does not enforce every source-law and provenance field, and its negative-control wrapper can count an unrelated nonzero exit as a successful rejection. Path exhaustion is internal to that automaton; inert stabilization supplies no cofinal physical screen refinement. The construction has no frozen receipt excluding target dependence. A global mutually exclusive response law, physical charged-source selection, recovery dynamics, family attachment, and a pole map are outside this model. A separate conditional nature/pole interface shows how a supplied chiral three-family carrier and exact renormalized kernel would transport the face operator to a charged singularity readout. One premise identifies the physical Yukawa response with the face response, and another identifies the Dyson readout. It therefore adds a precise downstream certificate format without deriving either attachment from this screen model.

Remark 6.11 (Hardware-to-QFT export boundary). A screen or hardware evidence bundle exports only the bounded observer-like object it actually instantiates: local state, ports and boundaries, readback, records, repair or feedback moves, and public evidence. Even a complete echosahedral selector receipt does not thereby export a source-selected G_6 action, chiral measure or constrained Hamiltonian, BV/ST restoration transcript, dressed-current amplitude, nonperturbative observable tower, or resonance sheet. The quantization-step implications for those typed inputs are proved in the compact and particle papers; construction of the inputs remains a separate producer problem.

On the certified echosahedral lineage, the source-selector theorem derives the unit lines, inverse pairs, proper A_5 action, and rank-three frame. On other declared cost/selector branches, the variational screen-sieve theorem provides a sufficient alternative. These results give the reference patch enough boundary structure to support nontrivial overlap comparison, symmetry tests, verifier-shadow behavior, and cross-patch routing while remaining small enough for explicit evidence records. The theorem selects the exposed interface architecture. The dimension of the observer's internal algebra belongs to a separate carrier theorem over ports, records, repair maps, and refinement.

Remark 6.12. The word "echosahedral" is intentionally implementation-facing. In a mathematical theorem, one uses the tuple above. In a public hardware evidence bundle, one may instantiate the tuple using a specific twelve-port body, wiring map, controller, firmware hash, readout protocol, and evidence manifest.

7 Toroidal Recurrence Is Local

Toroidal hardware supplies a local recurrence and mode-competition surface. A toroidal subchannel is a recurrent internal path inside a patch or between a bounded set of ports. It can recycle boundary information, support settling dynamics, and expose winding-like or phase-locking observables.

Global scalar order parameters are often misleading in recurrent local systems. A toroidal or ring-like patch may fail to show global alignment while selecting a stable winding class, a local coherence island, or a persistent twisted state. The validation metrics for toroidal subchannels should include:

1. local coupling matrices, with total brightness treated as a secondary summary;
2. recurrence and ring-diversity statistics, with global response treated as a secondary summary;
3. winding-sensitive or phase-lock-sensitive summaries when phase data are available;
4. matched controls that distinguish chamber-mediated dynamics from host-side filtering;
5. exact-verifier receipts for task-level claims.

Proposition 7.1 (Conditional phase-lock synchronization bridge)**.** *For every routed interface $e = ((i,a),(j,b))$, suppose a source-produced phase process provides an independently calibrated time parameter, frequency entrainment, a stable declared phase offset, and held-out controls. Suppose further that the locked phase selects a commensurability map for the exposed port packets and that every resulting write enters the consensus ledger through semantic-dependency-complete read sets, atomic revalidation, coherent union-collar payloads, and the quotient local-diamond and repair-completeness checks. Then the phase process supplies a physical synchronization parent for the federated consensus theorem.*

Proof. The phase certificate fixes when the two port readouts are commensurable. The remaining hypotheses are exactly the premises under which the accepted transaction relation terminates and has the declared schedule-independent quotient normal form. The phase process supplies the interface timing and comparison map; the consensus theorem supplies public agreement. □

Phase locking without the transaction, record, and clock packet is compatible with two oscillators that entrain while reporting different semantic records. It is therefore neither an observer test nor a consensus theorem by itself. No physical phase-lock receipt is asserted in this paper.

The OPH interpretation is simple: toroidal recurrence supplies local memory and mode competition inside the patch federation. Echosahedral symmetry supplies a reference and consensus geometry. The universe, at this level of description, is a federated repair system with many local carriers.

8 Patch Federation and Overlap Synchronization

Given a federation of echosahedral patches, overlaps are formed by routing ports into declared interface pairs or interface hyperedges. For an edge $e = \{(i, a), (j, b)\}$, the exposed packets are

$$x_{i,a} = \rho_{i,a}(s_i), \qquad x_{j,b} = \rho_{j,b}(s_j),$$

and the edge mismatch is a nonnegative function

$$\Phi_e(x_{i,a}, x_{j,b}) \geq 0.$$

The total visible mismatch is

$$\Phi(s) = \sum_{e \in E} \Phi_e\big(\rho_{i,a}(s_i), \rho_{j,b}(s_j)\big).$$

The local repair contract is the consensus-paper contract: accepted repairs do not increase the declared touched-overlap mismatch, and exact fixed-cutoff branches lower the relevant mismatch unless the local visible datum is repaired. The implementation is a bounded patch operation:

1. read the exposed overlap packets;
2. compare them through a declared commensurability map;
3. choose an allowed local update or rerank move;
4. write a record of the move;
5. expose the resulting port packet;
6. let the exact verifier decide any high-level task claim.

Proposition 8.1 (Federated synchronization contract)**.** *Suppose a finite patch federation has a declared mismatch functional Φ, a finite repair menu, and an accepted-repair rule such that every accepted repair lowers Φ unless the touched visible datum is locally repaired. Then every repair sequence terminates at a visible local normal form. If the union-collar gluing and repair-completeness hypotheses of the OPH consensus theorem hold together with protected-support-complete transactions, protected-conflict-complete dependencies, and an independently checked quotient local diamond, the terminal physical observable state is schedule-independent on that carrier. The carrier is thereby a finite constraint-code implementation surface; no QECC distance, min-cut formula, spectral mixing rate, or wall-clock liveness bound follows unless the corresponding extra certificate is supplied.*

Proof. The first sentence is the finite Lyapunov argument: Φ takes values in a finite ordered set of declared mismatch scores and strictly decreases on nontrivial accepted repairs. The second sentence consumes the quotient-local confluence package of the OPH consensus theorem. Atomic commits alone do not supply its protected-support, coherent-payload, or local-diamond receipts. This paper supplies the federated carrier and visible interface on which that theorem is read. □

The schedule-independent conclusion above is a same-source statement. Equality of endpoints reached from different interiors exposing the same protected boundary requires injectivity of the boundary map on the consistent quotient, as characterized in Ref. [1]; weak normalization and all-schedule liveness are separate obligations. The same reference shows that a total exact collar-preserving local repair exists only when every admissible collar value has a locally consistent extension. Neither condition follows from finite descent alone.

9 Records, Observers, and Checkpoints

An observer is not added as a metaphysical extra. In this paper an observer is an operational pattern in a patch or patch subfederation with persistent access to:

1. an observer-facing local algebra;
2. a record algebra;
3. a stable readout/update interface;
4. enough checkpoint data to define future observer-accessible probabilities.

Definition 9.1 (Federated observer checkpoint)**.** *For an observer-supporting patch subfederation O, a checkpoint at semantic cut D is*

$$\mathrm{Chk}_O(D) = (\mathcal{R}_O(D), \rho_O^{\mathrm{acc}}(D), \mathfrak{I}_O^{\mathrm{ext}}(D), \mathcal{L}_O^{\mathrm{sem}}(D), \mathfrak{B}_O(D)),$$

where $\mathcal{R}_O(D)$ is the observer-accessible record algebra, $\rho_O^{\mathrm{acc}}(D)$ is the state restricted to observer-accessible records and visible interfaces, $\mathfrak{I}_O^{\mathrm{ext}}(D)$ is the external port-interface tuple, $\mathcal{L}_O^{\mathrm{sem}}(D)$ is the semantic future-law class for the same physical instruments and event algebra, and $\mathfrak{B}_O(D)$ is the public or internal provenance bundle needed to replay the checkpoint at the declared accuracy. Worker IDs, queue positions, retry counters, repair-cycle indices, timestamps, and packet latencies live in $\mathfrak{B}_O(D)$ unless a branch declares them physical inputs; they are not observer identity or observer time.

Theorem 9.2 (Checkpoint continuation at fixed cutoff)**.** *If two federated observer checkpoints agree exactly on $\mathcal{R}_O(D)$, $\rho_O^{\mathrm{acc}}(D)$, $\mathfrak{I}_O^{\mathrm{ext}}(D)$, and $\mathcal{L}_O^{\mathrm{sem}}(D)$, then they induce the same future probability law on the observer-accessible event algebra. If their accessible states differ by trace distance at most ε, then the induced future history laws differ in total variation by at most ε.*

Proof. All future observer-accessible probabilities are computed by applying the same completely positive update maps and the same event-readout maps from the same semantic future-law class to the same accessible algebra and external interface data. Exact equality gives equality of all future event probabilities. In the approximate case, contractivity of trace distance under completely positive trace-preserving maps gives the stated total-variation bound. □

Definition 9.3 (Public checkpoint capacity interface)**.** *For a finite federated checkpoint family, the capacity-facing export is the packet*

$$\mathsf{PubChk}_{r,D} = (\{X_O\}, \{X_e\}, \{r_{Oe}\}, \mathsf{Reach}, \mathfrak{P}, \mathfrak{K}, \mathcal{H}_{\mathrm{cap},r,D}, \{P_x\}, \mathsf{Ext}, \mathsf{Ref}).$$

Here $X_O = \mathrm{At}\,\mathcal{R}_O$ is the local central-record atom set, X_e is the interface-record atom set, and $r_{Oe} : X_O \to X_e$ is the completed-instrument atom readout. The semantic event DAG Reach *witnesses endogenous public-record reachability; $\mathfrak{P}$ freezes the authorized collective, universal-local, or quorum publicness policy; and $\mathfrak{K}$ contains the source-derived joint checkpoint kernels with the required local marginals. The declared capacity carrier has $\dim \mathcal{H}_{\mathrm{cap},r,D} = D$, and P_x is the nonzero record projection for reachable public class x. The final two entries are capacity-extension and fixed-capacity refinement packets, kept as different types.*

The packet fields are an interface contract, not automatic outputs of a local checkpoint. In particular, the generic visible algebra maps $\pi_{i,e} : \mathcal{A}_i \to \mathcal{I}_e$ do not automatically supply the record-atom maps r_{Oe}, and observer-by-observer future-law marginals do not determine the joint kernels $\mathfrak{K}$. The source must therefore provide record-atom restriction maps, endogenous public-record reachability, the frozen publicness policy, a globally coupled checkpoint family, and a faithful representation of public records on the capacity carrier. The fixed-cutoff checkpoint and central-record theorems are their immediate parents. The correctable-code capacity and finite-size closure theorems are carried by Ref. [5].

On that synthesis surface the packet defines the multiplicative readback $M_0(q) = \alpha(G_q)$, where G_q is the compound confusability graph of reachable public records. When the complete terminal fiber scalarizes, $M_0(\mathfrak{U}_N) = \widehat{F}_{r,0}(e^N)$, and the universe-level equation is

$$\boxed{N = \log M_0(\mathfrak{U}_N)}.$$

Thus M_0 counts records and N is the logarithmic capacity; the checkpoint packet supplies the finite data needed to evaluate the count.

10 Central Records and Measurement

The measurement package is a finite central-record package. Records are exposed by observer-facing patch subfederations.

Let $\mathcal{Z}_{\mathrm{rec}}(t)$ be the commutative algebra generated by the completed, observer-accessible record projectors at cycle t. An event E is a projector in $\mathcal{Z}_{\mathrm{rec}}(t)$. The operational measurement rule is:

$$\Pr(E) = \mathrm{Tr}(\rho E), \qquad \rho \mapsto \frac{E\rho E}{\mathrm{Tr}(\rho E)} \quad \text{when } \Pr(E) > 0.$$

Theorem 10.1 (Federated central-record measurement)**.** *On a fixed-cutoff federated patch carrier, once a completed write/verify slice exposes a finite commutative central record algebra $\mathcal{Z}_{\mathrm{rec}}(t)$, the Born probability and Lüders conditioning rule above define the operational measurement package on the observer-accessible event surface. Re-reading the same completed record event has probability one, conditional on that record event.*

Proof. The theorem is the standard finite-dimensional central-record argument. Because all declared record events commute and belong to the observer-accessible center for the completed slice, they define an ordinary finite classical event algebra. Probabilities are Born traces on that event algebra, and conditioning on an event is the Lüders update. After conditioning on E, the same central projector E is true with probability one. Each finite-matrix step of this argument (commutativity and centrality of the record span, Born traces, the Lüders update with its fixed-point law, and the collapse of record conditioning to the normalized projector) is machine-checked in the companion Lean 4 event-algebra development [2]. □

10.1 Record-conditioned modular cap responses

For a quotient-visible record token i seen by observer O, let $P_{i,O}$ be its central, or declared approximately central, record projector with $\omega_O(P_{i,O}) > 0$. The conditioned record state is

$$\omega_{i,O}(A) := \frac{\omega_O(P_{i,O} A P_{i,O})}{\omega_O(P_{i,O})}.$$

At fixed cutoff this is the usual Lüders state

$$\rho_{i,O} := \frac{P_{i,O}\rho_O P_{i,O}}{\mathrm{Tr}(\rho_O P_{i,O})}.$$

For a cap C, let $M_{C,0,O}$ be the declared cap-response probe observable and set

$$M_{C,t,O} := \sigma_t^{C,O}(M_{C,0,O}), \qquad R_i(C,t,O) := \omega_{i,O}(M_{C,t,O}).$$

On the support-visible geometric branch, the dimensionless modular-parameter convention is the same 2π-normalized cap flow used by the spacetime and Einstein paper:

$$\sigma_t^{C,O} = \alpha_{\lambda_C(2\pi t)}.$$

The point-source cap-response model is a branch hypothesis or finite receipt, not an automatic consequence of a record projector. A passing source-localization branch must exhibit

$$R_i(C,t,O) = b(C,t,O) + a(i,C,t,O)\,\psi_{C,t,O}\left(\frac{\eta(X_i(t),n(C,t,O))}{R_H}\right) + \xi(i,C,t,O),$$

where gains, baselines, kernel shape, cap normal, modular normalization, and the total error budget are declared. The approximate centrality defect of $P_{i,O}$, calibration error, modular transport error, kernel mismatch, finite-record noise, and point-model error all contribute to the single localization error norm σ_{record} used by the compact localization theorem.

Two carrier presentations that induce the same quotient-visible record token, conditioned record state, cap probe, cap geometry, modular-parameter normalization, and response calibration must produce the same calibrated response vector and the same H^3 localization ball. A fitted point that does not clear held-out cap residuals, mixture controls, and the declared error bound is a failed or approximate point-source branch, not a populated-H^3 theorem receipt.

11 Edge Sectors and the Casimir Handoff

The edge heat-kernel/Casimir package is a fixed-cutoff theorem package tied to an overlap collar with a finite exposed sector algebra.

At fixed cutoff, let α label a finite set of exposed edge sectors on one declared overlap collar. Suppose the local thermalized edge dynamics has stationary weights

$$\pi_\beta(\alpha) = \frac{d_\alpha e^{-\beta C_2(\alpha)}}{Z(\beta)}.$$

For finite groups, $C_2(\alpha)$ is the declared sector penalty or finite-group Casimir surrogate. For compact groups, the Peter–Weyl lift belongs to the companion D10/compact-gauge handoff instead of hardware evidence.

Theorem 11.1 (Fixed-cutoff edge-sector handoff)**.** *If a declared finite overlap collar has sector labels α, degeneracies d_α, and a local repair/thermalization generator whose detailed-balance stationary law is $\pi_\beta(\alpha) \propto d_\alpha e^{-\beta C_2(\alpha)}$, then the collar exports the fixed-cutoff Casimir edge law required by the D10 handoff. The compact-group heat-kernel lift is a separate companion-branch step.*

Proof. This is a finite Markov or finite instrument stationary-measure statement on the declared sector algebra. The theorem exports only the finite overlap law. The compact-group lift uses the Peter–Weyl continuation and normalization conventions supplied on the companion compact/D10 surface. □

12 Bell/CHSH Event Surfaces

The Bell package is fixed-cutoff and event-surface-local. A federated carrier may contain two observer-facing wings L and R with commuting setting and outcome records on one compare slice. If the source-specified joint law is the usual two-wing quantum law, the CHSH and Tsirelson statements are carried on that declared event algebra.

Laboratory echosahedral hardware has its own evidence gate for Bell-style claims. The theorem package here is a finite event-algebra statement inside the OPH microphysics surface.

13 Relation to Yang–Mills

The Yang–Mills repair-gap argument belongs to the compact theorem surface. Echosahedral geometry supplies:

1. the fixed-cutoff patch, overlap, record, and repair interface;
2. the finite edge-sector Casimir law on declared collars;
3. the local repair semantics that the compact-gauge branch reads in controlled quotient form.

The finite cylinder system and its projective weak-* / GNS extraction belong to the compact/Yang–Mills theorem surface. Identification with the four-dimensional Yang–Mills state, reflection-positive OS reconstruction, noncollapse, and equality between the Yang–Mills gap and the repair gap are conditional on that surface's separately named receipts. Hardware evidence may test implementation discipline, but it does not supply the Clay-admissible construction. A public evidence bundle that is used for any continuum or Lorentzian claim must therefore include the multiresolution regulator certificate: factor manifest, presentation-circuit hash, refinement/expectation identity defects, transported-state errors, renormalized-observable tails, and positive-transfer or reflected-Gram receipts. Conventional free-field or lattice-gauge vacuum baselines used for calibration are reference ensembles only. They are not OPH-native vacuum promotion unless the quotient-ensemble selector, source Euclidean slab data, and transfer/reflection-positive reconstruction gate are supplied on the compact theorem surface.

14 Relation to Hadrons and Strong-Binding Execution

First-principles hadron masses require a working OPH strong-binding construction. This microphysics states the record and evidence requirements for such a construction. A construction fit for promotion must emit:

1. Ward-projected hadronic spectral data;
2. provenance tying every spectrum to body, controller, firmware, geometry, and scorebook;
3. finite-volume, continuum, chiral, and production-systematics fields where applicable;
4. exact replay or verification receipts for the pipeline stage being claimed;
5. a non-promotion boundary for surrogate or calibration runs.

Echosahedral or related hardware can be described as a candidate construction family only through public evidence bundles. Without those bundles, it is an architectural target and a design motivation. First-principles hadron theorem promotion requires the evidence listed above.

15 Validation Program

The validation program has two independent tracks.

15.1 Mathematical and Digital Calibration

The octahedral $\mathbb{Z}_2/S_3$ finite-group model is a digital calibration model. It gives exact finite groups, explicit patch covers, controlled defects, frustrated cycles, record writes, repair schedules, and negative controls.

> The calibration model checks the finite interface logic. The primary physical picture is the federated patch-carrier architecture.

15.2 Public Hardware Evidence

A public hardware run may support only the support level its evidence bundle can justify. The minimal claim form is:

> Module set M produced candidate enrichment or reproducible readout signature on benchmark T, under controls C, and exact verifier V accepted the reported hits.

The rejected form is:

> The optical chamber solved the hard problem or proved OPH.

Required gates include dark baseline, low-power sweep, coupling matrix, discharge-timing or MDD trace where applicable, ring-diversity or recurrence-sensitive metrics for toroidal bodies, duplicate-body checks, symmetric-reference or echosahedral shadow checks, exact-verifier receipts, and negative controls against hidden duplicate amplification or host-side filtering.

Material, plasma, and topological-phase claims need the same discipline. The evidence bundle must name the physical quotient, source action or repair ledger, readouts, controls, and promotion gates. A carrier that passes self-reading tests certifies the carrier branch only; it does not by itself certify a material selector, nuclear yield, delivered power, or condensed-matter order.

16 Finite Packet Source Bridge Schema

Cosmology-facing evidence bundles may not infer stress data from scalar source rows. A packet-source bridge exported from the microphysics layer must carry the primitive data needed by the finite covariant parent contract:

$$(\mathcal{C}_r, g_r, e_r, U_r, Z_r, \omega_r, p_r, f_r, \Phi_r, \mathcal{R}_r, \mathcal{G}_r, \pi_r, \mathsf{Read}_r).$$

In practical evidence bundles this means:

1. finite cell/face incidence, four-volumes, normals, areas, causal adjacency, and parallel transport maps;
2. metric and tetrad data with the declared $(-+++)$ convention;
3. packet sectors, local momenta, masses, positive invariant weights, occupations, and mass-shell residuals;

4. signed face fluxes and reaction-channel stoichiometry with transported four-momentum residuals;
5. quotient maps, local-frame covariance checks, and restriction/refinement maps;
6. readout fields for stress moments, variational stress, exchange currents, and gauge-invariant or rest-frame perturbation variables.

The corresponding finite evidence receipt is residual-based. A row labelled "recipient", a raw Newtonian/synchronous gauge comparison, a producer-declared detailed-balance boolean, or a frozen likelihood hash is not a substitute for primitive packet, stress, exchange, causal-response, and refinement evidence.

17 Finite Quotient Ensemble Compatibility

Finite quotient ensemble theorem surface. Fix a finite regulator r. The physical presentation space is Σ_r, the presentation redundancy groupoid is Γ_r, and the finite physical quotient is

$$Q_r = \Sigma_r/\Gamma_r, \qquad \pi_r : \Sigma_r \to Q_r.$$

The quotient removes nonphysical presentation data: gauge representatives, port relabelings, mesh labels, shard or worker identifiers, queue order, repair schedule identifiers, retry counters, timestamps unless declared semantic, hidden carrier coordinates, and inert ancillary labels. If only settled configurations carry probability, the probability space is the normal-form subset

$$N_r = n_r(Q_r).$$

The map n_r is a normal-form map, not a probability law. Any promoted physical branch inherits this firewall: it must declare the quotient-intrinsic source law or action before a normal form can be read as a selection or prediction claim.

Observable algebras and reference states. In the finite classical case the quotient observable algebra is

$$\mathcal{O}_r = \ell^\infty(Q_r).$$

In the finite quantum case the physical algebra is a declared quotient algebra $\mathcal{A}_r^{\mathrm{phys}}$ with state

$$\omega_r(A) = \mathrm{Tr}(\rho_r A).$$

When the reference object is obtained from a finite lifted carrier, the load-bearing data are not an abstract groupoid cardinality alone. They are a tracially pointed quotient

$$\left(\mathcal{A}_{r,b}^{\mathrm{phys}}, \tau_{r,b}^0\right), \qquad \mathcal{A}_{r,b}^{\mathrm{phys}} = z_{r,b} B(\widetilde{\mathcal{H}}_r)^{G_r} z_{r,b},$$

where $U_r : G_r \to U(\widetilde{\mathcal{H}}_r)$ is the compact gauge action and $z_{r,b}$ is the central projection for the declared boundary or superselection sector. The reference trace is

$$\tau_{r,b}^0(A) = \frac{\mathrm{Tr}_{\widetilde{\mathcal{H}}_r}(A)}{\mathrm{Tr}_{\widetilde{\mathcal{H}}_r}(z_{r,b})}.$$

If

$$z_{r,b}\widetilde{\mathcal{H}}_r \cong \bigoplus_\alpha V_\alpha \otimes M_\alpha,$$

with $d_\alpha = \dim V_\alpha$ and $m_\alpha = \dim M_\alpha$, then

$$\mathcal{A}^{\mathrm{phys}}_{r,b} \cong \bigoplus_\alpha I_{V_\alpha} \otimes B(M_\alpha), \qquad p_{r,\alpha} = \frac{d_\alpha m_\alpha}{\sum_\beta d_\beta m_\beta}.$$

These are the induced central-sector weights only after the carrier representation and boundary sector have been fixed.

OPH quotient ensemble. An OPH quotient ensemble is specified by a quotient-intrinsic base weight and action

$$m_r : Q_r \to \mathbb{R}_{>0}, \qquad S_r : Q_r \to \mathbb{R} \cup \{+\infty\},$$

and

$$w_r(q) = m_r(q)e^{-S_r(q)}, \qquad Z_r = \sum_{q\in Q_r} w_r(q), \qquad \mu_r(q) = Z_r^{-1} w_r(q).$$

Equivalently, one may state an intrinsic projective prior ν_r on Q_r and set $\mu_r = (n_r)_\#\nu_r$. Uniform quotient counting, uniform representative counting pushed to the quotient, groupoid weights, and tracial central-sector weights are different physical claims. The paper must declare which one is being used.

Normal-form projector non-selection. For any retraction $N : Q \to Q_{\mathrm{nf}}$ onto a subset $Q_{\mathrm{nf}} \subseteq Q$, that is, any map whose restriction to Q_{nf} is the identity, the induced map on laws

$$\mathcal{C}_Q(\mu) = N_\#\mu$$

is idempotent:

$$\mathcal{C}_Q^2 = \mathcal{C}_Q.$$

Every law supported on Q_{nf} is fixed; both statements use the retraction property. Therefore settlement or canonicalization never selects a unique physical probability law by itself.

Selection-gap corollary. Let $X \subseteq Q_{\mathrm{nf}}$ be a finite set of quotient-normal candidates distinguished by visible invariants. Normal-form data determine X and its quotient-visible invariants, but they do not choose a member of X. If two laws μ, ν are supported on X and concentrate on different candidates, both are fixed by $\mathcal{C}_Q$. Unique sector selection therefore requires source data: an intrinsic action with a unique minimizer, a declared physical ensemble, or a refinement-stable gap certificate. A defect or holonomy classification can classify possible sectors without choosing the physical sector, and a contraction or repair generator can certify convergence toward a declared target without creating the target law.

Finite MaxEnt quotient ensemble. For finite Q, positive m, and quotient observables $F_1, \dots, F_k$, maximizing

$$\mathcal{H}_m(\nu) = -\sum_q \nu(q) \log \frac{\nu(q)}{m(q)}$$

subject to

$$\sum_q \nu(q) = 1, \qquad \sum_q \nu(q) F_a(q) = c_a$$

has the full-support solution, when the feasible full-support surface is nonempty,

$$\mu(q) = \frac{m(q)\exp[-\sum_a \theta_a F_a(q)]}{Z(\theta)}.$$

Boundary optima obey the same formula after restricting to their support. On a finite noncommutative quotient algebra with faithful reference state σ_r,

$$\rho_r = \frac{\exp(\log\sigma_r - \sum_a \theta_a F_{r,a})}{\mathrm{Tr}\exp(\log\sigma_r - \sum_a \theta_a F_{r,a})}.$$

The finite constraint ledger must name every $F_{r,a}$, its units and support, the target expectation and source, sector or zero-mode treatment, refinement transformation, and proof that no run output or observational output entered the source definition.

Refinement compatibility and RG closure. For $s \succeq r$, let $c_{sr} : Q_s \to Q_r$ be the physical coarse map. Exact compatibility of weighted ensembles is equivalent to the fiber-sum identity

$$\sum_{q' : c_{sr}(q')=q} m_s(q')e^{-S_s(q')} = \alpha_{sr} m_r(q) e^{-S_r(q)}$$

for a constant $\alpha_{sr} > 0$ independent of q. Then

$$(c_{sr})_\# \mu_s = \mu_r.$$

If the one-step defects are

$$\delta_{k+1,k} = \|(c_{k+1,k})_\# \mu_{k+1} - \mu_k\|_{\mathrm{TV}},$$

then

$$\|(c_{nr})_\# \mu_n - \mu_r\|_{\mathrm{TV}} \le \sum_{k=r}^{n-1} \delta_{k+1,k}.$$

For exponential-family refinement, exact closure requires the fine conditional free energy

$$G_{sr,\theta}(q_r) = -\log \mathbb{E}_{m_s^0}\left[\exp[-\theta \cdot F_s(Q_s)] \mid c_{sr}(Q_s) = q_r\right]$$

to equal $\kappa_{sr}(\theta) + R_{sr}(\theta) \cdot F_r(q_r)$. If the residual is uniformly bounded by ε, the induced total-variation defect is bounded by $\tanh\varepsilon$.

Implementation invariance and representative lifting. If implementations A, B have quotient bijections $h_r : Q_r^A \to Q_r^B$ satisfying

$$m_r^B(h_r q) = m_r^A(q), \qquad S_r^B(h_r q) = S_r^A(q), \qquad h_r \circ c_{sr}^A = c_{sr}^B \circ h_s,$$

then

$$(h_r)_\# \mu_r^A = \mu_r^B.$$

For tracially pointed quantum quotients the corresponding equivalence is a trace-preserving quotient equivalence. It is invariant under unitary intertwiners preserving the gauge action and sector, and under inert trivial ancillas $A \mapsto A \otimes I_{\mathrm{anc}}$. It is not invariant under arbitrary changes of gauge-representation multiplicities.

If an implementation stores representatives, a representative-level law must be a conditional lift

$$\widetilde{\mu}_r(x) = \mu_r(\pi_r x)\kappa_r(x \mid \pi_r x), \qquad \sum_{x:\pi_r(x)=q} \kappa_r(x \mid q) = 1.$$

Then $(\pi_r)_\# \widetilde{\mu}_r = \mu_r$. Uniform representative sampling yields orbit-size weights and is physical only if representative counting is the declared base measure.

Quotient-lumpable kernels and sampler correctness. A representative kernel $\widetilde{P}(x, y)$ descends to Q_r only when

$$P_Q(q, q') = \sum_{y:\pi(y)=q'} \widetilde{P}(x, y)$$

is independent of the chosen representative $x \in \pi^{-1}(q)$. For $w(q) = m(q)e^{-S(q)}$ and proposal $R(q, q')$ with reciprocal support, the Metropolis–Hastings acceptance rule

$$a(q, q') = \min\left\{1, \frac{w(q')R(q', q)}{w(q)R(q, q')}\right\}$$

gives detailed balance

$$\mu(q)R(q, q')a(q, q') = \mu(q')R(q', q)a(q', q).$$

Repair-informed proposals must include the Hastings asymmetry term; otherwise the stationary law is generically changed.

Repair generators are not selectors. A repair generator of the form

$$L_{\text{rep}} = \sum_C c_C(I - E_C)$$

is a relaxation or sampling object after a law has been selected. Conditional expectations E_C are defined on $L^2(X_r, \pi_r)$, so the reference law π_r is input. On overlapping collars the expectations need not commute. The correct finite gap certificate is the Poincare constant

$$\kappa_r = \inf_{f \perp 1} \frac{\sum_C \|(I - E_C)f\|^2}{\|f\|^2}.$$

If local fiber rates have a positive lower bound γ_*, then

$$L_{\text{rep}} \geq \gamma_* \kappa_r (I - P_0).$$

Finite repair completeness gives $\kappa_r > 0$ at fixed regulator. A uniform refinement lower bound $\inf_r \kappa_r > 0$ is a separate theorem or receipt.

Finite evidence accuracy. For bounded coarse observables O, if

$$\|\widehat{\mu}_s - \mu_s\|_{\text{TV}} \leq \epsilon_{\text{samp}}$$

and the refinement defects sum to ϵ_{ref}, then

$$\left|\mathbb{E}_{\widehat{\mu}_s}[O \circ c_{sr}] - \mathbb{E}_{\mu_r}[O]\right| \leq 2\|O\|_\infty(\epsilon_{\text{samp}} + \epsilon_{\text{ref}}).$$

Continuum-facing observables require a realization map and correlation Cauchy bound in addition to a finite histogram.

Vacuum promotion gate. A stationary sampler is not a physical vacuum. For any faithful target law one can build a positive transfer operator with that law as ground state, so positivity alone is not a selector. Vacuum promotion requires source Euclidean slab data

$$\mathfrak{S}_r^E = (Q_r, m_r^0, J_r, V_r, a_{t,r})$$

whose conductance $J_r(q,q') = J_r(q',q) \geq 0$, local potential V_r, and slab thickness $a_{t,r}$ are derived without using the target law or sampler output. With connected event graph,

$$(H_r^E f)(q) = \frac{1}{m_r^0(q)} \sum_{q'} J_r(q,q')(f(q) - f(q')) + V_r(q) f(q)$$

is self-adjoint and bounded below on $L^2(Q_r, m_r^0)$; its finite Feynman–Kac semigroup is positivity improving. Perron–Frobenius gives a unique positive normalized ground state Ω_r, and the finite vacuum law is

$$\mu_r^{\mathrm{vac}}(q) = |\Omega_r(q)|^2 m_r^0(q).$$

For $T_r = e^{-a_{t,r}(H_r^E - E_{0,r})}$, the Doob kernel is stochastic and detailed-balanced with μ_r^{vac}. Continuum promotion additionally requires reflection positivity or equivalent reconstruction plus refinement compatibility of the transfer family.

Primordial and cosmological prediction firewall. A screen covariance contains incomplete radial information. The complete one-shell map

$$C_\ell = 4\pi \int_0^\infty \frac{dk}{k} \Delta_\zeta^2(k) j_\ell^2(k\chi_\star)$$

has an infinite-dimensional kernel that persists under positivity. OPH primordial promotion requires the source-only stress, single-clock, entropy-repair, curvature-evolution, adiabatic-mode, phase-coherence, physical mode, radial-null-space, and forward-projection receipts together with a scale-natural physical dilation intertwiner or complete radial cross-covariance tomography. A finite radial prior produces a conditional continuation. Observable CMB comparison also requires declared source, solver, dataset, covariance, nuisance, data-use, and pooled-reducer provenance.

Claim tiers and required receipts. Every ensemble-facing run records its ensemble id, claim tier, regulator, representative schema, gauge action, canonicalizer, base measure, action coefficients, coarse maps, zero-mode projector, amplitude convention, sampler, smoothing policy, source provenance, and explicit nonclaims. The seed belongs to the run receipt rather than the ensemble definition. The claim tiers are

$$\begin{aligned}
E0 &: \text{ seed noise, proposal noise, repair jitter,} \\
E1 &: \text{ conventional reference ensemble,} \\
E2 &: \text{ OPH-native quotient ensemble,} \\
E3 &: \text{ OPH vacuum,} \\
E4 &: \text{ OPH primordial field,} \\
E5 &: \text{ observable cosmological prediction.}
\end{aligned}$$

The evidence bundle must keep separate receipts for stationary-law schedule invariance, detailed balance of the aggregate kernel, and pathwise partition invariance. Deterministic replay of semantic random streams or a canonical serial chain is useful, but it is not pathwise partition invariance. Smoothing must preserve raw coefficients, raw spectra, smoothing kernels, smoothed coefficients, smoothed spectra, and hashes of each stage; it is not part of S_r unless explicitly declared.

18 Conclusion

OPH microphysics is a federation of finite observer patches. The sphere is a regulator and symmetry chart for observer-facing cuts. A_5-icosahedral and E_8-type structure belong to the geometry data and representation-closure data. The echosahedral patch is the reference local interface: a bounded multi-port patch with symmetry, records, readout, repair, and checkpoint data. Toroidal subchannels supply local recurrence and winding-sensitive dynamics. The mathematical exports are fixed-cutoff patch-net embedding, edge-sector Casimir handoff, central-record measurement, Bell/CHSH event surfaces, and checkpoint/restoration.

The carrier architecture is physical data up to its declared quotient. Hidden labels and material presentations may vary, while the visible incidence, response, repair, record, clock, and refinement signature may not be discarded. The local twelve-port theorem constrains the A_5-current branch. A separate carrier-to-support bridge constrains the spherical Lorentz/Einstein branch. The consensus normal form is their common finite hinge. Operational observer promotion requires readback, durable records, feedback or repair, prediction/control, and checkpoint continuation. The source-derived incidence nerve passes those observer controls together with twenty nonvacuous face cocycles and a refinement-natural oriented support limit. H^3, event-manifold, BW/KMS, physical-scale, and laboratory bridges are open. The finite current implication is exact under the explicit response and matter contracts, while physical matter typing, global-form selection, laboratory identification, equality with the distinct Tannaka current, scalar multiplicity, rank-45 family attachment, and continuum QFT realization are open.

The claim discipline is the main point. Hardware can guide the architecture and supply public evidence through hash-stable evidence bundles. The theorem surface stands on finite algebras and declared OPH branch assumptions.

A Evidence Bundle Sketch

A minimal hardware evidence bundle should use a structure like:

```
evidence/hardware/<bundle-id>/
  manifest.json
  README.md
  body/
    mesh_hashes.txt
    photos/
    measurements.csv
  controller/
    firmware_sha256.txt
    wiring_map.csv
  calibration/
    dark_scan.csv
    low_power_sweep.csv
    coupling_matrix.csv
    mdd_trace.csv
    ring_diversity.csv
  task/
    task.json
    scorebook.json
```

```
    candidates.jsonl
    verifier_receipts.jsonl
  controls/
    shuffle_replay.jsonl
    abba_controls.csv
    negative_controls.md
  claim.md
```

The manifest should state the strongest allowed claim and the non-claims. The paper may cite the bundle only at that claim level.

Algebraic audit bundles use a different namespace and claim level:

```
code/e8_triality_claim_statement/
  README.md
  claim_statement.json
```

This is a claim statement, not a certificate. A future separately named audit bundle may support a mathematical fixed-cutoff or regulator-chart claim only when the raw matrices, scripts, exact checks, and hashes are present. Neither surface is a hardware evidence bundle.

B Digital Calibration Compatibility

The octahedral $\mathbb{Z}_2/S_3$ build has the following reading rule:

1. it validates finite patch/overlap/record/repair bookkeeping;
2. it tests frustrated-cycle and defect behavior in an exact digital setting;
3. it calibrates the edge-sector law on a finite declared interface;
4. it supplies calibration data for the interface layer;
5. it leaves the physical carrier role with the federated echosahedral patch architecture.

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