BUREAU'S

HIGHER SECONDARY (+2)

BUSINESS MATHEMATICS AND STATISTICS

PART-I

(For +2 1st Year Commerce)

(Approved by the Council of Higher Secondary Education, Odisha,)

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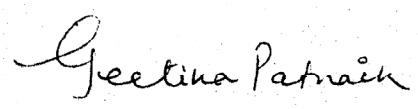
FOREWORD

The Odisha State Bureau of Textbook Preparation and Production, Bhubaneswar has made a pioneer attempt to publish text books for +2 Commerce Stream with an excellent team of teachers in different subjects.

The present book **"Business Mathematics and Statistics - Part I"** is meant for Higher Secondary Commerce students. This book has been written by a team of learned academicians namely Dr. Ranjit Mishra, Dr. Uma Chand Lal, Dr. R.C. Jena and Dr P.K. Parida and reviewed by eminent professor of the state Prof A. P. Nayak. I would like to record my sincere gratitude to all of them for accomplishing this maiden venture in time. The main purpose of developing this text book is to provide a thorough exposure to the students of Commerce in this subject. The book prepared according to the new syallabus prescribed by the CHSE, Odisha, and as per new pattern of questions, shall cater to the needs of young students. I owe my special thanks to Prof. A.P. Nayak for making necessary revision in the text and questions according to new pattern.

I believe that the students and teachers of commerce stream shall welcome and appreciate the book. I would also like to welcome constructive suggestions for further improvement of the book.



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PREFACE

Elementary knowledge in Mathematics & Statistics has assumed immense importance in every sphere of human activity. Its application is found in various diversified fields such as business, industry, management, economics, planning, agriculture, insurance, sociology, biomtry etc. and also in many professional courses. The Odisha State Bureau of Textbook Preparation and Production, Bhubaneswar through its Board of Writers and Reviewers, has presented this new edition of the book, "Business Mathematics and Statistics" for the first year of +2 Commerce students to have them a simple and comprehensive exposition to application of the quantitative techniques.

We, the writters are pleased to commend the readers the new edition of this book which is commensurate with the latest syllabus of CHSE, Odisha. The textbook is prepared to fully cater to the needs of the students both in terms of the course content as well as the level of knowledge required to excel in the examination. Further the textbook is throughly reviewed by experts to incorporate the requirements of the students.

The main features of the book are :

- The book has two parts, namely Mathematics and Statistics
- Simplicity of expression
- Sufficient illustrations to tackle practical problems
- Systematic presentation of subject matter
- Latest course content
- Meaningful focus on new question pattern

The book is unique in its presentatin because steps have been taken to keep pace with new syllabus and new pattern of questions. Sufficient Multiple Choice Questions and other objective type questions are provided along with their answers in addition to long questions.

We wish to thank the Odisha State Bureau of Textbook Preparation and Production, Bhubaneswar for their efforts and co-operation in the publication of the book in time.

Any suggestion for improvement of the book will be highly appreciated.

Board of writers

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- Logarithm Table
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SYLLABUS

BUSINESS MATHEMATICS AND STATISTICS
FIRST YEAR, PAPER- I

Objective

- To enable the students to learn basic concepts of Profit & Loss with their applications;
- To learn the concept, features, types of partnership and calculation of Profit sharing ratios;
- To develop an understanding about concept, properties, laws of logarithmic operations;
- To enable the students to learn the concept of simple & compound interest along with their calculation;
- To help the students in learning the concept of Annuity, Bills of Exchange and stock & Share,
- To provide a conceptual clarity about statistical framework and survey, and
- To enable the students to understand the concept of data, process of data collection, classification & tabulation of data;

Course Inputs

Unit-I Business Arithmetic:

(i) **Profit and Loss :** Concept, Calculation of Selling Price, Cost Price, Profit or Loss percentage, Concept & Types of Price, Discount.

(ii) **Partnership :** Concept, Characteristics, important terms, types of partnership, calculation of Profit Sharing Ratio.

(iii) **Logarithm:** Concept, Features, Properties, types & methods of Logarithm, Laws of Logarithmic Operation, Determination of Log & Antilog under the common logarithm system, interpolation of logarithms, Methods of Logarithm.

(iv) **Simple & Compound Interest :** Concept, Types, Calculation of simple interest, Concept & features of Compound Interest and calculation of compound interest. (Simple problems only)

Unit-II Business Arithmetic

(i) **Annuity :** Concept, Types, special application of annuity, techniques of calculation of Annuity Problems (Future value & Present value of Annuity), Sinking Fund, Loan, overdraft or borrowings.

(ii) **Discounting of Bills of Exchange:** Concept, Features, Types of Bills of Exchange, Important terms (Demand Bill, Time Bills, Terms of Bills Days of Grade, Bill Value, Discounted Period, Banker's Discount. Discounted Value, Present Value, True Discount, Banker's Gain.

(iii) **Stock & Share :** Concept, Features, Classification of Stock and Shares, Share Vs Stock, calculation of Dividend, Yield, income, cost of investment, Nominal and Market value of Shares, exdividend and cum dividend price of a share.

Unit-III Business Statistics :

(i) **Conceptual Framework of Statistics:** Meaning, Definition, Origin & Growth, (ii) Characteristics, Scope & functions of Statistics, (iii) Relationship of Statistics with other fields, (iv) Statistical Survey: meaning and its steps.

Unit-IV Business Statistics:

(i) **Data :** Types of Data - Primary and Secondary Data, (ii) Methods of Collection of Primary & Secondary data, (iii) Classification of Data and Tabulation of Data.

CHAPTER-1

PROFIT & LOSS

STRUCTURE

1.1 Meaning
1.2 Terms used
1.3 Important points
1.4 Standard Formulae
1.5 Applications
- 1.5.1 Calculation of selling price
- 1.5.2 Calculation of cost price
- 1.5.3 Percentage of profit or loss
- 1.5.3 Discounts
- 1.5.4 Types of Price / Invoice
- 1.5.5 Dishonest gain
- 1.5.6 Exercise

1.1 MEANING :

A business man either produces or purchases goods to sell them at a profit. The price at which an article is purchased is called cost price (C.P) and the price at which it is sold is called the selling price (S.P). The difference between the selling price and cost price is either profit or loss. The profit or loss can be expressed as follows:

Profit = selling price - cost price, when S.P > C.P.

Loss = cost price - selling price, when S.P<C.P

1.2 TERMS USED IN PROFIT & LOSS

(a) Cost price is the cash or cash equivalent value sacrificed by the purchaser to obtain some goods or services.

(b) Selling price is the price which the seller charges for selling one unit of goods or services.

(c) Marked Price is the price which is printed on the cover of the article, It is also called catalogue price or list price.

(d) Gross profit is the difference between selling price and cost of goods sold.

(cost of goods sold = opening stock + Net purchases + Expenses on purchase - closing stock).

(e) Net profit = Gross profit - Indirect Expenses

(Indirect Expenses = Factory expenses + office expenses + selling & Distribution expenses). These are also known as overheads).

1.3 IMPORTANT POINTS TO REMEMBER :

Profit or loss percentage is always calculated on cost price unless otherwise stated.

1.4 STANDARD FORMULAE TO FIND PROFIT/LOSS PERCENTAGE

(a) Profit percentage on cost price $= \frac{\text{Pr ofit}}{\text{Cost Pr ice}} \times 100$

(b) Loss percentage on cost price $= \frac{\text{loss}}{\text{Cost Pr ice}} \times 100$

(c) Profit percentage on selling price $= \frac{\text{Pr ofit}}{\text{Selling Pr ice}} \times 100$

(d) Loss percentage on selling price $= \frac{\text{loss}}{\text{selling price}} \times 100$

1.5 Applications

1.5.1 Calculation of selling price :

The price obtained by selling goods or services to a customer is called selling price. Any expenditure incurred relating to sales are to be deducted to get the net sales price.

Thus, SP = Selling Price - commission paid to the broker - any other expenditure incurred in connection with sale.

However, if the goods are expressed in marked price | list price | catalogue price, then selling price would be marked price - Trade Discount, where marked price is the price which is

printed on the face of an article or on the cover of its package.

So selling price = Marked Price - Trade Discount or SP=MP-TD

Trun over = Total cash value of sales

Selling price can also be calculated when :

i) Cost price and profit are known :

Selling Price = Cost price + Profit

ii) Cost Price and loss are knwon :

Selling Price = Cost price - Loss

iii) Cost price and profit percent are known :

$$\text{Selling price} = \text{Cost price} + \left(\frac{\text{Profit percent}}{100} \times \text{cost price}\right)$$

$$\left(\text{Cost Price} \times \frac{100 + \text{Profit percent}}{100}\right)$$

iv) Cost price and loss percent are known:

$$\text{Selling price} = \text{cost price} - \left(\frac{\text{loss percent}}{100} \times \text{cost price}\right)$$

or

$$\text{Selling Price} = \text{cost price} \times \left(\frac{100 - \text{loss percent}}{100}\right)$$

1.5.2 Calculation of cost price :

Price paid by the buyer to buy an article from the seller is called cost price. It also includes the expenses incurred in connection with the purchase.

Thus, cost price (C.P) = Purchase Price + carriage on purchase + octroi + any other expenses incurred in connection with purchases.

Again cost price can be calculated when :

i) Selling price along with profit per cent on selling price is given :

$$\text{Cost price} = \text{Selling price} - \left(\frac{\text{Profit per cent}}{100} \times \text{selling price}\right)$$

ii) Selling price along with loss per cent on selling price is given :

$$\text{Cost Price} = \text{Selling Price} + \left(\frac{\text{Loss per cent}}{100} \times \text{selling price}\right)$$

1.5.3 Profit or loss per cent :

Example: 1

Calculate profit or loss percentage under the following situations:

(i) When C.P. of an article is ₹240 and selling price is ₹300.

(ii) When cost price of the article is double the selling price of an article.

(iii) When there is a gain of 20% on selling price.

(iv) A man buys a radio for ₹1000 and sell & the same for ₹800.

(v) Cost price of 120 pencils = selling price 100 pencils.

(vi) When there is a loss of 20% on selling price.

Answer :

(i) Cost price of the article = ₹240

Selling price of the article = ₹300

So profit = Selloing price - Cost price

= ₹(300-240)

= ₹60

$$\therefore \text{Profit per cent} = \frac{\text{Profit}}{\text{Cost Price}} \times 100$$

$$= \frac{60}{240} \times 100 = 25\%$$

i.e., Profit = 25%

(ii) When cost price > Selling price = loss

Let the selling price be ₹100

Then cost price is = ₹200

$\therefore$ Loss = cost price - selling price

= ₹200 - ₹100 = ₹100

Hence percentage of loss = $\frac{\text{loss}}{\text{cost price}} \times 100$

$= \frac{100}{200} \times 100 = 50\%$

$\therefore$ Loss = 50%

(iii) Let selling price be ₹100

$\therefore$ Profit is 20

Hence, cost price= S.P - Profit

= 100-20

= 80

$\therefore$ Percentage of profit on cost price $= \frac{\text{profit}}{\text{C.P}} \times 100$

$= \frac{20}{80} \times 100$

$= 25\%$

(iv) As per question

S.P = ₹800

and C.P = ₹1000

$\therefore$ loss = C.P - S.P = ₹1000- ₹800 = ₹200

Hence percentage of loss = $\frac{\text{loss}}{\text{C.P}} \times 100$

$$= \frac{200}{1000} \times 100$$

$$= 20\%$$

(v) Let the cost price of 120 pencils be ₹100

and selling price of 100 pencils = ₹100

$\therefore$ Selling price of 120 pencils $= \frac{100}{100} \times 120 =$ ₹120

Hence profit = ₹120 - ₹100 = ₹20

and profit percentage $= \frac{\text{profit}}{\text{C.P}} \times 100$

$$= \frac{20}{100} \times 100 = 20\%$$

(vi) Let selling price be ₹100

$\therefore$ loss on S.P $= \frac{20}{100} \times 100 = 20\%$ or ₹20%

$\therefore$ cost price = S.P + loss

= ₹(100+20)

= ₹120

$\therefore$ loss percentage $= \frac{\text{loss}}{\text{C.P}} \times 100$

$$= \frac{20}{100} \times 100 = 16\frac{2}{3}\%$$

1.5.3 Discounts :

Discount is the reduction or rebate on the selling price or marked price of on article allowed by the seller to the buyer. It is usually expressed as a percentage. Discounts are of two types such as (i) Trade discount (ii) Cash discount.

Trade discount : It is allowed on marked price (M.P) or list price or catalogue price of an article.

Cash discount : It is allowed on selling price or on amount due from credit customer.

If trade discount and cash discount both are allowed by the seller, then first trade discount is calculated and then cash discount is calculated on the reduced balance as follows:

Marked Price – Trade Discount = Selling price

Selling price – Cash Discount = Net cash receipts

Successive Discounts : If a seller allows a series of discounts and each on preceeding reduced balance then, it is a case of successive discounts.

Example : 12%, 8% and 5%

Formula to calculate single equivalent rate of discount percentages (dx), when d_1, $d_2$and $d_3$are discount percentage respectively, is

$$dx = 100 - \frac{(100 - d_1)\ (100 - d_2)\ (100 - d_3)}{(10)^4}$$

Example : 2

An article's marked price is ₹5000.

The seller sells it at 12% discount. Find the buyer's cost price and amount of discount.

Solution : Let the M.P = ₹100

Discount = 12%

Cost price to the buyer = ₹(100-12) = ₹88

According to question M.P = ₹5000

When M.P	Discount	C.P for buyer
₹100	12	₹88
₹5000	$\frac{12}{100}\times 5000=600$	₹$\frac{88}{100}\times 5000=$₹4400

$\therefore$ cost price to the buyer = ₹4400

Discount = ₹600

Example : 3

At what price an article is to be marked to get a profit of 26% after allowing 10% trade discount, if the cost price of the article in 100 ?

Solution :

According to the question cost price = ₹100

Profit is 26% means ₹26

Net selling price = ₹(100+26) = ₹126

Which is after 10% trade discount.

Let marked price be x

Trade discount is 10% of $x = x \times \frac{10}{100} = \frac{x}{10}$

$\therefore$ Net selling price $= x - \frac{x}{10} = \frac{9x}{10}$

We know that C.P + Profit = Net selling price = ₹126

again M.P - T.D = Net selling price = $\frac{9x}{10}$

$\therefore x = \frac{126}{9/10} = 126 \times \frac{10}{9} =$ ₹140

$\therefore$ Marked price of the article = $\therefore$ ₹140

Example : 4

A seller allows 10% trade discount and 20% cash discount. Find the cash selling price if M.P of an article is ₹3000.

Solution :

Marked Price	= ₹3000
Less Trade discount 10%	300
Selling Price -	₹2700

Less cash discount 20%	540
Cash selling price	₹2160

Example : 5

The list price of a refrigerator is ₹17000. It is sold at a discount of 8%. If further off season discount of 10% is given, then find the selling price of the refrigerator.

Solution :

List price of the refrigerator	₹17,000
	1,360
Balance	₹15,640
Less off season discount @10%	₹1564
Selling price	₹14,076

Example : 6

Shopkeeper A gives two successive discount of 25% and 10% on his goods. Shopkeeper B gives successive discounts of 20% and 15%. From whom the customer would prefer to buy the goods ?

Solution :

From the shopkeeper A, single discount rate for the buyer is

d_1=25% and d_2=10%

$$\therefore\ dx = 100 - \frac{(100 - d_1)\ (100 - d_2)}{10^2}$$

Where d*x* = single equivalent rate of discount.

$$= 100 - \frac{(100 - 25)\ (100 - 10)}{100}$$

$$= 100 - \frac{(75)\ (90)}{100} = 32.5\%$$

Single equivalent discount rate from the sells B

when d_1= 20% and d_2 = 15%

$$dx = 100 - \frac{(100 - d_1)\ (100 - d_2)}{10^2}$$

$$= 100 - \frac{(100 - 20)\ (100 - 15)}{100}$$

$$= 100 - \frac{(80)\ (85)}{100} = 32\%$$

So the customer would prefer to buy from shopkeeper A as he gives higher discount.

Example:7

The Marked price of an article is ₹650. A customer pays ₹561.60 for it. He was allowed two successive discounts. If the first discount rate is 10%, find the 2nd discount rate.

Solution :

Marked Price of the article is ₹ 650

1st discount = 10% = ₹ 65

Balance after 1st discount ₹ 585

again let the 2nd discount is d_2

Price paid by customer

after 2nd discount ₹ $585 - d_2$

Price paid by customer

after 2nd discount ₹$585 - d_2$

But as per question price paid by customer = ₹561.60

$\therefore\ 585 - d_2 = 561.60$

or $d_2 = 585 - 561.60 = 23.40$

$$\therefore\ d_2\% = \frac{23.40}{585} \times 100 = 4\%$$

N.B.: Marked price can also be calculated when percentage of profit and the rate of discount is given by applying the following formula :

$$M.P = C.P \times \left\{\frac{100 + \text{Profit percent}}{100 - \text{Discount rate}}\right\}$$

Example : 8

A manufacturer makes a profit of 10% on cost after allowing trade discount of 20% if the cost of manufacture is ₹8,000, what is its list price ?

Solution :

Given C.P = ₹8,000, Profit percent = 10, Discount percent = 20

$$M.P = C.P \times \left(\frac{100 + \text{Profit percent}}{100 - \text{Discount rate}}\right)$$

$$= 8000 \times \left(\frac{100 + 10}{100 - 20}\right)$$

$$= 8000 \times \left(\frac{110}{80}\right) = ₹11,000$$

Proof :

M.P = ₹11,000

$$S.P = M.P - TD = ₹11,000 - \left(\frac{20}{100} \times 11000\right)$$

$\Rightarrow$ 11,000 - 2200 = ₹ 8,800

Again profit per cent is 10

When C.P is 100, S.P = ₹110

Or alternatively :

When S.P. = ₹110 C.P, ₹100

When S.P = 1 $C.P = \frac{100}{110} = \frac{10}{11}$

When S.P = 8,800 $C.P = \frac{10}{11} \times 8800 = ₹8,000$

Hence proved i.e., C.P is ₹8000

1.5.4 Types of price quotations/Invoice

Invoice may be defined as a list of goods sold with the price (prices) charged. It is prepared by the supplier and sent on supply of goods to the purchaser. It contains the information regarding the quantity, quality and price of the goods sold.

The following terms are usually used in making invoice:

(a) C.O.D. indicates cash on delivery, where many is paid on delivery of goods.

(b) F.O.R. means 'free on rail' This means that all charges upto the loading of goods in wagon will be paid by the seller.

(c) LOCO Invoice - This means only the manufacturer's price i.e., exfactory price or ex-godown price.

(d) FAS means 'free along side ship', this includes manufacturer's price, packing, carriage to railway station or dock, dock dues and duty.

(e) FOB means free on board. This means FAS along with boarding charges to ship or cargo plane but excluding the freight charges.

(f) C and F (cost and freight) This includes all expenses for carrying the goods to the destination except the insurance claim.

(g) C.I.F. This means cost, insurance and freight. This includes C & F, and insurance.

(h) Franco. In this case exporter bears all expenses for sending goods to the importer's place.

1.5.5 Dishonest Gain

Dishonest gain means cheating the supplier or customer in weight while purchasing or selling the goods.

$$\text{Percentage of gain} = \frac{\text{True weight} - \text{False weight}}{\text{False weight}} \times 100$$

Example : 9

A trader defrauds by means of false balance to the extent of 10% in buying and to the same extent in selling an article. What percent does he gain or lose on his outlay by defraud.

Solution

The trader defrauds 10% in buying, therefore he purchases the article worth ₹110 by paying ₹100. His selling price of article of ₹100 at ₹110.

cost price	selling price
₹100	₹110
₹110	$\frac{110}{100} \times 110 = 121$

He will sell the article at ₹121.

$\therefore$ Profit or Gain = (121-100) = ₹21

Example : 10

A dishonest dealer claims that he sells his goods at the cost price but was a false weight of 900 gm. instead of 1kg. weight. Find his gain per cent.

Solution

Let C.P of 1 kg. = ₹100

$\therefore$ C.P of 900gms = ₹90

since the dealer weights 900 gms instead of 1 kg.

$\therefore$ S.P of 900 gms = C.P of 1 kg = ₹100

Profit by selling 900 gm = ₹100 - ₹90 = ₹10

Now profit on ₹90 is ₹10

$$\therefore \text{ His profit or gain per cent} = \frac{10}{90} \times 100 = \frac{100}{9} = 11\frac{1}{9}\%$$

By short cut method

$$\% \text{ of dishonest gain} = \frac{\text{True weight - False weight}}{\text{False weight}} \times 100$$

$$= \frac{1000\text{gms} - 900\text{gms}}{900\text{gms}} \times 100$$

$$= \frac{100}{900} \times 100 = \frac{100}{9} = 11\frac{1}{9}\%$$

MISCELLANEOUS EXAMPLES :

Example : 11

Selling price of an article is ₹7000 and its loss per cont is $12\frac{1}{2}$ %, Calculate the cost price of the article.

Solution :

Let the C.P of the article is x

$\therefore$ loss is $12\frac{1}{2}\% = \frac{25}{2}\%$ on C.P

$\therefore$ loss $= x \times \frac{25}{200} = \frac{x}{8}$

$\therefore$ S.P = C.P - loss

$= x - \frac{x}{8} = \frac{7x}{8}$

$\therefore$ When S.P is $\frac{7x}{8}$, C.P is x

When S.P is 1 , C.P is $\frac{x \times 8}{7x}$

When S.P is 7000, C.P $= \frac{x \times 8}{7x} \times 7000$

$=$ ₹8000/-

C.P of the article is ₹8,000.

Example : 12

A Trader sold an article for ₹2250 and earned a profit of 20% on S.P. Find the C.P of the article.

Solution :

Selling price ₹2250

and profit is 20% on S.P

hence profit $= ₹2250 \times \frac{20}{100} = ₹450$

$\therefore$ C.P = S.P – Profit

$\Rightarrow$ C.P = ₹(2250 – 450)

= ₹1,800

Example : 13

If 7% of the sale price of a cycle is equal to 8% of its cost price and 9% of the sale price exceeds 10% of the cost price by ₹5, find the cost price and the sale price.

Solution :

Let S.P ₹x and C.P = ₹y

As per question

7% of x = 8% of y

or $\frac{7x}{100} = \frac{8y}{100}$

or $7x = 8y$ ... ... (i)

Again 9% of x - 10% of y = ₹5

or $9x - 10y = 500$ ... (ii)

Solving (i) and (ii) for x and y

$9\left(\frac{8}{7}y\right) - 10y = 500 \quad \because x = \frac{8}{7}y$

or $\frac{72y}{7} - 10y = 500$

or 72 y – 70y = 3500

$\therefore$ y $= \frac{3500}{2} =$ ₹17500

Again $x = \frac{8}{7} \times 17{,}500 =$ ₹20,000

$\therefore$ S.P = ₹20,000 and C.P = ₹17,500

Example : 14

A man sells an article at a loss of 10%. If he had sold it for ₹30 more, he would have gained 5%. Find the C.P of the article.

Solution :

Let the C.P of the article be ₹x

$\therefore$ S.P = C.P – loss

$$\text{S.P} = x - \frac{10}{100}x$$

$$\text{S.P} = x - \frac{x}{10} = \frac{9x}{10}$$

$$\text{again S.P at 5\% profit} = x + \frac{5}{100}.x$$

$$\text{S.P} = x + \frac{x}{20}$$

$$\text{S.P} = \frac{21x}{20}$$

$$\therefore \text{ difference in selling price} = \frac{21x}{20} - \frac{9x}{10} = \frac{3x}{20}$$

So when difference in two selling price is $\frac{3x}{20}$, C.P = x

When difference in two selling price is 1, C.P = $\frac{x}{3x/20}$

$\therefore$ When difference in two selling price is ₹30

$$\text{then C.P} = \frac{x \times 20}{3x} \times 30 = ₹200$$

Example : 15

A man bought a horse and a cow for ₹150,000. He sold the horse at 11% profit and cow at 26% profit, making 19% profit on the whole. Find the C.P of the horse and the cow separately.

Solution :

Let the C.P of the horse be x

$\therefore$ C.P of the cow is (₹150,000 - x)

19% is the profit on the whole

$$\therefore \text{Total profit} = \frac{19}{100} \times ₹150,000$$

$$= ₹28,500$$

again profit on horse = 11%

$$\text{i.e., } = \left[\frac{11}{100} \times x\right] = \frac{11x}{100}$$

and profit on cow is 26%

$$\text{i.e., } = \frac{26}{100} \times (150,000 - x)$$

As per question

Profit on horse + profit on cow = ₹28,500

$$\text{or } \frac{11x}{100} + \frac{26}{100} \times (150,000 - x) = ₹28,500$$

$$\text{or } \frac{11x + 26\ (150,000 - x)}{100} = ₹28,500$$

or $11x + 39,00,000 - 26x = 28,50,000$

or $-15x = 28,50,000 - 39,00,000$

$$\therefore -x = -\frac{10,50,000}{15}$$

$\therefore -x = -$ ₹70,000

$\therefore$ cost of horse ₹70,000

$\therefore$ cost of cow = ₹150,000 – ₹70,000 = ₹80,000

$\therefore$ C.P of Horse = ₹70,000

and C.P of cow = ₹80,000

Example : 16

A man sells his house at 5% profit if he sells it for ₹2,500 less, he will gain $3\frac{3}{4}\%$, Determine the cost of the house.

Solution :

Let the Cost price of the house ₹x

$\therefore$ Selling price $= x + \frac{5}{100}x = \frac{105x}{100}$

again selling price at $3\frac{3}{4}\%$ profit $= x + \frac{15}{400}x$

$$= \frac{415}{400}x$$

$\therefore$ Difference in two selling price

$$= \frac{105x}{100} - \frac{415x}{400} = \frac{420x - 415x}{400} = \frac{5x}{400}$$

$\therefore$ When Difference in two S.P of house is $\frac{5x}{400}$, then C.P is x

When Difference in S.P is 2,500 then C.P is $\frac{x \times 400}{5x} \times 2500$

$$= \frac{x \times 400}{5x} \times 2500 = ₹2,00,000$$

$\therefore$ Cost price of the house is ₹2,00,000

Example : 17

The marked price of an article is 25% above the selling price and its cost price is 40% below the marked price. Find the rate of discount allowed and profit.

Solution :

Let the S.P of the article is ₹100

$\therefore$ M.P of the article = $100 + \frac{25}{100}$ of $100 = ₹125$

$\therefore$ Rate of Discount:

When M.P is 125, Discount is 25

When M.P is 1, Discount is $\frac{25}{100}$

When M.P is 100, Discount is $\frac{25}{100} \times 100 = 20\%$

As per question,

Again C.P = M.P - Discount

$$= 125 - \left(\frac{20}{100} \times 125\right) = ₹75$$

$\therefore$ Profit = S.P – C.P = ₹(100–75) = ₹25

$\therefore$ Rate of profit : = $\frac{\text{profit}}{\text{C.P}} \times 100$

$$= \frac{25}{75} \times 100 = 33\frac{1}{3}\%$$

Example : 18

A man purchased a T.V and a Ratio. If he sells the T.V at 10% loss and Radio at 20% profit, he neither gains nor loses. If he sells the T.V at 5% gain and Radio at 5% loss, he gains ₹1,000 in all. Find the cost price of each.

Solution :

Let the cost price of the T.V and

Radio be ₹x and ₹y respectively

$\therefore$ Total cost price = ₹$(x+y)$

Selling price of T.V of 10% loss

$$= x + \frac{10}{100}x = \frac{9x}{10}$$

Selling price of Radio at 20% profit

$$\therefore \text{ S.P of Radio} = y + \frac{20}{100}y = \frac{6y}{5}$$

$$\therefore \text{ Total S.P} = \frac{9x}{10} + \frac{6y}{5} = \frac{9x + 12y}{10}$$

As there is no gain or loss

$$\therefore \text{ S.P} = \text{C.P}$$

hence $\frac{9x + 12y}{10} = x + y$

$$\Rightarrow 9x + 12y = 10x + 10y$$

$$\Rightarrow x = 2y \text{ ..... Eq. 1}$$

In 2nd case :

S.P of T.V at 5% gain

$$= x + \frac{5}{100}x = \frac{21x}{20}$$

and S.P of Radio at 5% loss

$$= y - \frac{5}{100}y = \frac{19y}{20}$$

$$\therefore \text{ Total S.P of T.V and Radio} = \frac{21x}{20} + \frac{19y}{20}$$

$$= \frac{21x + 19y}{20}$$

$$= \frac{21x + 19y}{20} - (x + y) = \frac{x - y}{20}$$

As per question

$$\frac{x-y}{20} = ₹1{,}000$$

$\therefore x - y = ₹20{,}000$ Eq.2

Now putting the value of x in Eq. 2.

$2y - y = ₹20{,}000$

$\Rightarrow y = ₹20{,}000$

and $x = 2y$ as per Eq.(1), hence $x = ₹40{,}000$

$\therefore$ C.P. of T.V = ₹40,000

& C.P of Radio = ₹20,000

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EXERCISE

GROUP-A

MULTIPLE CHOICE TYPE QUESTIONS:

Select the correct answer from the given alternatives

Q.1.

(i) By selling an article at ₹ 560 a man incurs is loss of 20%, then the cost price of the article is :

(a) ₹ 700 (b) ₹ 672 (c) ₹ 448 (d) ₹ 720

(ii) If the cost price of an article is ₹ 550 and selling price is ₹ 660, then profit percentage is :

(a) 10% (b) 15% (c) 20% (d) 25%

(iii) If a trader makes profit of $33\frac{1}{3}\%$ on cost price by selling an article, the percentage a profit on selling price is :

(a) $16\frac{2}{3}\%$ (b) 20% (c) 25% (d) $33\frac{1}{3}\%$

(iv) If cost price is ₹ 160 and selling price is ₹ 120, the loss percentage is:

(a) 20% (b) 25% (c) 30% (d) $33\frac{1}{3}\%$

(v) If marked price is ₹ 600 and trade-discount is 15%, the selling price is :

(a) ₹ 540 (b) ₹ 575 (c) ₹ 510 (d) ₹ 525

(vi) If the marked price of an article is ₹ 1000 and the trader allow a discount of 12% and 10% then selling price is :

(a) ₹ 780 (b) ₹ 792 (c) ₹ 782 (d) ₹ 790

(vii) A trader sold a radio at a loss of $6\frac{1}{4}$% on selling price. If the cost price of the radio is ₹ 3400, then selling price is :

(a) ₹ 3200 (b) ₹ 3400 (c) ₹ 3388.50 (d) ₹ 2890

(viii) If article is sold at a profit of ₹ 120. Which represents 20% of the selling price, then the cost price of the article is:

(a) ₹ 480 (b) ₹ 560 (c) ₹ 600 (d) ₹ 660

(ix) Trade discount is a rebate or allowance allowed to buyer, which is a deduction from :

(a) Selling Price (b) Cost Price (c) List Price (d) Net selling Price

(x) Cash discount is allowed on :

(a) cost price (b) selling price (d) marked price (d) Net selling price

Ans. (i) a, (ii) c, (iii) c, (iv) b, (v) c, (vi) b, (vii) a, (viii) a, (ix) c, (x) d

Q.1. (B) SHORT TYPE QUESTIONS.

Answer the following in one sentence each :

(a) What do you mean by profit and loss ?

(b) Define marked price.

(c) What is Trade discount ?

(d) What do you mean by successive discount ?

(e) Define cost price.

(f) What is cash discount ?

(g) What is selling price ?

(h) Why is cash discount allowed ?

(i) Which discount is not recorded in the books of accounts ?

(j) Which discount is deducted from selling price in order to findout net cash recipts ?

2.(*a*) Express in one word/term.

(i) A series of discount allowed by a wholesaler to retailer on product, is :

(ii) The price printed on the article.

(iii) The term representing the excess of revenue over the cost.

(iv) The price representing the cash value sacrificed to obtain goods.

Ans. (i) discount, (ii) Marked/list/catalogue price, (iii) profit, (iv) cost price

(b) Answer in one sentence each.

(i) When the selling price is increased from ₹110 to ₹120, what is the percentage increase in price ?

(ii) What is quotation ?

(iii) What is net selling price ?

(iv) What do you mean by an invoice ?

(c) Correct the underlined portion of the following sentences.

(i) The excess cost over revenue is <u>profit</u>.

(ii) Profit or loss percentage is usually calculated on <u>selling</u> price.

(iii) Trade Discount is always calculated on <u>selling</u> price.

(iv) For prompt payment by the credit customers. <u>trade</u> discount is allowed.

Ans. (i) loss, (ii) cost, (iii) Marked/list (iv) Cash

(d) Fill in the Blanks :

(i) Successive discount of 20% and 10% is equivalent to a single rate of discount of _____.

(ii) If the terms of cash discounts are cash, 4, $\frac{3}{20}$ and $\frac{n}{30}$, the maximum oredit period is _____ days.

(iii) If MP = ₹2000, T.D = 10%, C.D = 5% then not sales price = _____.

(iv) The reduction/allowance/rebate given by the whole saler to the retailer is called _____.

Ans. (i) 28%, (ii) 30, (iii) ₹1710, (iv) Trade Discount

Group B

3. Answer the following questions within two sentences each :-

   (i) What is meant by sales price ?

   (ii) What do you mean by single equivalent rate of discount ?

   (iii) Define catalogue price.

   (iv) Explain Turnover.

4. Answer the following within six sentences each.

   (i) What is an invoice ? What its contents ?

   (ii) Explain Tender & Quotations.

   (iii) Explain LOCO price and FRANCO price.

   (iv) What are FAS price and FOB price ?

   (v) What are included in C.F. price and CIF price ?

Group C

(Long type questions)

5. What percentage of profit is earned when banans are bought at ₹7 per dozen and sold at ₹65 per 100 ?

6. If cost price of 12 apples is equal to selling price of 10 apples, How much is the profit/loss percentage ?

7. A trader makes a profit of 25% an cost price. What percentage is this on selling price ?

8. A man sells two radios at ₹990 each. In one he gains 10% and in the other he losses 10%. What is his gain or loss per cent on the whole ?

9. A sells an article to B at a profit of 10%, B sells it to C at a profit of 20%. If C buys it for ₹330 Find cost price to A.

10. A man buys two watches for ₹13,500. He sells one watch at 10% loss and the other at 2% profit. On the whole he neither gains nor loses. Find the cost price of each watch.

11. A man bought a table & a chair for ₹15,000. He sold the table at 11% profit and the chair at 26% profit, making 19% profit on the whole. Find the cost price of table and the chair separately.

12. By selling an article for ₹12,000, the seller makes a profit of $\frac{1}{3}$ of its cost price. Find the cost price and its gain per cent.

13. Mr. Rajhanse sold an article at 5% profit. If he had sold it ₹15 less his loss would have been 5%. Find the cost price of the article.

14. A builder sells two houses at the same price, gaining 20% on one and $33\frac{1}{3}$% on the other. If his total profit on the sales of two houses is ₹50,000. Find the cost of each house.

15. If 7% of the selling price of an article is equal to 8% of its cost and 9% of the selling price exceeds 10% of cost by ₹1. Find the cost and selling price.

16. Mr Sharma sold a house for ₹2,12,800 at a loss of $12\frac{1}{2}$%. What did he pay for the house ?

17. A fruit seller purchased 25 baskets of fruit @₹200 per basket. The fruit of 5 baskets turn out rotten and thrown off. Find the selling price per basket, it a profit of $12\frac{1}{2}$%. is to be made.

18. By selling an article at ₹12,800, A trader loses 20%. What would be his new selling price if he wants to make a profit of 10%.

19. A manufacturer sells a Refrigerator to a retailer at profit of $12\frac{1}{2}$ %. The retailer sells it at a profit of $16\frac{2}{3}$ %, there by gaining ₹3,000. Find the selling price of the retailer & cost price to the manufacturer.

20. A book seller paid ₹493 for a book on which he had given a discount of 15%. Find the seller's list price.

21. A wholesaler allows 25% trade discount and a further discount of 5% for cash payment. If the list price of the article is ₹12,00. How much retailer has to pay ?

22. After allowing trade discount 10% and 3% cash discount a whole saler makes a profit of 20%. If selling price of the article is ₹837, at what price he has marked the article ?

23. An article's catalogue price is ₹4500, sold at a discount of 15% on the catalogue price there by making $27\frac{1}{2}$ % of profit. Find the cost price to seller.

24. Mr. Rabindra purchased two horses for ₹13, 000. He sold one horse at a profit of 20% and the other one at a loss of 12%. If the sale price of both the horses are same find the cost price of each horse.

25. Which is more profitable to the seller ?

    (i) a discount of 20% and 10%.

    (ii) a discount of 20%, 5% and 5%.

26. A seller makes a profit of 40% on cost after allowing a discount of 30%. If cost of production rises by 20% by what rate per cent he should reduce the rate of discount so as to make the same rate of profit, keeping his price unaltured.

27. A seller marks his goods 30% above cost price and allows 10% discount. Calculate his profit percentage; if he marks 60% more and allows a discount of 40%. What is his profit/ loss per cent ?

28. A retailer reduced the price of his goods by 25% below marked price, which he has originally fixed to allow 25% profit on selling price after allowing 10% cash discount. What percentage does he gain or lose if no discount is allowed in the later case ?

29. The list price of a washing machine is ₹30,000 with the following terms:

cash 5, $\frac{4}{10}$, $\frac{2}{20}$, $\frac{n}{30}$

Find the discount if payment is made

(a) Immediately

(b) On the 9th day.

(c) On the 18th day.

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CHAPTER-2

PARTNERSHIP

STRUCTURE

2.1 Meaning and defination of partnership

2.2 Characteristics/features of partnership

2.3 Partnership deed and its contents

2.4 Types of partnership

2.5 Calculation of profit sharing ratio

2.5.1 Admission of new partners

2.5.2 Retirement/Death of old partner

2.1 Meaning and definition of partnership

Partnership is an association of two or more persons to carry on some business with on objective to earn profit. Persons who enter into partnership are individually called partners and they collectively form a partnership firm.

Sec 4, of Indian Partnership Act, 1932 defines 'Partnership' as under :

'Partnership is the relation between persons who have agreed to share the profits of a business carried on by all or any of them acting for all.''

2.2 Characteristics features of partnership.

The following are the essential features of partnership.

(i) There must be an association of two or more persons.

At least two persons are needed to form a partnership. However the maximum number should not exceed ten in case of firm having banking business and twenty in case of any other business.

(ii) There must be an agreement

A partnership can arise only as a result of an agreement. The agreement may be expressed (i.e. oral or written) or implied. The written agreement is called partnership deed.

(iii) There must be a business

A partnership cannot be formed for any purpose other than carrying on a business.

(iv) Business carried on by all or any of them acting for all.

The business of partnership may be carried on by all the partners or any of them acting for all.

(v) Profit & losses of the business must be shared

Theobjective of a partnership is to earn profit. When profit is earned; it must be distributed among the partners in an agreed ratio.

However, if question is silent about profit sharing ratio the partners will share their profit in proportion to their capital contribution. And all the problems are solved on this basis in this chapter. But the provision of partnership Act is different from this approach.

2.3 Partnership deed and its contents :

A partnership agreement entered into in writing, containing all the terms and conditions in detail, is called ''Partnership Deed''.

(A) Besides the name of the firm, name of partners, nature and place of business etc., the deed contains;

(i) The amount of capital contributed by each partner.

(ii) Profit sharing ratio.

(iii) Rate at which interest on capital if any is to be paid.

(iv) Rate of interest on drawings if any, by partners to be charged.

(v) Salary, commission or remunation if any, to be paid to partners.

(vi) Rate of interest on loan advanced by a partner in addition to capital.

The partnership Act laid down certain provisions to be followed in the absence of partnership agreement ?

(a) All partners are entitled to share the profit of the business equally and also contribute to the losses equally, irrespective of their capital contribution.

(b) No salary or commission is to be paid to any partner for the conduct of the business.

(c) No interest is allowed on capital or charged on drawing.

(d) Any loan advanced by a partner is entitled to 6% interest per annum.

2.4 Types of Partnership :

Partnership is of two types :

(i) Simple ordinary partnership

(ii) Compound Partnership

When all partner's capital is invested for equal length of time, the partnership is called simple partnership.

If the capital of the partners is invested for different length of time it is called compound partnership.

Compound partnership arises due to admission or retirement or deathof a partner during the year. It may also arise due to introduction of further capital or drawings during the year by any partners.

Partners may be active partners, who contribute capital and take active part in the management, or may be demant or sleeping partners who contribute capital but donot part participate in the management of the firm.

2.5 Calculation of Profit sharing Ratio :

The following points should be noted for distribution of protits among partners:

(a) Where the profit/loss sharing ratio is determined by agreement, entered by them, the profit should be devided in that ratio.

(b) If profit sharing ratio is not mentioned, then for all practical purposes profit/losses are shared in proportion to capital contributed by them.

(c) If capital is invested by partners for unequal period or invested for different length of time period, then averange investment should be considered for sharing profit/losses.

2.6 Retirement/death of a partner

When any one of existing partners retire from partnership business or dies, his share is taken over by the remaining partners. So they will be gainer. So new profit sharing ratio of remaining partner is :

New Ratio = old ratio + gaining ratio.

Gaining Ratio is, therefore, the different between new ratio and old ratio among the partners.

Example : 1

x, y and z are partners in a partnership firm, sharing profits in the ratio, 2:3:5 respectvely. Calculate how a profit of ₹18,000 will be distributed among them.

Solution

Given profit = ₹18,000

Ratio of profit sharing = 2:3:5

$\therefore$ *x*'s share = $\frac{2}{10}$, y's share = $\frac{3}{10}$ & z' share $\frac{5}{10}$

$\therefore$ *x*'s share of profit = $\frac{2}{10} \times ₹1800 = ₹3,600$

y's share of profit = $\frac{3}{10} \times ₹1800 = ₹5,400$

z's share of profit = $\frac{5}{10} \times ₹1800 = ₹9,000$

Example : 2

A, B and C are partners in a business with capitals of ₹50,000, ₹60,000 and ₹70,000 respectively. Show how profit of ₹90,000 will be devided among them.

Solution

Total capital of all partners = ₹(50,000+60,000+70,000) = ₹1,80,000

$$\text{A's share} = \frac{₹50,000}{₹1,80,000} = \frac{5}{18} \text{ of the total capital}$$

$$\text{B's share} = \frac{₹60,000}{₹1,80,000} = \frac{6}{18} \text{ of the total capital}$$

$$\text{C's share} = \frac{₹70,000}{₹1,80,000} = \frac{7}{18} \text{ of the total capital}$$

$$\therefore \text{ A's share of profit} = \frac{5}{18} \times ₹90,000 = ₹25,000$$

$$\text{B's share of profit} = \frac{6}{18} \times ₹90,000 = ₹30,000$$

$$\text{C's share of profit} = \frac{7}{18} \times ₹90,000 = ₹35,000$$

Example : 3

Amar, Bharat and Chitta enter into a partnership business. Amar puts in ₹1,20,000 for 6 months, Bharat ₹80,000 for 7 months and chitta ₹60,000 for 8 months. Devide profit of ₹39,600 among them.

Solution

Since the partner's capital is invested for unequal length of time, first of all 1 month's investment of capital should be calculated to findout their profit sharing ratio.

∴ Amar's capital investment for one month

= ₹1,20,000 × 6 = ₹7,20,000

Bharat's capital investment for one month

= ₹80,000 × 7 = ₹5,60,000

& chitta's capital investment for one month

= ₹60,000 × 8 = ₹4,80,000

∴ their profit sharing ratio = 7,20,000 : 5,60,000 : 4,80,000 = 9:7:6

In the profit of ₹39,600,

$$\text{Amar's share} = \frac{9}{22} \times ₹39{,}600 = ₹16{,}200$$

$$\text{Bharat's share} = \frac{7}{22} \times ₹39{,}600 = ₹12{,}600$$

$$\text{Chita's share} = \frac{6}{22} \times ₹39{,}600 = ₹10{,}800$$

Example : 4

A is *a* working partner and B is *a* sleeping partner in *a* business. A invests ₹50,000 and B puts in ₹40,000. A gets 15% of profit for managing the business and the rest profit is devided in proportion to their capitals. What does each get, out of a profit of ₹22,500.

Solution

$$\text{A's share for managing the business} = \frac{15}{100} \times ₹22{,}500 = ₹3{,}375$$

The amount of profit left = ₹(₹22,500 - 3,375) = ₹19,125

It is devided in the ratio of their capitals

i.e., ₹50,000 : ₹40,000

5 : 4

$$\therefore \text{ A's share of profit} = \frac{5}{9} \times ₹19{,}125 = ₹10{,}625$$

$$\text{and B's share of profit} = \frac{4}{9} \times ₹19{,}125 = ₹8{,}500$$

$\therefore$ A's total share = ₹(₹3,375 + 10,625) = ₹14,000

$\therefore$ B's total share = ₹8,500

Example : 5

Ajaya and Bijoy start a business and Ajaya puts in ₹6000 more than Bijoy. Bijoy invests his capital for 5 months where as Ajaya for 4 months. Their total profit is ₹5280 and Ajaya gets ₹480 more than Bijoy from it. Find their capitals.

Solution

As per question Bijoy's profit $= \frac{5280 - 480}{2} = ₹2,400$

$\therefore$ Ajay's profit = ₹2400 + ₹480 = ₹2880

Now Ajay has invested his capital for 4 months

$\therefore$ Profit of Ajaya for one month $= \frac{₹2800}{4} = ₹720$

and Bijoy profit for 1 month $= \frac{₹2400}{5} = ₹480$

$\therefore$ Ratio of A's capital to B's capital

$= 720 : 480$

$= 3 : 2$

This shows that in a capital of ₹5, the difference in their share is ₹1

So when difference is ₹1, capital is ₹5

When difference is ₹6000, capital = 5 × ₹6000 = ₹30,000

Now Ajay's capital $= \frac{3}{5} \times ₹30,000$

$= ₹18,000$

and Bijoy's capital $= \frac{2}{5} \times ₹30,000$

$= ₹12,000$

ADMISSION OF NEW PARTNER

Situation - 1

If new partner's share is decided in advance or known, then old partner's will share balance profit in old ratio after new partner's share of profit.

Example : 6

P & Q are in partnership sharing profit and losses in the ratio 2 : 1. They admitted R giving $\frac{1}{6}$th of profit. Find their new profit sharing ratio.

Solution

P's share of profit = $\frac{2}{3}$

Q's share of profit = $\frac{1}{3}$

R is admitted and given $\frac{1}{6}$th of future profit.

$\therefore$ After R's share, balance of profit will be $1 - \frac{1}{6} = \frac{5}{6}$

$\therefore$ P's new share of profit = $\frac{5}{6} \times \frac{2}{3} = \frac{10}{18}$

Q's new share of profit = $\frac{5}{6} \times \frac{1}{3} = \frac{5}{18}$

$\therefore$ The new profit sharing ratio of P, Q & R

$= \frac{10}{18} : \frac{5}{18} : \frac{1}{6}$

or $\frac{10}{18} : \frac{5}{18} : \frac{3}{18} = 10 : 5 : 3$

Situation II

If new partner purchases his share from old partners in a particular ratio, then new profit sharing ratio will be calculated by deducting the proportion from old partner's profit sharing ratio.

Example : 7

Mohan & Nabin are partners in a business sharing profit and losses in the ratio of 10:8. They agreed to admit pradeep, who is to get $\frac{1}{2}$ share of profit acquiring $\frac{1}{3}$ from Mohan and $\frac{1}{6}$ from Nabin. Calculate new profit sharing ratio.

Solution

New partner pradeep acquires $\frac{1}{3}$ from Mohan's share

$\therefore$ Mohan's new share = old share - sacrifice of Mohan

$$= \frac{10}{18} - \frac{1}{3} = \frac{10-6}{18} = \frac{4}{18}$$

and New partner Pradeep acquires $\frac{1}{6}$ from Nabin's share

$$\text{So Nabin's new share} = \frac{8}{18} - \frac{1}{6} = \frac{8-3}{18} = \frac{5}{18}$$

$$\text{their new profit sharing ratio} = \frac{4}{18} : \frac{5}{18} : \frac{1}{2}$$

$$= \frac{4}{18} : \frac{5}{18} : \frac{9}{18}$$

$$= 4 : 5 : 9$$

Situation III

Some times to accommodate the new partner, old partners forego a fraction of their share in favour of the new partner. In this case new ratio of old partner = old ratio - sacrificing ratio.

Example : 8

M and N sharing profit and loses in the ratio 3:2. K was admitted into partnership. M sacrifices $\frac{1}{3}$ rd of his share and N sacrifices $\frac{1}{6}$ th of his share calculate the new profit sharing ratio.

Solution :

M's new profit sharing ratio = old ratio - sacrificing ratio

$$\Rightarrow \frac{3}{5} - \left(\frac{3}{5} \times \frac{1}{3}\right) \Rightarrow \frac{3}{5} - \frac{3}{15} = \frac{9-3}{15} = \frac{6}{15}$$

N's new profit sharing ratio $= \frac{2}{5} - \left(\frac{2}{5} \times \frac{1}{6}\right)$

$$= \frac{2}{5} - \frac{2}{30} = \frac{12-2}{30} = \frac{10}{30}$$

$\therefore$ K's share $= \left(\frac{1}{3} \times \frac{3}{5}\right)$ from M $\Rightarrow \left(\frac{1}{6} \times \frac{2}{5}\right)$ from N

$$= \frac{3}{15} + \frac{2}{30} = \frac{6+2}{30} = \frac{8}{30}$$

$\therefore$ New profit sharing ratio $= \frac{6}{15} : \frac{10}{30} : \frac{8}{30}$

$= 12 : 10 : 8$

IN CASE OF RETIREMENT :

If new ratio of continuing partners is not given in the question. It is assumed they will continue in the old ratio after retirement of one of the partner.

Example : 9

x, y and z are partners sharing profit and losses in the ratio of 7:5:3.

If x retires, New profit sharing ratio

between y & z = 5 : 3

If y retires, New profit sharing ratio

between *x* & z = 7 : 3

lilawise if z retires, New profit sharing

ratio between *x* & y = 7 : 5

However if any partner will purchase the retiring partner's share, then his profit sharing ratio will be enhanced by the retiring partners share.

For example, if z's share is purchased by x alone, then the new profit sharing ratio would be $\left(\frac{5}{15}+\frac{3}{15}\right):\frac{5}{15}$ (on z's retirement)

$$= \frac{10}{15}:\frac{5}{15}$$

$$= 10:5 \text{ or } 2:1$$

Again if z's share is purchased by y, then their new profit sharing ratio would be $\frac{7}{15}:\left(\frac{5}{15}+\frac{3}{15}\right)$

$$= \frac{7}{15}:\frac{8}{15}$$

$$= 7 : 8$$

Example : 10

A and B commenced business with capital of ₹60,000 and ₹90,000 respectively. After 6 months B withdraw his entire capital. Find the share of each partner, if profit at the end of the year amounts to ₹18,900.

Solution :

A's capital invested for 12 months

B's capital invested for 6 months.

∴ Investment of A's capital of ₹60,000 for 12 months

= Investment of ₹(60,000×12) i.e., ₹7,20,000 for 1 month and Investment of ₹90,000 for 6 month = Investment of ₹90,000×6) i.e., ₹5,40,000 for 1 month

$\therefore$ Ratio of their capital = 7,20,000 : 5,40,000

= 36 : 27

= 4 : 3

$\therefore$ A's share of profit $= \frac{4}{7} \times 18,900 = ₹10,800$

and B's share of profit $= \frac{3}{7} \times 18,900 = ₹8,100$

Example : 11

Mahendra and Narendra enter into partnership with capital ratio 5:6. At the end of 8 months Mahendra withdrawn. If they receive the profits in the ratio 5 : 9, find how long Narendra's capital was invested.

Solution :

Let Narendra's capital was invested for x months.

Capital ratio = 5 : 6

Time ratio (investment) = 8 : x

$\therefore$ Capital investment ratio for 1 month

= 40 : 6x

Profit sharing ratio after the period = 5 : 9

$\therefore$ 40 : 6x = 5 : 9

or 40 × 9 = 5 × 6x

$\therefore x = \frac{40 \times 9}{5 \times 6} = 12$ months

$\therefore$ Narendra's capital was invested for 12 months.

Example : 12

Arun started a business with ₹3,50,000 and Barun joins him with ₹6,00,000. When did Barun join if the profit were divided equally at the end of the year ?

Solution :

Arun's investment was for 12 months

∴ Arun's investment for 1 month = ₹3,50,000 × 12 = ₹42,00,000

Barun joined him with ₹6,00,000 after some time

Let Barun joined for x months

∴ Barun's investment for 1 month

$= 6,00,000 \times x$

$= 6,00,000\ x$

Since their profit is equal

One month investment of Arun = One month investment of Barun.

$\Rightarrow$ ₹42,00,000 = ₹6,00,000x

$$\therefore x = \frac{₹42,00,000}{₹6,00,000} = 7 \text{ months}$$

∴ Barun joins him after (12-7) = 5 months.

Example : 13

A, B,C are in partnership, share profits in their capital ratio.

A contributed capital of ₹3,50,000 and B ₹4,20,000. The profit at the end of the year amounted to ₹76,000, out of which C got ₹32,000. How much capital was contributed by C.

Solution :

As per question, profit sharing ratio of A and B

= ₹3,50,000 : ₹4,20,000 = 5 : 6

combined profit of (A+B) = ₹76,000 - ₹32,000 = ₹44,000

$\therefore$ As share of profit = $\left(\frac{5}{11} \times ₹44,000\right)$ = ₹20,000

Hence when profit is ₹20,000, capital contribution ₹3,50,000

When profit is 1, capital contribution = $\frac{₹3,50,000}{₹20,000}$

When profit is ₹32,000, capital contribution

$= \frac{3,50,000}{20,000} \times 32,000 = ₹5,60,000$

$\therefore$ C has contributed ₹5,60,000 as capital.

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EXERCISE
GROUP-A
MULTIPLE CHOICE TYPE QUESTIONS:

(i) The minimum number of persons needed to form a partnership business, is;

(a) 3 (b) 2 (c) 4 (d) 5

(ii) In case of partnership having banking business, the maximum number of members will be:

(a) 20 (b) 15 (c) 10 (d) 5

(iii) The partnership form of business is governed by partnership Act of :

(a) 1932 (b) 1912 (c) 1956 (d) 1961

(iv) In absence of any provision in the partnership deed, profit and loss among partners shall be shared :

(a) equally, (b) in the initial capital contribution ratio, (c) in the ratio of their average capital investment, (d) in the ratio of their capital at the end.

(v) In absence of any provision in the partnership deed, interest on loan payable to the partners is :

(a) 5% (b) 7% (c) 8% (d) 6%

(vi) Maximum number of parners in case of non-banking business is :

(a) 10 (b) 20 (c) 15 (d) 25

(vii) Three partners A, B and C invest ₹ 3,60,000, ₹ 4,50,000 & ₹ 5,40,000 respectively in a business. Out of profit of ₹ 3,75,000, C's share is :

(a) ₹ 1,25,000 (b) ₹ 1,00,000 (c) ₹ 1,50,000 (d) ₹ 1,60,000

ANSWER

(i) 2(b) (ii) 10 (c) (iii) 1932 (a) (iv) Equally (a)

(v) 6% (d) (vi) 20 (b) (vii) ₹ 1,50,000 (c)

2.(A) Express the following in one word | term.

(i) The document containing the terms and conditions of partnership.

(ii) The ratio in which the accumulated profit of partnership firm is devided among old partners while admitting a new partner.

(iii) The partner who only contributes capital, but doesnot take active part in the management of the business.

Ans. (i) Partnership deed, (ii) old profit sharing ratio, (iii) Sleeping or Dormant partner.

2.(B) Answer the following in one sentence each:-

(i) Define partnership

(ii) What is sacrifice ratio ?

(iii) What is gaining ratio ?

(iv) Who is an active partner ?

2 (C) Correct the underlined portion of the following sentences :

(i) The partner/partners who take active part in management of business is called <u>sleeping</u> partner.

(ii) In absence of agreement the interest on <u>drawings</u> is charged @6% per annum.

(iii) Dornant partners <u>participate</u> in the management of the partnership firm.

Ans.: (i) Active Partner, (ii) loan, (iii) do not (don't)

2(D) Fill in the Blanks:

(i) Partnership is governed by Partnership Act of _____.

(ii) The excess of old ratio over new ratio in partnership business is called _____ ratio.

(iii) At the time of retirement /death of a partner, normally _____ ratio is calculated.

(iv) Usually the liability of partners is _____ .

Ans.: (i) 1932, (ii) Sacrificing, (iii) Gaining, (iv) Unlimited.

GROUP B

Q.3. Answer the following within two sentences each:

(i) What do you mean by partnership form of business ?

(ii) Who is a partner ?

(iii) What is profit and loss sharing ratio ?

(iv) What do you mean by 'gaining ratio' ? $\frac{1}{10}$

Q.4. Answer the following within six sentences each.

(i) State any six contents by partnership deed ?

(ii) State the features of partnership.

(iii) Explain the provisions to be followed in the absence of partnership agreement.

(iv) What do you mean by sacrifice and gaining ratios in case of partnership business ?

GROUP C

Long type questions :

5. Ajay and Bijoy are partners with a total capital of ₹3,19,000. Ajay's capital was invested in the business for 5 months and Bijoy's for 7 months. If out of profit ₹75,400, Bijoy receives ₹40,600, Find the capital of each.

6. A and B were in partnership with capital ₹4,00,000 and ₹3,00,000 respectively. A's capital was in business for 9 months and B's for 8 months. How much each will receive if profit was ₹1,50,000.

7. Profit of ₹7,000 is devided among A, B & C so that A receives half as much as B, and B half as much as C. Then find the share of profit of C.

8. M and N are partners with 5:3 profit sharing ratio. They trok J into partnership by giving $\frac{1}{4}$ th share of profit. If J acquirs his shares equally from boty of them, what will be the new profit sharing ratio of M and N.

9. A partnership is formed by X, Y and Z, their respective capital being ₹56,000, ₹72,000 and ₹32,000. Z receives $\frac{1}{10}$ of the profit for his managerial service and rest of the profits are devided in proportion to their shares in capital. Show how profits of ₹64,000 be distributed.

10. Chita and Mita formed a partnership business with capitals ₹10,000 and ₹12,000 respectively. As per their deed 5% interest on capital will be provided. Chita as manager is to receive ₹500 P.M. as salary. Find how much chita will receive out of profits ₹28,000 in a year as they distribute profit equally.

11. Sun, Moon and star enter into a business with capitals of ₹25,000, ₹20,000 and ₹1,00,000 respectively. Interest is agreed to be allowed at 4%. Sun and Moon will received ₹2500 and ₹1200 p.m. as their salaries and balance profit to be devided in the ratio of 1:1:6. Distribute a profit of ₹69,800 among them and find how much each will receive.

12. X,Y & Z are in partnership sharing profit & losses in the ratio of their capitals. The ratio of capital between X and Y is 8:7 and between Z and Y is 9:8. If X received ₹5,200 more than Y in a particular year, find the profit to each partner and total profit.

13. Preeti and Pintu are partners, contributing ₹30,000 and ₹20,000 as their capitals. As per their agreement interest on capitals at the rate 5% should be allowed and 20% of the balance after interest be given to preeti as remuneration. Reminder to be distributed in proportion to their capitals. If both of them receive equal amount at the end of the year; Find the total profit.

14. A, B and C are in partnership with capital of ₹10,000, ₹8,000 and ₹6,000 respectively besides C's loan of ₹3,000 which carries interest at the rate of 6%. Interest on capital 5% p.a. is allowed. Each partner is allowed to draw salary of ₹500 per annum. 10% of the remaining profit after salary and interest is to be transferred to reserve fund, and balance distributed among them in proportion to their capitals. If at the end of the year B receives ₹1,674. Find other partner's share of profit.

15. A, B and C enter into a partnership. Their capitals are ₹20,000, ₹18,000 and ₹12,000 respectively. A and B get 12% and 8% profits respectively for managing the business and the rest is divided inproportion to their capitals. At the end of the year A receives together ₹648 more than B. Find their respective shares.

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CHAPTER-3

LOGARITHM

STRUCTURE

3.1 Meaning
3.2 Definition
3.3 Properties of Logarithm
3.4 Laws of Logarithm
3.5 Types of Logarithm/System of Logarithm
  3.5.1 Natural Logarithm
  3.5.2 Common Logarithm
3.6 Standard Form
3.7 Characteristics and Mantissa
3.8 Antilogarithm
3.9 Evoluation of Logarithm by Interpolation method
Exercise

3.1 MEANING

Logarithm is an important tool of mathematics which is used for simplifying complex numerical calclations. While working out the calculations related to multiplication, division and computations of power and roots, the mathematical tool logarithm plays an important role. Application of logarithm not only simplifies the cumbersome calclations but also saves calculation time. This tool was invented by an English Mathematician John Napier in 1550.

The concept of logarithm comes from the exponential equations where there is a base and a power. If 'a' is a positive real number other than '1' and 'x' is a national number such that $a^x = N$, it is an example of exponential equation leading to the evolution of logarithim concept.

Let us consider the exponential equation $a^x = N$. where the quantity 'a' is the base and 'x' is the magnitude of power. In such a case 'x' is considered to be the Logarithm of N to the base 'a'.

This is expressed as $x = \log_a N$

We read this as 'x' is logarithm of N to the base a.

3.2 DEFINITION

The logarithm of a number to a given positive real number base ($\neq 1$) is the index of the power to which the base must be raised inorder to make it equal to the given number.

If a is a positive real number ($\neq 1$) and $a^x = m$, then x is called the logarithm of the number m to the base a and is written as $\log_a m = x$.

EXAMPLES

Exponential expression	Logarithmic expression
(i) $2^2 = 4$	$\therefore \log_2 4 = 2$
(ii) $4^3 = 64$	$\therefore \log_4 64 = 3$
(iii) $10^3 = 1000$	$\therefore \log_{10} 1000 = 3$
(iv) $2^{-3} = \frac{1}{8}$	$\therefore \log_2 \frac{1}{8} = -3$

From the above i.e., (i), (ii), (iii) and (iv) we notice that there is deep relationship between exponential form and Logarithmic form. Infact the two forms are equivalent.

3.3 PROPERTIES OF LOGARITHM

(*i*) Logarithm of 1 to any base is zero.

We know that any thing power zero = 1

$\therefore a^0 = 1$

$\therefore \log_a 1 = 0$

Likewise $5^0 = 1$

$\therefore \log_5 1 = 0$ and so on.

(*ii*) The logarithm of any number to the base itself is 1.

We know that $a^1 = a$

$\therefore \log_a a = 1$

again As $9^1 = 9$

$\therefore \log_9 9 = 1$ and so on.

(*iii*) Logarithm of any number's reciprocal to its base = -1

Let $\log_a \frac{1}{a} = x \Rightarrow a^x = \frac{1}{a}$

or $a^x = a^{-1}$

$\therefore x = -1$

So, $\log_3 \frac{1}{3} = -1$

and $\log_{\frac{1}{5}} 5 = -1$ and so on.

(*iv*) The logarithm of zero to any base, greater than unity is $-\infty$ and smaller than unity is ∞.

If $a < 1$, $a^{\infty} = 0$ $\therefore \log_a 0 = \infty$

and if $a > 1$, $a^{-\infty} = 0$ $\therefore \log_a 0 = -\infty$

(*v*) The logarithm of infinity to the base greater than one is equal to infinity.

i.e., $\log_a \infty = \infty$ (when $a > 1$)

(*vi*) The logarithm of infinity to the base less than one is equal to minus infinity.

i.e., $\text{Log}_a \infty = -\infty$ (when $a < 1$)

Illstration - 1

Find the value of x if

(a) $\log_{100} 1 = x$

(b) $\log_{.01} 1 = x$

(c) $\log_5 x = 1$

(g) $\log_{.01} 0.01 = x$

(h) $\log_{\frac{1}{4}} 0 = x$

(i) $\log_4 0 = x$

(d) $\log_x \frac{1}{5} = -1$ (j) $\log_2 x = \infty$

(e) $\log_{.01} x = 1$ (k) $\log_2 \infty = x$

(f) $\log_{10} x = -1$ (l) $\log_5 x = x$

Solution :

(a) $\log_{100} 1 = x \Rightarrow 100^x = 1 = 100^0$

$\therefore x = 0.$

(b) $\log_{.01} 1 = x \Rightarrow .01^x = 1 = (.01)^0$

$\therefore x = 0.$

(c) $\log_5 x = 1 \Rightarrow 5^1 = x$

$\therefore x = 5$

(d) $\log_x \frac{1}{5} = -1 \Rightarrow x^{-1} = \frac{1}{5}$

$\Rightarrow \frac{1}{x} = \frac{1}{5}$

$\therefore x = 5$

(e) $\log_{.01} x = 1 \Rightarrow (.01)^1 = x$

$\therefore .01 = x$ or $x = .01$

(f) $\log_{10} x = -1 \Rightarrow 10^{-1} = x$

$\Rightarrow x = \frac{1}{10}$

$\therefore x = .1$

(g) $\log_{.01} .01 = x \Rightarrow (.01)^x = (.01)^1$

$\therefore x = 1$

(h) $\log_{\frac{1}{4}} 0 = x \Rightarrow \left(\frac{1}{4}\right)^x = 0 = \left(\frac{1}{4}\right)^{\infty}$

$\therefore x = \infty$ (since logarithm of 0 to any base less than unity is ∞)

(i) $\log_4 0 = x \Rightarrow 4^x = 0$

$\therefore x = -\infty$ (since logarithm of 0 to any base greater than unity is $-\infty$)

(j) $\log_x \infty = \infty \Rightarrow 2^{\infty} = x$

$\therefore x = \infty$

(k) $\log_2 \infty = x \Rightarrow 2^x = \infty$

$\therefore x = \infty$

(l) $(5)^{\log_5 x} = 5^1 \Rightarrow (5)^{\log_5 x} = (5)^{\log_5 5}$

$\Rightarrow \log_5 x = \log_5 5$

$\therefore x = 5$

3.4 LAWS OF LOGARITHM

3.4.1 Multiplication law :

The logarithm of the product of two numbers with respect to a given base is the sum of their individual logarithms with respect to the same base.

Mathematically if 'm' and 'n' are two positive numbers, then

$$\log_a (m \times n) = \log_a m + \log_a n$$

Proof :

Let $\log_a m = x$ and $\log_a n = y$

$\therefore a^x = m$ and $a^y = n$

Now, $m \times n = a^{x+y}$ (As per laws of indices)

or, $\text{Log}_a (m \times n) = (x+y) \text{ Log}_a a$ (taking logarithm in both the sides w.r.t. base a)

or $\text{Log}_a(m \times n) = x+y$

or $\text{Log}_a(m \times n) = \log_a m + \text{Log}_a n$ (Proved) (Putting the respective values of x and y)

3.4.2 Division law :

The logarithm of the ratio of two members with respect to a given base is the difference of the logarithms of the numerator and the denominator to the same base.

Mathematically $\boxed{\log_a \frac{m}{n} = \log_a m - \log_a n}$

Proof: Let $\text{Log} \frac{m}{n} = z$,

$\text{Log}_a m = x$ and $\text{Log}_a n = y$

$\therefore a^z = \frac{m}{n}$, $a^x = m$ and $a^{y=n}$

Now $a^z = \frac{m}{n}$

$\Rightarrow a^z = \frac{a^x}{a^y}$

$\Rightarrow a^z = a^{x-y}$

$\Rightarrow z = x-y$ (equating the exponents of a an either side)

$\Rightarrow \log_a \frac{m}{n} = \log_a m - \log_a n$ (Proved)

3.4.3 Involution law :

The logarithm of the number raised to an exponent is equal to the product of the exponent and the logarithm of the number to the to the same base.

Mathematically $\boxed{\log_a m^n = n \log_a m}$

Proof :

Let $\log_a m^n = z$ and $\log_a m = x$

$\therefore a^z = m^n$ and $a^x = m$

Now $a^z = m^n$

$\Rightarrow a^z = (a^x)^n$

$\Rightarrow a^z = a^{nx}$

$\Rightarrow z = nx$ (equating the exponents on either side)

$\Rightarrow \log_a m^n = n \log_a m$ (Proved)

3.4.4 Evolution law :

$$\text{Log}_a \sqrt[n]{m} = \frac{\log_a m}{n}$$

Proof :

Let $\log_a m = x \Rightarrow a^z = m$

So, $\sqrt[n]{a^x} = \sqrt[n]{m} \Rightarrow a^{\frac{x}{n}} = m^{\frac{1}{n}}$

Writing in Logarthmic form we get $\text{Log} m^{\frac{1}{n}} = \frac{1}{n} x$

Now substituting the value of x we get $Log_a m^{\frac{1}{n}} = \frac{1}{n} \log_a m = \frac{\log_a m}{n}$

$$\Rightarrow \boxed{\log_a \sqrt[n]{m} = \frac{\log_a m}{n}}$$

3.4.5 Change of base law

When the logarithm of a number on a particular base is given we can find out the logarithm of the same number on any other changed base easily by the following formula.

$\log_a m = \log_b m \times \log_a b$

Proof :

Let $\log_a m = x$, $\log_b n = y$ and $\log_a b = z$ By converting the equations to exponential form we get.

$m = a^x$ ..... (1)

$n = b^y$ ..... (2)

$b = a^z$ ..... (3)

From equation No (2) and (3) we get,

$m = b^y = (a^z)^y = a^{yz}$ ..... (4)

From equation no (1) and (4) we get, $m = a^x = a^{yz}$

$\Rightarrow \quad x = yz$

Substituting the vales of x, y and z,

$\log a^m = \log_b m \times \log_a b$ (Proved)

Illustration 2

Given $\log_{10} 2 = 0.3010$ and $\log_{10} 3 = 0.4771$

Find the value (*a*) Log 270 (b) Log45

(a) $\text{Log}270 = \log_{10}(3^3 \times 5 \times 2)$

$= 3 \log_{10} 3 + \log_{10} 5 + \log_{10} 2$

$= 3 \log_{10} 3 + \log_{10} \frac{10}{2} + \log_{10} 2$

$= 3 \times \log_{10} 3 + \log_{10} 10 - \log_{10} 2 + \log_{10} 2$

$= 3 \times 0.4771 + 1 - 0.3010 + 0.3010$ (substituting the value)

$= 1.4313 + 6990 + 0.3010$

$= 2.413$ (Ans.)

(b) $\text{Log}_{10} 45 = \log_{10}(3^2 \times 5)$

$= \log_{10} 3^2 + \log_{10} 5$

$= 2 \log_{10} 3 + \log_{10} \frac{10}{2}$

$= 2 \log_{10} 3 + \log_{10} 10 - \log_{10} 2$

$= 2 \times 0.4771 + 1 - 0.3010$

$= 0.9542 + 0.6990$

$= 1.6532$ (Ans.)

3.5 TYPES / SYSTEMS OF LOGARATHM

There are two systems of logarithm depending upon the type of base. These are (i) Natural logarithm and (ii) Common logarithm.

3.5.1 Natural logarithm :

This system of logarithms is named after John Napier, the inventor of logarithms; If the base of the logarithm is irrational numbe 'e',

(where $e = 1 + \frac{1}{\angle 1} + \frac{1}{\angle 2} + \frac{1}{\angle 3} + ... + \frac{1}{\angle r} + ...$ an infinite series whose values correct to 6 decimal places is found to be 2.718282 and the system is known as natral or Napierian system of logarithm. This system is used in pure mathematics (calculus) and theoretical calculations.

3.5.2 Common Logarithm :

This system of logarithm was first introduced by Henry Briggs. Under this system, the base of the logarithm is taken as '10'. Hence, the logarithm of a number calculated to the base 10 is called the common logarithm. Since the base of common logarithm is understood to be 10, it neednot be mentioned. Thus by log100, we mean $\log_{10} 100$ and hence log 100 = 2.

Illustration :3

Show that $\log_b a \times \log_c b \times \log_a c = 1$

Solution :

Let $\log_b a = x \Rightarrow b^x = a$

$\log_c b = y \Rightarrow c^y = b$

$\log_a c = z \Rightarrow a^z = c$

$\therefore\ a = b^x = (c^y)^x = c^{xy} = (a^z)^{xy} = a^{xyz}$

or $a^1 = a^{xyz}$

$\therefore\ xyz = 1$

or $\log_b a \times \log_c b \times \log_a c = 1$ (Proved)

Illustration : 4

Show that $\log_3.\log_2.\log_{\sqrt{3}} 81 = 1$

Solution

L.H.S = $\text{Log}_3.\log_2.\log_{\sqrt{3}}$

$= \text{Log}_3.\log_2.8\ \log_{\sqrt{3}}\sqrt{3}$

$= \text{Log}_3.\log_2.8 \quad (\because\ \log_{\sqrt{3}}\sqrt{3} = 1)$

$= \log_2.\log_2 2^3$

$= \log_3.\ 3\log_2 2$

$= \log_3 3 \quad (\because\ \log_2 2 = 1)$

$= 1$ Proved.

Illustration 5

If $\log_2 x + \log_4 x + \log_{16} x = \frac{21}{4}$, find x.

Solution

$\text{Log}_2 x = \log_{16} x \times \log_2 16$ (by change of base)

$\text{Log}_4 x = \log_{16} x \times \log_4 16$ (by change of base law)

replacing the values in the given expression

$$(\log_{16} x \times \log_2 16) + (\log_{16} x \times \log_4 16) + \log_{16} x = \frac{21}{4}$$

$$\text{or } (\log_{16} x \times 4\ \log_2 2) + (\log_{16} x \times 2\log_4 4) + \log_{16} x = \frac{21}{4}$$

or $4 \log_{16} x + 2 \log_{16} x + \log_{16} x = \frac{21}{4}$

$$7 \log_{16} x = \frac{21}{4}$$

$$\therefore \log_{16} x = \frac{21}{4} \times \frac{1}{7} = \frac{3}{4}$$

or $(16)^{\frac{3}{4}} = x$ (changing to exporential form)

or $\left(\sqrt[4]{16}\right)^3 = x$

$\therefore 2^3 = x \quad \therefore x = 8$ Ans.

Illustration 6

Simplify : $\text{Log}_2 1000 \times \log_5 2 \times \text{Log}_{10} 5$

Solution

$\text{Log}_2 1000 \times (\text{Log}_5 2 \times \text{Log}_{10} 5)$

$= \log_2 1000 \times \log_{10} 2 \quad (\therefore \log_5 2 \times \log_{10} 5 = \text{Log}_{10} 2)$

$= \log_{10} 1000$ (Applying change of base rule)

$= 3 \log_{10} 10 = 3$ (Ans.)

Illustration 7

If $a^2 + b^2 = 7ab$, Prove that

$$\log \frac{1}{3}(a+b) = \frac{1}{2}(\log a + \log b)$$

Solution

Given, $a^2 + b^2 = 7ab$

adding 2ab in both sides

$a^2 + b^2 + 2ab = 7ab + 2ab$

or $(a+b)^2 = 9ab$

or $(a+b)^2 = (3\sqrt{ab})^2$

or $a+b = 3\sqrt{ab}$

or $\frac{1}{3}(a+b) = \sqrt{ab}$

Taking log on both sides, we have,

$\log\frac{1}{3}(a+b) = \log(ab)^{\frac{1}{2}}$

or $\log\frac{1}{3}(a+b) = \frac{1}{2}[\log(ab)]$

or $\log\frac{1}{3}(a+b) = \frac{1}{2}[\log a + \log b)]$ Hence proved.

Illustration : 8

If $\log\frac{a+b}{2} = \frac{1}{2}(\log a + \log b)$

Prove that, $a^2+b^2=2ab$

Solution

Given, $\log\frac{a+b}{2} = \frac{1}{2}(\log a + \log b)$

or $\log\frac{a+b}{2} = \frac{1}{2}(\log ab)$ $(\because \log a+\log b = \log ab)$

or $\log\frac{a+b}{2} = \log(ab)^{\frac{1}{2}}$

or $\frac{a+b}{2} = (ab)^{\frac{1}{2}}$

Squaring both the sides

$$\left(\frac{a+b}{2}\right)^2 = ab$$

or $a^2+b^2+2ab = 4ab$

or $a^2+b^2 = 4ab - 2ab = 2ab$

$\therefore\ a^2+b^2=2ab$ Proved.

Illustration : 9

(i) Show that : $\log 2+\log\frac{8}{9}+\log\frac{15}{14}+\log\frac{21}{20}=\log 2$

L.H.S $= \log 2+\log 8-\log 9+\log 15-\log 14-\log 21-\log 20$

$= \log 2+\log 2^3-\log 3^2+\log(3\times 5)-\log(2\times 7)-\log(3\times 7)-\log(2^2\times 5)$

$= \log 2+3\log 2-2\log 3+\log 3+\log 5-\log 2-\log 7+\log 3+\log 7-2\log 2-\log 5$

$= 4\log 2-3\log 2-2\log 3+2\log 3+\log 5-\log 5-\log 7+\log 7$

$= \log 2$ (Proved)

(ii) Show that : $\log\frac{3}{2}+\log\frac{4}{3}+\log\frac{5}{4}+\log\frac{6}{5}=\log 3$

L.H.S $= \log 3-\log 2+\log 2^2-\log 3+\log 5-\log 2^2+\log(2\times 3)-\log 5$

$= \log 3-\log 2+2\log 2-\log 3+\log 5-2\log 2+\log 2+\log 3-\log 5$

$= 2\log 3-\log 3+3\log 2-3\log 2+\log 5-\log 5$

$= \log 3$ (Proved)

Illustration : 10

Prove that $9\log\frac{10}{9}+3\log\frac{81}{80}+2\log\frac{108}{125}=\log 2$

Solution

L.H.S $= 9(\log 10-\log 9)+3(\log 81-\log 80)+2(\log 108-\log 125)$

$= 9\log 10-9\log 9+3\log 81-3\log 80+2\log 108-2\log 125$

$= 9\log 10-9\log 3^2+3\log 3^4-3\log(2^4\times 5)+2\log(3^3\times 2^2)-2\log 5^3$

$= 9-18\log 3+12\log 3-3\log 2^4-3\log 5+2\log 3^3+2\log 2^2-2\log 5^3$

$= 9-18\log 3+12\log 3-12\log 2-3\log 5+6\log 3+4\log 2-6\log 5$

$= 9-8\log 2-9\log 5$

$= 9-9\log 2-9\log 5+\log 2$

$= 9-9(\log 2+\log 5)+\log 2$

$= 9-9(\log 10)+\log 2$

$= \log 2$ Proved.

Illustration : 11

Evaluate $\log_7 \sqrt{7\sqrt{7\sqrt{7.....\infty}}}$

Solution

Let $\sqrt{7\sqrt{7\sqrt{7.....\infty}}} = x$

Squaring both the sides :

$x^2 = 7\sqrt{7\sqrt{7\sqrt{7}}}$

or $x^2 = 7x$

or $x^2 - 7x = 0$

or $x^2 - 7x = 0$ or $x(x-7) = 0$

Hence either $x = 0$ or $x - 7 = 0$

We know that $x \neq 0$, so $x - 7 = 0$

Now $x = 7$

$\therefore \log_7 \sqrt{7\sqrt{7\sqrt{7.....\infty}}} = \log_7 x = \log_7 7 = 1$ (Ans)

Illustration : 12

Prove that (i) $2\log_2 7=7$

(ii) $3^{2+\log_3 5} = 45$

Solution

(i) Let $(2)^{\log_2 7} = x$

Changing to logarithmic form.

$\log_2 x = \log_2 7$

or $x = 7$

$\therefore\ 2^{\log_2 7} = 7$ Proved.

(ii) L.H.S $= 3^{2+\log_3 5}$

$= 3^2 \times 3^{\log_3 5}$

$= 9 \times 5$ ($\because 3^{\log_3 5} = 5$)

$= 45$ (Ans.)

3.6 STANDARD FORM OF A POSITIVE NUMBER.

Standard form is a number having one non-zero digit before the decimal point.

For example standard form of 123.0 is 1.230×10^2

A number can be converted to a standard form by keeping its first significant digit in the integral place and shifting the position of decimal point either to the left or right and multiplying the same with 10^x where x is an integer. This x is also determine the value of characteristics.

For example :

Standard form of $932.56 = 9.3256 \times 10^2$

Standard form of $.0093256 = 9.3256 \times 10^{-3}$

In the above examples the numbers are converted to a form with one significant digit i.e., '9' in the integral part. The decimal point in first case is shifted two places to the left and in the second case shifted to three places to the right. The converted numbers are multiplied by 10^x in order to retain the same value. Hence in the first case 10^2 and in the second case 10^{-3} is multiplied with the converted number.

Illustration : 13

Find the standard form of the following numbers :

(i) 12.41 (ii) 0.006932 (iii) 3.064

(iv) 639.32 (v) 0.6392 (vi) 0.03064

Solution

(i) Standard form of $12.41 = 1.241 \times 10^1$

(ii) Standard form of $0.006932 = 6.932 \times 10^{-3}$

(iii) Standard form of $3.064 = 3.064 \times 10^0$

(iv) Standard form of $639.32 = 6.3932 \times 10^2$

(v) Standard form of $0.6392 = 6.392 \times 10^{-1}$

(vi) Standard form of $0.03064 = 3.064 \times 10^{-2}$

Illustration : 14

Which of the following are not in standard form and why ?

(i) 0.538×10^{-1} (ii) 46.3×10^{-1}

(iii) 5.221×10^{25} (iv) 3.007×10^1

(v) 0.3988×10^0 (vi) 113.18×10^{-5}

(i) 0.538×10^{-1} is not in standard form. The standard form should have one significant digit in the integral part. The standard form should be in the form of 5.38×10^x.

(ii) 46.3×10^{-1} is not is standard form. The standard form should be in the form of 4.63×10^x

(iii) 5.221×10^{25} is in standard form as this number has one non-zero digit before the decimal point.

(iv) 3.007×10^1 is in standard form as it has one non-zero digit before the decimal point.

(v) 0.3988×10^0 is not in standard form. The standard form should be in the form of $3.988 \times 10^{x.}$

(vi) 113.18×10^{-5} is not in standard form. The standard form should be in the form of 1.1318×10^x.

3.7 CHARACTERISTICS AND MANTISSA :

The logarithmic value of a number consists of two parts i.e., integral part (the portion before the decimal point) and the decimal part (the portion after the decimal point). The integral part is known as 'characteristics' and the decimal part is known as 'mantissa'. For example, The logarithmic value of a number 4.32 indicates that 4 is characteristic part and .32 is the mantissa part.

3.7.1 Determination of characteristic part :

The characteristics part of a number may be positive or negative or zero.

The characteristic part of a number can be determined either by inspection or by converting to standard form. There are two methods to determine the characteristic which are explained below:

(a) Inspection method : Characteristic by inspection method can be determined with the help of following rules.

Rule-1 : This rule is applicable when the number consists of significant digits in the integral part which is equal to or greater than unity i.e., 1.

The characteristic of the logarithm of a number greater than unity is one less than the number of digit in the integral part. This is positive characteristic.

Mathematically, characteristic = + (No. of digits in the integral part -1)

For example, characteristic of 24.469 is + (2-1) = +1 (since there are two digits in the integral part).

Rule-2 : This rule is applicable when there is no significant digit in the integral part of a nmber. The number in such case is less than 1 (unity) but greater than zero.

Characteristic of such a number is one more than the number of zeroes immediabely after the decimal point. But this is negative characteristic. The negative characteristic is usually expressed in 'Bar form' viz, $\bar{2}$, $\bar{3}$, $\bar{6}$. etc.

Mathematically :

Characteristic = - (no. of zeroes immediately after the decimal point +1)

For example, characteristic of 0.0563 is - (1+1) = -2 (since there is one zero just after the decimal point)

Table for Rule 1

Characteristic of Numbers >1

Number > 1 Form... To	No. of Digits in the number (n)	Characteristis (n-1)
1 to 9	1	0
10 to 99	2	1
100 to 999	3	2
1000 to 9999	4	3
10000 to 99999	5	4
100, 000 to 999999	6	5

Hence we come to the conclusion that the characteristic of the logarithm of a number greater than unity is one less than the number of digits in its integral part and is positive.

Mathematically,

If N = Number, n = number of digits in the integral part, then characteristic = +(n-1)

Table for Rule-2

Characteristic of a Number < 1

Number >0<1 Form... To	No. of Zeros after decimal points.	Characteristis
.1 to .9	Nil	- (0+1)= -1 or $\bar{1}$ (1bar)
.01 to .09	1	- (1+1)= -2 or $\bar{2}$ (2bar)
.001 to .009	2	- (2+1)= -3 or $\bar{3}$ (3bar)
.0001 to .0009	3	- (3+1)= -4 or $\bar{4}$ (4bar)
.00001 to .00009	4	- (4+1)= -5 or $\bar{5}$ (5bar)
.000001 to .000009	5	- (5+1)= -6 or $\bar{6}$ (6bar)

So the characteristic of the logarithm of a number less than unity is one more than the number of zeros just after the decimal print, and is negativa. which is clear from the table for Rule-2.

(b) Standard form method :

For determining the characteristic of a number the number should be converted to standard form i.e., $m \times 10^x$ (where 'm' has only one significant digit in the integral past). Then the power of m, i.e., x is the characteristic of that number. This is a general rule for all number whether less than 1 or more than 1.

For example, The characteristic of 24.39 can be determined after converting 24.39 into standard form, i.e., 2.439×10^1.

Hence the characteristic is 1 (since 1 is the power of 10) similarly characteristic of 0.075 can be computed like this : $0.075 = 7.5 \times 10^{-2.}$

Hence -2 or $\bar{2}$ (Bar 2) is the characteristic.

Following examples will make this clear.

Number	Inspection method		Standard form method	
	Formula	Characteristic	(Standard form)	(Characteristic)
0.521	- (0+1)	-1 or $\bar{1}$ (Bar1)	5.21×10^{-1}	-1 or T (Bar I)
13.325	+ (2-1)	1	1.3325×10^1	+1
133.25	+ (3-1)	2	1.3325×10^2	+2
0.0133	- (1+1)	-2 or $\bar{2}$ (Bar 2)	1.33×10^{-2}	-2 or $\bar{2}$ (Bar 2)
0.0005	- (3+1)	-4 or $\bar{4}$ (Bar 4)	5.0×10^{-4}	-4 or $\bar{4}$ (Bar 4)
0.2003	- (0+1)	-1 or $\bar{1}$ (Bar 1)	2.003×10^{-1}	-1 or $\bar{1}$ (Bar 1)
2457	+ (4-1)	3	2.457×10^3	+3
2.457	+ (1-1)	0	2.457×10^0	0

3.7.2 Determination of Mantissa part :

While finding the logarithm of a number, the logarithm value consists of two parts, the integral part and the decimal part. Mantissa is the decimal part of the logarithm value and this can

be foundout from the logarithm table. The logarithm table is based on the common logarithm system and may be 4 or 5 figres representing the decimal part of logarithm correct to 4 decimal places or 5 decimal places.

The mantissa is always positive. All the numbers consisting of the same digits and placed in the same sequence have the same mantissa irrespective of the position of decimal point.

For example, 392, 39.2, 3.92, 0.392, 0.0392, digits 3, 9 and 2 in the same order. The mantissa of all the above numbers shall be same.

In order to determine the mantissa of a logarithm of a given nmber it is necessary to know the method of reading a logarithm table.

LOGARITHM TABLE :

In this book the four figure logarithm tables are used. In order to determine the mantissa, we have to ignore the decimal point if any in the given number. In the Log table the two figures or digits written in the first colmn represent the first two digits of the number.

Hence in case the number has only two significant digits, the mantissa is found against the digits of the first column with '0' as the heading. But if the number contains three significant digits, the 1st two digits are located in the 1st column of the log table and the third digit is located in the column heading, i.e., from 0 to 9. The intersecting value represent the mantissa.

This will be clear from the following table :

Extract of Log table (Four figure)

					Mean Difference				
	0	1	2	.....	1	2	3	4	5
10	0000	0043	0086		4	8	12	17	21
11	0414	0453	0492		4	8	11	15	19
12	0792	0828	0864		3	7	10	14	17
13	1139	1173	1206		3	6	10	13	16
14	1461	1492	1523		3	6	9	12	15
15	1761	1790	1818		3	6	8	11	14

Example :

Mantissa of 12	is	.0792
Mantissa of 1.2	is	.0792
Mantissa of 0.12	is	.0792

In all the above cases after ignoring decimal point we get 12. Mantissa is the interesecting value taking 12 of the first column with zero as the column heading.

Mantissa of 122	is	.0864
Mantissa of 12.2	is	.0864
Mantissa of 1.22	is	.0864
Mantissa of 0.122	is	.0864

In the above cases there are three significant digits. After ignoring decimal point we get 122. Now 12 is taken from the first column and 2 as the column heading to obtain the mantissa.

If the number consists of four significant digits the mantissa is calculated in the following manner.

(i) The mantissa of the first three digits are shown in the above examples.

(ii) The logarithm table contains 'Mean Difference' in its right part. The value obtained as in (i) is added with the nmber found in the same row in the 'Mean Difference' column headed by the number corresponding to the 4th digit of the number.

The following example will clarity it.

Example

(i) To determine the logarithm of 1123, first we have to take first two digits '11' from the first, i.e., the extreme left column and third digit from the head row and find the interesecting value which is 0.0492. In the next step we shall obtain the mean difference taking '11' of the 1st column and the fourth digit '3' from the 'Mean Difference' portion of the table, which is '11'. Corresponding to the above value this is considered as 0.0011.

Finally the mantissa of the given number 1123 can be determined by adding 0.0011 with 0.0492, i.e., 0.0492 + 0.0011 = 0.0503.

(ii) Similarly to determine the mantissa of 12.25, by ignoring the decimal point we obtain

1225. There are four digits. By taking first two digits 12 from the first column and third digit '2' as the heading, the intersection value in the logarithm table is 0.0864.

The mean difference taking '12' from the first column and fourth digits '5' as the heading of 'Mean Difference' portion we get 17. Now the mantissa shall be 0.0864+0.0017=0.0881.

Illustration : 15

What will be the characteristics of the logarithms of :

(i) 13300 (ii) 469.21 (iii) 4.56

(iv) 0.653 (v) 0.0025 (vi) 27.004

Soltion

(i) The number of digit before the decimal point is 5 so the characteristic is 5-1=4

So the characteristic is 5-1 = 4

Alternatively, the standard form is 1.3300×10^4

Hence the characteristic is 4

(ii) The number of digit before the decimal point is 3 so the characteristic is (3-1)=2

So the characteristic is (3-1) = 2

Alternatively, the standard form is 4.6921×10^2

Hence the characteristic is 2.

(iii) The number of digit before the decimal point is 1

So the characteristic is (1-1) = 0

Alternatively the standard form is 4.56×10^0

So the characteristic is 0.

(iv) The number 0.653 is less than 1 and there is no zero after the decimal point. Hence the charateristic of 0.653 will be -(0+1)= -1 or T.

Alternatively, standard form of 0.653

$$= \frac{0.653\times10}{10} = 6.53\times10^{-1}$$

Hence the characteristic is $\bar{1}$.

(v) The number 0.0025 is less than 1 and there are two zeros just after the decimal point.

Hence the characteristic is - (2+1) = -3 or = $\bar{3}$.

Alternatively, standard form of 0.0025

$$= \frac{0.0025 \times 1000}{1000} = 2.5 \times 10^{-3}$$

$\therefore$ Characteristic is $\bar{3}$.

(vi) The number 27.004 is greater than 1 and the number of digit before the decimal point is 2. hence the characteristic is (2-1) = 1.

Alternatively, the standard form of 27.004

$$= \frac{27.004 \times 10}{10} = 2.7004 \times 10^{1}$$

$\therefore$ Hence the Characteristic is 1.

Illustration : 16

Find the logarithm of the following numbers using log table.

(i) 6.037 (ii) 300 (iii) 1557 (iv) 1.5647

(v) 1245.65 (v) 0.00365 (vii) 0.06337 (viii) 0.5493

Soltion

(i) 0.8041 (ii) 2.4771 (iii) 3.1923 (iv) 0.1942

(v) 3.0951 (vi) $\bar{3}$.5623 (vii) $\bar{2}$.8019 (viii) $\bar{1}$.3542.

3.8 ANTILOGARITHM

The reciprocal of logarithm is called Antilogarithm. Antilogarithm in short is called 'Antilog'. It can be better understood from the following example.

(i) If log m = x

Then Antilog of x = m.

(ii) If log 10,000 = 4

Then Antilog of 4 = 10,000.

When it is required to findout a number whose logarithm is given. We need to determine. its antilogarithm. And when we want to calculate the antilogarithm of a given number, we have to refer to the Antilogarithm table. The process of finding antilogarithm of a given number is exactly the reverse of the procedure of finding logarithm.

3.8.1 Procedure of determining Antilog :

(a) While finding the antilog of a number, the integral part of the number shoulod be ignored. In other words we are required to take the mantissa part only and ignore the characteristic.

(b) The first two digits of the mantissa should be located from the first column of Antilog table.

(c) The third digit of the mantissa should be located from the top heading of the table and the corresponding intersecting value can be found from the body of the table.

(d) If there are more than three digits, the mean difference column should be referred in the same manner as in case of finding mantissa.

(e) After obtaining the table value of antilog table the decimal point is placed in the appropriate place depending upon the characteristic.

For example, In order to obtain antilog of 2.5467, first we need to ignore the characteristic i.e. 2. The mantissa i.e., 5467 shall be considered to get the antilog value from the antilog table. The antilog value of 0.5467 is 3522. Since the characteristic is 2, there should be 3 digits before the decimal point i.e., 352.2.

Hence the Antilog of 2.5467 = 352.2.

Alternatively, since we have foundout the antilog of mantissa part from the antilog table but not the antilog of characteristic, we can findout the Antilog of characteristic by inspection, For example antilog of 2 = 100, and 100 is having 3 digit so after 3 from left you can put the decimal point. So anti log of 0.5467 is 352.2.

Illustration : 17

Find the numbers whose logarithms are

(i) 2.4567 (ii) $\bar{1}$.4085 (iii) $\bar{2}$.6514 (iv) 0.3997

Solution

(i) We have to use mantissa, i.e., .4567.

For mantissa .45 row 6th column = 2858

Add Mean Difference for 7 = 5

$\therefore$ For mantissa .4567 the required figure = 2863 As the characteristic is 2. there must be three digits in the integral part opf the number.

Hence the antilog of 2.4567 = 286.3

(ii) The mantissa .4085 is taken into account to find out its value. The antilog table value of .4085 is 2562. Since the characteristic is $\bar{1}$, the decimal point should be placed just before 2562. Hence the number whose logarithm value is $\bar{1}.4085 = 0.2562$.

(iii) The mantissa .6514 is referred to antilog table and the table value is 4484. The decimal point should be placed having one zero after the decimal point, since its characteristic is $\bar{2}$. Hence antilog of $\bar{2}.6514 = 0.04481$.

(iv) Mantissa .3997 is referred to antilog table and found to be 2510 and since the characteristic is zero, there should be one significant digit in the integral part.

Hence antilog value of 0.3997 is 2.510.

Illustration : 18

If log 41.06 = 1.6134, Find the value of

(i) log .004106 (ii) Antilog 2.6134

Solution

(i) Given log 41.06 = 1.6134 $\Rightarrow$ A.L. of 1.6134 = 41.06

Hence A.L. of 2.6134 = 410.6 (The decimal print is placed after 3 significant digit, since the characteristic is 2)

3.9 EVALUATION OF LOGARITHM BY INTERPOLATION METHOD

The interpolation refers to finding the value of logarithm of a number by making change proportionately. It is based on the principle of proportional parts. The principle states that a change in the number results in proportionate change in the logarithm value. This will be

clarified by the following example.

Example :

Find log 3.8456, when log 3.845 = 0.5849 and log 3.846 = 0.5850.

Solution :

log 3.846 = 0.5850

log 3.845 = 0.5849

$\therefore$ Difference in the logarithm for a difference in number of .001 = .0001

" " " .0006 $= \frac{.0001 \times .0006}{.001}$

$= .00006$

$\therefore$ Log 3.8456 = 0.5849 + .00006

= 0.58496

MISCELLANEOUS ILLUSTRATIONS

Illustration : 19

If log 7 = 0.8451, log 3 = 0.4771, find log 21^3

Solution

$\text{Log}21^3 = 3 \log 21 = 3 \log (7 \times 3)$

$= 3 (\log 7 + \log 3)$

$= 3(0.8451 + 0.4771)$

$= 3(1.3222)$

$= 3.9666$ (Ans)

Illustration : 20

Given $\log_{10} 2 = 0.30103$, find the value of $\log_8 25$

Solution

By property of change of base, we have

$$\text{Log}_b m = \frac{\log_a m}{\log_a b}$$

$$\therefore \text{Log}_8 25 = \frac{\log_{10} 25}{\log_{10} 8} = \frac{\log 5^2}{\log 2^3}$$

$$\Rightarrow \frac{2\log 5}{3\log 2} = \frac{2\log \frac{10}{2}}{3\log 2}$$

$$= \frac{2(\log 10 - \log 2)}{3\log 2} = \frac{2 - 2\log 2}{3\log 2}$$

$$= \frac{2 - 0.60206}{0.90309} = 1.5482$$

$\therefore \text{Log}_8 25 = 1.5482$ (Ans)

Illustration : 21

Given log2 = 0.30103

Find the value of (i) $\log_{100} 128$

(ii) log400

Solution

(i) $\text{Log}_{100} 128 = \log_{10} 128 \times \log_{100} 10$ (By the law of change of base)

$$= \log_{10} 128 \times \frac{1}{\log_{10} 100}$$

$$= \frac{\log 128}{\log 100} = \frac{\log 2^7}{\log 10^2}$$

$$= \frac{7\log 2}{2\log 10} = \frac{7 \times 0.3010}{2 \times 1}$$

$$= \frac{2.1070}{2} = 1.0535$$

$\therefore$ $\text{Log}_{100}128 = 1.0535$ (Ans)

(ii) $\text{Log } 400 = \text{Log}(2^2 \times 10^2)$

$= \log 2^2 + 2\log 10$

$= 2\log 2 + 2\log 10$

$= 2 \times 0.3010 + 2 \times 1$

$= 2.6020$

$\therefore$ $\text{Log}400 = 2.6020$ (Ans.)

Illustration : 22

Find the value of the following with the help of log table.

(i) 4.785×0.624

(ii) $3.219 \times 0.0379 \times 11.11$

Solution

(i) Let $x = 4.785 \times 0.624$

Taking logs

$\log x = \log (4.785 \times 0.624)$

or $\log x = \log 4.785 + \log 0.624$

$= 0.6799 + \bar{1}.7984$

$= 0.4783$

$\therefore$ x = Antilog (0.4783)

$= 3.008$ (Ans)

(ii) Let $x = 3.219 \times 0.0379 \times 11.11$

Taking logs,

$\log x = \log (3.219 \times 0.0379\ 0 \times \log 11.11)$

or $\log x = \log 3.219 + \log 0.0379 + \log 11.11$

or $\log x = 0.5077 + 2.5786 + 1.0457$

$\therefore\ x = \text{Antilog } 0.1320$

$= 1.355$ (Ans.)

Illustration : 23

Using log table find the value of :

(i) $(3.125)^9$, (ii) $(36.21)^3$, (iii) $(3.57)^{-6}$, (iv) $(.006253)^{\frac{1}{5}}$

Solution

(i) Let $x = (3.125)^9$

Taking logs, $\log x = \log (3.125)^9$

$= 9 \log (3.125)$

$= 9 \times 0.4949$

$= 4.4541$

$\therefore\ x = \text{Antilog of } 4.4541$

$= 28450$ (Ans.)

(ii) Let $x = (36.21)^3$

Taking log of both sides

$\log x = \log (36.21)^3$

or $\log x = 3 \log (36.21)$

$= 3 \times 1.5588$

$= 4.6764$

$\therefore\ x = \text{Antilog of } 4.6764$

$= 47460$ (Ans)

(iii) Let $x = (3.57)^{-6}$

$\therefore$ Taking log of both the sides

$\log x = \log (3.57)^{-6}$

or $\log x = -6 \log (3.57)$

$= -6 \times 0.5527$

$= -(3.1162)$

$= -3-1+1- .1162$

$= -4 + .8838$

$= \bar{4}.8838$

$\therefore \quad x =$ Antilog of $\bar{4}.8838$

$= .0007651$ (Ans)

(iv) Let $x = (.006253)^{\frac{1}{5}}$

or $\log x = \log(.006253)^{\frac{1}{5}}$

or $\log x = \frac{1}{5} \log 0.006253$

$= \frac{1}{5} \times \bar{3}.7961$

$= (\bar{5}+2.7961)$

$= \bar{1}+.5592$

Now x = Antilog $(\bar{1}+.5592)$

$= 0.3624$ (Ans)

Illustration : 24

Using logarithm table compute the values of following :

(i) $\sqrt[3]{26.98}$ (ii) $\frac{1}{\sqrt[7]{36.21}}$

Soltion :

(i) Let $x = \sqrt[3]{26.98}$

Taking logs

$\log x = \log \sqrt[3]{26.98}$

or $\log x = \log (26.98)^{\frac{1}{3}}$

$= \frac{1}{3} \log 26.98$

$= \frac{1}{3} \times 1.4311 = 0.4770$

$\therefore \quad x =$ Antilog of 0.4770

$= 2.999$ (Ans)

(ii) Let $x = \frac{1}{\sqrt[7]{36.21}}$

or $\log x = \log \frac{1}{\sqrt[7]{36.21}}$

$= \log \frac{1}{(36.21)^{\frac{1}{7}}}$

$= \log (36.21)^{-\frac{1}{7}}$

$= -\frac{1}{7} \log 36.21$

$= -\frac{1}{7} \times 1.5588$

$= -0.2227$

$= -1 + 1 - 0.2227$

$= \bar{1}.7773$

$\therefore x =$ Antilog of $\bar{1}.7773 = 0.5988$ (Ans.)

Illustration : 25

Using log table, find the value of

(i) $\sqrt[4]{54.32} \times (0.9876)^3$ (ii) $\frac{1}{\sqrt[7]{.08176 \times 36.21}}$ (iii) $\left(\frac{21}{20}\right)^{100}$

Solution

(i) Let $x = \log \sqrt[4]{54.32} \times (0.9876)^3$

$\therefore \log x = \log\left(54.32^{\frac{1}{4}} \times 0.9876^{3}\right)$

or $\log x = \log\left(54.32^{\frac{1}{4}} \times 0.9876^{3}\right)$

$= \frac{1}{4}\log 54.32 + 3\log 0.9876$

$= \frac{1}{4} \times 1.735 + 3 \times \bar{1}.9946$

$= 0.4338 - 0.0162 = 0.4176$

$\therefore x = \text{Antilog } 0.4176 = 2.616$ (Ans.)

(ii) $x = \frac{1}{\sqrt[7]{.08176 \times 36.21}} = \left(\frac{1}{.08176 \times 36.21}\right)^{\frac{1}{7}}$

$\therefore \text{Log } x = \frac{1}{7}\left[\log 1 - \log(.08176 \times 36.21)\right]$

or $\text{Log } x = \frac{1}{7}\left[\log 1 - \log .08176 - \log 36.21)\right]$

$= \frac{1}{7}\left[0 - \bar{2}.9125 - 1.5588\right]$

$= \frac{1}{7}\left[0 + 2 - .9125 - 1.5588\right]$

$= \frac{1}{7}\left[-0.4713\right]$

$= -0.0673 = -1 + 1 - .0673$

$= \bar{1}.9327$

$\therefore x \text{ AL } (\bar{1}.9327) = 0.8565$ (Ans.)

(iii) Let $x = \left(\frac{15}{20}\right)^{100}$

Taking log of both the sides

$$\log x = \log \left(\frac{21}{20}\right)^{100} = \log \left(\frac{21}{20}\right)^{100}$$

$$\text{or } \log x = 100 \log \frac{1}{8}$$

$$= 100 \log 21 - 100 \log 20$$

$$= 100 \times 1.3222 - 100 \times 1.3010$$

$$= 132.22 - 130.10$$

$$= 2.12$$

$\therefore x$ = Antilog of 2.12

$x = 131.8$ (Ans)

Illustration : 26

If log 3 = 0.4771, Find the number of digits in 3^{23}.

Solution

Let $x = 3^{23}$

or $\log x = 23 \log 3$

$$= 23 \times 0.4771$$

$$= 10.9733$$

$\therefore x$ = Antilog 10.9733

Since the characteric is 10, the number of digits in the integral part would be 11.

Illustration : 27

If log 3 = 0.4771

Find the number of zeros after the decimal print in $(0.3)^{25}$.

Solution :

Since Log 3 = 0.4771 and log .3 = $\bar{1}$.4771

Now Let $x = (0.3)^{25}$

$\therefore$ Log x = 25 log 0.3

= 25 × $\bar{1}$.4771

= (25-x-1) + 25 (0.4771)

= -25 + 11.9275 = $\overline{14}$ + 0.9275

= $\overline{14}$.9275

Since characteristic is $\overline{14}$, The number of zeros after decimal point will be 13.= (14-1) (Ans.)

Illustration : 28

(i) If $3^{2x+1} \times 5^{2x} = 2^{3x}$, Find the value of x

(ii) If $6^{3-4x} \times 4^{x+5} = 8$, Find the value of x

Solution

(i) Applying logarithm on both the sides of the given equation, we have :

Log$(3^{2x+1} \times 5^{2x})$ = Log 2^{3x}

$\Rightarrow$ (2x+1) Log 3 + 2x Log5 = 3$x$log2

$\Rightarrow$ (2x+1) Log 3 + 2x Log $\frac{10}{2}$ = 3$x$log2

$\Rightarrow$ 2x log3+Log 3 + 2x Log10-2xlog2 = 3x log2

$\Rightarrow$ 2x × 0.4771+ 0.4771+2x × 1-2x×.3010 = 3x ×0.3010

$\Rightarrow$ 0.9542x + 0.4771 + 2x -2x (.3010) = 0.9030x

$\Rightarrow$ 2.9542x - 0.6020x - 0.9030x = - 0.4771

$\Rightarrow$ 1.4492x = - 0.4771

$\Rightarrow x = -\frac{0.4771}{1.4492} = -0.3292$ (Ans.)

(ii) Taking logarithm of both the sides, we have

Log $(6^{3-4x} \times 4^{x+5})$=log 8

$\Rightarrow \text{Log } 6^{3-4x} \times \log 4^{x+5} = \log 2^3$

$\Rightarrow (3 - 4x) \log 6 + (x + 5) \log 4 = 3\log 2$

$\Rightarrow 3 - 4x(0.7782) + (x+5)\ 0.6021 = 3 \times 0.3010$

$\Rightarrow 3 \times 0.7782 - 4x \times 0.7782 + 0.6021x + 5 \times 0.6021 = 0.9030$

$\Rightarrow 2.3346 - 3.1128x + 0.6021 + 3.0105 = 0.9030$

$\Rightarrow -2.5107x = 0.9030 - 2.3346 - 3.0105$

$\Rightarrow -2.5107x = -4.4421$

$\Rightarrow x = \dfrac{-4.4421}{-2.5107} = 1.7692$ (Ans.)

Illustration : 29

Given log 34.567=1.538668

Log 34.568 = 1.538681. Find log 345.6765

Now log 3456800=6.538681, log3456700=6.538668.

Difference 100 = .00013

Thus, for difference of 100 in the number, the change in mantissa is 13. Now by the method of interpolation we can obtain the corresponding change in the mantissa.

Change in the number	Change in the mantissa
100	13
65	$\dfrac{13}{100} \times 65 = 8.45$

Log 345.6700 = 2.538668 and for .0065 = 0.00000645

$\therefore$ log 345.6765 = 2.53867645 = 2.538676 (Ans.)

4.0 EXERCISE

GROUP -A

Q.1. Multiple choice type questions :

From the following alternatives choose and write the correct answer against each bit :

(i) Logarithm of 216 to the base 6 is :

(a) 2, (b) 4, (c) 3, (d) -2

(ii) The logarithm of 1728 to the base $2\sqrt{3}$ is :

(a) 4, (b) 6, (c) 2, (d) 3

(iii) Characteristic in logarithm of 0.002589 is :

(a) -4, (b) -3, (c) 4, (d) 3

(iv) If log 16 = 1.2041, Then log 4 is :

(a) 0.60205, (b) 0.40137, (c) 1.10205, (d) 2.0508

(v) The value of $\log_{343} 7$ is :

(a) $\frac{1}{3}$, (b) -3, (c) $-\frac{1}{3}$, (d) 3

(vi) The value of $\log_{10}(.0001)$ is :

(a) $\frac{1}{4}$, (b) $-\frac{1}{4}$, (c) -4, (d) 4

(vii) If $\log_x 4 = 0.04$, then the value of x is :

(a) 2, (b) .4, (c) 16, (d) 32

(viii) If log 5 = 0.6990 and log 3 = 0.4771. Then log 45 is :

(a) 1.8751, (b) 1.9085, (c) 2.1303, (d) 1.6532

Ans. (i) c, (ii) b, (iii) b, (iv) a, (v) a (vi) c, (vii) d, (viii) d

Q.2 (A)

Answer the following in one word / term each :

(i) Name the decimal part of the logarithm value.

(ii) Name the integral part of the logarithm value.

(iii) In what name is the logarithm whose base is '10'. Usually called ?

(iv) What is the characteristic part of 526.025 ?

(v) In what name is the logarithm whose base is 'e' called ?

Ans.: (i) Mantissa, (ii) characteristic, (iii) Common, (vi) 2, (v) Natural logarithm

Q.2 (B)

Answer the following in one sentence each :

(i) Define Logarithm

(ii) What is meant by common logarithm ?

(iii) What is natural logarithm ?

(iv) Define antilogarithm.

(v) What is standard form ?

Q.2 (C)

Correct the underlined portion of each of the following sentences :

(i) Logarithm of any number to the base it self is 10.

(ii) $\log_a(m \times n) = \log_a m \times \log_a n$

(iii) The characteristic of 0.0007 is 4.

(iv) Mantissa is always negative.

(v) Logarithm of intinity to the base greater than one is equal to zero.

(vi) $\log_n m = \dfrac{1}{\log_n m}$

Ans. (i) 1, (ii) $\log_a m + \log_a n$, (iii) $\bar{4}$, (iv) Positive, (v) ∞, (vi) $\log_m n$.

Q.2 (D)

Fill in the blanks :

(i) '$\text{Log}m^n = n \log m$' is ---------- law of logarithm.

(ii) Logarithm of 0.01 to the base 10 is ---------- .

(iii) Logarithm of 1 to any non zero base is ---------- .

(iv) The decimal part of logarithm value is ---------- .

(v) The value of logarithm will be ---------- in case of change of base.

Ans. (i) Involution law (ii) -2 (iii) 0, (iv) Mantissa (v) Same.

GROUP -B

Q.3 Answer the following within two sentences each :

(i) Find the value of $\log_5 125$.

(ii) Find the value of $\log_2 \frac{1}{8}$

(iii) Find the value of $\log_5 \frac{1}{3125}$

(iv) What is change of base laws ?

(v) What do you know about system of logarithm ?

(vi) Find the value of $\log_{.01} 0.1$.

(vii) Find the number whose logarithm is -4.5229.

(viii) Find the Antilog of 3.259.

LONG TYPE QUESTIONS

Q.4. Find the value of the following using logarithm :

(i) (*i*) $\frac{10.23^3}{502}$ (*ii*) $\frac{(21.2)^2}{300}$, (*iii*) $6.331 \times \sqrt{\frac{706}{983}}$

(ii) Evaluate $\frac{(38.56)^2 \times (4.381)^3}{(271.4) \times (73.45)^2}$

(iii) Evaluate $\sqrt{\left[\frac{294 \times 125}{42 \times 32}\right]^2}$

(iv) Evaluate $\frac{2\log 6 + 6\log 2}{4\log 2 + \log 27 - \log 9}$

Q.8. Without using log book, prove that :

(i) $\log\frac{41}{35}+\log 70-\log\frac{41}{2}+2\log 5=2$

(ii) $\log_2\log_2\log_2 16=1$

(iii) $7\log\frac{16}{15}+5\log\frac{25}{24}+3\log\frac{81}{80}+\log\frac{1}{2}=0$

(iv) $\log\frac{81}{16}-\log\frac{8}{9}+\log\frac{128}{243}=\log 3$

(v) $\log\frac{75}{16}-2\log\frac{5}{9}+\log\frac{32}{243}=\log 2$

Q.9. Find the value of x, if $(1.05)^2=\frac{10}{3}$

Q.10. Solve the following.

$(1+x)^{20}=1.8734$

Q.11. Solve the following.

$$\frac{(1.04)^x-1}{0.04}=15.023$$

Q.12. Solve $(1-x)^{10}=0.9045$

Q.13. Find x

$3180\times(1.06)^x=5070$

Q.14. Evaluate :

(i) $\sqrt[9]{432.7}$

(ii) $\sqrt[7]{(2.943)^2}$

(iii) $\sqrt[5]{0.7875}$

Q.15. Prove that :

$\text{Log}_y x \cdot \log_z y \cdot \log_x z = 1$

Q.16. Show that :

$\text{Log}_y x^3 \cdot \log_z y^3 \cdot \log_x z^3 = 27$

Q.17. Prove that :

$\left(\frac{21}{20}\right)^{100} > 110$, given $\log 2 = 0.3010$

Log3=0.4771, Log 7 = 0.8451 and Antilog 0.12 = 1.318.

Q.18. Prove that :

$X^{\log y - \log z} \cdot Y^{\log z - \log x} \cdot Z^{\log x - \log y} = 1$

Q.19. If $\log x = \log y = \log (x + y)$

Prove that $x = \frac{y}{y-1}$

Q.20. If $\frac{\log x}{y-z} = \frac{\log y}{y-x} = \frac{\log z}{x-y}$

Prove that $x^x \cdot y^y \cdot z^z = 1$.

Q.21. If $x^3 + y^3 = 0$ and $x+y \neq \phi$, prove that

$\text{Log}(x+y) = \frac{1}{2}(\log x + \log y + \log 3)$

Q.22. Evaluate : $\log_3 \sqrt{3\sqrt{3\sqrt{3.....\infty}}}$

Q.23. Prove that $\log_2 \log_{\sqrt{2}} \log_3 81 = 2$

Q.24. Find the number of digits in :

(i) 3^{19}, (ii) 2^{16}

Q.25. Find the number of zeroes just after the decimal point till the significant digit.

(i) 3^{-17} , (ii) 5^{-15}

ANSWER

4. (i) 2.131, (ii) 1.498, (iii) 5.37

5. 0.01407,

6. 9.076, 7. 2

7. 2

9. 24.67,

10. 0.319

11. 12.02

13. 7.977

14. 1.963

22. 3, 0

24. (i) 9 (ii) 5

25. (i) 8, (ii) 10

-o-

CHAPTER-4

SIMPLE INTEREST AND COMPOUND INTEREST

STRUCTURE

4.1 Meaning

4.2 Important terms

4.3 Simple interest - its calculation

4.4 Compound interest - its calculation

4.5 Varying rates of interest

4.6 Nominal and effective rate of interest

4.7 Exercise

4.1 MEANING

Every business undertaking needs money for runing the business. But always that money cannot be provided out of its owned fund/capital. So business undertakings, like individuals have to borrow fund from the outsiders. Outsiders may be individual persons, banks or any other financial institutions.

So there are two parties involved in calculation of interest i.e., the borrower and the lender. Initially the money which flows from lender to the borrower is called principal. And again after expiry of the period, the money which flows from the borrower to the lender is called amount. This amount is sum total of principal and an additional money. This additional money which is given by the borrower for use of that principal is called 'Interest'. This interest is of two types such as simple and compound.

4.2 IMPORTANT TERMS

The sum borrowed is called the principal, the time period for which it is borrowed is called as the period or term and the rate of interest charged on A100 per year is termed

as rate percent per annum. The Principal along with interest returned by the borrower is called as Amount.

Let P be the principal

T be the time or period in years

and R be the rate per cent per annum.

Then on A100 for 1 year interest will be R

$\therefore$ Interest on P for 1 year = $\frac{PR}{100}$ (since time=1 year)

But Interest on P for T years = $\frac{PRT}{100}$

$\therefore$ SI (Simple Interest) = $\frac{PRT}{100}$

$\therefore$ P = $\frac{SI \times 100}{RT}$

And R = $\frac{SI \times 100}{PT}$

Hence T = $\frac{SI \times 100}{PR}$

Now Amount = Principal + SI

or A = P + $\frac{PRT}{100}$

or A = P $\left[1 + \frac{RT}{100}\right]$

Remeber Amount is always greater than Principal.

4.3 CALCULATION OF SIMPLE INTEREST:

For calculation of simple interest three things are needed i.e., Principal (P), Rate percent (R) and Time period (T). Again, if simple interest and other two things are known, then the fourth can easily be calculated, which is illustrated as below:

Example-1

Find simple interest of an amount A5,000 for 9 months @5% per annum.

Solution:

$$SI = \frac{PRT}{100}$$

$$= \frac{5{,}000 \times 5 \times \frac{9}{12}}{100} = \frac{5{,}000 \times 5 \times 9}{100 \times 12}$$

= A187,50

Example-2

What sum of money invested at 6% simple intervest for 6 years amounts to A3400 ?

Solution:

As per question, we have, A = A3,400

T = 6 years and R = 6%, P=?

We know $A = P\left[1 + \frac{RT}{100}\right]$ (Principal+Interest)

or $3400 = P\left[1 + \frac{RT}{100}\right]$

or $3400 = P\left[1 + \frac{6 \times 6}{100}\right]$

or $3400 = P\frac{136}{100}$ $\quad \therefore P = 3400 \times \frac{100}{136}$

$\therefore$ P = A2500

Example-3

Anil invested A30,000 at simple interest for 3 years and received A41,700. Find the rate of interest.

Solution:

We know that S.I = $\frac{PRT}{100}$

or R = $\frac{S.I \times 100}{PT}$

Here S.I = Amount - Principal

S.I = A41,700 - A30,000

S.I = A11,400, P = A30,000, T = 3yrs

$\therefore$ R = $\frac{11,700 \times 100}{30,000 \times 3} = 13\%$

Example-4

In how many years a sum of money will double itself at 10% S.I.

Solution:

Let the sum | Principal be Ax

$\therefore$ Amount would be $2 \times x$ = A$2x$

S.I=Amount - Principal = A$(2x - x)$ = Ax

$\therefore$ P = Ax, S.I = Ax, R = 10%, T= ?

$\therefore$ S.I.= $\frac{PRT}{100} = \frac{x \times 10 \times T}{100}$

or x = $\frac{x \times 10 \times T}{100}$

$\therefore$ T = $\frac{x \times 100}{x \times 10}$ = 10 years

Example-5

A man deposited 58,300 in Bank. How much he would recive at the end of 5 years if the bank agrees to pay him simple interest at 8% per annun ?

Solution:

As per question, Principal = A58,300

number of years, i.e., Time T = 5

and Rate of Interest R = 8%

$$\therefore \text{S.I} = \frac{PRT}{100} = \frac{58,300 \times 8 \times 5}{100} = ₹23,320$$

Amount = Principal + Interest (S.I)

= ₹58,300 + ₹23,320

= ₹81,620

$\therefore$ The man will get back 81,620 at the end of 5 years.

Example-6

A certain sum at simple interest becomes ₹1300 in 3 years and ₹1500 in 5 years. Find the principal and interest rate per cent per year.

Solution:

Amount after 5 years = ₹1500

and Amount after 3 years = ₹1300

The difference in amount (A) is due to interest only, as Principal remains same in both the cases.

$\therefore$ Interest for 2 years = ₹200

$\therefore$ Interest for 1 years = ₹1100

so interest for 3 years = ₹300

$\therefore$ Principal = Amount - S.Interest

= ₹1300 - 300

= ₹1000

$$\therefore \text{Rate per cent} = \frac{₹100}{₹1000} \times 100 = 10\%$$

Example-7

A certain sum amounts to ₹18,750 in 5 years. If interest is $\frac{1}{4}$th of the Principal, find the rate per cent at which it is lent.

Solution:

Let the Principal be A1

As interest is $\frac{1}{4}$th of principal, amount would be $1+\frac{1}{4} = \frac{5}{4}$th of the principal.

$\therefore$ when amount is $\frac{5}{4}$, Principal 1

when amount is 18,750, Principal = $1 \times \frac{4}{5} \times 18,750$

$= $ A15,000

$\therefore$ Interest (S.I) = Amount - Principal

= A(18,750 - 15,000)

= A3,750

$\therefore$ Rate per cent = $\frac{\text{S.I} \times 100}{\text{Principal} \times \text{Time}} = \frac{\text{S.I} \times 100}{\text{P.T}}$

$= \frac{3,750 \times 100}{15,000 \times 5} = 5\%$

Example-8

HUDCO advanced equal sums to A, Band C for 3, 4 and 5 years respectively at 5% simple interest. It received in total A54,000 from all of them after expiry of their period. Find the principal amount lent to each.

Solution:

Let the Principal lent to each A,B & C be x.

So, Interest on x for 3 years at 5%

$\text{S.I} = \frac{\text{PRT}}{100} = \frac{x \times 5 \times 3}{100} = \frac{3x}{20}$

$\therefore$ Amount of A = $x + \frac{3x}{20} = \frac{23x}{20}$

Interest in x for 4 years at 5%

i.e, S.I = $\frac{x \times 5 \times 4}{100} = \frac{x}{5}$

$\therefore$ Amount of B = $x + \frac{x}{5} = \frac{6x}{5}$

Interest on x for 5 years at 5%

i.e., S.I= $\frac{x \times 5 \times 5}{100} = \frac{x}{4}$

$\therefore$ Amount of C = $x + \frac{x}{4} = \frac{5x}{4}$

$\therefore$ Total amount received by HUDCO

$= \frac{23x}{20} + \frac{6x}{5} + \frac{5x}{4} = \frac{23x + 24x + 25x}{20} = \frac{72x}{20}$

$\therefore$ As per question $\frac{72x}{20} = ₹54,000$

$\therefore$ x = $\frac{₹54,000}{72} \times 20 = ₹15,000$

$\therefore$ HUDCO had lent ₹15,000 each to A, B & C.

Example-9

The simple interest on a certain sum of money for 2 years at 12% per annum is ₹300 less than the simple interest on the same sum for 3 years at 10% p.a.. Find the sum.

Solution:

Let the sum be x.

S.I for 2 year @ 12% = $\frac{PRT}{100} = \frac{x \times 12 \times 2}{100} = \frac{6x}{25}$

again S.I for 3 year @10% = $\frac{x \times 10 \times 3}{100} = \frac{3x}{10}$

$\therefore$ Difference of S.I = $\frac{3x}{10} - \frac{6x}{25} = \frac{15x - 12x}{50}$

$= \frac{3x}{20}$

$\therefore$ When difference of S.I is $\frac{3x}{50}$, Principal x

$\therefore$ When difference of S.I is 1 = Principal $\frac{x}{3x} \times 50$

$\therefore$ When difference of S.I is 300 = Principal

$= \frac{x \times 50}{3x} \times 300 = ₹5000$

$\therefore$ Principal / Sum is ₹5,000/-

COMPOUND INTEREST

If interest is retained by the borrower when it becomes due and is added to the Principal, it is a case of compound interest. Thus, the borrower has to pay interest not only on the principal but also interest on the unpaid interest for the preccding period. So, the interest is considered as additional princial for the next period as it is retained by the borrower. If Mr. A borrows ₹100 from B for 3 years and rate of interest per annum being 10% on compound interest-terms, then total interest and principal would be as follows:

Interest on ₹100 at the end of 1st year = ₹10

So principal at the beginning of 2nd year will be,

Principal + Interest = ₹100 + ₹10 = ₹110

Interest for 2nd year will be $110 \times \frac{10}{100} \times 1 =$ ₹11

$\therefore$ Principal at the beginning of 3rd year will be, Principal + Interest

=A110 + A11= A121

$\therefore$ Interest for 3rd year $= 121 \times \frac{10}{100} \times 1 =$ A12.10

At the end of 3rd years A is to pay

A121 + A12.10 = A133.10

So, interest for all the 3 years will be

A10 + A11 + A12.10 = A33.10

Again, if P is the Principal

R is the rate of interest

n is the number of years.

The amount or A at the end of 1st year is

$$P + \frac{PR}{100} = P\left(1 + \frac{R}{100}\right)$$

A at the end of 2nd year

$$= P + P\left(1 + \frac{R}{100}\right) + \frac{R}{100}$$

$$= P\left(1 + \frac{R}{100}\right)^2$$

and A at the end of 3rd year

$$= P\left(1 + \frac{R}{100}\right)^3$$

like wise A at the end of n years

$$A = P\left(1 + \frac{R}{100}\right)^n = P(1 + i)^n, \text{ when } \frac{R}{100} = i$$

$\therefore$ Compound Interest = A - P

$$= P\left(1+\frac{R}{100}\right)^{n} - P$$

$$= P\left(1+\frac{R}{100}\right)^{n} - 1$$

$$\text{or C.I} = P\left(1+\frac{R}{100}\right)^{n} - 1$$

Remark:-

(i) When interest is compounded annually, the simple interest and compound interest for the 1st year to the same.

(ii) It interest is payable yearly or half yearly or quarterly or monthly or daily or continuously, we shall use the following formulac for calculation of amount (A).

Interest compourded	**Amount (A)**
Annually	$A = P(1+i)^{n}$
Half-yearly	$A = P\left(1+\frac{i}{2}\right)^{2n}$
Quarterly	$A = P\left(1+\frac{i}{4}\right)^{4n}$
Monthly	$A = P\left(1+\frac{i}{12}\right)^{12n}$
Daily	$A = P\left(1+\frac{i}{365}\right)^{365n}$
Continuously	$A = P.e^{n.i}$

(iii) It is easier to solve problems on compound interest with the use of logarithim. (See the chapter on Logarithim of this book).

Example-10

Find the compound interest of A3000 for 3 years at 12% p.a.

Solution:

We have the following information:

P=A3,000, n=3 years, R=12%

$\therefore i = \frac{R}{100} = \frac{12}{100} = 0.12, A = ?$

We know $A = P(1+i)^n$

or $A = 3,000\ (1+0.12)^3$

or $A = 3,000\ (1.12)^3$

using log in both sides

Log A = log 3,000 + 3 log (1.12)

or log A = 3.4771+3×0.0492

or log A = 3.4771+0.1477=3.6248

$\therefore$ A= A.L (3.6248)

$\therefore$ or A = A4,214.58

$\therefore$ Compound Interest = A - P

= A(4.214-3,000)= A1,214.58

Example-11

Find the compound interest of A1000 for 5 years at 4%. How much is it more than simple interest ?

Solution:

Compound Interest = A - P

When $A = P(1+i)^n$ Where $i = \frac{R}{100}$ (interst on A1)

or $A = 1,000(1+0.04)^5$ or $A = 1,000\ (1.04)^5$

Using log in both sides

Log A = log 1,000+5log 1.04

or Log A = 3+(5×0.0170)

= 3+0.0850

= 3.0850

$\therefore$ A=A.L (3.0850)

or A = A1216

$\therefore$ C.I= A - P = A1216 - A1,000= A216

Again simple Interest or S.I = $\frac{PRN}{100}$ (when N = Time)

$$= \frac{1{,}000 \times R \times 5}{100} = A200$$

Compound interest - Simple interest

= A216 - A200 = A16

Hence compound interest is A16 more than simple interest.

Example-12

What sum will amount to A2000 in 5 years at 10% p.a. compound interest ?

Solution:

The information provided here are

A= A2,000, n=5 years and R=10%

$i = \frac{10}{100} = .1$, P = ?

We know $A = P(1+i)^n$

$\therefore P = \frac{A}{(a+i)^n} = \frac{2{,}000}{(1+0.1)^5} = \frac{2000}{(1.1)^5}$

Taking log on both the sides

Log P = log 2,000 - 5 log 1.1

= 3.3010 - 5(0.0414)

= 3.3010 - .2070

= 3.0940

$\therefore$ P = A.L (3.0940) = A1241.70

$\therefore$ Principal / sum = 1241.70

Example-13

A sum of A10.000 becomes A11,025 at the rate of 5% per annum compound interest. Find the time.

Solution:

Let the period be n years.

As per question

11,025 = 10,000 (1+0.05)n ($\because$ R = 5%, i = 0.05)

or $\frac{11,025}{10,000} = (1.05)^n$

or $\frac{11,025}{10,000} = \left(\frac{105}{100}\right)^n$

or $\frac{441}{400} = \left(\frac{21}{20}\right)^n$

or $\left(\frac{21}{20}\right)^2 = \left(\frac{21}{20}\right)^n$

$\therefore$ n = 2

$\therefore$ Time is 2 years.

Example-14

In what time will a sum of money double itself at 10% p.a compound interest payable half yearly.

Solution:

Let P = Ax, Then A = A2x

$R = 10\%,\ i = \frac{R}{100} = \frac{10}{100} = 0.1$

We know $A = P\left(1+\frac{i}{2}\right)^{2n}$

or $2x = x\,(1+0.05)^{2n}$

or $\frac{2x}{x} = (1.05)^{2n}$

or $2 = (1.05)^{2n}$

Taking log in both sides

$\log 2 = 2n \log (1.05)$

or $.30103 = 2n \times 0.0212$

or $.30103 = .04240n$

$\therefore n = \frac{.30103}{.04240} = 7.1$ years.

Example-15

At what rate per cent of compound interest, A960 earn interest A240 in 13 years.

Solution:

$A = P + C.I$

$A = A960 + A240$

$= A1200$

$1200 = 960\,(1 + i)^{13}\left(i + \frac{R}{100}\right)$

or $\frac{1200}{960} = (1 + i)^{13}$

Putting log in both the sides

$\Rightarrow$ Log 1200 - log 960 = 13 log $(1 + i)$

or $\frac{3.0792-2.9823}{13}=\log(1+i)$

$\Rightarrow$ .00745 or .0075 = log (1 + i)

$\therefore$(1 + i) = A.L (.0075)

or (1 + i) = 1.017

$\therefore i$ = 1.017 -1 = 0.017

R = 0.017 × 100 = 1.7 ($\because$ R=100i)

VARYING RATES OF INTEREST:

When the rate of interest varies through out the period instead of being uniform or goes on changing every year then rate can be written as R_1, R_2, R_3..... and so on.

A will be = $P\left(1+\frac{R_1}{100}\right)\left(1+\frac{R_2}{100}\right)\left(1+\frac{R_3}{100}\right)$

If R_1= for 2 years, R_2= next 3 years and R_3=next 5 years

So, the period will be (2+3+5)=10years

then A = $P\left(1+\frac{R_1}{100}\right)^2\left(1+\frac{R_2}{100}\right)^3\left(1+\frac{R_3}{100}\right)^5$

Example-16

Find the compound interest on 10,000 in 3 years, if rate of interest is 4% for 1st year 5% for the second year and 6% for the third year.

Solution:

We have the following information

A = A10,000, R_1=4%, R_2=5% and R_3=6%

$\therefore$ A=10,000 $\left(1+\frac{4}{100}\right)^1\left(1+\frac{5}{100}\right)^1\left(1+\frac{6}{100}\right)^1$

or A = 10,000 (1.04) (1.05) (1.06)

Taking log on both the sides

Log A = log 10,000 + log 1.04 + log 1.05 + log 1.06

= 4.0000+0.0170+0.0212+0.0253

=4.0635

∴ A= Antilog of 4.0635

SO, A = A11,570

∴ Compound Interest = A11,570 - A10,000

= A1,570

Example-17

If the compound interest on a certain sum for 2 years at the rate of 4% per annum is A204, then find the simple interest on the same sum for the same time.

Solution:

As we know, $A = P(1+i)^n$

Now let the Principal be x.

$$\therefore A = x\left(1+\frac{4}{100}\right)^2 = x\,(1+0.04)^2$$

and Principal + Interest = Amount

$$\therefore x + 204 = x\,(1.04)^2$$

$$\text{or } x + 204 = x\left(\frac{104}{100}\right)^2$$

$$\text{or } 204 = x\left(\frac{26}{25}\right)^2 - x$$

$$\text{or } 204 = x\left\{\left(\frac{26}{25}\right)^2 - 1\right\}$$

$$\text{or } 204 = x\left\{\left(\frac{26}{25}+1\right)\left(\frac{26}{25}-1\right)\right\}$$

$$\text{or } 204 = x\left\{\frac{51}{25}\cdot\frac{1}{25}\right\}$$

$\therefore x = \frac{204 \times 25 \times 25}{51 \times 1}$

$\therefore x = 4 \times 25 \times 25$

$\therefore x =$ A2500

$\therefore$ Principal is A2500

$\therefore$ Simple interest on 2500 for 2 yrs at @4% P.a

$S.I = \frac{PRT}{100} = \frac{PRN}{100}$ (Here T = N)

$= \frac{2500 \times 4 \times 2}{100} =$ A200

$\therefore$ Simple interest on the same sum for same period at the same rate = A200

NOMINAL AND EFFECTIVE RATE OF INTEREST :

The annual rate of interest on principal if the interest is due annually is called nominal interest. The annual rate of interest earned, when interest is due for more than once in a year i.e., monthly, quarterly, half yearly etc; then interest is called effective rate of interest.

For example if 100 earns 10% p.a. interest then 10 is known as nominal rate of interest as it is due at the end of 1 year. But if payment is due half yearly basis then interest due after 1st half year will be = $\frac{100 \times 10 \times 1}{100 \times 2} =$ A5

So Principal for the 2nd half year = A100 + A5 = A105

$\therefore$ Interest for 2nd half year = $\frac{105 \times 10 \times 1}{100 \times 2}$ = A5.25

Total interest after 1 year will be A5.00 + A5.25 = A10.25

$\therefore$ Effective rate of interest = A10.25

Example-18

What principal would amount to A1000 at 6% p.a in 8 years compounded monthly ?

Solution:

n = 8 years = 8 × 12 = 96 months

rate p.a = 6%, rate p.m = $\frac{6}{12} = \frac{1}{2} = 0.5\%$

$A = P(1+i)^n$, $\therefore$ $1000 = P(1+.005)^{96}$

or $1000 = P(1.005)^{96}$

Taking log on both sides, log 1000 = log P + 96 log 1.005

3 = log P + 96 × .0021

or log P = 3 - 0.2016 or log P = 2.7984

$\therefore$ P = Antilog of 2.7984 = 628.7

$\therefore$ Principal = A628.7

Continuous Compounding :

Some financial institutions add the interest to principal continuously. So that the lender will earn more. This is called continuous compounding. When interest is continuously compounded, Amount, $A = P.e^{n.i}$

Where A = Amount

P = Principal

n = Number of years / Period

i = interest per unit sum

or $P = A.e^{-n.i}$

Example-19

What amount must be invested to realise A1000 after 10 years when interest at $3\frac{1}{2}$ per cent p.a is compounded continuously ?

Solution:

We have A = A10,000, n = 10 years

$i = \frac{3.5}{100} = 0.035$

The amount to be invested = Principal / Present value in case of continuous compounding.

We know $P = A.e^{-n.i}$

$\therefore P = 10{,}000.e^{-10\times0.035}$

$= 10{,}000.e^{-.35}$

Putting log on both sides : Log P = log 10,000 - 0.35 × log e

$\therefore$ log e = 0.4343

$\Rightarrow$ log p = 3 - (0.35×0.4343)

or log p = 2.848

$\therefore$ P = A.L (2.848) = A704.7

$\therefore$ The amount invested is A704.70

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GROUP A

4.7 Exercise

MULTIPLE CHOICE TYPE QUESTIONS:

1. (i) The rate percent of simple interest of A12,000 which will earn A270 in 9 months, is :

(a) 2% , (b) 3%, (c) 4%, (d) 5%

(ii) What sum of money invested at 6% simple interest for 6 months will be A515 ?

(a) A510, (b) A500, (c) A520, (d) A530

(iii) The time in which A1025 will earn S.I. of A287, when the rate of interest is 7%, is:

(a) 4 years, (b) 5 years, (c) 6 years, (d) 7 years

(iv) In how many years a principal trebles itself at 5% p.a compound interest.

(a) 18.3, (b) 18.6, (c) 18.5, (d) 18.8

(v) If a sum of money amounting to A8,464 in two years and A9733.00 in 3 years at compound interest, interest being compounded annually, rate per cent will be:

(a) 12%, (b) 13%, (c) 14%, (d) 15%

(vi) If A4096 invested for 3 years at $6\frac{1}{2}\%$ p.a, compounded annually, the compound interest will be:

(a) A850, (b) A817, (c) A837, (d) A840

Ans. (i) b, (ii) b, (iii) a, (iv) c, (v) d, (vi) a

2. (A) Express the following in one word/term each :

(i) The charge which is paid by the borrower to the lender for using his money.

(ii) The Principal along with interest returned by the borrower.

(iii) Interest calculated on the amount of the previous year.

(iv) The rate of interest charged on A100 per year.

Ans. (i) Interest, (ii) Amount, (iii) Compound interest, (iv) Rate percent.

(B) Answer the following in one sentence each.

(i) What is interest ?

(ii) What is Simple interest ?

(iii) What is compound interest ?

(iv) Write the formula to calculate simple interest.

(v) Write the formula to calculate compound interest.

(vi) What is the formula to calculate the amount, when interest is compounded continuously.

(vii) What is nominal rate of interest ?

(viii) What sum of money invested at 12% simple interest for 6 months will be A318 ?

(C) Correct the underlined portion of the following sentences:

(i) The Principal along with interest for the given period is compound interest.

(ii) The simple interest and compound interest on certain sum for the 1st year will be different, if interest is compounded annually.

(iii) The annual rate of interest on Principal, if interest is due annually is effective rate of interest.

(iv) The simple interest of P for N months at R% p.a is PNR.

Ans: (i) Amount (ii) Same (iii) Nominal (iv) $\frac{PNR}{1200}$.

(D) Fill in the Blanks.

(i) ---------- sum of money of 10% compound interest shall amounts to A665.50 in 3 years.

(ii) At ---------- rate of interest A1,000 will amount to A1102.50 after 2 years.

(iii) In ---------- years A1,000 will amount to A1350 at 7% simple interest.

Ans. (i) A500, (ii) 5%, (iii) 5%

GROUP B

3. Answer within two sentences each.

(i) What is Effective rate of interest ?

(ii) Explain compound rate of interest.

(iii) What is meant by amount.

(iv) What is Nominal rate of interest ?

4. Answer the following with in 6 sentences each.

Compare and contrast between:

(i) Simple interest and compound interest.

(ii) Effective rate of interest and nominal rate of interest.

(iii) Principal and Amount.

GROUP C

LONG TYPE QUESTIONS:

5. Rajan inverted A12,000 for 6 years and received A15,600. Find the rate of interest.

6. In what time will 66,000 amounts to 1,05,600 at 15% simple interest p.a ?

7. Kumar invested A20,000 for 2 years, A30,000 for 3 years and A40,000 for 4 years at simple interest. If he received a total simple interest of A31,900, find the rate of interest.

8. What sum will amount to 1,98,000 in 5 years at the same rate of simple interest at which A30,000 will amount to A41,700 in 3 years ?

9. The difference between simple and compound interest on a sum of money invested for 4 years at 5% p.a. is A15. Find the sum.

10. The difference between simple interest and compound interest on a sum of money for 2 years is A20, The simple interest on the same sum for 4 years is A2,000 Find the sum and rate of interest.

11. If A1,000 is invested for 4 years at 5% p.a compound interest, find the amount received.

12. In what time a sum of money will double itself at 12% p.a compound interest ?

13. In what time a sum of money triple itself at 8% p.a. compound interest ?

14. The population of a town was 10,000 in 2013. If it increases by 6% in 2014, 4% in 2015 and deer cash by 1% in 2016, find the population at the end of 2016.

15. A sum of money invested in compound interest amounts to A1,221 at the end of 5 years and A1,270 at the end of 7 years. Find the principal and rate of interest.

16. Cost price of a machine is A1,00,000. It's scrap value is A38,810 after charging depreciation

at 4% p.a on W.D.V method, Find its life.

Hints= $A = P(1 - i)^n$, where A= depreciated value

P = Cost price of Machine, i = interest rate per unit sum per year and n is period life of machine.

17. A sum of money at compound interest amounts to A6,720 in 2 years and A7,140 in 3 years. Find the rate of interest if compounded annually.

18. Find the principal, if it earns an interest of A237 at 5% p.a compound interest in 3 years.

19. What is the effective rate of interest corresponding to a nominal rate of 5% p.a, if interest is compounded quarterly ?

20. A man deposited A5,830 in a bank. How much he would receive at the end of five years if the bank agrees to pay him simple interest at 7% p.a ?

21. The compound Interest on a certain sum for 1st year at 4% is A25. Find the compound interest for 2 years at the same rate and on the same sum.

22. A sum of money double itself in $12\frac{1}{2}$ years at a certain rate per cent of simple interest How long it will take to double it self at the same rate of compound interest ?

23. A man borrowed A5,000 for 10 years at compound interest of 3% p.a for the first 2 years, 3.5% per annum for the next 3 years and 4% for the last 5 years. Find the amount.

24. The simple interest on a sum of money for 2 years at 4% is A450. Find the compound interest on this sum of money at the same rate for 1 year if interest is reckoned half yearly.

25. If the populaton of a town increases every year by 2% of that population at the beginning of that year, in how many years will the total increase of population be 40% ?

26. What amount must be invested to realise A1000 after 10 years when interest at $3\frac{1}{2}$% p.a is compounded continuously ?

ANSWER

Chapter-IV

(5) 5%	(6) 6.4 years	(7) 7.11%	(8) A1,20,000
(9) A10,000	(10) A12,500, 4%	(11) A1215.50	(12) 6.12 years
(13) 14.28	(14) A10,91,376	(15) Rate 2% and P=A1,106	(16) 25 years
(17) $6\frac{1}{4}$	(18) A1,500	(19) 5.09%	(20) A7,870.50
(21) A51	(22) 9 years	(23) A7,150	(24) A227.25
(25) 17 years (Approx)	(26) A704.70		

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CHAPTER–5

ANNUITY

STRUCTURE

5.1. Introduction

5.2. Meaning

5.3. Classification of Annuity

5.4. Present value of Annuity

5.5. Sinking fund

5.6. Application

EXERCISE

5.1 INTRODUCTIONS:

In our day to day life we come across a lot of transactions which include purchase and sale. This leads to receipts and payments. Some transactions involve a single payment, where as other transactions involve payments over a period at time. For example a motor car manufacturer instead of asking 5 lakh rupees at a time towards it cost, may ask to pay fifty thousand rupees as down payment and balance in equal (monthly, bimonthly, quarterly, annualy) instalments. A life insurance policy holder pays the insurance premium at regular intervals over a period of time as per contract. Insurance company pays in lump sum at the end of the period. Both the transactions consider compound rate of interest. Both the transactions have a common thing: periodicity of equal payments, which is known as annuity.

5.2 Meaning

The term annuity is derived from the latin word annum. Literally it means annual payment of a fixed amount. But in actual practice annuity means a series of payments of fixed amount at regular intervals. The period interval may be a year, a half year, a quarter or a month. The examples of annuities are salary, interest, rent, pension, insurance premium, recurring deposit in bank etc.

5.3 Classification of annuity

Annuities can be classified in to following three broad categories.

1. Contingent annuity
2. Annuity certain
3. Perpetuity

1. Contingent annuity

The payment of such annuity continues till the happening of some contingent event. (The date of happening of such event cannot be fore told). So number of instalments payable is uncertain.

Example – Life insurance premium payable on whole life policy is discontinued or stops payment on the death of the policy holder or annuitant.

2. Annuity certain

An annuity which is payable for a fixed number of years, say 5, 10, 15, years etc. is known as annuity certain. It is further divided into following three types.

i) Immediate annuity
ii) Annuity due
iii) Deferred annuity

When periodic payment of annuity starts at the end of each period, it is called immediate or ordinary annuity. If the payment of annuity starts at the beginning of each period, it is called annuity due. But in deferred annuity the payment starts after the expiry of certain period say, m years. If the first payment starts at the end of m+1 years, it is known as immediate deferred annuity. On the other hand, if annuity is deferred for m years and starts in the beginning of (m +1) years, it is called deferred annuity due.

3. Perpetuity

Annuity payable for ever is known as perpetuity or perpetual annuity. When annuity is payable at the end of each period, it is called immediate perpetuity. If annuity is payable in the beginning of each period, it is called perpetuity due.

A perpetual annuity may be deferred perpetuity, if payment starts after a certain period of time and goes on for ever. This deferred perpetuity may be deferred immediate perpetuity or deferred perpetuity due depending upon the time of payment.

[If the 1st payment starts at the end of $(m+1)^{th}$ year, it is known as deferred immediate perpetuity. On the other hand if payment starts at the beginning on $(m+1)^{th}$ year and continues for ever, is known as deferred perpetuity due.]

(Note – assuming that the deferment period is m)

Assumptions

Unless otherwise stated, the following points are to be assumed.

1. Annuity/ perpetuity means immediate annuity/ perpetuity.
2. Annuity certain or ordinary annuity means also immediate annuity.
3. The compound interest is reckoned if simple interest is not mentioned.
4. Payment period is one year.

5.4 Present value of an annuity

Present value of annuity refers to the aggregate present value of all future payments. It is the aggregate of all present discounted value of all future payments. For example- X will receive (a) R100 at the end of 1st, 2nd , 3rd, ... up to 10th year. Present discounted value of all such payments at specific compound rate of interest is known as present value.

5.5 Sinking fund

A sinking fund is a fund created by investing a fixed amount annually at regular periodical interval . The amount accumulates with compound interest. At the end of specific period the amount is utilised for a) redemption of a fixed liability i.e. debenture, bond, loan etc. (b) replacement of a fixed asset i.e. plant, machinery etc.

5.6 Application

The concept of annuity can be applied in solving a number of problems. The following are some fields to which annuity can be applied.

1) Loan repayment / instalment payments

A person may borrow some money from the bank to meet his financial requirements. The amount is to be repaid with interest over a period of time at equal periodical interval. Annuity helps in calculating that periodic amount.

Some times a person may purchase an asset under hire purchase or instalment purchase system. He pays some amount as down payment and the balance with interest in periodical equal payments over a period of time. Such instalment can be calculated with the help of annuity.

2. Sinking fund problems

A sinking fund is created by investing a fixed sum of money at regular intervals. The fund accumulates with compound interest. The specific amount can be paid at a future date. (e.g. for redemption of debenture or replacement of asset). The concept of annuity can be applied to determine periodical payment or the amount due at the end of such period.

3. Commuted value of pension

An employee on retirement from service receives monthly pension over the rest period of his life. The pension amount is constant. Some times he commutes a part of the pension amount for lump sum ready payment. Annuity helps for calculation of such lump sum.

4. Recurring deposit

Under recurring deposit scheme, a fixed sum of money is deposited in each and every month either in a bank or post office for a fixed period of time. The amount including interest is drawn at the end of the period i.e on maturity. Annuity helps for calculation of such amount.

5. Endowment fund problem

Endowment fund is created either by investing a lump sum amount at one time or by making periodic regular payment for a fixed period of time. This fund is utilised for

awarding prize, scholarship or other financial support for perpetual payment. Annuity helps to calculate such amount.

6. Calculation of depreciation

Depreciation is charged on fixed assets over its working life. Annuity method is one of the methods. Under this method the cost price is treated as an investment. It is assumed that the amount invested otherwise could earn interest. Depreciation is calculated considering both the cost price and the interest factor. Annuity approach helps in calculating the annual depreciation.

Formulae for calculation of amount of annuity

(A) Amount of Annuity (M)

1. For Immediate Annuity, $M = \frac{A}{i}\left[(1+i)^n - 1\right]$

2. For Annuity due, $M = \frac{A}{i}(1+i)\left[(1+i)^n - 1\right]$

3. For Deferred Annuity or immediate deferred annuity $M = \frac{A}{i}\left[(1+i)^n - 1\right]$

(Amount of annuity is not affected by deferment period)

4. For Deferred Annuity due $M = \frac{A}{i}(1+i)\left[(1+i)^n - 1\right]$

(Amount of annuity is not affected by deferment period)

(B) Present value of annuity (V)

1. For Ordinary Annuity / immediate Annuity / Annuity certain

$$V = \frac{A}{i}\left[1 - (1+i)^{-n}\right]$$

2. For Annuity due $V = \frac{A}{i}(1+i)\left[1 - (1+i)^{-n}\right]$

3. For Deferred Annuity or immediate deferred Annuity -

$$V = \frac{A}{i}(1+i)^{-m}\left[1 - (1+i)^{-n}\right]$$

4. For Deferred Annuity due

$$V = \frac{A}{i}(1+i)^{-(m-1)}\left[1-(1+i)^{-n}\right]$$

5. For Perpetuity or immediate perpetuity = V = $\frac{A}{i}$

6. For For Perpetuity due V = $\frac{A}{i}$ (1+i)

7. For Deferred immediate perpetuity V = $\frac{A}{i}(1+i)^{-m}$

8. For Deferred perpetuity due V = $\frac{A}{i}(1+i)^{-(m-1)}$

Where A = Annuity, i = Rate of Interest, n = Number of Years/Periods, M= Amount of annuity, V= Present value, m = deferred period

N B: Calculation of n and i

i) If annual instalments – n and i will not change

ii) If half yearly — n will be multiplied by 2, i will be divided by 2

iii) If quarterly —— n will be multiplied by 4, i will be divided by 4

Illustration – 1

Calculate the amount of ordinary annuity of R 1000 at 10% p.a. compounded annualy for 5 years.

Solution:

Here annuity = R1000, n = 5 years, i = R 0.1

$$M = \frac{A}{i}\left\{(1+i)^n - 1\right\}$$ (Applying the formula)

$$M = \frac{1,000}{0.1}\left\{(1+0.1)^5 - 1\right\}$$ (putting the values)

$$= 10.000\left\{(1.1)^5 - 1\right\}$$

Substituting the value of $(1.1)^5$

M = 10,000 (1.611 – 1)

= 10,000 (. 611) = R 6, 111

∴ Amount of annuity is R 6, 111.

To find $(1.1)^5$ - let x = $(1.1)^5$

logx = 5 log (1.1)

= 5 (0.414) = 0. 207

x = Antilog 0.207 = 1.611

Illustration – 2

Find the amount of annuity of R 3,000 payable at the end of every 6 months for 8 years at 6% p.a., compounded half yearly.

Solution :

Given - Amount of instalment (A) = R 3, 000

Rate of interest per rupee for 6 months = (6%p.a.) = 3% half yearly

$$\text{So, } i = \frac{6}{100} \times \frac{1}{2} = R\ 0.03$$

No of periods (n) = $8 \times 2 = 16$ half years

To find amount of annuity (M) ?

Applying the formula $M = \frac{A}{i}\{(1+i)^n - 1\}$

Putting the value $M = R\frac{3,000}{0.03}\{(1+0.03)^{16} - 1\}$

$= R\ 1,00,000\ \{(1.03)^{16} - 1)\}$

Substituting the value of 1.603 for $(1.03)^{16}$,

M = 1,00,000 (1.603 -1)

= R 1,00,000 (0.603)

= R 60,380

∴ Amount of annuity is R 60, 300

Let $x = (1.03)^{16}$

$\log x = 16 \log 1.03$

$= 16(0.0128)$

$= 0.2048$

x = Antilog 0.2048 = 1.603

Illustration – 3

A limited company intends to create a depreciation fund to replace assets worth R1 lakh at the end of the 15th year. Calculate the fixed amount to be retained out of profit every year if rate it interest is 3% [CHSE – 1999(A)]

Solution

Given - Amount of annuity (M) = R 1 lakh, Time period (n)= 15 years

Rate of interest = 3%.p.a., i = R 0.03 per rupee per year.

To find (A) = Annuity ?

$M = \frac{A}{i}\{(1+i)^n - 1\}$ (Applying the formula)

$\Rightarrow$ 1, 00,000 $= \frac{A}{0.03}\{(1. + 0.03)^{15} - 1\}$

$\Rightarrow$ 1,00,000(0.03) = A $\{(1.03)^{15} - 1\}$

Substituting the value $(1.03)^{15}$

1,00,000 (0.03) = A (1.556 – 1)

$\Rightarrow$ 3, 000 = A (0.556)

Let $x = (1.03)^{15}$

$\Rightarrow \log x = 15 \log 1.03$

$= 15\ (0.0128) = 0.192$

So x = Antilog 0.192 = 1.556

$\Rightarrow A = ₹\frac{3,000}{0.556} = ₹\frac{3,000}{556} \times 1000 = ₹\frac{750 \times 1,000}{139} =$ ₹ 5, 395.68

So Amount to be retained out of profit every year is ₹ 5,395.68

Illustration – 4

In how many years the annuity of ₹ 4, 000 payable half-yearly will amount to ₹47,900 at 8% p.a. compounded half yearly ?

Solution:

Given - Amount of annuity (M) = ₹47, 900, Annuity instalment (A) = ₹4, 000

Rate of interest = 8% p.a. = 4% per half year so i = ₹ 0.04 for one rupee for half year.

To find number of years ?

$M = \frac{A}{i}\{(1+i)^n - 1\}$ (Applying the formula)

$\Rightarrow$ ₹ 47,900 $= \frac{4,000}{0.04}\{(1+0.04)^n - 1\}$ (putting the value)

$\Rightarrow \frac{A47,900 \times 0.04}{4,000} = \{(1.04)^n - 1\}$

$\Rightarrow ₹\frac{1916}{4,000} = (1.04)^n - 1$

$\Rightarrow (1.04)^n = A\frac{479}{1,000} + 1 = A\frac{1479}{1,000} = 1.479$

Taking log of both the sides

$\log (1.04)^n = \log 1.479$

$\Rightarrow n \log 1.04 = \log 1.479$

$\Rightarrow \quad n = \frac{\log 1.479}{\log 1.04} = \frac{0.1700}{0.0170} = 10$half years

No of years $= \frac{10}{2} = 5$ years.

$\therefore$ The required number of year = 5 years.

Illustration– 5

At the beginning of each quarter, R 800 are deposited in to a saving account that pays interest at the rate of 8% .p.a. compounded quarterly. Find the balance in the account at the end of 3 years . {Given $(1.02)^{13}$=1.293} (CBSE)

Solution:

Given- Amount of instalment or annuity (A) = R 800

Rate of interest = 8% p.a. = 2% per quarter, So i = R 0.02

No. of period = (n) = 3 year $\times$ 4 quarter = 12 quarters

To find out Amount of annuity due (M) ?

$M = \frac{A}{i}(1 + i)\{(1 + i)^n - 1\}$ (Applying the formula)

$= R\frac{800}{0.02}(1+ 0.02)\{(1+0.02)^{12} - 1\}$ (putting the value)

$= R40, 000\{(1.02)^{13} - (1. 02)\}$

$= R40, 000 (1.293 – 1.02)$ {Given $(1.02)^{13} = 1.293$}

$= R40, 000 (0.273) = R0, 920$

So, the balance in the account at the end of 3 years is R10, 920.

Illustration - 6

A bank pays interest at the rate of 8% per annum compounded half yearly. Find how much should be deposited in the bank at the beginning of each half year in order to accumulate R 8,000 in 3 years. (CBSE-2002)

Given- Rate of interest = 8% p.a. = 4% per half year, So i = R 0.04

No. of periods = 3 years $\times$ 2 = 6 , Amount of annuity (M) = R 8, 000

To find out annuity (A) :

$M = \frac{A}{i}(1+i)\{(1+i)^n - 1\}$ (Applying the formula)

$\Rightarrow 8{,}000 = \frac{A}{0.04}(1+0.04)\{(1+0.04)^6 - 1\}$ (putting the values)

$\Rightarrow 8{,}000 = \frac{A}{0.04}(1.04)\{(104)^6 - 1\}$

$\Rightarrow 8{,}000\ (0.04) = A\{(1.04)^7 - 1.04\}$

$\Rightarrow 320 = A\{(1.04)^7 - 1.04\}$

$320 = A\,(1.315 - 1.04)$ { Substituting the value of $(1.04)^7$}

$\Rightarrow 320 = A\,(0.275)$

$\Rightarrow A = \frac{320}{0.275}$

Let $x = (1.04)^7$
$\log x = \log(1.04)^7 = 7 \log 1.04$
$= 7\,(0.170) = 0.119$
x = Antilog 0.119 = 1.315

$\Rightarrow A = R\frac{320 \times 1{,}000}{275} = R\frac{320 \times 40}{11} = R\ 1{,}163.63$

So amount to be deposited in the beginning of each half year is R 1, 163.63

Illustration- 7

Find the amount of annuity of R 6,000, the first being paid at the end of 6 years and continued till the end of 15 years if 12% p.a. compound interest is reckoned with ?

Solution:

Given - Amount of instalment or annuity (A) = R 6, 000

The deferred annuity consist of 5 periods of R 6,000 each

No. of years of payment (n) = 10 years, Rate of interest = 12% p.a., So i = R0.12

To find out Amount of ordinary deferred annuity (M)

$M = \frac{A}{i}\{(1+i)^n - 1\}$ (Applying the formula)

$= ₹ \frac{6,000}{12}\{(1+ 0.12)^{10} - 1\}$

Substituting the value of $(1.12)^{10}$

$M = ₹ \frac{6,000}{12}(3.105 - 1) = ₹ \frac{6,000 \times 100}{12}(2.105)$

$= ₹ 500 \times 100 (2.105)$

$= ₹ 50,000 (2.105)$

$= ₹ 1,05,250$

So the amount of deferred annuity is ₹ 1,05,250.

Let $x = (1.12)^{10}$
Log x = log $(1.12)^{10}$ = 10 log 1.12 = 10 (0.0492) = 0.492
x = Antilog 0.492 = 3.105
So $(1.12)^{10}$ = 3.105

Illustration- 8

A machine costs the company ₹97, 000 and effective life is estimated to be 12 years. A fund is created for replacing the machine at the end of its effective life period. If the scrap realises ₹ 2, 000 only, what amount should be retained out of profit at the end of each year to accumulate a compound interest at 5% per annum? [CHSE. 1989 (A)]

Solution:

Given- Cost of the machine ₹ 97, 000, Scrap value ₹ 2, 000

Life period of machine (n) = 12 years

Amount required (M) = ₹ 97, 000 – 2, 000 = ₹ 95, 000

Rate of interest = 5% p.a., So i = ₹ 0.05

To find amount of each annuity (A) ?

Here a fund is created means sinking fund is created. The amount of fund at the end of 12 years is ₹ 95,000.

$\therefore M = \frac{A}{i}\{(1+i)^{n} - 1\}$ (applying the formula)

$\Rightarrow 95, 000 = \frac{A}{0.05}\{1+0.05)^{12} - 1\}$ (putting the values)

$95,000 = \frac{A}{.05}(1.797 - 1)$ (substituting the value)

$\Rightarrow 95,000 (0.05) = A (.797)$

let $x = (1.05)^{12}$
log x = log $(1.05)^{12}$ = 12log 1.05
= 12 (0. 0212)
= 0.2544
x = Antilog 0.2554 = 1.797
So $(1.105)^{12}$ = 1.797]

$\Rightarrow$ R 4,750 = A (0. 797)

$\Rightarrow A = R\frac{4,750}{0.797} = R\ 5,959.85$

So every year R 5, 959.85 is to be invested in the sinking fund to replace the machine at the end of 12 years.

Illustration- 9

Find the present value of an ordinary annuity of R 1200 per annum for 12 years at 12 % per annum compounded annually.

Solution:

Given- Annuity (A) = R 1,200, Time, (n) = 12 Years

Rate of interest = 12% p.a., So i = R 0.12

To find - present value (v)

$V = \frac{A}{i}\{1-(1+i)^{-n}\}$ (Applying the formula)

$= R\frac{1200}{0.12}\{1-(1+0.12)^{-12}\}$ (putting the value)

$= R10,000\{1-\{1-\frac{1}{(1.12)^{12}}\}$

$V = R\ 10,000\left\{1-\frac{1}{3.894}\right\}$ {putting the value of $(1.12)^{12}$}

$= R\ 10,000\left(\frac{3894-1}{3.894}\right)$

$= R\ 10,000 \times \frac{2.894}{3.894}$

= R 7, 431.95

let $x = (1.12)^{12}$

$\log x = \log (1.12)^{12}$

= 12 log 1.12

= 12 (0.492)

= 0.5904

x = Antilog 0.5904 = 3.894

So present value of the annuity is R 7, 431.95

Illustration - 10

A man retires at the age of 60 years and his employer gives him a pension of R12,000 per year paid in half yearly instalments for the rest of his life. Reckoning his

expectation of life to be 13 years and the interest rate is 4% p.a. payable half yearly. What single sum is equivalent to his pension? [CHSE – 1990 (s)]

Solution:

Given - Annual pension or annuity (A) = R 12000 per year

Paid half yearly = R $\frac{12,000}{2}$ = R 6, 000, No. of period (n) = 13 years $\times$ 2 = 26 half years.

Rate of interest = 4%p.a, So i. = $\frac{0.04}{2}$ = R 0.02

To find out – Single equivalent payment (V) = Present value of annuity

$V = \frac{A}{i}\{1-(1+i)^{-n}\}$ (Applying the formula)

$= R\ \frac{6000}{0.02}\left[1-(1+0.02)^{-26}\right]$ (Putting the value)

$= R\ 3,00,000\left[1-(1+0.02)^{-26}\right]$

= R 3,00,000 (1– 0.5975)

{ putting the value of $(1.02)^{-26}$}

= R 3,00,000 (0.4025)

= R 1,20,750

So a single payment of R 1,20,750 is equivalent to his pension.

Let $x = (1.02)^{-26}$

Log x = log $(1.02)^{-26}$

= –26 log (1.02)

= –26 (0.0086)

= – 0.2236

= $\bar{1}$ 7764

x = Anti log of $\bar{1}$ .7764 = 0.5975

Illustration--11

A company borrowed R 1,00,000 on the condition that it will repay it with compound interest at 5% p.a. by annual instalment of R 10,000 each. In how many years the debt will be repaid? [CHSE 1992(A), 2004 (A)]

Solution:

Given - Amount of loan = present value (V) = R 1, 00, 000

Annual payment = Annuity (A) = R 10, 000,

Rate of interest = 5% p.a., So i = R 0.05

To find out Time period (n) = ?

$V = \frac{A}{i}\left[1-(1+i)^{-n}\right]$ (Applying the formula)

$1,00,000 = \frac{10,000}{0.05}\left[1-(1+0.05)^{-n}\right]$ (Putting the value)

$\Rightarrow$ 1, 00, 000 = 2, 00, 000 [1- $(1.05)^{-n}$]

$\Rightarrow \frac{1,00,000}{2,00,000} = 1-(1.05)^{-n}$

$\Rightarrow 0.5-1 = -(1.05)^{-n}$

$\Rightarrow -0.5 = -(1.05)^{-n}$

$\Rightarrow 0.5 = (1.05)^{-n}$

$\Rightarrow \log 0.5 = \log (1.05)^{-n}$

$\Rightarrow \log 0.5 = -n \log 1.05$

$\Rightarrow -n = \frac{\log 0.5}{\log 1.05}$

$\Rightarrow -n = \frac{\bar{1}.6990}{0.0212}$

$\Rightarrow -n = \frac{-0.3010}{0.0212}$

$\Rightarrow n = \frac{0.3010}{0.0212}$

$\Rightarrow$ n = 14.2 years.

So the debt can be repaid in a period of 14.2 years.

Illustration – 12

A loan of R 20, 000 is to be paid in 10 equal annual instalments. Find the amount of each instalment to cover principal and interest at 4% p.a. [CHSE 2002 (A), 1993 (A)]

Solution:

Given - Loan to be repaid = present value = V = R 20, 000

Time (n) = 10 years, Rate of interest = 4% p.a., So i = R 0.04

To find out amount of instalment or annuity (A)

$V = \frac{A}{i}\{1-(1+i)^{-n}\}$ (Applying the formula)

R20, 000 $= \frac{A}{0.04}\{1-(1+0.04)^{-10}\}$ (substituting the value)

$\Rightarrow$ R20,000 $\times (0.04) = A\{1-(1.04)^{-10}\}$

$\Rightarrow$ R800 $= A\{1-(1.04)^{-10}\}$

R 800 = A (1- 0.6761) {Putting the value of $(1.04)^{-10}$}

$\Rightarrow$ R 800 = A (0.3239)

$\Rightarrow \quad A = R\frac{800}{0.3239}$

= R 2469.90

So amount of each instalment is R 2469.90

Let $(1.04)^{-10} = x$

$\log x = \log (1.04)^{-10}$

$= -10 \log 1.04$

$= -10 (0.0170)$

$= -0.170$

$= \bar{1}.830$

Antilog $\bar{1}.830 = 0.6761$

Illustration-13

Find the present value of an annuity due to R 700 per annum payble at the beginning of each year for 2 years, allowed interest 6% per annum compounded annually. (CBSE-2000)

Solution

Given – Value of annuity (A) = R 700

Time (n) = 2 years, Rate of interest = 6% p.a., So i = R 0.06

To find-present value of annuity due.

$V = \frac{A}{i}(1+i)\left[1-(1+i)^{-n}\right]$ (Applying the formula)

or $V = R\frac{700}{0.06}(1+0.06)\left[1-(1+0.06)^{-2}\right]$ (Putting the values)

$= R\frac{700}{0.06}(1.06)\left[1-(1.06)^{-2}\right]$

Let $x = (1.06)^{-1}$

$\log x = \log (1.06)^{-1}$

$= -1 \log 1.06$

$= -1 (0.0253)$

$= \bar{1}.9747$

x = Antilog $\bar{1}.9747$

$= 0.9434$

$= R \frac{700}{0.06}[1.06 - (1.06)^{-1}]$

$= \frac{700}{0.06}(1.06 - 0.9434)$ {Putting the value of $(1.06)^{-1}$}

$= \frac{700}{0.06} \times (0.1166)$

= R 1360.33

So present value of given annuity is R 1360

Illustration - 14

Mr. Somesh Nanda purchased an article for R 78,900 under instalment payment. He pays equal instalment in the beginning of each quarter for $2\frac{1}{2}$ years. Money is worth 12% p.a. compounded quarterly. Find the amount of each instalment.

Solution

Given – Present value of the article R 78,900,Quarterly installments for $2\frac{1}{2}$ years

or number of periods (n) = $2\frac{1}{2} \times 4 = 10$,

Rate of interest per annum 12%, Rate of interest per quarter $\frac{12}{4}$ = 3%, So i = R 0 .03

To find amount of each instalment (A)

V = $\frac{A}{i}$ (1+ i) { 1- (1+i)$^{-n}$ } (Applying the formula)

let x = $(1.03)^{-9}$

log x = log $(1.03)^{-9}$

= - 9log 1.03

= -9(0.0128)

= - 0.1152

= $\bar{1}$ .8848

x = Antilog $\bar{1}$ .8848

= 0.767

R 78,900 = $\frac{A}{0.03}$ (1+ 0.03) { 1- (1+ 0.03)$^{-10}$ } (putting the values)

or R78,900 (0.03) = R A (1.03) {1– $(1.03)^{-10}$}

or R 2367 = A { 1.03 – $(1.03)^{-9}$}

{putting the value of $(1, 03)^{-9}$}

R 2,367 = A (1.03 – 0.767)

$\Rightarrow$ R 2,367 = A (0.263)

$$\Rightarrow \quad A = R\frac{2,367}{0.263}$$

= R 9000

So, amount of each quarterly instalment is R 9000.

Illustration - 15

Find the present value of annuity of R 5,000 commencing at the end of 6 year and continues for 15 years if money is worth 10%p.a.

Solution

Given :

It is a case of deferred annuity consisting of 10 payments. Ist payment is defered for 5 years.

Annual instalment/ Annuity (A) = Rs 5,000, 1st payment deferred (m) = 5 years

No of years to continue (n) = (15-5)=10 , Rate of interest = 10% p.a., So i = R 0.1

To find out present value of deferred annuity (V) =?

$$V = \frac{A}{i}(1+i)^{-m}\left[1-(1+i)^{-n}\right] \quad \text{(Applying the formula)}$$

$$= R\,\frac{5,000}{0.1}(1+0.1)^{-5}\left[1-(1+0.1)^{-10}\right] \quad \text{putting the value)}$$

= R 50,000 $(1.1)^{-5}$ {1 - $(1.1)^{-10}$}

= R 50,000 (0.6209) (1-0.3855)

{putting the value of $(1.1)^{-5}$ and $(1.1)^{-10}$}

= R 50,000(0.6209) (0.6145)

= R 19,077.15

So, present value of deferred annuity is R 19,077.15

Let $x = (1.1)^{-5}$

$\log x = \log (1.1)^{-5}$

$= -5 \log (1.1)$

$= -5\ (0.0414)$

$= -0.207$

$= \bar{1}.793$

$X = \text{Antilog } \bar{1}.793$

$= 0.6209$

let $y = (1.1)^{-10}$

$\text{Log } y = \log (1.1)^{-10}$

$= -10 \log 1.1$

$= -10\ (0.0414)$

$= -0.414$

$= \bar{1}.586$

$y = \text{Antilog } \bar{1}.586$

$= 0.3855$

Illustration – 16

Jogendra borrowed R 1,00,000 from finance Ltd. at 6% compound interest p.a. He has to repay the loan in 10 equal annual instalments. What will be amount of instalment if Jogendra starts repayment at the end of 3 years ?

Solution

Given- Rate of interest = 6% p.a., So i = R 0.06

Amount of loan or present value (V) = R 1,00,000

1st instalment starts at the end of 3 years, So deferred period (m) = 2 years

Period of payment (n) = 10 years

To find annuity (A) =?

$$V = \frac{A}{i}(1+i)^{-m}\left[1-(1+i)^{-n}\right]$$ (Applying the formula)

$$\text{R } 1,00,000 = \frac{A}{0.06}(1+0.06)^{-2}\left[1-(1+0.06)^{-10}\right]$$

(putting the values)

$$\Rightarrow \text{R } 1,00,000 = \frac{A}{0.06}(1.06)^{-2}\left[1-(1.06)^{-10}\right]$$

$$\text{R } 1,00,000 = \frac{A}{0.06}(0.8900)(1-0.5585)$$

{putting the value of $(1.06)^{-2}$ and $(1.06)^{-10}$}

= R 6,000 = A (0.8900) (0.4415)

$$\Rightarrow A = \text{R}\frac{6,000}{(0.8900)(0.4415)}$$

R 15,269.70

Hence, jogendra has to pay R 15,269.70 annualy to clear the loan.

Let $x = (1.06)^{-2}$

$\log x = \log (1.06)^{-2}$

$= -2\ (0.0253)$

$= -0.0506$

$= \bar{1}.9494$

$X = \text{antilog } \bar{1}.9494 = 0.8900$

Let $Y = (1.06)^{-10}$

$\log y = \log (1.06)^{-10}$

$= -10 \log (1.06)$

$= -10\ (0.0253)$

$= -0.253$

$= \bar{1}.747$

$Y = \text{Antilog } \bar{1}.747 = 0.5585$

Illustration– 17

Find the present value of deferred annuity due to commence 6 years hence, and continue for 17 years for an annuity of R 4,500 with compound interest of 5% p.a.

Solution

Given – Annuity (A) = R 4,500, Rate of interest 5% p.a,

So i = R 0.05

1st payment starts in the beginning of 6 years

So 1st payment deferred (m) = 5 years,

Period of annuity (n) = 17-5 =12 years

To find- present value of deferred annuity (V)

$$V = \frac{A}{i}(1+i)^{-(m-1)}\left[1-(1+i)^{-n}\right]$$ (Applying the formula)

$$= R\ \frac{4,500}{0.05}(1+0.05)^{-(5-1)}\left[1-(1+0.05)^{-12}\right]$$

R 90,000 $(1.05)^{-4}$ {1- $(1.05)^{-12}$}

{Putting the values of $(1.05)^{-4}$ and $(1.05)^{-12}$}

V = Rs 90,000 (0.8224) (1- 0.5567)

= R 90,000 (0.8224) (0.4433)

= R 32,811.29

So, present value of the deferred annuity is R 32,811.29

log x = log $1.05)^{-4}$

= - 4log (1.05)

= - 4 (0.0212)

= - 0.0848

= $\bar{1}$. 9152

X= Antilog $\bar{1}$. 9152 = 0.8224

log y = log $(1.05)^{-12}$

= - 12 log (1.05)

= - 12 (0.0212)

= - 0.2544

= $\bar{1}$ .7456

Y = Antilog $\bar{1}$ .7456

= 0.5567

Illustration – 18

What is the present value of perpetuity of R 1,000 a year at the rate of 10% p.a?

Solution

Given- Annual payment of perpetuity (A)= R1,000, Rate of interest =10% p.a., So i = R 0.1

To find present value of perpetuity (V)

Putting the formula $V = \frac{A}{i}$

= R $\frac{1,000}{0.1}$ (putting the value)

= R 10,000

Hence, present value of the perpetuity is R 10,000

Illustration - 19

A person wants to create on endowment fund for award of prize in the beginning of each year. Find the amount of fund to be set aside to give annual prize of R1,000 recking interest 10% p.a. compound.

Given – Annuity (A) = R 1, 000, Rate of interest = 10%p.a., So i = 0.1

To find present value (V) of perpetuity = ?

$V = \frac{A}{i} \times (1+ i)$ (Applying the formula)

$= \frac{1,000}{0.1}(1+0.1)$ (putting the value)

= R 10, 000 x 1.1

= R 11,000

So, the required fund is R 11, 000

Illustration – 20

Find the present value of deferred perpetuity of R 2,000 at 8% p.a. compound interest if it starts 6 years hence.

Solution

Given – Annuity – (A) = R 2, 000, Rate of interest = 8%p.a., So i = R 0.08

Deferred period (m) = 5 years, To find present value (V) = ?

$V = \frac{A}{i}(1 + i)^{-m}$ (Applying the formula)

$= R \ \frac{2,000}{0.08}(1+ 0.08)^{-5}$ (putting the values)

$= R \ 25,000 (1.08)^{-5}$

V = R 25,000 (0.6808) {putting the value of $(1.08)^{-5}$}

= R 17,020

Hence present value of given of given perpetuity is R17,020

let $x = (1.08)^{-5}$

$\log x = \log (1.08)^{-5}$

$= -5\log (1.08)$

$= -5 (.0334)$

$= - 0.1670$

$= \bar{1} . 8330$

$X = \text{Antilog } \bar{1} . 8330$

$= 0.6808$

Illustration – 21

Find the present value of deferred perpetuity due where the amount of annuity is R1500, commences 7 years hence. (Deferred period is 6 years)

Solution

Given – Annuity (A) = R 1500, Rate of interest =6%p.a., So i = R 0.06

Deferred period 6 years (m) = 6 years

To find present value of deferred perpetuity due (V)=?

$V = \frac{A}{i}(1+i)^{-(m-1)}$ (Applying the formula)

$= R\ \frac{1,500}{0.06}(1+0.06)^{-(6-1)}$ (putting the values)

$= R\ 25,000\ (1.06)^{-5}$

$V = R\ 25,000\ (0.7473)$ {putting the value of $(1,06)^{-5}$}

$= R\ 18,682.50$

So, present value of the deferred perpetuity due is R 18,682. 50

Let $x = (1.06)^{-5}$

$\log x = \log (1.06)^{-5}$

$= -5 \log 1.06$

$= -5\ (.0253)$

$= -0.1265$

$= \bar{1}.8735$

X = Antilog. $\bar{1}.8735$

= 0. 7473

EXERCISE

Q.1 From the given alternatives, choose and write serially the correct answer along with its serial number against each bit :

(a) In annuity certain annuities are payable:

(i) for a definite period of time

(ii) for ever.

(iii) till due happening of a contingent event

(iv) not payable at all.

(b) In immediate annuity annuities are payable:

(i) at the beginning of each period.

(ii) at the end of each period.

(iii) at the middle of each period.

(iv) any time during the period.

(c) In annuity due annuities are payable :

(i) at the end of each period.

(ii) at the middle of each period.

(iii) at the beginning of each period.

(iv) any time during the period.

(d) In case of contingent annuity, payments are made:

(i) at the beginning of each period.

(ii) at the middle of each period.

(iii) at the end of each period.

(iv) on the happening of a contingent event.

(e) Payment of pension for an indefinite period is known as :

(i) annuity certian

(ii) ordinary annuity

(iii) perpetual annuity

(iv) immediate annuity

(f) The person receiving the annuity is known as:

(i) annuitant

(ii) annuitator

(iii) annuity setter

(iv) annuity seeker

(g) The annuity which commences after certain period of time is known as:

(i) contingent annuity

(ii) time annuity

(iii) deferred annuity

(iv) perpetual annuity

(h) The meaning of the word annuity is:

(i) Payment of series of fixed amount at regular interval

(ii) receipt of series of fixed amount at regular interval

(iii) payment of single instalment

(iv) receipt of single instalment

(i) The ward annuity is devived from

(i) Italian word annum

(ii) Latin word annum

(iii) Greek word annum

(iv) Java word annum

(j) In contingent annuity, number of instalments is

(i) decided by annuity holder

(ii) decided by mutual agreement

(iii) known

(iv) un known

(k) Annuity payable for ever is known as:

(i) annuity due

(ii) immediate annuity

(iii) perpetuity

(iv) deferred annuity

(l) The annuity in which the first payment starts at the end of m+1 years, is called :

(i) deferred annuity due

(ii) immediate deferred annuity

(iii) past deferred annuity

(iv) deferred perpetuity

(m) In deferred immediate perpetuity, the 1st payment starts:

(i) at the end of $(m+1)^{th}$ year and continues for ever.

(ii) at the beginning of 'm+1' year and continues for ever

(iii) at the end at 'm' year and continues for ever

(iv) in the beginning if 'm' years and continues for ever

(n) The present value of a perpetuity of R10,000 at 10% is:

(i) A1,000

(ii) A10,000

(iii) A1,00,000

(iv) A2,00,000

(o) M invests A1,000 annually for 5 years at 10% p.a. compound interest. He will receive at the end of 5 years: {Given $(1.1)^5 = 1.6111$}

(i) A6,111

(ii) A6,222

(iii) A6,333

(iv) A6,444

(p) Payment starts in the beginning of $(m+1)^{th}$ year and continues for ever is known as:

(i) immediate annuity

(ii) ordinary annuity

(iii) deferred immediate perpetuity

(iv) deferred perpetuity due

(q) Incase it deferred annuity due, 1st payment starts:

(i) at the end of $(m+1)^{th}$ year

(ii) in the middle of $(m+1)^{th}$ year

(iii) in the begining of $(m+1)^{th}$ year

(iv) at any time after $(m+1)^{th}$ year

(r) Annuity certain does not include:

(i) immediate annuity

(ii) annuity due

(iii) deferred annuty

(iv) contingent annuity

(s) The fund created by investing fixed amount at regular intervals, is :

(i) sinking fund

(ii) annuity fund

(iii) perpetuity fund

(iv) endowment fund

(t) The relationship between amount of annuity and present value of annuity, is:

(i) amount of annuity = present value of annuity.

(ii) amount of annuity < present value of annuity.

(i) amount of annuity > present value of annuity.

(i) amount of annuity $\geq$ present value of annuity.

(u) In annuity certain annuities are payable :

(i) For evers

(ii) Till the happening of certain event

(iii) For a fixed period of time

(iv) After a fixed period of time

(v) The present value of perpetuity to of A800 a year @8% per annum, is :

(i) A 8,000

(ii) A 9,000

(iii) A 10,000

(iv) A 9,090

(w) M is a symbol for :

(i) present value of annuity

(ii) amount of annuity

(iii) perpetuity

(iv) contingent annuity

(x) 'V' is a symbol used for :

(i) amount of annuity

(ii) perpetuity

(iii) present value of annuity

(iv) contingent annuity

(y) n is a symbol used for :

(i) present value of annuity

(ii) number of instalment

(iii) amount of annuity

(iv) contingent annuity

(z) The present value of perpetuity can be calculated for using the formula:

(i) $V = \frac{A}{i}$

(ii) $V = \frac{A}{1+i}$

(iii) $V = \frac{A}{i}\left|1-(1+i)^{n}\right.$

(iv) $V = \frac{A}{i}\left|1-(1+i)^{-n}\right.$

Ans. (a) i, (b) ii, (c) iii, (d) iv, (e) iii, (f) i, (g) iii, (h) i, (i) ii, (j) iv, (k) iii (l) ii, (m) i, (n) iii, (o) i, (p) iv, (q) iii, (r) iv, (s) i, (t) iii, (u) iii, (v) iii, (w) ii, (x) iii, (y) ii, (z) i.

2. A- Express the following in one word/term each:

(i) Periodical payment of a fixed amount made for a definite period of time.

(ii) Periodical payment of a fixed amount payable at regular intervals.

(iii) The annuity in which payment continues till the happening of a contingent event.

(iv) Lump sum consideration against which annuity payment is granted.

(v) The annuity payable for ever.

B- Answer the following in one sentence each:

(i) What is annuity ?

(ii) What is perpetual annuity ?

(iii) What is deferred period ?

(iv) What is a meaning of contingent annuity ?

(v) What is present value of an annuity ?

C- Fill up the blanks.

(i) Annuity payment of fixed amount is made by ---------- at regular intervals.

(ii) Amount of annuity of A3,000 payable at the end of every 6 months for 8 years at 6% p.a. compounded half yearly is ----------.
given $(1.03)^{16} = 1.603$

(iii) Present value of perpetuity of A1,000 a year at the rate of 10% p.a. is -------

(iv) Present value of deferred perpetuity due (deferred period is 6 years) which commences 7 years hence and where amount of annuity is A1,500, is -----.
Given $(1.06)^{-5} = 0.7473$

(v) The formula to calculate present value of perpetuity is ----------.

D- Correct the underlined portion of each the following sentences :

(i) The formula to calculate present value of perpetuity is V= $\frac{A}{1+i}$

(ii) Annuity in which payments are made at the beginning of each period is called <u>immediate annuity</u>.

(iii) Annuity in which payments are made at the end of each period is called <u>annuity due</u>.

(iv) Annuity in which payments are deferred for 'm' years and start in the beginning of $(m+1)^{th}$ year is called <u>immediate deferred annuity</u>.

(v) Annuity in which annuities are payable for ever, is known as <u>deferred</u> annuity.

Ans. **A-** (i) Annuity certain, (ii) Annuity, (iii) contingent annuity, (iv) Present value, (v) Perpetual annuity

C- (i) Annuitant, (ii) A60,303, (iii) A 10,000, (iv) A18,682.50, (v) $\frac{A}{i}$

D- (i) $v = \frac{A}{i}$, (ii) annuity due, (iii) immediate annuity, (iv) deferred annuity due (v) perpetual

B-3. Answer the following questions within 2 sentences each.

a) What is annuity ? [CHSE 2000(A)]

b) What is contingent annuity ?

c) What is perpetuity? [CHSE 2800(A)]

d) Explain the term immediate annuity?

e) What do you mean by annuity due?

f) What is deferred annuity ?

4. Answer the following questions within 6 sentences each.

a) What is deferred annuity due?

b) Explain the term perpetuity?

c) What is deferred perpetuity due?

d) What is present value of annuity?

e) Give an example of present value of annuity?

f) What do you mean by amount of annuity?

g) What is sinking fund?

h) Name the fields to which annuity can be applied.

i) Explain the meaning of commuted value of pension.

C. 3. Calculate the amount of ordinary annuity of A 5,000 of 10% p.a. compounded annually for 10 years. [CBSE]

4. Find the amount of an annuity of A 10,000 payble of the end of each quarter for 6 years at 8% p.a. compounded quarterly. Use $(1.02)^{24} = 1.608$. [CBSE]

5. Find the amount of annuity of A 2,000 payble at the end of each quarter for 6 years, if the rate of interest is 8% p.a compounded quarterly. (Given $(1.02)^{24} = 1.608$) [CBSE]

6. Find the amount of annuity certain of A 400 payble at the end of each half year for 4 years, 8% p.a compounded half yearly.

7. A sinking fund is created for the redemption of debenture of ₹ 5,00,000 at the end of 25 years. How much money should be provided out of profit each year for the sinking fund if the investment can earn interest at 4% p.a compounded yearly ? [CHSE - 1992 (S)]

8. A fund is created for the redemption of debenture of ₹ 50,000 at the end of 10 years. How much money should be provided out of profit each year for the sinking fund, if investment can earn 4% p.a ? [CHSE - 1989 (S)]

9. A machine cost a company ₹ 97,000 and its effective life is estimated to be 12 yers. If the scrap realised ₹ 2,000 only. what amount is to be retained out of profit at the end of each year to accumulate at compound interest at 5% p.a to replace the machine ? [CHSE 1989 (A)]

10. The cost of a computer is ₹ 1,00,000 and its effective life period is estimated to be 20 years. After 20 years the computer is expected to cost more by 20% over its present cost. Find the sum to be invested every year at 5% p.a compound interest for 20 years to replace this computer. [CHSE - 1998 (A)]

11. In how many years an annuity of ₹ 6,000 will amount to ₹ 79,100 at 6% p.a compounded annually ?

12. Raman deposit ₹ 7,500 in a financial company in the begining of each year for 10 years. The deposit earns interest @ 7.5% p.a. compounded annually. How much money he will receive at the end of 10th year ? [Given Log $(1.075)^{11} = 2.221$]

13. What half yearly instalment will amount to ₹ 83,800 if it is paid in the begining of each half year for a period of 5 years? Interest being reckoned @ 12% p.a. compounded half yearly ?

14. Find the amount of an annuity of ₹ 10,000, the first being paid at the end of 6 years and continue till the end of 15 years, if 10% p.a compound interest is reckoned with.

15. Find the amount of an annuity of ₹ 10,000, the first being paid at the begining of 6 years and continue till the end of 15 years, if 10% p.a compound interest is reckoned with.

16. Mr Arup wants to creat a sinking fund of ₹ 5,00,000 for the marriage of his daughter

after 10 yers. How much money he has to deposit annually so as to earn 12% p.a. compounded annully ? Take $(1.12)^{10} = 3.105$

17. Find the present value of an annuity certain of A 500 per annum for 12 years at 15% per annum. [Use $(1.15)^{12} = 5.35$] (CBSE)

18. Find the present value of an annuity of A 300 per annum for 5 years at 3% compounded annually.

19. Anil borrows from Sunil A 25,000 on the condition to repay it with compound interest of 5% p.a. by annual instalment of A 2,500 each. In how many years will Anil repay the debt ? [CHSE - 2001 (A)]

20. A company borrows a loan of A 4,00,750 on the condition to repay it with compound interest at 6% p.a. by annual instalment of A 1,50,000 each. In how many instalments will the debt be paid off (CBSE)

21. A person wants to buy a scooter worth A 25,000. The contract is that he will pay A 5,000 immediately and the balance in 10 equal annual instalments with 10% p.a compound interest. How much he has to pay annually? [CHSE - 2003 A. 1993 (S)]

22. A man purchase a piece of land for A 12,29,000. The amount is to be paid in 10 equal annual instalment which includes compound interest at 10% p.a. Find the amount of instalment.

23. Find the present value of annuity of A 3,000 payable in the beginning of each half year for 10 years, interest being 4% p.a. compounded half yearly.

24. Find the present value of annuity due of A 800 p.a. payable at the beginning of each month at 10% compound interest per annum for 8 years.

25. Find the amount of instalment to be paid in the beginning of each month for 2 years to repay a loan of A 2,00,000, interest being reckoned 18% p.a. compounded monthly. Given $(1.015)^{-23} = 0.7125$

26. Find the present value of an annuity of A 9,000 which commences at the end of 4 years and continues for 10 years. Interest is compounded 6% p.a. annually.

27. Find the present value of immediate deferred annuity of A 2000 to continue 15 years with compound interest of 8% p.a., if payment of annuity starts after 6 years.

28 X borrowed A 80,000 at compound interest of 8% p.a. He has to repay in 16 equal annual instatments. What should be the instalment if X starts after paying 4 years.

29. What is the present value of a deferred annuity due if payment is made in 10 annual instalments of A 4,000 each, the first being made at the beginning of 3rd year? Interest is reckoned with 5% compound interest per annum ?

30. A man wishes to create an endowment fund to provide an annual prize of A 500 out of its income. If fund is invested at 4% p.a., find the amount of this fund.

[CHSE - 1989 (A)] (Hint - Apply V= $\frac{A}{i}$)

31 A man wishes to create an endowment fund to provide an annual prize of A 500 out of its income at the beginning of each year. If fund is invested at 4% p.a., find the amount of this fund. {Hint - Apply V= $\frac{A}{i}$ (1+i)}

32. Find the present value of perpetuity of A 1,200 at 8% compound interest per annum, if it commences 5 years hence.

33. Find the present value of perpetuity due of A 1200 at 8% compound interest per annum, if it commences in the beginning of each year and continues for 5 years.

34. X invested A 10,000 at the beginning of 1st year. The amount was invested for 10 years. In addition to it he invested A 1,000 at the end of each year for 10 years if interest is compounded at 10% p.a., how much will he get at the end of 10th year?

[Hint - Divide the question in to two parts.

Part 1 - Find the compound value of A 10,000 at the end of 10 years.

Part 2 - Find the amount of annuity of A 51000 where n = 10, i = 0.1

Add part 1 and part 2]

35. Mr. Somesh contributed a lumpsum on 31.12.2010 for award of annual scholarship of A 1,000. The rate of interest is 10% p.a. compounded annually. Find the value of lumpsum amount in the following cases if scholarship is paid on 31st December every year commencing with -

a) the year 2011

b) the year 2016

c) the year 2011 and continue for 5 years.

d) the year 2016 and continue for 5 years.

ANSWER

Q3. A79,700 **Q4.** A3,04,000 **Q.5.** A60,800 **Q6.** A3,680 **Q7.** A12,040.94 (Say 12,041)

Q8. A4,175.37 **Q9.** A5,959.85 **Q10.** A3,625.38. **Q11.** A10 years **Q12.** A1,14,600

Q13. A6,000 **Q14.** A1,59,400 **Q15.** 1,75,400 **Q16.** 28,503.56 **Q17.**A2,710.28

Q18. A1,370 **Q19.**A14.2years **Q.20.** 3years **Q21.** 3,255, **Q22.** A2,00,000

Q23. A50,040 **Q24.** A4,695.20 **Q25.** 9,917.36 **Q26.** 55,609.13 **Q27.**A11,650.19

Q28.A11,387.86 **Q29.** A28,019.58 **Q30.** A12,500 **Q31.** A13,000 **Q32.** A10,212,

Q33.A11,026.5, **Q34.** A41,880 **Q35.** a.A10,000 (b) A6209 (c) A3791 (d) A2583.83

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CHAPTER-6

DISCOUNTING OF BILL OF EXCHANGE

STRUCTURE

6.1 Introduction

6.2 Promissory note

6.3 Bills of exchange

6.4 Terms used in discounting

6.5 Discounting of the bill

6.6 Applications

EXERCISE

6.1 Introduction

In business houses, the transactions take place both in cash and credit. In the cash transaction the buyer pays the money and take delivery of the goods. In credit transaction the buyer will pay the money on a specific future date. A document is used to substitute this money. The buyer gives the undertaking in the document to the supplier to pay the money at a future agreed date. Such document may be a written promise or an order. Such document may be bills of exchange, promissory notes or hundies. These are known as credit instruments. These instruments are known as negotiable instruments.

If the instrument is written by the buyer himself promising to pay the money on a future agreed date, It is called primissory note. If the instrument is written by the seller and the same has been accepted by the buyer with his signature on the face, It is called bills of exchange. Hundies are like bills of exchange. The only difference is that the bills of exchange is written in English langauge where as Hundi is written in any regional language of India. How ever for the purpose of discounting, these documents are known as bills of exchange.

6.2 Promissory Note

As per sec. 4 of the Indian Negotiable Instrument Act 1881, "A promissory Note is an instrument in writing (not being a bank note or a currency note) containing an unconditional undertaking signed by the maker, to pay a certain sum of money only to or to the order of a certain person, or to the bearer of the instrument".

Specimen of a Promissory Note

A 20,000 Bhubaneswar,

December 11,2015

To,

Mr. James Bond

Bhadrak.

I promise to pay a sum of A 20,000 (Rupees Twenty thousand Only) to you or your order three months after date.

Signature of Mr. X

Here Mr. X is the maker or promisor, Mr. James Bond is the promisee or payee or creditor.

The promissory note does not require any acceptance since the debtor himself is the maker of the instrument.

6.3. Bills of Exchanges

As per sec.5 of the Indian Negotiable instrument Act, 1881 "A Bill of Exchange is an instrument in writing containing an unconditional order, signed by the maker, directing a certain person to pay a certain sum of money only to, or to the order of certain person or to the bearer of the instrument".

Specimen of a Bill of Exchange

A 25,000 Stamp ticket Chennai

December 11, 2015

(Date of drawing)

Three months after date pay to C or order a sum of A 25,000 (Rupees twenty five thousand only) for value of goods received.

(Signature of Mr. A)

To

Mr. B Accepted

Bhubaneswar (Signature of Mr. B)

21.12.15

(Date of Acceptance)

Here A is the creditor / drawer, B is the debtor / drawee, C is the payee.

On receiving the bill, B will put his signature on the face of the bill with the word accepted and return it to A. A will transfer it to C in settlement of dues.

After acceptance only it will be a proper bill of exchange. An unaccepted bill has no value at all.

Parties to a Bill of Exchange

A bill of exchange has the following three parties,

i) **Drawer** : The maker of the bill is known as drawer. In the above example 'A is the

drawer of the bill,

(ii) **Drawee :** The party who accepts the bill is known as drawee. In the above example 'B' is the drawee.

(iii) **Payee :**

The person to whom final payment will be made is known as payee. In the above example 'C' is the payee.

Since these instruments are negotable instruments, these can be

(i) retained by the holder till the date of maturity and realised on maturity;

(ii) endorsed in favour of third party by putting the signature at the back;

(iii) Sent to the bank for collection on due date.

(iv) discounted by the holder in the bank before due date i.e. date of maturity.

Feature of a bills of exchange

(i) A bill of exchange must be in writing .

(ii) It must contain an unconditional order to pay.

(iii) The period and the amount of the bill must be certain.

(iv) There are three parties, i.e. drawer, drawee and payee.

(v) It must be accepted by the drawee.

(vi) It must be properly stamped, dated and signed by the drawer.

(vii) It is a valid legal document. If the buyer fails to pay, the seller has the right to sue and recover the money.

Types of Bills of exchange

In the present context, the bill of exchange can be divided in to two broad catagories.

(i) Demand bill

(ii) Time bill

The demand bill is always payable on demand or payable on presentment. It is otherwise known as at sight bill. It is expressed as payable on demand.

Time bill is a bill of exchange when it is expressed as payable after a specific period. Time bill can be of two types:

(i) After date bill

(ii) After sight bill

After date bill : In such a bill, due date is calculated from the date of drawing. For example a bill drawn an 1st June 2016 payable 2 months after date. The due date is 4th August, 2016.

After sight bill :

In such a bill nominal due date is calculated from the date of acceptance of the bill. For example, a bill drawn on 1st April, 2016 for 2 months after sight, accepted on 4th April, 2016. The nominal due date is 4th June, 2016. Legal due date is 7th June, 2016.

Nominal due date :

This is the date on which actual term (period of the bill) of the bill expires.

(i) After date bill : Nominal due date = Date of drawing + period of the bill

(ii) After sight bill:

Nominal due date = Date of acceptance of the bill + period of the bill.

In this case days of grace is not taken in to consideration.

Legal due date/date of maturity:

Legal due date can be calculated after adding extra 3 days of grace to the nominal due date. Initially, three days of grace was allowed to the drawee as a gratituous favour. Now it has become a legal right of the drawee.

For example - A bill drawn on 1st August, 2015, payable 2 months after date, nominal due date is 1st October and, legal due date is 1st October, 2015 + 3 days of grace = 4th October, 2015.

Maturity of the bill and Days of Grace

The date on which the bill is payable is known as maturity date or date of maturity. If it is at sight bill or on presentation or demand bill it is payable on demand immediately on presentation.

If it is a time/term bill, it is payable after certain period of time, three days of grace is to be added for calculating date of maturity.

For the purpose of calculating the date of maturity, the expression after date means after date of drawal and after sight means after date of acceptance.

If the due date falls on public holiday i.e. Independence (15th August) day, Republic day (26th January), Gandhi Jayanti (2nd October), due date for payment will be previous working day. If due date falls on bank holiday or unforeseen one (emergency holiday), legal due date is shifted to its succeeding day.

The following examples will make it clear.

Illustration-1

A) **Term of the bill is expressed in days** - Due date calculation include the date of payment but exclude the date of transaction.

(i) A bill dated 13th May payable 60 days after date.

Date of maturity = 13th May + 60 days + 3 days of grace

= May 18 days + June 30 days + July 12 days + 3 days of Grace

= 15th July. (Nominal due date is 12th July and legal due date is 15th July)

(ii) A bill dated 13th May accepted on 20th May payable 60 days after sight.

Date of Matrurity = 20th May + 60 days + 3 days of grace

= May 11 days + June 30 days + July 19 days + 3 days of grace

= July 22 (Nominal due date is 19th July and legal due date is 22nd July)

Illustration-2

A bill dated 13th June payable 60 days after date.

Date of maturity = 13th June + 60 days + 3 days of grace

= June 17 days + July 31 days +August 12 days + 3 days of grace

= 1 2th August + 3 days of grace (12th August is nominal due date)

= 15th August

Since 15th August is a public holiday, 14th August is the date of maturity

14th August is legal due date.

B) Term of the bill is expressed in months -

Calculation will be made in term of calendar months ignoring the days of the month. If the period terminates in a month having no corresponding date, period will terminate on the last day of the month.

Illustration- 3

i) A bill drawn on 10th October, payable 3 months after date

Drawn on	10th October
Period/ Tenure	+3 months
Nominal due date	10th January of next year
	+3 days of grace
Legal due date	13th January of the next year

ii)A bill drawn on 31st January, 2016, payable one month after date :

Drawn on	31st January, 2016
Tenure / period	+1 month
Nominal due date	29th February, 2016
	+3 days of grace
Legal due date	3rd March, 2016

Illustration- 4

i) A bill drawn on 20th December, accepted on 23rd December, payable one month after sight.

Drawn on	20th December
Accepted on	23rd December
Tenure / Period	+1 month
Nominal due date	23rd January of next year
	+3 days, of grace

Legal due date 25th January (Since 26th January is a public holiday) of next year.

ii) A bill drawn on 31st January 2016, payable one month after date.

Drawn on	31st January, 2016
Tenure / Period	+1 month
Nominal due date	29th February, 2016
	+3 days of grace
Legal due date	3rd March, 2016

6.4. Discounting of the Bills

The bill amount is payable by the drawee on the date of maturity or legal due date. The drawer or the holder of bill of exchange sometimes is in need of money urgently. So he can get the bill discounted in the bank. The banker will not pay the face value of the bill. The banker will deduct interest at specific rate for the remaining period from the face value of the bill. He will pay the balance amount to the holder of the bill after deducting this interest from the bill value. This is known as discounting of the bill. The banker will get the face value of the bill from the drawee on due date. If the drawee will fail to pay the bill value,.bank will get the money from the person discounting it. Such interest is known as Banker's discount (BD). So interest on the bill value for the unexpired period is known as Banker's Discount.

For example,

Bill value = R 41,000, Present worth = R 40,000

Time = Three months, Rate of interest is 10%.

If the bill is discounted immediately the Banker's discount

will be R 41,000 $\times \frac{3}{12} \times \frac{10}{100}$ = R 1025.00 ... ... (i)

The discounted value of the = Bill Value - Banker's discount

= (R41.000 - R1025)= R39,975.

But the interest on R40,000 for 3 months @ 10% interest (R40,000 $\times \frac{10}{100} \times \frac{10}{100}$) is R1,000.

True discount = Interest on present worth = R1,000 ... (ii)

This excess of interest charged by the Bank is known as Banker's Gain.

Therefore,

Banker's gain = Banker's discount - True discount =R1025-R1000=R25 ... (iii)

1.5 Terms used in discounting

i) Bill Value or face value of Bill (B.V)

The value written on the face of the bill is known as face value of the bill. This is other wise known as maturity value or the value for which the bill is drawn. In the above example R41,000 is the Bill value or face value.

vi) Banker's discount (B.D)

Interest charged by the bank for discounting the bill is known as banker's discount. The amount of discount depends upon the rate of interest, unexpired period of the bill and bill amount. The banker's discount is the interest on bill value. On the above example R1025 is the banker's discount.

vii) True Discount (T.D)

Interest on present worth of the bill is known as true discount. It depends upon bill value, unexpired period and rate of simple interest. In the above example R1,000 is true discount.

viii) Present worth or present value of the bill (P.W / P.V.)

It means as on today the value of the bill due at a later date. In the language of bill of exchange it is the amount of money if invested, it will be equal to maturity value of the bill on due date. Here maturity value of bill = present worth + interest on present value. In the above example R40, 000 is present worth.

ix) Banker's Gain (B.G.)

The excess of Banker's discount over true discount is known as Banker's gain. It is the interest on true discount, in the above example banker's gain is R1,025-R1,000)R25**.**

x) Discounted value of the bill

The amount received by the holder of the bill after discounting the same with the bank is known as discounted value of the bill. It is the excess of bill value over banker's discount. In the above example R39, 975 is the discounted value of the bill.

xi) Discounting Period

The difference between legal due date and the date of discounting of the bill is known as discounting period. For example, a bill drawn on 28th September 2015, payable 3 months after date. The legal due date is 31st December, 2015. It is discounted on 19th October 2015. The discounting period is 19th October to 31st December 2015. It is equal to October 12 days + November 30 days + December 31 days = 73 days. This period is considered while calculating Banker's discount.

xii) Endorsement of the bills

The drawer can transfer the bill to another person before the date of maturity. The process of transfer of the bill is known as endorsement of the bill. The endorser endorses the bill putting his signature at the back of the bill. The person endorsing the bill is known as endorser. The person to whom the bill is endorsed is known as endorsee. Generally, the drawer endorses the bill in favour of his creditor.

xiii) Dishonour of the Bill

The bill of exchange is presented for payment to the drawee on legal due date. If he makes the payment, the bill is said to be honoured. If he fails to make the payment, the bill of exchange is said to be dishonoured.

xiv) Noting and Protesting

The holder of the bill wants some proof on dishonour of the bills. Later on the drawee may take the plea that the bill was not presented. When drawee fails to pay on'due date, the bill is taken to the Notary Public (appointed by the Government). The Notary Public present the bill for payment. If the payment is made, it is given to the payee. Otherwise fact of dishonour is noted on the face of the bill or on a separate paper attached to the bill. This is known as Noting through legal Act. The main advantage of noting is that

the fact of non-payment can be vindicated in the Court of Law. The Notary Public charges a small amount of service charge which is paid by the holder of the bill. Such charges is known as noting charges. The formal certificate issued by notary public about the dishonour of the bill is known as protesting.

All these amount will be recovered from the drawee.

xv) Dishonour of the discounted bill

The ownership and possession of the bills of exchange is transferred to the banker on discounting the same. On due date bank present the bill to the drawee for payment. If the bill is dishonoured, the bank debit the account of the holder / drawer / payee with the full amount plus noting charges, if any. For example, A had discounted a bill amounting R5,000 with the banker for R4,700. On due date the drawee failed to pay and the bank had to pay R100 as noting charges. The bank will debit account of A by R 5,100.

Distinction between

Bills of exchange

- Bills of exchange is an unconditional order to pay.
- Bills of exchange has three parties known as drawer, drawee, payee.
- It requires acceptance by the drawee before the due date of payment.
- Liabiling of the drawer is secondary.

Promissory note

- Promissory note is an unconditional promise to pay.
- Promissory note has two parties known as promisor, promisee.
- No acceptance is required in case of promissory notes.
- Liabiling of the promiser is primary.

Banker's Discount

- Banker's discount is the simple interest on the face value of the bill for the unexpired period.
- Banker's discount is always greater than true discount.
- Banker's discount give rise to banker's gain.

True Discount

- True discount is the simple interest on the present value of the bill for the unexpired period.
- True discount is always less than banker’s discount.
- There is no scope for banker’s gain.

Bill value	Present value
o Amount recorded on the face of the bill to be payable on maturity is known as bill value.	o Present value is the value if invested today will be equal to bill value on the date of maturity,
o Banker's discounts is calculated on the bill value.	o True discount is calculated on the present value
o Banker's gain takes place as discount is calculated on the bill value.	o Present value does hot give rise to banker's gain.

Various formulae for computine TD, B.D, P.W and B.G.

(A) True Discount (T.D.)

i) True Discount = Interest on present worth.

ii) $TD = \frac{P \times R \times T}{100}$ Where P = Present value / Present worth, R = Rate of Interest, T = Time in year

iii) $TD = \frac{BV \times R \times T}{100 + (R \times T)}$ (BV= Bill value)

iv) TD = Bill value - Present value / Present worth = (BV - PW)

vi) TD = BD-BG where BD = Banker's Discount

BG = Banker's Gain

vi) $TD = \sqrt{P \times B.G.}$ P = Present value/Present worth , BG = Banker's gain

B. Banker's Discount (B.D.)

(i) B.D. = Interest on bill value,

(ii) $B.D. = \frac{B.V. \times R \times T}{100}$ BV - Bill value, TD = True Discount, R = Rate of interest T = Time,

(iii) $\frac{B.V. \times T.D.}{P.V.}$

(iv) B.D = TD. + B.G. where, BG = Banker's Gain

(C) Present value / Present worth (P)

(i) P = B.V. - TD.

(ii) $P = \frac{B.V. \times 100}{100 + (R \times T)}$

(D) (i) B.G. = Interest on TD.

(ii) $B.G. = \frac{TD \times R \times T}{100}$

(iii) B.G. = B.D. - T.D.

(iv) $B.G. = \frac{B.V\,(RT)^2}{100\,(100 + RT)}$

(E) Bill Value

i) B.V = P + T.D.

ii) $BV = \frac{BD \times TD}{BD - TD}$

iii) $BV = \frac{P(100 + R \times T)}{100}$

iv) $BV = \frac{TD \times (100 + R \times T)}{R \times T}$

1.6. Applications :

(I) To Find the True Discount

Illustration - 5

Find the true discount on ₹ 10, 300 due 3months hence at 12% p.a.

Solution : Bill value (BV) = ₹ 10, 300, Time (T) = 3 months, Rate (R) = 12% p.a.

$$\text{True Discount} = \frac{BV \times R \times T}{100 + (R \times T)} = \frac{10,300 \times 12 \times \frac{3}{12}}{100 + 12 \times \frac{3}{12}} = \frac{10,300 \times 3}{100 + 3} = \frac{10,300}{103} \times 3 = ₹\ 300$$

Alternatively,

Let the Present value of the Bill be R 100

Interest on R100 for 3 months @ 12% p.a. = $100 \times \frac{3}{12} \times \frac{12}{100} =$ R 3

Bill Value = R100 +3 = R103

When Bill Value	True Discount	
103	3	
10,300	x	$x = \frac{3}{103} \times 10{,}300 =$ R 300

Illustration - 6

A bill of R 5,200 was drawn on 25th May 2016 payable 6 months after sight. It was accepted on 28th May 2016 and discounted on 8th July 2016. Find the True Discount if the rate of interest is 10% p.a.

Solution :

For the sight bill, date of maturity will be calculated from date of acceptance.

Date of Drawing 25.5.16

Date of Acceptance 28.5.16

Date of maturity = 28.05.16 + 6 months + 3 days of grace = 01.12.2016

Bill discounted on 08.07.2016

So unexpired period = Number of days from 08.07.2016 to 01.12.2016 = July 23 days + August 31 days + September 30 days + October 31 days + November 30 days + December 01 day = 146 days = $\frac{146}{365}$ year = $\frac{2}{5}$ year

Bill value (B.V.) = R 5, 200

Time (T) = $\frac{2}{5}$ year

True Discount = $\frac{BV \times R \times T}{100 + (R \times T)}$ (Applying the formula)

$$= \frac{5{,}200 \times 10 \times \frac{2}{5}}{100 + 10 \times \frac{2}{5}} \text{ (Substituting the value)}$$

$$= \frac{5{,}200 \times 4}{100 + 4}$$

$$= \frac{5{,}200 \times 4}{104}$$

= R 200

So true discount is R 200

Illustration - 7

Find the true discount on a bill of R 615 where present value is R 600. Bill is due 3months after date and the rate of interest is 10% p.a.

Solution

Bill Value (B.V) = R615, Present value (P) = R600, Rate of interest (R) = 10% p.a, Time (T) = 3 Months

So True discount (TD) = $\frac{P \times R \times T}{100}$ (Applying the formula)

$$= \frac{600 \times 10 \times \frac{3}{12}}{100} \text{ (Substituting the values)}$$

$$= \frac{50 \times 10 \times 3}{100} = R\ 15$$

Alternatively

True Discount(T.D) = Bill Value - Present Value

= R 615 - R 600 = R 15

So true discount is R 15

(II) To find Banker's Discount

Illustration - 8

Present value of a bill due later on is R 3,000 and banker's gain on the same bill is R7.50.

Find the true discount and BD.

Solution :

Present value (P) = R 3,000, Banker's gain (BG) = R 7.50

True Discount (TD) = $\sqrt{P \times BG}$ (Applying the formula)

$\sqrt{R\,3{,}000 \times R\,7.50}$ (Substituting the value)

= $\sqrt{R22{,}500}$

= R 150

So true discount is R 150

BD = R150 + R7.5 = R157.50

Illustration - 9

Find banker's discount on a bill of R 3, 200 payable 3 months after date. The rate of interest is 10% p.a.

Solution :

Bill value (BV) = R 3,200, Rate of interest (R) = 10% p.a., Time (T) = 3 months

Banker's discount (BD) $= \frac{BV \times R \times T}{100}$ (Applying the formula)

$= \frac{3,2000 \times 10 \times \frac{3}{12}}{100}$ (Substituting the value)

$= \frac{3,2000 \times 10 \times \frac{1}{4}}{100}$

$= \frac{800 \times 10}{100}$

= 80

So Banker's discount is R 80

Alternatively,

When Bill value R 100, Interest for 3 months = $\frac{3}{12}$ x 10 = $\frac{5}{2}$

When Bill value	Banker's Discount	
100	$\frac{5}{2}$	
3200	x	$x = \frac{5}{2} \times \frac{1}{100} \times 3200 = R\,80.00$

So Banker's discount is R 80.00

Illustration - 10

Calculate the banker's discount on a bill for R 51, 750, if true discount is R 6, 750.00

Solution:

Given Bill Value (BV) = R 51, 750, True Discount (TD) = R 6, 750

Present value (P) = B.V. - T.D. = R 51, 750 - R 6, 750 = R 45, 000

B.D. = $\frac{\text{B.V. x T.D.}}{\text{P.V.}}$ (Applying the formula)

= R $\frac{51,750 \times 6,750}{45,000}$ (Substituting the value)

= R 7, 762

So Banker's Discount is R 7, 762

Illustration - 11

Find the Banker's discount and True discount on R1950 due 6 months hence at $5\frac{1}{3}$ % per annum.

Solution

Given : Bill value (BV) = R 1950, Time (T) = 6 months

= $\frac{6}{12}$ or $\frac{1}{2}$ year, Rate (R) = $5\frac{1}{3}$ % p.a. or $\frac{16}{3}$ % p.a.

BD = $\frac{\text{B.V x R x T}}{100}$ (Applying the formula)

$= ₹ \frac{1950 \times \frac{16}{3} \times \frac{1}{2}}{100} = \frac{1950 \times 16 \times 1}{100 \times 3 \times 2}$ = Rs 52 (Substituting the value)

$TD = \frac{B.V \times R \times T}{100 + (R \times T)} = \frac{1950 \times \frac{16}{3} \times \frac{1}{2}}{100 + \left(\frac{16}{3} \times \frac{1}{2}\right)}$

$= ₹ \frac{650 \times 8}{100 + \frac{8}{3}}$

$= ₹ \frac{650 \times 8}{\frac{308}{3}}$

$= ₹ \frac{650 \times 8 \times 3}{308}$

= ₹ 50.65

So True discount is ₹50.65, Banker's discount is ₹52.

(III) To Find Banker Gain

Illustration-12

A billl for ₹7, 650 was accepted on 28th february 2015 for 9 months. It was discounted on 8th July 2015 at 5% p.a. Find True Discount, Banker's discount and Banker's Gain.

Solution

Given - Bill value (BV) = ₹ 7, 650, Drawn on 28th February 2015

Time (T) = 9 months, Rate of discount (R) = 5%

Date of maturity = 28th February, 2015 + 9 months + 3 days of grace

= 1st December, 2015

Date of discounting is 8th July, 2015

Unexpired period = Number of days from date of discounting to date of maturity.

= July 23 days + August 31 days + September 30 days + October 31 Days + November 30 days+ December 1 day = 146 days = $\frac{146}{365}$ year = $\frac{2}{5}$ year

Banker's Discount (BD) = $\frac{BV \times R \times T}{100}$ (Applying the formula)

$$= R\ \frac{7,650 \times 5 \times \frac{2}{5}}{100} \text{ (Substituting the value)}$$

$$= R\ \frac{7,650 \times 2}{100}$$

$$= R\ 153$$

$$\text{True Discount (TD)} = \frac{B.V. \times R \times T}{100 + (R \times T)} \text{ (Applying the formula)}$$

$$= R\ \frac{7,\ 650\ \times 5 \times \frac{2}{5}}{100\ + \left(5 \times \frac{2}{5}\right)} \text{ (Substituting the value)}$$

$$= R\ \frac{7,650 \times 2}{102} = R\ 150$$

Banker's Gain = BD - TD = R 153 - R150 = R 3

Alternatively

Calculate True discount, Banker's discount as above

$$\text{Banker's Gain} = \frac{TD \times R \times T}{100} \text{ (Applying the formula)}$$

$$= \frac{150 \times 5 \times \frac{2}{5}}{100} = \frac{150 \times 2}{100} = \text{Rs } 3 \text{ (Substituting the value)}$$

So Banker's discount, True discount and Banker's gain are R153, R150 and R3 respectvely.

(IV) To Find Present Value

Illustration -13

Find the present value of R1770 due 2 years hence at 9% interest per annum. Also find the True Discount.

Solution :

Given Bill value (BV) = R 1770, Time (T) = 2 years, Rate (R) = 9% p.a.

$$\text{True discount (TD)} = \frac{B.V. \times R \times T}{100 + (R \times T)} \text{ (Applying the formula)}$$

$$= R\ \frac{1,\ 770\ \times 9 \times 2}{100\ + (9 \times 2)} \text{ (Substituting the value)}$$

$$= R\ \frac{1,770 \times 9 \times 2}{118}$$

$$= R\ 270$$

Present Value = Bill value - True Discount = R1770 - R 270 = R1500

So present value and true discount ate R 1500 and R 270 respectively.

Alernatively

$$\text{Present value (P)} = \frac{\text{B.V.} \times 100}{100 + R \times T} \quad \text{(Applying the formula)}$$

$$= R\frac{1770 \times 100}{100 + (9 \times 2)} \quad \text{(Substituting the value)}$$

$$= R\ \frac{1770 \times 100}{118}$$

$$= R\ 1500$$

True discount = B.V - Present worth

$$= R\ 1770 - R\ 1500$$

$$= R\ 270$$

So Present value and true discount are R1500 and R 270 respectively.

Alternatively

Given - Bill value (BV) = R1770, Rate of Interest (R) = 9% p.a., Time (T) = 2 years.

Let the present value = R 100

$$\text{Interest @ 9\% p.a. for 2 years} = \frac{9}{100} \times 100 \times 2 = R\ 18$$

Bill value = Present value + Interest on present value

$$= R\ 100 + 18 = R\ 118$$

When Bill value	Present value	
118	100	
1770	x	$x = \frac{100}{118} \times 1770 = R\ 1500$

Present worth and True discount are R1500 and R 270 respectively.

Illustration -14

Find the present worth and True Discount of Rs 327 due in 18 months at 6% p.a. simple interest (CHSE-2002, 2004)

Solution

Given - Bill value (BV) = ₹ 327, Rate of the discount (R) = 6%

Time (T) = 18 months = $\frac{3}{2}$ years

present worth (P) = $\frac{BV \times 100}{100 + (R \times T)}$ (Applying the formula)

= ₹ $\frac{327 \times 100}{100 + (6 \times \frac{3}{2})}$ (Substituting the value)

= ₹ $\frac{327 \times 100}{100 + 9}$

= ₹ 300

TD = $\frac{P \times R \times T}{100}$ (Applying the formula)

= ₹ $\frac{300 \times 6 \times \frac{3}{2}}{100}$ (Substituting the value)

= ₹ $\frac{2700}{100}$

= ₹ 27

Present worth and True discount of the bill are ₹ 300 and ₹ 27 respectively.

Alternatively

Given - Bill value (BV) = ₹ 327, Rate of Discount = 6%

Time (T) = 18 months = $\frac{3}{2}$ years

Let the Present worth Rs 100

Interest for $\frac{3}{2}$ years = ₹ 6 x $\frac{3}{2}$ = ₹ 9

Bill value = Present worth + interest on present worth

= R (100 + 9) = R109

When Bill Value	Present worth	
R	R	
109	100	
327	x	$x = R\ \frac{100}{109} \times 327 = R300$

True Discount = Bill value - Present worth

= Rs 327 - Rs 300 = Rs 27

So present worth and True discount on the bill are R 300 and R27 respectively.

Illustration -15

The Banker's discount and true discount on a certain bill for 3 years is R54 and R36 respecively. Find the bill value and the rate.

Solution :

Given- Banker's discount (BD) = R 54, True discount (TD) = R 36

Time (T) = 3 years

Bill value = $\frac{BD \times TD}{BD - TD}$ (Applying the formula)

= $R\ \frac{54 \times 36}{54 - 36}$ (Substituting the value)

= R 108

Rate of Interest

Banker's discount for 3years = R 54

Banker's discount for 1years = $R\ \frac{54}{3}$ = Rs 18

When B.V. (Rs)	Interest for 1 year (R)	
108	18	
100	x	$x = \frac{18}{108} \times 100 = 16\frac{2}{3}\%$

So bill value and rate of discount are R 108 and $16\frac{2}{3}\%$ respectvely.

Illustration -16

The banker's discount and true discount on a certain bill at 5% simple interest is R60 and R 50 respectively for the same period of time. Find the bill value and the period of time.

Solution :

Given - Banker's discount (BD) = R 60, True discount (TD) = R 50

Rate of interest (R) = 5%

Bill value = $\frac{BD \times TD}{BD - TD}$ (Applying the formula)

= R $\frac{60 \times 50}{60 - 50}$ (Substituting the values)

= R 300

Interest on R 300 @ 5% p.a. for one year = $\frac{5}{100}$ x 300 = R 15

When BD (R)	Time in years	
15	1	
60	x	x = $\frac{60}{15}$ = 4 years

So the bill value and time period one R 300 and 4 years respectively.

Illustration -17

A bill was drawn on 7th May, 2015 payable 6 months after date. It was discounted on 17th June, 2015 at 5% and R1, 176 was paid. Find the bill value and banker's discount.

Solution :

Given - Bill drawn on 7th May, 2015, Time period 6 months after date.

Date of maturity 7th May, 2015 + 6 months + 3 days of grace = 10th November, 2015

Bill discounted on 17th June, 2015

Unexpired period June 13 days + July 31 days + August 31 days + September 30 days +

October 31 days + November 10 days = 146 days = $\frac{146}{365}$ year = $\frac{2}{5}$ year

Let the Bill value = R100

BD = $\frac{BV \times R \times T}{100}$ = R $\frac{100 \times 5 \times \frac{2}{5}}{100}$ = R 2

Discounted Value = Bill value - Banker's discount

= R100 - R 2

= R 98

When discount value (R)	Bill value (R)
98	100
1176	x

$x = \frac{100}{98} \times 1176 = R1200$

Banker's discount = Bill value - Discounted value

= R1, 200 - R 1, 176 = R 24

So Bill value and Banker's discount are R 1,200 and R 24 respectively.

Alternatively

Present value R 1, 176, Rate (R) = 5%. Time (T) = $\frac{2}{5}$ year

Bill value (BV) = $\frac{P.V \times (100 + R \times T)}{100}$ (Applying the formula)

= R $\frac{1,176 \times (100 + 5 \times \frac{2}{5})}{100}$ (Substituting the value)

= R 1, 199.52 or R 1,200

Banker's discount = Bill value - Present value

= R 1,200 - R1, 176

= R 24

So Bill value and Banker's discount are R 1,200 and R 24 respectively.

(V) Miscellaneous Illustrations

Illustration -18

A offers R 3,000 in cash for a portable T.V. B offers R 3,333 to pay after 2 years. Which offer is better, money being reckoned at 5% simple interest.

Solution:

To find out better offer, we have to find the present worth of R 3,333

Given - Bill value (BV) = R 3,333, Time (T) = 2 years, Rate of Interest (R) = 5%

Present worth = $\frac{BV \times 100}{100 + (R \times T)}$ (Applying the formula)

$= R \frac{3,333 \times 100}{100 + (5 \times 2)}$ (Substituting the value)

$= R \frac{3,333 \times 100}{110}$

$= R$ 3030

On the basis of present worth, A's offer is R 3,000 and B's offer is R 3,030.

So, B's offer is better

Alternatively,

We have to find the present worth of R 3,333 for knowing the better offer.

Given- Present cash R 3,000

To pay after 2 years R 3,333, rate of interest = 5%

Let the present worth R 100

Interest on R 100 @ 5% p.a. for 2 years = R100 x $\frac{5}{100}$ x 2 = R10

Amount due after 2years = R100 + R10 = R110

When amount due after 2years (R)	present worth (R)	
110	100	
3,333	x	x = $\frac{100}{110}$ x 3,333 = R 3,030

A's offer R 3,000

B's offer R 3,030

So B's offer is better.

Illustration -19

What is the actual rate of interest which a banker gets for the money when he discounts a bill legally due in 6months of 8% p.a. [CHSE-1990(s)]

Solution:

Given - Time (T) = 6 months = $\frac{1}{2}$ year, Rate of interest (R) = 8%

Let the bill value R100

Banker's discount = $\frac{BV \times R \times T}{100}$ (Applying thje formula)

$= R \frac{100 \times 8 \times \frac{1}{2}}{100} = R\ 4$ (Substituting the value)

So Discounted value (DV) = R 100 - R 4 = R 96

When discounted value R 96, Interest for $\frac{1}{2}$ year = R 4

So interest for 1 year = R 4 x 2 = R 8

When Discounted value (R)	Interest (R) For 1 year
96	8
100	x

$x = \frac{8}{96} \times 100 = 8\frac{1}{3}\%$

Effective rate of interest received by the bank is $8\frac{1}{3}\%$

Illustration - 20

Mr x purchased a bicycle for R1500 and sold it for R 3,300 at a credit of one year. Find the profit percentage reckoning 10% interest.

Solution :

Given - Selling price of bicycle is R 3,300 (BV). Amount is receivable after one year.

We have to find out present worth of R 3300

Time (T) = 1 year, Rate of interest (R) = 10%

Present worth of R 3,300 = $\frac{B.V \times 100}{100 + (R \times T)}$ (Applying the formula)

$= R \frac{3,300 \times 100}{100 + (10 \times 1)}$ (Substituting the values)

$= R \frac{3,300 \times 100}{110}$

= R 3,000

Profit is present worth of selling price - cost price = R 3,000 - R1,500 = R1,500

When Cost Price (R)	Profit (R)	
1500	1500	
100	x	$x = \frac{1500}{1500} \times 100 = 100\%$

Hence profit is 100%

Illustration - 21

A owes B R 6,000 payable 2years after date at 10% p.a. B owes A R 7,950 payable 6months after date at 12% p.a. They want to settle their account with immediate effect. How much should be paid by whom?

Solution :

Both A and B want to settle the account by cash immediately. Hence, we have to find out the present worth of their bills.

Given- For A

Bill value (BV) = R 6000, Time (T) = 2years. Rate of interest (R) = 10% p.a.

$$\text{Present worth} = \frac{\text{B.V.} \times 100}{100 + (\text{R} \times \text{T})} \text{ (Applying the formula)}$$

$$= \text{R}\frac{6{,}000 \times 100}{100 + (10 \times 2)} \text{ (Substituting the values)}$$

$$= \text{R}\frac{6{,}000 \times 100}{120}$$

$$= \text{R } 5{,}000 \quad \ldots \quad \ldots \quad \ldots \quad \text{(i)}$$

For B

Bill value (BV) = R 7,950, Time (T) = 6months = $\frac{1}{2}$ year, rate of interest (R) = 12% p.a.

$$\text{Present worth} = \frac{\text{B.V.} \times 100}{100 + (\text{R} \times \text{T})} \text{ (Applying the formula)}$$

$$= \text{R } \frac{7{,}950 \times 100}{100 + (12 \times \frac{1}{2})} \text{ (Substituting the value)}$$

$$= R\frac{7{,}950 \times 100}{106}$$

$$= R\ 7{,}500 \quad \ldots \quad \ldots \quad \text{(ii)}$$

B has to pay R7,500 - R5,000 = R2,500 to A for immediate settlement.

B can settle the account by paying R 2,500 to A immediately.

Illustration - 22

If the present value of billl due some times hence is R7,168 and the banker's gain is R7 Find the bill value.

Solution

Given present worth (P) - R7,168, Banker's gain (BG) = R7

True Discount (TD) = $\sqrt{P \times B.G}$ (Applying the formula)

= $\sqrt{7{,}168 \times 7}$ (Substituting the value)

= $\sqrt{50,176}$

= R 224

Bill value = Present worth + True discount

= R 7,168 + R 224

= R 7,392

So Bill value is R 7,392

Illustration -23

If the banker's discount on a bill amounting R2250 due sometime hence at 6% is R54, when was the sum due ?

Solution

Given- Bill value (BV) = R 2250

Rate of discount (R) = 6%, Banker's Discount (BD) = R 54

Banker's Discount (BD) = $\frac{BV \times R \times T}{100}$ (Applying the formula)

or R 54 = $\frac{2250 \times 6 \times T}{100}$ (substituting the values)

or R 54 = 135T

or T = $\frac{54}{135}$ year = $\frac{2}{5}$ year

Hence, the sum was due after $\frac{2}{5}$ year.

Alternatively

Given Bill Value(BV) = R 2250, Rate of discount(R) = 6% Banker's discount (BD) = R 54

When Bill value(R)	Bankers Discount for one year (R)	
100	6	
2250	x	$x = \frac{6}{100} \times 2250 = R135$

When Banker's Discount(Rs)	Time (Year)	
135	1	
54	x	$x = \frac{1}{135} \times 54 = \frac{2}{5}$ year

So the sum was due after $\frac{2}{5}$ year.

Illustration -24

The Banker's discount on R 800 at 6% p.a. is same as true discount on R 812 for the sametime and rate. Find the time.

Given Ist Bill - Bill amount (BV) = R 800, Rate (R) = 6% p.a

2nd Bill - Bill amount(BV) = R 812, Rate(R) = 6% p.a

Banker's Discount of the 1st Bill = True discount on the 2nd Bill

BD = $\frac{BV \times R \times T}{100} = R\frac{800 \times 6 \times T}{100}$ (Applying the formula and the substituting the values)

TD = $\frac{BV \times R \times T}{100 + (R \times T)} = R\frac{812 \times 6 \times T}{100 + (6 \times T)}$ (DO)

As per question $\frac{800 \times 6 \times T}{100} = \frac{812 \times 6 \times T}{100 + 6T}$

or 48T = $\frac{812 \times 6 \times T}{100 + 6T}$

or $8 = \frac{812}{100 + 6T}$

or $8(100 + 6T) = 812$

or $T = \frac{1}{4}$ year or 3 months.

Hence the time period is $\frac{1}{4}$ year.

Illustration -25

The true discount of a bill amounting R1,030 due sometimes hence is R30 at 9% p.a. When is the bill due ?

Solution :

Given- Bill value (BV) = R1,030, True Discount (TD) = R30,

Rate of interest (R) = 9% p.a.

True Discount = $\frac{B.V. \times R \times T}{100 + (R \times T)}$ (Applying the formula)

$30 = \frac{1030 \times 9 \times T}{100 + 9T}$ (substituting the values)

or $30 (100 + 9T) = 9270T$

or $3,000 + 270T = 9,270T$

or $T = \frac{1}{3}$

The bill is due after $1/3$ year.

Alternatively

Given - Bill value (BV) = R1030

True discount = R30, Rate of interest = 9% p.a.

Let the time period = x year.

when present value R100, interest for x year = R 9x

Bill value = Present value + Interest on present value

= R 100 + 9x

When Bill value (R)	True Discount (R)	
100 + 9x	9x	
1030	?	T.D. = $\frac{9x}{100+9x}$ x 1030 rupees.

As per question -

$$\frac{9x\ (1030)}{100+9x} = 30$$

or 30 (100 + 9x) = 9270x

or x = $\frac{1}{3}$

So, Bill is due after $\frac{1}{3}$ year.

Illustration - 26

The true discount on a bill of R 2,730 due 6 months hence is R130. Find the rate of interest.

Solution

Given- Bill value (BV) = R 2,730, True Discount (TD) = R130

Time (T) = 6 months = $\frac{1}{2}$ year.

We have to find out Rate of interest (R)

TD = $\frac{BV \times R \times T}{100+(R \times T)}$ (Applying the formula)

or 130 = $\frac{2,730 \times R \times \frac{1}{2}}{100+(R \times \frac{1}{2})}$ (Substituting the values)

or 130 = $\frac{\frac{2730R}{2}}{\frac{200+R}{2}}$

or 130 = $\frac{2730R}{200+R}$

or 2730R = 26000 + 130R

or 2600R = 26,000

or R = 10%

Hence rate of interest is 10%

Alternatively

Given - As above

let the present value R100, Interest at the rate of R% for $\frac{1}{2}$ year is R $\frac{R}{2}$

Bill value = Present value + interest on present value

$= R\ 100 + \frac{R}{2}$

When Bill value (R)	True Discount/ interest on present value (R)	
$100 + \frac{R}{2}$	$\frac{R}{2}$	
2730	x	$x = \frac{R}{2} \times \frac{2}{200 \times R} \times 2730 = \frac{2730\,R}{200 + R}$

As per question-

$\frac{2730\,R}{200 + R} = 130$

or 2730 R = 130 (200 + R)

or 2600 R = 26000

R = 10%

So rate of interest is 10%

Illustration - 27

The banker's discount on a bill amounting R 5,600 due 9 months hence is R 378. find the rate of interest.

Solution

Given- Bill value (BV) = R 5,600, Banker's discount (BD) = R 378

Time (T) = 9 months = $\frac{3}{4}$ year

Find rate of interest (R)

BD = $\frac{BV \times R \times T}{100}$ (Applying the formula)

378 = $\frac{5{,}600 \times R \times \frac{3}{4}}{100}$ (substituting the values)

or 378 = 42 R

or R = $\frac{378}{42}$ = 9% p.a.

So rate of interest is 9% p.a.

Alternatively

When B.V. R 5,600, Interest for 9 months = R 378

Interest for one year = R 378 x $\frac{4}{3}$ = R 504

When B.V. (R)	Interest / Banker's discount (R)	
5600	504	
100	x	x = $\frac{504}{5{,}600}$ x 100 = 9%

Hence rate of interest is 9% p.a.

Illustration - 28

The true discount on a bill due 2years hence at 9% p.a. simple interest is R1800. Find the amount of the bill and banker's gain.

Solution

Given - Time (T) = 2years

True discount (TD) = R1800, Rate of Interest (R) = 9%

To find Bill value (BV)?

$$\text{T.D.} = \frac{BV \times R \times T}{100 + (R \times T)}$$ (Applying the formula)

$$\text{or } R\ 1{,}800 = \frac{BV \times 9 \times 2}{100 + (9 \times 2)}$$ (Substituting the values)

$$\text{or } R1{,}800 = \frac{BV \times 18}{118}$$

$$\text{or } R100 = \frac{BV}{118}$$

or B.V = R11,800

Hence bill value = R 11,800

Banker's gain = Interest on true discount

Interest on R100 @9% p.a. for 2 years = 9 x 2 = R 18

When True discount (R)	Interest / Banker's gain (R)	
100	18	
1800	x	$x = \frac{18}{100} \times 1800 = R\ 324$

So bill value and banker's gain are R 11,800 and R 324 respectively.

Alternatively

Given - True discount (TD) = R1800, Rate of Interest (R) = 9%, Time (T) = 2years

Let the present value R100

Interest on R100 @ 9% p.a. for 2years = 9 x 2 = R18

Bill value = Present value + Interest on present value

= R100 + R18

= R118

When Interest on present value / True Discount (R)	Bill value (R)	
18	118	
1800	x	$x = R\frac{118}{18} \times 1800 = R11{,}800$

Calculate Banker's Gain as above

So Bill value and Banker's gain are R11,800 and R324 respectively.

Illustration - 29

The difference between the interest and true discount on a certain sum money for 6 months at 4% is R 2. Find the sum.

Solution

Given - Rate of interest (R) = 4% p.a., Time (T) = 6 months = $\frac{1}{2}$ year

Banker's Gain (BG) = R 2

Let the BV = R100

$$BD = \frac{BV \times R \times T}{100} \text{ (Applying the formula)}$$

$$= \frac{100 \times 4 \times \frac{1}{2}}{100} \text{ (substituting the value)}$$

= R 2

Now R 2 is the true discount on R 100 + 2 = R102

When amount (R)	True discount (R)	
102	2	
100	x	$x = \frac{2}{102} \times 100 = R\ \frac{100}{51}$

Banker's gain = Banker's discount - True discount

$$= R\ 2 - R\frac{100}{51} = R\frac{2}{51}$$

When Banker's Gain (R)	Bill Value (R)	
$\frac{2}{51}$	100	
2	x	$x = R100 \times \frac{51}{1} \times 2 = R5,100$

So, the sum is R5,100

Illustration -30

Find the Bill value of a 8 months bill when banker's discount at 6% p.a. is R 288

Solution

Given - Banker's Discount (BD) = R288, Rate of discount (R) = 6% p.a.

Time (T) = 8 months = $\frac{2}{3}$ year To find Bill value (BV) ?

BD = $\frac{BV \times R \times T}{100}$ (Applying the formula)

R 288 = R $\frac{BV \times 6 \times \frac{2}{3}}{100}$ (putting the value)

or R 288 = R $\frac{BV \times 4}{100}$

or BV = R 288 x 25 = R 7,200

So bill value is R7,200

Alternatively,

Given- Banker's discount R288, Rate of discount 6% p.a. Time 8 months = $\frac{2}{3}$ year

When bill value R100, interest on R100 @ 6% p.a. for $\frac{2}{3}$ years = 6 x $\frac{2}{3}$ = R4

When Banker's discount /

Interest on bill value (R)	Bill value (R)	
4	100	
288	x	x = R $\frac{100}{4}$ x 288 = R7,200

Bill value is R7,200.

Illustration - 31

The Banker's gain is 5% of true discount. Find the unexpired period of the bill on the date of discount, if the rate of interest is $7\frac{1}{2}$ %.

Solution

Given- Rate of interest (R) = $7\frac{1}{2}\% = \frac{15}{2}\%$ p.a.

Banker's gain (BG) = 5% of true discount.

Let True discount (TD) = R100

Banker's gain = 5% of TD = R 5

Banker's discount (BD) = TD + BG

= R100 + R5

= R105

Bill value (BV) = $\frac{BD \times TD}{BD - TD}$ (Applying the formula)

= R $\frac{105 \times 100}{105 - 100}$ (substituting the values)

= R 2100

Banker's discount = $\frac{BV \times R \times T}{100}$ (Applying the formula)

or 105 = $\frac{2100 \times \frac{15}{2} \times T}{100}$ (substituting the value)

or 105 = $\frac{21 \times 15}{2} T$

or 105 = $\frac{315T}{2}$

or T = $\frac{105 \times 2}{315} = \frac{2}{3}$ year = 8 months.

So the bill was discounted 8 months before its maturity date.

Illustration - 32

True discount on a certain sum of money is R48. Time period is 6 months. Rate of interest is 6%. Find i) sum, iii) banker's discount, iii) banker's gain.

Solution

Given- True discount (TD) = R 48, Time (T) = 6 months = $\frac{1}{2}$ year

Rate of interest (R) = 6%

True discount (TD) = $\frac{P \times R \times T}{100}$ (Applying the formula)

or R48 = R $\frac{P \times 6 \times \frac{1}{2}}{100}$ (substituting the value)

or R48 = R $\frac{P \times 3}{100}$

or Present worth = R1,600

Bill value / sum = P + TD

= R1600 + 48

= R1648

Banker's discount (BD) = $\frac{BV \times R \times T}{100}$ (Applying the formula)

= R $\frac{1,648 \times 6 \times \frac{1}{2}}{100}$ (substituting the value)

= R $\frac{1,648 \times 3}{100}$

= R 49.44

Banker's gain = BD - TD

= R49.44 - R48

= R1.44

So the sum, banker's discount, banker's gain are R1,648, R49.44 and R1.44 respectively.

Illustration - 33

The present value of a bill due 5 years hence is R1,200. But the present value of the same bill due 4 years hence is R1,260 at the same rate of interest. Find the present value.

Solution:

Given-		**1st case**	**2nd case**
	present worth/ Present value (P)	R1,200	R1,260
	Time (T)	5 years	4 years

Bill value (BV) = Same in both the case

$P = \frac{BV \times 100}{100 + (R \times T)}$ (R stands for rate of interest) (Applying the formula)

Substituting the value

1st case

$R1{,}200 = R\frac{BV \times 100}{100 + R \times 5}$

$\text{or } 12 = \frac{BV}{100 + 5R}$

or 1200 + 60R = BV ... (1)

2nd case

$R1{,}260 = R\frac{BV \times 100}{100 + R \times 4}$

$\text{or } 63 = \frac{BV \times 5}{100 + 4R}$

or R 6,300 + 252R = 5BV ... (2)

Multiplying equation (i) with 5 and subtracting equation (2) there from-

R 6,000 + 300R = 5BV

R 6,300 + 252R = 5BV

(-) R 300+ 48R = 0

or 48R = R 300

$\text{or } R = \frac{300}{48} = \frac{25}{4} = 6\frac{1}{4}\%.$

So Rate of interest is $6\frac{1}{4}\%$.

Alternatively,

Given -	1st case	2nd case
Present value / Present worth (pw)	R1,200	R1,260
Time (T)	5 years	4 years

Bill value is same in both the case.

We know Bill value = Present worth + itnterest on present worth

1st case

Bill Value = R1,200 + interest on R1,200 for 5 years

= R1,200 + interest on R (1,200 x 5) or R 6,000 for 1 year. ... (1)

2nd case :

Bill value = R 1,260 + interest on 1,260 for 4 years

= R 1,260 + interest on R(1,260 x 4) or R5,040 for 1 year ... (2)

As Bill value in both the case is same equation (1) = (2)

So R1,200 + interest on R6,000 for 1 year = R1,260 + interest on R5,040 for for 1 year.

Or interest on R960 for 1 year = R 60

Or interest on R 100 for 1 year = $R\ \frac{60}{960} \times 100 = R\frac{100}{16} = 6\frac{1}{4}\%$

So rate of interest is $6\frac{1}{4}\%$.

Illustration - 34

The difference between banker's discount and true discount on a bill due after 9 months at 8% p.a. is R 60. Find i) banker's discount, ii) true discount, iii) Face value of the bill.

Solution

Given - Rate of interest (R) = 8% p.a., Time (T) = 9 months = $\frac{3}{4}$ year.

Banker's discount (BD) = True discount (TD) + R 60

Find bill value (BV) = ?

Let true discount = R x

So Banker's discount = R x + 60

$$BD = \frac{BV \times R \times T}{100} \text{ (Applying the formula)}$$

$R\ x + 60 = \frac{BV \times 8 \times \frac{3}{4}}{100}$ (substituting the value)

or $R\ x + 60 = R\frac{BV \times 6}{100}$

or $R\ x + 60 = R\frac{3BV}{50}$

or $R\ 50x + R\ 3{,}000 = R\ 3BV$ ...(1)

$TD = \frac{BV \times R \times T}{100 + (R \times T)}$ (Applying the formula)

$R\ x = \frac{BV \times 8 \times \frac{3}{4}}{100 + (8 \times \frac{3}{4})}$ (substituting the value)

or $R\ x = R\frac{BV \times 6}{100 + 6}$

or $R\ x = R\frac{3BV}{53}$

or $R\ 53x = R\ 3BV$ ...(2)

Subtracting equation (1) from 2

$R\ 53x = R\ 3BV$

$(-)R50x + R3{,}000 = R3BV$

$R3x = R3{,}000$

or $x = R1{,}000$

So True discount is R1,000

Banker's discount = $R\ x + 60$

= R1000 + 60

= R1,060

Substituting the value of x in equation 2

$R\ 53x = R\ 3BV$

or $53{,}000 = R\ 3BV$

or $BV = R\frac{5300}{3} = R17{,}667$

So Banker's discount, True discount and Bill value are R1,060, R1,000 and R17,667 respectively.

EXERCISE

Q.1. From the following alternatives, choose and write serially the correct answer along with serial number against each bit:

(i) A promissory note contains:

a) an unconditional promise

b) an unconditional undertaking

c) a conditional promise

d) a conditional undertaking

(ii) Out of the following, the one which is a valid promissory note, is :

a) I owe you A5,000.

b) I promise to pay the bearer A5000

c) I promise to pay A or order A5000

d) I promise to pay A A5000, if I will marry his sister.

(iii) Negotiable instrument Act includes the following:

a) Cheque

b) Share

c) debenture

d) share warrant

(iv) A bill was drawn on 1st Feb.2016 payable 2 months after date. The date of maturity is :

a) April 1, 2016

b) April 2, 2016

c) April 4, 2016

d) April 5, 2016

(v) A bill drawn on 1st March 2016, accepted on 10th March, 2016. It was payable 3 months after sight. The date of maturity is:

a) June 4, 2016

b) June 5, 2016

c) June 13, 2016

d) June 14, 2016

(vi) A bill dated 10th May, payable 60 days after date. The date of maturity is:

a) July 10

b) July 11

c) July 12

d) July 13

(vii) A bill dated July 1, payable 90 days after date. The date of maturity is :

a) Oct 1

b) Oct 2

c) September 30

d) Oct 3

(viii) The nominal due date of a bill drawn on 14th August, 2015, payable after 3 months is :

a) 14th November, 2015

b) 15th November, 2015

c) 16th November, 2015

d) 17th November, 2015

(ix) True discount is interest on :

a) bill value

b) present value

c) stock value

d) banker value

(x) Banker's discount is interest on

a) present value

b) stock value

c) bill value

d) face value

(xi) Banker's gain is

a) interest on present value

b) interest on bill value

c) interest on banker's discount

d) interest on true discount

(xii) Bill value is equal to

a) P.V. + B.D.

b) P.V. + T.D.

c) P.V. - B.D.

d) P.V. - T.D.

(xiii) Number of parties to a bill of exchange is normally:

a) 2

b) 3

c) 4

d) 5

(xiv) A bill of exchange is a

a) unconditional promise

b) unconditional order

c) conditional promise

d) conditional order

(xv) A promissory note is a

a) unconditional order

b) unconditional undertaking

c) conditional order

d) conditional undertaking

(xvi) The banker's discount on a bill of A5,000 for 3 months of 8% p.a. is :

a) A400

b) A200

c) A100

d) A150

(xvii) The true discount on a bill of A5,100 for 3 months at 8% p.a is:

a) A120

b) A110

c) A100

d) A105

(xviii) The present value of a bill of A1650 due 2 years hence at 5% p.a., is :

a) A150

b) A1500

c) A1400

d) A1550

(xix) The face value of a six months bill (when banker's discount at 6% p.a. is A60), is:

a) A2,000

b) A6,000

c) A1,500

d) A1,500

(xx) When Banker's Discount is A525 and true discount is A500, then banker's gain is:

a) A250

b) A25

c) A150

d) A2550

(xxi) If the Banker's discount and true discount on a certain bill is A165 and A145 respectively, the bill amount is :

a) A1595

b) A1600

c) A1800

d) A1695

(xxii) Bill value is equal to :

a) $\frac{\text{B.D.+T.D.}}{\text{B.D.-T.D.}}$

b) $\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.-T.D.}}$

c) $\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.+T.D.}}$

d) $\frac{\text{B.D.} - \text{T.D.}}{\text{B.D.+T.D.}}$

(xxiii) If the present worth and banker's gain on a certain bill is A600 and A6 respectively, the true discount is :

a) A50

b) A55

c) A60

d) A65

(xxiv) If the banker's discount on a certain bill due 2 years hence is $\frac{11}{10}$ of true discount, the rate of interest is:

a) 6%

b) 5%

c) 7%

d) 8%

(xxv) If the bill value is A11,000, present value is A10,000 and Banker's Discount is 1,100, then Banker's gain is:

(a) A100, (b) A120, (c) A150, (d) A180

ANSWER

Q.1. (i) b, (ii) c, (iii) a, (iv) c, (v) c, (vi) c, (vii) a, (viii) a, (ix) b, (x) c, (xi) d, (xii) b, (xiii) b, (xiv) b, (xv) b, (xvi) c, (xvii) c, (xviii) b, (xix) a, (xx) b, (xxi) a, (xxii) b, (xxiii) c, (xxiv) b, (xxv) a

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Q.2.

A. Correct the underlined portion of each of of the following sentences:

a) Banker's Discount is calculated on the present value of the bill of exchange.

b) Banker's gain is calculated as a percentage on Banker's Discount.

c) Bill of exchange is an unconditional promise to pay.

d) In case of after sight bill, the legal due date is calculated from the date of drawing of the bill of exchange.

e) Banker's gain is simple interest on banker's discount.

f) The person to whom bill is payable is Drawee.

g) If legal due date of the bill falls on Independence day, due date of payment will be 16th August.

h) Present value = Bill value - Banker's Discount

i) True discount = Interest on Bill value.

j) TD = $\sqrt{\text{Present value} \times \text{Banker's Discount}}$

B. Fill in the Blanks.

a) ---------- Discount is calculated on the present value of bill.

b) Banker's gain is calculated as a percentage on ---------- Discount.

c) Promissory note is an unconditional ---------- to pay.

d) In case of after date bill legal due date is calculated from the date of ---------- of the bill.

e) Discount value of the bill = Bill value minus ---------- discount.

f) Banker's gain = Banker's Discount minus ---------- discount.

g) B.D. is ---------- than T.D.

h) In a bill of exchange the drawee is ---------- and the drawer is ---------- .

i) ---------- days of grace are allowed to calculate legal due date of a bill.

j) ---------- Bill is immediately paid on presentation.

C. Express each of the following in one word/term :

(i) Simple interest in Bill value of a bill.

(ii) Simple interest on true Discount.

(iii) Days added to nominal due date to find out the legal due date.

(iv) Excess of Banker's discount over true discount.

(v) The bill in which nominal due date is calculated from the date of acceptance.

D. Answer the following questions within one sentence each:

a) What is a demand bill ?

b) What is after sight bill ?

c) What is meant by nominal due date ?

d) What is legal due date ?

e) What is meant by present value of a bill ?

f) Define true discount ?

g) Define banker's discount ?

h) What is meant by banker's gain ?

i) What is discounted value of bill ?

j) What do you mean by days of grace ?

(k) What is the present value of a bill of A11,000 due one year hence at 10% p.a. ?

(l) If B.D. R 60, T.D. R 50, find the Bill value.

(m) Write the formula for Banker's Discount.

ANSWERS

Q.2.

A. (a) Bill value or face value, (b) True, (c) order, (d) acceptance, (e) True, (f) Payee, (g) 14th, (h) True, (i) Present, (j) gain

B. (a) True, (b) True, (c) Promise or undertaking, (d) drawing, (e) Banker's, (f) True, (g) greater, (h) debtor, creditor, (i) Three, (j) Demand

C. (i) Banker's discount

(ii) Banker's gain

(iii) 3 days of grace

(iv) Banker's gain

(v) After sight Bill

D. (k) A10,000, (l) A300,

(m) $\frac{\text{Bill value} \times \text{Rate of interest} \times \text{Time}}{100}$

-o-

Very short answer questions

3. Explain the following terms within 2 sentences each.

i) Bill of exchange
ii) Promissory note
iii) Days of grace
iv) Nominal due date
v) Legal due date
vi) Bill value
vii) Present value
viii) Banker's discount
ix) True discount
x) Banker's gain
xi) Notary public
xii) Noting charges
xiii) After sight bill
xiv) After date bill

4. Answer within 2 sentences each.

a) Name of the parties in a bill of exchange.
b) Who is a promisor ?
c) What is discounted value of a bill ?
d) If BV = R 1,000, TD= R 200, What is present value ?
e) If BV= R 100, BD= R 20 Discount bill value is _____.
f) Calculate the bill value, when PV = R 500, TD = R 20.
g) If BD = R 62, BG = R 12, TD = ?
h) $\frac{BD \times TD}{BD - TD}$ = ?
i) If legal date of maturity is a bank holiday, when the payment will be made?
j) How one can know that the bill is accepted.
k) What is the face value of a 4 months bill when banker's discount at 4% per annum is R 20.

5. Short answer questions

Answer each of the following within 6 sentences :

a) A bill of R1,800 discounted 3 months before due date at 3%. Find banker's discount.
[CHSE 1993(S)]

b) A bill of R1,000 discounted 73 days before due date at 10% p.a. find banker's discount.
[CHSE 1993(A)]

c) A bill of R 2,200 discounted at 10% p.a. before one year. Find true discount.

d) Give a specimen of bill of exchange.

e) Draw a specimen of promissory note.

f) State 3 features of bill of exchange.

g) What is a sight bill ?

h) What is banker's gain ?

i) How notary public helps in case of dishonour of a bill ?

j) Find the banker's discount, if Bill value 1,000, Rate of interest 10%, Time 6 months ?

k) Bill value R11,000, Rate of interest 10%, Time 1 year. Find out true discount.

l) Find the face value of the bill, if banker's discount on a 4 months bill at 4% is R20

m) Find the difference between true discount and banker's discount of R1,950 due 9 months hence at $5\frac{1}{3}$% p.a.

Q.6 Find the date of maturity of the bill

i). Bill drawn on 10th January, 2016, payable 2months after date.

ii). Bill drawn on 30th November, 2015, payable 3months after date.

iii). Bill drawn on 23rd November, 2015, payable 2months after date.

iv). Bill drawn on 13th June, 2015, payable 2months after date.

v). Bill drawn on 1st July, 2015, payable 90 days after date.

vi). Bill drawn on 7th February, 2016, payable 3months after sight, accepted on 10th February 2016.

vii). Bill drawn on 27th June, 2016, payable 3months after date. (30th September was Bank holiday.)

viii). Bill drawn on 25th January, 2016 and accepted on 30th January, 2016 payable one month after sight.

ix). Bill drawn on 17th May, 2016, accepted on 20th may 2016, payable 45 days after sight.

x). Bill drawn on 5th June, 2016, payable 2months after date. (8th August was Sunday)

Answer to question - 4

a) Drawer, Drawee, Payee

d) R 800

e) R 80

f) R520

g) R 50

h) Bill value

i) Payment will be made on the day following the bank holiday.

k) R1,500

Answer to question- 5

a) R13.50 b) R20 c) R200 j) R 50

k) R1,000 i) R1,500 m) 3

Answer to questine- 6

i. 13th March, 2016 ii. 3rd March, 2016 iii. 25th January, 2016

iv. 16th August, 2015 v. 1st October, 2015 vi. 13th May, 2016

vii. 1st October, 2016 viii. 3rd March, 2016 ix. 7th July, 2016

x. 9th August, 2016

Long Answer Questions

SECTION -1

1 Find the true discount on a sum of R1,750 due in 18 months at 6% p.a.

[CHSE 1999(A)]

2. Find true discount on a bill for R 645 due one year six months hence at 5% simple interest. [CHSE 1992(A)]

3. What is the discount on R 250 due 2 years hence if interest is 12.5% ?

[CHSE 1986(A)]

4 Find Banker's discount on a bill for R1800 discounted 3months before due at 3% p.a.. [CHSE 1993(S)]

5. Find banker's discount on a bill for R1,000 discounted 73 days before due date at 10% p.a. [CHSE 1993]

6. A bill for R14,300 drawn on 27th April 2015 at eight months. It was discounted on 6th August 2015 at 5% p.a. How much did the banker give for the bill?

7. A bill for R5050 was drawn on 13th February, 2015 for 5months. It was discounted on 4th may 2015 at 5% p.a. Find

a) Banker's discount

b) True Discount

c) Banker's gain.

8. Express the difference between B.D. and T.D. on R1170 due in 4months of $6\frac{1}{4}$% S.I [CHSE 1993 (A), 2006 (A)]

9. Find the difference between true discount and banker's discount of R1950 due 9months hence at $5\frac{1}{3}$% per annum. [CHSE 1991(S), 2003 (A)]

10. At the rate of 4% p.a., find banker's discount, true discount and banker's gain on bill of exchange for R650 due 4 months hence. [CHSE 1993 (A)]

11. Find the difference between simple interest and true discount on R575 in 3 years at 5% p.a. [CHSE 1992 (S)]

12. At a rate of 9% p.a., find banker's discount, true discount and banker's gain on a bills of exchange for R2,000 due four months hence. [CHSE 2000 (A)]

13. Find the present value of R416 due 8months hence at 6% p.a. [CHSE 1988(S)]

14. Find the present value and true discount on a bill of R735 due in 6months hence at 10% p.a. [CHSE 1988(A)]

15. The true discount and banker's gain on a certain bill of exchange due after a certain period are R600 and R50 respectively, find the face value of the bill. [CHSE 1990]

16. The true discount and banker's discount on a certain bill is R70 and R80 respectively, Find the face value of the bill. [CHSE 1990]

17. The present worth of a certain bill due later on is R200. The true discount on the bill is R10. Find the banker's discount and the extra gain banker will make in the transaction.

18. A owe's B R9,810 payable 9 months hence at 12%. B owes A R8,320 payable 6 months hence at 8%. They want to settle their account forthwith by cash payment. Who will pay and how much?

19. Sonu owes Munu R6,000 payable 2 years hence at 4% p.a. Munu owes Sonu R4,000 payable 3 months hence at 10% p.a. If they want to settle their account immediately with cash, what amount is to be paid and by whom?

20. Rajesh offeres R7,000 for T.V. Mukesh offers R7,526 payable after 9 months. Which is better offer and how much money being recockened at 8% p.a.
21. Mantu offers R9,000 for a computer. Chintu offers R9,108 payable after 7 months. What is better offer and how much if money is reckoned at 6% p.a.
22. The present worth of a bill due sometimes hence is R3,240. The banker's gain on the same bill is R10. Find the bill value.
23. The present value of a bill due after certain period is R8,000 and the banker's gain on the same bill is R5. Find the bill value.
24. What sum of money paid in cash will discharge a debt of R5,000 in two equal annual instalments at 8% p.a. ?

    [Hint: Find the present value of R2,500, Time 1year and R 2,500, Time 2years]
25. What sum of money paid in cash will settle a debt of R4,000 payable in two equal half yearly instalment at 10% p.a.
26. Time watch co. sold wrist watch for R1,660 at 6 month's credit. The cost price was R1,500. Find the gain percentage if interest at $7\frac{1}{2}$ % p.a. is reckoned.
27. What is the actual rate of interest a banker gets for discounting a bill at 9% p.a. legally due after 3 months ?
28. A bill was drawn on 23rd march 2016 for 3 months. It was discounted with the banker on 14th April 2016 at $6\frac{1}{4}$ % p.a. Find the actual rate of interest earned by the banker.
29. Banker's discount on R2,250 at 4% p.a. is equal to true discount on R2295 for the same period and same time. Find the time
30. Distinguish between : (a) Bill of exchange and promissory note
    (b) Banker's discount and true discount
    (c) Nominal due date and legal due date

SECTION- 2

1. A bill for R12,600 was drawn payable sometimes hence was discounted in the bank at 5% p.a. The bank paid R12,474. Find the discount period.
2. The banker's discount on R 4,500 at 4% per annum is equal to true discount on

R4,590 for the same rate and time. Find the time.

3. The True discount on R 2,106 at 6% per annum is same as simple interest on R2,025 for the same rate and time. Find the time.
4. When is the sum due if the discount on R463.50 be R13.50 at 6% p.a. [CHSE - 1996 (A)]
5. When is the sum due if the true discount on R672 is R14.25 at $6\frac{1}{2}$ % p.a.
6. Find the rate of interest on a bill amounting R5,200, the true discount and time being R200 and 6 months respectively.
7. If the True discount on R1,696 due at the end of 9 months is R96, find the rate of interest.
8. Find the rate of interest when banker's discount on a bill of R1,440 due 5 months hence is R42.
9. The Banker's discount on a bill of R900 due 3 months hence is R18. Find the rate of interest.
10. The true discount on a bill due 6 months hence at 7% is R700. Find the bill amount and banker's gain.
11. The true discount on a bill due after 8 months at 9% is R300. Find the bill amount and banker's gain.
12. What is the face value of a 4 months bill, when banker's discount at 4% per annum is R20. [CHSE - 2001(A)]
13. What is the face value of a 18 months bill, the banker discount being R90 discounted at 6% p.a.
14. The banker's gain on a bill due 4 months hence at 9% p.a. is R9. Find the bill amount.
15. The difference between banker's discount and true discount on a certain bill for 9months at 5% is R2.70. Find the bill amount.
16. The true discount on a certain bill is $\frac{4}{5}$ th of banker's discount. The rate of interest is 10% p.a. Find the time period.

17. If the banker's gain is $\frac{1}{10}$ th of true discount and the rate of interest being 5%, find the unexpired period of the bill.

18. The banker's discount and true discount on a certain sum of money at 3% simple interest are respectively ₹350 and ₹325 for the same period. Find the sum and the period. [CHSE - 1998 (S)]

19. A bill for ₹6,300 drawn on 27th march, 2016. It was discounted at 5% and the banker paid ₹6,237. When it was discounted.

20. True discount on a sum of money is ₹72. The unexpired period is 8 months and the rate of interest is 9% p.a. Find i) sum, ii) banker's discount, iii) banker's gain.

21. The difference between true discount and banker's discount on a bill legally due after 5 months @ $4\frac{1}{2}$ % is ₹15.75, find the discount and bill value. [CHSE - 1990(S)]

    (Hints: Let BV. ₹100, find BG,

    When BG $\frac{1125}{32600}$ BV ₹100.

    When BG. ₹15.75 - BV? Find TD, BD)

22. The difference between banker's discount and true discount on a bill due after 6 months hence at 10% is ₹30. Find i) Bill value ii) Banker's discount iii) True discount.

23. The present worth of a bill due after 3years is ₹450. But if the bill will be due after 2 years present worth would be ₹500. Find the rate percent.

24. The present worth of a bill due at the end of 4 years is ₹6,500, if it will be due at the end of 3 years the present worth would be ₹500 more. Find the rate percent.

25. The present worth of a bill due at the end of 3 years is ₹5,500. If the same bill will be due after 4 years, present worth would be less by ₹300. Find interest fate percent.

26. The difference between banker's discount and true discount on a bill due after 4 months at 6% p.a. is ₹2, find i) banker's discount, ii) true discount, iii) bill value.

27. The banker's gain on a bill due after 4 months at 9% p.a. is ₹9, find i) banker's discount, ii) true discount, iii) bill value.

28. True discount is $\frac{21}{20}$ of Banker's discount. Find the period of discount if rate of intrest was 5% p.a.

$$\left[\text{Hints}: \frac{\frac{BV \times 5 \times T}{100}}{\frac{BV \times 5 \times T}{100+(5 \times T)}} = \frac{21}{20}, \frac{100+5T}{100} = \frac{21}{20}\right]$$

29. The present value of a bill due after wards is R100. The true discount is R5, Find the banker's discount and banker's gain.
(Hints: BV= PV+TD = 105, TD is 5% of PV, BD is 5% of R105 = R5.25)

30. A bill drawn on 5th August payable 3months after date was discounted on 27th August at 10% and the amount was R1,470. Find the Bill value and banker's discount.

Answers of Long Answer Question

Section-1

1). R144.50 2. R45 3.R62.5 4. R13.50 5. R20 6. R14,014 7. a) R50.50, b) R50 c) R0.50 8. R0.50 9. R3 10. BD R8.67, TD R8.55, BG R0.12 11 R11.25 12. BD-R60, TD-R58.25, BG-R1.75 13.- R40014) P.V. R700 T.D. R35 15) R7800 16) R560 17). Banker's discount R10.50, Banker's gain R0.50 18) B will pay R1,000 to A 19) Munu will pay R1,653.12 (20) Mukesh's offer is better by R100 (21) Mantu's offer is better by R200 (22) R3,420 (23) R8,200, (24)R4469.98 (25). R3,722.94 (26) $6\frac{2}{3}$%(27) 9.21% (28) 6.33% (29) 6 months

Section-2

(1) $\frac{1}{5}$ year (2) 6months (3) $\frac{2}{3}$ year (4) 6months (5) $\frac{1}{3}$ year (6) 8% (7) 8% (8) 7% (9)8%.(10)BV: R20,700,BG:R24.50(11)BV:R5,300,BG:18

(12) R1,500 (13) R1,000(14)R10,300 (15) R1,992 (16)$2\frac{1}{2}$ years(17)2years(18) Sum-R4,550, Period- $2\frac{22}{39}$ years(19) 18th Sept. 2008(20)i)- R1,272, ii)- R76.32, iii)-R4.32. (21) BV: R45,640, BD: R855.75, TD: R840 (22) i) BV: R12,600 ii) BD: R630 iii) TD: R600 (23). $14\frac{2}{7}$% (24). 10% (25) $6\frac{42}{43}$%(26) i) Banker's discount R102, ii) True discount R100, iii) Bill value R5,100 (27) i) BD: R309, ii) TD: R300, iii) BV: R10,300 (28) Time - 1 year (29) BD: R5.25, BG: R 0.25 (30). BV: R1,500, BD: R30]

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Chapter-7

STOCK AND SHARES

STRUCTURE

7.1. Introduction

7.2. Meaning of Joint Stock Company

7.3. Types of Shares

7.4. Capital of a Company

7.5. Stock

7.6. Commission on Stock Transactions

7.7. Stamp Duty and Tax

7.8 Illustrations

EXERCISE

7.1 INTRODUCTION

The industrial revolution during seventeenth century and eighteenth century gave birth to the philosophy of large scale production. Such large scale production required heavy capital investment. Large capital investment was followed by heavy risk. The sole proprietor and partnership form of business could not meet the requirement due to shortage of funds and unlimited liability. This handicap gave birth to company form of business organisation. In company form of business, capital is contributed by large number of persons known as share holders. Companies Act was passed in 1850 in India. A comprehensive Companies Act was passed in 1956. The principle of limited liability was introduced in 1857.

7.2. JOINT STOCK COMPANY

According to Companies Act, 1956, “A company means a company formed and registered under Companies Act. 1956”.

According to James Stephenson, a company is an association of many persons who contribute money or money’s worth to a common stock and employ it in some trade or business and who share the profit and loss arising there from.

A company is an association of persons known as share holders. They contribute money in shape of shares. It has legal existance with personal succession. The capital is divided in to freely transferable shares. The share holder has limited liability.

The companies are two types.

i) Private limited company

A private company is formed with minimum 2 members. But maximum share holders cannot exceed 50. Normally it is a family affair. Share holders are friends and relatives. No public subscription can be invited. Shares cannot be transferred without the consent of other share holders.

ii) Public limited company

A public limited company can be formed with minimum 7 share holders. There is no maximum limit. Company issue prospectus to the public inviting application to subscribe for shares. The shares are freely transferable.

7.3 TYPES OF SHARES

A company issues normally two types of shares. These are

i) Preference shares-

Preference shares are the shares having preference over payment of dividend and return of capital at the time of liquidation of company. Dividend at a fixed rate is paid on preference shares before payment to equity share holders.

ii) Equity shares

Equity share holders are the real owners of the company. They enjoy voting rights. They control the affairs of the company. Dividend is paid on equity shares after paying preference dividend. Capital is returned to equity share holders only after payment to preference share holders in the event of liquidation of company.

7.4 CAPITAL OF A COMPANY

A company can raise finance by issuing shares, debentures, accepting public deposits, borrowing from financial institutions. Broadly it can be divided in to a) share capital, b) borrowed capital.

A. Share capital

Money collected by the company by issuing shares to the public is known as share capital. The capital requirement of a company is not same at all the times. So the company instead of raising all the capital at one time, it raises as per its requirement. The capital of a company can take the following forms.

i) Authorised / Registered / Nominal capital- This is the maximum amount of capital which a company is authorised to issue to the public. For example – The maximum amount of capital of the company is fixed at 50 lakhs shares of R10 each. So, R 5 crores is the authorised share capital.

ii) Issued Capital- It is that part of the authorised capital which is offered to the public for subscription. In the above example if 40 lakh shares of R10 each offered to the public for subscription, the issued capital will be R4 crores.

iii) Subscribed Capital-Subscribed capital is the capital subscribed for by the public or taken up by the public or applied by the public. Continuing the above example, if public apply for 30 lakh shares, subscribed capital will be 3 crores rupees. The application for share may be more or less than the number of share offered to the public. If applications are more shares than issued, it is known as over subscription. If applications are less than issued, it is under subscription. But capital to be allotted to the public cannot be more than the issued capital.

iv) Called up Capital- It is that part of the subscribed capital (allotted capital i.e. capital actually allotted to the applicant) which is called up the company for payment. Continuing the above example, if the company calls for R 5 per share, called up capital will be 40 lakh x R 5.

v) Paid up Capital- The amount of capital actually paid by the share holders towards their share capital (excluding calls-in-advance) is known as paid up capital. Taking the aforesaid example, if shares holders have paid one crore ninty five lakhs rupees, the paid up capital will be one crore ninty five lakhs rupees.

vi) Reserve Capital- It is part of the subscribed capital which cannot be called up except at winding up of the company. This is done to create confidence in the minds of the creditors.

vii) Calls-in-Arrear or unpaid Capital- This refers to that part of the called up capital which the share holder failed to pay. Continuing the earlier example calls in arrear is five lakh rupees. Share holder is liable to pay 5% interest on calls-in-arrear.

viii) Calls-in-advance or uncalled Capital-This refers to that part of the capital which the share holders have paid in advance without being asked to pay. The share holder is entitle to get 6% interest on calls-in-advance.

B. Borrowed capital

A company may raise fund from the public by issuing debentures or bonds. These are known as borrowed capital or debt capital.

Debenture

A debenture is an acknowledgement of debt. According to Thomas Evelyn. "A debenture is a document under company's seal, which provides for payment of a principal sum and interest there on at regular intervals, which is usually secured by a fix or floating charge on the company's property or undertaking and which acknowledge a loan to the company" So a debenture holder is a creditor of the company. He is entitled for a fixed rate of interest irrespective of the profitability of the company.

7.5 Stock

Stock is the total value of fully paid up shares. It refers to unit of capital which is formed by conversion of fully paid shares. The stock holders are issued with stock certificate. They enjoy the priviledges of share holder including voting rights. Fully paid shares can be converted in to stock and vice versa.

Distinction between share and stock

• Shares are fully paid or partly paid.	• Stock is always fully paid.
• A Share has a nominal value or face value. It is transferred only at nominal value.	• A stock has no nominal value. It can be transferred in fraction.
• Shares are issued for public subscription.	• Stock cannot be issued for public subscription.

- A share is always assigned with a distinct number.
- Any company can issue any type of shares.
- A share is always registered. It can't be transferred by mere delivery

- A stock has no distinct number.
- Companies limited by share can only issue stock.
- A stock may or may not be registered. Unregistered stock can be transferred by mere delivery.

Stock value (SV) and cash value (CV) -

The price stated on face of the stock (i.e. share or debenture certificate) is called face value or nominal value or stock value. It can be issued at any value. However it is normally stated at R100 / R 10.

The price at which it is purchased or sold is called market value, quoated value or cash value. The stock value is always constant. But the cash value changes depending upon market conditions. It is to be noted that-

(i) Issued at par

Here cash value = Stock value (For example - A stock of R100 issued at R 100)

(ii) Issued at premium

When cash value > stock value, it is issued at premium or above par.

(Premium = C.V – S.V)

(For example - A stock of R 100 issued at R 110),

iii) Issued at Discount

When cash value < stock value, it is issued at discount or below par.

(Discount = SV – CV)

(For Example- A stock R100 is issued at R 90)

Dividend and yield

Dividend is the part of the profit distributed by the company among the share holders. It is declared as a percentage on the stock value or face value of share. It has no link with cash value. For example a company declared a dividend of 10%. A share holder holding 50 shares of R100 each at R120 will get will get a dividend of R500 $\left(\frac{10}{100} \times 5000\right)$.

Yield refers to actual rate of return on investment. In other words it is the income on 100 rupees investment. We can calculate yield by using the following formula.

$$\text{Yield rate} = \frac{\text{Income}}{\text{Cash value}} \times 100$$

In the above example,

$$\text{Yield} = \frac{500}{6000} \times 100 = 8.33\%$$

7.6 COMMISSION ON STOCK TRANSACTIONS :

7.6.1 Brokerage-

Shares, Stocks, debentures, bonds etc. are purchased or sold in a stock exchange only through a authorised broker. The broker charges some commission on each purchase and sale. This charge is known as brokerage or broker's commission. Brokerage is charged as a percentage on purchase / sale price (i.e. cash value) of share / stock. But for the convenience of calculation, we take it as a percentage on stock value. Brokerage is added to the purchase price and subtracted from the selling price to calculate actual cost or actual sales price as the case may be. For example R100 stock is sold for R120 $\left(\text{Brokerage } \frac{1}{10}\right)$. The purchaser will pay $R\left(120 + \frac{1}{10}\right)$. and the seller will receive $R\left(120 - \frac{1}{10}\right)$.

7.6.2 Contango

Here the commission on sale of share / stock is paid by the seller only. The buyer does not pay any commission on purchase of shares / stock to the broker. So the sale price is reduced. But the cost price is not affected. For example A sold 8% R10, 000 stock at 99. Contago is R1. The seller will get R98 per share. The buyer will pay R 99 per share.

7.6.3 Backwardation

Here the buyer arranges the broker since he wants to posess the shares. He bears the entire brokerage or commission payable to the broker. The seller will not pay any commission. Continuing the above example the buyer will pay R100 (R99 + 1) and seller will get R 99 per share.

7.7 STAMP DUTY AND TAX

7.7.1 Stamp duty -Stamp duty is paid by both the purchaser and seller of the stock. It should be charged as a percentage on market price / cash price of the stock. However, for the convinience of calculation it is given in absolute rupees term only.

7.7.2 Securities transaction tax

Both the purchaser and seller have to pay securities transaction tax for purchase and sell of stock. It is calculated as a percentage on market price of the stock. The percentage is around 0.2% of the market value. However for the convenience of calculation we have given in absolute rupees term only.

In addition to it both buyer and seller pay service tax and transaction charges.

7.7.3 Cum dividend and ex-dividend quotation

The problem of ex-dividend and cum dividend arises when a stock is sold just before the declaration of dividend.

Cum-dividend

Buyer purchase the stock just before declaration of dividend. It means dividend has become due but not received by the stock holder. In such case market price included accrued dividend. The price will be higher to that extent. The buyer will be receiving the dividend. When the cost price include the accrued dividend, it is called cum dividend price.

Cost price = purchase price - accrued dividend

Ex-dividend

In such case market price does not include such dividend. The seller will receive the accured dividend as and when due. So the buyer will pay only the normal cash value of the stock.

The purchase price = cost price

Cum-dividend = Ex-dividend + Accrued dividend

IMPORTANT POINTS

i) 5% stock at 102 means

a) R100 stock can be purchased / sold at R102

b) R100 stock will give a divided of R 5

c) R102 investment will earn R 5 every year.

2. Sold 6% stock at 105 means.

a) Stock value of R100, can be sold at R105

b) R100 stock will give a dividend of R6

3. 4% stock at 10% premium means (or 4% stock at premium 10 or 4% stock at R 110)

a) Stock value R100, cash value R110

b) R100 stock will give a dividend of R4

c) R110 investment will earn R4 per annum.

4. 7% stock at R 90 means (or 7% stock at 10 discount)

a) Stock value R100, cash value R 90

b) R100 stock will give a dividend of R 7

c) R 90 investment will earn R 7 per annum.

5. 10% stock at 110, brokerage $\frac{1}{10}$ means

a) Stock value R100

b) Purchase price – R 110 + $\frac{1}{10}$

c) Sales price = R 110 - $\frac{1}{10}$

d) R100 stock will give a dividend of R10

e) R 110 + Brokerage will earn R10 per annum.

7.8 ILLUSTRATIONS

I. Calculation of investment of a given stock.

Illustration -1

Find the cost of R4,000, 4% stock at 97.

Solution:

Given- Stock value R4,000

Cash Price of R97 Stock Price = R 100

When stock value (R)	Cash price (R)	
100	97	
4, 000	x	$x = R\frac{97}{100} \times 4,000 = R\ 3,880$

So, cost price of R 4,000 stock is R 3,880

Illustration - 2

Find the cost price of R 10,000, 5% stock at 110, brokarage being $\frac{1}{10}$

Solution

Given - Stock value R 10,000, Cost price of R 100 stock = R 110, Brokerage $\frac{1}{10}$

Cost of R 100 stock = R 110 + Brokerage $\frac{1}{10}$ = R $\frac{1101}{10}$

When stock value (R)	Cost price (R)	
100	$\frac{1101}{10}$	
10,000	x	$x = \frac{1101}{10} \times \frac{1}{100} \times 10,000 = R\ 11,010$

So, cost price of R 10.000 stock is R 11,010

Illustration - 3

Calculate the amount required to purchase R 20, 000, 9% stock at 30 premium, brokerage $\frac{1}{2}$,stamp duty $\frac{1}{8}$, securities transaction tax $\frac{1}{4}$.

Solution:

Given - Stock value to be purchased R 20, 000

Cash value R130 (R100 + 30 Premium)

Brokerage $\frac{1}{2}$, Stamp duty $\frac{1}{8}$,Securities transaction tax $\frac{1}{4}$

Purchase price of R100 stock is R130 + $\frac{1}{2}$ + $\frac{1}{8}$ + $\frac{1}{4}$ (i.e cash value + Brokerage + stamp

duty + securities transaction tax) = R 130 $\frac{7}{8}$ = R$\frac{1047}{8}$

When stock value	Purchase amount	
R	R	
100	$\frac{1047}{8}$	
20, 000	x	x = R$\frac{1047}{8}$ x $\frac{1}{100}$ x 20, 000
		= R 26, 175

So, R 26, 175 is required to purchase R 20, 000 stock.

Illustration - 4

Find the amount of investment required for R 20, 800 10% stock at 10 discount, brokerage $\frac{1}{10}$, stamp duty R15 in total.

Solution:

Given - Stock value R 20, 000

Cash value = R100 - discount R10 = R 90, Brokerage $\frac{1}{10}$, stamp duty R15

Cash value or market value of R100 is R 90 + $\frac{1}{10}$ = R$\frac{901}{10}$.

When stock value	Amount of investment	
R	R	
100	$\frac{901}{10}$	
20, 000	x	x = R$\frac{901}{10}$ x 20, 000 = R18, 200

Total required investment = R18, 200 + stamp duty R15 = R18, 215

So total required investment to purchase R 20, 000 stock is R18, 215.

II. Calculation of sale proceeds of a given quantity of stock.

Illustration - 5

How much money will be realised by selling R 20, 000, 10% stock at 20 premium. (Brokerage R1)?

Solution:

Given- Stock value R 20, 000

Selling price = R100 + Premium R 20 - Brokerage R1 = R119

When stock value	Price to be realised	
R	R	
100	119	
20, 000	x	$x = R\frac{119}{100} \times 20,000 = R\ 23,800$

So, R 23,800 will be realised by selling R 20, 000 stock.

Illustration - 6

Find the sales proceed of R 8, 000 consols at R 75, brokerage $\frac{1}{2}$, stamp duty $\frac{1}{4}$, securities transaction tax $\frac{1}{4}$.

Solution:

Given- Stock value R 8,000, Cash value of R100 stock = R 75

Brokerage $\frac{1}{2}$, stamp duty $\frac{1}{4}$, securities transaction tax $\frac{1}{4}$.

Sales proceed of R 100 stock = $R75 - R\left(\frac{1}{2}+\frac{1}{4}+\frac{1}{4}\right) = R\ 74$

When stock value (R)	Sales proceed (R)	
100	74	
8, 000	x	$x = R\frac{74}{100} \times 8,000 = R\ 5,920$

So the sales proceeds of R 8000 consols is R 5,920

Illustration - 7

Find the sales proceed of R 50, 000 stock at par, brokerage $\frac{1}{2}$, securities transaction tax $\frac{1}{10}$, service tax $\frac{1}{20}$.

Solution

Stock value Rs 50,000, Brokerage $\frac{1}{2}$, Securities transaction tax $\frac{1}{10}$, Service tax $\frac{1}{20}$

sales proceed of R100 stock = R100 - (Brokerage $\frac{1}{2}$ + securities transaction tax $\frac{1}{10}$ + service tax $\frac{1}{20}$)

$= R\ 99\frac{7}{20} = R\frac{1987}{20}$.

When stock value (R)	Sales proceed (R)	
100	$\frac{1987}{20}$	
50, 000	x	$x = R\frac{1987}{20} \times \frac{1}{100} \times 50,000 = R49,675$

So sales proceed of R50, 000 stock is R49, 675.

Illustration - 8

How much money will be realised by selling R 40, 000, $9\frac{3}{4}$% stock at 30 premium.

Brokerage $\frac{1}{2}$, securities transaction tax R 50, service tax R 25, stamp duty R12 in toto,

Solution:

Given - Stock value R 40, 000

Cash value of R100 stock = R100 + R 30 premium = R130

Brokerage $\frac{1}{2}$, Securities transaction tax R 50, Service tax R 25

Stamp duty R12

Selling price of R100 stock = R130 - brokerage $R\frac{1}{2}$ = $R129\frac{1}{2}$ = $R\frac{259}{2}$.

When stock value (R)	Selling price (R)	
100	$R\frac{259}{2}$	
40, 000	x	$x = R\frac{259}{2} \times \frac{1}{100} \times 40,000 = R\ 51,800$

Money realised = selling price - (securities transaction tax + service tax + stamp duty)

= R 51, 800 - R (50 + 25 + 12)

= R 51, 800 - R 87

= R 51, 713.

So, R 51, 713 will be realised by selling R 40, 000 stock.

III. Amount of stock that can be purchased with a given sum

Illustration - 9

How much stock can be bought by investing R10, 050 at par (Brokerage $\frac{1}{2}$) ?

Solution:

Given- Investment R10, 050

Cost of R100 stock at par, Brokerage $\frac{1}{2}$ = R100 + $\frac{1}{2}$ = $R\frac{201}{2}$.

When investment (R)	stock to be purchased (R)	
$\frac{201}{2}$	100	
10, 050	x	$x = R100 \times \frac{2}{201} = R10,000$

So, R10, 000 stock can be brought by investing R10, 050.

Illustration -10

What amount of stock can be purchased by investing R 21, 837 in 5% stock at

$108\frac{1}{2}$, brokerage $\frac{1}{2}$? In addition to it Government charges are securiting transaction tax R 23, service tax R12, stamp duty R 2 payable.

Solution:

Given- Investment R 21, 837

Cost of R 100 stock R $108\frac{1}{2}$ + brokerage $\frac{1}{2}$ = R109

Government dues - Securities transaction tax : R 23, service tax: R12, Stamp duty : R 2

So, amount available for purchase of stock R 21, 837 - (R 23 + R12 + R2)

= R21, 800

When investment (R)	Stock to be purchased (R)	
109	100	
21, 800	x	$x = R\frac{100}{109} \times 21,800 = R\ 20,000$

So, R 20, 000 stock can be purchased by investing R 21, 837.

(iv) Amount of stock to be sold to get required sum of money

Illustration-11

How much 8% stock (brokerage being $\frac{1}{2}$) at par be sold to realise R17, 878, securities transanction tax R 20, service tax R10, stamp duty R 2.

Solution:

Given- Amount to be realised R17, 878

Amount to be realised on sale of R100 stock at par

= R100 - $\frac{1}{2}$ = R 99.5 = $R\frac{199}{2}$

Taxes- Securities transaction tax R 20, service tax R10, stamp duty R2

Total taxes R(20 + 10 + 2) of R32

Amount to be realised before payament of taxes

= R17, 878 + R 32 = R17, 910

When amount to be realised (R)	Stock to be sold (R)	
$\frac{199}{2}$	R100	
17, 910	x	$x = R100 \times \frac{2}{199} \times 17,910 = R18,000$

So, R18, 000 stock is to be sold to realise R17, 878

Illustration - 12

How much 8% consol at 109 be sold to realise R27, 125 $\left(\text{Brokerage } \frac{1}{2}\right)$?

Solution:

Given- Amount to be realised R 27, 125

Selling price of R100 consol R109, Brokerage $\frac{1}{2}$

So price to be realised on sale R100 consol R109 - $\frac{1}{2}$ = R108.5 = R $\frac{217}{2}$

When price to realise (R)	Value of consol to be sold (R)	
$\frac{217}{2}$	100	
27, 125	x	$x = R100 \times \frac{2}{217} \times 27,125$
		= R25, 000.

So, R 25, 000 stock (consol) are to be sold to realise R 27,125

Illustration - 13

How much stock at $87\frac{2}{5}$ be sold to realise R14,790(Brokerage $\frac{2}{5}$) ?

Solution

Given : Amount to be realise R 14,790, Selling price of R 100 stock R $87\frac{2}{5}$, Broker-

age $\frac{2}{5}$

Price to be realised by selling R 100 stock = R 87 $\frac{2}{5}$ - Brokerage $\frac{2}{5}$ = R 87

When price to be realised (R)	Stock to be sold (R)	
87	100	
14,790	x	x = R $\frac{100}{87}$ x 14,790 = R 17,000

So R 17,000 stock to be sold to realise R 14,790

Illustration -14

A man sells R12, 000, 8% stock at $120\frac{1}{4}$ and invests the proceed in 6% stock at $71\frac{3}{4}$. Find the amount of new stock purchased, brokerage being in both the cases $\frac{1}{4}$.

Solution:

Given: Sale of R 12, 000 8% stock at $120\frac{1}{4}$

Purchase of stock with the sales proceeds at $71\frac{3}{4}$,Brokerage in both the cases $\frac{1}{4}$

Price realised on sale of R100 stock = selling price - brokerage

= R$120\frac{1}{4}$ - R$\frac{1}{4}$. = R120

When stock value R	Price realised R	
100	120	
12, 000	x	x = R$\frac{120}{100}$ x 12, 000 = R14, 400

Purchase price of R100 stock at 71 $\frac{3}{4}$, Brokerage $\frac{1}{4}$

= R $71\frac{3}{4} + \frac{1}{4}$ = R 72

When purchase Price (R)	New stock purchased (R)	
72	100	
14, 400	x	$x = R\frac{100}{72} \times 14,400 = R\ 20,000$

So, amount of new stock purchased is R 20, 000

V. Calculation of profit or loss on purchase and sale of stock

Illustration -15

A person buys R 4, 500 stock at $93\frac{3}{4}$ and sells when the price rises to $97\frac{1}{4}$. Brokerage $\frac{1}{4}$ in both the cases. Find the profit.

Solution:

Given- R 4, 500 stock purchased at $93\frac{3}{4}$

The same stock sold at $97\frac{1}{4}$, Brokerage in both cases $\frac{1}{4}$

Purchase price of R100 stock = R $93\frac{3}{4}$ + Brokerage $\frac{1}{4}$ = R 94.

When stock value (R)	Purchase price (R)	
100	94	
4, 500	x	$x = R\frac{94}{100} \times 4,500 = R\ 4,230$

Selling price of R100 stock = R $97\frac{1}{4}$ - Brokerage $\frac{1}{4}$ = R 97

When stock value R	Selling price R	
100	97	
4, 500	x	$x = R\frac{97}{100} \times 4,500 = R\ 4,365$

Profit = selling price - purchase price

= R 4, 365 - R 4, 230

= R 135

So the profit on purchase and sale of R 4, 500 stock is R 135.

Illuistration-16

A man invests R 2,070 in 6% stock at 115 and sells it when price rises to R$119\frac{1}{2}$.

Find the gain.

Solution:

Given: Investment in 6% stock R 2,070

Purchase price of R100 stock R115

Selling price of R 100 stock R $119\frac{1}{2}$

Profit on R100 stock = Selling Price - Purchase Price

$$= R\ 119\frac{1}{2} - R115 \qquad = R4\frac{1}{2} = R\frac{9}{2}$$

When Investment (R)	Profit (R)	
115	$\frac{9}{2}$	
2,070	x	$x = R\frac{9}{2} \times \frac{1}{115} \times 2070 = R81$

So, Profit on purchase and sale of stock on an investment of R2,070 is R81

Illustration -17

A man buys R22, 500 stock at par. He sold the stock at R10 below par. Find the loss.

Solution:

Given- Purchased R22,500 stock at par.

Sold these stock at R 10 below par.

So selling price is R100 - 10 = R90 for a stock of R100

Loss for R100 stock = R100 - 90 = R10

When stock value (R)	Loss (R)	
100	10	
22, 500	x	$x = R\frac{10}{100}$ x 22, 500 = R2, 250

So loss suffered on purchase and sale of R22, 500 stock is R2, 250.

VI. Finding out amount of stock when the profit or loss is given.

Illustration-18

Mr. x invested a certain sum of money in 4% stock at $94\frac{1}{2}$. He sold them when the price rose to $99\frac{3}{4}$. He makes a gain of R85. Find i) amount of his investment ii) amount of stock bought by him. (Brokerage $\frac{1}{2}$ in both the ways.)

Solution:

Given- Purchase price $R94\frac{1}{2}$, Selling price $R99\frac{3}{4}$,

Brokerage $\frac{1}{2}$ on both purchase and sell, Profit on purchase and sale R 85

Purchase price of R100 stock = $R94\frac{1}{2}$ + Brokerage $\frac{1}{2}$ = R95

Selling price of R100 stock = $R99\frac{3}{4}$ - brokerage $R\frac{1}{2}$ = $R99\frac{1}{4}$

Profit on purchase and sale of R100 stock = $R99\frac{1}{4}$ - R95 = $R4\frac{1}{4}$ = $R\frac{17}{4}$

When profit (R)	Stock value (R)	
$\frac{17}{4}$	100	
85	x	x = R100 x $\frac{4}{17}$ x 85 = R2, 000

When stock purchased R	Investment R	
100	95	
2, 000	x	$x = R\frac{95}{100}$ x 2, 000 = R1, 900

So, investment of Mr. x is R1900 and the stock bought by him is R2, 000.

Illustration -19

A sold some stock and B brought it. Each paid a brokerage of $\frac{2}{5}$ p.c. If the broker earns R40, what is the amount of stock dealt with ?

Solution:

Given: Brokerage paid by both buyer and seller is $\frac{2}{5}$ %

Total brokerage earned (Both from buyer and seller) = R$\frac{2}{5}$ + R$\frac{2}{5}$ = R$\frac{4}{5}$

When brokerage earned (R)	Stock dealt (R)	
$\frac{4}{5}$	100	
40	x	x = R100 x $\frac{5}{4}$ x 40 = R5, 000

So the amount of stock dealt is R5, 000.

VII. To find the income from a specific quantity of stock or investment.

Illustration - 20

What annual income will be derived from

a) R2, 000 of $7\frac{1}{2}$ % stock at 90? b) R2, 000 of $7\frac{1}{2}$ % stock at 100?

c) R2, 000 of $7\frac{1}{2}$ % stock at 110?

Solution:

Calculation of income in all the cases will be same since stock values are same in three cases.

Given- Stock value = R2, 000, Dividend R $7\frac{1}{2}$ %

When stock value (R)	Income (R)	
100	$7\frac{1}{2} = \frac{15}{2}$	
2, 000	x	$x = R\frac{15}{2} \times \frac{1}{100} \times 2,000 = R150$

The income from R2, 000 stock is R150.

Illustration -21

Find the annual income from the following investments.

a) R9, 900 in 8% stock at 90. b) R9, 900 in 8% stock at par.

c) R9, 900 in 8% stock at 110, income tax being 10 paise in a rupee.

Solution:

a) Given - Investment R9, 900

Cash value of R100 stock = R90

Rate of dividend 8%

When cash value R	Income R	
90	8	
9900	x	$x = \frac{8}{90} \times 9900 = 880$

So the income by investing R9, 900 in 8% stock is R880

b)

Given: Investment = R9, 900

Cash value of R100 stock = R100, Rate of dividend 8%

When cash value R	Dividend R	
100	8	
9900	x	$x = R\frac{8}{100} \times 9,900 = R792$

So, the income by investing R9, 900 in 8% stock is R792.

c)

Given:Investment R9, 900, Cash value R110, stock value R100,Rate of dividend 8%

When cash value / Investment	Income	
R	R	
110	8	
9900	x	$x = R\frac{8}{110} \times 9900 = R720$

So the income by investment of R9, 900 in 8% stock is R720.

N.B.: We can also solve these problems first by calculating stock value and then the amount of dividend In example (c)

Investment	SV
110	100
1	$\frac{100}{100}$
9900	$\frac{10}{11} \times 9900 = 9000$

Dividen = R 9000 x $\frac{8}{100}$ = 720

Viii. To find rate of income or yield from a given investment

Illustration - 22

What rate percent can one earn by investing in 6 percent at 119. (Brokerage 1)

Solution:

Investment on R100 stock = Purchase price + Brokerage

= R119 + 1 = R120

When Investment	Income	
R	R	
120	6	
100	x	$x = R\frac{6}{120} \times 100 = 5\%$

Hence, rate of income on investment is 5%

Illustration - 23

A man invested R9, 600 in $7\frac{1}{2}$ stock at 119 (Brokerage 1). The income tax is 20%. Find the rate of return on investment.

Solution:

Investment on R100 stock = Purchase price + Brokerage

= R119 + R = R120

When investment	Income
R	R
120	$\frac{15}{2}$
9, 600	x

$x = R\frac{15}{2} \times \frac{1}{120} \times 9,600 = R600$

When Income	Income Tax	Net Income
R	R	R
100	20	80
600		x

$x = R\frac{80}{100} \times 600 = R480$

When investment	Net Income
R	R
9, 600	480
100	x

$x = R\frac{480}{9,600} \times 100 = 5\%$

So rate of return on investment is 5%

Illustration - 25

A man invests certain sum of money in 15% R100 shares at 10% premium. His annual dividend is R3, 000. Find the rate of return and the investment.

Solution:

Given- Purchase price of R100 share = R100 + Premium R10 = R110.

When dividend	Investment
R	R
15	110
3, 000	x

$x = R\frac{110}{15} \times 3,000 = R22,000$

When investment	Annual dividend	
R	R	
22, 000	3, 000	
100	x	$x = R\frac{3,000}{22,000} \times 100 = 13\frac{7}{11}\%$

So rate of return and investment are $13\frac{7}{11}\%$ and R22, 000 respectively.

IX. To find markete value / investment of the given yield.

Illustration - 25

Find the market value of 5% stock when the investment of R 2,500 gives an annual income of R150.

Given: Investment = R 2,500, Annual Income = R150, Rate of Interest on stock value 5%

When income	Investment	
R	R	
150	2500	
5	x	$x = R\frac{2500}{150} \times 5 = 83\frac{1}{3}$

So market value of 5% stock is $R83\frac{1}{3}$

Illustration - 26

Find the quotation price of 6% stock, when an investment of R5,560 yields a monthly income of R30. (Brokerage $\frac{2}{3}$)

Solution:

Given: Investment = R5, 560, Monthly Income = R30,

Rate of interest - 6% on stock value, Brokerage $\frac{2}{3}$

Annual income = R30 x 12 = R360

When income	Investment	
R	R	
360	5560	
6	x	$x = R\frac{5560}{360} \times 6 = R92\frac{2}{3}$

We will get the quotation price after subtracting the brokerage from the investment for R100 stock.

So, quotation price = $R\ 92\frac{2}{3} - \frac{2}{3} = R\ 92$

Quotation price of 6% stcok is R 92

X. To find better investment in given stocks.

Illustration - 27

Which is better investment ?

4% stock at 80 or 5% stock at 105 [CHSE - 1993 (A), 1986 (A)]

Solution:

Given- Purchase price of 4% stock is R80,

Purchase price of 5% stock is R105

Let the investment, in each case be R1, 680 (LCM of R 80 and 105)

1st case

When investment	Income	
R	R	
80	4	
1680	x	$x = R\frac{4}{80} \times 1680 = 84$

2nd case

When investment	Income	
R	R	
105	5	
1680	x	$x = R\frac{5}{105} \times 1680 = R80$

So income in the 1st case is better than the income in the 2nd case. Hence investment in 4% stock is better.

Alternatively,

We have find out the yield rate to find out better investment.

$$\text{Yield rate} = \frac{\text{Income from a stock}}{\text{Investment in a stock}} \times 100$$

Case- I Given stock value R100, Cash value R 80, Dividend 4%

$$\text{Yield} = \frac{4}{80} \times 100 = 5\%$$

Case II Given stock value R 100, Cash value R 105, Dividend 5%

$$\text{Yield} = \frac{5}{105} \times 100 = 4\frac{16}{21}\%$$

Yield from Ist investment is more than that from the 2nd one. Hence 4% stock at 80 is better investment.

Illustration - 28

Which is better investment, 8% stock at $89\frac{1}{2}$ or 9% stock at $109\frac{1}{2}$ (Brokerage $\frac{1}{2}$ in each case)

Solution:

Given- 1st case - Cost price of R100 stock is $R89\frac{1}{2}$ and income R8

2nd case - Cost price of R100 stock is $R109\frac{1}{2}$ and income R 9.

Brokerage in both the cases is $R\frac{1}{2}$

Let the investment in each case = R 990

1st case

Investment in R100 stock = Cost price + brokerage

$$= R\left(89\frac{1}{2} + \frac{1}{2}\right) = R90$$

When investment	Income	
R	R	
90	8	
990	x	$x = R\frac{8}{90} \times 990 = R88$

2nd Case

Investment in R100 stock = cost price + brokerage

$$= R\left(109\frac{1}{2} + \frac{1}{2}\right) = R110$$

When investment	Income	
R	R	
110	9	
990	x	$x = R\frac{9}{110} \times 990 = R81$

So income from 8% stock is more than the income in 9% stock. Hence, investment in 8% stock is better.

Illustration - 29

Which is more profitable- 6% stock at 90 or 9% stock at 20% premium ? How much is invested in each case when the difference in income is R903 ? (Brokerrage $\frac{1}{10}$)

Solution:

Given: Cost price of 6% stock is R90, Cost price of 9% stock is R120

Brokerage $\frac{1}{10}$ in each case.

Investment value of 6% R100 stock = $R90 + \frac{1}{10} = R\frac{901}{10}$

Investment value of 9% R100 stock = $R120 + \frac{1}{10} = R\frac{1201}{10}$

Let the investment in each case = $R\frac{901}{10} \times \frac{1201}{10}$ (LCM of both the investments)

1st case

When investment	Income	
R	R	
$\frac{901}{10}$	6	
$\frac{901}{10} \times \frac{1201}{10}$	x	$x = R6 \times \frac{10}{901} \times \frac{901 \times 1201}{10 \times 10} = R720.60$

2nd Case

When investment	Income	
R	R	
$\frac{1201}{10}$	9	
$\frac{901}{10} \times \frac{1201}{10}$	x	$x = R9 \times \frac{10}{1201} \times \frac{901 \times 1201}{10 \times 10} = R810.90$

So Investment is 9% stock is more profitable .

So, difference between the two income is R(810.90 - 720.60) = R90.30

When the difference in income	investment	
R	R	
90.30 or $\frac{903}{10}$	$\frac{901}{10} \times \frac{1201}{10}$	
903	x	$x = R\frac{901 \times 1201}{10 \times 10} \times \frac{10}{903} \times 903$
		= R1,08,210.10

Hence, investment in each case is R1,08,210.10.

XI. To find change in income for transfer of investment from one kind of stock to another.

Illustration - 30

Find the change in income of Amrit who transfers R5, 400 of $10\frac{1}{2}$ stock at par to a 10% stock at a discount of 10%.

Solution

Given: Amount of stock held is R5, 400

Dividend $10\frac{1}{2}$ %, Sold - at par, New stock - 10% stock at 10% discount.

Income from 1st stock

When stock value	Income
R	R
100	$10\frac{1}{2} = \frac{21}{2}$
5400	$\frac{21}{2} \times \frac{1}{100} \times 5400 =$ R567

1st stock is sold at par. Hence sale price is R5400.

Income from the 2nd stock.

When investment	Income	
R	R	
90	10	
5400	x	$x =$ R $\frac{10}{90} \times 5400 =$ R600

So change in income = (R600 - 567) increase = R33 increase.

Illustration - 31

Mr. Janardan sold R5, 000 of $5\frac{1}{2}$ % stock at 126. He invested the sales proceed in 3% stock at 61.50. How much new stock did he buy and what was the change in his income? (Brokerage R1 in both the ways.)

Solution:

Income from 1st stock

When stock value	Income	
R	R	
100	$5\frac{1}{2} = \frac{11}{2}$	
5, 000	x	$x =$ R$\frac{11}{2} \times \frac{1}{100} \times 5,000 =$ R275

Selling price of 1st stock

When stock value	Selling price	Brokerage	Price realised	
R	R	R	R	
100	126	1	125	
5, 000			x	$x = R\frac{125}{100} \times 5000$
				= R6, 250

Purchase of 2nd stock

When investmet	Stock purchased	
R	R	
Cash price R61.50 + R1 Brokerage = R62.50	100	
6,250	x	$x = R\frac{100}{62.50} \times 6, 250 = R10, 000$

Income from the 2nd stock

When stock vlaue (R)	Income(R)	
100	3	
10, 000	x	$x = R\frac{3}{100} \times 10, 000 = R300$

Change in income R(300 - 275) = R25 increase. So value of new stock purchased is R10,000 and increase in income is R25.

Illustration- 32

Chandan invested R7, 500 in 5% stock at 124.50. He sold the same at R132.50. He purchased 2.5% govt bond at 78.70 with the money realised. Find the change in income and the amount of new stock purchased. (Brokerage Re.0.50 in both the way i.e. in all 3 cases).

Solution:

Given: 1st investment R7, 500 in 5% stock at 124.50

Selling price of 1st investment is R132.50

Purchase of 2.5% govt. bonds at 78.70 with the price realised.

To find - (1) Amount of new stock, (2) Change in income.

Income from the 1st stock

When Investment	Income	
R	R	
124.50 + 0.50 = 125	5	
7, 500	x	$x = R\frac{5}{125} \times 7,500 = R300$

Sale proceeds of 1st stock

When Investment	Sales proceed realissed	
R	R	
125	132.50 - 0.50 = 132	
7, 500	x	$x = R\frac{132}{125} \times 7,500 = R7,920$

Purchase of 2nd stock

When Investment	Stock purchased	
R	R	
78.70 + 0.50 = 79.20	100	
7920	x	$x = R\frac{100}{79.20} \times 7920 = R10,000$

2nd Income

When stock value	Income	
R	R	
100	2.5	
10, 000	x	$x = R\frac{2.5}{100} \times 10,000 = R250$

Change in income R(300 - 250) = R50 decrease.

So amount of new stock purchased is R10, 000 and decrease in income is R50.

XII. To find the market value of the stock given the change in income.

Illustration - 33

Sailendra invested R12, 900 in 4% stock at 86. He sold the same at a price of 92. The sales proceed was invested in 5% stock. As a result his income was increased by

R90. At what price did he buy the later stock ?

Solution

Given- Investment R12, 900 in 4% stock at 86, Selling price of the stock is 92

By investing the sales proceed in 5% stock, income increased by R90

To find market price of 2nd stock?

1st Income

When Investment	Income	
R	R	
86	4	
12, 900	x	$x = R\frac{4}{86} \times 12,900 = R600$

Sales Proceed of the 1st stock

When purchase price of 1st stock	Selling price	
R	R	
86	92	
12, 900	x	$x = R\frac{92}{86} \times 12,900 = R13,800$

Income from 2nd stock = R(600+90) = R690

Market Price / Purchase price of 2nd stock

When Income	Investment	
R	R	
690	13, 800	
5	x	$x = R\frac{13,800}{690} \times 5 = R100$

So market price or quotation price of 2nd stock is R100 or at par.

Illustration - 34

Ashok holds R12, 000 of $4\frac{1}{2}$% stock and R8, 000 of $5\frac{1}{2}$% stock. He sold the stocks at R120 and R140 respectively. He invested the proceeds at $5\frac{1}{2}$% consols and his income increased by R44. At what price did he buy the new stock?

Solution:

Given: R12, 000 of $4\frac{1}{2}$% stock and R800 of stock $5\frac{1}{2}$% in hand

Sold R100 stock at R120 and R140 respectively.

Sales proceeds invested in $5\frac{1}{2}$% consols.

Income increased by R44

To find market price of new stock ?

Income from original stocks

	When stock value	Income	
	R	R	
1st stock -	100	$4\frac{1}{2} = \frac{9}{2}$	
	12, 000	x	$x = R\frac{9}{2} \times \frac{1}{100} \times 12,000 = R540$
2nd stock -	100	$5\frac{1}{2} = \frac{11}{2}$	
	8, 000	x	$x = R\frac{11}{2} \times \frac{1}{100} \times 8,000 = R440$

Total income from both the stocks = R(540 +440) = R 980

Income from new stock R(980 +44) = R 1024.

Calculation of sales proceeds of the stocks

	When stock value	Sale Proceeds	
	(R)	(R)	
1st stock -	100	120	
	12000	x	$x = R\frac{120}{100} \times 12,000 = R14,400$
2nd Stock	100	140	
	8,000	x	$x = \frac{140}{100} \times 8000 = R11200$

Total sales proceeds = R (14,400 + 11,200) = R 25,600

Market Price/ Quotation of new stock/ consol

When Income (R)	Investment (R)	
1024	25,600	
$5\frac{1}{5} = \frac{21}{5}$	x	$x = R\frac{25600}{1024} \times \frac{21}{5} = Rs\ 105$

So market price of new stock is R 105

XIII. To find the investment in various stocks given the capital.

Illustration - 35

A man invested R27, 500, partly in 3% stock at 78 and partly at 4% stock at 116. His income from both the investment is equal. What amount did he invest in each and what is his total income?

Solution

1st method

	When Income R	Investment R	
1st case- 3	78		
	1	x	$x = R\frac{78}{3} = R26$
2nd case- 4	116		
	1	x	$x = R\frac{116}{4} = R29$

So the ratio of investment = 26 : 29

Total of 26 and 29 = 55

$\therefore$ Investment in 3% stock at 78 = $\frac{26}{55}$ x 27, 500 = R13, 000

Investment in 4% stock at 116 = $\frac{29}{55}$ x 27,500 = R14, 500

	When Investment R	Income R	
1st case -	78	3	
	13, 000	x	$x = R\frac{3}{78}$ x 13, 000 = R500

2nd case-	116	4	
	14, 500	x	$x = R\frac{4}{116} \times 14{,}500 = R500$

Total income = R500 + 500 = R1000

Investment in 3% and 4% stock are R13, 000 and R14, 500 respectively and total income is R1,000.

2nd method

Let the Investment in 3% stock = x rupees

So investment in 4% stock = R(27, 500 - x)

	When Investment	Income	
	R	R	
1st case -	78	3	
	x	?	Income = $R\frac{3}{78}$ (x) = $R\frac{x}{26}$
2nd case	116	4	
	27, 500 - x	?	Income= $R\frac{4}{116}$ (27, 500 - x) = $R\frac{27,500-x}{29}$

As per the question the income from both the investments equal.

$\therefore\ R\frac{x}{26} = R\frac{27,500-x}{29}$

or 29x= R 27, 500 x 26 - 26x

or 29x + 26x = R 27, 500 x 26

or $x = R\frac{27,500 \times 26}{55}$ = R13, 000

Investment in 4% stock = R27, 500 - x

= R27, 500 - R13, 000

= R14, 500

	When Investiment	Income	
	R	R	
1st case-	78	3	
	13, 000	?	Income = $R\frac{3}{78} \times 13{,}000 = R500$

2nd case -	116	4	
	14, 500	?	Income = R$\frac{4}{116}$ x 14, 500 = R500

Total Income = R(500 + 500) = R1, 000

So Investment in 3% and 4% stock are R13,000 and R 14,500 respectively and total income is R1,000.

Illustration - 36

A person invests R31, 050 partly in 5% stock at 90 and partly in 7% stock at 115. Total income from both the investment is R1, 835. What is his investment in each ?

Solution:

Given- Total Investment in stocks is R31, 050,

Investment is partly in 5% stock and partly in 7% stock. Total Income is R1,835.

To find : Investment in each case ?

1st method

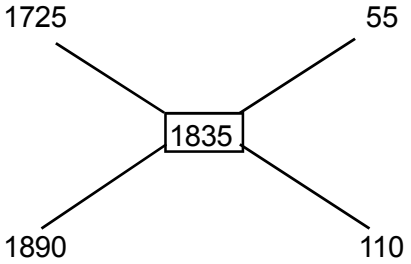
Total Income from total investment

	When investment	Income	
	R	R	
1st case-	90	5	
	31, 050	x	x = R$\frac{5}{90}$ x 31, 050 = R1, 725
2nd case-	115	7	
	31, 050	x	x = R$\frac{7}{115}$ x 31, 050 = R1, 890

But his actual income is R1, 835

Applying Alligation rule

Total income Difference



Thus ratio = 55 : 110 = 1 : 2

So investment in 5% stock = R31, 050 x $\frac{1}{3}$ = R10, 350

investment in 7% stock = R31, 050 x $\frac{2}{3}$ = R20, 700

So, investment in 5% and 7% stock is R10, 350 and R20, 700 respectively.

2nd method

Let the investment in 5% stock be Rx

So investment in 7% stock is R(31, 050 - x)

	When investment	Income	
	R	R	
1st case -	90	5	
	x	?	? = R$\frac{5}{90}$(x) = R$\frac{x}{18}$
2nd case -	115	7	
	31,050 - x	?	x = R $\frac{7}{115}$ (31, 050 - x)

As per question the total income from both the investments in R 1,835.

$\therefore$ R$\frac{x}{18}$ + R$\frac{7}{115}$ (31, 050 - x) = R1835

or $\frac{x}{18} + \frac{31,050 \text{ x } 7 - 7x}{115}$ = R1835

or $\frac{115x + 31,050 \times 7 \times 18 - 126x}{115 \text{ x } 18}$ = R1835

or $\frac{-11x}{115 \text{ x } 18} + \frac{31,050 \text{ x } 7 \text{ x } 18}{115 \text{ x } 18}$ = R1835

or $\frac{-11x}{115 \text{ x } 18}$ + 270 x 7 = R1835

or $\frac{-11x}{115 \text{ x } 18}$ = R1835 - R1890 = -R55

or x = R5 x 115 x 18 = R10, 350

Investment in 2nd stock is R(31, 050 - 10, 350) = R20, 700

So investment in 5% and 7% stock are R10, 350 and R 20, 700 respectively.

Illustration- 37

A person invested R93, 000 partly in 6% stock at 90 and partly in 10% stock at 120. What is his invesment in each stock, if the income from 2nd investment is R1, 000 more than that of 1st investment?

Solution:

Given: Total investment is R93, 000

Investment is partly in 6% at 90 and partly in 10% stock at 120.

Income from 2nd investment is R1,000 more than that of 1st investment.

Let the investment in 6% stock be x rupees. So investment in 10% stock is R(93,000-x)

Income from investment

	When investment	Income	
	R	R	
1st case	90	6	
	x	?	Income = $R\frac{6}{90}(x) = R\frac{x}{15}$
2nd case	120	10	
	(93, 000 - x)	?	Income = $R\frac{10}{120}(93,000 - x) = R\frac{93,000 - x}{12}$

As per qustion the income from 10% stock is R1,000 more than the income from 6% stock.

$$\therefore \; R\frac{93,000 - x}{12} - \frac{x}{15} = R1,000$$

$$\Rightarrow R\frac{4,65,000 - 5x - 4x}{60} = R1,000$$

$$\Rightarrow R\frac{4,65,000 - 9x}{60} = R1,000$$

$$\Rightarrow R\,(4,65,000 - 60,000) = 9x$$

$\Rightarrow$ x = R$\frac{4{,}05{,}000}{9}$ = R45,000

So investment in other stock / 2nd investment = R(93, 500 - 45, 000) = R 48, 000

So investment in 6% and 10% the stock are R45, 000 and R 48, 000 respectively.

Illustration - 38

Naresh invests R27, 000 partly in 6% stock at 90 (Income tax 10%) and balance is 9% stock at 120 (without income tax). His income in each case is equal. What is his investment in each stock?

Solution

Given- Total Investment is R27, 000
1st investment - 6% stock at 90 (Income Tax 10%)
2nd Investment- 9% stock at 120 (No Income Tax)
Income in both the investment is equal.
Let the investment in 1st case x rupees.
So investment in 2nd case = R27, 000 - x

To find out income

	When investment	Income	
	R	R	
1st case -	90	6	Income = R$\frac{6}{90}$(x) = R$\frac{x}{15}$
	x	?	
2nd case -	120	9	Income = R$\frac{9}{120}$ (27, 000 - x)
	27,000 - x	?	= R$\frac{3}{40}$(27, 000 -x)

6% stock is subject to 10% Income Tax

So net income from 6% stock = R$\frac{x}{15}$ x $\frac{90}{100}$ = R$\frac{3x}{50}$

As per question-

R$\frac{3x}{50}$ = R$\frac{3}{40}$ (27, 000 - x)

$\Rightarrow$ R$\frac{x}{5}$ = R$\frac{27,000 - x}{4}$

$\Rightarrow$ R4x = R1, 35, 000 - 5x

$\Rightarrow$ R9x = R1, 35, 000

$\Rightarrow$ x = R15, 000

Hence Investment in 9% stock = R (27, 000 - 15, 000) = R12, 000

So his investment in 6% stock and 9% stock is R15, 500 and R12, 000 respectively.

XIV. Calculation of Income of an individual holding of stock.

Illustration - 39

Himanshu holds 1000 shares of a company. The par value of the shares is R10. The company declares a dividend of 10%. Find the amount of dividend that Himansu will recieve during the year.

Solution :

Given - No of shares 1000, par value of share R 10, Rate of dividend 10%

Par value of shares = R1, 000 x 10 = R10, 000

Dividend received

When par value	Dividend	
R	R	
100	10	
10, 000	x	x = R$\frac{10}{100}$ x 10, 000 = R1, 000

So Himansu will receive R1, 000 as dividend during the year.

Illustration - 40

Babuli holds 14% 200 debentures. The par value of the debenture is R100. Find the amount of interest receivable by Babuli during the year.

Solution :

Par value of 200 debentures = R200 x 100 = R20, 000

Interest received

When par value	Interest received	
R	R	
100	14	
20, 000	x	$x = R\frac{14}{100} \times 20,000 = R2,800.00$

So Babuli will receive R2, 800 as interest during the year.

Illustration - 41

The market value of R10 shares is R22. Find the amount to be paid by a buyer purchasing 1500 shares. Find out the profit if the shares were purchased at par.

Solution

Given- No of shares purchased = 1500

Market value of each share = R22

To find out

1) Purchasing price at market value

2) Profit if shares purchased at par

Purchasing Price

When No of share purchased	Market value	
R	R	
1	22	
1500	x	$x = R22 \times 1500 = R33,000$

Profit on purchasing one share at par = R(22-10) = R12

So profit on 1500 shares = R12 $\times$ 1500 = R18, 000

So the buyer will pay R33, 000 by purchasing 1500 shares and he could gain R18,000 by purchasing the shares at par.

Illustration - 42

X buys 2000 shares of A Ltd. at R25 each. The book value being R10 and paid up value R5 each. The company declares a dividend of 20%. What percentage did X earn on his investment ?

Solution:

Given- No of share purchased = 2000

Market value of each share R25

Book value is R10, paid up value is R5 per share

To find rate of return on investment ?

Cost of one share is R25

Cost of 2000 share is R25 × 2000 = R50, 000

Paid up value of one share is R5

Paid up value of 2000 share is R5 × 2,000 = R 10, 000

Dividend is paid on the paid up value of shares.

When paid up value	Amount of Dividend
R	R
100	20
10, 000	?

$$\text{Dividend} = R\frac{20}{100} \times 10,000 = R2,000$$

When Investment	Income
R	R
50, 000	2, 000
100	?

$$\text{Dividend} = R\frac{2,000}{50,000} \times 100 = 4\%$$

So rate of return on investment is 4%

Illustration - 43

Total capital of H Ltd. is R25, 00, 000. The capital is dividend in to 5, 000 15% preference shares of R100 each and 20, 000 equity shares of R100 each. The company declared an annual dividend of R 2, 75, 000. What amount of dividend Namita will revieve holding 200 preference shares and 300 equity shares?

Solution:

Given- Total capital R25, 00, 000

Division- 5,000 15% preference shares of 100 each and 20, 000 equity shares of R100 each.

Amount of dividend declared R2, 75, 000

Namita is a holder of 200 preference shares and 300 equity shares.

To find dividend received by Namita ?

Dividend paid to preference share holders = $R\frac{5,000 \times 100 \times 15}{100}$ = R75,000

Balance dividend payable to equity share holders

= R(2,75,000 - 75,000) = R2,00,000

Dividend per equity share = $R\frac{2,00,000}{20,000}$ = R10

Dividend on 200 preference shares = R15 × 200 = R3, 000

Dividend on 300 equity shares = R10 × 300 = R3, 000

Total amount of dividend to Namita = R3, 000 + R3, 000 = R6, 000

So Namita will receive R6, 000 as dividend.

Illustration - 44

Jamuna sells 400 shares of R100 each at R125. The company declared a dividend of 15% on these shares. He purchased shares of R10 each at R12.50 each with the sales proceed realised. What is the difference in his income if new company declares a dividend of 18% ?

Solution

Given- Jamuna sells 400 shares of R100 each at R125

Dividend declared by the company = 15%

Shares of R10 each purchased for R12.50,with the sells proceeds

Dividend of new company 18%

To find- The difference in income ?

Present holding of Jamuna is ₹400 × 100 = ₹40, 000

Dividend from present holding = $\frac{15}{100} \times$ 40,000 = ₹6,000

Sales proceeds of present holding = ₹400 × 125 = ₹50,000

Stock value of new shares at ₹12.50 = ₹$\frac{10.00}{12.50} \times$ 50, 000 = ₹40,000

Dividend from new shares @18% = ₹40, 000 $\times \frac{18}{100}$ = ₹7, 200

So the difference in Income = ₹7, 200 - 6, 000 = ₹1, 200 (Increase)

Hence, the difference in income is ₹1, 200 (Increase)

EXERCISE- 1

1. From the alternatives given under each bit, choose and write serially the correct answer along with its serial number against each bit :

(a) A 9% stock yields 8%. The market value of the stock is : CHSE-2016 (A)

(i) A112.50

(ii) A100

(iii) A96.50

(iv) A92

(b) Which is best investment. CHSE-2009 (A)

(i) 4% at 80

(ii) 5% at 110

(iii) 3% at 75

(iv) 4% at 100

(c) The maximum amount of capital that a company can raise is :

(i) paid up capital

(ii) subscribed capital

(iii) authorised capital

(iv) called up capital

(d) Market value of A5,000, 5% stock at A90.

(i) A4,000

(ii) A4,500

(iii) A5,000

(iv) A4,800

(e) What is the cost of A4,000, 7% stock at 95.

(i) A4,280

(ii) A3,800

(iii) A4,000

(iv) A3,900

(f) How much money will be realised by selling A2,000, 5% stock at 97 ?

(i) A2,100

(ii) A1,900

(iii)A1,960

(iv) A1,800

(g) What is the selling price of A5,000, 8% stock at 5 premium.

(i) A5,400

(ii) A5,200

(iii) A5,300

(iv) A5,250

(h) How much money will be realised by selling $7\frac{1}{2}$ A6,000 stock at $95\frac{1}{4}$? (Brokerage $\frac{1}{4}$)

(i) A5,800

(ii) A5,700

(iii) A5,350

(iv) A5,650

(i) In share trading business, brokerage is paid by

(i) the seller

$\frac{1}{6}$

(ii) the purchaser

(iii) both the seller and the purchaser

(iv) none of the above.

(j) The yield from 4% stock at 80 is

(i) 4%

(ii) 5%

(iii) 6%

(iv) 4.5%

(k) Stock purchased at par means:

(i) cash value > stock value

(ii) cash value < stock value

(iii) cash value = stock value

(iv) none of the above.

(l) Stock purchased at premimum means

(i) cash value = stock value

(ii) cash value > stock value

(iii) cash value < stock value

(iv) none of the above.

(m) Commission payable by the speculative seller to the broker is known as:

(i) Backwardation

(ii) contango

(iii) stepwardation

(iv) forwardation

(n) Commission payable by the speculative buyer to the broker is known as:

(i) contango

(ii) forwardation

(iii) stepwardation

(iv) backwardation

(o) In security trading, stamp duty is paid by

(i) purchaser

(ii) seller

(iii) both by purchaser and seller

(iv) None of the above.

(p) In stock trading, securities transaction tax is paid by

(i) purchaser

(ii) seller

(iii) both by purchaser and seller

(iv) None of the above.

(q) Stock traded at discount means

(i) cash value = stock value

(ii) cash value < stock value

(iii) cash value > stock value

(iv) none of the above.

(r) How much 6% stock at $95\frac{1}{4}$ be sold to realise A9,500, (brokerage $\frac{1}{4}$)

(i) A9,000

(ii) A9,500

(iii) A10,000

(iv) A10,500

(s) A man purchases A4,600 stock at A91 and sells at A96. His profit is

(i) A200

(ii) A210

(iii) A225

(iv) A230

(t) Annual income from A6,500, 5% stock is

(i) A320

(ii) A325

(iii) A230

(iv) A350

(u) The annual income from 9% stock at 85 is A900. The stock value is

(i) A9,000

(ii) A10,000

(iii) A9,500

(iv) A11,000

(v) A company declares a dividend of 9%. Mr X has 550 shares of face value of A10 each. The market value is A25. How much dividend will he get ?

(i) A500

(ii) A495

(iii) A225

(iv) A1237.50

(w) Which is better investment : 4% stock at A80, 5% stock at A110.

(i) first

(ii) second

(iii) both first and second

(iv) none of the above

(x) The price realised on sale of 500, 6% share at $98\frac{1}{2}$ (Brokarage $\frac{1}{2}$) is:

(i) A50,000

(ii) A49,500

(iii) A49,000

(iv) A48,500

(y) In cum-dividend quotation, the next dividend is received by

(i) seller

(ii) purchaser

(iii) broker

(iv) both purchaser and seller

(z) Cum-dividend price is equal to

(i) Ex-dividend price - dividend

(ii) Ex-dividend price + dividend

(iii) Ex-dividend price

(iv) None of the above

Ans. (a) i, (b) i, (c) iii, (d) ii, (e) ii, (f) iii, (g) iv, (h) ii), (i) iii, (j) (ii), (k) iii, (l) ii, (m) ii, (n) iv, (o) iii, (p) iii, (q) ii, (r) iii, (s) iv, (t) ii, (u) ii, (v) ii, (w) i, (x) iii, (y) ii, (z) ii.

2. (A) Answer the following questions in one word/term.

(i) The percentage income an investment is called.

(ii) The commission charged by the broker on purchase or sell of shares.

(iii) The shares which have the preferential rights as to receipt of dividend and repayment of capital over equity shares.

(iv) The part of the shows capital which cannot be called up except in the event of winding up of the company.

(v) The value which shows the nominal value of the stock.

(vi) The value which shows the market value of stock.

B. Answer the following questions in one sentence each.

(i) What is a preference share ?

(ii) What do you mean by debenture ?

(iii) What is meant by yield.

(iv) Explain the meaning of the word ‘issued capital’.

(v) What is uncalled capital ?

(vi) What is unpaid capital ?

(vii) Explain the word ‘brokerage’.

(viii) What do you mean by authorised capital ?

(ix) What is subscribed capital ?

(x) What is called up capital ?

(xi) What is paid up capital ?

(xii) What is meant by cash value of a share ?

(xiii) State the meaning of the word 'contango' ?

C. Rectify the underlined portions of the following sentences.

(i) Income of the broker from A5,000, 10% stock at 120, brokarage $\frac{1}{4}$ is A500.

(ii) Income of the broker on A50,000, 5% stock at 110, backwardation 2% is A2,500.

(iii) Yield from 4% stock at A80 is 4%.

(iv) Yield from 5% stock at $109\frac{3}{4}$, brokerage $\frac{1}{4}$ is 5%.

(v) A1,000 will be realised by selling A1,000 5% stock at 95.

D. Fill in the blanks.

(i) The income of a broker from A80,000, 4% stock at 90, contango 3% is _____.

(ii) From the following _____% stock is better investment.

(a) 4% stock at 80

(b) 5% stock at 105.

(iii) Yield from $3\frac{3}{4}$% stock at 10% discount is _____.

(iv) A_____ will be realised on sale of A2,000, 5% stock at 98.

(v) Investment amounting to A_____ is required to purchase A8,000, 6% stock at 105.

Ans. (A) (i) Yield ; (ii) Brokerage ; (iii) Preference shares ; (iv) Reserve capital ; (v) Stock Value ; (vi) Cash Value

(C) A25, (ii) A1000, (iii) 5%, (iv) A4.55%, (v) A950.

(D) (i) A2,400, (ii) Sl.a, (iii) $4\frac{1}{6}$%, (iv) A1960, (v) A8,400

3. Answer the following within 2 sentences each.

a) Preference share (CHSE 2009)
b) Equity share
c) Debenture
d) Authorised capital
e) Yield
f) Subscribed capital
g) Issued capital
h) Called up capital
i) Un called capital
j) Paid up capital

k) Un paid capital l) Cash value

m) Brokerage n) Contango

4. Answer the following questions within 6 sentences each.

(a) Which is better investment :

i) 4% stock at 80 or 5% stock at 105? [CHSE 1986(A)]

ii) 5% stock at par or 6% stock at 130?

iii) 6% stock at 90 or 9% stock at 135 (Income tax 10%)

iv) 4% stock at 120 or 3% stock at 81? [CHSE 1986(S)]

v) 4% stock at 108 or $3\frac{1}{2}$% stock at 98? [CHSE 1985(A)]

vi) 3% stock at $111\frac{7}{8}$ or $3\frac{1}{2}$% stock at $114\frac{7}{8}$ (Brokerage $\frac{1}{8}$%) [CHSE 2008(A)]

vii) 4% stock at 110 or 3% stock 90? [CHSE 2003(A)]

viii) 4% stock at 82 or $4\frac{1}{2}$% stock at 95 [CHSE 1985(A)]

(b) How much 5% stock at 99 can be purchased from the sale proceeds of R7,200, 4% stock at 10% premium?

(c) Distinguish between stock and share.

(d) Find the rate of interest from –

i) $3\frac{3}{4}$% stock at 10% discount [CHSE 1985(A)]

ii) 4% stock at 80 [CHSE 1985(S), 1986(A)]

iii) 5% stock at $109\frac{3}{4}$, brokerage $\frac{1}{4}$.

(e) Find the quoted price if A 30, 000 4% stock are sold for A 27,000.

ANSWERS

(a) (i) 4% stock at 80 (ii) 5% stock at par (iii) 6% stock at 90 (iv) 3% stock at 81

(vi) 4% stock at 108 (vi)$3\frac{1}{2}$ stock at $114\frac{7}{8}$ (vii) 4% stock at 110 (viii) 4% stock at 82

(b) R8,000 ; (d) (i) $4.\frac{1}{6}$% (ii) 5% (iii) 4.55 ; (e) R90

Exercise-2

1. Find the cost price or investment required for

a) R7, 000, 5% stock at 105.

b) R1, 000, 5% stock at 5% premium [CHSE 1987(s)]

c) R3, 500 of 4% stock at 101 [CHSE 1989(s)]

d) R2, 200, $5\frac{1}{2}$% stock at par

e) R 4, 500, 4% stock at 5% below par

f) R1,500, $7\frac{3}{4}$% stock at 2% above par.

g) R3, 000, $4\frac{1}{2}$% stock at 8 premium $\left(\text{Brokerage } \frac{1}{4}\right)$

h) R15, 000, 9% stock at 15 discount (Brokerage 1)

i) R12, 000, $4\frac{1}{2}$% consols at 90 $\left(\text{Brokerage } \frac{1}{2}\right)$

j) 6% R5, 000 atock at $109\frac{7}{8}$ $\left(\text{Brokerage } \frac{1}{8}\right)$

2. How much money will be realised from the sale of

a) R1, 000, of 4% stock at 98? [CHSE 1986]

b) R7,200 of 4% stock at 10% premium? [CHSE 1989 (A)]

c) R5, 000, 5% stock at 10% premium $\left(\text{Brokerage } \frac{1}{10}\right)$ [CHSE 1992 (S)]

d) R5, 500, 3% stock at 3% discount? [CHSE 1989 (s)]

e) R 20, 000, $12\frac{1}{2}$ % stock at 20 below par? (Brokerage R2)

f) R25, 000, $12\frac{1}{2}$% stock at 15 above par? $\left(\text{Brokerage } \frac{1}{2}\right)$

g) 10% R10, 000 stock at 10% premium? Brokerage $\frac{1}{2}$, security transaction tax $\frac{1}{4}$

service tax $\frac{1}{4}$

h) $7\frac{3}{4}$% R15, 000 stock at 16 discount? Brokerage $\frac{1}{2}$ Securities transanction tax 30, service tax R15, stamp duty R5.

i) 9% R1,500 stock at $95\frac{1}{8}$? $\left(\text{Brokerage } \frac{1}{8}\right)$

j) 12% R14, 800 municipal bond at 10% discount? $\left(\text{Brokrage } \frac{1}{2}\right)$

3. What amount of stock can be bought by investing :

a) R15, 075 in 4% stock at par $\left(\text{Brokrage } \frac{1}{2}\right)$?

b) R16, 582.50 in 5% stock at par $\left(\text{Brokrage } \frac{1}{2}\right)$

c) R9, 200 in 9% stock at $14\frac{1}{2}$ premium $\left(\text{Brokrage } \frac{1}{2}\right)$?

d) R9, 296 in 8% stock at $15\frac{1}{2}$ above par, Brokerage $\frac{1}{2}$, securities transaction tax R10, service tax R5, stamp duty R1

e) R9, 843 in 4% consol at 4 below par ? $\left(\text{Brokrage } \frac{1}{2}\right)$

f) R10, 772.50 in 7% bond at 7 discount, Brokerage $\frac{1}{2}$?

Other expenses being securiting transanction tax R13, service tax R6, stamp duty R1. $\left(\text{Hints } - \frac{\text{R10, 772.50} - \text{R20}}{93.5} \times 100\right)$

4. How much stock should be sold to realise :

a) R1705.50 from 8% stock at 95 $\left(\text{Brokerage } \frac{1}{4}\right)$

b) R2049.60 from 9% bond of 98 $\left(\text{Brokerage } \frac{2}{5}\right)$

c) R3, 367.50 from $7\frac{1}{2}$% stock at $112\frac{3}{8}$, Brokerage $\frac{1}{4}$

d) R33, 619 from 8% stock at $112\frac{1}{2}$, brokerage $\frac{1}{4}$, securities transaction tax R35, service tax R17, stamp duty R4. (Hints-Amount to be realised R33,619+35+17+4)

5. A man sells R7, 500 of 8% stock at R 106 and invests the proceeds in a 11% stock at R139. Find the amount of new stock purchased. (Brokerage R1)

6. A man sells R 2, 700, 4% stock at $92\frac{1}{4}$. How much 6% stock at $107\frac{3}{4}$ can he purchase from the sales proceeds of the 1st stock. (Brokerage $\frac{1}{4}$)

7. A person sells his R80,000, 5% stock at a premium of R10.25. With the sales proceeds thus obtained, how much $3\frac{3}{4}$% stock at 2% discount can be purchased?

[CHSE 1999]

8. A man purchased R5, 000 stock at 96 and sold when the price rose to R98.50. Find his gain.

9. A man bought R4, 000 stock at par and sold them when the price quoted at 5 premium. Find his profit or loss.

10. A man invests R4, 704 to purchase 5% stock at $4\frac{1}{2}$ below par and sold the stock when the price rose to $98\frac{1}{2}$. Brokerage is $\frac{1}{2}$ in both the ways. Find the profit.

11. A person invested ₹11,040 in 7% stock at 92. He sold ₹8,000 stock at 97 and the balance at 88. Find the profit earned or loss suffered by him.

12. A man invested a certain sum of money in 7% Government bond at 104. He sold them when the price rose to ₹107, making a profit of ₹37.50. How much stock did he buy? What is the amount of his investment? (Brokerage $\frac{1}{4}$ in both the ways)

13. x invested certain amount in 3% stock at 2 premium. Suddenly the price of the stock fell and he sold at 3% discount making a loss of ₹240 in the process. Find i) his investment ii) the stock sold by him.

14. Mr. X invested certain sum of money in 4% stock at $104\frac{1}{2}$. The price went down. He had to sell the stock at 99 losing on income of ₹127.50. How much did he invest and what amount of stock did he buy? (Brokerage $1\frac{1}{2}$)

15. A man invested a certain sum in 8.5% stock at 92.5. He sold the stock when the price rose to $97\frac{3}{4}$. He made a profit of ₹27.50. What sum did he invest? How much stock did he buy? (Brokerage $1\frac{1}{4}$)

16. A sold certain amount of stock and B bought the same. Each pay a brokerage of $\frac{3}{4}$%. The broker earns ₹630. What is the amount of stock dealt with?

17. Find the annual income from the following investments:
    a) ₹1, 800 in 2% stock at 90 [CHSE 1986 (A)]
    b) ₹1, 800 in 2% stock at par.
    c) ₹1, 800 in 2% stock at 20% premium.

18. Find the annual income from the following stocks:
    a) ₹5,000, 5% stock at ₹95.
    b) ₹5,000, 5% stock at par.
    c) ₹5,000, 5% stock at 10% premium.

19. A man invested a certain sum in 8% stock at 108 and sold them at 115. He gains R100. How much stock did he buy? What was his investment? (Brokerage 1)
20. Find the rate of income from investment from the following stocks :
    a) 5% stock at 80.
    b) $6\frac{1}{2}$ stock at apr.
    c) $7\frac{1}{2}$ stock at $124\frac{1}{4}$ (Brokerage $\frac{3}{4}$)
    d) $4\frac{1}{2}$% stock at 10% discount
    e) 5% stock at $94\frac{1}{2}$ (Brokerage $\frac{1}{2}$)
    f) 4% stock at 80 [CHSE 19959A0, 1993(A)
21. Find the yield.
    a) R20,000 distributed on 10, 000 shares of R10 each R8 paid up. [Ans. 25%]
    b) R20,000 distributed on 10,000 shares of R10 each purchased at a premium of 20%
    c) R20,000 distributed on 10,000 shares of R10 each purchased at 10% discount.
22. A man invest R9,500 in 5% stock. If he gets an annual income of R450 after deduction of income tax of 10 paise in a rupee, what is the stock price?
23. X invests certain amount in 6% stock. But he gets a return of 8% on his investment. Find the quotation price of the stock.
24. What is market price of 6% stock, if an investment of R12, 000 gives an annual income of R600?
25. The income derived by X in 3% stock is half the income of what he gets in 5% stock at 90. What is the quotation price of 3% stock?
    (Hints- When income $\frac{5}{2}$ investment 90, when income 3, find investment)
26. A person by investing in 4% gets 5% for his money. What is the price of the stock?

[CHSE 2005(A), 2002(A)]

27. What is the price of 5% stock if it yields a return of 4% on money invested?
[CHSE 1988(A)]

28. A man invests equal sum in 6% stock and 8% stock. He gets a return of 9% in the total sum. The cost price of 6% stock is ₹80. What is the quotation price of 8% stock.

$$\left[\begin{array}{l}\text{Hints - Let R100 each invested in both the stocks.}\\ \text{Income from 6\% stock R}\frac{15}{2}\text{, income from 8\% stock R}\frac{21}{2}\end{array}\right]$$

29. Which is better investment-

a) 4% stock at 120 or 3% at 81? [CHSE 1986(S)]

b) 4% stock at 108 or $3\frac{1}{2}$% at 98? [CHSE 1985(A)]

c) 4% stock at 82 or $4\frac{1}{2}$% at 95? [CHSE 1988(S)]

d) 4% stock at 110 or 3% stock at 90? [CHSE 1993(S)]

e) 8% stock at 96 or 9% at 108? [CHSE 1994(S)]

f) 4% stock at $121\frac{9}{10}$ or 3% at $91\frac{9}{10}$? [Brokeragge $\frac{1}{10}$]
[CHSE 1996(A)]

30. As a commerce student, advise Mr. X wheat her to invest his money in 4% at 76 or $4\frac{1}{2}$% at 95 or 5% at 110 to get maximum benefit.

31. Out of the following investments which is least profitable.
$4\frac{1}{2}$% stock at $69\frac{1}{2}$, 7% stock at $119\frac{1}{2}$ and 6% debenture at $94\frac{1}{2}$.
(Brokerage $\frac{1}{2}$ in all cases)

32. Which is more profitable - $4\frac{1}{2}$% stock at 69 (Brokerage 1) or $5\frac{1}{2}$% stock at $79\frac{1}{2}$, (Brokerage $\frac{1}{2}$). find out the difference in income between them for ₹ 100 investment

33. Which is more profitable?

    4% consols at $69\frac{1}{2}$ or 7% debenture at $11\frac{1}{2}$ premium? How much money is invested in each case when difference in income is 105? (Brokerage $\frac{1}{2}$ in both the cases)

34. Mr. X invested equal sum in $4\frac{1}{2}$% stock at 76 and 6% stock 96. He got R25 more from one stock than the other. What is his investment in each case?

35. Which is better investment - $3\frac{1}{2}$% at 72 free of income tax or $5\frac{1}{2}$% stock at 108 subject to income tax of 10%. What amount of investment will yield R5 more than the other stock?

36. Amit sells out R4, 200 of 4% stock at 90. He invests the proceed in 6% stock at 126. Find the change in his income.

37. A person sold R4, 000 of 3% stock at 80 and invested the proceeds there of in 5% stock at 128. What was the change in his income? [CHSE 1993(A)]

38. A person sells Rs6, 000, 7% stock at R$108\frac{1}{6}$ and with the money realise purchase 6% stock at $95\frac{5}{6}$. Find the change in his income. Brokerage being $\frac{1}{6}$% each transaction. [CHSE 2003(A)]

39. A man invested R1890 in $4\frac{1}{2}$% debentures at R$104\frac{9}{10}$ and sells out when they are at $107\frac{3}{5}$ and invests the proceeds in 4% consols at $85\frac{9}{10}$. What is the change in his income, brokerage being $\frac{1}{10}$ in all transaction? [CHSE 1986(A)]

40. How much money did Naresh invest in $4\frac{1}{2}$% at 94, when his income is R108. Later on he sold the stock at 98 and invested the proceeds in 7% stock at 147. What is the change in his income?

41. Mr. Anand invested in 10% stock an amount of R7, 500 at rate of R$124\frac{9}{10}$ and sold

them at R$120\frac{1}{10}$. With the sales proceeds he purchased 9% stock at R$119\frac{9}{10}$. Find the change in his income. (Brokerage $\frac{1}{10}$% is to be charged on each transaction)

[CHSE 2004(A)]

42. Dutta transferred his stock from $4\frac{1}{2}$% at $73\frac{1}{2}$ to $5\frac{1}{2}$% stock at $85\frac{1}{2}$, there by increasing his annual income by R29. What amount of original stock held by him? (Brokerage being $\frac{1}{2}$ in both the Ways).

[Hint: Take the LCM of 73 and 86 for investment. Find the change in income with this investment. Compare it with the increase in income to find the investment aoumt. Find the original stock with the investment]

43. How much did I invest in 4% stock at 80 so that when I sold R1,000 stock at 83 and the balance at $77\frac{1}{2}$, there by gaining R 10. Find the difference in income if I had invested the proceeds in 9% stock at 145.

[Ans: Investment R1,440, difference in income R18 increase]

[Hint: Find the stock value of balance sale. Find the investment. Find the sales proceed. Find the Income and difference in income]

44. Dinesh sold his 6% stock at R108 and invested the proceeds in 8% stock. As a result his income was increased by 50%. At waht price did he purchase the later stock.
45. Muna invested R4, 800 in 6% stock at 120. later on he sold the stock at a certain price. The proceeds was invested by him in 7% debenture at 140. As a result his income was increased by R20. At what price did he sell the 1st stock?
46. Somesh gets same income by investing R1, 800 in $4\frac{1}{2}$% stock as he receives from R1,500, 6% debentures at 100. At what price did he buy the 1st stock.
47. A man invested R12, 000 in 8% stock at 80. He sold the stock when priced increased to R90. The same amount was invested in 12% stock. By such act his income was increased by R135. What is the purchase price of the 2nd stock.

48. By investing R30, 000 partly in a 5% stock at par and the rest in $3\frac{1}{2}$% stock at 75, a total yearly income of R1462.50 is obtained. Find the amount of investment in each stock. [CHSE 1991(S)]
49. Having R8, 370 to invest a man put part of it in 3% stock at 96 and reminder in 4% stock at 120. His dividend from each investment is same. Find the amount invested in each stock. [CHSE 1991(A)]
50. A man invested R10, 200 partly in 3% stock at $79\frac{3}{4}$ and partly in 4% stock at $109\frac{3}{4}$ (brokerage $\frac{1}{4}$% in each case). If the total income earned is R380, find the investment in each stock. [CHSE- 1990(A), 2001 (A), 2005(A), 2008 (A)]
51. A person invested R2, 200 partly in 10% stock at 120 and partly in 12% stock at 96. Find his investment in each if the income derived from both the investment be R200. [CHSE 1993 (S)]
52. Jameer invested R61, 000 partly in 6% stock of par and partly in 8% stock at 120. His income from 2nd investment is R900 more than the 1st investment. What did he invest in each case.
53. Somesh invests R35, 000 partly in 5% stock at 90 (Income tax 10%) and partly in 10% stock (no income tax) at 150. His Income in each case is equal. Find his investment in each case.
54. Jameer invests R51, 000 partly in 9% stock at 90 (no income tax) and partly in 15% stock at 120 (Income tax 10%). His income is equal in each case. Find his investment in each case.
55. Find the amount of dividend the share holder will get

Par value of share	Rate of dividend	No of Shares.
R10 _____	12% _____	100
R20 _____	15% _____	150
R50 _____	9% _____	230
R100 _____	10% _____	300

56. The market price of R10 share is 32. Find the amount to be paid by a buyer for purchasing 1000 shares. Find the profit if shares were purchased at par.

57. Sailesh buys 3, 000 shares of R10 each at R30. The paid up value is R5 per share. The company declares a dividend of 20%. What percentage of income will Sailesh earn on his investment?

58. Total capital of H ltd is R30,00,000 which is dividend in 6, 000 12% preference shares of R100 each and 24,000 equity shares of R100 each. The company declared an annual dividend of R3, 12, 000. Naresh is a holder of 300 preference shares and 400 equity shares. What amount of dividend will be receive?

59. Madhu sells 600 shares of R100 each at R120. The company declared a dividend of 15% on these shares. He purchased shares of R100 each at R144. The company declared a dividend of 21%. Find the difference in his income.

60. A company declares a dividend of 9%. What is the price of my shares if I earns 12% on my investment.

61. What income will I get by investing R1008 in purchasing R10 shares at R12 per share, (The paid up value being R5 per share) if the company declares a dividend of 14%.

62. Which is best investment, 4% stock at 120, 5% stock at 130 or 6% stock at 150.

Answers

(1) (a) R 7350 (b) R 1050 (c) R 3535 (d) R 2200 (e) R 4275 (f) R 1530 (g) R 3247.50 (h) R12,900 (i) R 10,860 (j) R 5500 (2) (a) R 980 (b) R 7,920 (c) R 5,495 (d)R 5,335 (e) R15,600 (f) R 28,625 (g) R 10,900 (h) R 12,475 (i) R1,425 (j) R 13,246 (3)(a)R 15,000 (b) R16,500 (c) R 8000 (d) R 8000 (e) R 10,200 (f) R 11,500 (4) (a) R 1,800 (b) R 2,100 (c) R 3000 (d) R 30,000 (5) R 5,625 (6) R 2,300 (7) R 90,000 (8) R 125 (9) R 200 (10) R 98 (11) R 240 (12) Stock R1,500, Investment R 1,563.75 (13) Investment R 4896, Stock R 4800 (14) nvestment R 1590, Stock R 1,500 (15) Investment R 937.50, stock R 1,000 (16) R 42,000 17) (a) R 40 (b) R 36 (c) R 30 18) (a) R 250 (b) R 250 (c) R 250 19) Stock R 2000, Investment R 2,180 20) a) 6.25% b) 6.5% c) 6% d) 5% e)5 $\frac{5}{19}$% f) 5% 21) a) 25% b)$16\frac{2}{3}$% c) $22\frac{2}{9}$% 22) R 95 23) R 75 24) R 120 25) R 108 26) R 80 27) R 125 28) $66\frac{2}{3}$ 29) (a) 3% at 81, b) 4% at 108, c) 4% at 82 e) both are equal f) 4% at $121\frac{9}{10}$ 30) 4% at 76 31) 7% stock 32) $5\frac{1}{2}$ stock, difference $\frac{25}{56}$ in R100 investment 33) 7% stock, R 19,600 34) R 7,600 35) . $3\frac{1}{2}$%, R1800 36) R 12 increase 37) R 5 increase 38) R 15 decrease 39) R 9 increase 40) Investment R 2256, R 4 increase 41) Rs 60 decrease 42) R 17,200 43) Investment R 1440, difference in income R 18 increase 44) R 96 45) R 130 46) R 90 47) R 120 48) R 18,750, R 11,250 49) R 4320, R4050 50) R 8000, R 2,200 51) R 1800 R 400 52) R 25,000 53) 20,000 54) 27,000 55) R 120, R 450, R 1035, R 3,000 56) R 32,000, R 22,000 57) 3.33% 58) R 7,600 59) R 1,500 60) R 75 61) R 58.80 62) 6% stock at 150

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CHAPTER-8

Conceptual Framework of Statistics

STRUCTURE

8.1 Introduction
8.2 Origin and Growth of Statistics
8.3 Meaning of Statistics
8.4 Definition of Statistics
8.5 Characteristics and Nature of Statistics
8.6 Questions

8.1 INTRODUCTION

We use statistics in our day to day life knowingly or unknowingly. Early in the morning we go through the newspapers or view the news on the television screen about production, distribution, marketing, transportation, communication, politics, economics, sport and games, weather etc. All the news are backed by some facts or figures for the purpose of quick attention of the television viewers or news paper readers. Then we share with our friends or peers about the news we have collected earlier in the form of figures with logical facts. Thus we use and find statistics in our everyday life.

The word 'statistics' has been derived from the Latin word 'Status' which means a political state. Again it resembles with the Italian word 'Statista', German word 'Statistik' and French word 'Statistique' with the same meaning of a political state.

Statistics was related to the affairs of political state in the early stage of its implementation and use. Kings, rulers, and emperors used statistics as an instrument of their administration and military establishments. Statistics was then developed as a 'Science of Kings' or 'Political Arithmetic'.

For the first time Egypt used statistics by conducting 'Census of Populations' perhaps in the year 3050 BC to finance the erection of pyramids. Statistics in India are dated back to Chandra Gupta Maurya's kingdom under Chanakya to collect data on births and deaths. We can find the use of Statistics in the history of all most all the kingdoms and states of ancient world.

Statistics is all pervasive in nature due to increase in its scope and curriculum. It is being widely used in all most all fields of human life and knowledge, such as business, economics, social sciences, physical and natural sciences. All credit goes to R.A. Fisher who evolved the modern concept of statistics in 1749 and called it as "Mathematics to Observational Data."

8.2 Origin and Growth of Statistics

Statistics is not a new discipline of knowledge. It is as old as human civilisation. It has been in use since the existence of human being on earth. It was used in a limited manner and not widespread and popular as it is today.

8.2.1 Origin of Statistics

The science of statistics originated from two main sources, i.e.(i) Government records and (ii) Mathematics.

(i) Government records : Government records are foundations of statistics on which it is built. The various governmental agencies collect and record regularly the facts and figres for execution of welfare programmes of the Government. To take note, in ancient Egypt, the Police prepared a registered list all heads of families. In ancient Judea arround 3000 BC, Population census was on several occasions. The first Roman census was taken in 435 BC for taking statistics of defence power, tax paying ability, birth rate and mortality rate. Statistical data were collected for the governments and statistics was called the 'Science of Kings' or the 'Science of Statecraft'. A large number of handbooks on statistics were published by different governments since sixteenth century.

(ii) Mathematics : Statistics is believed to be branch of applied mathematics. The statistical methods are used in drawing inferences about a universe/ population from sample study. The sample study is based on the theory of probability. Famous mathematicians like James Bernoulli, Danel Bernoulli, Laplace and Karl Gauses discovered and developed the theory of probability.

The great mathematician Jacques Quettlet (1796-1874) discovered the fundamental principles "the consistency of great numbers". This principle is the basis of sampling. Statistical techniques have been developed too much during the ninteenth and mid-twentith century by the renowned mathematicians. Sir Francis Galton developed the concept of regression, Karl Pearson developed the chi-square test of goodness of fit, Sir Ronal A Fisher made more contributions in the field of experimental designs.

8.2.2 Growth of statistics

Statistics has achieved a phenomenal development in several fields now a days. It is now regarded as one of the most important tools of taking decisions in the face of uncertainty. In fact, statistics has been used in almost all branches of sciences. The growth of statistics has been attributed to three factors, i.e. (i) utility of statistics, (ii) Increased demand for statistics, (iii) decreased cost of statistics.

(i) Utility of statistics- Statistical data and statistical methods give accurate results in making various decisions in the face of uncatainty.

(ii) Increased Demand for Statistics- The demand for statistics has been increasing since the nineteenth century due to the development in the field of business and commerce, governmental activities and science. In the era of globalisation the business activities have become very complex and statistics help in solving the problems of the modern business and commerce.

The governmental activities use statistics for the assessment and implementation of welfare policies. Today, there is hardly any sphere of the Government in which statistics are not being used. The increase in the functions of the modern welfare government has increased the demand for statistics.

There is remarkable change and advancement in the field of science due to continuous research. Since statistics is used as a tool of research, the demand for statistics has increased.

(iii) Decreasing cost of statistics- The time and cost of collection of statistical data has been reduced. The use of electronic equipments like calculations, computers, etc. reduces the cost of analysis of data. This has led to the increasing use of statistics in various fields.

8.3 Meaning of Statistics :

Statistics means data or figures supported by related facts, further application and analysis, interpretation and reporting through some suitable methods. To a layman, statistics are only data about a problem under study. The data are to be arranged for the ultimate use of the statistician or the organiser of the study.

Statistics is not confined only to State as per the old concept. Now it has a broad meaning and denotes a set of numerical data about a particular phenomenon irrespective of the State. For example purchases, production, sales, profit, savings, investment, national income, population, weather etc. with numerical figures are statistics Examples :

(a) The annual purchase of raw material is ₹ 5,00,000.

(b) The monthly production is 5,000 units.

(c) The total automobile sales in Bhubaneswar in the month of January 2016 is 2000 units.

(d) The annual net profit of the soletrader is is ₹ 2,40,000.

(e) The highest and lowest temperature of Bhubaneswar.

on 29th February 2016 is 15^0c and 29^0 celcious.

The above statements giving some numerical figure about a factor or phenomenon are called 'statistics'.

8.4 Definition of Statistics

Statistics has been defined in two different senses, i.e. plural and singular by many authors. In the plural sense statistics is defined as **numerical data** while in the singular sense, it is the **statistical methods applied to the numerical data for the purpose of analysis.**

Different authors have defined statistics differently. Some important definitions of statistics as used in singular and plural senses are given below :

8.4.1 In the Singular Sense :

The term statistics in the singular sense has been defined by different authors. Statistics in the singular is the statistical methods. It is scientific in nature. It is in the form of tools and techniques

applied in the processing of statistical information and final reporting on the study. The methods of collection, analysis and interpretation of data are called statistics in the singular sense. A few definitions are explained below :

Proof. A.L. Bowly has given a few definitions. Once he defines "Statistics may be called science of counting." It is a very narrow definition and covers only a simple aspect of science, i.e. 'collection of data'. The other aspects of statistics like presentation, analysis and interpretation etc are completely overlooked here. Again he has said "Statistics may be rightly called the science of average" which is also not widely acceptable. He has only used one device, i.e.., 'average', ignoring the other devices like dispession, skewness, kurtosis, correlation etc. His third definition of statistics as statistical methods is that "Statistics is the science of measurements of social organism, regarded as a whole in all its manifestations." This definition is also inadequate in its scope and covers only 'sociology'.

Boddington defines statistics as "the science of estimates and probabilities". It is an incomplete definition because estimates and probabilities are only a part of statistics and not the whole of statistics.

Birsten and Levin define statistics as "the science of statistics can be viewed as the application of scientific method in the analysis of numerical data for the purpose of making rational decisions." Croxton and Cowden give a very simple and precise definition of statistics. According to them "Statistics may be defined as the collection, presentation, analysis and interpretation of numerical data."

Seligman defines it as "statistics is the science which deals with the method of collecting, classifying, presenting, comparing and interpreting numerical data collected to throw some light on any sphere of enquiry." This is a satisfactory definition as it touches the most essential functions of statistics.

From the above definitions. Statistics may be defined as the science of collection, organisation, presentation, analysis, interpretation and reporting of numerical data.

8.4.2 In the Plural Sense :

"Statistics are numerical statements of facts in any department of enquiry, placed in relation to each other" (A.L. Bowley).

Webster has defined statistics as "The classified facts representing the conditions of the people in a state-especially those facts which can be stated in numbers or in tables of numbers or in any tabular or classified arrangements."

"Statistics are quantified data affected to a marked extent by multiplicity of causes"- Yule & kendall.

The above definitions give a narrow concept of statistics as numerical facts. Such facts can be compared and interpreted in tabular arrangements. The facts have some sort of homogeneity and they relate to the conditions of human life in political, social and natural fields of enquiry.

The definition given by professor Horace Secrist is found to be most comprehensive and meaningful. He defined statistics "as aggregate of facts affected to a marked extent by multiplicity of causes, numerically expressed, enumerated, or estimated according to a reasonable standard of accuracy, collected in a systematic manner for a predetermined purpose and placed in relation to each other."

According to the definition given by Horace Secrist, it is clear that statement facts will be called statistics when they are aggregated.

8.5 Characteristics and Nature of Statistics

8.5.1 Characteristics of statistics

From the analysis of above definitions, we derive the following chief characteristics of statistics. These are :

(1) Aggregate of facts : The main characteristics of statistics is aggregate of facts. A single figure can not make statistics. So the age of father 50 years in a family is not statistics. If we take the age of mother, children, grand father and grand mother for the purpose of studying a phenomena, then it will be statistics. A set of facts on figures will constitute statistics.

(2) It is affected to a marked extent by multiplicity of causes : A statistical datum can not be statistics. In order to be statistics, the data should be variable and the data should vary considerably by a number of causes. For example, production of rice in Odisha during 2015-16 depends on factors like quality of seed, fertiliser, rainfall, temparature, soil etc. The variability nature of data makes it statistics. The data change frequently and continuously due to change in time, place and situations.

3) It should be numerically expressed : The data in order to be called statistics should be numerically expressed. Numerical data help in the counting or measurement. The facts should be expressed in the form of quantities. Statistics constitute only numerical information.

The qualitative facts such as honesty, beauty, sincerity or intelligence do not come under statistics. These qualitative information in order to be called statistics, should be converted to quantitative form.

4. It should be enumerated or estimated according to a reasonable standard of accuracy : Facts and figures about a problem under study can be derived by actual counting or measurement or estimation., We can enumerate the number students reading in different classes of a college. But we can not count the number of people attending a public meeting of the President of India. Here one can only estimate the number of people attending the meeting. Similarly, one can measure the length of clothes in meter, being produced in a factory. But we can exactly measure the distance between two cities. The distance measured may be a few meters more or less than the distance estimated. It is important that a reasonable standard of accuracy should be attained when we are unable to enumecate or count a particular phenomenon.

5. It should be collected in a systematic manner: Statistical data may be collected in a systematic manner. A suitable plan should be prepared before collecting data. One should decide whether primary data or secondary data are to be used. If primary data are to be used, one should decide how to collect the primary data-either by sending enumerators on by sending

questionnaires to the respondents etc. Data collected in a haphazand manner will result in wrong conclusion.

6. It should be collected for a predetermined purpose : The purpose of collection of data must be decided in advance. The purpose of collection of data must be specific and well difined. For example, if the purpose of the study is to collect data on demand of rice in Odisha, it would not serve any purpose unless one knows whether he wants to collect data on coarse rice or fine rice, and the type of consumers who demand different types of rice.

7. It should be placed in relation to each other : This characteristic of statistics refers to the comparative study of numerical data. If numerical facts are not comparable with one another then these are not statistics. Statistical data are compared period-wise, regionwise, literacy-wise, income-wise etc. Valid comparisons can be made only if the data are homogeneous, i.e. relate to the same prenomenon or study. Like things/ data are compared with each other. The comparison of the average annual income of business men and beggars in a city, height of trees with the height of animals is meaningless.

It can be concluded that in the absence of any one of the above characteristics, numerical data can not be called statistics. The job of the statistician is to make the data meaningful and useful to the user of those data. Therefore, it can be said that **statistics are numerical data but all numerical data are not statistics.**

8.5.2 Nature of statistics

Whether statistics is a science or an art ? Statisticians advocate in two different ways. Some says statistics is a science. While other says statistics is an art.

Science is described as a systematised body of knowledge based on cause and effect relationship and attempts to generalise the conclusions in the form of scientific principles. Prof. A.L. Bowley described statistics as a science while Prof. Tippet calls it both as science as well as

an art. As a science, statistics studies the problem in a systematic manner. As an art, statistics solve problems in real life. Science is like a light house that provides light to the ship in the mid-sea to find out their own way but doesn't indicate the direction in which they should go. On the other hand, art is the skill of handling the problems so as to achieve the given objective. It is the ways and means of handling and analysing the data to arrive at a conclusion.

In the past century, the natural scientists do not recognise statistics as a distinct science. But now almost all sciences are statistical in nature. It is noteworthy to point out that statistics is not an exact science like the natural sciences. It is an inexact science like the social sciences. Statistical phenomena are affected to a great extent by multiplicity of causes which cannot be measured accurately.Thus it is confirmed that statistics is science only in a limited sense. The science of statistics give less precise findings and conclusions in studying problems compared to the natural sciences.

It is known that science is a body of knowledge and art is an action.Statistics may be regarded as an art or skill. It involves the application of scientific methods to the social and economic problems of the people.

Hence, statistics is both a science and an art.

8.6 Conceptual Framework of Statistics Question & Answers

Q.1. From the following alternatives write the appropriate answer along with its serial number against each bit.

(i) In its present usage, the word statistics is

(a) a decade old (b) a century old (c) five centuries old. (d) as old as civilisation

(ii) Statistics can (a) prove anything, (b) disprove anything, (c) neither prove not disprove anything, just a tool, (d) can prove something.

(iii) The real giant in the development of statistical theory is

(a) Fisher, (b) Gauess, (c) Bowley, (d) Karl Pearson

(iv) Statistics can be best considered as (a) an art, (b) a science, (c) both art as well as science, (d) neither art nor science

(v) In the development of statistical methods the greater contribution is that of (a) mathematicians, (b) economists, (c) scientists, (d) businessmen.

(vi) The word from which statistics has been derived, is : (a) Status, (b) Statistik, (c) Statista, (d) all of these.

(vii) In the ancient period 'statistics' was regarded as as a function of (a) sales of a company (b) economics, (c) political state, (d) personal attribute.

(viii) One of the followings, which can be expressed in numerial form, is :

(a) beauty, (b) honesty, (c) marks, (d) poverty

(ix) 'Statistics is the science of estimates and probability' is said by (a) Croxton and Cowden (b) Boddington, (c) AL Bowley, (iv) Seligman.

Ans. (i) d, (ii) c, (iii) a, (iv) c, (v) a, (vi) d, (vii) c, (viii) c, (ix) b, (x) d

(x) One of the followings, which can not be recorded by statistics directly, is (a) height, (b) population, (c) age, (d) intelligence.

2.(a) Express the following in one word or term each :

(i) Statistics used in singular sense

(ii) Statistics used in plural sence.

(iii) Related facts and figures affected to a marked extent.

(iv) The method of collecting, organising, classifying, presenting and interpreting numerical data.

Ans.: (i) Statistical methods (ii) data. (iii) statistics (iv) statistical methods.

2.(b) Answer the following in one sentence each

(i) Define statistics in singular sense.

(ii) Define statistics in plural sense.

(iii) What is meant by nature of statistics ?

(iv) What is aggregate of facts in statistics ?

2.(c) Fill in the gap.

(i) Statistics is derived from the Latin word ----------.

(ii) The word statistics is used in ---------- senses.

(iii) The word statistics refer to ---------- information.

(iv) Statistics is derived from the German word '----------'.

(v) Statistics is derived from the Italian word '----------'.

(vi) Statistics may rightly be called the science of ----------.

Ans.: (i) Status (ii) Two, (iii) Quantitrative (iv) statistik (v) Statista (vi) averages

2(d) Correct the underlined portion of the following statements.

(i) Statistics has been derived from the German word Status.

(ii) Statistics has been derived from the Latin word Statistik.

(iii) In singular sense, statistics is defined as statistical data.

(iv) In plural sense, statistics is defined us statistcal methods.

(v) Statistics has been derived from the Italian word Statistik.

Ans. (i) Statistik (ii) Status (iii) Plural (iv) Singular (v) Statista

Q.3 Answer the following within two sentences each:

(i) How do you consider statistics as an 'art' ?

(ii) How do you consider statistics as a science ?

(iii) State any two features of statisticses used in singular sense.

(iv) State any two features of statistics as used in plural sense.

(v) How are statistics placed in relation to each other ? Give one example.

(vi) Which technique is followed in collecting statistical data ?

(vii) Why do you call statistics as science of human welfare ?

Q.4. Give an idea about the origin and growth of statistics.

Q.5. Define statistics and state its features.

Q.6. Explain the nature of statistics.

Q.7. "Science without statistics bears no fruit and statistics without science has no root." Comment.

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CHAPTER-9

Scope & Importance of Statistics

STRUCTURE

9.1 Introduction

9.2 Scope and importance of statistics

9.3 Relationship of Statistics with other fields

9.4 Functions of Statistics

9.5 Limitations of Statistics

9.6 Distrust of Statistics

9.7 QUESTIONS

9.1 Introduction

Statistics was associated with only administrative and political activities of the states in the olden days. The scope of statistics refers to the area where statistics can be applied in studying different phenomena. However, over the years its scope has been so widened in the modern age that it cannot be expressed. In fact, now-a-days statistics has embraced all branches of science and all spheres of human activities. The ever expanding application of statistics reveals its importance. So the scope and importance of statistics can be explained as an undivided concept. It should be understood that the scope of statistics is also the importance of statistics.

9.2 Scope and importance of statistics

The scope and importance of statistics from the view points of its application in different areas are explained below :

(1) Statistics and State : The statistical techniques and methods were initially used by the State to collect information regarding public administration such as that of crimes, military power, population census, tax revenue. These statistics were the by-products of administrative activities. But now, statistics are not treated in its previous sense of administrative by-products. The functions of statistics have changed in recent years from that of simply maintaining law and order to promoting human welfare and development. The States collect statistics on several problems. Statistics help the States in formulation of policies. All the departments of the Government depend on statistical figures and data for their smooth functioning. Statistics are indispensable during the period of both peace and war time, normal and crisis time. A State can not take remedial or preventive measures to the problems in the absence of factual data. All the Governments in the world collect and use statistical data for their strategic planning and implementation. The state is the biggest collector and use of statistical data in any economy.

(2) Statistics in business : With the growth, development, diversification, modernisation accompanied with ever-increasing competition, the business activities have become more complex. So the modern business houses use more and more statistics in their decision making. In the earlier days these were very small and confined to limited area of operation. The proprietor of a small business then might have acted as manager, accountant, salesman, purchaser etc. The proprietor had personal contact with his customers, creditors and was able to know the needs of the customers. But with the growth in the size of the business firms, it has become impossible for the owners to have personal contact with all the parties. Besides, globalisation makes the management of the business more challenging. Moreover, most of the production is made in anticipation of the demand now-a-days. The market survey for the products is to be made with utmost care. A business man is to deal in an atmosphere of uncretainty. The different problems of the business can be solved by using statistics and statistical methods.

The business activities consist of purchase, production, finance, personnel/human resources, marketing, customer care, research and development, quality control etc. Statistical methods help in collecting abundant quantitative information in respect of the above areas of business for decision making.

(3) Statistics and Economics : Statistical data and statistical methods are of immense help in the proper understanding of economic problems and in the formulation of economic policies. Statistics is used by economists to study on production, consumption, distribution, income, saving and investment etc. of wealth. Professor Alfred Marshall, the world famous economist remarked in 1890 that "Statistics are the straw out of which I, like every other economist, have to make bricks." This statement proves the importance of Statistics in Economics. Statistics of production help in the adjustment of supply and demand for products. Statistics of consumption enable a statistician to know the pattern of spending the income of the people, their standard of living, their tax paying capacity etc. Statistical tools are of immense help in analysing the problems of production functions, cost findings and consumption functions. A few uses of statistics in the different fields of economics are given below :

(a) Statistics studies gross national product and input output analysis.

(b) Statistics helps in the fields of money, banking short-term credit, consumer finance, public finance etc.

(c) Statistics studies business cycles, long term growth and short term flutuations.

(d) Statistics studies the market prices in competition, oligropoly and monopoly conditions.

(e) Statistics help in analysing population, land economics, economic geography also.

(f) Statistics help in solving various economic problems of poverty, unemployment, distribution of wealth etc.

4. Statistics and Natural sciences : Statistics appear to be very much essential in the field of natural sciences like astronomy, meteorology. botany, zoology medicine. Astronomers make use of statistics in their studies of stars, planets and other celestial bodies. We daily come accross the statistics of meteorology in the television screen and news papers. The doctors mostly rely on the actual data like body temperature, pulse rate, blood pressure, blood sugar of a patient from time to time. In biology, study on different animals and plants heavily depend on statistics. The impact of temperature, rainfall, soil on plant life is studied successfully with the help

of statistics. Statistics also makes the study meaningful on the animals, about their fertility, longevity, health and diseases. Prof. Karl Pearson's formula of coefficient of correlation helps in studying the theory of heredity.

5. Statistics and physical sciences : Physical sciences like geology, physics, astrophysics, chemistry, engineering etc. were first using statistical methods and techniques. Afterwards, the use of statistics was made in other sciences. Now the physical sciences are using statistical methods extensively for various purposes. For example, in a study of a physical science, the degree of accuracy can be measured with the help of statistical techniques like degree of confidence, confidence limits, confidence intervals, sampling estimation and design of experiments etc. The natural scientists make series of experiments to prove their findings and results with the help of statistics. There is no area of physical science which does not use statistics.

6. Statistics and social sciences : Statistics is frequently used in the field of social sciences like sociology, political science, history, economics, education, commerce, philosophy and psychology etc. Bowley, Croxton & Cowden and W.I. King have defined statistics in the sense of social science. The methods of statistics are useful in every sphere of human activities. Sociology uses statistics in knowing birth and death rates, marriage, population growth through different surveys. In psychology, statistical methods are used in scaling the mental tests, determing the intelligent quotient (1Q), measuring the reliability and validity of test scores etc. In political science, prepoll and exit poll surveys are made during the period of election to State Legislative Assembly and Parliament. In the field of history, statistical analysis is made on the political and social life of people in the past. Statistics is extremely useful in the field of education for both Government and private bodies. Governments implements modern educational policies on the basis of statistical findings and evaluation.

7. Statistics and Research : Statistics is found to be extensively used in research and development in various fields like agriculture, medicine, public health, business, etc. Most of the development in knowledge has taken place because of the experiments conducted with the help of statistical methods. For example, statistical surveys and experiments are conducted on the

yield of agricultural produces with respect to application of fertilizer, use of high yielding variety (HYV) seeds. In the field of medicine, surveys are made on the efficacy of a medicine on a particular disease. In the field of business and industry, a number of research studies are undertaken on the marketability of product, labour turnover etc. In fact, there is hardly any research work today that one can find where statistics are not used. The implications and findings of most of the research studies are published with the help of statistical methods and techniques.

The universal applicability of statistics is enough to indicate its importance.

9.3 Relationship of statistics with other fields/ Universality of Statistics.

Statistics is really a tool which can be used in the fields of scientific, socio-economic and political studies. Statistics is a body of methods and techniques to obtain knowledge and and information from various areas. Statistics has access to all subject matters and all facets of human life. Statistics is used in the fields of industry, commerce, trade, economics, physics, chemistry, biology, mathematics agronomy, astronomy, astrophysics, psychology, education, political science, sociology, philosophy, meteorology and medicine etc. Hardly any field of knowledge is left from the purview of statistics.

Apart from the above fields, statistics are useful to bankers, insurers, social workers, trade unions, stock marketers, non-governmental organisations (NGO), opinion polls and public utility bodies etc.

Banks need data on consumers, deposits, advances, nonperforming assets (NPA) for future policy formulation. Daily cash requirements is assessed through statistical techniques.

Insurers need data on life table, premium policies, claims, bonus, surrender value etc. Social workers need data to chalk out programmes of social services at different places for the neglected people.

Trade unions need data on wage rate, other monetary and fringe benefits to workers in an industry.

Stock marketers need data to know the trend of prices of stocks, yield/income for the investors.

Nongovernmental organisations need data on common people about their health, education, nutrition, shelter etc.

Politicians and their followers are keenly interested in knowing their prospect of winning an election. By taking a sample of voters through opinion polls they can prepare the strategy to win the mind of the voters.

Public utility companies such as water, electricity, railways, local bodies need statistical data to provide services to the beneficiaries.

The earlier discussions on the application of statistics are not exhaustive. Simply, they suggest the diversity of applications of various methods and ideas of statistics.

In fact, the applications of statistics are so widespread that presently statistics has been transformed from the science of statecraft to the science of universal applicability. It has become an instrument in enhancing human welfare. It is the factor which enables to solve the problems of mankind almost in every corner of life. Most of us make use of statistics consciously or unconsciously in taking day to day decisions. Infact, the knowledge in statistics is essential for the responsible citizens of a nation. H.G. Wells very strongly pointed out a century ago that "Statistical thinking will one day be as necessary for efficient citizenship as the ability to read and write."

In the words of Wallis and Roberts, one is to remember that statistical application though universal in its ideas and techniques it must be modified/tailored in application. It is dangerous to apply to any problem in a cook book style using the same recipe over and over without careful study of the ingredients of each new problem.

9.4 Functions of Statistics

The functions of statistics can be clearly understood from the definitions of statistics described earlier. The important functions of statistics are as follows:

(a) Presents facts in precise and definite form : Statistics present the facts of a study in numerical expressions. Expression of facts in numbers, is more convincing. The general

statements of study can be expressed in a precise and definite form by statistics. Statistics presents the facts in precise and definite form, i.e. numerical expression. For example the annual production and procure of rice can be expressed in figures, like tons, quintals etc.

The reader will have a clear idea of the situation with the following example. When the intelligence of students is discussed, we take the marks secured by him on each subject in the last qualifying examination.

So the number is the medium of expression of a phenomenon. Numbers make the fact meaningful.

(b) Simplifies complex data : Statistics also simplifies the mass data which are complex in nature. The data so collected for the study of a problem are prepared in a few significant figures. Statistics help in condensing the mass data into a meaningful, informative figure. The statistical techniques of average, percentage, dispersion, skewness can reduce the complex data to a few single figure. The average annual production of agricultural produce, average marks secured by a student are easy to remember.

c) Comparison of figures : Statistics helps in comparing the condensed figures with one another. If data are not compared, then the collection of data is meaningless. Further analysis and interpretation will not be possible, statistical methods present an overall information from the mass data. This overall information on a particular problem is meaningful. The effect of prices on the demand of a product can be easily measured and compared by the help of statistical methods. For example an average rise of ₹ 10 in prices per kilogram of a product will definitely reduce the demand for the product by 2 kg on an average in a day. Similarly, if we want to express the average foodgrain production of 2015-16 which definitely increase over the production of 2014-15. it is not more informative. But if we express the average foodgrain production of 2015-16 will increase above 50 million tons over the production of 2014-15, it is informative and meaningful.

d) Formulating and testing hypothesis : Statistics helps in formulating and testing hypothesis and developing new theories. For example, we take a hypothesis that quinine is

effective in curing malaria, a coin thrown is fair, credit squeeze is effective in checking inflation, and then we test the veracity or truthfulness of such hypothesis through appropriate statistical methods and techniques. Formulating a hyphothesis means putting a proposal that certain things may happen and testing a hypothesis means verifying the happening of the event so proposed. Statistic performs the function of testing and formulating hypothesis in different subjects and areas.

e) Prediction of plans and policies : Planning and policy formulation in any organisation is made well in advance of the time of their execution. Statistical techniques help in predicting the plans and policies. Statistics help in forecasting the future events. For example TISCO is to decide the production of steel and iron products in the year 2016-17, 2017-18, 2018-19, it must know the expected demand for its product TISCO can take a subjective judgement and can guess. However, it will be prudent on the part of TISCO to analyse the sales data for the past few years through statistical surveys and estimate the sales volume or demand for sale for the future years.

f) Measurement of uncertainty : It is essential to take decisions in an environment of uncertainty and risk in the fields of social sciences. The chance of happening or not happening of an event is studied in statistics. Statistics provides some techniques i.e. theory of probability, linear programming, theoretical distribution, extrapolation, regression analysis etc. for the measurement of chances of happening of an uncertain event and effect of such event. Statistics is capable of measuring the happening of uncertain events.

g) Framing of suitable policies : Statistics is the source of providing basic materials to frame suitable policies. For example. data about financial inclusion of citizens, its distribution by age and gender, literacy, and other socio economic characteristics help in determining the future needs for banking facilities, self employment facilities, opportunities for opening micro, small and medium enterprises (MSME), financial literacy, training for new entrepreneurs etc. It will not be an exaggeration to say that at the time of war or peace or any problem of economics, social or political nature, statistics provide data on the basis of which suitable policies are framed. Statistics

not only helps in framing suitable policies on socio economic issues, but also it sees how the policies are implemented.

(h) Studies relationship between variables : Statistics studies relationship between variables of certain phenomena. The relationship between height and weight, demand and supply, price and demand, rainfall and temperature, rainfall and foodgrain production etc. can be studied by the statistical techniques like coefficient of correlation, coefficiant of variation, coefficient of association, regression analysis etc.

(i) Enrichment of human knowledge and experience : Statistics provide various relevant information about facts which were unknown to the mankind previously. For example, the extinction of animals and birds due to certain causes can only be studied by taking the statistical information. Statistics enlarges and enriches human experience by conducting research over a long period time. Dr. A.L. Bowley has rightly explained that proper function of statistics is to enlarge human experience.

To sum up the functions of Statistics, Robert W. Burgess has beautifully described as "The fundamental gospel of statistics is to push back the domain of ignorance, rule of thumb, arbitrary and premature decision, tradition and dogmatism and to increase the domain in which decisions are made and principles are formulated on the basis of analysed quantitative facts".

9.5 LIMITATIONS OF STATISTICS

Despite the various functions and usefulness of statistics and its universal applicability, one should not carry the impression that statistics is flawless. It is not a magical device to provide correct answer to any problem. Unless the data are properly collected, analysed, presented and interpreted, there is fair chance of arriving at a wrong conclusion. Therefore it is necessary to know the limitations of statistics, which are discussed below:

(i) Statistics do not study individuals- According to Horace Secrist, statistics is aggregate of facts affected to a marked extent by multiplicity of causes. The study of individuals lies outside

the scope of statistics. Data collected for an individual is not statistics but when those are collected for masses or many, then it is statistics. For example, the price of a single commodity, the wage of a single worker, or the marks secured by a single student does not constitute statistics. When we take the data for large number of people, it is statistics. But the average price of a commodity, average wage of workers or the average mark secured by a student has statistical relevance.

(ii) Statistics don't study qualitative characteristic: Statistics are numerical statement of facts. It studies only the characteristics which are expressed in quantitative form. For example, facts on production, consumption, income, expenditure, investment are expressed in number. So they come under the purview of statistics. While the qualitative characteristics like honesty, intelligence, beauty, efficiency, poverty, health are not measurable quantitatively. These are subjective concepts. These are not coming directly under the study of statistics. These qualitative characteristics are to be quantified indirectly in order to be studied under statistics. For example, poverty can be quantified indirectly by the parameters of daily expenditure or calorie in take by an individual. The intelligence of a student can be measured in terms of marks obtained in the last qualifying examination or intelligent quotient (IQ).

(iii) Statistical findings are true only on average : The conclusions drawn and findings obtained are not universally true. They are true only under certain conditions or on an average. Because statistics is an inexact science, the results vary from situation to situation according to the need of precision and degree of accuracy. For example, in measuring the distance between two cities, a few metres can be ignored which is not true. But statistically the distance so measured is true. The distance measured between the two cities is true only on average. Similarly in other cases, the findings are the approximate results, not the actual results.

(iv) Statistics do not reveal the entire story of a phenomenon : Statistics study one aspect or some aspects of a phenomenon but not all the aspects. It can not reveal the entire story of a phenomenon. Any phenomenon is affected by multiple factors. Stastics is unable to study all these factors. The conclusion drawn by statistics is not based on the entire background of a

phenomenon. Some factors like culture, religion, philosophy, conditions of life may be left while taking decisions in a statistical study.

(v) Statistics can be misused : The greatest weakness of statistics is that it is likely to be misused. When statistical conclusions are drawn on incomplete information, the conclusion will be fallacious. For example, Government sends a team of investigators to a village for a survey on launching mid day meal to the villagers on the basis of age. The investigators met an old man of ninty years of age wooing his grandson of nine months old in the sunny morning of a wintery day. On the basis of the average age of the two villagers, the investigators recommended to the Government to provide solid foods. It is a wrong conclusion given by statistics because a few data has been taken for the study. Statistics are like clay and they can be moulded in any manner to prove right or wrong conclusion by the investigator. W.I. King pointed out that "one of the shortcomings of statistics is that they do not bear on their face the label of their quality." Moreover, any layman cannot deal with statistics correctly. To deal with statistics, proper skill, experience and knowledge is necessary to draw sensible conclusion from the data.

(vi) Statistics is only one of the methods of studying a phenomenon : Statistical methods and techniques are not able to provide the best solution under all circumstances. Frequently, it is necessary to consider a problem from the angle of culture, religion, tradition, custom and religion of people in a nation. Statistics will not help much in the above conditions. There are alternative methods and tools to deal with those problems.

(vii) Statistical methods are not always beyond doubt : According to Prof. L.R. Cannon Statistics deals with only measurable aspects of a phenomenon and leave the aspects which cannot be measured. For example, Statistics measures the food grain production with its different techniques. but it has no technique or method to deal with the causes of such production in quality and quantity. Therefore, statistics can seldom give the complete solution to a given problem. They provide a basis for judgement and not the whole judgement.

(viii) Statistical laws are not exact : The fundamental laws of statistics are (i) law of inertia of large numbers, and (ii) law of statistical regularly. The laws are probability oriented. The

conclusions derived from the statistical studies are not as precise and representative as physical sciences. The statistician can estimate the food grain production of a particular year on the basis of probability, interpolation/extrapolation but he cannot claim that his estimation is exactly 100% correct.

9.6 Distrust of Statistics

Lack of trust in statistical statements and conclusions, is taken as the distrust of statistics.The people who come across it have no faith on it. They disbelieve in any statistical report or statement or findings. They have lack of trust in statistics. Statistics deal with data and figures which are innocent in nature. Those data and figures can be misused by unscrupulous persons. Some times the enumerators are biased to prove any thing in favour of the body organising the study. For example, many years ago, the Agriculture minister of U.P. declared in their State Legislative Assembly with data and figures that composts have been dug up to supply organic manure so that there will be no shortage of fertilities for agriculture hence forth in the State. The actual data and figures have been manipulated upward starting from the Gram Panchayat to the District Collector and finally by the Minister. The leader of the Opposition Party shrewdly made a comparative statement that their Assembly is on the composts as the land area of U.P. is less than the land area of composts dug up.

The people disbelieve/ distrust statistics because of the followings :

a) As figures and data are innocent, they are manipulated to serve their private and personal purpose.

b) The statistician analysing and interpreting data may be inexperienced, unskilled and ignorant.

c) The collection of data by enumerators may be incomplete, and biased.

d) Wrong method of statistics may be used. For example out of the five measures of central tendencies whether mean or median or mode or geometic mean or harmonic mean is appropriate for measuring the average speed of a train or an aeroplane.

e) The limitations and weaknesses of statistics make the people to disbelieve it.

The people start distrusting statistics and condemns statistics by passing sarcastic comments on it as :

a) Statistics can prove anything.

b) Statistics is a tissue of falsehood.

c) Statistics is a clay from which one can make either God or Devil.

d) An ounce of truth can produce tonnes of statistics.

e) Statistics are lies of the first order.

d) According to Disraeli, there are three types of lies is lies, i.e. damnlies and statistics. Statistics is the highest degree of lie.

Statistics is very liable to be misused by ignorant and unscrupulos persons to give fallacious conclusion. The people of ordinary prudence denounce statistics without going deep into the causes start condemning. The people should bear in mind that the fault is not with the statistical methods but with the persons who use those methods.

9.7 QUESTIONS

1. Multiple choice questions.

(a) Out of the following, the one which is a function of statistics, is (i) Testing hypothesis, (ii) ignoring individuals, (iii) Not considering qualitative data (iv) Misusing statistics.

(b) Out of the following, the one which is not a function of statistics, is :

(i) Formulation of plans and policies, (ii) ignoring individuals (iii) Testing hypothesis (iv) Simplification of complex data.

(c) Out of the following, the one which is a limitation of statistics, is : (i) Simplifies complex data, (ii) Statistical records may be biased, (iii) Presents facts in numerical form, (iv) measures results.

(d) Out of the following, the one which is not a limitation of statistics, is (i) Studies relationship

with other fields, (ii) Statistical results may be misused, (iii) Statistical finding may be biased, (iv) Statistics ignores individuals.

(e) Statistical methods can be applied to the field connected with: (i) sociology, (ii) economics, (iii) anthropology, (iv) All of the above.

(f) Out of the following, the one which is a distrust of statisticsis, is: (i) Statistics ignores individuals, (ii) Statistics is aggregate of facts, (ii) Statistical laws are true only on average (iv) an ounce of truth can produce tonnes of Statistics.

Ans: (a) i, (b) ii, (c) ii, (d) i, (e) iv, (f) iv

Q.2. (a) Answer the following in one word or term each :

(i) State any one statistical device to compare statistical data.

(ii) Name a technique to measure uncertainty.

(iii) Name the biggest user of statistical data.

(iv) Name the characteristics not directly studied in statistics.

(v) State the term used for the application of statisties in all fields of human life.

Ans. (i) average, (b) Theory of probability, (iii) State, (iv) qualitative, (v) universality.

2.(b) Answer the following in one sentence each :

(i) Why are complex data simplified ?

(ii) In what form statistics come into existence ?

(iii) Who can misuse statistics ?

(iv) Why don't statistics study honesty ?

(v) State one of the causes of the growth of statistics.

(vi) What are the types of lies ?

2 (c) Fill in the gaps :

(i) Statistics may be misused by ---------- persons.

(ii) In the field of ----------, statistical methods are used in measuring inflation, price and demand.

(iii) Statistics mainly deal with figures which are ----------.

(iv) Data are innocent and donot bear the ---------- of quality on their face.

(v) In the development of statistics methods, greater contribution is made by ----------.

Ans. (i) intexperienced/biased (ii) Economics

(iii) innocent, (iv) label, (v) Mathematicians

2.(d) Correct the underlined portion of the following sentences :

(i) The real giant in the development of statistics is Karl Pearson.

(ii) Statistics can best be considered as an art.

(iii) Statistical laws are universally applicable.

Ans. (i) Fisher, (ii) an art and a science, (iii) not applicable.

Q.3. Answer within two sentences each :

(a) How are statistics related to research ?

(b) Bring out the relationship of statistics with the State.

(c) How statistics related to business ?

(d) Explain the importance of statistics in education.

(e) What is the use of statistics by insurance companies ?

(f) State any two functions of statistics.

Q.4. Answer within six sentences each:

a) State any three functions of statistics.

b) What is the importance of statistics to politicians ?

c) How is statistics related to Government ?

d) Explain 'distrust' of statistics.

e) State any two limitations of statistics.

Long Questions :

5. Discuss the utility of statistics to the State, an economist and the industrialist.
6. Explain the utility of statistics as a managerial tool. Also discuss its limitations.
7. Discuss the application of statistics in various fields.
8. Discuss the functions of statistics.
9. Give few examples of misuse of statistics : How can the wrong use of statistics be avoided ?
10. Explain the reasons for distrust of statistics.

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CHAPTER-10

Statistical Survey : Meaning and its steps

STRUCTURE

10.1 Introduction

10.2 Meaning of Statistical Survey

10.3 Organisation of Statistical Survey : Steps / Stages

10.3.1 Planning the survey

10.3.2 Executing the Survey

10.4 QUESTIONS

10.1 Introduction

A statistical analysis is based related facts and figures. The data about a particular study are essential for statistical analysis. Governments, local bodies, business units, individuals, researchers, collect data with the help of statistical methods for carrying out their day to day work efficiently. The rate of collection of data is increasing day by day because better decisions are possible with the help of analysis of data. Statistical data are collected, analysed and interpreted in the context of population/universe and its characteristics. In statistics, 'universe' is used in a different sense. 'Population' on 'universe' is the total number of units and 'characteristics' is an attribute or quality of a population. For example in a study of wage structure of labourers of a company, the population is the total number of labourers in the company and the characteristic to be studied is the wage of each labourer. There are also other characteristics of the population, such as age, sex, incomes, literacy experience etc. These characteristics are called variables or attributes.

Commonly one on more characteristics can be taken in a particular study from one population/universe. Collection of data is the first step for any statistical enquiry or investigation. The necessary information and data for a particular study can be collected in two methods i.e. either by (i) survey or, (ii) experiment.

We shall discuss about the statistical survey relating to social sciences only. The entire process of collecting data through statistical survey involves three sequential steps, i.e. (a) organisation of statistical survey, (b) collecting data through different sources and (c) Types of investigation, i.e., census or sample.

10.2 Meaning of Statistical Survey :

An enquiry for the search of facts and figures about a particular subject of study is called 'survey' or 'investigation'. Statistical survey or enquiry means search for information and data about a study with the help of statistical methods. It is a technical job which requires specialised knowledge, skill and intelligence. This is the most popular device of collecting and analysing the desired data with a view to preparing a final report. Statistical survey is a process of collecting data from the existing population unit.

A statistical survey may be of two types, i.e. (i) general purpose survey, and (ii) special purpose survey. In a general purpose survey we may obtain data for multifarious purposes. For example, a population census is conducted by the Central Government once in every ten years. This census survey in India gives information not only about the total population but also information about sex ratio, literacy, employment, income distribution, caste ratio etc. All these are called the attributes in the census survey.

A special purpose survey is one in which data collected are useful in analysing a single problem only. For example, the state Government in the recent past made an economic survey to revise the number of people below the poverty line (BPL). The BPL survey is made to distribute the ration cards to the poor people. BPL survey is a special purpose survey. Similarly, survey on city dwellers, owners of four-wheeled vehicles, general graduate/ technical graduate job seekers, literacy, housing projects for slum dwellers etc.

10.3 Organisation of Statistical survey : Stages/steps.

A Statistical survey is a time consuming and comprehensive process, Organisation of a survey passes through the following two main steps :

(i) Planning the survey

(ii) Executing the survey

10.3.1 Planning the Survey

Planning is the foundation stone of survey. It is of utmost importance as the quality of survey depends on its proper planning. Proper planning gives a better quality survey. Planning of survey precedes the execution of the survey. The following matters require careful considerations for planning a statistical survey :

(a) Purpose of the survey.

(b) Scope of the survey.

(c) Unit of data collection and interpretation.

(d) Sources of data/ Types of data

(e) Choice of a frame

(f) Degree of accuracy required.

(g) Other considerations

These above matters can not be considered in isolation from one another. There is a clearcut need of integrated consideration of these matters.

(a) Purpose of the survey : The first step in planning the survey is to specify the objectives of the survey. A clear statement of the problem under study is to be defined. For example if our purpose is to study the changes in price level over a period of time, we have to collect data on commodity prices either in wholesale or retail market.

(b) Scope of the survey : The next step is to decide the scope of the survey. It is the

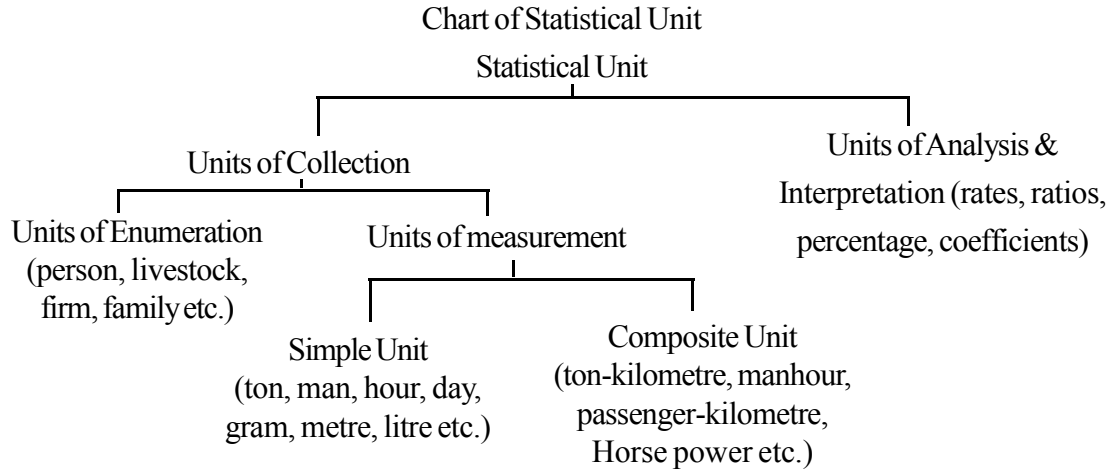
coverage of subject matter, the geographical area and the time of completion. The scope of survey depends on the availability of time, money, man power, and purpose of the survey. Scope of survey is decided before starting it.

(c) Unit of data collection and Interpretation : The investigator collects the data and records them in terms of 'statistical unit'. 'The unit in which the variable or attribute is counted or measured for the purpose of collection, analysis and interpretation of statistical data is called statistical unit". It is the basis of collection and analysis of data in a statistical survey. For example in a census survey the statistical unit is "a person". Unit is the basis of all the measures like gram, kilogram, tonne, quintals in weight and meter, kilometer in length/distance etc. These are called physical units which are used conventionally being well defined and fixed. The units are not same for all the surveys. They differ from one survey to another survey. Where customary physical units are not available, the investigator artificially determine a unit and defines it properly.

There are two types of statistical units such as (i) units of collection and (ii) units of analysis and interpretation.

(i) Units of collection - The data collected in terms of there basic units. Units of collection are further subdivided into two types, i.e. (a) Units of enumeration and (b) Units of measurement. Units of enumeration is the basic unit on which the data are to be collected. For example. A person, a livestock, a firm, a family. You might be aware of the cost of living index number in which 'family' is taken as the unit of enumeration.

Unit of measurement is the unit in terms of which the data are measured or quantified. Examples are : a kilogram, ton, kilometre, ton-kilometre. Such units may be either (a) simple unit or (b) composite/compound un it. A simple unit represents a single condition without any other qualification on adjective. Examples are a kilometre, ton, man, hour, week, month, year, gram, litre, mitre etc. A composite unit is one where some qualifying word is added to a simple unit. Examples are : a ton-kilometre, man-hour, skilled worker, retail price, horse-power, man-power, passenger-kilometres etc.



(ii) Units of Analysis and Interpretation- The units of analysis and interpretation include rates, ratios, percentage, coefficients etc. The units are necessary for comparison and interpretation of findings and conclusions in a survey.

(d) Sources/Types of data : The next step of planning the survey is to decide the sources/ types of data. The sources of data may be either primary on secondary. If the investigator collects data from the origin of the incident, it is called primary data. On the other hand, if he collects the data from published on unpublished sources, those data are called secondary data. Sometimes both types of data are used if one type of data is not available. The sources of collection of data depends much on the purpose and scope of the investigation. Regarding the sources of collection of data, details are discussed in a subsequent chapter.

(e) Techniques of data collection : Basically two techniques are followed for the collection of data : These are : (i) census technique or complete enumeration, and (ii) sample technique or selective enumeration. In case of census enumeration method, each and every individual (item) of the population/universe/group is covered and information is collected for each separately Indian population census carried out every ten yearly by Central Government is an example of census technique. In case of sample technique, only a representative part of the universe is selected and data are collected from each unit of such selected group/universe. This may be done because the group may be too large or the number of items on which the data collected are too large. The method, which are adopted in both techniques of collection of data are the questionnaire method,

the interview method and the method of direct observation. In case of sampling technique, usually "random sampling" method is adopted where the chance of inclusion of each item in the universe is equal.

(f) Choice of a Frame : The frame or structure of the population is to be set for any statistical survey. A 'frame' means a list of the units of the population and their descriptions under the study. For example, in a survey of capital investment in real estate by different banks in the twin city of Bhubaneswar- Cuttack, we have to collect the complete list of all banks and other financial institutions and their addresses. The list of names and addresses of the banks are called the 'frame' here.The entire process of survey depends upon the frame of the population/universe and study. The of a population is to be prepared by the administrative machinery. It should be complete, accurate, upto date and adequate.

(g) Degree of Accuracy Required : The statistical enumerator or investigator while collecting data from different sources is to decide the degree of accuracy in data collection. Absolute degree of accuracy is impossible in any statistical investigation. Higher the degree of accuracy, more is its cost and completion time. A 100% accuracy may not be realistic. The degree of accuracy necessary for a study depends much or the purpose of enquiry. The unbiased, clerical errors may be eliminated altogether on reduced to minimum, so that the investigator will be successful in achieving a higher degree of accuracy in data collection. Sometimes the investigator thinks it proper to collect data quickly by using approximate figures rather than spending a lot of time and cost for attaining a higher degree of accuracy. However one should not sacrifice the required degree of accuracy to reduce the cost and time of the survey.

(h) Types of Enquiry : The investigator should keep in mind the types of enquiry depending upon the object and scope of enquiry. Such enquiries may be (a) formal, semi-formal or non-formal, (b) confidential or non-confidential, (c) regular or adhoc, (d) initial or repetitive, (d) direct or indirect, (f) census or sample etc.

A formal or official survey is conducted by Central or State or Local-self Governments. A semiformal enquiry is conducted by Government sponsored bodies and a non-formal or nonofficial enquiry is undertaken by private organisations and individuals.

In a confidential survey, reports and results are kept secret and not made open to the public, while a non confidential survey report is open through different media for the knowledge of the general public.

A regular survey is made at regular intervals over a period of time, while an adhoc study is undertaken when necessary for a special purpose.

An initial survey is made for the first time, while a repetitive survey is made in continuation with the previous enquiries.

A direct survey is one where the data are capable of direct quantitative expression such as income, expenditure, supply and demand etc. The indirect study is made for qualitative aspects of the society like honesty. intelligence, beauty, literacy, poverty etc.

The above matters are only common sense to be kept in mind of the investigator while planning the survey. These are not to be neglected by him. Otherwise, the results of the study will be misleading.

10.3.2 Executing the Survey

Planning preceeds execution. After the planning of the statistical survey, the next step is to execute the survey. The different steps of execution of the survey are :

(a) Setting up an administrative structure of organisation.

(b) Designing the forms.

(c) Selecting, training and supervising of field staff.

(d) Control over the quality of field work.

(e) Follow-up of non-response,

(f) Processing and editing of data.

(g) Preparation of report

(a) Setting up an Administrative Structure : A well defined organisation structure is necessary to administer the survey. An administrative structure will vary from one survey to another survey because of nature and scope of the enquiry. When an enquiry is undertaken over

a large geographical area, supervision is to be decentralised and administration is exerted from the regional offices.

(b) Design of forms : The forms the survey should be designed as per the requirement of the organisation. Specially, the design of the questionnaire is very important. The reply to the questions should be short, precise and clear. The questions should be set in such a manner that the respondents can give their answers by 'yes' or 'no' or 'at best' design in one sentence.

(c) Selecting, Training and supervising the field staff : In a survey, data are collected by sending enumerations/investigators to the field. These are to be selected through different tests and imparted proper training before starting their job. It is necessary to be watchful towards the performances of the investigators. The field investigators are to work under the field supervisors. A field supervisor is a superior staff and he keeps eyes on the works of the enumerators.

(d) Control over the quality of Field Work : A statistical control is to be undertaken over the field work. A system of field check by supervisors will make the survey meaningful and result oriented. In the control process of the field work, errorneous enumerators are to be cautioned and dishonest and prejudiced enumerators are to be removed.

(e) Follow-up of Non-Response : Sometimes respondents are not interested to supply the information to the enumerators. These respondents do not send the replies or answers of the questions set in the questionnaire. The cases of non-respondents are tackled by setting up a suitable follow-up process. They are to be convinced of the utility of survey benefits to him, and benefits to the society. The enumerators are not allowed to take substitutes of non-respondents.

(f) Processing and Editing of Data : Data collected from fields are shifted to the office for processing and editing. The data so collected are classified, coded, tabulated. There are chances of errors arising at each and every step of data processing. The superior officers are to be cautious while editing the data. Now-a-days a great deal of survey work is done by computers and it saves a lot of time. Most of the work of classification, tabulation and analysis is done through computers.

(g) Preparation of Report : The findings of the survey is written in the form of a report after collection and analysis of data. The report is the finished product and collection and analysis

of data are the rawmaterials in a survey. The preparation of report constitutes the final step of the survey. The final report of a survey contains purpose of the survey, descriptions of the coverage, collection of information, numerical results and degree of accuracy attained etc.

Reports are generally of two types : general report and technical report. General reports contain the results of the survey. These are useful to those who are primarily concerned with the result. Technical reports contain the details of the sample plan, the process of codification. classification, tabulation, analysis and the final results achieved.

10.4 QUESTIONS

1. Multiple Choice Questions.

(a) Statistical survey undergoes :

(i) two stages, (ii) three stages (iii) four stages (iv) five stages

(b) The process of collecting data through statistical survey involves :

(i) three steps (ii) four steps (iii) five steps (iv) six steps

(c) The techniques of data collection are :

(i) one (ii) two (iii) threes (iv) four

(d) Collection of data in the statistical survey is the :

(i) first stage (ii) 2nd stage, (iii) 3rd stage, (iv) 4th stage

(e) Analysis of data in the statistical survey is the :

(i) 1st stage (ii) 2nd stage, (iii) 3rd stage, (iv) 4th stage

(f) Units of collection of data may be divided into :

(i) two types (ii) three types (iii) four types (iv) five types

(g) The scope of the survey depends on (i) the objective, (ii) availability of time (iii) availability of resources (iv) All of these

(h) Units of analysis and interpretation include

(i) rates, (ii) ratio and percentage (iii) coefficients (iv) All of these

(i) Data are generally obtained from :

(i) Primary sources, (ii) secondary sources (iii) both primary and secondary sources (iv) None of these

(j) A frame should be (i) accurate (ii) complete, (iii) adequate, (iv) All of these

Ans.: (a) i, (b) i, (c) ii, (d) i, (e) ii, (f) i, (g) iv, (h) iv, (i) ii, (j) iv

Q.2. (a) Explain the following in one word/term each :

(i) Data originally collected in the process of investigation.

(ii) Data collected from published and unpublished sources.

(iii) The technique in which each and every individual of the group is covered and information is collected from each.

(iv) The technique in which only a representative part of the group is covered.

(v) Listing of all units in the population under study

(vi) A device for obtaining the desired data.

(vii) The individual items in a population.

Ans. (i) Primary data (ii) Secendary data (iii) Census technique (iv) Sample technique, (v) Frame, (vi) Survey (vii) Elementary units

(b) Answer the following in one sentence each:

(i) What do you mean by census technique ?

(ii) What is meant by sample technique ?

(iii) What are the sources of collection of data ?

(iv) What is meant by unit of analysis and interpretation ?

(v) What are composite units ?

(vi) Define statistical unit.

(vii) What are arbitrary units ?

(viii) What is the frame of a survey ?

(c) Fill in the gaps.

(i) A statical unit should be ----------.

(ii) If the purpose and object of enquiry are not defined in concrete terms, then it leads to ---------- conclusion.

(iii) A statistical unit may be unit of collection or a unit of ----------.

(iv) Data can be obtained through a statistical ----------.

(v) A statisticl survey my be of general purpose or ---------- purpose survey.

(vi) A clear and detail ---------- is essential in planning a survey.

(vii) After the data have been collected and analysed, it is usually necessary to embody the result of the survey in the form of a ----------.

Ans.(i) flexible, (ii) fallacious, (iii) analysis, (iv) survey, (v) special, (vi) statement of the problem, (vii) report.

(d) Correct the underlined portion of the following sentences :

(i) Setting up an administrative organisation is one of the steps of <u>planning</u> a statistical survey.

(ii) Deciding the statistical unit is a step in <u>executing</u> a statistical enquiry.

(iii) Statistical data can be collected either through a survey or through an <u>investigation</u>.

(iv) A perfect survey is a <u>reality</u>.

(v) There is <u>no</u> difference between survey and experiment.

(vi) A listing of <u>selected</u> units from the population is called frame.

(vii) One of the <u>smallest</u> problems in the survey is that of non-response.

(viii) One of the biggest solutions in the survey is that of non-response.

(ix) Preparation of survey report is purely and simply a clerical job.

Ans.: (i) executing, (ii) planning, (iii) experiment, (iv) myth/impossible, (v) some, (vi) all, (vii) biggest, (viii) problems, (ix) managerial.

3. Answer the following within two sentences each :

(i) What do you mean by organisation of statistical data ?

(ii) What do you mean by analysis of data ?

(iii) State the stages of statistical investigation.

(iv) What are the coverages of statistical enquiry ?

(v) What is a frame ?

(vi) What are the units of analysis ?

4. Answer within six sentences each:

(i) What is meant by organisationof statistical data ?

(ii) What are the features of a good statistical unit ?

(iii) What are the planning stages of a survey ?

(iv) State the executing stage of a survey

(v) Explain a frame.

Long Questions :

5. Explain the preliminary considerations of planning a statistical survey.

6. What are the statistical units ? Make a classification of statistical units ?

7. What is a statistical enquiry ? Explain the main stages of statistical enquiry.

8. Explain the different types of statistical enquiry ?

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CHAPTER-11

DATA AND COLLECTION OF DATA

STRUCTURE

11.1 Introduction
11.2 Collection of primary data
- its meaning, merits and demerits
11.3 Collection of secondary data
- its meaning, merits and demerits
11.4 Distinction between primary data and secondary data
11.5 Methods of collection of primary data
11.6 Drafting the questionnaire
11.7 Sources of secondary data
11.8 Precautions to be taken in the use of secondary data.
EXERCISE

11.1 INTRODUCTION

In the previous chapter we have defined statistics as aggregate of facts which are numerically expressed. In order to understand the concept of aggregate of facts which are inter related, let us take a set of numerical expressions relating to sales of a business organisation.

Table-1

Year (1)	Sales (in lakhs of rupees) (2)
2011	445
2012	471
2013	502
2014	575
2015	620

In the above table, column (2) shows the amount of sales of the organisation at different points of time. The values give information about sales of the organisation and helps to understand

sales performance over the years 2011 to 2015. These values together are called '**data**'. Therefore, data are tools which provide information and enable us to understand their behaviour.

The phenomenon which assumes the different values in the data is known as '**variable**'. In table 1. the sales vary from year to year and hence sales is called a variable. In other words the values of a variable are known as data.

11.2 COLLECTION OF PRIMARY DATA

In a statistical enquiry, collection of data is a very important task. The sources from which the data are to be collected should be such that useful and valid conclusions are drawn from them. The data may be collected from the following two sources :

a) The enumerator, who collects the data, may conduct the enquiry originally.

b) He may collect the data from sources which had already been collected by some one.

The data which are collected by enumerator for the first time (originally) are known as 'primary data'. This first hand information usually gives correct and proper information about the phenomenon. These primary data may be defined as follows:

"Data which are gathered originally for a certain purpose are known as primary data"

- Horace Secrist.

"Data originally collected in the process of investigation are known as primary data"

-Vessel.

Merits of primary data :

(i) These data are original, accurate and reliable as these are first hand information.

(ii) There is scope for cross checking by the investigator.

(iii) The response is encouraging as the enumerator directly contacts the informant.

Demerits of primary data:

(i) Collection of primary data is more expensive and time consuming

(ii) It is not suitable when the field of enquiry is very large.

(iii) Personal bias and prejudice of the investigator may a effect the data.

11.3 COLLECTION OF SECONDARY DATA

If the data, which have already been collected by some other agency, is collected by the investigator, then it is known as secondary data. It is called secondary data because the investigator as a second person is using the data collected by some one else. Thus secondary data may be defined as follows:

"The data which are used in an investigation, but which have been gathered originally by some one else for some other purpose, are known as secondary data" - M.M. Blair.

Secondary data may be collected from a published source or unpublished source. The published sources may be official publications of Central or State government, reports of financial institutions, news papers and periodicals etc.

The unpublished sources may be records of business organisations, records of Central or State government departments, findings of research scholars etc.

Merits of secondary data :

(i) Collection of secondary data is less expensive and less time consuming.

(ii) It is quite convenient to collect data from published source.

(iii) When the field of enquiry is large, this method is more suitable than that of primary data.

Demerits of secondary data

(i) The degree of accuracy and suitability is less than that of the primary data.

(ii) Personal bias and prejudice of the investigator can influence the data.

(iii) Secondary data relating to abnormal events like war, political instability may not be reliable.

11.4 DISTINCTION BETWEEN PRIMARY DATA AND SECONDARY DATA.

Basis	Primary data	Secondary data
1. Character	These are original in character as these are collected for the first time.	These are not original in character as these are already collected by some other agency.
2. Form	These are collected in the form of raw materials.	These are collected in the form of finished products as these are organised and presented by some one else.
3. Cost	Collection of primary data is always more expensive.	Collection of secondary data is less expensive as it is already collected and organised.
4. Time	As primary data is collected by enumerator from the original source, it is more time consuming.	Collection of secondary data in time saving as it has already been collected.
5. Reliability	Primary data are more accurate and reliable as these are collected exclusively for a definite purpose.	Secondary data need to be used carefully as these are collected earlier and may or may not be accurate and sitable for another purpose.
6. Personal bias	Primary data are influenced by personal bias and prejudice of the investigator.	Secondary data may not be influenced to that extent by personal bias.

11.5 METHODS OF COLLECTION OF PRIMARY DATA

The statistical data may be collected either from primary source or it may be obtained from an already collected source. When secondary data is used, its collection is not very difficult. But, when the primary data is used, the collection of such raw data needs various steps to be followed. Suitable method is to be used to collect primary data so as to make the data accurate and appropriate. The methods commonly used for collection of primary data are given below:

(i) Direct personal investigation.

(ii) Indirect oral interviews.

(iii) Report from local correspondents

(iv) Mailed questionnaire Method

(v) Schedules sent through enumerators

(vi) Telephonic Interviews

(i) Direct personal investigation:

This method is used when the investigator has direct access to the respondents or informants or the persons from whom information is to be collected. Under this method, the investigator has to go to the field personally for making enquiries and for collecting required information from the informants or respondents. The investigator conducts face to face interviews with the informants and tries his best to collect the required information. The success of this method depends on the competancy of the investigator. The investigator should have acquintance with the locality, people, the local language and the cultural tradition so that he can effectively persuade the informants to provide the desired information. As the scope of the enquiry is restricted, it is suited for intensive enquiry rather than extensive enquiry.

Merits :

(a) Data collected are first hand and original.

(b) Personal contact with the respondents encourages them to supply desired information unhesitantly.

(c) The data collected are correct and accurate.

(d) Supplementary information may be collected which may prove useful when analysing and interpreting the results.

(e) The investigator can handle the delicate situations effectively by his skill, intelligence and insight.

Demerits :

(a) This is an expensive and time consuming method.

(b) The personal bias, prejudice and whims of the investigator may adversely affect the findings of the survey.

(c) Unless the investigator is intelligent, tactful and honest, the data obtained may not be reliable.

(d) This method is not suitable if the field of investigation is too wide.

Suitability:

(a) This method is suitable, if the number of persons to be interviewed or area to be covered is small.

(b) Only original data are required.

(c) If accurate and reliable data are required.

(d) If data to be collected are required to be confidential.

(ii) Indirect oral interviews:

Under this method data are collected from persons other than the respondents. This indirect method is adopted in those cases where direct personal investigation is not practicable. When the informant is reluctant or unwilling to furnish requisite information, the investigator collects information from persons who are directly or indirectly concerned with the requisite data. For example, in an enquiry regarding addiction to drugs, gambling, smoking etc. people will be reluctant to give correct information. In that case the correct information may be obtained from persons who know them. The procedure which is generally adopted in this method is that the investigator prepares a small list of questions relating to the enquiry, puts them to different persons known as witnesses, and records their information or statement. The witnesses should not be prejudiced. This method is very popular. It is also adopted by enquiry committees or commissions appointed by the Government.

Merits:

(a) It is a popular method in practice.

(b) If the enumerator applies adequate skill, correct and reliable data may be obtained.

(c) As compared to direct personal investigation method, it is less expensive.

Demerits:

(a) The success of investigation under this method depends on the honesty and efficiency of the enumerator.

(b) The third party's information may be biased and fabricated.

(c) Sometimes the information is supplied not by a single person but by a number of

persons. Hence the findings may not be correct.

Suitability:

This method is suitable:

(a) If direct sources are not easily available.

(b) If information from direct source is not reliable.

(c) If the informant is reluctant to furnish information. (like drug hiabits).

(d) If the subject matter of the enquiry is confidential.

(iii) Report from local correspondents:

Under this method local agencies or correspondents are appointed by the investigator in different parts of the field of enquiry. The correspondents collect the information and submit the reports periodically to the control office for further processing. This method is usually adopted by newspaper or periodical agencies and Government, who collect information regularly in different fields like sports, strikes, riots, accidents, economic and business events etc.

Merits :

(a) It covers a vast area.

(b) It is cheap and economical

(c) This method takes less time to cover vast area.

(d) Data can be collected continuously at regular intervals.

Demerits :

(a) There may be personal bias and prejudice of the correspondents.

(b) Different correspondents adopt different ways of collecting information.

(c) Data may not be always reliable.

Suitability:

This method is suitable;

(a) If the area of coverage is very wide

(b) If the information is required continuously at regular intervals.

(c) If data like registration of birth, death, diseases etc are to be recorded continuously.

(d) If only rough information is required.

(iv) Mailed questionnaire Method:

Under this method a printed questionnaire is sent to each individual directly by mail with a request to complete and return it by a specified date. A questionnaire is a list of questions to be answered by the respondent himself in his own handwriting. For quick and better response, the return postage expenses are borne by the investigator by sending a self addressed stamped envelope. Drafting of questionnaire is very important so as to collect adequate and accurate information. On-line survey is also becoming popular where the investigator collects information from the informants through short messaging service i.e.SMS.

Merits :

a) A vast area can be covered.

b) The investigator can have access to people in the remote area.

c) It provides the respondents sufficient time to give thoughtful and accurate answers.

d) The data are reliable as these are original and authentic in nature.

Demerits:

a) If the informant is not sufficiently educated, the method will not be effective.

b) Information collected is less reliable as there is no check of accuracy and truth of the information.

c) There is uncertainty about the response of the informants.

Suitability: This method is suitable:

a) If the informants are educated.

b) If the investigator wants quick response with less cost.

c) If the investigator has no direct access to the people, especially people in remote areas and where language is a problem of understanding.

(v) Schedules sent through enumerators:

Under this method schedules are sent through enumerators or interviewers to obtain answers to the questions in the schedule in a face to face situation with the respondent. A schedule also contains a list questions which are asked by the enumerator personally. The enumerator records the replies to these questions and submits these for final analysis. Population census is conducted by this technique. The success of this method depends upon the honesty, integrity and intelligence of the enumerator.

Merits :

a) This method gives fairly reliable information as the answers are collected by highly trained enumerators.

b) Investigation over a very wide area is possible under this method.

c) This method is effective even if the respondents are illiterate.

d) It provides a high degree of response.

Demerits:

a) This method is quite expensive and time consuming.

b) The correctness of the information depends on the honesty, insight, intelligence, patience, and courage of the enumerator.

c) Due to variations in the individual personalities of enumerators, the information collected by different enumerators may show variations.

Suitability:

a) This method is suitable for projects and investigation where a high degree of response is desired.

b) This method is also suitable for research organisations, public enterprises and government surveys.

c) It is suitable where adequate finance is available and trained and educated enumerators are available.

(vi) Telephonic Interview

Under this method the investigator asks questions over the telephone and collects the desired information. He also clarifies the doubts of the informants through telephone

and obtains correct answers. These days this method is very effective because many possess telephone and they can be easily contacted.

Merits:

a) This method is cheaper.

b) It is conducted for a shorter time and hence it is time saving.

c) Unlike personal interviews, respondents are not reluctant to answer.

Demerits:

a) Many people may not like to respond the telephone interviews.

b) As there is no personal contact, the information collected may not be accurate and reliable.

c) During leisure time there may be many phone calls and the informant may not be available.

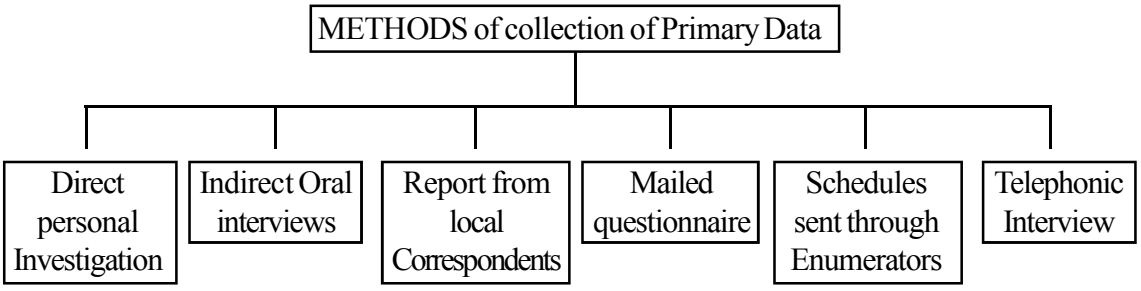
Suitability :

This method is quite suitable

a) To collect feedbacks by service providers.

b) To cover wide area with lessor cost.

Methods of collection of primary data at a glance



11.6 DRAFTING THE QUESTIONNAIRE

A questionnaire is a document containing a list of questions which are to be answered by the respondents. To obtain correct and adequate information from these answers, a questionnaire should be designed with great care, skill, wisdom, efficiency and experience.

In this connection the following general principles of drafting a questionnaire should be borne in mind:

PRINCIPLES OF DRAFTING QUESTIONNAIRE:

(i) The questionnaire should be small in size:

A questionnaire should contain only relevant and important questions so that the respondents will not hesitate to answer. Usually it should contain fifteen to twenty number of questions. If the nature and scope of the enquiry require more number of questions, the questionnaire may be divided into two or three parts.

(ii) The questions should be arranged in a logical order:

The arrangement of questions should be in a logical sequence so that spontaneous and natural reply of the respondents are obtained. For example, before asking about number of children he should be asked about his marital status.

(iii) Questions should not be ambiguous:

Multi-meaning words should be avoided so that the different respondents will not interpret differently to the same question.

(iv) Questions relating to personal or private matters should be avoided:

Direct questions which will hurt the sentiments, pride or prestige of an individual should not be included. Questions relating to income, savings, social evils should be indirectly and tactfully asked.

(v) Adequate instructions to the respondents should be given:

Sufficient instructions relating to answering the questions, time of submitting the questionnaire, address at which it is to be submitted etc. should be clearly specified.

(vi) Questions should be of objective type:

Questions should be short or objective type so that it will be easy for the respondents to answer Multiple choice questions may be given or questions with 'Yes' or 'No' answers may be preferred.

(vii) Questions involving mathematical calculations should be avoided:

Questions requiring mathematical calculations like ratios, percentages, logarithms

etc will cause difficulty to respondents to answer and hence these should be avoided as for as possible.

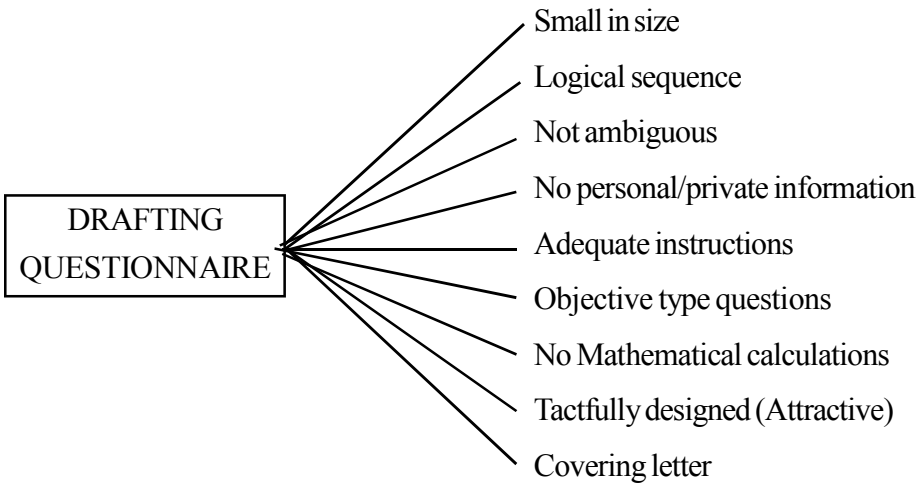
(viii) Questionnaire should be tactfully designed:

Questionnaire should be attractive and appealing to respondents.Both the subject-matter and get up of the questionnaire should be designed in an attractive manner. It should be so designed that it should provide internal check on the accuracy of the information supplied by the respondents.

(ix) A covering letter should be accompanied along with the questionnaire:

For quick and better response, a cover letter should be attached. The cover letter should specify the objective and scope of the survey, the units of measurement, the degree of accuracy desired, the mode of returning the quesionnaire, the procedure for maintaining the secrecy of the information and awards or incentives or gifts for supplying information.

The principles of drafting questionnaire at a glance:



11.7 SOURCES OF SECONDARY DATA :

It may not be always possible to collect data from primary sources. Further, fairly reliable published data are now available in publications of the governments or private organisations. So the statistical enumerator may go for using data which have already

been collected by some other agency for some purpose. The sources of secondary data may be classified into the following two broad groups:

(a) Published sources

(b) Unpublished sources

(a) Published sources :

When data are collected and presented in published form as statistical reports, they are known as published sources of secondary data. The important publications of various national and international organisations are stated below:

(i) International reports and publications:

The reports and official publications of the United Nations Organisation (UNO) , World Health Organisation (WHO), International Labour Organisation (ILO), International Monetary Fund (IMF), World Bank etc are valuable sources of secondary data.

(ii) Government reports and publications:

A large number of reports and publications of Central and State Governments on various topics like, price, wages, production, consumption, population etc. are also available as authentic and reliable sources of secondary data. Economic survey, national income statistics, publication by central statistical organisation etc. are some of the examples of sources of secondary data.

(iii) Semi-Governemnt reports and publications:

Semi government organistions like municipal corportions, district boards, Block and Panchayat Samitis also provide fairly reliable and useful information in their reports and publications.

(iv) Publications of Commercial and Financial Institutions:

Institutions like Indian Chamber of Commerce, Institute of Chartered Accountants of India, Stock Exchanges, Co-operative Societies etc publish reports on economic, business and on other matters.

(v) News papers and periodicals:

Periodicals like Commerce, Yojana, Business India etc and news-papers like

Economic Times, The Financial Express etc publish a large number of useful data. It is an important source of current data on socio economic problems.

(vi) Research Publications:

Research Publications of individual scholars of various universities and research organisations like ICA, NCERT, Trade Unions etc. provide useful findings which are important sources of secondary data.

(vii) Reports of Committees and Commissions:

Enquiry committes and commissions by Central and State Governments, publish reports which are valuable for others.

(b) Unpublished Sources:

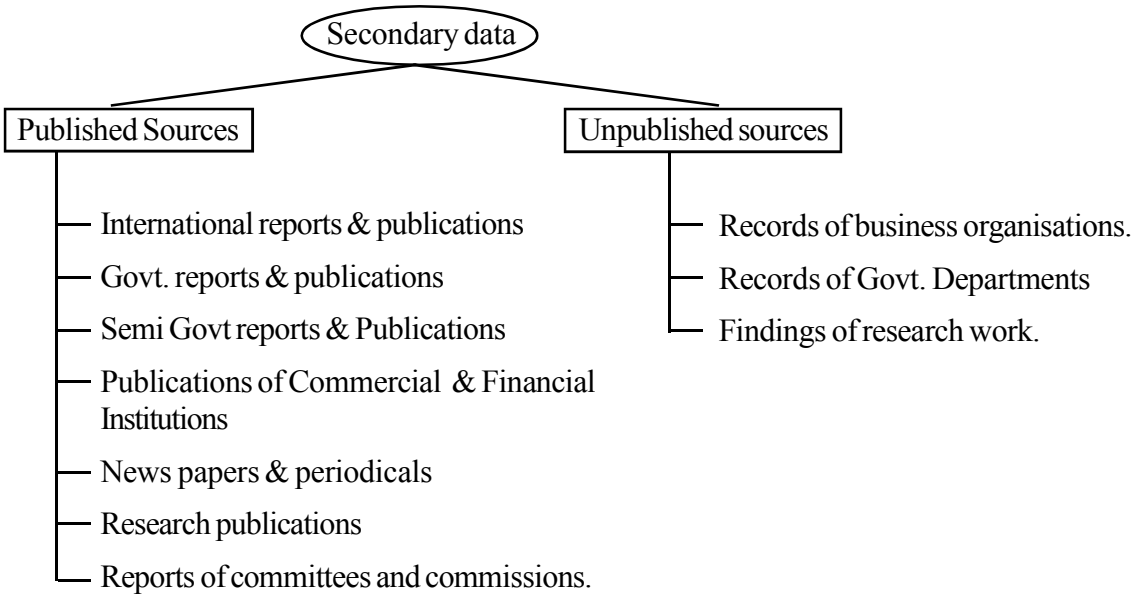
Sometimes statistical data are collected from unpublished sources like:

(i) Records of business organisations,

(ii) Records of various departmental offices of Central and State Governments.

(iii) Data from research work carried by research scholars, research association, chamber of commerce, trade association etc.

Sources of secondary data at a glance.



11.8 Precautions to be taken for the use of secondary data :

Before using secondary data, the investigator should be cautious as to the suitability, reliability and adequacy of such data.

(i) The investigator should compare its nature and scope of given enquiry with that of the available data. He should be aware of the time, unit, purpose of original data collection. If he finds the data acceptable for his purpose, then only he should use such secondary data in his investigation.

(ii) The investigator must verify the methods of collection adopted, the ability and efficiency of the persons who had collected, the degree of accuracy accepted etc. If he is satisfied with the suitability of such data for his enquiry, then only he should accept the data.

(iii) The secondary source of data may be suitable and reliable, but it may not have adequate coverage. If the original data relate to a small area or period, it may not serve the purpose of investigation.

On the whole, the secondary data should be used only after sufficient scrutiny as to suitability, reliability and adequacy of such data.

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EXERCISE

A. 1. Choose and write the correct answer from the given alternatives:

(a) Primary data are those which are collected:

(i) From newspapers.

(ii) From Central Government's reports.

(iii) Through mailed questionnaires.

(iv) From international reports.

(b) The data which have already been collected by some other agency are called:

(i) Primary data

(ii) Raw data

(iii) First hand data

(iv) Secondary data

(c) Population census is conducted by:

(i) Mailed questionnaire method

(ii) Direct personal investigation method

(iii) Indirect oral interviews method

(iv) Schedules sent through enumerators method

(d) Under mailed questionnaire method the list of questions is.

(i) Asked by the enumerator to the informant.

(ii) Asked by the investigator personally

(iii) Mailed to the informants to be answered.

(iv) Asked indirectly to other persons for answer.

(e) One of the principles of drafting questionnaire, is :

(i) It should be lengthy

(ii) It should contains descriptive questions

(iii) It should have sufficient instructions

(iv) It should relate to personal matters

2. (a) Express the following in one word/term:

(i) The plenomenon which assumes different values of data.

(ii) The person from whom requisite information is collected.

(iii) The document containing a list of questions which is mailed to the respondents for answering

(iv) The source of data which is available in published form.

(b) Answer each of the following questions in one sentence:

(i) What are the two types of data ?

(ii) When is schedules sent through enumerator method of data collection, suitable ?

(iii) Point out a demerit of indirect oral interviews method of collection of primary data.

(iv) Name any two published souces of secondary data.

(c) Correct the following underlined portions of the sentences:

(i) News papers and periodicals are unpublished sources of secondary data.

(ii) Mailed questionnaire method is suitable for illiterate people.

(iii) Data collected for the first time is known as secondary data.

(iv) The direct personal investigation method of data collection is less time consuming.

(d) Fill in the blanks:

(i) Report of IMF is a _____ source of secondary data.

(ii) _____ sent through enumerators method is suitable when respondents are illiterate.

(iii) News paper agencies regularly collected information through reports from _____ correspondents.

(iv) In _____ survey information is collected through short messaging service.

3. Answer the following questions within two sentences each.

(i) Name any two methods of collecting primary data.

(ii) Write any two merits of primary data.

(iii) Mention any two merits of secondary data.

(iv) Write any two merits of direct personal investigaton method of data collection.

(v) Point out any two demerits of direct personal investigation method of data collection.

(vi) State any two situations where direct personal investigation method is suitable.

(vii) Mention any two principles of drafting a good questionnaire.

(viii) Write any two merits of indirect oral interviews method of data collection.

(ix) State any two merits of report from local correspondents' method of data collection.

(x) Mention any two demerits of mailed questionnaire method of data collection.

(xi) Point out any two merits of schedules sent through enumerators method of data collection.

(xi) Name the two sources of secondary data.

(xii) Name any two published sources of secondary data.

(xiii) What do you mean by secondary data ?

(xiv) When is telephonic survey method most suitable ?

(xv) Define 'primary data'

(xvi) Define 'secondary data'.

(xvii) State the features of 'primary data'.

4. Answer the following questions within six sentences each:

(a) When will you go for primary data ?

(b) Under what circumstances is direct personal investigation method most suitable ?

(c) Under what circumstances is mailed questionnaire method most suitable ?

(d) Explain the direct personal investigation method of data collection.

(e) Explain schedules sent through enumerators method of data collection.

(f) Point out any three distinctions between primary data and secondary data.

(g) Name any three sources each for obtaining published secondary data and unpublished secondary data.

(h) Which method of collection of primary data is preferred for extensive investigation and why ?

C. LONG QUESTIONS:

5. What are primary and secondary data ? Distinguish between primary data and secondary data.

6. Explain any three methods of collecting primary data.

7. Discuss the principles of drafting a good questionnaire.

8. What are primary and secondary data ? Discuss their relative merits and demerits.

9. State the sources of secondary data. Discuss the precautions to be observed while using secondary data.

10. Explain the mailed questionnaire method of collecting primary data.

ANSWERS

1. a. (iii), b. (iv), c. (ii), d. (iii), e. (iii), f. (iii)

2. (a) (i) Variable, (ii) Informant/respondent, (iii) Questionaire (iv) Published

   (b) (i) Primary data and secondary data.

   (ii) When the respondents are illiterate.

   (iii) If the third party witness is dishonest, the data cannot be relied upon.

   (iv) a. Report of IMF

   b. Newspapers and periodicals

   (c) (i) Published (ii) Literate

   (iii) Primary (iv) more.

   (d) (i) Published (ii) Schedules (iii) Local (iv) Online

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CHAPTER-12

CLASSIFICATION AND TABULATION OF DATA

STRUCTURE

12.1.1 Introduction
12.1.2 Meaning and characteristics of classification of data
12.1.3 Objectives of classification
12.1.4 Types of classification
12.1.5 Discrete and continuous variable
12.1.6 Frequency distribution
12.1.7 Bivariate Frequency distribution

12.1.1 INTRODUCTION

In the previous chapter we have discussed about collection of data. These data are huge, highly disorganised and are in a raw form. These raw data need to be arranged in a systematic manner so as to draw meaningful conclusions. So proper organisation and presentation of such raw data is essential for undertaking statistical analysis and interpretation. To organise these data, we must arrange them in a classified form. The raw data are summarised and made comprehensible by the process of classification. Organisation or classification of data is the second stage in statistical investigation or survey.

12.1.2 MEANING & CHARACTERISTICS OF CLASSIFICATION OF DATA:

Meaning of classification:

Classification of data is the process of arranging data into groups or classes according to their common features.

Prof Horace Secrist defines classification as "classification is the process of arranging data into sequences and groups according to their common charateristics, or separating them into different but related parts."

Data classification is the process of sorting and categorizing data into various types, forms or any other distinct classes.

In classification, the data having common features are grouped together into a class and the whole data are subdivided into a number of classes. For example, the successful students of college can be classified into different groups on the basis of the percentage of marks secured by them.

13.1.2 Characteristics of classification (Rules for classification):

For an ideal classification of data, the following rules or principles may be observed:

(a) **Un ambiguity :** The classes should be clearly defined so that there is no confusion in understanding the meaning of the classes. For example, if the people of a locality are classified into literates and illiterates, the meaning of literates and illiterates should be rigidly defined.

(b) **Homogeneity :** The data should be classified on the basis of homogeneous features. The data in a group should have resemblence or similarity.

(c) Exhaustive and mutually exclusive: The classification should be exhaustive so that all the observations are included in the classes. It should also be mutually exclusive so that the classes donot overlap each other.

(d) Stable classification : A classification should be stable so that it is consistently used in a statistical analysis. It enables comparison of data at different points of time. For example, results of two censuses can be compared if the classification in each is stable.

(e) Flexibility : The classification should be flexible so that new and changed conditions can be easily incorporated. There should be enough scope for adjustment of new data under changed situations.

12.1.3 OBJECTIVES OF CLASSIFICATION

The objectives or functions of classification of data are as follows :

(i) To condense the data :

Classification enables to condense the huge raw data in such a manner that it becomes

easy to sort out the relevant and significant features.

(ii) To facilitate comparison :

The stable nature of classification enables us to make a meaningful comparison of data.

(iii) To study the relationship :

Classification enables us to study the relationship among two or more criteria of a given set of data.

(iv) To highlight the significant features:

Classification presents comprehensible data in such a manner that the significant features of the whole data are highlighted.

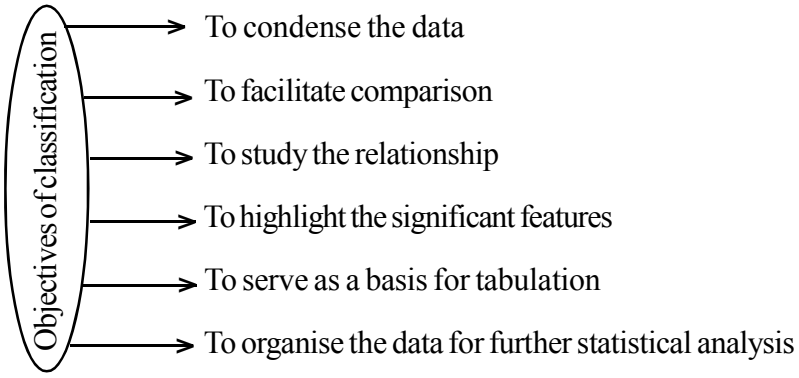
(v) To serve as a basis for tabulation :

Classification is a pre-requisite for its next step. i.e. tabulation.

(vi) To organise the data for further statistical analysis:

Classification creates condensed data which make subsequent statistical treatment more convenient and accurate.

Objectives of classification at a glance :



12.1.4 TYPES OF CLASSIFICATION:

The classes or groups of classification are made depending on the objective and requirement of the statistical survey. Generally the classification is of the following four types :

(i) Chronological (Ascending or descending order of time)

(ii) Geographical (Area-wise)

(iii) Qualitative (On the basis of qualities or attributes)

(iv) Qantitative (on the basis of numerical values)

(i) Chronological classification :

Under chronological classification the data are grouped either in ascending or descending order of ocurrence of time, such as years, months, weeks etc. As the values are expressed against time, the series of the values for different points of time is called time series.

Example :

(i) Sales of a business organisation

Year	**Sales** (In lakhs of rupees)
2011	351
2012	392
2013	409
2014	425
2015	487
2016	500

(ii) Population of India

Year	**Population** (in crores)
2011	121
2012	123
2013	125
2014	126
2015	128
2016	131

(ii) Geographical classification:

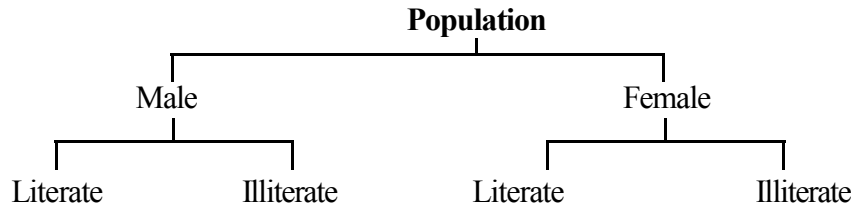
Under geographical classification the data are classified on the basis of geographical locations such as countries, states, districts, cities, areas etc. This classification is suitable for population survey, agricultural production survey etc. It is also known as spatial classification as the basis of classification is place.

Districtwise population in Odisha

Districts	**Persons**
Cuttack	26,18,708
Khurdha	22,46,341
Puri	16,97,983
Sambalpur	10,44,410
Koraput	13,76,934
Malkangiri	6,12,727

(iii) Qualitative classification:

Under qualitative classification, the data are classified on the basis of characteristics or attributes such as literacy, religion etc. This classification is suitable where quantitative classification is not possible. The classification may be dichotomous or manifold. Under dichotomous classification the data are classified into presence or absence of a particular attribute. Example of dichotomous classification is as under :



(iv) Quantitative classification:

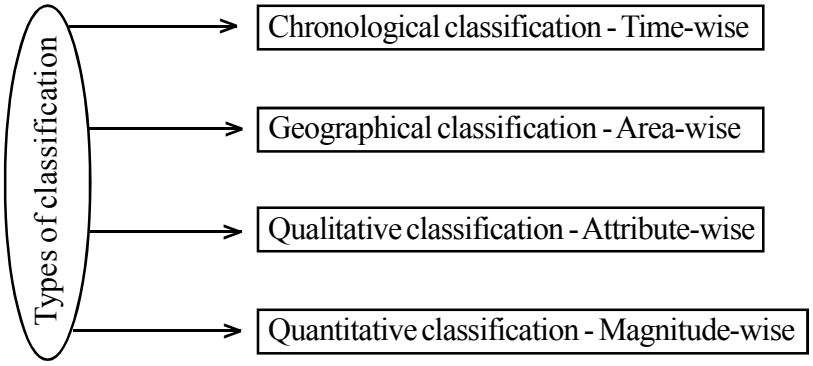
Under quantitative classification the data are classified into groups or classes which are capable of quantitative measurements. The data relating to a phenomenon or variable are divided into different classes of quantitative expression. Examples are classes relating to measurement of height, weight, production, sales, marks, income, expenditure, profit. Such classification are based on figures.

Example :

Marks secured	Number of students
10-20	7
20-30	10
30-40	12
40-50	1

In quantitative classification, there are two elements namely (i) the variable (which changes) i.e. marks in the above example and (ii) the frequency i.e. the number of students. In the above example, 10 students have secured marks between 20-30. Different variables are measured in different units, like incomes are measured in rupees, marks are measured in figures, weights are measured in bs. or kg. etc.

Types of classification at a glance:



12.1.5 DISCRETE AND CONTINUOUS VARIABLE

A phenomenon assumes different values in a set of observations. In other words the values vary and hence it is called a variable. The variable can be of two types namely,

(i) Discrete variable and

(ii) Continuous variable

(i) A Discrete variable:

A discrete variable can take only certain values. It usually changes by finite numbers. It does not take any intermediate values between them. It does not assume any other values between one value and the next. For example, if we take the sizes of a shoe, it can be 5 or 6 or 7 or 8 and so on, but it can not be 5.1, 5.11, 5.10, 5.2 etc. in case of discrete variable. If the variable jumps from $\frac{1}{4}$ to $\frac{1}{8}$, there can not be any intermediate value or values in between $\frac{1}{4}$ to $\frac{1}{8}$. Thus in discrete variable, only integral values are taken, where the unit is complete and indivisible.

(ii) Continuous variable :

A continuous variable can take all possible values in a given range. If we take the weights of persons between 50 kgs to 60 kgs, it can be any possible values within this range of 50-60 kgs. It can be 50.2 kgs, 50.7kgs, 55.92 kgs etc. The values can assume infinite gradations and hence the variable in said to be continuous. Thus continuous variable can take all the possible values (integral or fractional) in a given specified range.

12.1.6 FREQUENCY DISTRIBUTION :

In arranging the raw data in an ascending or descending order, the values of a variable may be repeated. In stead of writing the same value several times we may like to write the number of occurrences against that value. Frequency of a variable is the number of times a particular value of the variable occurs in a distribution. Frequency distribution is a table in which the data are grouped into several groups or classes and the number of occurrences of the groups or classes, are recorded.

For better understanding we have to arrange the values of the variable in a systematic manner and in a logical sequence. Such logical and systematic arrangement of data is known as seriation. Seriation enables us to facilitate further statistical analysis and interpretation of data. The essential features of seriation are as under :

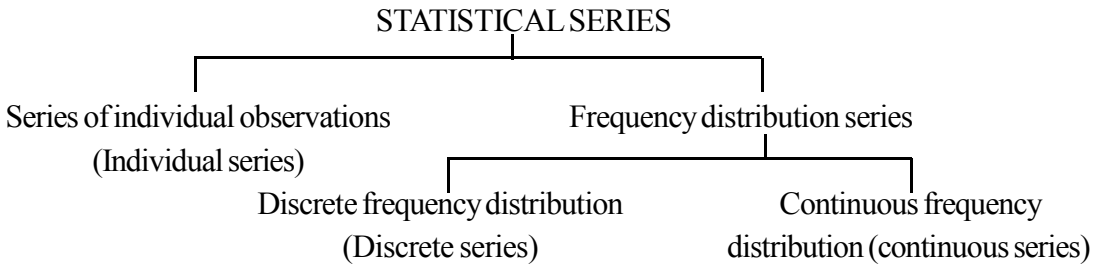
(i) It is an arrangement of different values of a variable in an ascending or descending order.

(ii) There must be certain logical basis of arrangement.

(iii) It is done after classification of data.

(iv) It displays the values of the variable along with their related frequencies.

The different types of series associated with classification of data are indicated below:



- **Series of individual observations (Individual series):**

When the values of a variable are listed individually, the set of observations together is known as a series of individual observations. The marks secured by 20 students are mentioned below individually which forms a series of individual observations or individual series :

30	35	40	35	42	48	52	52	35	40
48	35	42	48	48	52	52	58	58	60

In the above series the individual mark of each student is recorded. The marks such as 35, 42, 45 etc are repeated but they are listed separately. Construction of an individual series is suitable for small number of values, but it is difficult and not comprehensible for large number of values. To make the data comprehensible and accessible, the observations should be classified into discrete or continuous frequency distribution.

- **Discrete frequency distributon (Discrete series)**

In a discrete or ungrouped frequency distribution, similar values of the variable are put together. The number of times each value of the variable occurs is counted and recorded against the values as frequencies. This process makes the data comprehensive and understandable and avoids repeated writing of similar values in the distribution.

Tally Bars technique:

Tally Bars or Tally Marks technique is a very easy and useful technique to comprehend bulk individual values of the data. This technique of forming discrete frequency distribution has the following three steps.

1st step :

All distinct values of the variable should be listed in the first column without repetition under the head 'values of the variable' (i.e. age, marks, wages etc)

2nd step :

The second column contains vertical bars (/) called Tally Bars or Tally Marks. The number of each value of the variable occurs is counted, and recorded against the respective values with the help of these tally bars. In marking tally bars, a block of 5 bars is taken. Suppose the counted frequency is five, it is written as '𝍸', that is four vertical bars are crossed by another tally bar. For example, to record a frequency of 14 with the help of tally bars, we write 𝍸, 𝍸, ////.

3rd step :

The third column is frequency column. It shows the number of occurrences of each value of the variable as counted by the tally bars. Let us prepare a discrete frequency distribution from the following individual marks of 20 students.

30, 35, 48, 35, 48, 48, 52, 52, 35, 48,

48, 35, 48, 48, 48, 52, 52, 58, 58, 52.

FREQUENCY DISTRIBUTION OF MARKS OF 20 STUDENTS

Marks	Tally Bars	Frequency
30	/	1
35	////	4
48	~~////~~ ///	8
52	~~////~~	5
58	//	2
Total		20

- **Continuous frequency distribution (Continuous series):**

In a continuous frequency distribution, the entire values of the variable is condensed to suitable number of groups or classes or class intervals. The class-intervals are usually continuous and are arranged in ascending or descending order. The classes or groups are framed according to the requirement and relevance of the statistical survey. The following example is a table of continuous or grouped frequency distribution :

DISTRIBUTION OF MARKS OF 100 STUDENTS

Marks (×)	Number of students (f)
0-10	10
10-20	12
20-30	14
30-40	20
40-50	38
50-60	4
60-70	2

Basic concepts in the formation of continuous series :

Before forming a continuous series, the following basic concepts should be taken into consideration. However, the statistician has the discretion to apply the concepts in classification as per his skill and requirements.

(i) Class intervals:

The groups or classes into which the entire data is classified are known as class-intervals. For example, 10-20 marks is a class-interval or a class.

(ii) Magnitude of a class or class interval:

The length of the class interval is called the magnitude (difference) of the class interval . In a class of 10-20, the magnitude of the class is 10 (20 minus 10).

(iii) Class limits :

The two values specifying the class are called the class limits of that particular class. The upper or larger value in called upper limit and the lower or smaller value is called lower limit of the class. In a class of 10-20, 20 is the upper limit and 10 is the lower limit.

(iv) Class frequency:

Number of observations included in a particular group or class is known as the frequency of the class or the class frequency. If a class 10-20, a class frequency of 7, it implies that the data has seven values between 10-20.

(v) Mid-value or mid points

The middle value of a class interval is the mid-value or mid-point. The value obtained by dividing the sum of lower limit and upper limit of a class by two, is known as the mid-value of the class.

$$\text{Mid-value} = \frac{\text{Lower limit} + \text{Upper limit}}{2}$$

$$\text{In a class 10-20, midvalue} = \frac{10+20}{2} = 15$$

(vi) Number of class intervals :

The number of classes into which the entire data is to be divided, is suggested by Prof. Sturges, known as Sturges' rule. If N is the total frequency and K is the number of classes, then

$$K = -1 + 3.222 \log_{10} N$$

For example, if the total frequency is 10,000, then the number of classes (K) = 1+3.222 $\log_{10}$10000=14 (Approx)

Types of continuous series :

The classification of a frequency distribution into different classes can be studied under the following heads :

(i) Exclusive type of classes:

In exclussive type of classes the upper limiting value of a class is excluded from the same class and included in the next class. In other words the upper limit of a class becomes the lower limit of the next class. For example, in a class interval 20-30, the value 30 is not included in the class 20-30 but included in the next class 30-40. The examples in the following table will explain the meaning of exclusive type classes clearly:

Age (in years)	Age (in years)	Age (in years)
10-20	$10 \leq x < 20$	10 and above but below 20
20-30	$20 \leq x < 30$	20 and above but below 30
30-40	$30 \leq x < 40$	30 and above but below 40
40-50	$40 \leq x < 50$	40 and above but below 50

(ii) Inclusive type of classes :

In inclusive type of classes both upper and lower limits are included in the class. In a class 20-29, 29 is included in the same class 20-29 whereas the next class is 30-39. The meaning of inclusive type of classes is explained below :

Age (in years)	Age (in years)
10-19	10 and above upto 19
20-29	20 and above upto 29
30-39	30 and above upto 39
40-49	40 and above upto 49

In inclusive type of classes there is a gap between upper limit of a class and lower limit of the next class which causes difficulty in certain statistical measures. This necessitates conversion of inclusive type of classes to exclusive type of classes. This conversion makes the distribution continuous. A technique for correction for continuity is applied and new classes of exclusive type are obtained.

In the above example the gap between the upper limit of any class and lower limit of the succeeding class is one i.e. (20-19) or (30-29) or (40-39). A correction factor is determined from this gap and adjusted with the old upper and lower limits. The new class limits are obtained which makes the distribution continuous. The process can be observed from the following example:

$$\text{Correction factor} = \frac{\text{Difference between upper limit of one class and lower limit of next class}}{2}$$

New upper limit = Old upper limit + correction factor

New lower limit = Old lower limit-correction factor

Age (in years)	Age (in years) with new class limits
10-19	(10-0.5) - (19+0.5) i.e. 9.5 - 19.5
20-29	(20-0.5) - (29+0.5) i.e. 19.5 - 29.5
30-39	(30-0.5) - (39+0.5) i.e. 29.5 - 39.5
40-49	(40-0.5) - (49+0.5) i.e. 39.5 - 49.5

$$\text{Correction factor} = \frac{(20-19) \text{ or } (30-29) \text{ or } (40-39)}{2} = \frac{1}{2} = 0.5$$

(iii) Open end classes :

In open classes, either the lower limit of a class or upper limit of a class is not specifically mentioned. Usually the lower limit of the first class and upper limit of the last class are found open. Below 10, above 40 are examples of open end classes. In such cases mid values cannot be accurately determined. Hence statistical measures based on mid-values cannot be found out without some assumptions. It is a practice to accept the magnitude of the first class as the same, as that of the succeeding class and that of the last class as the same, as that of the preceding class.

Age (in years)

Below 10 $\rightarrow$ Open end class = 0-10

10-20

20-30

30-40

Above 40 $\rightarrow$ Open end class = 40-50

Such assumption may lead to some error in further algebraic treatment, hence it should be avoided as for as possible.

(iv) Continuous series with un-equal class-intervals:

Continuous series may have class-intervals with unequal magnitudes. Nature and type of data with wide fluctuations may compel formation of such unequal class-intervals. Following two examples are continuous series with unequal class-intervals.

Age	Number of workers
10-15	8
15-20	12
20-30	26
30-40	32
40-60	24
60-70	18
Total	**120**

In the above example the magnitude of the class-intervals 10-15, 15-20 is 5, where as the magnitude of the class-intervals 20-30, 30-40, 60-70 is 10 and that of 40-60 is 20.

The magnitudes of the class-intervals can be made equal by two rearrangement processes:

(i) One or more groups or classes may be merged to form a single group of class.

(ii) One group or class may be divided into more number of groups or classes In such case the frequencies are also apportioned proportionately.

The re-arrangements of the given example of unequal class intervals are shown below:

Age	**Number of workers**
0-10	0
10-20	20
20-30	26
30-40	32
40-50	12
50-60	12
60-70	18

In the above example the class (10-15) and (15-20) and (15-20) are merged to form one class interval i.e. (10-20) with frequency of (8+12) i.e. 20. Similarly the class-interval 40-60 is divided into two class intervals of 40-50 and 50-60 with equal apportionment of frequency i.e. 12 and 12 (24÷2).

(v) Continuous series represented by mid-values:

In such continuous series instead of class-intervals their mid-values are given. But it is necessary to rewrite the class-intervals with upper and lower limits for further statistical analysis. Let us take an example of such mid-value form of frequency distribution and then try to bring the regular class intervals from them.

Age (Mid-values)	**Number of workers**
5	10
15	30
25	40
35	20
Total	100

If the mid-values show a constant increase or decrease in their values, then the following process may be adopted to find out the original class-intervals.

Half of the difference between any two consecutive mid-values is to be subtracted from the mid-values to get the respective lower limits and is to be added to the mid-values to get the respective upper limits of the class intervals.

In the above example half of the difference of two consecutive mid values $= \frac{15-5}{2} = 5$

So the rearrangement of class intervals are as below:

Age (with class intervals)	Number of workers (f)
(5-5)-(5+5)=0-10	10
(15-5)-(15+5)=10-20	30
(25-5)-(25+5)=20-30	40
(35-5)-(35+5)=30-40	20
Total	100

(vi) Continuous series with cumulative frequency:

The cumulative frequencies can be obtained by adding the individual frequencies of the classes prior to the present class. These frequencies appear when the values of the variable are

expressed as below (less than) or above (more than) forms. These cumulative frequencies are of two types i.e.

(i) Less than cumulative frequency and

(ii) More than cumulative frequency.

Examples of continuous series with the above two types of cumulative frequencies are given below:

Table A_1	
Marks	**Frequency (f)**
10-20	2
20-30	7
30-40	12
40-50	8
50-60	1

	Table A_2	
Marks	**Lessthan c.f.**	**Frequency**
Less than 10	0	0
Less than 20	2	2
Less than 30	2+7	9
Less than 40	9+12	21
Less than 50	21+8	29
Less than 60	29+1	30

	Table A_3	
Marks	**More than Cumulative frequences (c.f)**	**Frequency**
More than 10	2+28	30
More than 20	7+21	28
More than 30	12+9	21
More than 40	8+1	9
More than 50	1	1

In table A_1 the class frequences are stated against the class intervals which are not expressed in forms of 'less than' or 'more than'. But the marks are expresed in the form of 'less than' as shown in Table A_2, the frequencies are added to state the cumulative frequencies against the 'less than' type values. Similarly when the marks are expresed in 'more than' type as shown in

Table A_3, we start with total frequencies and go on sub tracting the previous frequencies. These also become frequencies of more than type of classes.

At times, we may be required to find out the indvidual frequencies from the cumulative frequencies. The individual class frequencies can be obtained by deducting the previous cumulative frequency from the respective cumulative frequency. The process is illustrated below :

Example (less than series of table A_2

Marks	Frequency
10-20	2+0=2
20-30	9-2=7
30-40	31-9=12
40-50	29-21=8
50-60	30-29=1
Total	30

Example (More than series of Table A_3

Marks	Frequency
10-20	30-28=2
20-30	22-21=7
30-40	21-9=12
40-50	9-1=8
50-60	1-0=1
Total	30

12.1.7 BIVARIATE FREQUENCY DISTRIBUTION

The frequency distribution of a single variable is called univariate frequency distribution. When the frequency distribution of two variables are shown simultaneously, it is called bivariate frequency distribution. If there are m classes for X-variable and n classes for Y-variable, then there will be mxn cells in the two-way table and each cell will show frequency for a pair of values (x,y). This kind of frequency is seen in correlation analysis. An example of bivariate frequency distribution is given below:

Age of wives (years)	Age of husbands (years)				
	30-35	35-40	40-45	45-50	
20-25	5	9	3	-	17
25-30	-	10	25	-	35
30-35	-	2	12	2	16
35-40	-	-	4	8	12
Total	5	21	44	10	80

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TABULATION OF DATA

STRUCTURE

12.2.1 Meaning and importance
12.2.2 Characteristics
12.2.3 Objectives of tabulation
12.2.4 Distinction between classification and tabulation
12.2.5 Essentials of good tabulation
12.2.6 Parts of a table
12.2.7 Types of tables
12.2.8 Examples, Exercise
EXERCISE

12.2.1 MEANING AND IMPORTANCE:

Tabulation is a process of presenting data in a condensed form. It is the systematic presentation of information in rows and columns. The horizontal arrangements are called rows and the vertical arrangements are called columns. In tabulation, the information in the data are presented in a summarised way within a minimum space without affecting the quality of the data.

Tabulation involves preparation of statistical table which is the logical listing of related quantitative data in vertical columns and horizontal rows in a meaningful manner.

According to Corner, "Tabulation involves the orderly and systematic presentation of numerical data in a form designed to elucidate the problem under consideration."

According to D. Gregory and H Ward, "Tabulation is the process of condensing classified data in the form of a table so that it may be more easily understood, and so that any comparisons involved may be more readily made."

Tabulation is an important aspect of statistical analysis. It facilitates futher statistical analysis and interprotation. It arranges the data in such a manner that the tabulated data becomes suitable for comparison, diagrammatic representation and further statistical analysis. It is the base for subsequent statistical applications. It follows classification of statistical data and forms a part of organisation of data, i.e. the second stage of statistical enquiry or survey.

12.2.2 CHARACTERISTICS :

- From the above definitions, essential features of tabulation may be discussed as under :

(i) It is a systematic arrangement of quantitative statistical data.

(ii) The data is arranged in rows and columns.

(iii) Explanatory notes must be added to it to make the data clear.

(iv) Tabulation is done after classifying the data.

12.2.3 OBJECTIVES OF TABULATION :

The main objectives of tabulation of data are discussed below:

(i) Tabulation simplifies the complex data which becomes easy to understand.

(ii) Systematic arrangement of data facilitates easy comparison.

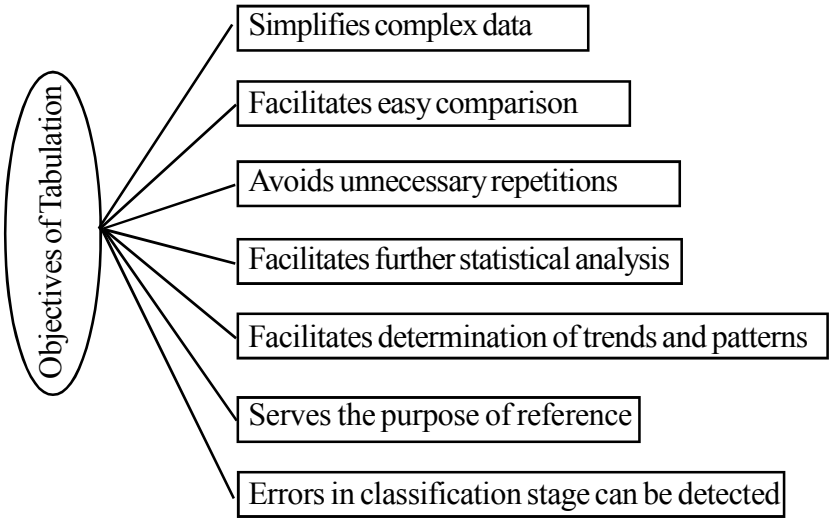
(iii) It avoids unnecessary repetitions of data as it condenses the data in a systematic manner.

(iv) It facilitates further statistical analysis of data like average, dispersion, skewness correlation etc.

(v) It also facilitates to determine trends and patterns of movement of data.

(vi) It serves the purpose of reference, specially when secondary data are used.

(vii) Errors if any, at the classification stage can be detected through tabulation:



12.2.4 DISTINCTION BETWEEN CLASSIFICATION AND TABULATION:

Classification and tabulation are the final stages of collection and compilation. There are some differences between them. These differences are stated below:

	Classification	Tabulaton
Step	Classification is the first step of the process after collection of data.	Tabulation is the next step after classification. Classification is the base for tabulation.
Arrangement	It is the process of arrangement of the data is different groups or classes.	It is the process of systematic arrangement of data in rows and columns.
Nature	It is a statistical analysis and presentation.	It is a statistical representation of data.

12.2.5 ESSENTIALS OF A GOOD TABULATION:

Before constructing a statistical table one should have clear understanding as to the nature and purpose of the data to be arranged. Practical experience helps a lot in making this table more meaningful and useful. Following are the essential features of designing a good table.

(i) The statistical table should be simple and comprehensive as its main objective is to simplify the complexity of data.

(ii) It should be designed in an attractive manner.

(iii) Unnecessary complication in the table should be avoided.

(iv) It should be so designed that meaningful comparison becomes possible. One of its objective is that it facilitates comparative analysis of data.

(v) The tables should have absolute clarity and be free from ambiguity.

(vi) It should ensure economy of space and time.

(vii) It should indicate the trend and pattern of data shown therein.

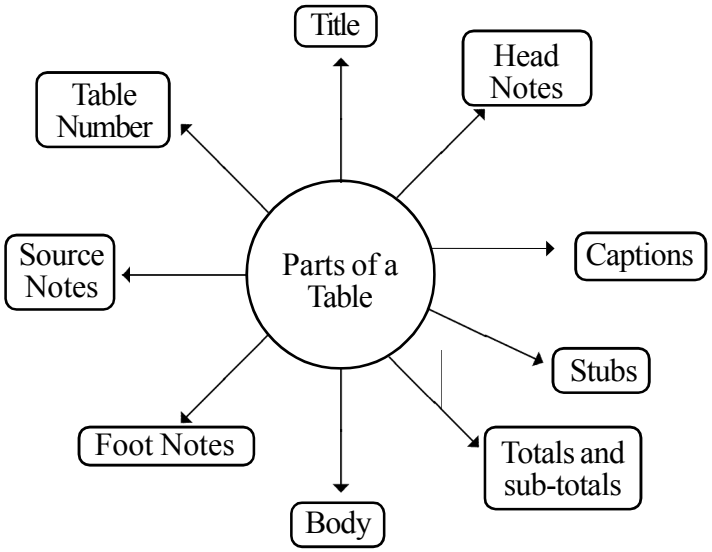
(viii) It should be capable of detection of errors, of any, at classification stage.

12.2.6 PARTS OF A TABLE:

Construction of various parts of table depends on the purpose of investigation and the nature of the data. The design of a table is very important for easy accessibility and meaningful conclusions. A good statistical table should have the following parts.

(i) Table number
(ii) Table title
(iii) Caption and Stubs
(iv) Body of the table
(v) Totals and sub-totals.
(vi) Head Notes
(vii) Foot Notes
(viii) Source Notes

Parts of a table at a glance:



The above mentioned parts of a table are explained below:

(i) **Table number :** A serial number is allotted to each table for easy identification and reference. This table number is usually written at the top of the table, on the left-hand side along with the title of the table. For example,

Table 12.04 COMPUTATION OF LINEAR TREND

(ii) **Table title :** Every table is given a self explanatory, brief title or name on the top of the table. The title explains the contents of the table.

(iii) **Captions and stubs:** The heading of the vertical values or column is known as captions. There may be sub headings under one column. If necessary, relative units like ₹, ratio etc. may be mentioned along with the captions. The captions should be precise and meaningful.

Stubs are headings of the rows or horizontal values. Usually the stubs are given at the extreme left-hand side of rows. Stubs should be precise and appropriate.

(iv) Body of the table: The body of the table is the most important portion of the table. It contains the numerical information meant for the purpose of investigation or analysis. These data are arranged in rows and columns.

Totals and sub-totals : Figures of columns and sub columns and of rows are to be totalled and sub-totalled.

(v) Head Notes : It is a brief statement given below the title of the table which explains the contents of the table. For examples, In rupees in lakhs, In billions of tonnes etc. The unit of measurement is usually written in brackets.

(vi) Foot-notes : Foot-note is a brief statement to clarify any confusion or ambiguity in the content of the table. It is usually stated below the body of the table. Any exceptional situation is explained in the foot-note.

(vii) Source notes: Source note indicates the sources from which data are collected. This note is also stated below the body of the table. It is given to verify the authenticity of data and to make reference of the information in the data. For secondary data, source of information is always given.

A specimen of a statistical table is given below:

Table 12.04 Faculty, sex and course-wise distribution of students

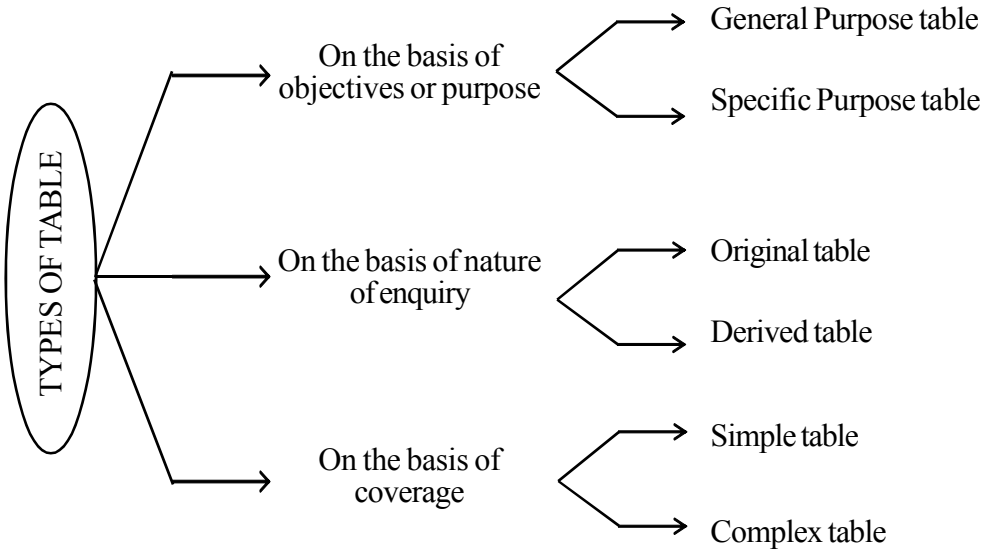
Faculty (caption) / Courses (Stub)	Commerce			Science			Total		
	No. of Boys	No. of Girls	Total	No. of Boys	No. of Girls	Total	No. of Boys	No. of Girls	Total
Management									
C.A.									
Costing									

Foot note : Management students include part-time management students.

12.2.7 TYPES OF TABLES :

On the basis of objectives, nature and scope of enquiry and the extent of coverage, different types of statistical tables are prepared. The types of table can be broadly classified as below:

Types of table at a glance:



(a) Classification on the basis of objectives or purpose:

On the basis of the purpose for which the tables are designed, the tables can be of two types, namely general purpose table and specific purpose table.

(i) General purpose table: General purpose tables provide information for general use. These tables are suitable for records and quick reference. The tables are usually not meant for comparison or establishment of relationship. It is often referred to as reference table. The tables published by Government agencies i.e. tables in Economic Survey, RBI reports, Census reports etc. are general purpose tables.

(ii) Specific purpose tables: Specific purpose tables are constructed for a particular purpose. These tables are analytical by nature and enables comparative studies. From these tables ratios, percentages, coefficients etc. are derived to facilitate comparisons. These tables are also known as interpretative tables or summary tables. These tables are derived from the general purpose tables.

(b) Classification on the basis of nature of enquiry:

On the basis of the nature of enquiry, the tables can be of two types, namely original table and derived table.

(i) Original table: The original table is also known as primary table. It contains the original data or actual figures. The table containing a time series is an original table.

(ii) Derived table: The derived table contains information expressed in terms of ratios, percentages, averages etc. It contains information derived from the original data. The table showing trend values, seasonal and cyclical variations are derived tables.

(c) Classification on the basis of coverage:

On the basis of coverage of the number of characteristics the tables can be of two types namely, simple table and complex table.

(i) Simple table : Simple table or one way table furnishes information about a single characteristic. For example, the sales of an organisation over a particular period of time are shown a simple table as below:

Sales of X Ltd. over 2010-2016

Year	Sales (in lakhs of rupees)
2010	537
2011	600
2012	612
2013	658
2014	702
2015	740
2016	788

(ii) Complex table: The table containing information relating to two or more characteristics is known as a complex table. When two characterists are shown in the same table simultaneously it is known as two way table or bivariate table. Here caption or the stub is divided into two sub-groups. As the information is about two inter-related characteristics, a bivariate form of table is constructed.

At times more than two inter-related characteristics are shown in the same table. These tables are known as manifold tables. Examples of two way table and manifold table are given below:

Two way table

Age of wives \ Age of husbands	25-30	30-40	40-50
20-25	5	15	2
25-30	2	7	7
30-35	-	2	10

Manifold table

Courses \ Faculty	Commerce		Science		Total	
	Boys	Girls	Boys	Girls	Boys	Girls
Management	5	7	10	7	15	14
C.A.	11	3	4	2	15	5
Costing	4	2	3	1	7	3

12.2.8. After discusing about the various aspects of classification and tabulation, let us workout some practical problems on them:

Example-1

Prepare a discrete frequency distribution from the following data with the help of tally bars.

6,7,5,7,3,4,3,6,4,5,3,4,6,4,4,5,5,4,3,4,5,3,4,5,3,4,

6,3,4,4,3,4,5,4,3,4,5,6,4,3,4,3,4,5,7,6,5,7,4,3.

Solution :

FREQUENCY DISTRIBUTION TABLE

Values (x)	Tally Bars	Frequency (f)
3	𝍸 𝍸 //	12
4	𝍸 𝍸 𝍸 ///	18
5	𝍸 𝍸	10
6	𝍸 /	6
7	////	4
Total		∑f=50

Example-2

Prepare a continueus frequency distribution from the following data taking magnitude of the class as 10.

Marks (x) :

8	44	9	42	27	20
12	38	41	33	35	37
27	9	21	20	25	28
32	24	39	13	14	15
48	17	45	22	24	25

Solution :

FREQUENCY DISTRIBUTION TABLE

Marks	Tally Bars	Frequency (f)
0-10	///	3
10-20	~~////~~	5
20-30	~~////~~ ~~////~~ /	11
30-40	~~////~~ /	6
40-50	~~////~~	5
Total		$\sum f=30$

Example-3

Construct an inclusinve type of continuous series from the following data with magnitude of class interval as 9:

Marks (x) :	10	44	9	42	27	20
	12	30	41	33	35	37
	27	9	20	20	25	28
	32	8	39	13	14	15
	48	24	40	22	24	25

Solution :

Marks	Tally Bars	Frequency (f)
0-9	///	3
10-19	~~////~~	5
20-29	~~////~~ ~~////~~ /	11
30-39	~~////~~ /	6
40-49	~~////~~	5
Total		$\sum f=30$

Example- 4

Convert the following inclusive type of continuous series into exclusive type of continuous series :

Daily wages (in rupees)	Number of workers (f)
100-199	10
200-299	38
300-399	25
400-499	18
500-599	9
Total	100

Solution :

The distribution in the question is an inclusive type of continuous series. Now to convert it to exclusive type of continuous series, the following correction factor is to be determined.

$$\text{Correction factor} = \frac{200-199}{2} = \frac{1}{2} = 0.5$$

The correction factor, 0.5 is subtracted from lower limits, and added to upper limits. Then the new class intervals become exclusive type of continuous series as shown below:

Wages (in ruppees)	Number of workers (f)
99.5-199.5	10
199.5-299.5	38
299.5-399.5	25
399.5-499.5	18
499.5-599.5	9
Total	$\sum f=100$

Example- 5

Construct the class intervals from the following mid-values;

Mid-values :	150	250	350	450	550
Frequency:	10	38	25	18	9

Solution :

The mid-values are increasing uniformly by 100. Half of this increase is equal to (100÷2) 50. Now by subtracting 50 from mid-values, we get lower limits and adding 50 to mid-values, we get upper limits. Thus the class intervals of the mid-values are as follows:

Mid-values	Calculation of class intervals	Class intervals	Frequency
150	(150-50)-(150+50)	100-200	10
250	(250-50)-(250+50)	200-300	38
350	(350-50)-(350+50)	300-400	25
450	(450-50)-(450+50)	400-500	18
550	(550-50)-(550+50)	500-600	9

Example-6

Construct a normal continuous frequency distribution from the following cumulative frequency series :

(A)

Less than Series

Marks	No. of students
Below 5	2
Below 10	12
Below 15	47
Below 20	68
Below 25	85
Below 30	100

(B)

More than Series

Marks	No. of students
Above 10	50
Above 20	42
Above 30	30
Above 40	23
Above 50	18
Above 60	5

Solution:

(A)

Normal continuous series

Marks	No. of students
0 - 5	2
5 - 10	12 - 2 = 10
10 - 15	47 - 12 = 35
15 - 20	68 - 47 = 21
20 - 25	85 - 68 = 17
25 - 30	100 - 85 = 15

(B)

Normal continuous series

Marks	No. of students
10 - 20	50 - 42 = 8
20 - 30	42 - 30 = 12
30 - 40	30 - 23 = 7
40 - 50	23 - 18 = 5
50 - 60	18 - 5 = 13
60 - 70	5 - 0 = 5

Example- 7

Convert the following continuous series with unequal magnitude in to a continuous series of equal magnitude:

Age (in years)	No. of persons
0-10	7
10-20	12
20-40	32
40-50	21
50-70	28

Solution:

In class-intervals 0-10, 10-20, 40-20, 40-50, the magnitude is 10 and in class intervals 20-40 and 50-70, the magnitude is 20. We shall prefer to maintain the magnitude of class intervals as 10. Accordingly the new classes will be 0-10, 10-20, 20-30, 30-40, 40-50, 50-60, 60-70 The frequencies of the splitted class interval will also be accordingly determined. Now, the new distribution with equal magnitude will be as follows:

Age (in years)	Number of persons	Calculation
0-10	7	7
10-20	12	12
20-30	16	32÷2=16
30-40	16	32÷2=16
40-50	21	21
50-60	14	28÷2=14
60-70	14	28÷2=14

Example 8

Represent the following data by a bivariate frequency distribution:

Age of husbands :	24	26	27	28	24	29	28	25	26	28	27	24
Age of wives :	17	18	19	20	18	20	21	20	19	23	20	17

Solution

Let us take age of husbands as X and age of wives as Y and display their corresponding ages in the bivariate frequency distribution table.

Age of husbands (x) / Age of wives (y)	24	25	26	27	28	29
17	// (2)					
18	/ (1)		/ (1)			
19			/ (1)	1(1)		
20		/ (1)			//(2)	/ (1)
21					/ (1)	
23					/ (1)	

Example- 9

Draw a blank table showing number of students studying in +2 1st year and 2nd year classes in a college in Arts, Science and Commerce streams during the year, 2016-17:

Solution

Table showing of number of students in different streams.

Stream / Class	+2 Science	+2 Commerce	+2 Arts	Total
First Year				
Second Year				
Total				

Example- 10

Tabulate the following data:

The students strength during last 4 years is given below:

2012-13	1000
2013-14	970
2014-15	920
2015-16	900

During the year 2012-13, the number of students in Arts, Science and Commerce were 256, 512, 232 respectively.

During the year 2013-14, the number of students in Arts, Science and Commerce were 250, 512, 208 respectively.

During the year 2014-15, the number of students in Arts, Science and Commerce were 200, 512, 208 respectively.

During the year 2015-16, the number of students in Arts, Science and Commerce are 188, 512, 200 respectively.

Solution :

Year	Number of students			
	Arts	**Science**	**Commerce**	**Total**
2012-13	256	512	232	1000
2013-14	250	512	208	970
2014-15	200	512	208	920
2015-16	188	512	200	900

Example- 11

From the following data construct a bivariate frequency distribution table:

120 male workers and 80 female workers get daily wages between ₹ 100 and ₹ 149.

60 male workers and 50 female workers get daily wages between ₹ 150 and ₹ 199.

160 male workers and 110 female workers get daily wages between ₹ 200 and ₹ 249.

Solution

The following table shows the values of daily wages in the columns and the number of workers (sex-wise) in the rows.

Table showing daily wages of workers (sex-wise)

Daily wages(₹)	Number of workers		
	Male workers	Female workers	Total
100 - 150	120	80	200
150-200	60	50	110
200-250	160	110	270
Total	340	240	580

Example- 12

Tabulate the following information in a suitable form :

In a factory, in the year 2015 the total number of workers were 1000, out of which 750 were members of a trade union. The number of female workers were 130 out of which only 30 were members of the trade union.

In the same factory, in the year, 2016 the number of union workers were 810 out of which 750 were men. The number of workers who were not member of the trade union were 190 and of which 100 were female workers.

Solution :

Year / Trade Union	2015			2016		
	Males	Females	Total	Males	Females	Total
Members	720 (750-30)	30	750	750	60 (810-750)	810
Non-members	150 (250-100)	100 (130-30)	250	90 (190-100)	100	190
Total	870	130	1000	840	160	1000

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EXERCISE

A.1. From the given alternatives, choose and write the correct answer along with its serial number against each bit:

(i) Classification of data refers to arrangement of data in:

(a) Groups

(b) Tables

(c) Rows

(d) Columns

(ii) Arrangement of data in rows and columns refers to:

(a) Classification of data

(b) Tabulation of data

(c) Seriation of data

(d) Editing of data

(iii) Geographical classification of data refers to classification of data:

(a) On the basis of areas

(b) On the basis of time

(c) On the basis of attributes

(d) On the basis of magnitude

(iv) The horizontal group headings of a table is known as:

(a) Captain

(b) Stub

(c) Sub-caption

(d) Body

(v) The number of occurrences of a value of a variable is known as

(a) Class interval

(b) Class limit

(c) Class mark

(d) Class frequency

(vi) Classification on the basis of literacy is a :

(a) Geographical classification

(b) Chronological classification

(c) Qualitative classification

(d) Quantitative classification

(vii) The difference between upper and lower limits of a class is known as:

(a) Class limit

(b) Class frequency

(c) Magnitude of the class

(d) Class interval

(viii) When the upper limit of a class is same as the lower limit of the next class, the series is known as:

(a) Inclusive continuous series

(b) Exclusive continuous series

(c) Open and continuous series

(d) Cumulative continuous series

(ix) The vertical column headings of a table is known as:

(a) Stub

(b) Caption

(c) Head note

(d) Foot note

(x) A statistical table with more than two characteristics is known as :

(a) Simple table

(b) Two- fold table

(c) Manifold table

(d) One way table

2.(a) Express the following in one word/term:

(i) Number of times a particular value of a variable occurs.

(ii) The process of arranging data in groups or classes according to their common features.

(iii) The process of arranging data in rows and columns.

(iv) A type of table which provides information for general use and reference.

(v) A type of table which is constructed for a particular purpose.

(vi) The value obtained by dividing the sum of upper and lower limits of a class interval by two.

(vii) The class of a continuous series where one of the limits is not specified.

(viii) The type of classification where data are classified on the basis of occurrence of time.

(b) Answer the following question is one sentence:

(i) What is classification of data ?

(ii) What is is tabulation of data ?

(iii) What do you mean by chronological classification ?

(iv) What do you mean by two fold table ?

(v) What is a frequency distribution ?

(vi) Name any two parts of a statistical table.

(vii) Give an example of inclusive type of continuous series.

(viii) Give an example of geographical classification of data.

(c) Write the correct answer for the underlined portions of the following sentences:

(i) Systematic arrangement of data in rows and columns is known as <u>classification</u> of data.

(ii) A brief explanatory statement given below the body of a table to clarify confusions is known as <u>head</u> note.

(iii) <u>Caption</u> refers to horizontal row heading of statistical table.

(iv) Classification succeeds tabulation.

(v) In a discrete series the bulk of data is reduced to groups or classes.

(vi) In an inclusive type of class-interval the upper limit is excluded from the same class-inverval.

(vii) Sturges' rule provides a rule of thumb for calculating the proper magnitude of classes.

(viii) In chronological classification, the data are classified on the basis of area.

(d) Fill in the blanks:

(i) The number of observations in a particular class is called the _____ of the class.

(ii) The difference between the upper and lower limits of a class gives _____ of the class.

(iii) If the midpoints of the classes are 8,16,24,32 and so on, then the magnitude of the class intervals is _____.

(iv) In chronological classification, the statistical data are classified on the basis of _____.

(v) In a statistical table, captions refer to the _____.

(vi) _____ precedes tabulation.

(vii) _____ is a process of orderly arrangement of data in rows and columns.

(viii) _____ is a process of arrangement of data in different classes.

B 3. Anser the following questions within two sentences:

(i) State any two objectives of classification.

(ii) Name the types of classification.

(iii) What do you mean by classification of data ?

(iv) What is a frequency distribution ?

(v) Give two examples of qualitative classification.

(vi) Give an example of chronological classification.

(vii) What do you mean by series of individual observations ?

(viii) What do you mean by tabulation of data ?

(ix) What do you understand by general purpose tables ?

(x) Name any four parts of a statistical table.

(xi) What do you mean by tally bars ?

(xii) State any two objectives of tabulation of data.

(xiii) What is a manifold table ?

(xiv) What is meant by cumulative frequency ?

4. **Answer the following questions within six sentences:**

(i) State the objectives of classification of data.

(ii) State the objectives of tabulation of data.

(iii) Distinguish between classification and tabulation.

(iv) Distinguish between inclusive type and exclusive type of continuous series.

(v) Explain geographical classification with example.

(vi) Explain chronological classification with example.

(vii) Distinguish between simple table and complex table.

(viii) Distinguish between general purpose table and specific purpose table.

(ix) What is a bivariate frequency distribution ?

(x) Give a specimen of a statistical table showing its different parts.

(xi) Give one example each of inclusive and exclusive type continuous series.

(xii) Give a specimen of twofold table.

(xiii) Distingush between discrete series and continuous series.

(xiv) Construct class intervals from the following mid-values :

Mid values : 24 28 32 36 40

LONG TYPE QUESTIONS :

5. What is meant by classification ? Explain the different types of classification.
6. Define classification. Explain its objectives.
7. Define tabulation. Explain its objectives.
8. What is classification ? How does it differ from tabulation ?
9. What is tabulation ? Explain the different parts of a table.
10. Construct a blank table showing the number of students, sex-wise appearing in +2 1st year Arts, Science and Commerce.
11. Discuss the functions and importance of tabulation.
12. Prepare a discrete series from the following data :

X : 20	16	12	16	14	12	18	14	12	14
18	14	18	18	10	10	12	16	16	16
20	20	16	14	12	14	14	18	14	14
12	16	12	20	18	16	12	10	14	14
14	14	16	14	10	14	16	12	16	18

13. From the following distribution prepare the continuous series class-intervals:

Mid values :	15	25	35	45	55	65	75
Frequency:	10	20	32	18	12	6	2

14. From the following distribution prepare continuous series with normal frequencies:

(A)

Values (x)	Cumulative frequencies
Below 10	7
Below 20	17
Below 30	32
Below 40	47
Below 50	50

(B)

Values (x)	Cumulative frequencies (c.f)
Above 90	4
Above 80	18
Above 70	38
Above 60	49
Above 50	63
Above 40	70

15. Construct a continuous series from the following data with a class internal of 10 marks.

Marks: 46, 74, 57, 44, 60, 48, 63, 50, 65, 56, 35, 68, 55, 40, 55, 63, 78, 19, 17, 79, 44, 73, 69, 64, 71, 89, 94, 77, 25, 36, 47, 29, 39, 40, 50, 60, 74, 70, 80, 10, 20, 30, 34, 84, 92, 98, 97, 14, 16, 54.

16. Convert the following inclusive type of continuous series to exclussive type of continuous series:

Marks (x) :	10-19	20-29	30-39	40-49	50-59	60-69	70-79
No of students(f):	7	13	25	38	12	3	2

17. Give one example each for the following data:

a) Exclusive type of continuous series

b) Inclusive type of continuous series

c) Open-end class-interval type of continuous series

d) Unequal class-intervals type of continuous series

e) Series with cumulative frequencies.

(both less than and more than series)

18. From the following data construct a two way table:

1000 male workers and 800 female workers get daily wages between ₹100 and ₹199.

700 male workers and 400 female workers get daily wages between ₹ 200 and ₹ 299.

1500 male workers and 1000 female workers get daily wages between ₹ 300 and ₹ 399.

1800 male workers and 800 female workers get daily wages between ₹ 400 and ₹ 499.

19. Convert the following into an ordinary frequency distribution.

6 students got less than 10 marks,

27 students got less than 20 marks,

42 students got less than 30 marks,

50 students got less than 40 marks.

20. From the following distribution prepare 'less than' and 'more than' cumulative frequency distributions:

Weight (In kgs):	30-40	40-50	50-60	60-70	80-90
No of persons:	10	17	33	20	12

21. From the following ages in years of married husbands and wives, prepare a bivariate frequency table to represent the data:

Age of husband:	25	26	27	24	27	28	25	26	26	27
Age of Wife:	18	19	20	18	20	23	20	22	21	21
Age of husband:	24	28	25	26	27	26	24	27	26	27
Age of Wife:	20	22	20	21	23	22	19	21	22	23.

22. Marks obtained by 20 students is Accounting and Business Mathematics are given below. Construct a bivariate frequency table:

Serial Number	**Marks in Accounting**	**Marks in Business Mathematics**
1	60	85
2	61	82
3	60	84
4	63	85
5	61	80
6	64	86
7	62	81
8	60	82
9	61	83
10	63	85
11	64	86
12	65	86
13	60	81
14	61	82
15	62	83
16	61	82
17	64	83
18	61	84
19	62	82
20	60	83

23. Explain the relative importance of classification and tabulation.

24. What is meant by a bivariate series ? Construct a bivariate discrete series with imaginary figures.

ANSWERS

A.1. (i) a (ii) b (iii) a (iv) b (v) d (vi) c (vii)c (viii) b (ix) b (x) b.

2.(a) (i) Frequency (ii) Classification (iii) Tabulation (iv) General Purpose Table

(v) Specific Purpose Table (vi) Mid-value (vii) Open end class

(viii) Chronological classification

(b) (i) Tabulation (ii) Foot (iii) Stub (iv) Precedes

(v) Continuous (vi) Exclusive (vii) Number (viii) Time

(d) (i) Frequency (ii) Magnitude (iii) 4 (iv) Time (v) Vertical

(vi) Classification (vii) Tabulation (viii) Classification.

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Appendix 1—Numerical Tables

TABLE 1 : LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	.0000	0043	0086	0120	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	.0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	.0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	.1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	.1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	.2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	.2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	.2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	.2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	.3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	.3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	.3424	3444	3464	3483	3502	3522	3541	3562	3579	3598	2	4	6	8	10	12	14	15	17
23	.3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	.3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	.3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
27	.4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	.4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	.4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	.4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	.4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	.5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	.5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	.5315	5315	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	.5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	.5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	.5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	.5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	.5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	.6021	6031	6042	6053	6065	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	.6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	.6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	.6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	.6435	6444	6454	6465	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	.6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	.6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	.6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	.6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	.6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	.6990	6993	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	.7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	.7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	.7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7

TABLE 1 : LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
54	.7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	.7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	.7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	.7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	.7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	.7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	.7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	.7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	.7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	.7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	.8062	8069	8075	8082	8989	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	.8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	.8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	.8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	.8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	.8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	.8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	.8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	.8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	.8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	.8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	.8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	.8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	.8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	.8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	.8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	.9031	9036	9042	9047	9053	9058	6063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	.9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	.9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	.9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	.9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	.9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	.9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	.9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	.9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	.9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	.9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	.9590	9595	9600	9605	9609	9614	9619	9624	9628	9933	0	1	1	2	2	3	3	4	4
92	.9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	.9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	.9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	.9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	.9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	.9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	.9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	.9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

TABLE II : ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2513	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

TABLE II : ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9227	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20