

2022 形式语言自动机期末模拟试卷

****题型仅供参考，与期末考试不一定相同，**

1. Give a DFA accepting the language that meets the following requirements over the alphabet $\{0,1\}$.

The number of 0s is even and don't end in 01

2. Give a NFA accepting the following language. $\{xwx^R \mid x, w \in \{0,1\}^+\}$

3. Write a regular expression accepting the strings that represent a number divisible by 5 in binary.

4. Prove that the language $\{a^m b^n c^{2k} d^{2z} \mid z \neq m + n + k\}$ is not regular with pumping lemma.

5. Convert to a DFA the following NFA:

| | | 0 | 1 | 2 |
|-------|----|-------------|-------------|----------|
| Start | q0 | {q0, q1} | {q0, q2} | {q0, q2} |
| | q1 | {q0, q3} | \emptyset | {q2} |
| | q2 | \emptyset | {q1, q3} | {q1, q2} |
| * | q3 | {q2, q3} | {q3} | {q0} |

6. Give a context-free grammar over $\{1,2,3,+,*,(,),\emptyset,\epsilon\}$ for all regular expressions over alphabet $\{1,2,3\}$.

7. Construct CNF equivalent to the following grammar:

$S \rightarrow aBB \mid bAA$

$B \rightarrow aBa \mid aa \mid \epsilon$

$A \rightarrow bbA \mid \epsilon$

8. Design a PDA for $L(M) = \{1^n 0^n \mid n \geq 1\} \cup \{1^n 0^{2n} \mid n \geq 1\}$

9. Prove the language $L = \{x\#y \mid x, y \in \{0,1\}^* \text{ and } y \text{ is a substring of } x\}$ is not CFL with pumping lemma; Alphabet $\{0,1,\#\}$.

10. Design Turing machine to compute n^2 . (start from 0^n to 0^{n^2})

命题人：计算学部讲师团形式语言与自动机命题组
命制时间：2022. 5. 4

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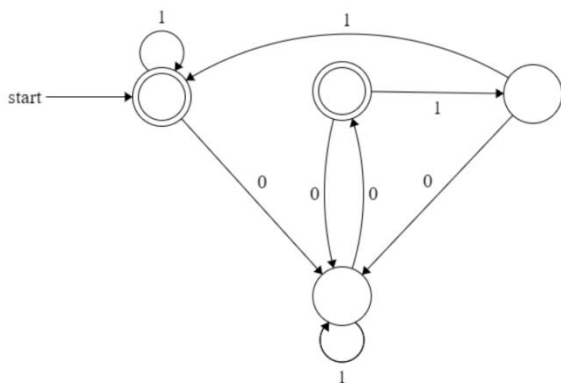
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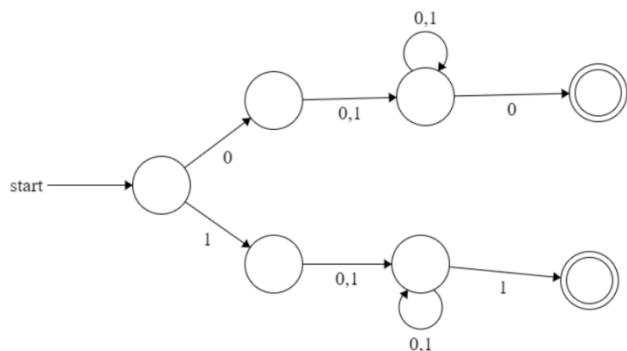
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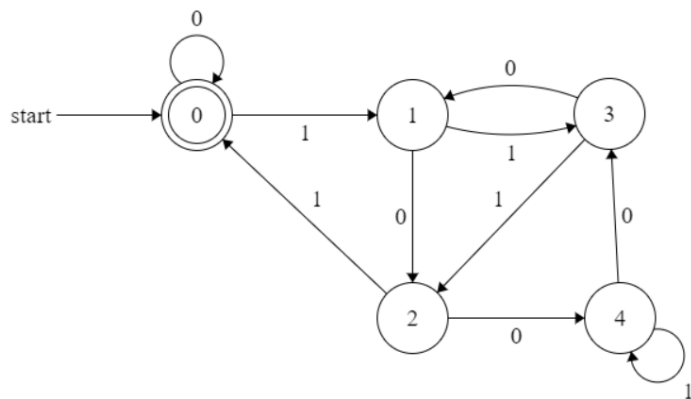


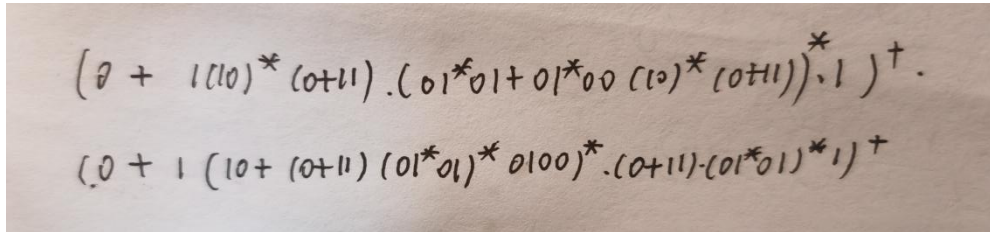
2. Give a NFA accepting the following language. $\{xwx^R \mid x, w \in \{0,1\}^+\}$



3. Write a regular expression accepting the strings that represent a number divisible by 5 in binary.

思路：设计 dfa，再转 re





4. Prove that the language $\{a^m b^n c^{2k} d^{2z} \mid z \neq m + n + k\}$ is not regular with pumping lemma.

思路一. 直接使用泵引理

可取 $m = N, n = N, k = N, z = 3N + N!$,

则分为 xyz 后, $y = a^s, 1 < s < n$,

则对于 $xy^f z$, $m = N + (f-1)s, n = N, k = N, z = 3N + N!$,

取 $(f-1) = \frac{N!}{s}$ 即可

思路二. 利用封闭性和泵引理

$L1 = \{a^m b^n c^{2k} d^{2z} \mid z, m, n, k \text{ 非负}\}$

$L2 = \{a^m b^n c^{2k} d^{2z} \mid z = m + n + k\}$

$L3 = \{a^m b^n c^{2k} d^{2z} \mid z \neq m + n + k\}$

由泵引理易证 $L2$ 非正则, 则若 $L3$ 正则, 由 $L1 - L3 = L2$ 可知 $L2$ 为正则, 矛盾! 所以 $L3$ 非正则

5. Convert to a DFA the following NFA:

| | | 0 | 1 | 2 |
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| | q2 | \emptyset | {q1, q3} | {q1, q2} |
| * | q3 | {q2, q3} | {q3} | {q0} |

| | 0 | 1 | 2 |
|----------------------------|--------------------------|--------------------------|---------------------|
| $S \rightarrow \{9_2\}$ 1 | $\{9_0, 9_1\}$ | $\{9_0, 9_2\}$ | $\{9_0, 9_2\}$ |
| $\{9_0, 9_1\}$ 2 | $\{9_0, 9_1, 9_3\}$ | $\{9_0, 9_2\}$ | $\{9_0, 9_2\}$ |
| $\{9_0, 9_2\}$ 3 | $\{9_0, 9_1\}$ | $\{9_0, 9_1, 9_2, 9_3\}$ | $\{9_0, 9_1, 9_2\}$ |
| $\{9_0, 9_1, 9_2\}$ 4 | $\{9_0, 9_1, 9_3\}$ | $\{9_0, 9_1, 9_2, 9_3\}$ | $\{9_0, 9_1, 9_2\}$ |
| $\{9_0, 9_1, 9_3\}$ 5 | $\{9_0, 9_1, 9_2, 9_3\}$ | $\{9_0, 9_2, 9_3\}$ | $\{9_0, 9_2\}$ |
| $\{9_0, 9_2, 9_3\}$ 6 | $\{9_0, 9_1, 9_2, 9_3\}$ | $\{9_0, 9_1, 9_2, 9_3\}$ | $\{9_0, 9_1, 9_2\}$ |
| $\{9_0, 9_1, 9_2, 9_3\}$ 7 | $\{9_0, 9_1, 9_2, 9_3\}$ | $\{9_0, 9_1, 9_2, 9_3\}$ | $\{9_0, 9_1, 9_2\}$ |

6. Give a context-free grammar over $\{1, 2, 3, +, *, (,), \emptyset, \varepsilon\}$ for all regular expressions over alphabet $\{1, 2, 3\}$.

答案:

这题考察通过正则表达式的定义来构造 CFG

$$S \rightarrow \emptyset | \varepsilon | 1 | 2 | 3 | S + S | S^* | S(S)$$

7. Construct CNF equivalent to the following grammar:

$$S \rightarrow aBB|bAA$$

$$B \rightarrow aBa|aa|\varepsilon$$

$$A \rightarrow bbA|\varepsilon$$

答案:

首先去除空产生式:

观察 A 和 B 是可空的, 所以对 A 和 B 进行替换

$$S \rightarrow a|aB|aBB|b|bA|bAA$$

$$B \rightarrow aBa|aa$$

$$A \rightarrow bbA|bb$$

接着将其转化为乔姆斯基范式 ($A \rightarrow BC$ 或者 $A \rightarrow a$ 的形式)

$$S \rightarrow a|S_1B|S_1S_2|b|S_3A|S_3S_4$$

$$S_1 \rightarrow a$$

$$S_2 \rightarrow BB$$

$$S_3 \rightarrow b$$

$$S_4 \rightarrow AA$$

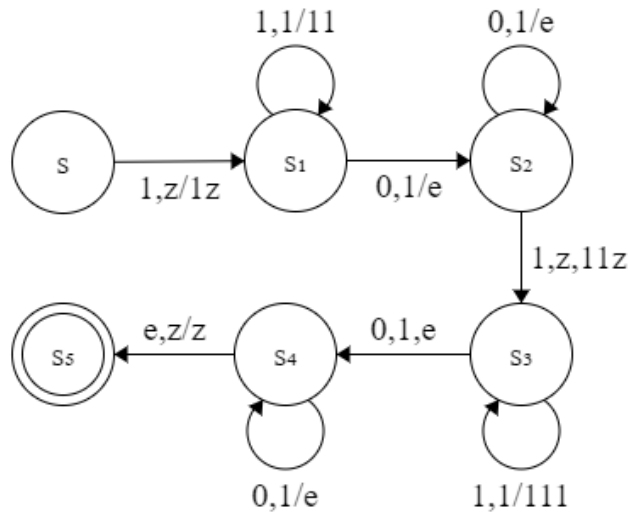
$$B \rightarrow S_1B_2|S_1S_1$$

$$B_2 \rightarrow BS_1$$

$$A \rightarrow S_3A_2|S_3S_3$$

$A_2 \rightarrow S_3A$

8. Design a PDA for $L(M) = \{1^n 0^n | n \geq 1\} \{1^n 0^{2n} | n \geq 1\}$



答案:

其中, $S-S_2$ 是判断 $\{1^n 0^n | n \geq 1\}$ 的过程, S_2-S_4 是判断 $\{1^n 0^{2n} | n \geq 1\}$ 的过程, S_5 为判断结束的最终状态。

9. Prove the language $L = \{x\#y | x, y \in \{0,1\}^* \text{ and } y \text{ is a substring of } x\}$ is not CFL with pumping lemma;

答案:

假设 L 是 CFL, N 为泵引理所说的正整数, 取字符串 $1^N 0^N \# 1^N 0^N$ 在 L 中

由泵引理存在 $z = uvwxy$ 满足 (1) $|vwx| \leq N$; (2) $|vx| \geq 1$ (3) $|vu^i wx^i y| \in L, i = 0, 1, 2, \dots$

若 vwx 在 $\#$ 前取 $i=0$, 显然不成立

若 vwx 在 $\#$ 后取 $i \geq 2$, 也不成立

若 vwx 包含 $\#$ 号

若 $\#$ 在 vx 中, 取 $i=0$ 新字符串不包含 $\#$ 显然不成立

若 $\#$ 不在 vx 中取,

若 $|x| \neq 0$, 则 $x_1 \neq 0$

取 $i=0$ 由于 $|vwx| \leq N$ 字符串变为 $1^N 0^{N-x_1} \# 1^{N-x_2} 0^N$, 此时 $1^{N-x_2} 0^N$ 不是 $1^N 0^{N-x_1}$ 的子串, 也不成立

若 $|v| \neq 0$, 则 $x_2 \neq 0$

取 $i=2$ 由于 $|vwx| \leq N$ 字符串变为 $1^N 0^{N+x_1} \# 1^{N+x_2} 0^N$, 此时 $1^{N+x_2} 0^N$ 不是 $1^N 0^{N+x_1}$ 的子串, 也不成立

所以 L 不是 CFG

10. Design Turing machine to compute n^2 . (start from 0^n to 0^{n^2})

答案: 起初是 $00 \dots 00$

S0→S1 变为 00...00A

S1→S3 将 A 左边的第一个 0 变成 1 之后返回到 A 的位置，开始一次加 n 操作，
S3, S4, S5 循环是将 A 右边 0 的个数加上 A 左边字符的个数，用 3 暂时代替 1，2 暂时代替 0，代表该数字已经被复制到右边。

变化过程： 00...0011..11A00..00

00...0033..33A00..0000..00 (此时 1 已经全部复制到右边)

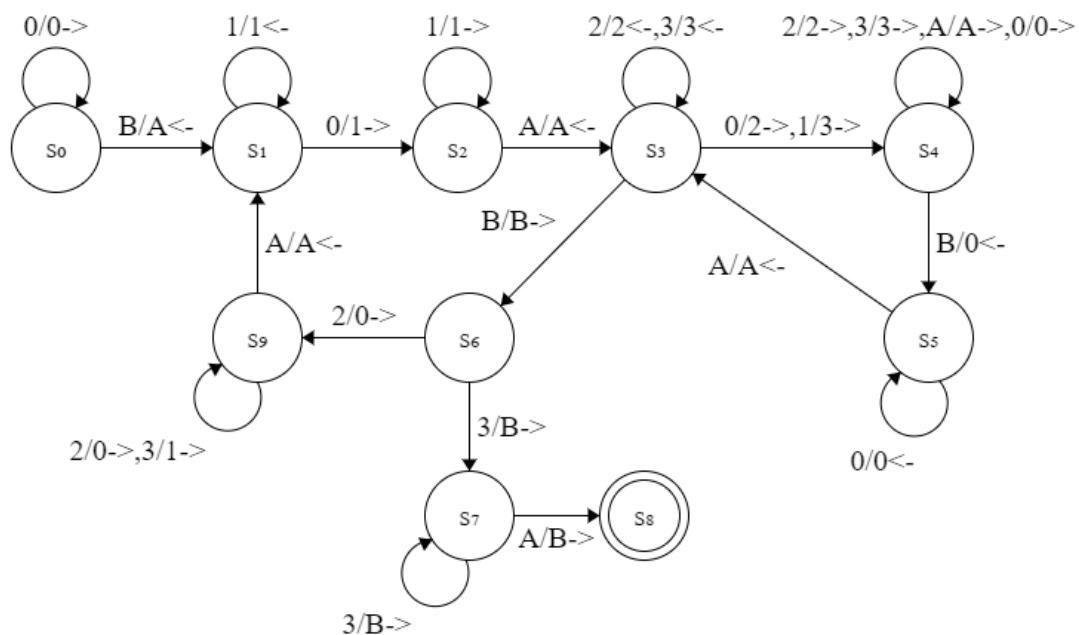
22...2233..33A00..0000..00 (此时 A 右边的 0 已经全部复制到右边)

S3→S6 发现 S 右边的 0，1 已经全部被替换为 2，3

S6→S7 发现 串此时为 33...33A00..0，说明右边已经进行了 n 次加 n 的操作，可以结束了，这时把 3 和 A 从串里边删去就行了

S6→S9→S1 把 2，3 还原为 0，1 开始下一轮的加 n 操作

M=({s1, s2, s3, s4, s5, s6, s7, s8, s9}, {0}, {A, 1, 2, 3}, δ, s0, B, {s8}) 如下图：



(其实 A 状态是可以省略的，但是为了理解和讨论方便，我还是加上了)

命题人：计算学部讲师团形式语言自动机命题组
命制时间：2022. 5. 4

主管
领导
审核
签字

1. [10 points] Design a DFA for $L = \{w \in \{0, 1\}^* \mid w \text{ has exactly three 0s.}\}$

2. [10 points] Design an NFA for the language:

$$L = \{w \in \{a, b, c\}^* \mid w \text{ starts with } ac \text{ and ends with } cb.\}$$

3. [10 points] Design regular expressions for languages over $\Sigma = \{a, b\}$.

(1) All strings that do not end with aba .

(2) $L = \{w \mid w \text{ has no more than 5 } a\text{'s.}\}$

4. [10 points] Prove that the language $L = \{w \in \{a, b\}^* \mid w = w^R\}$ is not regular with pumping lemma.

5. [10 points] Consider the following ε -NFA.

| | ε | a | b | c |
|-----------------|---------------|-------------|-------------|-------------|
| $\rightarrow p$ | $\{q, r\}$ | \emptyset | $\{q\}$ | $\{r\}$ |
| q | \emptyset | $\{p\}$ | $\{r\}$ | $\{p, q\}$ |
| $*r$ | \emptyset | \emptyset | \emptyset | \emptyset |

(1) Compute the ε -closure of each state.

(2) Give all the strings of length three or less accepted by the automaton.

(3) Convert the automaton to a DFA by subset construction. (diagram of transition function)

6. [10 points] Give a CFG for $L = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j + k\}$.

7. [10 points] Find a grammar equivalent to

$$S \rightarrow AB \mid CA$$

$$A \rightarrow a$$

$$B \rightarrow BC \mid AB$$

$$C \rightarrow aB \mid b$$

with no useless symbols.

8. [10 points] Design a PDA for $L_{eq} = \{w \in \{0, 1\}^* \mid w \text{ contains the same number of 0's and 1's}\}$.

9. [10 points] Prove or disprove: if L_1 is CFL and $L_1 \cup L_2$ is also CFL, then L_2 must be CFL.

10. [10 points] Design Turing machine for the language $\{0^{2n}1^n \mid n \geq 0\}$.

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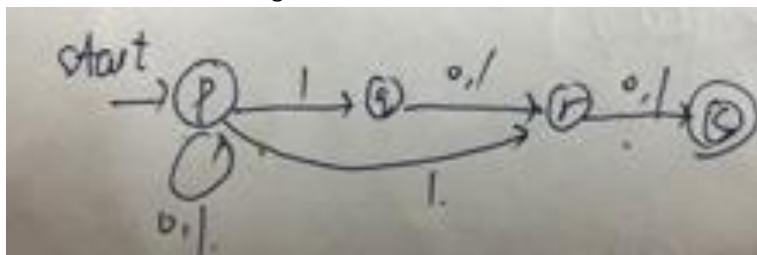
姓名

学号

班号

哈尔滨工业大学 2019 年《形式语言与自动机》期末试题

1. Design a DFA for the language $L = \{w \in \{0,1\}^* \mid w \text{ contains both } 01 \text{ and } 10 \text{ as substrings}\}$.
2. Design a NFA within four states for the language $\{a\}^* \cup \{ab\}^*$.
3. Design regular expressions for language over $\Sigma = \{0,1\}$.
 - (1). All strings contain the substring 001.
 - (2). All strings except the string 001.
4. Prove that $L = \{0^m 1^n \mid m/n \text{ is an integer}\}$ is not regular with pumping lemma.
5. Convert the following NFA into DFA with subset construction.



6. Give a context-free grammar for $L = \{a^i b^j c^i + j \mid i, j \geq 0\}$
7. Let L be the language generated by the grammar G below

$$S \rightarrow AB|BBB$$

$$A \rightarrow Bb|\epsilon$$

$$B \rightarrow aB|A$$
 - (1). 消除空产生式
 - (2). 消除单元产生式
 - (3). 转换到 CNF
8. Design a PDA for $L = \{w \in \{a,b\}^* \mid w \text{ has more } a\text{'s than } b\text{'s}\}$
9. Prove : for every context free language L , the language $L' = \{0|w||w \in L\}$ is also context free.
10. Design a Turing Machine that computes the following function $f: 0n \rightarrow \text{Binary}(n)$
 Where integer $n \geq 1$ and $\text{binary}(n)$ is the binary representation of n .
 For example: $f(03) = 11$ $f(05) = 101$.