Research Proposal for LSE Statistics

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1 Project Objectives

Market microstructure was investigated by Albert S. Kyle[6] where he showed how informed traders (IT) behave as monopolistic agents. He derived a linear relationship between the price impact of market orders and the limit order book depth. Glosten[4] solves the equilibrium price schedule for the IT by describing the dynamics between three groups of traders, elaborated further in **Section 2.2**. Goettler et al.[5] adapts Christine A. Parlour's [7] market microstructure framework to allow for potentially infinite period trading. These frameworks provide our basis of understanding for observable events in the market.

This project will carry out research in market microstructure dynamics over multiple periods. While the frameworks show how equilibrium prices can be established, the rate at which they converge and the conditions that affect such a rate have not been studied explicitly. Goettler et al.[5] have studied the effect of tick size on order book depth and market liquidity. Cont et al[12] have also been able to describe limit order book dynamics empirically and using SDEs.

This proposal investigates the limitations dealers face when trading with order books with differing tick sizes, in the presence of partially informed traders. I outline how to confirm some of Cetin and Waelbrock's findings[2], and how to examine the effect of tick sizes on order book depth and private signals.

The proposal is organized as follows. In **Section 2** we describe the limit order book, tick size, and the results afforded by Parlour et al.[5]. We recount the Glosten framework for order book dynamics as presented in [2, 4]. In **Section 3** We then propose an experiment that will determine when there is a statistically significant departure in spread size. Combining this detection scheme with numerical results shown from [9], we will present questions and further methodology for research.

2 Background

2.1 The Limit Order Book and The Parlour Market Framework

Goettler et al. present an infinite horizon of the Parlour framework [7]. They start with a discrete grid of N bid and N ask prices, $p^{-(N)}, p^{-(N-1)}, \ldots, p^{-1}, p^{1}, \ldots, p^{(N-1)}, p^{(N)}$ with discrete constant tick size d, where the bid prices are represented by the negative index and the asks are posted on the positive part of the index. At time t, each price level, p^i, has an associated outstanding size limit orders, ℓ_t^i. Such that the limit order book is a vector of these outstanding orders, $L_t = \left\{\ell_t^i\right\}_{(N-1)}^{N-1}$. at positions $-N$ and N there are infinite orders to truncate the prices.

In each period, t, a new trader enters the market and is represented by the tuple $\{z_t, \beta_t\}$ where $z_t \in \mathbb{N}_0$ represents the maximum number of shares they wish to buy(sell), β_t represents the premium above(below) the traders believes the asset is worth.

Definition 1.2: Consider a trade of size $\bar{x}$ at t. Then,

1. The wealth accrued to market order submitter is

$$W_t^m = \bar{x}(\beta_t + v_t - P_t(\bar{x})), \tag{1}$$

   where $P_t(\bar{x})$ is the average execution price.

2. The surplus accruing to limit order submitters taking the other side of the transactions is

$$W_t^l = \bar{x}(P_t(\bar{x}) - (v_t + \beta_{(t)}^l)). \tag{2}$$

   where $\beta_{(t)}^l$ is the share weighted private premiums of the limit order submitters.

From this formulation we see that the welfare gain(loss) from market order submitters is contingent on the tick size where smaller tick sizes lead to smaller losses and transaction costs. Likewise, welfare gained(lost) by the market maker is gained(lost) by the limit order trader on the other side. From this model, we see that their is an incentive for limit order traders to participate in an order book with large ticks, and market order traders prefer smaller ticks. Following the Glosten Market participant framework in the next section, we treat ITs as market order submitters and uninformed liquidity suppliers as limit order submitters. The Glosten framework will help us understand how the order book changes on the arrival of private signals. In **Section 3** we will be able to propose experiments to observe, the effect of tick sizes on order book depth as a proxy for market impact.

2.2 Glosten Market participant framework

The Glosten framework[4], takes place in a single period and has three classes of investors. 1. the competitive liquidity supplier, 2. competitive dealer and 3. $N \geq 1$ ITs who know liquidation value V of the asset. The initial limit order book is generated by liquidity suppliers that places limit orders. $h : \mathbb{R} \to \mathbb{R}$ is the function faced by market order when hitting a limit order level y. In the Glosten framework the market maker already has $Z \sim N(0, \sigma^2)$ number of shares of the asset, assumed to be independent of V. The market maker's role of smoothly moving the price to the true liquidation value to V is done through buying and selling shares from liquidity providers that create the initial order book.

Cetin[2] et. al show that the liquidation value with N ITs,

$$V = E^v \left[\frac{h(Z + Nx^*)}{N} + \frac{N-1}{N^2 x^*} \int_0^{Nx^*} h(Z+u)du \right] \tag{3}$$

was determined from first order conditions where x^* is the optimal demand for a single IT. E^v denotes the expectation operator of the investor where $V = v$.

Section 5.2 Figure 9 of [2] shows that for many ITs, $N \geq 5$, the bid ask spread increases with the arrival of unbounded (e.g., log-normal) signals. Despite the rationale that for a fixed V an increasing N would result in decreasing x^*, traders may have minimum order sizes, and are likely not be fully informed as to the number of other insiders that have received the private signal. Cetin et al. also states that market makers would change the market impact of ahead of a move to V as a result of a private signal. In the following section we detail simple experiments to verify the claim.

3 Research Plan

We propose an initial experiment to evaluate one of the observations in **Section 3**. Using Crobat [8] with ACoPrA[10] we can record instantaneous changes in order book depth on free exchanges. We would seek to determine whether a period in which spreads increase and there is a marked move in price to V would be preceded by an increase in order book depth. Similar to Dufrense [3] we take order book depth as a proxy for price impact.

Our detection scheme would implement a Radon Nikodym derivative to detect a change of measure from $\lambda_0\mu \to \lambda_1\mu, \forall \lambda_1\mu > \lambda_0\mu$ for a Compound Poisson random variable $\lambda\mu$ as described in [11] for:

1. rate of order book depth increase(decrease),
2. effective spread, and
3. rate of market order arrival increase(decrease).

The rate of limit order arrivals (1), λ in orders/second would be the rate of arrivals for limit order insertion net of cancellations on a single side (i.e., bid or ask) of the market. μ would be the mean size of net limit orders in an interval. The base distribution for the effective spread(2) would need to be further understood, but [1] implies that it may be Poisson-like. The rate of market order arrival (3) would be identical to the work I have previously done in [9], where the base rate is the rate of market order arrivals per second.

The detection scheme would declare periods where a marked shift to V is observed on a combination of CUSUM alarms. The initial experiment would reveal how market participants act on private signals prior to their arrival. After confirming Cetin's findings, we can apply results from [5] to compare order book fragility with increasing tick size. Contingent on positive results, we could also determine which order book may be more sensitive to private signals. Order books can be compared by examining the time series changepoints and determining whether the points are correlated, and then examining their co-linearity during statistically large increases in the bid ask spread.

The research plan requires some further understanding of the locally stationary distributions of observable market microstructure statistics. We will investigate methods to expand Cetin's observations to include limit order insertion from either market makers with private signals or ITs.

References

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