6. 0.055 °c
7. 16.49×10^{-2} m
8. (a) 1.53×10^{-2}m; (b) 1.77×10^{-2} m
9. $\dfrac{2\pi s^2}{\rho g}$
10. 4.447 cm
11. 1183415 dyne / cm^2

F. (1) decreases (2) surface tension (3) smaller (4) surface tension (5) 2.4 pa.

G. (1) False (2) True (3) True (4) True (5) False

15

Fluid Motion

15.1 Introduction :

The flow of fluid such as water in a pipe, blood in blood vessels, sailing of clouds, motion of smoke, flood water in river etc. are few common day to day phenomena. In such fluid motions although each particle of fluid obeys Newton's Laws of motion, but the study of the entirety of the fluid flow is very complex.

The flow of fluid can be broadly classified as (i) steady (streamline) flow and (ii) turbulent flow.

15.2 Steady flow :

The flow of a fluid is said to be steady, if the fluid particles attain definite velocities at various points in the fluid

i.e.

$$\ddot{u}(x_1, y_1, z_1, t) = \ddot{u}_1 = \text{constant}$$

$$\ddot{u}(x_2, y_2, z_2, t) = \ddot{u}_2 = \text{constant}$$

$$\ddot{u}(x_n, y_n, z_n, t) = \ddot{u}_n = \text{constant} \quad ...(15.2.1)$$

such that $\ddot{u}_1$, $\ddot{u}_2$ ----- $\ddot{u}_n$ may be in general different or some/all of them may be equal.

The curve traced out by joining the points (x_1, y_1, z_1, t), (x_2, y_2, z_2, t) ---- (x_n, y_n, z_n, t) such that the tangents drawn to the curve at these points lie in the directions of $\ddot{u}_1$, $\ddot{u}_2$ ----- $\ddot{u}_n$ respectively, is called a streamline. Thus :

"A streamline is defined as a curve in the fluid such that a tangent drawn at any point of it gives the direction of fluid velocity at that point."

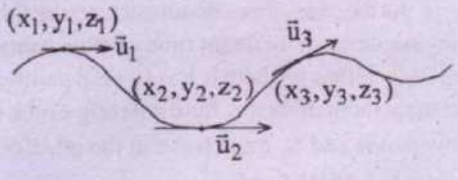


fig. 15.1

Properties of streamlines :

i) A streamline represents the actual path of a fluid particle in steady flow of fluid.

ii) Streamlines are parallel to fluid velocity at each point.

iii) Two streamlines never intersect each other. Because if they do so then it will imply two fluid velocities (along the two possible tangents) at the same points, which is unphysical.

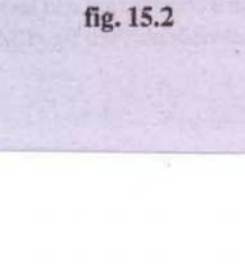


fig. 15.2

Tube of flow :

If we consider an area 's' in a fluid in steady flow and draw streamlines from all the points of the periphery s, then these streamlines enclose a tube of which s is the cross-section. Thus

"A tube of flow is a bundle of streamlines emanating from every point of the periphery of an area of cross-section."

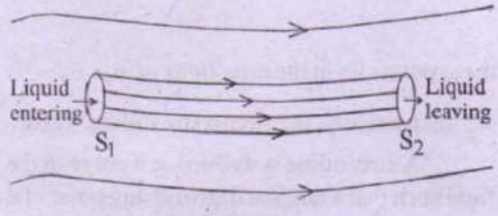


fig.15.3

As the streamlines donot intersect the fluid flowing through different tube of flow cannot intermix, although there is no physical partition between them. It means fluid entering a tube of flow at one end S_1 must leave at the other end S_2 (see fig. 15.3).

When there is an obstacle the tube of flow splits into two portions at a point called stagnation point on the upstream side of the obstacle. These portions rejoin at a second stagnation point on the down stream side. The velocity at the stagnation point is zero. (see fig. 15.4).

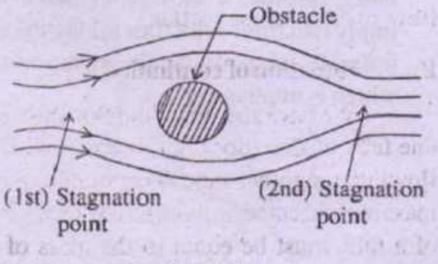


fig. 15.4

The cross-section of all flow tubes decreases near a constriction and increases again as the channel widens. (see fig. 15.5)

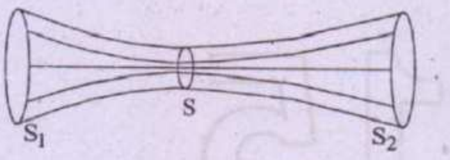


fig. 15.5

(ii) Turbulent flow :

If the velocity of fluid particles at any point in a fluid does not remain constant, then fluid flow is called turbulent.

Fluid flow remains steady for small velocities, but as the velocity increases the flow becomes turbulent.

The maximum velocity (ϑ_c) with which a fluid particle can flow along a streamline is called **critical velocity**. The value of this critical velocity depends on the density (ρ), dimension of the channel (D) and the coefficient of viscosity of the fluid. It is given as

$$\vartheta_c = N_R \, \eta / \rho D \qquad ...(15.2.2)$$

The above equation can be derived by dimensional method. Let

$$\vartheta_c \quad \alpha \, \eta^a$$

$$\alpha \, \rho^b$$

$$\alpha \, D^c$$

Then $\vartheta_c = N_R \, \eta^a \, \rho^b \, D^c \qquad ...(15.2.3)$

Where 'N_R' is a dimensionless constant; called Reynold number.

Equating the dimensions on both sides of eqn. (15.2.3) we have

$$LT^{-1} = \left(ML^{-1}T^{-1}\right)^a \left(ML^{-3}\right)^b \, (L)^c$$

$$\Rightarrow \quad -a - 3b + c = 1 \qquad ...(15.2.4)$$

$$a + b = 0 \qquad ...(15.2.5)$$

$$-a = -1 \qquad ...(15.2.6)$$

Solving the above equations we find

$a = 1, b = -1, c = -1$

Hence

$$\vartheta_C = N_R \frac{\eta}{\rho D} \qquad \text{...(15.2.7)}$$

It is experimentally observed that

i) When $2000 > N_R > 0$ flow is steady

ii) When $N_R > 3000$ flow is turbulent

iii) When $3000 > N_R > 2000$ flow is sometimes steady and sometimes turbulent.

e.g. (a) If we consider flow of water in a tube of diameter 1 cm (i) for $2000 > N_R > 0$, $\vartheta_C \leq \frac{2000 \times .01}{1 \times 1} = 20$ cm/s (ii) for $N_R > 3000$, $\vartheta_C \leq 30$ cm/s.

(b) If we consider flow of air in a tube of diameter 1 cm for $\vartheta_C = 30$ cm, $N_R = \frac{\rho \vartheta_C D}{\eta}$

$$= \frac{0.0013 \times 30 \times 1}{181 \times 10^{-6}} = 215 < 2000$$

Hence flow remain steady.

15.3 Ideal fluid :

As pointed out earlier in sec. 15.1, the study of fluid motion as a whole is complex. But, however, in many situations of practical importance the study of fluid motion can be made simple by idealised models. So we introduce the concept of ideal fluid.

An ideal fluid is characterised by the following properties.

i) It is incompressible. Its density is uniform throughout and remains constant at all times. This assumption is reasonable for almost all liquids but true for gases flowing at low pressure difference only.

ii) It is non- viscous i.e. no internal friction exists between consecutive layers.

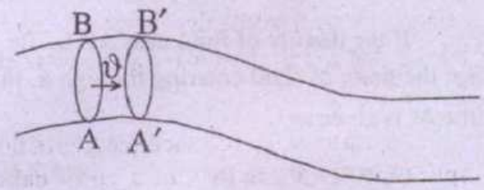
iii) The flow is steady

iv) The flow is irrotational i.e. fluid particles donot possess angular velocity.

A. Rate of flow :

Consider a fluid flowing steadily through a cross - section AB of area 'a'. Let the fluid particles over this cross-section possess velocity ϑ. Then the fluid particles lying over AB at the beginning of the interval shall be over the cross - section A'B' at the end of the interval Δt. The fluid particles entering at the end of the interval Δt, shall be within the elementary cylinder AB B'A'. Hence volume of fluid flowing through AB in time Δt is

$\Delta V = a.\ \vartheta.\ \Delta t$ ...(15.3.1)



This gives

$$\text{rate of flow} = \lim_{\Delta t \to 0} \frac{\Delta V}{\Delta t} = \frac{dV}{dt} = a\vartheta$$

= (area of cross-section) (velocity of flow over the cross section) ...(15.3.2)

B. Equation of continuity :

We have seen that fluid flowing through one tube of flow does not intermix with fluid flowing in another tube. This implies that the mass of fluid entering through any cross-section of a tube must be equal to the mass of fluid flowing out through any other cross-section of the tube. This leads to a mathematical equation called equation of continuity.

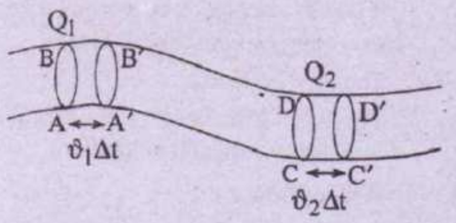


fig. 15.7

Consider two cross-sections AB and CD (fig.15.7) of area a_1 and a_2 respectively, of a tube of flow. Let the speeds of the fluid particles over these cross-sections be ϑ_1 and ϑ_2 respectively. There is no flow across the side wall of the tube, because at every point on the wall the velocity is tangent to the wall. The volume of fluid entering through cross-section S_1 in a time Δt is

$$V_1 = a_1\ \vartheta_1\ \Delta t \qquad ...(15.3.3)$$

If the density of fluid near AB be ρ_1, then the mass of fluid entering through a_1 in time Δt is given as

$$m_1 = a_1 \cdot \vartheta_1\ \Delta t\ \rho_1 \qquad ...(15.3.4)$$

Similarly, the volume of fluid going out in time Δt through CD shall be

$$V_2 = a_2 \cdot \vartheta_2\ \Delta t \qquad ...(15.3.5)$$

and mass of fluid going out through CD, in time Δt is

$$m_2 = a_2 \cdot \vartheta_2\ \Delta t\ \rho_2 \qquad ...(15.3.6)$$

Where ρ_2 is the density of fluid near CD.

Since mass is to be conserved so from (15.3.4) & (15.3.6)

$$a_1 \cdot \vartheta_1\ \Delta t\ \rho_1 = a_2 \cdot \vartheta_2\ \Delta t\ \rho_2 \qquad ...(15.3.7)$$

$$\Rightarrow \quad a_1\ \vartheta_1\ \rho_1 = a_2\ \vartheta_2\ \rho_2 \qquad ...(15.3.8)$$

If the fluid is incompressible (ideal fluid) then $\rho_1 = \rho_2$ and eqn. (15.3.8) reduces to

$$a_1\ \vartheta_1 = a_2\ \vartheta_2 \qquad ...(15.3.9)$$

Eqn. (15.3.8) is known as the **equation of continuity** for an incompressible liquid stated as ; "The product of area of cross - section and speed (over that cross-section) remains the same at all points of a tube of flow."

Ex. 15.3.1 What is the volume of water that flows per minute across a pipe of diameter 2 cm if the speed of water is 0.1 m/s.

Soln.

Rate of flow = $a\,\vartheta$

Volume of water flowing in 1 min = $a.\,\vartheta \times 60s$

$$= \pi\,(1 \times 10^{-2})^2\ m^2 \times 0.1\ m/s \times 60s$$

$$= 1.885 \times 10^{-3}\ m^3 = 1885\ cm^3.$$

Ex. 15.3.2 Water is flowing through a pipe of non-uniform cross - section. The speed of flow is 0.25 m/s at a section of radius 3 cm. Find the speed at the section of radius 2 cm.

Soln.

By equation of continuity

$$a_1\ \vartheta_1 = a_2\ \vartheta_2$$

$$\Rightarrow \quad \pi\,(3 \times 10^{-2})^2 .\ 0.25 = \pi\,(2 \times 10^{-2})^2 x \vartheta_2$$

$$\Rightarrow \quad \vartheta_2 = \frac{(3x10^{-2})^2 x0.25}{(2x10^{-2})^2}\ \frac{m}{s} = 0.5625\ m/s$$

$$\vartheta_2 = 56.25 \times 10^{-2}\ m/s.$$

C. Energy of a flowing fluid :

The energy of a flowing fluid consists of (i) kinetic energy (ii) potential energy and (iii) pressure energy.

i) **Kinetic energy :**

It is the energy possessed by a fluid by virtue of its velocity.

If m be mass of an element of a fluid flowing with velocity $\vec{\vartheta}$. then

$$\text{K.E.} = \tfrac{1}{2} m \vartheta^2 \qquad ...(15.3.10)$$

$$\text{K.E. per unit mass} = \tfrac{1}{2} \vartheta^2 \qquad ...(15.3.11)$$

$$\text{K.E. per unit volume} = \tfrac{1}{2} \rho \vartheta^2 \qquad ...(15.3.12)$$

ii) **Potential energy :**

It is the energy possessed by a fluid by virtue of its position.

If m be mass of an element of fluid situated at a height h above the reference level (ground). then

$$\text{P.E.} = mgh \qquad ...(15.3.13)$$

$$\text{P.E. per unit mass} = gh \qquad ...(15.3.14)$$

$$\text{P.E. per unit volume} = \rho gh \qquad ...(15.3.15)$$

iii) **Pressure energy :**

It is the energy possessed by a fluid by virtue of its pressure :

As an example consider fluid being pushed into a vessel 'V' through a side tube by applying some pressure with a piston c (or otherwise) Let the side tube have area of cross-section 'a' (fig.15.8)

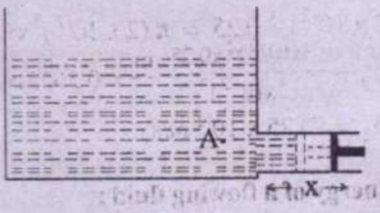


fig.15.8

If P be pressure at a point 'A' then work done against this pressure is

$$W = P.\,a.x = PV \qquad ...(15.3.16)$$

This is stored as energy of fluid. We call this energy as pressure energy (Pr.E)

$$\therefore \text{ pressure energy} = PV = \frac{mP}{\rho} \qquad ...(15.3.17)$$

$$\text{pressure energy per unit mass} = \frac{P}{\rho} \qquad ...(15.3.18)$$

$$\text{pressure energy per unit volume} = P \qquad ...(15.3.19)$$

Therefore, total energy of a fluid is

$$E = \text{K.E.} + \text{P.E.} + \text{Pr. E}$$

$$\Rightarrow \quad E = \frac{1}{2} m \vartheta^2 + mgh + \frac{mP}{\rho} \qquad ...(15.3.20)$$

Total energy per unit mass =

$$\frac{1}{2} \vartheta^2 + gh + \frac{P}{\rho} \qquad ...(15.3.21)$$

Total energy per unit volume =

$$\frac{1}{2} \rho \vartheta^2 + \rho gh + P \qquad ...(15.3.22)$$

15.4 Bernoulli's theorem (equation)

Study of equation of continuity (15.3.9) indicates that when fluid flows through a horizontal tube of varying cross-section the velocity of fluid changes i.e. the fluid is accelerated. The force required to produce this acceleration is due to the pressure difference between different regions. If the tube is not horizontal, there must also occur a pressure difference which in turn would imply velocity variation. Thus there exists a relation between pressure difference, velocity variation and variation of height of fluid. Bernoulli's equation essentially relates the pressure difference between two points in a flowing fluid with velocity changes and elevation changes. This relation, known as Bernoulli's theorem is thus stated as **"the sum total of the kinetic energy, potential energy, and pressure energy of a**

small amount of incompressible non-viscous fluid, flowing in streamlines remains constant."

Proof : In order to prove the theorem we consider an ideal fluid (incompressible and non-viscous) flowing steadily in a tube of varying cross-section and apply work-energy theorem.

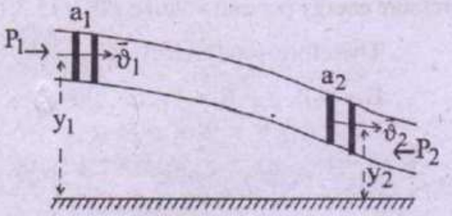


fig.15.9

Consider a tube of flow (see fig. 15.9). Let 'a_1' and 'a_2' be any arbitrary cross-sections of the tube of flow. The heights of a_1 and a_2 w.r. to some reference level be y_1 and y_2 respectively. Fluid is entering through the cross-section 'a_1' and flowing out through the cross-section 'a_2'. Let the velocity and pressure over these cross-sections be (ϑ_1, P_1) and (ϑ_2, P_2) respectively. Then mass of fluid entering through 'a_1' in time Δt is

$$m_1 = a_1\ (\vartheta_1 \Delta t).\rho_1 \qquad ...(15.4.1)$$

and mass of fluid flowing out through a_2 in time 'Δt' is

$$m_2 = a_2\ (\vartheta_2 \Delta t)\,\rho_2 \qquad ...(15.4.2)$$

As mass is conserved and liquid is assumed to be incompressible so $m_1 = m_2 = m$ and $\rho_1 = \rho_2 = \rho$ This gives

$$a_1.\ (\vartheta_1 \Delta t).\rho = a_2.\ (\vartheta_2 \Delta t)\,\rho$$

$$\Rightarrow a_1\vartheta_1 = a_2\vartheta_2 = \frac{m}{\rho.\Delta t} \qquad ...(15.4.3)$$

The kinetic energy of this fluid (entering in time Δt) is

$$T_1 = \frac{1}{2} m\vartheta_1^2 \qquad ...(15.4.4)$$

and K.E. of the fluid (flowing out in time Δt) is

$$T_2 = \frac{1}{2} m\vartheta_2^2 \qquad ...(15.4.5)$$

Now the work done by the pressure P_1 (near 'a_1') upon the fluid is

$$\Delta W_1 = \vec{F}_1 . \overrightarrow{\Delta X_1} = P_1 a_1 \hat{n}_1 . \hat{n}_1 \vartheta_1 \Delta t$$

$$\Rightarrow \quad \Delta W_1 = P_1 a_1\ \vartheta_1 \Delta t \qquad ...(15.4.6)$$

and work done by the fluid against the pressure P_2 (near 'a_2') is

$$\Delta W_2 = \vec{F}_2 . \overrightarrow{\Delta X_2} = P_2 a_2 (-\hat{n}_2).\hat{n}_2 \vartheta_2 \Delta t$$

$$\Rightarrow \quad \Delta W_2 = -\ P_2 a_2\ \vartheta_2 \Delta t \qquad ...(15.4.7)$$

Where $\hat{n}_1$ and $\hat{n}_2$ are normal to the cross-sections a_1 and a_2 respectively.

In addition work is also done against force of gravity due to the change of elevation of fluid. This work is given by

$$\Delta W_3 = \int_{y_1}^{y_2} (-mg\ \hat{y}).\ \overrightarrow{dy}\ =\ -\ mg \int_{y_1}^{y_2} dy$$

$$\Rightarrow \quad \Delta W_3 = -mg\ (y_2 - y_1)\ = -\ \Delta U$$
$$= -\ (\text{change in P.E}) \quad ...(15.4.8)$$

Therefore, the total work done on the fluid is

$$W = \Delta W_1 + \Delta W_2 + \Delta W_3$$

$$\Rightarrow \quad W = P_1 a_1 \vartheta_1 \Delta t - P_2 a_2 \vartheta_2 \Delta t - mg\,(y_2 - y_1) \qquad ...(15.4.9)$$

Using eqns. (15.4.3) in eqn. (15.4.9) we obtain

$$W = P_1 \frac{m}{\rho} - P_2 \frac{m}{\rho} - mg\,(y_2 - y_1) \qquad ...(15.4.10)$$

Now applying work - energy theorem the total work done on the system is equal to the change in the kinetic energy - we obtain

$$P_1 \frac{m}{\rho} - P_2 \frac{m}{\rho} - mg\,(y_2 - y_1) = \frac{1}{2}\, m\,\vartheta_2^2 - \frac{1}{2}\, m\,\vartheta_1^2 \qquad ...(15.4.11)$$

On re-arrangement we obtain

$$\frac{1}{2}\, m\,\vartheta_1^2 + mgy_1 + \frac{m}{\rho} P_1$$

$$= \frac{1}{2}\, m\,\vartheta_2^2 + mgy_2 + \frac{m}{\rho} P_2 \qquad ...(15.4.12)$$

Since subscripts 1 and 2 refer to any arbitrary two cross-sections along the tube of flow so we must have

$$\frac{1}{2}\, m\,\vartheta^2 + mgy + \frac{m}{\rho} P = \text{constant} = C \qquad ...(15.4.13)$$

In the form (15.4.13) Bernoulli's equation gives the conservation of energy of a flowing fluid. From eqn. (15.4.13) we could also obtain

i) $\frac{1}{2}\,\vartheta^2 + gy + \frac{P}{\rho} = \text{constant} = \frac{C}{m} = C_1$ (say) ...(15.4.14)

i.e. sum total of K.E, P.E and Pr. E per unit mass is constant.

ii) $\frac{1}{2}\,\rho\,\vartheta^2 + \rho\, gy + P = \text{constant} = \frac{C}{V} = C_2$ (say) ...(15.4.15)

i.e. sum total K.E, P.E and Pr.E per unit volume is constant.

iii) $\frac{\vartheta^2}{2g} + y + \frac{P}{\rho g} = \text{constant} = \frac{C}{mg} = C_3$ (say) ...(15.4.16)

i.e. sum total of velocity head, potential head and pressure head is constant.

Eqn. 15.4.13, 15.4.14, 15.4.15 and 15.4.16 are just different forms of Bernoullis theorem.

Applications :

i) **Hydrostatics :** Bernoulli's equation reduce to equations of hydrostatics when it is applied to fluid at rest.

e.g with $\vartheta_1 = \vartheta_2 = 0$, we have

$$P_1 + \rho g y_1 = P_2 + \rho g y_2$$

$$\Rightarrow \quad P_1 = P_2 + \rho g\,(y_2 - y_1)$$

$$\text{or } P_1 = P_2 + \rho g h$$

This is the pressure depth relation in hydrostatics.

ii) **Atomiser** (sprayer) :

An atomiser is essentially as shown in fig. 15.10

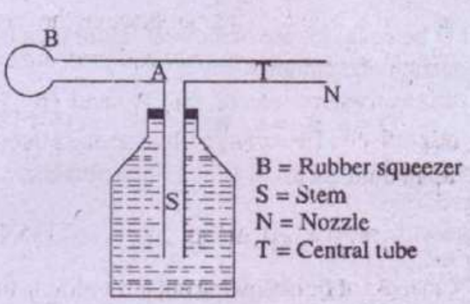


fig. 15.10

When B is squeezed, air blows through the central tube T with high velocity causing low pressure at the mouth A of the stem 'S' (by Bernoulli's theorem). Hence liquid rises up and is carried away by the air blowing through the central tube. When the liquid passes through the nozzle, it acquires high velocity.

iii) **Venturimeter :**

It is a device for measuring the rate of flow of liquids generally water through pipes.

It consists of two truncated conical tubes A and B connected together by a short neck T.

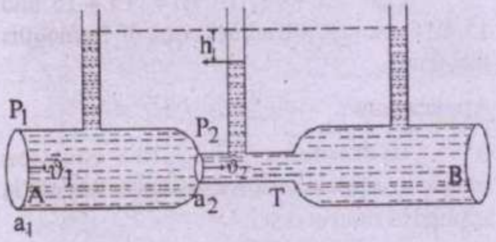


fig. 15.11

It is connected horizontally with the pipes in which the rate of flow is to be measured.

Applying Bernoulli's equation to end A and neck T.

$$\frac{P_1}{\rho} + gy_1 + \frac{1}{2}\vartheta_1^2 = \frac{P_2}{\rho} + gy_1 + \frac{1}{2}\vartheta_2^2$$

$$\Rightarrow \frac{1}{2}(\vartheta_2^2 - \vartheta_1^2) = \frac{P_1 - P_2}{\rho} \qquad ...(15.4.17)$$

If Q be equal to rate of flow of liquid then by equation of continuity

$$Q = a_1\vartheta_1 = a_2\vartheta_2 \qquad ...(15.4.18)$$

Using eqn. 15.4.18 in 15.4.17 one obtains.

$$\frac{1}{2}\left(\frac{Q^2}{a_2^2} - \frac{Q^2}{a_1^2}\right) = \frac{P_1 - P_2}{\rho}$$

$$\Rightarrow \quad Q^2 = \frac{2a_1^2\, a_2^2}{a_1^2 - a_2^2}\left(\frac{P_1 - P_2}{\rho}\right)$$

$$\Rightarrow \quad Q = a_1 a_2 \sqrt{\frac{2(P_1 - P_2)}{\rho(a_1^2 - a_2^2)}} \qquad ...(15.4.19)$$

If 'h' be difference of heights of the liquid in the two limbs (see fig. 15.11) then

$$Q = a_1 a_2 \sqrt{\frac{2h\rho g}{\rho(a_1^2 - a_2^2)}}$$

$$\Rightarrow \quad Q = a_1 a_2 \sqrt{\frac{2hg}{a_1^2 - a_2^2}} \qquad ...(15.4.20)$$

Equation 15.4.20 gives the rate of flow of liquid. Since $Q = a_1\vartheta_1 = a_2\vartheta_2$, so one also obtains

$$\vartheta_1 = a_2\sqrt{\frac{2hg}{a_1^2 - a_2^2}} \qquad ...(15.4.21)$$

$$\vartheta_2 = a_1\sqrt{\frac{2hg}{a_1^2 - a_2^2}} \qquad ...(15.4.22)$$

Where ϑ_1 is velocity at the entry point and ϑ_2 is the velocity at exit point.

iv) Efflux of liquids (Toricelli's theorem)

Bernoulli's equation can be used to find the speed of efflux of liquid through an orifice on the side wall of a reservoir.

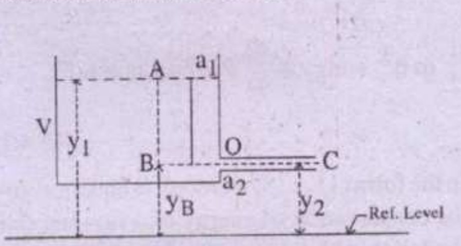


fig. 15.12

Consider a reservoir V, with an orifice 'o' on the side wall. Let at any instant the free surface be at a height b above the orifice. Let the free surface have area of cross-section a_1 and orifice have area of cross-section a_2 $(a_1 >> a_2)$. Follow a streamline ABC as shown in fig. 15.12 and apply Bernoulli's equation. Then we obtain,

$$\frac{P}{\rho} + gy_1 + \frac{1}{2}\vartheta^2 = \frac{P_B}{\rho} + gy_B + \frac{1}{2}\vartheta_B^2$$

$$= \frac{P_a}{\rho} + gy_2 + \frac{1}{2}\vartheta_C^2 \qquad ...(15.4.23)$$

Where P is pressure at A, lying on the topmost (free) surface; P_B is pressure at B, and P_a is the atmospheric pressure at C. ϑ, ϑ_B and ϑ_C correspond to values of velocities at free surface, at point B and at C. respectively. From (15.4.23) we obtain

$$\frac{P-P_a}{\rho} + g(y_1 - y_2) = \frac{1}{2}(\vartheta_C^2 - \vartheta^2) \quad ...(15.4.24)$$

Since by equation of continuity $a_1\ \vartheta = a_2\ \vartheta_C$,

so $\vartheta = \frac{a_2}{a_1}\ \vartheta_C$ and equation (15.4.24) reduces

to $$\frac{P-P_a}{\rho} + g(y_1 - y_2) = \frac{1}{2}\vartheta_C^2\left(1-\frac{a_2^2}{a_1^2}\right)$$

$$\Rightarrow \quad \vartheta_C^2\left(1-\frac{a_2^2}{a_1^2}\right) = 2g(y_1 - y_2) + \frac{2}{\rho}(P-P_a)$$

$$\Rightarrow \quad \vartheta_C^2\left(1-\frac{a_2^2}{a_1^2}\right) = 2gh + \frac{2}{\rho}(P-P_a) \quad ...(15.4.25)$$

Eqn. (15.4.25) gives the value of velocity ϑ_C at the orifice. However as the streamlines converge as they approach the orifice, the cross-section of the stream continues to diminish for a short distance outside the tank. Therefore it is the area of smallest cross-section (a_o) known as **vena contracta**, which must be used in eqn. (15.4.25) i.e. a_2 should be replaced by $a_o = 0.65\ a_2$.

Eqn. (15.4.25) will also give different values of ϑ_C, depending on the conditions of reservoir. e.g.

i) **Reservoir open to atmosphere :**

Then $P=P_a$ and eqn. (15.4.25) reduces to

$$\vartheta_C^2\left(1-\frac{a_2^2}{a_1^2}\right) = 2gh \quad ...(15.4.26)$$

If $a_2 << a_1$, then

$$\vartheta_C = \sqrt{2gh} \quad ...(15.4.27)$$

That is the speed efflux is same as the speed of a body falling freely under gravity through a height h.

ii) **Closed reservoir**

Then $P \neq P_a$, but $a_2 << a_1$. Hence eqn. (15.4.25) gives

$$\vartheta_C^2 = 2gh + \frac{2}{\rho}(P-P_a) \quad ...(15.4.28)$$

But if $2(P-P_a)/\rho >> 2gh$, then

$$\vartheta_C = \sqrt{\frac{2}{\rho}(P-P_a)} \quad ...(15.4.29)$$

v) **Pitot tube :**

It is a simple device used to measure the speed of flow at any depth in a flowing fluid, which is not confined in a pipe. e.g. speed of flow of water in a river or speed of air under the wings of an aeroplane.

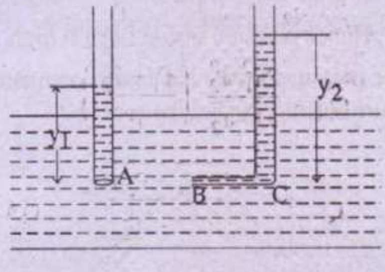


fig. 15.13

It consists of a vertical tube with a narrow opening at A, kept perpendicular to the direction of flow of fluid and another L-shaped tube with a rarrow opening at B, kept parallel to the direction of flow. Ends, A & B are at the same horizontal level and are very close to each other. The fluid entering to L-shaped tube stops at the bend 'c'. So the speed of flow of the fluid becomes zero at the bend c (called as stagnation point).

Let P_1 and P_2 be pressures at A and C respectively; and y_1, y_2 be the heights of liquid in the two tubes (see fig. 15.13).

Let ϑ_1 be the speed of flow of fluid near A. Then applying. Bernoulli's equation (to a streamline AC)

$$\frac{P_1}{\rho} + \frac{1}{2}\vartheta^2 = \frac{P_2}{\rho} \qquad ...(15.4.30)$$

$$\Rightarrow \quad \frac{1}{2}\rho\vartheta^2 = P_2 - P_1 = \rho g(y_2 - y_1) = \rho gh$$

$$\Rightarrow \quad \vartheta^2 = 2gh$$

$$\vartheta = \sqrt{2gh} \qquad ...(15.4.31)$$

vi) Blowing off of the roof of a house during tornado.

During tornado the wind blows with high speed (ϑ_O) outside the roof. This causes low pressure (P_O) outside the house. On the other hand speed of wind inside the house (ϑ_i) is very small. Hence pressure inside (P_i) is high. This higher pressure inside the house compared to pressure outside, pushes the roof up.

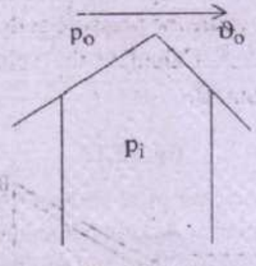


fig. 15.14

vii) Lifting of an aeroplane :

A section of the wing of an aeroplane is shown in fig. 15.15. The orientation of the wing relative to the flow direction causes the flow lines to crowd together above the wing. The air speed in this region is greater than the air speed below the wing. Therefore as per Bernoulli's equation the pressure in the region above the wing shall be less than in the region below the wing. Because of this pressure difference a net upward force acts on the aeroplane and it is lifted upward. This is called dynamic lifting.

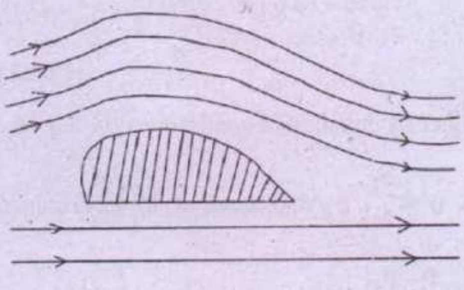


fig. 15.15

viii) If air is blown under high pressure through a nozzle, in between two pith balls, suspended from a rigid support (see fig. 15.16) the two balls are pulled towards each other. Because when air is blown with high speed, a low pressure is created around the nozzle. Hence the balls are pulled towards the nozzle as air rushes towards this zone.

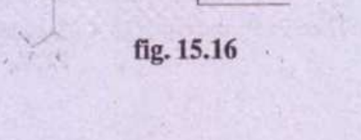


fig. 15.16

ix) **Burning of Bunsen burner :**

As the gas rushes out of the nozzle with high speed, it creates a low pressure near the nozzle. Therefore air rushes towards the nozzle through the side hole and gets mixed with the gas, helping it to burn.

x) **Curved flight of a spinning ball :**

When spin bowlers in cricket deliver the ball, the ball changes its plane of motion in air.

Suppose a ball spinning about the vertical direction is going ahead with some velocity in the horizontal direction in still air. If we work in a frame in which centre of the ball is at rest, then in this frame air moves past the ball at speed ϑ.

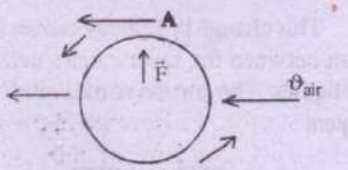
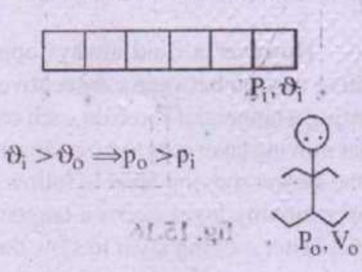


fig. 15.17

The air that goes from A side of the ball is dragged by the spin of the ball and its speed increases. The air that goes from B side of the ball suffers an opposite drag and its speed decreases. Therefore, by Bernoulli's theorem, pressure on A side is reduced while on B side it is increased. As a result a resultant force $\vec{F}$ acts on the ball from the B-side to the A-side due to this pressure difference. This force causes the deviation of plane of motion.

xi) A person standing near a train (or a fast moving vehicle) is pulled towards the vehicle.



As the vehicle moves with high speed, the pressure near the vehicle diminishes. i.e. pressure in between the person and vehicle falls. Hence air from outside rushes and pushes the person towards the vehicle.

Ex. 15.4.1 Consider an open tank filled with water to a height of 6 m. A hole of cross-sectional area 1.0 cm^2 is made 4 m. below the water level. calculate the velocity of efflux of water from the hole

Soln.

$$\vartheta = \sqrt{2gh} = \sqrt{2x9.8x4} = 8.85 \text{ m/s.}$$

Ex. 15.4.2 Water stand at a height of 3m. in an enclosed tank with compressed air at the top having a gauge pressure of 3 x 10^5 N/m^2. A hole of 4 cm^2 is made at the bottom. Calculate the velocity and discharge rate of water from the hole.

Soln.

$$\vartheta_c^2 = 2\,gh + \frac{2}{\rho}\,(P\text{-}P_a)$$

$$= 2 \times 9.8 \times 3 + \frac{2x3x10^5}{10^3} = 658.8 \text{ m}^2\text{/s}^2$$

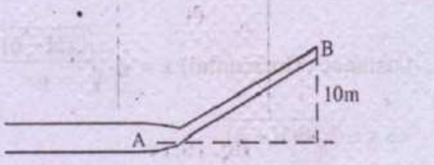
$$\vartheta_c = 25.67 \text{ m/s} = \text{speed of discharge}$$

$$\text{Rate of flow} = a\,\vartheta_c = 0.0102668 \text{ m}^3\text{/s}$$

$$= 102.67 \times 10^{-4} \text{ m}^3\text{/s}$$

Ex. 15.4.3 The cross sectional area of a pipe at a point A is twice that at a point B; which is 10m above A. The speed of water in the pipe at A is 5 m/s, while the pressure at this point is 1.5 x 10^5 N/m^2. Calculate the velocity and pressure at point B.

Soln.



By equation of continuity $a_A \vartheta_A = a_B \vartheta_B$

$$\Rightarrow \vartheta_B = \frac{a_A}{a_B} \vartheta_A = 2\vartheta_A$$

$$\Rightarrow \vartheta_B = 2 \times 5 \text{ m/s} = 10 \text{ m/s}$$

By Bernoulli's eqn.

$$\frac{P_A}{\rho} + \frac{1}{2}\vartheta_A^2 + gh_A = \frac{P_B}{\rho} + \frac{1}{2}\vartheta_B^2 + gh_B$$

$$\Rightarrow P_B = \rho\left[\frac{P_A}{\rho} + \frac{1}{2}\left(\vartheta_A^2 - \vartheta_B^2\right) + g(h_A - h_B)\right]$$

$$= P_A + \frac{1}{2}\rho\left(\vartheta_A^2 - \vartheta_B^2\right) - \rho gh$$

$$= 1.5 \times 10^5 \text{ N/m}^2 - \frac{3}{2}.10^3.(5)^2$$

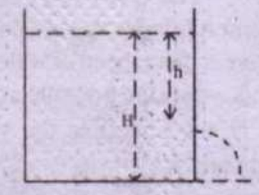
$$-10^3 \text{x} 9.8 \text{x} 10$$

$$P_B = 1.45 \text{x} 10^4 \text{ N/m}^2$$

Ex. 15.4.4 A tank is filled with water to a height H. A hole is pinched in the wall at a depth h below the water surface. find the distance x form the foot of the wall at which the stream strikes the floor.

Soln.

Velocity of efflux $\vartheta = \sqrt{2gh}$



Distance (Horizontal) $x = \vartheta\sqrt{\frac{2(H-h)}{g}}$

$\Rightarrow x = 2\sqrt{h(H-h)}$

15.5. Viscosity :

When a fluid flows at a steady rate its different layers are seen to flow with different velocities. For example, consider the flow of a liquid in a tube. The layer of liquid in contact with the wall of the tube is at rest due to adhesive force between the liquid molecules and molecules of the wall of the tube. But as we move towards the centre of the tube the velocity gradually increases.

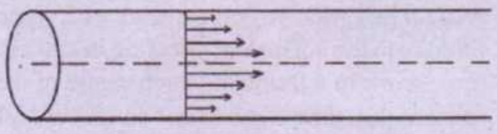


fig. 1.518

This change in velocity causes a relative motion between the layers and a deformation of the liquids. The portion of the liquid 'C' which is cubical in shape at one instant, assumes a rhomboidal shape later at R. Thus the phenomenon is similar to shear stress and shear strain.

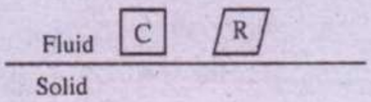


fig. 15.19

Similarly if a solid surface is kept in contact with a liquid and is moved the liquid in contact is dragged with the solid, while liquid far away are almost at rest. For example, when a boat moves slowly on a calm river, water in contact with the boat is dragged, while water layer in contact with the bed of the river is at rest.

However a fluid always opposes this relative motion between consecutive layer by exerting a tangential force on each other. (The faster moving layer tries to exert tangential force on the slower moving layer to follow it and the slower moving layer exerts a tangential force on the faster moving layer to slow down it).

The property of a fluid to oppose relative motion between its consecutive layers is called viscosity. The force between the consecutive layers which tends to annul the relative motion between them is known as force of viscosity or viscous drag and it acts tangentially to the layer in a direction opposite to the direction of flow. This force may be looked upon as internal friction between the layers.

Viscous force (drag) and coefficient of viscosity

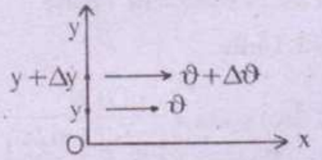


fig. 15.20

Consider a fluid flowing in a tube along X-direction. Let the velocity of two layers, separated by a distance Δy be ϑ and $\vartheta + d\vartheta$ (see fig. 15.20). Then $\frac{\Delta\vartheta}{\Delta y}$ may be taken as **average velocity gradient** between the two layers. The velocity gradient is the limiting value of average velocity gradient when $\Delta y \to 0$ i.e.

$$\text{Velocity gradient} = \underset{\Delta y \to 0}{\text{Lm}} \frac{\Delta\vartheta}{\Delta y} = \frac{d\vartheta}{dy} \qquad ...(15.5.1)$$

The direction of velocity gradient at a point is always along the perpendicular to the direction of velocity of layer at the point.

Now we re-write eqn. (15.5.1) as

$$\frac{d\vartheta}{dy} = \frac{d}{dy}\left(\frac{dx}{dt}\right) = \frac{d}{dt}\left(\frac{dx}{dy}\right) \qquad ...(15.5.2)$$

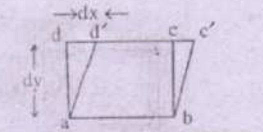


fig. 15.21

Now $\frac{dx}{dy} = \frac{dd'}{ad}$ = shear strain.

Therefore velocity gradient can be looked upon as time rate of change of shear strain.

According to Newton's observations for most of the fluids the magnitude of viscous force $\left(|\vec{F}|\right)$ between two consecutive layers is directly proportional to (i) the surface area 'A' of the fluid layer and (ii) the velocity gradient $\frac{d\vartheta}{dy}$ i.e.

$$|\vec{F}| \quad \alpha \ A$$

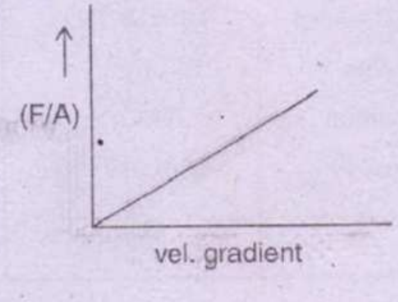
$$\alpha \ \frac{d\vartheta}{dy}$$

$$\Rightarrow \quad |\vec{F}| = -\eta \, A \, \frac{d\vartheta}{dy} \qquad ...(15.5.3)$$

Where, η is the constant of proportionality, called coefficient of viscosity; whose value depends on the property of the fluid. The negative sign is included as the force is an opposing force. Equation (15.5.3) can be re-written as (disregarding - ve sign).

$$\eta = \frac{F/A}{(d\vartheta/dy)} = \frac{\text{shear stress}}{\text{rate of change of shear strain}}$$

This justifies our earlier statement that viscosity is similar to shear stress and shear strain. The fluids obeying eqn. (15.5.3) are sometimes called Newtonian fluids. For Newtonian fluids, the graph (F/A) vs vel. gradient is a straight line



From eqn. (15.5.3) it also follows that when $A=1$, $d\vartheta/dy=1$, $F=\eta$. Hence we define **"Coefficient of viscosity of a fluid is the tangential force per unit area required to produce a unit velocity gradient between its layer".**

OR

"Coefficient of viscosity of a fluid is the tangential force required to maintain unit relative velocity between two layers separated by unit distance."

The reciprocal of coefficient of viscosity is called fluidity. The ratio of coefficient of viscosity to the density of the fluid (η/ρ) is called its kinematic viscosity.

Dimension of η

$$[\eta] = \frac{[F]}{[A][d\vartheta/dy]} = \frac{MLT^{-2}}{L^2.LT^{-1}/L}$$

$$= ML^{-1}T^{-1} \qquad ...(15.5.4)$$

Unit of η

i) **C.G.S. Unit**

$$1 \text{ poise} = \frac{1\text{dyne}}{1\text{cm}^2.\left(\frac{\text{cm/s}}{\text{cm}}\right)} = 1 \text{ dyn.s.cm}^{-2}$$

"Coefficient of viscosity of a fluid is said to be 1 poise if a tangential force of 1 dyne per cm^2 is required to maintain a relative velocity of 1 cm/sec between two layers separated by 1 cm".

Some times we use smaller units like

1 centi poise = 10^{-2} poise

1 micro poise = 10^{-6} poise

ii) **S.I. Units**

$$1 \text{ deca poise} = \frac{1\text{ N}}{1\text{ m}^2.\left(\frac{1\text{ m/s}}{\text{m}}\right)}$$

$= N.S.m^{-2}$ = Pa-s.

"Coefficient of viscosity of a fluid is said to be 1 decapoise if a tangential force of 1 N per m^2 is required to maintain a relative velocity of 1 m/s between two layers separated by 1m".

Table No. 15.1 (η of selected fluids at 20^{0}C)

Liquid	η in $N.S.m^{-2}$ (decapoise)	Gas	η in $N.S.m^{-2}$ (decapoise)
Benzene	6.6×10^{-4}	Air	1.8×10^{-5}
Crude Oil	7.2×10^{-3}	Carbon-di-Oxide	1.47×10^{-5}
Ether	2.34×10^{-4}	Helium	1.97×10^{-5}
Ethyl alcohol	1.19×10^{-3}	Hydrogen	0.9×10^{-5}
Gasoline	2.9×10^{-4}	Methane	1.34×10^{-5}
Glycerine	0.83	Nitrogen	1.76×10^{-5}
Mercury	1.56×10^{-3}	Oxygen	2.01×10^{-5}
Water	1×10^{-3}		

Temperature dependence of 'η' (Coefficient of Viscosity)

'η' of a liquid decreases with increase of temperature. 'η' of gas increases with increase of temperature. Table 15.2 gives values of 'η' of few selected fluids at different temperatures.

fluid	η in poise 0°C	20°C	40°C	60°C	80°C	100°C
Castor Oil	53	9.86	2.31	0.8	0.3	0.17
Water	1.792×10^{-2}	1.005×10^{-2}	0.656×10^{-2}	0.469×10^{-2}	0.357×10^{-2}	0.284×10^{-2}
Blood	—	3.015×10^{-2}	2.084×10^{-2}	—	—	—
Air	171×10^{-6}	181×10^{-6}	190×10^{-6}	200×10^{-6}	209×10^{-6}	218×10^{-6}

15.5.(a) Flow through a narrow tube (Poiseuile's law)

When a viscous fluid flows steadily in a narrow tube, the volume of fluid flowing per unit time is given by poisseuille's formula

$$\frac{dV}{dt}=\frac{\pi.\Delta P.r^4}{8\eta\ell}=\frac{\pi(P_1-P_2)\,r^4}{8\eta\ell} \quad ...(15.5.4)$$

Where, $\frac{dV}{dt}$ is the rate of flow of fluid.

$\Delta P = P_1 - P_2$ = Pressure difference between the ends of the tube.

ℓ = length of the tube

r = radius of the tube

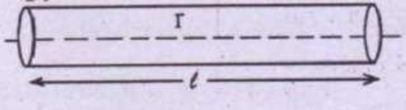


fig. 15.22

The deduction of the eqn. (15.5.4) is beyond the scope of this book. However we can deduce this by dimensional method.

The volume of fluid flowing per unit time $\left(=\frac{dV}{dt}\right)$ should depend on flow velocity which in turn depends on the pressure gradient $(=\Delta P/\ell)$. $\frac{dV}{dt}$ also depends on radius of the tube and coefficient of viscosity (η). Hence let

$$\frac{dV}{dt} \quad \alpha \ r^a$$

$$\alpha \ (\Delta P/\ell)^b$$

$$\alpha \ \eta^c$$

Therefore,

$$\frac{dV}{dt}=\alpha\, r^a\left(\frac{\Delta P}{\ell}\right)^b\eta^c \quad ...(15.5.5)$$

Where 'α' is a dimensionless constant. Applying principle of homogeneity of dimensions on both sides

$$L^3T^{-1}=L^a\left(\frac{ML^{-1}T^{-2}}{L}\right)^b\ (ML^{-1}T^{-1})^c$$

$$=L^{a-2b-c}\,M^{b+c}\,T^{-2b-c}$$

Equating dimensions of similar quantities on both sides

$b + c = 0$ ...(15.5.6)

$a - 2b - c = 3$ ...(15.5.7)

$-2b - c = -1$ ...(15.5.8)

Solving equations (15.5.6) to (15.5.8) we obtain

$a = 4,\ b = 1,\ c = -1$

So eqn. (15.5.5) reduces to.

$$\frac{dV}{dt} = \alpha r^4 . \frac{\Delta P}{\ell} . \eta^{-1} = \alpha \frac{\Delta P}{\eta \ell} r^4 \qquad ...(15.5.9)$$

Experiment shows that $\alpha = \frac{\pi}{8}$. Hence

$$\frac{dV}{dt} = \frac{\pi . \Delta P . r^4}{8 \eta \ell} = \frac{\pi (P_1 - P_2)}{8 \eta \ell}$$

...(15.5.10)

Thus poisseuille's formula is derived.

The velocity of flow at a distance y from centre is

$$\vartheta = \frac{\Delta P (r^2 - y^2)}{4 \eta \ell} \qquad ...(15.5.11)$$

15.5.(b) Stoke's Law :

When a spherical body of radius r, moves at a speed ϑ through a fluid of infinite extent, the viscous drag on the sphere is given by

$$F = 6 \pi \eta r \vartheta \qquad ...(15.5.12)$$

Where η = coefficient of viscocity of the fluid.

Derivation of stoke's law

We adopt for simplicity, a dimensional method. Observations indicate that F depends on the size of the sphere (r), the instantaneous speed (ϑ) and the coefficient of viscosity (η). Let

$$F \propto r^a$$
$$\propto \vartheta^b$$
$$\propto \eta^c$$

Then

$$F = \alpha r^a \vartheta^b \eta^c \qquad ...(15.5.13)$$

Where 'α' is a dimensionless constant. Equating the dimensions on both sides we have

$$MLT^{-2} = L^a (LT^{-1})^b (ML^{-1}T^{-1})^c \qquad ...(15.5.14)$$

Equating similar dimensions on both sides

$$c = 1 \qquad ...(15.5.15)$$
$$a + b - c = 1 \qquad ...(15.5.15)$$
$$-b - c = -2 \qquad ...(15.5.17)$$

Solving eqns. (15.5.15) to (15.5.17) we obtain

$a = 1,\ b = 1,\ c = 1$

Hence

$$F = \alpha \eta r \vartheta \qquad ...(15.5.18)$$

Experiments suggest that $\alpha = 6\pi$

So, $F = 6 \pi \eta r \vartheta$

Thus stoke's law is deduced.

15.5.(c) Terminal Velocity

When an object of mass m falls vertically downward in a viscous medium the forces acting on it are

i) Viscous drag $\vec{F} = F \hat{j}$, acting vertically upwards.

ii) Weight of the body $\vec{W} = -W \hat{j}$, acting vertically downward.

iii) The buoyancy force $\vec{F}_B = F_B \hat{j}$, acting vertically upward.

Hence net force acting on the object is

$$\vec{F}_{net} = \vec{F} + \vec{W} + \vec{F}_B = (F - W + F_B) \hat{j} \qquad ...(15.5.18)$$

According to Newton's 2nd law

$$\vec{F}_{net} = (F - W + F_B) \hat{j} = m \vec{a} \qquad ...(15.5.19)$$

Where $\vec{a}$ is the acceleration of the object in the fluid.

In the eqn. (15.5.18) F_B and W are constant in time; but F shall depend on the speed of the body. When the body starts from rest its speed gradually increases; resulting in gradual increase of F. For small values of F, $W - F_B > F$ giving $F - W + F_B < 0$. so that body moves downwards with an acceleration. But as the speed increases so that F increases considerably, $W - F_B$ becomes equal to F, giving a = 0. Under this condition the body attains dynamic equilibrium. The speed (velocity) at which the body attains dynamic equilibrium is called its terminal velocity (speed).

15.5(d)Determination of using stoke's Law :

Stoke's Law can be used to determine the coefficient of viscosity (η) of certain highly viscous liquid.

The apparatus used (shown in fig. 15.23) consists of a tall glass jar. The liquid whose 'η' is to be determined fills the jar. Three thread marks A,B and C are given such that AB = BC, well below the top surface of the liquid. Two stop watches are used to note the times t_1 and t_2 taken by a sphere to cover the distance AB and AC - respectively. The thread marks are adjusted until $t_2 = 2t_1$.

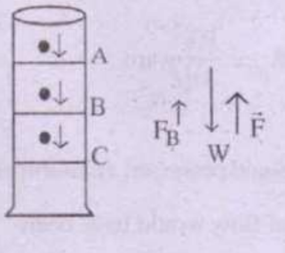


fig.15.23

Theory : When it is observed that the time $t_2 = 2t_1$, the sphere has definitely attained terminal velocity and we must have

$$W = F + F_B$$

$$\Rightarrow \quad \frac{4}{3}\pi r^3 \rho g = 6\pi \eta r \vartheta + \frac{4}{3}\pi r^3 \sigma g \qquad ...(15.5.20)$$

Where 'ρ' is density of the material of the sphere, and σ is the density of the liquid.

Then eqn. (15.5.20) gives

$$6\pi\eta r \vartheta = \frac{4}{3}\pi r^3 (\rho - \sigma) g \ .$$

$$\Rightarrow \quad \eta = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\vartheta} \qquad ...(15.5.21)$$

The terminal velocity ϑ is given by

$$\vartheta = \frac{AB}{t_1} = \frac{AC}{t_2} \qquad ...(15.5.22)$$

The radius of the sphere is measured by using a screw gauge, the terminal velocity is calculated by noting time t_1 and t_2 by the help of a stop watch. Then using the values of ρ, σ, g, r and ϑ we obtain value of 'η'.

Practical applications of Viscosity :

i) Aeroplanes, jet planes, have pin-pointed shapes to minimise the force of viscosity.

ii) Inks used in pens should have proper value of 'η' so that it neither overflows nor get stuck up.

iii) Oils of high grade are used as lubricants in summer.

iv) Efficiency of our body is greatly affected due to change in viscosity of our blood flowing through blood vessels.

Ex. 15.5.1 A flat bottomed boat with a bottom area of $50 m^2$ is dragged along a canal with a speed of 2m/s. The bottom of the boat is 0.2 m above the bottom of the canal. The coefficient of viscosity of water is 1.0×10^{-3} Pa. S. Compute the viscous drag on it.

Soln.

Given $A = 50\ m^2$, $dy = 0.2\ m$, $d\vartheta = 2 m/s$. (Assuming that at the bottom of canal $\vartheta = 0$)

Viscous drag $F = \eta A \dfrac{d\vartheta}{dy}$

$$= 1.0 \text{x} 10^{-3}\ \text{Pa.s. x } 50 m^2 \text{ x } \frac{2 m/s}{0.2 m}$$

$$F = 0.5 N$$

Ex. 15.5.2 Water at 20°C flows through a pipe of radius 1.0cm. If the flow velocity at the centre is 10.cm/s, find the pressure drop along 2-m section of pipe due to viscosity.

Soln.

Given $r = 1.0$ cm, $\ell = 2m = 200$ cm.

velocity at the centre (y=0) = 10 cm/s.

We have (see eqn. 15.5.11)

$$\vartheta = \frac{\Delta P (r^2 - y^2)}{4 \eta \ell}$$

$$\Rightarrow \quad 10\ cm/s = \frac{\Delta P.(1^2 - 0) cm^2}{4 \text{ x } \eta \text{ x } 200}$$

$$\Rightarrow \quad \Delta P = 10 \text{ x } 4 \text{ x } 200 \text{ x } \eta$$

$$= 10 \text{x} 4 \text{x} 200 \text{x} 1.005 \text{ x } 10^{-2}\ dyne/cm^2$$

$$\Delta P = 80.4\ dyne/cm^2 = 8.04 N/m^2 = 8.04 Pa.$$

$\Rightarrow$ Pressure drop = 8.04 Pa.

Ex. 15.5.3 Water is conveyed through a pipe 8 cm in diameter and 4 kilometre in length at the rate of 120 litres per minute. Calculate the pressure difference required to maintain the flow.

Soln.

Given $d = 8\ cm \Rightarrow r = 4\ cm$

$$\ell = 4\ km = 4 \text{ x } 10^5\ cm$$

Rate of flow = V = 120 litres = $120 \text{ x } 10^3 cm^3$

So

$$V = \frac{\pi P r^4}{8 \eta \ell} \Rightarrow P = \frac{8 \eta \ell V}{\pi r^4}$$

$$\Rightarrow P = \frac{8 \text{x} (1.005 \text{x} 10^{-2}) \text{x} (4 \text{x} 10^5) \text{x} (120 \text{x} 10^3)}{\pi \text{ x } (4)^4}$$

$dyne/cm^2$

$$P = 0.8 \text{ x } 10^5\ dyne/cm^2 = 8 \text{ x } 10^3\ Pa.$$

Ex.15.5.4 A viscous liquid flows in a tube with laminar flow. Prove that the volume rate of flow is the same as if the velocity were uniform at all points of a cross-section and equal to half the velocity at the axis.

Soln.

Let the tube have radius = r

Let its length = ℓ and viscosity of liquid = η

Let the pressure difference between its ends be = P.

$$\text{Then rate of flow } V = \frac{\pi P r^4}{8 \eta \ell} \qquad ...(1)$$

Now velocity 'ϑ' at a distance y from the axis is

$$\vartheta = \frac{P(r^2 - y^2)}{4 \eta \ell} \qquad ...(2)$$

So velocity of liquid at the axis (y = 0)

$$\vartheta_a = \frac{P.r^2}{4 \eta \ell} \qquad ...(3)$$

Had the liquid possessed a uniform velocity $\dfrac{\vartheta_a}{2}$ the rate of flow would have been

$$V' = \pi r^2 . \frac{\vartheta_a}{2} = \pi r^2 . \frac{Pr^2}{8 \eta \ell}.$$

$$\Rightarrow \quad V' = \frac{\pi P r^4}{8 \eta \ell} \qquad ...(4)$$

Thus we see that $V = V'$. Hence the proof

Ex. 15.5.5 An air bubble of radius 1 cm is found to rise in a cylindrical vessel of large radius at a steady rate of 0.2 cm/s. If the density of liquid is 1470 kg/m^3, find its viscosity. ($g = 9.8$ m/s^2).

Soln.

Given

radius of bubble $r = 1$cm

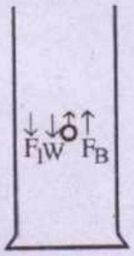
terminal velocity $\vartheta = 0.2$ cm/s upward

density of liquid $\sigma = 1470$ kg/m^3

$= 1.47$ gm/cm^3

Hence

$$\eta = \frac{2}{9} \cdot \frac{r^2 (\sigma - \rho) g}{\vartheta}$$



Where σ = density of liquid

ρ = density of air

since $\sigma >>> \rho$ we put $\rho = 0$

$$\therefore \quad \eta = \frac{2}{9} \cdot \frac{r^2 . \sigma . g}{\vartheta}$$

$$= \frac{2}{9} \cdot \frac{(1\text{cm})^2 x (1.47\text{gm}/\text{cm}^3) x 980(\text{cm}/\text{s}^2)}{(0.2\text{cm}/\text{s})}$$

= 1600.67 poises = 160.067 decapoise.

Summary

1. Critical velocity V_c is the maximum velocity with which a fluid particle can flow along a stream line.

Critical velocity $V_c = N_R \dfrac{\eta}{\rho D}$;

N_R - Reynold No.

$D \rightarrow$ Dimension of the channel and $\eta \rightarrow$ coefficient of viscosity and ρ is density of fluid

2. Equation of continuity :

If V_1 is the speed of a fluid at any point of the tube having crossectional area a_1 and v_2 is the speed of the fluid at another point having area a_2 then

$$a_1 v_1 = a_2 v_2$$

3. Bernoulli's the orem :

The sum total of the kinetic energy, potential energy and pressure energy of a small amount of incompressible non-viscous fluid, flowing in stream lines remains constant.

$$\frac{1}{2} mv^2 + mgy + \frac{m}{\rho} P = \text{constant}$$

where m is the mass and ρ is the density of the fluid, 'P' is the pressure of the fluid at a height from the reference level.

4. Viscosity :

The property of a fluid to oppose any relative motion between its layers is called viscosity.

The force of viscosity acting on any layer of the fluid due to the other is

$$F = -\eta A \frac{dv}{dy}$$

where η is the coefficient of viscosity, A is the cross sectional area of the fluid and $\dfrac{dv}{dy}$ is the velocity gradient.

5. Stokes law :

The viscons drag F acting on a spherical rigid and smooth solid of radius r moving through a fluid of coefficient of viscosity n, with a velovity v is

$$F = 6\pi \eta r v$$

6. Terminal velocity :

The velocity at which the body attains dynamic equilibrium is called its terminal velocity. It is that constant velocity with which a body moves as it is dropped through a fluid. Terminal velocity v attained by a spherical rigid body of radius r and density ρ as it is dropped through a fluid of density σ and coefficient of viscosity η is :

$$V = \frac{2}{9}\frac{r^2 (\rho - \sigma) g}{\eta}$$

Physical quantity	Dimensional Formula	SI unit
1. Reynold's Number	No	No
2. Coefficient of viscosity (n)	$[ML^{-1}T^{-1}]$	decapoise or pa.s

MODEL QUESTIONS

A. Multiple Choice Type Questions:

1. A tank is filled with water to a height H. The tank has a hole at a depth h below the water surface. The value of h so that water strikes at maximum distance from the tank is

   a) H b) H/3

   c) $\frac{3}{2}$ H d) H/2

2. Streamline flow in a liquid is one in which

   a) all particles of the liquid move with same speed

   b) all particle move with a constant speed.

   c) the velocity at every point in the flow remains constant.

   d) the particles have same speed at the inlet and outlet.

3. Pressure energy of a liquid of mass m and density ρ is

   a) $m\rho$ b) mP/ρ

   c) ρ/m d) $m\rho/P$

4. Fire-fighters have a jet attached to the head of their water pipes in order to

   a) increase the mass of water flowing per second

   b) increase the volume of water flowing per second

   c) increase the velocity of water flowing out

   d) avoid wastage of water.

5. Rain drops donot hurt our head nor make holes on the ground because they move with

   a) constant acceleration

   b) variable acceleration

   c) Variable speed

   d) constant terminal velocity.

6. Viscosity of gas

   a) increases with rise of temperature

   b) increases with fall of temperature

   c) does not depend on temperature

   d) none of the above.

7. Viscosity of liquid

   a) increases with rise temperature

   b) increases with fall of temperature

   c) does not depend in temperature

   d) none of the above.

8. The terminal velocity of a sphere of radius r, in a viscous liquid is ϑ. The corresponding terminal velocity of a sphere of radius 2r and of the same density will be

   a) 4ϑ b) 2ϑ

   c) $\vartheta/2$ d) $\vartheta/4$

9. The force that lifts aeroplane in air is produced

   a) by the wind striking against the wings

   b) by the buoyancy

   c) by the asymmerical streamlines of air around the wings .

   d) by all the above factors.

10. The clouds float in the atmosphere on account of their

    a) low viscosity

    b) low density

    c) low pressure

    d) low temperature

11. Two light balls A and B are suspended by light strings from a horizontal rigid support. Air is blown between A and B. Now

    a) A and B will come closer

    b) A and B will be separated by a greater distance

    c) both A and B will start moving to the right

    d) A and B will be unaffected

12. A rubber ball and a steel ball of the same size are gently dropped in a very tall cylinder containing highly viscous liquid. Now
    a) rubber ball will reach the bottom first
    b) rubber ball will never reach the bottom
    c) steel ball will reach the bottom first
    d) steel ball will never reach the bottom

B. Very short answer type Questions :

1. What is Reynold's number ?
2. Write equation of continuity for an incompressible liquid.
3. Write Bernoulli's equation.
4. Write the expression for pressure energy for unit volume.
5. Write expression for kinetic energy per unit volume.
6. What is stream line flow ?
7. Define a streamline.
8. Why cannot two streamlines intersect ?
9. Define tube of flow.
10. What principle is utilised in an atomiser ?
11. Write Toricelli's formula for speed of efflux.
12. Define critical velocity.
13. Define poise.
14. Define 'η'.
15. Write unit of 'η' in S.I. system.
16. Write the dimension of 'η'.
17. State stoke's law.
18. Write poisseuille's formula.
19. What is terminal velocity ?
20. Is the velocity of efflux different for sea water and fresh water ?

C. Short Answer Type Questions :

1. Explain the significance of Reynold number.
2. Give two applications of Bernoulli's equation.
3. Explain lifting of aeroplane
4. Establish equation of continuity
5. Explain why roofs are blown off.
6. Explain the floating of tiny droplets of water in a fog.
7. Why does water come out with greater velocity near jets ?
8. Why do two ships moving parallel to each other have a tendency to move closer.

D. Unsolved problems :

1. The speed of blood in a major artery of radius 0.35 cm is 0.04 m/s. The artery is divided into 50 capillaries, each of radius 0.1 cm. Calculate the speed of blood through a capillary.
2. A tank of large cross-sectional area is filled with water to a depth of 3.5 m. A hole of area 2 cm^2 is made 2.5 m below the water level. Calculate (a) speed of efflux (b) the mass of water that flows out per second.
3. What are the absolute pressure and the gauge pressure required in the main water lines so that water from a fire house can reach a height of 30 m ?
4. The reading of a pressure meter attached with a closed water pipe is 3.5×10^5 N/m^2. On opening the valve of the pipe, the reading of pressure meter reducess to 3×10^5 N/m^2. Calculate the speed of water flowing in the pipe.

   (Hints- Use Bernoulli's equation)
5. Calculate the velocity with which rain drops reach us if diameter of the rain drops is 1mm and viscocity of air is 18×10^{-6} S.I. units.
6. A vessel fitted with a horizontal capillary tube near its bottom is alternately filled with two liquids of sp. gr. 0.8 and 0.9. It empties

itself in 40 and 30 minutes respectively. Calculate the ratio of their coefficient of viscosities.

7. Show that a vessel filled with a liquid of density ρ upto a height h is emptied by a capillary tube exponentially with time according to equation

   $x = h(1-e^{-kt})$, where k is a constant depending on the radius of the vessel and tube, coefficient of viscosity and density of the liquid and length of the tube. Assume the tube to be connected at the bottom of the vessel.

8. A metal plate 100 sq. cm in area rests on a layer of caster oil 2mm. thick, whose coefficient of viscosity is 1.55 S.I. units. Calculate the horizontal force required to move the plate with a speed of 3 cm/s.

[Hints : $F = -\eta A \frac{d\vartheta}{dy}$

Here $\frac{d\vartheta}{dy} = \frac{3\text{x}10^{-2}\text{m}/\text{s}}{2\text{x}10^{-3}\text{m}} = 15\ S^{-1}$]

E. Long Answer Type Questions :

1. State and prove Bernoulli's theorem
2. Write Bernoulli's equation. On the basis of it explain four applications of Bernoulli's equation.
3. State stoke's law and deduce it by dimensional methods.
4. Define terminal velocity. Derive on expression for it. Explain how can you use this concept to determine η.
5. Define 'η'. Give its units. Discuss its application in our day to day life.

F. Fill in blanks Type

1. The pressure energy of a liquid of mass m and density ρ is ....
2. Rain drops donot hurt our head nor make holes on the ground because they move with..........
3. Vescosity of liquids .... with rise of temperature.
4. Viscosity of gases .... with rise of temperature.
5. The clouds float in the admosphere on account of their ....
6. The terminal velocity of a ball falling in a viscous liquid is directly propertional to the .... of the radius.

G. True - False Type

1. The blood pressure in humans is greater at the feet than at brain.
2. The size of the needle of a syringe controls flow rate better than the thumb pressure exerted by a doctor while administering an injection.
3. The force required by a man to move his limbs immersed in water is smaller than the force for the same movement in air.
4. In order to keep a price of paper horizontal, we should blow over, not under it.
5. A fluid flowing out of a small hole in a vessel results in a backward thrust on the vessel.

ANSWERS

A. Multiple Choice Questions :

(1) d, (2) c, (3) b, (4) c, (5) d, (6) a, (7) b, (8) a, (9) c, (10) b, (11) a, (12) c

B. Very Short Type Questions :

1. see text
2. see text
3. see text
4. see text
5. see text
6. see text
7. see text
8. see text
9. see text
10. Bernoulli's equation
11. $\vartheta = \sqrt{2gh}$
12. see text
13. see text
14. see text
15. decapoise
16. $ML^{-1}T^{-1}$
17. See text
18. $v = \frac{\pi P r^4}{8\eta l}$
19. See text
20. No.

C. Short Answer Type Questions :

1. Reynold's number $N_R = \vartheta_c \, . \, R.D/\eta$

a) $2000 \geq N_R > 0$ flow is steady

b) $N_R > 3000$ flow is turbulent

c) $3000 > N_R > 2000$ flow is sometimes steady, sometimes turbulent.

2. (i) Bunsen burner (ii) Atomiser or sprayer

3. See text

4. See text

5. See text

6. The net force on the droplets due to buoyancy, its weight and viscous drag is zero. Hence the tiny droplets float].

7. [$\vartheta_2 = \frac{a_1}{a_2}\vartheta_1$, since $a_1 >> a_2$, so $\vartheta_2 >> \vartheta_1$].

8. [The liquid in between the two ships attain high speed. Hence by equation of Bernoulli, a low pressure is created in the region. Hence ships are pushed towards each other].

D. :

1. 0.98 cm/s
2. (a) 7m/s (b) 1.4 kg/s
3. 2.94×10^6 N/m^2, 1.93×10^5 N/m^2
4. 10 m/s
5. 30.2 m/s
6. 1.185 : 1
7.
8. 0.2325 N

F. (1) $^{mb}/_{\rho}$ (2) constant terminal velocity (3) decreases (4) increases (5) lowdensity (6) square

G. (1) True (2) True (3) False (4) True (5) True

16

Heat Phenomena

16.1 Introduction to Heat and internal energy

The discorder part of internal energy of a system is called its thermal energy. It is the sum total of the Kinetic energy of the molecules, constituting the body, possessed by virtue of their vibrational, translational and rotational motion.

The part of the thermal energy which is transferred from one body to another or from one part to another part of the body is called Heat. Thus heat is the energy in transit. Once it is transferred it becomes internal energy of the receiving body. The expressions like "heat in a body" or "heat of a body" are meaningless and misleading. It is to be noted that no mechanical work is done during the (spontaneous) transfer of heat.

16.2 Work and Heat (Mechanical equivalent of heat)

In mechanics work is defined as the scalar product of force and displacement ($W= \vec{F}.\vec{S}$). The mechanical energy of a system increases when work is done on it (by another body or system). Thus work is the energy that is transferred from one system to another, when one exerts a force on the other. We have also seen that heat is the energy in transit. Thus it is imperative that heat and work should be interrelated. For example, when we rub our hand heat is generated, indicating mechanical work being converted to heat. Similarly in heat engines heat is converted to work.

Joule observed that when heat (Q) is converted to mechanical work (W) or vice versa, they are proportional to each other.

i.e. $W \ \alpha \ Q$

$$\Rightarrow W = J Q \qquad ...(16.2.1)$$

Where the proportionality constant J is called **mechanical equivalent of heat**. Joule has determined the value of J to be 4.186 J/cal. This constant J is also known as Joule's Constant. Equation (16.2.1) implies that when 1 calorie of heat is converted to work, 4.186 J of work is obtained or when 4.186 J of work is done on a body, 1 calorie of heat is generated.

Units of Heat :

A. Heat Units

(i) Calorie :

It is the quantity of heat required to raise the temperature of one gram of water from 14.5^0C to 15.5^0 C (ordinarily speaking through 1^0 C.)

(ii) Kilo-Calorie :

It is the quantity of heat required to raise the temperature of 1 kg. of water from 14.5^0 C to 15.5^0 C (ordinarily speaking through 1^0 C),

B. Energy Units :

Since heat is a form of energy, it can also be expressed in energy units e.g. by equation (16.2.1)

$$1 \text{ cal} = 4.186 \times 10^7 \text{ ergs}$$

$$\Rightarrow 1 \text{ erg} = \frac{10^{-7}}{4.186} \text{cal}$$

Similarly

$$1 \text{ joule} = \frac{1}{4.186} \text{cal} = \frac{10^{-3}}{4.186} \text{K Cal}$$

Note : In SI system energy in any form is measured in joule. In that case w=Q and J=1.

16.3 Temperature

From a lay-man's point of view "the sensation of degree of hotness or coldness which one experiences on touching a body is called temperature". But this is a subjective definition of temperature which is not reliable. For example a finger dipped in hot water when touches tepid (i.e. luke warm) water it feels cold; whereas if the finger was dipped in cold water, the same tepid water would be felt warm.

This warrants an operational definition of temperature. One such operational but qualitative definition is - "**temperature is a property that determines the direction of (spontaneous) energy flow between two bodies in (thermal) contact.**"

We call this a qualitative definition because when heat flows from a body 'A' to another body 'B' we can only say 'A' is hotter than 'B' ; but we cannot say how much hot or cold A and B are.

Thermal equilibrium, Zeroth Law of Thermodynamics and temperature

For a rigorous and quantitative definition of temperature, we must define it in terms operations which invlove quantities that can be measured objectively.

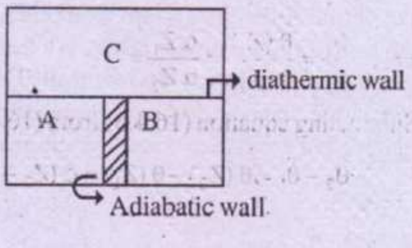
It is first of all important to note that temperature is a fundamental property of a macroscopic system. It is meaningless to talk about temperature of a single particle or a system of few particles. Secondly one can define temperature for a macroscopic system, when it is in thermal equilibrium with the surrounding. The meaning of thermal equilibrium is as explained below.

We know that every system possesses measurable properties like length (L), pressure (P), volume (V), resistance (R) etc., which vary with degree of hotness or coldness. When a system exists in a given state these quantities (determining the state of the system) have definite values. The values of these quantities completely specify the state of the system and are called as state-variable or state co-ordinates of a system. When the state coordinates of a system are constant in time, the system is said to be in thermal equilibrium.

If two systems A and B are separated by an adiabatic wall, they do not influence each other's state-coordinates. On the contrary when A and B are separated by a diathermic wall, they influence each other's state-coordinates and we say 'A' and 'B' are in thermal contanct. Eventually a state is reached when no further change in state coordinates of A and B takes place, implying that A and B are in thermal equilibrium with each other. Thus "**two systems in thermal contact, so that they influence each other's state-coordinates, are said to be in thermal equilibrium, if their state-coordinates donot change.**"

Zeroth law of thermodynamics states that *"if two systems A and B are separately in thermal equilibrium with a third system (C), then they (A & B) must also be in thermal equilibrium."*

Fig.:



Thus Zeroth law of thermodynamics implies that two systems have the same measure of a characteristic property which determines whether or not two systems will be in thermal equilibrium. This point of view leads one to define :

" Temperature of a system is a scalar quantity and may be represented by a number. Setting up a temperature scale is merely an adoption of a set of rules for assigning numbers of temperature. When two bodies are in thermal equilibrium, we say they have same temperature."

16.4 Thermometers and Scales of temperature

A thermometer is a device, used to measure the temperature of a body which is in thermal equilibrium with it.

For this purpose one chooses a substance and looks for a measurable property of the substance which monotonically (linearly) changes with temperature. Such a property is called thermometric property, and the substance is called thermometric substance. The temperature (θ) can thus be defined as a chosen monotonic function of the thermometric property (Z) of the thermometric substance.

i.e $$\theta = \theta(Z) = \alpha Z \qquad ...(16.4.1)$$

Where 'α' is a constant of proportionality.

If Z_1 and Z_2 are measured values of the thermometric property, then temperatures are given by

$$\theta_1 = \theta(Z_1) = \alpha Z_1 \qquad ...(16.4.2)$$

$$\theta_2 = \theta(Z_2) = \alpha Z_2 \qquad ...(16.4.3)$$

$$\Rightarrow \quad \frac{\theta_1}{\theta_2} = \frac{\theta(Z_1)}{\theta(Z_2)} = \frac{\alpha Z_1}{\alpha Z_2} = \frac{Z_1}{Z_2} \qquad ...(16.4.4)$$

Subtracting equation (16.4.3) from (16.4.2)

$$\theta_2 - \theta_1 = \theta(Z_2) - \theta(Z_1) = \alpha(Z_2 - Z_1) \qquad ...(16.4.5)$$

This implies if $Z_1 = Z$, $Z_2 = Z + \Delta Z$, so that $\theta_1 = \theta$ and $\theta_2 = \theta_1 + \Delta\theta$, then we obtain

$$(\theta + \Delta\theta) - \theta = \theta(Z + \Delta Z) - \theta(Z) = \alpha[(Z + \Delta Z) - Z]$$

$$\Rightarrow \quad \Delta\theta = \alpha\,\Delta Z \qquad ...(16.4.6)$$

Equation 16.4.6 implies that equal temperature intervals correspond to equal changes in thermometric property Z; irrespective of the starting temperature.

However for absolute measurement, two standard reference point Z_O and Z_S are warranted.

Let one standard point be Z_O, so that

$$\theta_o = \theta(Z_o) = \alpha Z_o \qquad ...(16.4.7)$$

Then

$$\alpha = \frac{\theta_o}{Z_o} = \frac{\theta(Z_o)}{Z_o} \qquad ...(16.4.8)$$

Using eqn. (16.4.8) in eqn. (16.4.1) we obtain

$$\theta = \theta(Z) = \frac{Z}{Z_o}\theta(Z_o) = \frac{Z}{Z_o}\theta_o \qquad ...(16.4.9)$$

The standard fixed point 'θ_0' used in modern thermometry is the triple point of water θ_{tr} to which the arbitrary number given is 273.16 ^{0}K

i.e. $$\theta_o = \theta_{tr} = \theta(Z_{tr}) = 273.16\ ^0K$$

The other standard fixed point is the steam point θ_s at normal atmospheric pressure. Then from (16.4.9).

$$\theta_s = \frac{Z_s}{Z_o}\theta_o \qquad ...(16.4.10)$$

This gives

$$\theta_s - \theta_o = \frac{Z_s - Z_o}{Z_o}\theta_o$$

$$\Rightarrow \quad \frac{\theta_o}{Z_o} = \frac{\theta_s - \theta_o}{Z_s - Z_o} \qquad ...(16.4.11)$$

Using eqn. 16.4.11 in 16.4.9

$$\theta = \left(\frac{\theta_s - \theta_0}{Z_s - Z_0}\right).Z$$

and

$$\theta - \theta_0 = \frac{\theta_s - \theta_0}{Z_s - Z_0}(Z - Z_0) \qquad ...(16.4.12)$$

For the sake of illustration we consider below few thermometers.

A. Liquid Thermometer :

In case of liquid thermometer, the thermometric property is the length that varies monotonically with temperature. Then by equation 16.4.9.

$$\theta = \frac{\ell}{\ell_0}\theta_0$$

and

$$\theta - \theta_0 = \frac{\ell - \ell_0}{\ell_0}\theta_0 \qquad ...(16.4.13)$$

If θ_s corresponds to (Steam temperature) some other standard temperature, then

$$\theta_s - \theta_0 = \frac{\ell_s - \ell_0}{\ell_0}\theta_0 \qquad ...(16.4.14)$$

From eqn. (16.4.14) we have

$$\frac{\theta_0}{\ell_0} = \frac{\theta_s - \theta_0}{\ell_s - \ell_0} \qquad ...(16.4.15)$$

Using (16.4.15) in eqn. (16.4.13)

$$\theta - \theta_0 = \frac{\theta_s - \theta_0}{\ell_s - \ell_0}(\ell - \ell_0) \qquad ...(16.4.16)$$

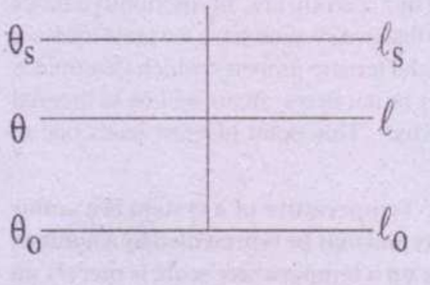


Fig.16.1

If the segment ($\ell_s - \ell_0$) is divided into N-equal divisions then each division 'd' is equal to $(\ell_s - \ell_0) / N$. Then

$$\theta - \theta_0 = \frac{\theta_s - \theta_0}{N.d}(\ell - \ell_0) \qquad ...(16.4.17)$$

If $\ell - \ell_0 = n\,d$, then

$$\theta - \theta_0 = \left(\frac{\theta_s - \theta_0}{N}\right).n \qquad ...(16.4.18)$$

In eqn. (16.4.18), $(\theta_s - \theta_0)/N = \Delta\theta$, correspond to temperature rise per division of the thermometers. Therefore temperature 'θ' above the reference temperature θ_0 i.e. $\theta - \theta_0$ is n-times the value of each division.

Example : Mercury in glass the rmometer.

B. Constant volume gas thermometer :

In case of constant volume gas thermometer the thermometric property "pressure" varies monotonically with temperature. Hence

$$\theta = \frac{P_\theta}{P_0}.(273.16)^0K \qquad ...(16.4.19)$$

where P_θ is the pressure at the temperature θ.

Example : Joly's constant volume air thermometer, Regnault's constant pressure air thermometer.

Scales of temperature

Different scales of temperature in use are

(i) Kelvin scale (ii) Celsius scale (iii) Fahrenheit scale.

Kelvin Scale

The temperature scale defined by the equation

$$\theta = 273.16 \times \lim_{P_o \to 0} \frac{P_\theta}{P_o} \qquad ...(16.4.20)$$

is called the ideal scale. In this scale the lowest fixed point is the absolute zero and triple point of water is 273.16^0 K. the ice point is 273.15^0K and steam point is 373.15^0K. The temperature interval between ice point and steam point is divided into 100 equal divisions.

Celsius Scale : (Named after Anders Celsius)

In this scale, zero point is shifted so that temperature of triple point of water is 0.01^0 C.

i.e. $\theta_c = \theta_k - 273.15 \qquad ...(16.4.21)$

The steam-point on celsius scale is therefore given by

$$373.15 - 273.15 = 100^0\text{C}$$

The temperature interval between steam point and ice-point is divided into 100 equal divisions. So

1 div. of Celsius scale = 1 div. of Kelvin scale.

The absolute zero on Kelvin scale corresponds to $\theta_c = 0 - 273.15 = -273.15^o$C on celcius scale.

Fahrenheit Scale :(Named after Gabriel Fahrenhe't)

In this scale ice point is chosen to be 32^0F, and steam point is chosen to be 212^0 C. The temperature interval between steam point and ice point is divided into 180 equal divisions.

i.e $100\,^0\text{C} = 180\,^0\text{F}$

$$(\theta_F)_{ice} = (\theta_c)_{ice} + 32 \qquad ...(16.4.22)$$

Note : There is also another scale, known as Reaumur (introduced by french philosopher Reumer) With melting print of ice is taken at 0° and boiling point of water as 80°, under normal atmospheric pressure, dividing the interval into 80 equal parts.

Relation between temperature Scales :

It is observed from (16.4.21) & (16.4.22) that

$$\theta_c = \theta_k - 273.15$$

$$(\theta_F)_{ice} = (\theta_c)_{ice} + 32$$

$$\Rightarrow \quad (\theta_c)_{ice} = (\theta_k)_{ice} - 273.15$$

$$= 273.15 - 273.15 = 0^0\text{C}$$

$$(\theta_F)_{steam} = (\theta_F)_{ice} + 180$$

$$(\theta_c)_{steam} = (\theta_c)_{ice} + 100$$

$$(\theta_k)_{steam} = (\theta_k)_{ice} + 100$$

$$\Rightarrow \quad 180^o\text{F} = 100^o\text{C} = 100^o\text{K}$$

$$\Rightarrow \quad 1^o\text{F} = \tfrac{5}{9}{}^o\text{C} = \tfrac{5}{9}{}^o\text{K}$$

$$1^o\text{C} = \tfrac{9}{5}{}^o\text{F} = 1^o\text{K}$$

Therefore if θ_F and θ_c be temperatures on Fahrenheit and Celsius scale respectively, then

$$\theta_F = \tfrac{9}{5}\theta_c + 32$$

$$\Rightarrow \quad \theta_C = \tfrac{5}{9}(\theta_F - 32) = \theta_k - 273.15$$

$$\Rightarrow \quad \frac{\theta_c}{100} = \frac{\theta_F - 32}{180} = \frac{\theta_k - 273.15}{100} \qquad ...(16.4,23)$$

Ex.16.4.1 Suppose a constant volume gas thermometer has a pressure of 1.50×10^4 Pa at the triple point of water and a pressure of 2.05×10^4 Pa at the normal boiling point. What is the temperature T_B at the normal boiling point.

Soln.

$$\theta = (273.16^\circ K) \times \frac{P_\theta}{P_o}$$

$$= 273.16 \times \frac{2.05 \times 10^4}{1.50 \times 10^4} {}^\circ K$$

$$\theta = 373^\circ K$$

Ex.16.4.2 Normal temperature of a human body is 98.40 K. Express it in celsius temperature.

Soln.

$$\theta_c = \frac{5}{9}(\theta_F - 32)$$

$$= \frac{5}{9}(98.4 - 32) = \frac{5}{9} \times 66.4$$

$$\Rightarrow \quad \theta_c = 36.89\ {}^\circ C$$

Ex.16.4.3 At what temperature (a) Fahrenheit and Celsius (b) Fahrenheit and Kelvin scale give the same reading.

Soln.

(a) Let 'x' be the reading. Then

$$x = \frac{5}{9}(x - 32)$$

$$\Rightarrow \quad \frac{4}{9}x = -\frac{32 \times 5}{9}$$

$$\Rightarrow \quad x = -40$$

$$\Rightarrow \quad x = -40^\circ F = -40\ {}^\circ C$$

(b) Let x be the reading, then

$$\theta_k - 273.15 = \frac{5}{9}(\theta_F - 32)$$

$$\Rightarrow \quad x - 273.15 = \frac{5}{9}(x - 32)$$

$$\Rightarrow \quad \frac{4}{9}x = 273.15 - \frac{5 \times 32}{9}$$

$$\Rightarrow \quad x = \frac{9}{4}\left[273.15 - \frac{160}{9}\right]$$

$$\Rightarrow \quad x = 574.59\ {}^\circ K = 574.59\ {}^\circ F$$

Ex.16.4.4 At what temperature on celsius scale the reading on fahrenheit scale will be twice that on celsius scale.

Soln.

Let 'x' be temperature on celsius scale then 2x shall be reading on Fahrenheit scale.

$$\Rightarrow \quad x = \frac{5}{9}(2x - 32)$$

$$\Rightarrow \quad \frac{x}{9} = \frac{160}{9}$$

$$\Rightarrow \quad x = 160^\circ C$$

Ex.16.4.5 A thermometer is designed with ice point marked as 15^0 and steam point marked as 105^0. It reads the temperature of a body as 78^0. What will be its reading on celsius scale ?

Soln.

Now

$$\frac{\theta_c}{100} = \frac{\theta_u - 15}{90}$$

$$\Rightarrow \quad \theta_c = \frac{10}{9}(78 - 15) = 70^\circ C$$

16.5 Specific, Heat, Thermal Capacity, Water equivalent, molar sp. heat

A. Specific heat (or Specific heat capacity)

It is observed that (i) when two bodies of same material but of different masses (m_1 & m_2) are heated through same range of temperature different amounts of heat are required, greater mass requiring larger amount of heat. (ii) when two bodies of same mass and same material are heated through different range of temperature different amounts of heat is required, higher range requiring larger amount of heat. (iii) when two bodies of same mass but of different materials are heated through same range of temperature different amounts of heat are required. These observations indicate that heat (ΔQ) required to raise the temperature through $\Delta\theta$, for a body of mass m, must satisfy :

$$\Delta Q \ \alpha\, m$$

$$\alpha\, \Delta\theta$$

and

$$\Delta Q = c(\theta).m.\ \Delta\theta \qquad ...(16.5.1)$$

where the constant of proportionality 'c' depends on the thermal property of the material. This constant of proportionality is called specific heat. In general $c = c(\theta)$ i.e. specific heat is a function of temperature. So we define average specific heat as follows :

Suppose a body of mass m is heated through the temperature range θ_1 to θ_2 in n-steps. The total heat supplied is ΔQ (say). Then

$$\Delta\theta = \frac{\theta_2 - \theta_1}{n}$$

$$\Rightarrow \quad \theta_2 = \theta_1 + n\,\Delta\theta \qquad ...(16.5.2)$$

We make 'n' sufficiently large so that $\Delta\theta$ is very small and specific heat 'c' remains constant over this interval. Now considering heating in steps.

$\Delta Q_1 = m.c_1.\Delta\theta$ (c_1 = sp. heat over range θ_1 to $\theta_1 + \Delta\theta$)

$\Delta Q_2 = m.c_2.\Delta\theta$ (c_2 = sp. heat over range $\theta_1 + \Delta\theta$ to $\theta_1 + 2\Delta\theta$)

$\Delta Q_n = m.c_n.\Delta\theta$ (c_n = sp. heat over range $\theta_1 + (n-1)\Delta\theta$ to $\theta_2 = \theta_1 + n\Delta\theta$)

On adding vertically

$$\Delta Q = \Delta Q_1 + \Delta Q_2 + ...+ \Delta Q_n = m(c_1 + c_2 + ...+ c_n)\Delta\theta$$

$$\Rightarrow \quad \Delta Q = m\left(\frac{c_1 + c_2 + ...+ c_n}{n}\right).n\Delta\theta \quad ...(16.5.3)$$

Defining

$$\langle c \rangle = \overline{c} = \frac{c_1 + c_2 + ...+ c_n}{n}$$

= average sp.heat over range θ_1 to θ_2 ...(16.5.4)

and using eqn. (16.5.4) in eqn. (16.5.3) we obtain

$$\Delta Q = m\,\langle c \rangle.(\theta_2 - \theta_1) \qquad ...(16.5.5)$$

Specific heat at any temperature θ is defined as

$$c(\theta) = \frac{1}{m} \lim_{\Delta\theta \to 0} \frac{\Delta Q}{\Delta\theta} = \frac{1}{m}\frac{dQ}{d\theta} \qquad ...(16.5.6)$$

giving

$$\int dQ = Q = m\int_{\theta_1}^{\theta_2} c(\theta)\, d\theta \qquad ...(16.5.7)$$

However 'c' varies very slightly with temperature, so ordinarily we treat specific heat as nearly constant and thus put

$$Q = mc\int_{\theta_1}^{\theta_2} d\theta = mc(\theta_2 - \theta_1) \qquad ...(16.5.8)$$

Eqn. (16.5.8) leads to define "Specific heat of a substance is the quantity of heat required to raise temperature of unit mass of it through unit range."

Dimension of Specific heat :

$$[c] = \frac{[Q]}{[m][\Delta\theta]} = \frac{M L^2 T^{-2}}{M\ {}^0K} = L^2\ T^{-2}\ {}^0K$$

Unit of specific heat : The various units are as given below

SI unit ofspecific heat is $Jkg^{-1}\,K^{-1}$ or $Jkg^{-1}\,{}^0C^{-1}$

other unit still in use is $calg^{-1}\,{}^0C^{-1}$.

$1 calg^{-1}\,{}^0C^{-1} = 4.186 \times 10^3\ Jkg^{-1}K^{-1}$.

Specific heat of water is taken as $1 calg^{-1}\,{}^0C^{-1}$

B. Thermal Capacity (Heat Capacity)

Heat capacity of a body is defined as the amount of heat required to raise the temperature of the body through unit range.

Since $\Delta Q = m.c.\Delta\theta$

So Heat Capacity $c_T = \dfrac{\Delta Q}{\Delta\theta} = mc$ ...(16.5.9)

i.e. Heat Capacity = mass x specific heat

Its diemension is $[c_T] = M\,L^2\,T^{-2}\,{}^oK^{-1}$

Its units are

(i) Energy Units : (a) C.G.S. = $1\ erg\,/\,{}^0C$

(b) S.I. = $1\ J\,/\,{}^0K$

C. Water Equivalent :

Water equivalent of a body is defined as the mass of water which rises through the same range of temperature as the body when same amount of heat is supplied to water as well as to the body.

Suppose a body of mass m, specific heat c, requires ΔQ amount of heat to rise through a temperature range θ_1 to θ_2. Let W be mass of water which rises through the temperature range θ_1 to θ_2 when ΔQ heat is supplied. Then

$$\Delta Q = m\,c\,(\theta_2 - \theta_1) = W.c_w(\theta_2 - \theta_1)$$

$$\Rightarrow \quad W = m\,\frac{c}{c_w} \qquad ...(16.5.10)$$

Thus we find that water equivalent of a body is equal to the product of mass of the body and ratio of its specific heat to specific heat of water. If specific heat is expressed in cals $gm^{-1}\,{}^0C^{-1}$, then $c_w = 1$ cal $gm^{-1}\,{}^0\,C^{-1}$ and water equivalent then becomes numerically equal to the heat capacity. But water equivalent is expressed in mass units and has the dimension of mass.

D. Molar Specific Heat

Molar specific heat of a substance is defined as amount of heat required to raise the temperature of 1 mole. of the substance through unit range.

$$C = Mc \qquad ...(16.5.11)$$

Where C is molar specific heat, c is specific heat corresponding to unit mass, M is molecular mass.

Suppose a body of mass 'm' has specific heat c, then

Heat capacity $c_T = mc$ ...(16.5.12)

Molar specific heat $C = Mc = \dfrac{m}{n}c$ ...(16.5.13)

where n is the number of moles present in the body. Using (16.5.12) in (16.5.13) we obtain

$$C = \frac{c_T}{n} \qquad ...(16.5.14)$$

Molar specific heat at constant volume (C_V)

It is defined as the amount of heat required to raise the temperature of 1 mole of gas through unit range at constant volume.

i.e. If dQ be heat supplied to the body of mass 'm' containing 'n' moles to raise the temperture through $d\theta$, at constant volume, then

$$C_v = \frac{1}{n}\left(\frac{dQ}{d\theta}\right)_v \qquad ...(16.5.15)$$

$$\Rightarrow \quad dQ = n\,C_v\,d\theta$$

C_V is expressed in cals / mole ^{0}C, or jouls / mole ^{0}K.

Specific heat at constant volume per unit mass is then given by

$$c_V = \frac{C_V}{M} = \frac{1}{M\,n}\left(\frac{dQ}{d\theta}\right)_V = \frac{1}{m}\left(\frac{dQ}{d\theta}\right)_V \qquad ...(16.5.16)$$

Molar specific heat at constant pressure :

It is defined as the amount of heat required to raise the temperature of 1 mole of the substance through unit range at constant pressure. i.e.

$$C_P = \frac{1}{n}\left(\frac{dQ}{d\theta}\right)_P \qquad ...(16.5.17)$$

Where dQ is the heat supplied to a body of mass 'm' containing n moles of the substance at constant pressure.

Specific heat at constant pressure (per unit mass) is given by

$$c_P = \frac{C_P}{M} = \frac{1}{M\,n}\left(\frac{dQ}{d\theta}\right)_P = \frac{1}{m}\left(\frac{dQ}{d\theta}\right)_P \qquad ...(16.5.18)$$

For solids and liquids the difference between Cp and Cv (or c_p & c_V) is negligible. But for gases Cp and Cv differ appreciably.

Ex.16.5.1 A piece of lead weighing 500 g. gives out 1200 calorie of heat when it is cooled from 90^0 C to 10^0 C. Find its specific heat capacity and water equivalent.

Soln.

$$Q = m.\ c.\ (\theta_1 - \theta_2)$$

$$\Rightarrow\ 1200 \text{ cals} = 500 \text{ g. c. } (90-10)\ ^0\text{C}$$

$$\Rightarrow\ c = \frac{1200}{500\times 80}\,\frac{\text{cals}}{\text{g}\ ^0\text{C}} = \frac{12}{400}\,\frac{\text{cals}}{\text{g}\ ^0\text{C}} = 0.03\,\frac{\text{cals}}{\text{g}^0\text{C}}$$

Water equivalent W= mc = 500 x 0.03 =15 g.

Ex.16.5.2 A liquid of specific heat capacity 0.5 cal/(g ^{0}C) at 60^{0}C is mixed with another liquid of specific heat capapcity 0.3 cal/(g ^{0}C) at 20^{0}C. After mixing, the temperature of the mixture becomes 30^{0}C. In what proportion, by weight are the two liquids mixed ?

Soln.

Given liquid No. 1 has sp. heat = c_1 = 0.5 cal/g ^{0} C

Initial temp. of liq. No. 1 = 60^0 C

Let its mass be = m_1

Liquid No. 2 has sp. heat = c_2 = 0.3 cal/ g^{0}C

Initial temp. of liq. No. 2 = 20^0 C

Let its mass be = m_2

Now heat loss by liq. no. 1 = Heat gain by liq. No.2

$$\Rightarrow\ m_1 c_1 (60-30) = m_2 c_2 (30-20)$$

$$\Rightarrow\ \frac{m_1}{m_2} = \frac{c_2\times 10}{c_1\times 30} = \frac{0.3\times 10}{0.5\times 30} = \frac{1}{5}$$

Thus their weights are in the ratio 1 : 5.

Table No. 16.1 (Specific heat of some materials)

Material	sp. heat in cal/gm^0 C	Material	So. heat in cal/gm^0 C
Aluminium	0.217	Lead	0.031
Asbestes	0.196	Magnesium	0.248
Beryllium	0.436	Silicon	0.168
Bismuth	0.03	Silver	0.056
Boric	0.2	Sodium	0.294
Cadmium	0.055	Sodium Chloride	0.21
Carbon, graphite	0.16	Tungsten	0.034
Concrete	0.21	Wood	0.693
Copper	0.093	Zinc	0.084
Diamond	0.119	Benzene	0.387
Ethanol	0.55	Bromine	0.107
Ethyleneglycol	0.528	Ethylalcohol	0.58
Germanium	0.077	Glycerine	0.597
Glass	0.1-0.2	Methylalcohol	0.602
Gold	0.031	Metcury	0.033
Ice	0.5	Water	1.0
Iron	0.107	Steam	0.48

Table No. 16.2 (C_p, C_V for some gases)

Gas	Molar mass in gm / mole	Molar sp. heat in cal/mole °C		C_p / C_V
		C_p	C_v	
Argon	39.9	4.993	2.986	1.672
Helium	4	4.993	2.986	1.672
Hydrogen	2	6.832	4.893	1.402
Neon	22	5.041	3.034	1.662
Nitrogen	28	6.952	4.969	1.399
Oxygen	32	7.023	5.017	1.4

16.6 Calorimetry :

A system (or a body) is said to be thermally isolated if there is no heat exchange between the system (or the body) and the surrounding.

When different parts of a system are at different temperatures, heat exchanges between its parts continue until the parts attain thermal equilibrium and the mixture reach an equilibrium temperature. In this process, accroding to law of conservation of energy (when there is no heat exchange with surroundings)

Heat lost by hot object (s) = Heat gained by cold object (s). ...(16.6.1)

The technique of measuring the thermal properties of a substance by using the principle "Heat gained by cold objects is equal to the heat lost by hot objects" is called calorimetry.

Calorimeter :

A calorimeter consists of a metallie vessel (generally made of copper) with a stirrer of the same material. The vessel is kept inside a wooden box to thermally isolate it from the surroundings. There is an opening in the top cover (see fig. 16.2) to insert a thermometer for measuring temperature.

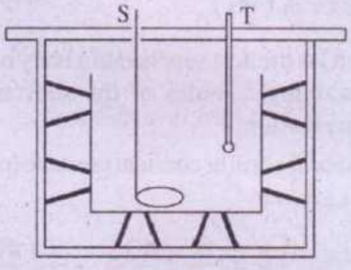


Fig. 16.2

Determination of sp. heat of a solid :

Specific heat of a solid can be determined by using a calorimeter.

Working Formula :

Let M be mass of solid, m_1 be mass of calorimeter and stirrer and m_2 be mass of cold water, calorimeter and stirrer. Then mass of cold water $m = m_2 - m_1$.

Let S be specific heat of solid, S_1 be specific heat of material of calorimeter & stirrer and S_ω be specific heat of water.

Let θ be initial temperature of solid (hot), θ_1 be initial temperature of calorimeter containing cold water and θ_2 be final temperature of the mixture.

Then heat lost by solid = $M.\, S\,(\theta - \theta_2)$

Heat gained by calorimeter, stirrer and water

$$= m_1 S_1(\theta_2 - \theta_1) + (m_2 - m_1)S_\omega(\theta_2 - \theta_1)$$

$$= [m_1 S_1 + (m_2 - m_1)S_\omega](\theta_2 - \theta_1)$$

Since heat loss = heat gain

So

$$M.S(\theta - \theta_2) = [m_1 s_1 + (m_2 - m_1)S_\omega](\theta_2 - \theta_1)$$

$$\Rightarrow \quad S = \frac{[m_1 s_1 + (m_2 - m_1)s_\omega](\theta_2 - \theta_1)}{M(\theta - \theta_2)} \qquad ...(16.6.2)$$

Experimental Procedure :

A clean, dry and empty calorimeter along with stirrer is weighed by a physical balance and its mass m_1 is recorded. It is then filled with water up to $\frac{2}{3}$rd of its capacity and its mass m_2 is taken. Its temperature θ_1 is recorded. Then the calorimeter is kept within the wooden jacket. The solid whose specific heat is to be determined is weighed and its mass M is recorded. Then the solid is heated in a steam chamber by suspending the solid within it . Its steady temperature θ is recorded with the help of a thermometer. After a steady temperature is attained, the string is cut with a sharp knife so that the solid drops into the liquid in the calorimeter. Then constant stirring is made and final steady temperature (θ_2) of the mixture is noted.

By inserting data into the working formula 16.6.2, S is computed.

16.7 Phase Change :

Matter can exist in four different states (or phases) : solid, liquid, gas (or vapour) and plasma. This fourth state of matter plsma, can occur only at high temperature and under unusual physical conditions (whose study is beyond the scope of this book). So we shall consider only the solid, liquid and gaseous states of matter. **Transition from one phase (state) to another phase is called phase change.** This phase change takes place at a definite temperature but with abosption or rejection of heat.

The change of state from solid to liquid is called fusion and from liquid to solid is called freezing. At a given pressure, this phase change occurs at a definite temperature. The temperature at which a solid begins to melt, so that solid and liquid phase co-exist in thermal equilibrium is called melting point. The melting point of a substance depends on pressure, which can be studied from the phase diagram - the graph between pressure and phase change temperature - as shown below.

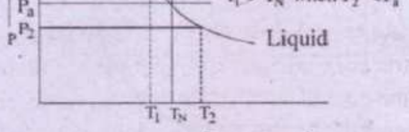


Fig. 16.3. (a)

(fusion curve for substance which contract on melting)

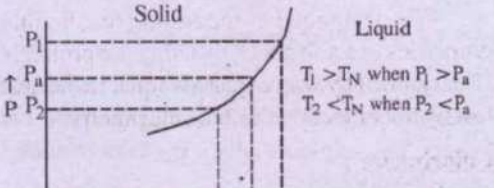


Fig.16.3. (b)

(fusion curve for substances which expand on melting)

The fusion curve (a) shows that for substances which contract on melting (like ice), increase of pressure reduces the melting point. e.g. for temperature $T_1 < T_N$ ice remains as ice at pressure P_a while ice melts at temperature T_1 at a pressure $P_1 > Pa$, conversely when the pressure is decreased to $P_2 < P_a$ the melting point is increased to $T_2 > T_N$. On the contrary for substance which expand on melting, increase of pressure increases the melting point and vice versa.

The change of state from liquid to vapour state is called vaporisation and from vapour to liquid is called condensation. The **temperature at which the liquid phase and vapour co-exist in thermal equilibrium is called boiling point.** Boiling point also varies with pressure and can be studied from the vaporisation curve.

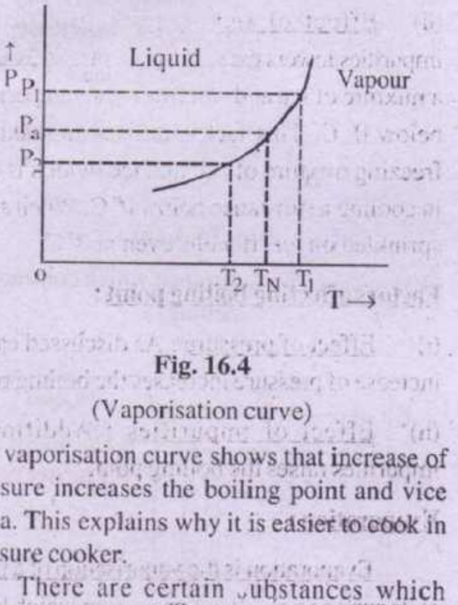


Fig. 16.4

(Vaporisation curve)

The vaporisation curve shows that increase of pressure increases the boiling point and vice versa. This explains why it is easier to cook in pressure cooker.

There are certain substances which normally pass from a solid to vapour state and vice-versa. The change of phase from solid to vapour directly is called sublimation. The corresponding phase-diagram (sublimation curve) is (as shown in fig. 16.5) similar to vaporisation curve.

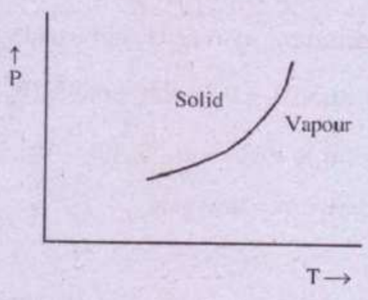


Fig. 16.5

(sublimation curve)

Tripple Point :

It is a point on the phase-diagram giving values of pressure (P_{tr}) and temperature (T_{tr}) at which solid, liquid and vapour states of the substances can co-exist. Fig. 16.6 illustrates this feature. In table no. 16.3 triple points for few substances have been mentioned.

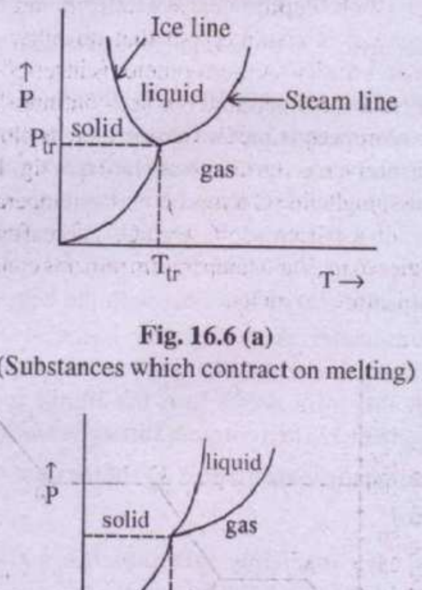


Fig. 16.6 (a)

(Substances which contract on melting)

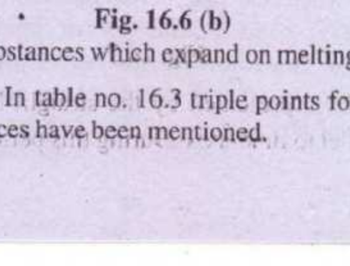


Fig. 16.6 (b)

(Substances which expand on melting)

Table No. 16.3

Substance	T_{tr} in ° K	P_{tr} in 10^5 Pa
H_2	13.84	0.0704
N_2	63.18	0.125
O_2	54.36	0.00152
CO_2	216.55	5.17
H_2O	273.16	0.00610

Phase change of water :

In order to bring about an idea about the phase change, it is instructive to study the phase change of water.

Suppose ice at -25°C is crushed and placed in a container, surrounded by a heating coil which supplies heat at a uniform rate. The container is insulated, so that no other heat reaches the ice. A thermometer is inserted into the ice and temperature is noted continuosly. A graph between time and temperature is plotted. The graph has the nature as shown in fig. 16.7. The straight line OA represents the temperature rise of ice from -25°C to 0°C. Thereafter ice begins to melt and temperature remains constant until entire ice melts.

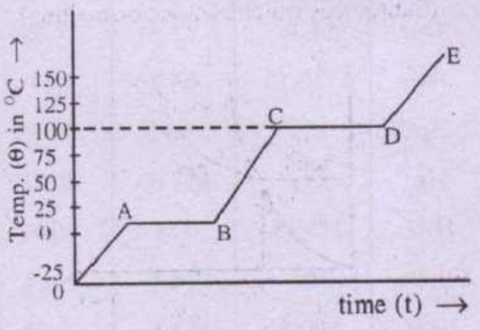


Fig. 16.7

This is represented by the straight line AB, parallel to time-axis. During this period water and ice co-exist in thermal equilibrium. Then temperature of water rises from 0° C to 100° C. As soon as temperature becomes 100° C, water begins to boil and vaporisation takes place. This continues until whole of water is vaporised. This is represented by the straight line CD. During this period water and steam co-exist. Then temperature begins to rise (shown by st. line DE) and steam is now called "Superheated steam".

Factors affecting melting point :

(i) Effect of pressure : As discussed earlier increase of pressure reduces the melting point of substances (like ice) whose volume decreases on melting. But in case of substances which expand on melting (like wax), increase of pressure raises the melting point.

(ii) Effect of impurities : Addition of impurities lowers the melting point. For example a mixture of ice and salt freezes at temperature below 0° C. This fact is utilised in making a freezing mixture of salt and ice, which is used in cooling a substance below 0° C. When salt is sprinkled on ice, it melts even at 0° C.

Factors affecting boiling point :

(i) Effect of pressure : As discussed earlier increase of pressure increases the boiling point.

(ii) Effect of impurities : Addition of impurities raises the boiling point.

Evaporation :

Evaporation is the vaporisation of a liquid at its surface only. It is a process in which liquid molecules on the surface escape and become vaporised. The following facts about evaporation may be noted :

(i) Evaporation can take place at all temperatures. (ii) The rate of evaporation depends on the material, surface area, amount of ventilation and pressure exerted on the surface. (iii) Evaporation causes cooling. Because when the surface molecules, which are more energetic, escape away, the average K.E. of the remaining liquid molecules decreases and hence its temperature decreases.

Regelation : ***"Refreezing after melting under pressure is called regelation".***

For example if a copper wire carrying heavy weights at its ends is hung over a block of ice, the copper wire passes through the block of ice but the ice block is not cut into two halves. This happens because under pressure ice under the wire melts and copper wire passes. But the molten ice above the wire refreezes as there is no further pressure above the wire.

This process of Regelation explains the formation of snow balls, ice skating etc. as explained below.

(i) Ice skating : The narrow blades of the skate exerts high pressure on the ice so that ice melts under pressure. The thin layer of water thus created serves as a lubricant and the skate moves easily without friction.

(ii) Snow ball : When snow is compressed by hands, some of the ice crystals melt due to increase of pressure. When the presure is released, refreezing occurs. As a result snow particles stick together forming a ball.

Latent Heat :

As discussed earlier during phase change temperature remains constant. So heat used (or given up) during phase change of unit mass of the substance is called its latent heat (or heat of transformation).

If 'm' units of mass takes in (or gives up) Q amount of heat during phase change, then latent heat 'L' is given by

$$L = \frac{Q}{m} \quad ...(16.7.1)$$

$$\Rightarrow Q = mL \quad ...(16.7.2)$$

Latent heat is a scalar quantity. Its dimension is $L^2 T^{-2}$ and its units are cals / g, erg / g, k cal / kg and joules / kg etc.

Depending on the kind of phase change latent heat can be of two types (i) latent heat of fusion (L_f) (ii) latent heat of vaporisation (L_v).

The quantity of heat taken in by unit mass of a substance during its phase change from solid to liquid at its melting point is called latent heat of fusion i.e. $L_f = \frac{Q}{m}$.

The quantity of heat taken in by unit mass of a substance during its phase change from liquid to vapour state at its boiling point is called latent heat of vaporisation i.e. $L_v = Q / m$.

Tables 16.4 and 16.5 respectively show values of L_f and L_v for few substances.

Table No. 16.4 (Latent heat of fusion)

Substance	Normal m.p in ^{0}K	L_f in J/g	L_f in cal/g
H_2	13.84	58.6	14.00
N_2	63.18	25.5	6.09
Cu	1356	134	32.01
O_2	54.36	13.8	3.30
Au	1336.15	64.5	15.41
C_2H_5OH	159	104.2	24.89
Hg	234	11.8	2.82
H_2O	273.15	335	80.02
Sulphur	392	38.1	9.10
Lead	600.5	24.5	5.85
Sb	903.65	165	39.42
As	1233.95	88.3	21.09

Table No. 16.5 (Latent heat of vaporisation)

Substance	Normal boiling point in ºK	L_V in J/g	L_v in cal/g
H_2	20.26	452	107.98
N_2	77.34	201	48.02
O_2	90.18	213	50.88
C_2H_5OH	351	854	204.01
Hg	630	272	64.98
H_2O	373.15	2260	540
S	717.75	326	77.88
Pb	2023	871	208.07
Sb	1713	561	134.02
Ag	2466	2336	558.05
Au	2933	1578	376.97
Cu	1460	5069	1210.94

Determination of Latent heat :-

(i) Latent heat of fusion of ice :

Latent heat of fusion of ice can be determined by making use of a calorimeter, shown in fig. 16.8

Working formula :

Let 'm' be mass of calorimeter and stirrer

'm_1' be mass of warm water, calorimeter and stirrer

=> mass of warm water = $m_1 - m = M$.

Let S be sp. heat of material of calorimeter

S_i be sp. heat of ice

θ_1 be initial temperature of warm water

θ_2 be final temperature of mixture after ice is dropped and melted.

θ_0 be temperature of dry ice

m_2 be mass of calorimeter, stirrer and mixture.

=> mass of ice = $M_i = m_2 - m_1$

Now heat loss = heat gain

$$\Rightarrow \quad m\,S(\theta_1-\theta_2)+M.1.(\theta_1-\theta_2)=M_iS_i(0-\theta_0)$$

$$+M_iL_f+M_i.1.(\theta_2-0) \qquad ...(16.7.3)$$

Assuming dry ice to have temperature $\theta_0 = 0^0C$. Eqn. 16.7.3, reduces to

$$(m\,S+M)(\theta_1-\theta_2)=M_iL_f+M_i\theta_2$$

$$\Rightarrow \quad L_f=\frac{(mS+M)(\theta_1-\theta_2)-M_i\theta_2}{M_i} \qquad ...(16.7.4)$$

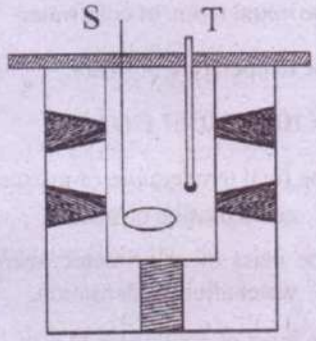


Fig : 16.8
(Determination of latent heat of fusion of ice)

Procedure :

The calorimeter and stirrer are weighed. Then some warm water is taken in the calorimeter and weighed again. Initial temperature of warm water is recorded by the help of the thermometer T. Then some dry ice is taken, its temperature is recorded and dropped into the warm water kept in the calorimeter. Then constant stirring is done till the whole of ice melts and a steady temperature is attained.

This is recorded by the thermometer T. The mass of calorimeter, stirrer and mixture is measured. Using the values in eqn. (16.7.4) L_f is determined.

(ii) Latent heat of vaporisation of water :

Latent heat of vaporisation can be determined by making use of apparatus shown in fig. 16.9.

Working formula :

Let 'm' be mass of calorimeter and stirrer

'm_1' be mass of calorimeter, stirrer and water.

'S' be specific heat of calorimeter

θ_1 be initial temp. of cold water

θ_s be temperature of steam

$\theta_s = 100 + 0.0367\,(760 - P)$

θ_2 be final temperature of mixture after condensation of steam

m_2 be mass of calorimeter, stirrer and water after condensation.

Then mass of cold water $M = m_1 - m$

mass of steam $M_v = m_2 - m_1$

Since heat gain = heat loss

$$\Rightarrow \; M.1.\,(\theta_2 - \theta_1) + mS(\theta_2 - \theta_1) = M_v L_v + M_v.1.(\theta_s - \theta_2)$$

$$\Rightarrow \; (M + mS)\,(\theta_2 - \theta_1) = M_v L_v + M_v(\theta_s - \theta_2)$$

$$\Rightarrow \; L_v = \frac{(M + mS)(\theta_2 - \theta_1) - M_v(\theta_s - \theta_2)}{M_v}$$

$$\Rightarrow \; L_v = \frac{(M + mS)}{M_v}(\theta_2 - \theta_1) - (\theta_s - \theta_2) \quad \text{...(16.7.5)}$$

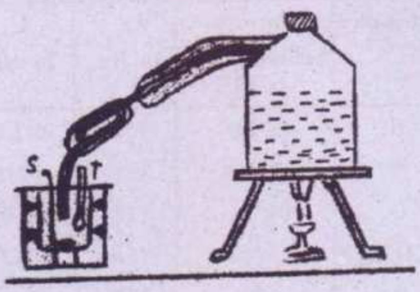


Fig. 16.9

(Determination of Latent heat of vaporisation)

Procedure :

Empty and dry calorimeter along with stirrer is weighed. Then calorimeter is filled with some cold water and again weighed. Initial temperature (θ_1) of cold water is noted and then steam from a steam chamber is passed through the cold water. Final temperature (θ_2) of the mixture is noted. Then mass of the mixture is determined. Using eqn. (16.7.5) latent heat of vaporisation is determined.

Ex.16.7.1 Calculate the amount of heat required to convert 10 gm of ice at - 40° C to steam at 120°C (sp.heat of steam S = 0.46 cals/g)

Soln.

Heat required $Q = Q_1 + Q_2 + Q_3 + Q_4 + Q_5$

Q_1 = Heat required for ice at - 40°C → ice at 0°C

Q_2 = Heat required for ice at 0°C → water at 0°C

Q_3 = Heat required for water at 0°C → water 100°C

Q_4 = Heat required for water at 100°C → steam at 100°C

Q_5 = Heat required for steam at 100°C → steam at 120°C

$Q_1 = 10 \times 0.5 \times 40 = 200$ cal

$Q_2 = 10 \times 80 = 800$ cal

$Q_3 = 10 \times 1 \times 100 = 1000$ cal

$Q_4 = 10 \times 540 = 5400$ cal.

$Q_5 = 10 \times 0.46 \times 20 = 92$ cal

=> $Q = 7492$ cal.

Ex.16.7.2 80 g of ice at -10°C is dropped into water at 0° C. If 5 g. of water freeze, calculate the specific heat capacity of ice.

Soln.

5 g. of water give up heat = 5×80 cal

This heat is absorbed by ice at - 10°C and attain temperature 0°C.

=> $5 \times 80 \text{ cal} = 80 \text{ g} \times S \times 10^0\text{C}$

=> $S = 0.5 \text{ cal} / (\text{g } {}^0\text{C})$.

Ex.16.7.3 The latent heat of vaporisation of alcohal is 220 cal / g at 0°C. Calculate the amount of alcohol to be evaporated in order to freeze 11 g of water at 0°C. (latent heat of ice is 80 cal/g)

Soln.

Heat taken in by alcohal for vaporisation
= Heat given up by water during freezing

$$\Rightarrow \quad 220 \frac{\text{cal}}{\text{g}} \times m \text{ g} = 11 \text{g} \times 80 \frac{\text{cal}}{\text{g}}$$

=> $m = 4$ g.

Ex.16.7.4 What is the result of mixing 0.1 kg of ice at 0° C and 0.1 kg of water at 60° C ?

Soln.

Amount of heat given up by water in going from 60° C to 0° C = $100 \times 60 = 6000$ cal

Heat required for melting ice is

$= 100 \times 80 \text{ cal} = 8000$ cal.

Hence whole of ice cannot melt.

Ice that can melt is $\frac{6000}{80} = 75$ g.

So final mixture shall be 175 g of water at 0°C and 25 g of ice at 0°C.

16.8 Thermal Expansion :

It is commonly experienced that a body expands on heating and contracts on cooling, in all dimensions simultaneously. The change in any dimension is directly proportional to its original dimension and also the change in temperature, when temperature change is small.

$$\text{i.e.} \quad \Delta L(T) \propto L(T)$$

$$\propto \Delta T \text{ (for small } \Delta T)$$

$$\Rightarrow \quad \Delta L(T) = \lim_{\Delta T \to 0} \alpha(T) L(T) \Delta T \qquad ...(16.8.1)$$

where $L(T)$ is the measure of a dimension at temperature T, ΔT is the change in temperature. $\Delta L(T) = [L(T + \Delta T) - L(T)]$ is the change in the measure of the dimension and $\alpha(T)$ is the constant of proportionality, whose value depends on the nature of the material and the temperature of the body in general.

From materials having no preferential directions (isotropic body) every linear dimension changes according to eqn. (16.8.1). However there are certain exceptional cases, wood, for example which have different expansion properties along the grain and across the grain and some crystals can have different properties along different crystal-axis directions. However we exclude these exceptional cases.

Since the amount of change in any dimension depends on the original dimension so in case of solids there can be three types of expansion (i) linear expansion (ii) superficial expansion (iii) volume (cubical) expansion. In case of liquids and gases there shall be only volume expansion.

I. Thermal expansion of solids :

(a) Linear expansion :

When we consider a long and thin rod, the expansion of its length is appreciable while the changes in lateral dimensions is not appreciable. Therefore we have a linear expansion.

Thus the change in length of any dimension is called linear expansion or contraction. It follows from (16.8.1) that

$$\Delta L(T) = \lim_{\Delta T \to 0} \alpha(T) L(T) \Delta T$$

$$\Rightarrow \quad \alpha(T) = \lim_{\Delta T \to 0} \frac{1}{L(T)} \frac{\Delta L(T)}{\Delta T} = \frac{1}{L(T)} \cdot \frac{d L(T)}{dT} \qquad ...(16.8.2)$$

As said earlier $\alpha(T)$ in general is a function of temperature. In particular it depends on temperature at low temperatures and remains nearly constant above room temperature (see fig. 16.10)

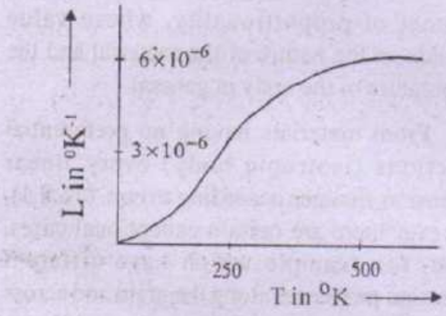


Fig. 16.10

It is named as the temperature coefficient of linear expansion or simply coefficient of linear expansion. Its temperature dependence is given by a relation like

$$\alpha(T) = B_1 e^{-a/T} \qquad ...(16.8.3)$$

where T is absolute temperature, B_1 and a are constants. Thus we can define (from 16.8.2) : "Coefficient of linear expansion at any temperature T, is the ratio of the rate of change (increase) of length with respect to temperature to the original length L(T) at that temperature T"

From eqn. (16.8.2) we find that when $\alpha(T)$ slowly varies average value of $\alpha(T)$ can be defined as

$$\langle \alpha(T) \rangle = \overline{\alpha} = \frac{\int_{T_0}^{T} \alpha(T) dT}{T - T_0}$$

$$= \frac{\int_{L(T_0)}^{L(T)} \frac{1}{L(T)} \cdot \frac{dL(T)}{dT} dT}{T - T_0}$$

$$\Rightarrow \quad \langle \alpha(T) \rangle = \overline{\alpha} = \frac{\int_{L(T_0)}^{L(T)} \frac{dL(T)}{L(T)}}{T - T_0}$$

$$= \frac{\log [L(T) / L(T_0)]}{T - T_0} \qquad ...(16.8.4)$$

This gives

$$L(T) = L(T_0)\, e^{\overline{\alpha}(T - T_0)} \qquad ...(16.8.5)$$

Differentiating both sides of (16.8.5) w.r.to T

$$\frac{d L(T)}{dT} = L(T_0)\, e^{\overline{\alpha}(T - T_0)} . \overline{\alpha} = \overline{\alpha} L(T)$$

$$\Rightarrow \quad \overline{\alpha} = \frac{1}{L(T)} \frac{d L(T)}{dT} \qquad ...(16.8.6)$$

Eqns. (16.8.6) and (16.8.2) agree. Hence the definition given by eqn. (16.8.2) can be treated as average value of coefficient of linear expansion, when temperature range is wide enough.

Now from (16.8.5) one finds

$$L(T) = L(T_0) [1 + \overline{\alpha}(T - T_0) + \text{terms involving higher powers of } \overline{\alpha}] \qquad ...(16.8.7)$$

Since $\overline{\alpha}$ is of the order of 10^{-6} so we can put

$$L(T) \approx L(T_0)\ [1 + \overline{\alpha}(T - T_0)] \qquad ...(16.8.8)$$

Eqn. (16.8.8) gives

$$\overline{\alpha} = \frac{L(T) - L(T_0)}{L(T_0)(T - T_0)} \qquad ...(16.8.9)$$

Eqn. (16.8.9) leads one to define : **The mean (average) coefficient of linear expansion is the increase (change) in length per unit original length per degree rise (change) of temperature.**

Coefficient of linear expansion has dimension $[\alpha] = {}^0K^{-1}$ and its units are ${}^0C^{-1}$ or ${}^0K^{-1}$. In table 16.6 values of α for some substances are given.

Substance	α in $10^{-5}\ {}^0C^{-1}$	Substance	α in $10^{-5}\ {}^0C^{-1}$
Aluminium	2.3	Gold	1.42
Brass	1.9	Platinum	0.9
Copper	1.7	Wood along grain	0.3 - 0.6
Germanium	0.6	Wood across grain	3.5 - 6.0
Glass	0.-4 - 0.9	Lead	2.9
Silicon	0.24	Steel	1.17
Silver	1.9	Iron (soft)	1.2

(b) Superficial Expansion :

When a thin sheet, whose surface area is very large compared to its thickness, is heated, the change in area is appreciable and change in thickness is negligible

It is observed that the change in area $\Delta A(T)$ at any temperature T is directly proportional to the original area at that temperature and to the change in temperature, when tempeerature changes are small.

i.e $\Delta A(T) \propto A(T)$

$\propto \Delta T$ (for small ΔT)

where, $\Delta A(T) = [A(T+\Delta T) - A(T)]$, and ΔT is the change in temperature. This gives

$$\Delta A(T) = \lim_{\Delta T \to 0} \beta(T).A(T).\Delta T \quad ...(16.8.10)$$

leading to

$$\beta(T) = \lim_{\Delta T \to 0} \frac{1}{A(T)} \frac{\Delta A(T)}{\Delta T} = \frac{1}{A(T)} \frac{dA(T)}{dT} \quad ...(16.8.11)$$

$\beta(T)$ is called the temperature coefficient of superficial expansion at temperature T. Thus one can define (from eqn. 16.8.11) **"Coefficient of superficial expansion at any temperature T is defined as the ratio of the rate of cahnge of area with respect to temperature, to the original are A (T) at that temperature."**

In general $\beta(T)$ depends on temeprature T, in the same manner as $\alpha(T)$ i.e. $\beta(T) = B_2 e^{-b/T}$, where B_2 and b are constant. But above room temperature, $\beta(T)$ is nearly constant or slowly varies with temperature. Hence proceeding in the same manner, as done in case of linear coefficient of expansion $(\alpha(T))$, we can find

$$A(T) = A(T_0)\, e^{\bar{\beta}(T-T_0)} \quad ...(16.8.12)$$

Over the region of temperature T_0 to T, where $\beta(T)$ is assumed to vary slowly with temperature. On differentiation of (16.8.12) one easily finds

$$\bar{\beta} = \frac{1}{A(T)} \frac{dA(T)}{\Delta T} \quad ...(16.8.13)$$

From eqn. (16.8.12) one also finds

$$A(T) = A(T_0)\,[1 + \bar{\beta}\,(T - T_0) + \text{higher orders of } \bar{\beta}]$$

Since $\bar{\beta}$ is of order of 10^{-6}, so

$$A(T) \simeq A(T_0)\,[1 + \bar{\beta}\,(T - T_0)] \quad ...(16.8.14)$$

This gives

$$\bar{\beta} = \frac{A(T) - A(T_0)}{A(T_0).(T - T_0)} \quad ...(16.8.15)$$

Eqn. (16.8.15) leads one to define **"The mean (average) coefficient of superficial expansion is the increase (change) in area per unit original area per degree rise (change) in temperature".**

(c) <u>Cubical Expansion</u> :

When cahnge in all the dimensions is appreciable, cubical (or volume) expansion is said to take place. Proceeding in a similar manner one would then obtain

$$\Delta V(T) = \lim_{\Delta T \to 0} \gamma(T).\, V(T).\, \Delta T \qquad ...(16.8.16)$$

$$\gamma(T) = \lim_{\Delta T \to 0} \frac{1}{V(T)} \frac{\Delta V(T)}{\Delta T} = \frac{1}{V(T)} \frac{dV(T)}{dT} \qquad ...(16.8.17)$$

where $\gamma(T)$ is the coefficient of volume expansion. In general $\gamma(T)$ is a fuction of temperature, in particular at low temperature. Its time dependence is given by

$$\gamma(T) = B_3\, e^{-c/T} \qquad ...(16.8.18)$$

where B_3 and c are constants. From eqn. (16.8.17) one defines : **"Coefficient of volume expansion at temperature T, is the ratio of rate of change of volume w.r.to temperature to the original volume V (T) at that temperature T."**

Above room temperature where $\gamma(T)$ varies slowly with temperature, we have

$$V(T) = V(T_0)\, e^{\bar{\gamma}(T-T_0)} \qquad ...(16.8.19)$$

giving

$$\bar{\gamma} = \frac{1}{V(T)} \frac{dV(T)}{dT}$$

$$V(T) \simeq V(T_0)\,[1 + \bar{\gamma}(T - T_0)] \qquad ...(16.8.20)$$

$$\bar{\gamma} = \frac{V(T) - V(T_0)}{V(T_0)(T - T_0)} \qquad ...(16.8.21)$$

Eqn. (16.8.21) leads one to define **"Mean (average) coefficient of volume expansion is the cahange in volume per unit original volume per degree change in temperature."**

It is important to note that if there is a hole in a solid body, the volume of the hole increases when the body expands, as if the hole were a solid of the same material as the body.

<u>Relation between</u> $\bar{\alpha}, \bar{\beta}, \bar{\gamma}$

We can relate $\bar{\alpha}$, $\bar{\beta}$ and $\bar{\gamma}$ by considering a regular body like a rectangular block or a cube.

(i) Consider a rectangular block of sides ℓ_{10}, ℓ_{20} and ℓ_{30} at temperature T_0. Then its surface area and volume at temperature T_0 are given as

$$A_0 = 2(\ell_{10}\ell_{20} + \ell_{10}\ell_{30} + \ell_{20}\ell_{30}) \qquad ...(16.8.22)$$

$$V_0 = \ell_{10}.\ell_{20}.\ell_{30} \qquad ...(16.8.23)$$

If the block is heated to temperature T, let its sides be ℓ_{1T}, ℓ_{2T} and ℓ_{3T} respectively. Then by eqn. (16.8.5)

$$\ell_{1T} = \ell_{10}.e^{\bar{\alpha}(T-T_0)}$$

$$\ell_{2T} = \ell_{20}\, e^{\bar{\alpha}(T-T_0)}$$

$$\ell_{3T} = \ell_{30}\, e^{\bar{\alpha}(T-T_0)} \qquad ...(16.8.24)$$

The new area and volume are then given as

$$A_T = 2(\ell_{1T}\ell_{2T} + \ell_{1T}\ell_{3T} + \ell_{2T}.\ell_{3T})$$

$$= 2(\ell_{10}\ell_{20} + \ell_{10}\ell_{30} + \ell_{20}.\ell_{30})\, e^{2\bar{\alpha}(T-T_0)}$$

$$\Rightarrow \quad A_T = A_0\, e^{2\bar{\alpha}(T-T_0)} \qquad ...(16.8.25)$$

and

$$V_T = \ell_{1T}\,\ell_{2T}\,\ell_{3T} = \ell_{10}\,\ell_{20}\,\ell_{30}\;e^{3\overline{\alpha}(T-T_0)}$$

$$\Rightarrow\quad V_T = V_0\;e^{3\overline{\alpha}(T-T_0)} \qquad \text{...(16.8.26)}$$

But by definitions (16.8.12) and (16.8.19) we have

$$A_T = A_0\;e^{\overline{\beta}(T-T_0)} \qquad \text{...(16.8.27)}$$

$$V_T = V_0\;e^{\overline{\gamma}(T-T_0)} \qquad \text{...(16.8.28)}$$

Comparision of (16.8.26) & (16.8.27) and eqn. (16.8.26) & (16.8.28) give

$$\overline{\beta} = 2\,\overline{\alpha} \text{ and } \overline{\gamma} = 3\,\overline{\alpha} \qquad \text{...(16.8.29)}$$

i.e $\overline{\alpha} : \overline{\beta} : \overline{\gamma} = 1 : 2 : 3$

Applications of thermal expansion :

(i) In laying down the rail track, a gap is left in between the two consecutive pieces of rails. Because in summer the rails expand and if there is no gap, the rails may bend.

(ii) Concrete floors are not laid in one piece. They are laid in the form of small sections, with joints in between. Because during summer the concrete floors will expand and if there is no joint, it may crack.

(iii) When a drop of water, suddenly falls on a glass cover of a lamp, it cracks. Because, the temperature of the portion where water drop falls, causes contraction of that portion. This creates a crack.

(iv) The telephone and telegraph wires get slackened in summer and become tight in winter.

(v) Sometimes it is difficult to unscrew the metal cap of a bottle. But if we slightly heat it due to larger expansion of metal cap then the glass (body), the metal cap is easily unscrewed.

(vi) Metal tyre of wooden wheel of a cart is made slightly smaller than the size of the wheel. The tyre is heated so that it expands. While it is hot it is dressed on the wheel. On cooling it contracts and forms a firm grip on the wheel.

I Thermal expansion of liquids :

In case of liquids there is only volume expansion. But while measuring volume expansion of liquids one has to take into account the expansion of the contrainer. Because of expansion of the container itself (say a measuring flask) the expansion of liquid noted is less than its actual expansion. So

$$\Delta V_r(T) = \Delta V_c(T) + \Delta V_a(T)$$

where $\Delta V_r(T)$ = Real expansion of the lqiuid

$\Delta V_c(T)$ = Expansion of container

$\Delta V_a(T)$ = Apparent(observed) expnasion of liquid.

So we write

$$V_\ell(T_0)\left[\overline{\gamma}_r(T-T_0)\right] = V_c(T_0)\left[\overline{\gamma}_c(T-T_0)\right] + V_\ell(T_0)\left[\overline{\gamma}_a(T-T_0)\right]$$

$$\Rightarrow\quad \overline{\gamma}_r = \overline{\gamma}_a + \frac{V_c(T_0)}{V_\ell(T_0)}\,\overline{\gamma}_c \qquad \text{...(16.8.45)}$$

where V_c = Volume of container

V_ℓ = Volume of liquid

$\dot{\gamma}_r$ = Coefficient of real expansion of liquid

γ_a = Coefficient of apparent expansion of liquid

γ_c = Coefficient of volume expansion of container

If the liquid completely fills the container, or nearly fills the container so that $\frac{V_c}{V_\ell} \simeq 1$, then

$$\bar{\gamma}_r = \bar{\gamma}_c + \bar{\gamma}_a \qquad ...(16.8.44)$$

Density variation with temperature :

By definition

$$\rho = \frac{m}{V}$$

$$\Rightarrow \quad \rho(T_0) = \frac{m}{V(T_0)}$$

$$\rho(T) = \frac{m}{V(T)}$$

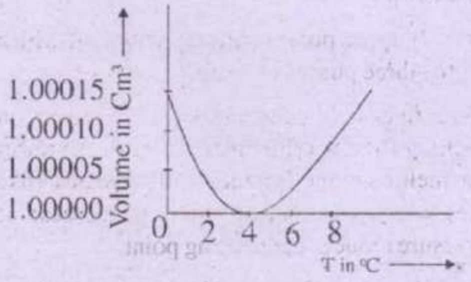
$$\Rightarrow \quad \frac{\rho(T)}{\rho(T_0)} = \frac{V(T_0)}{V(T)} = \frac{V(T_0)}{V(T_0)\, e^{\bar{\gamma}(T-T_c)}}$$

$$\Rightarrow \quad \rho(T) = \rho(T_0).\ e^{-\bar{\gamma}(T-T_c)} \qquad ...(16.8.45)$$

$$\text{or } \rho(T) \cong \rho(T_0)\left[1-\bar{\gamma}(T-T_0)\right] \qquad ...(16.8.46)$$

Anomalous expansion of water :

The change in volume of water as temperature increases is different from other general liquids; which can be inferred from fig. 16.11. It has lowest volume at 4° C. Hence it has highest density at 4° C.



Ex.16.8.1 A steel measuring tape is 5m. long when calibrated at 20^0 C. What is its length in summer when its temperature is 35^0 C. $[\alpha_{steel} = 1.2 \times 10^{-5}\ {}^0C^{-1}]$

Soln.

$$\Delta L = L_0.\ \alpha.\ \Delta T = 5 \times 1.2 \times 10^{-5} \times 15$$

$$= 90 \times 10^{-5} = 0.0009 \text{ m.}$$

Hence length of the tape = 5.0009 m.

Ex.16.8.2 The difference in lengths of two rods of different materials remains constant at all temperatures. Show that the ratio of their lengths is equal to the inverse ratio of their coefficient of linear expansion.

Soln.

$$\Delta L_0 = L_{10} - L_{20}$$

$$\Delta L = L_1 - L_2$$

$$\Delta L - \Delta L_0 = (L_1 - L_{10}) - (L_2 - L_{20}) = 0$$

$$\Rightarrow \quad L_{10}.\alpha_1.\Delta T = L_{20}.\alpha_2.\Delta T$$

$$\Rightarrow \quad \frac{L_{10}}{L_{20}} = \frac{\alpha_2}{\alpha_1} = \frac{1/\alpha_1}{1/\alpha_2} \qquad \text{Proved}$$

Ex.16.8.3 A brass disc has a diameter of 30 cm at 20^0 C. What is its area at 50^0 C. ($\alpha_{Brass} = 18 \times 10^{-6}\ {}^0C^{-1}$)

Soln. $A_0 = \pi d_0^2 / 4 = 706.86 \text{ cm}^2$

$$A = A_0[1+\alpha(T-T_0)] = A_0[1+\alpha.30]$$

$$= \quad 707.24 \text{ cm}^2.$$

Ex.16.8.4 A circular hole in an aluminium plate has a diameter 2.5 cm at 0^0 C. What is its diameter, when it is raised to 10^0 C. $[\alpha_{AL} = 23 \times 10^{-6}\ {}^0K^{-1}]$

Soln.

$$\ell = \ell_0[1+\bar{\alpha}(T-T_c)]$$

$$= 2.5[1+\bar{\alpha}(100-0)]$$

$$= 2.5[1 + \bar{\alpha} \times 100]$$

$$\ell = 2.506 \text{ cm.}$$

Ex.16.8.5 A solid occupies 1000 cm^3 at 20^0 C. Its volume becomes 1016 cm^3 at 320^0 C. Calculate coefficient of cubical expansion.

Soln.

$$\bar{\gamma} = \frac{V(T) - V(T_0)}{V(T_0)(T - T_0)}$$

$$= \frac{(1016 - 1000) \text{ cm}^3}{1000 \text{ cm}^3 (320 - 20)}$$

$$= \frac{16}{300 \times 10^3} \, {}^0C^{-1}$$

$$\Rightarrow \bar{\gamma} = 5.33 \times 10^{-5} \, {}^0C^{-1}$$

Summary

1. When work (w) is converted to heat (Q) and vice versa,

   W = JQ (J = 4.186 J cal^{-1}, called mechanical equivalent of heat)

2. In SI units both work and heat are measured in joule. In this system J=1 and hence W = Q

3. Temperature determines the direction of heat flow between two bodies in thermal contact.

4. **Zeroth law :**

   Two system in thermal equilibrium with a third system are in thermal equilibrium with each other.

5. A thermometer is a temperature measuring device that makes use of some measurable property that changes with temperature.

6. Relations between kelvin scale, celsius scale and fahrenheit scale :

$$\frac{\theta_c}{100} = \frac{\theta_F - 32}{180} = \frac{\theta_k - 273.15}{100}$$

7. Heat required to raise the temperature of a body of mass 'm' by $\Delta\theta$ is

$$\Delta Q = m\,c\,\Delta\theta$$

where 'c' is the specific heat of the body which depends on the thermal property of its material.

8. Molar specific heat

$$C = Mc$$

where M is the molecular mass.

9. a) Molar specific heat at constant volume (C_v)

$$C_v = \frac{1}{n}\left(\frac{dQ}{d\theta}\right)_v$$

b) Molar specific heat at constant pressure

$$C_p = \frac{1}{n}\left(\frac{dQ}{d\theta}\right)_P$$

10. Principle of calorimetry :

    Heat lost by all the hot bodies = Heat gained by all the cold bodies

11. a) Melting point is a temperature at which solid and liquid phase co-exist in thermal equilibrium.

b) Boiling point is a temperature at which liquid and vapour phase coexist in thermal equlibrium.

c) Tripple point is a temperature at which all the three phases of a substance co-exist.

12. a) In case of substances which expand on melting (like wax) increase of pressure raises the melting point. But incase of substance (like ice) which contract on melting, increase of pressure reduces the melting point.

b) Increase of pressure increases the boiling point.

c) During change of phase, there is no

change in temperature even if heat is added to the system.

13. Amount of heat required to change the phase of a substance is called latent heat.

If Q amount of heat is supplied during the change of phase of a substance of mass 'm' then

$$Q = mL$$

L is called specific latent heat.

14. Coefficient of linear expansion,

$$\alpha = \frac{L(T) - L(T_0)}{L(T_0)\,(T - T_0)} = \frac{\Delta L}{L\,\Delta T}$$

where $\Delta L = L(T) - L(T_0)$,

$$\Delta T = T - T_0,\ L = L(T_0)$$

15. Coefficient of superficial expansion :

$$\beta = \frac{A(T) - A(T_0)}{A(T_0)\,(T - T_0)} = \frac{\Delta A}{A\,\Delta T}$$

16. Coefficient of Cubical expansion :

$$\gamma = \frac{V(T) - V(T_0)}{V(T_0)\,(T - T_0)} = \frac{\Delta V}{V\,\Delta T}$$

17. Relation between α, β and γ.

$$\alpha : \beta : \gamma = 1 : 2 : 3$$

18. In case of liquid there is only volume expansion. While measuring volume expansion of liquid on has to take into account the expansion of its container.

MODEL QUESTIONS

A. Multiple Choice Type Questions :

1. What is the change in internal energy when 1 gm of ice at 0^0 C is converted into water at 0^0 C with negligible change in volume ?

   (a) 80 J (b) 0.8 J

   (c) 0.34 KJ (d) 41.8 J

2. The temperature of a body rises by 25^0C. The increase on Fahrenheit scale would be

   (a) 25^0 F (b) 45^0 F

   (c) 100^0 F (d) 180^0 F

3. A bullet of mass 10 g moving with speed 210 m/s is suddenly stopped. If its kinetic energy is totally converted into heat, how much of heat in calories would be generated ?

   (a) 5.25×10^4 (b) 525

   (c) 52.5 (d) 5.25×10^{-3}

4. 50 g of ice is added to 150 g of water at 500 C. The resulting temperature will be

   (a) 0^0 C (b) 10.5^0 C

   (c) 17.5^0 C (d) 20^0 C

5. Fire can be extinguished more quickly by

   (a) ice

   (b) water at room temperature

   (c) boiling water

   (d) water at 0^0 C

6. The boiling point of water decreases when surrounding

   (a) temperature increases

   (b) temperature decreases

   (c) pressure increases

   (d) pressure decreases

7. A metal plate has an elliptical hole in it. The temperature of the plate is increased. The ratio of major and minor axis will

   (a) decrease

   (b) increase

   (c) remain same

   (d) non of the above

8. 100 g of ice at 0^0 C is added to 100 g of water at 8^0 C. The final state is

   (a) mixture of ice and water at 0^0 C

   (b) only water at 0^0 C

   (c) only ice ice at 0^0 C

   (d) water at more than ^{0} C

9. A bi-metallic strip is made of two strips of different materials A and B, with coefficient of linear expansion $\alpha_A > \alpha_B$. If the strip is heated

   (a) It will bend but will not elongate

   (b) it will bend so that A is on the outerside of the arc

   (c) it will bend with B on the outerside of the arc

   (d) it will only elongate with an apparent coefficient of expansion $(\alpha_A + \alpha_B)/2$.

10. The temperature of a block of iron is 140^0F. Its temperature on celsius scale is

    (a) 180^0 (b) 320^0

    (c) 60^0 (d) 140^0

11. Which of the following produces more severe burns ?

    (a) boiling water (b) air at 100^0 C

    (c) sun rays (d) none of these

12. Substance having the highest specific heat among gold, copper, kerosene oil, and water is
    (a) copper (b) kerosene oil
    (c) water (d) gold

13. Heat Q is produced when mechanical work W is done. These are related by
    (a) $Q = \frac{W}{J}$ (b) $Q = WJ$
    (c) $Q = \frac{J}{W}$ (d) none of the above

14. Latent heat of fusion of ice is
    (a) 80 cal/g (b) 100 cal/g
    (c) 540 cal/g (d) None of the above

15. The coefficient of linear expansion of platinum is $9 \times 10^{-6}\ {}^0C^{-1}$. Its coefficient of volume expansion is
    (a) $3 \times 10^{-6}\ {}^0C^{-1}$
    (b) $4.5 \times 10^{-6}\ {}^0C^{-1}$
    (c) $9 \times 10^{-6}\ {}^0C^{-1}$
    (d) $27 \times 10^{-6}\ {}^0C^{-1}$

16. When a copper ball is heated, the largest percentage increase will be in its
    (a) mass (b) radius
    (c) area (d) volume

17. A solid iron ball has a spherical cavity inside it. If the ball is heated, the volume of a cavity will
    (a) remain unchanged
    (b) decrease
    (c) increase
    (d) none of the above

18. The density of mercury is $13.6 \times 10^3\ kg/m^3$ at 0^0 C. If coefficient of cubical expansion of mercury is $18 \times 10^{-5}\ {}^0C^{-1}$ its density at 50^0 C is
    (a) $13.72 \times 10^3\ kg/m^3$
    (b) $13.6 \times 10^3\ kg/m^3$
    (c) $13.48 \times 10^3\ kg/m^3$
    (d) $13.4 \times 10^3\ kg/m^3$

B. Very Short Answer Type Questions :

1. Define temperature.
2. Define specific heat.
3. Which produces severe burn, steam or boiling water.
4. How much heat is needed to covert 5 g of ice at 0^0 C to water at 0^0 C?
5. Will the coefficient of linear expansion of two steel rods vary if their length and diameter are unequal ?
6. Define water equivalent of a body.
7. State the units of specific heat.
8. State units of latent heat.
9. State the units of water equivalent.
10. Give the value of J.
11. What is the relation between thermal capacity and water equivalent of a body?
12. Give unit of thermal capacity.
13. Give the values of latent heat of fusion of ice and latent heat of vaporisation of water.
14. Give the unit of coefficient of linear expansion.
15. Give the dimension of coefficient of cubical expansion.

16. Give the dimension of latent heat.
17. Give the dimension of specific heat.
18. Give the dimension of water equivalent.
19. Write the relation between α, β and γ.
20. What is meant by thermal energy ?
21. Does the temperature of a body depend on frame from which it is observed ?
22. Can the bulb of a thermometer be made of an adiabatic wall ?
23. If mercury and glass had equal coefficient of volume expansion, could we make a mercury thermometer in a glass tube ?
24. Is it possible for two bodies to be in thermal equilibrium if they are not in contact ?
25. The coefficient of volume expansion of a solid is $5.4 \times 10^{-5}\ {}^{0}C^{-1}$. What is the coefficient of linear expansion ?
26. The temperature of a body rises by 25°c. What will be this rise in temperature as measured in kelvin scale ?
27. Write the dimensional formula of temperture gradient.

C. Short Answer Type Questions :

1. If 10 g of water $8^{0}C$ is mixed with 1 g of ice at $0^{0}C$ what will be the final temperature of the mixture ?
2. Calculate the amount of heat required to convert 10 g of ice at $-10^{0}C$ to water at $5^{0}C$.
3. How much heat is required to convert 1 g of ice at $0^{0}C$ to water at $100^{0}C$?
4. How many grams of water at $20^{0}C$ will evolve 1 k. cal of heat by becoming ice at $0^{0}C$?
5. Explain why steam produces more severe burn than boiling water ?
6. Explain why melting point of ice decreases whereas that of wax increases with increase of pressure ?
7. What is the height from which a piece of ice should be dropped so that it completely melts on hitting the ground ?
8. Give reasons why temperature of ice is lowered when salt is added to it.
9. Why is iron tyre of a cart wheel so tightly fitted ?
10. An isotropic solid, cube has side length 100 cm at 30^{0} C and 100.04 cm at 50^{0} C. Compute α, β and γ.
11. A sheet of copper ($\alpha = 1.7 \times 10^{-5}\ {}^{0}C^{-1}$) has an area of 1 m^2 at 0^{0} C. Find its area at 30^{0} C.
12. Establish relation between α and β.
13. Establish relation between α and γ.
14. Why do water pipes burst in cold countries ?
15. Why do we leave some space in between two rails of a railway line ?
16. Why donot we lay floors in one piece ?
17. Why pendulum clocks run slow in summer and faster in winter ?
18. Why do marine animals live deep inside a lake when the surface of the lake freezes ?
19. Why is water used in hot liquid bag ?
20. If two bodies are in thermal equilibrium in one frame will they be in thermal equilibrium in all frames ?
21. Does the temperature of a body dpend on the frame from which it is observed ?

D. Unsolved Problems :

1. If two tempratures differ by 25 degrees on celsius, scale, what is the difference on Fahrenheit scale ?

2. The U.F.P. and L.F.P. of a thermometer are wrongly marked as 96^0C and -2^0C respectively. What is the reading of this thermometer if a correct thermometer reads 50^0C ?

3. Equal volumes of mercury and glass have the same thermal capacity. The densities of mercury and glass are 13,600 kg/ m^3 and 2500 kg / m^3 respectively. If the specific heat capacity of mercury is 130 J kg^{-1} $^0K^{-1}$, calculate the specific heat capacity of glass.

4. What volume of water has same thermal capacity as 1 m^3 of ice ? [density of ice = 920 kg / m^3]

5. When a body of mass 2 kg absorbs 200 cal. of heat its temperature rises from 300^0C to 400^0 C. Calculate the specific heat capacity, thermal capacity and water equivalent.

6. How much energy is required to convert 1 litre of water at 288^0 K into steam ?

7. Equal masses of two liquids of specific heat capacities 840 J. Kg^{-1} $^0K^{-1}$ and 1260 J Kg^{-1} $^0K^{-1}$ and respective temperature 60^0C and 30^0 C are mixed. Find the final temperature of the mixture.

8. Equal amount of ice at 0^0 C and water at 90^0C are mixed. Find the final temperature.

9. A bimetallic strip is heated through t^0C. If α_1 and α_2 be the linear coefficients of expansion for the two metals, then the strip bends into an arc of radius R. The thickness of each strip is d. Find the value of R.

10. A pendulum clock, made of brass $[\alpha = 19\times10^{-6}\ ^0C^{-1}]$ records time correctly at 20^0C. If the temperature is increased by 10^0C, how much time the clock loses in a day.

11. A lead bullet, travelling at 350 m/s, strikes a target and is brought to rest. What would be the rise in temperature of the bullet if there are no heat loss to the surroundings? Does the bullet melt ?

12. Two meter scales, one of steel and the other of aluminium, agree at 20^0C. Calculate the ratio aluminium - centimeter/ steel - centimeter at (a) 0^0 C (b) 40^0 C (c) 100^0C , α for steel = $1.1\times10^{-5}\ ^0C^{-1}$ and α for aluminium = $2.3\times10^{-5}\ ^0C^{-1}$.

13. Show that the moment of inerita of a solid body of any shape changes with temperature as $I = I_0(1+2\alpha\theta)$ where I_0 is the moment of inertia at 0^0C and α is the coefficient of linear expansion of the solid.

14. A pendulum clock gives correct time at 20^0C and at a place where $g = 9.8\ m/s^2$. The pendulum consists of a light steel rod connected to a heavy ball. It is taken to a different place where $g = 9.788\ m/s^2$. At what temperature will it give correct time? Coefficient of linear expansion of steel = $12\times10^{-6}\ ^0C^{-1}$.

E. Long Answer Type Questions :

1. Define coefficient linear, superfical and volume expansion of a substance and establish a relation between them.

2. Define coefficient of linear expansion and cubical expansion. Establish the reltion between them. Discuss about cubical expansion of a liquid.

3. Discuss about cubical expansion of a liquid. Explain the terms coefficient of real cubical expansion and apparent cubical expansion. Establish relation between them.

4. Define specific heat, thermal capacity and water equivalent. Give a method to determine specific heat of a solid.

5. Discuss phase change of matter taking water as an example.

F. Answer as directed

1. The Celsius and Fahrenheit scales do not agree at any temperature (yes/no)

2. Thermometers with spherical bulbs are less sensitive then the thermometers with cylindrical bulbs because the surface area of the sphere is .... than the surface area of the cylinder. (fill in the blanks)

3. The temperature at which two bodies appear equally hot or cold when touched by a person is .... (fill in the blank).

4. If the ratio of densities of two substances is 5 : 6 and that of specific heats is 3 : 5 then the ratio of heat capacities per unit volume is .... (fill in the blank)

5. What is the specific heat of water at the time of boiling ?

6. A body cools from 91^0C to 89^0C in 5 minutes when the room temperature is 20^0C. How much time it will take to cool from 71^0C to 69^0C ?

7. The difference between the length of a certain brass rod and that of a steel rod is claimed to be constant at all temperatures. (yes/no)

8. The scale of temperature on which there are no negative temperature is called .... (fill in blank)

9. A clock with a brass pendulum keeps correct time at 20^0C. It .... each day if the temperature is 40^0C. (Fill in the blank)

10. A perfect gas at 27^0C. is heated at constant pressure so as to double its volume. What will be the temperature of the gas ? (fill in the blank)

ANSWERS

A. Multiple Choice Type Questions :

1. (c), 2. (b), 3. (c), 4. (c), 5. (c), 6. (c), 7. (b), 8. (a), 9. (b), 10. (c), 11. (a), 12. (c), 13. (a), 14. (a), 15. (d), 16. (d), 17. (c), 18. (c).

B. Very Short Answer Type Questions :

1. See text
2. See text
3. Steam
4. 400 cals
5. No
6. See text
7. $\text{cal g}^{-1}\,{}^0\text{C}^{-1}$, $\text{Kcal kg}^{-1}\,{}^0\text{K}^{-1}$, $\text{J kg}^{-1}\,{}^0\text{C}^{-1}$
8. Cal / g , K Cal / kg, J / g , J / kg
9. g, kg
10. 4.186 J / cal
11. They are numerically equal
12. $\text{cal }{}^0\text{C}^{-1}$, $\text{Kcal }{}^0\text{C}^{-1}$, $\text{J }{}^0\text{K}^{-1}$, $\text{erg }{}^0\text{C}^{-1}$
13. 80 cals / g, 540 cals / g
14. ${}^0\text{C}^{-1}$, ${}^0\text{K}^{-1}$
15. K^{-1}
16. $\text{L}^2\,\text{T}^{-2}$
17. $\text{L}^2\,\text{T}^{-2}\,{}^0\text{K}^{-1}$
18. M
19. $\alpha : \beta : \gamma = 1 : 2 : 3$
20. See text
21. No
22. No
23. No
24. No
25. $1.8 \times 10^{-5}\,{}^0\text{C}^{-1}$
26. $25\,{}^0\text{K}$
27. KL^{-1}

C. Short Answer Type Questions :

1. 0^0 C
2. 900 cal
3. 180 cal
4. 10 g
5. 1 g of steam gives up 540 cal of heat whereas 1 g of boiling water gives up 1 cal of heat.
6. Ice contracts on melting but wax expands on melting.
7. 3.43×10^4 m

8. When salt is added a part of ice melts and for this latent heat is extracted from the mixture. Further when common salt dissolves, the heat of solution required is again extracted from the mixture. Therefore temperature of ice falls below 0^0 C.

9. During summer the iron tyre shall expand, so it is tightly fitted.

10. $2\times10^{-5}\ {}^0C^{-1},\ 4\times10^{-5}\ {}^0C^{-1},\ 6\times10^{-5}\ {}^0C^{-1}$

11. 1. 001 m^2

12. Consider a thin square plate of side length ℓ_o at temp. t_0

$$A_o = \ell_o^{\,2},\ A = \ell^2,\ \ell = \ell_o\, e^{\alpha(t-t_o)}$$

$$\Rightarrow\ A = \ell_o^{\,2}\, e^{2\alpha(t-t_o)} = A_o\, e^{\beta(t-t_o)}$$

$$\Rightarrow\ \beta = 2\alpha$$

13. Consider a cube of side length ℓ_0 at temp. to

$$V_o = \ell_o^3,\ V = \ell^3,\ \ell = \ell_o\, e^{\alpha(t-t_o)}$$

$$\Rightarrow\ V = \ell^3 = \ell_o^{\,3}\, e^{3\alpha(t-t_o)} = V_o\, e^{\gamma(t-t_o)}$$

$$\therefore\ \gamma = 3\alpha$$

14. Water expands on freezing. Hence pipes burst.

15. See text

16. See text

17. $T = 2\pi\sqrt{\ell/g}$. In summer 'ℓ' increases so T increases, hence time recorded is less.

Hence we say pendulum clock runs slower in summer.

18. Since the density of water is maximum at 4^0 C, water at the bottom of lake remains at 4^0 C even if surface water freezes. So marine animals live deep inside a lake.

19. In comparision with other liquids water has high specific heat capacity and as such absorbs more amount of heat. Hence our body can get more amount of heat from water. Further it is easily available and a poor conductor.

20. Yes

21. No

D. Unsolved Problems :

1. 45^0 F

2. 47^0 C

3. 707. 2 J $kg^{-1}\ {}^0K^{-1}$

4. 0. 46 m^3

5. 42 J $kg^{-1}\ {}^0K^{-1}$, 84 J $^0K^{-1}$, 0.02 kg

6. 6.25×10^5 cal

7. 42^0 C

8. 5^0 C

9. $\dfrac{d}{(\alpha_2-\alpha_1)t}$

[Hints :

$$\ell_1=\ell_{10}(1+\alpha_1 t),\ \ell_2=\ell_{20}(1+\alpha_2 t)$$

But $\ell_1=R\theta,\ \ell_2=(R+d)\theta$ and $\ell_{10}=\ell_{20}$

$$\Rightarrow\quad (R+d)\theta=\ell_{20}(1+\alpha_2 t)$$

$$R\theta=\ell_{10}(1+\alpha_1 t)$$

$$\Rightarrow\quad \frac{R+d}{R}=\frac{1+\alpha_2 t}{1+\alpha_1 t}$$

$$\Rightarrow\quad \frac{d}{R}=\frac{(\alpha_2-\alpha_1)t}{1+\alpha_1 t}$$

$$\Rightarrow\quad R=\frac{d}{(\alpha_2-\alpha_1)t}+\frac{\alpha_1.d.t}{(\alpha_2-\alpha_1)t}\approx\frac{d}{(\alpha_2-\alpha_1)t}$$

10. 8.25

11. 472^0C

12. 0.99957; 1.00024; 1.00096

13. Hints : $I=mK^2\Rightarrow I_0=mK_0^2$

$$K=K_0(1+\alpha\theta)\Rightarrow K^2=K_0^2(1+\alpha\theta)^2\approx K_0^2(1+2\alpha\theta)$$

$$\therefore\ I=mK^2=mK_0^2(1+2\alpha\theta)=I_0(1+2\alpha\theta)$$

14. $-82^\circ C$

F. (1) No. (2) Less (3) 37^0C (4) 1 : 2 (5) Inifinite (6) 7 minutes (7) Yes (8) Absolute scale (9) Loses (10) 327^0C.

* * *

17

Thermodynamics

17.1 Introduction

Thermodynamics is the branch of physics that deals with conversion of heat into other forms of energy and vice-versa. It is based on certain laws like (i) Zeroth Law (ii) First law (iii) Second law and (iv) Third law of thermodynamics which are formulated on the basis of experimental observations.

The **Zeroth law** of thermodynamics deals with thermal equilibrium and introduces the concept of temperature. The **First law** of thermodynamics deals with energy conservation in thermodynamic process. The **Second law** of thermodynamics determines the direction in which a thermodynamic process will proceed. The **Third law** of thermodynamics speaks about the unattainability of absolute zero.

Thermodynamic principles can be developed in a macroscopic way without going into the details (microscopic way); although it is possible to arrive at the thermodynamic principles in a microscopic manner, as done in statistical mechanics.

Before coming to the discussions of thermodynamics laws, it is necessary to aquaint oneself with certain useful concepts like thermodynamic system, state of a system, thermodynamic coordinates, thermodynamic process, thermodynamic equilibrium etc; and hence they are discussed in the following sections.

17.2 Thermodynamic System :

Generally a system is a well defined and specified region of the universe, having a boundary, which may be real or imaginary, may be fixed or movable. **A thermodynamic system is one that can interact with its surroundings in at least two ways, one of which must be transfer of heat.**

The region surrounding the system is called surroundings or environment. The surrounding may or may not affect the system. A system which is uniform throughout is called a homogenenous system. (e.g. a pure liquid, solid or gas or mixture of gases). If the system is not uniform throughout then it is called a heterogeneous system (e.g. a mixture of immiscible gases).

A thermodynamic system can be also classified as (i) closed system (ii) open system (iii) isolated system.

(i) CLOSED SYSTEM :

A system which does not exchange matter with the environment is called a closed system, i.e. mass of a closed system remains constant. However energy may be exchanged with the environment.

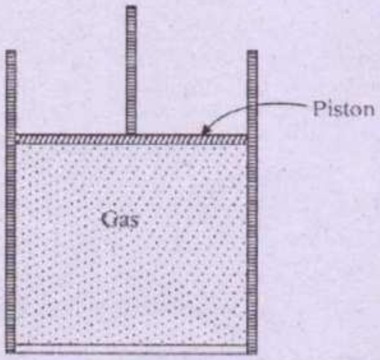


Fig. 17.1

A gas in a vessel fitted with a tight-fitting piston is an example of a closed system. In this case gas is the system, and walls of the cylinder is its boundary. When gas is heated, the gas expands so that piston moves up and hence the boundary moves. But the mass of gas remains unaltered, although heat energy crosses through the boundary.

(ii) OPEN SYSTEM

A system which exchanges matter with the surroundings is called an Open System. Energy may also be transported through the boundary. Plants and animals can be considered as open systems as they exchange food, oxygen, waste-products etc. with the surroundings.

(iii) ISOLATED SYSTEM

A system which does not exchange matter or energy with the surroundings i.e. does not interact with the surroundings is called an isolated System.

Properties of a System :

The macroscopic properties of a system can be classified as *(i) extensive properties (ii) intensive properties.*

(i) Extensive Properties :

Properties like mass, volume, internal energy etc. which depend on amount of matter contained in the system are called as extensive properties. These properties are additive i.e. total value of an extensive property is equal to the sum of its values for different parts of the system.

(ii) Intensive Properties :

Properties like temperature, pressure, density, viscosity, surface tension etc. which do not depend on the amount of matter contained in the system are called intensive properties.

An extensive property per unit amount can represent the correponding intensive property for example, mass is an extensive property but density (= mass/volume) is an intensive property.

17.3 State of a System

and

Thermodynamic Coordinates

The state of a system is specified by variables like (a) composition (b) pressure P. (c) Volume (V), (d) temperature T, (e) internal energy U etc. For a homogeneous system composition is fixed, hence P, V, T completely specify a homogeneous system. These P, V and T are called **Thermodynamic Coordinates or State Coordinates** of a system. These thermodynamic coordinates are related and the equation expressing their relation is called **Equation of State**.

e.g. $P = P(V, T)$ ...(17.3.1)

In case of an ideal gas, the equation of state is

$$PV = nRT$$

One can draw a graph between P and V and use the equation (17.3.1) to find T for every point on the curve. This curve is popularly known as **P-V diagram** or **indicator diagram.** (See fig. 17.2)

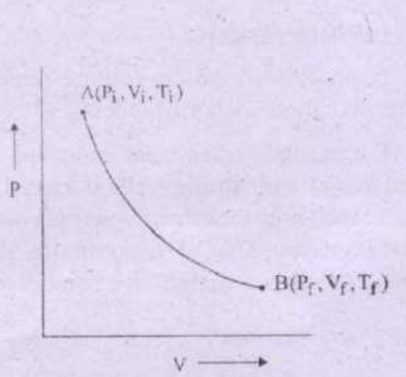


Fig : 17.2

Fig.17.2

The P-V diagram actually represents a thermodynamic process.

Thermodynamic equilibrium

If the macroscopic properties of a system remain constant in time, then the system is said to be in thermodynamic equilibrium.

Thermodynamic equilibrium implies simultaneous existence of three equilibria :

(i) Thermal equilibrium

(ii) Chemical equilibrium and

(iii) Mechanical equilibrium

A system is said to be in thermal equilirbium if the temperature of the system is same throughout, so that no heat exchange takes place between parts of the system and this is also the temperature of the surrounding.

A system is said to be in chemical equilibrium if the composition of the system is uniform throughout.

A system is said to be in mechanical equilibrium if there is no unbalanced force between the system and environment or in the interior of the system.

17.4 Thermodynamic Process :

When a system changes from one equilibrium state to another equilibrium state the system is said to have undergone a thermodynamic process and this causes a change (however small) in the thermodynamic coordinates.

Generally a thermodynamic process passes through a number of intermediate states. If the intermediate states are equilibrium states then the process is called an equilibriun process. On the otherhand if it passes through non-equilibrium states it is called a non-equilirbrium process.

If the process of transition from one intermediate state to another is very slow, but sufficient enough so that the thermodynamic coordinates undergo change, then the process is called a **Quasistatic Process**. In a quasistatic process the state variables undergo change very slowly.

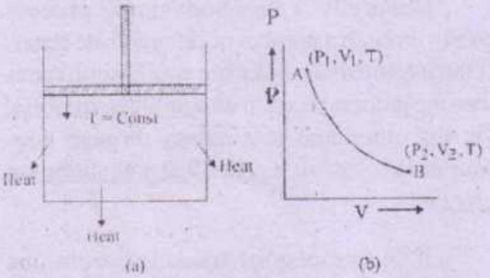
Any process that can be made to proceed in the reverse direction (i.e. final state to initial state) through the same intermediate states as in the direct process (i.e. initial to final state) is called a **Reversible Process**. For example slow compression of a gas is reversible. A process which cannot be reversed through the same intermediate states as in the direct process is called an **Irreversible Process**. For example processes involving friction, electrical heating of a coil, explosion of a bomb, rushing out of a gas through a nozzle etc. are irreversible.

There can be several types of thermodynamic process. However we shall consider few simple process as given below :

(i) *Isothermal Process* :

A process in which the temperature of the system remains constant is called an Isothermal process.

For example when a gas contained in a barrel, with conducting walls is compressed by an air-tight piston gradually and slowly, the liberated heat moves out. The temperature of the gas remains unaltered; although the system moves from state A (P_1, V_1, T) to state B (P_2, V_2, T) (See fig. 17.3 (b)).



(a) (b)

Fig. 17.3

(ii) Isobaric Process :

A process in which the pressure on the system is maintained constant is called isobaric process.

For example when a gas contained in a barrel with a conducting base and fitted with a freely movable piston is kept in thermal contact with a heat reservoir, the gas expands due to inflow of heat. But the pressure remains constant. The P-V diagram for an isobaric process is a straight line parallel to the V-axis, as shown in fig. 17.4.

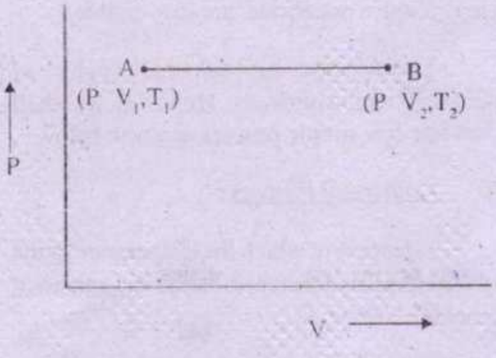


Fig. 17.4

(iii) *Isochoric Process :*

A process in which volume remains constant is called isochoric process.

For example when a gas contained in a closed vessel with strong walls is heated, its pressure and temperature increases but volume remains constant. The P-V diagram (fig. 17.5) for this process is a straight line parallel to P-axis.

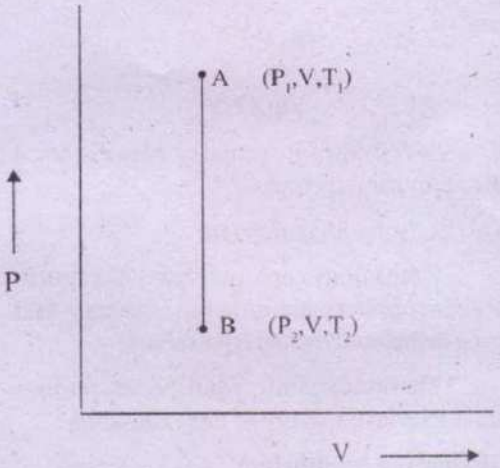


Fig. 17.5

(iv) *Adiabatic Process :*

A process in which no heat enters or leaves the system (i.e. there is no heat exchange between the system and surroundings) is called Adiabatic Process.

For example when a gas contained in a barrel with insulating walls and fitted with an air-tight piston is compressed, its pressure (P), temperature (T) and volume (V) change but the heat generated during compression remains confined to the gas within the barrel. This increases the temperature of the gas. The P-V diagram for this process is as shown in fig. 17.6.

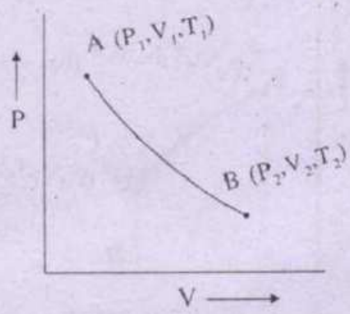


Fig. 17.6

(v) *Cyclic Process :*

A process in which the initial and final states of a system remains same is called a cyclic process. The corresponding P-V- diagram is as shown in fig. 17.7.

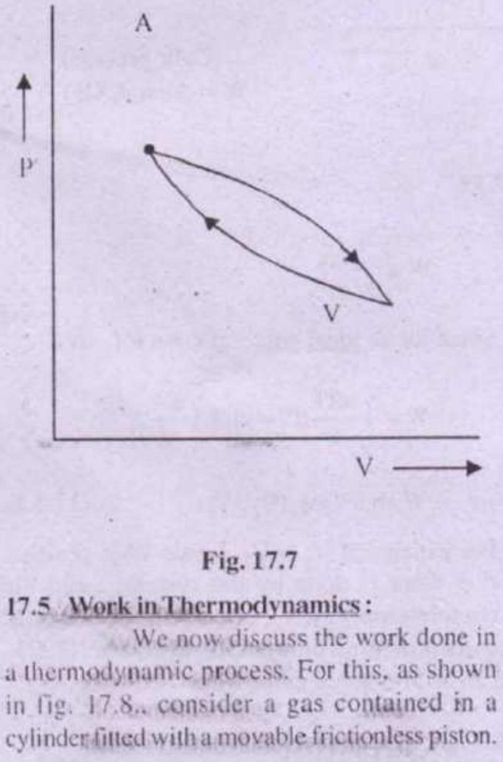


Fig. 17.7

17.5 Work in Thermodynamics :

We now discuss the work done in a thermodynamic process. For this, as shown in fig. 17.8., consider a gas contained in a cylinder fitted with a movable frictionless piston.

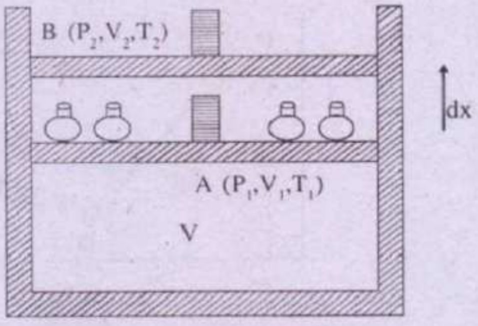


Fig. 17.8

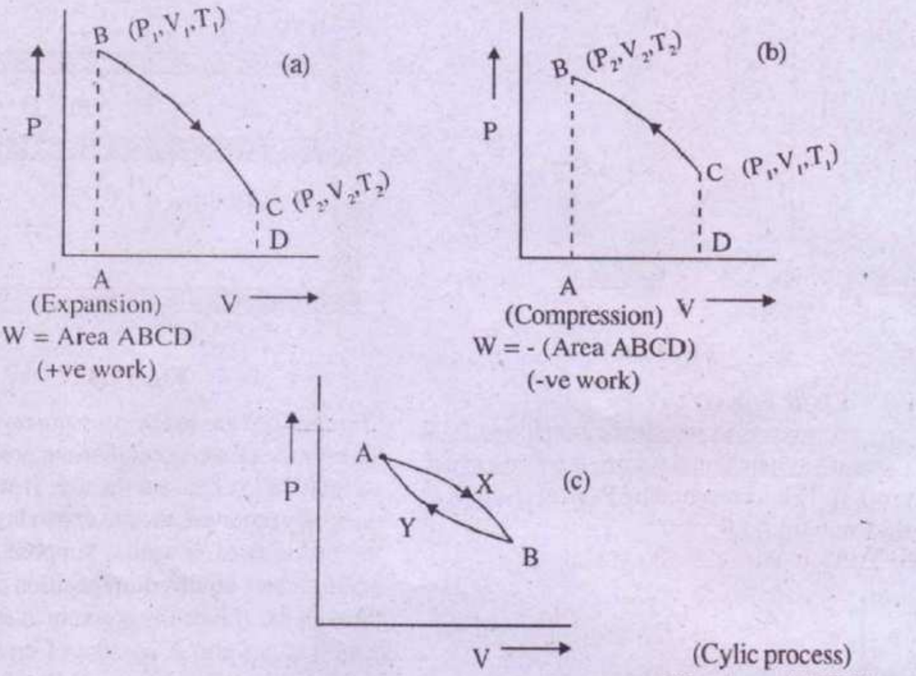
The confined gas exerts pressure on the piston. To keep the piston in equilibrium position some weights 'W' is kept on the top. If weights are gradually removed, the gas expands slowly and the piston rises upwards. Suppose the piston attains a new equilibrium position after rising through dx. If P be the pressure exerted by the confined gas and A be area of cross-section, then work done by the gas on the piston is

$$dW = PA\,dx = P\,dV \qquad ...(17.5.1)$$

where $dV = A\,dx$ is the infinitesimal change in the volume of the gas. This work is called the **Thermodynamic work** done by the gas. Total work done for a finite change of volume from V_1 to V_2 is given by

$$W = \int_{V_1}^{V_2} PdV \qquad ...(17.5.2)$$

The integral on r.h.s of (17.5 .2) can be evaluated only when pressure (P) is known as a function of Volume (V). Customarily this relationship is represented by plotting P as a function of V (i.e. plotting P-V diagram for the process). Then the work $W = \int_{V_1}^{V_2} PdV$ is the area under the curve between the limits V_1 and V_2 (See fig. 17.9)



(Expansion)
W = Area ABCD
(+ve work)

(Compression)
W = - (Area ABCD)
(-ve work)

(Cylic process)
W = Area AXBYA

Fig. 17.19

By convention, work done by a system is considered positive ; because force and displacement are in the same direction. On the otherhand, work done on a system is taken as negative, because force and displacement are in opposite direction.

(i) Work in Isothermal Process :- (Ideal gas)

As said earlier work (W) done in a thermodynamic process can be calculated if pressure (P) is known as a function of volume (V). So we consider n moles of an ideal gas confined in a cylinder, kept in contact with a heat reservior at temperature (T). When heat gradually enters, the gas expands slowly but the temperature remains constant (=T). The initial state (P_1, V_1, T) changes to final state (P_2, V_2, T). So work done by the system (gas) on the piston is

$$W = \int_{V_1}^{V_2} P dV$$

Since for an ideal gas $pV = n RT$, so

$$W = \int_{V_1}^{V_2} \frac{nRT}{V} dV = nRT \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\Rightarrow \quad W = nRT\log_e(V_2/V_1) \qquad ...(17.5.3)$$

For expansion $V_2 > V_1$, hence W is positive (i.e. work is done by the system) ; and for compression $V_2 < V_1$, hence W is negative (ie work is done on the system). Equation (17.5.3) can be also given in the form

$$W = nRT\log_e\left(\frac{P_1}{P_2}\right) \qquad ...(17.5.4)$$

(ii) Wrok in Isobaric Process :

In an isobaric process, when n moles of an ideal gas undergoes a change from initial state (P, V_1, T_1) to the final state (P, V_2, T_2) the work done is given by

$$W = \int_{V_1}^{V_2} PdV = P\int_{V_1}^{V_2} dV = P(V_2 - V_1) \quad ...(17.5.5)$$

Since $PV_1 = n\,R\,T_1$, $PV_2 = n\,RT_2$, so eqn (17.5.6) reduces to

$$W = n\,R\,(T_2 - T_1) \quad ...(17.5.6)$$

$$\Rightarrow \quad W = -n(C_p - C_v)(T_1 - T_2) = nC_v(T_1 - T_2) - nC_p(T_1 - T_2) \quad ...(17.5.6a)$$

(iii) Work in Isochroic Process :

In an isocheric process volume (V) remains constant. Therefore work done by the system or on the system is zero.

(iv) Work in Adiabatic Process :

Consider n moles of an ideal gas undergoing a transition from initial state (P_1, V_1, T_1) to final state (P_2, V_2, T_2), then the work done is given by

$$W = \int_{V_1}^{V_2} PdV$$

But for an ideal gas undergoing adiabatic change

$$PV^{\gamma} = \text{constant} = A \quad ...(17.5.7)$$

So using eqn (17.5.7) in the above

$$W = \int_{V_1}^{V_2} \frac{A}{V^{\gamma}} dV = A\left(\left.\frac{V^{-\gamma+1}}{-\gamma+1}\right|_{V_1}^{V_2}\right)$$

$$\Rightarrow \quad W = \frac{A}{1-\gamma}\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right) \quad ...(17.5.8)$$

Now $P_1V_1^{\gamma} = A = P_2V_2^{\gamma}$ ...(17.5.9)

So

$$W = \frac{P_2V_2 - P_1V_1}{1-\gamma} \quad ...(17.5.10)$$

Again since for an ideal gas PV = nRT

So $P_1V_1 = n\,RT_1$, $P_2V_2 = n\,RT_2$ ...(17.5.11)

Using (17.5.11) in (17.5.10)

$$W = \frac{nR(T_2 - T_1)}{1-\gamma} \quad ...(17.5.12)$$

But $\frac{R}{1-\gamma} = \frac{C_p - C_v}{1-\frac{C_p}{C_v}} = -C_v$

$$\therefore W = \frac{nR(T_2 - T_1)}{1-\gamma} = \frac{-nR(T_1 - T_2)}{1-\gamma} = nC_v(T_1 - T_2) \quad ...(17.5.13)$$

Work depends on Path :

It is to be noted that work done in a thermodynamic process depends on the path along which the change takes place.

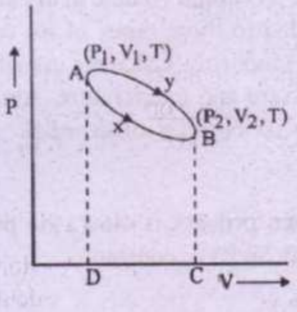


Fig. 17.10

e.g If a system undergoes a transition from initial state (P_1, V_1, T_1) to final state (P_2, V_2, T_2) along the path AXB, then work done is the area AXBCDA; whereas if the transition is along the path AYB, then work done is equal to the area AYBCDA. We note that

Area AYBCDA $\neq$ Area AXBCDA

Ex.17.5.1 A motor car tyre is pumped to a pressure of two atmosphere at 15^0 C when it suddenly bursts. Calculate the resulting drop in temperature of the escaping air ($\gamma = 1.4$)

Soln.

As it suddenly bursts, it is adiabatic.

Enclosed air is at pressure $p_1 = 2\,p_a$, and temp. $T_1 = (273 + 15)^0$ K, where p_a is atm. pressure. After bursting the air pressure $p_2 = p_a$ and temp. T_2. So

$$T_1^\gamma p_1^{1-\gamma} = T_2^\gamma p_2^{1-\gamma}$$

$$\Rightarrow \quad (288)^\gamma (2p_a)^{1-\gamma} = (T_2)^\gamma (p_a)^{1-\gamma}$$

$$\Rightarrow \quad (T_2)^\gamma = 2^{1-\gamma} \cdot (288)^\gamma$$

$$\Rightarrow \quad T_2 = 2^{\frac{1-\gamma}{\gamma}} \times 288 = 288 \times 2^{(0.4/1.4)}$$

$$\Rightarrow \quad T_2 = 236.3^0\ \text{k}$$

Drop in temp = (288 – 236.3) ^{0}K = 51.7^0 K

Ex.17.5.2 A certain volume of dry air at 20^0 C is expanded to three times of its volume (i) slowly (ii) suddenly. Calculate in each case the final pressure and temperature. Atmospheric pressure= 10^5 N/m^2, γ of air = 1.4.

Soln.

When process is slow, the process is isothermal. So PV = constant

$$\therefore P_1V_1 = P_2V_2$$

$$\Rightarrow \quad (10^5 N/m^2)V_1 = P_2(3V_1) \qquad (\because V_2 = 3V_1)$$

$$\Rightarrow \quad P_2 = \frac{10^5}{3} N/m^2 = 3.3 \times 10^4 N/m^2$$

(ii) When process is sudden, it is adiabatic.

$$\text{So, } T_1^\gamma p_1^{1-\gamma} = T_2^\gamma p_2^{1-\gamma}$$

$$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$$

$$\Rightarrow \quad T_1\ V_1^{\gamma-1} = T_2\ (3V_1)^{\gamma-1}$$

$$\Rightarrow \quad T_2 = T_1\ (\tfrac{1}{3})^{\gamma-1} = 293 \times (\tfrac{1}{3})^{0.4}$$

$$T_2 = 188.807^0\ K = -84.2^0\ C$$

Ex.17.5.3 A gas originally at N.T.P. is expanded isothermally, to twice its volume. Calculate the final pressure and temperature of the gas.

Soln.

$$P_1V_1 = P_2V_2$$

$$P_2 = P_1\frac{V_1}{V_2} = P_1\left(\frac{V_1}{2V_1}\right) = \frac{P_1}{2}$$

$$\therefore P_2 = \frac{P_a}{2} = \frac{1.013 \times 10^5 \times N/m^2}{2}$$

$$P_2 = 0.506 \times 10^5 N/m^2$$

$$T_2 = 273^0 K$$

17.6 Internal energy (U)

The sum total of the kinetic energy and potential energy of all the molecules /atoms constituting a system is the internal energy of the system.

The kinetic energy arises due to translational, rotational and vibrational motion of the atoms/molecules and potential energy arises due to interatomic/intermolecular force of attraction.

In case of monoatomic ideal gas the internal energy is equal to the total kinetic energy of the gas molecules (as P.E. is assumed to be zero due to non-interacting nature of gas molecules) and is given as

$$U = \frac{3}{2}\,N\,k\,T$$

where 'N' is number of monoatomic molecules.

However in general internal energy of a system depends on the parameters of the system. Only the **change in internal energy is measurable but internal energy (absolute value) is not a measurable quantity.**

It is worthwhile to note that internal energy of a real gas does depend on pressure (P), volume (V) and temperature (T). But for an ideal gas internal energy is only a function of temperature (T).

17.7 First Law of Thermodynamics :

It states that if dQ amount of heat is supplied to a thermodynamic system capable of doing work dW, then the change in internal energy dU of the system is given by the relation

$$dU = dQ - dW$$

or

$$dQ = dU + dW \qquad ...(17.7.1)$$

Equation (17.7.1) represents (i) a statement of conservation of energy and is applicable to any system, however complicated and (ii) it treats heat and work on equal footing.

It is also important to note from eqn (17.7.1) that if we take a system from an initial state 'i' to a final state 'f' by several different processes, the balance dQ - dW, should be identical in all processes.

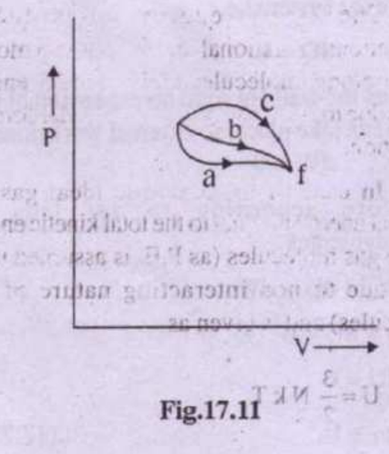


Fig.17.11

e.g. in the above processes a, b, c the change in internal energy $U_f - U_i = dU = dQ - dW$ is same. Although different amounts of work and heat might be involved in the processes a, b and c, but their difference is constant. i.e. $dQ_a - dW_a = dQ_b - dW_b = dQ_c - dW_c = dU$.

One is not allowed to write

$dQ = Q_f - Q_i$ or $dW = W_f - W_i$; but one can write $dU = U_f - U_i$ and this dU is only measurable.

It is also essential to note the following while applying the first law.

i) All quantites Q, U and W must be expressed in same units.

ii) Work done by a system is to be taken as positive and work done on the system is to be taken as negative.

iii) Heat taken by (given to) a system is taken as positive and heat given out by (drawn from) a system is taken as negative.

Finally, when a thermodynamic process proceeds smoothly, then for a finite process one can write eqn. (17.7.1) as

$$U_f - U_i = Q - W \qquad ...(17.7.2)$$

Applications of first law of thermodynamics

We now cansider the application of first law of thermodynamics.

(i) *Isolated system* :

An isolated system is one that does no work and into which there is no heat flow (i.e. the system has no interaction with the surrounding). Then for such a system

$$W = Q = 0$$

Therefore 1st law gives

$$U_f - U_i = Q - W = 0$$

$$\Rightarrow \quad \Delta U = 0 \qquad ...(17.7.3)$$

This implies that the internal energy of an isolated system remains constant. The internal energy of an isolated system cannot be changed by any process (mechanical, electrical, chemical, nuclear or biological) taking place within the system.

(ii) *Isothermal process :*

When a system undergoes an isothermal process temperature of the system remains constant. As a result the internal energy of the system remains constant. Therefore from the first law we have

$$U_f - U_i = 0 = Q - W$$

$$\Rightarrow \quad Q = W \qquad \text{...(17.7.4)}$$

Equation (17.7.4) implies that when work is done by a system isothermally on its surroundings equal amount of heat flows into the system from outside (heat reservoir). Similarly when work is done on the system isothermally equal amount of heat is rejected by the system to outside (heat sink).

(iii) *Isochoric Process :*

In an isochoric process volume remains constant. Hence no work is done by the system or on the system i.e. $W = 0$. Therefore from first law we have

$$U_f - U_i = Q = \Delta U \qquad \text{...(17.7.5)}$$

Equation (17.7.5) shows that the heat added to the system at constant volume is equal to the increase in internal energy and hence increase in its temperature. Also heat drawn from the system at constant volume, is equal to the decrease in internal energy of the system and hence decrease in its temperature.

(iv) *Adiabatic Process :*

In an adiabatic process no heat enters or leaves the system, i.e. $Q = 0$. So from 1st law

$$U_f - U_i = -W = \Delta U \qquad \text{...(17.7.6)}$$

Equation (17.7.6) shows that when work is done by the system adiabatically (W-positive) there is a decrease of the internal energy of the system; hence fall in the temperature of the system. On the other hand when work is done on the system adiabatically (W-negative) there is an increase in the internal energy of the system, hence rise in temperature. For example when we open the valve of a cycle tube the air within is forced out. Here work is done by the pressure within the tube (W, positive), hence one feels cooling. On the contrary when we push in air by a pump into the tube (work is done on the system), the valve warms up.

(v) *Adiabatic free expansion :*

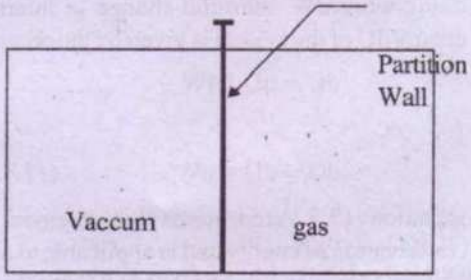


Fig. 17.12

Consider a box with rigid and insulating walls having two compartments in it. One of it contains gas and the other is evacuated. On removing the partition the gas expands. But since the walls are insulating no heat can enter or leave the system i.e.

$$dQ = 0$$

Futher as the wall are rigid no expansion of the system can take place, so external work done is zero i.e. $dW = 0$

Therefore according to 1st law of thermodynamics

$$dQ = dU + dW$$

$$\Rightarrow \quad 0 = dU + 0$$

$$\Rightarrow \quad dU = 0$$

$$\Rightarrow \quad U_f = U_i \qquad \text{...(17.7.7)}$$

Hence in an abiabatic free expansion, the internal energy of the system remains unchanged. This in turn implies that the temperature of the gas remains unaltered.

(vi) *Cyclic Process :*

In a cyclic process initial and final states are same. Therefore

$$U_f - U_i = \Delta U = 0$$

and 1st law yields

$$Q = W \qquad ...(17.7.8)$$

It means that during a cyclic process work done by the system is equal to the heat absorbed by the system from the heat reservoir; each being equal to the area enclosed by the closed curve in the P - V diagram.

vii) *Relation between C_P & C_V :*

As discussed in Sec. 16.5, Gram-molecular specific heat at constant volume (C_V) is defined as the amount of heat required to raise the temperature of 1 gm. mole of gas at constant volume through unit range (1 degree).

i.e. $$C_V = \frac{1}{n}\left(\frac{dQ}{dT}\right)_V = Mc_V \qquad ...(17.7.9)$$

where dQ is the heat supplied to n gm.moles of gas at constant volume to raise its temperature by dT.

The gram-molecular specific heat at constant pressure (C_P) is defined as the amout of heat required to raise the temperature of 1 gm. mole of gas of constant pressurethrough unit range (1degree).

i.e $$C_P = \frac{1}{n}\left(\frac{dQ}{dT}\right)_P = Mc_P \qquad ...(17.7.10)$$

where dQ is the heat supplied to n gm. mole of gas at constant pressure to raise its temperature by dT.

Now consider n gm-moles of an ideal gas to which dQ amount of heat is supplied at constant volume, so that its temperature rises by dT.

Then by 1st law

$$dQ = dU + dW = dU + PdV = dU$$

($\because$ dV = 0 in this case)

Therefore

$$\left(\frac{dQ}{dT}\right)_v = \left(\frac{dU}{dT}\right)_v$$

and using this in eqn (17.7.9)

$$C_v = \frac{1}{n}\left(\frac{dU}{dT}\right)_v \qquad ...(17.7.11)$$

$$\Rightarrow \quad dU = n\,C_v dT \qquad ...(17.7.12)$$

Again consider n gm-moles of the same ideal gas to which dQ' amount of heat is supplied at constant pressure so that its temperature rises by dT. Then by 1st law

$$dQ' = dU + P\,dV \qquad ...(17.7.13)$$

Using (17.7.12) in r.h.s of (17.7.13)

$$dQ' = n\,C_v\,dT + P\,dV \qquad ...(17.7.14)$$

But for an ideal gas

$$PV = n\,R\,T$$

So at constant pressure

$$P\,dV = n\,R\,dT \qquad ...(17.7.15)$$

Using eqn. (17.7.15) in (17.7.14)

$$dQ' = n\,C_v\,dT + n\,R\,dT \qquad ...(17.7.16)$$

But by definition (17.7.10)

$$dQ' = n\,C_p\,dT$$

So using this on l.h.s of (17.7.16)

$$n\,C_p\,dT = n\,C_v\,dT + n\,R\,dT$$

$$\Rightarrow \quad C_P - C_V = R \qquad ...(17.7.17)$$

Equation (17.7.17) holds when C_p, C_v and R are expressed in heat units or energy units. But if C_p and C_v are expressed in heat units and R is expressed in energy units then

$$C_P - C_V = \frac{R}{J} \qquad ...(17.7.18)$$

It is important to note that

$$c_P - c_V = \frac{R}{M} \qquad ...(17.7.19)$$

or $$c_P - c_V = \frac{R}{JM} \qquad ...(17.7.20)$$

Thus eqns (17.7.17) & (17.7.19) indicate that although $C_P - C_V$ is a constant (=R) the difference $c_P - c_V$ is not a constant, but depends on the molecular mass.

(viii) ***Equation of state for an ideal gas undergoing adiabatic process*** :

Consider n gm-moles of an ideal gas undergoing an abiabatic change so that dT is the change in its temperature. Using 1st law

$$dQ = 0 = dU + dW = dU + P\,dV \qquad ...(17.7.21)$$

Using equation (17.7.12) in r.h.s of (17.7.21) we obtain

$$n\,C_V\,dT + P\,dV = 0 \qquad (17.7.22)$$

Now for an ideal gas

$$P\,V = n\,RT$$

$$\Rightarrow \quad P\,dV + V\,dP = n\,R\,dT$$

$$\Rightarrow \quad dT = \frac{P\,dV + V\,dP}{n\,R} \qquad ...(17.7.23)$$

Using (17.7.23) in (17.7.22)

$$n\,C_V \frac{P\,dV + V\,dP}{n\,R} + P\,dV = 0$$

$$\Rightarrow \quad n\,C_V\,P\,dV + n\,C_V V\,dP + n\,R\,P\,dV = 0$$

Since $C_p - C_v = R$, so using this in the above

$$n\,C_V\,P\,dV + n\,C_V V\,dP + n\,(C_p - C_v)\,P\,dV = 0$$

$$\Rightarrow \quad n\,C_V\,V\,dP + n\,C_P P\,dV = 0$$

$$\Rightarrow \quad n\,C_p\,P\,dV = -n\,C_v V\,dP$$

$$\Rightarrow \quad \frac{C_p}{C_v}\cdot\frac{dV}{V} = -\frac{dP}{P} \qquad ...(17.7.24)$$

Defining the abiabatic gas constant

$$\gamma = \frac{C_P}{C_v} \qquad ...(17.7.25)$$

and using it we have

$$\gamma\frac{dV}{V} = -\frac{dP}{P} \qquad ...(17.7.26)$$

Integrating both sides of eqn. (17.7.26) we obtain

$$\gamma \log V = -\log P + \log A \qquad ...(17.7.27)$$

where log A is an integration constant - Rewriting (17.7.27) we obtain

$$\log PV^{\gamma} = \log A$$

$$\Rightarrow \quad PV^{\gamma} = A = \text{constant} \qquad ...(17.7.28)$$

From (17.7.28) we can also obtain other forms. Since PV = n RT

$$\therefore\ PV^{\gamma} = \frac{nRT}{V}V^{\gamma} = A$$

$$\Rightarrow \quad T\,V^{\gamma-1} = \frac{A}{nR} = B = \text{ constant for a given amount of gas} \qquad ...(17.7.29)$$

Similarly

$$P\,V^{\gamma} = P\left(\frac{nRT}{P}\right)^{\gamma} = P^{1-\gamma}T^{\gamma}(nR)^{\gamma}$$

$$\Rightarrow \quad P^{1-\gamma}\,T^{\gamma} = \frac{A}{(nR)^{\gamma}} = B' = \text{ constant for a given amount of gas} \qquad ...(17.7.30)$$

Equations (17.7.28), (17.7.29) and (17.7.30) are all equations of state for an ideal gas undergoing adiabatic change.

Comparison between an isotherm and adiabatic curve

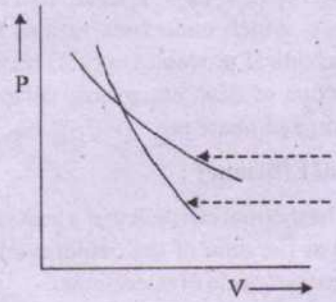


Fig. 17.13

For an isothermal change

$$PV = nRT = \text{constant}$$

$$\Rightarrow \left(\frac{dP}{dV}\right)_T V + P = 0$$

$$\Rightarrow \left(\frac{dP}{dV}\right)_T = \left(\frac{dP}{dV}\right)_{\text{isotherm}} = -\frac{P}{V} \quad ...(17.7.31)$$

For an adiabtic change

$$PV^{\gamma} = \text{constant}$$

$$\Rightarrow \left(\frac{dP}{dV}\right)_Q V^{\gamma} + \gamma PV^{\gamma-1} = 0$$

$$\Rightarrow \left(\frac{dP}{dV}\right)_{\text{adibatic}} = \left(\frac{dP}{dV}\right)_Q = -\gamma\frac{P}{V} \quad ...(17.7.32)$$

Since $\gamma = \frac{C_P}{C_V} > 1$, so eqns (17.7.31) and (17.7.32) imply that an adiabtic is steeper than an isotherm i.e. slope of an adiabatic is greater than the slope of an isotherm (for an ideal gas).

Limitations of 1st law :

First law of thermodynamics suffers from the following limitations.

i) It does not tell us whether any particulr process can actually take place or not.

ii) It does not predict the direction of heat flow.

iii) According to 1st law one could convert the whole (100%) of heat into work, which does not actually occur in nature.

Distinction between Isothermal and Adiabatic Process

Features	Isothermal	Adiabatic
1. Internal energy	1. No change $dU = 0$	1. Changes, $dU \neq 0$
2. Heat	2. Involved $dQ \neq 0$	2. Not involved $dQ = 0$
3. Expansion	3. Work done is positive, hence dQ is positive	3. Work done is positive hence dU is decreased (- Ve)
4. Compression	4. Work done is negative, hence dQ is negative	4. Work done is negative, hence dU is increased (+ Ve)
5. Temperature	5. Remains constant	5. Changed

Ex.17.7.1 One gram of water (1 cm^3) becomes 1671 cm^3 of steam when boiled at a pressure of 1 atm. The heat of vaporisation at this pressure is 2256 J g^{-1}. Compute the external work and the increase in internal energy.

Soln.

$$W = P(V_v - V_L) = (1.013 \times 10^5 N/m^2) x$$
$$(1671 - 1) \times 10^{-6} m^3$$
$$= 169 J$$

By 1st law

$$dQ = dU + dW$$

or $\quad Q = (U_f - U_i) + W$

$$U_f - U_i = Q - W = mL - W$$

$$= 2256\text{ J} - 169\text{ J} = 2087\text{ J}$$

$\therefore \Delta U = 2087\text{ J}$

Ex.17.7.2 A monoatomic ideal gas undergoes isobaric expansion when 6000J of heat energy is given to it. Calculate the work done by the gas and increase in its internal energy.

Soln.

$$Q = n\,C_P\,\Delta T = 6000\text{ J}$$

$$\Delta U = n\,C_V\,\Delta T$$

$$Q = \Delta U + W$$

$$\Rightarrow \quad W = Q - \Delta U = n\,(C_p - C_V)\,dT$$

$$W = n\,R\,dT$$

For monoatomic gas $C_v = \frac{3}{2}R,\ C_P = \frac{5}{2}R$

$$\frac{Q}{\Delta U} = \frac{nC_P\Delta T}{nC_v\Delta T} = \frac{C_P}{C_v} = \frac{5}{3}$$

$$\Rightarrow \quad \Delta U = \frac{3}{5}Q = \frac{3}{5} \times 6000\text{J} = 3600\text{J}$$

$$\therefore W = Q - \Delta U = (6000 - 3600)$$

$$\Rightarrow \quad W = 2400\text{ J}$$

17.8 Conversion of heat into work :

We have noted earlier that first law of thermodynamics treats heat and work on equal footing. It is observed that work can be converted into heat (e.g. heat is liberated due to friction between two surfaces). But for conversion of heat into work we require a device called, in general, heat engine.

Heat Engine :

It is a device that draws heat from a heat reservoir and converts as much as possible heat energy into mechanical energy (work) through a cycle of operations.

It consists of three essential components (i) infinite heat reservoir maintained at a temperature T_a (ii) infinite heat sink, maintained at a temperature T_b ($T_a > T_b$) and (iii) a working substance, which undergoes various thermal and mechanical processes such as addition or substraction of heat, expansion, compression and change of phase etc.

Thermal Efficiency :

The thermal efficiency of a heat engine is defined as the ratio of the output work to the input heat per cycle of operations.

$$\text{i.e.} \quad \eta = \frac{W}{Q_1} \qquad \text{...(17.8.1)}$$

Suppose an engine takes heat Q_1 from heat reservoir, converts a part W of it into work and rejects an amount Q_2 of heat into the heat sink. Then for each cycle

$$U_f - U_i = 0 = (Q_1 - |Q_2|) - W \equiv Q_1 + Q_2 - W$$

where W is the work done by the system, and Q_2 is a negative number.

This gives

$$W = Q_1 - |Q_2|$$

Hence efficiency

$$\eta = \frac{W}{Q_1} = 1 - \frac{|Q_2|}{Q_1} = 1 + \frac{Q_2}{Q_1} \qquad \text{...(17.8.2)}$$

Equation (17.8.2) shows that efficiency of heat engine is always less than 1 or 100% unless Q_2 (heat rejected) is zero.

Carnot Engine (Carnot Cycle) :

It is an ideal heat engine which transfers heat into mechanical work through a cycle of reversible operations, namely an isothermal expansion, an adiabatic expansion, an isothermal compression and an adiabatic compression.

Design: It consists of the following components.

(i) Cylinder : (Body of the engine)

It is a barrel having perfectly insulating walls and a perfectly conducting base. It is fitted with a perfectly insulating piston capable of moving without friction.

(ii) Heat Source : (reservoir)

It is a hot body of infinitely large thermal capacity such that when heat is withdrawn from it its temperature 'T_1' does not change.

(iii) Heat Sink :

It is a body of infinitely large thermal capacity such that when heat is added to it, its temperature T_2 does not change. T_2 is always kept less than T_1.

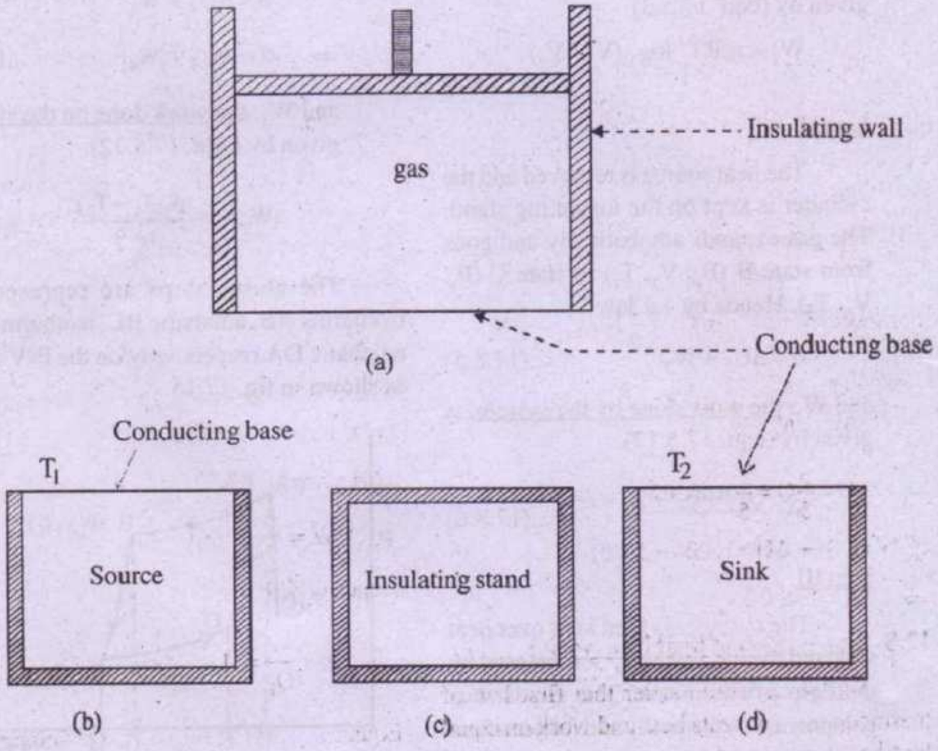


Fig. 17.14

(iv) Insulating Stand :

It is a stand made of perfectly insulating material.

(v) Working Substance :

An ideal gas is used as the working substance.

Operations :

The carnot engine operates through the repetitions of the following four reversible processes.

i) Step - I

The cylinder containing the working

substance is kept in contact with the heat source. The working substance draws heat Q_1 and expands isothermally until it changes form state A (P_1, V_1, T_1) to state B (P_2, V_2, T_1). Using first law of thermodynamics

$$Q_1 = \Delta U_1 (= 0) + W_1 = W_1 \qquad ...(17.8.3)$$

($\Delta U_1 = 0$, since it is an isothermal process)

and W_1, the work done by the system, is given by (eqn. 17.5.3)

$$W_1 = n\,RT_1 \log_e (V_2 / V_1) \qquad ...(17.8.4)$$

ii) Step II

The heat source is removed and the cylinder is kept on the insulating stand. The gas expands adiabatically and goes from state B (P_2, V_2, T_1) to state C (P_3, V_3, T_2). Hence by 1st law

$$0 = \Delta U_2 + W_2 \qquad ...(17.8.5)$$

and W_2, the work done by the system, is given by (eqn. 17.5.12).

$$W_2 = \frac{nR(T_2 - T_1)}{1-\gamma} \qquad ...(17.8.6)$$

iii) Step III

The cylinder is then kept over heat sink and the gas is slowly compressed by the help of the piston. The gas is thus compressed isothermally and it rejects heat $|Q_2|$ (developed due to compression) to the sink. The gas thus changes from state C (P_3, V_3, T_2) to state D (P_4, V_4, T_2). Hence by 1st law

$$Q_2 = \Delta U_3 (= 0) + W_3 \qquad ...(17.8.7)$$

$$\Rightarrow \quad -|Q_2| = -|W_3| \qquad ...17.8.7(a)$$

and W_3, the work done on the system is given by (eqn. 17.5.3)

$$W_3 = -nRT_2 \log_e \left(\frac{V_4}{V_3}\right) \qquad ...(17.8.8)$$

iv) Step - IV

Then the heat sink is removed and the cylinder is kept on the insulating stand. The gas is then compressed adiabatically until it regains its initial state A (P_1, V_1, T_1). Hence by 1st law

$$0 = \Delta U_4 + W_4 \qquad ...(17.8.9)$$

$$\Rightarrow \quad 0 = \Delta U_4 - |W_4| \qquad ...17.8.9(a)$$

and W_4, the work done on the system, is given by (eqn. 17.5.12)

$$W_4 = -\frac{nR(T_1 - T_2)}{1-\gamma} \qquad ...(17.8.10)$$

The above steps are represented by isotherms AB, adiabatic BC, isotherm CD and adiabatic DA respectively on the P-V diagram as shown in fig. 17.15

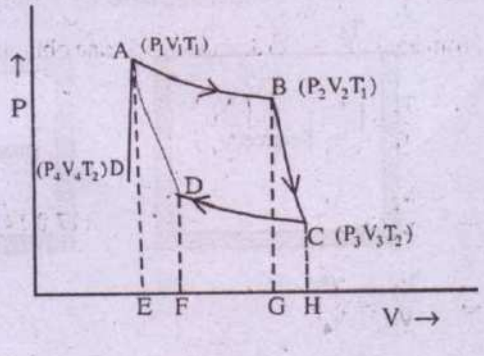


Fig.17.15

W_1 = Area ABGEA; W_2 = Area BCHGB

W_3 = Area CDFHC ; W_4 = Area DAEFD

Calculation of Efficiency

Net work done by the system is given by

$$W = W_1 + W_2 - W_3 - W_4$$

$$= nRT_1 \log_e(V_2/V_1) + \frac{nR(T_2 - T_1)}{1-\gamma}$$

$$+ nRT_2 \log_e(V_4/V_3) + \frac{nR(T_1 - T_2)}{1-\gamma}$$

$$\Rightarrow \quad W = nRT_1 \log_e(V_2/V_1) + nRT_2 \log_e(V_4/V_3) \qquad ...(17.8.11)$$

Since B and C lie on the same adiabatic, so

$$T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} \qquad ...(17.8.12)$$

Similarly since A and D lie on the same adiabatic

$$T_2 V_4^{\gamma-1} = T_1 V_1^{\gamma-1}$$

$$\Rightarrow \quad \frac{T_2}{T_1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1} \qquad ...(17.8.13)$$

From eqns (17.8.12) and (17.8.13) one obtains

$$\frac{T_2}{T_1} = \left(\frac{V_2}{V_3}\right)^{\gamma-1} = \left(\frac{V_1}{V_4}\right)^{\gamma-1}$$

$$\Rightarrow \quad \frac{V_2}{V_3} = \frac{V_1}{V_4} \qquad ...(17.8.14)$$

$$\Rightarrow \quad \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

$$\Rightarrow \quad \log_e\left(\frac{V_2}{V_1}\right) = \log_e\left(\frac{V_3}{V_4}\right) = -\log_e\left(\frac{V_4}{V_3}\right) \qquad ...(17.8.15)$$

Using (17.8.15) in (17.8.11)

$$W = nR(T_1 - T_2)\log_e(V_2/V_1) \qquad ...(17.8.16)$$

Therefore efficiency

$$\eta = \frac{W}{Q_1} = \frac{W}{W_1} = \frac{nR(T_1 - T_2)\log_e(V_2/V_1)}{nRT_1 \log_e(V_2/V_1)}$$

$$\Rightarrow \quad \eta = 1 - \frac{T_2}{T_1} \qquad ...(17.8.17)$$

Further we also find, by applying first law of thermodynamics to the whole cycle

$$Q_1 - |Q_2| = W \quad (\because \Delta U = 0 \text{ for the cycle})$$

giving

$$\eta = \frac{W}{Q_1} = 1 - \frac{|Q_2|}{Q_1} \qquad ...(17.8.18)$$

comparing equations (17.8.17) and (17.8.18) we find that

$$\left|\frac{Q_2}{Q_1}\right| = \frac{T_2}{T_1} \qquad ...(17.8.19)$$

Conclusions:

The following conclusions can be drawn

(i) Efficiency 'η' is independent of the nature of the working substance

(ii) 'η' depends on the source temperature (T_1) and sink temperature (T_2).

(iii) **$\eta \neq 1$ or 100%, since then we will have $T_2 = 0^0$ K, which is not attainable.**

(iv) **Further since $\eta \neq 1$, $Q_2 \neq 0$ i.e. it is not possible to operate an engine without rejecting heat.**

(v) **Conversion of heat into work is not possible without temperature difference between source and sink. Because when $T_1 = T_2$, $\eta = 0$.**

(vi) **Since in any real engine some energy is dissipated (due to irreversible processes involved) so efficiency of any other engine is always lesss than that of carnot** engine. This leads to **Carnot's theorem, stated as " among all heat engines, working between same temperatures carnot engine has the maximum efficiency".**

Carnot Refrigerator :

Since each step in the carnot engine is reversible, the entire cycle may be reversed, converting the engine into a refrigerator, aborbing heat at lower temperature and rejecting heat at higher temperature.

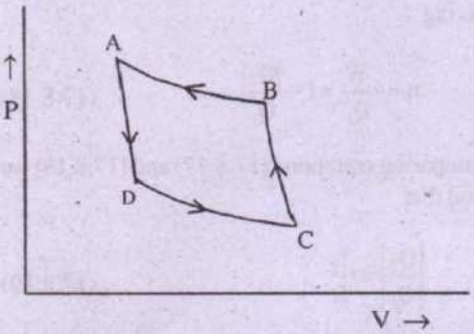


Fig.17.16

Then coefficient of performance (COP) of the refrigerator is given by

$$K = \left|\frac{Q_2}{W}\right| = \frac{|Q_2|}{|Q_1| - |Q_2|} \quad ...(17.8.20)$$

$$\Rightarrow \quad K = \frac{|Q_2|/|Q_1|}{1 - \frac{|Q_2|}{|Q_1|}} = \frac{T_2/T_1}{1 - T_2/T_1}$$

(where in r.h.s we have used eqn 17.8.19)

$$\Rightarrow \quad K = \frac{T_2}{T_1 - T_2} \quad ...(17.8.21)$$

The above analysis shows that when temperature difference $T_1 - T_2$ is small K is much greater than unity; in that case heat can be pumped out from lower to higher temperature with only a little expenditure of work. But greater the temperature difference, smaller is K, and more work is required to pump out same amount of heat.

The thermal efficiency 'e' is defined as

$$\frac{1}{e} = 1 + K = \frac{|Q_1|}{|Q_1| - |Q_2|}$$

$$\Rightarrow \quad e = \frac{1}{1+K} = \frac{|Q_1| - |Q_2|}{Q_1} < 1$$

Also $e = \frac{1}{1+K} = \frac{T_1 - T_2}{T_1} < 1$

17.9 Second Law of Thermodynamics :

The limitations of first law of thermodynamics needed the formulation of a Second law which has been stated in different forms by different physicists.

i) Clausius Statement :

Heat cannot flow from a cold body to a hot body without performance of external work.

The above statement can be visualised if we consider the working of a refrifterator. From eqn. (17.8.20) and (17.8.21) we find that

$$COP = K = \frac{|Q_2|}{|W|} = \frac{T_2}{T_1 - T_2}$$

So $K \to \infty$ when $|W| \to 0$. Since $|W| = |Q_1| - |Q_2|$ it means $K \to \infty$ as $|Q_1| \to |Q_2|$ i.e. heat extracted from cold reservoir is completely transferred to the hot reservoir without performance of any external work on the system. This process is never seen to occur in nature. Hence Clausius statement forbids spontaneous flow of heat from cold body to hot body.

ii) Kelvin's statement :

It is impossible to draw heat from a body by cooling it below the coldest of its surroundings.

The meaning of the above statement can be understood if we consider the Carnot engine. When heat is extracted from a body its temperature gradually decreases. When its temperature becomes equal to the sink temperature η becomes zero i.e. engine stops working.

iii) **Planck's Statement :**

It is impossible to draw heat from a body and convert the whole of it into work.

The above statement can be justified by considering a Carnot engine. Thus when $Q_2=0$, efficiency $\eta = 1$ implying $T_2 = 0^0$ K. But absolute zero cannot be attained. Hence $Q_2 \neq 0$ i.e. some heat has to be rejected.

17.10 Thermodynamic Scale of Temperature

We have seen that the efficiency of a Carnot engine operating between reservoirs at two different given temperatures is independent of the nature of the working substance and is a function of the two temperatures of the reservoirs (heat source and heat sink). This fact was used by Kelvin to describe a temperature scale.

The efficiency of the Carnot engine absorbing heat Q_1 from heat source at a definite temperature θ_1 in some scale and rejecting heat Q_2 to the heat sink at another temperature θ_2 $(\theta_2 < \theta_1)$ in the same scale is given by

$$\eta = 1 - \frac{|Q_2|}{|Q_1|} = \text{constant}$$

Hence $\left|\frac{Q_2}{Q_1}\right|$ is a constant for all carnot engines, working between two given temperatures θ_1 and θ_2. Kelvin proposed that the ratio of the temperatures θ_1 and θ_2 be defined as equal to this constant ratio -

i.e. $$\frac{\theta_2}{\theta_1} = \frac{|Q_2|}{|Q_1|} \qquad ...(17.10.1)$$

Equation (17.10.1) appears to be identical with (17.8.19); but there is a subtle and crucial difference. In eqn. (17.8.19) the temperatures are those based on ideal gas thermometer, whereas (17.10.1) defines a temperature scale, based on Carnot Cycle and Second Law of thermodynamics, which is completely independent of the nature of any particular substance. Thus Kelvin temperature scale is truly absolute and is a thermodynamic temperature scale.

However to fix the size of the temperature scale we assign arbitrary value of 273.16 °K to the temperature of triple point of water. So for a Carnot engine operating between heat source temperature θ and heat sink temperature θ_t (corresponding to triple point of water), we have by (17.10.1)

$$\frac{\theta}{\theta_t} = \frac{\theta}{273.16} = \frac{|Q|}{|Q_t|}$$

$$\Rightarrow \quad \theta = 273.16\ \frac{|Q|}{|Q_t|} \qquad ...(17.10.2)$$

But $|Q|/|Q_t|$ is also equal to the ratio of the temperature on the ideal gas scale.

i.e. $$\frac{\theta}{\theta_t} = \frac{\theta}{273.16^0\text{K}} = \frac{T}{273.16^0\text{K}} \qquad ...(17.10.3)$$

$$\Rightarrow \quad \theta = T$$

i.e Kelvin Scale and ideal gas scale of temperatures are identical.

Absolute Zero :

It follows from eqn. (17.10.2) that smaller the value of Q, lower is the temperature θ on the absolute scale. The smallest possible value of Q is zero and the corresponding temperature θ is called **Absolute Zero.**

Thus the temperature at which a system undergoes a reversible isothermal process without transfer of heat is called absolute zero. In otherwords at absolute zero, an isotherm and an adiabatic are identical.

Further 'θ' cannot be negative; because then efficiency

$$\eta = 1 + \frac{\theta}{\theta_1} > 1$$

But $\eta > 1$ violates Carnot's theorm and hence second law of thermodynamics.

Ex.17.8.1 Find the efficiency of the Carnot engine operating between temperatures 227°C and 27°C. If it receives 500 cal of heat per cycle, find the work done and heat rejected per cycle.

Soln.

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{300}{500} = 1 - \frac{3}{5} = \frac{2}{5} \equiv 40\%$$

$$\eta \equiv 40\%$$

$$\Rightarrow \quad \eta = \frac{W}{Q_1} = \frac{2}{5}$$

$$\Rightarrow \quad W = \frac{2}{5}Q_1 = \frac{2}{5} \times 500 \text{ cal} = 200 \text{ cal}$$

Workdone = W = 840 J.

$$\frac{Q_2}{Q_1} = \frac{T_2}{T_1} = \frac{300}{500} = \frac{3}{5}$$

$$\Rightarrow \quad Q_2 = \frac{3}{5} \times Q_1 = \frac{3}{5} \times 500 \text{ cal} = 300 \text{ cal.}$$

Hence heat rejected $Q_2 = 300$ cal.

Ex.17.8.2 The efficiency of a Carnot engine is 0.25. If the temperature of the sink is reduced by 30°C, the efficiency increases by 30%. Find the temperature of the source.

Soln.

$$\eta = 1 - \frac{T_2}{T_1} = 0.25$$

$$\Rightarrow \quad T_2 = 0.75 \times T_1 \qquad \text{...(1)}$$

When temperature of sink is reduced by 30°C

$$\eta' = 1 - \frac{T_2 - 30}{T_1} = \eta + 0.3\eta = 1.3 \times \eta = 1.3 \times 0.25$$

$$\Rightarrow \quad 1 - \frac{T_2 - 30}{T_1} = 0.325$$

$$\Rightarrow \quad T_2 - 30 = 0.675 \times T_1 \qquad \text{...(2)}$$

Subtracting (2) from (1)

$$30 = 0.075 \times T_1$$

$$\Rightarrow \quad T_1 = \frac{30 \times 1000}{75} = 400^0\text{K} \qquad \text{Ans.}$$

Ex.17.8.3 A carnot engine X receives heat at temperature T_1 (= 800° K) and rejects heat at temperature T_2. Another carnot engine Y; coupled to A receives heat rejected by X, and rejects heat at temperature T_3 (= 300° K). Calculate the temperature T_2 when (i) the work outputs of two engines are equal (ii) the efficiency of two engines are equal.

Soln.

$$\eta_x = \frac{W_x}{Q_{x1}} = 1 - \frac{Q_{x2}}{Q_{x1}} = 1 - \frac{T_2}{T_1} \qquad \text{...(1)}$$

$$\Rightarrow \frac{Q_{x2}}{Q_{x1}} = \frac{T_2}{T_1} \qquad \text{...(2)}$$

$$\eta_y = \frac{W_y}{Q_{x2}} = 1 - \frac{Q_{y2}}{Q_{x1}} = 1 - \frac{T_3}{T_2} \qquad ...(3)$$

(i) If $W_x = W_y$, then

$$\frac{\eta_x}{\eta_y} = \frac{Q_{x2}}{Q_{x1}} = \frac{(T_1 - T_2)/T_1}{(T_2 - T_3)/T_2} \qquad ...(4)$$

From (2) & (4)

$$\frac{T_2}{T_1} = \frac{(T_1 - T_2)T_2}{(T_2 - T_3)T_1}$$

$$\Rightarrow \quad T_2 - T_3 = T_1 - T_2$$

$$\Rightarrow \quad 2T_2 = T_1 + T_3$$

$$\Rightarrow \quad T_2 = \frac{T_1 + T_3}{2} \qquad ...(5)$$

$$\Rightarrow \quad T_2 = \frac{800 + 300}{2}\ {}^0K = 550^0K$$

(ii) If $\eta_x = \eta_y$

$$\Rightarrow \quad \frac{T_1 - T_2}{T_1} = \frac{T_2 - T_3}{T_2}$$

$$\Rightarrow \quad T_1T_2 - T_2^2 = T_1T_2 - T_1T_3$$

$$\Rightarrow \quad T_2^2 = T_1T_3$$

$$\Rightarrow \quad T_2 = \sqrt{T_1T_3}$$

$$\therefore \quad T_2 = \sqrt{800 \times 300}\ {}^0K = 489.9^0K$$

Summary

1. Thermodynamic co-ordinates(or,state co-ordinates) arevariables used to specify the state of a system example: pressure, volume, temperature.

2. In adiabatic change, dQ= 0, as no heat enters or leaves the system.

3. In isothermal change, dv = 0, as temp of the system remaind constant. (This is also meant for cyclic process).

4. First law of Thermodynamics :

$$dQ = dU + dW$$

Where Q = Quantity of heat

U = Internal energy

W = Work done

5. (i) Work done in adiabatic process =

$$\frac{P_2V_2 - P_1V_1}{1 - \gamma}$$

Where γ = Ratio of specific heats of the gas 1 and 2 refer to the initial & final states respetively.

(ii) Work done in isothermal process =

$$RT\log_e (V_2 / V_1)$$

6. Mayer's Relation : $C_p - C_V = R$

Where $C_p$and C_V are specific heats of a gas at constant pressure and volume respectively.

7. Molar specific heat = (Molecular weight of the gas) x specific heat.

8. Efficiency of a heat engine = $n = 1 - \frac{Q_2}{Q_1}$

$$= 1 - \frac{T_2}{T}$$

Where T_1 and T_2 are temperatures of source and sink respectively in abolute scale.

9. Second law of thermodynamics gives the condition for heat flow.

MODEL QUESTIONS

A. Multiple Choice Type Question :

1. For a perfect gas under adiabatic expansion there occurs
   - (a) no change in internal energy
   - (b) no external work is done
   - (c) change in internal energy is greater than the external work done
   - (d) change in internal energy is equal to the external work done.
2. A frictionless heat engine can be 100% efficient only if its exhaust temperature is
   - (a) equal to the input temperature
   - (b) less than input temperature
   - (c) 0^0 C
   - (d) 0^0 K
3. Which of the following carnot engine has maximum efficiency
   - (a) operating between 10^0 K & 100^0K
   - (b) operating between 100^0K & 300^0K
   - (c) operating between 200^0K & 800^0K
   - (d) operating between 100^0K & 500^0K
4. Compressed air in a tube of a wheel of a scooter at normal temperature starts coming out from a puncture. The air inside
   - (a) starts becoming hotter
   - (b) remains at same temperature
   - (c) starts becoming cooler
   - (d) none of the above.
5. The first law of thermodynamics is a statement of
   - (a) conservation of heat
   - (b) conservation of work
   - (c) conservation of momentum
   - (d) conservation of energy
6. If heat is supplied to an ideal gas in an isothermal process
   - (a) the internal energy of the gas will increase
   - (b) the gas will do positive work
   - (c) the gas will do negative work
   - (d) the said process is not possible
7. The 2nd law of thermodynamics states that
   - (a) heat is neither created nor destroyed
   - (b) heat can be converted to other forms of energy
   - (c) heat flows from a hot object to a cold one.
   - (d) mechanical energy is converted to heat energy.
8. The door of an operating refrigerator in a closed room is opened. After some time, the temperature of the room will be
   - (a) lowered
   - (b) raised
   - (c) unaffected
   - (d) lowered or unaffected depending on the temperature of the room.
9. When a gas expands adiabatically, its temperature
   - (a) increases
   - (b) decreases
   - (c) does not change
   - (d) none of the above.
10. For a perfect gas undergoing adiabatic process the relation between pressure P and volume V is
    - (a) $PV^{\gamma-1}$= constant
    - (b) PV^{γ} = constant
    - (c) $PV^{1/\gamma}$ = constant
    - (d) $PV^{(1-\gamma)/\gamma}$ = constant

11. For an adiabatic expansion of a perfect gas the value of $\frac{\Delta p}{p}$ is equal to

(a) $\frac{1}{2}\frac{\Delta V}{V}$ (b) $-\frac{\Delta V}{V}$

(c) $-\gamma\frac{\Delta V}{V}$ (d) $-\gamma^2\frac{\Delta V}{V}$

12. A gas performs the most work when it expands

(a) isothermally

(b) adiabatically

(c) isobarically

(d) at non-uniform rate

13. The amount of mechanical work to be done to completely melt 1 gm of ice is

(a) 4.2 J (b) 42 J

(c) 80 J (d) 336 J

14. In an adiabatic change, the system

(a) takes heat from the surroundings

(b) gives heat to the surroundings

(c) exchanges no heat with the surroundings

15. During an adiabatic expansion, the increase in volume is asociated with

(a) decrease in pressure and decrease in temperature

(b) increase in pressure and increase in temperature

(c) decrease in pressure and increase in temperature

(d) increase in pressure and increase in temperature.

16. A thermosflask contains hot tea. It is vigorously shaken. If the tea is considered as the system, then its temperature will

(a) rise

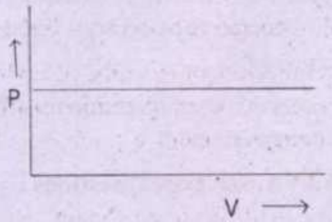
(b) fall

(c) not change

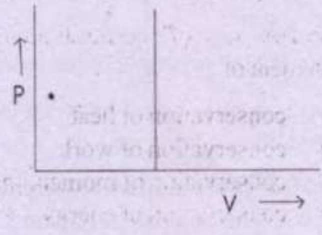
(d) first fall and then rise

B. Very Short Answer Type Questions :

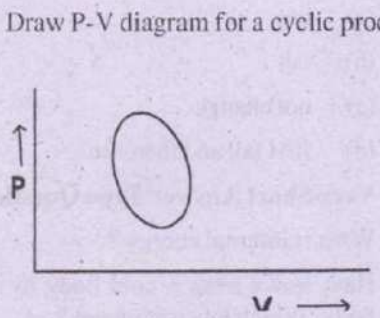
1. What is internal energy ?
2. Heat flows from a cold body to a hot body. Which law is violated ?
3. Is the refrigerator a heat engine ?
4. Which law of thermodynamics gives concept of temperature ?
5. Which law of thermodynamics defines a temperature scale ?
6. Write down the equation of state of an adiabatic process, for a perfect gas.
7. Give an example of reversible process.
8. Give few examples of irreversible process.
9. State 1st law of thermodynamics.
10. State 2nd law of thermodynamics.
11. State zeroth law of thermodynamics.
12. Draw P-V diagram for isobaric process.



13. Draw P-V diagram for an isochoric process.



14. Draw P-V diagram for a cyclic process.



15. What is cyclic process ?

16. What is an isothermal process ?

17. What is an adiabatic process ?

18. In P-V diagram which has a greater slope-isothermal or adiabatic ?

19. Express 30^0 C in Kelvin Scale .

20. A piece of lead is hammered. (i) Does the internal energy increase ? (ii) Does heat enter the lead from outside ?

21. A hot piece of iron is immersed in cold water (i) Has the internal energy of water increased ? (ii) Has any work been done by iron ?

22. Why are break drums of a car heated ?

23. An ideal gas is compressed isothermally, will its internal energy increase or decrease ?

24. Write down the expression for efficiency of carnot engine.

C. Short Anser Type Questions :

1. Why a sink is necessary for operation of a heat engine ?

2. Distinguish between a refrigerator and a heat engine.

3. Describe the principle of refrigerator.

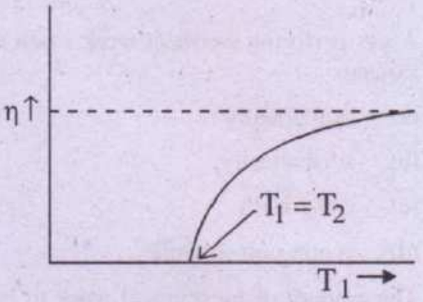
4. State two differences between adiabatic and isothermal process.

5. What are the minimum requirements for the working of a carnot engine ?

6. Discuss if rusting of iron is an irreversible or reversible process.

7. Is waterfall a reversible or irreversible process ? Discuss.

8. Draw a graph between efficiency(η) and source temperature (T_1) when sink temperature is constant.



9. State Clausius statement of 2nd law of thermodynamics.

10. If an inflated tyre bursts, the air coming out is cooled - Explain.

11. Water at the base of a waterfall is warmer than at the top. Why ?

12. Coffee kept in a thermos flask is vigorously shaken. Discuss the result.

13. Milk is added to a cup of tea and mixed with a spoon. Is this a reversible process? Explain your answer.

14. Why is C_p greater than C_V ?

15. If hot air rises, then why is it cooler at the top of a mountain than near the sea level?

16. Can a room be cooled by leaving the door of the refrigerator open ?

17. Ocean contains large amount of heat energy. Can we drive a ship across the ocean by utilising its energy ?

18. State limitations of 1st law of thermodynamics.

19. Explain how Zeroth law of thermodynamics defines temprature.

20. Should the internal energy of a system necessarily increase if heat is added to it ?

21. The final volume of a system is equal to the initial volume in a certain process. Is the work done by the system necessarily zero ? Is it necessarily non-zero ?

22. The outer surface of a cylinder containing a gas is rubbed vigorously by a polishing machine. The cylinder and its gas becomes warm. Is the energy transferred to the gas heat or work ?

23. When ice melts (decreasing in its volume), is the internal energy change greater or less than the heat added ?

D. Unsolved Problems :

1. 224 litres of an ideal gas at temperature of 27^0C and pressure 1 atm. is compressed isothermally to a volume of 56 litres. Calculate the final pressure and work done on the gas.

2. 800 CC. of an ideal gas at NTP is compressed adiabatically to 200 CC. Calculate the final pressure and work done on it. (Given $\gamma = 1.4$)

3. A tyre pumped to a pressure of 3 atm. suddenly bursts. Calculate the fall in temperature if the air before expansion is 27^0 C (Given $\gamma = 1.4$)

4. A gram molecule of a gas at 127^0 C expands isothermally until its volume is doubled. Find the amount of work done and heat absorbed.

5. 1 kg. of water is converted to steam at 100^0 C at a constant pressure of 1.01 x 10^5 Pa. Calculate the change in internal energy (density of water = 10^3 kg/m^3, density of steam = 0.598 kg/m^3, $L_\theta = 2.26$ x 10^6 J/kg)

6. Calculate the work done when 1 mole of a perfect gas is completely pressed adiabatically. The initial pressure and volume of the gas are 10^5 N/m^2 and 5 litres respectively. The final volume of the gas is 2 litres. Molar specific heat of gas at constant volume is 3R / 2.

7. If number of degrees of freedom is denoted by f and the ratio of the two molar specific heats of an ideal gas be γ, then prove that $\gamma = 1 + \frac{2}{f}$

8. Specific heat capacity at constant volume for argon is 0.075 cal/(gm ^{0}K). Calculate the molar weight and also the mass of one molecule (Given R = 2 cal/ (mol ^{0}K), $N_a = 6$ x 10^{23} / mole)

9. When 200 cal. of heat is supplied to a certain amount of gas at constant pressure, 80 cal. is used up for external work. Calculate γ.

10. A carnot engine operates between 27^0 C and 227^0 C. What is the maximum output work per kilo calories of heat input ?

11. A carnot engine receives 100 cal. of heat from the source at 127^0C and rejects 75 cal. to the sink in each cycle of operation. Calculate the temperature of the sink and efficiency of the engine.

12. The efficiency of a carnot engine is 40%. If sink temperature is 27^0C, calculate the source temperature.

13. The efficiency of a carnot engine is 30%. If the temperature of both source and sink are increased by 10^0C, the efficiency reduces to 29%. Determine the initial temperature of source and sink.

14. A carnot engine operating between 400^0 K and 300^0 K delivers 4.2KW power. Calculate the quantity of heat absorbed and rejected in each second.

15. An ideal refrigerator transfers 100 cal heat per second from a chamber at - 3^0C to the outside at 27^0C. Calculate (a) the average power consumed (b) coefficient of performance (COP).

16. The COP of refrigerator is 4. How much water at 0^0 C can it convert to ice at 0^0 C, when it receives 1 KWh energy. ($L_f = 80$ cal/g)

E. Long Answer Type Questions :

1. Describe a carnot's engine. Derive expression for its efficiency.

2. Distinguish between isothermal and adiabatic process. Derive expression for the work done in isothermal and adiabatic processes.

3. State and explain first law of thermodynamics. Hence prove that $C_P - C_V = R$.

4. State and explain first law of thermodynamics hence deduce $PV^\gamma =$ constant, for a perfect gas undergoing adiabatic change.

5. State second law of thermodynamics. Explain how it defines a scale of temperature.

F. Answer as directed

1. No heat is removed or added in an adiabatic process. (Yes/No)

2. Isothermal and adiabatic processes are identical at absolute zero (Yes/No)

3. What does the area under a curve on P-V diagram represent ?

4. The work done along any adiabatic between two isothermals is idependent of the particular adiabatic. (Yes/No)

5. The value of J in Mks system is .... (Fill in the blank)

6. The difference between c_p and c_v for solids, liquids and gases is .... (positive/negative/zero) choose the correct one.

7. What is the adiabatic elasticity of hydrogen gas ($\gamma = 1.4$) at NTP ?

8. What is the specific heat during an adiabatic change ?

9. What is the shape of isothermal curves ?

10. A waterfall is 42 meter high. Assuming that the kinetic energy of the falling water gets converted into heat, what is the rise in temperature of water ?

ANSWERS

A. Multiple Choice Type Questions :

1. (d), 2. (d), 3. (a), 4. (c), 5. (d), 6. (b), 7. (c), 8. (b), 9. (b), 10. (b), 11. (c), 12. (c), 13. (d), 14. (c), 15. (a), 16. (a),

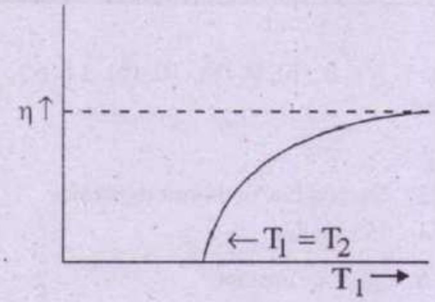
B. Very Short Answer Type Questions :

1. See text.
2. Second law of thermodynamics
3. No, it is a reversed heat engine
4. Zeroth law
5. 2nd law
6. pV^{γ} = constant
7. Slow expansion of gas
8. Diffusion of gas; sudden expansion of gas
9. See text.
10. See text
11. See text.
12. fig.
13. fig.
14. fig.
15. See text
16. See text
17. See text
18. Adiabatic
19. 303.16 °K
20. (i) Yes, (ii) No
21. (i) Yes, (ii) No
22. Due to friction
23. No change
24. See text

C. Short Answer Type Questions :

1. According to 2nd law of thermodynamics an engine cannot work without rejecting a part of heat. Hence a sink is required.
2. (i) In case of heat engine heat is converted to work by the engine, rejecting a part of it at lower temperature. In case of refrigerator heat is given up to the surrounding at higher temperature and work is done on the system by external agency. (ii) As the refrigerator works efficiency gradually decreases. But in case of heat engine efficiency almost remains constant.
3. See text
4. (i) In case of isothermal process internal energy remains constant, but in case of adiabatic process it changes. (ii) In case of isothermal heat is either taken in or given out; but in case of adiabatic heat is not involved.
5. (i) Heat source of infinite extent at temp. T_1 (ii) Heat sink of infinite extent at temp. T_2 ($T_1 > T_2$) (iii) Cylinder (container) with insulating walls, but conducting bottom (iv) Insulating pad.
6. Rusting of iron is an electro-chemical reaction in which iron is oxidized to iron oxide. Hence it is an irreversible process.
7. It is an irreversible process. Here P.E. is converted to K.E. and then K.E. is converted to heat and sound.

8.



9. See text

10. See text

11. P.E. of water at the top is converted to heat at the bottom. This causes increase in internal energy hence temperature increase.

12. By shaking work has been done on the coffee, but no heat has been added. Hence dQ $= 0 = \Delta U + \Delta W \Rightarrow \Delta U = -\Delta W$, as ΔW is -ve so ΔU is +ve, hence increase in temperature.

13. No. The process can not be reversed back and milk can not be drawn out.

14. When heat is added at constant volume $dQ = dU = n\,C_V\,dT$ and when heat is added at constant pressure $dQ' = dU + pdV = n\,C_P\,dT$. Thus $\eta(C_P - C_V)dT = PdV > 0 \Rightarrow C_P > C_V$.

15. Pressure decreases with height. When hot air rises it suffers adiabatic expansion. From 1st law $dQ = 0 = \Delta U + \Delta W \Rightarrow \Delta U = -\Delta W$ (as ΔW is +ve). This causes a decrease in internal energy. So the temperature falls.

16. No. A refrigerator rejects heat into the room and extracts heat from the freezing chamber. This is made possible by doing work on the system by the compressor. But when the door is open, the room and the refrigerator combine to be one system.

17. No. This is not possible as there is no sink available. Here $T_1 = T_2 \Rightarrow \eta = 1 - \frac{T_2}{T_1} = 0$.

18. See text

19. See text.

20. No. In an isothermal process $\Delta U = 0$. In other processes ΔU may not be zero.

21. No. In a cyclic process work done is equal to the area enclosed by the P-V diagram. But if the cycle is completed by reversing the direct path then area enclosed is zero, hence work done is zero.

22. Heat. During polishing work against friction is converted to heat. This heat is transferred to the gas and increases its temperature.

23. Ice melts at constant temperature (i.e. the process is isothermal) But $\Delta U \neq 0$, $\Delta U = \Delta Q - p\,dV = mL - p\,dV$. Since dV is '-ve', so $\Delta U > \Delta Q$ i.e. internal energy change is greater than the heat added.

D. Unsolved Problems :

1. 4 atm. ; -3.46×10^4 J.
2. 6.964 atm. , -150.295 J.
3. $T_2 = 219.2\,^0$K, fall in temp. = $80.8\,^0$K
4. 2306.8 J, 549.24 cal
5. 2.091×10^6 J
6. 972 J
8. 40 gm, 6.67×10^{-23} gm
9. 1.67
10. 1680 J
11. 27^0 C, 25%
12. 227^0 C
13. 290^0 K, 203^0 K
14. 4000 cal, 3000 cal.
15. (a) 46.67 W, (b) 0
16. 42.86 kg.

F. (1) Yes (2) Yes (3) work done on or by the system (4) Yes (5) 4200 joule/kcal (6) positive (7) 1.4×10^5 newton/met^2 (8) zero (9) Rectangular hyperbolae (10) 0.098^0C.

18

Heat Transfer

Heat can be transferred from one place to another by three different ways, namely, (i) conduction (ii) convection and (iii) radiation. Conduction usually takes place in solids, convection in liquids and radiation requires no medium. We shall discuss these processes separately.

18.1 Thermal Conduction :

It is a process in which heat is transferred from one point to another through a substance in the direction of fall of temperature without body motion of the particles (atoms / molecules) constituting the substance, except for their vibration about their mean position of rest.

The molecules at the hot end acquire more energy and begin to vibrate about their mean positions. During their vibrations (in all possible directions) they collide with their neighbours and share some of their energy of motion. These neighbours then begin to vibrate vigorously and pass on the energy by collision. Thus energy is transferred from the hot end to the farthest cold end. However the mean positions of rest of the atoms / molecules donot change.

As we know free electrons in a metal take part in conduction of electric current. The free electrons also play an effective rolè in conduction of heat. That is why metals are also good conductors of heat.

Steady state and temperture gradient

During conduction heat flows from hot end towards the cold-end. As this heat flows the temperature of different points goes on increasing gradually. But then a stage comes when the temperatures at different points donot change although the temperatures are different at different points. **Thus when the temperature remains constant at different points (although their values are different at different points), then the body is said to have attained a steady state."**

This is represented mathematically as

$$\theta = \theta(x) \text{ and } \frac{d\theta}{dx} = \text{const.}$$

where 'θ' is the temperature at position x.

When steady state is not attained we call the state as variable state and this is represented as

$$\theta = \theta(x, t)$$

In the variable state the heat supplied $Q = Q_1 + Q_2 + Q_3$, where Q_1 is heat absorbed for raising temperature, Q_2 is heat radiated out and Q_3 is heat conducted.

The space rate of variation of temperature along the direction of heat flow, is called ***temperature gradient*****.** Thus if x-be direction of heat flow, then $\frac{d\theta}{dx}$ is the temperature gradient. Its value at any point can be obtained from the slope of tangent to the $\theta\ V_s\ x$ curve at the point (see fig. 18.1) i.e.

$$\left(\frac{d\theta}{dx}\right)_{x=x_o} = \tan\phi$$

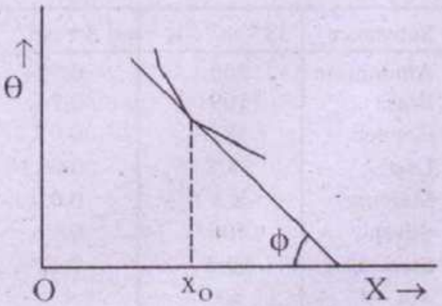


Fig.18.1

Under steady state conditons $\frac{d\theta}{dx}$ is constant. This implies $\theta\ V_s\ x$ curve is a straight line.

Thermal Conduitivity :

It is observed experimentally that when there is no radiation loss from the sides of substance (lagged bar) the amount of heat flowing in steady state through the cross-section of a substance by conduction depends on area of cross-section (A), temperature gradient $\left(\frac{d\theta}{dx}\right)$ near the cross-section and the time of flow as

$$dQ \ \alpha \ A$$

$$\alpha \ \frac{d\theta}{dx}$$

$$\alpha \ dt$$

$$\Rightarrow \quad dQ = -KA\frac{d\theta}{dx}dt \qquad ...(18.1.1)$$

The negative sign is used because temperature θ decreases as x increases, the origin being fixed near the hot end; so that dQ becomes positive.

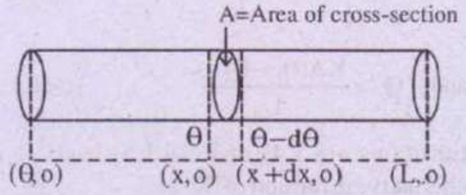


Fig.18.2

The heat current 'H' = Rate of flow of heat

$$= \frac{dQ}{dt} = -KA\frac{d\theta}{dx} \qquad ...(18.1.2)$$

The constant of proportionality 'K' in eqn (18.1.1) and (18.1.2) is called thermal conductivity and it depends on the property of the substance only and does not depend on the shape or size of the substance. **Thus thermal conductivity is defined as the amount of heat flowing per second per unit area of cross-section per unit temperature gradient.**

If we consider an isotropic bar of unifrom area of cross-section (A fixed) then eqn. (18.1.2) gives

$$\int_0^L H\,dx = -KA\int_{\theta_1}^{\theta_2} d\theta \qquad ...(18.1.3)$$

Now under steady state since $\frac{d\theta}{dx}$ is constant, so for a bar of uniform area of cross-section $H = -kA\frac{d\theta}{dx}$ is constant. Therefore for a uniform bar, in the steady state eqn. (18.1.3) reduces to

$$H\int_0^L dx = -KA\int_{\theta_1}^{\theta_2} d\theta$$

$$\Rightarrow \; H = \frac{Q}{t} = \frac{KA(\theta_1 - \theta_2)}{L} \qquad ...(18.1.4)$$

$$\text{and} \quad Q = \frac{KA(\theta_1 - \theta_2)t}{L} \qquad ...(18.1.5)$$

Equations (18.1.4) and (18.1.5) leads to an alternative difinition of 'K' as

"Thermal conductivity of a material is defined as the amount of heat that flows per unit time through the opposite faces of a unit cube of the material under steady state, when the temperature difference between the faces is unit."

For most materials thermal conductivity increases slightly with increase of temperature. But since the variation is very small it is oftern taken to be constant. The numerical values of K, for various materials at room temperature are given in table 18.1

Dimension and Unit of K

(a) Dimension : From eqn (18.1.1) we find

$$[K] = \frac{[Q]}{[A]\left[\frac{d\theta}{dx}\right][t]} = \frac{M L^{-2} T^{-2}}{L^2 \left({}^{\circ}K L^{-1} \right) T}$$

$$\Rightarrow [K] = M L T^{-3}\, {}^{0}K^{-1}$$

(b) Unit of K : Since heat can be expressed in heat units as well as energy units, so thermal conductivity (K) can be expressed in heat and energy units as stated below.

	Energy Unit	**Heat Unit**
(i) S.I.	$J\,s^{-1}\,m^{-1}\,{}^{0}K^{-1}$	$K\,cal\,s^{-1}\,m^{-1}\,{}^{0}K^{-1}$
(ii) C.G.S	$erg\,s^{-1}\,cm^{-1}\,{}^{0}C^{-1}$	$Cal\,s^{-1}cm^{-1}\,{}^{0}C^{-1}$

The heat units and energy units are related as

A. $1\,J\,S^{-1}\,m\text{-}1\,{}^{0}K^{-1} = \frac{1}{4.2}\,cal\,s^{-1}\,m^{-1}\,{}^{0}K^{-1}$

$$= \frac{10^{-3}}{4.2}\,K\,cal\,s^{-1}\,m^{-1}\,{}^{0}K^{-1}$$

B. $1\,erg\,s^{-1}\,Cm^{-1}\,{}^{0}C^{-1} =$

$$\frac{1}{4.2 \times 10^{7}}\,cal\;\; s^{-1}\,m^{-1}\,K^{-1}$$

C. $1\,J\,s^{-1}$ m-1 0K-1 $= \frac{1}{418.4}\,cal.\,s^{-1}cm^{-1}\,{}^{0}C^{-1}$

Table 18.1 (K - at room temp. ~ 20° C)

Substance	$JS^{-1}\,m^{-1}\,{}^{0}K^{-1}$	cal S^{-1} cm^{-1} ${}^{0}C^{-1}$
Aluminium	209	0.504
Brass	109	0.26
Copper	385	0.92
Lead	34.7	0.083
Mercury	8.3	0.02
Silver	406	0.97
Steel	50.2	0.12
Tin	63	
Water	0.585	
Asbestos	0.08	
Brick	0.15-0.9	
Concrete	0.8	0.002
Cork	0.04	0.0001
Felt	0.04	0.0001
Glass	0.8	0.002
Ice	1.6	0.004
Rock wool	0.04	0.0001
Styrofoam	0.01	0.00002
Wood	0.023	0.000056
Air	0.024	0.000057
Argon	0.016	0.000039
Helium	0.14	0.00034
Hydrogen	0.14	0.00033
Oxygen	0.023	0.000056

Thermal Conductance and thermal resistance

Equation (18.1.2) can be re-written as

$$|H| = K\,A\frac{d\theta}{dx} = \frac{KA}{\Delta x}.\Delta\theta$$

and in the steady state eqn. (18.1.4) can be re-written as

$$H = \frac{KA(\theta_1 - \theta_2)}{L} = \left(\frac{KA}{L}\right).\Delta\theta$$

i.e $H = C_Q.\Delta\theta$ ...(18.1.6)

where, $\Delta\theta = \theta_1 - \theta_2$ = change in temperature > 0, and $C_Q = \frac{KA}{L} = \frac{KA}{\Delta x} = \frac{H}{\Delta\theta}$ = Thermal conductance. Thus, **thermal conductance is the heat current per unit temperature difference.**

The reciprocal of thermal conductance is called thermal resistance (R_Q)

i.e. $R_Q = \frac{L}{KA} = \frac{\Delta x}{KA} = \frac{\Delta\theta}{H}$

The dimension of thermal resistance is given as

$$[R_Q] = \frac{{}^0K}{ML^2\,T^{-3}} = M^{-1}\,L^{-2}\,T^{+3}\,{}^0K$$

and is expressed in $\text{cal}^{-1}\,\text{s}\,{}^0\text{C}$, $(\text{K cal})^{-1}\,\text{s}\,{}^0\text{C}$, $\text{erg}^{-1}\,\text{s}\,{}^0\text{C}$, $\text{J}^{-1}\,\text{s}\,{}^0\text{K}$ etc.

Determination of 'K' (Searle's method) :

Searle's method is suitable for the determination of the absolute value of thermal conductivity (K) of a material, if it is available in the form of a rod.

Apparatus :

The apparatus consists of a rod PQ, being properly insulated from the surrounding. The end P is heated by a steam chamber (S).

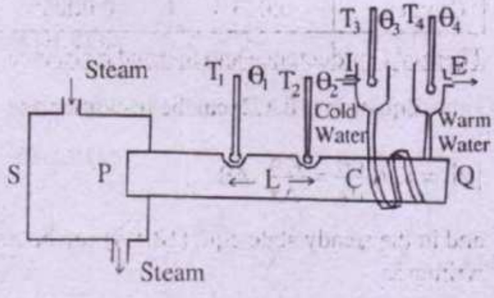


Fig. 18.3

A copper pipe (C) is wound near the end Q. By injecting cold water at I, a steady flow of water is maintained within the copper pipe; and warm water leaves the pipe near exit (E), which is collected. Two thermometers T_1 & T_2 are fitted in the grooves, filled with mercury for good thermal contact. Two other - thermometers T_3 & T_4 are provided to measure the temperatures of cold water and warm water respectively.

Theory of Experiment :

When steady state is reached the thermometers T_1 and T_2 record steady temperatures θ_1 and θ_2 respectively. Therefore if A be area of cross-section of the bar, L be separation between the two grooves, then the heat Q flowing across any cross-section in time t is given by

$$Q = \frac{K.A\,(\theta_1 - \theta_2)t}{L} \quad ...(18.1.7)$$

This heat 'Q' is used up in heating the water of mass m, which flows out in time t, through the exit E. Hence

$$Q = m.\,C_w.\,(\theta_4 - \theta_3) = \frac{K.A\,(\theta_1 - \theta_2)t}{L}$$

$$\Rightarrow \quad K = \frac{m.\,C_w\,(\theta_4 - \theta_3)L}{A(\theta_1 - \theta_2).t} \quad ...(18.1.8)$$

where C_W is the specific heat of water. If we use C.G.S heat units ($\text{cal g}^{-1}\,{}^0\text{C}^{-1}$) or S.I. heat units ($\text{K cal kg}^{-1}\,{}^0\text{k}^{-1}$) then $C_W = 1$ and we have

$$K = \frac{m\,(\theta_4 - \theta_3)L}{A(\theta_1 - \theta_2).t} \quad ...(18.1.9)$$

Procedure :

Experiemntal arrangement as shown in fig 18.3 is made. Steam is allowed to flow into the steam chamber. When steady state is reached (indicated by steady temperatures on T_1 and T_2), water is allowed to flow through the copper tube at the inlet for a period t. Then water collected near the exit E, is weighed and temperatures θ_3 and θ_4 are recorded from thermometers T_3 and T_4 respectively. These values are then put in the eqn. (18.1.8) or (18.1.9) as the case may be and 'K' is determined.

Conduction of heat through a composite block :

A. Consider a block of uniform cross-sectional area (A), composed of two different types of materials of thermal conductivity K_1 and K_2.

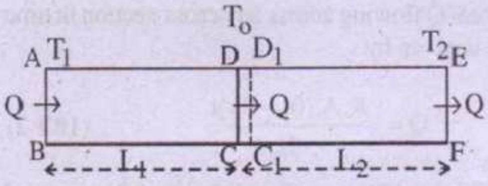


Fig. 18.4

Let the block ABCD have thermal conductivity K_1 and that of block $C_1 D_1$ EF have K_2. The composite bar is in the steady state condition. So heat current flowing through any cross-section of the block ABCD is

$$H_1 = \frac{T_1 - T_0}{R_1} \qquad ...(18.1.10)$$

and heat current through any cross-section of block C_1D_1EF is

$$H_2 = \frac{T_0 - T_2}{R_2} \qquad ...(18.1.11)$$

where $R_1 = \frac{L_1}{K_1 A}$ and $R_2 = \frac{L_2}{K_2 A}$.

Since the bar is of uniform cross-section so under steady conditions H_1 = constant and H_2 = constant. Therefore the heat current through the cross-section CD should be identical with the heat current through C_1D_1, (As CD and C_1D_1 are identical and belong to both the blocks). Hence

$$H_1 = H_2 = H$$

$$\Rightarrow \quad H = \frac{T_1 - T_0}{R_1} = \frac{T_0 - T_2}{R_2} \qquad ...(18.1.12)$$

$$\Rightarrow \quad T_1 - T_0 = H R_1$$

$$T_0 - T_2 = H R_2$$

Adding vertically

$$T_1 - T_2 = H (R_1 + R_2)$$

$$\Rightarrow \quad H = \frac{T_1 - T_2}{R_1 + R_2} \qquad ...(18.1.13)$$

Or

$$H = \frac{A(T_1 - T_2)}{\left(\frac{L_1}{K_1} + \frac{L_2}{K_2}\right)} \qquad ...(18.1.14)$$

Suppose the composite slab is replaced by a single slab of thickness $L_1 + L_2$ and area of cross-section A, thermal conductivity K, and thermal resistance R. If the faces are maintained at same two temperatures T_1 and T_2 $(T_1 > T_2)$, then

$$H = \frac{T_1 - T_2}{R} \qquad ...(18.1.15)$$

Comparing eqns (18.1.15) and (18.1.13) we find

$$R = R_1 + R_2 \qquad ...(18.1.16)$$

$$\Rightarrow \quad \frac{L_1 + L_2}{KA} = \frac{L_1}{K_1 A} + \frac{L_2}{K_2 A}$$

$$\Rightarrow \quad \frac{L_1 + L_2}{K} = \frac{L_1}{K_1} + \frac{L_2}{K_2} \qquad ...(18.1.17\_$$

If $L_1 = L_2$, then eqn. (18.1.17) reduces to

$$\frac{2}{K} = \frac{1}{K_1} + \frac{1}{K_2} \qquad ...(18.1.18)$$

Further eqn. (18.1.12) gives

$$\frac{T_1 - T_0}{R_1} = \frac{T_0 - T_2}{R_2}$$

$$\Rightarrow \quad T_0 = \frac{T_1R_1 + T_2R_2}{R_1 + R_2} \qquad ...(18.1.19)$$

Eqn. (18.1.19) gives the temperature at the interface.

If there are more than two slabs, each of same cross-setional area, constituting the block, then

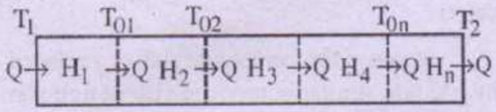


Fig. 18.5

$$H_1 = H_2 = ... = H_n = H$$

$$\Rightarrow \quad \frac{T_1 - T_{01}}{R_1} = \frac{T_{01} - T_{02}}{R_2} = ... = \frac{T_{0,n-1} - T_2}{R_n} = H$$

$$\Rightarrow \quad T_1 - T_{01} = H\,R_1$$

$$T_{01} - T_{02} = H\,R_2$$

....

....

$$T_{0,n-2} - T_{0,n-1} = H\,R_{n-1}$$

$$T_{0,n-1} - T_2 = H\,R_n \qquad ...(18.1.21)$$

Adding vertically

$$T_1 - T_2 = H\,(R_1 + R_2 + ... + R_n)$$

$$\Rightarrow \quad H = \frac{T_1 - T_2}{R_1 + R_2 + ... + R_n} \qquad ...(18.1.22)$$

Thus the effective thermal resistance is

$$R = R_1 + R_2 + ... + R_n = \sum_{i=1}^{n} R_i \qquad ...(18.1.23)$$

This gives

$$\frac{L_1 + L_2 + ... + L_n}{KA} = \sum_i \frac{L_i}{K_i A}$$

$$\Rightarrow \quad \frac{\sum_i L_i}{K} = \sum_i \frac{L_i}{K_i} \qquad ...(18.1.24)$$

and $$H = A(T_1 - T_2) \Bigg/ \left(\sum_i L_i / K_i \right) \qquad ...(18.1.25)$$

The intermediate temperatures are obtained as

$$T_{0,\,n-1} = T_2 + H\,R_n$$

$$T_{0,\,n-2} = T_{0,n-1} + H\,R_{n-1}$$

$$= T_2 + H(R_n + R_{n-1})$$

$$T_{0,\,n-3} = T_2 + H(R_n + R_{n-1} + R_{n-2})$$

....

....

$$T_{0,\,n-j} = T_2 + H(R_n + R_{n-1} + ... + R_{n-j+1})$$

....

$$T_{01} = T_2 + H(R_n + R_{n-1} + ... + R_2)$$

Thus writing the above in reverse order, the intermediate temperature are given as

$$T_{01} = T_2 + H(R_n + R_{n-1} + ... + R_2)$$

$$T_{02} = T_2 + H(R_n + R_{n-1} + ... + R_3)$$

....

$$T_{0j} = T_2 + H(R_n + R_{n-1} + ... + R_{j+1})$$

....

$$T_{0,n-1} = T_2 + HR \qquad ....(18.1:26)$$

B. Suppose a block consists of two bars of thermal conductivity K_1 and K_2 and also of different area of cross-section (fig. 18.6).

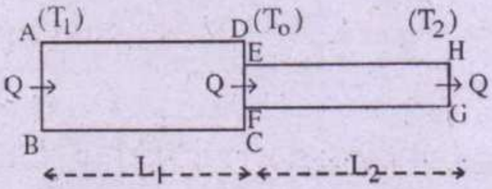


Fig. 18.6

Let block ABCD have thermal conductivity K_1 and area of cross-section A_1 and that of the block EFGH be K_2 and A_2 respectively. The composite bar is in the steady state conditions. So heat current flowing through any cross-section of the block ABCD is

$$H_1 = \frac{T_1 - T_0}{R_1} \qquad ...(18.1.27)$$

where $R_1 = L_1 / K_1A_1$

and heat current through any cross-section of the block EFGH is

$$H_2 = \frac{T_0 - T_2}{R_2} \qquad ...(18.1.28)$$

where $R_2 = L_2 / K_2A_2$

Since each block ABCD and EFGH are of uniform cross-sections H_1 = constant and H_2 = constant. As cross- sections CD and EF are in contact so

$$H_1 = H_2 = H \qquad ...(18.1.29)$$

$$\Rightarrow \quad \frac{T_1 - T_0}{R_1} = \frac{T_0 - T_2}{R_2} = H \qquad ...(18.1.30)$$

This gives

$$H = \frac{T_1 - T_2}{R_1 + R_2} \qquad ...(18.1.31)$$

and intermediate temperature

$$T_0 = \frac{T_1R_2 + T_2R_1}{R_1 + R_2} \qquad ...(18.1.32)$$

Equation (18.1.32) gives the temperature of the interface and eqn. (18.1.31) gives the heat current.

Suppose the composite slab is replaced by a single similar composite slab of uniform material of thermal conductivity 'K' such that the end temperatures are same. Same heat current flows in it, then

$$H = \frac{T_1 - T_2}{\frac{L_1}{KA_1} + \frac{L_2}{KA_2}} = \frac{T_1 - T_2}{R} \qquad ...(18.1.33)$$

Comparing (18.1.31) and (18.1.33)

$$R = R_1 + R_2$$

$$\Rightarrow \quad \frac{L_1}{KA_1} + \frac{L_2}{KA_2} = \frac{L_1}{K_1A_1} + \frac{L_2}{K_2A_2}$$

$$\Rightarrow \quad \frac{1}{K} = \frac{L_1 / K_1A_1 + L_2 / K_2A_2}{\frac{L_1}{A_1} + \frac{L_2}{A_2}} \qquad ...(18.1.34)$$

Equation (18.1.34) gives the effective thermal conductivity.

If there are a number of slabs of varying cross-sections, then

$$R = R_1 + R_2 + \ldots\ldots R_n$$

$$\Rightarrow \quad \frac{1}{K} = \frac{\sum_i \frac{L_i}{K_i A_i}}{\sum_i \frac{L_i}{A_i}} \qquad \ldots(18.1.35)$$

and

$$H = \frac{T_1 - T_2}{\sum_i \frac{L_i}{K_i A}} = \frac{T_1 - T_2}{R} \qquad \ldots(18.1.36)$$

The intermediate temperatures are given as

$$T_{01} = T_2 + H(R_n + R_{n-1} + \ldots + R_2)$$

$$T_{02} = T_2 + H(R_n + R_{n-1} + \ldots + R_3)$$

....

....

$$T_{0j} = T_2 + H(R_n + R_{n-1} + \ldots + R_{j+1})$$

....

$$T_{0,\, n-1} = T_2 + H R_n \qquad \ldots(18.1.37)$$

C. Suppose bars of different thermal conductivity (K) are connected in parallel. Then under steady state

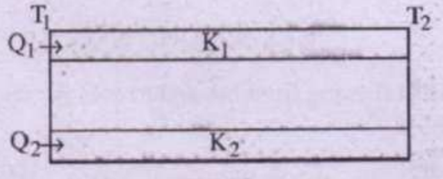


Fig.18.7

$$H_1 = \frac{K_1 A_1 (T_1 - T_2)}{L_1}; \quad H_2 = \frac{K_2 A_2 (T_1 - T_2)}{L_2}$$

$$\Rightarrow \quad H_1 = \frac{T_1 - T_2}{R_1}; \quad H_2 = \frac{T_1 - T_2}{R_2}$$

Net heat current $H = H_1 + H_2$

$$\Rightarrow \quad H = (T_1 - T_2)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \equiv \frac{T_1 - T_2}{R}$$

$$\Rightarrow \quad \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} \qquad \ldots(18.1.38)$$

If a number of rods are joined in parallel then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \qquad \ldots(18.1.39)$$

Applications of thermal Conductivity :

(1) Cooking untensils are provided with wooden handle; as wood is poor conductor of heat.

(2) Eskimos make double walled houses of blocks of ice. As air is a bad conductor of heat air within the two blocks does not conduct away heat from within the house to the cold surrounding.

(3) Ice is packed in gunny bags or sawdust. This is because gunny bags or saw dust being poor conductor of heat, heat from outside (the hot surrounding) is not conducted to the ice through the gunny bag or saw dust and as a result ice does not melt.

(4) Birds swell their feather in winter. By way of swelling their feather, air gets enclosed within the body and this prevents heat to be conducted away from the body to outside.

(5) In winter, a metallic handle of a door appears colder than the remaining part of the door. Because in winter body temperature is higher than the temperature of the surrounding. So when we touch the metallic handle, it being good conductor, heat is conducted away from our body, so that we feel cold. On the

otherhand wood being bad conductor of heat, heat does not flow from our body, and as a result we donot feel cold.

(6) Saw dust is poorer conductor than wood from which it is prepared. Because in saw dust air is enclosed which makes it a poorer conductor than wood.

(7). Refrigerator is provided with insulating walls. The insulating wall prevents heat to be conducted into the refrigerator from the surrounding.

(8) In Davy's safety lamp, copper wire gauge is used. Copper being a very good conductor of heat it conducts the heat and does not allow the heat to reach the combustible gas.

(9) In thermos flask, the double wall enclosing a vaccum keeps the temperature of the content steady.

(10) Two blankets are warmer because they contain a thin layer of air between them, which is poorer conductor than wool. Hence two blankets are warmer than a single blanket of twice the thickness of each.

Ex.18.1.1 A rod has length 1m and cross-sectional area 2 cm^2 and thermal conductivity 0.92 cal / (cm. s. ^{0}C). If hot end is at temperature 200^{0}C and cold end is at temperature 0^{0}C, find assuming steady state (a) temperature gradient (b) rate of heat flow, (c) temprature at a point 10 cm from the hot end, (d) thermal resistance.

Soln.

Given,

length of the rod $\ell = 1\text{m} = 100\text{ cm}$

Area of cross-section $A = 2\text{ cm}^2$

Thermal conductivity $K = 0.92\text{ cal/(cm.s.}^0\text{C)}$

Hot end temperature $\theta_1 = 200^0\text{C}$

Cold end temperature $\theta_2 = 0^0\text{C}$

(a) Temperature gradient =

$$\frac{\theta_1 - \theta_2}{\ell} = \frac{200\ ^0\text{C}}{100\text{ cm}} = 2^0\text{c / Cm}$$

(b) Rate of heat flow =

$$\frac{dQ}{dt} = KA\frac{\theta_1 - \theta_2}{\ell} = 0.92 \times 2 \times 2\text{ cal / s}$$

$$= 3.68\text{ cal / s}$$

(c) Temperature θ at 10 cm away from hot end then

$$\frac{\theta_1 - \theta}{10} = \frac{\theta_1 - \theta_2}{\ell} = 2\ ^0\text{C / Cm}$$

$$\theta = \theta_1 - 20\ ^0\text{C} = (200 - 20)\ ^0\text{C} = 180\ ^0\text{C}$$

(d) Thermal resistance =

$$\frac{\ell}{KA} = \frac{100\ \text{cm}}{0.92 \times 2}\ \text{S. c / cal}$$

$$= 54.35\ \text{S}\ ^0\text{C / cal.}$$

Ex.18.1.2 The opposite faces of a cubical block of metal 20 cm side are kept in contact with steam and melting ice. Determine the rate at which ice melts. The conductivity of metal is 0.2 and Latent heat of ice is 80 cal / g.

Soln.

Given $\ell = 20\text{ cm}, K = 0.2\text{ cal / (cm.s.}^0\text{C)}$

$$L_{ice} = 80\text{ cal / g}$$

Heat flowing from hot end to cold per second is

$$\frac{dQ}{dt} = H = KA\frac{\theta_1 - \theta_2}{\ell} = 0.2 \times (20)^2 \times \frac{100 - 0}{20}$$

$$= 400\text{ cal / S}$$

Ice melting per second is m =

$$\frac{dQ / dt}{L} = \frac{400}{80} = 5\text{ g}.$$

Ex.18.1.3 A wooden box of dimensions 1m × 0.6m × 0.6m and wall thickness 2×10^{-2}m contains ice at 0^0C. If the outside temperature is 40^0C and thermal conductivity of wood is 0.168 W m^{-1} $^0K^{-1}$, estimate the mass of ice which melts per hour.

Soln.

Given $a = 1\text{m},\ b = 0.6\text{ m},\ c = 0.6\text{ m}$

Wall thickness $d = 2 \times 10^{-2}$ m.

$\theta_1 = 40^0\text{ C},\ \ \theta_2 = 0^0\text{ C}$

Heat enter through all six faces. Hence

$$Q = K.\ 2\,(ab + ac + bc).\ \frac{\theta_1 - \theta_2}{d}.t = m\,L$$

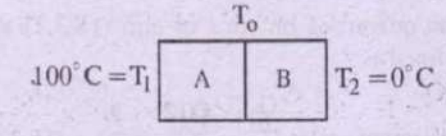
Hence ice melting in 1hour is

$$m = \frac{0.168 \times 2(0.6 + 0.6 + 0.36) \times \frac{40}{2 \times 10^{-2}} \times 3600}{80 \times 10^3 \times 4.2}\ \text{kg}$$

$m = 11.232$ kg.

Ex.18.1.4 Two plates A and B have thermal conductivities 84 W m^{-1} $^0K^{-1}$ and 126 W m^{-1} $^0K^{-1}$ respectively. They have same surface area and same thickness. They are placed in contact along their surfaces. If the temperatures of the outer surfaces of A and B are 100^0C and 0^0C respectively, find the temperature of the surface of contact in steady state.

Soln.



Intermediate temperature

$$T_0 = \frac{T_1R_2 + T_2R_1}{R_1 + R_2} = \frac{T_1\frac{L}{K_2A} + T_2\frac{L}{K_1A}}{\frac{L}{K_1A} + \frac{L}{K_2A}}$$

$$\Rightarrow\ T_0 = \frac{T_1/K_2 + T_2/K_1}{\frac{1}{K_1} + \frac{1}{K_2}} = \frac{T_1K_1 + T_2K_2}{K_2 + K_1}$$

$$= \frac{100 \times 84}{84 + 126} = \frac{100 \times 84}{210} = 40\ ^0\text{C}.$$

18.2 Convection :

Convection is the process of transfer of heat by the bulk motion of a fluid. It occurs in the presence of gravity.

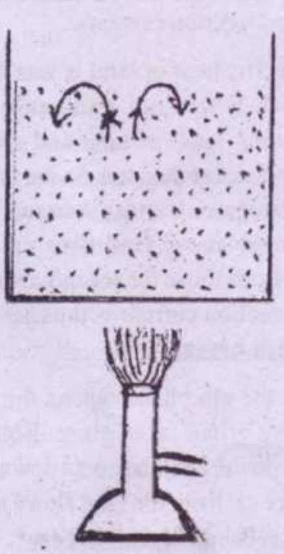


Fig.18.8

When a liquid is heated, the mass of liquid near the heat source gets heated up first. Its temperature increases so that volume of the mass of element increases. Consequently its density decreases. Thus the heated mass rises up and cooler liquid comes down.

When a liquid is heated from the top, the top portion of the liquid only gets heated, the lower portion does not get heated i.e. there is no convection current.

Thus gravity plays a role in natural convection.

Examples of natural Convection :

1) When we heat water in a pan, convection process carries heat from the bottom of the pan to the top.

2) Due to convection, warm and impure air of a room is expelled out and cold fresh air gets in.

3) A wind is a convection current which results from the uneven heating of the atmospheres.

4) The <u>land breeze</u> and <u>see breeze</u> are due to convection currents.

Specific heat of land is less than that of water. When the sun rises and shines on neighbouring mass of land and sea, the land warms up quicker than sea. So during day time, the land becomes warmer than sea. As a result the warm air above land rises up and colder dense air from above the sea moves in to replace it. A convection current is thus set up. This is called **"<u>Sea breeze</u>".**

On the otherhand during the night time the reverse effect takes place. Both land and sea cools down, but land cools down faster than sea. Hence air from the land flows towards the sea. This is called **"<u>Land breeze</u>".**

18.3 Radiation :

Radiation is a process in which heat (energy) is transferred from one place to another without heating the intervening medium.

The energy emitted by a body in the form of radiation on account of its temperature is called thermal radiation.

Thus the word **"radiation"** is used in two meanings as given above (i) It refers to process by which energy is emitted by a body and is transmitted in space and falls on another body (ii) It also refers to energy itself which is being transmitted in space.

Properties of thermal radiations :

1) It travels in empty space in a straight line with the speed of light.

2) The intensity of heat radiation varies inversely as the square of the distance from the source.

3) It obeys the same laws of reflection and refraction as light does.

4) It exihibits the phenomenon of interference, diffraction and polarisation.

5) It does not heat the medium through which it passes.

6) Heat radiations donot pass through all substances. Substances through which radiations can pass through are called <u>diathermanous</u> and through which it cannot pass are called as <u>*athermanous*</u>.

Reflectance, Absorptance, Transmittance

When head radiation falls on a substance, it gets partly reflected, partly absorbed and partly transmitted.

It Q_i be amount of heat radiation incident on a body, out of which Q_r is reflected, Q_t is transmitted and Q_a is absorbed, then

$$Q_i = Q_r + Q_t + Q_a$$

$$\Rightarrow \ 1 = \frac{Q_r}{Q_i} + \frac{Q_t}{Q_i} + \frac{Q_a}{Q_i} \qquad ...(18.3.1)$$

The quantities on r.h.s of eqn (18.3.1) are defined as :

$$\text{Reflectance (r)} = \frac{Q_r}{Q_i} = \frac{Q_r/t}{Q_i/t} = \frac{P_r}{P_i} \quad ...(18.3.2)$$

$$\text{Transmittance (t)} = \frac{Q_t}{Q_i} = \frac{Q_t/t}{Q_i/t} = \frac{P_t}{P_i} \qquad ...(18.3.3)$$

$$\text{Absorptance (a)} = \frac{Q_a}{Q_i} = \frac{Q_a/t}{Q_i/t} = \frac{P_a}{P_i} \quad ...(18.3.4)$$

Thus,

(i) Reflectance of a body is defined as the ratio of the part of heat radiation reflected (Q_r) by the body to the total heat radiation incident (Q_i) on it.

OR

Reflectance of a body is the ratio of radiation power reflected (P_r) by the body to the radiation power incident (P_i) on it.

(ii) Transmittance of a body is defined as the ratio of the heat radiation transmitted (Q_t) by the body to the heat radiation incident (Q_i) on it.

OR

Transmittance of a body is defined as the ratio of the radiation power transmitted by the body (P_t) to the radiation power incident (P_i) on it.

(iii) Absorptance of a body is defined as the ratio of heat radiation absorbed (Q_a) by the body to the heat radiation incident (Q_i) on it.

OR

Absorptance of a body is defined as the ratio of the radiation power absorbed (P_a) by the body to the radiation power incident (P_i) on it.

Equation (18.3.1) in combination with 18.3.2, 18.3.3 & 18.3.4 gives

$$r + t + a = 1 \quad ...(18.3.5)$$

Special Cases :

(i) For athermanous substances $t = 0$, so

$$r + a = 1$$

This implies that a good reflector is a bad absorber.

(ii) For perfect reflectors $r = 1$, $t = 0$ and $a = 0$.

(iii) For perfect black body $a=1$, $r = 0$ and $t=0$.

Black Body :

A body which completely absorbs all radiations incident upon it, irrespective of wavelength, is called a perfect blackbody.

i.e. $a = 1$ for a perfect black body.

A black body when heated emits radiations of all wavelengths at that temperature. The wavelength range of emitted radiation is independent of the material of the body and depends only on the temperature of the body. The intensities of radiations, at a given temperature varies with wavelength as shown in fig. 18.9.

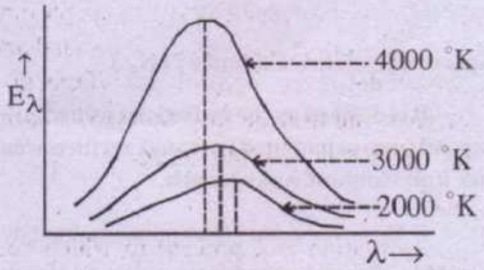


Fig. 18.9

An ideal black body is not realized in practice (nature). However (i) a surface coated with lampblack or platinum black absorbs 98% of the incident radiation. But it does not emit full radiation spectrum. So lampblack can be treated as perfect black body for absorption purposes (ii) A hollow chamber with a very small opening and maintained at a constant temperature can be treated as a perfect black body for absorption as well as emission purposes (see fig. 18.10)

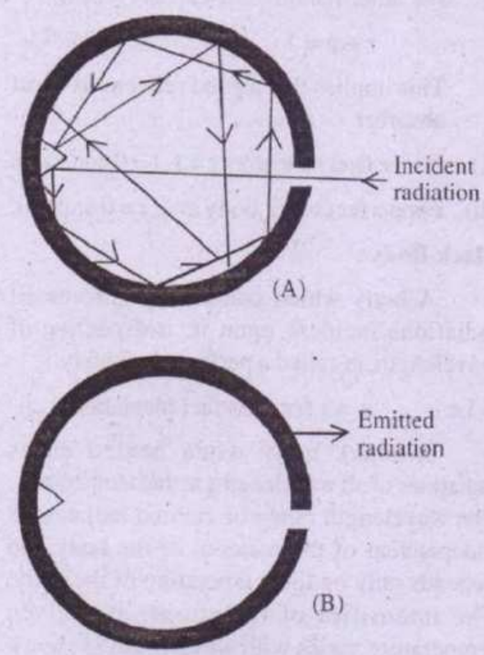


Fig. 18.10

Spectral (radiant) Exitance : (N_λ)

It is defined as the radiation emitted per second (power radiated) per unit surface area per unit range of wave length.

If P_λ be the power (radiation) emitted by a surface of area A, over a range of wave length λ to $\lambda + d\lambda$, then

$$N_\lambda = \frac{P_\lambda}{A.d\lambda} \qquad ...(18.3.6)$$

where $P_\lambda = E_\lambda / t$; E_λ being radiation emitted in the wavelength range λ to $\lambda + d\lambda$ over a time t. For a perfect transmitter or a perfect reflector $N_\lambda = 0$.

Spectral exitance has the dimension $M L^{-1} T^{-3}$. In S.I. units it is expressed in watt m^{-3} or $J S^{-1} m^{-3}$. In C.G.S. units it is expressed in erg $S^{-1} cm^{-3}$.

Total radiant exitance (N)

It is defined as the power radiated (emitted) per unit surface area over all wavelength.

$$\text{i.e.} \quad N = \frac{P}{A} = \frac{E/t}{A} \qquad ...(18.3.7)$$

where $P = E / t$ is the power radiated over all wave lengths by a surface of area A, and E is the total radiant energy emitted in time t by the surface area A. Thus

$$N = \int_0^\infty N_\lambda d\lambda \qquad ...(18.3.8)$$

Total radiant exitance has the dimension $M T^{-3}$. In S.I. units it is expressed in watt m^{-2} or $J S^{-1} m^{-2}$. In C.G.S. units it is expressed in erg $S^{-1} cm^{-2}$.

Spectral emissivity : (e_λ)

It is defined as the ratio of the power emitted within a waveband λ to $\lambda + d\lambda$ to the power emitted by a blackbody of same surface area within the same wave band.

$$\text{i.e.} \quad e_\lambda = \frac{P_\lambda}{P_\lambda^B} = \frac{A\, N_\lambda\, d\lambda}{A\, N_\lambda^B\, d\lambda} = \frac{N_\lambda}{N_\lambda^B} \qquad ...(18.3.9)$$

where on r.h.s. we have used (18.3.6). Equation (18.3.9) shows that spectral emissivity of a body is equivalent to relative spectral exitance w.r.to black body. Thus e_λ is dimensionless and unitless.

For a perfect transmitter or reflector $N_\lambda = 0$ so $e_\lambda = 0$. For a perfect emitter $N_\lambda = N_\lambda^B$ and hence $e_\lambda = 1$. For all other bodies $e_\lambda < 1$.

Total emissivity (e) (total radiant emittance)

Total emissivity or simply called missivity of a body is defined as the ratio of the heat radiation emitted (E) by the body in a given time interval (dt) to the heat radiation emitted (E_B) by a blackbody of same surface area in the same time interval (dt).

i.e. $$e = \frac{E}{E_B} = \frac{P\ dt}{P_B\ dt} = \frac{N\ A\ dt}{N_B\ A\ dt} \quad ...(18.3.10)$$

For a perfect black body e = 1.

Spectral absorptivitiy (a_λ) :

It is defined as the ratio of the radiation (power) absorbed within a given waveband (λ to $\lambda + d\lambda$) to the radiation (power) incident on the body in the same waveband.

i.e. $$a_\lambda = \frac{P_\lambda^a}{P_\lambda^i} = \frac{P_\lambda^a\ dt}{P_\lambda^i\ dt} = \frac{E_\lambda^a}{E_\lambda^i} \quad ...(18.3.11)$$

Where,

P_λ^a = radiation power absorbed within waveband λ to $\lambda + d\lambda$.

P_λ^i = radiation power incident within the waveband λ to $\lambda + d\lambda$.

Thus a_λ is dimensionless and unitless. For a perfect blackbody $a_\lambda = 1$, $P_\lambda^i = P_\lambda^a$. For a perfect transmitter or reflector $a_\lambda = 0$ and $P_\lambda^a = 0$. For other bodies $1 > a_\lambda > 0$.

The <u>total absorptivity</u> (or simply absorptivity / absorptance) as has been already defined is given as

$$a = \int_0^\infty a_\lambda\ d\lambda \quad ...(18.3.12)$$

Prevost's theory of heat exchange :

It states that all objects continuously emit heat radiation to the surrounding and receive heat radiations from the surrounding at all finite temperatures.

<u>Examples</u> : (i) One feels cold when standing near a block of ice. Because the person radiates more than he receives. (ii) One feels warm when standing near a heater. Because he receives more radiation than he wants.

Kirchoff's Law of heat radiation :

It states that the ratio of the spectral exitance and spectral absorptance for a given waveband and temperature is constant for all bodies and is equal to the spectral exitance of a perfect blackbody, within the same waveband and at same temperature.

i.e. $$\left(\frac{N_\lambda}{a_\lambda}\right)_T = \left(N_\lambda^B\right)_T \quad ...(18.3.13)$$

This gives

$$\left(\frac{N_\lambda}{N_\lambda{}^B}\right)_T = \left(a_\lambda\right)_T$$

and using eqn. (18.3.9) on ℓ.h.s. we obtain

$$(e_\lambda)_T = (a_\lambda)_T \quad ...(18.3.14)$$

Equation (18.3.14) leads one to state Kirchoff's law of radiation as **"for a specified waveband and temperature the spectral emossivity and spectral absorptivity are equal."**

<u>Proof</u> :

Consider a body of surface area A, maintained at constant temperature T. Let P_λ^i be the power incident on the body within the waveband λ to $\lambda + d\lambda$. Let the body absorb power P_λ^a and emit power P_λ. Then by equation (18.3.11) and (18.3.6)

$$P_\lambda^a = a_\lambda P_\lambda^i \qquad ...(18.3.15)$$

and

$$P_\lambda = A\,N_\lambda\,d\lambda \qquad ...(18.3.16)$$

Since the body is maintained at constant temperature so the power absorbed must be equal to the power emitted.

i.e. $P_\lambda^a = P_\lambda$

$$\Rightarrow \quad a P_\lambda^i = A\,N_\lambda\,d\lambda \qquad ...(18.3.17)$$

Equation (18.3.17) must hold good for all types of bodies. So this should also hold good for a blackbody. Since for a blackbody $a_\lambda = 1$, so eqn. (18.3.17) yields

$$P_\lambda^i = A\,N_\lambda^B\,d\lambda \qquad ...(18.3.18)$$

Using r.h.s. of eqn. (18.3.18) in the l.h.s of eqn. (18.3.17) we obtain

$$a_\lambda . A\,N_\lambda^B\,d\lambda = A\,N_\lambda\,d\lambda$$

$$\Rightarrow \quad \frac{N_\lambda}{a_\lambda} = N_\lambda^B = \text{constant} \qquad ...(18.3.19)$$

Or

$$\frac{N_\lambda}{N_\lambda^B} = e_\lambda = a_\lambda \qquad ...(18.3.20)$$

Equations (18.3.19) and (18.3.20) one the statements of Kirchoff's law of radiation.

Applications of Kirchoff's law of Radiation :

(i) A good absorber is a good emitter.

It means if a body absorbs certain colour of radiation efficiently, then it emits radiation of that colour efficiently. For example if a metal ball with a dark spot is heated and left in a dark room, then the black spot glows brilliantly, than other portions.

(ii) A red glass heated in a furnace and then taken to a dark room appears green.

A red glass appear red (when cold) because it absorbs all colours except red. Therefore when heated it emits all colours except red.

(iii) Sodium, when heated emits yellow radiation. When white light is passed through sodium vapour, it absorbs yellow colour selectively.

(iv) In solar spectrum a large number of dark lines called Fraunhoffer lines are observed. This is explained as follows:

The central hot core of sum emits continuous radiation. This core is surrounded by various elements in vapour state. These vapours selectively absorb those colours (which they might have emitted on heating). Therefore the solar spectrum is devoid of these colours, causing dark lines. This helped in indicating the presence of hydrogen and helium in the atmosphere.

Stefan's Law :

It states that the total radiant exitance of a perfect block body is proportional to the fourth power of its absolute temperature.

i.e. $N_B \propto T^4$

$$\Rightarrow \quad N_B = \sigma\,T^4 \qquad ...(18.3.21)$$

Using ean. (18.3.7) in l.h.s. of eqn. (18.3.21) we obtain

$$\frac{P_B}{A} = N_B = \sigma\,T^4 \qquad ...(18.3.22)$$

Equation (18.3.22) leads one to state Stefan's law as : **"The power emitted by a blackbody per unit area is directly proportional to the fourth power of absolute temperature of a black body."**

Since by eqn. (18.3.10) we have $N = e N_B$, so eqn. (18.3.21) can give

$$N = e\, N_B = e\,\sigma\, T^4 \qquad ...(18.3.23)$$

$$\Rightarrow \quad P = e P_B = e\,\sigma A\, T^4 \qquad ...(18.3.24)$$

Equations (18.3.23) & (18.3.24) hold good for any body other than black body, with 'e' as emissivity of the body.

When a body at temperature T is surrounded by another body (or surrounding) at temperature T_0, the power lost by the body at temperature T (using Prevost's theory of heat exchange) is given as

$$-P = \sigma e A\,(T^4 - T_0^4) \qquad ...(18.3.25)$$

'- ve' sign in l.h.s. indicates loss.

The constant 'σ' is called as Stefan's constant, and its value is $5.7 \times 10^{-8}\ Wm^{-2}(^0K)^{-4}$. It has the dimension $M\,T^{-3}\,{}^0K^{-4}$.

Newton's law of cooling :

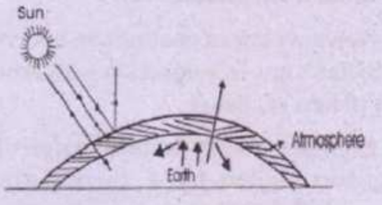
The law states that the rate of loss of heat by a body is directly proportional to the temperature difference between the body and the surrounding, when temperature differences are small.

i.e. $-\frac{dQ}{dt}\ \alpha\ (\theta - \theta_0)$

where θ is the temperature of source (body) and θ_0 is the temperature of the surrounding.

Green House Effect

The green house effect is a natural process that warms the earths surface. Earths natural green house effect is critical to supporting life. Without natural green house effect, the average temperature at earths surface would have been much below the freezing point of water, rather than the present average of 15° c. Thus green house effect is responsible for maintaining a temperature suitable for living organisims.



Let us try to understand the mechanism behind this green house effect.

The solar energy reaches the earths upper atmosphere in the form of radiation of very short wave length predominately in the visible or near visible part of electromagnetic spectrum. Nearly one fourth of this incoming radiations are reflected back by clouds and gases. Another one fourth of radiations are absorbed by the atmosphere.

Nearly half of the incoming radiation thus falls on earths surface heating it. To maintain an energy balance, earth radiates almost same amount of heat back to the space. Since earth is much colder in comparision to the sun, it radiates at much longer wave length (infrared region) in comparision to incoming solar radiation. A major fraction of this radiation emitted by different elements of earths surface is absorbed by the atmospheric gases (such as carbon dioxise, methame etc.) The molecules of these gases radiate energy back to the earths surface, heating it up once again.

The two most abundant gases in the atmospere nitrogen and oxygen exert almost no green house effect where as gases of more complex molecules are responsible for it. Carbon dioxide, Methane, Nitrous xide, CFCS are some of the gases present in the atmosphere causing green house effect.

Human activities primarity burning of fossilfuels and deforestration have intensified green house effect by increasing the level of green house gases. This in turn leads to global warming.

Derivation from Stefan's law :

Newton's law of cooling can be deduced from Stefan's law in conjuction with Prevost's theory of heat exchange.

Consider a body at temperature T °K, being surrounded by a surrounding at temperature T_0 °K. Then the powerloss (or heat loss per second) by the body is given by

$$-P = -\frac{dQ}{dt} = \sigma e A (T^4 - T_0^4) \quad ...(18.3.27)$$

$$\Rightarrow \quad -\frac{dQ}{dt} = \sigma e A (T^2 + T_0^2)(T + T_0)(T - T_0) \quad ...(18.3.28)$$

Let $T = T_0 + \delta$ (with δ small)

Then $T^2 + T_0^2 = 2T_0^2 + 2T_0\delta + \delta^2$

$$T + T_0 = 2T_0 + \delta \quad ...(18.3.29)$$

Using (18.3.29) in (18.3.28)

$$-\frac{dQ}{dt} = \sigma e A (2T_0 + 2T_0\,\delta + \delta^2)(2T_0 + \delta)\,\delta$$

Since δ is small, so retaining upto 1st order in δ we obtain

$$-\frac{dQ}{dt} = \sigma e A\, 4\, T_0^3\, \delta$$

$$\Rightarrow \quad -\frac{dQ}{dt} = 4\sigma e A T_0^3 (T - T_0) \quad ...(18.3.30)$$

The factor $B = 4\sigma e A T_0^3$ is constant for a given body in a given surrounding. Hence

$$-\frac{dQ}{dt} \propto (T - T_0) \quad ...(18.3.31)$$

Now $T = 273.16 + \theta$

$$T_0 = 273.16 + \theta_0$$

So $T - T_0 = \theta - \theta_0$

Hence

$$-\frac{dQ}{dt} \propto (\theta - \theta_0) \quad ...(18.3.32)$$

Equation (18.3.31) or (18.3.32) is the content of Newton's law of cooling.

Determination of specific heat of a liquid :

Suppose a mass m of a liquid of specific heat s, kept in a calorimeter of mass M and specific heat s_c, loses heat dQ in time dt. Let its temperature fall by $d\theta$. Then by Newton's law of cooling.

$$-dQ = (ms + Ms_c)(-d\theta) = B(\theta - \theta_0)dt$$

$$\Rightarrow \quad \frac{d\theta}{\theta - \theta_0} = -\frac{B}{(ms + Ms_c)}dt \quad ...(18.3.33)$$

Integrating both sides within proper limits

$$\int_{\theta_1}^{\theta_2} \frac{d\theta}{\theta - \theta_0} = -\frac{B}{(ms + Ms_c)} \int_{t_1}^{t_2} dt$$

$$\Rightarrow \quad \log_e\left(\frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}\right) = -\frac{B}{(ms + Ms_c)}(t_2 - t_1) \quad ...(18.3.34)$$

If there are two liquids of masses m_1 and m_2 with specific heats s_1 and s_2 respectively, kept in the same calorimeter one after the other, and are allowed to cool from temperature θ_1 to θ_2, then

$$\log_e\left(\frac{\theta_2 - \theta_0}{\theta_1 - \theta_0}\right) = -\frac{B}{m_1 s_1 + Ms_c}(t_2 - t_1) \quad ...(18.3.35)$$

and

$$\log_e\left(\frac{\theta_2-\theta_0}{\theta_1-\theta_0}\right)=-\frac{B}{m_2s_2+Ms_c}\,(t_2'-t_1') \qquad ...(18.3.36)$$

The l.h.s. of eqn. 18.3.35 & 18.3.36 are equal, hence

$$\frac{B}{m_1s_1+Ms_c}(t_2-t_1)=\frac{B}{m_2s_2+Ms_c}(t_2'-t_1')$$

This gives

$$s_2=\frac{t_2'-t_1'}{t_2-t_1}\left(\frac{m_1s_1-Ms_C}{m_2}\right)-\frac{M}{m_2} \qquad ...(18.3.37)$$

Eqn. 18.3.37 can be used to determine s_2 if s_1 is known.

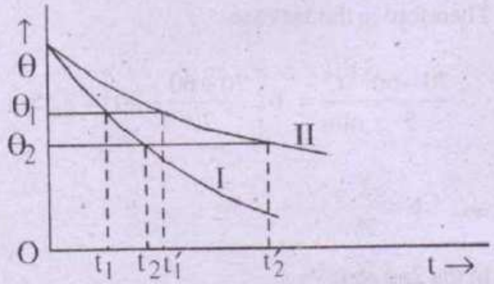


Fig.18.11

Procedure :

A known hot liquid (preferably water) is taken in a calorimeter (previously weighed) and its temperature (θ) at definte time intervals are recorded. Then a graph between temperature and time is plotted. The experiment is repeated for the second (unknown) liquid and the graph is plotted. Then $\Delta t'=t_2'-t_1'$ and $\Delta t=t_2-t_1$ are determined from the graph and used in eqn. (18.3.37) to obtain s_2.

Ex.18.3.1 When the temperature of a body is raised from 27° C to a higher value, the radiation emitted becomes 4 times. Find the final temperature.

Soln.

$$P=\sigma\, e\, A\, T^4$$

$$\Rightarrow \quad \frac{P_1}{P_2}=\left(\frac{T_1}{T_2}\right)^4=\left(\frac{300}{T_2}\right)^4=\frac{1}{4}$$

$$\Rightarrow \quad T_2^4=(300)^4\times 4$$

$$\therefore \quad T_2=300\sqrt{2}=424.2\ ^0\mathrm{K}=157.2\ ^0\mathrm{C}.$$

Ex.18.3.2 Calculate the value of Stefan's constant if the radiant emittance of a perfect blackbody at 327° C is 7380 W/ m².

Soln.

$$P_B=\sigma T^4$$

$$\Rightarrow \quad \sigma=\frac{P_B}{T^4}=\frac{7380}{(600)^4}=5.694\times10^{-9}\,\frac{W}{m^2\ ^0K^4}$$

Ex.18.3.3 Two bodies A and B having emissivity 0.4 and 0.1 respectively have the same radiant emittance. If the temperature of A is 127° C, find the temperature of B.

Soln.

$$\sigma e_A\, T_1^4=\sigma e_B\, T_2^4$$

$$\Rightarrow \quad \frac{e_A}{e_B}=\left(\frac{T_2}{T_1}\right)^4=\frac{0.4}{0.1}=4$$

$$T_2^4=4.\ T_1^4=4.(400)^4\ K^4$$

$$T_2=565.6\ ^0\mathrm{K}=292.6\ ^0\mathrm{C}.$$

Ex.18.3.3 If the temperature of a body is raised from 27° C to 177° C, what would be the ratio of the final to initial radiations emitted ?

Soln.

$$\frac{P_1}{P_2}=\frac{\sigma e T_1^4}{\sigma e T_2^4}=\left(\frac{T_1}{T_2}\right)^4$$

$$\Rightarrow\ \frac{P_f}{P_i}=\left(\frac{T_2}{T_1}\right)^4=\left(\frac{450}{300}\right)^4=\left(\frac{3}{2}\right)^4$$

$$\frac{P_f}{P_i}=\frac{81}{16}$$

Ex.18.3.4 If the solar energy received by unit area of the earth per second normally called solar constant is S. the radius of the sun is r and the mean distance of earth from the Sun is R, show that the absolute temperature of the surface of the sun, assuming it to be perfect blackbody, is given by

$$T=\left(\frac{S}{\sigma}\right)^{\frac{1}{4}}\cdot\left(\frac{R}{r}\right)^{\frac{1}{2}}$$

Soln.

Power radiated by sun $P_1=\sigma A T^4=\sigma(4\pi r^2)T^4$ (assuming it to be a perfect black body) Power received by unit area of earth is

$$S=\frac{P_1}{4\pi R^2}=\frac{\sigma(4\pi r^2)T^4}{4\pi R^2}$$

$$\Rightarrow\ T^4=\frac{S}{\sigma}\cdot\frac{R^2}{r^2}$$

$$T=\left(\frac{S}{\sigma}\right)^{\frac{1}{4}}\left(\frac{R}{r}\right)^{\frac{1}{2}}$$

Ex.18.3.5 A liquid cools from 70⁰C to 60⁰C in 5 minutes. Calculate the time taken by the liquid to cool from 60⁰C to 50⁰C, if the temperature of the surrounding is constant at 30⁰C.

Soln.

By Newton's law of cooling

$$-\frac{dQ}{dt}=B(\theta-\theta_0)=\frac{ms(-d\theta)}{dt}$$

$$\Rightarrow\ -\frac{d\theta}{dt}=\frac{B}{ms}(\theta-\theta_0)=b(\theta-\theta_0)$$

$$\Rightarrow\ \left\langle-\frac{d\theta}{dt}\right\rangle=b\langle\theta-\theta_0\rangle$$

$$\Rightarrow\ \frac{\theta_1-\theta_2}{t_1-t_2}=b\,\frac{(\theta_1-\theta_0)+(\theta_2-\theta_0)}{2}$$

$$\Rightarrow\ \frac{\theta_1-\theta_2}{t_1-t_2}=b\left[\frac{\theta_1+\theta_2}{2}-\theta_0\right]$$

Therefore in the 1st case

$$\frac{70-60}{5}\frac{{}^0C}{\min}=b\left[\frac{70+60}{2}-30\right]=b.35$$

$$\Rightarrow\ b=\frac{2}{35}$$

In the 2nd case

$$\therefore\ \frac{\theta_1'-\theta_2'}{t_2'-t_1'}=b\left(\frac{\theta_1'+\theta_2'}{2}-\theta_c\right)$$

$$\Rightarrow\ \frac{60-50}{t_2'-t_1'}=\frac{2}{35}\left(\frac{60+50}{2}-30\right)=\frac{2}{35}\times 25$$

$$\therefore\ t_2'-t_1'=\frac{10\times 35}{2\times 25}=7\text{ min.}$$

OR

$$\log\left(\frac{\theta_1-\theta_0}{\theta_2-\theta_c}\right)=\frac{B}{ms}(t_2-t_1)=b(t_2-t_1)$$

$$\Rightarrow \quad \log \frac{70-30}{60-30} = b\,(5\,\text{min}) \qquad \Rightarrow \quad t_2' - t_1' = \frac{1}{b}\log_e\left(\frac{60-30}{50-30}\right)$$

$$\Rightarrow \quad b = \frac{1}{5}\log\frac{40}{30} = \frac{1}{5}\log_e\left(\frac{4}{3}\right) = 0.058 \qquad = \frac{1}{0.058} \times \log_e\frac{30}{20} = 7.047\,\text{min}.$$

$$\log\left(\frac{\theta_1' - \theta_0}{\theta_2' - \theta_0}\right) = b(t_2' - t_1')$$

SUMMARY

1. Three modes of heat transfer are conduction, Convection and Radiation.
2. In conduction heat is transferred from one point to another through a maternal medium without involving transfer of any portion of the medium.

   Heat current (H) is

   $$\frac{dQ}{dt} = -KA\,\frac{d\theta}{dx}$$

   Where K is the thermal conductivity of the maternal and $\frac{d\theta}{dx}$ is termperature gradient.
3. To find the amount of heat 'Q' that flows in time 't' through a bar of length L and cross-sectional area 'A' with its ends maintained at temperatures Q_1 and Q_2, we can use the formula:

   $$Q = \frac{KA\,(\theta_1 - \theta_2)t}{L}$$
4. Convection is a mode of heat transfer by actual flow of matter. It is possible only in fluids. Gravity plays a role in natural convection.
5. Radiation is a process in which heat is transmitted from one place to another without requiring any intervening medium.
6. Stefain's law:

   Power emitted by a body of surface area 'A' at absolute temperature 'T' is

   $P = e\sigma AT^4$

   Where 'e' is the emissivity and τ is the Stefanis constant

   In case of a black body $e = 1$

   $\therefore P = \sigma AT^4$
7. Black body is one that absorbs all the radiation incident upon it, irrespective of wavelength.
8. Black body radiation a temperature 'T' has a characteristic distribution of emergy versus wavelength of radiation that depends only on T and is independent of the size, shape and material of the body.
9. Wien's displacement law states that the wavelength with maximum energy of radiation is,

   $\lambda_{max}\,T = 2.9\times10^{-3}\,m^0K$

MODEL QUESTIONS

A. Multiple Choice Type Questions :

1. The thermal conductivity of a rod depends on

(a) length

(b) mass

(c) area of cross-section

(d) material of the rod

2. In a room containing air heat can go from one place to another

(a) by conduction only

(b) by convection only

(c) by radiation only

(d) by all three methods

3. A solid at temperature T °K is kept in an evacuated chamber at temperature T_0 °K ($T_0 > T$). The rate of increase of temperature of the body is proportional to

(a) $T_0 - T$ (b) $T_0^2 - T^2$

(c) $T_0^3 - T^3$ (d) $T_0^4 - T^4$

4. Newton's law of cooling is a special case of

(a) Kirchoff's law

(b) Prevost theory of heat exchange

(c) Stefan's law

(d) Planck's law

5. Two bodies A and B having equal surface area are maintained at temperatures 10°C and 20°C. The thermal radiation emitted in a given time by A & B are in the ratio

(a) 1 : 15 (b) 1 : 2

(c) 1 : 4 (d) 1 : 16

6. One end of the rod is kept in a furnance. In steady state the temperature of the rod

(a) increases

(b) decreases

(c) remains constant

(d) is non uniform

7. Metals are good conductors of heat because

(a) they have a reflecting surface

(b) the atoms collide very frequently

(c) they contain free electrons

(d) they are relatively far apart.

8. Under steady state, the temperature of a body

(a) increases with time

(b) decreases with time

(c) does not change with time and it remains same at all points

(d) does not change with time but is different at different point of body.

9. It is hotter at some distance over the fire than in front of it because

(a) air conducts upwards only

(b) heat is radiated upwards only

(c) convection of heat occurs upwards only

(d) non of the above.

10. Convection is that mode of transmission in which heat travels from one particle of the medium to another

(a) by actual contact of the particles

(b) by actual motion of the particles

(c) by conduction

(d) by radiation.

B. Very Short-Answer Type Questions :

1. Give the unit of thermal conductivity.
2. Which of the following is the best conductor of heat

   (i) iron (ii) copper (iii) silver
3. What is the speed with which radiation propagate in vaccum?
4. State Newton's law of cooling.
5. State Stefan's law of radiation.
6. Define spectral radiant exitance.
7. Define total radiant exitance.
8. Define spectral emissivity.
9. Define total emissivity.
10. Define a black body.
11. Define absorptance.
12. Define transmittance.
13. Define reflectance.
14. State unit of Stefan's constant.
15. State Kirchoff's law of heat radiation.
16. At what temperature would a block of wood and a block of metal be felt equally cold or equally hot when touched.
17. State Prevost's theory of heat radiation.
18. Define steady state.
19. Define variable state.
20. What is the value of absorption coefficient of blackbody.

C. Short-Answer Type Questions :

1. Explain why iron is colder to touch than wood in winter ?
2. Explain why do we pack ice in gunny bags ?
3. Why do we have wooden handles for untensil ?
4. Why are woolen clothes warmer than others ?
5. Explain why a thatched-roof house is comfortable to live both in summer and winter ?
6. Explain sea-breeze and land-breeze.
7. Why are there small holes at the lower part of the lamp ?
8. Why do we feel warmer in night, when cloud cover the sky than when sky is clear?
9. Why does a thick glass breaks when hot water is poured on it ?
10. Why do we feel warmer by using two blankets than one blanket of thickness equal to their combined thickness ?
11. Explain why a man near a furnance feel warmer.
12. Explain why a man near a block of ice feel colder ?
13. Explain why the bottom of cooking vessel is made black ?
14. Explain why white clothes are preferred to black in summer ?
15. After sometime of switching on electric heater, the temperature of the heater coil becomes constant although current continuously flows in it. Explain.
16. Why do birds sit with their feathers swollen in winter ?
17. Why do eskimos live in double walled ice houses ?
18. Why does a steel chair appear colder than a wooden chair in winter ?
19. Why stainless steel cooking pans are preferred with extra copper bottom ?
20. Can we boil water inside an earth satellite by convection. Explain your answer.

21. Does a body at 20^0 C radiate in a room, where the room temperature is 30^0 C ? If yes, why does its temperature does not fall further.

22. Why does blowing over a spoonful of hot tea cool it ? Does evaporation play a role? Does radiation play a role ?

23. An ordinary electric fan does not cool the air, still it gives comfort in summer. Explain.

24. Why does one prefer to stand in shade than in the sun in summer ?

D. Unsolved Problems :

1. A metal rod of length 2m and diameter 2 cm, is covered with non-conducting substance. One end of the rod is maintained at 100^0 C and the other end enters a vessel containing ice at 0^0 C. If 25 g of ice melts in 10 minutes, calculate the thermal conductivity of the metal.

2. A layer of ice of thickness 10 cm is formed on the surface of a pond. If the temperature of air above it is at -10^0 C, how long will it take for next, 1mm of ice to form ? Given

$$K_{ice} = 0.008 \text{ cal}/(\text{cm.s.}^0\text{C}) \quad \text{and}$$

$$\rho_{ice} = 1\text{g}/\text{cm}^3$$

3. Calculate the heat lost by a room having floor area of 4m x 5m and 3m height, per hour through its four brick walls. The temperatures inside and outside are 15^0C and -10^0 C respectively. The walls are 0.5 m thick and the thermal conductivity of brick is $0.84 \text{ W m}^{-1}\,{}^0\text{K}^{-1}$.

4. A wooden box 2 cm thick having dimensions 1m x 0.5 m x 0.5m is packed with ice at 0^0 C. If the air temperature is 40^0 C and the thermal conductivity of wood is $0.168 \text{ W m}^{-1} \text{k}^{-1}$, calculate the mass of ice melting per hour.

5. A layer of ice of 1cm thickness is formed over the surface of the pond. The temperature above ice is 4^0 C. Calculate the rate of heat loss in the pond per sq. meter. Given

$$K_{ice} = 0.008 \text{ cal}/(\text{cm.s.}^0\text{C})$$

6. A body having surface area 5 cm^2 and a temperature of 727^0 C radiates 300 J. of energy each minute. What is its emissivity ? Given $\sigma = 5.67 \times 10^{-8}$ W $\text{m}^{-2}\,\text{K}^{-4}$.

7. The outer surface of a copper sphere of radius 5 cm is coated black. How much time is required for the sphere to cool down from 1000^0 K to 300^0 K. Density of copper = $9 \times 10^3 \text{ kg}/\text{m}^3$, sp.heat of copper = 40 KJ / kg.

8. The operating temperature of a tungsten filament in an incandescent lamp is 2450^0K; and its emissivity is 0.30. Find the surface area of the filament of a 25 watt lamp.

9. A certain incandescent light bulb operates at 3000^0 K. The total surface area of the filament is 0.05 cm^2 and the emissivity is 0.3. What electric power must be supplied to the filament ?

10. A small blackened copper sphere of radius 2cm is placed in an evacuated enclosure, whose walls are kept at 100^0C. At what rate must energy be supplied to the sphere to keep its temperature constant at 127^0 C.

E. Long Answer Type Questions :

1. Explain variable state and steady state as heat flows through a bar. Describe searle's method for determination of K.

2. State and prove Kirchoff's law of heat radiation. Give some of its uses.

3. State Stefan's law. Hence derive Newton's law of cooling from it.

F. Answer as directed

1. A hot body loses heat in vacuum by radiation. (Yes/No.)
2. A black body is a good absorber of heat, but it is a bad radiator. (Yes/No.)
3. The thermal conductivity of air being less than that of felt, we prefer felt to air for thermal insulation. (Yes/No)
4. The unit of the coefficient of thermal conductivity in SI system is .... (Fill in the blank)
5. The fall in temperature per unit length of a material in the steady state of conduction is called .... (Fill in the blank)
6. Land and sea breezes are due to the process of .... (Fill in the blank)
7. For proper ventilation of a building, windows must be open near the bottom and the top of the walls so as to let pass .... (Fill in the blank)
8. What type of spectrum you will obtain from a black body radiation ?
9. Temperature of the upper part of the flame is equal to the temperature on the sides (Yes/No)
10. Coefficient of thermal conductivity is .... for good conductors and .... for bad conductors. (Fill in the blanks)
11. When the temperature of a luminous body is increased then the frequency corresponding to maximum energy of radiation .... (Fill in the blank)

ANSWERS

A. Multiple Choice Type Questions :

1. (d), 2. (d), 3. (d), 4. (a), 5. (a), 6. (d), 7. (c), 8. (d), 9. (c), 10. (b).

B. Very Short Answer Type Questions :

1. See text
2. Silver
3. $c \approx 3\times10^8 m/s$
4. See text
5. See text
6. See text
7. See text
8. See text.
9. See text
10. See text
11. See text
12. See text
13. See text
14. See text
15. See text
16. At temp. of human body
17. See text
18. See text
19. See text
20. See text.

C. Short Answer Type Questions :

1. See text
2. See text
3. See text
4. Woolen clothes have a layer of dry air in its texture. Air being bad conductor of heat, it does not allow heat of body to escape. So we feel warmer.
5. Straw being bad conductor of heat, it does not allow heat to go out from room to outside during winter and from outside to the room during summer. Hence it is comfortable.
6. See text
7. When lamp burns the air inside becomes hot and rises up. Cooler air from outside enter the lamp through the holes below the help burning.
8. Earth absorbs heat radiation during day time and radiates during night. When clouds cover the sky, the heat radiated by earth is reflected-back while the heat escapes when the sky is clear. So we feel warter in a cloudy night.
9. When hot water falls on one side of a thick glass, that side expands. But the opposite side is still cold (as glass is a bad conductor of heat), and thus does not expand. Hence this causes a crack. But in case of a thin glass heat reaches the other side hence both sides expand almost equally, hence no crack.
10. See text
11. See text

12. See text
13. This increases the absorption of heat, hence cooking becomes quicker.
14. White clothes absorb very little heat radiation. But black clothes absorb maximum heat radiation.
15. After the coil attains a constant temperature it emits all radiations generated by elecetrical power consumption.
16. See text
17. See text
18. See text
19. Because copper is a better conductor than steel.

D. Unsolved Problems :

1. 2 .122 cal / (cm. S. ^{0}C)
2. 1005 sec
3. 8.165 x 10^6 J
4. 9.0 kg
5. 320 cal / s
6. 0.176
7. 127.11 x 10^3 s
8. 0.408 cm^2
9. 6. 89 W
10. 1.78 W

F. (1) Yes (2) No (3) No (4) Watt/m/k (5) Temperature gradient (6) convection (7) in cool air near the bottum and hot air out near the roof. (8) continuous spectrum (9) No. (10) large ; small (11) Increases

19

Molecular Properties of Matter

19.1 Molecular Properties of Matter

Anything which occupies a space is called Matter. Matter exits in solid, liquid and gaseous form. While explaining various observed (macroscopic) properties of matter it is assumed that matter is composed of molecules. From observations of the average effect of an assembly of molecules, we build a model or theory of the arrangement and behaviour of the molecules. The results predicted by the model or theory is then tested by experiment. The degree of agreement of predictions and experimental observations tells about the degree of merit of the proposed model or theory and indicates about the possible modifications. Thus we observe macroscopic events and attempt to draw conclusions about microscopic events by proposing plausible models.

19.2 Kinetic Theory of Gases

A gas does not have any fixed shape or fixed volume. The attraction between any two molecules being very weak, the molecules move freely and randomly. Hence one builds up a kinetic molecular model for gas, which is popularly known as **"Kinetic theory of gas"**. It aims at relating the macroscopic properties like pressure (P), Volume (V), temperature (T) with microscopic properties like linear momentum, kinetic energy etc. of molecules.

19.2(a) Postulates of Kinetic Theory of Gas

The Kinetic theory of gas is based on the following basic postulates.

1. **Constituents** : (a) An ideal gas consists of large number of identical molecules.

(b) The molecules are supposed to be rigid and spherical. The size of a molecule is negligible in comparison with the average distance between molecules. The volume of the molecule is also negligible in comparison with the volume of the container.

2. **Motion :** (a) Molecules are constantly in random motion. (b) They move in straight lines with all possible speed. (c) All directions are equally probable. (d) They move in such a manner that C.O.M. of the gas is at rest w.r.to the container, i.e. the gas as a whole is in equilibrium. (e) While in motion they collide with each other and also on the wall of the container. The collision is elastic and time of collision is small compared to the time taken by molecule to travel a free path (i.e. the distance between any two consecutive collisions).

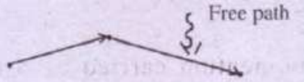


Fig.19.1

3. **Force on Molecules** : (a) No appreciable force exists between any two molecules as intermolecular force is weak and short ranged;

and gravitational force is also small due to small mass of the molecules. (b) As no force acts on molecules, the molecules move with a constant linear momentum before colliding with a wall. (c) Also because of absence of force, the energy possessed by a molecule is totally kinetic.

19.2 (b) Pressure of an ideal gas

Consider a cubical container with each side of length L, so that its volume is

$$V = L^3 \qquad ...(19.2.1)$$

Suppose there are n-moles of gas in the container. Then the total number of gas molecules within the container is

$$N = n.\ N_A \qquad ...(19.2.2)$$

where $N_A = 6.02 \times 10^{23}$ mole^{-1}, is the Avogadro's number.

Let the molecules be sorted into groups according to the velocities possessed by them.

Suppose N_1 molecules possess velocity $\vec{\vartheta}_1$

N_2 molecules possess velocity $\vec{\vartheta}_2$

N_i molecules possess velocity $\vec{\vartheta}_i$

such that

$$\vec{\vartheta}_i = \hat{i}\,\vartheta_{ix} + \hat{j}\,\vartheta_{iy} + \hat{k}\,\vartheta_{iz} \qquad ...(19.2.3)$$

so that

$$\vartheta_i = \sqrt{\vartheta_{ix}^2 + \vartheta_{iy}^2 + \vartheta_{iz}^2} \qquad ...(19.2.4)$$

and

$$\sum_i N_i = N = nN_A \qquad ...(19.2.5)$$

Then momentum carried by a molecule travelling with velocity $\vec{\vartheta}_i^I$ before collision is given by

$$\vec{P}_i^I = m\vec{\vartheta}_i^I = m(\hat{i}\,\vartheta_{ix} + \hat{j}\,\vartheta_{iy} + \hat{k}\,\vartheta_{iz}) \qquad ...(19.2.6)$$

where 'm' is the mass of each molecule.

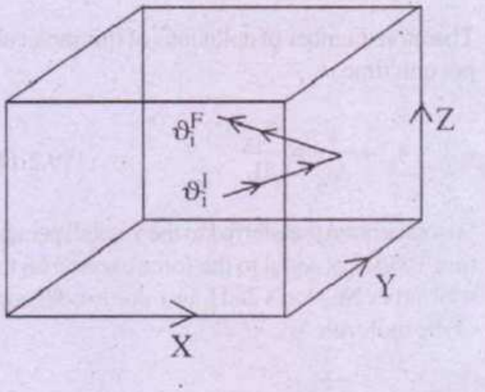


Fig. 19.2

Let this molecule collide with the yz - wall. Then the x-component of momentum will change its direction after collision.

i.e. $$\vec{P}_i^F = m\vec{\vartheta}_i^F = m(-\hat{i}\,\vartheta_{ix} + \hat{j}\,\vartheta_{iy} + \hat{k}\,\vartheta_{iz}) \qquad ...(19.2.7)$$

Therefore change in momentum of the molecule (under consideration) after collision with yz-wall is

$$\overrightarrow{\delta P_i} = \vec{P}_i^F - \vec{P}_i^I = -2m\vartheta_{ix}\hat{i} \qquad ...(19.2.8)$$

Since no external force is acting on the system (of gass and wall) the linear momentum of the system must be conserved. Therefore change in momentum of the wall or momentum transferred to the wall during collision of the molecule is

$$\overrightarrow{\Delta P_i} = -\overrightarrow{\delta P_i} = 2m\vartheta_{ix}\hat{i} \qquad ...(19.2.9)$$

$$\Rightarrow \quad \Delta p_{ix} = 2\,m\,\vartheta_{ix} \qquad ...19.2.9(a)$$

$$\Delta p_{iy} = 0 = \Delta\,p_{iz} \qquad ...19.2.9(b)$$

Now time between two consecutive collisions of the molecule (under consideration) on the same wall is

$$\Delta t_i = \frac{2L}{\vartheta_{ix}} \qquad \text{...(19.2.10)}$$

Therefore number of collisions of this molecule per unit time is

$$n_c = \frac{1}{\Delta t_i} = \frac{\vartheta_{ix}}{2L} \qquad \text{...(19.2.11)}$$

So momentum transferred to the yz-wall per unit time (which is equal to the force exerted on the wall as per Newton's 2nd Law) due to collisions of the molecule is

$$f_i = \frac{\Delta \vec{p}_i}{\Delta t_i} = \frac{m\vartheta_{ix}^2}{L}\hat{i} \qquad \text{...(19.2.12)}$$

$$\Rightarrow \quad f_{ix} = \frac{m\vartheta_{ix}^2}{L} \qquad \text{...(19.2.13)}$$

So considering the group of molecules possessing velocity ϑ_i, the force exerted on the YZ-wall is

$$F_{ix} = N_i f_{ix} = \frac{m\,N_i\,\vartheta_{ix}^2}{L} \qquad \text{...(19.2.14)}$$

Similarly considering other groups of molecules, the net force on the YZ-wall due to collision of all the gas molecules is

$$F_x = \sum_i F_{ix} = \frac{m}{L}\sum_i N_i\,\vartheta_{ix}^2 = \frac{mN}{L}\left(\frac{\sum_i N_i\vartheta_{ix}^2}{N}\right)$$

$$\Rightarrow \quad F_x = \frac{mN}{L}\langle\vartheta_x^2\rangle \qquad \text{...(19.2.15)}$$

where

$$\langle\vartheta_x^2\rangle = \frac{\sum_i N_i\vartheta_{ix}^2}{N} = \frac{\sum_i N_i\vartheta_{ix}^2}{\sum_i N_i} \qquad \text{....(19.2.16)}$$

Therefore pressure on the YZ-wall due to collisions of gas molecules is

$$P_x = \frac{F_x}{L^2} = \frac{mN}{L^3}\langle\vartheta_x^2\rangle \qquad \text{...(19.2.17)}$$

Similarly pressure (P_y) on XZ-wall and pressure (P_z) on XY-wall are given as

$$P_y = \frac{mN}{L^3}\langle\vartheta_y^2\rangle \qquad \text{...(19.2.18)}$$

$$P_z = \frac{mN}{L^3}\langle\vartheta_z^2\rangle \qquad \text{...(19.2.19)}$$

As all directions are equally probable, so

$$P_x = P_y = P_z = P$$

$$\Rightarrow \quad P = \frac{1}{3}(P_x + P_y + P_z)$$

$$\Rightarrow \quad P = \frac{1}{3}\,\frac{mN}{L^3}(\langle\vartheta_x^2\rangle + \langle\vartheta_y^2\rangle + \langle\vartheta_z^2\rangle) \qquad \text{...(19.2.20)}$$

Now

$$\langle\vartheta_x^2\rangle + \langle\vartheta_y^2\rangle + \langle\vartheta_z^2\rangle = \frac{\sum_i N_i\vartheta_{ix}^2}{\sum_i N_i} + \frac{\sum_i N_i\vartheta_{iy}^2}{\sum N_i}$$

$$+\frac{\sum_i N_i\vartheta_{iz}^2}{\sum N_i}$$

$$= \frac{\sum\limits_i N_i\left(\vartheta_{ix}^2 + \vartheta_{iy}^2 + \vartheta_{iz}^2\right)}{\sum N_i}$$

$$\Rightarrow \quad \langle\vartheta_x^2\rangle + \langle\vartheta_y^2\rangle + \langle\vartheta_z^2\rangle = \frac{\sum\limits_i N_i\vartheta_i^2}{\sum\limits_i N_i} = \langle\vartheta^2\rangle \qquad ...(19.2.21)$$

Using eqn. (19.2.21) in (19.2.20) we obtain

$$P = \frac{1}{3}\frac{mN}{L^3}\langle\vartheta^2\rangle = \frac{1}{3}\frac{mN}{V}\langle\vartheta^2\rangle \;...(19.2.22)$$

Thus the pressure on the wall of the container is given by

$$P = \frac{1}{3}\frac{mN}{V}\langle\vartheta^2\rangle \qquad ...(a)$$

$$= \frac{1}{3}\frac{M}{V}\langle\vartheta^2\rangle \qquad ...(b)$$

$$= \frac{1}{3}\rho\langle\vartheta^2\rangle \qquad ...(c)$$

$$= \frac{1}{3}\rho \;\; \vartheta_{rms}^2 \qquad ...(d)$$

$$= \frac{2}{3}\frac{N}{V}\left(\tfrac{1}{2}m\langle\vartheta^2\rangle\right) = \frac{2}{3}\;\frac{N}{V}\;\langle E_k\rangle \;\; ...(e) \qquad ...(19.2.23)$$

where M is the total mass of gas in the container and ρ is the density of gas.

19.2 (b1) Avogadro's Number :

Avogadro's number (Also known as Loschmidt constant) is a historical term, which gives the number of atoms in one gram-molecule of atomic hydrogen (ie, one gram of hydrogen). At present it is redefined as the number of atoms in 12 grams of the isotope carbon C-12. However these two have almost the same value. They can be taken as same numerically, to the first approximation.

In general, Avogadro's number is the number of atoms or molecules in one mole of any substance. As such, Avogadro's number is just a number, i.e, it is dimensionless, unitless. But since the mole is now officially recognised as a unit, Avogadro's number has become Avogadro's constant, with a unit of mole^{-1}.

Avogadro's constant is a constant, having the value 6.0221 x 10^{23} and symbol N_A. Avagadro's constant plays a very vital role in physical sciences. This can be evident if we look at the following familiar equations.

Gas constant = $R = K_B N_A = 8.315$ Jmol^{-1}K^{-1}

Faraday constant $= N_A e = 96,485.332$ C^1 mol^{-1}

Where e = elementary charge

$$1u = \text{Unit mass} = \frac{M_u}{N_A} = 1.661 \times 10^{-27}\text{ kg}$$

Where M_u = Molar mass constant.

Thus it is found that different physical can starts and properties are related by using Avogadro's constant. Here N_A takes the role of a scaling factor between the macroscopic and microscopic (of the order of the atomic scale) observations.

19.2 (c) Root - Mean- Square (RMS) speed

It is defined as the square root of the mean of the squares of the velocities.

$$\text{i.e.} \quad \vartheta_{rms} = \sqrt{\langle\vartheta^2\rangle} = \sqrt{\frac{\sum_i N_i\vartheta_i^2}{\sum\limits_i N_i}}$$

$$\Rightarrow \quad \vartheta_{rms}^2 = \langle\vartheta^2\rangle$$

From eqn. 19.2.23 (c) we obtain

$$\vartheta_{rms} = \sqrt{\frac{3P}{\rho}} \qquad ...(19.2.24)$$

Since for an ideal gas PV = nRT, so

$$\vartheta_{rms} = \sqrt{\frac{3PV}{\rho V}} = \sqrt{\frac{3nRT}{M}} = \sqrt{\frac{3nRT}{m.n.N_A}}$$

$$\Rightarrow \quad \vartheta_{rms} = \sqrt{\frac{3RT}{mN_A}} \quad ...(19.2.25)$$

$$\Rightarrow \quad \vartheta_{rms} = \sqrt{\frac{3RT}{M_0}} = \sqrt{\frac{3(R/N_A)T}{m}} = \sqrt{\frac{3KT}{m}} \quad ...(19.2.26)$$

where M_0 is the gram-molecular mass, N_A is Avogadro's number and $K = \frac{R}{N_A}$ is Boltzmann's constant.

Equation (19.2.26) shows that

$$\vartheta_{rms} \alpha \sqrt{T}$$

$$\alpha \frac{1}{M_0}$$

19.2(d) Kinetic interpretation of temperature

The total translational K.E. of all the molecules in a container is given as

$$E_k = \frac{1}{2} m \sum_i N_i \vartheta_i^2 = \frac{1}{2} mN \left(\frac{\sum_i N_i \vartheta_i^2}{N} \right)$$

$$\Rightarrow \quad E_k = \frac{1}{2} m\, N\, \vartheta_{rms}^2 \quad ...(19.2.27)$$

Using eqn. (19.2.26) in (19.2.27)

$$\langle E_k \rangle = \frac{E_k}{N} = \frac{1}{2} m.\ \frac{3KT}{m} = \frac{3}{2} kT^* \quad ...(19.2.28)$$

$\frac{3}{2}KT$, because we have considered the molecules to have translational motion only.

Equation (19.2.28) shows :

(i) $\langle E_k \rangle \ \alpha\ T$ i.e. average kinetic energy per molecule of a given sample of gas is directly proportional to the absolute temperature of the sample. Thus $T = f(\langle E_k \rangle)$ for a given sample of gas. So if a mixture of gases A and B are kept in a container, then the mean K.E. of gas A and B are same.

i.e $\langle E_k \rangle = \frac{1}{2} m_A\, \vartheta_{A(rms)}^2 = \frac{1}{2} m_B \vartheta_{B(rms)}^2 = \frac{3}{2} kT$

Also for a given sample of gas

$$\langle E_k \rangle = \frac{1}{2} m\, \vartheta_{rms}^2 = \frac{3}{2} kT$$

$$\Rightarrow \quad \vartheta_{rms}^2 \ \alpha\ T$$

$$\text{i.e } T = F(\vartheta_{rms})$$

(ii) If $T = 0$, then eqn. 19.2.28 shows that $\langle E_k \rangle = 0$ and $\vartheta_{rms} = 0$. This means that at absolute zero the average K.E. = 0 and there is no molecular motion. Hence absolute zero is not attainable.

Thus absolute zero is that temperature at which the molecular motion ceases.

19.2(e) Deduction of gas law

(i) **Boyle's law :**

From eqn. 19.2.23 (a) we have

$$PV = \frac{1}{3} mN\, \vartheta_{rms}^2$$

So at constant temperature since ϑ_{rms}^2 is constant so r.h.s. of the above equation is constant and we have

* However it is worthwhile to note that the average Kinetic energy given by (19.2.28) is

$$PV = \text{constant}$$

i.e $P \alpha \frac{1}{V}$

or $V \alpha \frac{1}{P}$

i.e **at constant temperature volume of a certain mass of gas varies inversely as the pressure.**

(ii) **Charle's Law :**

Also from eqn. 19.2.23 (a) we have

$$V = \frac{1}{3}\frac{mN}{P}\vartheta_{rms}^2$$

So at constant pressure, for a given sample of gas

$$V \alpha \vartheta_{rms}^2$$

Since $\vartheta_{rms}^2 \alpha T$, so

$V \alpha T$ at constant P.

i.e **at constant pressure the volume of a given mass of gas varies directly as the absolute temperature.**

(iii) **Regnault's Law :**

Rewriting eqn. 19.2.23

$$P = \frac{1}{3}\frac{mN}{V}\vartheta_{rms}^2$$

So at constant volume, for a given sample of gas $P \alpha \vartheta_{rms}^2$

Since $\vartheta_{rms}^2 \alpha T$, so

$P \alpha T$ at constant volume V.

i.e **at constant volume, the pressure of a given mass of gas varies directly as the absolute temperature of the gas.**

(iv) **Avogadro's Law :**

It states that equal volumes of all gases under the same condition of temperature and pressure contain same number of molecules.

Consider two samples of ideal gas No.1 and No. 2, each of volume V. Let the number of molecules of No. 1 and No. 2 type be N_1 and N_2 respectively. Then pressure of the gases are given by eqn 19.2.23 as

$$P_1 = \frac{1}{3}\frac{m_1N_1}{V}\langle\vartheta_1^2\rangle$$

$$P_2 = \frac{1}{3}\frac{m_2N_2}{V}\langle\vartheta_2^2\rangle$$

Since $P_1 = P_2$, so

$$m_1 N_1 \langle\vartheta_1^2\rangle = m_2N_2 \langle\vartheta_2^2\rangle$$

$$\Rightarrow \quad N_1(\tfrac{1}{2} m_1 \langle\vartheta_1^2\rangle) = N_2 (\tfrac{1}{2}m_2\langle\vartheta_2^2\rangle) \qquad ...(19.2.29)$$

But as discussed in sec 19.2.(d) mean K.E. per molecule of gases at same temperature is same. So $\frac{1}{2} m_1 \langle\vartheta_1^2\rangle = \frac{1}{2}m_2\langle\vartheta_2^2\rangle$ as the two gases are kept at same temperature. Hence from eqn (19.2.29) we have $N_1 = N_2$. This proves Avogadro's Law.

(v) **Graham's Law of diffusion :**

This law states that the rate of diffusion of a gas is inversely proportional to the square root of its density.

i.e $\frac{R_1}{R_2} = \sqrt{\frac{\rho_2}{\rho_1}}$

Where R_1, R_2 denote rates of diffusion of No.1 and No. 2 gas respectively

It is quite evident that

$$R_1 \propto \vartheta_{1(\text{rms})}$$

$$R_2 \propto \vartheta_{2(\text{rms})}$$

So. $$\frac{R_1}{R_2} = \frac{\vartheta_{1(\text{rms})}}{\vartheta_{2(\text{rms})}} \qquad ...(19.2.30)$$

But since diffusion takes place at same pressure so by eqution19.2.23(d)

$$P = \frac{1}{3}\rho_1\vartheta^2_{1(\text{rms})} = \frac{1}{3}\rho_2\vartheta^2_{2(\text{rms})}$$

$$\Rightarrow \qquad \frac{\vartheta_{1(\text{rms})}}{\vartheta_{2(\text{rms})}} = \sqrt{\frac{\rho_2}{\rho_1}}$$

Therefore

$$\frac{R_1}{R_2} = \sqrt{\frac{\rho_2}{\rho_1}} \qquad ...(19.2.31)$$

This proves Graham's Law of diffusion.

(vi) **Dalton's Law of Partial pressure :**

It states that when two or more gases (which do not interact chemically) are present in the same container, the partial pressure of the mixture is equal to the sum of the partial pressures of individual gases.

Consider a container of volume V, containing the gases. Let N_1, N_2, N_3 be numbers of No. 1, No. 2, and No. 3 type respectively, and m_1, m_2, m_3 be masses of each molecular types respectively. Let $\langle\vartheta_1^2\rangle, \langle\vartheta_2^2\rangle, \langle\vartheta_3^2\rangle$ be their mean square velocities respectively. Then the partial pressures of the gases, defined as the pressure due to a gas had it occupied the whole volume, are given as

$$P_1 = \frac{1}{3}\frac{m_1N_1}{V}\langle\vartheta_1^2\rangle$$

$$P_2 = \frac{1}{3}\frac{m_2N_2}{V}\langle\vartheta_2^2\rangle$$

$$P_3 = \frac{1}{3}\frac{m_3N_3}{V}\langle\vartheta_3^2\rangle \qquad ...(19.2.32)$$

Since the gas mixture is at a given temperature so their average K.E. per molecule is same.

i.e. $$\frac{1}{2}m_1\langle\vartheta_1^2\rangle = \frac{1}{2}m_2\langle\vartheta_2^2\rangle$$

$$= \frac{1}{2}m_3\langle\vartheta_3^2\rangle = \frac{3}{2}KT \qquad ...(19.2.33)$$

Using (19.2.33) in (19.2.32)

$$P_1 = \frac{N_1KT}{V}$$

$$P_2 = \frac{N_2KT}{V}$$

$$P_3 = \frac{N_3KT}{V}$$

Therefore

$$P_1 + P_2 + P_3 = (N_1 + N_2 + N_3)\ \frac{KT}{V}$$

$$= \frac{NKT}{V} \qquad ...(19.2.34)$$

Now the mixture contains $N = N_1 + N_2 + N_3$ molecules at temperature T. Therefore by eqn.19.2.23 (e)

$$P = \frac{2}{3}\ \frac{N}{V}\ \langle E_k\rangle$$

$$= \frac{2}{3}\ \frac{N}{V}\ (\frac{3}{2}KT)$$

$$\Rightarrow P = \frac{NKT}{V} \qquad ...(19.2.35)$$

Comparison of eqns. (19.2.34) and (19.2.35) shows that

$$P = P_1 + P_2 + P_3$$

This proves Dalton's Law of partial pressure.

19.2(f) Degrees of Freedom :

The degree of freedom of a system or a body is equal to the number of independent parameters required to specify its position and configuration at any instant of time.

Examples :

(i) A particle constrained to move along a straight line requires one coordinate to specify its position. Hence it is said to have **one** degree of freedom.

(ii) A particle moving on a plane requires two coordinates to specify its position. Hence it is said to have **two** degrees of freedom.

(iii) A bird flying in sky requires three coordinates to specify its position hence it is said to have **three** degrees of freedom.

However the constraints reduce the number of degrees of freedom.

i.e. $n = n_0 - n_c$

where n_0 is the total degrees of freedom, n_c is the number of constraints and n is the actual degrees of freedom. For example a bird free to fly has three degrees of freedom. But when we force it to move on a plane (either by tying its leg or otherwise. This is called an example of constraint) it has two degrees of freedom.

$$n = n_0 - n_c = 3 - 1 = 2$$

Degrees of Freedom of Gas Molecule

(a) **Monoatomic gas** : It can have only translational motion. So it has **three** degrees of freedom.

(b) **Diatomic Gas** : Such a molecule can have translational as well as rotational motion.

So **three** coordinates are needed to describe translation of C.O.M. and **three** coordinates are required to describe rotation about three mutually perpendicular axes. However, experiments suggest that no rotation occurs about the axis joining the two atom. So at ordinary temperature (250^0K – 750^0K) a diatomic molecule can have **three** translational and **two** rotational degrees of freedom totalling to five degrees of freedom. At high temperatures ($T > 750^0K$) the atoms may vibrate. Hence at high temperatures, it has $3+2+1 = 6$ degrees of freedom. At very low temperatures ($0 - 25^0$k) it has only **three** translational degrees of freedom.

19.2(g) Law of Equipartition of Energy :

It states that in any dynamical system in thermal equilibrium at absolute temperature T, the energy is equally divided among all the degrees of freedom and is equal to $\frac{1}{2}$ KT. (K = Boltzman's constant = 1.38×10^{-23} J. mole-1 oK^{-1})

The above law implies that if a gas molecule has 'f' degrees of freedom, then

(i) K.E. of each molecule $= \frac{f}{2}kT$ ...(19.2.36)

(ii) K.E. of 1 gm. mole $= \frac{f}{2}N_AkT = \frac{f}{2}RT$ ...(19.2.37)

(iii) K.E. of 1 gm. of gas $= \frac{f}{2}RT/M_0$ ...(19.2.38)

19.2(h) Maxwell's Speed Distribution law :

We have seen that ϑ_{rms} of a given sample of gas maintained at a definite temperature varies as $\vartheta_{rms} \alpha \sqrt{T}$. But it does not mean that all molecules move with this speed. Some molecules move with speed less than ϑ_{rms} and some with speed more than ϑ_{rms}. If dN be number of molecules with speeds between ϑ and $\vartheta + d\vartheta$, then

$$dN = 4\pi N\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} \vartheta^2 \; e^{-m\vartheta^2/2kT} \, d\vartheta \equiv f(\vartheta)\, d\vartheta \qquad ...(19.2.39)$$

So that

$$f(\upsilon) = 4\pi N\left(\frac{m}{2\pi KT}\right)^{3/2} V^2 e^{-mV^2/2KT}$$

where $f(\vartheta)$ is the distribution function and is shown graphically in fig. 19.3.

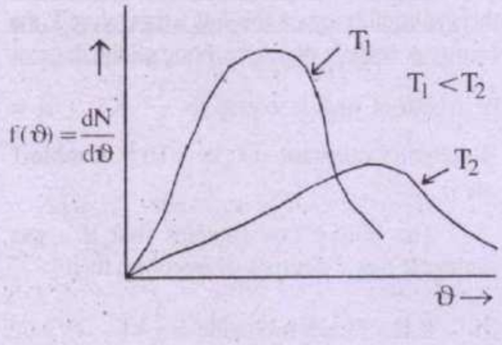


Fig.19.3

(i) **Mean Speed** : The mean speed of the molecules of a gas at temperature T is given by

$$\langle\vartheta\rangle = \frac{\sum\limits_i N_i \vartheta_i}{\sum\limits_i N_i} = \frac{1}{N}\int_0^\infty \vartheta \, dN$$

Using (19.2.39) on r.h.s and integrating one obtains

$$\langle\vartheta\rangle = \sqrt{\frac{8KT}{m\pi}} \qquad ...(19.2.40)$$

Thus the relation between rms speed and mean speed using equns 19.2.40 & 19.2.26 is

$$\frac{\langle\upsilon\rangle}{\upsilon_{rms}} = \left(\sqrt{\frac{8KT}{m\pi}} / \sqrt{\frac{3KT}{m}}\right) = \sqrt{\frac{8}{3\pi}} = .921 \qquad .....(19.2.41)$$

(ii) **Most Probable speed** : The speed with which maximum number of molecules move is called most probable speed. This is obtained by putting

$$\frac{df(\vartheta)}{d\vartheta} = 0$$

This yields from eq 19.2.39 :

$$\vartheta_m = \sqrt{\frac{2KT}{m}} \qquad ...(19.2.41)$$

(iii) **r.m.s speed**

$$\vartheta_{rms} = \sqrt{\frac{\sum\limits_i N_i \vartheta_i^2}{\sum\limits_i N_i}} = \sqrt{\frac{1}{N}\int_0^\infty \vartheta^2 dN}$$

$$= \sqrt{3KT/m}$$

19.3. Specific heat of gas :

Specific heat of a substance is defined as the amount of heat required to raise the temperature of unit mass of the substance through 1^0 C (or 1^0k).

The above definition holds good for solids and liquids. Because the whole of the supplied heat is used to raise the temperature as there is almost no change in volume or pressure.

But in case of gases, when heat is supplied the volume and pressure may change. Therefore we need to define two specific heats for gases. Accordingly we have (i) specific heat at constant volume and (ii) specific heat at constant pressure.

As defined in (17.7.9) and (17.7.10)

$$C_v = \frac{1}{n}\left(\frac{dQ}{dT}\right)_v = Mc_v$$

$$C_p = \frac{1}{n}\left(\frac{dQ}{dT}\right)_p = Mc_p$$

where c_v and c_p correspond to unit mass.

Relation between C_p and C_v

Consider n. gm. moles of gas. Let dQ heat raise its temperature from T to T+dT at constant volume. Then by 1st law of thermodynamics

$$dQ = dU + pdV = dU \qquad (\because dV = 0)$$

$$= E_{T+dT} - E_T \qquad ...(19.3.1)$$

where E_T is the energy of the gas at temperature T. By law of equipartition of energy by 19.2.37.

$$E_T = n\,N_A\,f\left(\frac{1}{2}kT\right) \qquad ...(19.3.2)$$

where N_A = Avogadro's number
f = number of degrees of freedom
K = Boltzman's constant.

Using (19.3.2) in (19.3.1)

$$dQ = nN_A f\,\frac{1}{2}K\,(T+dT) - nN_A f\left(\frac{1}{2}KT\right)$$

$$= \frac{1}{2}nN_A f\,K\,dT$$

$$\Rightarrow \quad \left(\frac{dQ}{dT}\right)_V = \frac{1}{2}nN_A f\,K = \frac{1}{2}n\,f\,R$$

$$(\because R = N_A K)$$

So for one 1 gm. mole

$$C_v = \frac{1}{n}\left(\frac{dQ}{dT}\right)_V = \frac{1}{2}f\,R \qquad ...(19.3.3)$$

Now considering supply of heat dQ' at constant pressure, so that temperature rises from T to T+dT

$$dQ' = dU + pdV = (E_{T+dT} - E_T) + pdV$$

$$\Rightarrow \quad dQ' = \frac{1}{2}\,n\,N_A f\,K\,dT + pdV$$

(by using eq 19.3.2)

$$dQ' = \frac{1}{2}\,n\,f\,R\,dT + pdV \qquad ...(19.3.4)$$

For a perfect gas PV = nRT

$$\therefore\ pdV = nR\,dT \quad \text{(at constant pressure)} \qquad ...(19.3.5)$$

Using (19.3.5) in (19.3.4)

$$dQ' = \frac{1}{2}\,n\,f\,R\,dT + n\,R\,dT$$

giving

$$C_p = \frac{1}{n}\left(\frac{dQ'}{dT}\right)_p = \frac{1}{2}\,fR + R = C_v + R$$

$$\Rightarrow \quad C_p - C_V = R \qquad ...(19.3.6)$$

where C_p, C_V and R are all expressed in same unit.

If C_p and C_V are expressed in heat units and R is expressed in energy units then

$$C_p - C_V = \frac{R}{J} \qquad ...(19.3.7)$$

(ii) **Specific heat of a mixture of gasses**

Consider a mixture of gases No. 1 and No. 2. Let gas No. 1 certain n_1 gm. moles and No. 2 gas contain n_2 gm. moles. Let each molecule of No 1 gas have 'f_1' degrees of freedom, and that of No. 2 gas 'f_2'. Let the specific heats of the two gases be (C_{v1}, C_{p1}) and (C_{v2}, C_{p2}) respectively.

Let dQ amount of heat be supplied to the mixture at constant volume, so that its temperature rises through dT^0 k. Then

$$dQ = dQ_1 + dQ_2 = dU_1 + \cancelto{0}{(PdV_1)} + dU_2 + \cancelto{0}{(PdV_2)}$$

$$= (E^{(1)}{}_{T+dT} - E^{(1)}{}_T) + (E^{(2)}{}_{T+dT} - E^{(2)}{}_T) \qquad ...(19.3.8)$$

$$\text{Now}\quad E_T^{(1)} = n_1 N_A f_1\left(\frac{1}{2}KT\right) = \frac{n_1 f_1}{2}RT$$

$$E_T^{(2)} = n_2 N_A f_2\left(\frac{1}{2}KT\right) = \frac{n_2 f_2}{2}RT$$

$$\text{So}\quad dQ = \frac{1}{2}n_1 f_1 R dt + \frac{1}{2}n_2 f_2 R dT$$

$$\Rightarrow \quad \left(\frac{dQ}{dT}\right)_V = \left(\frac{n_1 f_1}{2} + \frac{n_2 f_2}{2}\right) R$$

$$\Rightarrow \quad \left(\frac{dQ}{dT}\right)_V = n_1 C_{V1} + n_2 C_{V2} \quad ...(19.3.9)$$

This gives

$$\frac{1}{n_1 + n_2}\left(\frac{dQ}{dT}\right)_V = \frac{n_1 C_{V1} + n_2 C_{V2}}{n_1 + n_2}$$

But by definition l.h.s = C_V of the gas mixture.

Hence

$$C_V = \frac{n_1 C_{V1} + n_2 C_{V2}}{n_1 + n_2} \quad ...(19.3.10)$$

Similarly considering heating of gas at constant pressure, one can show

$$C_P = \frac{n_1 C_{P1} + n_2 C_{P2}}{n_1 + n_2} \quad ...(19.3.11)$$

Hence adiabatic gas constant for the mixture is

$$\frac{C_P}{C_v} = \frac{n_1 C_{P1} + n_2 C_{P2}}{n_1 C_{V1} + n_2 C_{V2}} \quad ...(19.3.12)$$

Now

$$\frac{C_P}{C_v} = \gamma = \frac{C_V + R}{C_V} = 1 + \frac{R}{C_V}$$

$$\Rightarrow \quad C_V = \frac{R}{\gamma - 1}$$

$$C_P = C_V + R = \frac{R}{\gamma - 1} + R = R\frac{\gamma}{\gamma - 1}$$

Using these results we have from (19.3.12)

$$\gamma = \frac{n_1 R \frac{\gamma_1}{\gamma_1 - 1} + n_2 R \frac{\gamma_2}{\gamma_2 - 1}}{n_1 \frac{R}{\gamma_1 - 1} + n_2 \frac{R}{\gamma_2 - 1}}$$

$$\Rightarrow \quad \gamma = \frac{\frac{n_1 \gamma_1}{\gamma_1 - 1} + \frac{n_2 \gamma_2}{\gamma_2 - 1}}{\frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1}} \quad ...(19.3.13)$$

This gives

$$\frac{n_1 + n_2}{\gamma - 1} = \frac{n_1}{\gamma_1 - 1} + \frac{n_2}{\gamma_2 - 1} \quad ...(19.3.14)$$

19.4 mean free path :

Clausius could detect that there are two flaws in the kinetic theory of gases. He solved the deficiency by proposing that :

(i) Molecules have finite dimension, though very small

(ii) There are collisions between the molecules.

It is obvious that molecules with finite size will collide with other moleules while moving forward straight, after a short interval, so that the magnitude and direction of the velocity of individual molecules will change. This results in a molecule's motion as a series of short zig-zag lines which, of course, are confined within the boundaries of the vessel, containing the gas. This concept modifies the postulates of an ideal gas.

Assuming that the gas-molecules do not exert fore an each other, the path travelled by a molecule will be a straight line with a uniform velocity between two consecutive collisions. Hence this is called as free path. It is to be noted that the free paths will not be equal in length throughout. Hence its mean value will be of practical importance. This leads to the idea of a mean free path, which can be mathematically written as :

$$\text{Mean free path} = \lambda = \frac{S}{N} \quad .... (19.4.1)$$

Where S = Total path travelled in N collisions by a molecule.

Maxwell, basing on the law of distribution of velocities, has obtained a correct expression for mean free path. Basing on the mathematical expression : it is found that the mean free path varies (i) inversely to the square of the molecular diameter, i.e, λ will be large, if the size of the molecular is small (ii) λ is inversely proportional to the density of the gas. In other words it is inversely proportional to the pressure of the gas and directly proportional to the absolute temperature of the gas.

Ex. 19.2.1 If the r.m.s. speed of nitrogen molecules is 490 m/s at 273^0 K, find the rms speed of hydrogen molecules at the same temperature.

Soln.

At same temperature

$$\frac{1}{2}m_1\langle\vartheta_1^2\rangle=\frac{1}{2}m_2\langle\vartheta_2^2\rangle$$

$$\Rightarrow \quad \frac{1}{2}m_N\langle\vartheta_N^2\rangle=\frac{1}{2}m_H\langle\vartheta_H^2\rangle$$

$$\Rightarrow \quad \sqrt{\langle\vartheta_H^2\rangle}=\sqrt{\frac{m_N}{m_H}}\ \sqrt{\langle\vartheta_N^2\rangle}=\sqrt{\frac{m_N}{m_H}}\vartheta_{N(rms)}$$

$$\vartheta_{H(rms)}=\sqrt{14}.(490)=1830\text{m}/\text{s}$$

Ex.19.2.2 Calculate the number of molecules in each cubic meter of a gas at 1 atm, and 27^0 C.

Soln.

$$P=\frac{1}{3}\ \frac{mN}{V}\langle\vartheta^2\rangle=\frac{1}{3}\ \frac{mN}{V}\ \frac{3KT}{m}$$

$$\Rightarrow \quad P=\frac{NKT}{V} \quad \Rightarrow \quad N=\frac{PV}{KT}$$

$$\therefore N=\frac{(1.0\times10^5\text{N}/\text{m}^2)(1\text{m}^3)}{(1.38\times10^{-23}\text{J.K}^{-1})(300^0\text{K})}=2.4\times10^{25}$$

Ex.19.2.3 Find the rms speed of oxygen molecules in a gas at 300^0 K.

Soln.

$$\vartheta_{rms}=\sqrt{\frac{3KT}{m}}=\sqrt{\frac{3N_AKT}{N_Am}}=\sqrt{\frac{3RT}{M_O}}$$

$$=\sqrt{\frac{3\times(8.3\text{J}/\text{mol}^{-0}\text{K})\times(300^0\text{K})}{32\times10^{-3}\text{Kg}/\text{mol}}}=483\text{m}/\text{s}$$

Ex.19.2.4 Calculate the Kinetic energy of an oxygen molecule at 270 C. Given Baltzmann's constant K = 1.38×10^{-23} J. mol^{-1}. k^{-1}.

Soln.

The oxygen molecule is at temperature

$T = 27^0 \text{ C} = 300^0 \text{ K}$

Hence it has 5 degrees of freedom.

$E = 5/2 \text{ kT} = 2.5 \times 1.38 \times 10^{-23} \times 300$

$E = 1.035 \times 10^{-20}$ J

Ex.19.2.5 Calculate the kinetic energy of one mole of organ at 127^0 C. Given K =1.38×10^{-23} J $\text{mol}^{-1}\ {}^0\text{K}^{-1}$ and $N_A = 6.022 \times 10^{23}\ \text{mole}^{-1}$.

Soln.

$$E_k=\frac{3}{2}N_AKT=1.5\times6.022$$

$$\times10^{23}\times1.38\times10^{-23}\text{J}\times400^0\text{k}$$

$$\therefore E_k=4.99\times10^3\text{J}$$

Ex.19.2.6 Prove that for a perfect gas having f-degrees of freedom $\frac{C_P}{C_V}=1+\frac{2}{f}$.

Soln.

$$C_P-C_V=R$$

$$\Rightarrow \quad \frac{C_P}{C_V}-1=\frac{R}{C_V}$$

But $C_V=\frac{1}{2}fR$

$$\therefore \quad \frac{C_P}{C_V}=1+\frac{R}{C_V}=1+\frac{R}{\frac{1}{2}fR}=1+\frac{2}{f}.$$

Summary :

1. Pressure exerted by a gas

$$P = \frac{1}{3}\frac{mN}{V}\left\langle \upsilon^2 \right\rangle$$

where $\left\langle \upsilon^2 \right\rangle$ = Mean Square velocity

$$= \frac{\upsilon_1^2 + \upsilon_2^2 + \ldots \upsilon_N^2}{N}$$

m = mass of a gas-molecule

N = Total no of moleculea in the gas

V = Volume of the gas

2. Root mean square speed =

$$\upsilon_{rms} = \sqrt{\left\langle \upsilon^2 \right\rangle}$$

3. $\upsilon_{rms} = \sqrt{\frac{\pi P}{\rho}}$ where ρ = Density of gas

$= \sqrt{\frac{3RT}{M}}$ where M = Molecular weight of the gas.

4. Mean kinetic energy = $\left\langle E_k \right\rangle = \frac{3}{2}KT$

if the molecules have translational motion only.

K = Boltz mann constant = R / NA

5. $\upsilon^2_{rms} \propto T$, T being the absolute temperture.

6. No of degrees of freedom :

a) For monoatonic gas = 3

b) For diatomic gas = 5 at ordinary temperature (250° k - 750° k)

= 6 at high temperature (> 750° k)

7. Law of eqllipartion energy :

Energy per molecule per each degree of freedom = $\frac{1}{2}$ KT

8. Mean speed = $\left\langle \upsilon \right\rangle = \sqrt{\frac{8KT}{\pi m}}$

9. $\left\langle \upsilon \right\rangle = 0.921\ \upsilon_{rms}$

10. Mean free path $\lambda = \frac{S}{N}$

MODEL QUESTIONS

Where S = Total path travelled in N collisions by a molecule.

A. Multiple Choice Type Questions :

1. Which of the following quantities is same for molecules of all gases at a given temperature?

(a) mass (b) speed

(c) momentum (d) kinetic energy

2. A gas behaves very similar to an ideal gas at

(a) low pressure and low temperature

(b) low pressure and high temperature

(c) high pressure and low temperature

(d) high pressure and high temperature

3. The pressure of an ideal gas is written as $p = \frac{2E}{3V}$. Here E refers to

(a) translational K.E.

(b) rotational K.E.

(c) vibrational K.E.

(d) total K.E.

4. The energy of a given sample of ideal gas depends on its

(a) volume (b) pressure

(c) density (d) temperature

5. Which of the following gases has maximum r.m.s. speed

(a) hydrogen (b) nitrogen

(b) oxygen (d) carbon-dioxide

6. The mean square speed of the molecules of a gas at absolute temperature T is proportional to

(a) $\frac{1}{T}$ (b) $\sqrt{T}$

(c) T (d) T^4

7. The pressure of a gas kept in an isothermal container is 200k Pa. If half the gas is removed from it, the pressure will be

(a) 100 k Pa (b) 200 k Pa

(c) 400 k Pa (d) 800 k Pa

8. The rms speed of oxygen molecules is ϑ. If the temperature is doubled and the oxygen molecules dissociate into oxygen atoms, the rms speed will become

(a) ϑ (b) $\vartheta\sqrt{2}$

(b) 2ϑ (d) 4ϑ

9. 'P' is the pressure and 'd' is the density of a gas at constant temperature. Then

(a) $P\alpha\frac{1}{d}$ (b) $P\alpha\frac{1}{d^2}$

(c) $P\alpha d$ (d) $P\alpha d^2$

10. Unit of universal gas constant is

(a) erg. cm/K (b) erg . K

(c) erg./mol./K (d) erg. /K

B. Very Short Answer Type Questions :

1. Define r.m.s speed of a gas molecule.
2. Give the degrees of freedom of flying bird.
3. Give the degrees of freedom of a diatomic molecule.
4. What is the value of C_P / C_V for oxygen?
5. Give the value of Avogadro's number.
6. Give the diamension of R.
7. What kind of internal energy does a 'perfect gas possess ?

8. Is $C_P - C_V$ of CO_2 equal to that of N_2 gas.

9. Write the energy of mono atomic molecule.

10. Calculate C_V of N_2 gas.

11. Write the expression for pressure due to a gas contained in a vessel.

12. State the law of equipartition of energy.

13. Does the gas at absolute zero possess zero energy ?

14. How does r.m.s. speed depend on temperature ?

15. Write the value of Baltzmann's constant (k).

C. Short Answer Type Questions :

1. Define C_V .

2. Distinguish between mean speed and rms speed.

3. Three molecules of a gas have speeds 1m/s, 2m/s, 3m/s respectively. Calculate their mean speed and r.m.s speed.

4. What is the Kinetic interpretation of temperature ?

5. On the basis of kinetic theory explain why expansion causes cooling ?

6. Equal masses of mono-atomic and diatomic gases at the same temperature are given equal quantities of heat. Which gas will undergo a larger temperature rise and why ?

7. Explain why C_P is greater than C_V for a gas.

8. Define C_P.

9. When we place a gas cylinder on a van and the van moves, does the kinetic energy of the molecules increase ? Does the temperature increase ?

10. While gas from a cooking gas cylinder is used, the pressure does not fall appreciably until the last few minutes. Why ?

11. Do you expect the gas in cooking gas cylinder to obey ideal gas equation ?

12. Can we define the temperature of vacuum ? The temperature of a single molecule.

13. Comment on the statement - the temperature of all the molecules in a sample of gas is same.

14. A gas is kept in a rigid cubical container. If a load of 10 kg is put on the top of the container, does the pressure increase ?

15. A gas is kept in an enclosure. The pressure of the gas is reduced by pumping out some gas. Will the temperature of the gas decrease by Charle's law ?

D. Unsolved Problems :

1. Calculate the r.m.s speed of Hydrogen molecules at (i) 20^0 C (ii) 40^0 C.

2. Calculate the kinetic energy of an oxygen molecule at 27^0 C. Given $k = 1.37 \times 10^{-16}$ erg. $mol^{-1}\ {}^0k^{-1}$.

3. Calculate the K.E. of 1 mole of argon at 127^0 C. Given $k = 1.36 \times 10^{-16}$, $N_A = 6.022 \times 10^{-23}$ $mole^{-1}$.

4. Calculate the temperature at which r.m.s. speed of hydrogen molecules shall be 2000 m/s.

5. 0.014 kg of nitrogen is enclosed in a vessel at a temperature of 27^0 C. How much heat has to be transferred to the gas to double the r.m.s. speed of its molecules.

6. Compute the number of molecules in a gas contained in a volume of 1.0 cm^3 at a pressure of 1.03×10^{-3} atm. and a temperature of 200^0 k.

7. Calculate the number of molecules in 1.0

cm^3 of an ideal gas at 27^0 C and a pressure of 10 mm of mercury. The average translational kinetic energy of the molecule at 27^0 C is 6.20×10^{-21} J/(mole. kg.)

8. Calculate $C_P - C_V$ for one gram of helium gas. Given molecular mass of helium 4 and $J = 4.186 \times 10^{-1}$ erg./cal.

9. Calculate the value of the two specific heats with the aid of the following data. Density of hydrogen at NTP = 0.09 gm/ litre, speed of sound in hydrogen at 0^0C = 1262 m/s.

10. The escape speed on earth is 1.12×10^4 m/s. At what temperature would the r.m.s speed of (i) He atom (i) H_2 - molecule (iii) N_2 molecule. attain the escape speed.

E. Long Answer Type Questions :

1. Give the basic postulates of kinetic theory of gas, and derive an expression for pressure of a gas.

2. Discuss the kinetic interpretation of temperature. Find expression for K.E. per gm. mole of the gas and K.E. per 1 gm of gas.

3. Establish the relation $\frac{C_P}{C_V} = 1 + \frac{2}{f}$, where 'f' is the degrees of freedom of the gas.

4. Write down the postulates of kinetic theory. Obtain the gas laws from the expression for pressure.

5. Discuss about degrees of freedom. State law of equipartition of energy. Hence deduce expression for K.E. of a gas molecule, of 1 gm of gas and 1 gm. mole of gas.

F. Answer as directed

1. Two different gases at same temperature have equal kinetic energy (Yes/No)

2. 16 gm of oxygen gas and x gm of hydrogen gas occupy the same volume at same temperature and pressure. Then x is .... (Fill in the blank)

3. The average KE of gas molecules is E at 0^0C what is its value at 273^0C ?

4. Pressure coefficient and volume coefficient of an ideal gas are .... (Fill in the blank)

5. The degrees of freedom of a diatomic gas molecule are .... out of which .... are translatory and .... are rotatory. (Fill in the blanks)

6. The velocity with which the maximum number of molecules of the gas move is called .... (Fill in the blank)

7. Four molecules of a gas have velocity 2, 4, 6 and 8 km/sec. respectively. What is the ratio of their root mean velocity and average speed ?

8. The translational degrees of freedom of an ant moving freely in a plane is .... (Fill in the blank)

9. To double the volume of a given mass of an ideal gas at 27^0C, keeping the pressure constant, one must raise the temperature in degree centrigrade to .... (Fill in the blank)

10. In an ideal gas of density d, the pressure exerted at a temperature T is P. Then is P proprotional to d ? (Yes/No)

ANSWERS

A. Multiple Choice Type Questions :

1. (d), 2. (b), 3. (a), 4. (d), 5. (a), 6. (c), 7. (a), 8. (c), 9. (c), 10. (c).

B. Very Short Type Questions :

1. See text
2. three
3. See text
4. 1.4
5. 6.022×10^{23} mole^{-1}
6. $ML^2 T^{-2}\, {}^0k^{-1}$
7. Kinetic tyre
8. If C_P & C_V correspond to unit mass, then $C_P - C_V$ is not constant. If C_P & C_V correspond to 1 gm. mole, then $C_P - C_V$ is constant.
9. $\frac{3}{2}$ kT
10. $\frac{5}{2}$ R
11. See text
12. See text
13. No. At T = 0, K.E. = 0, but it may possess P.E.
14. $\vartheta_{rms} \ \alpha \sqrt{T}$
15. $1.38 \times 10^{-23}\, JK^{-1}$

C. Short Type Questions :

1. See text
2. See text
3. 2 m/s, 2.16 m/s
4. See text
5. When a gas suddenly expands, work is done by the gas at the expense of its own energy. So its K.E. decreases. Hence its temperature decreases.
6. Monatomic gas will undergo larger temperature change. For monoatomic gas $Q = +\frac{3}{2}RT.N$ for diatomic gas $Q = \frac{5}{2}RT'.N$. Hence $T > T'$.
7. See text
8. See text

D. Unsolved Problems :

1. 1911.08 m/s; 1975.23 m/s
2. total K.E. = 1.028×10^{-13} erg. Translational K.E. = 6.165×10^{-14} erg.
3. 4.99×10^3 J
4. $320.8\ {}^0k$

5. 9348.75

6. 3.737×10^{16}

7. 3.22×10^{17}

8. 0.496 cals/gm

[Hints: 1 gm of He = $\frac{1}{4}$ gm. mole of He

$$\therefore\ n\,(C_p - C_v) = \frac{nR}{J}$$

$$= \frac{1}{4} \times \frac{8.31 \times 10^7\,\text{erg}/(\text{mole.}^\circ\,\text{k})}{4.186 \times 10^7\,\text{erg}/\text{cal}}$$

$$= 0.496\ \text{cal/gm.}\ ]$$

9. Cp = 3.358, Cv = 2.373 C.G.S. unit

[Hints: $\vartheta = \sqrt{\frac{\gamma P}{\rho}} \Rightarrow \gamma = \frac{\vartheta^2 \rho}{P} = 1.415$

$$\frac{C_p}{C_v} = \gamma = 1.415$$

$$C_p - C_v = \frac{R}{J} = \frac{PV}{TJ} = 0.985$$

$$\therefore (1.415 - 1)C_v = 0.985$$

$\Rightarrow C_v = 2.373$ C.G.S. units. Hence Cp = 3.358 C.G.S. units]

10. $T_{He} = 2.012 \times 10^{4}\,{}^{0}$K; $T_{H_2} = 1.006 \times 10^{4}$ K, $T_{N_2} = 7.044 \times 10^{4}\,{}^{0}$ K.

F. (1) Yes (2) 1 gm (3) 2E (4) Equal (5) Five, three, two (6) Most probable velocity (7) 1.095 (8) 3 (9) 327°C (10) Yes

I
LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						5	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	16	20	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	18	22	26	29	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	6	10	13	16	19	23	26	29
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	14	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	6	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	3	5	8	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	17	19	21
						2672	2695	2718	2742	2765	2	4	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3224	3345	3365	3385	3402	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4320	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	6	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6002	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

II
LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1 2 3	4 5 6	7 8 9
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1 2 3	3 4 5	6 7 8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1 2 3	3 4 5	6 7 8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1 2 2	3 4 5	6 7 7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	8316	1 2 2	3 4 5	6 6 7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1 2 2	3 4 5	6 6 7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1 2 2	3 4 5	5 6 7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1 2 2	3 4 5	5 6 7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1 2 2	3 4 5	5 6 7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1 1 2	3 4 4	5 6 7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1 1 2	3 4 4	5 6 7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1 1 2	3 4 4	5 6 6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1 1 2	3 4 4	5 6 6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1 1 2	3 3 4	5 6 6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1 1 2	3 3 4	5 5 6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1 1 2	3 3 4	5 5 6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1 1 2	3 3 4	5 5 6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1 1 2	3 3 4	5 5 5
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1 1 2	3 3 4	5 5 6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1 1 2	3 3 4	4 5 6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1 1 2	2 3 4	4 5 6
70	8451	8457	8463	8476	8476	8482	8488	8494	8500	8501	1 1 2	2 3 4	4 5 6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1 1 2	2 3 4	4 5 5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1 1 2	2 3 4	4 5 5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1 1 2	2 3 4	4 5 5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1 1 2	2 3 4	4 5 5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1 1 2	2 3 3	4 5 5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1 1 2	2 3 3	4 5 5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1 1 2	2 3 3	4 4 5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1 1 2	2 3 3	4 4 5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1 1 2	2 3 3	4 4 5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1 1 2	2 3 3	4 4 5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1 1 2	2 3 3	4 4 5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1 1 2	2 3 3	4 4 5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1 1 2	2 3 3	4 4 5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1 1 2	2 3 3	4 4 5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1 1 2	2 3 3	4 4 5
86	9345	9350	9355	3360	9365	9370	9375	9380	9385	9390	1 1 2	2 3 3	4 4 5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0 1 1	2 2 3	3 4 4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0 1 1	2 2 3	3 4 4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0 1 1	2 2 3	3 4 4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0 1 1	2 2 3	3 4 4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0 1 1	2 2 3	3 4 4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0 1 1	2 2 3	3 4 4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0 1 1	2 2 3	3 4 4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0 1 1	2 2 3	3 4 4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0 1 1	2 2 3	3 4 4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0 1 1	2 2 3	3 4 4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9998	0 1 1	2 2 3	3 4 4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0 1 1	2 2 3	3 4 4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0 1 1	2 2 3	3 3 4

III
ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	3
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2383	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2788	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6

IV
ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	10
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	12
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	9	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	8	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	16	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	15	17	19
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	10	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

V
NATURAL SINES

Degrees	0′ 0°.0	6′ 0°.0	12′ 0°.0	18′ 0°.0	24′ 0°.0	30′ 0°.0	36′ 0°.0	42′ 0°.0	48′ 0°.0	54′ 0°.0	1	2	3	4	5
0	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	.0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	.0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	.0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	.1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	.1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	.1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	.1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	.1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	.1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	.2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	.2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	.2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	.2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	.2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	.2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	.3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	.3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	.3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	.3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	.3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	.3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	.4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	.4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	.4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	.4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	.4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	.4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	.5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	.5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	.5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	.5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	.5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	.5736	5750	5764	5779	5793	5807	5821	5835	5120	5864	2	5	7	10	12
36	.5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	.6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	.6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	.6293	6307	6320	6334	6347	6361	6474	6388	6401	6414	2	4	7	9	11
40	.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	.6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	.6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	.6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	.6947	6959	6972	6984	6697	7009	7022	7034	7046	7059	2	4	6	8	10

VI
NATURAL SINES

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences				
											1	2	3	4	5
45	.7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	.7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	.7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	.7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	.7547	7558	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	.7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	.7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	.7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	.7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	.8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	.8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	.8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	.8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	.8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	.8572	8582	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	.8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	.9746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	.8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	.8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	.8988	8996	9903	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	.9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	.9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	.9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	.9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	.9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	.9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	.9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	.9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	.9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	.9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	.9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	.9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	.9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	.9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	.9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	.9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	.9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	.9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	.9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	.9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	.9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	.9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	.9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	.9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	.9998	9999	9999	9999	9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0
90	1.000														

VII
NATURAL COSINES

(Numbers in difference columns to be subtracted, not added)

Degrees	0′ 0°.0	6′ 0°.0	12′ 0°.0	18′ 0°.0	24′ 0°.0	30′ 0°.0	36′ 0°.0	42′ 0°.0	48′ 0°.0	54′ 0°.0	Mean Difference				
											1	2	3	4	5
0	1.000	1.000	1.000	1.000	1.000	1.000	.9999	.9999	.9999	.9999	0	0	0	0	0
1	.9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	.9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	.9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	.9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	.9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
6	.9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	.9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	.9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	.9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	.9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	.9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	.9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	.9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	.9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	.9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	.9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	.9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	3	4
18	.9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	.9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	.9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	.9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	.9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	.9205	9198	9191	9184	9178	9171	9164	9157	9130	9143	1	2	3	5	6
24	.9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	6
25	.9063	9036	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	.8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	.8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
28	.8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	.8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	.8660	8652	8643	8634	8625	8616	8607	8199	8500	8581	1	3	4	6	7
31	.8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	.8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	.8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	.8290	8291	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	.8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	.8090	8080	8070	8059	8049	8039	8028	8118	8007	7907	2	3	5	7	9
37	.7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	.7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	.7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	.7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	.7547	7536	7524	7513	7501	7490	7478	7466	7455	7448	2	4	6	8	10
42	.7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	.7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	.7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

VIII
NATURAL COSINES

(Numbers in difference columns to be subtracted, not added)

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences				
											1	2	3	4	5
45	.7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	.6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	.6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	.6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	.6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
50	.6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	.6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	.6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	12
53	.6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
54	.5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	.5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	.5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	.5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	.5299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	.5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60	.5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3	5	8	10	13
61	.4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	.4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	.4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
64	.4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	.4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	.4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
67	.3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	14
68	.3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	.3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70	.3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	.3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	.3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	.2924	2967	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	.2756	2780	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	.2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	.2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	.2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	.2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	.1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
80	.1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
81	.1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	.1392	1374	1357	1340	1323	1305	1288	1271	1251	1236	3	6	9	12	14
83	.1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	.1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	.0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	15
86	.0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	.0523	0506	0448	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	.0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	.0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15
90	.0000														

IX
NATURAL TANGENTS

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences				
											1	2	3	4	5
0	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	.0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	.0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	.1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	.1228	1246	1263	1281	1299	1217	1334	1352	1370	1388	3	6	9	12	15
8	.1405	1423	1441	1459	1477	1493	1512	1530	1548	1566	3	6	9	12	15
9	.1384	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	.1944	1962	1980	1988	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	.2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	.2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	.2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	.2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	.2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	.3057	3076	3096	3115	3134	3153	3132	3191	3211	3230	3	6	10	13	16
18	.3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	.3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	.3640	3659	3679	3699	3219	3739	3759	3779	3799	3819	3	7	10	13	17
21	.3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	.4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	.4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	.4452	4473	4494	4515	4536	4457	4578	4599	4621	4642	4	7	11	14	18
25	.4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	.4877	4899	4921	4942	4964	4686	5008	5029	5051	5073	4	7	11	15	18
27	.5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	.5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	.5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	.6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	.6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	.6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	.6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	.7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	.7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	.7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	.8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	.8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	.9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	.9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	.9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

X
NATURAL TANGENTS

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences				
											1	2	3	4	5
45	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1.0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1.0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1.1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1.1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1.1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1.2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1.2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1.3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1.3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1.4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1.4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1.5399	5458	5517	5577	5687	5697	5757	5818	5880	5941	10	20	30	40	50
58	1.6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1.6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1.7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1.8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1.8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1.9626	9711	9797	9883	9970	2.0057	2.0145	2.0233	2.0323	2.0413	15	29	44	58	73
64	2.0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2.1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2.2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2.3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	53	40	60	79	99
68	2.4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2.6031	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	109
70	2.7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2.9042	9208	9375	9544	9714	9887	3.0061	3.0237	3.0415	3.0595	53	58	87	116	145
72	3.0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3.2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3.4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3.7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4.0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4.3315	3662	4015	4374	4737	5107	5483	5864	6252	6646	Mean differences cease to be sufficiently accurate				
78	4.7046	7453	7867	8288	8716	9152	9594	5.0045	5.0504	5.0970					
79	5.1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5.6713	7297	7894	8502	9124	9758	6.0405	6.1066	6.1742	6.2432					
81	6.3138	3859	4596	5350	6122	6912	7720	8548	9395	7.0264					
82	7.1154	2066	3002	3962	4947	5958	6996	8062	9158	8.0285					
83	8.1443	2636	3863	5126	6427	7769	9152	9.0579	9.2052	9.3572					
84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20					
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95					
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46					
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27					
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08					
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	57.30					
90	00														

NOTES