

6.1

解：系统的频率特性为：
$$G(j\omega) = \frac{36}{(j\omega)^2 + j13\omega + 36}$$

6.2

解：系统的闭环频率特性为：
$$\Phi(j\omega) = \frac{1}{2 + j\omega} = \frac{1}{\sqrt{4 + \omega^2}} e^{-j \arctg \frac{\omega}{2}}$$

系统在输入信号 $r(t)$ 的作用下，系统的稳态输出为：

$$\begin{aligned} c_{ss}(t) &= \frac{1}{\sqrt{5}} \sin\left(t + 30^\circ - \arctg \frac{1}{2}\right) - \frac{1}{2\sqrt{2}} \cos(2t - 45^\circ - \arctg 1) \\ &= \frac{1}{\sqrt{5}} \sin\left(t + 30^\circ - \arctg \frac{1}{2}\right) - \frac{1}{2\sqrt{2}} \sin(2t) \end{aligned}$$

系统的稳态误差：

$$\begin{aligned} e_{ss}(t) &= r(t) - c_{ss}(t) \\ &= \sin(t + 30^\circ) - \cos(2t - 45^\circ) - \frac{1}{\sqrt{5}} \sin\left(t + 30^\circ - \arctg \frac{1}{2}\right) + \frac{1}{2\sqrt{2}} \sin(2t) \\ &= \frac{\sqrt{10}}{5} \sin(t + 30^\circ + \arctg \frac{1}{3}) - \frac{\sqrt{10}}{4} \cos(2t - 45^\circ + \arctg \frac{1}{3}) \\ &= 0.632 \sin(t + 48.435^\circ) - 0.79 \cos(2t - 26.565^\circ) \end{aligned}$$

6.3

系统的闭环传递函数为：
$$\Phi(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

系统的幅频特性为：
$$A(\omega) = \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_n^2}\right)^2 + 4\zeta^2 \frac{\omega^2}{\omega_n^2}}}$$

相频特性：
$$\varphi(\omega) = -\arctg \frac{2\zeta \frac{\omega}{\omega_n}}{1 - \frac{\omega^2}{\omega_n^2}}$$

由系统的输入和稳态输出可知：

$$A(1) = 1 \quad \varphi(1) = -45^\circ$$

解得：
$$\omega_n = \sqrt{2 + \sqrt{2}} = 1.848, \quad \zeta = \frac{\sqrt{4 + 2\sqrt{2}}}{4} = 0.653$$

6-10 绘制下列传递函数的对数幅频渐近特性曲线

$$(3) \quad G(s) = \frac{8(\frac{s}{0.1} + 1)}{s(s^2 + s + 1)(\frac{s}{2} + 1)}$$

$$(4) \quad G(s) = \frac{1}{s(1 + 0.5s)(1 + 0.4s)}$$

$$(5) \quad G(s) = \frac{32(s + 2)}{s(s^2 + 4s + 16)}$$

解:

(3) 开环系统由以下典型环节组成: $\frac{8}{s}$, $\frac{s}{0.1} + 1$, $\frac{1}{s^2 + s + 1}$, $\frac{1}{\frac{s}{2} + 1}$

确定转折频率和斜率变化:

$\frac{s}{0.1} + 1$ 的转折频率为 $\omega_1 = 0.1$, 斜率变化 20dB/dec

$\frac{1}{s^2 + s + 1}$ 的转折频率为 $\omega_2 = 1$, 斜率变化 -40dB/dec

$\frac{1}{\frac{s}{2} + 1}$ 的转折频率为 $\omega_3 = 2$, 斜率变化 -20dB/dec

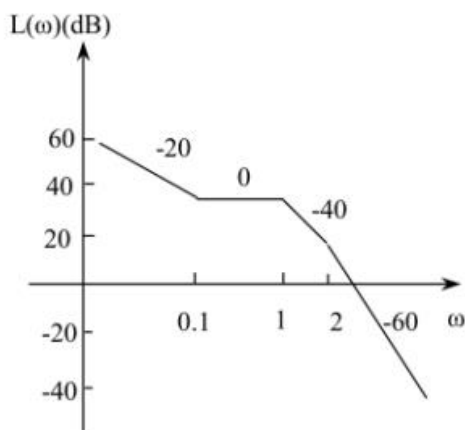
绘制低频段渐近特性曲线 ($\omega < \omega_1$),

因为 $v=1$, 所以低频渐近线斜率 $k=-20\text{dB/dec}$, 直线上一点为 $\omega_0=1$,

$$L_a(\omega_0) = 20 \lg K = 18\text{dB}$$

绘制频段 $\omega > \omega_1$ 渐近特性曲线:

$$\begin{aligned} \omega_1 \leq \omega < \omega_2, \quad k &= 0\text{dB/dec} \\ \omega_2 \leq \omega < \omega_3, \quad k &= -40\text{dB/dec} \\ \omega_3 \leq \omega, \quad k &= -60\text{dB/dec} \end{aligned}$$



(4) 这是 1 型系统

$$\text{基本环节: } \frac{1}{T_1 s} \cdot \frac{1}{T_2 s + 1} \cdot \frac{1}{T_3 s + 1}$$

$$\text{其中: } \frac{1}{T_1} = 1; \quad \frac{1}{T_2} = 2; \quad \frac{1}{T_3} = 2.5$$

图略。

(5) 1 型系统 $G(s) = \frac{32(s+2)}{s(s^2+4s+16)} = \frac{32 \times 2(0.5s+1)}{s \times 16(1+\frac{1}{4}s+\frac{1}{16}s^2)} = \frac{4(0.5s+1)}{s(1+\frac{1}{4}s+\frac{1}{4^2}s^2)}$

基本环节: $\frac{K_1}{T_1 s} \cdot T_2 s + 1 \cdot \frac{1}{1+2\zeta \frac{1}{\omega_n} s + \frac{1}{\omega_n^2} s^2}$

其中: $K_1=4, \frac{1}{T_1} = 1; \frac{1}{T_2} = 2; \omega_n = 4; 2\zeta = 1 \Rightarrow \zeta = 0.5$

$$20 \log K_1 = 20 \log 4 = 12 \text{ dB}; \zeta = 0.5 \Rightarrow M_m = \frac{1}{2\zeta \sqrt{1-\zeta^2}} = 1.15 \text{ dB}$$

图略。