

# **Synthesizing data from pretest-posttest-control-group designs in mediation meta-analysis**

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Good afternoon. The research I'm gonna present today is about the feasibility and the finite-sample performance of mediation meta-analysis under pretest-posttest-control group designs.



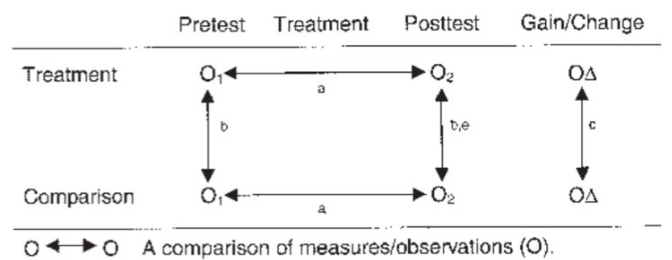
## Pretest-posttest Control Group Design

Let's look at this design first.

## ► Introduction

- Pretest-posttest-control-group (PPCG) designs

- Commonly used and highly recommended (Morris, 2008)



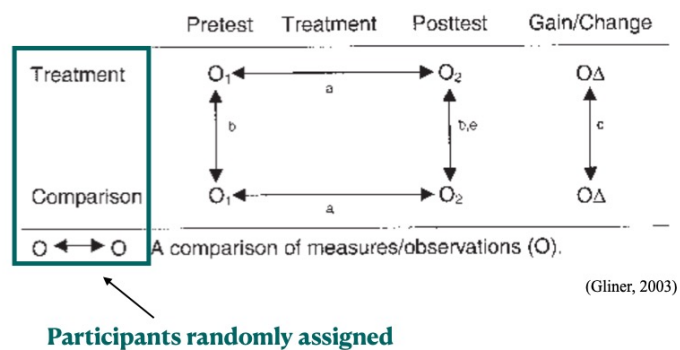
(Gliner, 2003)

PPCG design is one of the most commonly used design in clinical psychology.

## ► Introduction

- Pretest-posttest-control-group (PPCG) designs

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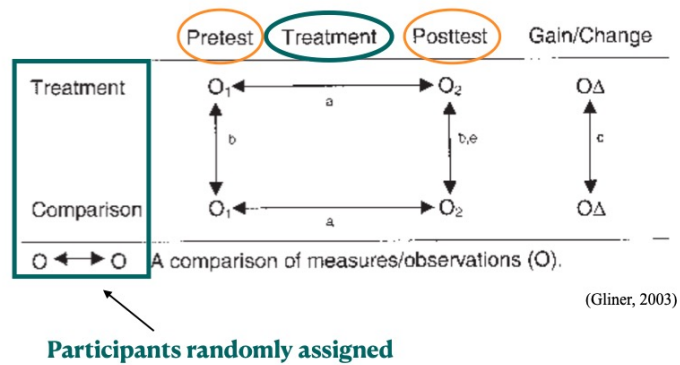


In this design, participants are first randomly assigned to either the treatment group or the control group.

## ► Introduction

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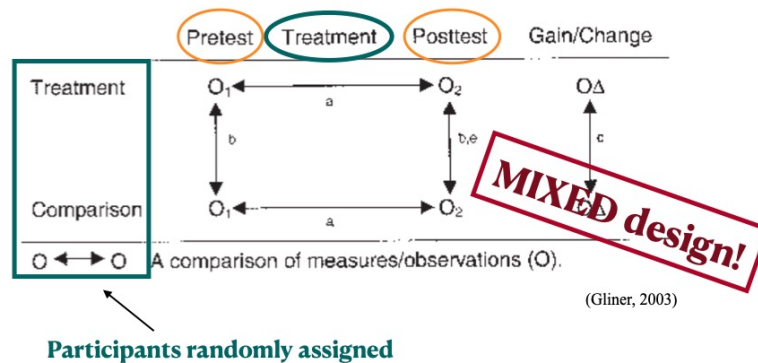


Then the outcome or the mediator is measured both before and after the treatment.

## ► Introduction

- Pretest-posttest-control-group (PPCG) designs

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So PPCG design is a mixed design, and there's a within-subject correlation between pretest and posttest data.



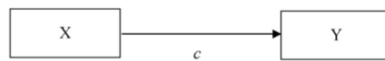
## Mediation meta-analysis

Now, what is mediation meta-analysis?

## ► Introduction

- Meta-analysis

- Originally used to investigate bivariate effect(s)



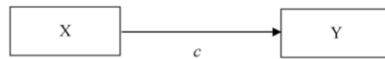
Originally, meta-analysis is conducted based on bivariate models.



## ► Introduction

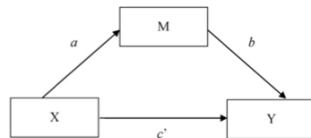
- Meta-analysis

- Originally used to investigate bivariate effect(s)



- Mediation Meta-analysis (MMA)

- Synthesizing mediating mechanism(s)



Similarly. Mediation meta-analysis, or MMA, is just a meta-analytic approach based on a mediation model. Typically conducted using meta-analytic SEM techniques.

## ► Introduction

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- Currently on MMA
  - Used in many other important areas:

MMA has recently been used in many critical areas in clinical psychology,

## ► Introduction

- Currently on MMA

- Used in many other important areas:



(Such as chronic pain)

## ► Introduction

- Currently on MMA

- Used in many other important areas:



(insomnia and so on)

## ► Introduction

- Currently on MMA

- Used in many other important areas:



## ► Introduction

- Currently on MMA

- Used in many other important areas:



## ► Introduction

- Currently on MMA

- Used in many other important areas:



## ► Introduction

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- The feasibility of MMA under PPCG designs?

Although multivariate meta-analytic techniques, such as One-stage MASEM, have been well developed and widely adopted, there are still many concerns for MMA under PPCG designs, both theoretically and practically.



## ► Introduction

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- The feasibility of MMA under PPCG designs?

1. The multinormality assumption (Jak & Cheung, 2020)

First, because the independent variable  $X$  is binary in PPCG designs, concerns arise about the multi-normality assumption in meta-analytic SEM techniques.

## ► Introduction

- The feasibility of MMA under PPCG designs?

1. The multinormality assumption (Jak & Cheung, 2020)

- Multinormality assumption of **residuals/errors** (Bollen, 1989, p.127 ; Curran, 2003)

About this, it's actually been demonstrated in the literature of SEM, that, when X is randomized, the normality assumption on X can be relaxed as long as the residuals of other variables conditional on X are multi-normally distributed. That's why this assumption is also called the multi-normality assumption of residuals/errors.

## ► Introduction

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- The feasibility of MMA under PPCG designs?

1. The multinormality assumption (Jak & Cheung, 2020)

- Multinormality assumption of **residuals/errors** (Bollen, 1989, p.127 ; Curran, 2003)
- Unexamined for MMA under finite samples

While this has been tested in individual SEMs, it has not been tested in meta-analytic SEM under finite-sample.

## ► Introduction

- The feasibility of MMA under PPCG designs?

1. The multinormality assumption (Jak & Cheung, 2020)

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- Unexamined for MMA under finite samples

2. Differently defined treatment & control conditions

Second, the treatment and control conditions are often differently defined in different primary studies. For example, one study may define the treatment condition as an 8-week therapy, whereas another may define it as a 4-week therapy. With such crucial heterogeneity in study-level characteristics, it is infeasible to simply average the effect sizes in a meta-analysis.

## ► Introduction

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2. Differently defined treatment & control conditions

- A latent continuous variable? (Pustejovsky, 2014)

Some researchers suggested that the heterogeneity is actually in the continuous latent variable underlying X, which should be the target variable, and should be considered in the calculation of effect sizes in the meta-analysis. But the problem is it's quite difficult to identify an exact value for it.

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2. Differently defined treatment & control conditions

- A latent continuous variable? (Pustejovsky, 2014)
- Group difference - binary in nature (Borenstein & Hedges, 2019)

Instead, we believe that in practice, the effect of interest is actually the group differences between the treatment group and the control group, so X can be regarded as **binary in nature**. In this case, the X related effect sizes can be quantified using point-biserial correlations.

## ► Introduction

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- Unexamined for MMA under finite samples

2. Differently defined treatment & control conditions

- A latent continuous variable? (Pustejovsky, 2014)
- Group difference - binary in nature (Borenstein & Hedges, 2019)
- Meta-regression and/or subgroup meta-analysis

Under this viewpoint, the heterogeneity can be addressed simply by meta-regression, or by subgroup meta-analysis. That is to model it as a moderator of the meta-analysis.

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- Meta-regression and/or subgroup meta-analysis

3. Violation of the homogeneous variance assumption

- Robustness of MMA under such violation

Third, in practice, the variance of posttest scores in the treatment group is often larger than that of the pretest scores, because of individual differences regarding the treatment effectiveness.

In this case, the homogeneous variance assumption is violated.

It's an inevitable context in reality, so it must be considered when assessing meta-analytic approaches.



## ► Introduction

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- How to conduct MMA under PPCG designs?

In addition, it should also be further clarified about how exactly to conduct MMA under PPCG designs. Or what type of data to use.

## ► Introduction

- How to conduct MMA under PPCG designs?

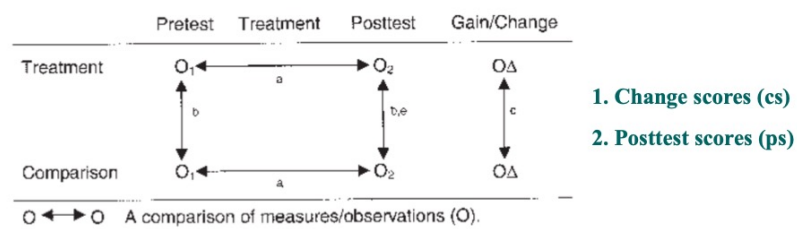
- Bivariate correlations as data input
- For MMA:  $r_{XM}$ ,  $r_{XY}$ , and  $r_{MY}$

Currently, bivariate correlations are required as data input for meta-analytic SEM techniques.

## ► Introduction

- How to conduct MMA under PPCG designs?

- Bivariate correlations as data input
- For MMA:  $r_{XM}$ ,  $r_{XY}$ , and  $r_{MY}$

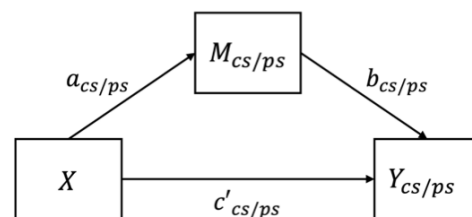


Under PPCG designs, these correlations can be computed using either change scores or posttest scores, depending on the research objective.

## ► Introduction

- How to conduct MMA under PPCG designs?

- Change-score MMA (CSMMA): pretest-posttest changes
- Posttest-score MMA (PSMMA): posttest performance



If the objective is about pretest-posttest changes, change-scores should be used, and the MMA is called a CSMMA in short.

Or, if the objective is to compare posttest performances, posttest scores should be used. In this case the MMA is called a posttest-score MMA or PSMMA.

## ► A trade-off

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- **Same conclusions regarding the existence of effects**

In the population, these two approaches would yield to the same conclusion about the existence or the direction of the treatment effect,

## ► A trade-off

- Same conclusions regarding the existence of effects
- Differ in statistical power

	Within-subject correlation	Number of primary studies
CSMMA	✓	Smaller
PSMMA	✗	Larger

but they may differ in finite-sample performances.

because the within-subject correlation is accounted for in CSMMA but not in PSMMA by definition, CSMMA may have a higher statistical power than PSMMA.

However, primary studies may not always report the sufficient information for conducting CSMMA, so CSMMA may have a smaller number of primary studies than that of PSMMA.

## ► A trade-off

- Same conclusions regarding the existence of effects
- Differ in statistical power

	Within-subject correlation	Number of primary studies
CSMMA	✓	Smaller
PSMMA	✗	Larger

### 4. Controlling within-subject correlation vs. sample size

So, there's a tradeoff here. It's currently unclear whether the advantage of considering the within-subject correlation outweighs the disadvantage of having a smaller number of primary studies.

## ► Simulation Study

- Study 1:  
How does MMA under PPCG designs perform with **EQUAL** group variances
- Study 2:  
How does MMA under PPCG designs perform with **UNEQUAL** group variances
- Introduced heterogeneity in path coefficients in both Studies
- Manipulated
  1. within-subject correlation ( $\rho$ )
  2. number of primary studies ( $K$ )

We conducted two simulation studies to examine the aforementioned issues. Specifically, we compared the finite-sample performance of CSMMA and PSMMA under PPCG designs, with heterogeneity introduced in path coefficients, which corresponds to the first two concerns.

In Study 1, we assumed equal variances, but in Study 2, the posttest variance in the treatment group was inflated, violating the homogeneous variance assumption, corresponding to the third concern.

In addition, we manipulated the magnitude of within-subject correlation  $\rho$  and the number of primary studies  $K$ , to examine the tradeoff we just talked about.



## ► Simulation Study

### • Simulation settings

Manipulated factors	Parameter settings
1. Mean standardized regression coefficient: $a_{s,cs} = \{0, 0.3\}$ $b_{s,cs} = \{0, 0.3\}$ $c'_{s,cs} = \{0, 0.3\}$	1. Probability of being assigned to one group: $\pi = 0.5$
2. Within-subject correlation: $\rho_{12} = \{0.45, 0.9\}$	2. Sample sizes: $\mu_N = 80, \sigma_N = 23$
3. The moderating effect: $\beta_{c'_{s,cs}} = \{0, 0.1\}$	3. Heterogeneity in random-effects model: $\tau = 0.1$
4. Number of primary studies: $K = \{5, 10, 30\}$	4. Pretest variances of both groups and posttest variance of the control group: $\sigma_{M_{1,C}}^2 = \sigma_{M_{1,T}}^2 = \sigma_{Y_{1,C}}^2 = \sigma_{Y_{1,T}}^2 = \sigma_{M_{2,C}}^2 = \sigma_{Y_{2,C}}^2 = 1$
5. Posttest variance of the treatment group: $\sigma_{M_{2,T}}^2 = \sigma_{Y_{2,T}}^2 = 1$ (Study 1) $\sigma_{M_{2,T}}^2 = \sigma_{Y_{2,T}}^2 = 1.5$ (Study 2)	5. Pretest means of both groups and posttest means of the control group: $\mu_{M_{1,C}} = \mu_{M_{1,T}} = \mu_{Y_{1,C}} = \mu_{Y_{1,T}} = \mu_{M_{2,C}} = \mu_{Y_{2,C}} = 0$
	6. Posttest means of the treatment group: $\mu_{M_{2,C}} = MD_K^M, \mu_{Y_{2,T}} = MD_K^Y$

The simulation settings are listed here. We won't delve into the details of model settings due to time limit.

## ► Simulation Study

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- True path coefficients for CSMMA and PSMMA
  - Data were generated using change-score parameters
  - A large sample ( $N = 1000$ ,  $K = 10000$ ) to simulate posttest-score parameters

Although It should be noted that the data generating model was based on change-score coefficients, which are in nature true values for CSMMA. But for PSMMA, the true posttest-score parameters were simulated from a large sample.

## ► Simulation Results

- In both studies, bias and type I error rates ✓
  - Finite-sample MMA under PPCG designs ✓
  - Heterogeneous path coefficients ✓
  - Inflated posttest variances ✓

So the results showed that, in both study 1 and study 2, estimation bias and type I error rates of CSMMA and PSMMA remained favorable under all conditions, with heterogeneity introduced in path coefficients.

this answered to the first three concerns, and provided support for the feasibility of MMA under PPCG designs, with a randomized X.

## ► Simulation Results

- Power: CSMMA > PSMMA

	$\rho_{12}$	$K$	Study 1		Study 2	
			CSMMA	PSMMA	CSMMA	PSMMA
Statistical Power	0.45	5	0.962	<b>0.280</b>	0.957	<b>0.439</b>
		10	1	<b>0.560</b>	1	<b>0.770</b>
		30	1	0.971	1	0.998
	0.9	5	0.962	<b>0.006</b>	0.955	<b>0.060</b>
		10	1	<b>0.024</b>	1	<b>0.285</b>
		30	1	<b>0.088</b>	1	0.889

In addition, CSMMA had higher power than PSMMA under all conditions in study 1 and 2, although the inflation of posttest variance was actually in favor of PSMMA.

But overall, the power of PSMMA was smaller,

## ► Simulation Results

- Power: CSMMA > PSMMA

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- Why is the power of PSMMA as small as 0.006?
- The true posttest-score coefficient on the b path

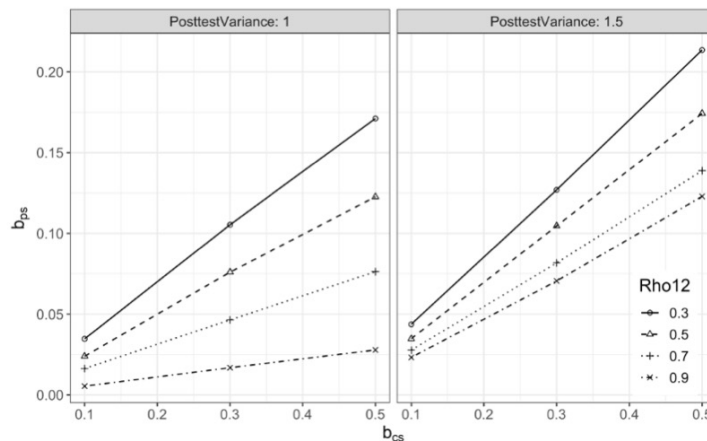
Especially when Rho was large and equaled to 0.9. This was much smaller than we had expected. So the next question is Why does PSMMA had such a small power.

Then, we went through the results of parameter estimation, and found that when Rho equaled to 0.9, the true posttest-score coefficients on the b path were so small that the estimates of it were too small to reach significance in PSMMA.

So apparently, the within-subject correlation is an important factor here.

## ► Simulation Study

- The relationship between change-score and posttest-score parameters



So, we conducted a large-sample simulation, to explore how within-subject correlation impacted the relationship between change-score and posttest-score coefficients .

The X axis represents the preset change-score parameters and the Y axis is the generated posttest-score parameters.

The lines on the top is when Rho equaled to 0.3, and at the bottom is when Rho equaled to 0.9.

As we can see, for a given change-score parameter, the corresponding true posttest-score parameter on the b path decreased with a larger within-subject correlation.

## ► Conclusions

- CSMMA vs. PSMMA under finite samples
  1. The multinormality assumption (Jak & Cheung, 2020) ✓
  2. Differently defined treatment & control conditions ✓
  3. Violation of the heterogeneous variance assumption ✓
  4. Power: CSMMA > PSMMA
    - Within-subject correlation

To sum it up, in this study, we compared the finite-sample performance of two approaches to mediation meta-analysis under PPCG designs and addressed the aforementioned concerns about the feasibility of MMA. Moreover, we found that CSMMA had higher power than PSMMA, because within-subject correlation played a crucial role in the magnitude of true change-score and posttest-score parameters.



# THANK YOU

That's all for this presentation. Thank you.



$r_{MY}$  : directly reported

$r_{XM}$ ,  $r_{XY}$  : point-biserial correlations

$$r_{pb,cs/ps} = \frac{\hat{\mu}_{T,cs/ps} - \hat{\mu}_{C,cs/ps}}{\hat{\sigma}_{cb,cs/ps}} \sqrt{\frac{N_T N_C}{(N_T + N_C)^2}}$$

$$\hat{\sigma}_{cb,cs/ps} = \sqrt{\frac{N_T(\hat{\sigma}_{T,cs/ps}^2 + D_{T,cs/ps}^2) + N_C(\hat{\sigma}_{C,cs/ps}^2 + D_{C,cs/ps}^2)}{N_T + N_C}}$$

$$b_{s,cs,k} = \frac{r_{MY,k} - r_{XM,k}r_{XY,k}}{1 - r_{XM,k}^2}$$

$$= \frac{\sigma_X^2 \text{cov}(MY) - \text{cov}(XM)\text{cov}(XY)}{\sigma_Y(\sigma_X^2 \sigma_M - \sigma_X)}$$