An Executable Observer for Observer Patch Holography: Specification, Memory and Agreed Records

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Abstract

We construct finite observers whose records are stored on overlapping patches of a spherical federation. A local repair writes the disagreement between two ports into a private memory. Its coherent evolution realizes an exchange of loads with an agreement energy. We prove a size-independent bound on the loss of a prepared adjacent pair and exhibit a local measurement that distinguishes the sign of the interaction. A second patch can detect phase transported by the bound excitation. In the Interval Echo candidate, a finite patch carries its own clock. With a calibrated handoff rule, the traveller and home patches read the predicted clock difference at reunion. Separate measurements of the two owners' energy responses determine the current through their shared boundary. These results connect overlap repair to matter, clocks and conservation through records that finite observers can compare. The coupling and experimental services are specified inputs.

1 The executable model and what a test of it is

Observer Patch Holography proposes that observers recover the specification of their world from shared records and realize that specification within the same world [2, 3]. An executable model makes the proposal concrete. It names a carrier, the local observers on it and the records they hold, a law that is specified before any run, the preparations, and complete instruments: every outcome of a reading belongs to the model, including the outcomes that count against a claim. This paper specifies one such model with three candidate laws on a common carrier. Its main candidate law, Interval Echo, runs the seam interaction of the reference candidate, A-contrast-memory-spherical-v1, at $\kappa = 8$ and adds a clock that a finite observer carries. The paper proves the central properties of that seam interaction, reports the record probabilities computed from it, and reports the clock, agreement and response measurements of Interval Echo.

The model. The carrier is the twelve-port icosahedral cell of the carrier paper and the spherical federations of such cells [7]. One cell is one native observer, and its twelve ports are its interfaces; larger observers are finite, possibly overlapping patches of cells. Every port holds a load 0, 1 or 2, and every seam, the link between two neighbouring ports, has a private memory of eleven states shared by its two ends. The law of the reference candidate is the contrast-memory law: on every

seam the native seam event of the carrier is written into the memory, a phase that depends on the recorded contrast of the two loads is applied, and the write is reversed. On the loads this is

$$H_\kappa = \frac{1}{2}\sum_e (I - \mathrm{SWAP}_e) + \kappa\, C, \qquad C = \sum_{e=\{a,b\}} (n_a - n_b)^2 + \sum_v (6 - d_v)\, n_v^2 = 6\sum_v n_v^2 - 2\sum_{e=\{a,b\}} n_a n_b, \tag{1}$$

where SWAP_e exchanges the loads of the two ports of the seam e, n_v is the load of the port v and d_v the number of its seams. The coupling κ is one dimensionless number, the agreement energy in units of the exchange. A seam whose two ports both hold a load is an agreed seam, and each agreed seam lowers C by 2. Interval Echo uses this law at $\kappa = 8$ as its elementary interaction and adds a clock that an observer patch carries with it (Section 13). The scalar channel, the second candidate, has registers of its own (Section 14).

What a test of the model is. A test compares distributions of admissible observer records with what observers in our world record. The state of the computation is no physical universe and contains no spacetime. Ports, seams, cells and federations are hardware of the computation. No test asks that the carrier be a three-dimensional lattice; a difference between microscopic state encodings is no failure of the model, and a resemblance between them is no success. Physical space, time, fields and particles are effective descriptions that the observers inside the computation use for the records they agree on. In the theory, observers who inhabit such a computation can describe their experience as a relativistic continuum spacetime with one time and three space dimensions. For every fixed finite admissible menu of observer readouts on bounded wave times, a smooth static $(1+3)$-dimensional Lorentzian metric carries a free scalar field whose complete predictions agree with theirs within any prescribed error, beyond the stated source and clock errors [2], and the spacetime paper gives the rank-three spatial response, its $1+3$ Lorentz carrier and the limits of the causal orders and normalized counts of record populations [5]. Both results hold under their stated premises. What a candidate law has to supply is those premises, with its own local clocks and records. It does not have to rebuild three dimensions inside its state. Section 15 lists the finite theorems of the theory whose premises the outputs of the model supply, with what follows for its observers, and names the outputs that the other theorems consume. Section 17 lists the supplied parts and the limits.

How observers enter. Every physical claim of this paper is a probability of agreed records. The observers are the cells and finite patches of cells, and a seam has an interface at each end. At each end a private pointer receives a copy of the predicate "both ends hold one load", and a record is admitted when both copies are valid and agree (Section 6). State vectors, levels of the law, the evolution parameter and positions in a sequence of events are tools of the calculation. A statement that uses them is labelled as audit, and no reader uses them. A bound that holds for every value of the evolution parameter holds at every reading in any observer's own history, and it is written in that form. No common clock is assumed: each port counts the events of its own seams, and that count is a local clock of its observer (Section 7). Under Interval Echo an observer patch carries its clock with it as it moves (Section 13).

Proved and computed. Proofs are given for the following statements. The complete local law is given on all 99 states of a seam, and on blank memory it reduces exactly to the seam term of (1), for every real κ (Theorem 3.2). The law binds a pair of loads with constants that do not depend on the number of cells (Theorem 5.2). A local emitter writes the pair within a stated error (Proposition 6.1), a reading performed while the law acts has a stated error (Proposition 6.3), and

the records depend on the order of events at each port alone (Proposition 7.1). For the second candidate law the pair of a ring, the conservation of its excitation and the single line of its records are proved (Proposition 14.2, Theorem 14.3). The other results are computed: the record probabilities on one cell and on federations of 20, 80 and 320 cells, the behaviour under schedules of events, the bound objects, the port response, the readings of the seam memory, the laws of two readings of a pair, and the experiments on a cage, whose readings are computed from sampled counts. Every computed number of the reference candidate is given with the size of the sector it was computed in (Section 16), and the measurements of Interval Echo are computed from sampled material histories and paired preparations on the federations (Section 13).

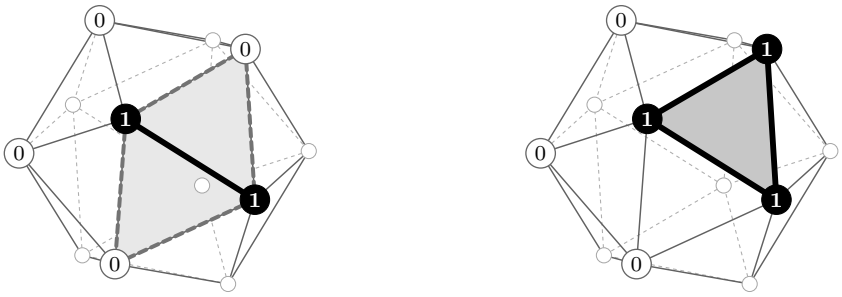
Status. Interval Echo is the main candidate law, A-contrast-memory-spherical-v1 is the reference candidate whose seam interaction it uses, and the scalar channel is the second candidate. The seam interaction reproduces particle stability: bound objects persist in free evolution, read on agreed seam records, and the same law without the agreement energy loses them (Section 8). Interval Echo inherits this in its free sector. Its observer patches carry clocks: a travelling clock, compared at reunion with the clock kept at home, runs slow by the relativistic factor computed from the observers' own records, and overlapping observers agree on every shared checkpoint reading (Section 13). The scalar channel holds an object on a ring of four seams. Its excitation is kept for every interval, which is particle stability for a second object, and its seam record carries a prepared clock that an observer reads in units of its own reference (Section 14). The scalar channel acts on a different state space, and its results are not added to those of the other two laws. The model is a candidate executable observer, and its specification may change. This paper determines no selection of the couplings. Section 17 lists the supplied parts and the limits of the construction, and Section 18 the predictions that a realization of the model can test.

Relation to known interactions. On loads 0 and 1 the seam term is the ferromagnetic XXZ interaction with anisotropy $1+2\kappa$, and the binding is of the droplet kind known for that interaction [9, 10]. The paper claims no interaction of its own. Its content is the realization of the law by a private memory with every memory state kept, the constants on this carrier with loads up to two, and the tests on agreed records.

Sections 2 to 4 fix the carrier, the local law and the law on loads. Section 5 proves the binding theorem, and Sections 6 and 7 give the emitter, the reader and the law as local events. Section 8 reports particle stability with its controls, Section 9 two readings of a pair and what each leaves, and Sections 10 to 12 the bound objects, the port response and the unrecovered collar. Section 13 gives the main candidate law, Interval Echo, with its observer clock, the agreement of overlapping observers, its port currents and an information projection. Section 14 gives the second candidate law with its cage, and Section 15 joins the outputs of the model to the finite theorems of the theory.

2 The carrier

A cell is one twelve-port icosahedral carrier, the echosahedron of the carrier paper [7]. It has twelve ports, thirty seams and twenty faces with the incidence of the icosahedron (Figure 1). Five seams meet at every port, the five neighbours of a port form a cycle, every seam lies on two faces, and every triangle of the seam graph is a face. A port has 5, 5 and 1 ports at seam distance one, two and three; the port at distance three is its antipode. Sixty rotations preserve the oriented incidence. They form the alternating group on five letters, and the port functions decompose under it into representations of dimension 1, 3, 5 and 3, written $1 \oplus 3 \oplus 5 \oplus 3'$.



(a) pair: two loads on the two ends of one seam; one agreed seam, cost 10

(b) face object: three loads on one face; three agreed seams, cost 12

Figure 1: One cell: twelve ports, thirty seams and twenty faces with the incidence of the icosahedron, in an orthographic view with the far seams dashed. Filled ports hold one load. (a) A pair. The dashed heavy seams are the four seams on which the pair is read after one exchange; they are the other sides of the two faces on its seam. (b) A face object.

A federation is a set of cells joined by external seams. An external seam joins two ports of different cells, and no port carries more than one, so the external seams form a partial matching on the ports. Every cell keeps its thirty internal seams. A port has five seams, or six if it carries an external seam. In the spherical federations used here every cell has three external ports (Table 1). One cell is the federation without external seams. The routing, that is the choice of the external ports and of their partners, follows the cell-centre and tangent-frame convention of the spherical federation [7]. It is a supplied part of the construction (Section 17).

Table 1: The carriers. The last two columns give the sizes of the sectors used in the computations: two unit loads on distinct ports, and two loads with doubled loads included.

Carrier	Cells	Ports	Seams	External seams	Ports with five / six seams	Unit pairs	Two loads
one cell	1	12	30	0	12 / 0	66	78
level 0	20	240	630	30	180 / 60	28,680	28,920
level 1	80	960	2,520	120	720 / 240	460,320	461,280
level 2	320	3,840	10,080	480	2,880 / 960	7,370,880	7,374,720

The binding theorem uses four facts of the incidence. Call two seams *linked* when they are two sides of one face.

Lemma 2.1 (Incidence facts)**.** *On every federation of complete cells whose external seams form a partial matching on the ports:*

(i) *every port has five or six seams;*

(ii) *the two ends of an internal seam have exactly two common neighbours, the third ports of its two faces, and the two ends of an external seam have none;*

(iii) *two distinct ports that share no seam have at most two common neighbours;*

(iv) *an external seam is linked to no seam, and within one cell the graph of the linked relation is the line graph of the dodecahedron dual to the cell. Its adjacency matrix has no eigenvalue below* -2.

Proof. (i) holds by the definition of a federation. For (ii) and (iii) note that a port w in a cell other than the cells of two ports u and v cannot be a neighbour of both, since it would carry two external seams, and that a port w in the cell of u is a neighbour of a port v of another cell only if $\{w, v\}$ is the external seam of v. Hence the ends of an internal seam have their common neighbours in their own cell, where the icosahedron gives exactly two. For an external seam $\{u, v\}$ a common neighbour in the cell of u would be joined to v by a second external seam of v, so there is none. Two ports of one cell that share no seam are at distance two or three in the cell, with two or no common neighbours there and none outside. Two ports of different cells that share no seam have at most one common neighbour in each of their two cells, the external partner of the other port. (iv) By (ii) every face lies in one cell, and the three sides of a face of the icosahedron are the three edges at a vertex of the dual dodecahedron, which gives the line graph. The adjacency matrix of a line graph is $N^{\mathsf{T}}N - 2I$ with N the vertex-edge incidence matrix of the underlying graph, and $N^{\mathsf{T}}N \geq 0$. □

The hypotheses and the four facts were also counted on the three federations of Table 1, and they hold on each.

3 State, private memory and the local law

3.1 State

Every port v holds a load $n_v \in \{0, 1, 2\}$, a system with the three basis states $|0\rangle, |1\rangle, |2\rangle$. Every seam has a private memory with eleven basis states: one blank state $|\varnothing\rangle$ and ten labels $|c, d\rangle$ with a coin $c \in \{0, 1\}$ and a contrast $d \in \{-2, \ldots, 2\}$. The memory is coherent working memory at the two ends of the seam. It is no public record: nothing outside the seam reads it, and the law returns it to blank. The local space of one seam is $\mathbb{C}^9 \otimes \mathbb{C}^{11}$, of dimension 99, and the law is defined on all of it.

A port is an interface of its cell, and the cell is one native observer. A port holds a load, takes part in the events of its own five or six seams, shares one private memory with the port at the other end of each of them, and counts its events.

3.2 The native seam event and the contrast write

For loads (a, b) on the two ports of a seam put $s = a + b$ and $p = s \bmod 2$. The native seam event with coin c has the output

$$y_c(a, b) = (\lfloor s/2 \rfloor + c\,p,\ \lfloor s/2 \rfloor + (1 - c)\,p). \tag{2}$$

The two ports receive the floor and the ceiling of half the total, and for an odd total the coin decides which port receives the larger part. This is the native seam rule of the carrier [7]. The output conserves the total, and it is the same for the inputs (a, b) and (b, a).

The contrast write records the coin and the contrast of the inputs and leaves the output on the ports:

$$W|a, b\rangle = \frac{1}{\sqrt{2}} \sum_{c=0}^{1} |c, a - b\rangle \otimes |y_c(a, b)\rangle. \tag{3}$$

On the sum of the blank sector $|\varnothing\rangle \otimes \mathbb{C}^9$ and the record sector $\mathbb{C}^{10} \otimes \mathbb{C}^9$ define the echo

$$U = \begin{pmatrix} 0 & W^* \\ W & I - WW^* \end{pmatrix}. \tag{4}$$

Lemma 3.1. *W is an isometry on all nine inputs, and U is a Hermitian unitary on all 99 states.*

Proof. The vectors $|c, a-b\rangle \otimes |y_c(a,b)\rangle$ for the two coins are orthogonal, so $W|a,b\rangle$ has norm one. If two inputs give the same label and the same output for some coin, they have the same difference and, since the output conserves the total, the same sum; they are equal. Hence $W^*W = I$. Then WW^* is a projector, $W^*(I - WW^*) = 0$, and

$$U^2 = \begin{pmatrix} W^*W & W^*(I-WW^*) \\ (I-WW^*)W & WW^* + (I-WW^*)^2 \end{pmatrix} = I.$$

U is Hermitian by inspection. □

3.3 The memory generator and the echo

On the ten labels let $R|c,d\rangle = |c,-d\rangle$ and $D|c,d\rangle = d\,|c,d\rangle$, and put

$$M_\kappa = \frac{I-R}{2} + \kappa D^2, \qquad G_\kappa = 0_\varnothing \oplus (M_\kappa \otimes I), \qquad h_\kappa = U\,G_\kappa\,U. \tag{5}$$

R reverses the recorded contrast and D^2 charges its square. The generator G_κ acts on the memory alone and vanishes on the blank state. The local term h_κ is the law of one seam.

Since $U^2 = I$, one event of angle τ is

$$e^{-i\tau h_\kappa} = U\,e^{-i\tau G_\kappa}\,U. \tag{6}$$

On a blank memory the first factor writes, the middle factor is a phase on the label, and the last factor reverses the write: this is the echo (Figure 2). The reverse write acts on the coherent private memory alone. No admitted public record is unwritten by it.

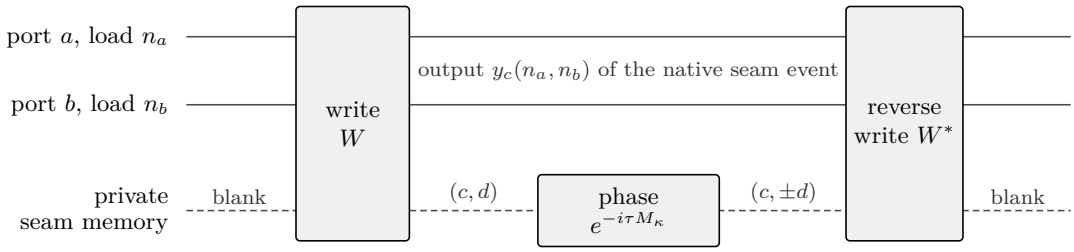


Figure 2: The echo on one seam. The write records the coin c and the contrast $d = n_a - n_b$ in the private memory and leaves the output of the native seam event on the two ports. The phase acts on the memory alone. The reverse write returns the memory to blank. On the loads the three steps together are one event of the seam term of (1) (Theorem 3.2).

Theorem 3.2 (Exact reduction)**.** *Let J embed the loads of a seam into the blank memory, $Jx = |\varnothing\rangle \otimes x$. For every real κ*

$$h_\kappa J = J\Big[\frac{I - \mathrm{SWAP}}{2} + \kappa\,(n_a - n_b)^2\Big]. \tag{7}$$

The local space is the orthogonal sum of three subspaces that h_κ maps into themselves: the blank code $\operatorname{ran} J$*, of dimension* 9*, on which h_κ is the seam term* (7)*; the written code* $\operatorname{ran} W$*, of dimension*

9, *on which* $h_\kappa = 0$*; and the other records* $(\operatorname{ran} W)^\perp$ *inside the record sector, of dimension* 81*, on which* $h_\kappa = M_\kappa \otimes I$*.*

Proof. Two intertwining identities hold on every input:

$$(R \otimes I)\, W = W \,\mathrm{SWAP}, \qquad (D^2 \otimes I)\, W = W\, (n_a - n_b)^2. \tag{8}$$

The second holds because the label of $W|a,b\rangle$ carries $d = a - b$. For the first, $(R \otimes I)W|a,b\rangle = 2^{-1/2} \sum_c |c, b-a\rangle \otimes |y_c(a,b)\rangle$ and $W|b,a\rangle = 2^{-1/2} \sum_c |c, b-a\rangle \otimes |y_c(b,a)\rangle$, and the outputs agree by the symmetry of (2). Write $h = \frac{1}{2}(I - \mathrm{SWAP}) + \kappa(n_a - n_b)^2$. Then $(M_\kappa \otimes I)W = Wh$, and

$$h_\kappa J x = U G_\kappa(0, Wx) = U(0, Whx) = (W^*Whx,\, (I - WW^*)Whx) = (hx, 0) = Jhx.$$

For the second statement, $M_\kappa \otimes I$ is Hermitian and maps $\operatorname{ran} W$ into itself, so it commutes with the projector WW^*. For $z = Wx$ in the written code, $U(0,z) = (x,0)$ and $G_\kappa(x,0) = 0$, so $h_\kappa z = 0$. For z in the record sector with $W^*z = 0$, $U(0,z) = (0,z)$, $G_\kappa(0,z) = (0, (M_\kappa \otimes I)z)$ is again orthogonal to $\operatorname{ran} W$, and U fixes it. The three subspaces are orthogonal and their dimensions add to 99. □

The identities are polynomials of degree one in κ. Both coefficient matrices were checked in exact arithmetic on all 99 states, including the inputs $(2,0)$, $(0,2)$, $(2,1)$, $(1,2)$ and $(2,2)$ and every memory state that the echo never visits.

Corollary 3.3 (Overlapping seams)**.** *Let* H_{joint} *be the sum of the local terms of all seams and of the degree terms below, acting on all loads and all memories, and let* J_{all} *embed the loads with every memory blank. Then* $H_{\mathrm{joint}} J_{\mathrm{all}} = J_{\mathrm{all}} H_\kappa$ *and* $e^{-itH_{\mathrm{joint}}} J_{\mathrm{all}} = J_{\mathrm{all}} e^{-itH_\kappa}$ *for every real* t*.*

Proof. The term of a seam acts on the loads of its two ports and on its own memory. On a vector of $\operatorname{ran} J_{\mathrm{all}}$ it acts by (7) and leaves every memory blank, although neighbouring seams share a port and their terms do not commute. Summing over the seams gives the first identity. H_{joint} is Hermitian and maps $\operatorname{ran} J_{\mathrm{all}}$ into itself, which gives the second. □

The corollary concerns the sum of the local terms with a continuous parameter. A finite program of echoes on seams that share ports is a different object with its own schedule; Section 7 treats it. At $\kappa = 0$ the echo is the exchange alone. That law is the control of Section 8.

3.4 The degree term

The seam terms alone give $\sum_e (n_a - n_b)^2 = \sum_v d_v n_v^2 - 2\sum_e n_a n_b$, which favours ports with five seams over ports with six: a pair on a seam between two ports with six seams and two separate loads on ports with five seams would have the same cost. A port with five seams therefore carries one private reference register, a load system prepared in $|0\rangle$, with an eleven-state memory of its own. On the port and its reference the law uses the echo with the contrast part of the generator alone, $U(0_\varnothing \oplus \kappa D^2 \otimes I)U$. By Theorem 3.2 it reduces on the blank memory to $\kappa(n_v - n_{\mathrm{ref}})^2$, which is diagonal, keeps the reference at zero and equals κn_v^2 there. The exchange part is left out on this wire, because it would move load into the reference. Every port then counts six, which gives the term $\sum_v (6 - d_v) n_v^2$ of (1).

3.5 What depends on the written output

Proposition 3.4 (Symmetry of the output)**.** *Replace the native output in* (3) *by any outputs* $y'_c(a,b)$ *that conserve the total. If* $y'_c(a,b) = y'_c(b,a)$ *for all inputs and both coins, Lemma 3.1 and Theorem 3.2 hold with the same right-hand side in* (7)*.*

Proof. The proofs used three properties of W: the isometry, which needs the conservation of the total, and the two identities (8). The second identity uses the label alone, and the first is the stated symmetry. □

The reduction was checked in exact arithmetic for four such replacements: the sorted input with the coin ignored, the most concentrated pair of the same total, the native output with the coin reversed, and one coin branch used for both coins. All four give the seam term of (7). A write that keeps the order of its inputs, $y'_c(a,b) = (a,b)$, violates the first identity of (8), and for it (7) fails. The equalization of the loads, the fair coin and the value of the output therefore play no role in the induced law: the write is undone exactly. The native seam event contributes its symmetry under exchange of the two ports, and the binding of Section 5 comes from the supplied memory generator.

4 The law on loads

4.1 Terms, costs and conserved quantities

By Corollary 3.3 the model with blank memories evolves the loads by $H_\kappa = K + \kappa C$ of (1), with the exchange $K = \frac{1}{2}\sum_e (I - \mathrm{SWAP}_e)$ and the contrast cost C. Each $\frac{1}{2}(I - \mathrm{SWAP}_e)$ is a projector, so $K \geq 0$. The term κC is the agreement energy. In C a load counts 6, a doubled load counts 24, and every agreed seam takes 2 off. For two loads this gives $C = 10$ for a pair on a seam, $C = 12$ for two separate unit loads and $C = 24$ for one doubled load.

The exchanges permute the loads of the ports and C is diagonal. The law therefore conserves the number of loads $\sum_v n_v$ and the multiset of loads, hence also $\sum_v n_v^2$. A doubled load stays doubled, and unit loads on distinct ports stay unit loads on distinct ports. No event of the law lowers the sum of the squared loads: the law is reversible and has no settling. The sector of N unit loads on V ports has $\binom{V}{N}$ states. The sector of two loads has $V(V+1)/2$ states, the $V(V-1)/2$ unit pairs and the V doubled loads, and the two parts do not mix.

4.2 One load

On one load K is half the graph Laplacian L of the carrier and $C = 6$:

$$H_\kappa\big|_{\text{one load}} = \tfrac{1}{2}L + 6\kappa. \tag{9}$$

The agreement energy is a constant here, so the law of one load is the same for every κ. On one cell the levels of $L/2$ are 0 for the uniform mode, $(5-\sqrt{5})/2$ three times, 3 five times and $(5+\sqrt{5})/2$ three times, the representations 1, 3, 5 and 3′ of the rotations. The threefold level $(5-\sqrt{5})/2$ is the slow band of the carrier paper: its eigenspace is the range of the incidence Gram matrix of the twelve port directions, which has rank three, and it is the slowest response of the uniform repair mean [7]. In the theory this rank-three response is the material from which the observers' effective description with three space dimensions is built [5]. The cell is no piece of space. On the federation of 20 cells the twenty lowest levels of one load lie between 0 and 0.17023, one for each cell, and the next level is 1.38197.

4.3 Loads 0 and 1: the XXZ form

On loads 0 and 1, with $n = (I - Z)/2$ and the Pauli matrices X, Y, Z of the two ports, the seam term of (7) is

$$\frac{I - \mathrm{SWAP}}{2} + \kappa(n_a - n_b)^2 = \frac{(1 + 2\kappa)(I - Z_a Z_b) - X_a X_b - Y_a Y_b}{4}, \tag{10}$$

since $\mathrm{SWAP} = \frac{1}{2}(I + X_a X_b + Y_a Y_b + Z_a Z_b)$ and $(n_a - n_b)^2 = \frac{1}{2}(I - Z_a Z_b)$. This is the ferromagnetic XXZ interaction with anisotropy $\Delta = 1 + 2\kappa$. Its bound clusters of overturned spins are the droplet states of the XXZ chain [9] and of XXZ systems on general graphs [10]. Those results give the context. The constants below are proved directly for this carrier and for the complete sector of two loads with doubled loads.

5 The binding theorem

5.1 Ordered blocks

Lemma 5.1 (Ordered blocks)**.** *Let H be Hermitian on a finite-dimensional space, P an orthogonal projector of rank m and $Q = I - P$. Suppose*

$$PHP \le \mu_P\, P, \qquad QHQ \ge \mu_Q\, Q, \qquad \gamma = \mu_Q - \mu_P > 0, \qquad \|QHP\| \le \beta.$$

Let $\lambda_1 \le \lambda_2 \le \dots$ be the eigenvalues of H. Then:

(a) *$\lambda_m \le \mu_P < \mu_Q \le \lambda_{m+1}$. Let Π be the spectral projector of H on its m lowest eigenvalues.*

(b) *$\|Q\,\Pi\| \le \beta/\gamma$ and $\|(I - \Pi)\,P\| \le \beta/\gamma$.*

(c) *For every unit vector $\psi \in \operatorname{ran} P$ and every real t, $\|Q\, e^{-itH}\psi\| \le 2\beta/\gamma$.*

Proof. (a) By the min-max principle λ_m is the least value, over subspaces of dimension m, of the largest expectation of H on the subspace. The subspace $\operatorname{ran} P$ gives $\lambda_m \le \mu_P$. Also λ_{m+1} is the largest value, over subspaces of codimension m, of the least expectation of H on the subspace, and $\operatorname{ran} Q$ gives $\lambda_{m+1} \ge \mu_Q$. The coupling between the blocks does not enter.

(b) Put $X = Q\Pi$, regarded as a map from $\operatorname{ran}\Pi$ to $\operatorname{ran} Q$, and let H_{low} be the restriction of H to $\operatorname{ran}\Pi$, with spectrum at most μ_P, and $H_Q = QHQ$ on $\operatorname{ran} Q$, with spectrum at least μ_Q. Since $\operatorname{ran}\Pi$ is invariant under H,

$$X H_{\mathrm{low}} = QH\Pi = QH(P + Q)\Pi = (QHP)\,P\Pi + H_Q X,$$

so $H_Q X - X H_{\mathrm{low}} = Y$ with $Y = -(QHP)\,P\Pi$ and $\|Y\| \le \beta$. Put $A = H_Q - \mu_P \ge \gamma$ and $B = H_{\mathrm{low}} - \mu_P \le 0$. Then $AX - XB = Y$ and

$$\frac{d}{ds}\Big(e^{-sA} X e^{sB}\Big) = -e^{-sA}(AX - XB)e^{sB} = -e^{-sA} Y e^{sB}.$$

Since $\|e^{-sA}\| \le e^{-s\gamma}$ and $\|e^{sB}\| \le 1$ for $s \ge 0$, integration from 0 to ∞ gives $X = \int_0^\infty e^{-sA} Y e^{sB}\, ds$ and $\|X\| \le \|Y\|/\gamma \le \beta/\gamma$. This is the Sylvester equation with ordered spectra [12]; the bound is the one-sided form of the rotation estimates of Davis and Kahan [11]. For the second bound put $Z = (I - \Pi)P$, a map from $\operatorname{ran} P$ to $\operatorname{ran}(I - \Pi)$, and let H_{high} be the restriction of H to $\operatorname{ran}(I - \Pi)$, with spectrum at least μ_Q, and $H_P = PHP$ on $\operatorname{ran} P$. Then

$$H_{\mathrm{high}} Z - Z H_P = (I - \Pi)HP - (I - \Pi)PHP = (I - \Pi)\,(QHP),$$

and the same argument gives $\|Z\| \le \beta/\gamma$.

(c) Split $\psi = \Pi\psi + (I-\Pi)\psi$. Both parts evolve inside their spectral subspaces, so

$$Q\, e^{-itH}\psi = Q\,\Pi\, e^{-itH}\Pi\psi + Q\, e^{-itH}(I-\Pi)P\psi,$$

and the two terms have norm at most β/γ each by (b). □

The first term of (c) is the dressing of the band: the eigenvectors of the lowest band are no pure vectors of $\operatorname{ran} P$. The second term is the part of a state prepared in $\operatorname{ran} P$ that lies outside the band. Both are kept. The weight outside the band, $\|(I-\Pi)e^{-itH}\psi\|^2 \le \beta^2/\gamma^2$, does not change with t. By convexity (c) holds for mixtures supported in $\operatorname{ran} P$, and a preparation within norm distance ϵ of a unit vector of $\operatorname{ran} P$ has $\|Qe^{-itH}\psi\| \le 2\beta/\gamma + \epsilon$. No spectral projector is an operation of an observer; Π appears in the proof alone.

5.2 The theorem

In the sector of two loads write $|\{u,v\}\rangle$ for unit loads on two distinct ports u, v and $|2u\rangle$ for a doubled load. Let P project on the span of the pairs on a seam, the states $|\{a,b\}\rangle$ with $\{a,b\}$ a seam, and let $Q = I - P$. The range of Q contains every separated pair and every doubled load.

Theorem 5.2 (Binding)**.** *On every federation of complete cells whose external seams form a partial matching on the ports, in the sector of two loads and for every $\kappa \ge 0$,*

$$PH_\kappa P \le (10\kappa + 6)\, P, \qquad QH_\kappa Q \ge 12\kappa\, Q, \qquad \|QH_\kappa P\| \le \sqrt{10}. \tag{11}$$

For $\kappa > 3$, every state ρ supported in $\operatorname{ran} P$ and every real value t of the evolution parameter,

$$\operatorname{Tr}(Q\, e^{-itH_\kappa}\rho\, e^{itH_\kappa}) \le \frac{40}{(2\kappa-6)^2}. \tag{12}$$

The constants do not depend on the number of cells or on the routing.

Proof. For a seam $e = \{a,b\}$ the projector $\frac{1}{2}(I - \mathrm{SWAP}_e)$ annihilates a basis state with $n_a = n_b$ and maps a basis state with $n_a \ne n_b$ to half the difference of the state and its exchanged image. Hence, with $[u \sim v]$ equal to one if $\{u,v\}$ is a seam and to zero otherwise,

$$\langle\{u,v\}|K|\{u,v\}\rangle = \tfrac{1}{2}(d_u + d_v) - [u \sim v], \tag{13}$$

$$\langle\{u',v\}|K|\{u,v\}\rangle = -\tfrac{1}{2} \quad \text{for } u' \sim u,\ u' \ne v, \tag{14}$$

every other entry of K between unit pairs vanishes, and K maps unit pairs to unit pairs and doubled loads to doubled loads.

The block of pairs. A pair on a seam has $C = 10$ and, by (13) and Lemma 2.1(i), a diagonal entry of K of at most 5. By (14) two pairs on seams are joined by K exactly when the moved load lands on a common neighbour of the two ends, that is when the two seams are linked. So $PKP = T - \frac{1}{2}M$ with a diagonal matrix $T \le 5$ and the adjacency matrix M of the linked relation. By Lemma 2.1(iv) M is a direct sum over the cells of matrices with no eigenvalue below -2 and of zeros on the external seams, so $-\frac{1}{2}M \le 1$ and $PKP \le 6$.

The complement. On $\operatorname{ran} Q$ the cost C is 12 or 24, and $QKQ \ge 0$ because $K \ge 0$.

The coupling. C is diagonal, so $QH_\kappa P = QKP$. The column of a pair on a seam $\{u,v\}$ has entries $-\frac{1}{2}$ at the separated pairs $\{u',v\}$ and $\{u,v'\}$, where u' is a neighbour of u other than v that is no neighbour of v, and likewise for v'. Their number is $d_u + d_v - 2 - 2c$ with c the number of

common neighbours of u and v. By Lemma 2.1(ii) it is $d_u + d_v - 6 \le 6$ for an internal seam and 10 for an external seam, so the absolute entries of a column sum to at most 5. The row of a separated pair $\{x, y\}$ has entries $-\frac{1}{2}$ at the pairs $\{w, y\}$ and $\{x, w\}$ with w a common neighbour of x and y, so its absolute entries sum to the number of common neighbours, at most 2 by Lemma 2.1(iii). Rows of doubled loads vanish. A matrix whose columns have absolute sums at most 5 and whose rows have absolute sums at most 2 has norm at most $\sqrt{5 \cdot 2}$.

This proves (11). For $\kappa > 3$ Lemma 5.1 applies with $\mu_P = 10\kappa + 6$, $\mu_Q = 12\kappa$, $\gamma = 2\kappa - 6$ and $\beta = \sqrt{10}$, and its part (c) with the remark on mixtures gives (12). □

The statement on records. The projectors on the pairs on the individual seams are mutually orthogonal in the sector of two loads, and the complete seam reading of Section 6.2 has one outcome for each seam and the outcome "no pair". The left-hand side of (12) is the probability of "no pair". A pair that starts on a seam is therefore read as a pair on some seam with probability at least $1 - 40/(2\kappa - 6)^2$ at every reading. The value t is audit. No translation invariance, preferred centre or global observer enters the proof.

What the theorem leaves free. The bound concerns the two loads being read on one seam. It does not say on which seam, and Section 8 shows that the pair is read on other seams than the first. It does not require every configuration along the evolution to be a pair on a seam. For a pair written on a seam $\{a, b\}$ the probability of "no pair" is $\frac{1}{4}(d_a + d_b - 2 - 2c)\, t^2$ to leading order in t, with c as in the proof: t^2 on one cell and $\frac{5}{4}t^2$ on a seam whose ends have six and five seams. In the sector of unit pairs the mean of C is 10 plus twice that probability, so the agreement energy rises while the pair dresses and the exchange compensates it; H_κ is conserved. A law of strict descent of C would exclude this mechanism.

5.3 Constants on the carriers

The largest level of PKP and the norm of QKP were computed on the carriers of Table 1; Table 2 compares them with the bounds of the proof. With the computed constants, part (c) of Lemma 5.1 gives $17.3/(2\kappa - 5.70)^2$ on the federations: 0.163 at $\kappa = 8$ and 0.00028 at $\kappa = 128$, against 0.4 and 0.00064 from (12).

Table 2: Constants of the binding theorem. The first column is the largest level of the exchange on the pairs on seams, the second the largest singular value of its coupling to the other states of two loads.

Carrier	Largest level of PKP	$\|QKP\|$
one cell	5.0000	2.0000
20 cells	5.7008	2.0821
80 cells	5.7000	2.0811
320 cells	5.7000	2.0822
every federation of the theorem	at most 6	at most $\sqrt{10} = 3.1623$

Proposition 5.3 (One cell)**.** *On one cell, in the sector of two loads, the largest level of $PH_\kappa P$ is $10\kappa+5$, the lowest level of $QH_\kappa Q$ is $\min(12\kappa+4-\sqrt{6},\ 24\kappa)$, and $\|QH_\kappa P\| = 2$. For $\kappa > (1+\sqrt{6})/2$ a state supported in $\operatorname{ran} P$ is read as "no pair" with probability at most $16/(2\kappa - 1 - \sqrt{6})^2$ at every reading.*

Proof. Every port has five seams, so $PKP = 4 - \frac{1}{2}M$ with M the adjacency matrix of the line graph of the dodecahedron. This graph has thirty vertices and the incidence matrix N of the dodecahedron has twenty rows, so $N^{\mathsf{T}}N$ is singular and -2 is an eigenvalue of M: the largest level of PKP is 5. The range of Q is the sum of the separated unit pairs, with $C = 12$, and the doubled loads, with $C = 24$, and K does not join the two. On the doubled loads K is $L/2$ with lowest level 0. On the thirty pairs at distance two and the six antipodal pairs, (13) gives the diagonal 5; a load of a pair at distance two moves to four separated positions at distance two and to two antipodal positions, and a load of an antipodal pair moves to ten positions at distance two. A vector equal to x on the pairs at distance two and to y on the antipodal pairs is an eigenvector with eigenvalue λ when $3x - y = \lambda x$ and $5y - 5x = \lambda y$, that is $\lambda^2 - 8\lambda + 10 = 0$. The root $\lambda = 4 - \sqrt{6}$ has $y = (\sqrt{6} - 1)x$, an eigenvector with positive entries. The compression of K to the separated pairs has no positive entries off the diagonal, so its lowest eigenvector can be chosen with nonnegative entries and cannot be orthogonal to a positive one: $4 - \sqrt{6}$ is the lowest level. Each column of QKP has four entries $-\frac{1}{2}$, each row of a pair at distance two has four and each antipodal row none, so $\|QKP\| \le 2$, and the uniform vector on the thirty pairs on seams attains 2. The last statement is Lemma 5.1 with $\gamma = 2\kappa - 1 - \sqrt{6}$ and $\beta = 2$; for these κ the first term of the minimum is the smaller one. □

At $\kappa = 8$ the bound of Proposition 5.3 is 0.102 and at $\kappa = 128$ it is 0.00025.

6 Emitter, reader and agreed records

6.1 The local emitter

The emitter at a seam $e = \{a, b\}$ is one fuel state $|F\rangle$ that carries the resource of two loads and is coupled to the pair $|e\rangle = |\{a, b\}\rangle$ on that seam. Its energy is the diagonal entry of the law at the pair,

$$E_F = 10\kappa + \tfrac{1}{2}(d_a + d_b - 2), \qquad H_{\text{emit}} = H_\kappa + E_F|F\rangle\langle F| + g\big(|F\rangle\langle e| + |e\rangle\langle F|\big), \tag{15}$$

and the coupling g acts for the duration $\pi/(2g)$ while the law acts throughout. No eigenvector of the law is installed, and fuel left in the emitter is an outcome of every later reading.

Proposition 6.1 (Emitter)**.** *Let ψ be the state after the pulse from the initial state $|F\rangle$. Then*

$$\big\|\psi + i\, e^{-iE_F\pi/(2g)}|e\rangle\big\| \le \frac{\sqrt{d_a + d_b - 2}}{2g}.$$

Proof. Let $r = (H_\kappa - E_F)|e\rangle$. By (13) and (14) it is the sum of $-\frac{1}{2}$ times the $d_a + d_b - 2$ distinct states reached from the pair by one exchange, so $\|r\|^2 = (d_a + d_b - 2)/4$, and r is orthogonal to $|e\rangle$ and $|F\rangle$. Put $H_{\text{emit}} = H_0 + |r\rangle\langle e| + |e\rangle\langle r|$. Under H_0 the states $|F\rangle$ and $|e\rangle$ have the same energy E_F and are coupled to each other alone, so $\psi_0(s) = e^{-iE_F s}\big(\cos(gs)|F\rangle - i\sin(gs)|e\rangle\big)$ and $\psi_0(\pi/2g) = -i\, e^{-iE_F\pi/(2g)}|e\rangle$. By Duhamel's formula

$$\psi - \psi_0\big(\tfrac{\pi}{2g}\big) = -i\int_0^{\pi/(2g)} e^{-i(\pi/(2g)-s)H_{\text{emit}}}\big(|r\rangle\langle e| + |e\rangle\langle r|\big)\psi_0(s)\, ds,$$

and the integrand has norm $|\sin(gs)|\,\|r\|$. The integral of $|\sin(gs)|$ over the pulse is $1/g$. □

Corollary 6.2. *On a federation as in Theorem 5.2 with $\kappa > 3$, at every reading after the pulse the probability of the outcomes "no pair on a seam, a doubled load, or fuel" is at most*

$$\Big[\frac{2\sqrt{10}}{2\kappa - 6} + \frac{\sqrt{d_a + d_b - 2}}{2g}\Big]^2 \le \Big[\frac{2\sqrt{10}}{2\kappa - 6} + \frac{\sqrt{10}}{2g}\Big]^2.$$

At $\kappa = 128$ and $g = 64$ the right-hand side is $64009/25600000 = 0.0025003515625$ on every seam of every such federation.

Proof. After the pulse the coupling is off and the evolution preserves the norm distance of Proposition 6.1. The remark after Lemma 5.1 applies with that distance as ϵ, and $d_a + d_b \le 12$. □

With the same constants the weight outside the band of pairs or in the fuel is at most $(\sqrt{10}/250 + \sqrt{10}/128)^2 = 35721/25600000$, constant after the pulse. The duration and the coupling of the pulse, the fuel and the switch are resources of the apparatus. Errors of timing or initialization add to these bounds.

6.2 The reader and the complete outcome set

At a seam $e = \{a, b\}$ the predicate

$$b_e = \mathbf{1}\{n_a = n_b = 1\},$$

"both ends hold one load", is read at both ends. The reader of a seam has eleven bits: a value bit and a validity bit for the pointer at each of the two ports and for a courier, two decision bits, and three public record bits. It acts in four stages. The pointer at the first port receives a copy of b_e and is set valid. The pointer at the second port does the same. The courier copies the first pointer and carries it to the second port. The comparator then writes the decision: *agreement* if the second pointer and the courier are both valid and hold the same value, *mismatch* if both are valid and differ, *unavailable* if one of them is invalid; a reader that has not run is *pending*. On agreement, and then alone, the comparator writes the agreed value into the three public bits. An *agreed record* of the seam is the public value one with the decision agreement.

Each stage is an exclusive-or gate and therefore a permutation of all 2^{11} words of the reader that is its own inverse; the reader is reversible on every input, clean or not. The four stages were composed and inverted on all 2^{11} words for both values of b_e and for four variants: the nominal reader, a courier that flips the message (decision mismatch), a second pointer that is never written (decision unavailable) and both pointers holding the wrong value. The last variant ends in agreement on a wrong value. Agreement by itself does not prove that a reading was correct; it is the condition under which a record is admitted. After admission the public record has no generator of its own. It is a supplied protected memory, and its three copies give no protection against a common corruption of all three.

In the sector of two loads the predicates of different seams exclude each other, so the complete reading of all seams has one outcome for each seam and one further outcome, "no pair". On the federation of 20 cells these are 630 records and "no pair". Separated loads, doubled loads and fuel left in the emitter all fall under "no pair". Their separate weights are audit of the computation and no information that the readers of the seams hold.

The seam memory of the law stays private and unread. Seam records are committed at the reading. Every reported reading uses a fresh run, and a reading is never used to hold the object in place: after an ideal complete reading the loads are in the mixture $\rho \mapsto \sum_e P_e \rho P_e + Q \rho Q$ with P_e the projector on the pair on the seam e, which is a different state from the unread one.

6.3 Reading while the law acts

A reading of finite duration is performed with the law acting. Stage j of the reader of a seam e is the involution $S_{e,j}$ and has the generator

$$\Lambda_{e,j} = \frac{\pi}{2\tau_j}(I - S_{e,j}), \qquad e^{-i\tau_j \Lambda_{e,j}} = S_{e,j},$$

because $\frac{1}{2}(I - S_{e,j})$ projects on the eigenvalue -1 of $S_{e,j}$. During stage j the complete generator is $H_\kappa + \sum_e \Lambda_{e,j}$ together with the energy of the fuel; the reading lasts $\tau_{\mathrm{read}} = \sum_j \tau_j$.

Proposition 6.3 (Reader with the law acting)**.** *In the sector of two loads of a carrier whose ports have at most d seams, let U_{act} be the complete evolution over the reading of any set of seams and U_{ref} the same evolution with the exchange K removed. Then U_{ref} is the ideal reading at the start of the reading followed by a phase that is diagonal in the loads, and*

$$\|U_{\mathrm{act}} - U_{\mathrm{ref}}\| \le 2d\,\tau_{\mathrm{read}}.$$

The distribution of the complete public records differs from that of the ideal reading at the start by at most $2d\,\tau_{\mathrm{read}}$ in total variation, for every κ and every number of seams read at once.

Proof. The stage generators are controlled by the diagonal predicates b_e and act on distinct registers. They commute with each other, with κC and with the energy of the fuel. Without K the evolution of stage j is therefore the product of the $S_{e,j}$ and of a diagonal phase, and the phases can be collected after the stages. Duhamel's formula gives $\|U_{\mathrm{act}} - U_{\mathrm{ref}}\| \le \|K\|\,\tau_{\mathrm{read}}$. By (13) a diagonal entry of K is at most d on a unit pair, and it is at most $d/2$ on a doubled load. In both cases the absolute entries off the diagonal in the same row sum to the diagonal entry, so $\|K\| \le 2d$. For unit vectors the trace distance of the two outputs, and with it the total variation distance of any complete measurement, is at most the norm distance. □

On the federations $d = 6$, and for $\tau_{\mathrm{read}} = 1/16384$ the bound is $3/4096 = 0.000732421875$. The comparison is with the ideal reading at the start of the reading; the proposition does not claim that the two pointers copy an unchanged load.

One such reading was executed with every state kept: the reader of one seam of one cell, on the emitted state at $\kappa = 128$ and $g = 64$ after an evolution of four units, with four stages of duration $1/65536$ each. The joint space of the 78 states of two loads, the fuel and the 2^{11} reader words has dimension 161,792. The state differs from the reference by 2.44×10^{-4} in norm, against the bound $10\,\tau_{\mathrm{read}} = 6.10 \times 10^{-4}$ of the cell, and the public records by 2.09×10^{-7} in total variation. The agreed record of the seam has probability 0.19502961, against 0.19502982 for the ideal reading; this is the probability for one seam at one reading and no probability that a pair is read somewhere. The decisions have weight 1.2×10^{-10} on mismatch and 4.7×10^{-11} on unavailable.

6.4 Error budget

Table 3 collects the bounds for the emitter at $\kappa = 128$ and $g = 64$ and a complete reading of duration $1/16384$ on any federation of Theorem 5.2. After one complete reading every admitted branch is a pair on a seam, to which the theorem applies again, and the outcome "no pair" keeps its weight; the last line follows. Further readings need fresh records and add their own errors. No error-free unlimited observation is claimed.

Table 3: Bounds on the probability of every outcome other than an agreed record of a pair, at $\kappa = 128$, $g = 64$ and $\tau_{\mathrm{read}} = 1/16384$, on every federation of Theorem 5.2.

Step	Added	Bound
emitter and free evolution, ideal reading		0.0025003515625
reading with the law acting	3/4096	0.0032327734375
free evolution after one complete reading, ideal second reading	0.00064	0.0038727734375

7 Events and clocks

7.1 The law as local events

One event is one seam acting once with an angle τ. As the echo (6) on blank memories it acts on the loads as $\exp(-i\tau[\frac{1}{2}(I - \mathrm{SWAP}_e) + \kappa(n_a - n_b)^2])$. Since $(n_a - n_b)^2 = n_a^2 + n_b^2 - 2n_a n_b$ and $n_a^2 + n_b^2$ commutes with the exchange of the seam, this is the product of

$$E_e(\tau) = \exp\Big(-i\tau\Big[\frac{I - \mathrm{SWAP}_e}{2} - 2\kappa\, n_a n_b\Big]\Big) \tag{16}$$

and of the phase $\exp(-i\tau\kappa(n_a^2 + n_b^2))$ of the two ports on their own loads. In the law with a continuous parameter the port phases of all seams and the degree term add to $6\kappa\sum_v n_v^2$, which the law conserves, and they change no record. In a finite program the phase of a port does not commute with the events of its other seams, so the two forms are different programs. The programs of this section use the event (16), the seam term without its port part, and need no reference registers. In the memory it is the echo whose phase reads the product of the two loads from the recorded contrast d and the total s of the written output, $n_a n_b = \frac{1}{4}(s^2 - d^2)$; the reduction of Theorem 3.2 holds for it on all 99 states, with $-2\kappa n_a n_b$ in place of $\kappa(n_a - n_b)^2$.

A program is a sequence of events. A port takes part in the events of its own seams and counts them, and that count is a local clock of its observer. No common count is assumed. The position of an event in the sequence is audit.

Proposition 7.1 (Each port's own order)**.** *Two programs in which every port sees the events of its seams in the same order give the same state, and therefore the same probabilities of all records.*

Proof. Events on two seams without a common port act on different loads and memories and commute. Let e be the first event of the first program. In the second program no event on a seam that shares a port with e can come before the first occurrence of e, since the shared port would see a different first event. That occurrence can therefore be commuted to the front. Removing it from both programs leaves two programs with the same property, and induction on the length finishes the proof. □

No order between events on seams that share no port enters a record. The statement was also executed: a program of 3,000 events and a second one obtained from it by 12,141 exchanges of neighbouring events without a common port give the same seam records on one cell, with largest difference 0, and 19,722 exchanges on 20 cells give a largest difference of 1.1×10^{-41}.

7.2 Schedules

Table 4 reports four kinds of program at $\kappa = 8$, started from a pair on one seam. In a round every seam acts once, in an order drawn afresh for each round. In the rounds with two rates the

seams between ports of one half of the carrier act twice in every round. In the last two programs every event draws its seam at random, with equal weights or with double weight on that half. The readings are the seam records after every thirty events on one cell and after every 630 events on 20 cells; the total angle per seam is 32 on one cell and 8 on 20 cells.

Table 4: The law as local events at $\kappa = 8$: largest probability of "no pair" over all readings along the program, and the range of the ports' own counts at the end of the programs with angle 1/64.

Program	One cell, $\tau = 1/64$	One cell, $\tau = 1/256$	20 cells, $\tau = 1/64$	Ports' own counts (one cell; 20 cells)
rounds, every seam once	0.0217	0.0137	0.0188	all 10240; 2560 to 3072
rounds, one half twice	0.0470		0.0268	10240 to 16384; 2560 to 6144
random seam, equal rates	0.5048	0.1897	0.1989	10063 to 10428; 2419 to 3184
random seam, two rates	0.5206	0.1522	0.2859	8170 to 13366; 1616 to 4275

Rounds keep the pair, with equal and with unequal rates. The ports' own counts then differ by a factor of up to 2.6, and no common count exists. Seams that act independently at random lose the pair at these angles. The loss falls with the angle: on one cell the largest probability of "no pair" is 0.505 at $\tau = 1/64$, 0.190 and 0.184 for two programs at 1/256, and 0.085 at 1/1024, a factor 2.2 for a factor 4 in the angle.

Seams without a common port can act in parallel. One cell has five classes of six such seams, and on a federation the same five classes hold in every cell and the external seams form a sixth. One symmetric period applies all classes except the last for half the angle, the last for the full angle, and the others in reverse order for half the angle, a splitting of second order [13]. Its error against the law with a continuous parameter falls with the square of the angle: at $\kappa = 2$ the state of a pair differs in norm by 0.337, 0.0215 and 0.00134 at angles 1/8, 1/32 and 1/128 on one cell after a total angle 16, and by 0.0916 and 0.00578 at 1/8 and 1/32 on 20 cells after a total angle 4. No closed constant for this error is claimed. On records at $\kappa = 8$, for a pair started on one seam, the layered program gives a probability of "no pair" of 0.0064 on one cell after a total angle 32 at angle 1/256, in both orders of the classes, which is the value of the continuous law, and of 0.0076 and 0.0075 in the two orders on 20 cells after a total angle 8 at angle 1/64, against 0.0076.

7.3 What the binding rests on

Two members of the memory family differ in what an event leaves behind. In the law the memory is unread and returns to blank. In the committed member the contrast of the acting seam, which of its ends holds a load, is committed to a public record after every event. On one cell at $\kappa = 8$ with rounds of angle 1/64 the committed member does not hold the pair: the probability of "no pair" is 0.015, 0.11 and 0.32 after 64, 512 and 2048 rounds, while the unread member has a largest value of 0.018 over all readings. Records of the port loads written during the evolution at a rate γ per port act in the same direction. On one cell, from the lowest level of two unit loads, the probability of a pair on a seam at parameter values 0, 128, 256, 384, 512 is 0.859, 0.865, 0.869, 0.870, 0.869 at $\kappa = 2$ and $\gamma = 0.001$, and 0.859, 0.844, 0.778, 0.717, 0.666 at $\gamma = 0.01$; at $\kappa = 4$ and $\gamma = 0.01$ it is 0.951, 0.941, 0.915, 0.889, 0.863. A uniformly random pair has 0.455.

The binding rests on the regular phase between successive events of a seam. A port that serves each of its seams once per round supplies that regularity with its own count and needs no other clock. Independent random service and records committed at every event disturb it. The theorems

of Section 5 are proved for the law with a continuous parameter, the limit of small angle, and the results of Section 8 are stated for that law.

8 Particle stability

8.1 The claim on records

Particle stability is an observable that the seam interaction reproduces, under the reference candidate and in the free sector of Interval Echo, which uses it at $\kappa = 8$. The claim is about agreed seam records.

Observers and records. The two ends of every seam report one agreed record, "both ends loaded". The complete set of outcomes is one record for each seam and the outcome "no pair"; fuel left in the emitter counts as "no pair".

Claim. A pair of loads written on one seam by a local emitter is read as a pair on some seam with probability at least one minus the bound, whenever the observers read. It is read on other seams than the first. Nothing holds it except the law. Under the same law without the agreement energy the record is lost.

Clock. The claim uses none. The bound holds at every value of the evolution parameter, hence at every reading in any observer's own history. The sampled values of the parameter below are audit.

What counts against the claim. "No pair" above the bound at a reading; no level of two loads below the energy of two free loads, an audit on the spectrum; or a changed law with the same alphabet, routing and symmetry that gives the same answer on the whole property.

The words "the pair moves" mean "the pair is read on other seams". A seam is a place in the carrier, which is hardware of the computation, and the statement is no statement about a position in physical space.

8.2 Readings

The emitter of Section 6 with $g = 64$ writes a pair on an internal seam; on the federations one end of that seam carries an external seam. The law then acts, and the seam records are read at the parameter values $0, 4, \ldots, 32$, each in a fresh run. Table 5 and Figure 3 give the largest probabilities over these readings. The bound is that of Lemma 5.1 with the constants of the carrier from Table 2 and the emitter's error, $\left[2\beta/(2\kappa - \mu) + \sqrt{d_a + d_b - 2}/(2g)\right]^2$, with μ the largest level of PKP and $\beta = \|QKP\|$.

Table 5: Particle stability on agreed seam records. The probabilities are the largest values over the readings of each run.

Carrier	κ	Bound	Largest P(no pair)	Largest P(pair on another seam)
one cell	8	0.148790	0.00956834	0.985996
one cell	128	0.001447	0.00001881	0.991270
20 cells	8	0.182975	0.00962164	0.977029
20 cells	128	0.001606	0.00002270	0.933194
80 cells	8	0.182787	0.00964389	0.977130
80 cells	128	0.001605	0.00002270	0.933194
320 cells	8	0.182965	0.00963435	0.977091

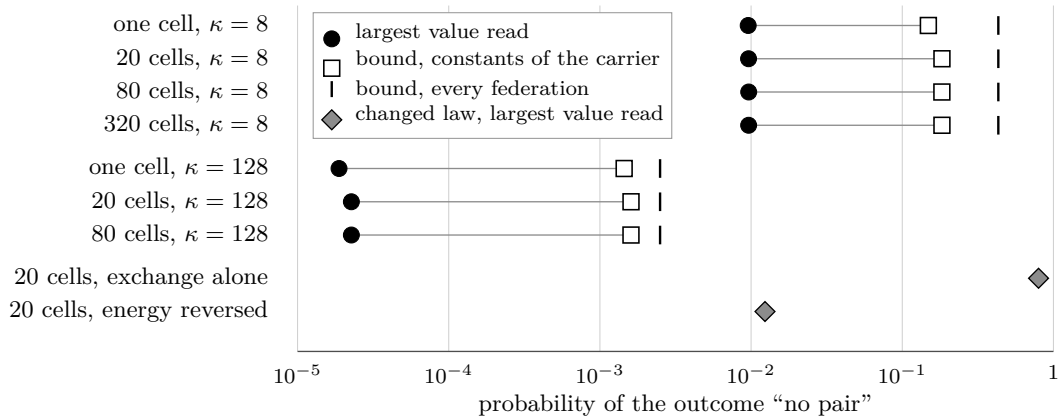


Figure 3: The largest probability of “no pair” over the readings of each run of Table 5, on a logarithmic axis, with the bound for the constants of the carrier and the bound of Corollary 6.2 for every federation. The two lowest lines are the changed laws of Table 6, which have no bound of this kind.

The readings lie a factor of 15 to 77 below the bounds and do not change with the number of cells. At long parameter values on 20 cells and $\kappa = 8$ the probability of “no pair” is 0.0089, 0.0088 and 0.0084 at 64, 256 and 1024, and the probability of a pair on another seam than the first is 0.676, 0.926 and 0.977.

Single readings. On 20 cells at $\kappa = 8$ the reading at parameter value 32 returns “no pair” with probability 0.007660. Of 100,000 sampled single readings 786 return “no pair”, the pair is read on 59 different seams, and 1,538 readings find it on the first seam, where the probability is 0.01531.

Audit on the spectrum. Count energies from 12κ, the energy of two free loads in the uniform mode. The lowest level of two loads lies at -14.07 on 20 cells and at -14.24 on one cell, and it is a pair on a seam with probability 0.987 and 0.986. The pair is bound by the law and by no trap: the level lies below the energy at which two loads are free. At $\kappa = 0.5$, 1 and 2 on 20 cells the lowest level lies at -0.355, -1.002 and -2.596 and is a pair on a seam with probability 0.570, 0.712 and 0.861.

8.3 Controls

Changed laws. Table 6 repeats the run on 20 cells for two laws with the same alphabet, routing and symmetry. The exchange law alone, $\kappa = 0$, loses the pair. Its lowest level of two unit loads is the uniform state at 0, in which a pair on a seam has the probability $630/28680 = 0.022$ of a uniformly random pair. With the agreement energy reversed, $-\kappa$ in place of κ, the pair is kept in free evolution. Lemma 5.1 explains this: free persistence follows from the isolation of the band of pairs, and with the reversed sign the pairs form the highest band. What the sign of the law adds is the level below two free loads. With the reversed sign every level of two unit loads lies above 0 and the lowest holds no pair. The property of the claim is persistence together with binding, and both changed laws give another answer on it.

A state without the property. Two loads on ports of different cells, under the law, are read as an agreed pair with probability at most 0.00004 at $\kappa = 8$ over the same readings.

Planted laws. Three planted changes test the reader and count for nothing else. Without any exchange the pair stays on its seam, with a probability of 0 on another seam, and the case is excluded because nothing is read elsewhere. One loss channel from the first seam raises the largest

Table 6: Changed laws on 20 cells with $\kappa = 8$. The lowest level is that of two unit loads, counted from the energy of two free loads.

Law	Largest P(no pair)	Lowest level	Pair on a seam in it
the law	0.0096	-14.07	0.987
exchange alone, $\kappa = 0$	0.7968	0	0.022
agreement energy reversed	0.0124	$+0.0066$	0.00002

probability of "no pair" from 0.0096 to 0.038. With the agreement energy removed on the thirty seams of the emitter's cell the reader shows 0.81.

Changes of the test. The four checks of the claim (the pair is kept within the bound, it is read on other seams, a level lies below two free loads, the control loses the pair) were also run under six deliberate changes of the law, the emitter and the control, on one cell and on 20 cells. The law passes the four checks, and each check fails under at least one change.

8.4 The face object

Three loads on the three ports of one face form the face object (Figure 1). Its record is the conjunction of the agreed records of the three seams of a face. Started on one face at $\kappa = 8$ and read at the same parameter values, the outcome "no agreed face" has probability at most 0.0178 on one cell and 0.0174 on 20 cells, and the object is read on another face with probability up to 0.980 and 0.944. Under the exchange law alone "no agreed face" reaches 0.80 on one cell and 0.97 on 20 cells. The sectors of three unit loads have 220 and 2,275,280 states.

8.5 Unequal seams

With every seam term multiplied by a strength $1 + \delta_e$, $|\delta_e| \le w$, drawn independently, one draw for each w, the lowest level of two unit loads on 20 cells at $\kappa = 2$ stays below the energy of two free loads and stays a pair: -2.596 with a pair on a seam at probability 0.861 for $w = 0$, -2.668 with 0.868 for $w = 0.1$, and -2.796 with 0.879 for $w = 0.2$. The binding uses no symmetry between the seams.

9 What a reading leaves

The reader of Section 6 keeps one record for every seam. A second instrument keeps one answer for the whole cell, and the two leave different states behind.

9.1 The presence reading

On one cell with the fuel, let P_e be the projector on the pair on the seam e, $P = \sum_e P_e$ the projector on the thirty pairs on seams, and $Q = I - P$ the projector on the other 49 states. The presence reading runs the four stages of the reader on every seam, combines the thirty public words by a tree of reversible OR gates of depth five, copies the root into a fresh public register, and runs the tree and the readers backwards (Figure 4). Every gate is an involution on its register. The scratch registers return to blank for every basis state of the loads, and by linearity for every state. Orthogonal record labels are copied and nothing else. One read allocates 720 bits. With one switch

set, the public word of every seam is copied into an archive of its own before the reversal, which gives the reader with seam records in the same circuit.

For a state X of the loads, a positive answer acts as

$$X \longmapsto PXP \qquad \text{and} \qquad X \longmapsto \sum_{e} P_e X P_e \tag{17}$$

for the presence reading and for the reader with seam records, and a negative answer acts as $X \mapsto QXQ$ for both. Both find a pair with the same probability. The presence reading leaves the pair spread over the seams as it was. The reader with seam records places it on one seam. A state supported on one answer block is left unchanged by the presence reading. On the uniform superposition of the thirty pairs the two positive outputs differ by $29/30$ in trace distance, the largest value over all inputs.

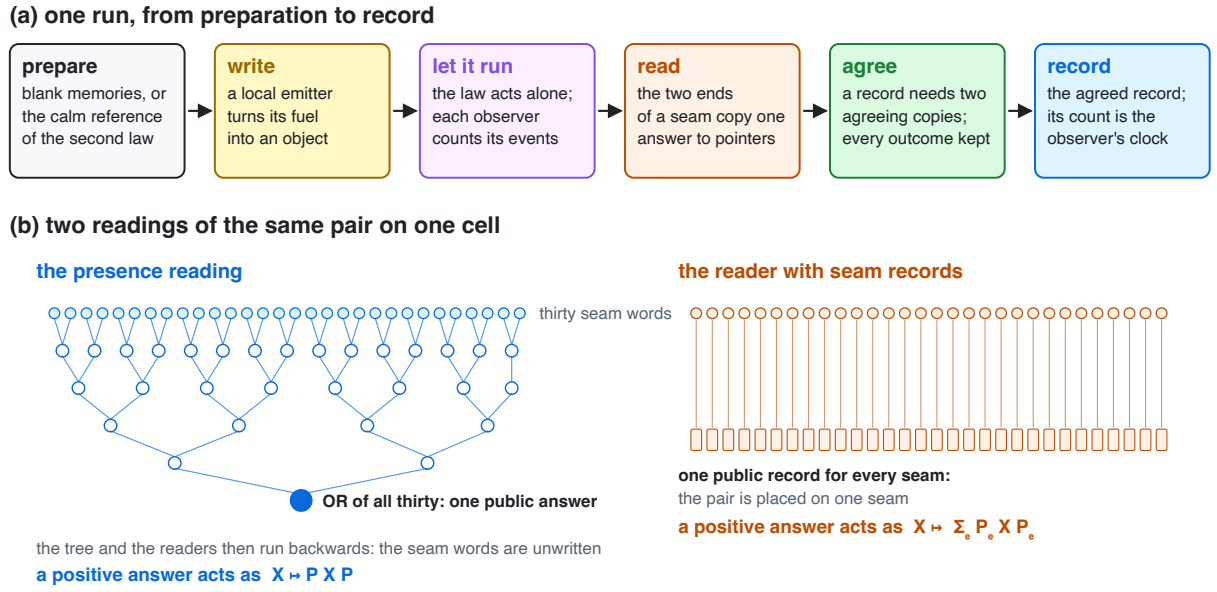


Figure 4: Observers, instruments and records. (a) One run, the same for the reference candidate and the scalar channel: a preparation, a local write by an emitter, the law acting alone while every observer counts its own events, a reading copied into two pointers, agreement of the two copies, and the agreed record whose count is the observer's clock. (b) The presence reading combines the thirty seam words by a reversible tree, copies one answer and unwrites the words; the reader with seam records keeps thirty records. Table 7 gives the laws of the later records.

9.2 Two readings in one history

The emitter writes the pair on the seam $(0, 1)$ of one cell at $\kappa = 8$ and $g = 64$. After four units of the exchange the observers take a first reading, the presence reading or the reader with seam records. After four further units they take a final reading with seam records. The seam of the first reading is hidden in both histories before they are compared, so that both show the same public alphabet: one first answer and one final record. The first answer has the same law in both histories. The laws of the complete histories differ (Table 7).

Table 7: Two readings of a pair on one cell: the probability of a positive first answer, equal for both instruments, and the total variation between the laws of the two public histories.

Run	First answer positive	Total variation
contrast-memory law, $\kappa = 8$	0.992329	0.440513
exchange alone, $\kappa = 0$	0.647174	0.196148
contrast of the reversed sign, $\kappa = -8$	0.987884	0.249676
$\kappa = 8$, reports of the first seam mismatch or are not available	0.747346	0.260336
$\kappa = 8$, every predicate flipped in agreement	1	0.446650

These numbers take a reading as instant, with the diagonal part of the law acting during its window. With the exchange acting through both read windows of length $w = 1/16384$, the law of one experiment differs from this calculation by at most $24w$ in total variation, and a comparison of two experiments by at most $48w$. One event is certified in exact arithmetic with every such error counted: a positive first answer followed by a final record on the emitting seam has the probabilities 0.0182640 under the presence reading and 0.1029264 under the reader with seam records, and the total variation between the two public histories exceeds 0.0817.

The pair of Section 8 is one object spread over seams. An instrument that asks where it is places it, and later records show the difference. The event of the binding theorem, a pair on some seam, has an instrument of its own that measures that event and nothing more.

10 Bound objects

10.1 Costs

An object is a set of N unit loads. Its cost in C is

$$C = 6N - 2\,(\text{number of agreed seams}). \tag{18}$$

Table 8 lists, for each N on one cell, the objects with the most agreed seams. The first three belong to the three kinds of centre of the cell, its 12 ports, 30 seams and 20 faces, the 62 points in which the rotation axes meet the cell. The last column is the least amount by which the cheapest object of N loads undercuts two cheapest objects of k and $N - k$ loads. It is at least 2 throughout: at strong coupling no object gains by falling apart. No object is protected against merging. The cost per load falls from 6 to 1, and the whole cell costs 12, the same as one face. What protects an object in this law is the conserved number of loads and the gap of the law. The law holds no winding and no flux.

10.2 Bindings at finite coupling

At finite κ the lowest level E_N of N unit loads on one cell is a single state, invariant under the rotations of the cell. Table 9 gives its binding $B_N = 6\kappa N - E_N$ against N free loads in the uniform mode. These are levels of the law in units of the exchange and audit on the spectrum. The bindings follow the numbers of agreed seams 1, 3, 5, 7, 10 of Table 8: at $\kappa = 8$ the lowest levels hold 0.986, 2.985, 4.986, 6.989 and 9.995 agreed seams on average. The binding of the pair lies between 0 and one sixth of the cost 12κ of its two loads. In the lowest level of the pair on one cell a pair on a seam has probability 0.60, 0.72, 0.86, 0.99 and 1.00 at $\kappa = 0.5$, 1, 2, 8 and 128.

Table 8: The cheapest objects of N unit loads on one cell. The last column is the least gain over the splits into two parts.

N	Shape	How many	Agreed seams	Cost	Gain against a split
1	one port	12	0	6	
2	one seam	30	1	10	2
3	one face	20	3	12	4
4	two faces on a seam	30	5	14	4
5	three faces round a port	60	7	16	4
6	a port and its five neighbours	12	10	16	6
7	that cap and one more port	60	12	18	4
8	the cell less a face and a port	30	15	18	6
9	the cell less a face	20	18	18	6
10	the cell less a seam	30	21	18	6
11	the cell less a port	12	25	16	8
12	the whole cell	1	30	12	10

Table 9: Binding B_N of the lowest level of N unit loads on one cell against N free loads, in units of the exchange.

κ	B_2	B_3	B_4	B_5	B_6
0.5	0.526	1.556	3.049	4.965	7.277
1	1.185	3.546	6.864	10.905	15.816
2	2.779	8.554	15.927	24.206	35.305
8	14.236	43.728	75.212	107.678	155.076
128	254.016	763.513	1275.013	1787.511	2555.005

10.3 Within a cell and between cells

Within a cell a pair passes from its seam to a linked seam by one exchange: one of its loads moves to the third port of a face on the seam (Figure 1). This is the term $-\frac{1}{2}M$ of the proof of Theorem 5.2, of first order in the exchange. External seams are linked to no seam. A pair passes from an internal seam to the external seam beside it through one separated configuration, which costs 2κ more; the element of K^2 between the two pairs is $\frac{1}{4}$, so the amplitude of this passage is $1/(8\kappa)$. The cells of the federation of 20 are of two kinds, which differ in the places of their three external ports, and the lowest level of a pair inside a cell differs between the two kinds by 0.016, 0.024, 0.032, 0.037 and 0.040 at $\kappa = 1$, 2, 4, 8 and 16. The amplitude $1/(8\kappa)$ is 0.125, 0.063, 0.031, 0.016 and 0.008 at these couplings, so from $\kappa = 4$ upward the difference between the cells exceeds it. As an audit on the state, the participation number of the cells in the load density of the runs of Table 5 on 20 cells is 1.02 at parameter value 32 for $\kappa = 8$, 1.00 for $\kappa = 128$, and 1.05 at 1024 for $\kappa = 8$: the pair is read on the seams of one cell for long. At $\kappa = 2$ a pair started on one seam is read as a pair on a seam with probability 0.870, 0.872 and 0.875 at 8, 128 and 2048, while that number grows from 1.07 to 2.34.

11 The port response

The gauge paper of the theory derives the compact Lie type $\mathfrak{u}(1) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(3)$ from a complete response of the twelve ports: twelve independent generators, closed under commutators, on which the rotations of the cell act through the response itself [4]. This section reports what the contrast-memory law gives for that premise on one cell. Everything in it is derivation and audit on the operators of the law. No observer reads these matrices, and the statement on records is the number of directions that change at least one record probability, which leaves out a phase common to all states. Section 13 reports the port currents that Interval Echo selects by the rotations of the cell, which give twelve closed directions on one load and on the bound pair.

11.1 Services and the response space

A port service is a weak operation at one port that lasts long against the inverse spacings of the levels of the law. It acts through the part of its generator F that is diagonal in the levels. The response space is the sum of the two triplets that are odd under the antipodal map: for one load the slow level 3 and the fast level $3'$ of (9), with projectors P_3 and $P_{3'}$, where the service acts as $\Phi = -i\,(P_3 F P_3 + P_{3'} F P_{3'})$. The plain services of the law are the load n_p of a port, the exchange $\frac{1}{2}(I - \mathrm{SWAP}_e)$ of a seam with its own strength, and the twist J_p of a port, the circulation round its five neighbours.

Table 10: Dimension of the real Lie algebra generated by port services on the two odd triplets of one cell, for one load and for the two lowest odd triplets of the bound pair at $\kappa = 8$.

Services	One load	Bound pair
the load of each port	17	17
the exchange of each seam	17	
the twist of each port	6	6
load and twist as one service of each port	17	17
loads, twists and exchanges together	18	
load weighted by the energy below the top level, with the twist (19)	12	17
load weighted from the pair's own $3'$ level, with the twist		12

11.2 Seventeen under the plain services

The plain services give seventeen (Table 10): two copies of $\mathfrak{su}(3)$, one on each triplet, and one common number. Sixteen of the seventeen directions change a record under the port loads. This is the count and the algebra that the gauge paper reports for a response built from exchanges [4].

The reason is a lock between the two triplets. Every operator at a port with rational entries acts on the fast triplet as the conjugate under $\sqrt{5} \mapsto -\sqrt{5}$ of its action on the slow triplet. Five symmetric operators at a port were computed: the load of the port, the load of its ring of neighbours, the exchange between port and ring, and the first and second exchange along the ring. Each has entries $x + y\sqrt{5}$ on the slow triplet and $x - y\sqrt{5}$ on the fast one with rational x and y. A service whose symmetric part is no number on one triplet is therefore no number on the other, and the

closure contains $\mathfrak{su}(3)$ twice. Twelve needs a symmetric part that is a number on one triplet and no number on the other, which no service with rational entries has.

11.3 Twelve with one declared service

The declared service weights the load of a port by the energy below the top level of one load and adds the twist:

$$F_p = \tfrac{1}{2}\{n_p,\ E_{\mathrm{top}} - \tfrac{1}{2}L\} + J_p, \qquad E_{\mathrm{top}} = \tfrac{1}{2}(5+\sqrt{5}). \tag{19}$$

Its twelve generators are independent and close under commutators in dimension twelve: $\mathfrak{u}(3)$ on the slow triplet and $\mathfrak{so}(3)$ on the fast one, the Lie type $\mathfrak{u}(1) \oplus \mathfrak{su}(3) \oplus \mathfrak{su}(2)$. The trace form is positive, with eigenvalues between 0.5 and 7.24. All 66 brackets lie in the span, with largest residual 9×10^{-15}. With Φ_p the response of F_p and U_g the action of a rotation g on the response space, the 720 identities $U_g \Phi_p U_g^{-1} = \Phi_{g(p)}$ for the 60 rotations hold with largest residual 2×10^{-15}, and the generators of the rotations lie in the span with largest residual 2×10^{-14}. The twelve services sum to the number $\sqrt{5}$ on the slow triplet and to 0 on the fast one. With the load unweighted the same construction gives seventeen. The weighting, with its irrational coefficient tied to a level of the law, and the twist are supplied; the events of the law provide neither.

11.4 The bound pair and two exact couplings

On the two lowest odd triplets of the bound pair the plain services give seventeen at $\kappa = 8$, 2 and $(\sqrt{5}-1)/2$. Twelve comes out when the load is weighted from the pair's own $3'$ level, and seventeen when it is weighted from the top of the bound levels. The reference that needs no knowledge of the target for one load needs it for the pair.

Two couplings have exact features. No selection of the coupling is claimed from them. At $\kappa = 1$ the four triplet levels of the pair, counted from 12κ, are $2-\sqrt{5}$, $(7-\sqrt{5})/2$, $2+\sqrt{5}$ and $(7+\sqrt{5})/2$. On the second and the fourth the load of every port is the number $1/6$ and the twist vanishes, to 10^{-14}, and the plain services on the two lowest odd triplets give nine, all of $\mathfrak{u}(3)$ on the slow triplet and a number on the fast one. At $\kappa = (\sqrt{5}-1)/2$ a pair on a seam costs exactly what two free loads in the slow level cost, $5-\sqrt{5}$ above 12κ, and the lowest level of the pair, written as a symmetric function of two ports, has no component on the slow level, to 10^{-16}.

12 The unrecovered collar

12.1 Conservation, neutrality and levels

The collar of a seam is its private memory. Recovery is the return of the memory to blank, and a seam whose memory is not blank is an unrecovered collar. Theorem 3.2 describes it completely. The local term maps the blank code into itself and the record sector into itself, so the projector on "memory blank" commutes with the law: the recovery state of every seam is conserved. In the complete law on one cell with one memory kept, the largest element between "blank" and "not blank" is 0 with 0, 1 and 2 loads. From a recovered start the law never makes a collar unrecovered, and an unrecovered collar is supplied. On the records the squared contrast d^2 commutes with the projector on the written code and with M_κ, and so with the local term. Every seam therefore carries a label that no event changes: its memory is blank, or it holds a record of size $|d| = 0$, 1 or 2.

The collar holds no load. With no load anywhere the local term acts on the ten records of a seam as M_κ, with the levels

$$0, \qquad \kappa, \qquad \kappa+1, \qquad 4\kappa, \qquad 4\kappa+1,$$

each twice: 0 for the records of zero contrast, κ and $\kappa+1$ for the symmetric and the antisymmetric record of contrast ±1, and 4κ and $4\kappa+1$ for contrast ±2. At $\kappa = 8$ these are 0, 8, 9, 32, 33, and at $\kappa = 0$ they are 0 and 1. A record of nonzero contrast on a seam whose ports hold no contrast lies at least κ above the recovered seam. The symmetric and the antisymmetric record of one collar are one exchange unit apart for every κ.

The collar acts on loads. On records that the loads of the seam cannot have written, the seam term is M_κ alone and exchanges nothing, and on a record that they can have written it vanishes. With one load on the cell, every level of the law on the carrier without that seam, raised by κ, is a level of the law with the collar. A load on a port of the seam lies lower than a load elsewhere. Beside a record of zero contrast it has one seam fewer to disagree with: at $\kappa = 8$ the lowest such level is 41.81, below the lowest recovered level 48. Beside a record of contrast ±1 a load elsewhere starts at 56.38, the level of a free load raised by the level κ of the record. Below it lie two levels at 41.94, on which the record is the one that a write of the loads produces and the seam term vanishes, and six levels between 49.81 and 50.94, on which the record does not fit the load. On all eight the load sits on a port of the seam with probability above 0.97. The load binds to the collar, by about 2κ where the record fits and by about κ where it does not. With two loads the lowest levels with and without a collar are 74.48 and 81.76.

12.2 The level and the coupling as readings

The level of a collar is read from records. A local fuel at level 0 is converted into a contrast record of the collar by a pulse of area $\pi/4$ with coupling 64 and the complete law acting. After q cycles of a protocol, $q = 0, \dots, 32$, a second pulse with one of four phase settings 0, $\pi/2$, π, $3\pi/2$ reads “fuel present”, true or false. With the four probabilities $p_\theta(q)$ the observer forms $Z(q) = \frac{1}{4}[p_0 - p_\pi - i\,(p_{\pi/2} - p_{3\pi/2})]$ and the phase of $Z(q)/Z(0)$, whose slope in q is the phase rate per cycle. The observer is not given the length of a cycle. It reads the rates r_s and r_a of the symmetric and of the antisymmetric record of the same collar. Their difference is the length of the cycle in units of the exchange, and $r_s/(r_a - r_s)$ is the coupling (Table 11). The two records of one collar are a clock that the observer reads from its own records.

Table 11: The level of an unrecovered collar as a reading, on one cell with a cycle of 1/16 exchange units.

Run	Rates per cycle	Cycle read	Coupling read
no load, probability law	0.500000, 0.562500	0.062500	8.00000
no load, 10^6 sampled trials per setting	0.499998, 0.562485	0.062486	8.0017
no load, law without the agreement energy	0.000000, 0.062500	0.062500	0.00000
one load, started off the seam	0.502209, 0.564749	0.062540	8.030

A load on the cell shifts the reading, as the last line shows, and with a load started on a port of the seam the reading of the coupling is lost.

Testing a reference for the coupling. On one cell of the twenty-cell sphere, place two units at independent, uniformly drawn port addresses. On a chosen seam a fair controller either accepts the input or accepts only its even component under exchange. Each rejected preparation gives a public outcome. After the native write, four contacts empty the two read ports into exterior spares. The accepted memory state, resolved at a fixed coin value, assigns probabilities $(204, 40, 20, 2, 1)/267$ to the five levels in the order above. A single positive-temperature Gibbs reference would imply

$$\beta = \log(p_{1+}/p_{1-}), \qquad \kappa = \beta^{-1} \log(p_0/p_{1+}), \qquad p_{2+} = p_0(p_{1+}/p_0)^4.$$

Here $\beta = \log 2$ in exchange units. The first three populations imply $\kappa = \log_2(204/40)$. The held even population at magnitude two is $132651/20000$ times the prediction. The parity ratio itself is two at both magnitudes. Thus this phase-sensitive reference resolves a parity imbalance at both magnitudes while failing the common-temperature identification of the contrast energies. Its construction uses supplied local preparation and isolated contacts.

More generally, sample the two emission addresses from any joint law invariant under the rotations of the twelve-port cell and retain the same seam and parity instrument. Conditional on the first address, the return probability d and the probability e at each of its five neighbors obey $d + 5e \le 1$. The even-mode weights are proportional to $(8 + 2d + 4e,\ 2(1 - d - e),\ d)$, giving, for $R = p_0/p_{1+} \ge 11/2$,

$$D = \frac{p_{2+}p_0^3}{p_{1+}^4} \ \ge \ \frac{(2R - 11)R^3}{20}.$$

A Gibbs reference would require $D = 1$ and $\beta = \log 2$, which bounds the inferred $\kappa = \log_2 R$ by 2.47463 and excludes 8 for this family.

Coherence cannot produce the thermal weights for $\kappa = 8$ at this temperature either. Choose one of the thirty internal seams uniformly, independently of the two-load state, or fix the seam in a rotation-invariant state. A double, an adjacent pair and a nonadjacent pair have nonzero contrast on five, eight and ten seams respectively. The contrast test is diagonal in the load basis, so its probability x is at least $1/6$ for every pure or mixed state under this sampling. The fair bypass accepts at least $x/2$ with nonzero contrast, while the zero-contrast weight is at most $1 - x$. The nonzero fraction among accepted records is therefore at least $x/(2 - x) \ge 1/11$. The five Gibbs weights at $\beta = \log 2$, $\kappa = 8$ assign only 0.5825% to nonzero contrast.

A standard from neighboring repair records. Two independently supplied qutrits each start uniformly and receive idle, downward or upward offers with probabilities $1/2, 1/4, 1/4$. A saturated offer rejects that history. Finite patch buffers retain signed differences from surviving pairs, with fresh loads from supplied reservoirs. Let w_j be the probability of the individual signed word $+j$. Require the same signed word from all m other seams at the endpoints of the read seam. An independent pair from the same source admits the exchange-even outcome on a zero word and the exchange-odd outcome on one specified signed unit. The exact even weights at magnitudes zero and one are proportional to $w_0^m w_0$ and $w_1^m w_0$, and the odd weight at magnitude one is proportional to $w_1^m w_1$. Taking negative logarithms of the mode weights, relative to the zero mode and in units of the exchange splitting, therefore gives $\kappa = m$ exactly.

A protected complete cell has $m = 8$ on every seam. With symmetric offers, 17 is the first horizon whose magnitude-two phase at exposure π agrees with the quadratic prediction within 10^{-11}. The error is 2.41×10^{-12}. This level tests the contrast profile and was used to choose the horizon. For state-independent upward and downward offer probabilities $p, q > 0$, the positive stationary load weights conditional on survival are proportional to $(q, \sqrt{2pq}, p)$. At either parity the magnitude-two population is $[(p + q)^2/(4pq)]^m$ times its quadratic prediction. This factor equals

one exactly when $p = q$, independently of the idle probability, and is $(9/8)^8$ for $p = 2q$ in the protected cell. Keeping the symmetric source while admitting odd parity by an independent fair coin gives $\kappa \simeq 4$. The common comparator and the use of the protected cell as a standard for every seam are design choices made after the earlier pair experiments. Counting neighbors on the deployed sphere instead gives $m = 8, 9, 10$ according to the seam address.

An exact preparation follows by requiring the initial weights (a, b, a) to be stationary under one surviving offer, which gives $r(2 + r) = 2 + 2r$ and $r = b/a = \sqrt{2}$. An isolated single-load exchange pulse of the prescribed angle $2 \arcsin \sqrt{\sqrt{2} - 1}$ selects the middle load with probability $\sqrt{2} - 1$, and a fair choice of the two remaining loads completes the initial distribution. At every finite offer depth this preparation gives $w_1/w_0 = 1/\sqrt{2}$ and $w_2/w_0 = 1/4$, so the eight-witness comparator yields energies 8 and 32 exactly and, with the specified coin, spectator-load and blank-memory actions, the full 99-state seam generator.

12.3 Relation to the dark-sector carrier of the theory

The dark-sector paper of the theory takes a recovery defect on an overlap collar as the carrier of its non-luminous source: the defect carries the charge of the collar's generator, has no electromagnetic readout, and its stress vanishes at full recovery [8]. The unrecovered collar of A-contrast-memory-spherical-v1 has the corresponding properties inside the model. It is conserved, it holds no load, its generator vanishes on the recovered seam, and its level above the recovered seam is set by κ. This paper establishes no physical reading of the collar as that carrier.

An isolated seam gives a material probe of this distinction. With opposite coin amplitudes on a contrast-two record, the native write has zero overlap with that memory for each of the nine two-port load inputs. The two reversal parities have energies 32 and 33 at $\kappa = 8$. At exposure $\pi/32$ a balanced contact interferometer returns bright with probability one for either sign of material probe on the reversal-even, coin-odd state. Choosing even coin parity at the same empty-seam energy gives probability zero for either sign. These two null material responses identify a finite-energy seam memory. Its fixed address supplies no inertial mass, and an independent electromagnetic identification is required for physical neutrality.

A second neutral preparation uses a mobile excess and vacancy over a unit-filled 20-cell host with every seam free. An energy filter admits the lowest 24 lines of the pair's bound band. Its bandwidth is 1.23782×10^{-5} in exchange units. The full ordered-sector Hamiltonian gives at least 99.36999% probability for an adjacent regular excess and gateway vacancy at any time on this subspace. At 10^6 exchange times, three filtered launches place the adjacent pair at least two cells from its source with probabilities 6.8287%, 12.6035% and 18.8158%. The time-zero probabilities are below 1.6×10^{-12}. Twelve of the twenty cells carry nearly all the low-cluster weight. The experiment measures transport across owners. An inertial dispersion and a physical electromagnetic charge measurement on that mobile branch remain open. The spectrum and all terminal counts are archived in `model/experiments/material_inertia/pair_spectrum/`.

13 The main candidate law: Interval Echo

Interval Echo is the main candidate law of the model. Its elementary interaction is the contrast-memory seam at $\kappa = 8$: a port holds 0, 1 or 2 loads, every seam carries the private memory of eleven states, the write and its reverse form the self-inverse echo on all 99 states of a seam (Lemma 3.1), and on blank memory the seam generator is $\frac{1}{2}(I - \mathrm{SWAP}) + 8(n_a - n_b)^2$ (Theorem 3.2). The theorems proved above for this seam cover $\kappa = 8$, and they and the results computed at $\kappa = 8$

hold for Interval Echo in its free sector: the echo, the binding of a pair, the rank-three band of one load, the readers and the two readings of a pair. By (9) the law of one load is $\frac{1}{2}L + 48$, and its slow band carries all three directions of the Gram matrix of the port directions. With the clock idle, an observer patch that receives only anonymous calibration probabilities of one load infers the thirty seams and a Gram matrix of rank three and predicts 288 held probabilities to below 10^{-10}, while the best Gram matrix of rank two has a relative error of 0.577. What Interval Echo adds is a clock that a finite observer carries. The measurements below are computed from sampled material histories on the federations [1].

A wave prediction from a collar. The sampled rates of the empty-port collar in Table 11 give $\widehat{\kappa} = 8.001733$ in exchange units. A separate experiment prepares one unit in a period-four channel of written memories on the 320-cell sphere and releases it to free evolution. For wave number q the channel's four-site generator predicts a frequency relative to the middle-band midpoint of $\frac{1}{2}|\widehat{\kappa}|\sqrt{3 - \sqrt{6 + 2\cos q}}$. With the scale fixed by the collar, the largest discrepancy across the rest line, four held wave numbers and one short-wavelength challenge is 0.03514%. This checks the independently read scale of the prepared one-dimensional channel. The neighboring-record standard above supplies a possible microscopic origin for that scale.

The patch clock. A travelling observer patch carries a hand of five positions, an elapsed count of nine bits and an interval memory of three bits, which holds how many of its own repair opportunities elapsed since its last verified handoff. With this memory the patch owns 12 consecutive cells at a checkpoint and 13 during a transfer. A departure probe and an arrival probe verify each handoff, and the patch publishes the result on a signed seam and reads it back before it uses a fresh repair outcome. A handoff closes an interval. A positive closing mark then passes a bounded first-return walk, with fair signed steps, uniform binary addresses and at most twelve trials, which suppresses or releases it. Only the confirmed movement, the patch registers and local fair draws enter this decision; the clock reads no speed and no target table. At reunion both owners read the same digits on one common seam. The closing filter is calibrated on the reunion ratio of a supplied reciprocal relay journey. The guide, the custody transfers, the gate selection, the relative cadence and the read windows are supplied services, and the hand size, the widths of the two counts and the walk budget are designed.

Moving clocks run slow. The observers time the journeys themselves. A home patch takes half the round trip of a reciprocal light pulse as the one-way light time d. The receipt of a pulse that the traveller's arrival at the far end triggers, less d, gives the outward duration T_1, the rest of the trip is the return duration T_2, and d/T_1 and d/T_2 are the leg speeds in units of the signal speed. The relativistic forecast for the ratio of the traveller's elapsed count to the home count at reunion is the ratio of the traveller's proper time to the home time,

$$\frac{\sqrt{T_1^2 - d^2} + \sqrt{T_2^2 - d^2}}{T_1 + T_2}. \tag{20}$$

For journeys with leg speeds $(1/2, 1/6)$, $(1/6, 1/2)$ and $(1/4, 1/4)$ the measured ratio minus the forecast is -0.00105 ± 0.00068, $+0.00019 \pm 0.00052$ and $+0.00087 \pm 0.00064$, each from 524,288 paired journeys with one standard error. The time dilation itself is 3.2 to 4.4 percent, a ratio of about 0.956 on the fast journeys, so it is resolved at dozens of standard errors, and the clock follows the forecast within about one part in a thousand. Embeddings in federations of 1,280 and 5,120 cells give the same material sequence of handoffs.

Capacity. On handoffs of eight services the interval memory of three bits overflows and passes the closing mark. Giving three fields of the clock one more bit each, at the cost of one more cell at each checkpoint, extends the working range from 7 to 15 services and restores the forecast: the four-bit clock reads 0.99005 ± 0.00193 against 0.99216. On alternating handoffs of 2 and 6 services both widths read 0.95535 ± 0.00190, and on alternating handoffs of 6 and 2 services 0.95650 ± 0.00189, against 0.95602. Each comparison uses 8,192 paired preparations.

Overlapping observers. On the federation of 320 cells, 320 single-cell observers and 960 overlapping three-cell observers read 23 signed repair records and track 34 conserved mobile doubles through 2,399 transfers. Every shared checkpoint reading agrees. The preparation probabilities, the matching schedule, the tag protection and the gate windows belong to the experiment.

Coherent repair under local precedence. During a protected sweep, each seam applies its strict repair instrument once all higher-priority seams at its endpoints have completed. The outcome and completion bit stay on that seam. Incomparable visits act on disjoint material registers, so exchanging their order preserves every branch operator. Every fair schedule therefore gives the same instrument for the supplied precedence. On a four-seam path, five legal orders agreed on all 3,888 load and occupancy words. In one common outcome record, two local unwrite probes distinguished opposite prepared phases by joint even parity with probability zero or one. Probing a seam before its predecessors completed gave one half for either phase. The claim applies to the protected repair sweep with free propagation and clock service suspended.

Port currents. On one cell the port currents selected by the rotations of the cell give twelve independent responses, on one load and on the bound pair, that close under all 66 pairwise commutators. The 720 conjugations of a current by a rotation return the current of the image port with residuals below 10^{-11}. A signed pulse at one port changes the distribution of the bound pair's seam readings by up to 0.11 in total variation. Prepared overlap transport generates all 60 proper rechartings on 20 and 80 cells, with a matching current implementer of unit gain. These supply the spanning set, the commutator closure and the proper rotations by overlap histories that the premises of the finite gauge theorem ask for [4]. The premises also ask for a complete menu of the physical controls, which these measurements do not establish: a bare port-load response added to the menu enlarges the visible closure to 17 directions. The oriented cell, the preparation, the protected bands and the unit gain of the service are supplied.

Least-informative assignment. The state-selection principle of the theory adopts, on a declared feasible family, reference, observer cover and positive weights, the information projection that minimizes the weighted relative entropy to the reference [2]. On the federation of 20 cells two overlapping two-cell observers, with a shared fair record and a private mismatch probability $1/4$, realize the unique weighted projection to a uniform reference in both orders: each owner reads $(3/8, 1/8, 1/8, 3/8)$.

Scope. Of the premises of the finite theorems, Interval Echo supplies observer clocks, the three directions and the time dilation of moving clocks from records that the observers themselves make. A comparison of clocks on held clock constructions and directions, the transport of the three directions by a moving observer's full records, a jointly authenticated causal order with its count-volume limit, one common measurement of current response and stress on one bound excitation, the completeness of the control menu, the confluence of all accepted repairs on the federation and

an identification of physical particles by the law as specified lie outside what these measurements establish.

13.1 Matter-wave dispersion on a prepared route

On the 320-cell federation an ordinary load can travel through 194 successive cells by reversing written seam coins. Preparing four ports in each cell gives a route of 776 ports. All terms act freely after release. A stationary local three-port reference resolves the frequency of one moving band. With the speed and gap fixed independently, four held frequencies differ from the long-wave Klein–Gordon relation $\omega^2 = m^2 + c^2k^2$ by 0.10% to 1.61%. At the microscopic challenge wave number the discrepancy reaches 18.79%.

The same ordered 194 cells can be connected through the same entry and exit gateway ports using a shortest route *within* each cell. The carrier, intercell links, free law and degree references are the same in both runs. Changing only the written internal route shortens the path to 578 ports and places 62 levels in the native energy interval $(-5, -3)$, which contains none on the four-port route. The mean weight of those 62 modes in the middle half of the path is 48.28%. Both routes transmit a sharp load to the next cell. The observed gap and dispersion describe the prepared medium. A route-independent matter law or a mass for the signed excitation requires further evidence.

13.2 Signed loads coupled to a written field

A different interaction on the same ports tests whether a signed load can source a disturbance outside its cell. Take one load per port as the neutral reference. An excess has charge $q = +1$ and a vacancy has $q = -1$ in the register's units. A written seam has two head orientations. A material hop reverses its head, and a circulating face can reverse all its heads at fixed material loads. On the tested written sector the experimental law assigns each allowed transition $i \leftrightarrow j$ the term $a(|i\rangle - |j\rangle)(\langle i| - \langle j|)$, with $a = 4$ for a material hop and $a = 1/2$ for a face reversal. The diagonal terms and face process specify an extension of Interval Echo. The freely persistent two-load pair in Section 8 evolves under the original interaction.

Transport and field response. Place a load and its remote compensator on a prepared guide. After a half-transfer across a cell boundary the two regional load records are $(q, 0)$ or $(0, q)$ with equal probabilities. Summing the changes in oriented boundary heads gives the same charge on each outcome for both signs on nine guides. A control that hops the material while holding its head fixed disagrees on half the outcomes. The load and head alphabets supply the integer step. In a separate release on a coherent field ground the source material stays in the launch cell. A distant material probe changes its occupation when the sign of the source changes. On the 320-cell federation, the half-difference of the two probabilities exceeds 0.05/2061 at a receiver 22 cell steps away. Cutting the emitter face erases the shift. This threshold defines the reported reach.

Two further measurements constrain the electromagnetic interpretation. With fixed charges the equal-amplitude ground on each connected component of face configurations has zero face-flip energy. Beyond a shared contact the remaining projected ground energy is a sum of two one-port costs. Subtracting those costs gives zero static interaction. On ordered guides of six and twelve faces the probe traces give the angular frequency $\omega(k) = 1 - \cos k$ in native time units. On the longer guide doubling the first wave number multiplies the frequency by 3.942. This branch is quadratic at long wavelength. The experiment concerns an ordered guide and supplies no bulk transverse light dispersion.

Persistence and conversion. On all 630 intercell seams of the 20-, 80- and 320-cell federations one can prepare an adjacent excess and vacancy with oriented surrounding heads that block every hop and face reversal. Both signs were tested at each seam. This net-neutral dipole is an exact stationary state at the energy of the neutral preparation. Reversing one neighbouring head can enable separation. A sharp isolated load with a remote compensator has at least one outgoing hop. The free generator separately conserves the counts of both signs, which excludes spontaneous conversion of such a pair to neutral material. A separately applied conversion pulse can launch a neutral signal. Its detection gives no photon multiplicity.

Formation energy and the clock. With the one-load degree reference, the bottom of the mobile one-end band on the repeating three-port medium is $10 - 2\sqrt{17} = 1.753789$ in generator units above the neutral medium. The separated-pair threshold is twice this value. A supplied pair source resolves a line at 3.509882 on a 48-cell ring. The central-port occupation of a free zero-momentum preparation oscillates at angular frequency 16.492423, and every nonzero translation-invariant internal one-end beat on this medium is at least 4. A different written preparation on the same 320-cell federation admits a trial pair at least 55 port links apart with combined energy at most 0.126165 above the neutral ground. The band edge therefore prices the end in a particular medium.

For comparison, the measured cyclotron ratio of bare carbon to an electron, two ratios of doubly ionized rubidium to hydrocarbon ions, and rubidium photon recoil determine the laboratory electron rest energy without using its tabulated mass [14, 15, 16]. With bounded ionization and chemical corrections the reconstructed value rounds to 510.999 keV. These observations supply an independent laboratory comparison. An energy in keV for the simulated end requires a preparation-independent rest energy and a transition in the same model identified with a measured physical frequency.

14 A second candidate law: the scalar channel and its cage

The same carrier holds the second candidate law, with other registers. Its results stand beside those of the contrast-memory seam and of Interval Echo and are not added to them. Its executable package and the cage experiment run it on one cell [1].

14.1 The law

A wedge is a pair of seams at one port. A port with five seams has ten wedges and a cell has 120. Every seam and every wedge carries one mode that holds one object or none. Let C be the signed seam-by-wedge incidence, with $+1$ on the first and -1 on the second seam of a wedge. The law of one object and the law of many are

$$D = \begin{pmatrix} m & c\,C \\ c\,C^{\top} & -m \end{pmatrix}, \qquad H_0 = \sum_{i,j} D_{ij}\, c_i^{\dagger} c_j, \qquad m = 1, \quad c = \tfrac{1}{\sqrt{2}}. \tag{21}$$

An object on a seam and an object in transit at a port sit at opposite levels, so the mixed terms of D^2 cancel: $D^2 = m^2 + c^2\,\mathrm{diag}(CC^{\top}, C^{\top}C)$. Every frequency ω of the law obeys $\omega^2 = m^2 + \lambda$ with λ a level of $K = c^2 CC^{\top}$, half the Laplacian of the graph of seams. On one cell K has the levels 0, $(5-\sqrt{5})/2$ three times, 3 five times, $(5+\sqrt{5})/2$ three times and 5 eighteen times, and the first four are the levels of $L/2$ of Section 4. The reference state has every level of D below zero filled, 120 objects on one cell, and one object more costs at least m. On ordinary registers the 150 modes of

a cell, with one fuel and one parity reference at every port, are written on 432 two-state registers with 260 loop conditions. The reference is prepared by loop measurements whose signs are kept, a fill of the transits and twelve local ramps, and lies 0.0058 from the filled lower branch in operator norm.

14.2 The cage

Let Q be the port-by-seam incidence without signs and d_a the number of seams at the port a.

Lemma 14.1 (Incidence identity)**.** *On every graph of ports and seams, $CC^{\top} = \operatorname{diag}(d_a + d_b) - Q^{\top}Q$.*

Proof. The entry of $CC^{\top}$ at two seams sums, over the wedges that contain both, the product of their signs. A seam ab lies in $(d_a - 1) + (d_b - 1)$ wedges. Two different seams share at most one port, and two seams that share a port lie in exactly one wedge, with opposite signs. $Q^{\top}Q$ has 2 on the diagonal and 1 for two seams that share a port. □

Proposition 14.2 (The pair of a ring)**.** *Let four ports a, b, c, d with five seams each form a ring, that is, let ab, bc, cd and da be seams. Put*

$$e = \tfrac{1}{2}\big(|ab\rangle - |bc\rangle + |cd\rangle - |da\rangle\big), \qquad e' = C^{\top}e/\sqrt{10}.$$

Then $Qe = 0$, $CC^{\top}e = 10\,e$, and

$$D\,(e,\ e') = (e,\ e')\begin{pmatrix} m & \sigma \\ \sigma & -m \end{pmatrix}, \qquad \sigma = \sqrt{10}\,c, \qquad E = \sqrt{m^2 + \sigma^2}. \tag{22}$$

The pair lies on 4 seams and 28 wedges at the four ports of the ring.

Proof. At every port of the ring a seam with $+$ meets a seam with $-$, so $Qe = 0$, and Lemma 14.1 gives $CC^{\top}e = 10\,e$. Hence $\|C^{\top}e\|^2 = 10$, $D\,(e, 0) = m\,(e, 0) + \sigma\,(0, e')$ and $D\,(0, e') = \sigma\,(e, 0) - m\,(0, e')$. □

On the cell $\sigma = \sqrt{5}$ and $E = \sqrt{6}$. The law moves the profile e to e' and back and to no other place: the signs of the hop make a cage (Figure 5). The profiles with $Qe = 0$ form a space of dimension 18, the level 5 of K. The cell has 30 rings, their profiles span that space, and no profile with $Qe = 0$ lives on fewer than four seams. On the federations of 20, 80 and 320 cells 92, 312 and 1,272 rings have five seams at each of their ports, each checked in integers. A ring through a port with an external seam is no cage: an object placed on its profile keeps 0.07 of its weight in the pair.

Let $\phi_{\pm}$ be the modes of the pair at $\pm E$ and let $a_{\pm}$ remove an object from them. In the reference ϕ_{-} holds an object and ϕ_{+} is empty.

Theorem 14.3 (The excitation of a cage)**.** *Let $X^{\dagger} = a_{+}^{\dagger}a_{-}$ and $P_X = n_{+}(1 - n_{-})$. Then*

$$[H_0, X^{\dagger}] = 2E\,X^{\dagger}, \qquad [H_0, P_X] = 0. \tag{23}$$

For every state of the carrier the probability of the excitation is the same at every reading. The expectation of every operator built from $a_{\pm}$ and their adjoints with as many creators as annihilators is a constant plus one term at the frequency $2E$.

Proof. For a law of the form (21), $[H_0, a^{\dagger}(\phi)] = a^{\dagger}(D\phi)$, so $[H_0, a_{+}^{\dagger}] = E\,a_{+}^{\dagger}$ and $[H_0, a_{-}] = E\,a_{-}$. This gives the first identity, and n_{+} and n_{-} commute with H_0. An operator of the stated kind is a combination of 1, n_{+}, n_{-}, $n_{+}n_{-}$, $X^{\dagger}$ and X and of their products with the commuting numbers; the first four are constant under the law, and $X^{\dagger}$ and X carry e^{2iEt} and e^{-2iEt}. □

The excitation leaves the number of objects as it is and lies $2E$ above the reference. For the object of the pair placed on e the occupation of the profile is $1-(\sigma^2/E^2)\sin^2(Et)$: it starts from rest and swings at the one line $2E$, the difference of the two levels.

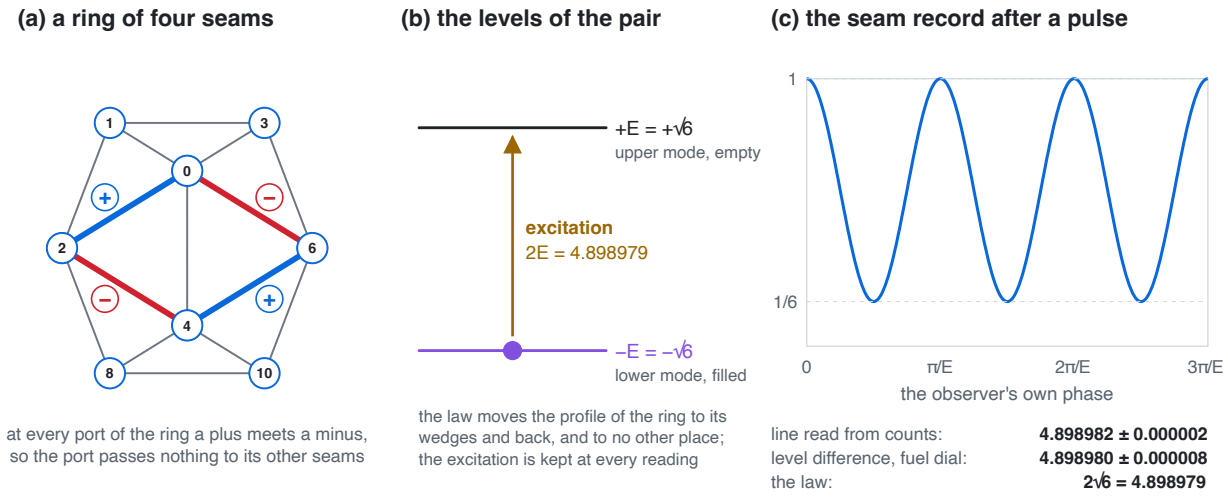


Figure 5: The cage of the scalar channel. (a) A ring of four seams on one cell with the signs $+,-,+,-$; at every port of the ring a plus meets a minus. (b) The two levels $\pm E$ of the pair of the ring and the excitation that lifts the object of the lower mode by $2E$. (c) The occupation of the profile after the coherent pulse, $1-(\sigma^2/E^2)\sin^2(Et)$, against the observer's own phase, with the line and the level difference read from counts (Table 12).

14.3 What the observer of a cage reads

The observer of a cage holds two operators on the profile: the number $n(e)$ as a potential, and a converter between e and a local fuel at level zero. On the registers $n(e)=Q_e/8$ is a sum of 17 words with $Q_e^2=8Q_e$, and the converter is a sum of 8 words. Both commute with all 260 loop conditions and touch the registers of the four ports of the ring. The potential $-2E\,n(e)$ for the time $\pi/(2\sqrt{2E(E-m)})$ moves the object of the lower mode onto e, and the potential $-(2E^2/m)\,n(e)$ for the time $\pi m/(2\sigma E)$ moves it into the upper mode. The observer's clock is the phase of a circulation of three wedges at one of its ports, which sits at the level $-m$ exactly; the observer reads that phase through its fuel with four phase settings and states every frequency in units of this reference.

Three experiments are executed on one cell with the law running through every step. In the first, the coherent pulse is followed by a wait of q steps, and one trial reads the observer's reference and then, with the fuel used as found, the occupation of the profile. In the second, a fuel whose level is set by a dial is coupled weakly to the profile, with the fuel prepared full and prepared empty; the difference of the two runs is the survival of one object in the fuel, whatever the state of the pair is, and the fuel empties where the dial meets a level of the pair. In the third, one of three preparations is followed by a wait of 0, 8, 32, 128 or 512 steps and by one of four analyzers; the four counts give the occupation matrix of the pair and from it the probability of the excitation. Every reading is computed from sampled counts, 10^8 trials for each setting of the first and third experiment (Table 12).

The values of the last column for the excitation are computed in the prepared reference, which is why they differ from 1 and from $(1+m/E)/2=0.704$. An estimate from counts is linear in the

Table 12: Readings on a cage of one cell, from counts. Frequencies are in units of the observer's own reference.

Reading	From counts	The law
line of the seam record, steps of equal length	4.898982 ± 0.000002	$2\sqrt{6} = 4.898979$
the same line, steps of unequal length	4.898976 ± 0.000002	4.898979
level difference of the pair, from the fuel dial	4.898980 ± 0.000008	4.898979
amplitude of the line; with no pulse	0.4161; 0.0019	
excitation at five waits after the sharp pulse	0.9999 to 1.0002	0.999954
excitation at five waits after the coherent pulse	0.7059 to 0.7060	0.705932
excitation at five waits with no pulse	0.000004	0.000004
changed law: share of the variance in its best single line	0.507	
changed law: largest distance of its counts from the record law of the law	0.555	

counts and may leave the range of a probability by its sampling error. With the two modes of the cage stopped during the wait while the reference runs, the amplitude of the line is below 10^{-5}, and with the converter switched off no record of a full fuel occurs. The changed law adds seam hopping round every port, with the coefficient one, in every step of a run. It keeps the alphabet, the routing, the number of objects and the symmetries of the cell, and the observer's reference keeps its level under it. It does not keep the pair: the lower mode holds 0.79, 0.29, 0.21 and 0.14 of its weight in the pair at the parameter values 1, 4, 16 and 64.

14.4 What the cage establishes

Particle stability holds for a second object: the excitation of a cage is kept for every interval by Theorem 14.3, a local detector reads it, and a changed law with the same alphabet, routing and symmetry loses it. The cage carries a prepared clock: after a finite pulse its seam record swings with no drive at one line, which the observer reads in units of its own reference and finds equal to the level difference read separately. The equality is the frequency condition of two levels, and it holds for every m and c. The pulse supplies the phase and the work of the swing, $E + m$ on the ideal reference. The reference by itself has a constant seam record, and so has a sharp excitation. The observer takes as given that its reference advances by between 1/16 and 1/4 in every step, since counts cannot show a missed full turn. The level of a cage is flat: a cage has no group speed, and the law holds no term between two objects.

15 Where the outputs of the model meet the finite theorems

The theory is a library of finite theorems, each proved under stated premises and many of them machine-checked in Lean [1]. The model produces records. A theorem applies to the model where an output of the model has the form of its premise, and its conclusion is then a statement about the model's observers. The list below gives the theorems for which this holds. Every entry is a statement about records: the state of the simulation contains no spacetime, and no entry compares that state with the world.

The carrier. *Theorem:* the Gram matrix G of the twelve port directions has rank three and

$G^2 = 4G$, and $G/4$ is the projector on the band $5-\sqrt{5}$ of the port Laplacian L; machine-checked in `Lean/Screen/PortFrameGram.lean` and `Lean/Screen/PortGramRepairBand.lean` [7]. *The model supplies* the cell, which is that table, and a law under which one load moves with $L/2$ (Section 4). *It follows* that the slowest motion of a load has exactly three independent directions, the range of G. The first four levels of the scalar channel are the same levels (Section 14).

The seam rule. *Theorem:* four requirements leave one seam rule: keep the total, end where $n_a^2 + n_b^2$ is smallest, break a tie by a coin that does not look at the loads, treat the two ports alike [7]; the rule leaves a difference g of at most one and lowers $n_a^2 + n_b^2$ by $\frac{1}{2}[(n_a - n_b)^2 - g^2]$ (`Lean/ObserverPatchHolography/DirectedSeamRepair.lean`). *The model supplies* the write W of Section 3, which is that rule with the coin in superposition. *It follows* that the event which every seam writes and unwrites is the one rule with those properties, and that its classical moves settle after a number of steps that is polynomial in the number of ports for bounded loads. The echo unwrites the event and conserves the multiset of loads, so this bound is a statement about the moves and no statement about the law.

The projective instrument. *Theorem:* a Kraus map with effect P that leaves every state certain for P unchanged is the map $X \mapsto PXP$, and conversely (`Lean/EventAlgebra/InstrumentSelectionByCertainStates.lean`). *The model supplies* the presence reading of Section 9: it has the effect P and leaves every state on one answer block unchanged, on the ideal circuit and up to the known diagonal phase of the read window. *It follows* that the presence reading is the projective instrument of the event "a pair on some seam". The reader with its seam hidden has the same effect and fails the condition on the uniform superposition of the thirty pairs, so by the converse it is another instrument. That is the difference between the two histories of Table 7.

Records. *Theorem:* orthogonal record projectors generate a commutative algebra with Born weights and Lüders conditioning, an immediate reread returns the record with probability one [6], and two different states that one isometry copies from the same blank are orthogonal (`Lean/EventAlgebra/NoBroadcastingAdapter.lean`). *The model supplies* readers with complete outcome sets that copy orthogonal labels, public records that the law never erases, and a seam label that no event changes. *It follows* that the agreed records of the model form a public algebra with certain rereads, and that a reading which keeps one answer for the cell has to unwrite the seam words, as the presence reading does.

Local operations. *Theorem:* a complete operation on one factor of a product leaves the reduced state of every other factor unchanged [6]. *The model supplies* seam events and seam readings that act on the registers of one seam. *It follows* that no event or reading changes the record law of an observer that shares no register with it.

The generator. *Theorem:* a continuous one-parameter group of automorphisms of a finite matrix block is conjugation by e^{-itH}, with H unique up to an additive constant [3]. *The model supplies* two such flows, the free evolution of the contrast-memory seam, which Interval Echo uses at $\kappa = 8$, and that of the scalar channel. *It follows* that each free evolution is fixed by its flow up to an energy offset. The parameter t is no observer's clock.

Frequencies. *Theorem:* a reading that intertwines the frequencies of its source with its own shows no frequency that the source lacks (`Lean/QFT/ModularClockComparison.lean`). *The model supplies* the pair of a cage, whose frequencies are 0 and $\pm 2E$ (Theorem 14.3). *It follows* that a record of the pair under the free law holds the one line $2E$, which the observer reads from counts at 4.8990 of its reference (Table 12).

Clocks. *Theorem:* two shared readings fix the positive affine map between two clocks, and it is unique (`Lean/Time/ClockReadout.lean`). *The model supplies* ports that count their own events and share the records of their seams, and under Interval Echo two observer patches that read both elapsed counts on one common seam at reunion (Section 13). *It follows* that two shared records relate the counts of two observers and a third checks the relation. No common clock enters.

Rereads. *Theorem:* for a state in the range of an event projector, n rereads of the event over a total parameter t all return it with probability at least $1 - 2t^2\|H\|^2/n$ [3]. *The model supplies* the presence reading and the law. *It follows* that frequent presence readings would hold a pair at any coupling. Every reading of Section 8 therefore uses a fresh run, and the binding theorem needs no reading.

The port response. *Theorem:* a faithful closed twelve-dimensional response with a positive trace form, on which the carrier rotations act by inner automorphisms, has the Lie type $\mathfrak{u}(1) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(3)$ [4]. *The model supplies* the response of one load on one cell under the declared service of Section 11. *It follows* that this response has that type, and the direct computation gives $\mathfrak{u}(3) \oplus \mathfrak{so}(3)$, the same type. Under the plain services the response has seventeen directions and the premise fails. Interval Echo supplies twelve independent port currents selected by the rotations, on one load and on the bound pair, closed under commutators and covariant under the rotations (Section 13); that this menu is complete for a physical observer is not established.

Theorems that consume other outputs. Other finite theorems of the theory consume outputs that the model does not produce in the required form. The settling theorems of the consensus paper consume accepted repairs that lower a well-founded measure and close every fork [6]. The free evolutions of the model are unitary and conserve their loads and objects, so for them those theorems apply to the classical seam rule and make no statement about the echo. In the repair experiment of Interval Echo on one cell a seam write is accepted when a sharp reading finds both ends holding one load, and each accepted write lowers the sum of the squared loads. Two such repairs competing for one doubled load leave distinguishable records; a prepared local priority, read without being moved, makes both request orders end in one common terminal state in all six combinations of sharp preparation and priority. No confluence of all accepted repairs on the federation is proved. The proposition on smooth effective field descriptions consumes readouts that are polynomials in the Weyl operators of a free field on a radial cylinder over the screen [5]; the readouts of the model are instruments on finite registers. The causal and count limits consume an authenticated order of events with its placement and a refinement family. The matched-regional Einstein theorem consumes entropy, area and stress readouts of one region in a common frame. The Maxwell selection theorems consume a gauge-invariant action on seam potentials, and the theorems on defect classes and statistics a five-level code with a spin doublet. The dark-sector theorems consume a modular generator with a recovery defect, where the unrecovered collar supplies a conserved label [8]. Each of these names an output that a member of the model has to produce before the theorem applies to it.

16 Resources and the scope of the computations

A literal allocation of the reference candidate on the federation of 20 cells has 240 load systems of three states, 630 seam memories of eleven states, 180 reference registers of three states with 180 further memories of eleven states for the degree term, one fuel system of two states and 630 reader

blocks of eleven bits. Its dimension is

$$3^{240} \cdot 11^{630} \cdot (3 \cdot 11)^{180} \cdot 2 \cdot 2^{11 \cdot 630},$$

about 10,399 bits. The computations of this paper use sectors of this space that the law maps into themselves. By Corollary 3.3 the memories stay blank, and the sector of two loads has $V(V+1)/2$ states on V ports (Table 1); the unit pairs form an invariant part of it, and the emitter adds one state. The sector of three unit loads has 220 states on one cell and 2,275,280 on 20 cells. The complete reading of Section 6 has 161,792 states. These sizes are polynomial in the number of ports because the number of loads is fixed. The space of many loads, and the allocation above, grow exponentially with the number of ports.

The statements for every federation, Theorem 5.2, Corollary 6.2 and Proposition 6.3, are operator estimates and no claim that the allocation of a whole federation with its readers was propagated.

The construction encodes no constraint instance, decodes no witness from its records and reaches no fixed point, since the law conserves the multiset of loads. It is no solver for general constraint problems and claims no polynomial bound for one.

17 Supplied parts and limits of the construction

17.1 Supplied

The following parts are design choices of the construction.

- The coupling κ. Computed values are 0, 0.5, 1, 2, 4, 8, 16, 64 and 128, with 8 and 128 in the main tables.
- The coherent generator M_κ on the private seam memory, and the blank initial state of every memory.
- The reference registers of the degree term on ports with five seams.
- The emitter with its fuel, coupling and duration, the timing of the reader, and the public record as a protected memory.
- The evolution parameter of the runs with a continuous parameter. As local events the pair needs every seam to act once per round, or the limit of small angle, and the event programs use the event (16) (Section 7).
- The routing of the federation by the cell-centre and tangent-frame convention.
- The weighted port service (19) of Section 11, and the unrecovered collar with its fuel and converter in Section 12.
- The tree of the presence reading with its wiring and pulse schedule (Section 9).
- For the scalar channel: its two coefficients m and c, the prepared reference with its ramps, the apparatus registers and the penalties of the register form, the two operators on a profile, the pulses with their strengths and times, the fuel dial, and the bounds on a step of the observer's reference (Section 14).
- For Interval Echo: the guide, custody transfers, gate selection, relative cadence and read windows of the clock, the prepared hands, the designed sizes of the hand, the two counts and the walk budget, and the calibrated closing filter; the preparations, matching schedule, tag protection and gate windows of the overlap measurements; and the oriented cell, preparation, protected bands and unit gain of the port-current service (Section 13).

17.2 Limits

The construction does not supply the following.

- A selection of κ. The collar reading of Section 12 returns the coupling from records; no equation fixes its value.
- A particle–antiparticle assignment in the contrast-memory law. One load per port supports an excess and a vacancy, but their free evolution conserves both counts. In the scalar channel an object above the filled reference and a missing object below it are the two signs, and the law holds no term between two objects.
- A physical electromagnetic connection. In the written-field extension of Section 13.2, head changes carry signed flux and a distant material probe responds. A static Coulomb interaction and a transverse Maxwell branch have no realization in the tested ground and ordered channel, respectively.
- The class of an object as a record. Objects are protected by the number of loads and the gap alone (Section 10).
- The twelve port currents on the bound object from the reference law's own services (Section 11). Interval Echo obtains twelve from a supplied service selected by the rotations, and a complete menu of the physical controls is not identified (Section 13).
- One limiting speed for all objects. The bound pair is a Galilean composite: on a chain, with the seam term (10), one load has the branch $\Delta - \cos k$ and the pair has $E(K) = \Delta - \cos^2(K/2)/\Delta$ with $\Delta = 1 + 2\kappa$, which is quadratic at small K. At $\kappa = 8$ the kinetic ratios of free clusters differ from their rest-energy ratios, so under the free law, which Interval Echo inherits, rest energy and inertia of free clusters follow two dictionaries. The scalar channel has the relation $\omega^2 = m^2 + \lambda$ by the form of its law, and its cage has a flat level and does not move.
- A clock that starts with no supplied phase. The swing of a cage is written by a pulse that supplies its phase and its work.
- A refinement map that preserves seams. The port-ancestry map between the levels of the federation does not preserve seam adjacency [7]. The test of a refinement is agreement of the distributions of admissible observer records between levels; agreement of state encodings is no part of it.
- A comparison of clocks on held clock constructions and directions. The clocks of Section 7 are counts of events, and the patch clock of Interval Echo follows the relativistic proper-time forecast on the journeys of Section 13.
- The transport of the three directions by a moving observer's full records.
- The regional response that the Einstein theorem of the spacetime paper consumes [5].
- A physical reading of the unrecovered collar as the dark-sector carrier of the theory (Section 12).

These limits are measured against the test of Section 1. Observers who inhabit the computation can describe their experience as a relativistic continuum spacetime with one time and three space dimensions, by the proposition on smooth effective field descriptions of the flagship and the reconstruction of the spacetime paper, under their stated premises [2, 5]. Of those premises the model supplies the carrier with its rank-three band, count clocks compared through shared readings (Section 15) and, under Interval Echo, clocks carried by observer patches that run slow by the relativistic factor when they move (Section 13); the field readouts, a jointly authenticated order of

events and the refinement family that the two results also consume are outputs it does not produce. The model is not required to contain three dimensions in its state, and no item above asks for that.

18 What the model predicts on records

Every result of this paper is a probability of records under a stated preparation. Each is therefore a prediction for any realization of the model, on a quantum processor or in an independent simulation: the realization prepares, runs and reads as specified and compares its counts with the numbers of Table 13. The table names for each prediction the outcome that counts against it. These are predictions about the model. A comparison with a laboratory system requires in addition that the same preparation, intervention and readout be identified in that system.

Table 13: Predictions of the model on records, and the outcome that counts against each.

Experiment	Prediction	Counts against it
A pair written on one seam, read later, $\kappa > 3$	no pair with probability at most $40/(2\kappa-6)^2$ on every federation; at most 0.0097 at $\kappa = 8$ on 1 to 320 cells	a larger probability at some reading
The same law without the agreement energy	no pair with probability up to 0.80 on 20 cells	a pair that is kept
Rounds of seam events in any order, with unequal rates of the ports	the pair is kept	a loss of the pair under rounds
Two readings of a pair, $\kappa = 8$, one cell	equal first answers; later records that differ by more than 0.0817 in total variation; one event at 0.0183 against 0.1029	equal laws of the later records
An unrecovered collar with empty ports	read unrecovered at every reading; the rates of its two records return the coupling	a recovery; another ratio of the rates
The seam record of a cage after the coherent pulse	one line at $2\sqrt{6} = 4.8990$ of the observer's reference, with more than 0.9999 of the variance	a second line, or a line elsewhere
The fuel dial on the same cage	the level difference 4.8990, equal to the line	a value away from the line
The excitation of a cage after the sharp pulse	read above 0.99 at waits of 0 to 512 steps, the same at every wait	a decrease with the wait
The cage under seam hopping round every port	no single line; the excitation is lost	a single line, or a kept excitation
Interval Echo: a travelling patch on journeys with leg speeds $(1/2, 1/6)$, $(1/6, 1/2)$ and $(1/4, 1/4)$	a ratio of its elapsed count to the home count at reunion within about 0.001 of the forecast (20)	a residual far outside its standard error
Interval Echo: overlapping single-cell and three-cell observers on 320 cells	equal readings at every shared checkpoint	a shared checkpoint read differently

19 Conclusion

The reference candidate A-contrast-memory-spherical-v1 is a finite specification with every part stated: a carrier of icosahedral cells, loads on ports, a private memory on every seam, one local term for each seam, an emitter, a reader with a complete outcome set, and the schedules under which the law is run. The echo with the contrast-memory generator is exactly the exchange with an energy of agreement, and the written output enters through its symmetry alone. The law binds a pair of loads with constants that hold for every number of cells, the agreed records show the pair on changing seams, and the law without the agreement energy loses it. The same law has bound objects on seams and faces, a port response of seventeen directions that becomes twelve under one declared service, and a conserved neutral excitation of the seam memory. Two readings of the pair, one that keeps a single answer for the cell and one that keeps the seam, have the same first statistics and different later records. The main candidate law, Interval Echo, runs this seam at $\kappa = 8$ and lets a finite observer patch carry its own clock. A travelling clock, compared at reunion with the clock kept at home, runs slow by the relativistic factor computed from the observers' own records, within about one part in a thousand. Overlapping observers agree on every shared checkpoint reading, port currents selected by the rotations close on twelve directions, and two overlapping observers realize the unique information projection to a uniform reference. The second candidate law on the same carrier holds an object on four seams by the signs of its hop: its excitation is kept for every interval, and its seam record carries a prepared clock that an observer reads in units of its own reference. All three laws are candidate native repair laws, and their couplings are supplied.

The written-field extension identifies a candidate for the charge and field response of an electron. Its signed material load carries the change in boundary flux, alters a distant material probe and responds to an orbital magnetic twist. The four-face field doublet changes how that charge moves, yet an addressed exchange carries the material origins past one another while leaving the doublet labels at their original gateways. Physical spin attached to the travelling charge, fermionic exchange of the whole excitation and its rest energy in keV are distinct, unverified identifications. On the original Interval Echo law the long-wave dispersion measured on a prepared four-port route depends on how the same gateways are connected inside each cell (Section 13.1).

20 Reproducibility

The reference candidate is the package `model/candidate_laws/contrast_memory_spherical_v1` of the public repository [1]. The command

```
python -m model.candidate_laws.contrast_memory_spherical_v1.verify
```

recomputes the reference candidate from the carrier arrays of the package and compares the outcome with the stored results; with the option `--full` it also recomputes the federations of 80 and 320 cells and the event programs. It checks the identities of Section 3 in exact arithmetic on all 99 local states, counts the incidence facts of Lemma 2.1 on the federations, computes the constants of Table 2, and recomputes the record probabilities of Sections 6, 7 and 8, the objects and bindings of Section 10, the port response of Section 11 and the collar levels and readings of Section 12. Exact identities must hold exactly, and floating values are compared within declared tolerances. Sampled readings use fixed seeds. The proofs of Sections 2 to 7 are given in full in the text and do not depend on the computations.

The commands

```
python -m model.candidate_laws.contrast_memory_spherical_v1.presence
```

```
python -m model.candidate_laws.contrast_memory_spherical_v1.presence_certificate
python -m model.candidate_laws.scalar_channel
python -m model.experiments.cage
```

recompute the laws of two readings of Section 9, the certified event in exact arithmetic, the scalar channel of Section 14 and its cage. The counts of the experiments on a cage are stored with the package. A replay recomputes the record laws from the law and the prepared reference, checks that the stored counts are draws from them, and recomputes every reading from the counts.

The command

```
python -m model --full
```

replays the main candidate law, Interval Echo, and compares the outcome with the stored results: the material clock transfers on 20, 80 and 320 cells, the reunion cohorts, the overlap measurements, the repair fork, the port currents and the information projection of Section 13. The journeys timed by light pulses are archived with their mean records and standard errors, from which the reported residuals follow.

The signed-load experiments of Section 13.2 are documented in `model/candidate_laws/interval_echo/openwave/electron_field/`. The route spectra and local frequency records of Section 13.1 are in `row_30_route/` and `row_30_autonomous/` under `model/candidate_laws/interval_echo/openwave/`.

Credited prior work

The results of this paper build on the following established work. Each entry names an established result as it is used here and cites its sources.

- Algebraic graph theory and symmetric spectral methods [17, 19].
- Finite-dimensional states, quantum channels and partial traces [18].

Statements and declarations

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Data availability. The model documents, executable sources and computed results supporting this study are available in the public OPH repository [1].

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