But $\frac{1}{\lambda}$ = wave number = $\overline{\upsilon}$

so that

wave number = $\overline{\upsilon}$

$$= \frac{me^4}{8\varepsilon_o^2 ch^3}\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right) \quad ...16.7.4$$

Rydberg constant (=R) is mathematically defined as :

$$R = \frac{me^4}{8\varepsilon_o^2 ch^3},$$

so that eq.(16.7.4) gives :

$$\overline{\upsilon} = R\left(\frac{1}{n_f^2}-\frac{1}{n_i^2}\right) \quad ...16.7.5$$

This is known as Rydberg formula. The dimensional analysis of (16.7.5) reveals that R has the dimension of $\overline{\upsilon}$, which is (length^{-1}).

The above equation (16.7.5) specifies different series hydrogen spectra and in each series there are several special lines. The first five series of hydrogen spectra are :

1. Lyman series
2. Balmer series
3. Paschen series
4. Brackett series
5. P fund series

1. Lyman series : In this series all spectral lines correspond to transition of electron from a higher excited state to lower state having n_f=1

Put $n_f = 1$

$n_i = 2, 3, 4 .... \infty$

Then the Wave number is given by :

$$\overline{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{1^2}-\frac{1}{n_i^2}\right] \quad ...16.7.6$$

For the highest wave number

$$\overline{\upsilon} = \frac{1}{\lambda} = R\left[1-\frac{1}{2^2}\right]$$

which gives λ = 1216 A^0

(Putting R = 1.097 x 10^7 m^{-1})

For the lowest wave number,

$$\overline{\upsilon} = \frac{1}{\lambda} = R\left[1-\frac{1}{\infty^2}\right]$$

$$= R$$

so that $\overline{\upsilon}$ = 912 A^0

Since the wavelength-range is from 1216 to 912 A^0, it corresponds to the ultraviolet region of the spectrum and not visible to us.

2. Balmer series : This corresponds to transition of electrons from higher states to n_f=2.

Put $n_f = 2$

and $n_i = 3, 4, 5, ... \infty$

so that
$$\overline{\upsilon} = \frac{1}{\lambda} = R\left(\frac{1}{2^2}-\frac{1}{n_i^2}\right) \quad ...16.7.7$$

Here $\lambda_{highest}$ = 6561 A^o

and λ_{lowest} = 3646 A^o

The first four of the spectral lines are contained in the visible spectrum.

3. Paschen series : This corresponds to transition to electrons from higher states to n_f=3.

Put $n_f = 3$

and $n_i = 4, 5, 6, ... \infty$

Thus
$$\overline{\upsilon} = \frac{1}{\lambda} = R\left(\frac{1}{3^2}-\frac{1}{n_i^2}\right) \quad ...16.7.8$$

Then $\lambda_{highest} = 18752\ A^o$

$$\lambda_{lowest} = 8204\ A^o$$

The series lies in the infrared region of the spectrum and is beyond our vision.

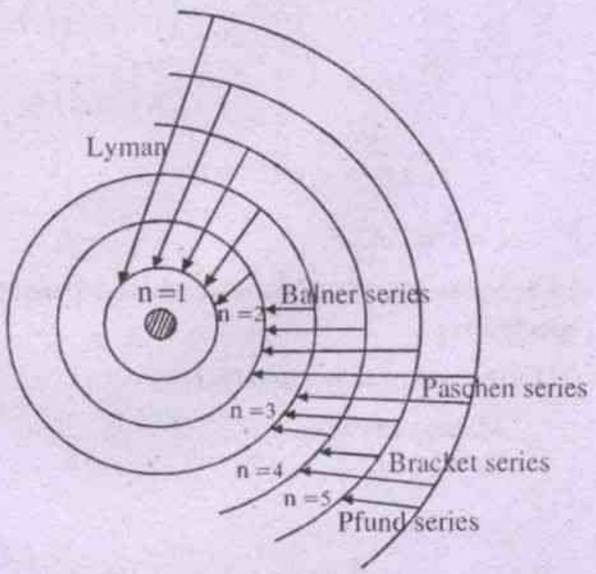


Fig. 16.6 Emission of different series of hydrogen

4. Brackett series : This corresponds to transition of electrons from higher states to n_f=4.

Put $n_f = 4$

and $n_i = 5, 6, 7, ...$

$$\overline{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{16} - \frac{1}{n_i^2}\right] \quad ...16.7.9$$

Then $\lambda_{highest} = \dfrac{1}{R\left[\frac{1}{16} - \frac{1}{25}\right]}$

$$= 40514\ A^0$$

and $\lambda_{lowest} = \dfrac{1}{R\left[\frac{1}{16} - \frac{1}{\infty}\right]}$

$$= \frac{16}{R}$$

$$= 14585\ A^0$$

This series also lies in the infrared region.

5. Pfund series : This corresponds to transition of electrons from higher states to n_f=5.

Put $n_f = 5$

and $n_i = 6, 7, 8, ...$

so that $$\overline{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{25} - \frac{1}{n_i^2}\right] \quad ...16.7.10$$

Then $\lambda_{highest} = \dfrac{1}{R\left[\frac{1}{25} - \frac{1}{36}\right]}$

$$= 74583\ A^0$$

$$\lambda_{lowest} = \frac{1}{R\left[\frac{1}{25} - \frac{1}{\infty}\right]}$$

$$= 22789\ A^0$$

These spectral lines lie in the far-infrared region of the spectrum.

All the above five series can be represented in the enery-level diagram, as shown below :

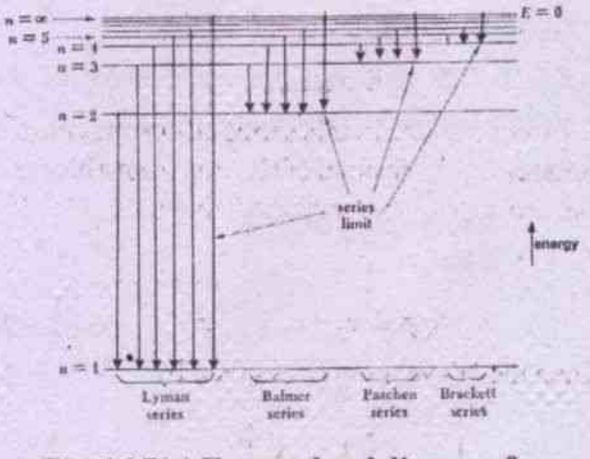


Fig. 16.7(a) Energy-level diagram for hydrogen atom

The Atomic spectrum of hydrogen-atom can be represented diagramatically as :

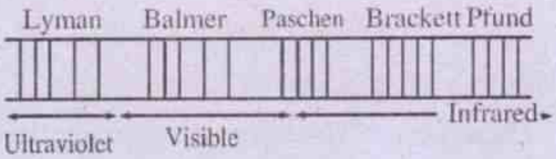


Fig. 16.7(b)

Ex.16.7.1: Calculate the wavelength of the emitted radiation when the hydrogen atom is excited from its ground state (n = 1), where its energy-level is -21.8×10^{-19} J to the higher level (n = 2) of energy -5.4×10^{-19} J and then falls back to the ground state.

Soln.

Let E = change in energy

$$= h\nu$$

$$= \frac{hc}{\lambda}$$

$$\therefore \quad \lambda = \frac{hc}{E}$$

$$= \frac{(6.6\times10^{-34})(3\times10^{8})}{(-5.4\times10^{-19})-(-21.8\times10^{-19})}$$

$$= 1.2 \times 10^{-7} \text{ meter}$$

This wavelength is in the ultraviolet region (or, Lyman series)

Ex.16.7.2: Find out the wavelength of the second member of the Balmer series.

Soln.

Use the eq. (16.7.7) :

$$\lambda = \frac{1}{R\left(\frac{1}{4}-\frac{1}{n_i^2}\right)} \qquad (\because n_f = 2)$$

$$= \frac{1}{R\left(\frac{1}{4}-\frac{1}{16}\right)} \qquad (\because n_i = 4)$$

$$= \frac{1}{R(3/16)} \qquad \begin{pmatrix} n=3 \\ M-\text{shell} \end{pmatrix}$$

$$= \frac{16}{3R} \qquad \begin{pmatrix} n=2 \\ L-\text{shell} \end{pmatrix} \begin{pmatrix} n=1 \\ K-\text{shell} \end{pmatrix}$$

$$= \frac{16}{3\times1.097\times10^{7}}$$

$$= 4861 \text{ A}^0$$

16.8: Some terms associated with hydrogen spectrum :

(i) Fine structure constant (α) :

Mathematically,

$$\alpha = \frac{e^2}{4\pi\varepsilon_o\left(\frac{h}{2\pi}\right)C} \qquad ...16.8.1$$

It is a dimensionless quantity. Its numerical value is $\frac{1}{137}$. It is related to orbital velocity of the electrons as :

$$\upsilon_n = \left(\frac{C}{n}\right)(\alpha) \qquad ...16.8.2$$

Hence the velocity of the innermost electron is $\frac{1}{137}$ th part of velocity of light.

(ii) Excitation potential :

It is the energy required to raise the electron from the ground state of the atom to a higher energy level (i.e., n > 1). [Whenever an electron is in an excited state (n = 2, 3, 4 ...), its natural tendency will be to jump to the ground state (n = 1), so that its potential energy will be minimised, the fall in the energy being emitted as a photon-leading to a spectral line.]

Ionisation potential is that excitation-potential when the higher energy level will have the quantum number $n = \infty$.

16.9: Limitations of Bohr's Atomic model :

Bohr's theory successfully explained the origin of discrete spectral lines of atoms. But it suffered from the following drawbacks :

(i) Bohr supposed that the electron's orbits are circular. But elliptical orbits are also possible. So the theory does not satisfy the atomic structure in all cases.

(ii) It is observed that there is fine structure of spectral lines of hydrogen atom, i.e., Group of closely packed spectral lines. Bohr's model is too simple to explain this.

(iii) Bohr did not take into account the motion of the nucleus and electrons about their common center of mass.

(iv) It does not say anything about the relative intensities of the spectral lines.

(v) Even though the orbital electrons show wave-perturbations, Bohr based his model on planetary model. Hence the distribution of charge of electrons in an orbit is different from that of Bohr's theory.

(vi) Bohr's theory is silent about the time, for which an electron will stay in an orbit.

16.10: Photoelectric effect :

16.10.1 Hertz's observations:

Herz in 1887, while experimenting on the production of electromagnetic waves by means of spark discharge, observed that high voltage spark across the detector loop were enhanced when the emitter plate was illuminated by ultraviolet light from an arc lamp.

When Ultra Violet falls on the metal surface, some electrons near the surface absorb enough energy from the incident radiation to overcome the attaction of the positive ions in the material of the surface. After gaining enough energy from the incident light, the electrons escape from the metal surface into the surrounding space.

16.10.2 Hallwach's and Lenard's observations:

Wihelm Hallwachs in 1888 and Philipp lenard in 1890 investigated the phenomenon of photoelectric emission.

Lenard observed that when U.V. radiations are allowed to fall on the emitter plate of an evacuated glass tube enclosing two electrodes (metal plates), current flows in the circuit shown in fig. 16.8(a). When U.V. radiations is stopped, the current flow stopped. This observation indicates that when U.V. Radiations fall on the metal (emitter) surface E electrons are ejected from it which flow towards the postitive (collector) plate P. Thus electrons flow through the evacuated glass tube, resulting in the current flow, called photo current. Hallwachs and Lenard carefully studied the dependence of photo current on plate potential, frequency of light incident, and intensity of light. They also discovered many other aspects as described below.

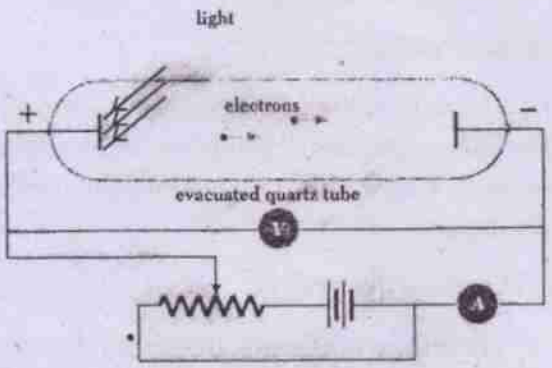


Fig. 16.8(a)

When suitable electromagnetic radiations (like visible light, X-rays, γ -rays, infrared or ultra violet rays) fall on metals, electrons (called photo-electrons) are ejected from the metals. This phenomenon is known as photo-electric effect.

The historical background of this effect can be traced to the year 1887, when Hertz observed that current moves easily in a spark-gap, when one of the terminals of the gap is illuminated by ultraviolet light.

Next year, in 1888, Hallwachs discovered that, if a negatively charged zinc plate is illuminated by ultra-violet rays, it loses its charge.

In 1890, Lenard and Sir J.J. Thomson experimentally found that when light of certain wavelength is incident on a metal plate, slow-moving electrons will come out of the plate.

All these happenings clearly indicate the existence of photoelectric emission.

Several experiments have established the following facts about photo-electric effect.

(i) Rate of emission :

The rate of emission of photoelectrons is directly proportional to the intensity of the incident light. Hence, to increase the number of photoelectrons per second from a metal surface, the intensity of the incident light has to be increased proportionately.

(ii) Kinetic energy :

The velocity (and hence kinetic energy) of the photoelectrons depends on the incident frequency. This is independent of the intensity of the incident radiation.

(iii) Threshold frequency :

There is a minimum frequency for the incident radiation, called the threshold frequency, below which no photoelectrons will be emitted from the surface of the material.

(iv) Time-lag :

When radiation is incident on a metal-surface, photo-electrons are immediately released. Thus there is no time-lag between irradiation of a metal-surface and emission of photoelectrons. Hence photoelectric emission is an instantaneous process.

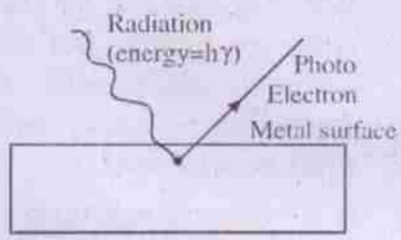


Fig. 16.8 Photo electric emission

16.11: Einstein's Photo electric equation (1905) ; work function :

To explain the above facts, Einstein used the quantum theory of radiation, given first by Planck. According to this theory, electro-magnetic wave (like X-rays, γ -rays, light rays etc.) consists of bundles of energy called quanta or photons. Energy of a photon is given by :

$$E = h\gamma \qquad ...16.11.1$$

where h is the Planck's constant

γ is frequency of radiation.

When electromagnetic radiation falls on a metal surface, part of the incident energy is used towards work function. (Work function - w- of a material is the amount of energy required for an electron to be just liberated from the surface of the material). The balance incident energy of the photon is used for imparting kinetic energy ($\frac{1}{2}m\upsilon^2$) to the just liberated electron (which is called photo electron).

Mathematically,

$$E\,(= h\gamma) = W + \tfrac{1}{2}m\upsilon^2 \qquad ...16.11.2$$

where m = mass of electron

W = work function of the material

We can write $W = h\gamma_0$ ...16.11.3

where γ_0 = Theshold frequency

(It may be noted that work-function arises, because work or energy is required to take a free electron out of a metal against the attractive forces of the surrounding positive ions. In thermionic emission we also encounter work function, since this phenomenon is concerned with electrons, breaking away from the metal).

Substituting eq. (16.11.3) in (16.11.2), we obtain :

$$h\gamma = h\gamma_o + \frac{1}{2}m\upsilon^2$$

$$\text{or,}\quad \frac{1}{2}m\upsilon^2 = h(\gamma - \gamma_o) \qquad ...16.11.4$$

This equation is called Einstein's photo electric equation. This shows that if the frequency of the incident radiation (γ) is less than the threshold frequency (γ_o), there will not be any photo-electric emission from the surface of the material. We also find from this equation that the velocity of the photo-electron depends on the frequency of the incident radiation and the work-function of the metal.

Further the number of photo-electrons emitted by the surface will depend on the number of photons striking it. Thus, the greater the number of photons striking the material (i.e., greater the intensity of the incident radiations), the greater will be the number of emitted photo-electrons.

16.12: Experimental study of photo-electric emission :

The experimental set-up to verify the laws of photoelectric emission is given below :

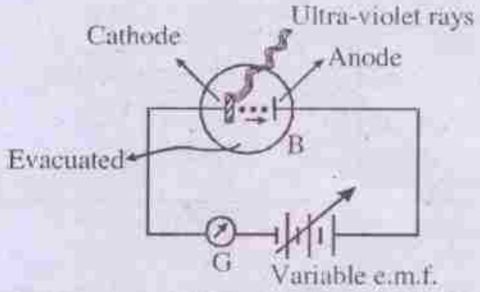


Fig. 16.9 Circuit diagram for studying photoelectric effect

G = Galvanometer

B = Evacuated Quartz Bulb

The electrodes (cathode and anode) are made up of zinc. A variable emf (with a suitable potential-divider arrangement) is applied to the electrodes. A sensitive galvanometer G (or, micro ammeter) is connected to the circuit, so that when photoelectrons are ejected from the cathode, being irradiated by ultra-violet rays, the photo current will be measured. (Since the electrons are emitted by light - i.e., photon - the electrons are called photoelectrons).

It was observed that for zinc, magnesium etc as cathode, ultra violet rays were used for photoelectric emission. But for alkali metals like sodium, potassium and rubidium, less energetiic rays, like visible rays, were able to cause this effect. Thus it is proved that photo-electric current depends on (i) nature of the emitter (i.e, its work function) and (ii) quality of rays (i.e., its energy-content, as expressed by its photon). It was further verified that the strength of the photo-electric current was determined by the intensity of the incident rays.

I. Intensity effect :

1. Suppose, Anode is having a +ve potential with respect to cathode. For this we may keep cathode at zero potential and the +ve potential of anode is gradually increased. Then the current increases steadily (AB) and then attains a saturation value (BC), since there are no more photo-electrons left in the evacuated bulb to be attracted and accepted by anode.

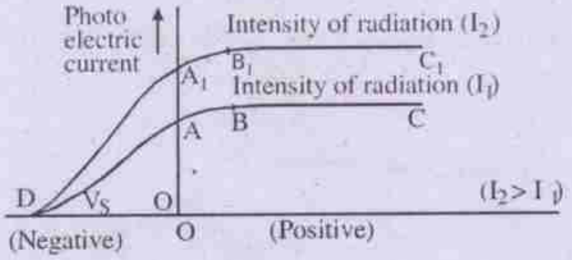


Fig. 16.10 A study of photo-electric effect

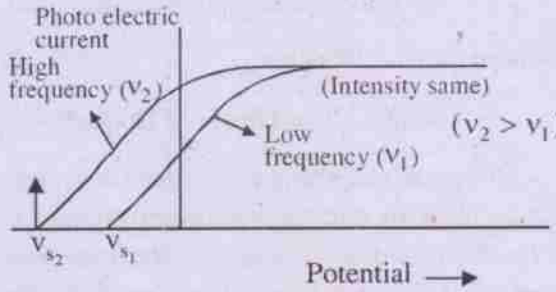


Fig. 16.10 (A) Study of photo-electric effect

2. Suppose, Cathode is having a +ve potential with respect to Anode. Then because of this, photo-electrons, which are emitted by the Cathode, will be pulled back to Cathode, before reaching the anode. Hence photo-electric current will fall.

If the cathode potential is made more and more +ve, the current will ultimately fall to zero. Then the potential difference between cathode and anode will give the stopping potential, (V_s), as this value stops the current (AD). Thus we can write : -

$$(KE)_{max} = \frac{1}{2} m\upsilon_{max}^2 = eV_s \quad ...16.12.1$$

DABC represents the curve for the intensity of incident radiation, (I_1, say).

If the intensity of incident radiation is raised to I_2 and the above procedure is repeated, we get the curve $DA_1B_1C_1$.

A comparision between the two curves shows that the stopping potential does not change, because even though the intensity has changed, we use the same incident radiation, thus fixing the frequency.

II. Frequency effect :

Now we use different types of incident rays (say, ultra-violet rays and violet rays - one at a time). Thus the study is made by changing the frequency. However we keep the intensity constant for which we use Lux meter to measure intensity.

The above procedure (Ref I) is followed. The graphical representation shows :

a) The stopping potential Vs increases with increase of frequency of the incident radiation.

b) The maximum kinetic energy of photo-electrons depends on the frequency of incident radiation.

c) The photo current increases, as frequency increases, till the saturation value is reached - which is same for both.

d) At the stopping potential, the photo-current continues to remain zero, even if we increase intensity of incident radiation.

16.13: Verification of Einstein's Photo-electric equation :

We can use the above experimental set-up (Fig. 16.9), to verify Einstein's photoelectric equation. Here light of different frequencies are made to fall on the cathode. However, for each frequency, the stopping potential is to be noted.

Now we plot a graph between the frequency of light (γ) and the stopping potential (eV_s). Then we get a straight line graph.

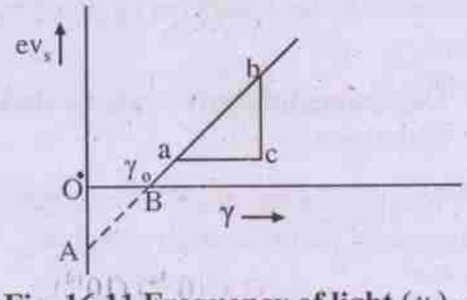


Fig. 16.11 Frequency of light (γ) ~

stopping potential (eV_s)

The straight line graph verifies Einstein's photoelectric equation, as can be seen below :

We know, Einstein's photoelectric eqn. is :

$$h\gamma = h\gamma_o + \frac{1}{2}m\upsilon^2$$

From eq. (16.12.1), we have :

$$(KE)_{max} = \frac{1}{2}m\upsilon_x^2$$

$$= eV_s$$

Substituting this eqn. above :

$$h\gamma = h\gamma_o + eV_s$$

$$\therefore \quad eV_s = h\gamma - h\gamma_o \qquad ...16.13.1$$

This is in the form of a straight line equation :

$$y = mx + c \qquad ...16.13.2$$

Comparing the above two equations, we can write :

$$\text{slope } (=m) = \frac{bc}{ac} = h \qquad ...16.13.3$$

and the intercept (= c) = OA = $-h\gamma_o$

Eq. (16.13.3) may be used to experimentally determine Planck's constant.

Ex.16.13.1: Calculate the amount of energy carried by electromagnetic radiation of wavelength 3×10^{-6} m.

Soln.

We know :

$$\gamma = \frac{c}{\lambda}$$

where c = velocity of light

$$= \frac{3\times10^8}{3\times10^{-6}}$$

$$= 10^{14} \text{ Hertz.}$$

Further $E = h\gamma$

$$= (6.63 \times 10^{-34})(10^{14})$$

$$= 6.63 \times 10^{-20} \text{ Joule}$$

Ex.16.13.2: Calculate the threshold frequency and wavelength for metal sodium, for which the work-function is 2.0 ev.

Soln.

W = work-function = 2 ev

$$= 2 \times (1.6 \times 10^{-19})$$

$$= 3.2 \times 10^{-19} \text{ J}$$

Since E = Energy

$$= h\gamma = w,$$

We get $\gamma = \frac{W}{h}$

$$= \frac{3.2\times10^{-19}}{6.63\times10^{-34}}$$

$$= 4.8 \times 10^{14} \text{ Hertz}$$

The corresponding wavelength is

$$\lambda = \frac{c}{\gamma}$$

$$= \frac{3\times10^8}{4.8\times10^{14}}$$

$$= 6.2 \times 10^{-7} \text{ m.}$$

Ex.16.13.3: Sodium has a work function of 2.0 ev. Calculate the maximum energy and speed of the emitted electrons, when sodium is illuminated by radiation of wavelength 150 nm.

Soln.

Incident Photon energy = $h\gamma$

$$= h\frac{c}{\lambda}$$

$$= \frac{(6.6\times10^{-34})(3\times10^8)}{(150\times10^{-9})}$$

$$(\because 1\text{nm} = 10^{-9}\text{m})$$

$$= 13.2 \times 10^{-19} \text{ J}$$

Maximum kinetic energy = hγ – W

where W = work function.

= $(13.2 \times 10^{-19}) - (2 \times 1.6 \times 10^{-19})$

= 10×10^{-19}

= 10^{-18} J

$$\therefore \quad \frac{1}{2}m\upsilon_{max}^2 = 10^{-18}$$

$$\text{or,} \quad \upsilon_{max} = \sqrt{\frac{10^{-18} \times 2}{9.1 \times 10^{-31}}}$$

= 1.5×10^6 met/sec.

16.14: Photo-electric cell :

It is a device which converts light energy into electrical energy. It is of three types :

1. Photo emissive cell
2. Photo voltaic cell
3. Photo conductive cell.

1. Photo emissive cell :

It is based on photo-electric effect. It is divided into two types :

a. Vacuum type
b. Gas filled type

(a) Vacuum type :

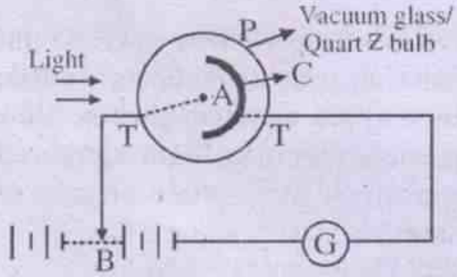


Fig. 16.12 Vacuum type photo emissive cell

C = Cathode of large surface area, coated with a suitable alkali metal to increase its sensitivity.

A = A small cylindrical anode

P = Evacuated glass or Quartz bulb, which contains A & C

B = Battery with a potential divider arrangement

Cathode and Anode are connected to -ve and +ve ends of B, respectively.

G = Microammeter (to detect the photo-electric current)

Light, having frequency more than the threshold frequency, is made to fall on cathode. Then the photo-electric current is produced.

By increasing the potential of the anode, the current rises. However, after reaching a certain value (called, as saturation), the current does not increase, even if potential of the Anode is raised. The value of the saturation-current is directly dependent on the intensity of the incident light.

The advantages of this type of cell is that (i) there is no time-lag between incidence of light and emission of photoelectric current.

(ii) The photo-electric current is proportional to the intensity of illumination.

(b) Gas-filled type :

In the vacuum type cell, the photo-electric current produced is small. So, to increase it, some gas (for example, inert gas-argon, neon) is filled in the tube under a few millimeter of pressure. When the potential of the anode is increased beyond the ionisation potential of the gas, the gas is ionised which results in increasing the photo-electric current.

2. Photo voltaic cell :

There are certain materials, which develop e.m.f., when light falls on them. The emf, so generated, is proportional to the intensity of the incident light.

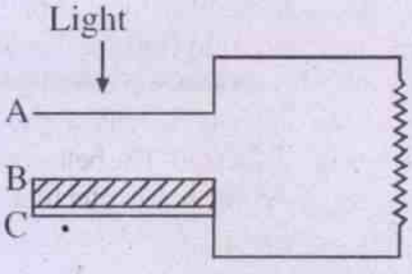


Fig. 16.13 Photo voltaic cell

A = Thin semi-transparent metal film

B = Thin film of cuprous oxide (Cu_2O)

C = Copper plate

R = Low Resistance

When light, falling on A is transmitted to B, electrons flow from B to C, thus generating photo electricity.

The advantages of this cell is that, we are not using an external battery, as in the case of photo emissive cell. Further this type of cell is robust and cheap.

3. Photo conductive cell (or, Selenium cell):

There are certain materials, the resistance of which changes when light falls on them. For example, if selenium is subjected to this process, i.e, when light falls on it, its resistance decreases. It will decrease further, if the illumination is increased.

This property is used for the construction of photo-conductive cell.

In this case, an external battery is required.

In many cases, cadmium sulphide (Cd S), a photo-conductive material, is used because it has greater light-sensitivity than selenium.

These cells are used in exposure-meters of cameras.

The disadvantages of this cell are (i) there is considerable time-lag between the change in resistance and illumination (ii) the sensitivity is not proportional to the illumination.

16.15: Uses of photocells:

(1) Burglar Alarm :

At the entrance of the house, a photo-cell is placed, on which light (infra-red and hence invisible) falls. When a person crosses the path of light, photo-electric current stops, as the incident light is intercepted. The bell, connected to the burglar alarm, becomes effective.

(2) Fire Alarm :

Photo cells are placed at different parts of the building. When, due to some reason, fire breaks, more light is incident on the photo-cells, thus, increasing photo-current. This results in the ringing of the bells.

(3) Reproduction of sound in 'talkies' (Talking Pictures)

(4) Television :

Its scaning system uses photo-cells.

(5) Luxmeter:

It measures light-intensity with the help of photo cells.

(6) In phototelegraphy, i.e., transmission of pictures to distant places.

(7) In meteorology as day light recorders.

(8) For measuring the complexion of persons.

(9) In automatic light switches for switching on and off the street light.

(10) In the determination of temperature of stars.

(11) In photoelectric sorters :

Photocells can sort out articles into different categories according to quality.

16.16: Dual nature of light :

Light behaves sometimes as a wave and some times as a particle. This behaviour of light is known as its dual nature.

That light acts like a wave is evidenced by the phenomena of interference, diffraction, polarisation, colour of thin films etc. However, the particle nature of light aptly explains phenomena such as photo-electric effect, Raman effect and Compton effect.

Thus we find that light has dual nature (wave and particle - hence, sometimes called as 'wavicle').

While light, as explained above, has dual nature, it is found that matter, similarly, has dual nature, i.e., it has the properties of particle and wave. For example, electron (a particle) has not only mass (just as any type of matter) but also a wave length (just as any type of wave or radiation).

Thus looking at these pictures it is concluded that "The radiant energy in the form of wave must have particle nature and the particles in motion should exhibit wave properties".

In fact, even though wave or particle has dual nature, it should be remembered that light or particle cannot have the wave and particle nature simultaneously - there is a peculiar relation between them. For example, if we design an experiment in which the wave character of a particle (say, electron) is revealed strongly, then its particle nature will be blurred. Alternatively. if an experiment is devised to have the particle-character of the particle (i.e., say, electron), then its wave-nature will be no more clear but very much blurred. Hence it is held that "matter and light are like coins having two faces, but they display either face at will, but not both of them simultaneously."

16.17: Wave nature of Matter De Broglie relations :

Let us consider a particle of mass m. Then its energy equivalence can be obtained by using Einstein's equation

$$E = mc^2 \qquad ...16.17.1$$

where c = velocity of light

E = Energy associated with the particle

Further, from Quantum Mechanics, we have

$$E = h\gamma \qquad ...16.17.2$$

Again, we know for a photon (i.e., quantum of light)

$$\gamma = \frac{c}{\lambda} \qquad ...16.17.3$$

Substituting eq. (16.17.3) in (16.17.2), we get :

$$E = \frac{hc}{\lambda} \qquad ...16.17.4$$

Equating 16.17.1 & 16.17.4, we obtain,

$$mc^2 = \frac{hc}{\lambda}$$

$$\text{or,} \quad \lambda = \frac{h}{mc} \qquad ...16.17.5$$

Since momentum (P) is the product of mass and velocity, we write, for a photon :

$$P = mc$$

so that (16.17.5) gives :

$$\lambda = \frac{h}{P} \qquad ...16.17.6$$

Eq (16.17.6) is meant for photon. However, this can be written for all particles, in genral. Of course, in that case, the velocity of the particle may be written as υ, so that

$$P = m\upsilon$$

Substituting this in eq. (16.17.6), we get a general eqn. :

$$\lambda = \frac{h}{m\upsilon} \qquad ...16.17.7$$

This is de Broglie relation. Here λ represents the wave-nature, whereas m represents the particle-nature. Thus eq. (16.17.7) is containing the wave-particle duality.

Eq. (16.17.7) shows that when the mass of a particle is large, then its wavelength will be small. Hence ordinary material particles will have such small wavelengths that they cannot be detected.

Ex. 16.17.1: Find the wavelength of an electron whose speed is 9×10^7 m/sec.

Soln.

From (16.17.7) we have

$$\lambda = \frac{h}{m\upsilon}$$

$$= \frac{6.63 \times 10^{-34}}{(9.1 \times 10^{-31})(9\times10^{7})}$$

$$= \frac{6.63}{9.1 \times 9} \times 10^{-10}$$

$$= 8.09 \times 10^{-12} \text{ m}$$

16.18 Davisson- Germer Experiment:

One possible way of producing a beam of electrons is by means of an electrons gun, which emits electrons with certain definite velocity v. In case when $v << c$ (i.e. in the non-relativistic limit)

$$\frac{1}{2} m_0 v^2 = e\phi \qquad \text{-----(16.18.1)}$$

Where, ϕ is the accelerating potential in volts

= P.D. between last grid oand cathode

$e = 1.602 \times 10^{-19}\, coul.$

$m_0 = 9.11 \times 10^{-31}\, kg$ =rest mass of electron

v is in m/s

Therefore the electron will have De Broglie wavelength given as

$$\lambda = \frac{h}{m_0 v} = \frac{h}{\sqrt{m_0^2 v^2}} = \frac{h}{\sqrt{2 m_0 e\phi}} \quad \text{---(16.18.2)}$$

Where eqn. 16.18.1 has been used in obtaining 16.18.2. Putting the value h, m_0 *and* e in the above equation we obtain

$$\lambda = \frac{1.2 \times 10^{-9}}{\sqrt{\phi}} m = \frac{1.2 \times 10^{-7}}{\sqrt{\phi}} cm \quad \text{--(16.18.3)}$$

Equation 16.18.3 shows that in order to exhibit wave properties in the most pronounced form one must impart to them the longest possible wavelength λ and it is possible by decreasing ϕ. But a certain amount of energy equal to work function ($\sim eV$) is expended in the ejection of electrons from metal. Therefore the smallest possisble potential ϕ at which the beam will be relatively monochromatic is $15 \sim 20V$. At ths potential the De Broglie wavelength associated with the electrons will be $\lambda \sim 10^{-8} cm \sim A^0$ and this is in the range of wavelength of soft X-rays.

Davisson and Germer tried to establish the wave nature of electrons by showing that the electrons are diffracted by a crystal in a way similar to X-rays are diffracted by a crystal.

For this purpose they designed the experimental arrangement as given below:

Experimental arrangement:

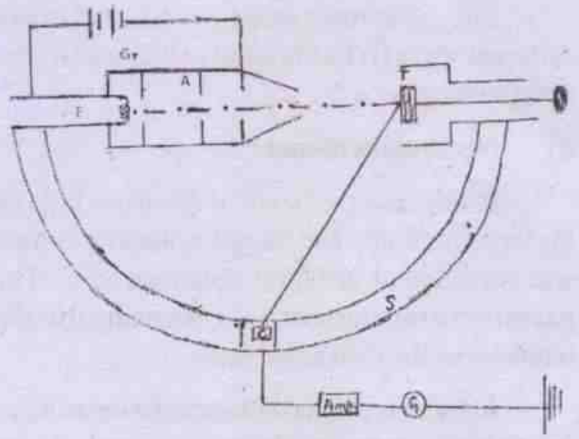


Fig.16.14

An electron beam is produced from the electron gun. The filament F is heated to dull red and electrons are emitted from the filament F due to thermionic emission. These electrons are made to pass through a system of electrodes with central holes, maintained at increasing potentials. By this arrangement electrons emerge as a well collimated beam.

These monoenergetic electrons fall on a target T (a single crystal of baked nickel). Some of the scattered electrons are collected by the Faraday Cylinder C, called as collector. The collectror current is amplified and measured with a sensitive galvanometer.

The collector can be moved on a graduated scale S to receive electrons. The collector has two walls insulated from each other. A retarding potential is applied between the inner and outer walls of the collector, such that only fast moving electrons (primary electrons) comingfrom the electron gun may enter into the collector, and not the slower (secondary electrons) produced by collision from the target. The slower electrons will be reflected from the retarding potential.

the whole apparatus is enclosed in a highly evacuated and degassed chamber.

Observations:

The experiment may be conducted in two different ways (i) For normal incidence (ii) For oblique incidence

(i) **Normal incidence:**

In this case the beam of electrons falls on the target normally and the galvanometer current was recorded at different positions of C. The galvanometer current is a measure of the intensity of the diffracted beam.

A graph is plotted between the colatitudes (angle between the incident beam and the beam entering the collector) and the galvanometer current. The curves were down for several voltage electrons.

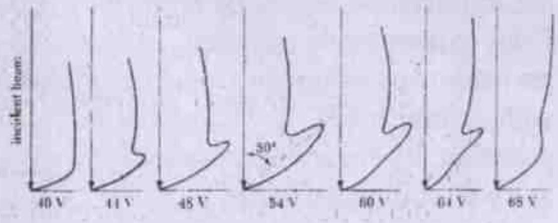


Fig. 16.15

It is observed that a bump begins in the curve for 44 volt electrons. This bump moves upwards as the voltage increases and attains the greatest development for 54 volts at a colatitudes of 50^0. Above 54 volts the bump again diminishes. The bump at 54 volts offers the evidence for the existence of electro waves.

Since the crystal can be thought to be a plane diffraction grating; hence

$$n\lambda = s\sin\theta \qquad \text{------(16.18.4)}$$

where 's' is the lattice spacing. Since $s = 2.15A^0$ so in the first order

$n=1$ and $\lambda = 2.15\sin 50^0 = 1.65A^0$ --(16.18.5)

From De Broglie's calculation

$$\lambda = \frac{1.2\times10^{-7}}{\sqrt{\phi}} = \frac{1.2\times10^{-7}}{\sqrt{54}} = 1.66A^0 \quad \text{—(16.18.6)}$$

This agreement clearly shows that electrons have wavelike characteristics.

(ii) **Oblique incidence:**

In this case the crystal is oriented so that the angle of incidence is 10^0 and the position of the collector is also chosen corresponding to the angle of reflection of 10^0. The De broglie wavelength of the electron is changed by changing the accelerating potential ...

The electrons suffer Bragg's reflection so that they satisfy

$$n\lambda = 2s\sin\theta \qquad \text{----(16.18.7)}$$

Since $\lambda = \frac{1.2\times10^{-7}}{\sqrt{\phi}} \Rightarrow \lambda \propto \frac{1}{\sqrt{\phi}}$ or $\frac{1}{\lambda} \propto \sqrt{\phi}$

Therefore a graph between galvanometer current 'i' and $\sqrt{\phi}$ is equivalent to the graph i vs. $(1/\lambda)$. The graph is shown in fig. 16.16.

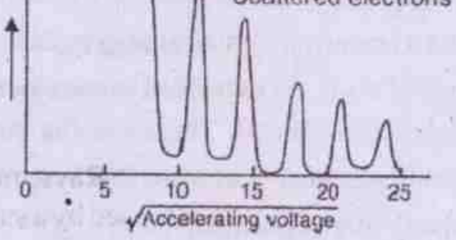


Fig.16.16

From De Broglie relation

$$\lambda = \frac{1.2\times10^{-7}}{\sqrt{\phi}} \Rightarrow \frac{1}{\lambda} \propto \sqrt{\phi} \quad \text{---(16.18.8)}$$

From Bragg's relation

$$n\lambda = 2s\sin\theta \Rightarrow n \propto \frac{1}{\lambda} \quad \text{----(16.18.9)}$$

From eqns. 16.18.8 & 9 we get $n \propto \sqrt{\phi}$

This indicates that the order of the spectrum is directly proportional to $\sqrt{\phi}$. The experimental curve shows distinct maximum at regular intervals of $\sqrt{\phi}$ confirming De Broglie theory.

16.19 X-Rays

X-Rays are electromagnetic waves of short wavelength (-of the order of 1 Angstrom unit).

Production : X-Rays can be produced by using a Coolidge tube.

H = Hard Glass bulb having a vacuum of 10^{-6} cm of mercury.
F = Filament (Tungsten) cathode.
D = Molybdenum cylinder
C = Copper Block, the front surface being at 45^0 to the electron beam.
P,S = Primary and Secondary coil of step-up transformer
E = Electron-beam.
B = Battery
mA= mill - ammeter
Rh = Rheostat
A = Ammeter
L = Lead - wall ($\approx$ 5 mm thick)
T = Target (Molybdenum / Tungsten) - Anode
W = Window

For the production of X-Rays, rapidly moving electrons are to be stopped by a suitable target.

A coolidge tube is an evacuated large glass bulb. In this case, a step-up transformer is used to have high voltage. The anode (T) is maintained at a large potential difference with respect to the Cathode (F). When current is supplied from the battery (B), the filament gets heated and emits electrons. These electrons are accelerated highly due to the large P.D. and hit the target with large impact. Thus X-Rays are emitted from the target, which is inclined at 45^0 with the horizontal, and pass out through the window W.

Quality of X-Rays depends on the P.D. between the anode and cathode mostly. For this, the step-up transformer plays an important role.

However, intensity of X-Rays depends on the rate of emission of thermo-electrons from the cathode. This is controlled by the filament-current. Hence for controlling the intensity of the X-Rays, we use the rheostat.

In this way, intensity (as controlled by the transformer) and quality (as controlled by the rheostat) are made independent of each other. This is a significant achievement, since, in the earlier gas-tubes for production of X-Rays, increase in quality was always accompanied with increase in intensity. Separate control of quality and intensity of X-Rays, as produced by coolidge tube, is hence a distinct advantage.

Since production of X-Rays involves heat-generation to a large extent, the target needs to be cooled by air or water cooling arrangement.

The electrons, emitted from the filament, strike the target only during the +ve half-cycle of a.c. However, during the -ve half-cycle, the target becomes negative with respect to the cathode. Hence the thermo-electrons are repelled. Thus the X-Ray tube acts as its own rectifier. As the operation of the tube involves danger to the operator, it is essential to enclose the coolidge-tube in a box, lined with lead.

Types of X-Rays :

X-Rays can be divided into 2 types.

(1) Soft X-Rays. (Wavelength $\approx$ 4A^0 or above)

(2) Hard X-Rays. (Wavelength $\approx 1A^0$)

(1) Soft X-Rays :

They have smaller energy and low penetrating power. They are produced in the coolidge-tube, when low potential difference is applied and there is high pressure.

(2) Hard X-Rays :

They have higher energy and large penetrating power. They are produced under high potential difference and low pressure.

Properties of X-Rays :

These properties can be divided under the following heads :

(A) Properties similar to light rays

(B) Properties similar to cathode rays.

(A) Properties similar to light rays :

(1) They are electromagnetic waves of very short wavelength ($100 \sim 0.6\ A^0$); whereas light waves have a wavelength range of ($4000 \sim 7000\ A^0$). This is the basic difference between X-Rays and light.

(2) They are invisible and travel straight, with the velocity of light.

(3) They exhibit wave-like properties, such as reflection, refraction, diffraction, interference, polarisation under special circumstances.

(4) They are not deviated by electric or magnetic fields.

(5) They produce fluorescence, when they fall on materials like zinc sulphide, calcium tungstate, barium platinocyanide etc.

(6) Like light rays, they show continuous spectrum.

(B) Properties of X-Rays, similar to cathode rays:

(1) They have penetrating power. If the matter is dense, then their penetration is less. They do not pass through bones. If such objects are found in their path, they cast shadows (as in the case of light).

(2) They ionise the gas through which they pass.

(3) When they fall on metals, they liberate electrons (as in photoelectric effect). In some cases, they emit secondary X-Rays.

(4) They affect photographic plates.

(5) Long exposure of X-Rays are harmful to human body.

Uses of X-Rays :

The uses of X-Rays can be classified in three categories.

(1) Radiography

(2) Radiotherapy

(3) Scientific research

(1) Radiography :

(a) Surgery : X-Rays can easily pass through low-density matters like skin, flesh and blood; but are obstructed by high-density substances like bone. This property is used in surgery for detection of fractures, presence of foreign matter like bullet in the human body, formation of bone or stones and progress of a healing bone of human body.

(b) Engineering : X-Ray radiographs are used for detecting faults, cracks, flaws, gas-pockets and air-bubbles in finished products, buildings and bridges etc.

(c) Trade and Detective Department : They are used to distinguish between artificial and real diamonds, gems etc. The homogeneity of different materials can be tested with it. Detection of opium, explosives, contraband goods can be made with the help of X-Rays. Gold, silver etc. carried secretly by smugglers, can be also spotted.

(d) Industry : Pearls in oysters, homogeneity in timber, uniformity in materials, defects in rubber tyres, wood etc. can be tested by x-rays.

(e) Arts : Changes in old oil paintings can be examined with X-Rays.

(2) Radiotherapy :

Intractable skin-diseases and malignant growths in human body can be treated by X-Rays. These rays are made to fall on that part of the body, which is affected by cancer, to destroy the cells.

(3) Scientific research :

X-Rays have found wide application in the scientific investigation of the internal structure of crystals, constitution and properties of atoms and arrangement of atoms/molecules in complex substances.

16.20 Bragg's law :

We know that atoms are arranged regularly in a crystal. When a beam of X-Rays is incident on a crystal, then a plane of atoms of the crystal acts like a plane mirror. Since the planes of atoms are transparent to X-Rays, reflection takes place at successive parallel planes. Then the reflected beams combine in accordance with the principle of wave-optics.

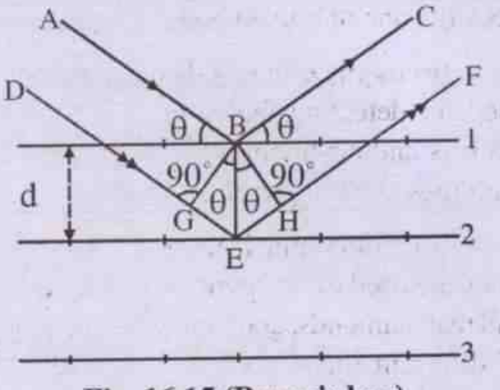


Fig. 16.15 (Bragg's law)

1, 2, 3 = Cleavage planes in a crystal, where the dots indicate the position of atoms.

AB, DE = Beams of X-Rays, incident on the crystal at a glancing angle θ (AB is parallel to DE)

d = distance between two successive layers

B, E = Two atoms, where X-Ray diffraction is shown.

BC, EF = Diffracted rays

When a beam of X-Rays fall on the crystal, the beam is scattered by the atoms B and E in random directions. However constructive interference can take place only in case of those rays, for which the path difference is an integral multiple of wavelength λ.

The path difference between the rays ABC and DEF

$$= GE + EH$$

In the $\perp$ rt angled Δ le BGE, $GE = d \sin\theta$

Similarly, $EH = d \sin\theta$

$\therefore$ The total path differnce $= 2d \sin\theta$

For constructive interference, $2d \sin\theta = n\lambda$

where $n = 1, 2, 3, ...$ ...16.19.1

This is Bragg's law.

Here the values of n determine the order of diffraction.

16.21 Spectrum of X-Rays :

The spectrum of X-Rays can be studied under two heads.

(1) Characteristic spectrum

(2) Continuous spectrum.

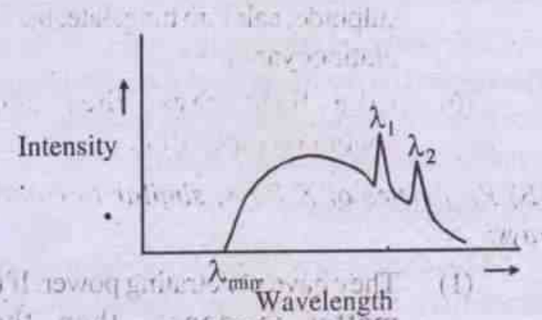


Fig. 16.16 (X-Ray spectrum)

This figure depicts the spectrum in general of the X-Rays, as produced by a coolidge tube. This shows that the spectrum is continuous; but it is crossed by sharp lines (called characteristic) indicated by λ_1, λ_2. This figure further presents that there is a limit to the wavelength (λ_{min}), below which no X-Rays are emitted.

(1) Characteristic X-Rays :

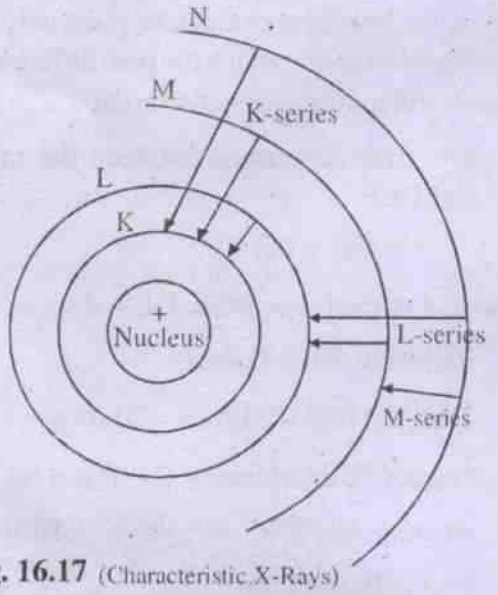


Fig. 16.17 (Characteristic X-Rays)

According to the modern concept of physics, based on quantum mechanics, an atom consists of a centrally positively charged nucleus, around which electrons revolve in circular/elliptical orbits (or, shells) having definite radii. The first four shells are named as K, L, M, N; K being the inner most (i.e, nearest to the nucleus).

Since the electrons in the K-shell are most tightly bound to the nucleus (being nearest to it), maximum force is required to knock out an electron out of it. In an X-Ray tube, the cathode ray (ie, electrons from the cathode) impinge on the anode (or, anticathode) with such tremendous velocity, that it enters well inside the atoms of the anti cathode and ejects out electrons from the inner shells (K, L, M, N, etc.) depending on the energy of the incident impinging electron. The vacancy, thus caused, in the shell is filled by the outer electrons. The balance of the energy (which is equal to the energy-difference between the two sells) leads to the emission of an X-Ray photon.

The characteristic X-Rays are assigned nomenclature, as follows :

Name of the series	Characteristic spectrum-line	Shells involved during the electron jump	Remark
K	K_α	K - L	Similar nomenclatures
	K_β	K - M	can be used for
	K_γ	K - N	L-series,
			M-series etc.

The wavelengths of the lines of these series depend on the material of the anticathode. Since they are characteristic of the material, they are named as characteristic X-Rays.

16.22 Moseley's law :

Moseley's law (1913) is related to characteristic X-Ray spectrum. Mathematically, it is represented by :

$$\sqrt{\gamma} \ \alpha \ (Z - b)$$

or, $$\sqrt{\gamma} = K(Z - b) \qquad ...16.21.1$$

where γ = Frequency of the spectral line

Z = Atomic number of the element, from which X-Ray is emitted

K and b = constants.

However K and b are different for different series.

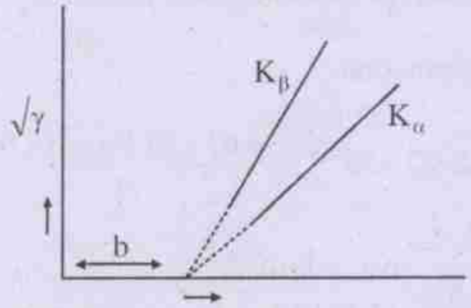


Fig. 16.18 Atomic Number (Z) ⟶
(Moseley's law)

The above equation represents a straight line. It shows that, higher the atomic number of the element, greater will be the frequency of the emitted X-Rays, having larger energy-content.

Moseley found that, if instead of atomic number we use atomic weight of the elements, the graphs will not be straight line. There will be considerable departure. Thus he concluded that, while considering characteristic X-Rays, we should be concerned with the atomic number instead of atomic weight.

This has great significance in the arrangement of elements in periodic table. Moseley changed the position of certain elements in the periodic table of Mendeleev, basing upon the value of atomic number. He further pointed out that Z = 43 and 72 should be left vacant, as these elements were not yet discovered at that time. Later on, they were discovered and found as : Technetium (Z = 43) and Hafnium (Z = 72). Moseley's law has significant achievement, by correcting the anomalies in Mendeleev's periodic table.

16.23 Continuous x-rays :

Let a voltage V be applied to the Coolidge-tube. Then the energy of an electron, emitted from the filament, will be given by :

Energy = eV

where e = charge of an electron.

When this electron strikes the target, either its entire energy (eV) or a part of it will be lost. The energy, so lost, will appear as X-Rays. The frequency of the emitted X-Ray photon will be determined by the amount of energy, received from the incident electron.

It is found that normally the whole amount of energy eV is not transferred in a single collision. On the other hand, a sequence of collisions with different atoms of the target takes place, transferring bits of energy, thus, leading to the emission of a large number of X-Ray photons of varying frequencies, before the incident electron finally comes to rest. This is the cause behind continuous X-Ray spectrum. Even though the X-Ray spectrum is continuous, it is observed that there is a minimum wavelength - i.e., a wavelength, below which there is no X-Ray emission. This can be explained with the help of Planck's law.

By Planck's law, we can write,

Energy of a photon = $h\nu$

where υ is the frequency of the emitted X-Ray photon.

Equating this with the energy carried by the incident electron and transferred to the emitted X-Ray photon, we have :

$$h\nu = eV \qquad ...16.22.1$$

If the entire energy is transferred in a single collision, we can write this equation as :

$$h\upsilon_{max} = eV$$

$$\text{or,} \quad \upsilon_{max} = \frac{eV}{h}$$

$$\text{But } \lambda_{min} = \frac{C}{\upsilon_{max}}$$

where C = velocity of light

$$= \frac{Ch}{eV} \quad \text{Hence } \lambda_{min} \alpha \frac{1}{V}$$

...16.22.2

This shows that the minimim wavelength limit (λ_m) is inversely proportional to the accelerating potential (V).

EXAMPLES

Ex.16.1 The wavelength of the first spectral line of an element is 5893 A⁰. Show that its first excitation potential is 2.1 ev.

Soln.

We have,

$$\text{Energy} = h\upsilon$$

$$= \frac{hc}{\lambda}$$

$$= \frac{(6.63\times10^{-34})\times(3\times10^{8})}{5893\times10^{-10}}$$

$$= 0.00337 \times 10^{-16} \text{ joule}$$

$$= 2.1 \text{ ev}$$

Ex.16.2 The ionisation potential of hydrogen atom is 13.6 volts. When it is in its lowest energy-level, it is ionised by absorbing a photon, due to which an electron is released with a kinetic energy of 1.91 ev. Calculate the wave length of the ionised photon.

Soln.

Kinetic energy of the emitted electron =

$$\begin{pmatrix}\text{Energy of the}\\ \text{absorbed photon}\end{pmatrix} - \begin{pmatrix}\text{Maximum energy needed}\\ \text{to ionise the atom}\end{pmatrix}$$

or, $$1.91 = \begin{pmatrix}\text{Energy of the}\\ \text{absorbed photon}\end{pmatrix} - 13.6$$

$\therefore$ Energy of the absorbed photon = 15.51 ev.

$$= 15.51 \times (1.6 \times 10^{-19}) \text{ Joule}$$

$$= 2.482 \times 10^{-18} \text{ Joule}$$

But we know,

energy of the incident photon in joules = $\frac{hc}{\lambda}$

On substitution :

$$2.482\times10^{-18} = \frac{(6.62\times10^{-34})\times(3\times10^{8})}{\lambda}$$

$$\therefore \quad \lambda = \frac{(6.62\times3)10^{-26}}{(2.482)10^{-18}}$$

$$= 8 \times 10^{-8} \text{ meter}$$

$$= 800 \text{ A}^0$$

Ex. 16.3 Hydrogen atom in its ground state is excited by a monochromatic radiation of wavelength 975 A⁰. How many spectral lines are possible in the emission spectrum.

Soln.

We know, Energy supplied (by an incident photon) =

$$\Delta E = \frac{hc}{\lambda}$$

$$= \frac{(6.63\times10^{-34})\times(3\times10^{8})}{975\times10^{-10}} \text{ Joule}$$

$$= 20.24 \times 10^{-19} \text{ Joule}$$

$$= \frac{20.4\times10^{-19}}{1.6\times10^{-19}}$$

$$= 12.75 \text{ ev.}$$

We know also, $E_n = \frac{-13.6}{n^2}$ in ev for hydrogen atom.

$$\therefore \; E_1 = \begin{pmatrix}\text{Energy of hydrogen atom}\\ \text{in 1st energy - level}\end{pmatrix} = -13.6 \text{ ev}$$

Similarly,

$$E_2 = \frac{-13.6}{2^2} = -3.4ev$$

$$E_3 = \frac{-13.6}{9} = -1.51ev$$

$$E_4 = \frac{-13.6}{16} = -0.85ev$$

When a photon is incident on the atom, it is excited by a gain of energy (equal to the energy of the incident photon).

∴ Energy of the atom after absorption of energy (equal to 12.75) ev = - 13.6 + 12.75

= - 0.85 ev

This shows that the electron of the hydrogen atom will be excited to the fourth energy level (n = 4).

Hence the transitions, now allowed, from the excited state (n = 4), will be :

(i) n = 4 to n = 3

(ii) n = 4 to n = 2

(iii) n = 4 to n = 1

(iv) n = 3 to n = 2

(v) n = 3 to n = 1

(vi) n = 2 to n = 1

Total transitions = 6

Thus six spectral lines are possible

Ex.16.4 A hydrogen atom at rest in the second excited state emits photon, when it jumps to the ground state. Find out the momentum of the emitted photon and de-Broglie wavelength of the hydrogen atom. (Given : energy of the ground state = - 13.6 ev).

Soln.

We know, $E_n = \frac{-13.6}{n^2}$ ev

Energy of the emitted photon =

$$E_2 - E_1$$

$$= \left[\frac{-13.6}{(3)^2}\right] - \left[\frac{-13.6}{1^2}\right]$$

(∵ for 2nd excited state, n = 3)

$$= 13.6 - \frac{13.6}{9}$$

$$= 12.09 \text{ ev}$$

For a photon :

Energy = E

= PC where P = momentum

∴ P = E / C

$$= \frac{12.09 \times (1.6 \times 10^{-19})}{3 \times 10^8}$$

$$= 6.45 \times 10^{-27} \text{ kg.m. S}^{-1}$$

Now by de-Broglie's relation :

$$\lambda = h / p$$

$$= \frac{6.62 \times 10^{-34}}{6.45 \times 10^{-27}}$$

$$= 1.03 \times 10^{-7} \text{ meter}$$

$$= 1030 \text{ A}^0$$

Ex.16.5 The energy transition in hydrogen atom occurs from n = 3 to n = 2 energy level. What is the wavelength of the emitted wave.

Soln.

$$\frac{1}{\lambda} = R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right)$$

$$= (1.096 \times 10^7)\left[\frac{1}{4} - \frac{1}{9}\right]$$

$$\therefore \quad \lambda = \frac{36}{(1.096 \times 10^7) \times 5}$$

$= 6.563 \times 10^{-7}$ met

$= 6563\ A^0$

Ex.16.6 Given that,

When light of frequency 2.6×10^{15} Hertz falls on a metal surface, the ejected photoelectrons are fully retarded by a reverse potential of 6.5 V.

Further the photo-electrons emitted by a light of frequency 4.6×10^{15} Hertz from that metal surface are fully retarded by a reverse potential of 14.8 V.

Find out Planck's constant.

Soln.

We have

$$h\gamma_1 = \left(\frac{1}{2} m\upsilon_{max}^2\right)_1 + W$$

where, W = work function of the metal

and $$h\gamma_2 = \left(\frac{1}{2} m\upsilon_{max}^2\right)_2 + W$$

Subtracting,

$$h(\gamma_2 - \gamma_1) = \frac{1}{2} m\left[\left(\upsilon_{max}^2\right)_2 - \left(\upsilon_{max}^2\right)_1\right]$$

$$= eV_2 - eV_1$$

$$\therefore \quad h = e\frac{[V_2 - V_1]}{[\gamma_2 - \gamma_1]}$$

$$= \frac{(1.6 \times 10^{-19})(14.8 - 6.5)}{(4.6 - 2.6) \times 10^{15}}$$

$= 6.63 \times 10^{-34}$ Joules. sec.

Ex.16.7 Find out the kinetic energy of an electron whose de Broglie wavelength is $5000A^0$.

Soln.

We have, $$P = \frac{h}{\lambda}$$

$$= \frac{6.63 \times 10^{-34}}{5000 \times 10^{-10}}$$

$= 1.33 \times 10^{-27}$ kg. m/sec.

Further, $$K.E. = \frac{P^2}{2m}$$

$$= \frac{(1.33 \times 10^{-27})^2}{2 \times (9.1 \times 10^{-31})}$$

$= 9.7 \times 10^{-25}$ J

$= 6 \times 10^{-6}$ ev

SUMMARY

1. Atomic model :

Any atomic model should explain the structure of an atom.

2. Thomson's Atom model :

Atom consists of a sphere filled uniformly with positive charges. The electrons are embedded in it in regular fashion.

3. Rutherford's Atom model :

The entire mass and positive charge of the atom is concentrated in an extremely small central core, called the nucleus. The electrons revolve round it in circular orbits.

4. Bohr Atom model :

I Postulate :

The electrons revolve round the nucleus only in circular orbits. The centripetal force is provided by electrostatic attraction.

II Postulate :

Electrons revolve only in those orbits, in which their angular momentum is an integral multiple of $h/2\pi$. This is the quantisation of atomic model.

$$m\upsilon r = \frac{nh}{2\pi} \quad \text{where } n = 1, 2, 3, ...$$

III Postulate :

An electron, revolving in circular orbit, does not radiate energy. But it radiates energy, when it jumps from high energy orbit to low energy orbit. The radiation of energy takes place in the form of Photon.

$$E_2 - E_1 = h\gamma$$

5. Radius of Bohr's orbit :

$$r_n = \frac{\varepsilon_o n^2 h^2}{\pi m e^2 z}$$

$$= \frac{\varepsilon_o n^2 h^2}{\pi m e^2} \quad \text{(for hydrogen)}$$

$$r_o = \frac{\varepsilon_o h^2}{\pi m e^2} = 0.53\ A^0 \quad (\text{for } n = 1)$$

6. Velocity of electron in Bohr's orbit :

$$\upsilon_n = \frac{e^2}{2\varepsilon_o nh}$$

7. Energy of Bohr's orbit :

$$E_n = -\frac{ze^2}{8\pi\varepsilon_o r_n}$$

$$= -\frac{mz^2e^4}{8\varepsilon_o^2 n^2 h^2}$$

8. Rydberg constant :

$$R = \frac{me^4}{8\varepsilon_o^2 ch^3}$$

$$= 1.097 \times 10^7\ m^{-1}$$

9. $E_n = -\dfrac{Rchz^2}{n^2}$

For hydrogen $E_n = -\dfrac{13.6}{n^2}$ ev

10. Ground state :

This is the lowest energy state of an atom. In this case, all the electrons of the atom remain in their respective orbits.

11. Excited state :

When energy is supplied to an atom, the atom is said to be in an excited state. Then the electrons jump to higher energy-states, or orbits. But the electrons can remain in the higher states for a period of 10^{-8} sec (i.e., the normal life time of the atom), after which they come back to their normal state, after emitting the balance amount of energy in the form of electromagnetic radiations, leading to discrete atomic spectral lines.

12. Frequency of emitted radiation = υ

$$= Rc\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]$$

13. Wavelength of emitted radiation = υ

$$= \frac{1}{Rz^2\left[\frac{1}{n_f^2} - \frac{1}{n_i^2}\right]}$$

14. Wave number = $\bar{\gamma} = \dfrac{1}{\lambda}$

15. Spectrum of hydrogen atom :

(i) Lyman series (Ultraviolet region)

$$\bar{\upsilon} = \frac{1}{\lambda} = R\left[\frac{1}{1^2} - \frac{1}{n_i^2}\right]$$

where $n_i = 2, 3, 4, ...$

(ii) Balmer series (Partly in visible region)

$$\overline{\upsilon} = R\left[\frac{1}{2^2} - \frac{1}{n_i^2}\right]$$

where $n_i = 3, 4, 5, ...$

(iii) Paschen series (Infrared region)

$$\overline{\upsilon} = R\left[\frac{1}{3^2} - \frac{1}{n_i^2}\right]$$

where $n_i = 4, 5, 6, ...$

(iv) Brackett series (Infrared region)

$$\overline{\upsilon} = R\left[\frac{1}{4^2} - \frac{1}{n_i^2}\right]$$

where $n_i = 5, 6, 7, 8, ...$

(v) Pfund series (far-Infrared region)

$$\overline{\upsilon} = R\left[\frac{1}{5^2} - \frac{1}{n_i^2}\right]$$

where $n_i = 6, 7, 8, ...$

16. *Energy-level diagram for Hydrogen atom :*

$$E_n = -\frac{13.6}{n^2} ev$$

$$\therefore \quad E_1 = -13.6 ev$$

$$E_2 = -3.4 ev \text{ etc.}$$

17. *Ionisation potential :*

The minimum energy absorbed by an atom from an outside source to ionise an atom is called the ionisation potential of the atom.

For hydrogen atom, ionisation potential = 13.6 volt.

18. *Bohr's correspondence principle :*

"The predictions of the quantum theory for the behaviour of any physical system must coincide with the predictions of the classical theory in the limit, in which the quantum numbers specifying the state of the system becomes very large."

19. The single electron systems, such as He^+, Li^{++}, are called hydrogen like atoms. For these systems, Bohr's theory can be applied.

20. When an electron jumps from one quantum state to another, its energy and angular momentum changes. However its intrinsic properties, like charge, rest mass and spin do not change.

21. *Planck's quantum theory of radiation :*

Radiation consists of tiny packets of energy, called photons, on quanta of light. Each photon has a definite energy given by $h\gamma$, $2h\gamma$, $3h\gamma$..... (i.e, integral multiples of $h\gamma$).

Rest mass of photon = 0

Momentum of photon = $\frac{h}{\lambda}$

22. Photo-electric effect :

The process of emission of electron from a material due to illumination by light (or, radiation) of suitable wave length is called photoelectric effect.

23. Threshold frequency :

There is a certain minimum frequency (called as threshold frequency) for a given material, below which no photoelectric emission takes place.

24. Stopping potential :

The anode potential (negative), for which photo-electric current is zero, is called the stopping potential.

25. Work function :

For any metal, work-function is defined as the energy, required to liberate an electron from it, without imparting any kinetic energy to it.

26. Einstein's photo-electric equation :

$$h\gamma = W + \frac{1}{2}m\upsilon^2$$

where γ = frequency of the incident radiation

W = work function of the metal

m = mass of the electron

υ = velocity of the photo electron.

27. Kinetic energy of the photoelectron :

$$(E_k)_{maximum} = h(\upsilon - \upsilon_o)$$

where $\upsilon_o = \frac{W}{h}$

or, $\frac{1}{2}m\upsilon^2{}_{max} = h(\gamma - \gamma_o)$

28. The maximum kinetic energy of photo-electron is given by the product of the electronic charge and stopping potential ($\frac{1}{2}m\upsilon_x^2 = eV_o$), wehre V_0 = stopping potential.

29. Photoelectric cell :

It is a device which converts light energy directly into electric energy.

30. Bohr's law of complimentary radiation :

It states that both the aspects - wave and particle - are complimentary to each other.

31. de Broglie relation :

$$P = \frac{h}{\lambda}$$

where P = momentum of a particle

λ = wavelength, associated with the particle.

32. Matter-wave's are probability-waves.

33. de Broglie wavelength of electron (Non-relativistic case) :

Consider an electron, accelerated from rest through a potential difference V and thus acquires a velocity υ.

Then $\frac{1}{2}m_o\upsilon^2 = eV$

$$\upsilon = \sqrt{\frac{2eV}{m_o}}$$

Hence $\lambda = \frac{h}{m_o\upsilon}$

$$= \frac{h}{\sqrt{2m_oeV}}$$

MODEL QUESTIONS

A. Multiple choice type questions :

1. The dimension of an atom is of the order of :

a) 10^{-6} m b) 10^{-8} m

c) 10^{-10} m d) 10^{-14} m

2. In which of the orbits will the electron of a hydrogen atom have maximum energy :

a) first b) second

c) third d) completely detached

3. Which is quantised in Bohr's atomic model

a) total energy of the electron

b) angular momentum of the electron

c) linear velocity of the electron

d) angular velocity of the electron

4. The number of photoelectrons emitted by a photo sensitive material is proportional to

a) wavelength

b) frequency

c) intensity

d) colour of light

5. The velocity of photoelectrons increases with the increase of

a) intensity of radiation

b) frequency of incident radiation

c) angle of incident radiation

d) none of these

6. The photo electric effect occurs only when the incident light has more than a certain minimum

a) frequency b) wavelength

c) velocity d) charge

7. The energy of photon corresponding to maximum wavelength of visible light is

a) 1 eV b) 1.8 eV

c) 13.1 eV d) 7 eV

8. The dimension of planck's constant are similar to those of

a) energy b) mass

c) frequency d) angular momentum

9. When photoelectric emission is taking place, increasing the intensity of light will :

a) have no effect

b) increase the number of electrons released.

c) increase the maximum energy per electron.

d) cause a time delay in the emission of electrons.

10. A photon of requency γ is incident on a metal surface of threshold frequency γ_o. The kinetic energy of the emitted photoelectrons is

a) $h(\gamma - \gamma_o)$ b) $h\gamma$

c) $h\gamma_o$ d) $h(\gamma + \gamma_o)$

11. The work function for photoelectric effect

a) depends upon the frequency of incident light

b) is same for all metals

c) is different for different metals

d) none of these

12. A metal surface ejects electrons when hit by green light but none when hit by yellow light. The electrons will be ejected when the surface is hit by

a) Red light b) Blue light

c) Heat rays d) Infrared light

13. A Photocell is illuminated by a small bright source placed one meter away. When the same source of light is placed two meters away, the electrons emitted by the photo-cathode

a) Each carries one quarter of its previous energy.

b) Each carries one quarter of its previous momenta.

c) Are half as numerous

d) Are one quarter as numerous

14. Given that a photon of light of wavelength 1000 A^0 has an energy equal to 1.23 eV, when light of wavelength 5000 A^0 and intensity I_0 falls in a photoelectric cell and the saturating current is 0.40×10^{-6} ampere and the stopping potential is 1.36 volt. Then the work function is

a) 0.43 eV b) 1.10 eV

c) 1.36 eV d) 2.47 eV

15. Photo electric effect supports

a) Newton's corpuscular nature of light

b) Huygen's wave theory of light

c) Max Well's electromagnetic theory of light.

d) Einstein's quantum theory of light

16. A radio transmitter operates at a frequency of 880 Khz and a power of 10 kW. The number of photons emitted per second are

a) 1.71×10^{31}

b) 1327×10^{34}

c) 13.27×10^{34}

d) 0.075×10^{-34}

17. In order to increase the kinetic energy of ejected photo electrons, there should be an increase in

a) intensity of radiation

b) wavelength of radiation

c) frequency of radiation

d) both the wavelength and intensity of radiation.

18. Work functions of tungsten and sodium are 4.4. eV and 2.3 eV respectively. If threshold wavelength of sodium is 5460 A^0, then the threshold wavelength of tungsten is

a) 11360 A^0 b) 8000 A^0

c) 6000 A^0 d) 1840 A^0

19. Light of frequency 1.5 times the threshold frequency is incident on photosensitive material. If the frequency is halved and intensity is doubled, the photo current becomes

a) quadrupled b) doubled

c) halved d) zero

20. A quantum will have more energy if

a) the wavelength is larger

b) the frequency is higher

c) the amplitude is higher

d) the velocity is higher

21. The threshold frequency of potassium is 3×10^{14} Hz. The work function is

a) 1.0×10^{-19} J b) 2×10^{-19} J

c) 4×10^{-19} J d) 0.5×10^{-19} J

22. Light of wavelength 4000 A^0 is incident on a metal plate whose work function is 2 eV. The maximum kinetic energy of the emitted photoelectrons would be

a) 2.0 eV b) 1.5 eV

c) 1.1 eV d) 0.5 eV

23. Mater waves

a) are electromagnetic waves

b) are transverse waves

c) are longitudinal waves

d) exhibit diffraction.

24. If the kinetic energy of the moving particle is E, then the de Broglie wavelength is

a) $\lambda = h\sqrt{2mE}$ b) $\lambda = \frac{\sqrt{2mE}}{h}$

c) $\lambda = \frac{h}{\sqrt{2mE}}$ d) $\lambda = \frac{hE}{\sqrt{2mE}}$

25. de Broglie wavelength associated with a moving particle of velocity υ is given by :

a) $\lambda = hm\upsilon$ b) $\lambda = \frac{h}{m\upsilon}$

c) $\lambda = \frac{\upsilon}{mh}$ d) $\lambda = \frac{h}{\sqrt{1-\upsilon^2/c^2}}$

26. Neglecting variation of mass with energy, the wavelength associated with an electron having kinetic energy E is proportional to

a) $\sqrt{E}$ b) E

c) $\frac{1}{\sqrt{E}}$ d) $\frac{1}{E^2}$

27. From the following, moving with the same velocity, the one which has largest wavelength λ is

a) An electron

b) A proton

c) An α - particle

d) All have same de Broglie wavelength

28. The energy of a photon is 10 eV. The momentum of photon is

a) 5.33×10^{-25} kg. m/sec

b) 5.33×10^{-27} kg. m/sec

c) 5.33×10^{-29} kg. m/sec

d) 5.33×10^{-23} kg. m/sec

29. The energy of a photon corresponding to the visible light of a maximum wavelength is approximately

a) 1 eV b) 1.6 eV

c) 3.2 eV d) 7 eV

30. A proton and an α-particle are accelerated through the same potential difference. The ratio of their de Broglie wavelengths ($\lambda_P / \lambda_\alpha$) is

a) 1 b) 2

c) $\sqrt{8}$ d) $\frac{1}{\sqrt{8}}$

31. If the Planck's constant is larger than its present value, the de Broglie wavelength associated with material particles would have been

a) unchanged

b) larger

c) smaller

d) larger for some particles and smaller for others

32. When electrons are accelerated through potential difference of V volts, the de Broglie wavelength associated is given by

a) $\sqrt{\frac{150}{V}} A^o$ b) $\sqrt{\frac{150}{V}} m$

c) $\frac{150}{\sqrt{V}} A^o$ d) $\frac{\sqrt{150}}{V} A^o$

33. A material particle with a rest mass m_0 is moving with speed of light C. The de Broglie wavelength associated is given by

a) h/m_0c b) m_0c/h

c) 0 d) ∞

34. A particle of mass 10^{-31} kg is moving with a velocity equal to 10^5 m/sec. The wavelength of the particle is equal to

a) 0 b) 6.6×10^{-8} m

c) 0.66 m d) 1.5×10^7 m

35. The momentum of a photon in kg. m/sec of frequency 10^9 cycles per second is

a) 7.3×10^{-29} b) 2.2×10^{-33}

c) 6.6×10^{-26} d) 3.1

36. The de Broglie wavelength of an electron is 1.224 A^0. The energy of electron is

a) 1 eV b) 10 eV

c) 100 eV d) 1224 eV

37. In Bohr's atom, the number of de Broglie waves associated with an electron moving in n^{th} permitted orbit is

a) n b) 2n

c) n / 2 d) n^2

38. In a hydrogen atom an electron is moving in the nth permitted orbit of radius r_n. The de Broglie wavelength associated with this electron is

a) $2\pi r_n$ b) $\frac{2\pi r_n}{n}$

c) $\frac{n}{2\pi r_n}$ d) $\frac{r_n}{n}$

39. The de Broglie wavelength of a molecule of thermal energy KT (K = Bottz mann constant and T = absolute temperature) is

a) $\lambda = \frac{h}{2mKT}$ b) $\frac{h}{\sqrt{2mKT}}$

c) $h\sqrt{2mKT}$ d). $\frac{h}{4m^2K^2T^2}$

40. In the hydrogen atom an electron is moving in nth orbit. The circumference s of the orbit and the de Broglie wavelength λ of the moving electron are related by the equation

a) $s = n\lambda$ b) $s = n/\lambda$

c) $s = \lambda/n$ d) $s = \frac{1}{n\lambda}$

41. Matter waves are similar in nature to

a) X-Rays

b) γ -rays

c) cathode rays

d) none of the above

42. The wavelength of matter wave is independent of

a) mass b) velocity

c) momentum d) charge

43. An electron accelerated by a P.D. of V volts possesses a de Broglie wavelength λ. If the accelerating potential is increased by a factor of 4, the de Broglie wavelength of the electron will

a) remain unchanged

b) become four times

c) become halved

d) become double

44. The relativistic expression for the wavelength of electrons moving with very high speed is

a) $\lambda = \frac{h}{mc}$

b) $\lambda = \frac{h}{m\upsilon}$

c) $\lambda = \frac{h}{m_o\upsilon}\sqrt{1-\upsilon^2/c^2}$

d) $\lambda = \frac{h}{m_o\sqrt{\frac{1-\upsilon^2}{c^2}}-\upsilon}$

45. The stopping potential for electron beam of de Broglie wavelength of 1 A^0 is

a) 12.3 volt b) 123 volt

c) 151 volt d) 1505 volt

46. To produce hard X-Rays in Coolidge tube we should increase

a) current in filament

b) potential difference across the filament

c) potential difference across cathode and anti cathode.

d) none of these

47. The maximum frequency of X-Rays is determined by

a) current in filament

b) potential difference across the filament

c) potential difference across cathode and anti cathode.

d) current from high tension

48. The voltage applied across an X-Ray tube is nearly

a) 10 volts b) 100 volts

c) 10,000 volts d) 10^6 volts

49. The characteristic of the target element that determines the frequency of characteristic X-Ray is

a) its mass number

b) its atomic number

c) its melting point

d) its conductivity

50. Ionisation in a gas cannot be produced by

a) heat

b) X-Rays

c) Radioactive emanations

d) gold leaf electroscope

51. In obtaining an X-Ray photograph of our hand we use the principle of

a) shadow photography

b) image formation by an optical system

c) photoelectric effect

d) ionisation

52. A metal block is exposed to beams of X-Rays of different wavelengths. Which of the X-Rays, having following wavelengths, penetrate most

a) 1 A^0 b) 2 A^0

c) 4 A^0 d) 3 A^0

53. The X-Ray beam coming from an x-ray tube will be

a) Monochromatic

b) Having all wavelengths smaller than a certain maximum wavelength

c) Having all wavelengths larger than a certain minimum wavelength.

d) Having all wavelengths lying between a minimum and a maximum wavelength.

54. An X-Ray tube is run at 50,000 volts. The current flowing in it is 20 mA. The power of the tube is

a) 1000 watt b) 200 watt

c) 20,000 watt d) 20 watt

55. The minimum wavelength of the continuous X-Ray radiation is

a) Ve / h b) ch / Ve

c) $\frac{ve}{hc}$ d) $\frac{ec}{vh}$

56. The energy of X-Ray photon is 3.3×10^{16} J. Its frequency per second would be

a) 5×10^{17} b) 5×10^{-18}

c) 6.62×10^{18} d) 2×10^{-18}

57. In an X-Ray tube, the intensity of emitted X-Ray beam is increased by

a) In creasing the filament current

b) Decreasing the filament current

c) Increasing the target potential

d) Decreasing the target potential

58. If the potential difference between the cathode and target of coolidge tube is 1.2×10^5 volt, then the minimum wavelength of continuous X-Rays is

a) $10A^0$ b) $1A^0$

c) $0.1A^0$ d) $0.01A^0$

59. A metal block is exposed to beams of X-Rays of different wavelengths. X-Rays, of which wavelength, penetrates most

a) 2A.U b) 4A.U

c) 6A.U d) 8A.U

60. An X-Ray tube is run at 50,000 volts. The current flowing in it is 40 mA. The power of the tube is

a) 2000 watts b) 200 watts

c) 20,000 watts d) 20 watts

61. When X-Rays fall on a crystal of latice constant d and the glancing angle is θ, then of the following equations, the one which gives diffraction maximum of the n^{th} order in accordance with Bragg's formula is

a) $2d \sin\theta = n\lambda$ b) $2d \sin\theta = \lambda$

c) $2d \sin\theta = \lambda / n$ d) $2n \sin\theta = d$

62. When high speed electrons hit a target

a) Only heat is produced

b) Only continuous X-Rays are emitted

c) Only continuous and characteristic X-Rays are emitted.

d) Heat is produced and simultaneously continous and characteristic X-Rays are emitted.

63. An electric field can deflect

a) X-Rays b) Neutrons

c) $\alpha -$ particles d) $\gamma -$ rays

64. In radiotheraphy X-Rays are used to

a) Detect bone - fractures

b) Treat cancer by controlled exposure

c) Detect heart diseases

d) Detect fault in radio-receiving circuits

65. The X-Rays are

a) Stream of electrons

b) Stream of positively charged particles

c) Electromagnetic radiation

d) Stream of uncharged particles

66. In order to study internal atomic structure of crystal, we use

a) X-Rays

b) Ultraviolet rays

c) Infrared radiations

d) Yellow light

67. The minimum frequency γ of continuous X-Rays is related to the applied potential difference v as

a) $\gamma \alpha \sqrt{v}$ b) $\gamma \alpha v$

c) $\gamma \alpha v^{3/2}$ d) $\gamma \alpha v^2$

68. X-Rays are

a) Electrons

b) Protons

c) Light of small wavelength

d) Light of high wavelength

69. The continuous X-Rays spectrum produced by an X-Ray machine at constant voltage has, which of the following

a) A maximum wavelength

b) A minimum wavelength

c) A single wavelength

d) A minimum frequency

70. X-Rays are produced due to

a) Break-up of molecules

b) Change in atomic energy level

c) Change in nuclear energy level

d) Radioactive disintegration

71. Hydrogen atom does not emit X-Rays because

a) It has single electron

b) It is very small in size

c) Its energy levels are too far apart

d) Its energy levels are too close to each other.

72. The characteristic X-Ray radiation is emitted when

a) The electrons are accelerated to a fixed energy.

b) The source of electrons emits a monoenergetic beam.

c) The bombarding electrons knock out electrons from the inner shell of the target atoms and one of the outer electrons falls into this vacancy.

d) The valence electrons in the target atoms are removed as a result of the collision.

73. X-Rays which can pentrate through longer distances in substance are called

a) soft X-Rays

b) Continuous X-Rays

c) Hard X-Rays

d) None of the above

74. The penetrating power of X-Rays increased with the

a) Increase in its velocity

b) Increase in its intensity

c) Decrease in its velocity

d) Increase in its frequency

75. Which one of the following statements about X-Rays is not an accepted fact

a) They are generated when fast moving electrons strike a metal target.

b) They can penetrate through a thin sheet of aluminium.

c) When they traverse a space across which there is a magnetic field and electric field perpendicular to one another, they are undeviated.

d) When they traverse a space across which there is a magnetic and electric field parallel to one another they describe a circular path.

76. Generation of X-Rays is an illustration of the
   a) Phenomenon of conversion of K.E. into radiant energy.
   b) Principle of conservation of mass
   c) Principle of conservation of momentum.
   d) Phenomenon of conversion of matter into energy.

77. X-Rays passing through a strong magnetic uniform field
   a) Get deflected in the direction of the field.
   b) Get deflected in the direction opposite to that of the field.
   c) Get deflected in the direction perpendicular to that of the field.
   d) Do not get deflected at all.

78. A metal sphere is suspended in the region of a uniform electric field by an insulated vertical string. X-Rays fall on the metal sphere. The sphere
   a) Remains undeflected
   b) Is deflected in the direction of the electric field.
   c) Is deflected in a direction opposite to that of the electric field.
   d) Starts executing S.H.M.

79. A direct X-Ray photograph of the intestines is not generally taken by the radiologists because
   a) Intestines would burst on exposure to X-Rays.
   b) X-Rays would not pass through the intestines.
   c) X-Rays will pass through the intestines without casting a good shadow for any useful diagnosis.
   d) A very small exposure of X-Rays causes cancer in the intestines

80. To increase the penetrating power of X-Rays in a coolidge-tube
   a) Temperature of cathode should be increased.
   b) Pressure of gas should be increased
   c) Acceleration potential should be increased.
   d) All the above

81. X-Rays produce
   a) Photoelectronic effect
   b) Diffraction
   c) Compton effect
   d) All of the above

82. According to classical theory the proposed circular path of an electron in Rutherford atom model will be
   a) Circular b) Straight line
   c) Parabolic d) Spiral

83. For a given value of n, the number of electrons in an orbit is
   a) n b) n^2
   c) $2n^2$ d) 2n

84. The hydrogen atom can give spectral lines in the series, Lyman, Balmer and Paschen. Which of the following statements is correct.
   a) Lyman series is in the infrared region
   b) Balmer series is in the visible region (Partly).
   c) Balmer series is (solely) in the ultraviolet region.
   d) Paschen series is in the visible region.

85. Fraunhoffer lines are observed in the spectrum of

a) hydrogen discharge tube

b) carbon arc

c) the sun

d) sodium vapour lamp

86. The potential energy of the electron in hydrogen atom is $-ke^2/r$. Its kinetic energy is :

a) $-\frac{ke^2}{2r}$ b) $+\frac{ke^2}{2r}$

c) $-\frac{ke^2}{r}$ d) $\frac{ke^2}{r}$

87. According to Bohr's atomic model

a) the electron radiates energy only when it jumps to inner orbit.

b) an atom has heavy, positively charged nucleus.

c) the electron can move only in particular orbits.

d) all the above statements are true.

88. In a black and white television, pictures on the screen are produced due to bombardment of

a) X-Ray photons on a white screen

b) X-Ray photons on a white fluorescent screen.

c) electrons on a white screen

d) electrons on a fluorescent white screen.

89. If the series limit wavelength of the Lyman series for hydrogen atom is 912 A^0, then the series limit wavelength of the Balmer series of hydrogen atom is

a) 912 A^0 b) 912 x 2 A^0

c) 912 x 4 A^0 d) $\frac{912}{2}$ A^0

90. The first spectral line of sodium atom is 5890A^0. The first excitation potential of sodium is

a) 3.7 v b) 2.1 v

c) 10.2 v d) 4.1 v

91. Infrared spectrum lies between

a) Radiowave and micro wave regions

b) Microwave and visible regions

c) Visible and ultraviolet regions

d) Ultraviolet and X-Ray regions

92. Of the following transitions in the hydrogen atom, the one which gives an emission line of highest frequency is

a) n = 1 to n = 2 b) n = 3 to n = 10

c) n = 10 to n = 3 d) n = 2 to n = 1

93. For light of wavelength 5000 A^0, photon energy is nearly 2.5 ev. For X-Ray of wavelength 1 A^0, the photon energy will be close to

a) $[2.5 \div 500]$ ev

b) $[2.5 \div (5000)^2]$ ev

c) $[2.5 \times 5000]$ ev

d) $[2.5 \times (5000)^2]$ ev

94. The kinetic energy of the electron in an orbit of radius r in hydrogen atom is (e=electronic charge)

a) $\frac{e^2}{\text{[illegible]}}$ b) $\frac{e^2}{2r}$

c) $\frac{e^2}{r}$ d) $\frac{e^2}{2r^2}$

95. Band spectrum is produced by

a) H b) H_2

c) He d) Na

96. Ionisation potential of hydrogen atom is 13.6 ev. Hydrogen atoms in the ground state are excited by monochromatic radiation of photon energy 12.1ev. The spectral lines emitted by the hydrogen atoms according to Bohr's theory will be

a) one
b) two
c) three
d) four

97. When the electron in the hydrogen atom jumps from the 2nd orbit to the 1st orbit, the wavelength of the radiation emitted is λ. When the electron jumps from the 3rd orbit to the 1st orbit, the wavelength of the emitted radiation would be

a) $\frac{27}{32}\lambda$
b) $\frac{32}{27}\lambda$
c) $\frac{2}{3}\lambda$
d) $\frac{3}{2}\lambda$

98. The energy of an electron in the n^{th} orbit of the singly ionised helium atom is

a) $\frac{-13.6}{n^2}$ ev
b) $\frac{-13.6\times 2}{n^2}$ ev
c) $\frac{-13.6\times 4}{n^2}$ ev
d) $\frac{-13.6}{2n^2}$ ev

99. What is deduced from the fact that when α-particles are fired at a fine gold leaf it is found that most of them pass through with little or no deflection

a) α-particles have a high penetrating power.
b) α-particles are +vely charged
c) Gold atoms are nearly all empty space
d) α-particles cause gold atoms to disintegrate.

100. According to Bohr's postulates which of the following quantities takes discrete values

a) kinetic energy
b) potential energy
c) Angular momentum
d) momentum

101. According to Bohr's model of the hydrogen atom

a) Total energy of electron is quantised
b) Angular momentum of the electron is quantised.
c) Linear velocity of the electron is quantised.
d) Angular velocity of the electron is quantised.

102. According to classical theory, Rutherford atom was

a) stable
b) unstable
c) semi-stable
d) meta-stable

103. The radius of the atom is of the order of

a) 10^{-6}m
b) 10^{-8}m
c) 10^{-10}m
d) 10^{-12}m

104. The speed of an electron in the orbit of the hydrogen atom in the ground state is

a) c
b) c/2
c) c/10
d) c/137

105. In Rutherford's expt, the number of α-particles scattered through an angle of 60^0 by a silver foil is 200 per minute. When the silver foil is replaced by a copper foil of the same thickness, the number of α-particles scattered through an angle of 60^0 per minute is

a) $\frac{200\times Z_{cu}}{Z_{Aq}}$
b) $200\left(\frac{Z_{cu}}{Z_{Aq}}\right)^2$
c) $200\times\frac{Z_{Aq}}{Z_{cu}}$
d) $200\times\left(\frac{Z_{Aq}}{Z_{cu}}\right)^2$

106. The binding energy of electron in the lowest orbit of hydrogen atom is 13.6 ev. The energies required in electron-volt to remove an electron from three lowest orbit of hydrogen atom in ev are

a) 13.6,6.8,8.4 b) 13.6,10.2,3.4

c) 13.6,27.2,40.8 d) 13.6,3.4,1.5

107. Ionisation energy of an hydrogen atom in its ground state is

a) 13.6 ev b) 13.6 joule

c) 13.6 erg d) 13.6 watt.sec

108. The ionisation potential of hydrogen atom is 13.6 volts. The energy required to remove an electron in the n=2 state of the hydrogen atom is

a) 27.2 ev b) 13.6 ev

c) 6.8 ev d) 3.4 ev

109. The difference in angular momentum associated with the electron in the two successive orbits of hydrogen atom is

a) h/π b) $h/2\pi$

c) h/2 d) $(n-1)\,h/2\pi$

110. In the following transistions of the hydrogen atom, the one which gives the absorption line of highest frequency is

a) n=1 to n=2 b) n=3 to n=8

c) n=2 to n=1 d) n=8 to n=3

111. As the quantum number increases, the difference of energy between censecutive energy-levels

a) remains the same

b) decreases

c) increases

d) first decreases and then increases

112. A continuous band of radiation having all wavelengths from about 1000 A^0 to 10,000 A^0 is passed through a gas of monoatomic hydrogen. In the emission spectrum, one can observe the entire

a) Lyman series b) Balmer series

c) Paschen series d) Pfund series

113. The work that must be done to remove an electron from an atom is called its

a) electron affinity

b) ionisation energy

c) energy band

d) heat of vaporisation

114. The radius of the orbit is given by

a) $r=\frac{4\pi^2 mZe^2}{n^2h^2}$ b) $r=\frac{n^2h^2Ze^2}{4\pi^2 m}$

c) $r=\frac{4\pi^2 m^2}{n^2h^2Ze^2}$ d) $r=\frac{n^2h^2}{4\pi^2 mZe^2}$

115. Rydberg constant is

a) Same for all elements

b) Different for different elements

c) A universal constant

d) Is different for lighter elements but same for heavier elements.

116. When an electron jumps from a higher energy state to a lower energy state with an energy difference of ΔE electron volt, then the wavelength of the line emitted is given approximately by

a) $\frac{12375}{\Delta E}$ cms b) $\frac{12375}{\Delta E}$ meters

c) $\frac{12375}{\Delta E}$ Angstrom d) $\frac{12375}{\Delta E}$ microns

117. Which of the following sources give discrete emission spectrum

   a) Incandescent electric bulb

   b) sun

   c) Mercury vapour lamp

   d) candle

118. While electron in hydrogen atom revolves in a stationary orbit it

   a) radiates light but its velocity is unchanged.

   b) does not radiate light and velocity remains unchanged.

   c) Does not radiate though its velocity changes.

   d) Radiates light with the change of energy.

119. Generally the approximate limits of visible spectrum are

   a) 1,000 to 4,000 A^0

   b) 4,000 to 7,000 A^0

   c) 7,000 to 10,000 A^0

   d) 10,000 to 13,000 A^0

120. If the mass of the electron is reduced to half, the Rydberg constant

   a) Remains unchanged

   b) Becomes half

   c) Becomes double

   d) Becomes one-fourth

121. According to Bohr's theory the radius of an electron-orbit, as described by n and Z, is proportional to

   a) Z^2n^2 b) Z^2/n^2

   c) Z^2/n d) n^2/Z

122. Give another name for the 'smallest particle of an element capable of taking part in a chemical reaction' :

   a) Electron b) Proton

   c) Atom d) Molecule

123. Given the electronic charge

   $e=1.6\times10^{-19}$ coulomb

   electronic mass $m=9.1\times10^{-31}$ kg

   $h=6.6\times10^{-34}$ Joule sec

   $c=3\times10^8$ m/sec

   The radius of the first orbit of electron in hydrogen atom is

   a) 5.3×10^1 A^0 b) 5.3×10^0 A^0

   c) 5.3×10^{-1} A^0 d) 5.3×10^{-2} A^0

124. In Q.no. 123, the velocity of the electron in the first orbit is

   a) 2.2×10^4 m/sec b) 2.2×10^5 m/sec

   c) 2.2×10^6 m/sec d) 2.2×10^7 m/sec

125. In Q.no. 123, the time period of revolution of the electron in the first Bohr orbit is

   a) 1.5×10^{-4} sec b) 1.5×10^{-16} sec

   c) 1.5×10^{-18} sec d) 1.5×10^{-20} sec

126. In Q.no. 123, the frequency of revolution of the electron in the first Bohr's orbit is

   a) 6.6×10^{15} b) 6.6×10^{13}

   c) 6.6×10^{17} d) 6.6×10^{19}

127. Electrons in the atom are held in the atom by

   a) Coulomb forces

   b) Nuclear forces

   c) Van der waal's forces

   d) Gravitational forces

128. A substance which emits light of certain colour at a given temperature can also absorb the same colour at that temperature. This is a statement of

a) Snell's law b) Fresnel's law
c) Kirchoff's law d) Newton's law

129. The ratio of areas within the electron-orbits for the first excited state to the second excited state for the hydrogen atom is

a) 2:1 b) 4:1
c) 8:1 d) 16:81

130. In an atom, two electrons move round the nucleus in circular orbits of radii R and 4R. The ratio of the times taken by them to complete one revolution is

a) 1/4 b) 4/1
c) 8/1 d) 1/8

131. The binding energy of the hydrogen atom (the energy binding the electron to the nucleus) is

a) 1 ev b) Infinite
c) 1 3.6 ev d) Zero

132. In the Bohr model of the atom, electrons revolve round in orbits, known as

a) stationary circular orbits
b) stationary elliptical orbits
c) Radiating circular orbits
d) Radiating elliptical orbits

133. The frequency of revolution of an electron in the n^{th} circular Bohr orbit of hydrogen is

a) Directly proportional to n^2
b) Inversely proportional to n
c) Inversely proportional to n^3
d) Directly proportional to n^3

134. The hydrogen atoms in a sample are in excited state, described by n=3. The number of spectral lines in emission spectrum will be

a) 1 b) 2
c) 3 d) 6

135. When α-particles are passed through thin foils

a) They all pass straight through
b) They are all deflected
c) Most of them are deflected
d) Most of them pass straight through

136. α-particles that come closer to nuclei

a) Are deflected more
b) Are deflected less
c) Make more collisions
d) Are slowed down more

137. Consider an electron in the n^{th} orbit of a hydrogen atom in the Bohr model. The circumference of the orbit can be expressed in terms of the de Broglie wavelength λ of that electron as

a) $(0.529)\, n\lambda$ b) $\sqrt{n}\,\lambda$
c) $(13.6)\,\lambda$ d) $n\lambda$

138. A hydrogen atom remains in its ground state when electron

a) Resides inside the nucleus
b) Escapes from the atom
c) Is in its first orbital
d) Does not orbit round, but is stationary

139. Any model of the atom must incorporate certain basic requirements. Which one of the following is not a basic requirement

a) The atom is electrically neutral

b) The atom has mass

c) The atom is stable

d) The size of the atom varies from element to element.

140. According to Bohr's theory the product of velocity of electron in the n^{th} orbit and the radius of the n^{th} orbit is

a) Inversely Proportional to the mass of the electron.

b) Directly proportional to the mass of the electron.

c) Independent of the mass of the electron

d) Directly proportional to the square of mass of the electron.

B. Very short answer type questions :

1. The atomic nucleus scatters α-particles at large angles but not the electrons. Explain.

2. Explain why the emission spectra of hydrogen possesses many lines even though it has only one electron.

3. Find out the regions in which Lyman and Paschen series of hydrogen spectrum fall.

4. If the frequency of the incident light is doubled on a metallic plate, will the kinetic energy of electrons be also doubled ?

5. How is photoelectric emission different from thermionic emission ?

6. Find out the de Broglie wavelength of 150 eV electron.

7. State limitations of Rutherford's atomic model.

8. How the quality and intensity of X-Rays are maintained separately in an X-Ray Machine ?

9. Derive an expression for the speed of electron in the n^{th} orbit.

10. What is a photon ? [CBSE 2001 C]

11. The wavelength of e.m radiation is doubled; how will the energy of a photon change ? [CBSE 1998 C, 93]

12. What is the energy in joules, associated with a photon of wavelength $4000A^0$? [CBSE 1994]

13. What is the stopping potential applied to a photocell if the maximum kinetic energy of a photoelectron is 5 eV ? [CBSE AI 2008 , CBSE 2001, 1996]

14. Express de Broglie wavelength associated with electron in terms of the accelerating voltage V. [CBSE AI 2007]

15. Write the expression for Bhor radius in hydrogen atom. [CBSE Delhi 2010]

16. The radius of the innermost orbit of hydrogen atom is 5.1×10^{-10} m. [CBSE Delhi 2010]

17. Define ionization energy. What is its value for hydrogen atom ? [CBSE AI 2010]

C. Numericals :

1. An electron in a hydrogen atom moves with a uniform speed in its stationary circular orbit of radius 5.3×10^{-11} m. Find (i) its speed and (ii) energy in its ground state.

2. A 10 kg satellite circles the earth once in two hours in an orbit of radius 8000 km. If Bohr's theory applies to the satellite-earth system, calculate the quantum number n.

3. The second member of the Balmer series

has a wavelength of 486.1nm. Calculate the wavelength of the first member.

4. A doubly-ionised Lithium atom is hydrogen-like with atomic number 3. (i) Find the wavelength of radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit. The ionisation energy of the hydrogen atom is 13.6 eV. (ii) How many spectral lines are observed in the emission of spectra of the above excited system ?

5. The wavelength of photon is 4000 A^0. Calculate its energy.

6. Light of wavelength 5000 A^0 falls on a sensitive surface. If the surface has received 10^{-7} joule of energy, then what is the number of photons falling on the surface ?

7. Calculate the de Broglie wavelength of an electron whose kinetic energy is 50 eV. $h = 6.62 \times 10^{-34}$ Joule. Sec. $m_0 = 9.1 \times 10^{-31}$ kg.

8. An electron and a proton are accel erated through the same potential difference. Find the ratio of their de Broglie wavelengths (λ_e / λ_p) in terms of their mass.

9. Use Bohr's postulate for permitted orbits to prove that the circumference of the nth permitted or bit for the electron can contain exactly n wavelengths of the de Broglie wavelength associated with the electron in that orbit.

[CBSE Sample Paper]

10. What is the longest wavelength photon that can ionize a hydrogen atom in its ground state ? Specify the type of radiation. [CBSE 2007]

11. In Bohr's theory of hydrogen atom, calculate the energy of the photon emitted during a transition of the electron from the first excited state to the ground state. Write in which region the spectrum lies.

[CBSE 2008 C]

D. Long Answer type Questions :

1. Explain Rutherford's scattering of α-particles. What information did it convey about atomic structure ?

2. What are the postulates of Bohr's theory of hydrogen atom ? Derive an expression for the energy of an electron in its n^{th} orbit.

3. Explain the meaning of the terms : energy level, excitation potential, ground state. Write down the names of the series obtained in hydrogen spectrum. Drawing the excited energy diagram of hydrogen atom, show these series on an energy-level diagram.

4. Write down and explain Einstein's photo-electric equation.

5. Describe experiments performed to illustrate photo-electric effect. Explain their significance.

6. What are the different types of photo-electric cells ? Describe them. What are their practical uses ?

7. Explain dualistic nature of light and matter. In a simple way, derive de Broglie's relation for matter-wave.

8. Discuss different atomic models and compare their merits and demerits.

9. a) Explain with a neat diagram how X-Rays are produced.

   b) Write about their Properties and use.

E. Answer as directed :

1. The threshold frequency for photoelectric emission from platinum is greater than that of zinc which in turn is greater than that of sodium. An incident radiation of frequency $\gamma > \gamma_0$ (zn) _____ cause photoemission from sodium but _____

cause photoemission from platinum.

2. The threshold wavelength of a photo cathode to emit photoelectrons is λ_0. The average KE of the photoelectron is ____, when a radiation of wavelength λ is incident upon it.

3. The ratio of the areas within the electronic orbits for the first excited state to the ground state is ____.

4. In Rutherford's experiment of scattering of α-particles, transfer of maximum energy is possible only when the scattering angle is ____.

5. What is the trajectory of scattered α-particles ?

6. The toal energy of an electron in an atom is always negative (True/False)

7. If the electron is in n = 5 state, then the number of possible transitions it can make is ____.

8. Choose the correct one : An atom emits a photon when one of its orbital electrons (a) jumps from a lower to a higher orbit (b) jumps from higher to lower orbit.

9. Choose the correct one : Band spectrum is obtained whenever the incandescent vapours at low pressure of the excited substance are in their : (a) atomic state (b) molecular state (c) nuclear state (d) none of these.

F. Correct the following sentences :

1. The energy of an electron in the first orbit of hydrogen atom is -14 eV.

2. The trajectory of α-particle in Rutherford's scattering experiment is parabolic.

3. Emission spectra are due to transition of electrons from lower orbit to higher orbit.

4. Balmer series in H-spectra arises due to transition of electrons from higher orbits to third orbit.

5. Einstein's photoelectric equation is $h\upsilon = m v_m^2/2$, where υ is the frequency of incident light, v_m is the maximum velocity of ejected electron.

6. The de Broglie relation is $p = h\lambda$, where p is the momentum of the particle, h is the Planck's constant and λ is the wavelength associated with the particle.

7. The energy associated with a radiation is given by $E = h/\upsilon$.

ANSWERS

A. Multiple Choice Type Questions :

1. (c)	**2.** (d)	**3.** (b)	**4.** (c)	**5.** (b)	**6.** (a)	**7.** (b)	**8.** (d)
9. (b)	**10.**(a)	**11.**(c)	**12.**(b)	**13.**(d)	**14.**(b)	**15.**(d)	**16.**(a)
17.(c)	**18.**(d)	**19.**(d)	**20.**(b)	**21.**(b)	**22.**(c)	**23.**(d)	**24.**(c)
25.(b)	**26.**(c)	**27.**(a)	**28.**(b)	**29.**(b)	**30.**(c)	**31.**(b)	**32.**(a)
33.(c)	**34.**(b)	**35.**(b)	**36.**(c)	**37.**(a)	**38.**(b)	**39.**(b)	**40.**(a)
41.(d)	**42.**(d)	**43.**(c)	**44.**(c)	**45.**(c)	**46.**(c)	**47.**(c)	**48.(c)**
49.(b)	**50.**(d)	**51.**(a)	**52.**(a)	**53.**(c)	**54.**(a)	**55.**(b)	**56.(a)**
57.(a)	**58.**(c)	**59.**(a)	**60.**(a)	**61.**(a)	**62.**(d)	**63.**(c)	**64.**(b)
65.(c)	**66.**(a)	**67.**(b)	**68.**(c)	**69.**(b)	**70.**(b)	**71.**(d)	**72.**(c)
73.(c)	**74.**(d)	**75.**(d)	**76.**(a)	**77.**(d)	**78.**(b)	**79.**(c)	**80.**(c)
81.(d)	**82.**(d)	**83.**(c)	**84.**(b)	**85.**(c)	**86.**(c)	**87.**(d)	**88.**(d)
89.(c)	**90.**(b)	**91.**(b)	**92.**(d)	**93.**(c)	**94.**(b)	**95.**(b)	**96.**(c)
97.(a)	**98.**(c)	**99.**(c)	**100.**(c)	**101.**(b)	**102.**(b)	**103.**(c)	**104.**(d)
105.(b)	**106.**(d)	**107.**(a)	**108.**(d)	**109.**(b)	**110.**(a)	**111.**(b)	**112.**(b)
113.(b)	**114.**(d)	**115.**(a)	**116.**(c)	**117.**(c)	**118.**(c)	**119.**(b)	**120.**(b)
121.(d)	**122.**(c)	**123.**(c)	**124.**(c)	**125.**(b)	**126.**(a)	**127.**(a)	**128.**(c)
129.(d)	**130.**(d)	**131.**(c)	**132.**(a)	**133.**(c)	**134.**(c)	**135.**(d)	**136.**(a)
137.(d)	**138.**(c)	**139.**(d)	**140.**(a)				

B. Very short type questions :

6. 1 A^0

C. Short Answer type questions :

1. 6.95×10^8 m/Sec.
- 13.6 ev

2. 5.3×10^{21}

3. 656.24 nm

4. 113.74 A^0, 3 spectral lines $3 \rightarrow 1,\ 3 \rightarrow 2,\ 2 \rightarrow 1$

5. 4.95×10^{-19} Joule

6. 2.5×10^{11}

7. 1.73 A^0

8. $\sqrt{m_p / m_e}$

10. 0.910×10^{-10}m,

11. 10.2 eV, belongs to Lyman series.

E. **1.** will certainly : may or may not **2.** $hc\left[\left(\frac{1}{\lambda}-\frac{1}{\lambda_0}\right)\right]$ **3.** 16:1 **4.** 90^0 **5.** Hyperbola **6.** True

7.10 **8.** (b) **9.** (b)

17

Nuclear Physics

In nuclear physics, we study the physics of the atomic nucleus and subatomic particles, mostly in the context of nuclear energy.

Rutherford (1911) performed experiments by bombarding alpha particles on thin metallic goldsheet. After careful observation he propounded an atomic model, known as "Rutherford Atomic Model". According to this model atom has two parts, i) Nucleus and ii) Extra nuclear part.

Nucleus is a hard core having a strong positive charge, which is situated at the center of an atom. The radius of the nucleus is of the order of one fermi ($\approx 10^{-15}$ m). When we compare this dimension of the nucleus with that of an atom ($\approx 10^{-10}$ m), we find that the nucleus is 10^{-5} times smaller. Hence the nucleus is an extremely small part of an atom - the extra nuclear part (i.e., the space beyond the nucleus). of an atom being very very large and electrons revolve round the nucleus with very high speed in fixed, stationary orbits. There is enough empty space between the nucleus and the orbiting electrons. It is to be mentioned that the nuclueus is very heavy and contains almost all the mass of an atom, whereas the orbiting electrons are very light and have almost negligible mass, when compared with that of the nucleus.

17.1 Structure of nucleus :

Protons and neutrons are the basic building blocks of a nucleus. They are also called nucleons.

A proton has a positive charge of same magnitude as that of an electron. It has a mass(m_p) which is about 1836 times the electronic mass(m_e).

Charge of proton : $e = 1.6 \times 10^{-19}$C.

Mass of the proton:

$m_p = 1.6726 \times 10^{-27}$Kg $= 1.007273$u.

['u' called unified atomic mass unit and u= 1.66 x 10^{-27}Kg].

A neutron is electrically neutral. Its mass is almost equal but slightly more than the mass of the proton.

$m_n = 1.6749 \times 10^{-27} Kg = 1.00866u$. A neutron is stable inside the nucleus but is unstable infree state.

The number of protons in the nucleus determines the atomic number (z) of the nuclide. The number of neutrons in the nucleus is denoted by N. the sum of number of protons (Z) and number of neutrons (N) is called mass number or nucleon number (A).

$$A=N+Z \quad ...17.1.1$$

Mass number (A) is an integer.

$${}^{A}_{Z}X$$

Where X is the Chemical symbol of the element.

[Eg. $^{35}_{17}Cl$: it represents Chlorine with atomic number Z=17 and mass number A=35 no. of neutrons N=A-Z=18]

17.2 Size of the nucleus :

It is found that nucleus is not a point-particle; but it has finite volume, even though it may be small. The shape of most of the nuclei is spherical. We know that as the number of nucleons inside a nucleus increases, its volume correspondingly increases. Hence volume and mass number are related to each other. Mathematically :

Volume α mass number

i.e. $V \alpha A$

or, $\frac{4}{3}\pi R^3 \alpha A$ where R = Radius of the nucleus

$\therefore R^3 \alpha A$

or $R \alpha A^{1/3}$

$$\therefore R = r_o A^{1/3} \qquad ...17.2.1$$

where r_o = proportionality constant, having the dimension of length. It is called nuclear radius parameter. It is experimentally calculated that

$$r_o = 1.2 \times 10^{-15} m = 1.2 \text{ fm}$$

$(\because 1 \text{ fermi} = 10^{-15} m)$

Eq. 17.2.1 shows that radius of a nucleus is proportional to the cube root of the mass number.

17.3 Nuclear density : $\int nu = \frac{Nuclear\ mass}{Nucleus\ Volume}$

$$\int nu = \frac{Am_n}{\frac{4}{3}\pi R^3}$$

Where m_n is the mass of a nucleon $(\because m_p \approx m_n)$ which is about 1.67×10^{-27} Kg; Am_n is the mass of the nucleus.

$$\int nu = \frac{3A_{m_n}}{4\pi r_0^3 A} (\because R = r_0 A^{1/3})$$

$$\int nu = \frac{3_{m_n}}{4\pi r_0^3} \qquad ...17.3.1$$

As the density of the nucleus is ind.endent of (A), the value is almost same for all nuclei. Putting the standard values of m_n and r_0 we can see that

$$\int nu = 1.82 \times 10^{17} kgm^{-3}$$

It is unusually a large quantity; indicating that the nuclear matter is in highly compressed state.

17.4. Nuclear angular momentum and magnetic moment : Since the nuclear particles are in motion, they possess angular momentum. Further a circulating charge gives rise to a current. Thus a magnetic moment is associated with this current. As in the case of electrons the angular momentum is quantised.

The magnetic moment of a proton

$$= 2.79\mu_N$$

where $\mu_N = \frac{eh}{2m_p}$

= nuclear magneton

and the magnetic moment of a neutron = $-1.91\mu_N$

17.5. Mass : The total mass of the nucleus is less than the total mass of its constituents. This difference in mass is known as the mass defect which in the basic reason to explain why the constituents of the nucleus are held tightly together.

17.6 Nuclear force :

There are four types of fundamental forces in nature. These are

1. Gravitational
2. Electromagnetic

3. Strong nuclear

4. Weak nuclear

The stability of the nucleus cannot be explained on the basis of gravitational and electromagnetic interaction between the nucleons. The gravitational interaction is the weakest of all the forces and hence cannot be responsible to keep the nucleons tight together. The electromagnetic interaction is charge dependent. Hence neutrons do not take part in such interactions. More over protons repel under electrostatic force. Weak nuclear force comes into play during -decay inside the nucleus.

In order to explain the stability of the nucleus a strong attractive nuclear force operating between the nucleons was suggested by Yukawa. This nuclear force has the following characteristic properties.

1. Nuclar force is strong and attractive.

It is the strongest of all the forces. It is about 1040 times stronger than gravitational force and about 100 times stronger than electromagnetic force. it is strong enough to overcome electrostatic repulsion between protons.

2. Nuclear force is a short range force.

This force is appreciate only within a distance of 10^{-15}m and vanishes for all practical purposes at distances greater than 10^{-15}m.

3. Nuclear force is charge independent. The nuclear forces betweentwo protons or between two nutrons or between a proton and neutron are same.

4. The Nuclear force is spin dependent and assymmetric stronger between two nucleons having parallel spin and is weaker for antiparallel spin particles.

5. Nuclear force has the property of saturation; i.e.; in a nucleus any one nucleon interacts with a limited number of other nucleons nearest to it.

6. Nuclear force is non central i.e., it does not act along the line joining the two nucleons.

7. Nuclear forces involve exchange of meson between the interacting pair of the nucleons of a nucleus. This has been explained by Yukawa (1935), who formulated a theory by introducing a new particle, called $\pi -$ meson. All nucleons (protons or neutrons) are surrounded by a cloud, created by one or more $\pi -$ mesons. However the core of all nucleons are identical. There is a continuous exchange of these mesons between neighbouring nucleons. This exchange of mesons gives rise to nuclear force.

To illustrate yukawa's theory, we give the following examples :

(i) when π^+ meson jumps from a proton to neutron, the proton is converted into neutron and the neutron is converted to proton :

$$p - \pi^+ \rightarrow n$$

$$\text{and} \quad n + \pi^+ \rightarrow p$$

(ii) When a π^- meson jumps from a neutron to proton, the neutron is converted into proton and proton is converted into neutron.

$$n - \pi^- \rightarrow p$$

$$\text{and} \quad p + \pi^- \rightarrow n$$

(iii) When we consider two protons (instead of a neutron and proton in the above two cases), neutral meson (π^0) is exchanged between them.

(iv) Similarly when we consider two neutrons, neutral meson is exchanged between them.

In all the above cases, it is observed that, on the whole, in a given nuclear structure, the number of proton and neutrons does not change, even though continuous activity goes on with the help of mesons.

17.7 Isotope, Isotone, Isobar, Isomer and Mirror Nuclide :

(a) Isotope : Nuclei having same atomic number but different mass number, are called isotopes.

(i) Isotopes have the same number of protons but different number of neutrons in their respective nucleus.

(ii) They occupy the same position in the periodic table.

(iii) They cannot be separated by chemical process.

(iv) They have different nuclear properties

Example : Sodium has three isotopes $\left({}_{11}Na^{22}, {}_{11}Na^{23}, {}_{11}Na^{24}\right)$

${}_{11}Na^{22} \rightarrow$ Decays by Beta - emission

${}_{11}Na^{23} \rightarrow$ stable

${}_{11}Na^{24} \rightarrow$ Decays by positron emission.

other examples of isotopes :

(i) ${}_{8}0^{16}, {}_{8}0^{17}, {}_{8}0^{18}$

(ii) ${}_{17}Cl^{35}, {}_{17}Cl^{37}$

(iii) ${}_{3}Li^{6}, {}_{3}Li^{7}$

(b) Isotone : nuclei have the same number of neutrons :

Example : (i) ${}_{1}H^{2}, {}_{2}H^{3}$

(ii) ${}_{1}H^{3}, {}_{2}H^{4}$

(iii) ${}_{7}N^{17}, {}_{8}O^{18}, {}_{9}F^{19}$

(iv) ${}_{11}Na^{23}, {}_{12}Mg^{24}$

(v) ${}_{6}C^{14}, {}_{7}N^{15}$

(c) Isobar : Atoms having some mass - number, but different atomic number are called isobars.

(i) The number of nucleons is same. Hence atomic weight is same.

(ii) The number of revolving electrons are different in each case.

(iii) Their Chemical properties are different. No stable isobars are found among the higher elements.

Example :

(i) ${}_{1}H^{3}$ ${}_{2}He^{3}$

(ii) ${}_{32}Ge^{76}, {}_{34}Se^{76}$

(d) Isomer : Atoms have the same mass number (A) and same atomic number (z), but their radioactive properties are different.

Example ; Uranium $X_2 \left({}_{91}U^{234}\right)$

Uranium $Z_1 \left({}_{91}U^{234}\right)$

(e) Mirror Nuclide : Two isobars are called mirror nuclides, if the number of protons (z) of one of the nuclei is equal to number of nutrons (N) of the other.

Example : (i) ${}_{7}N^{13} \because {}_{6}C^{13}$

(ii) ${}_{6}C^{11}, {}_{5}B^{11}$

17.8 Mass-Energy conservation and nuclear energy : Mass-defect :

Before Albert Einstein's theory, it was supposed that mass and energy are completely different entities. Accordingly we had two conservation- laws : (i) conservation of mass or matter and (ii) conservation of energy.

According to conservation of matter, the total quantity of matter in the universe remains constant. Similarly the conservation of energy states that the total amount of energy remains constant and conserved in the universe. It can be neither created nor destroyed, even though it can be changed from one form to another.

However Einstein unified these two separate conservation laws and propounded the principle of equivalence of mass and energy. This states that the mass of a body is a measure of the quantity of energy contained in it. The mass of a material body can be converted into energy completely. Similarly vice versa is also equally correct, i.e. energy, in its turn, can also be transformed into matter.

Einstein's principle is mathematically expressed as :

$$E = mc^2$$

Where 'm' is the mass annihilated and c is the speed of light.

This equation can be applied to nuclear physics. It is found that the actual mass of any nucleus is different from the total mass of its constituents. The difference between these two masses is known as mass - defect.

To understand this, let us consider a specific case, say, deuterium ($_1H^2$).

Deuterium has one proton and one neutron. In terms of atomic mass unit (u),

Mass of 1 proton = 1.008 142 u

and " 1 neutron = 1.008982 u

$\therefore$ Total mass of the constituents

= 1.008142

+ 1.008982

= 2.017124 u ...(i)

But the mass of Deuterium (Nucleus)

= 2.014735 u ...(ii)

Comparing (i) and (ii), we note that the mass of deuterium (ii) is less than the mass of the constituents (i).

$\therefore$ The difference in mass (= mass defect) = Δm (say)

= 2.014735 - 2.017124

= - 0.002389 u

This mass defect accounts for the energy, which the nucleus possesses, because from the mass defect we can calculate energy, by using Einstein's equation :

$$\Delta E = \Delta m . c^2$$

Mass defect can be expressed in a general from : consider a nucleus, having mass number A and atomic number Z.

Then no of protons = Z

and no of neutrons = A - Z

$\therefore$ Total mass of the constituent nucleons

$$= Zm_p + (A - Z)m_n$$

Let mass of nucleus = M

$\therefore$ Mass defect of the nucleus = Δm

$$= Zm_p + (A - Z)m_n - M \quad ...17.8.1$$

Mass defect of an atom is obtained by adding mass of the electrons (Zm_e) to the Right Hand Side of eq. 17.8.1.

17.9 Packing fraction :

Packing fraction = mass defect per nucleon.

Mathematically, Packing fraction = $\frac{\Delta m}{A}$

Packing fraction measures the stability of a nucleus. If the packing fraction is negative, the nucleus will be stable.

17.10 Binding energy :

Binding energy of a nucleus is defined as the energy required to break up a nucleus into its constituent protons and neutrons and place them far apart. In other words, it represents the energy required to bind the protons and neutrons to form a nucleus.

Mathematically,

Binding energy of an atom

$= [Zm_p + Zm_e + (A-Z)m_n$

$-\text{Mass of atom}]c^2$ ...17.10.1

Binding energy of nucleus

$= [Zm_p + (A-Z)m_n$

$-\text{Mass of atom}]c^2$ ...17.10.2

17.11 Biding energy per nucleon

Binding energy per nuceon

$$= \frac{\text{Binding energy}}{\text{Mass number}} = \bar{E}_B(\text{say})$$

(We divide by mass number, since mass number is equal to the no of nucleons).

Binding energy per nucleon gives the average energy, necessary to take away one nucleon from the nucleus. This binding energy pernucleon is the measure of stability of the nucleus.

17.12 Binding energy curve

The binding energy per nucleon expresses the stability of a nucleus. So it is important to draw a Binding energy curve between mass number A and binding energy per nucleon.

For this we may take the help of a table, with data covering a widespread spectrum of elements. In the following table a few important elements have been enlisted :

Nuclei	Binding energy (Mev)	Binding energy per nucleon (Mev)
$_1H^2$	2.22	1.11
$_2He^4$	28.3	7.075
$_3Li^7$	37.66	5.38
$_{17}Cl^{35}$	287.7	8.22
$_{27}Fe^{56}$	492	8.8
$_{92}U^{235}$	1786	7.6

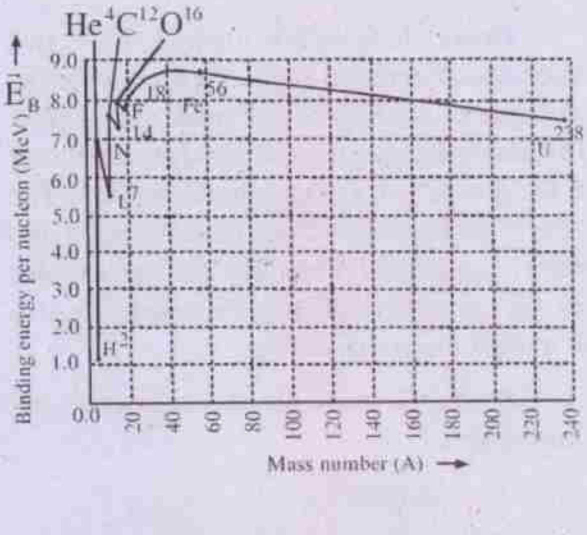


Fig. 17.1 : Binding Energy Curve $(\bar{E}_B \sim A)$

(1) The Binding energy per nucleon $(\overline{E}_B)$ for very light nuclei like $_1H^2$ is very small ($\approx$ 1Mev).

(2) In the region A<20, the variation of $\overline{E}_B$ is very irregular. There are subsidiary peaks at $_2He^4$, $_6C^{12}$ and $_8O^{16}$. These peaks show that the corresponding nuclei are more stable and better bound than their neighbours.

(3) For A>20, $\overline{E}_B$ increases slowly from 8 to 8.8 MeV/nucleon approximately. $\overline{E}_B$ has maximum value of 8.8 MeV/nucleon for $_{26}Fe^{58}$. So iron-nucleus is the strongest bound.

(4) $\overline{E}_B$ for most of the elements is about 8. But it is less for lighter and heavier nuclei.

(5) A>120, $\overline{E}_B$ decreases. For uranium, it is 7.6 MeV/nucleon. This means that heavy nuclei cannot control the coulomb repulsion between protons. These nuclei break up to give out powerful radiation. So they are unstable and become radioactive sources. As these heavier nuclei split into lighter nuclei, this causes fission.

(6) Nuclei, with A<58, have low $\overline{E}_B$. So they try to be stable by fusing together and releasing energy. This explains the process of fusion in these cases. Note that in both the processes of fission and fusion, achievement of greater Binding energy results in liberation of energy.

Example 17.1 What is atomic mass unit ? Express its value in different units.

Solution :

The subatomic particles have small mass. Hence unified mass units symbolised by u, is used.

$$1u = \frac{1}{12} \times \text{Mass of pure carbon atom } {}_6C^{12}$$

But Mass of one carbon atom $= \frac{0.012}{N}$ kg

where N = Avogadro's number

$$= 6.02 \times 10^{23}$$

$$\therefore 1u = \frac{1}{12} \times \frac{0.012}{6.02 \times 10^{23}}$$

$$= 1.66 \times 10^{-27} \text{ kg}$$

Energy equivalent : since $E = mc^2$

We get $E = 1.66 \times 10^{-27} \times (3 \times 10^8)^2$ J

$= 1.49 \times 10^{-10}$ J

But we know 1.6×10^{-19} J = 1 eV

$$\therefore E = \frac{1.49 \times 10^{-10}}{1.6 \times 10^{-19}} = 0.931 \times 10^9 \text{eV}$$

$= 931$ MeV

Hence energy of IU is

E $= 1.49 \times 10^{-10}$ J

otherwise

E $= 931$ MeV.

17.2 Calculate the binding energy per nucleon of ${}_{17}Cl^{35}$

Given : Mass of chlorine nucleus

= 34.96 u

Mass of proton = 1.007825 u

Mass of neutron = 1.008665 u

Solution : No. of protons = 17

No. of neutrons = 35 - 17 = 18

∴ Binding energy per nucleon

$$= \left[\frac{(17 \times 1.007825) + (1.008665 \times 18) - 34.96}{35}\right] \times 931$$

= 8.48 MeV/nucleon.

17.13 Nuclear Reaction :

In a broader sense nuclear reaction means the process that occurs when a nuclear particle such as proton, neutron. deuteron, α – particle, a nucleus etc. come in close contact (within 10^{-15}m) with another nucleus, resulting in any one or more of the rearrangements of momenta, energy and nucleons.

For example we can have

(i) *Elastic scattering :* In this process same particles are scattered in different directions and there is no loss of energy. Such a reaction is represented as

$$a + X \rightarrow a + X \quad ...17.13.1$$

e.g. ${}_6C^{12} + {}_0n^1 \rightarrow {}_6C^{12} + {}_0n^1$

(ii) *Inelastic scattering :* In this process the same particles are scattered in different directions with different energy, as there is loss of energy due to collision. This reaction is represented as

$$a + X \rightarrow X^* + a \quad ...17.13.2$$

(iii) *Radiative capture :* In this process the incident particle is absorbed or captured by the target nucleus to form an excited compound

nucleus which disintegrates to produce one or more γ (photons) rays and goes down to the ground state.

$$a + X \rightarrow Y^* \rightarrow Y + \gamma \qquad ...17.13.3$$

(one or more)

(iv) Disintegration (transmutation) : In this process the final particles are different from the initial particles. This is represented as

$$a + X \rightarrow C^* \rightarrow Y + b \qquad ...17.13.4$$

or

X (a,b) Y.

In the restricted sense this reaction is commonly termed as *nuclear reaction.* This disintegration is supposed to occur in two stages. In the first stage an unstable compound nucleus of life-time nearly ~ 10^{-16} S is formed. A compound nucleus could be formed in various ways.

e.g.

$${}_7N^{13} + {}_0n^1 \rightarrow {}_7N^{14^*} \qquad (10.5 \text{ MeV})$$

$${}_6C^{13} + {}_1H^1 \rightarrow {}_7N^{14^*} \qquad (7.5 \text{ MeV})$$

$${}_6C^{12} + {}_1H^2 \rightarrow {}_7N^{14^*} \qquad (10.3 \text{ MeV})$$

$${}_5B^{10} + {}_2He^4 \rightarrow {}_7N^{14^*} \qquad (11.6 \text{ MeV})$$

In the second stage the compound nucleus decays. The decay can occur in one or more different ways depending on its excitation energy. For example a 12 MeV ${}_7N^{14^*}$ can decay via reactions like

$${}_7N^{14^*} \rightarrow {}_7N^{13} + {}_0n^1$$

$$\rightarrow {}_6C^{13} + {}_1H^1$$

$$\rightarrow {}_6C^{12} + {}_1H^2$$

$$\rightarrow {}_5B^{10} + {}_2He^4$$

$$\rightarrow {}_7N^{14} + \gamma$$

(one or more)

A few examples of nuclear reactions are given below

(i) (α, p) reaction :

$${}_2He^4 + {}_5B^{10} \rightarrow {}_7N^{14^*} \rightarrow {}_6C^{13} + {}_1H^1$$

symbolically represented as

$${}_5B^{10}(\alpha, p)\ {}_6C^{13}$$

(ii) (p, α) reaction :

$${}_1H^1 + {}_5B^{11} \rightarrow {}_6C^{12^*} \rightarrow {}_4Be^8 + {}_2He^4$$

symbolically represented as

$${}_5B^{11}(p, \alpha)\ {}_4Be^8$$

(iii) (n, α) reaction :

$${}_0n^1 + {}_5B^{10} \rightarrow {}_5Be^{11^*} \rightarrow {}_3Li^7 + {}_2He^4$$

symbolically represented as

$${}_5B^{10}(n, \alpha)\ {}_3Li^7$$

(iv) (d, α) reaction :

$${}_1H^2 + {}_7N^{14} \rightarrow {}_8O^{16^*} \rightarrow {}_6C^{12} + {}_2He^4$$

symbolically represented as

$${}_7N^{14}(d, \alpha)\ {}_6C^{12}$$

(v) (γ, n) reaction (photo disintegration)

$$\gamma + {}_4Be^9 \rightarrow {}_4Be^{9^*} \rightarrow {}_4Be^8 + {}_0n^1$$

symbolically represented as

$${}_4Be^9(\gamma, n)\ {}_4Be^8$$

17.13(A) Laws Governing Nuclear Reactions

Nuclear reactions take place obeying the following laws :

(i) Conservation of energy

The total energy including the rest mass energies of all the nuclei taking part in the nuclear reaction and their kinetic energies must equal the sum of rest-mass energies and kinetic energies of the products and the energy released.

i.e.

(Rest-mass energy + K.E.) of initial particles = (Rest-mass energy + K.E.) of product particles + energy released.

(ii) ***Conservation of linear momentum :***

The sum of the linear momentum vectors of the reacting (initial) particles must be equal to the sum of the linear momentum vectors of the product (final) particles.

i.e. $\vec{p}_x + \vec{p}_a = \vec{p}_y + \vec{p}_b$

(iii) ***Conservation of angular momentum :***

In any nuclear reaction the total angular momentum of the initial nuclei (reacting nuclei) is equal to the total angular momentum of the product nuclei (final nuclei)

i.e. $\vec{J}_x + \vec{J}_a = \vec{J}_y + \vec{J}_b$

where $\vec{J} = \vec{L} + \vec{S}$, with $\vec{L}$ representing orbital angular momentum and $\vec{S}$ representing spin angular momentum.

(iv) ***Conservation of charge :***

The sum total of charge of initial (reacting) nuclei is equal to the sum total of charge of the final (product) nuclei.

i.e $Q_X + Q_a = Q_y + Q_b$

This conservation law is also called conservation of atomic number.

(v) ***Conservation of mass number :***

The total number of nucleons in the nuclei in a nuclear reaction remains unaltered after reaction.

i.e

total number of reacting nucleons
= total number of product nucleons

(vi) ***Conservation of spin :***

Spin is conserved in a nuclear reaction (i.e. statistics remain unaltered).

(vii) ***Conservation of Parity :***

Parity remains same before and after reaction, except for β-decay.

(viii) ***Conservation of isotopic spin :***

Isotopic spin is same before and after reaction.

$I_x + I_a = I_y + I_b$

17.13 (B) Energetics of nuclear reaction (Q-value of nuclear reaction) :

Consider a nuclear reaction X(a,b) Y represented by the equation

$$a + X \rightarrow Y + b$$

where,

X is the target nucleus, of rest mass M_X^o

a is the incident particle of rest mass m_a^o

Y is the product or recoil nucleus of rest-mass M_Y^o

b is the product particle of rest-mass m_b^o

Thus **before reaction**

energy of a = restmass energy + K.E.

$$= m_a^o C^2 + E_k^a$$

energy of X = $M_X^o C^2 + 0$

Hence total energy before reaction is

$$E_i = (m_a^o + M_x^o)C^2 + E_k^a \qquad 17.13.5$$

Then **after reaction**

energy of Y = $M_y^o C^2 + E_k^y$

energy of b = $m_b^o C^2 + E_k^b$

Hence total energy after reaction

$$E_f = (m_b^o + M_y^o)C^2 + E_k^y + E_k^b \qquad 17.13.6$$

As energy is conserved in nuclear reaction, so we must have

$$E_i = E_f$$

giving

$$(m_a^o + M_x^o)C^2 + E_k^a = (m_b^o + M_y^o)C^2 + E_k^y + E_k^b \quad ...17.13.7$$

Eqn. 17.13.7 gives

$$E_k^y + E_k^b - E_k^a = \left[(m_a^o + M_x^o) - (m_b^o + M_y^o)\right]C^2 \quad ...17.13.8$$

The quantity on l.h.s. of eqn. 17.13.8 is the difference between the kinetic energy of the products of reaction and that of the (initial) interacting particles. This quantity is usally called Q-value. Hence

$$Q = \text{final K.E. - initial K.E.}$$

$$= E_k^y + E_k^b - E_k^a$$

$$= \left[(m_a^o + M_x^o) - (m_b^o + M_y^o)\right]C^2$$

$$\Rightarrow \quad Q = \Delta M \,.\, C^2 \quad ...17.13.9$$

Where ΔM = sum of initial rest masses - sum of final rest masses

Case (i) Endothermic (endoergic) reaction :

A nuclear reaction in which Q is negative is called endothermic. Eqn. 17.13.9 gives

$$m_a^o + M_x^o < M_y^o + m_b^o$$

i.e. sum of the rest-masses of product particles is greater than the sum of the rest-masses of the reactant particles.

Caste (ii) Exothermic (exoergic) reaction :

A nuclear reaction in which Q is positive is called exothermic. Eqn. 17.13.9 gives

$$m_a^o + M_x^o > M_y^o + m_b^o$$

i.e. sum of the rest-masses of the product particles is less than the sum of the rest masses of the reactant particles.

17.13 (C) Cross-Section

One of the most important parameter in the study of nuclear reactions is reaction cross-section; symbolised as σ. This cross-section (σ) is a measure of the probability that a bombarding particle will interact in a certain way with a target particle.

One imagines that each target presents a certain area called its cross-section, to the incident particles. Any incident particle directed at this area interacts with the target particle. Hence greater the cross-section greater is the likelihood of an interaction. The interaction cross-section of a target particle varies with the nature of the process involved and with the energy of the incident particle. It may be greater or less than the geometrical cross-section.

Suppose we have a slab of some material whose area is A and whose thickness is dx. If the material contains 'n' atoms per unit volume, then there are a total of nAdx nuclei in the slab. Each nucleus has cross-section σ, for some particular interaction, so that aggregate cross-section of all the nuclei in the slab is $nA\sigma dx$. If there are N incident particles in a bombarding beam, the number dN that interact with the nuclei in the slab is therefore specified by

$$\frac{\text{No. of interacting particles}}{\text{No. of incident particles}} = \frac{\text{aggregate cross-section}}{\text{target area}}$$

$$\Rightarrow \quad \frac{dN}{N} = \frac{nA\sigma\, dx}{A} = n\sigma\, dx$$

$$\Rightarrow \quad \sigma = \frac{dN}{N.n.dx} \quad ...17.13.10$$

Eqn. 17.13.10 shows that 'σ' has the dimension of area. Since nuclear radius is ~10^{-14} to 10^{-15}m, so σ is ~10^{-28} m^2. The common unit for cross-section is a barn.

$$1 \text{ barn} = 10^{-28} \text{ m}^2$$

17.13 (D) Discovery of Neutron :

The nuclear model of atom suggested for two electrons is He-atom. However its atomic weight measurements required four protons, if protons were the only constituents of nucleus; contrary to the requirement of two protons to fulfil the charge neutrality of the atom. This indicated that there might be two neutral particles having mass of same order as proton, inside the nucleus.

In 1930, German physicists W. Bothe and H. Becker while bombarding Beryllium with α-particles from a sample of polonium, detected highly penetrating neutral radiations of energy ~ 7 MeV. Quite naturally they assumed the radiation to be Gamma rays.

Later I. Curie and F. Joliot observed that when these emitted rays were passed through a slab of paraffin (a hydrogen rich substance), protons of energy ~ 5.7 MeV were knocked out. Curie and Joliot concluded that protons were knocked out by γ-rays and suggested the reaction

$$_2He^4 + _4Be^9 \rightarrow _6C^{13} + \gamma$$

But calculations indicated that in order to knock out protons of energy ~ 5.7 MeV, the energy of γ-rays should be ~ 55 MeV, contrary to experimental value of ~ 7 MeV.

In 1932 James Chadwick proposed an alternative hypothesis. He assumed that the radiation consisted of neutral particles whose mass is of the same order as protons. He named this particle "neutron". So according to Chadwick

$$_2He^4 + _4Be^9 \rightarrow _6C^{12} + _0n^1$$

Properties of neutrons

(i) It has no charge and mass is equal to 1.6747×10^{-27} kg $\approx$ 1.0086643 u. (slightly greater than the proton mass)

(ii) As it is charge neutral, it does not ionise gaseous medium while passing through it.

(iii) They can pass through electron cloud without deflection.

(iv) Neutrons lose energy on passing through matter as a result of collision with nuclei.

(v) Inside the nucleus neutron is stable; but free neutron is unstable and decays.

(vi) As neutron is charge neutral, it can penetrate into a target. So neutrons are better probes.

Ex 17.13.1: Complete the following nuclear reactions

(i) $_1H^2 + _1H^3 \rightarrow ? + _2He^4$

(ii) $_5B^{10} + _1H^1 \rightarrow _6C^{10} + ?$

(iii) $_1H^2 + _{11}Na^{22} \rightarrow ? + _2He^4$

(iv) $_2He^4 + ? \rightarrow _8O^{17} + _1H^1$

Soln.

(i) $_1H^2 + _1H^3 \rightarrow _0n^1 + _2He^4$

(ii) $_5B^{10} + _1H^1 \rightarrow _6C^{10} + _0n^1$

(iii) $_1H^2 + _{11}Na^{22} \rightarrow _{10}Ne^{20} + _2He^4$

(iv) $_2He^4 + _7N^{14} \rightarrow _8O^{17} + _1H^1$

Ex. 17.13.2 : Calculate the energy released in the reaction

$$_3Li^7 + _1H^1 \rightarrow _2He^4 + _2He^4$$

Given mass of $_3Li^7$ = 7.0160 u, mass of $_2He^4$ = 4.0026 u and mass of $_1H^1$ = 1.007825 u.

Soln.

$$Q = \Delta M.C^2 = \left[(m_{H}^{0} + m_{Li}^{0}) - 2m_{He}^{0}\right]C^2$$

$$= [(1.007825 + 7.016) - 2 \times 4.0026] \times 931 \text{MeV}$$

$$\simeq 17.34 \text{ MeV}.$$

Ex. 17.13.3 : Calculate the Q-value in MeV of the following reactions

(i) $_1H^2 + _1H^2 \rightarrow _1H^3 + _1H^1$

(ii) $_1H^2 + _1H^2 \rightarrow _2He^3 + _0n^1$

Given mass of $_1H^2$ = 2.014102 u, $_1H^3$ = 3.016050 u, $_1H^1$ = 1.007825 u, $_2He^3$ = 3.016030 u, $_0n^1$ = 1.008665 u

Soln.

(i) $Q = \Delta M.C^2$

$= [2 \times 2.014102 - (3.016050 + 1.007825)]$ x 931 MeV

$= 4.03$ MeV

(ii) $Q = [2 \times 2.014102 - (3.016030 + 1.008665)]$ x 931 MeV

$= 3.27$ MeV

17.14 : Nuclear Fission

Nuclear fission is a special type of nuclear process by which a heavy nucleus splits into two or more lighter nuclei of intermediate mass numbers (comparable size) either spontaneously or through the absorption of projectile like neutrons, protons, deuterons, α-particles and even electrons and γ-rays. But nuclear fission induced by neutron has acquired practical importance.

e.g.

(i) Spontaneous fission :

$$_{92}U^{235} \rightarrow _{56}Ba^{144} + _{36}Kr^{89} + 2_0n^1 + Q$$

(ii) Induced fission :

$$_{92}U^{235} + _0n^1 \rightarrow _{92}U^{236*}$$
$$\rightarrow _{56}Ba^{141} + _{36}Kr^{92} + 3_0n^1 + Q$$
$$\rightarrow _{56}Sb^{133} + _{41}Nb^{99} + 4_0n^1 + Q$$
$$\rightarrow _{54}Xe^{140} + _{38}Sr^{94} + 2_0n^1 + Q$$

However the probability of spontaneous fission in a nucleus with A < 240 is generally low.

A fission process takes place in three distinct stages as illustrated in fig. 17.2.

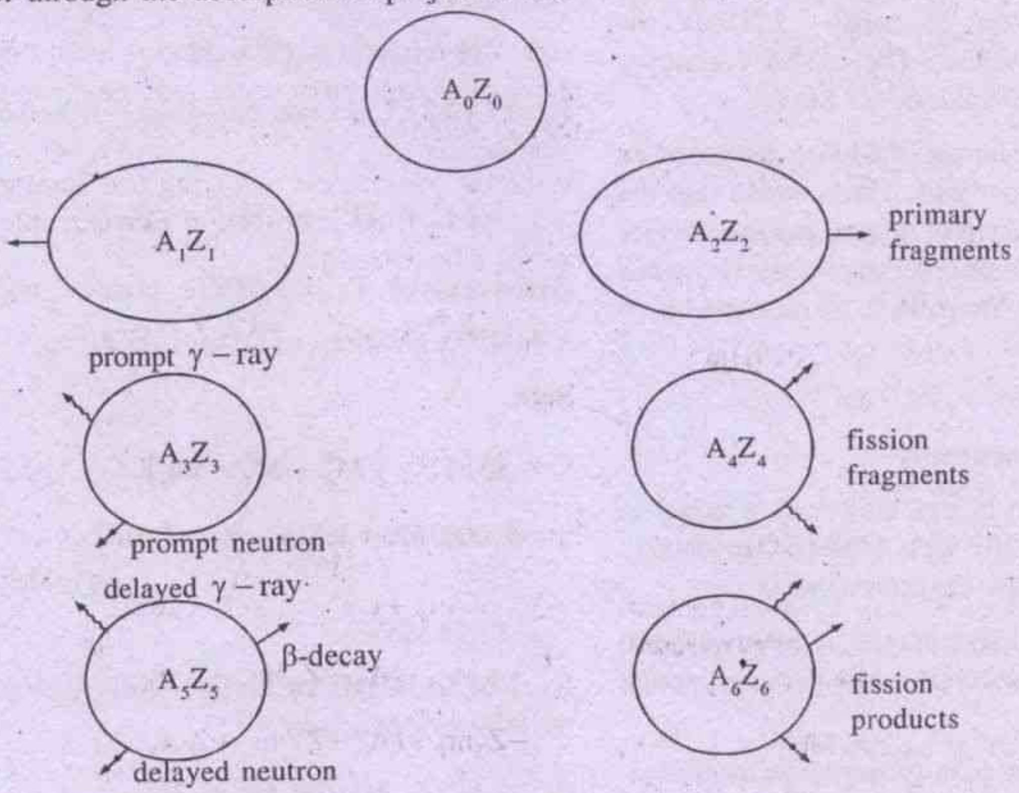


Fig 17.2

In the first stage the compound nucleus (A_0, Z_0) splits into the primary fragments (A_1Z_1) & (A_2Z_2) which are in unstable state. In the second stage the primary fragments decay emitting prompt neutrons and prompt γ-rays In the third stage the fission fragments (A_3Z_3) and (A_4Z_4) decay by emitting β-particles, γ-rays (called delayed γ-rays), neutrons (called delayed neutrons) and form stable fission products (A_5,Z_5) and (A_6,Z_6)

Energy released in fission :

As shown in sec. 17.13 (B), the Q-value in a nuclear reaction is given as (see eqn. 17.13.9)

$$Q = \Delta M . C^2$$

where

ΔM = (sum of rest-masses of initial nuclei) -

(sum of rest-masses of final products)

With a view to giving an idea about the order of energy released we consider a fission process

$${}_0n^1 + {}_{92}U^{235} \rightarrow {}_{56}Ba^{144} + {}_{36}Kr^{89} + 3{}_0n^1 + Q$$

Then

$$\Delta M = (M_U^0 + m_n^0) - (M_{Ba}^0 + M_{Kr}^0 + 3m_n^0)$$

putting the values of various masses

$$\Delta M = [(235.0439 + 1.0087) - (140.9139 + 91.8973 + 3.0261)]u$$

$$= 0.2153 \text{ u}$$

Hence energy released*

$$Q = 0.2153 \times 931 \text{ MeV} \simeq 200 \text{ MeV}$$

Since 1 gm-atom (234 g) of Uranium contain 6.022×10^{23} Uranium atom, so energy released from 1 gm-atom of Uranium is

$$E = 6.022 \times 10^{23} \times 200 \text{ MeV}$$

and energy released from 1 g of Uranium is

$$E_g = \frac{6.022 \times 10^{23} \times 200}{234} \text{MeV}$$

$$= 5.147 \times 10^{23} \text{ MeV}$$

$$= \frac{6.022 \times 10^{23} \times 200 \times 1.6 \times 10^{-13}}{234} \text{J}$$

$$= 8.235 \times 10^{10} \text{ J}$$

$$= \frac{6.022 \times 200 \times 1.6}{234 \times 4.2} \times 10^{10} \text{Cal}$$

$$= 1.961 \times 10^{10} \text{ cal} \quad \text{...17.14.1}$$

The above calculation shows how enermous amount of energy is released in a fission process.

* The amount of energy released in a fission process or in general in a nuclear reaction may also be calculated if reference is made to binding energy curve. The B.E. per nucleon in the vicinity of A = 236 is about 7.6 MeV, while in the vicinity of A = 140, it is about 8.5 MeV. So energy released per fission is about 236 (8.5 - 7.6) $\approx$ 200 MeV.

The reliability of the conclusion in the above paragraph can be better understood from the following exercise as given below.

(A) Nuclear reaction involving two nuclear particles in final states only. For example consider the reaction

$${}_{Z_1}X_1^{A_1} + {}_{Z_2}X_2^{A_2} \rightarrow {}_{Z_3}Y_1^{A_3} + {}_{Z_4}Y_2^{A_4}$$

Then

$$Q = \left(M_{X_1}^0 + M_{X_2}^0 - M_{Y_1}^0 - M_{Y_2}^0\right)C^2 \quad \text{...A.1}$$

Introducing mass defect eqn. A.1 reduces to

$$Q = \big[Z_1 m_P + (A_1 - Z_1)m_n - \Delta M_{X_1} + Z_2 m_P + (A_2 - Z_2)m_n - \Delta M_{X_2} - Z_3 m_P - (A_3 - Z_3)m_n + \Delta M_{Y_1} - Z_4 m_P - (A_4 - Z_4)m_n + \Delta M_{Y_2}\big]C^2 \quad \text{...A.2}$$

Since $Z_1 + Z_2 = Z_3 + Z_4$

$A_1 + A_2 = A_3 + A_4$

So eqn. A2 reduces to

$$Q = (\Delta M_{Y_1} + \Delta M_{Y_2} - \Delta M_{X_1} - \Delta M_{X_2})C^2$$

$$= (B.E)_{Y_1} + (B.E)_{Y_2} - (B.E)_{X_1} - (B.E)_{X_2} \quad ...A.3$$

When any of the nuclear particle is a free nucleon (like free neutron) the B.E. for that is set to be zero.

For example consider the reactions

(i) $_1H^2 + _7N^{14} \rightarrow _6C^{12} + _2He^4$

$$Q = (M^0_{H^2} + M^0_{C^{12}} - M^0_{He^4})C^2$$

$$= Q = (\Delta M_{He^4} + \Delta M_{C^{12}} - \Delta M_{H^2} - \Delta M_{N^{14}})C^2$$

$$\Rightarrow Q = (B.E)_{C^{12}} + (B.E)_{He^4} - (B.E)_{H^2} - (B.E)_{N^{14}}$$

But in

(ii) $_0n^1 + _5B^{10} \rightarrow _3Li^7 + _2He^4$

$$Q = (m^0_n + m^0_{B^{10}} - M^0_{Li^7} - M^0_{He^4})C^2$$

$$= [m^0_n + (5m_p + 5m_n - \Delta M_B)$$

$$-(3m_p + 4m_n - \Delta M_{Li})$$

$$-(2m_p + 2m_n - \Delta M_{He})]C^2$$

$$= (\Delta M_{Li} + \Delta M_{He} - \Delta M_B)C^2$$

$$= (B.E)_{Li} + (B.E)_{He} - (B.E)_B$$

(B) Nuclear fission involving more than two nuclear particles in the final state.

e.g.

$$_{Z_1}X_1^{A_1} + _{Z_2}X_2^{A_2} \rightarrow _{Z_3}Y_1^{A_3} + _{Z_4}Y_2^{A_4} + a_{Z_5}y^{A_5}$$

Then

$$Q = (M^0_{X_1} + M^0_{X_2} - M^0_{Y_1} - M^0_{Y_2} - M^0_y)C^2 \quad ...B.1$$

Introducing man defect

$$Q = [Z_1m_P + (A_1 - Z_1)m_n - \Delta M_{X_1}$$

$$+Z_2m_P + (A_2 - Z_2)m_n - \Delta M_{X_2}$$

$$-Z_3m_P - (A_3 - Z_3)m_n + \Delta M_{Y_1}$$

$$-Z_4m_P - (A_4 - Z_4)m_n + \Delta M_{Y_2}$$

$$-aZ_5m_P - a(A_5 - Z_5)m_n + a\Delta M_y]C^2 \quad ...B.2$$

Since $Z_1 + Z_2 = Z_3 + Z_4 + aZ_5$

$A_1 + A_2 = A_3 + A_4 + aA_5$

So

$$Q = (\Delta M_{Y_1} + \Delta M_{Y_2} + a\Delta My - \Delta M_{X_1} - \Delta M_{X_2})C^2$$

$$\Rightarrow Q = (B.E)_{Y_1} + (B.E)_{Y_2}$$

$$+ a(B.E)_y - (B.E)_{X_1} - (B.E)_{X_2} \quad ...B.3$$

When any of the nuclear particle is a free nucleon, the corresponding B.E. shall be put equal to zero. for Example consider the reaction

$$_{92}U^{235} + _0n^1 \rightarrow _{56}Ba^{141} + _{36}Kr^{92} + 3_0n^1$$

Then

$$Q = (M^0_u + M^0_n - M^0_{Ba} - M^0_{Kr} - 3M^0_n)C^2$$

$$= [(92m_p + 143m_n - \Delta m_u) + m^0_n$$

$$-(56m_p + 85m_n - \Delta M_{Ba})$$

$$-(36m_p + 56m_n - \Delta M_{Kr}) - 3m^0_n]C^2$$

$$= (\Delta M_{Ba} + \Delta M_{Kr} - \Delta M_u)C^2$$

$$\Rightarrow Q = (B.E)_{Ba} + (B.E)_{Kr} - (B.E)_u$$

(C) Nuclear fission involving nuclear particles as well as non-nuclear particles.

e.g.

$${}_{Z_1}X_1^{A_1} + {}_{Z_2}X_2^{A_2} \rightarrow {}_{Z_3}Y_1^{A_3}$$

$$+ {}_{Z_4}Y_2^{A_4} + a_{Z_5}y_2^{A_4} + b_x e^0$$

Then

$$Q = (M^0_{X_1} + M^0_{x_2} - M^0_{Y_1} - M^0_{Y_2} - bm^0_e)C^2 \quad \text{...C.1}$$

$$= \big[(Z_1M_P + (A_1 - Z_2)M_n - \Delta M_{X_1})$$

$$+ (Z_2M_P + (A_2 - Z_2)M_n - \Delta M_{X_2})$$

$$- (Z_3M_P + (A_3 - Z_3)M_n - \Delta M_{Y_1})$$

$$- (Z_4M_P + (A_4 - Z_4)M_n - \Delta M_{Y_2})$$

$$- a(Z_5M_P + (A_5 - Z_5)M_n - \Delta M_y)$$

$$- bm^0_e\big]C^2 \quad \text{...C.2}$$

Since $Z_1 + Z_2 = Z_3 + Z_4 + aZ_5 + bx$

$$A_1 + A_2 = A_3 + A_4 + aA_5$$

So

$$Q = \big[(bx)m_p - (bx)m_n - \Delta M_{X_1} - \Delta M_{X_2}$$

$$+ \Delta M_{Y_1} + \Delta M_{Y_2} + a\Delta M_y\big]C^2 - bm^0_e C^2$$

$$Q = (\Delta M_{Y_1} + \Delta M_{Y_2} + a\Delta M_y - \Delta M_{X_1}$$

$$- \Delta M_{X_2})C^2 - bx(m_n - m_p)C^2 - bm^0_e C^2 \quad \text{...C.3}$$

$$\Rightarrow Q = (B.E)_{Y_1} + (B.E)_{Y_2} + a(B.E)_y - (B.E)_{X_1}$$

$$- (B.E)_{X_2} - bx(m_n - m_p)C^2 - bm^0_e C^2 \quad \text{...C.4}$$

Eqn. C.4 shows that when non-nuclear particles are involved, the Q-value involves the last two terms $-bx(m_n - m_p)C^2 - bm^0_e C^2$ over above the B.E. terms for example consider the reaction

$${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{58}Ce^{140} + {}_{40}Zr^{94} + 2\,{}_0n^1 + 6\,{}_{-1}e^0$$

Then

$$Q = (B.E)_{C_e} + (B.E)_{Z_r} - (B.E)_U$$

$$- 6(-1)(m_n - m_p)C^2 - 6m^0_e C^2$$

(D) Nuclear fission involving nuclear, non-nuclear particles and gamma rays.

e.g.

$${}_{Z_1}X_1^{A_1} + {}_{Z_2}X_2^{A_2} \rightarrow {}_{Z_3}Y_1^{A_3} + {}_{Z_4}Y_2^{A_4}$$

$$+ a_{Z_5}y^{A_5} + b_x c^0 + \gamma$$

Then

$$Q = (E_R^{Y_1} + E_R^{Y_2} + E_R^{y} + E_R^{e}$$

$$- E_R^{X_1} - E_R^{X_2}) - E_r$$

$$Q = (B.E)_{Y_1} + (B.E)_{Y_2} + a(B.E_y$$

$$- (B.E)_{X_1} - (B.E)_{X_2} - E_r$$

Chain Reaction :

When the neutrons produced in a fission can be made to induce fission in the rest of the nuclei present, the reaction is called chain-reaction.

All the neutrons produced in a chain reaction may or may not take part in further fission. Some of them may leak out and some may be lost in non-fission nuclear processes (being absorbed). However to maintain the chain reaction at least one neutron be available to induce and maintain the reaction. The rate at which the reaction will continue depends upon a factor called multiplication factor (K) defined as

K = (No. of neutrons available for further fission)/ (No. of neutrons initially present (in the just previous generation)) ...17.14.2

Thus if in one generation number of neutrons available be N, then in the next generation number of neutrons available is KN.

So the number of neutrons increases by dN = KN - N = N (K - 1). So the rate of increase of neutrons will be N(K-1). If τ be the average time between successive neutron generation ($\tau \sim 10^{-8}$s), then

$$\frac{dN}{dt}=\frac{N(K-1)}{\tau}$$

$$\Rightarrow \quad \frac{dN}{N}=\frac{k-1}{\tau}dt$$

On integration this gives

$$N = N_o e^{\frac{K-1}{\tau}t} \qquad ...17.14.3$$

Where N_0 is the number of neutrons at time t=0 (initial time). Eqn. 17.14.3 shows that

(i) If K>1 then the number of neutrons increases with each generation leading to an **uncontrolled chain reaction.**

(ii) If K<1, then the number of neutrons decreases with each generation and the chain reaction stops.

(iii) If K=1, then $N=N_0$ always and the chain reaction is maintained at a steady rate. This is called **controlled chain reaction.**

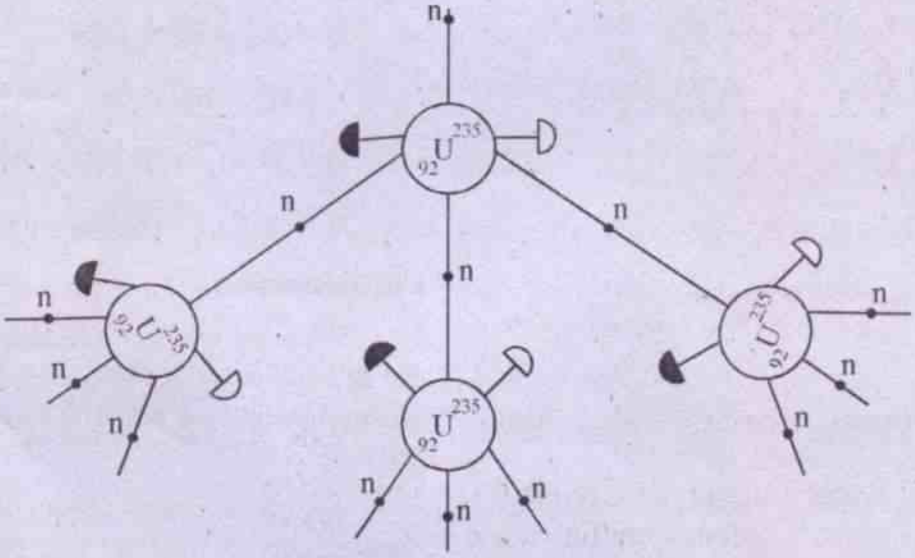


Fig. 17.3

The possibility of a chain reaction depends on the balance among the following four competing processes.

(i) fission with the emission of more than one neutron.

(ii) non-fission capture of neutrons by fissile materials.

(iii) non-fission capture of neutrons by non-fissile materials.

(iv) complete loss or escape of neutrons (referred as leakage).

In each of the processes (ii) to (iv) above, neutrons are lost and in (i) neutrons are generated. If the loss is less than or equal to the surplus neutrons produced then the chain reaction occurs. The loss in process (ii) and (iii) can be reduced by appropriately choosing the materials of the reactor; but the loss in process (iv) can be diminished by suitably choosing the size of fissionable material.

The optimum size in which controlled chain reaction can be maintained is called **critical size.** A prompt neutron can be slowed down to cause further fission, if it travels through 10 cm of air. So the size of the sample (fissionable material) should be 10 cm in any direction. The smallest mass of fissionable material in which controlled chain reaction can be set up is called **critical mass.**

Nuclear Reactor :

A nuclear reactor is a device from which energy can be obtained in a controlled and sustained manner by inducing a controlled chain reaction involving fission of heavy elements.

A nuclear reactor works on the principle that when multiplication factor K (defined in eqn. 17.14.2) is equal to unity, controlled chain reaction is set up and energy is continuously available.

Basic elements of a reactor :

All types of nuclear reactors contain the following components.

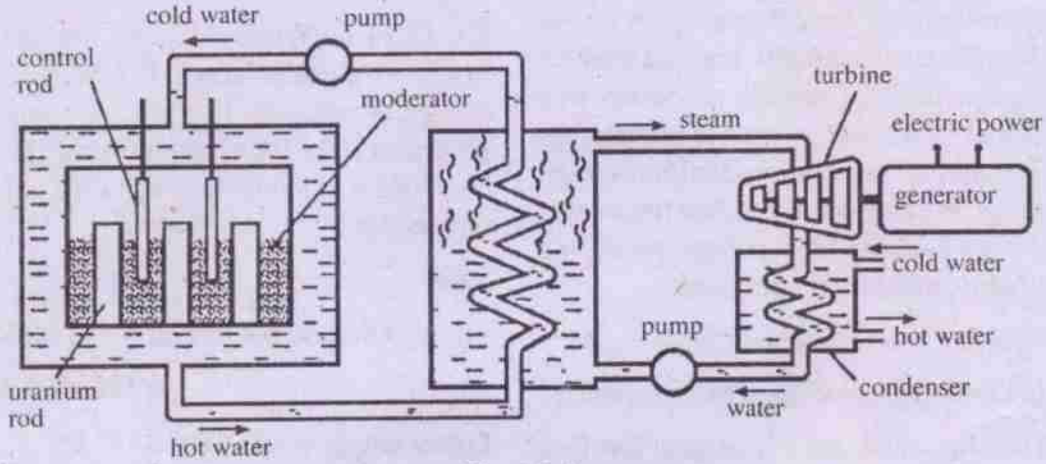


Fig. 17.4

(i) **Fuel :** A material that fissions and supplies neutrons for further fission, such as $_{92}U^{233}$, $_{92}U^{235}$, $_{90}Th^{232}$, $_{94}Pu^{239}$, $_{94}Pu^{240}$, $_{94}Pu^{241}$

(ii) **Moderator :** Heavy water (D_2O), graphite rod, beryllium, paraffin, carbon etc. are used as moderators. These materials contain large number of H-atoms, so that they can slow down the prompt neutrons. Moderators have low atomic weight, low neutron absorption cross-section and high inelastic scattering cross-section.

(iii) **Neutron reflector :** Materaials having high scattering cross-section for neutrons are used as reflectors on the surface of the reactor.

(iv) **Cooling system :** Commonly air, CO_2, He, liquid sodium, water or any other suitable oil are used as coolants. The cooling system removes the heat evolved in the reactor core.

(v) **Control rods :** These are made up of materials having large neutron absorption cross-section such as : boron, steel, cadmium. By pushing or pulling out the rods, the rate of reaction can be controlled.

(vi) **Safety system :** The entire reactor is housed inside a room having thick concrete walls. Sometimes the entire reactor is surrounded by a water tank. The working personnels use lead approns.

Types of nuclear reactor :

Reactors are broadly classified according to the type of fuel, moderator and heat transfer agents used.

With respect to arrangements of fuel and moderator, the reactors are classified as (i) homogenous (ii) heterogeneous reactors. In homogeneous type, the fuel and moderator are finely divided and uniformly mixed together. In heterogeneous type the fuel and moderator are in separate elements as blocks.

Depending on the energy of neutrons the reactors may be (i) thermal (ii) intermediate and

(iii) fast reactors. In fast reactors the fast neutrons are directly used and moderator is completely removed.

Depending on the purpose, the reactors are classified as (i) Power reactor (ii) Test and research reactor (iii) Breeder reactor (iv) Isotope producing reactor etc. In a power reactor, energy available is transformed into useful power form such as electricity. Test and research reactors are used for testing processes like dimensional stability or instability of materials under irradiation and other radiation damage phenomena. In a breeder reactor the fissionable materials are bred and in an isotope producing reactor radio-isotopes are produced.

Uses : Nuclear reactors have extensive uses.

(i) Utilised in generation of electric power.

(ii) Used in production of isotopes. The flux of neutrons produced in fission is used to induce nuclear reaction like

$$_{11}Na^{23} + {}_0n^1 \rightarrow {}_{11}Na^{24}$$

(iii) It is also used in scientific research. The neutrons produced in the reactor are used as probes to study various nuclear reactions.

Atom bomb

As said in sec. 17.14 in a nuclear fission chain reaction, when $K > 1$, the reaction becomes uncontrolled chain reaction. It is calculated that a prompt neutron takes about 10^{-8} s. to induce another fission, the number of fissions continue in a geometric progression, resulting in disintegration of huge amount of fissionable material and hence release of huge amount of energy.

For uncontrolled chain reaction to take place, the mass of fissionable material must be more than critical mass (critical mass for $_{92}U^{235}$ is ~ 5 kg). Since A/V (= Area / Volume) is minimum for sphere therefore a spherical shape provides the most efficient geometry for maximising K. For a sphere $A/V = 3/R$. So the fraction of neutron that escapes without interaction with the fissile material decreases as R increases.

The reaction in an atom bomb is initiated either by atmospheric neutron or by neutron available due to spontaneous fission of $_{92}U^{235}$.

Ex. 17.14.1 : When a neutron induces fission in $_{92}U^{235}$ nucleus, about 185 MeV of usable energy is released. If a reactor continuously generates 100 MW of power using $_{92}U^{235}$, then how long will it take for 1 kg of the uranium to be used up ?

Soln.

Energy released per fission = 185 MeV

$= 185 \times 1.6 \times 10^{-13}$ J

Power output = 100 MW = 10^8 J/S

So no. of fissions required per second is

$$= \frac{10^8}{185 \times 10^{-13} \times 1.6} = \frac{10^{21}}{1.6 \times 185}$$

No. of U-nuclei present in 1 kg-atom of U^{235} is $6.023 \times 10^{23} \times 10^3$

So no. of nuclei present in 1 kg of U^{235} is

$$= \frac{6.023 \times 10^{26}}{235}$$

So time required for fission of 1kg of U^{235} is

$$t = \frac{6.023 \times 10^{26}}{235} \Big/ \frac{10^{21}}{1.6 \times 185} = 7.586 \times 10^5 \text{ sec}$$

t = 8.781 year.

Ex. 17.14.2 : A U^{235} nucleus liberates an energy of 200 MeV per fission. How much energy is released when a Uranium bomb containing 1.5 kg is exploded.

Soln.

No. of nuclei in 1.5 kg of U^{235} is

$$= \frac{6.023 \times 10^{26}}{235} \times 1.5$$

So energy released

$$= \frac{6.023 \times 10^{26}}{235} \times 1.5 \times 200 \text{ MeV}$$

$= 7.69 \times 10^{26}$ MeV

$= 12.3 \times 107$ J.

Ex. 17.14.3 : A nuclear reactor generates power at 50% efficiency by fission of ${}_{92}U^{235}$ into two equal fragments of ${}_{46}Pd^{116}$ with the emission of two gamma rays of 5.2 MeV each and three neutrons. The average B.E. per nucleon of ${}_{92}U^{235}$ and ${}_{46}Pd^{116}$ is 7.2 MeV and 8.2 MeV, respectively. Calculate the energy released in one fission event. Also estimate the amount of U^{235} consumed per hour to produce 1600 megawatt power.

Soln.

The reaction is ${}_{92}U^{235} \rightarrow 2\,{}_{46}Pd^{116} + 3\,{}_{0}n^{1} + 2\gamma$

(i) So $Q = 2(B.E)_{Pd^{116}} - (B.E)_{U^{235}} - E_{\gamma}$

$\Rightarrow Q = 2 \times 8.2 \times 116 - 7.2 \times 235 - 10.4$

$= 200$ MeV

(ii) Efficiency is 50%. So in each fission the available energy is $200 \times 0.5 = 100$ MeV.

The reactor produces 1600 megawatt power.

So number of fissions per second is

$$n = \frac{1600 \times 10^{6}}{1.6 \times 10^{-19}} \times \frac{1}{100 \times 10^{6}}$$

and

Number of fissions per hour is

$$N = 3600\, n = \frac{3600 \times 1600 \times 10^{6}}{1.6 \times 10^{-19}} \times \frac{1}{100 \times 10^{6}}$$

$N = 36 \times 10^{22}$

So amount of Uranium consumed

$$= \frac{235 \times 36 \times 10^{22}}{6.02 \times 10^{23}} = 140.53\text{g}$$

17.15 : Nuclear Fusion

A reaction in which two light nuclei combine to form a single heavier nucleus, with the release of energy is called **nuclear fusion.**

Fusion reaction is possible when the two lighter nuclei have sufficient K.E. to overcome the repulsive force between the two nuclei. It is calculated that if the light nuclei are at temperature ~ 10^{18} K, then the fusion reaction can take place. At these high temperatures atoms are completely in the ionised state and still it is neutral. This state is called **plasma state**. Therefore ordinarily fusion is not possible to be arranged in the laboratory.

Examples :

(1) ${}_{1}H^{1} + {}_{1}H^{2} \rightarrow {}_{2}He^{3} + \gamma$

(2) ${}_{1}H^{2} + {}_{1}H^{2} \rightarrow {}_{2}He^{3} + {}_{0}n^{1} + Q$ (= 3.25 MeV)

(3) ${}_{1}H^{2} + {}_{1}H^{2} \rightarrow {}_{1}H^{3} + {}_{1}H^{1} + Q(= 40 \text{ MeV})$

Since nuclear fusion can take place at very high temperature, so it is also called **thermo nuclear reaction.**

Source of Stellar energy

Sun is the source of energy. It is believed that the cause of this energy is nuclear fusion. Four hydrogen nuclei combine to produce a helium nucleus.

$$4\,{}_{1}H^{1} \rightarrow {}_{2}He^{4} + 2\,{}_{+1}e^{0} + 2\nu_{e} + Q$$

Two possible cycles are suggested for the above reaction to occur.

(i) Carbon-nitrogen cycle :

$${}_{6}C^{12} + {}_{1}H^{1} \rightarrow {}_{7}N^{13*} + \gamma$$
$${}_{7}N^{13*} \rightarrow {}_{6}C^{13} + {}_{+1}e^{0} + \nu_{e} \text{ (neutrino)}$$
$${}_{6}C^{13} + {}_{1}H^{1} \rightarrow {}_{7}N^{14} + \gamma$$
$${}_{7}N^{14} + {}_{1}H^{1} \rightarrow {}_{8}O^{15*} + \gamma$$
$${}_{8}O^{15*} \rightarrow {}_{7}N^{15} + {}_{+1}e^{0} + \nu_{e}$$
$${}_{7}N^{15} + {}_{1}H^{1} \rightarrow {}_{6}C^{12} + {}_{2}He^{4}$$
$$\overline{4\,{}_{1}H^{1} \rightarrow {}_{2}He^{4} + 2\,{}_{+1}e^{0} + 2\nu_{e} + Q}$$

Using the available data

Mas of $4\ {}_1H^1 = 4.031300$ u

${}_2He^4 = 4.002603$ u

$2\ {}_{+1}e^0 = 0.001098$ u

$\Delta M = 0.027564$ u

energy released $= 0.027564 \times 931$ MeV

$= 25.7$ MeV

Here ${}_6C^{12}$ acts as a catalyst.

(ii) Proton-Proton Cycle :

$$2_1H^1 + 2_1H^1 \rightarrow 2_2He^2 \rightarrow 2_1H^2 + 2_1e^0 + 2\nu_e + 1.86 \text{ MeV}$$

$$2_1H^2 + 2_1H^1 \rightarrow 2_2He^{3*} \rightarrow 2_2He^3 + 10.99 \text{ MeV}$$

$$_2He^3 + _2He^3 \rightarrow _4Be^6 \rightarrow _2He^4 + _1H^1 + _1H^1 + 12.85 \text{ MeV}$$

$$4_1H^1 \rightarrow _2He^4 + 2_{+1}e^0 + 2\nu_e + 25.7 \text{ MeV}$$

Hydrogen bomb

It is an explosive device which makes use of principle of nuclear fusion.

The high temperature necessary for the fusion to take place, is obtained by detonation of an atom bomb.

The calculation given below indicate how a H-bomb is more powerful than the atom bomb.

1 gm-atom of Hydrogen contains 6.023 x 10^{23} H-atoms. Energy released in fusion of 4 H-atoms is 25.7 MeV. Therefore the energy released when 1 gm-atom hydrogen fuse is

$$= \frac{6.023 \times 10^{23}}{4} \times 25.7 \text{ MeV}$$

So energy released when 1g of H-atom fuse is

$$= \frac{6.023 \times 10^{23}}{4 \times 1} \times 25.7 \text{ MeV}$$

$= 38.698 \times 10^{23}$ MeV

$= 6.3 \times 10^{11}$ J

$= 1.5 \times 10^{11}$ Cals.

Thus the energy released from 1g. of hydrogen is nearly 8 times that released from 1g. of uranium.

Ex. 17.15.1 : Calculate the energy released in the fusion reaction

$${}_1H^2 + {}_1H^2 \rightarrow {}_2He^4$$

Given mass of ${}_1H^2 = 2.014102$ u

${}_2He^4 = 4.0026$ u

Soln.

$$Q = \Delta M.C^2 = (2 \times 2.014102 - 4.0026) \times 931 \text{ MeV}$$

$= 23.84$ MeV

Ex. 17.15.2 : One neutron and one proton combine to form a deuteron. Calculate the energy liberated during the process. Given mass of neutron = 1.008665 u, mass of proton = 1.007825 u and mass of ${}_1H^2 = 2.014102$ u.

Soln.

$$Q = \Delta M.C^2$$

$$= [(1.008665 + 1.007825) - 2.014102] \times 931 \text{ MeV}$$

$= 2.22$ MeV

$= 3.56 \times 10^{-13}$ J

17.16 : Radio Activity

In year 1896 H. Becquerel discovered that Uranium compound spontaneously emitted radiations which affected photographic plate wrapped in black paper. This radiation could ionise gas also. In 1898 M. Curie also found that Polonium and Radium spontaneously emitted similar radiations. In 1899, Rutherford analysed and found that the radiations consisted of α-particles ($_2He^4$) and β-particles ($_{-1}e^0$). In 1900, Villard discovered that the radiations also consisted of γ-rays. Later on positron emission and electron capture were also observed.

The phenomenon of spontaneous disintegration of a nucleus (naturally occuring or artificially prepared) into a fragment with the emission or absorption of corpuscular particle or emission of electro-magnetic radiation is called radioactivity.

When a naturally occuring nucleus disintegrates spontaneously we call it **natural radioactivity**. When a nucleus artificially prepared exhibits radioactivity we call it **artificial radioactivity**.

Types of Radioactive Decay

Alpha-decay (α-decay) :

In this process the product nucleus has mass number less by 4 units and atomic number less by 2 units, with respect to the parent nucleus. The emitted radiation possesses mass number 4 and atomic number 2. This process is represented as

$$_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2He^4$$

Example

$$_{83}Bi^{212} \rightarrow {}_{81}T\ell^{208} + {}_2He^4$$

This decay occurs due to largeness of the nucleus.

Beta-decay (β-decay) :

In this process the product nucleus has same mass number as the parent nucleus, but atomic number changes by one unit. In this process either a neutron is converted to a proton or a proton is converted to a neutron; with emission of electron or a positron or capture of an electron, and emission of an antineutrino ($\overline{\nu}_e$) or a neutrino (ν_e).

i.e.

(i) $_ZX^A \rightarrow {}_{Z+1}Y^A + {}_{-1}e^0 + \overline{\nu}_e$ (β^- decay)

e.g. $_{27}Co^{60} \rightarrow {}_{28}Ni^{60} + {}_{-1}e^0 + \overline{\nu}_e$

(ii) $_ZX^A \rightarrow {}_{Z-1}Y^A + {}_{+1}e^0 + \nu_e$ (β^+ decay)

e.g. $_{11}Na^{22} \rightarrow {}_{10}Ne^{22} + {}_{+1}e^0 + \nu_e$

(iii) $_ZX^A + {}_{-1}e^0 \rightarrow {}_{Z-1}Y^A + \nu_e$ (el^n capture)

e.g. $_{18}Ar^{37} + {}_{-1}e^0 \rightarrow {}_{17}C\ell^{37} + \nu_e$

A nucleus lying above the region of stability [i.e. region of neutron excess (N/Z > 1)] becomes stable by reducing the number of neutrons in it, so that N/Z gets reduced. This is achieved through β^- decay.

Similarly a nucleus lying below the stability region becomes stable by increasing its N/Z ratio. This is achieved by β^+ decay (positron emission) or electron capture.

Gamma-decay (γ-decay) :

In the process the product nucleus has the same mass number and same atomic number with only change in the energy state.

$$_ZX^{A^*} \rightarrow {}_ZX^A + {}_0\gamma^0$$

Properties of Radioactive radiations :

The various properties of α, β and γ-radiations can be inferred from the following comparative chart, prepared featurewise.

Features	α-rays	β-rays	γ-rays
1. Nature	material particle and stable (Helium Nucleus)	material particle and stable(Fast moving electron)	e.m. radiation and stable (Photon)
2. Charge	+vely charged and has charge $+2e = 3.2 \times 10^{-19}$ C	-vely charged for β^- decay & +vely charged for β^+ decay 1.6×10^{-19} C	neutral
3. Rest Mass	6.6×10^{-27} kg	9.1×10^{-31} kg	nil
4. Speed	14×10^6 ms^{-1} to 23×10^6 ms^{-1}	11×10^6 ms^{-1} to 29.6×10^6 ms^{-1}	3×10^8 ms^{-1}
5. Penetrating power	small (little)	medium (moderate)	large (high)
6. Ionising power	High (About 100 times that of β-particle)	Moderate (medium) (About 100 times that of γ - rays)	small (little)
7. Effect on photographic plate	feeble	moderate	strong
8. Effect of Electric & magnetic field	Deflect in the direction of field	Deflected opposite to the direction of field	No deflection
9. Effect on human skin	causes incurable burns	causes incurable burns	causes incurable burns

Laws of Radioactivity

The following laws are obeyed during radioactivity.

(i) Radioactivity is due to the disintegration of nucleus.

(ii) Rate of disintegration is not affected by the external conditions like temperature and pressure etc.

(iii) The law of conservation of charge and nucleon number hold god in the process of radioactivity.

(iv) In α-decay the atomic number and mass number of the parent nucleus decreases by two and four units respectively. In β-decay, atomic number changes (either increased or decreased) by one unit and mass number is unchanged. In γ-decay both mass number and atomic number are unaltered.

(vii) Statistical law :

Rate of disintegration of a radio-active substance, at any instant is directly proportional to the number of radioactive nuclei (atoms) present at that instant.

i.e. $$-\frac{dN(t)}{dt} \propto N(t)$$

$$\Rightarrow \frac{dN(t)}{dt} = -\lambda N(t)$$

$$\Rightarrow \frac{dN(t)}{N(t)} = -\lambda \, dt$$

Where λ is a constant of proportionality and is known as **decay constant** or **disintegration constant**.

On integration this gives $\int_{N_0}^{N}\frac{dN}{N(t)}=-\lambda\int_0^t dt$

$$\Rightarrow l_n\frac{N(t)}{N_0}=-\lambda t$$

$$\Rightarrow N(t)=N_0e^{-\lambda t} \qquad ...17.16.1$$

where, N_0 is the number of nuclei at time $t = 0$ (initial time). Equation 17.16.1 gives the number of nuclei surviving after time 't' and this equation is called readioactive decay law or decay equation. This equation shows that the number of active nuclide decreases exponentially with time.

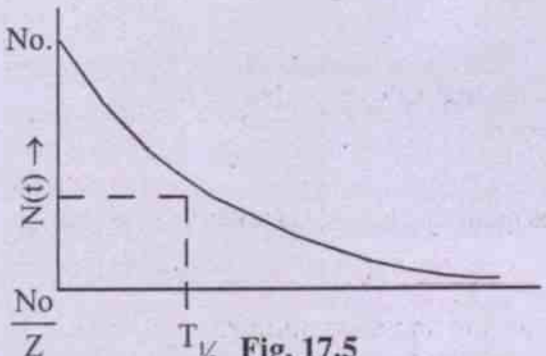


Fig. 17.5

An infinite time is theoretically needed to complete the disintigration of the sample. (Practicaly long time)

Radioactive decay constant

In equation 17.16.1, 'λ' is called decay constant, as it indicates at what rate the radioactive nuclei shall decay. We note from eqn. 17.16.1 that if $t=\frac{1}{\lambda}$, then

$$N(t)=No.\ e^{-1}=\frac{N_0}{e} \qquad ...17.16.2$$

This exercise shows : *the radioactive decay constant is the reciprocal of the time in which the number of radioactive nuclei (atoms) are reduced to N_0/e (i.e. 1/e times the initial number of parent nuclei).* 'λ' is different for different radioactive nuclei. Unit of decay constant is $(\text{second})^{-1}$.

Half life :

Half-life of a radioactive substance is defined as the time during which the number of atoms (nuclei) of the substance are reduced to half of their original (initial) value.

Thus if $T_{1/2}$ is the half life of a radioactive sample, then at $t=T_{1/2}$, $N(t)=No/2$. Putting these values in eqn. 17.16.2:

$$\frac{N_0}{2}=N_0.e^{-\lambda T_{1/2}}$$

This gives

$$T_{1/2}=\frac{\log_e 2}{\lambda}=\frac{0.693}{\lambda} \qquad ...17.16.3$$

Equation 17.16.3 gives the relation between half-life and decay constant.

We further note from 17.16.1

$$\frac{N_o}{N}=e^{\lambda t}$$

$$\Rightarrow \quad \lambda t=\log_e\left(\frac{N_o}{N}\right)$$

$$\Rightarrow \quad t=\frac{1}{\lambda}.\log_e(N_o/N) \qquad ...17.16.4$$

Using eqn. 17.16.3 in 17.16.4 one obtains

$$t=\frac{T_{1/2}}{0.693}.\log_e(N_o/N) \qquad ...17.16.5$$

or

$$t=\frac{2.3026}{0.693}T_{1/2}\log_{10}\left(\frac{N_o}{N}\right)$$

$$\Rightarrow t=3.322\ T_{1/2}\log_{10}(N_o/N) \qquad ...17.16.6$$

Again we note that after **one half-life period** $\left(t = T_{1/2}\right)$ the number of atoms (nuclei) present is

$$N_1 = \frac{N_o}{2} = N_o\left(\frac{1}{2}\right)^1$$

After **two half-life periods** (i.e. after $t=2T_{1/2}$)

$$N_2 = \frac{N_1}{2} = \frac{1}{2}.\frac{N_o}{2} = N_o\left(\frac{1}{2}\right)^2$$

After **three half-life periods** (i.e. after $t=3T_{1/2}$)

$$N_3 = \frac{N_2}{2} = \frac{1}{2}N_o\left(\frac{1}{2}\right)^2 = N_o\left(\frac{1}{2}\right)^3$$

This shows that after **n** half-life period the number of nuclei present is

$$N = N_o\left(\frac{1}{2}\right)^n \quad ...17.16.7$$

Applying eqn. 17.16.7 we find that after a time $t = 10\tau_{1/2}$, only 0.01% of the original nuclei is present. Thus although eqn. 17.6.1 shows that after infinite time the nuclei totally disintegrates, it is practically not true.

Average life (Mean life)

Average or mean life T_{av} of a radioelement is the average life time of all the atoms (nuclei) in a given sample and is defined as the fatio of the total life time of all the atoms (nuclei) to the total number of atoms (nuclei)

i.e.

$$T_{av} = \frac{\text{Sum of life times of all atoms}}{\text{Total number of atoms}} \quad ...17.16.8$$

$$\therefore T_{av} = \frac{\int_0^\infty t\,dN}{N_0} \quad ...17.16.9$$

Where dN is the number of atoms having a life time 't' and N_0 is the number of atoms present initially in the sample.

But $N=N_0e^{-\lambda t}$ and $dN = -\lambda N_0 e^{-\lambda t}dt$.

$$\therefore T_{av} = \lambda\int_0^\lambda te^{-\lambda t} \quad ...17.16.10$$

Integrating by parts we can show that

$$T_{av} = \frac{1}{\lambda} \quad ...17.16.11$$

since $T_{1/2} = \frac{0.693}{\lambda}$

$$\therefore \quad \overline{T} = \frac{T_{1/2}}{0.693} = 1.44\ T_{1/2} \quad ...17.16.12$$

NOTE :

(i) The decay equation $N(t) = N_o e^{-\lambda t}$ applies to the behaviour of a single, pure radioactive element. Mixtures of radioactive substances and radio-samples having products of initial disintegrations that are themselves radioactive donot obey the above law.

(ii) When λ is small (i.e. $T_{1/2}$ large) the radioactive sample decays rapidly. If λ is large (i.e. $T_{1/2}$ small) the radioactive sample decays slowly.

(iii) Half life of a radioactive material cannot be changed by physical or chemical changes.

Activity (or strength)

The activity or strength A(t) of a radioactive sample at any instant 't' is defined as the number of nuclei disintegrating in unit time at the instant 't'.

i.e.

$$A(t) = -\frac{dN}{dt} = \lambda N(t) \quad ...17.16.13$$

Equation 17.16.13 shows that activity of a substance is the product of disintegration constant and number of nuclei present.

Since $N = N_o e^{-\lambda t}$

So $A(t) = \lambda N_o e^{-\lambda t} = A_o e^{-\lambda t}$ ...17.16.14

Equation 17.16.14 shows that A(t) vs t curve shall be of the same type as N(t) vs t curve.

Again $T_{1/2} = \dfrac{\log_e 2}{\lambda}$

$$\Rightarrow \lambda = \frac{\log_e 2}{T_{1/2}}$$

$$\therefore \quad A(t) = A_o e^{-\lambda t} = A_o e^{-(\log_e 2).\frac{t}{T_{1/2}}}$$

$$= A_o\left(e^{\log_e 2}\right)^{-\frac{t}{T_{1/2}}} = A_o(2)^{-\frac{t}{T_{1/2}}}$$

$$\Rightarrow A(t) = A_o\left(\frac{1}{2}\right)^{\frac{t}{T_{1/2}}} \quad ...17.16.15$$

This shall give

(i) After **one** half-life time (i.e. $T_{1/2}$)

$$A_1 = A_o.\left(\frac{1}{2}\right)$$

(ii) After **two** half - life time (i.e. $t = 2T_{1/2}$)

$$A_2 = A_o\left(\frac{1}{2}\right)^2$$

(iii) After **three** half - life time (i.e. $t = 3T_{1/2}$)

$$A_3 = A_o\left(\frac{1}{2}\right)^3$$

(iv) After **n** half - life time (i.e. $t = nT_{1/2}$)

$$A_n = A_o\left(\frac{1}{2}\right)^n \quad ...17.16.16$$

Unit of Activity :

The S.I. unit of activity is becquerel. It is named after the scientist Antonie-Henri Bacquerel.

$$1 \text{ becquerel} = 1 \text{ Bq} = \frac{1 \text{ decay}}{\text{S}}$$

The activity of a ratioactive element is said to be 1 becquerel (1 Bq) if its average rate of decay is one disintegration per second.

Activity is also measured in non-S.I. unit like Curie(Ci), Rutherford (Rd)

$$1 \text{ Ci} = 3.7 \times 10^{10} \frac{\text{decays}}{\text{Second}}$$

$$1 \text{ Rd} = 10^6 \frac{\text{decays}}{\text{Second}}$$

*** Estimate of Earth's age :**

Radio-activity can be used to estimate the age of earth by knowing half-life of the radioactive substance from earth's sample.

For example ${}_{92}U^{238}$ undergoes a chain of decay with the stable end product lead $\left({}_{82}Pb^{206}\right)$. Suppose when we analyse the composition of rocks, we find Uranium and lead are present in the ratio 3:1. Assuming that initially no lead was present and lead is formed due to radioactive decay of uranium, we find that

$$\frac{N}{N_o} = \frac{3}{4}$$

So time 't' is given by (see eqn.17.16.6)

$$t = 3.322 \times T_{1/2} \times \log_{10}\left(\frac{N_o}{N}\right)$$

$$= 3.322 \times T_{1/2} \times \log_{10}\left(\frac{4}{3}\right)$$

$$\Rightarrow t = 3.322 \times 0.125 \times T_{1/2}$$

It is to be noted that the yield of radio-isotopes produced through bombardment of nuclei by α - particle is low. But yield of radio-isotope produced through bombardment of nuclei by neutrons or deuterons is high.

***Uses of Radio-isotopes** (Uses of artificial radio-activity)

The uses of radio isotopes are extensively found in medicine, industry and agriculture as given below.

(A) **As tracer :** Radio-isotopes are mixed with its stable isotopes, as their chemical properties are same. The presence or distribution of the radio-isotopes in a biological or physical system is then traced by detecting the radiations emitted by them.

Examples : (i) Radio-carbon $\left({}_{6}C^{14}\right)$ is being used for research in photosynthesis. It also helps in knowing the age of a fossil, by carbon dating process (ii) ${}_{15}P^{32}$ is mixed with phosphorous manure to study the intake food from soil by the plants (iii) The absorption of NaCl in the human body can be studied by feeding the person ${}^{24}_{11}NaCl$ alongwith normal NaCl.

(B) **In medicine :** Radio-isotopes are used in medical diagnosis and therapy.

(a) **Diagnosis :** (i) The position and size of a tumour in the brain can be located by injecting radio-iodine with organic dyes, as tumour tissues obsorb more radio isotopes. (ii) Radio-iodine $\left({}_{53}I^{131}\right)$ can be used to locate the disorders in thyroid gland. (iii) Cr^{51}, Fe^{59} are used in locating haemorrhage.

(b) **Therapy :** (i) Radio-gold is used in the treatment of leukaemia (ii) Co^{60} is used in the treatment of cancer. (iii) Radio-bismuth is used in the treatment of syphilis.

(C) **In industry :**

(i) Radio-carbon $\left({}_{6}C^{14}\right)$ is used to study wear and tear of the parts of an engine.

(ii) Radio-cobalt (Co^{60}) is used for testing faults in the castings by taking photographs with γ - rays emitted from Co^{60}.

(D) **In agriculture :**

(i) Radio- phosphorous $({}_{15}P^{32})$ in the form of phosphates is used as manure to improve plant growth and to increase the yield of crops.

(ii) Crop mutation is possible with radio-isotopes. Crops with high-yield resistance to diseases are being obtained by exposing seeds to γ - rays.

(iii) Male insects are sterilised by exposing them to α- radiations of radio-isotopes, so that they can not further spread their families.

(iv) Radio-carbon when injected increases photosynthesis.

Ex. 17.16.1 Radioactive phosphorous - 32, has half-life of 14 days. A source containing this isotope has an initial activity 10 μCi.

(a) What is the activity of the source after 42-days.

(b) What time elapses before the activity falls to 2.5 μCi.

Soln.

(a) $A = A_{o}\left(\frac{1}{2}\right)^{n}$

Where 'n' is no. of half-lives.

Now 42 days include $^{40}/_{14} = 3$ half- lives

Hence $A = A_o\left(\frac{1}{2}\right)^3 = \frac{A_o}{8} = \frac{10}{8}\ \mu Ci$

$= 1.25\ \mu Ci$

(b) $A_o = 10\ \mu Ci,\ A = 2.5\ \mu Ci$

$\therefore \frac{A}{A_o} = \left(\frac{1}{2}\right)^n$

$\Rightarrow \frac{2.5}{10} = \left(\frac{1}{2}\right)^n \Rightarrow \frac{10}{2.5} = (2)^n$

$\Rightarrow 4 = (2)^n$

$\Rightarrow n = 2$

So after 2-half-lives i.e. after 28 days the activity falls to 2.5 μCi.

Ex. 17.16.2 10^{-3} kg of a radio-isotope (atomic mass 226) emits 3.72×10^{10} α-particles in a second. Calculate the half-life of the isotope. If 4.2×10^2 joules is released in one hour, in this process what is the average energy of the α-particle ?

Soln.

$$N(t) = N_o e^{-\lambda t}$$

$$\Rightarrow \frac{dN}{dt} = \lambda N$$

$$\Rightarrow (\lambda)^{-1} = \frac{N}{dN/dt}$$

Given $\frac{dN}{dt} = 3.72 \times 10^{10}$

Now $N = \frac{6.023 \times 10^{23}}{226} = 2.665 \times 10^{21}$

$\therefore (\lambda)^{-1} = \frac{2.665 \times 10^{21}}{3.72 \times 10^{10}} = 7.164 \times 10^{10}$

$$\Rightarrow \frac{T_{1/2}}{0.693} = 7.164 \times 10^{10} \text{sec}$$

$$\Rightarrow T_{1/2} = 4.965 \times 10^{10} \text{ sec}$$

$$= 1574.276 \text{ years.}$$

No. of α-particles emitted per hr $= 3.72 \times 10^{10} \times 3600$

So average energy per α-particle

$$= \frac{4.2 \times 10^2 J}{3.72 \times 10^{10} \times 3600}$$

$$= 3.14 \times 10^{-12} J$$

Ex. 17.16.3 Find the half-life of U^{238}, if one gram of it emits 1.23×10^4 α - particles /s. [one year = 3.155×10^7 s].

Soln.

No. of U^{238} atoms in one 1g is

$$N = \frac{6.023 \times 10^{23}}{238}$$

$$\Rightarrow N = 2.531 \times 10^{21}$$

Now $\frac{dN}{dt} = \lambda N = 1.23 \times 10^4 s^{-1}$

$$\Rightarrow \frac{1}{\lambda} = \frac{N}{(dN/dt)} = \frac{T_{1/2}}{0.693}$$

$$\Rightarrow T_{1/2} = 0.693 \times \frac{N}{(dN/dt)}$$

$$\therefore T_{1/2} = \frac{0.693 \times 2.531 \times 10^{21}}{1.23 \times 10^4} S = 1.426 \times 10^{17} S$$

$$\Rightarrow T_{1/2} = 4.519 \times 10^9 \text{ years}$$

Ex. 17.16.4 A radioactive element undergoes disintegration for an interval of time equal to its mean life.

(a) What fraction of element remains ?

(b) What fraction had disintegrated ?

Soln.

(a) fraction remaining $= \frac{N}{N_o} = e^{-\lambda t} = e^{-\lambda T_{av}}$

But $T_{av} = \frac{1}{\lambda}$

So $\frac{N}{N_o} = e^{-1} = \frac{1}{e} = 0.3679$

(b) fraction disintegrated $= 1 - \frac{N}{N_o} = 0.6321$

Ex. 17.16.5 At a given instant there are 25% undecayed radio-active nuclei in a sample. After 10 s the number of undecayed nuclei reduces to 12.5%. Calculate (1) mean life of the nuclei and (ii) the time in which the number of the undecayed nuclei will further reduce to 6.25% of the reduced number.

Soln.

(i) Let $N_1(t) = \frac{25}{100} N_o = N_o e^{-\lambda t}$

then $N_2(t+10) = \frac{125}{1000} N_o = N_o e^{-\lambda(t+10)}$

$\Rightarrow \; 2 = e^{10\lambda}$

$\Rightarrow \; \lambda = \frac{\log_e 2}{10} = \frac{1}{T_{av}}$

$\Rightarrow \; T_{av} = \frac{10}{\log_e 2} = 14.43 \text{ s}$

(ii) Further it is reduced to 6.25% of reduced number i.e.

$$N_3(t+10+t') = \frac{6.25}{100} \times N_2(t+10)$$

$$= N_0 e^{-\lambda(t+10+t')}$$

$$\Rightarrow \quad \frac{6.25}{100} \times \frac{12.5}{100} N_0 = N_0 e^{-\lambda t} e^{-10\lambda} e^{-\lambda t'}$$

$$= \frac{25}{100} N_0 \times 2^{-1} \times e^{-\lambda t'}$$

$$\Rightarrow \quad \frac{6.25 \times 12.5 \times 2}{100 \times 25} = e^{-\lambda t'}$$

$$\Rightarrow \quad e^{\lambda t'} = \frac{100 \times 25}{6.25 \times 12.5 \times 2}$$

$$\lambda t' = \log_e \left(\frac{100 \times 25}{6.25 \times 12.5 \times 2} \right) = 2.773$$

$$t' = \frac{1}{\lambda} \times 2.773 = T_{av} \times 2.773$$

$$= 40.008 \text{ s}$$

Ex.17.16.6 : If 1 gm of R_a^{226} emits 11.6×10^{17} - particles per year and half life of R_a^{226} be 1600 years, calculate the value of Avogadro's number.

Soln.

Let N_{av} be Avogadro's number

Then $N = \frac{N_{av}}{226}$

Decay rate $\frac{dN}{dt} = \lambda N = \frac{0.693}{T_{1/2}} . N$

$$\Rightarrow \quad 11.6 \times 10^{17} = \frac{0.693}{1600} \times \frac{N_{av}}{226}$$

$$\Rightarrow \quad N_{av} = \frac{11.6 \times 10^{17} \times 1600 \times 226}{0.693}$$

$$= 6.052 \times 10^{23}$$

Ex.17.16.7 : A piece of ancient wood shows an activity of 3.9 disintegrations per second per gram of C^{14}. Calculate the age of wood. Half life of C^{14} = 5568 year, Activity of fresh C^{14} = 15.6 disintegrations / sec / gram.

Soln.

Activity $A(t) = \lambda N(t) = 3.9$

$A_0 = \lambda N_0 = 15.6$

$$\frac{A}{A_0} = \frac{N}{N_0} = \frac{3.9}{15.6} = \frac{1}{4} = e^{-\lambda t}$$

$$\Rightarrow \quad e^{\lambda t} = 4$$

$$t = \frac{1}{\lambda} \log_e 4 = \frac{2}{\lambda} \log_e 2 = 2 \times \left(\frac{0.693}{\lambda} \right)$$

$= 2 \times T_{1/2} = 2 \times 5568$ yrs.

$t = 11136$ yrs.

Ex. 17.16.8 : If λ is the decay constant of a nucleus, find the probability that a nucleus will decay in time 't' and will not decay in time t.

Soln.

No. of atoms surviving after time t is

$$N = N_0 e^{-\lambda t}$$

So no. of atoms decaying in time t is

$$N_0 - N = N_0(1 - e^{-\lambda t})$$

Probability that an atom decay in time t is

$$P_d = \frac{\text{No. of atoms decayed}}{\text{Total no. of atoms}}$$

$$= \frac{N_0(1 - e^{-\lambda t})}{N_0} = 1 - e^{-\lambda t}$$

and probability of not decaying in time t is

$$P_s = 1 - (1 - e^{-\lambda t}) = e^{-\lambda t} = \frac{N(t)}{N_0}$$

17.17: Cyclotron

The **cyclotron,** first devised by E. O. Lawrence and M. S. Livingstone, is a device to speed up charged particles in successive steps under the combined influence of electric and magnetic fields along spiral path.

Construction:-

It consists essentially of two flat semi-circular hollow metal boxes D_1 D_2, which are called as

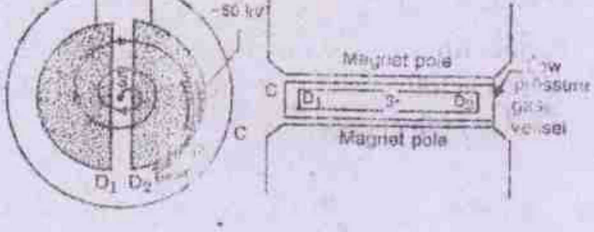


Fig.17.17.1 Fixed frequency cyclotron

dees (D). These dees (D_1, D_2) have their diametric edges parallel and slightly separated from each other so as to produce a narrow gap in between them.

The dees are placed in an evacuated chamber 'C', and are connected to an rf-oscillator so that a high frequency (=10 M Hz) alternating potential is applied between the dees acting as electrodes. As a result the potential between the dees alternates rapidly and the electric field in the gap is first directed to one dee and then to the other. The space within each dee is however field-free (zero field). The chamber 'C' is mounted horizontally between the pole pieces N, S of a huge electromagnet that provides an intense and uniform vertical magnetic field of several tesla. The diameters of the poles are slightly greater than that of the dees so that the field is uniform over the whole of the dees.

The ion source 'S' is located near the mid-point of the gap between the dees. The ions, after being accelerated, are brought out of the chamber through a window P by a charge deflecting plate to bombard the' target.

Principle (/Theory):-

Let a positive ion, carrying charge q, leave the ion-source between the dees and enter the negative dee at the instant with velocity v_0, normal to the magnetic field B. The force acting on the particle is then

$$\vec{F} = q\vec{v_0} \times \vec{B} \qquad \text{............17.17.1}$$

Since B is perpendicular to v_0, so

$$F = qv_0B \qquad \text{............17.17.2}$$

Under the action of this force, the particle of mass m pursues a circular path of radius r_0, given by

$$qv_0B = mv_0^2/r_0 \qquad \text{..... 17.17.3}$$

So that $\quad \mathbf{r_0 = mv_0/qB} \qquad$ 17.17.4

When the particle is inside the dee, its speed remains constant; but after describing the semicircular path inside the dee, as it emerges to reach the gap, the electric field of the rf-source changes direction synchronously. The particle thus gets a kick and is further accelerated and enters the other dee with larger speed ($v > v_0$). As a result it moves in a semicircular path of comparatively larger radius r, given by

$$\mathbf{r = mv/qB} \quad \text{........ 17.17.5}$$

This process is repeated again and again and the speed hence the kinetic energy increases every time. The time for the ion to travel a semicircle is

$$T/2 = \pi r/v = \pi m/q B \quad \text{...... 17.17.6}$$

This shows that this time is independent of both speed of the ion and the radius r of the path. It follows from eqn. 17.17.6, that the frequency of revolution of the ion in the circular orbit is

$$f = 1/T = (2\pi r/v)^{-1} = (2\pi m/q B)^{-1} = q B/2\pi m \quad \text{.......... 17.17.7}$$

This shows that the frequency is independent of both the speed of the ion and the radius r of the path in the dees. So if the electric field reverses regularly at a frequency exactly equal to f, then the field in the gap is always in the right direction to accelerate the ion by an impulse each time the ion crosses the gap. Hence it is required

$$qB/2\pi m = f = f^* \quad \text{......... 17.17.8}$$

This is called the basic resonance equation for a fixed frequency cyclotron; f being the frequency of the oscillator output.

Thus the revolving ion is steadily speeded up describing a flat spiral path of increasing radius. Finally it reaches periphery of the dees and is brought out of the chamber through the window 'p' by a negatively charged deflector to impinge on the properly mounted target.

The kinetic energy associated with the ion is

$$W_k = mv^2/2 \quad \text{............. 17.17.9}$$

Giving, $(W_k)\max = m\, v_m^2/2 = (q^2 B^2/2m)\, r_m^2$ 17.17.10

Thus the maximum kinetic energy that can be imparted to the ion depends on squares of (i) the radius of the dees and (ii) magnetic field strength of the electromagnet.

Advantages:

It can energize proton, deuteron, a- particle to several million eV (10 - 50 M eV) without requiring a high voltage source. Thus no accelerating tube capable of withstanding high potential difference is necessary.

Limitations:

The relativistic effect is not taken into account. At high speed we have

$m = m_0/\sqrt{(1 - v^2/c^2)}$ Where m_0 is the rest mass.

So the frequency f is given by

$$f = q B / 2\pi\ m = B q c^2 / 2\pi (m_0 c^2 + W_k) \quad \text{17.17.11}$$

where W_k is the K.E of the ion corresponding to speed v. The above shows that as speed of ion increases the W_k increases and hence frequency 'f decreases. So in a fixed frequency cyclotron, the particle as it spirals out from the source will lag in phase behind the accelerating dee voltage. It would then cease to be accelerated. It is therefore necessary to suitably modify the oscillator frequency to ensure resonance acceleration in the gaps. This cannot be done without upsetting the condition for low energy particles.

The relativistic change is more pronounced for electrons than for positive ions. This makes the cyclotron less useful for accelerating electrons to high energy.

Further limitation is that we need larger size of the cyclotron (as W_k is proportional to r^2) to achieve high energy.

SUMMARY

1. The nucleus contains protons and neutrons - which are called nucleons. An electron cannot remain inside the nucleus as de broglie wave length of electron is much greater than the nuclear size.
2. Neutron is slightly heavier than proton.
3. Fermi ($\approx 10^{-15}$ m) is convenient for measuring length in nuclear physics.
4. Atomic number (A) = No. of protons (Z) + No. of neutrons (N).
5. Radius of nucleus $R = r_0 A^{1/3}$
6. Density of nuclei is independent of mass number. So it is same for all atoms.

   $\int nu : 10^{17} kgm^{-3}$.
7. Relative strength of gravitational, coulomb and nuclear force is :

   $$F_g : F_e : F_n :: 1 : 10^{36} : 10^{38}$$
8. Theoretically all the four forces operate inside the nucleus. But strong nuclear force dominates.
9. After Yukawa predicted meson theoritically, it was discovered in cosmic rays later on.
10. Nuclear force is charge independent. So

    $$F_{n-n} = F_{n-p} = F_{p-p}$$
11. The nuclear force between two nucleons having parallel spins is greater than that between two nucleons having anti-prallel spins.
12. The mass of nucleus is less than the sum of masses of its nucleons in free state. This difference of mass is called mass-defect (Δm).
13. Packing fraction $= \frac{\Delta m}{A}$

    Nuclei with negative packing fraction are more stable.
14. Binding energy (E_B) is the external energy required disrupt a stable nucleus into its constituent nucleons.

    $E_B = \Delta m.C^2 = \Delta m \times 931.5$ (in MeV)

    Greater binding energy per nucleon greater is the stability.
15. Nuclide is a species of nucleus. Sometimes it is used for "nucleus".
16. Nuclear mass is measured in terms of unified atomic mass unit(u)

    $1u = \frac{1}{12}$ mass of one atom of $^{12}_{6}C$ including mass of electrons.

    $1u = 1.660566 \times 10^{-27}$ Mev.
17. The energy equivalent of 1u=931 Mev.
18. When an unstable nucleus emits an α – particle its atomic number decreases by 2 and atomic mass decreases by 4.
19. β rays are streams of electrons and positrons. These electrons and positrons do not pre-exit in the nucleus. Either a proton transforms into a nutron or a neutron transforms into proton. This is called β - decay.

    Beta minus decay: $n \rightarrow P + e^- + \bar{v}$

    Beta plus decay : $P \rightarrow n + e^+ + v$

    The emission of β^- (electron) results in the increase in atomic no. by one unit where as emission of β^+ (positron) results in the decreases in atomic no. by one unit. There is no change in mass number.
20. the activity of a sample at any instant is the rate at which the nuclei of its constituent atoms decay.

    $$A = -\frac{dN}{dt}$$

21. Statistical law: $\frac{dN}{dt} = -\lambda N$. [$\therefore A = \lambda N$]

22. If N_0 is the no. of nuclide present initially then at any instant.

$$N = N_0 \, e^{-\lambda t}$$

In terms of activity : $A = A_0 e^{-\lambda t}$

23. Half lite $T_{1/2} = \frac{ln\ 2}{\lambda} = \frac{0.693}{\lambda}$

24. Average life $T_{av} = \frac{T_{1/2}}{0.693}$

25. If 'n' is the number of half lives, then

$$\frac{N}{N_0} = (\frac{1}{2})^n \text{ or } \frac{A}{A_0} = (\frac{1}{2})^n$$

MODEL QUESTIONS

A. Multiple Choice Type Questions :

1. The atomic nucleus contains :
   a. protons and electrons
   b. neutrons and electrons
   c. protons and neutrons
   d. protons, neutrons and electrons

2. The nuclear dimension is of the order of
   a. 10^{-10} m b. 10^{-6} m
   c. 10^{-14} m d. 10^{-12} m

3. For nuclear fission, which equation is used
   a. $E = m$ b. $E = mC^2$
   c. $E = m/C^2$ d. $E = mc$

4. The mass of proton is
   a. 1.67×10^{-24} kg
   b. 1.67×10^{-27} kg
   c. 1.67×10^{-30} kg
   d. 1.67×10^{-19} kg

5. The mean binding energy per nucleon in the nucleus of an atom is nearly
   a. 8 eV b. 8 keV
   c. 8 MeV d. 9 GeV

6. One requires an energy E_n to remove a nucleon from the nucleus and an energy E_e to remove an electron from an atom. Then
   a. $E_n = E_e$ b. $E_n < E_e$
   c. $E_n > E_e$ d. $E_n \geq E_e$

7. Nuclear force exists between
   a. neutron-neutron
   b. neutron-proton
   c. proton-proton
   d. all of the above

8. Cosmic rays, as they arrive at the top of the atmosphere, consists mainly of :
   a. High energy electrons
   b. Heavy atoms
   c. Heavy nucleus
   d. Protons

9. When the mass equal to 1 amu is converted completely into energy, the energy produced is
   a. 1.5×10^{-18} J b. 1.5×10^{-14} J
   c. 1.5×10^{-12} J d. 1.5×10^{-10} J

10. 1 atomic mass unit (amu) is
    a. 1 kg
    b. 1 gm
    c. mass of one atom of ${}_6C^{12}$
    d. 1/12 th of mass of one atom of ${}_6C^{12}$

11. Mass of one atom of ${}_8O^{16}$ = 15.994915 a.m.u.
    Mass of proton = 1.007825 amu
    Mass of neutron = 1.008665 amu
    The binding energy per nucleon of ${}_8O^{16}$ nucleus is
    a. 0.137005 MeV
    b. 127.62 MeV
    c. 7.98 MeV
    d. 119.64 MeV

12. Isotopes possess
    a. same physical and chemical properties
    b. same physical but different chemical properties
    c. different physical but same chemical properties
    d. different physical and chemical properties

13. In the nuclear reaction

$_{92}U^{234} \rightarrow _{90}Th^{230} + X + Q;$

'X' is

a. $_1H^2$ b. $_2He^4$

c. $_3Li^6$ d. $_4He^2$

14. In a working nuclear reactor, cadmium rods are used to

a. speed up neutrons

b. slow down neutrons

c. absorbs some neutrons

d. absorbs all neutrons

15. Heavy water is one of the substance used as moderator in reactors. Which of the following is also used as moderator ?

a. sea water b. graphite

c. cobalt d. titanium

16. Fusion reactions take place at a high temperature because

a. atoms are ionised at high temperature

b. molecules break up at high temperature

c. nuclei break up at high temperature

d. kinetic energy is high enough to overcome repulsion between nuclei

17. $_{92}U^{238}$ on absorbing neutron goes over to $_{92}U^{239}$. This nucleus emits an electron and goes over to Neptunium, which on further emitting an electron goes over to plutonium. The resulting plutonium can be represented as

a. $_{94}Pu^{239}$ b. $_{92}Pu^{239}$

c. $_{94}Pu^{240}$ d. $_{92}Pu^{241}$

18. In the nuclear reaction

$_2He^4 + _7N^{14} \rightarrow X + _1H^1;$

'X' is

a. nitrogen of mass 16

b. nitrogen of mass 17

c. oxygen of mass 16

d. oxygen of mass 17

19. When $_3Li^7$ captures a proton, the element formed is

a. Helium b. Calcium

c. Barium d. Sodium

20. In nuclear reactor we have conservation of

a. mass only

b. energy only

c. momentum only

d. mass, energy and momentum

21. In what sequence the radioactive radiations are emitted in the following nuclear reaction

$_ZX^A \rightarrow _{Z+1}Y^A \rightarrow _{Z-1}K^{A-4} \rightarrow _{Z-1}K^{A-4}$

a. α, β, γ b. β, α, γ

c. γ, α, β d. β, γ, α

22. Neutrino emitted in β-decay has

a. no charge b. spin equal to $\frac{1}{2}$

c. zero rest mass d. all of these

23. The age of a rock can be estimated by

a. absorption of γ-rays, when they pass through it

b. knowing the gravimetric composition of the compound of silicon in the rock

c. radioactive dating

d. knowing the elastic properties of rock.

24. When an atomic nucleus emits γ-rays, then
    a. atomic number is changed
    b. mass number is changed
    c. mass of nucleus decreases
    d. mass of nucleus increases

25. The function of a moderator in a nuclear reactor is to
    a. absorb dangerous γ-rays
    b. react with Uranium to release energy
    c. release energy by combustion
    d. slow down fast neutrons so as to have controlled fission.

26. $_{92}U^{238}$ decays successively to form $_{92}U^{234}$. What are the radiations emitted in these three steps
    a. β, α, β b. α, β, α
    c. α, β, β d. β, α, α

27. In fission reactor heavy water is used as
    a. coolant
    b. moderator
    c. heat exchanger
    d. controller of reaction rate

28. Energy in Sun is produced as a result of
    a. fission
    b. combustion
    c. explosion
    d. thermonuclear reaction

29. What percentage of original radio-active atoms is left after five half-lives ?
    a. 20% b. 10%
    c. 5% d. 3.25%

30. 1 curie is equal to
    a. 3.7×10^{10} disintegrations / sec
    b. 4.7×10^{10} disintegrations / sec
    c. 2.7×10^{11} disintegrations / sec
    d. 3×10^{9} disintegrations / sec

31. 1 Rutherford is equal to
    a. 10^{7} disintegrations / sec
    b. 10^{8} disintegrations / sec
    c. 10^{6} disintegrations / sec
    d. 10^{9} disintegrations / sec

32. 1 becquerrel is equal to
    a. 2 decays / sec
    b. 1 decay / sec
    c. 5 decays / sec
    d. 10^{4} decays / sec

B. Very Short Answer Type

1. What is mass of proton in amu ?
2. Mention two properties of neutrons.
3. What is an isotope ?
4. How binding energy and mass defect are related ?
5. What is mass defect ?
6. How radius of a nucleus is related with its mass number ?
7. Complete the following nuclear reactions
   (i) $_{8}O^{15} + _{0}n^{1} \rightarrow _{6}C^{12} + ?$
   (ii) $_{7}N^{14} + ? \rightarrow _{6}C^{14} + _{1}H^{1}$
8. State two laws obeyed in nuclear reaction.
9. What are β-rays ?
10. What is the nature of charge in α-rays ?
11. What is the speed of γ-ray in vaccum ?
12. Why γ-rays are not deflected in a magnetic field ?
13. What is the function of a moderator in a nuclear reactor ?
14. Among α, β and γ-rays which one is an e.m. wave ?
15. Write the name of a substance used as a moderator ?
16. Write the relation between half-life and mean life of a radioactive nucleus.
17. Write the relation between decay constant and half-life of a radioactive nucleus.

18. What is a fission reaction ?
19. What is fusion reaction ?
20. What is the source of solar energy ?
21. Name the process in which two nuclei combine to form a new nucleus.
22. Write the name of two units of radio-activity.
23. What is the energy equivalence of 1kg of matter.
24. Write the name of a coolant used in nuclear reactor.
25. Which of the two is more energetic-fission of U^{235} or fusion of hydrogen nuclei ?
26. $_{92}U^{235}$ decays to thorium, by emitting an α-particle. What is the mass number and atomic number of thorium ?
27. Among alpha, beta and gamma radiations, which get affected by electric field ? [CBSE AI 2004]
28. What will be the ratio of two nuclei of mass numbers A_1 and A_2 ? [CBSE 1993, 95]
29. What is the nuclear radius of F^{125} if that of Al^{27} is 3.6 fermi ? [CBSE AI 2008]
30. The binding energy per nucleon of the two nuclei A and B are 4 MeV and 8.2 MeV respectively. Which of the two nuclei is more stable ? [CBSE 1999]

C. Short Answer Type

1. Which of the fundamental forces strongest ? Mention their relative strength.
2. Prove that the nuclear density is independent of mass number.
3. What do you mean by Binding energy per nucleon ? What are the conclusions drawn from binding energy curve ?
4. In what way isotope differs from isobar ?
5. What is an atomic mass scale ? Where is it used ?
6. Distinguish between fission and fusion.
7. Explain controlled chain reaction.
8. Write four properties of γ-rays.
9. Explain chain reaction.
10. Explain why fusion reaction cannot be possible in the laboratory.
11. What is radio-isotope ?
12. Find out the decay constant of Radon whose half-life is 1620 years.
13. What is the cause of natural radio-activity ?
14. Why should a fissionable material have a minimum critical mass to sustain chain reaction ?
15. Describe briefly uses of radio-isotopes as tracers in medicine.
16. Explain the release of energy in Sun.
17. Explain the difference between isotope and radio-isotope.
18. State the use of radio-isotope as tracer in agriculture and industry.
19. Define half-life and mean-life of a radioactive nuclei.
20. Name the units used to measure radio-activity.
21. Calculate the energy released when 10kg

of $_{92}U^{235}$ undergoes the reaction

$$_{92}U^{235} + _0n^1 \rightarrow _{56}Ba^{141} + _{36}Kr^{92} + 3_0n^1 + Q$$

22. What percentage of a given mass of radioactive substance will be left undecayed after four half periods ?

[CBSE 1997]

23. Write the nuclear decay process for β-decay of $_{12}P^{32}$. [CBSE 2004]

24. Explain with example, whether the neutron-proton ratio in a nucleus increases or decreases due to -decay.

[CBSE AI 2003]

25. A radioactive nucleus A undergoes a series of decay according to following scheme:

$$A \xrightarrow{\alpha} A_1 \xrightarrow{\beta-} A_2 \xrightarrow{\alpha} A_3 \xrightarrow{\gamma} A_4$$

The mass number and atomic number of A are 180 and 72 respectively. What are these numbers for A_4 ?

[CBSE AI 2002 C]

D. Unsolved Problems

1. Compare the nuclear radius of $_2He^4$ and $_6C^{12}$.

2. Calculate the binding energy per nucleon in the lithium $_3Li^7$ from the following data

Mass of proton = 1.00759 amu

neutron = 1.00898 amu

electron = 0.00594 amu

lithium atom = 7.01818 amu

3. Find out the binding energy of helium nucleus, if the loss of mass for helium nucleus is 0.0304 amu.

4. How many electrons, protons and neutrons are there in 12 gm of $_6C^{12}$? Avogadro's number = 6×10^{23} at 1gm at wt.

5. 0.1 gm atom of radio-active isotope $_ZX^A$ (half-life 5 days) is taken. How many number of atoms will decay during eleventh day.

6. 1 g of radioactive substance takes 50 second to lose 1 centigram. Find its decay constant and half-life period.

7. The half-life of radon is 3.8 days. After how many days will $\frac{1}{20}$th of radon sample be left over ?

8. Calculate the weight in grams of one curie of Pb^{214} from its half-life of 26.8 minutes.

9. The half-life of cobalt radio-isotope is 5.3 years. What strength will a milli-curie source of the isotope have after a period of one year ?

10. What is the activity of one gram of $_{86}Ra^{226}$ whose half-life is 1622 years ?

11. If 3×10^{-9} kg of $_{79}Au^{200}$ has an activity of 58.9 ci, what is its half-life ?

12. Calculate the total energy liberated in the fission reaction

$$_{92}U^{236} \rightarrow X^{117} + Y^{117} + 2n^1 + Q$$

Given that binding energy per nucleon of X and Y is 8.5 MeV and of U^{236} is 7.6 MeV.

13. If mass of U^{235} = 235.12142 u and mass

of U^{236} = 236.12305 u and mass of neutron = 1.008665 u, then calculate the energy required to remove one neutron for U^{236}.

14. Assuming that about 20 MeV of energy is released per fusion reaction.

$${}_1H^2 + {}_1H^3 \rightarrow {}_0n^1 + {}_2He^4$$

Calculate the mass of ${}_1H^2$ consumed per day in a fusion reactor of power 1 megawatt.

15. Energy produced from the fusion reaction

$$2\,({}_1H^2) = {}_2He^4 + Q$$

is to be used for the production of power. Assuming the efficiency of the process to be 30%, calculate the mass of deuterium that will be consumed in a day for an output of 50 MW.

Given m $({}_1H^2)$ = 2.01478 u,

m $({}_2He^4)$ = 4.00388 u

16. In the reaction

$${}_{92}U^{235} + {}_0n^1 \rightarrow {}_{42}Mo^{95} + {}_{57}La^{139} + 2\,{}_0n^1 + 7\,{}_{-1}e^0$$

Calculate the energy released

m (U^{235}) = 235.0439 u;

m $({}_0n^1)$ = 1.0087 u;

m (Mo^{95}) = 94.9058 u;

m (La^{139}) = 138.9061 u

17. Binding energy per nucleon of C^{12} is 7.68 MeV and that of C^{13} is 7.47 MeV. Calculate the energy required to remove a neutron from C^{13}.

18. If 200 MeV is released per fission of U^{235} atom, find the number of fission per second required to release 1 KW power.

19. A free neutron decays to a proton and electron and antineutrino $(n \rightarrow \rho + e^- + \bar{\nu}_e)$. Calculate the energy produced in this reaction in MeV.

Given m $({}_{-1}e^0)$ = 9 x 10^{-31} kg,

m $({}_0n^1)$ = 1.6747 x 10^{-27} kg

m $({}_1H^1)$ = 1.6725 x 10^{-27} kg

20. A radioactive istope has half-life of 5 years. After how much time is its activity reduced to 3.125% of its original activity? [CBSE AI 2008]

21. A radioactive sample contains 2.2 mg of pure ${}_6C^{11}$ which has half-lifeperiod of 1224 seconds. Calculate (i) the number of atoms present initially. (ii) the activity when 5 μg of the sample will be left. [CBSE AI 2005]

22. The half-life of ${}_{92}U^{238}$ against α-decay is 4.5×10^9 years. Calculate the activity of 1 g sample of ${}_{92}U^{238}$.

[CBSE AI 2005]

E. Long Answer Type

1.(a) What is neutron ? Describe its properties.

(b) What is proton ? Describe its properties.

2. What is atomic mass unit ? Derive the relation between atomic mass unit and million electron volt (MeV).

3. What is nuclear force ? Describe the characteristics of nuclear force.

4. Discuss in detail the properties of Nucleus.

5. What do you mean by nuclear reaction ? Find expression for Q-value in nuclear reaction.

6. Discuss chain reaction in fission. Explain uncontrolled and controlled chain reaction.

7. What is natural radioactivity ? Name the radioactive rays and state their distinguishing properties.

8. What are radio-isotopes ? Describe the uses of radio-isotopes.

F. Answer as directed :

1. One atomic mass unit is equal to ____ kg.

2. Are fission fragments radiactive ? (Yes/No.)

3. Which process is responsible for the source of stellar energy ?

4. How is the size of the nucleus related to the power of mass number ?

5. Atoms having the same ____ but different ____ are called isotopes.

6. The number of neutrons in ${}_{17}Cl^{37}$ is ____.

7. What is the rest energy in ev involved in a mass of one atomic mass unit ?

8. If the radiation, as emitted by a natural radioactive substance, is not affected by electric field, What is that radiation called ?

9. What is the half life period of lead ?

10. The last member in any natural radioactive series is ____.

G. Correct the following sentences :

1. Atoms having different atomic number but same mass number are isotopes.

2. Atomic nucleus contains protons and electrons.

3. 1 atomic mass unit (amu) is 1/10 th of mass of one atom of ${}_6C^{12}$.

4. Nuclear force is weaker than electromagnetic force.

5. In unclear reaction mass number is conserved but not atomic number.

6. ${}_8O^{15} + {}_0n^1 \rightarrow {}_6C^{13} + {}_2He^4$

7. γ-rays are deflected in magnetic field.

8. $1\ MeV = 1.6 \times 10^{-15} J$

9. Half life ($T_{1/2}$) and decay constant (λ) of a radioactive substance are related as $T_{1/2} = 0.693/\lambda^2$.

10. Average life (T_{av}) and decay constant (λ) of a radioactive substance are related as $T_{av} = 2/\lambda$.

11. 1 curie = 3.7×10^8 decays/second.

ANSWERS

A. Multiple Choice Type Questions :

1. (c)	**2.** (c)	**3.** (b)	**4.** (b)	**5.** (c)	**6.** (c)	**7.** (d)	**8.** (d)
9. (d)	**10.** (d)	**11.** (a)	**12.** (c)	**13.** (b)	**14.** (c)	**15.** (b)	**16.** (d)
17. (a)	**18.** (d)	**19.** (a)	**20.** (d)	**21.** (b)	**22.** (d)	**23.** (c)	**24.** (c)
25. (d)	**26.** (c)	**27.** (b)	**28.** (d)	**29.** (d)	**30.** (a)	**31.** (c)	**32.** (b)

B. Very Short Answer type Questions :

1. 1.007825 u

2. (i) charge neutral (ii) does not ionise gaseous medium while passing through it

3. see text

4. see text

5. see text

6. $R = R_0 A^{1/3}$

7. (i) ${}_2He^4$ (ii) ${}_0n^1$

8. see text

9. electrons

10. Two units of positive charge

11. 3×10^8 m/s

12. γ -rays are charge neutral

13. see text

14. γ -rays

15. heavy water

16. $T_{1/2} = \dfrac{0.693}{\lambda} = 0.693\, T_{av}$

17. $T_{1/2} = \dfrac{0.693}{\lambda}$

18. see text

19. see text

20. Thermo-nuclear reaction in sun

21. Fusion

22. Becquerel, Curie

23. 9×10^{16} J

24. liquid sodium

25. Fusion of hydrogen nuclei

26. ${}_{90}Th^{231}$

D. Unsolved Problems :

1. 7 : 10

2. 5.6 MeV

3. 28.3 MeV

4. Each 36×10^{23}

5. 1.93×10^{21}

6. 57.46 min

7. 16.43 days

8. 305.03×10^{-10} g

9. 0.877 milli lurie

10. 3.608×10^{10} decays/sec

11. 2.873×10^3 s

12. 195.4 MeV

13. 6.55 MeV

14. 0.0897 g

15. 0.025 kg

16. 207.8923 MeV

17. 4.95 MeV

18. 3.125×10^{13}

19. 0.73125 MeV

20. 25 years,

21. 12.046×10^{49} atoms, 1.55×10^{14} becquerel

22. 1.237×10^4 becquerel

F. Answer as directed :

1. 1.67×10^{-27} kg **2.** Yes **3.** Fusion **4.** $\dfrac{1}{3}$

5. Atomic number ; mass number **6.** 20

7. 931×10^6 **8.** γ - rays **9.** Infinite **10.** Lead

18

Electron Emission

18.1 Electron emission :

Electrons may be obtained from atoms in all states of matter : solid, liquid and gaseous. However, we are here concerned with : *liberation of electrons from metal surfaces, known as electron emission.*

Metals are used for emission of electrons because they have plenty of "free electrons". However, these electrons are free only to move from one atom to another within the metal but they cannot leave the metal surface. If any electron comes out, the metal is left with a net positive charge which pulls the electron back. Thus a free electron, at the surface of a metal, feels a force that prevents it form leaving the surface. This amounts to saying that the metallic surface offers a barrier to its free electrons, known as surface barrier. The free electrons need additional energy from an external agency to overcome the surface barrier for their emission from the metal surface. *The minimum extra energy needed for emission of an electron from a metal surface is known as the work function of that metal.* Work function of pure metals varies roughly from 2 to 6 eV.

18.2 Type of electron emission :

A metal can emit an electron if it receives an energy, from an external source, equal to or greater than its work function. Depending on the source of external energy, the electron emission is of four types as follows :

i) *Field emission*

Suppose a strong electrostatic field of the order of a million volts per centimeter from a positive voltage source is applied to a metal surface. This intense positive field attracts the free electrons and pull them out of this metal. Greater this intensity of the field, larger is the number of electrons emitted from the surface. This phenomenon is known as field emission.

ii) *Secondary emission*

In secondary emission, a metal is bombarded with high-speed particles which knock out its free electrons from its surface. Depending on the type and energy of the incident particle, one or more electrons are emitted.

iii) *Photoelectric emission*

Light or any other radiation of suitable frequency falling on a metal surface causes emission of electrons known as photoelectric emission.

iv) *Thermionic emission*

In thermionic emission electrons are "boiled out" from a metal in the form of a filament maintained at a high temperature. This effect was discovered by Thomas A. Edison.

The process of electron emission from a metal, maintained at a high temperature is known as thermionic emission. The emitted electrons are called thermions. The energy acquired by the electrons for their emission and motion comes from the thermal source. Diode, Triode and other vacuum tubes are based on thermionic emission.

18.3 Richardson-Dushmann equation:

Richardson and Dushmann derived an equation which gives a quantitative estimation of emission current density as

$$J_s = AT^2 e^{-b/T} \qquad ...18.3.1$$

where J_S = emission current density i.e., current per square meter of the emitting surface

T = Absolute temperature of the emitter

e = 2.718 (base of natural log)

A and b are constants depending on the type of the emitter and

$$b = \frac{\varphi e}{K}$$

where φ is the work function of the emitter in electron volt, e is the electronic charge, K is the Boltzmann's constant.

$$\text{Emission current, } I = J_S \cdot S \qquad ...18.3.2$$

where S is the area of the emitting surface of the emitter.

From eq. (18.3.2) it is clear that, emission current depends on

i) surface area of the emitter

ii) Temperature of the emitter (The product $T^2e^{-b/T}$ in the eq. (18.3.1) makes emission current to increase rapidly beyond the square of the temperature with increase in temperature)

iii) Wrok function of the emitter (emission current increases exponentially with decrease in work function).

18.4 Thermoionic emitter :

The filament or the wire used for electron emission is known as an emitter or cathode. The cathode is heated in an evacuated space to emit electrons. A thermoionic emitter should have following characteristics :

i) Low work function :

The electrons of the emitter of low work function need comparatively lower energy for their emission and hence the operating temperature of the cathode is considerably lowered.

ii) High melting point :

Electron emission takes place at a high temperature (> 1500^0 C). If the cathode is of high melting point it will not melt before the emission of electrons from it.

iii) High mechanical strength :

Cathode is at negative potential and hence positive ions, formed due to the impact of emitted electrons with molecules (even in a highly evacuated tube some residual air molecules are inveriably present) of a vacuum tube, bombard with it. The cathode or the emitter should hence have high mechanical strength to withstand the bombardment of positive ions.

Commonly used thermoionic emitters are tungsten, thoriated tungsten (tungsten mixed with a small quantity of thorium) and oxide coated metals such as nickel ribbon coated with barium and strontium oxides. Oxide coating lowers the work function.

Methods of heating the Cathode :

There are two ways of heating the cathode in vacuum tubes. Accordingly we have either directly heated cathode or indirectly heated cathode. In directly heated cathode, electric current is passed through a wire, called filament, which serves as the electron emitter. Fig. (18.1a) shows the structure of directly heated cathode while fig (18.1b) shows its symbol.

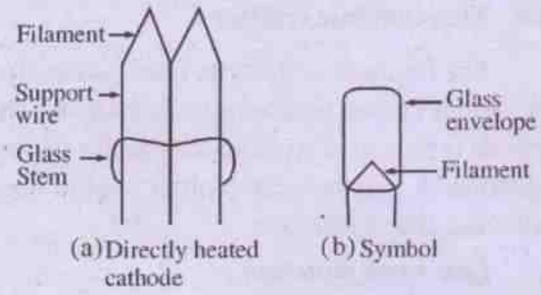


Fig. 18.1

In an indirectly heated cathode, electric current is passed through a separate heater element which is enclosed by a cylindrical cathode (Fig. 18.2). Thus heat is indirectly transferred by radiation from heater element to cathode.

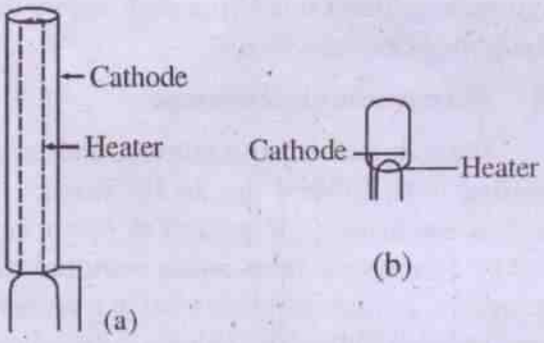


Fig. 18.2 (a) Indirectly heated cathode and (b) schematic symbol

Indirectly heated cathodes are preferred in vacuum tubes due to following advantages.

i) Indirectly heated cathode can be maintained to a desired potential as it is completely separated from heater circuit.

ii) Small heater voltage fluctuations do not affect much the electron emission from an indirectly heated cathode and hence hum (a type of noise) is not produced.

and iii) A.C. can be used in the heater circuit.

18.5 Vacuum tubes :

A vacuum tube is an electronic device in which electrons flow through a highly evacuated glass or metal tube. In all types of vacuum tubes the two common electrodes are :

i) Cathode - the emitter of electrons

ii) Anode or plate - the collector of electrons

The flow of electrons inside a vacuum tube is unidirectional - from cathode to anode. Hence current flows from anode to cathode. This behaviour resembles with the working of a valve in a water lift pump. That is why in early days, vacuum tubes were popularly known as valves.

A vacuum tube is highly evacuated (air pressure ~ 10^{-3} mm of mercury) due to the following reasons.

i) In the presence of air, the heated filament will oxidize and burn.

ii) The cathode emits more electrons in vacuum.

iii) If air is present, its molecules will be ionized and these ions will move towards the electrodes besides the emitted electrons which is undesirable.

Types of vacuum tubes :

Depending on the number of electrodes used, vacuum tubes are of following types :

i) Vacuum diode (contains two electrodes)

ii) Vacuum triode (contains three electrodes)

iii) Vacuum tetrode (contains four electrodes)

iv) Vacuum pentode (contains five electrodes)

18.6 Vacuum diode :

In 1904, Sir J.A. Fleming, an English Physicist, invented first vacuum diode, called

Fleming's valve. Many improvements have been made in the vacuum diode since the invention of Fleming's diode.

Construction :

A vacuum diode consists of two electrodes, a cathode and an anode.

a) ***Cathode :***

The cathode is in the form of a nickel cylinder coated with barium and strontium oxides and is heated indirectly.

b) ***Anode or plate :***

It is also in cylindrical form surrounding the cathode and it is made up of nickel or molybdenum.

These two electrodes are enclosed in a highly evacuated glass or metal envelope and pin connections are brought out for application of necessary potentials to electrodes (Fig. 18.3).

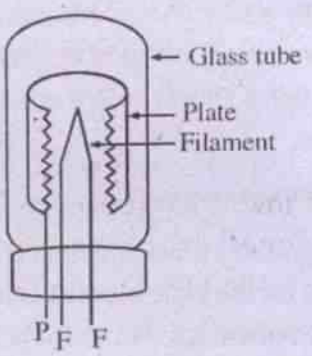


Fig. 18.3 Diode

Working of a diode :

When the cathode is heated by the heater element, thermionic electrons are emitted from the cathode.

Space charge :

If the plate is open or at zero or negative potential with respect to cathode, the electrons emitted from heated cathode start accumulating near the cathode and forms a negatively charged cloud of electrons. This cloud of electrons is called space charge [Fig. 18.4 (a)].

As the electrons leave the cathode surface, it becomes positively charged. The combined effect of positively charged cathode and negative space charge is to send a few electrons from space charge back to cathode. But in the meanwhile more electrons are emitted by the cathode. Soon a dynamic equilibrium is established when the number of electrons emitted is equal to the number of electrons attracted back to cathode. At a given operating temperature of the cathode, the number of electrons constituting the space charge becomes constant. The space charge now becomes a source of electrons that can be attracted by the plate if it is at positive potential w.r.t. cathode Fig 18.4 (b).

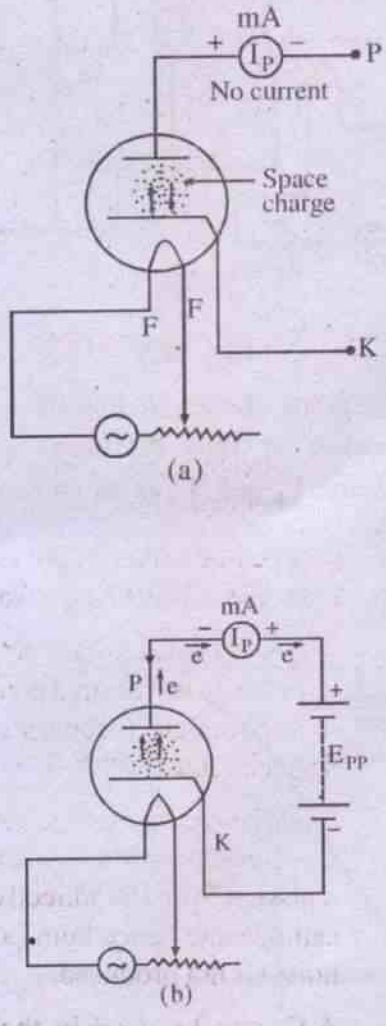


Fig. 18.4

The attracted electrons by the plate sets up a current in the plate circuit known as plate current. With increase in positive potential of the plate, plate current increases until saturation is reached.

18.7 Characteristics of vacuum diode :

The relation between plate current in a diode and plate-to-cathode voltage at a given cathode temperature can be represented by a curve known as plate characteristic of diode. It can be studied using the circuit (fig. 18.5).

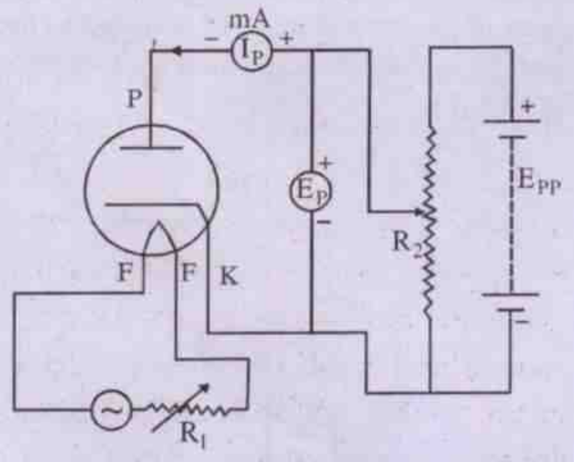


Fig. 18.5

The diode characteristics for a typical diode valve at two different cathode temperatures T_1 and T_2 is as shown in the Fig.(18.6).

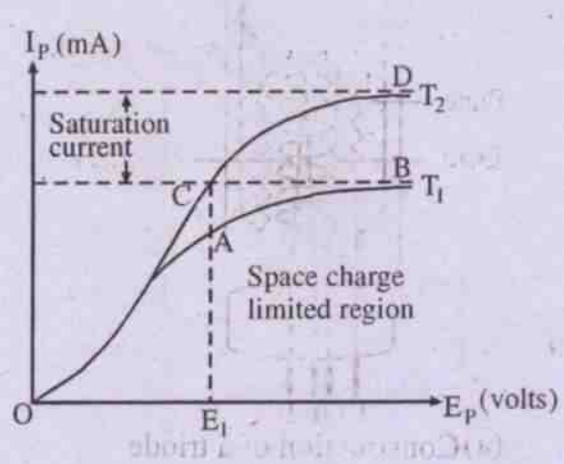


Fig. 18.6

These are non-linear curves that can be subdevided into two regions :

i) Space charge limited region [OA and OC in the Fig. (18.6)].

ii) Saturation regions [AB and CD in the Fig. (18.6)].

i) Space charge limited region

In this region applied plate potential E_p is not sufficient to collect all the electrons that are emitted by the cathode and the charges uncollected by the plate remain in between plate and cathode as space charge. In this space charge limited region, the relation between the plate current I_p and plate voltage E_p is given by Child's law or three halves power law as

$$I_P = KE_P^{3/2} \qquad ...18.7.1$$

where K is a constant that depends on the shape of the electrodes and geometry of the valve. In the space charge limited region, the plate current I_p is controlled by plate - cathode voltage E_p but is almost independent of cathode temperature.

ii) Saturation region :

It begins at a plate potential ($E_p = E_1$) that is sufficient for the plate to collect almost all the electrons emitted by the cathode and hence space charge is eliminated in the saturation region. In this region the plate current approximately remains constant even though the plate potential is increased. The only way to increase the plate current is to increase the rate of emission of electrons i.e. to increase the cathode temperature. Since the plate current is governed by the temperature of the cathode, this region is also called temperature limited region and the plate current is given by

$$I_P = A'T^2e^{-b/T} \qquad ...18.7.2$$

where A' and b are constants for a given diode value.

18.8 Plate resistance of a diode :

The internal resistance offered to the flow of current by a diode is known as its plate resistance. It is mainly due to space charge and it differs at different plate voltages and hence it is non-ohmic. Plate resistance of a diode also depends on the nature of applied plate voltage. Accordingly, a diode has two types of resistance, viz., d.c. plate resistance and a.c. plate resistance.

i) d.c. plate resistance :

The resistance offered by a diode valve to direct current is known as d.c. plate resistance. It varies with d.c. plate voltage.

ii) a.c. plate resistance (r_P) :

The ratio of a small change in plate voltage (ΔE_p) across a diode to the resulting change in plate current (ΔI_P) is known as a.c. plate resistance.

$$r_P = \frac{\Delta E_p}{\Delta I_P}$$

In the saturation region r_P is infinity.

Example 18.7.1 The plate current in a diode is 1.0 mA at a plate voltage of 50V. When the plate voltage is 200 V, what is its plate current if it is operating in the space charge limited region.

Solution :

$I_{P_1} = 1$ mA, $E_{p_1} = 50$ V, $E_{p_2} = 200$ V

$I_{P_2} = ?$

According to child's law

$$\frac{I_{P_2}}{I_{P_1}} = \left(\frac{E_{P_2}}{E_{P_1}}\right)^{3/2}$$

$$= 8$$

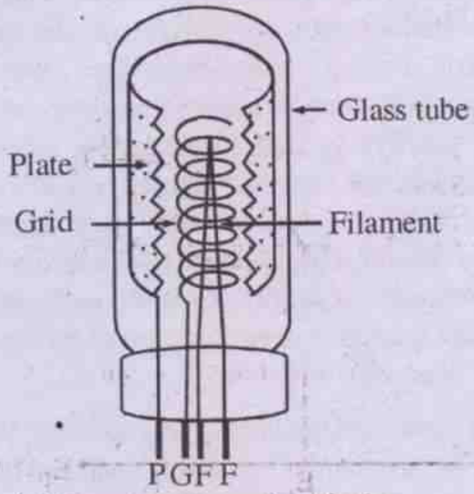
$$\therefore I_{P_2} = 8 \text{ mA}$$

18.9 Vacuum Triode :

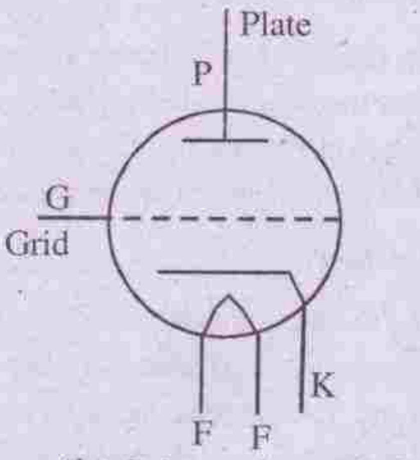
In 1906, Dr. Lee De Forest, an American Scientist, invented triode by placing a third electrode, called control grid, between the cathode and the plate of a vacuum diode.

Construction :

As its name implies, a triode consists of three electrodes, viz., i) cathode ii) plate and iii) control grid. The cathode is located at the centre of the tube and is surrounded by the control grid which is in turn surrounded by the plate. The cathode and the plate have similar construction as in a diode. The control grid is in the form of a spiral of fine wire usually made of nickel, molybdenum or iron and it is situated very close to the cathode. The spacings between the turns of the spiral are wide enough so that passage of electrons from the cathode to plate is not directly hindered by the grid. The assembly of the three electrodes is enclosed in a highly evacuated envelope of glass or metal. The construction and schematic symbol of a triode are shown in Fig. (18.7 a) and Fig. (18.7 b) respectively.



(a) Construction of a triode



(b) Schematic symbol

Fig. 18.7

Working of a triode :

The cathode and the plate of a triode serve the same purpose as in a diode. The cathode emits the electrons by thermionic process. If the plate is at positive potential with respect to the cathode, electrons are attracted towards the plate through the openings of the control grid.

Function of the control grid :

Since the control grid is nearer to the cathode than the anode, a voltage applied to the grid has a much larger effect (due to the greater electric field intensity) on the anode current than the same voltage applied to the anode. The grid, thus, has a controlling effect on the anode current. Thus a triode needs three operating voltages. The anode is normally connected to a high positive voltage to attract the electrons emitted by the cathode. A relatively low (a.c. or d.c.) voltage is connected to the filament or heater to bring the cathode to its proper emitting temperature for supply of electrons. Finally, a voltage is applied to control grid to govern the flow of anode current.

To assign bias (operating voltage) to the grid, let us consider separately each one of the following three possibilities :

i) ***Zero grid bias :***

The positive voltage of the plate attracts the electrons through the openings of the grid wire giving rise to plate current. The action is similar to that of a diode but due to shielding action of grid, the plate current is somewhat less than it would be with the grid entirely removed.

ii) ***Positive grid bias :***

The grid potential now aids the plate voltage and produces stronger electrostatic field at the cathode, resulting in a larger plate current. causing a huge plate dissipation and hence the plate may become excessively hot. Due to positive potentiial of the grid, some electrons are also attracted towards it, giving rise to grid current in the grid circuit. Grid current is undesireable because it produces power loss in the grid circuit and grid dissipation produced by the grid current may damage the grid structure seriously. Moreover, in the presence of signal in the grid circuit, variation of plate current, with positive grid bias, becomes non linear and it is not of practical importance.

iii) ***Negative grid bias :***

In this case, electrons moving towards the plate experience a retarding force and hence plate current decreases. If negative potential of the grid is increased continuously (for constant positive plate potential), the plate current continuously decreases and ultimately becomes zero. *The smallest negative potential between the grid and the cathode that is just capable of cutting off the plate current is called cut-off grid bias.* Cut-off grid bias is the characteristic of a tube but depends on plate voltage. If the grid bias voltage is more negative than this cut-off grid bias, the triode will not function.

Triode valves are, thus, generally operated at small negative grid voltages (less than cut-off bias) with respect to the cathode.

18.10 Characteristics of a triode :

The relationship between plate current I_p, plate voltage E_p and grid voltage E_g under *normal operating conditions can be represented by a set of graphs known as triode characteristics.*

Triode characteristics are of two types (i) Static characteristics and (ii) Dynamic characteristics.

i) ***Static characteristics :***

A triode is said to be in static condition when there is no load in its plate circuit, no signal in its grid circuit and various d.c. voltages are applied to the triode electrodes. The curves obtained under static conditions are known as static characteristics.

ii) ***Dynamic characteristics :***

When there is a signal in the grid circuit, load in the plate circuit and various d.c. voltages applied to the triode electrodes, plate current of a triode does not remain static or constant but varies with plate potential. The curves drawn under these conditions are known as dynamic characteristics. The dynamic characteristics represent the actual operating condition of triode.

18.11 Static characteristics of a triode :

A circuit for obtaining the static characteristics of a triode is shown in the Fig.(18.8).

Temperature of the cathode remaining constant, plate current I_p of a triode depends on plate voltage E_p and grid voltage E_g i.e.

$$I_p = f(E_p, E_g)$$

To represent the three variables I_p, E_p and E_g at a time in a graph, we need three dimensional space. Static characteristics can be drawn on a paper which is two dimensional by keeping one of the variables constant while the other two vary. Accordingly, the static characteristics are of three types :

i) Plate characteristics :

$I_p = f(E_p)$ keeping E_g constant.

ii) Mutual characteristics :

$I_p = f(E_g)$ keeping E_p constant.

iii) Constant current characteristics :

$E_p = f(E_g)$ keeping I_p constant.

Since I_p changes with change of E_g, we can keep I_p constant while changing E_g by appropriately changing E_p.

1. Plate characteristics :

The family of curves showing variation of plate current with plate voltage at fixed values

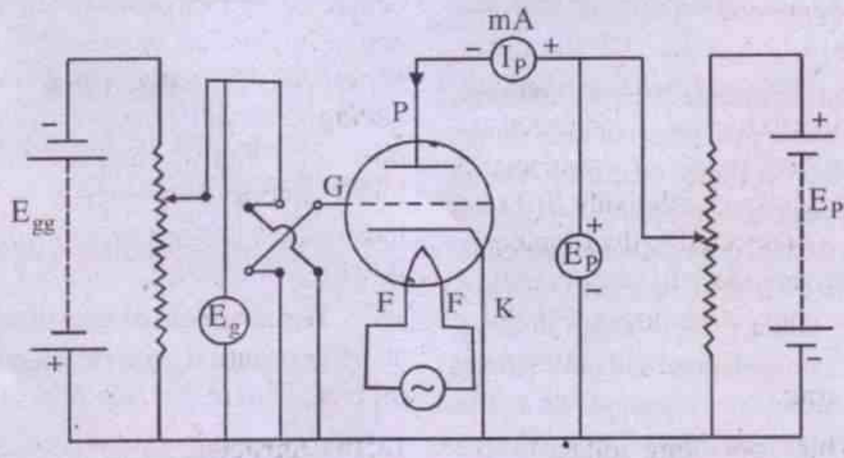


Fig. 18.8

Circuit to study triode characteristics

of grid voltage are known as plate characteristics of a triode.

Static plate characteristics of a triode can be obtained using the circuit of the Fig. (18.8). Keeping grid voltage fixed at 0 V, plate voltage is increased from 0 V in steps of 10 V and corresponding values of plate current are measured. The process is repeated with Eg = -2V, -4V, -6V etc. Thus a family of plate characteristics is obtained at different grid voltages as shown in the Fig. (18.9).

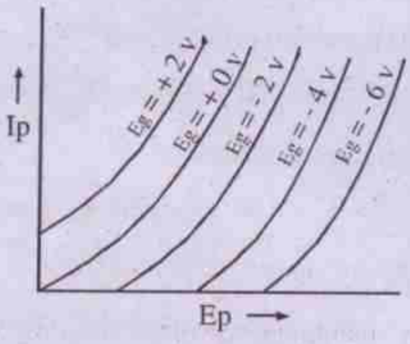


Fig. 18.9

The points to be noted from these characteristics are :

i) I_p increases as E_p increases but I_p starts at different values of E_p for different curves. This shows that when negative voltage of the grid increases, plate voltage required to start the plate current also increases.

ii) All the plots are curved at lower portion, but fairly linear over much of their range. At low plate voltage, electrons having passed through the grid are slowed down and in most cases may not reach the plate. Hence plate current increases slowly. However, when plate voltage is large, electrons are accelerated and plate current increases almost at a constant rate so that the curve is almost linear.

iii) The curves are approximately equally spaced for equal differences of grid voltages.

2. **Mutual characteristics :**

The family of curves showing the variation of plate current with grid voltage at fixed values of plate voltage are called mutual characteristics of a triode.

Static mutual characteristics of a triode can be drawn using the circuit of the Fig.(18.8). Keeping plate voltage fixed at 50V, negative grid voltage is increased from 0V in steps of 2V (i.e. Eg = 0V, -2V, -4V, etc.) until plate current becomes zero. The process is repeated for E_p = 100 V, 150 V and 200 V. The plot of these observations are the family of curves shown in the fig.(18.10) and are called mutual characteristics of a triode.

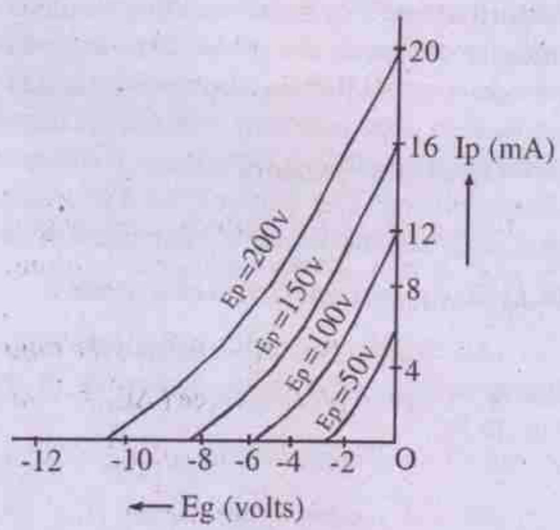


Fig. 18.10

The points to be noted from these characteristics are :

i) The curves are almost linear.

ii) For a given plate voltage, the curve intersects E_g axis which gives cut-off grid bias.

iii) The cut-off grid bias is different for different curves and cut-off grid bias increases with increase in plate voltage.

3. Constant current characteristics :

The family of curves showing the variation of plate voltage and grid voltage at constat plate current.

Constant current characteristics can be drawn using the circuit of fig. (18.8). As the utility of these curves are relatively unimportant, these are never drawn. However, these characteristics can be drawn either from plate characteristics or mutual characteristics.

18.12 Triode constants or parameters :

The behaviour of triode valve in a circuit is determined by three useful parameters known as tube parameters or tube constants. These are amplification factor, a.c. plate resistance and transconductance. Different triode valves will have different sets of values of tube constants as they depend on the design of the triode such as shape of electrodes, spacing between electrodes etc.

i) Amplification factor (μ)

It is a measure of the effectiveness of grid voltage relative to plate voltage in controlling the plate current.

Amplification factor is defined as the ratio of small change in plate voltage (ΔE_P) to the corresponding change in grid voltage (ΔE_g) of a triode at a constant plate current (I_P).

$$\mu = -\left(\frac{\Delta E_P}{\Delta E_g}\right)_{I_P} \qquad \text{...18.12.1}$$

The significance of minus sign is that, to maintain a constant plate current, an increase in plate potential should be followed by a decrease in grid potential. Amplification factor has no unit.

ii) a.c. plate resistance or dynamic plate resistance (r_P)

It is the opposition offered by the tube to the flow of electrons from cathode to plate when a time varying voltage is applied to electrodes.

The a.c. plate resistance is defined as the ratio of small change in plate voltage (ΔE_P) to the resulting small change in plate current (ΔI_P) at constant grid voltage (E_g).

$$r_P = \left(\frac{\Delta E_P}{\Delta I_P}\right)_{E_g} \qquad \text{...18.12.2}$$

Unit of r_P is ohm. r_P varies from 100 Ω to 100 kΩ.

The reciprocal of a.c. plate resistance is termed as the plate conductance (g_p)

$$\therefore \quad g_P = \frac{1}{r_P} \qquad \text{...18.12.3}$$

iii) Transconductance or mutual conductance (g_m) :

By changing grid voltage, the conducting ability of plate circuit can be changed and the conductance is transferred from grid circuit to plate circuit and hence the name trans-conductance.

A change in grid voltage produces a change in plate current. It is a mutual relation and hence the name mutual conductance.

Mutual conductance indicates the effectiveness of grid potential in changing the plate current and hence it is the most important parameter out of the three valve constants.

Mutual conductance is defined as the ratio of a small change in plate current (ΔI_P) to the small change in grid voltage (ΔE_g) at constant plate voltage (E_p).

$$g_m = \left(\frac{\Delta I_P}{\Delta E_g}\right)_{E_P} \qquad \text{...18.12.4}$$

Unit of g_m is mho. Typical values of g_m are a few millimho.

18.13 Relation between the triode parameters :

The plate current I_p is a function of the plate and grid voltages i.e.,

$$I_p = f(E_p, E_g)$$

If the plate and grid voltages are changed by small amounts, the resulting change in plate current dI_p is given by

$$dI_P = \left(\frac{\partial I_P}{\partial E_P}\right)_{E_g} dE_P + \left(\frac{\partial I_P}{\partial E_g}\right)_{E_P} dE_P \quad ...18.13.1$$

Using eqs. (18.12.2) and (18.12.4) in eq. (18.13.1) we have

$$dI_P = \frac{1}{r_P} dE_P + g_m dE_g \quad ...18.13.2$$

If the variatioon of plate and grid potentials be such that the plate current remains constant i.e. $dI_p = 0$, then eq. (18.13.2) reduces to

$$-\left(\frac{dE_P}{dE_g}\right)_{I_P} = r_P \times g_m$$

or, $\mu = r_P \times g_m$ using eq. (18.12.1) ...18.13.3

Thus, if two of the valve constants are known, the third can be determined from eq. (18.13.3).

Example 18.13.1 The slopes of the mutual and anode characteristics of a triode are 4 mA/V and 0.1 mA/V respectively. Calculate the three tube parameters.

Solution :

Slope of mutual characteristic is

$$\left(\frac{\partial I_P}{\partial E_g}\right)_{E_P} = g_m = 4\frac{mA}{V} = 4 \text{ millimhos.}$$

Slope of anode characteristic is

$$\left(\frac{\partial I_P}{\partial E_g}\right)_{E_P} = \frac{1}{r_P} = 0.1\frac{mA}{V} = 0.1 \text{ millimho}$$

or, $r_P = 10K\Omega$

Amplifications factor, $\mu = r_P \times g_m$

$= 10K\Omega \times 4$ millimho

$= 40$.

Example 18.13.2 A valve in which 1V change in grid voltage produces 2mA change in plate current when plate potential is kept constant. Find mutual conductance of the triode.

Solution :

$$\text{Mutual conductance } g_m = \left(\frac{\Delta I_P}{\Delta E_g}\right)_{E_P}$$

$$= \frac{2\times10^{-3}}{1}$$

$$= 2 \text{ m mho.}$$

Example 18.13.3 In a valve, a change in plate voltage from 215V to 255V, when its grid is at - 8V, produces a change in plate current from 5mA to 9mA. Find the a.c. anode resistance.

Solution :

$$\text{a.c. anode resistance } r_P = \left(\frac{\Delta E_P}{\Delta I_P}\right)_{E_g}$$

$$= \frac{(255-215)V}{(9-5)mA}$$

$$= \frac{40}{4} K\Omega$$

$$= 10K\Omega$$

18.14 Determination of triode constants from its characteristics :

Tube constants can be determined using either plate characteristics or mutual characteristics of a triode.

i) ***Determination of tube constants from plate characteristics :***

Fig(18.11) shows a typical plate characteristics of a triode.

$$\text{a.c. plate resistance } r_P = \left(\frac{\Delta E_P}{\Delta I_P}\right)_{E_g}$$

$$= \left(\frac{GF}{ED}\right)_{E_p=E_{p_1}}$$

$$\text{Mutual conductance } g_m = \left(\frac{\Delta I_P}{\Delta E_g}\right)_{E_P}$$

$$= \left\{\frac{AC}{(E_{g_2} - E_{g_1})}\right\}_{E_p=OF}$$

$$= \left(\frac{AC}{\Delta E_g}\right)_{E_p=OF}$$

$$\mu = r_P . g_m$$

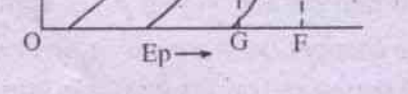


Fig. 18.11

Thus substituting the values of GF, ED, AC, and $(E_{g2} - E_{g1})$, the tube constants r_P, g_m and μ can be determined from fig. (18.11).

ii) Determination of tube constants from mutual characteristics :

Fig. (18.12) shows a typical mutual characteristics of a triode.

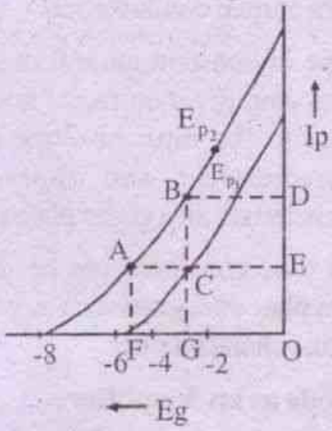


Fig. 18.12

Two horizontal and two vertical dotted lines are drawn as shown in the fig. (18.12)

$$\Delta E_P = E_{P_2} - E_{P_1}$$

$$\Delta E_g = FG$$

$$\Delta I_P = DE$$

$$= \left(\frac{\Delta E_P}{\Delta I_P}\right)_{E_g}$$

$$r_P = \left(\frac{\Delta E_P}{DE}\right)_{E_g=OG}$$

$$= \left(\frac{\Delta I_P}{\Delta E_g}\right)_{E_P}$$

$$g_m = \left(\frac{DE}{FG}\right)_{E_p=E_{P_2}}$$

$$= \left(\frac{\Delta E_P}{\Delta E_g}\right)_{I_P}$$

$$\mu = \left(\frac{\Delta E_P}{FG}\right)_{I_p=OE}$$

Important points to note :

1. Slope of plate characteristic of a triode gives reciprocal of a.c. plate resistance.
2. Slope of mutual characteristic of a triode gives mutual conductance.
3. Slope of constant current characteristic gives amplification factor and it is also equal to the ratio of slope of mutual characteristic and that of plate characteristic at a given plate current.
4. The tube constants can be determined from plate characteristics as well as from mutual characteristics.

18.15 Triode as an Amplifier :

The important application of a triode is as an amplifier. *An amplifier is a device which raises the strength of a weak input signal.* A varying voltage is usually called as the signal voltage or simply signal. The signal contains a message or informtion.

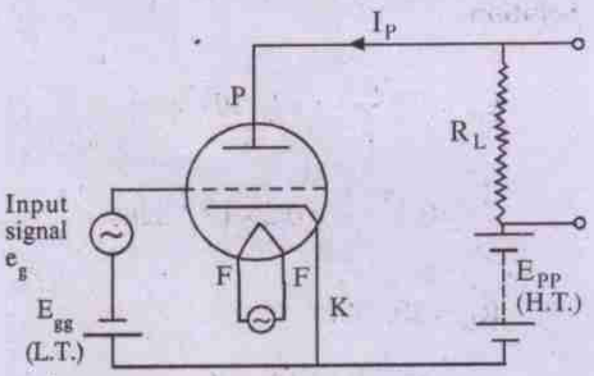


Fig.18.13

Fig.(18.13) is the circuit of a basic triode amplifier. A weak signal $e_g = e_g(t)$ is super imposed with grid bias and the amplified output is obtained across the load R_L connected in the plate circuit. The high tension (H.T) battery E_{PP} provides a positive voltage E_p to the plate w.r.t. cathode. In the grid circuit, there is a low tension (L.T.) supply E_{gg} called grid bias. The value of grid bias is so chosen that the effective potential at grid remains negative w.r.t. cathode throughout so that triode does faithful amplification. (In faithful amplification, the shape of the output remains same as that of input signal but magnitude becomes as large as possible.)

Operation :

The instantaneous potential difference between the cathode and grid is

$$E_g = E_{gg} + e_g(t)$$

Consider, now the mutual characteristic of the triode as shown in the fig.(18.14) to understand the amplification process of the triode.

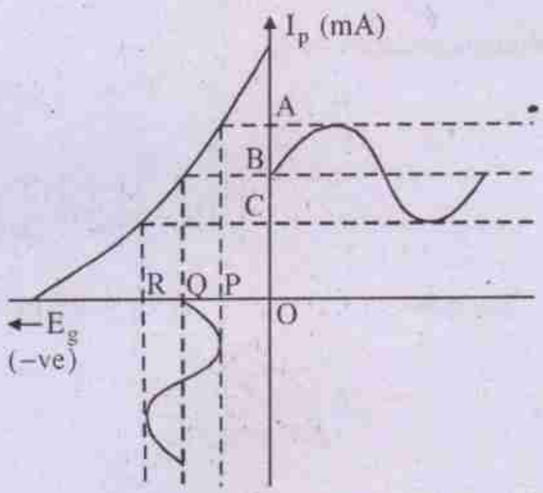


Fig. 18.14

When there is no signal in the grid circuit i.e. $e_g = 0$, $E_g = E_{gg}$. This corresponds to the point Q on the grid voltage. Corresponding plate current is given by the point B on the I_p axis.

When an a.c. signal e_g is applied, its voltage changes from $+e_g$ to $-e_g$ in one half of its time period and during this time, grid voltage fluctuates from $E_{gg} + e_g$ to $E_{gg} - e_g$ i.e. from P to R. Plate current now varies between the points A and C in the wave form as shown in the fig. (18.14) replicating the input signal. This changing plate current passing through a large load resistance R_L causes an output a.c. voltage across R_L which is much greater than the input a.c. signal. Thus triode acts as an amplifier.

Voltage amplification or voltage gain (A_v) :

It is the ratio of output voltage to input voltage. The a.c. voltage across the load resistance R_L is the output voltage, $e_o = \Delta I_P . R_L$

$$A_v = \frac{\text{output voltage } (e_o)}{\text{input voltage } (e_g)}$$

$$= \frac{\Delta I_P . R_L}{e_g} \qquad ...18.15.1$$

where $$\Delta I_P = \frac{\Delta E_P}{r_P + R_L}$$

$$= \frac{\mu e_g}{r_P + R_L} \qquad ...18.15.2$$

$$\left(\because \mu = \frac{\Delta E_P}{\Delta E_g} = \frac{\Delta E_P}{e_g} \right)$$

Using eq. (18.15.2) in eq. (18.15.1), we have

$$A_v = \frac{\mu e_g R_L}{(r_P + R_L) e_g}$$

$$= \frac{\mu R_L}{r_P + R_L} \qquad ...18.15.3$$

$$= \frac{\mu}{1 + r_P / R_L} \qquad ...18.15.4$$

When $R_L \to \infty$, $A_v \to \mu$ i.e. the maximum voltage amplification of a triode amplifier is equal to its amplification factor when its load resistance R_L is equal to infinity. But in practice, R_L can never be infinite in which case the plate circuit will be open.

Hence $A_v < \mu$.

Example 18.15.1 A triode has an amplification factor of 40 and slope of its plate characteristic is 100 μ mho. What is its voltage gain for an anode load of 40 KΩ ?

Solution :

$\mu = 40,$

$\frac{1}{r_P} = 100 \times 10^{-6} \mu$ mho or $r_P = 10{,}000\Omega$

$R_L = 40{,}000$

Voltage amplification

$$A_v = \frac{\mu R_L}{r_P + R_L}$$

$$= \frac{40 \times 40 \times 10^3}{50 \times 10^3} = 32$$

Example 18.15.2 The slopes of the mutual and anode characteristics of a triode are 1.2 mA/V and 0.2 mA/V, respectively. Calculate the voltage gain for a load resistance of 20,000 ohm.

Solution :

$$g_m = 1.2 \frac{\text{mA}}{\text{V}} = 1.2 \times 10^{-3} \text{mho}$$

$$\frac{1}{r_P} = 0.2 \frac{\text{mA}}{\text{V}} = 0.2 \times 10^{-3} \text{mho}$$

$$R_L = 20{,}000\Omega$$

$$\mu = g_m \times r_P$$

$$= \frac{1.2}{0.2} = 6$$

Voltage gain $$A_v = \frac{\mu R_L}{r_P + R_L}$$

$$= \frac{6 \times 20{,}000}{5000 + 20{,}000}$$

$$= \frac{6 \times 20}{25} = 4.8$$

Example 18.15.3 A triode amplifier has mutual conductance 0.5 mA/V and plate resistance, $r_P = 20K\Omega$. Find (i) the voltage amplification for a load resistance of $40\ K\ \Omega$ and (ii) the power output when a sinusoidal signal of peak value 2V is applied to the grid.

Solution :

$$g_m = 0.5\frac{mA}{V} = 0.5\times10^{-3}\text{mho}$$

$$r_P = 20K\Omega,\ R_L = 40K\Omega,\ e_g = 2V$$

$$\mu = r_P \times g_m = 0.5\times10^{-3}\times20\times10^3 = 10$$

$$\Delta I_P = \frac{\mu e_g}{r_P + R_L} = \frac{10\times2}{60\times10^3} = \frac{1}{3}\times10^{-3}$$

$$\text{Power output} = (\Delta I_P)^2 \times R_L$$

$$= \frac{1}{3}\times\frac{1}{3}\times10^{-6}\times40\times10^3$$

$$= 4.4\ \text{mW}.$$

18.16 Distinction between a triode and a step up transformer :

A triode and a step up transformer, both produce voltage amplification. A transformer does so at the cost of its current. If the voltage is stepped up by a certain ratio in a step up transfermor, the current in it is stepped down in the same ratio so that there is no power gain. However in a triode amplifier, there is voltage gain as well as power gain. Thus a loud speaker can be driven by a triode amplifier but not by a step up transformer. Incidentally, the power gain in an amplifier is the useful power gain and it is obtained at the cost of power supplied by the battery or energy source.

18.17 Vacuum diode as a rectifier :

A rectifier is an electronic device which converts alternating current into direct current.

Direct current has one-way path of electric current. A diode conducts only in one direction and this characteristic of diode renders it suitable to be used as a rectifier.

Rectifiers are of two categories :

i) ***Half wave rectifier :***

It conducts only during one half cycle of input a.c.

ii) ***Full wave rectifier :***

It conducts during the full cycle of input a.c. Usually a.c. supply to be rectified is supplied through a transformer for the following reasons :

(a) A transformer allows us to step up or step down the a.c. voltage.

(b) It isolates the rectifier circuit from power line and thus reduces the risk of electric shock.

(i) Half wave rectifier :

Circuit :

Fig (18.15 a) is the half wave rectifier circuit. It makes use of a single diode. The input a.c. voltage to be rectified is applied to the primary of a transformer.

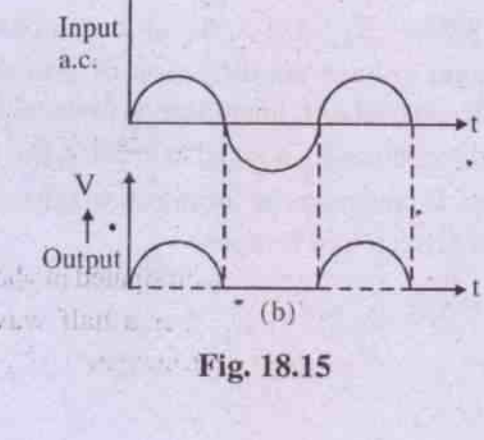


Fig. 18.15

The diode and the load R_L are connected in series with the secondary of the transformer.

Operation :

When a.c. input is applied, the plate P of the diode is at positive and negative potential alternately with respect to cathode K. For the positive half cycle of input a.c., the plate P is at positive potential with respect to cathode K so that the diode conducts. The plate current flowing from plate to cathode inside the diode, through R_L and secondary of the transformer completes the circuit. Output voltage thus occurs across load R_L and its shape as per the shape of the input voltage as shown in the fig. (18.15 b).

During negative half cycle of input a.c. voltage, plate is at negative potential w.r.t. cathode. So the diode does not conduct and no output voltage occurs across R_L. From the fig. (18.15 b), it is clear that output appears only during one half cycle of input a.c. and hence the circuit is called half wave rectifier.

The ratio of d.c. power output to the a.c. power input in a rectifier is known as rectification efficiency (η).

$$\eta = \frac{\text{d.c. output power}}{\text{a.c. input power}}$$

For a half wave rectifier, average current

$$I_{av} = I_{d.c.} = \frac{I_o}{\pi}$$

where I_0 is the peak value of the current

$\therefore$ d.c. output power, $P_{d.c.} = I^2{}_{d.c.} . R_L$

$$= \left(\frac{I_o}{\pi}\right)^2 R_L$$

a.c. input power :

The a.c. input power is dissipated in plate resistance r_P and load R_L. For a half wave rectifier, the r.m.s value of the current.

$$I_{r.m.s.} = \frac{I_o}{2}$$

$\therefore$ a.c. input power, $P_{a.c.} = I^2{}_{r.m.s.}(r_P + R_L)$

$$= \frac{I_o^2}{4}(r_P + R_L)$$

$$\eta = \frac{P_{d.c.}}{P_{a.c.}} = \frac{\left(\frac{I_o}{\pi}\right)^2 R_L}{\frac{I_o^2}{4}(r_P + R_L)}$$

$$= \frac{0.406}{1 + \frac{r_P}{R_L}} \qquad ...18.17.1$$

we se that,

$\eta_{max} = 40.6\,\%$

when $\frac{r_P}{R_L} << 1$.

In a half wave rectifier, maximum 40.6% of an a.c. input power can be converted into d.c. power. Due to such poor effciency, half wave rectifier is rarely used.

(ii) Full wave rectifier :

Circuit :

Fig. (18.16 a) is the full wave rectifier. It uses two diodes. The input a.c. is applied to the primary of centre-tapped transfermer. The plates of two diodes D_1 and D_2 are connected to two ends A and B respectively of the secondary of the transformer. The two cathodes K_1 and K_2 of the two diodes are connected together and the common junction C is connected to the end of load R_L while the other end of R_L is connected to centre-tap T of the secondary of the transformer. With this arrangement, transformer secondary voltage is divided into two halves, each half appears between the plate and cathode of each diode.

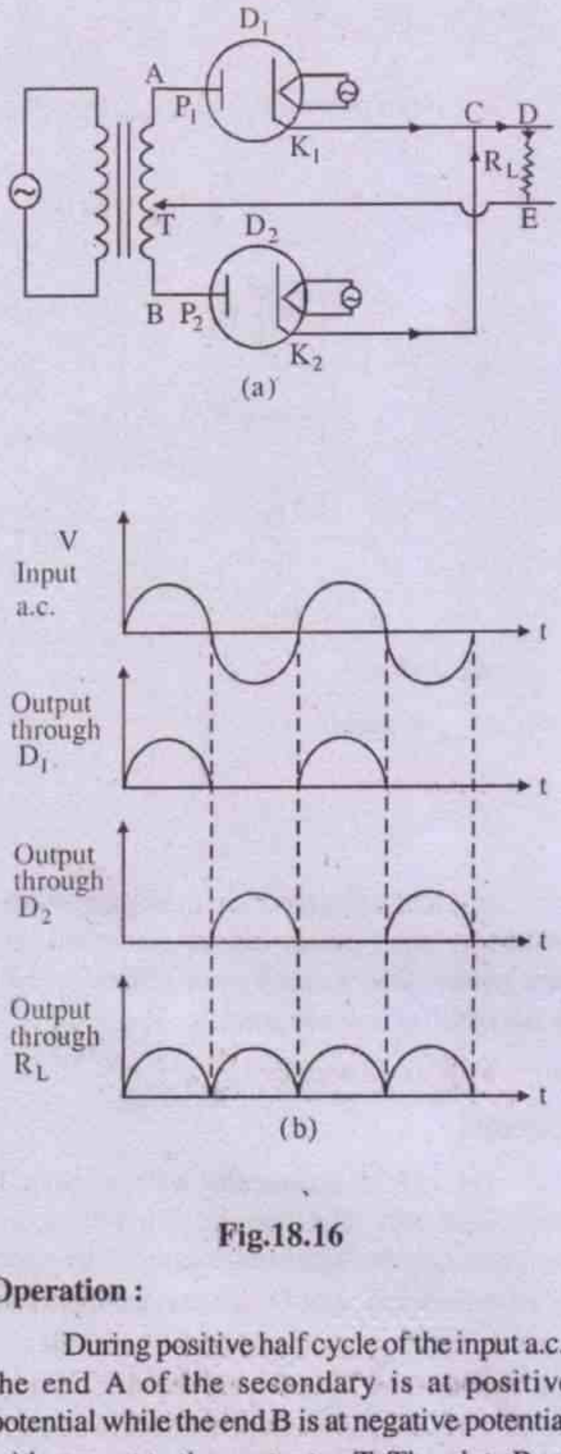


Fig.18.16

Operation :

During positive half cycle of the input a.c., the end A of the secondary is at positive potential while the end B is at negative potential with respect to the centre tap T. The plate P_1 of diode D_1 is at positive potential with respect to cathode K_1 while the plate P_2 of diode D_2 is at negative potential w.r.t. cathode K_2. Therefore, diode D_1 conducts but the diode D_2 does not conduct. The current flows through the diode D_1 and the load in the direction $K_1CDETAK_1$ and the output voltage appears across the load R_L.

During negative half cycle of the input a.c., the end B of the secondary is at positive potential and the end A at negative potential with respect to the centre tap T. Diode D_2 thus conducts while diode D_1 does not. The current flows through D_2 and load R_L in the direction $K_2CDETBK_2$. Thus the current through the load R_L is in the same direction as during the +ve half cycle i.e. from D to E and the output appears across the load R_L with same polarity. Thus output appears across R_L for the full cycle of the input a.c. as shown in the fig.(18.16 b).

Output obtained from the above full wave rectifier circuit is not a pure d.c. (fig. 18.16 b) but it is pulsating d.c. which is a mixture of a.c. and d.c. A rectifier circuit is therefore followed by filter circuit which consists of capacitors and inductors. A filter circuit allows the d.c. component to pass through the load R_L while the a.c. component gets by passed. Thus a rectifier-filter circuit combination gives a steady d.c. output.

Rectification efficiency of a full wave rectifier :

As the output current from a full wave rectifier (without filter circuit) is pulsating, the average output current of a full wave rectifier is

$$I_{a.v.} = I_{d.c.} = \frac{2I_o}{\pi}$$

$\therefore$ d.c. output power, $P_{d.c.} = I^2{}_{d.c.} . R_L$

$$= \left(\frac{2I_o}{\pi}\right)^2 . R_L$$

R.M.S. value of the output current of the full wave rectifier is

$$I_{rms} = \frac{I_o}{\sqrt{2}}$$

Input power dissipasion occurs both in plate resistance r_P and load R_L which are in series and hence input a.c. power, $P_{a.c.} = I^2{}_{r.m.s}(r_P + R_L)$

Rectification efficiency of a full wave rectifier is,

$$\eta = \frac{P_{d.c.}}{P_{a.c.}}$$

$$= \left(\frac{2I_o}{\pi}\right)^2 .R_L \Bigg/ \frac{I_o^2}{2}(r_P + R_L)$$

$$= \frac{0.812}{1 + r_P / R_L} \qquad ...18.17.2$$

If $r_P \to 0$ or $R_L \to \infty$ or $\frac{r_P}{R_L} << 1$,

rectification efficiency is maximum and η_{max} = 81.2%. It is double the efficiency of half wave rectifier.

Example 18.17.1 A vacuum diode whose internal resistance is 200 ohm is to supply to a 1000 ohm load from a 300 volt source. Calculate (i) peak current through load (ii) d.c. component of the current (iii) a.c. component of the current through the load (iv) d.c. potential difference between cathode and anode of the diode (v) input a.c. power (vi) output d.c. power and (vii) rectification efficiency.

Solution :

Internal resistance $r_P = 200\Omega$

Load $R_L = 1000\Omega$

Supply voltage $E_{rms} = 300$ V

i) Peak current, $I_o = \frac{E_o}{r_P + R_L}$

$$= \frac{300\sqrt{2}}{200 + 1000} = 0.354 \text{ A}$$

ii) d.c. component, $I_{d.c.} = \frac{I_o}{\pi} = 0.113$ A

iii) a.c. component, $I_{a.c.} = \left[I_{rms}^2 - I_{d.c.}^2\right]^{1/2}$

$$\left[0.177^2 - 0.113^2\right]^{1/2}$$

$$(\because I_{rms} = \tfrac{I_o}{2})$$

$$= 0.136 \text{ A}$$

iv) d.c. pot. diff. $= I_{d.c.} \times r_P$

$$= 0.113 \times 200 = 22.6 \text{ V}$$

v) Input a.c. power, $P_{a.c.} = I_{rms}^2(r_P + R_L)$

$$= 0.177^2\,(200 + 1000)$$

$$= 37.59 \text{ watt}$$

vi) Output d.c. power, $P_{d.c.} = I_{d.c.}^2 . R_L$

$$= (0.113)^2 \times 1000$$

$$= 12.77 \text{ watt}$$

vii) Rectification efficiency $\eta = \frac{P_{d.c.}}{P_{a.c.}} \times 100$

$$= 33.97\ \%$$

Example 18.17.2 A full wave rectifier consists of two diodes. Resistance of each diode is 500 ohms. The output of the rectifier supplies power to a 2000 ohm resistance load. The primary to secondary turns ratio of centre-tapped transformer is 11:28. If the input a.c. supply is 220V, find (i) d.c. in the load (ii) the d.c. in each tube (iii) the a.c. component of voltage across the load (iv) the d.c. output power and (v) the efficiency of rectification.

Solution :

Let V_p and N_p denote the r.m.s. voltage and the number turns of the primary of the transfer respectively and corresponding values of the secondary be V_S and N_S.

$$\therefore \quad \frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{28}{11} = \frac{V_S}{220}$$

$$\therefore \quad V_S = 20 \times 28 = 560V$$

Potential across each diode $= \frac{560}{2} = 280V$

Peak voltage applied to the plate of each odiode,

$$E_0 = 280\sqrt{2}V$$

Peak current through the load, $I_o = \frac{E_o}{r_P + R_L}$

$$I_o = \frac{280\sqrt{2}}{500+2000} = \frac{396}{2500} = 0.158A$$

i) d.c. in the load, $I_{d.c.} = \frac{2I_o}{\pi} = 0.100A$

ii) current supplied by each tube

$$\frac{0.1}{2} = 0.05A$$

iii) $I_{rms} = \frac{I_o}{\sqrt{2}} = 0.112A$

a.c. component of voltage across the load

$$= I'_{rms}.R_L = \left[I^2_{r.m.s.} - I^2_{d.c.}\right]^{1/2} \times R_L$$

$$= 100.9 \text{ V}$$

iv) d.c. power output, $P_{d.c.} = I^2_{d.c.}.R_L$

$$= 0.01 \text{ x } 2000 = 20 \text{ V}$$

v) Rectification efficiency

$$\eta = \frac{81.2}{1+\frac{r_P}{R_L}}\% = 64.96\ \%$$

SUMMARY

1. Thermionic emission :

The emission of electrons from a metallic surface due to thermal energy is called thermionic emission.

2. Richardson-Dushmann equation :

The emission current density,

$$J_S = A\ T^2\ e^{-\phi e/KT}$$

3. Diode valve :

It is a vacuum tube having two electrodes, namely, cathode and plate. It conducts current in only one direction from plate to cathode when the former is at positive potential with respect to the later.

4. Cathode should be made up of a material having high melting point and low work function. Cathode is made up of tungsten, thoriated tungsten or nickel coated with barium or strontium oxides to lower the work function.

5. Child's law :

In a diode valve, the space charge limited current varies directly as three half power of the plate potential.

$$I_P = K\ E_P^{3/2}$$

6. Diode can be used as a rectifier. Rectifiers are of two types (i) half wave rectifier and (ii) full wave rectifier. Rectification efficiency of

i) half wave rectifier is, $\eta = \frac{40.6}{1+r_P/R_L}\%$

ii) full wave rectifier is, $\eta = \frac{81.2}{1+r_P/R_L}\%$

7. Triode :

It is a vacuum tube consisting of three electrodes, namely, cathode, grid and plate. The working potentials of the triode are : grid must be at slightly negative potential and plate must be at positive potential w.r.t. cathode.

8. The plate current in a triode is given by Child's law as

$$I_P = K\left(E_P + \mu E_g\right)^{3/2}$$

Thus I_p is controlled by plate potential and grid potential. But the effect is more by changing grid potential, since grid is nearer to cathode than plate. Hence grid is called control grid.

9. Cut off grid bias :

It is the minimum magnitude of negative grid potential for which plate current becomes zero. Cut off grid bias increases with increase in plate potential and is given by

$$E_g = -E_P / \mu$$

10. Static characteristics of a triode are of three types :

i) Plate characteristics :

It is a curve between I_p and E_p keeping E_g constant. Reciprocal of slope of plate characteristic gives a.c. plate resistance (r_P)

ii) Mutual characteristics :

It is a graph between I_p versus E_g keeping E_p constant. Its slope gives transconductance or mutual conductance (g_m).

iii) Constant current characteristics :

It is a graph between E_p and E_g keeping I_p constant. Its slope gives amplification factor (μ).

11. Relation between μ, r_P and g_m is

$$\mu = r_P \times g_m$$

12. Amplification :

The process of increasing the amplitude of input signal is called amplification.

13. Triode can be used as an amplifier. Its voltage gain $A_v = \dfrac{\mu R_L}{r_P + R_L}$

14. Triode can be used as an oscillator which converts d.c. to a.c. at a desired frequency.

SOLVED NUMERICAL EXAMPLES :

Example 1 A diode valve working in a space charge limited condition has a plate current of 4 mA at a plate voltage of 400 V. What is its plate current at the plate voltage of 200V.

Solution :

At the plate voltage $E_{P_1} = 400\text{V}$,

$I_{P_1} = 4\text{ mA}$

At the plate voltage $E_{P_2} = 200\text{V}$

$I_{P_2} = ?$

$$\frac{I_{P_1}}{I_{P_2}} = \left(\frac{E_{P_1}}{E_{P_2}}\right)^{3/2}$$

$$\frac{4}{I_{P_2}} = \left(\frac{400}{200}\right)^{3/2} = 2\sqrt{2}$$

$$\therefore\ I_{P_2} = \sqrt{2} = 1.414\text{mA}$$

Example 2 The plate current characteristic of a triode is represented by the following expression

$$I_P = 0.002\left(E_P + 20E_g\right)^{3/2}\text{mA}$$

where I_p is the plate current, E_p and E_g are the plate and grid voltages in volts respectively. Determine mathematically the values of (i) mutual conductance (ii) amplification factor and (iii) the plate resistance of the triode at the point where E_p = 250 V, E_g = - 2.5 V.

Solution :

The operating point is $E_p = 250$ V

$E_g = -2.5$ V

$$I_P = 0.002(E_P + 20E_g)^{3/2} \text{mA} \quad ...(1)$$

i) Mutual conductance :

Taking partial derivative of eq. (1) w.r.t. E_g keeping E_p constant, we have

$$g_m = \left(\frac{\partial I_P}{\partial E_g}\right)_{E_P=250V}$$

$$= 0.002 \times 10^{-3} \times \frac{3}{2}(E_P + 20E_g)^{1/2} \text{ x } 20$$

$$= 60 \times 10^{-6}(250 + 20\text{x} - 2.5)^{1/2} \text{A / V}$$

$= 848.5$ μ mho

ii) Amplification factor :

Taking partial derivative of eq. (1) with respect to E_g keeping I_p constant, we have

$$0 = 0.002 \times 10^{-3} \times \frac{3}{2}(E_P + 20E_g)^{1/2} \times \left[\left(\frac{\partial E_P}{\partial E_g}\right)_{I_P} + 20\right]$$

or $$\left(\frac{\partial E_P}{\partial E_g}\right)_{I_P} + 20 = 0$$

or $$\mu = -\left(\frac{\partial E_P}{\partial E_g}\right)_{I_P} = 20$$

iii) Plate resistance,

$$r_P = \frac{\mu}{g_m}$$

$$= \frac{20}{848.5 \times 10^{-6}} = 23.6 \text{ k}\Omega$$

Example 3 The voltage amplification of a triode is 20 when anode load is 50 kΩ and 25 with a 75 kΩ load. Determine valve constants of the triode.

Solution :

Voltage amplification of a triode

$$A_v = \frac{\mu R_L}{r_P + R_L}$$

$A_v = 20$ for $R_L = 50$ KΩ

and $A_v = 25$ for $R_L = 75$ KΩ

$$\therefore \quad 20 = \frac{\mu \times 50}{r_P + 50}$$

and $$25 = \frac{\mu \times 75}{r_P + 75}$$

Solving $r_P = 75\text{k}\Omega$, $\mu = 50$

$$\therefore \quad g_m = \frac{\mu}{r_P} = \frac{50}{75 \times 10^3}$$

$= 6.67 \times 10^{-4}$ mho

Example 4 In a triode, find the relation between plate resistance and load resistance so that output power is maximum. Also find the maximum power output which can be obtained for an a.c. signal of 1 volt by a triode amplifier having plate resistance $r_P = 2\text{k}\Omega$ and amplification factor $\mu = 20$.

Solution :

Plate current, $I_P = \dfrac{\mu E_g}{r_P + R_L}$

Output power across the load, $P = R_L I_P^2$

$$= \frac{\mu^2 E_g^2 R_L}{(r_P + R_L)^2}$$

The condition for maximum power across the load is

$$\frac{\partial P}{\partial R_L} = 0$$

or, $$\mu^2 E_g^2 \left\{ \frac{1}{(r_P + R_L)^2} - \frac{2R_L}{(r_P + R_L)^3} \right\} = 0$$

or, $$\frac{1}{(r_P + R_L)^2} = \frac{2R_L}{(r_P + R_L)^3}$$

or, $$r_P + R_L = 2R_L$$

or, $$r_P = R_L$$

This is known as impedance matching.

$$P_{max} = \frac{\mu^2 E_g^2 r_P}{(2r_P)^2}$$

$$= \frac{20^2 \times 1}{4 \times 2 \times 10^3} = 50 \text{ mW.}$$

Example 5 The plate of a triode of $\mu = 20$ and $g_m = 1.0$m mho is given a potential of 300 volt. Find the grid voltage for which plate current reduces to zero. Find also plate resistance.

Solution :

According to Child's law for a triode

$$I_P = K(E_P + \mu E_g)^{3/2}$$

$$I_P = 0 \text{ if } E_g = -\frac{E_P}{\mu}$$

$$= -\frac{300}{20} = -15 \text{ volts}$$

Plate resistance, $r_P = \dfrac{\mu}{g_m} = \dfrac{20}{1 \times 10^{-3}} = 20\text{k}\Omega$

Example 6 Plate current in a triode $I_P \propto (E_P + \mu E_g)^{3/2}$. Show that mutual conductance is proportional to cube root of the plate current.

Solution :

Plate current $I_P = K(E_P + \mu E_g)^{3/2}$

where K is a constant.

$$g_m = \left(\frac{\partial I_P}{\partial E_g}\right)_{E_P} = \frac{3}{2} K (E_P + \mu E_g)^{1/2} \mu$$

$$g_m \propto (E_P + \mu E_g)^{1/2}$$

$$\therefore \quad g_m \propto I_P^{1/3}$$

Example 7 A plate current of 10 mA is obtained when a potential of 60 volts is applied across a diode tube. Assuming Child's law $I_P \propto E_P^{3/2}$ to hold good, find the a.c. plate resistance in the given operating condition.

Solution :

Child's law $I_P = K\, E_P^{3/2}$ ...(1)

$$\frac{dI_P}{dE_P} = K \frac{3}{2} E_P^{1/2}$$

or $\dfrac{1}{r_P} = \dfrac{3}{2} K E_P^{1/2}$ ...(2)

Dividing eq. (1) with eq. (2) we have

$$I_P \cdot r_P = \frac{2}{3} E_P$$

or $$r_P = \frac{2}{3}\frac{E_P}{I_P}$$

$$= \frac{2}{3} \times \frac{60}{10 \times 10^{-3}} = 4\text{k}\Omega$$

MODEL QUESTIONS

A. Multiple Choice Type Questions :

1. In thermionic emission, the thermionic current varies with temperature of the filament as

a) T b) T^2

c) T^{-1} d) T^{-2}

2. The space charge limited current in a diode is 2 mA at 20 V. At what voltage its value will be 16 mA.

a) 160 V b) 20 V

c) 40 V d) 80 V

3. The a.c. plate resistance of a diode in the saturation current region is

a) zero b) constant

c) finite d) infinite

4. Under space charge limited conditon, the plate current of a diode

a) increases with increase in plate potential.

b) increases with increase in filament temperature.

c) both of the above are correct

d) None of the above is correct.

5. A diode as a rectifier converts

a) a.c. to steady d.c.

b) d.c. to a.c.

c) a.c. to pulsating d.c.

d) low voltage to high voltage

6. If the plate voltage of a diode valve is increased from zero to a large valve, the plate current

a) increases continuously

b) first increase and then decreases

c) first increases and then remains constant.

d) remains constant.

7. Which of the following is correct

a) Diode can be used as a rectifier.

b) Triode can be used as an amplifier.

c) Both of the above are correct.

d) None of the above are correct.

8. The saturation current in a diode depends on

a) Cathode temperature

b) plate voltage

c) separation between cathoe and plate

d) material of the plate

9. In a full wave rectifier circuit operating from 50 Hz mains frequency, the fundamental frequency of its out put is,

a) 50 Hz b) 100 Hz

c) 25 Hz d) $50\sqrt{2}$Hz

10. In a half wave rectifier, the rms value of a.c. component is

a) equal to d.c. valve

b) more than d.c. valve

c) less than d.c. valve

d) zero

11. There is an increase in thermionic current when the

a) area of the filament is increased

b) temperature of the filament is increased.

c) Cathode is made oxide coated

d) All the above are correct.

12. When the grid of a triode is at highly negative potential with respect to its cathode

    a) its plate current becomes zero

    b) its plate resistance becomes infinite

    c) both of the above are correct

    d) None of the above are correct.

13. The relation between triode constants is

    a) $\mu \times r_P = g_m$

    b) $r_P \times g_m = \mu$

    c) $\mu \times g_m = r_P$

    d) $\mu \times g_m \times r_P = 1$

14. The static amplification factor of a triode valve depends on

    a) temperature of the cathode

    b) temperature of anode

    c) supplied plate voltage

    d) relative positions of cathode, grid and anode.

15. For a given plate voltage, the plate current in a triode valve is maximum when the potential of

    a) the grid is positive and plate is negative.

    b) the grid is positive and plate is positive.

    c) the grid is negative and plate is positive.

    d) the grid is zero and the plate is positive.

16. When a triode is used as an amplifier, the phase difference between the input signal voltage and the output is

    a) 0 b) π

    c) $\pi/2$ d) can not determined

17. The amplification factor of a triode is 10. Its plate current will remain constant if its plate potential is increased by 20 volts and its grid potential

    a) decreased by 2 volts

    b) decreased by 20 volts

    c) increased by 2 volts

    d) increased by 20 volts

18. The slopes of anode and mutual characteristics of a triode valve are 0.04 mA / Volt and 2 mA / V respectively. The amplification factor of the valve is

    a) 8×10^{-8} b) 0.08

    c) 50 d) 20

19. A triode is operated at 250 V and grid potential is kept at -10 V. The plate current is 10 mA. If the plate voltage is reduced to 200 V and the grid voltage is set to - 5V, the plate current remains same as 10 mA. The amplification factor of the triode is

    a) 10. b) 20

    c) 25 d) 50

20. A triode is used as an amplifier, with $r_P = 20k\Omega$, $R_L = 100k\Omega$ and $\mu = 60$. What is the voltage gain of the amplifier?

    a) 10 b) 50

    c) 100 d) 20

21. Three amplifier stages ceach with a gain of 10 are cascaded. The net gain will be

    a) 30 b) 200

    c) 100 d) 1000

22. In a triode, cathode, grid and plate are at 0, -2 and 80 V respectively. An electron is emitted from the cathode with energy of 3 eV. The energy of the electron reaching the plate is

    a) 85 eV b) 80 eV

    c) 83 eV d) 3 eV

23. The amplification factor of a triode is 20 and load resistance is equal to plate resistance. What is the output voltage if input voltage is 10 mV ?

a) 1 mV b) 100 mV

c) 200 mV d) 400 mV

24. If the plate resistance is half of load resistance, what will be the voltage gain if the amplification factor is 30 ?

a) 20 b) 45

c) 30 d) 15

B. Very Short Answer Type Questions :

1. What is thermionic emission ? [CHSE 89 S]
2. By which equation, thermionic emission is explained. [CHSE 00]
3. What is meant by work function of a metal ? [CHSE 00 Instant]
4. What is the significance of work function in thermionic emission ? [CHSE 95 S]
5. Write Richardon-Dushmann equation of thermionic emission.
6. What are different methods of obtaining electron emission ?
7. What do you mean by surface barrier ?
8. What are the materials used for thermionic emitters ?
9. What are the advantages of indirectly heated cathode over directly heated cathode ?
10. State the two principal electrodes that are present in every vacuum tube.
11. Specify the common use of a diode valve. [CHSE 98 A]
12. How much is the maximum efficiency of a (i) half wave rectifier and (ii) full wave rectifier ?
13. In a diode, plate is at -10 volt and cathode is at - 180 volt. Will it conduct ? Explain.
14. Name different electrodes in a triode. [CHSE 89 S]
15. What is the function of a grid in a triode ? [CHSE 97 S, 99 A]
16. Why is the grid of a triode valve perforated ? [CHSE 86 S]
17. State two uses of a vacuum triode. [CHSE 87 S, 85 A]
18. Draw a graph showing mutual characteristics of a triode. [CHSE 97 S, 86 A]
19. Draw a graph showing plate characteristics of a triode.
20. Define transconductance. State its units. [CHSE 89 S]

C. Short Answer Type Questions :

1. Draw the circuit diagram of a diode valve used as a rectifier. [CHSE 96 S]
2. How does the control grid affect the operation of a triode valve. [CHSE 90 A]
3. Grid should be at slightly negative potential w.r.t. cathode in a triode. Discuss.
4. Discuss operating potentials of electrodes in a triode.
5. Name the constants of a triode and state the relation between them. [CHSE 91 S, 97 A]
6. What do you mean by static characteristics of a triode ?

7. Slope of each characteristic curve of a triode gives one parameter. Name the curves and parameters they give.

8. Draw the circuit diagram for the use of a triode as an amplifier.

[CHSE 91 S]

9. Draw VI characteristics of a vacuum diode and explain the reasons for the difference in its nature in different regions.

[CHSE 01]

10. Explain the formation of space charge. What is the effect of cathode temperature on space charge ?

11. Does vacuum diode obey ohm's law ? Which law is applicable under space charge limited condition ?

12. Give the Child's law for plate current in a triode.

13. Why cathode is heated in vacuum ?

14. What do you mean by amplification and faithful amplification ?

15. The actual gain of a triode is less than the amplification factor. Why ?

16. Why is control grid of a triode always held at negative potential w.r.t. cathode ?

D. Numerical Problems :

1. A diode operating in space charge limited region has an anode voltage of 100 V when the current is 125 mA. What is the anode voltage if the current is 64 mA.

2. The slopes of anode and mutual characteristic of a triode valve are 0.02 and 1.0 mA/V respectively. Calculate amplification factor.

3. A triode has a mutual conductance of 2 mA/V and a plate resistance of $20k\Omega$. Find the load resistance which must be applied to secure a voltage amplification of 20.

4. A triode valve operates at plate potential of 200 Volts and grid voltage of - 4.0 volts. If the plate potential is decreased to 150 volts, grid potential is increased to -2 Volts, plate current remains unchanged. Calculate amplification factor.

5. Dynamic plate resistance (r_P) of a triode is $8k\Omega$. Find the change in plate current if the plate voltage is changed from 180 V to 220 V.

6. The gain factor of an amplifier is increased from 10 to 12 as the load resistance is changed from $4k\Omega$ to $8k\Omega$. Calculate (a) amplification factor and (b) plate resistance.

7. A triode registers a plate current 5 mA at the plate voltage 150 V and grid voltage -2 volt. If the grid voltage be changed to -3.5 V, the plate current falls to 3.2 mA but that can be restored back to 5 mA if the plate voltage is increased to 195 volt. Find the constants of the triode valve.

8. A voltage 200 sin 100 π t volt is supplied to a half-wave vacuum tube rectifier. If $r_P = 1k\Omega$ and $R_L = 10k\Omega$, calculate (a) peak current (b) average current (c) r.m.s. valve of the current (d) a.c. power input (e) d.c. power output and (f) rectification efficiency of the rectifier.

E. Long Answer Type Questions :

1. What is a diode valve ? Show how a diode valve can be used as a rectifier ?

[CHSE 95 S, 97 S]

2. Describe the construction and working of a triode valve. Describe a cricuit diagram for the study of its characteristics.

[CHSE 98 A]

3. Draw the characteristics of a triode. Discuss its different regions. Define the

three tube constants of a triode and establish a relation among them.

4. With a circuit diagram, explain how the static characteristic curves of a triode are drawn ?

   Show how the tube constants are estimated with the help of these curves. [CHSE 01]

5. What do you understand by the term amplifier ? Explain the action of a triode amplifier.

6. What do you mean by faithful amplification ? How will you achieve it in a vacuum tube amplifier ? Derive an expression for voltage gain of a triode amplifier.

F. True-False Type Questions :

1. Cathodes are coated with borium oxide to lower the thermionic work function.

2. A single diode can be used for full-wave rectifier.

3. The effect of ionisation in a gas is to inerease the electric space charge.

4. The triode valve cannot be used at high frequencies because of existence of inter-electrode capacitance between plate and grid.

5. The space charge limited current varies directly as square root of plate potential.

G. Fill-in-Blank Type Questions :

1. The emission current density from a heated filament obeys _____ law.

2. The space charge limited current in a diode valve obeys _____ law.

3. The output voltage of a full-wave rectifier is _____ voltage.

4. The grid in a triode is usually maintained at a ____ potential w.r. to cathode.

5. The efficiency of a full-wave rectifier is nearly _____ when $r_p << R_L$.

6. The relation between cutoff grid bias and plate potential is _____.

7. The relation between amplification factor (μ), a.c plate registance (r_p) and trans conductance (g_m) is _____.

H. Correct the following sentence:

1. In thermionic emission current varies with temperature of the filament as $T^{1/2}$.

2. A diode as a rectifier converts a.c to steady d.c.

3. The relation between triode constants is $\mu \times g_m = r_p$.

4. When the triode is used as an amplifier the phase difference between input signal voltage and output is $\pi/2$.

5. The emission current density from a heated filament obeys Child's law.

6. Minimum number of diodes for a full-wave rectifier is one.

7. The space charge limited current varies directly as the square root of plate current.

ANSWERS

A. Multiple Choice Type Questions :

1. (b)	**2.** (d)	**3.** (d)	**4.** (a)	**5.** (c)	**6.** (c)	**7.** (c)	**8.** (a)
9. (b)	**10.** (b)	**11.** (d)	**12.** (c)	**13.** (b)	**14.** (d)	**15.** (b)	**16.** (b)
17. (a)	**18.** (c)	**19.** (a)	**20.** (b)	**21.** (d)	**22.** (c)	**23.** (b)	**24.** (a)

D. Numerical Problems :

1. 64 Volts
2. 50
3. 20kΩ
4. 25
5. 5 mA
6. (a) 15, (b) 2kΩ
7. $g_m = 1.2$ m mho, $r_P = 25k\Omega$, $\mu = 30$
8. a) 18.18 mA

   b) 5.78 mA

   c) 9.09 mA

   d) 0.91 mW

   e) 0.34 mW

   f) 36.9 %

F. **1.** True **2.** False **3.** False **4.** True **5.** False

G. **1.** Richardson - Dushman law **2.** Child's law **3.** Pulsating **4.** Negative **5.** 81.2% **6.** Eg $= - E_p/\mu$ **7.** $\mu = r_p \times g_m$

19

Solids

Matter commonly exists in three states, viz., solid, liquid and gaseous. (Plasma is the fourth state of matter which exists under special conditions) A solid object has the tendency of maintaining a definite shape and volume due to strong intermolecular force.

19.1: Structure of solids :

All solids are classified into two main groups - crystalline and amorphous.

i) Crystalline solids :

In crystalline solids, the atoms are packed in a regular periodic geometrical pattern. This pattern is repeated throughout the three dimensions of the crystal. Examples of crystalline solids are sodium chloride, diamond, mica, quartz etc.

Characteristics of crystalline solids :

a) Crystalline solids have definte sharp melting points.

b) They are anisotripic i.e. their physical properties like thermal conductivity, refractive index etc. are different in different directions.

ii) Amorphous solids :

In amorphous solids atoms are arranged in a random fashion in three dimensions i.e. the atoms are not in regular periodic geometric pattern. Glass, wax, pitch, rubber etc. are some examples of amorphous solids.

Characteristics of amorphous solids :

a) Amorphous solids have no sharp melting points.

b) They are isotropic i.e. their physical properties like thermal conductivity, electrical conductivity, refractive index etc. are same in all directions.

19. 2 : Binding in solids :

The atoms and molecules in a solid are held together by the electrostatic force of attraction, between negative charge of electrons and the positive charge of nuclei, known as cohesive force. Gravitational force and magnetic force have negligible effect in such binding.

The electrostatic force visualised as links formed between the oppositely charged particles to form a stable solid is called bond. The participating neutral atoms contribute energies in forming bonds, thereby making total energy of the solid less than that of the constituent atoms in free state. The difference in these energies (i.e., sum of energies of atoms in free state minus energy of the solid) is known as cohesive energy or binding energy of the solid. The larger the binding energy, the more stable is the solid.

The fundamental cause of atomic combination by forming bonds is due to tendency of atoms to attain rare gas configuration which is most stable configuration. The common types of bonds are

i) Ionic or electrovalent or heteropolar bond

ii) Covalent or homopolar bond

iii) Metallic bond

and iv) Vander Waal or Molecular bond.

19. 3 : Ionic or heteropolar bond :

The bond formed by the transfer of one or more electrons from one atom to another is called ionic bond. The crystals formed due to ionic bond are called ionic crystals. Some examples of ionic crystals are common salt (NaCl), sodium sulphate ($NaSo_4$), copper sulphate ($CuSo_4$) etc.

Let us see how ionic bond is formed in Nacl crystal. The atomic number of sodium is 11 and its electronic configuration is $1s^2$, $2s^2$ $2p^6$ $3s^1$. The atomic number of chlorine is 17 and its electronic configuration is $1s^2$, $2s^2$, $2p^6$, $3s^2$, $3p^5$. If sodium atom loses one electron, it acquires stable configuration like that of an inert gas whereas chlorine atom acquires stable configuration if it gets one additional electron. Hence when Na and cl atoms come close together, sodium atom has a tendency of losing one electron while chlorine atom has the tendency of receiving one electron so that both of them acquire stable structures. In doing so, Na^+ and cl^- ions are formed. The oppositely charged ions experience a force of attraction while similarly charged ions repel each other. But these ions arrange themselves in such a way that the force of attraction between unlike ions is greater than the force of repulsion between like ions. This is possible when Na^+ and cl^- ions are placed alternatively at lattice points as shown in the fig. (19.1).

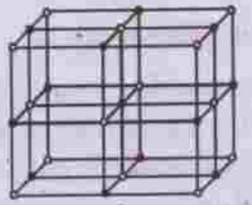


Fig. 19.1

Properties of ionic crystals :

i) The ionic bonds are fairly strong. Hence ionic crystals are hard, brittle and have high density.

ii) They have high melting points and high latent heat of fusion because of their high cohesive energy.

iii) Ionic crystals are bad conductors of electricity as they do not have free electrons.

iv) Ionic crystals are highly soluble in water and seperate into positive and negative ions. Hence their solutions in water conduct electricity.

v) The ionic crystals are generally transparent and some are coloured.

19. 4 : Covalent bond :

The force which binds atoms of same or different elements by mutual sharing of electrons is called a covalent bond.

The atoms involved in bond formation contribute equal number of electrons for sharing. The shared electrons become common property of both the atoms and form a bond between them.

Covalent binding in observed in diatomic molecules like H_2, N_2, O_2, F_2, Cl_2 etc., in organic solids made up of atoms of carbon and hydrogen, in tetravalent elements like carbon, silicon, germanimum and in compounds like H_2O, NH_3 etc.

For understanding how covalent bond is formed let us consider the case of chlorine molecule. It is diatomic and each chlorine atom has seven electrons (2, 8, 7) in its valence shell. Both the chlorine atoms contribute one electron each to share two electrons. Thus the valence shells of both the chlorine atoms are completely full and achieve the stable noble gas configuration as shown in the fig. 19.2.

$$: \ddot{\underset{\cdot\cdot}{Cl}} : \ddot{\underset{\cdot\cdot}{Cl}} :$$

Fig. 19.2

Properties of covalent crystals :

i) Covalent bond is stronger than ionic bond. So covalent crystals are hard and tough.

ii) Covalent crystals possess high melting point and high cohesive energy but not as high as ionic solids have.

iii) The electrical conductivity of covalent crystals varies over a wide range. Some like diamond are excellent insulators while germanium, silicon etc are semiconductros. Granite has higher conductivity than semiconductors.

iv) Covalent crystals are insoluble in ordinary liquids like water.

19. 5 : Metallic bond :

In metals, the valence electrons are loosely bound to the nucleus. In the bulk of the metal, the valence electrons become free to move from atom to atom. In a metal, thus, positive ions are embeded in a sea of electrons. The attractive force between positive ions and electrons is much greater than the repulsive force between the electrons and between positive ions. Hence atoms are bound together in a metal.

Properties of metals :

i) Metals have high thermal and electrical conductivity due to presence of free electrons in them.

ii) The free electrons are capable of absorbing and re-emitting electromagnetic radiations giving lustre to the metals.

iii) Metallic binding being devoid of directional bond, metals can be drawn into wires, beaten into sheets or twisted.

iv) Metals are opaque as the free electrons absorb photons in the visible region.

19.6 : Van der Waals bond or Molecular bond

The outer shells of inert gases like neon, argon, krypton etc. are completely filled with electrons. In such cases there is no possibility of transfer of electrons giving rise to ionic binding or sharing of electrons giving rise to covalent binding. When the temperature of the substances is decreased to a very low value, they condense into liquids and then to solids. Obviously there is a force of attraction between inert gas molecules which was first conceived by Van der Waal. *The bond formed due to a very weak force of electrostatic attraction between molecules of inert gas crystals, (like neon, organ etc.) and in molecular crystals (like paraffin, benzene etc.) is known as Van der Waal's bond.*

Properties of molecular crystals :

i) Molecular crystals have very low melting and boiling points due to very weak binding.

ii) They are bad conductors of heat and electricity.

iii) They are compressible and deformable.

iv) They are soft and insoluble.

v) They have very small cohesive energy.

19. 7 : Energy bands in solids :

Solids are classified into three groups depending on their capability of electrical conductivity. Mechanism of conduction in solids can be best understood quantitatively, on the basis of band theory of solids.

The electrons of an isolated atom possess discrete energy states such as 1s, 2s, 2p, 3s etc. These states are filled with electrons according to Pauli's exclusion principle. As an example, let us consider an isolated single sodium atom in its lowest energy state i.e., ground state. It has 11 electrons. Its electronic configuration is $1s^2$, $2s^2$, $2p^6$, $3s^1$. The levels 1s, 2s 2p are completely filled while 3s is partially filled with one electron though it can accomodate 2 electrons. All the energy levels above 3s level of sodium atom are completely empty.

Now consider a group of N isolated sodium atoms which are well separated from each other as in sodium vapour and hence they are non interacting. In the group, there are 11N electrons out of which 2N electrons in 1s state, 2N electrons in 2s state, 6N electrons in 2p state and N electrons in 3s state. Electrons in their respective states have identical energies. The 2N states of 1s, 2N states of 2s and 6N states of 2p are completely filled by the electrons while only N out of the 2N states of 3s are filled by the electrons and the remaining N states are empty. This idea is illustrated in the figs. (19.3 a) and (19.3 b) assuming N = 1 and N = 3 respectively.

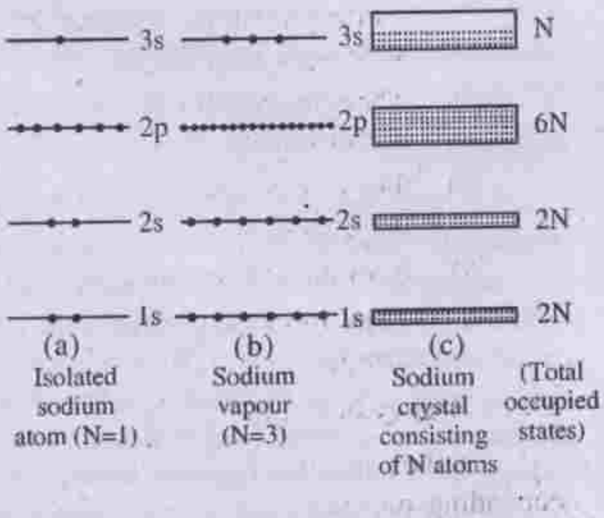


Fig. 19.3

Now consider sodium crystal consisting of N atoms. According to Pauli's exclusion principle, all the electrons in an energy level are in different states. As a result, all the 1s electrons in the crystal fill up the 2N states (since there are N atoms in the crystal and 2 electrons in s level). Similarly all the 2s and 3s electrons each fill up 2N states. All the 2p electrons fill up the 6N states. In a crystal the atoms are closely packed. The outer 3s electron of one atom starts interacting with 3s electrons of neighbouring atoms. Because of these interactions, the 2N states of 3s level do not all posses identical energies. A sharply defined 3s level in an isolated atom is now to be replaced by a group (or band) of 2N closely packed energy levels for 3s level in a crystal. These 2N levels are so numerous and so close together that they form a continuous energies band and it is called the 3s band. Similarly 2p level, 2s level and 1s level are to be replaced by 2p band, 2s band and 1s band respectively. But their band spreads decrease as one goes to lower order levels since the inner electrons being more tightly bound and also being shielded by the outer electrons, interact weakly with electrons of other atoms.

Fig. (19.3 c) shows schematically the splitting of energy levels into bands for a sodium atom in a crystal. 3s band contains 2N states with slightly different energies, N of them are occupied while rest N states are empty. 2p band contains 6N states with slightly different energies and all these states are filled with electrons. Similarly for other inner bands. The difference between the highest energy in a band and the lowest enrgy in the next higher band is called the band gap between the bands.

The highest occupied band at OK is called **valence band** which is occupied by valence electrons. The band above the valence band is known as conduction band. The electrons in the conduction band are free electrons. The energy gap between conduction band and valence band is known as forbidden energy gap or simply band gap, since no electron can stay in the forbidden energy gap, there being no allowed energy state in this region. Thus, to push an electron from valence band the minimum energy required is equal to forbidden energy gap so that it will go to conduction band.

19. 8 : Classification of solids on the basis of band theory :

Solids can be classified into three groups viz., conductros, semiconductros and insulators on the basis of width of the forbidden energy gap (Δ) of band theory.

Broadly solids are characterised as

i) Conductors if $\Delta = 0$

ii) Semiconductors if $\Delta < 2$ eV

iii) Insulators if $\Delta > 2$ eV

i) Conductors :

A solid can be a conductor under the following two circumstances :

a) If the conduction band is partially filled :

Fig. (19.4 a) shows the type of band structure exhibited by the solids whose atoms have odd number of electrons in their outermost subshells. Sodium is an example of this kind. At obsolute zero, half of energy levels of conduction band are occupied and other half of energy levels are completely empty (fig. 19.4 a). The highest occupied level of conduction band at OK is known as Fermi level and its energy is known as Fermi energy. When an electric field is applied, the electrons receive energy and get excited into empty energy levels just above the Fermi level in the conduction band and drift, giving rise to electrical conductivity. Other examples of this type of band structure are potassium, lithium etc.

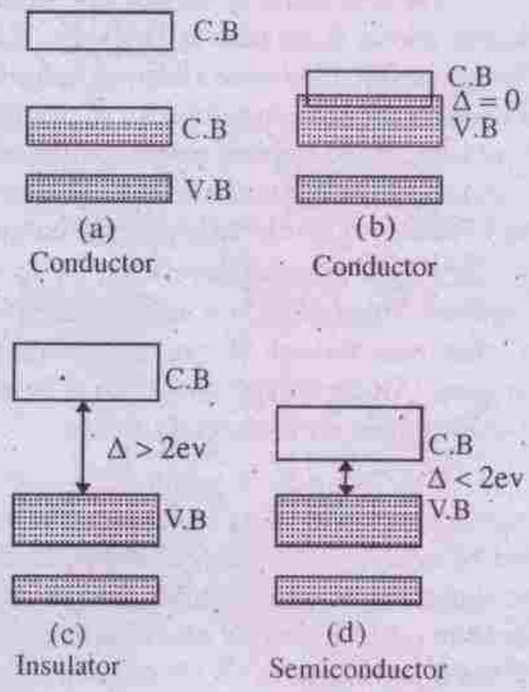


Fig. 19.4

b) If the conductions band is completely empty and overlap with fully filled valence band (fig. 19.4 b) :

In this case the forbidden energy gap is zero. Even a feeble electric field enables, the electrons to shift from valence band to conduction band and drift, constituting electric current. Such solids conduct electricity and are good conductors. Some of the examples of such conductors are Be ($1s^2, 2s^2$), Mg ($1s^2, 2s^2, 2p^6, 3s^2$) Zn ($1s^2, 2s^2, 2p^6, 3s^2, 3p^6, 3d^{10}, 4s^2$) etc.

ii) Insulators :

Solids having their valence bands completely filled with electrons at absolute zero and conductions bands completely empty with a very large forbiddes energy gaps are called insulators (fig. 19.4 c). When an electric field is applied to an insulatator, electrons in the valence band cannot gain sufficient energy to reach the conduction band for conduction. Diamond is an example which has a forbidden band gap of 5.4 eV. Diamond slab of thickness about 1 cm needs a potential difference of about

10^5 volts for its electrical conductivity. Diamond, thus, does not conduct when an ordinary battery is connected to it and hence it is an insulator.

iii) Semiconductors :

Solids having their valence bands completely filled with electrons at ok and conduction bands completely empty with a narrow forbidden band gap are called semiconductors. The band structure of a semiconductor is similar to that of an insulator except for the narrow band gap (fig. 19.4 d). Some examples of semiconductors are silicon which has a band gap of 1.17 ev and germanium having a band gap of 0.74 ev. At room temperature, thermal collisions may push some of the electrons from the valence band to empty conduction band creating empty states in valence band. These electrons and some electrons in valence band (due to creation of empty states in it) move when a potential difference is applied across the semiconductor. As the number of electrons available for conduction is very small, the conductivity is quite small as compared to conductors but greater than that of insulators. Such solids are called semiconductors.

In semiconductors, with increase in temperature, the number of electrons pushed from valence band to conduction band increases. Thus conductivity of semi conductors and also of insulators increases with increase in temperature.

19.9 : Types of semiconductors :

As discussed earlier, a semiconductor is a substance which has an almost filled valence band and a nearly empty conduction band with a very small forbidden band gap (~ 1ev) separating the two bands. At OK, a semiconductor behaves as an insulator. At room temperature, the resistivity of a semiconductor varies from 10^{-5} to 10^{4} Ω m and hence the conductiviy of the semiconductor is too small to have any practical utility. But its conductivity can be increased by adding a small amount of impurity. Thus, depending on whether any impurity is added or not, we have two types of semiconductors : (i) Pure or intrinsic semiconductor and (ii) Impure or extrinsic semiconductor.

i) Pure or Intrinsic semiconductor :

A semiconductor in which charge carriers are generated for its conductivity due to thermal energy alone is known as a pure or intrinsic semiconductor.

At room temperature, in an intrinsic semiconductor, some electrons in valence band gain energy to jump into the conduction band and become free and create vacancies in the valence band. These vacancies are termed as holes. A hole is equivalent to a positive charge equal in magnitude to that of an electron. Thus in a pure semiconductor number of electrons in conduction band is equal to number of holes in valence band.

The electrons and holes generated due to thermal energy are known as minority charge carriers and they move at random in the crystal. When an electric field is applied across an intrinsic semiconductor, electrons and holes move in opposite directions giving rise to electron current (I_e) and hole current (I_h) respectively in the same direction. The total conventional current (I) through the intrinsic semiconductor is equal to sum of electron current and holecurrent

$$I = I_e + I_h$$

In case of covalent semiconductors like Ge and Si, the generation of electrons and holes and their movement can be easily explained by considering their crystal structure as follows :

Fig. (19.5 a) shows a two dimensional representation of a germanium crystal. The valency of Ge is 4. At ok, all the valence electrons of a Ge-atom form covalent bonds with valence electrons of neighbouring 4 Ge-atoms thereby leaving no free electrons for conduction. At room temperature some valence electrons acquire thermal energy to break their covalent bonds and become free for conduction leaving behind vacant states of electrons called holes. Electron-hole pairs are thus generated and two such pairs are shown in the fig. (19.5b). When a hole is created, an adjacent valence electron having sufficient thermal energy may fall into the hole position. As a result a bond is reconstructed but a new hole is created in the earlier position of the electron.

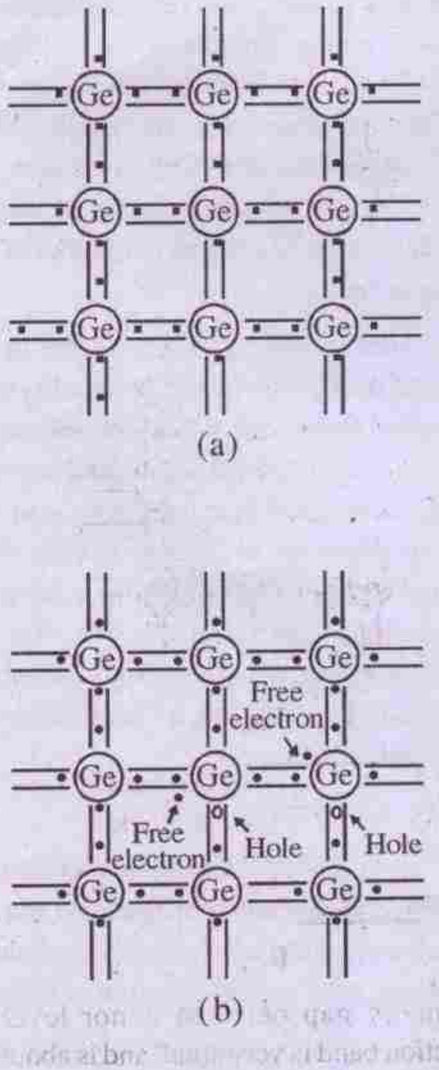


Fig. 19.5

This can be viewed as the movement of the (vacancy) hole from one position to another opposite to the direction of electron movement. For this reason, the hole is regarded as a positive charge of same magnitude as that of an electron.

At room temperature only one out of 10^9 covalent bonds breaks. Therefore, the conductivity of an intrinsic semiconductor is so small that, it can not be used for practical purposes.

ii) Impure or extrinsic semiconductor :

Due to low conductivity, intrinsic semiconductors have little practical utility. The most efficient and convenient way of generating free electrons or holes is to add a small amount of selected impurity to the semiconductor crystal thereby increasing its conductivity.

The process of increasing the conductivity of the semiconductor by adding a small amount of suitable impurity is called doping. The impurity that is added is known as dopant. A semiconductor containing the impurity atoms to enhance its conductivity is called extrinsic semiconductor. Impurities frequently employed for Ge and Si are the elements of group III and V of the periodic table i.e. trivalent and pentavalent impurities. Commonly used trivalent impurities are boron, gallium, indium and aluminium and pentavalent impurities are phosphorus, antimony and arsenic.

19.10: Types of Extrinsic semiconductors :

Doping of an entrinsic semiconductor results in an extrinsic semiconductor. Depending on the type of dopant used, extrinsic semiconductors are of two types viz., n-type and p-type. Doping the semiconductor with pentavalent impurity results in n-type while doping with trivalent impurity gives rise to p-type. However, the atoms of doping elements and that of semiconductors should have comparable sizes. Table 19.1 illustrates this.

Table 19.1

Dopant	Symbol	Atomic Number	Valence electrons	Semiconductor with which doped	Type of extrinsic semiconductor
Antimony	Sb	51	5	Ge z = 32	n-type
Arsenic	As	33	5	Ge	n-type
Phosphorus	P	15	5	Si z = 14	n-type
Aluminium	Al	13	3	Si	p-type
Boron	B	5	3	Si	p-type
Gallium	Ga	31	3	Ge	p-type
Indium	In	49	3	Ge	p-type

i) n-type semiconductor :

The doping elements in n-type semiconductor are pentavalents such as arsenic, antinomy and phosphorus. In each of these atoms there are 5 electrons in their outermost orbits.

Consider a small amount of phosphorus as an impurity in a Si crystal. In the crystal lattice, a phosphorus atom is surrounded by Si atoms. Four of the five valence electrons of phosphorus atom form covalent bonds with neighbouring four Si atoms, leaving one surplus electron. This surplus electron is loosely bound to Si nucleus and can easily be detached by thermal energy. At room temperature, these extra electrons become as free as the electrons in a metal and these are donated by dopant atoms. Such dopants are called donors. The mechanism of how electrons get free in n-type semiconductor is illustrated in fig. 19.6. Thus in n-type semiconductor majority charge carriers are electrons and minority charge carriers are thermally generated holes.

The mechanism in which electrons are available for conduction can also be explained on the basis of energy band theory as follows :

The energy state corresponding to fifth surplus valence electron of the dopant is formed in the forbidden energy gap, just below the conduction band of Si. This state is known as donor level (fig. 19.7).

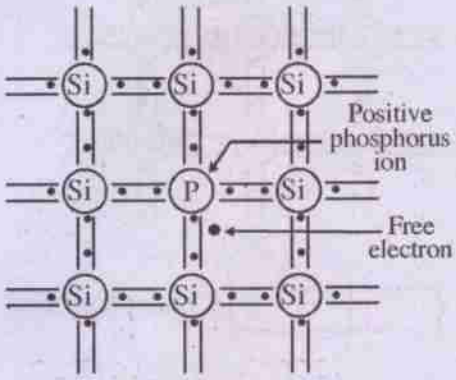


Fig. 19.6

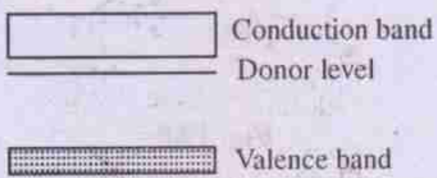


Fig. 19.7

The energy gap between donor level and conduction band is very small and is about 0.01 eV. At room temperature, the electrons in the donor level get sufficient energy to jump into the conduction band and become free for conduction. The impurity atoms (which donate electrons) now become immobile positive ions and do not take part in conduction.

ii) p-type semiconductor :

When a Ge or Si crystal is doped with one of the trivalent impurities such as boron, indium or aluminium, p-type semiconductor is formed.

Consider a small amount of indium as impurity, doped with a Ge-crystal. Each impurity atom with 3 valence electrons replaces a Ge-atom and hence it has a deficiency of one bonding electron. This deficiency of one electron acts like a positive charge and is called a hole (fig. 19.8a).

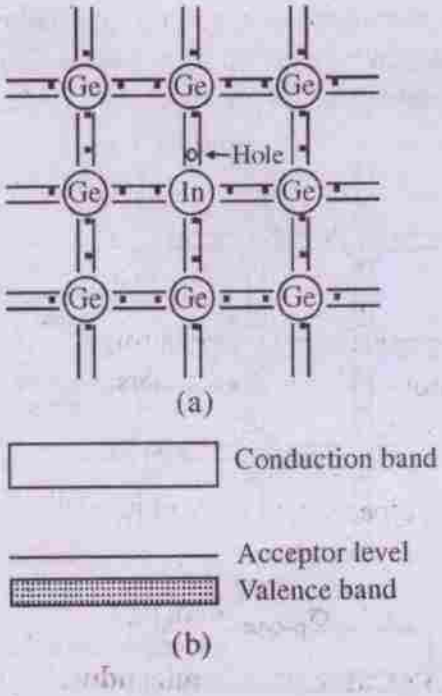


Fig. 19.8

A valence electron of the neighbouring atom needs a small amount of energy to drop into this hole thereby creating a new hole in the neighbouring atom which it left. Thus the hole can move from one atom to another due to acceptance of electrons by dopant atoms. Such dopants are called acceptors. In p-type semiconductor majority charge carriers are holes and minority charge carriers are thermally generated electrons.

The conductivity of p-type semiconductor can also be explained on this basis of energy band theory as follows :

The energy state corresponding to deficit valence electron is formed in the forbidden energy gap of Ge-crystal just above the valence band. This energy state is called acceptor level (fig. 19.8 b).

The energy gap between valence band and acceptor level is very small and is about 0.01 ev. Electrons from the valence band can readily be agitated thermally to move into the vacant acceptor level. In doing so holes are produced in the valence band. The trivalent indium atom accepts the electron and gets negatively charged and these become immobile negative ions and do not contribute to electric current. The holes are mobile and produce current when electric field is applied.

It should be noted that doping does not really add or take away charges from the semiconductor. Hence n-type and p-type semiconductors are neutral. However, doping redistributies the valence electrons so that more free charges are available.

19.11: Effect of temperature on extrinsic semiconductor :

With increase in temperature of the extrinsic semiconductor, minority charge carriers increase (due to breakage of bonds) and they may become almost equal with majority charge carriers which are electrically opposite in nature. The extrinsic semiconductor thus behaves as almost an intrinsic semiconductor with increase in temperature. Thus extrinsic semiconductor has positive temperature coefficient of resistance (α) while intrinsic semiconductor has negative α.

19.12 : Electrical conductivity in semiconductors :

Before finding an expression for electrical conductivity, let us define the following terms.

Mobility (μ) :

Mobility of a carrier is defined as the average drift speed (v) per unit electric field (E)

$$\mu = \frac{v}{E}$$

Current density (J) :

It is defined as the current (I) per unit cross sectional area (A)

$$J = \frac{I}{A}$$

Conductivity (σ) :

It is defined as the current density per unit electric field

$$\sigma = \frac{J}{E}$$

When an electric field E is applied to an intrinsic semiconductor, electrons and holes move in opposite directions in the semiconductor but currents due to them are in the same direction. Hence, total current in the semiconductor is

$$I = I_e + I_h$$

where I_e is the electron current and I_h is the hole current. These currents are related to drift speeds by the relations

$$I_e = n_e A e v_e$$

$$I_h = n_h A e v_n$$

where n_e is the free electron density and n_h is the free hole density. v_e and v_n are the drift speeds of electron and hole respectively

$$\therefore \quad I = A e (n_e v_e + n_h v_n)$$

$$J = \frac{I}{A} = \sigma E = e(n_e v_e + n_h v_n)$$

$$\sigma = e\left(n_e \frac{v_e}{E} + n_h \frac{v_n}{E}\right)$$

$$= e(n_e \mu_e + n_h \mu_h)$$

where $\mu_e = \frac{v_e}{E}$ is the mobility of electrons

$\mu_h = \frac{v_n}{E}$ is the mobility of holes

$$\sigma = \sigma_e + \sigma_h$$

where $\sigma_e = e n_e \mu_e$ is the conductivity of the semiconductor due to electrons.

$\sigma_h = e n_h \mu_h$ is the conductivity of the semiconductor due to holes.

In an intrinsic semiconductor

$$n_e = n_h = n \text{ (say)}$$

$$\therefore \quad \sigma_{int.} = en(\mu_e + \mu_h)$$

In an n-type semiconductor, $n_e >> n_h$ i.e. electrons are majority carriers.

$$\therefore \quad \sigma_{n-type} \approx e n_e \mu_e$$

In a p-type semiconductor, $n_h >> n_e$ i.e. holes are majority carriers

$$\therefore \quad \sigma_{p-type} \approx e n_h \mu_h$$

19.13 : Currents in semiconductors :

i) Forward and reverse currents :

In n-type semiconductor, majority charge carriers are electrons and minority charge carriers are holes. A p-type semiconductor has holes as majority charge carriers and electrons as minority charge carriers.

When a voltage is applied to an extrinsic semiconductor, a large current known as forward current due to motion of majority charge carriers whereas a small current known as reverse current or leakage current flows in the reverse direction. In case of n-type semiconductor, forward current is due to electrons while reverse current is due to holes. Similarly, forward current is due to holes and reverse current is due to electrons in p-type semiconductors.

Further, the reverse current increases with increase of temperature as more and more number of covalent bonds break due to increase of thermal energy. Hence temperature is an important factor for the operation of semiconductor devices.

ii) Drift current :

Drift current is due to motion of charges in response to the electric field due to an applied potential difference. It is the usual way of producing current in any electrical or electronic device. It can be due to the flow (drift) of electrons or holes. Forward and reverse currents in an extrinsic semiconductor due to applied voltage are drift currents.

iii) Diffusion current :

Diffusion current is due to diffusion of charge carriers from a higher concentration region to a lower concentration region of the crystal lattice in a solid semiconductor. For diffusion current, an applied voltage is not necessary. Either electrons or holes can diffuse through a solid semiconductor when one side has higher concentration of either of the charge carriers. When an n-type material and a p-type material come in contact with each other, electrons from the n-side diffuse into the p-side and holes from the p-side diffuse into the n-side.

19.14 : P-n junction :

Technically fabricated interface of p-type and n-type materials is called a p-n junction. It is not formed by just bringing a p-type semiconductor in contact with an n-type semiconductor. It is a single crystal of the solid semiconductor, doped suitably with donor and acceptor dopants so that opposite free charges appear on the two sides of the juction. p-n junction is a useful semiconductor device because the juction voltage controls the flow of current through it. A p-n junction by itself is a semiconductor diode having two electrodes, one each on the p-side and the n-side.

19.15 : Function a of p-n junction :

For the sake of understanding the basic function of a p-n junction, let us consider an idealized situation in a diagram with left half as p-type and right half as n-type semiconductor as shown in the fig. (19.9)

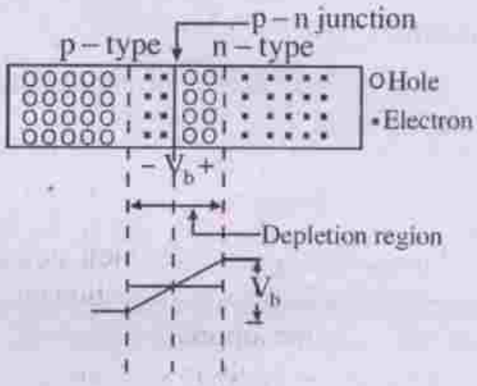


Fig. 19.9

A p-type semiconductor contains an excess of holes whereas an n-type contains an excess of electrons. As in a p-n junction, p-type and n-type materials are grown in the same crystal, some of holes from p-region will diffuse into the n-region and some of electrons diffuse from the n-side to the p-side simultaneously. Diffusion of holes and electrons is limited to a very narrow region spreading on either side of the junction. Before diffusion, p-type and n-type semiconductors were neutral. After diffusion of electrons into the p-side, it becomes negatively charged while due to diffusion of holes into the n-side, it becomes positively charged. This creates an electric field at the junction directed from the n-side to the p-side. Any hole near the junction is pushed by this electric field towards the p-region while any free electron near the junction is pushed towards the n-side. Thus there are no free charges in a small region near the junction. As the free charges are depleted, this region is called the depletion region.

The electric field in the depletion region makes the potential of the n-sider higher than that of the p-side. The variation in potential is sketched in fig. (19.9). This potential prevents further flow of electrons from n-side to p-side and holes from p-side to n-side and behaves as a barrier for flow of charges. Hence it is known as potential barrier (V_b). which is a characteristic property of the junction. For Si junction V_b is 0.7V and it is 0.3V for Ge junction. (It should be noted that outside of the potential barrier, on either side of the junction, the material is still neutral.)

19.16 : P-n junction diode :

Diode is a device which conducts electric current freely in one direction but does not conduct in the opposite direction. P-n junction conducts freely in one direction and hence it is known as p-n junction diode or semiconductor diode or crystal diode as it is grown on a single semiconductor crystal.

To investigate the conductivity of a p-n junction, let us consider two ways of biasing it viz., (i) forward biasing and (ii) reverse biasing.

i) Forward bias :

A p-n junction (fig. 19.10 a) is said to be forward biased when an external voltage is applied in such a way that the potential barrier decreases permitting easy flow of drift current. To apply forward bias, the p-side is to be connected to the positive terminal and the n-side to the negative terminal of the external battery as shown in the fig. (19.10b). The external electric field $\vec{E}$ due to the applied forward bias is from p-side to n-side

Fig. 19.10 (a) unbiased

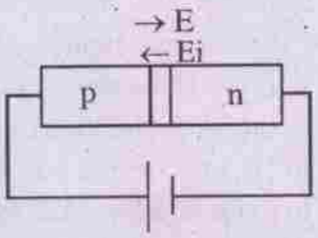


Fig. 19.10 (b) Forward biased

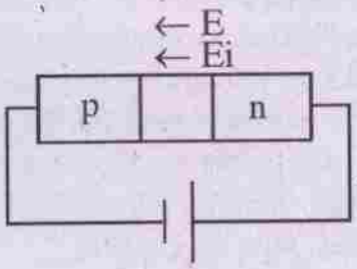


Fig. 19.10 (c) Reverse biased

whereas the internal electric field $\vec{E}_a$ due to potential barrier V_b is from n-side to p-side. Forward bias, thus, reduces the potential barrier, narrows down the depletion region (Fig. 10 b) and makes the junction resistance very low.

If $E > E_i$ i.e. applied voltage is greater than barrier voltage, the barrier voltage and the depletion region are eliminated, establishing a low resistance path in the circuit. Current now easily flows through the junction and it is known as forward current. With forward bias

a) potential barrier decreases

b) depletion region narrows down

c) junction offers a very low resistance

d) current flows easily

ii) Reverse bias :

P-n junction is said to be reverse biased when the external voltage is applied in such a way that the potential barrier increases letting a very feeble reverse current to flow. For reverse bias, p-side is to be connected to negative terminal and n-side to positive terminal of the external battery as shown in the fig. (19.10 c). The external electric field ($\vec{E}$) is thus in the same direction as the internal electric field ($\vec{E}_i$) due to the potential barrier i.e. from n to p side and therefore the height of the potential barrier increases and the depletion region widens (Fig 19.10 c). The increased potential barrier prevents the flow of majority charge carriers but favours the flow of minority charge carriers through the junction. Hence a very small reverse current (of the order of a few μ A, negligible for practical purposes) flows establishing a high resistance path in reverse bias. With reverse bias

a) potential barrier increases

b) depletion region widens

c) junction offers a very high resistance

d) a very low reverse current passes.

19.17 : Volt-amp characteristic of a p-n junction :

The graphical representation of current through a p-n junction with applied voltage is called Volt-amp or V-i characteristic of the p-n junction. Fig. 19.11 shows the typical characteristic curve of a p-n junction using the circuit of fig. (19.12) in which the diode is symbolised as , the arrow pointing in the direction in which the current can pass freely. For a p-n junction diode, the arrow points from the p-side to the n-side.

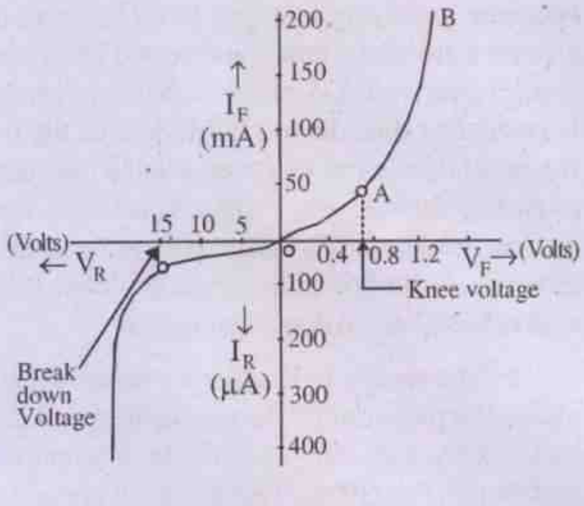


Fig. 19.11

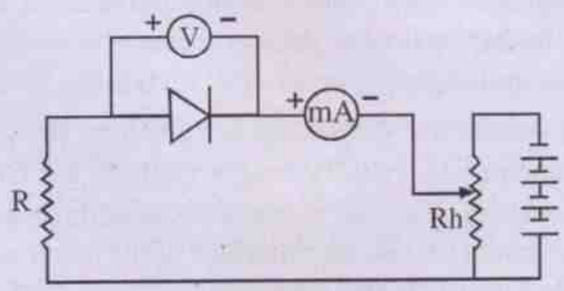


Fig. 19.12 (a) Circuit for forward bias

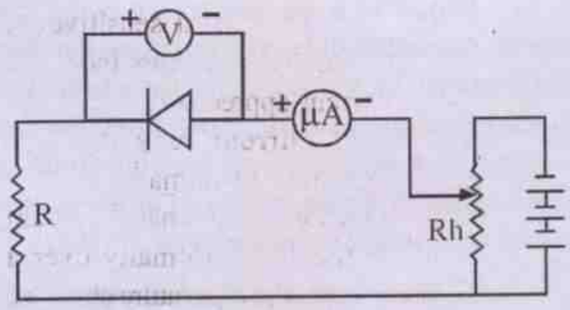


Fig. 19.12 (b) Circuit for reverse bias

When the external voltage is zero, the potential barrier (V_b) does not permit the current to flow. With increase in forward bias Fig. (19.12 a) forward current first increases slowly and then rapidly. The upward surge of the current occurs at a forward voltage known as knee voltage. At this voltage barrier potential is completely eliminated and the junction offers a very low resistance.

When reverse bias is applied fig. (19.12b), the current is very low and practically remains constant with increase in reverse voltage. Hence this current is called reverse saturation current. If reverse voltage reaches a value known as break down voltage (about 15 volts) current increases very sharply due to breakage of a large number of covalent bonds releasing a large number of minority charge carriers. This effect is used in voltage regulators.

19.18 : Crystal diode versus vacuum diode :

Advantages :

a) A junction diode is robust, cheap and very small in size. Its use in an instrument makes it portable.

b) It has no heating element. Hence power consumption is low and it has a long life.

c) It can be operated with low d.c. voltages and currents which can be supplied even by dry cells.

d) As there is no filament, its response time is very small.

Disadvantages :

a) It is extremely heat sensitive. A slight rise in temperature increases the current appreciably. Increased flow of current may produce enough heat to damage the p-n junction. On the otherhand, vacuum diode functions normally over a wide range of temperature changes.

b) It can handle only small currents and low inverse voltages as compared to vacuum diode.

c) It cannot withstand overload even for a short period. On the otherhand vacuum diode can withstand overload for a short period.

19.19 : P-n junction as a rectifier :

A rectifier is a device which converts an a.c. to d.c. A p-n junction can be used as a rectifier because it permits current in one direction only when it is in forward bias. In reverse bias, current is too small to be practically significant.

The following two rectifier circuits can be used :

a) Half-wave rectifier

b) Full-wave rectifier

i) Half-wave rectifier :

The rectifier conducts only during one half-cycle of input a.c. supply while other half-cycle is suppressed and hence the name half-wave rectifier.

Circuit :

Fig. (19.13 a) shows the circuit of half wave rectifier which requires a single diode. A.C. supply to be rectified is given through a transformer and its advantages are two fold :

a) It allows us to step up or step down the a.c. input voltage as per requirement.

b) The transformer isolates the rectifier circuit from power line and thus reduces the risk of electric shock.

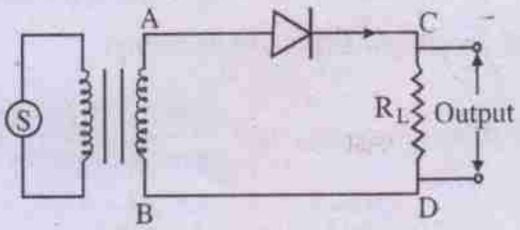


Fig. 19.13 (a)

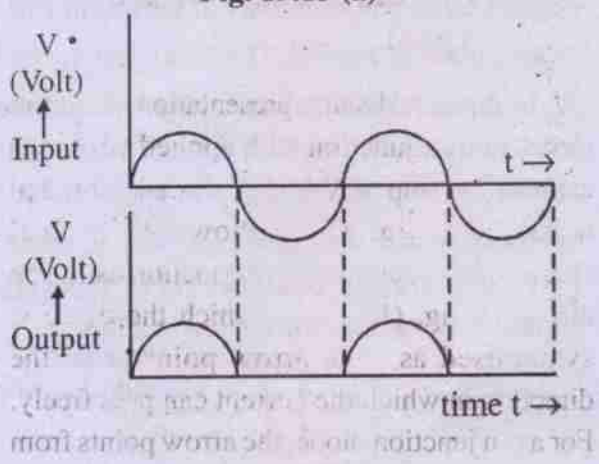


Fig. 19.13 (b)

The diode and load R_L are connected in series with the secondary of the transformer.

Operation :

Fig. (19.13 b) shows the input a.c. supply. During positive half-cycle of input a.c., the secondary terminal A of the transformer becomes positive w.r.t. its end B. This makes the diode forward biased and the current flows through the load R_L from C to D. During negative half cycle, the terminal A becomes negative w.r.t. its end B. The diode is now in reverse bias and hence it does not conduct. The current through the load R_L is always uni-directional and output is obtained across R_L only during the positive half cycles of input a.c. (fig. 19.13 b). But it is pulsating d.c. These pulsations are smoothened with the help of filter circuits (not shown in the fig. 19.13 a).

Disadvantages :

a) An elaborate filtering circuit is required to smoothen pulsating d.c.

b) Output is low as it is obtained only during half of the input a.c. supply.

Efficiency of a half wave rectifier :

$$\text{Rectifier efficiency, } \eta = \frac{\text{output d.c. power}}{\text{input a.c. power}}$$

Let $V = V_o \sin \omega t$ be input a.c. that appears across the secondary of the transformer of the circuit of the Fig. (19.13 a). Let r_f and R_L be diode resistance in forward bias and the load resistance respectively. The output appears across R_L during the positive half cycles only and it is given by $V = V_o \sin \omega t$ for $t = 0$ to T/2 and $V = 0$ for T/2 to T. As the current is pulsating, the average current,

$$I_{av.} = I_{d.c.} = \frac{1}{T}\int_o^{T/2} \frac{V_o \sin \omega t dt}{(r_f + R_L)}$$

$$= \frac{V_o}{T(r_f + R_L)}\left[\frac{-\cos \omega t}{\omega}\right]_o^{T/2}$$

$$= \frac{V_o}{\omega T(r_f + R_L)}[-\cos \pi + \cos o]$$

$$= \frac{I_o}{\pi}$$

$$\text{where } \omega T = 2\pi \text{ and } I_o = \frac{V_o}{r_f + R_L}$$

Output d.c. power across R_L,

$$P_{d.c.} = I^2 d.c. R_L$$

$$= \frac{I_o^2}{\pi^2} R_L$$

Input a.c. power, $P_{a.c.} = I_{rms}^2 (r_f + R_L)$

For half wave rectifier $I_{rms} = \frac{I_o}{2}$

$$\therefore \quad P_{a.c.} = \frac{I_o^2}{4}(r_f + R_L)$$

$$\eta = \frac{P_{d.c.}}{P_{a.c.}}$$

$$= \frac{I_o^2}{\pi^2}.R_L \Big/ \frac{I_o^2}{4}(r_f + R_L)$$

$$= \frac{4}{\pi^2}.\frac{R_L}{r_f + R_L}$$

$$= \frac{0.406}{1 + r_f / R_L}$$

The efficiency is maximum if $R_L >> r_f$.

$\therefore \ \eta_{max} \approx 40.6\%$ for half wave rectifier.

Half wave rectifier can thus convert upto 40.6% of a.c. power into d.c. power.

ii) Full wave rectifier :

In a full wave rectifier, current flows through the load in the same direction during both half cycles of input a.c. It makes use of two diodes which work alternately.

Circuit :

Fig. (19.14 a) shows the circuit of a full wave rectifier. It employes two diodes D_1 and D_2 and a centre tapped transformer. The centre tapped secondary winding AB is connected with the two diodes D_1 and D_2 such that D_1 utilizes the input a.c. voltage appearing across the upper half (OA) while D_2 uses the voltage appearing across the lower half (OB) of the secondary winding. Load R_L is connected in series with the common terminal of the two n-sides of the diodes and terminal of the centre tap.

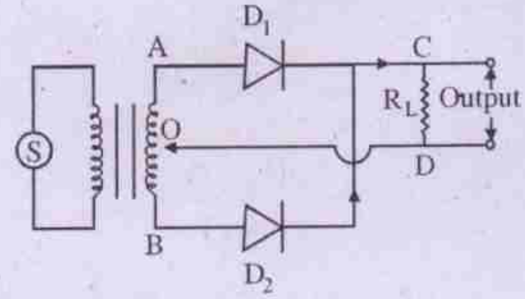


Fig. 19.14 (a)

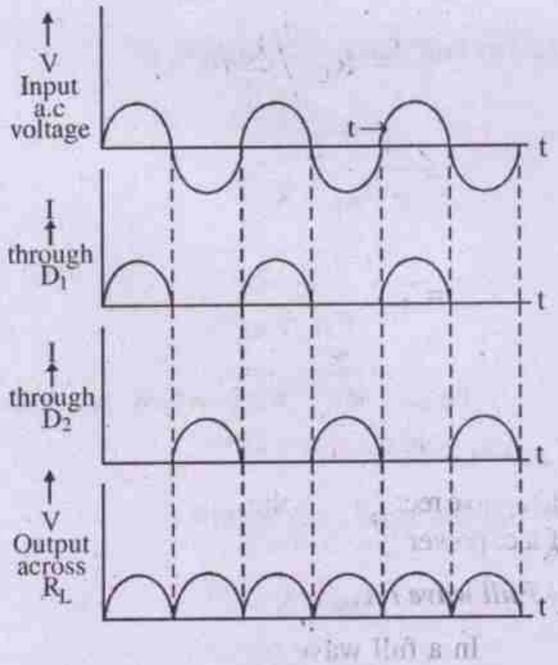


Fig. 19.14 (b)

Operation :

During the positive half cycle of input a.c. voltage, let the secondary terminal A be positive and the end B be negative with respect to centre tap O. The diode D_1 is now forward biased while D_2 is reverse biased. Diode D_1 therefore conducts and the current passes through the load R_L from C to D while D_2 does not conduct. During negative half cycle of input a.c., the terminal A is negative and terminal B is positive w.r.t. to centre tap O. Then D_1 is reverse biased while D_2 is forward biased. Current now flows through D_2 and R_L from C to D but not through D_1. The current direction in both the half cycles through the load R_L is in one direction i.e. from C to D. For input a.c. voltage, the output voltage is a series of unidirectional pulses which are smothened to get steady d.c. voltage using filter circuit. Fig. (19.14 b) shows the currents through D_1 and D_2 and output voltage across R_L.

Efficiency of a full wave rectifier :

Fig. (19.14 b) shows output voltage across R_L of full wave rectifier. Let input a.c. voltage be represented by $V = V_o \sin \omega t$ for t = 0 to T and output voltage be represented by $V = V_o \sin \omega t$ for t = 0 to T/2 and this shape is repeated at every half period T/2.

Instantaneous output current

$$I = \frac{V_o \sin \omega t}{r_f + R_L}$$

where r_f is the resistance of each diode while they are in forward bias.

$$I_{av} = I_{d.c.} = \frac{1}{T/2}\int_o^{T/2} \frac{V_o \sin \omega t}{(r_f + R_L)} dt$$

$$= \frac{2V_o}{T(r_f + R_L)}\left[\frac{-\cos \omega t}{\omega}\right]_o^{T/2}$$

$$= \frac{2V_o}{\omega T(r_f + R_L)}\left[-\cos\frac{\omega T}{2} + \cos o\right]$$

$$= \frac{2I_o}{\pi}$$

where $\omega T = 2\pi$ and $\frac{V_o}{r_f + R_L} = I_o$, peak current.

$\therefore$ Output d.c. power across R_L,

$$P_{d.c.} = I_{d.c.}^2 R_L$$

$$= \left(\frac{2I_o}{\pi}\right)^2 \times R_L$$

Input a.c. power, $P_{a.c.} = I_{rms}^2(r_f + R_L)$

But for the input sinusoidal a.c.

$$I_{rms} = \frac{I_o}{\sqrt{2}}$$

$$\therefore \quad P_{a.c.} = \frac{I_o^2}{2}(r_f + R_L)$$

Rectification efficiency of the full wave rectifier,

$$\eta = \frac{P_{d.c.}}{P_{a.c.}}$$

$$= \left(\frac{2I_o}{\pi}\right)^2 R_L \Bigg/ \frac{I_o^2}{2}(r_f + R_L)$$

$$= \frac{8}{\pi^2} \cdot \frac{R_L}{r_f + R_L}$$

$$= \frac{0.812}{1 + \frac{r_f}{R_L}}$$

The efficiency is maximum if r_f is negligible in comparison to R_L.

$\therefore$ Maximum efficiency, $\eta_{max} \approx 81.2\%$

Full wave rectifier has a maximum efficiency which is double that of half wave rectifier.

19.20 : Ripple factor :

The output of a rectifier is pulsating d.c. It consists of a d.c. component and an a.c. component or ripples. *The ratio of r.m.s. value of a.c. component to d.c. component in the rectifier output is known as ripple factor.*

Ripple factor, $r = \frac{I_{a.c.}}{I_{d.c.}}$

The effective value of total load current is given by

$$I_{rms} = \sqrt{I_{d.c}^2 + I_{a.c}^2}$$

$$r = \frac{\sqrt{I_{rms}^2 - I_{d.c.}^2}}{I_{d.c.}}$$

$$= \sqrt{\frac{I_{rms}^2}{I_{d.c}^2} - 1}$$

(a) For half wave rectification,

$$I_{rms} = I_o/2$$

$$I_{d.c.} = I_o/\pi$$

$$\therefore \quad r = \sqrt{\frac{(I_o/2)^2}{(I_o/\pi)^2} - 1} = 1.21$$

Thus, in a half wave rectifier, $I_{a.c.} > I_{d.c.}$

(b) For full wave rectification :

$$I_{rms} = I_o/\sqrt{2}$$

$$I_{d.c.} = 2I_o/\pi$$

$$\therefore \quad r = \sqrt{\frac{(I_o/\sqrt{2})^2}{(I_o/\pi)^2} - 1} = 0.48$$

Thus, for a full wave rectifier, $I_{a.c.} < I_{d.c.}$

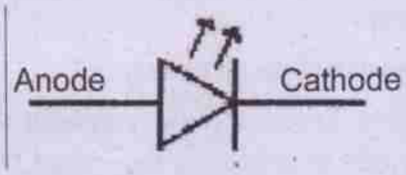
19.21: Special purpose p-n junction diode

There are some specially designed p-n junction diodes, with interesting behaviours and wide variety of uses. Some of these are LED, Photodiode, Solar cell and Zener diode. We will ahve brief disscussion about these.

19.21.1 LED:

A light emitting diode (LED) is a special purpose p-n junction diode. When carries are injected across a forward biased junction, it emits incoherent light. Most of the LEDs are realised using a highly doped p-n junction.

Symbol:



Principle:-

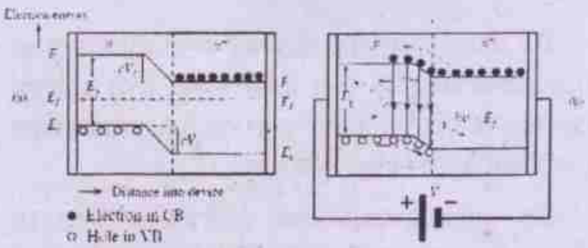


Fig.1

Fig.1(a) shows an unbiased p-n junction energy band diagram. The depletion region extends maily into the p-side. There is a potential barrier from E_c on the n-side to the E_c on the p-side called built-in voltage V_0. This is the potential barrier. It prevents the excess free electrons on the n-side form diffusing into p-side. When a potential V is applied across the juction, so that it is forward biased, the built in potential is reduced from V0 to V0= V. This allows the electrons from the n-side to get injected into the p-side. Thse electrons recombine with the holes on the p-side. This recombination results in spontaneous emission of photons. This effect is called electroluminescence.

These photons should be allowed to escape from the device without being reabsorbed. The recombination is of two types:

1. Direct recombination
2. Indirect recombination

Direct Recombination:-

When the minumum energy of the conduction band lies directly above the maximum energy of the valence band (see fig.2 (a) there is direct recombination. Direct recombination occurs spontaneously. GaAs is an example of a direct band-gap material.

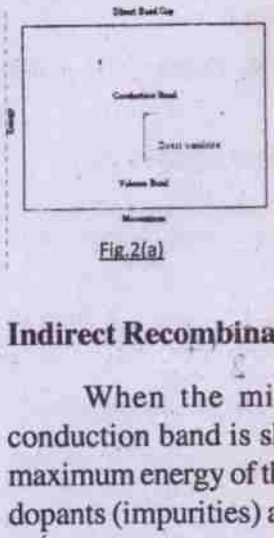


Fig.2(a)

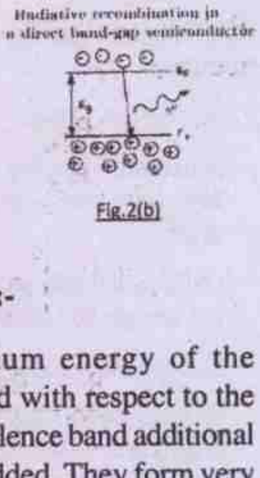


Fig.2(b)

Indirect Recombination:-

When the minumum energy of the conduction band is shifted with respect to the maximum energy of the valence band additional dopants (impurities) are added. They form very shallow donor states. These donor states serve as recombination centres. This is called indirect (non-radiative) recombination. Fig.3(b) shows how the Nitrogen serves as a recombination centre in GaAsP.4

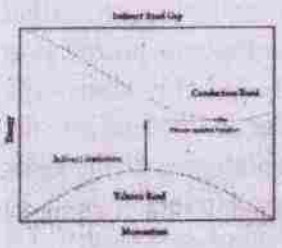


Fig.3(a)

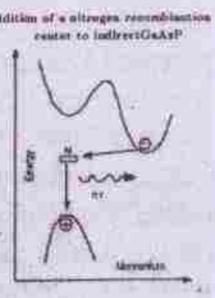


Fig.3(b)

The emitted photon energy is approximately equal to the band gap energy of the semiconductor. i.e.

$$E_g = h\nu = \frac{hc}{\lambda}$$

where, Eg is the band gap energy, h is Plank's constant, c is the speed of light and λ is the wave length of radiation. Thus, a semiconductor of 2 eV band-gap would emit 414 nm, in the violet. See the list given below to know how are emitted radiation depends on the semiconductor.

Following is a list of semiconductor materials and the corresponding colours:

- Aluminium gallium arsenide (AIGaAs)- red and infrared
- Aluminium gallium phosphide (AIGaP)- green
- Aluminium gallium indum phosphide (AIGaInP)- high-brihgness oranger-red, orange, yellow, and green
- Gallium arsenide phosphide (GaAsP)- red, orange-red, orange and yellow
- Gallium phosphide (GaP)- red, yellow and green
- Gallium nitride (GaN)- green, pure green (or emerald green) and blue also white (if is has an AIGaN Quantum Barrier)
- Indium gallium nitride (InGaN)-450 nm - 740nm - near ultraviolet, bluishgreen and blue.
- Silicon carbide (SiC) as substrate-blue
- Silicon (Si) as substrate - blue (under development)
- Sapphire(Al_2O_3) as substrate - blue
- Zinc selenide (ZnSe)-blue
- Diamond (C)- ultraviolet
- Almunium nitrate (AIN), aluminium gallium nitride (AIGaN), aluminium gallium indium (AIGaInN)-near to far ultraviolet (down to 210 nm)

LED efficiency:- A very important metric of an LED is the external quantum efficiency next. it quantifies the efficiency of the conversion of electrical energy into emitted optical energy. **It is denied as the ratio of light output to the input electrical power. i.e.**

$$n_{ext} = P_{out}(\text{optical})/IV$$

It is also defined as the product of internal radiactive efficiency and extraction efficiency. For indirect band gap semiconductor next is generally less than 1%, whereas for a direct band gap material it could be substantial.

n_{ext} = rate of radiation recombination/Total recombination

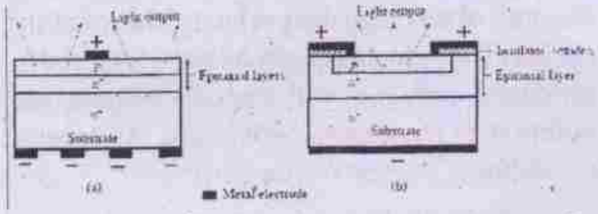
The internal efficiency is a function of the quality of the material and the structure and composition of the layer.

LED Construction (Structure):-

The Led structure plays a crucial role in emitting light from LED surface. The LEDs are structured to ensure most of the combination takes place on the surface by the following two ways:

- By increasing the doping concentration of the substrate, So that additional free charge carriers (electrons) move to the top, recombine and emit light at the surface.
- By increasing the diffusion length $L = \sqrt{D\tau}$, where D is the diffusion coefficient and τ is the carrier life time. But when increased beyond a critical length there is a chance of re-absorption of the photon into the device.

The LED has to be structured so that photons generated from the device are emitted without being reabsorbed. To meet this one p-layer of Gallium arsenide (GaAs) or Gallium arsenide phosphide (GaAsP). This n-type layer is grown on a substrate. LEDs are uqually built on an n-type substrate, with an electrode attached to the p-type layer attached on its surface. Suitable semiconducting material is chosen according to the required wavelength of emitted radiation. Metal anode and cathode are connected at p-side and n-side respectively.



Uses of LED:-

LEDs are mainly used in:

1. signal lamps 2. Display devices 3. Digital watches 4. Medical appliances 5. Remote controls 6. The infrared LEDs are used in optical fibre communications.

Advantages:-

1. LEDs produce more light per watt than incandescent bulbs.
2. LEDs can emit light of an intended colour without use of colour filters that traditional lighting methods require.
3. They require low operational voltage.
4. When dimming is reuired theLEDs do not change colour unlike ordinary incandescent lighting.
5. LEDs do not burn out when frequently switched on and off unlike ordinary lighting.
6. LEDs have very long life.
7. They emit light which is very nearly monchromatic.
8. LEDs can be very small and are easily populated onto printed circuit boards.

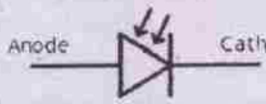
Disadvantages:-

1. At present LEDs are more costly.
2. Adequate heat-sinking is required to maintain long life.
3. LEDs must be supplied with correct current. This can involve series resistors or current-regulated power supplies.
4. Blue LEDs and white LEDs are capable of exceeding safe limits and can cause damages to the eye.

19.21.2: Photodiode

A photodiiode is light sensitive electronic device capable of consuming light energy and generating a voltage or current signal. It works on the principle of photo generation. In photodiodes both light and voltage are used as energy sources.

Symbol:- A photodiode is represented by the symbol as given below.



The arrows indicate the light rays failling on diode. The two terminals are shown as **anode** and **cathode.**

Types of photodiodes:-

There are mainlythree types of photodiodes:

1. P-N junction photodiode

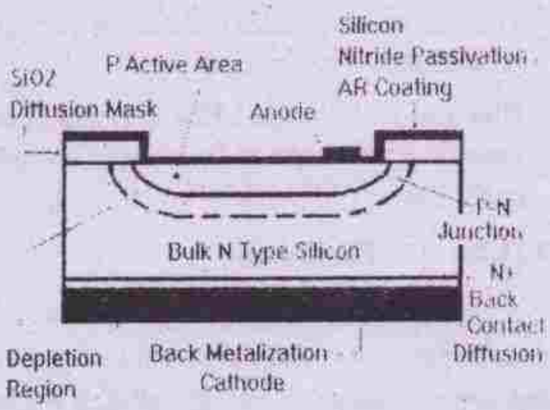
2. PIN photodiode

3. Avalanche photodiode

Normal PN junction photodiode is used in low frequency and low sensitivity is needed avalanche or PIN photodiodes are used.

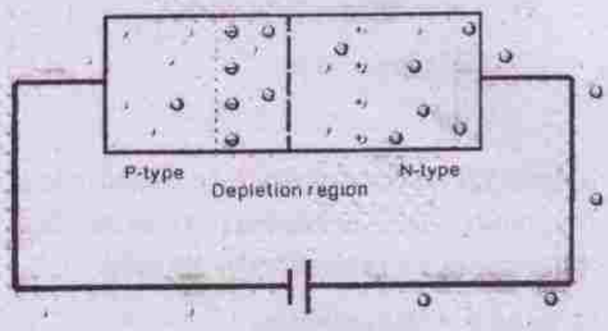
Structure of photodiode:-

A normal PN junction photodiode is made by sandwilching a p-type semiconductor into an n-type semiconductor. All sides of PN junction diode is enclosed in a metallic case or painted black except for one side on which radiation is allowed to fall.



Principle of Operation:-

When a PN junction is illuminated with light, with energy greater than the band gap of semiconductor material, the valence electrons gain enough energy and break bonding with the parent atom and become free electrons. These free electrons move freely from one place to another place by carrying the electric current. When the valence electron leaves the valence shell an empty space is created which call a 'hole'. Thus, both free electrons and holes are generated as pairs (e-h pair). This mechanism of generating e-h pairs by using light energy is known as **inner photoelectric** effect and the mode of operation is called photo conductive mode.



PN-Junction photodiode

The minority carries in depletion region experience force due to depletion region electric field and the external electric field due to reverse biasing. For example, free electrons in the depletion region experience repulsive and attractive force from the electon and holes present at the edge of depletion region at p--side and p-side. As a result, free elections move towards the n-region. When free electrons reach n-region they are attracted towards the positive terminal of the battery. In the same way holes move in opposite direction. The strong depletion region electric field and the external electric field increase the drift velocity of the free electrons. Because of this high drift velocity, the minority carriers (free electrons and holes) generated in the depletion region will cross the p-n junction before they recombine with atoms. As a result the minority current increases.

When no light is applied to the reverse biased photodiode, it carries a small reverse current due to external voltage. This small current under the absence of light is called **dark current,** and dentoed by I_λ.

The current equation for photodiode is given as

$$I = I_s + I_0(1 - e^{V/nV_t})$$

Where n is 1 for Ge and 2 for Si, Vt is the voltage equivalent of temperature, Is is the short circuit current proportional to the light intensity, I0 is reverse saturation current of diode, V is positive for forward bias and negative for reverse bias. Photo curent is diffusion current.

There can be two types of photon excitations (i) Instrinsic (ii) Extrinsic. Instrinsic excitations occur when an electron in valence band is excited by a hgh energy photon to conduction band. Alternatively a photon may excite an electron in donor level to conduction band or a valence band electron may go into acceptor state. Such excitations are called as extrinsic excitations.

The minimum energy of light required for photo generation due to intrinsic excitation is the forbidden gap energy E_g. The wave length associated with this critical energy is known as crritical wavelength given as:

$$\lambda = 1.24 / E_g (in\ eV)$$

Photons with wavelength greater than critical wavelength cannot generate new charge carrier pairs.

responsibility:- It is defined as the ratio of photo generated current to incident light power. It is measured in units of amp/watt.

Responsivity $R = I_s/P_{in}$

Where Is is the photo current and Pin is the incident light power.

Quantum efficiency:- It is defined as fraction of incident photon contributing to photo current. It is unit less as it is a fraction.

$$Q = N_E/N_{Ph}$$

Where N_E is the numbers of generated carriers/unit time and N_{Ph} is the numbers of incident photons/unit time.

Dark current:- It is the current through the diode for zero illumination. It is non zero due to back ground radiation and thermally excited minority saturation current.

The relation between Responsivity (R) and Quantum efficiency (Q) is given as:

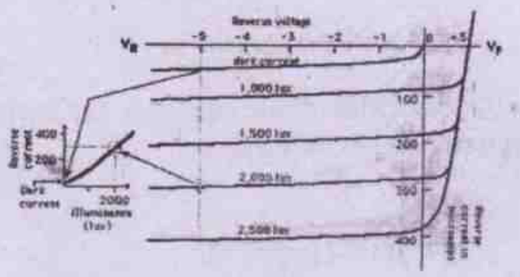
$$R = \frac{I_s}{P_{in}} = \frac{N_{E'}}{N_{Phhv}} = \frac{Q_e}{hv} \Rightarrow Q = Rhv / e$$

Hence the current in a photo diode is given as .

$$I = I0[e\frac{ev}{kT} - 1] - \frac{QeP}{hv}$$

VI Characteristics of photo diode

The VI characteristics of a photo diode is shown in the figure.



A phtodiode is always operated in reverse bias mode. the above figure shows that photo current is almost independent of reverse bias voltage.

Uses (applications):-

1. Uses as photo detectros.

2. It couples two circuits optically although they are electrically isolated.

3. Used consumer electronics.

4. They are used in cameras as phto sensors, slotted optical switch, in scintillators e.t.c.

Advantages:-

1. Generates:- This portion is for the students for better knowledge low noise.

2. High gain, 3. High sensitivity to light, 4. Small size, 5. Low cost 6. Long life time

Limitations:-

1. High response time.

2. Low sensitivity to temperature

19.21.3 Solar Cell

A solar cell is a solid state device, made of p-n junction diode which converts light (radiant) energy in photons into electrical energy using **Photovoltaic effect.**

Construction:-

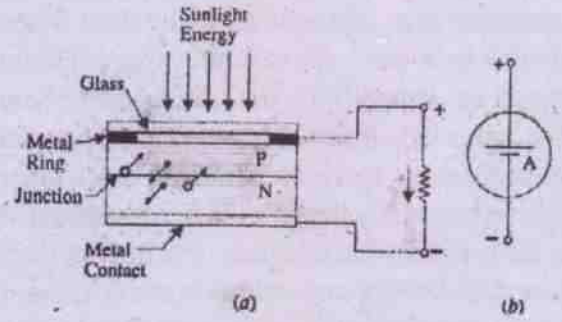


Fig.12.21.1

A solar cell consists of a p-n junction diode packed in a can with glass window on top so that light may fall upon p and n sections. The materials used for the construction of solar cells should have the following characteristics:

1. Must have band gap from 1 eV to 1.8 eV.

2. It must have high optical absorption.

3. It must have high electrical conductivity.

4. The raw material must be abundantly available and its cost must be low.

so materials used in solar cells are Sillicon (Si), Gallium Arsenide (GaAs), Indium Arsendide (InAs), Cadmium Arsenide (CdAs), Cadmium Telurium (CdTe) and Copper Indium selenium ($CulnSe_2$). The most common of them is **Sillicon** which has a band gap of 1.12 eV at 300^0K temperature and fulfils the above conditions. The p- and n- sections are made thin so that electrons or holes generated can diffuse into the junction before any recombination takes place. More over they are heavily doped to obtain large photo voltage. The peripheral part of p-layer is in contact with a nickel plated ring which acts as positive output terminal. Similarly a metal contact at the bottom serves as the negative output terminal.

Working Principle of Solar Cell:-

The light radiation of proper frequency is allowed to fall on the diode. If the incident photon energy (hv) is more than the forbidden band gap Eg the the electro-hole pairs are generated in p- and n- sides of junction. These electro-hole pairs on reaching the depletion region are separated by the strong barrier field. The minority carriers (electrons) in the p-side slide down the barrier potential to reach the n-side and similarly the holes in the n-sides move to the p-side of the junction. The flow of these minority carriers constitute minority current, whose magnitude depends on the illumination and exposed surface area.

Equivalent circuit of a solar cell:-

To understand the behaviour of a solar cell, it is useful to create a model which is electrically

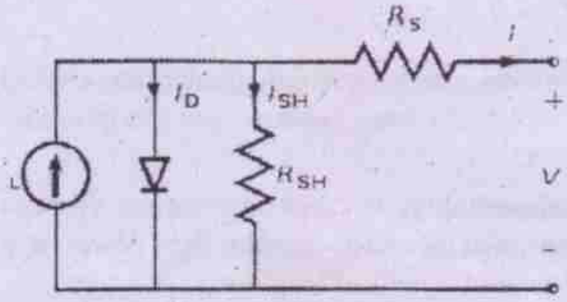


Fig. 19.21.2

equivalent, and is based on discrete ideal electrical components whose behaviour is well defined. An ideal solar cell (in practice no solar cell is idea)may be modelled as shown above. The above circuit gives:

$$I = I_L - I_D - I_{SH}$$

Where,

I = Outpur current (ampere)

I_L = Photogenerated current (ampere)

I_D = diode current (ampere)

I_{SH} = shunt current (ampere)

The current through them is governed by the voltage across them:

$$V_j = V + IR_S$$

Where,

V_j = Voltage across the diode and shunt R_{SH} (Volt)

V= Voltage across the output terminal(volt)

I= output current (ampere)

RS = Series resistance (Ω)

By the **Shockley** diode equation, the current diverted through the diode is:

$$I_D = I_0\{\exp[\frac{V_j}{nV_T}]-1\}$$

Where,

I0 = reverse saturation current (ampere)

n= diode ideality factor (1 for an ideal diode)

q= elementary charge

k = Boltzmann's constant

T = absolute temperatue

VT= kT/q, the thermal voltage. At 250C, VT ≈ 0.0259 volt

By Ohm's law, the current diverted through the shunt resistor is

$I_{SH} = \frac{V_j}{R_{SH}}$, where R_{SH} is the shunt resistance

Substituting these into the first equation gives the output current as

$$I = I_L - I_0\{\exp[\frac{V + IR_S}{nV_T}] - 1\} - \frac{V + IR_S}{R_{SH}}] **$$

** This portion is for the students for better knowledge

Uses:-

1. It may used to charge batteries.
2. Used in light meters.
3. Used in calculators and wrist watches.
4. Used in spacecraft and supply power to remote areas.

Advantage:-

1. No pollution associated with it.
2. It last for a long time.
3. Much less maintenance cost.

Disadvantages:-

1. High cost of installation
2. Low efficiency
3. During cloudy day, it cannot function

19.21.4 Zener diode:-

A zener diode is a specially designed p-n junction diode which operates in the zener breakdown region of reverse voltage characteristics (see sec. 19.17).

Symbol:- It is represented by the symbol as given below. It is similar to a normal diode except that the line representing the cathode is bent at both the ends to look like a Z for Zener.

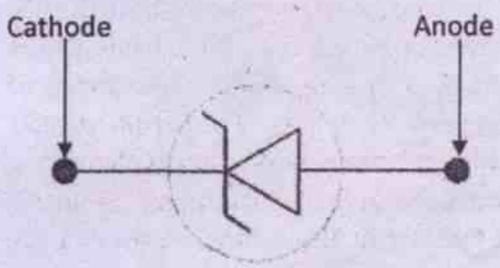


Fig.1

Structure (/construction):-

It is made of heavily doped (by acceptor and donor impurities) silicon (or germanium) p-n junction diode. Silicon is preferred to germanium because of its higher temperature and current capability. This is reverse biased and operated in zener breakdown region. As this is heavily doped, the depletion region is thin (<10-6m) and junction field is very high $(\sim 5\times10^6 V/m)$.

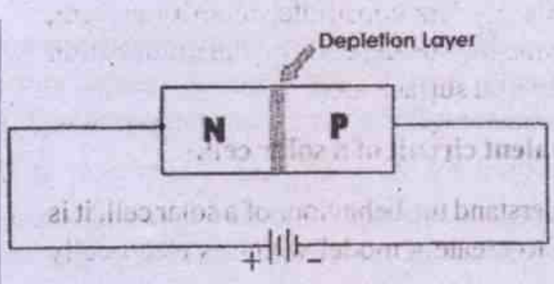


Fig.2

Theory (/Principle):-

In such a heavily doped diode, when reverse bias is increased beyond certain limit (zener breakdown voltage), the resulting electric field at the junction exerts a large force on a bound electron to tear it out of its covalent bond. this results in the production of large number of electron-hole pairs which, in turn results in increasing the reverse current sharply. This process is termed as zener breakdown. For heavily doped diodes zener breakdown occurs at voltages below 6V. But this breakdown voltage can be changed by changing the doping concentration.

V - I Characteristics of zener diode:-

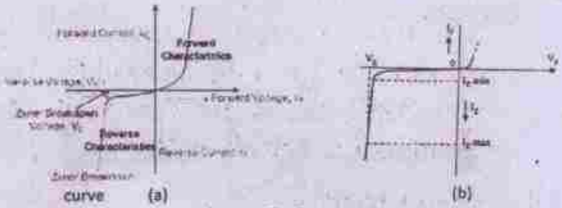


Fig. 3

The forward biased V-I characteristics of a zener diode is similar to that of an ordinary p-n junction diode. Under reverse bias; below the break down voltage, the zener diode also shows same behaviour as an ordinary diode. But as the breakdown voltageis reached the reverse current increases sharply. the important points on the reverse characteristics are:

V_z = Zener breakdown voltage

$I_{z.min}$ = minimum current to sustain breakdown

$I_{z.max}$ = maximum zener current limited by, maximum power dissipation.

Since its reverse characteristics isnot exactly vertical, the diode possesses some resistance called zener dynamic impedance (Z_Z), defined by $Z_Z = \Delta V_Z / \Delta I_Z$. However it is negligible as compared to the external resistance connected in the circuit. so it is neglected assuming that the characteristic is truly vertical. It implies that VZ remains constant even when IZ increases conderably. The complete equivalent circuit of a zener diode is shown in fig.4 (a) and the approximate one in fig. 4(b) as given below.

Zener diodes are available having Zener voltage of 2.4V to 200V. This voltage is temperature dependent. Their power dissipation is given by the product $V_Z I_Z$. Maximum ratings vary form 150mW to 50W.

Uses:-

1. Used as voltage regulator.
2. Used as a fixed reference voltage in a network for biasing and comparison purposes and for calibrating voltmeters.
3. Used as peak clippers.
4. For meter protection against damage from accidental application of excessive voltage.

Zener diode as Voltage regulator:-

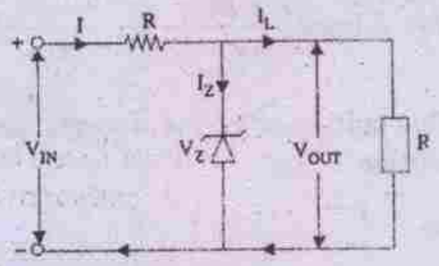


Fig.5

It is a measure of a circuit's ability to maintain a constant output voltage even when either input voltage or load current varies. A zeer diode, when working in the breakdown region, can serve as a voltage regulator. In fig.5 V_{IN} is the input dc voltage whose variations are to be regulated. The zener diode is reverse connected across Vin. when $V_{IN} > V_Z$, it conducts and draws relatively large current through the series resistance R. The load resistance R_L across which a constant voltage

V_{OUT} is required is connected in parallel with the diode. The total current I passing through R equals the sum of diode current and load current i.e., $I = I_Z + I_L$. Under all conditions, $V_{OUT} = V_Z$. Hence, $V_{IN} = I_R + V_{OUT} = I_R + V_Z$

<u>Case 1:</u> Suppose R is kept fixed but supply voltage V_{IN} is increased slightly. It will cause an increase in I. This increase in I will be absorbed by the Zener diode without affecting I_L. The increase in V_{IN} will be dropped across R thereby keeping V_{OUT} constant. Conversely if V_{IN} falls, the diode takes a smaller current and voltage drop across R is reduced, thus again V_{OUT} is constant. Thus when V_{IN} changes, I and IR drop change in such a way that V_{OUT} $(=V_Z)$ remains constant.

<u>Case 2.</u> In this case, V_{IN} is fixed but I_L is changed. When I_L increases, diode current I_Z decreases thereby keeps I and IR drop constant. In this way V_{OUT} remains constant. When I_L decreases, I_Z increases in order to keep I and IR drop constant. Again V_{OUT} $(=V_Z)$ remains unchanged because $V_{OUT} = V_{IN} - IR = V_{IN} - (I_Z + I_L)R$

Giving

$$R = \frac{V_{IN} - V_{OUT}}{I_Z + I_L}$$

It may also be noted that diode current IZ attains its maximum value, IL becomes zero and we have

$$R = \frac{V_{IN} - V_{OUT}}{I_Z(\max)}$$

SUMMARY

1. Crystalline solids have atoms arranged in a regular periodic geometrical pattern.

   They are anisotropic and have sharp melting points.

2. Amorphous solids have atoms arranged in a random manner.

   They are isotropic and do not have sharp melting points.

3. Ionic bond :

   It is formed by transfer of one or more electrons from one atom to another. Ex : Nacl, $NaSo_4$.

4. Covalent bond :

   It is formed by mutual sharing of electrons. Ex : H_2, N_2, Ge, Si etc.

5. Metallic bond :

   It is formed due to stronger force of attraction between electrons and the positive ions than force of repulsion between electrons and between positive ions. Ex : copper, aluminium.

6. Molecular bond :

   It is formed due to a weak force of attraction between the molecules whose outermost shells are saturated with electrons. Ex : crystals of Neon, organ etc.

7. Energy band :

   An energy level in an isolated atom can be replaced by a group of levels of slightly differing energies of an atom in a crystal, known as energy band.

8. Valence band :

   Top most fully occupied band is known as valence band. It is formed due to valence electrons.

9. Conduction band :

   Above the valence band, an unoccupied band is known as conduction band.

10. Forbidden energy gap :

    It is the energy gap between valence and conduction bands.

    Forbidden energy gap

    i) of conductor is 0

    ii) of semiconductor is small ~ 1eV.

    iii) of insulators is large > 2 eV.

11. Semiconductor :

Substances of which electrical conductivity is in between conductors and insulators are called semiconductors. Semiconductors are of two types :

i) Intrinsic semiconductor :

It is a pure semiconductor. Ex. : Ge, Si

ii) Extrinsic semiconductor :

It is an impure semiconductor. It is again of two types.

(a) n-type semiconductor :

Impurities are pentavalents called donors. Majority charge carriers are electrons and minority charge carriers are holes.

(b) p-type semiconductor :

Impurities are trivalents called acceptors. Majority charge carriers are holes and minority charge carriers are electrons.

Mobility of electrons is more than holes Conductivity of an extrinsic semiconductor is

$$\sigma = e(n_e\mu_e + n_h\mu_h)$$

12. Junction diode :

It is a single crystal having p-type material on one side and n-type material on the other. The junction at which p-type and n-type semiconductors meet is called p-n junction.

13. Forward biasing :

If p-type is connected to positive terminal and n-type to negative terminal of a battery, the p-n junction is said to be forward biased. In forward biasing

a) depletion region decreases or eliminated.

b) registance offered by the junction is very low.

c) current flow is large due to majority charge carriers.

14. Reverse biasing :

In this case p-type is connected to negative terminal and n-type to positive terminal of the battery. In reverse biasing

a) depletion region widens

b) resistance offered by the junction is very high

c) current flow is very small due to flow of minority charge carriers.

15. Barrier potential :

Unbiased junction diode will have diffusion of charges due to thermal agitation at the junction creating a potential barrier. It is 0.3 V for Ge diode and 0.7 V for Si diode.

16. Diode can be used as a rectifier which converts a.c. to d.c. Rectifiers are of two kinds :

a) Half wave rectifier :

It allows current to pass through during only one half cycle of input a.c. and other half cycle is suppressed. Maximum efficiency of half wave rectifier is 40.6 %.

b) Full wave rectifier :

It allows current to pass through full cycle of input a.c. Its rectification efficiency is

$$\eta = \frac{81.2}{1+\frac{r_f}{R_L}}\%$$

- Its maximum efficiency is 81.2% which is double that of half wave rectifier.

SOLVED NUMERICAL PROBLEMS

Ex.1: A battery of emf 2V is used across an intrinsic silicon block of length 0.1m, at a temperature 300 K and area of cross section 1.0 x 10^{-4} m^2. Find electron and hole currents if mobility of electrons is 0.135, that of holes is 0.048 and free electron density is 1.5 x 10^{16}.

Soln.

Free electron density $n_e = n_h = 1.5 \times 10^{16}$

Emf $V = 2V$, $\ell = 0.1m$

Area of cross section $A = 1.0 \times 10^{-4} m^2$

Electron mobility $\mu_e = 0.135$, $\mu_h = 0.048$

Electric field intensity $E = \frac{V}{\ell}$

Mobility of the charge carrier, $\mu = \frac{v}{E}$

where v is the drift velocity of the charge carrier $= \mu E = \frac{\mu V}{\ell}$

Electron current, $I_e = n_e A_e V_e$

$$= \frac{n_e A \mu_e Ve}{\ell}$$

$$= \frac{1.5 \times 10^{16} \times 10^{-4} \times 1.6 \times 10^{-19} \times 0.135 \times 2}{0.1}$$

$= 6.48 \times 10^{-7}$ A

Hole current, $I_h = \frac{n_h Ae\mu_h V}{\ell}$

$$= \frac{1.5 \times 10^{16} \times 10^{-4} \times 1.6 \times 10^{-19} \times 0.048 \times 2}{0.1}$$

$= 2.30 \times 10^{-7}$ A

Ex. 2 : A semiconductor is known to have an electron concentration of 8 x 10^{13} per cm^3 and hole concentration of 5 x 10^{12} per cm^3.

a) Is the semiconductor n type or p type

b) What is the resistivity of the sample if the electron mobility is 23000 cm^2 / Vs and hole mobility is 100 cm^2 / Vs ?

Soln.

a) Electron concentration, $n_e = 8 \times 10^{19} / m^3$

hole concentratiion, $n_h = 5 \times 10^{18} / m^3$

$\because n_e >> n_h$, the semiconductor is n-type.

b) Electron mobility, $\mu_e = 2.3 \frac{m^2}{Vs}$

hole mobility, $\mu_h = 0.01 \frac{m^2}{Vs}$

cunductivity, $\sigma = e(n_e \mu_e + n_h \mu_h)$

$= 1.6 \times 10^{-19} (8 \times 10^{19} \times 2.3 + 5 \times 10^{18} \times 0.01)$

$= 29.45$ mho/m

Resistivity $= \frac{1}{\sigma} = 3.4 \times 10^{-2} \Omega m$

Ex.3 : The band gap of an extrinsic semiconductor is 1.98 eV. Calculate the wave length of radiation that is emitted when an electron and hole in this material recombine directly.

Soln.

Band gap, Δ =1.98 eV

$= 1.98 \times 1.6 \times 10^{-19}$ J

$$\Delta = h\nu = \frac{hC}{\lambda}$$

Wavelegth of radiation emitted, λ

$$= \frac{hC}{\Delta}$$

$$= \frac{6.6 \times 10^{-34} \times 3 \times 10^{8}}{1.98 \times 1.6 \times 10^{-19}}$$

$= 6250 \times 10^{-10}$ m

$= 6250$ A^0

Ex.4 : Assume that the silicon diode in the following circuit (fig. 19.15) requires a minimum current of 1mA to be above the knee point (0.7 V) of its V-I characteristics. Also assume that voltage across the diode is independent of the current above the knee point.

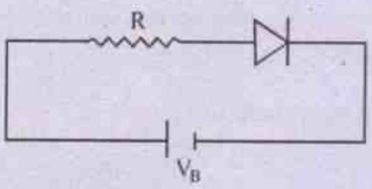


Fig. 19.15

i) If $V_B = 5V$, what should be maximum value of R so that the voltage is above the knee point ?

ii) If $V_B = 5V$, what should be the value of R to establish a current of 5 mA in the circuit ?

iii) What is the power dissipated in the resistance R and in the diode if $V_B = 6V$ and current in the circuit is 5 mA.

Soln.

i) Effective bias voltage = 5 - 0.7 = 4.3 V

$$R_{max} = \frac{\text{Effective bias}}{I_{min.}}$$

$$= \frac{4.3V}{1\times10^{-3}A} = 4.3\ K\Omega$$

ii) $R = \frac{4.3}{5\times10^{-3}} = 860\ \Omega$

iii) Voltage across resistance R, V_R
$= 6 - 0.7 = 5.3$ V

Power dissipated in the resistance R =

$= V_R . I$

$= 5.3 \times 5 \times 10^{-3} = 26.5$ mW

Power dissipated in the diode

$= 0.7 \times 5 \times 10^{-3} = 3.5$ mW.

Ex.5 : A 2 volt battery is connected across A and B as shown in the fig. 19.16. Assume that the resistance of each diode is zero in forward bias and infinity in reverse bias. Find the current supplied by the battery if the positive terminal of the battery connected to

i) the point A and negative terminal to B

ii) the point B and negative terminal to A.

Soln.

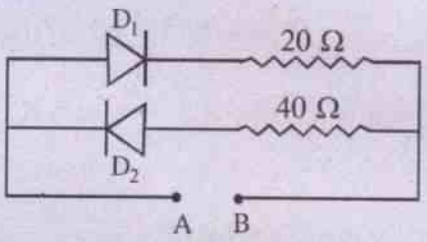


Fig. 19.16

i) When positive terminal of the battery is connected to A and negative terminal to be B, diode D_1 is forward biased and diode D_2 is reverse biased. Current through D_1 is

$$I = \frac{2}{20} = 0.1A$$

ii) D_2 is forward biased and D_1 is reverse biased when positive terminal of the battery is connected to B and negative terminal to A

$$\therefore \quad I = \frac{2}{40} = 0.05A$$

Ex. 6 : In a p-n junction, the depletion region is 0.4 μm wide and an electric field of 1.75×10^6 V/ m exists in it. (a) Find the height of potential barrier. (b) What should be minimum kinetic energy of a conduction electron which can diffuse from n-side to p-side. (c) If an electron of energy 0.3 eV enters p-side with what energy the electron diffuses from p-side to n-side.

Soln.

Electric field, $E = 1.75 \times 10^6\ Vm^{-1}$

Depletion region, $d = 0.4 \times 10^{-6}$ m

a) Potential barrier, $V = Ed$

$= 0.7$ volt

b) Electron loses K.E when it diffuses from n-side to p-side.

The minimum energy, the electron should have for diffusion = V x q

$= 0.7 \times e = 0.7$ ev.

c) Electron gains K.E. when it diffuses from p-side to n-side.

$\therefore$ Total K.E. = Initial K.E. + gain in K.E.

$= 0.3$ ev $+ 0.7$ ev

$= 1.0$ ev.

Ex. 7 : In a full wave crystal diode rectifier, the load resistance is 1k Ω. If forward dynamic resistance of the diode is 10 Ω, the voltage across half of the secondary winding is of amplitude 220v and frequency equal to 50 Hz, calculate (i) peak, average and rms values of current, (ii) rectifier efficiency and (iii) frequency of the riples in the output.

Soln

peak voltage, $E_o = 220$ v

Load resistances, $R_L = 1.0 \times 10^3\ \Omega$

forward resistance $r_f = 10\ \Omega$

Input frequency f = 50 Hz.

i) Peak currence, $I_o = \dfrac{E_o}{r_f + R_L}$

$$= \frac{220}{1010} = 217.82 \text{ mA}$$

$$I_{av} = \frac{2I_o}{\pi} = 138.74 \text{ mA}$$

$$I_{rms} = \frac{I_o}{\sqrt{2}} = 154.05 \text{ mA}$$

ii) Rectification efficiency $\mu = \dfrac{81.2}{1 + \frac{r_f}{R_L}}\ \%$

$$= \frac{81.2}{1 + \frac{10}{1000}} = 80.4\%$$

iii) Ripple frequency in the output

$= 2 \times f$

$= 2 \times 50 = 100$ Hz.

MODEL QUESTIONS

A. Multiple Choice Questions :

1. The type of bonding in a Germanium crystal is

a) ionic b) metallic

c) covalent d) vander waal's

2. In an intrinsic semiconductor, the forbidden energy gap between valence band and a conduction band is of the order of

a) 1 ev b) 4 ev

c) 1 kev d) 1 Mev

3. The forbidden energy gap in conductors, semiconductors and insulations are Δ_1, Δ_2 and Δ_3 respectively. The relations among them is

a) $\Delta_1 = \Delta_2 = \Delta_3$ b) $\Delta_1 > \Delta_2 > \Delta_3$

c) $\Delta_1 < \Delta_2 > \Delta_3$ d) $\Delta_1 < \Delta_2 < \Delta_3$

4. The level formed due to impurity atom, in the forbidden energy gap, very near to conduction band in n-type semiconductor is called

a) acceptor level

b) donor level

c) conduction level

d) forbidden level

5. Acceptor level in a p-type semiconductor is

a) nearer to valence band

b) nearer to conduction band

c) at the middle of the gap between conduction and valence bands.

d) None of the above

6. In a semiconductor

a) there are no free electrons at OK

b) the number of free electrons increases with increase in temperature.

c) the number of free electrons is less than that in a conductor.

d) all the above

7. The impurity atoms with which pure germanium may be doped with it to form a p-type semiconductor are those of

a) boron b) aluminium

c) gallium d) all the above

8. A semiconductor is doped with an acceptor impurity.

a) The hole concentration increases

b) The electron concentration decreases

c) both of the above are correct

d) None of the above are correct

9. Electric conduction in a semiconductor takes place due to

a) Electrons only

b) holes only

c) both electrons and holes

d) neither electrons nor holes

10. Let n_h and n_e be the number of holes and conduction electrons in an istrinsic semiconductor. Then

a) $n_h = n_e$ b) $n_h < n_e$

c) $n_h > n_e$ d) $n_h \neq n_e$

11. An n-type semiconductor is
    a) negatively charged
    b) positively charged
    c) uncharged
    d) its charge increases with increase in temperature.
12. When pure silicon is doped with trivalent impurity like boron, the conduction is due to
    a) Electrons b) holes
    c) protons d) positrons
13. If silicon atom is doped with a donor impurity, the donor atoms should be
    a) trivalent b) pentavalent
    c) tetravalent d) none of the above
14. Majority charge carriers in extrinsic semiconductors are
    a) holes in n-type and electrons in p-type
    b) holes both in n-type and p-type
    c) electron in n-type and holes in p-type
    d) electrons in both in n-type and p-type
15. The depletion layer in a p-n junction is caused by
    a) drift of holes
    b) diffusion of charge carriers
    c) migration of impurity atoms
    d) drift of electrons
16. In the depletion region of an unbiased p-n junction diode, there are
    a) only electrons
    b) only holes
    c) both electrons and holes
    d) only fixed ions.
17. In a semiconductor diode, p-side is earthed and n-side is applied a potential of -3v, then the diode is
    a) forward biased
    b) reverse biased
    c) unbiased
    d) depletion region widens
18. A diode commonly used as
    a) an amplifier b) an oscillator
    c) a modulator d) a rectifier
19. In a full wave rectifier, the minimum number of diodes required is
    a) 1 b) 2
    c) 3 d) 4
20. In a full wave rectifier, input frequency is 50 Hz. The frequency of the ripples in the output is
    a) 50 Hz b) 100 Hz
    c) 25 Hz d) 2500 Hz

B. Very short answer type questions

1. Give the name of a crystal which has covalent binding. [CHSE 98A]
2. Why do metals have lustre [CHSE 87A]
3. Name two solides : one for ionic binding and one for covalent binding. [CHSE 96A]
4. Give one example of vander waal's binding. [CHSE 90A]

5. State the type of binding in

(i) Nacl (ii) Germanium crystal

[CHSE 91A]

6. For which substances, the conduction band overlaps the valence band ?

[CHSE 96A]

7. State two properties of solids which have vander waal's type of bonding.

[CHSE 86S]

8. When does a pure semiconductor behave as an insulator. [CHSE 97A]

9. What is doping ? [CHSE 97S]

10. When trivalent impurities are added to germanium, what type of semiconductor is obtained ? [CHSE 90S]

11. When pentavalent impurities are added to germanium, what type of semiconductor is obtained ? [CHSE 91A]

12. Addition of which impurity to germinium crystal converts it to an n-type semiconductor. [CHSE 91S]

13. Name the charge carrier in a p-type semiconductor [CHSE 91S]

14. Which are majority charge carriers in an n-type semiconductor ? [CHSE 2000]

15. Name a material as a dopant to convert pure silicon to a p-type semiconductor [CHSE 00 Instant]

16. How does the conductivity of a pure germanium change with increase in temperature ? [CHSE 00 Instant]

17. Under what conditions silicon can be an insulator ? Is it realisable ? [CHSE 01]

18. What are minority charge carriers in an n-type germanium ?

19. Name the minority charge carriers in a p-type semiconductor ?

20. If a p-n junctions is forward biased, what changes occur in the depletion region [CHSE 94A]

21. What is the barrier potential of a germanium p-n junction diode.

[CHSE 97A]

22. What changes occur in depletion region of a p-n junction diode, when is it reverse biased. [CHSE 94S]

23. Can we measure barrier potential of a junction diode with the help of a voltmeter.

24. State whether the output of a half wave rectifier has an a.c component more than d.c component or less than d.c component.

25. What is the order of energy gap in a semiconductor ? [CBSE 1998C]

26. At what temperature would an intrinsic semiconductor behave like a perfect insulator. [CBSE Delhi 2009]

27. How does the energy gap in a semiconductor vary, when doped with a pentavalent impurity ? [CBSE 2000]

28. What type of extrinsic semiconductor is formed when (i) germanium is doped with indium ? (ii) sillicon is doped with dismuth ? [CBSE 2003]

29. How does the width of depletion region of a p-n junction vary if the reverse bias applied to it is decreased ?

[CBSE AI 2008, 2002]

30. In a semiconductor the concentration of electrons is $8 \times 10^{13} cm^{-3}$ and that of holes is $5 \times 10^{12} cm^{-3}$. Is it a p-type or n-type semiconductor ? [CBSE Sample Paper]

31. The energy gaps in the energy band diagrams of a conductor, semicondctor and insulator are E_1, E_2 and E_3 respectively. Arrange them in increasing order. [CBSE 2007]

C. Short answer type questions

1. What is covalent bond ?

[CHSE 85A,86A]

2. State the type of bonding in

(i) Nacl and (ii) dimond [CHSE 89A]

3. Give two important properties of covalent compounds. [CHSE 96A]

4. Distinguish between ionic bond and covalent bond. [CHSE 94A, 99A]

5. Distinguish between semiconductors and conductors. [CHSE 98A, 96A]

6. Distinguish between intrinsic and extrinsic semiconductors. [CHSE 95S, 00]

7. Distinguish between N-type and P-type semiconductors. [CHSE 97S, 96S]

8. Explain energy band in a p-type Germanium. [CHSE 96S]

9. What is approximately the energy difference between valence band and conduction band of semiconductors and insulators ? [CHSE 95S]

10. Distinguish between crystalline and amorphous solids.

11. What are energy bands ?

12. What is a valence band ?

13. What is a conduction band ?

14. What is a forbidden band ?

15. What is the order of energy of forbidden band in (a) dimond (b) silicon (c) Germanium (d) aluminium.

16. Distinguish between a metal, an insulator and a semiconductor.

17. What is the total current through an intrinsic semiconductor due to ?

18. Name two donor impurities and two accepetor impurities.

19. Why are metallic bodies always opaque ?

20. What is meant by depletion region in a p-n junction diode ? How it is affected by biasing. [CHSE 00 Instant]

21. What do you an mean by a p-n junction.

[CHSE 85A]

22. Draw the circuit of a half wave rectifier using junction diode. [CHSE 92A]

23. Draw a circuit diagram of a full wave rectifier using junction diode.

24. What are chief merits of junction diode over vacuum diode.

25. Which has a greater resistance : -a forward biased p-n junction or a reverse biased p-n junction.

26. What is a rectifier ?

27. Compare the forward and reverse current static volt-amp characteristics of a semiconductor diode.

28. The barrier potential across a p-n junction cannot be measured by placing a voltmeter across the diode terminals. Explain.

29. Distinguish between drift current and diffusion current in a semicunductor device.

30. In a full wave junction rectifier, the forward resistance is equal to load resistance. What is its maximum rectification efficiency ?

31. Draw a p-n junction with reverse bias.

[CBSE 1995]

32. Draw energy band diagram of a p-type semiconductor. [CBSE 2004, AI 1998]

33. Draw a labeled diagram of full wave rectifier using p-n junction diodes.

[CBSE 1995]

D. Numerical Problems

1. A doped semiconductor has impurity levels 0.01 ev above the valence band. (a) Is the material n-type or p-type. (b) Find maximum wave length of light which can create a hole in the valence band.

2. Calculate number of states per grammole of sodium in (a) 2S band and (b) 3S band (c) In these bands how many states are empty.

3. The band gap between the valence and the conduction bands in Germanium is 0.7 ev. If an electron in the conduction band combines with a hole in the valence band and the excess energy is released in the form of electromagnetic radiation. Find the maximum wavelength that can be emitted in the process.

4. In a p-n junction, the depletion region is 2 μ m vide and an electric field of 1.0 x 10^5 v/m exists in it. (a) Find the height of the potential barrier (b) What should be the minimum kinetic energy of a conduction electron which can diffuse from n-side to p-side ?

5. The potential barrier of 0.25 v exists across an unbiased p-n junction. What minimum K.E. a hole should have to diffuse from p-side to n-side if (a) the junction is unbiased, (b) the junction is forward biased at 0.15 v and (c) the junction is reverse - biased at 0.15v.

6. Calculate the intrinsic resistivity of germanium if intrinsic carrier density is 2.4 x 10^{19} per m^3 and hole and electron mobilities are 0.2 and 0.4 m^2 $volt^{-1}$ sec^{-1} respectively.

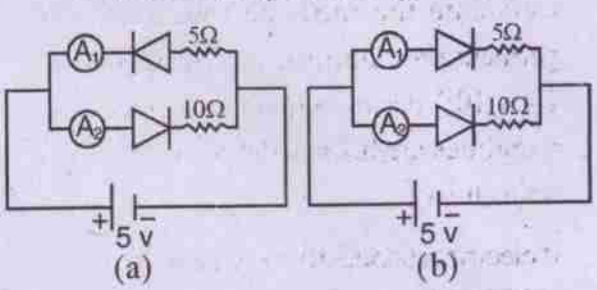
7. If electrical conductivity of germanium at 27^{0}C is 2.5 mho/m, calculate the current density if the electric field intensity is 2000 volt/m.

8. The electrical conductivity of an intrinsic semiconductor increases when electromagnetic radiation of wavelength shorter than 12400 A^0 falls on it. Find the forbidden band gap of the semiconductor.

9. The input voltage applied to a half wave rectifier is 220 sin $100\pi t$ volt. The load resistance is 900 ohm and forward resistance is 100 ohm. Calculate (a)maximum value of load current (b) d.c load current (c) r.m.s value of current (d) rectification efficiency (e) frequency of the rectified d.c.

10. The input voltage to a full wave rectifier is 220 sin $100\pi t$ volt. If the load resistance is 1100 Ω and forward resistance is neglible, Find (a) peak value of load current, (b) d.c load current (c) r.m.s value of current (d) rectification efficiency (e) frequency of rectified output voltage.

11. Find the ammeter readings of A_1 and A_2 in the following circuits (a) and (b)



12. Pure silicon at 300°K has equal electron and hole concentration of $1.5\times10^{16}m^{-3}$. Doping by indium increases the hole concentration to $4.5\times10^{22}m^{-3}$. (i) Calculate the new electron concentration in the doped silicon. [CBSE 1997 C]

13. A semiconductor has equal electron and hole concentrations of $6\times10^{8}m^{-3}$. On doping with certain impurity, electron concentration increases to $9\times10^{12}m^{-3}$ (i) Identify the new semiconductor obtained after doping. (ii) Calculate the hole concentration. [CBSE Sample Paper]

E. Long answer type questions

1. Discuss different types of binding. State their properties. Give an example of each type of binding.
2. Discuss the classification of solids as conductors, insulators and semiconductors on the basis of band theory. [CHSE 95A]
3. What do you mean by a semiconductor ? Discuss how can you improve its conductivity on the basis of band theory.
4. How bands are formed in a crystal. Assuming sodium crystal has a mass of 0.01 gm mole, each band of it consists of how many maximum number of energy levels.
5. Discuss working of a p-n junction. Draw circuit diagram to draw volt-amp characteristic of a junction diode and discuss its different regions. Which regions are useful for voltage regulation and rectification ?
6. What is a rectifier ? Explain with a neat circuit diagram how a p-n junction diode acts as a rectifier.

F. True-False Type Questions :

1. Covalent binding is observed in germanium.
2. Ionic bond is due to transfer of electron from one atom to another.

3. If Δ_1, Δ_2 and Δ_3 be the forbidden energy gap in conductors, semiconductors and insulators respectively, then $\Delta_1 > \Delta_2 > \Delta_3$

4. In p-type semiconductor electrons are the major carriers of current.

5. If n_h and n_e be the number of holes and conduction electrons respectively in an intrinsic semiconductor, then $n_h = n_e$

6. A semiconductor diode conducts when it is reverse biased.

G. Fill-in-Blank Type Questions :

1. In n-type semiconductor _____ are the major carriers of current.

2. The minimum number of diodes required for a full-wave rectifier is ______.

3. In a full wave rectifier, the input frequency is 50Hz. The frequency of the ripples in the output is _______.

4. H_2 is formed due to _____ bonding

5. The forbidden energy gap for a conductor is _____.

6. Above the valence band, an unoccupied band is known as _____ band.

H. Correct the following sentences :

1. The type of bonding in germanium is ionic.

2. The type of bonding in sodium chloride is covalent.

3. Electrons are majorty carriers in p-type semiconductor.

4. Holes are majority carriers in n-type semiconductor.

5. In an intrinsic semiconductor electrons are the only charge carriers.

6. Addition of travalent impurity to germanium crystal converts it to an n-type semiconductor.

7. The conductivity of a semiconductor increases with rise of temperature.

ANSWER

A. Multiple Choice Questions :

1. (c) **2.** (a) **3.** (d) **4.** (b) **5.** (a) **6.** (d) **7.** (d) **8.** (c)
9. (c) **10.** (a) **11.** (c) **12.** (b) **13.** (b) **14.** (c) **15.** (b) **16.** (d)
17. (a) **18.** (d) **19.** (b) **20.** (b).

D. Numerical Problems :

1. (a) p-type (b) 124 x 10^{-6}m [Hint $\lambda_{max} = \dfrac{hc}{\Delta}$ when $\Delta = 0.01$ ev]

2. (a) 12.04 x 10^{23} (b) 12.04 x 10^{23} (c) 0 and 6.02 x 10^{23}.

3. 17714 A^0.

4. (a) 0.2 v (b) 0.2 ev

5. (a) 0.25 ev (b) 0.10 ev (c) 0.40 ev

6. 0.43 ohm.m

7. 5.0 x 10^3 A m^{-2}

8. 1 ev

9. (a) 0.22A (b) 0.07 A (c) 0.11 A (d) 36.54% (e) 50 Hz.

10. (a) 0.2A (b) 0.127A (c) 0.141 A (d) 81.2 % (e) 100 Hz.

11. (a) $A_1 = 0$, $A_2 = 0.5A$ (b) $A_1 = 1.0$ A, $A_2 = 0.5A$

12. free electrons,

13. (i)n-type, (ii) $4 \times 10^4 m^{-3}$

F. **1.** True **2.** True **3.** False **4.** False **5.** True **6.** False

G. **1.** Electrons **2.** Two **3.** 100 Hz **4.** Covalent **5.** Zero **6.** Conduction.

20

Transistors

Basic Transistor was invented by John Bardeen, William Schockley and W.H. Brattain of Bell Telephone Laboratories, U.S.A. in 1948. It is the solid state version of vacuum triode. The name "Transistor" originated from the fact that it transfers a signal from a low resistance region to a high resistance region. Broadly transistors are of two categories (i) unipolar or field effect transistors and (ii) bipolar or junction transistors.

In unipolar transistor, the conductivity is due to only one type of majority charge carriers i.e. either due to electrons or holes. The electric field causes the charge carriers to move in the transistor and hence the name field effect transistor (FET). Further discussion of FET is beyond our scope.

In a bipolar transistor, the conductivity is due to both types of majority and minority charge carriers and hence the name. A bipolar transistor has two p-n junctions. Hence it is also known as junction transistor.

20.1: Junction transistor :

A junction transistor or simply a transistor is a logical extension of junction diode consisting of two p-n junctions. It is fabricated from a single crystal semiconductor (e.g. Ge or Si) in which a p-type thin layer is sandwiched between two n-type layers or an n-type thin layer is sandwiched between two p-type layers. Accordingly junction transistors are of two types. (i) n-p-n transistor (Fig. 20.1 a) and (ii) p-n-p transistor (Fig. 20.1 b). Thus a transistor has three regions with two p-n junctions. The three regions of transistor are called (i) emitter (e) (ii) base (b) and (iii) collector (c).

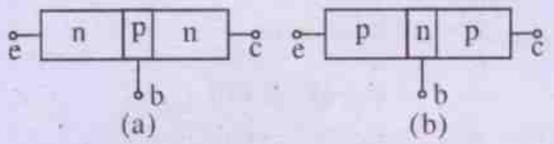


Fig. 20.1

i) Emitter :

It is one of the two outer layers of the transistor, usually represented on the left side. It is heavily doped. It emits majority charge carriers into base.

ii) Base :

It is the middle layer of the transistor. It is very thin ($\approx 1\mu m$) as compared to other two outer layers. Base is lightly doped and it forwards most of the emitter injected charge carriers to the collector.

iii) Collector :

It is the outer layer on the right side and it is moderately doped i.e. its doping level is intermediate between heavy doping of emitter and light doping of base. It collects the charge carriers from the base.

A transistor, thus, is a three terminal device having two p-n junctions viz., (i) emitter-base junction and (ii) collector-base junction.

Fig. (20.2) shows the symbols used for transistors. The arrow on the emitter line shows the direction of the emitter current.

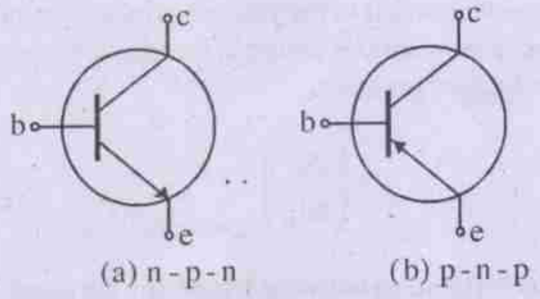


Fig. 20.2

20.2 : Biasing of a transistor :

Maintaining suitable potential differences across the two junctions of a transistor is called biasing. In normal operation of a transistor, the emitter-base junction is always forward-biased whereas the collector-base junction is reverse biased.

20.3 : Working of an n-p-n transistor :

Fig. (20.3) shows an n-p-n transistor with forward bias to emitter base junction and reverse bias to collector base junction. The forward bias causes the electrons in the n-type emitter to flow towards the base constituting emitter current I_e.

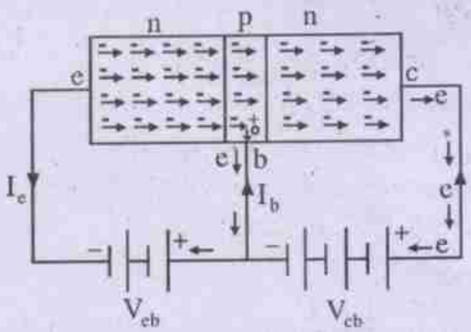


Fig. 20.3

As these electrons flow through the p-type base, they tend to combine with holes. As the base is lightly doped and very thin, only a few electrons (about 2%) combine with holes of the base and the atoms on receiving the electrons get negatively charged. The positive terminal of the battery V_{eb} attracts these electrons, forming base current I_b and new holes in the base. The remaining (about 98%) electrons coming from the emitter cross over the base into the collector and they move under the reverse bias through the collector constituting collector current I_c. It is clear that emitter current is the sum of collector and base currents i.e.,

$$I_e = I_b + I_c$$

20.4: Working of a p-n-p transistor :

Fig. (20.4) shows a p-n-p transistor with forward bias to emitter base junction and reverse bias to collector base junction. The forward bias causes holes to move from the p-type emittor to the n-type base constituting emitter current I_e. Due to thinness and light doping, most of the holes (about 98%) cross the base and enter the collector while the rest of the holes (about 2%) combine with electrons of the base. As soon as a hole combines with an electron of the base,

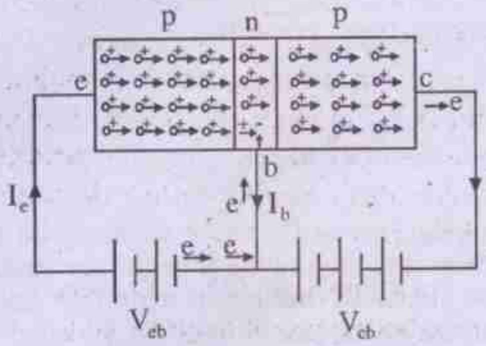


Fig. 20.4

the atom gets positively charged and to neutralize it a fresh electron leaves the negative terminal of the battery V_{eb} and enters the base. This causes a very small base current I_b. The holes that entered the collector move, under the reverse bias, towards collector terminal c causing collector current I_c. Both these currents I_b and I_c combine to form the emitter current i.e.,

$$I_e = I_b + I_c$$

20.5: Modes of operation of a transistor :

When a transfor is connected in a circuit, anyone of its terminals must be common to both input and output circuits. Accordingly, there are three modes of operation of a transistor.

i) common base operation (base terminal is common to both input and output circuits)

ii) common emitter operation

iii) common collector operation

Out of these three modes of operations, the first two are used as amplifiers.

20.6: A.C. current gains or Transistor parameters :

i) Current gain α :

It is defined as the ratio of change in collector current (ΔI_c) to change in emitter current (ΔI_e) at constant collector base voltage in common base operation

$$\alpha = \left(\frac{\Delta I_c}{\Delta I_e}\right)_{V_{cb}=\text{constant}} \quad ...20.6.1$$

obviously $\alpha < 1$. Its value depends on the thickness and the doping level of the base. It can be increased (but not to more than unity) by decreasing the base current. This is achieved by making the base thinner and doping it very lightly. Practical values of α are between 0.95 to 0.99.

ii) Current gain β:

It is defined as the ratio of change in collector current (ΔI_c) to change in base current (ΔI_b) at constant collector emitter voltage in common emitter operation.

$$\beta = \left(\frac{\Delta I_c}{\Delta I_b}\right)_{V_{ce}=\text{constant}} \quad ...20.6.2$$

Typical values of β lie in the range 15 to 100.

iii) Current gain γ :

It is defined as the ratio of change in emitter current to the change in base current at a constant emitter collector voltage in common collector operation.

$$\gamma = \left(\frac{\Delta I_e}{\Delta I_b}\right)_{V_{ec}=\text{constant}} \quad ...20.6.3$$

20.7: Relation between transistor parameters

i) Relation between α and β :

We know in a transistor

$$I_e = I_b + I_c$$

or, $$\Delta I_e = \Delta I_b + \Delta I_c$$

or, $$\frac{\Delta I_e}{\Delta I_c} = \frac{\Delta I_b}{\Delta I_c} + 1$$

$$\frac{1}{\alpha} = \frac{1}{\beta} + 1$$

$$\frac{1}{\beta} = \frac{1}{\alpha} - 1$$

$$= \frac{1-\alpha}{\alpha}$$

or, $$\beta = \frac{\alpha}{1-\alpha} \quad ...20.7.1$$

Again $$\frac{1}{\alpha} = \frac{1}{\beta} + 1$$

$$= \frac{1+\beta}{\beta}$$

Thus, $$\alpha = \frac{\beta}{1+\beta} \quad ...20.7.2$$

ii) Relation between α and γ :

$$\Delta I_e = \Delta I_b + \Delta I_c$$

or, $$1 = \frac{\Delta I_b}{\Delta I_e} + \frac{\Delta I_c}{\Delta I_e}$$

$$= \frac{1}{\gamma} + \alpha$$

or, $$\frac{1}{\gamma} = 1 - \alpha$$

or, $$\gamma = \frac{1}{1-\alpha} \qquad ...20.7.3$$

or, $$\alpha = 1 - \frac{1}{\gamma}$$

Hence, $$\alpha = \frac{\gamma - 1}{\gamma} \qquad ...20.7.4$$

iii) Relation between β and γ :

$$\Delta I_e = \Delta I_b + \Delta I_c$$

or, $$\frac{\Delta I_e}{\Delta I_b} = 1 + \frac{\Delta I_c}{\Delta I_b}$$

Thus, $$\gamma = 1 + \beta \qquad ...20.7.5$$

Ex. 20.7.1 : In certain npn transistor when there is a change in base current of 105 μA, collector current changes, by 2.05 mA. Find the values of α, β and change in emitter current.

Soln.

$$\Delta I_b = 105 \times 10^{-3}\,\text{mA}$$

$$\Delta I_c = 2.05\,\text{mA}$$

$$\therefore \quad \beta = \frac{\Delta I_c}{\Delta I_b} = \frac{2.05}{105 \times 10^{-3}} = 19.5$$

$$\alpha = \frac{\beta}{1+\beta} = \frac{19.5}{20.5} = 0.95$$

$$\Delta I_e = \Delta I_b + \Delta I_c$$

$$= 105\,\mu\text{A} + 2050\,\mu\text{A}$$

$$= 2155\,\mu\text{A} = 2.155\,\text{mA}$$

20.8: Transistor characteristics :

Graphical forms of the relations among various current and voltage variations (when signal in the input circuit and load in the output circuit are not present) of a transistor are known as the transistor static characteristics. By considering any two as independent variables while keeping other variables at constant values, it is possible to draw different families of characteristic curves. However, only two sets of characteristic curves known as the input and output characteristics for the common base and common emitter modes of operations, are of practical importance. The performance of a transistor is determined from its characteristic curves.

Input characteristics :

The plot of the input current against input voltage with the output voltage as a parameter for a particular mode of operation gives the input characteristics for that mode.

Output characteristics :

The plot of output current versus output voltage with the input current as a parameter for a given mode of operation gives the output characteristics for that mode.

20.9: Common base (CB) characteristics of a transistor :

The circuit arrangement for tracing static characteristic curves of a p-n-p transistor in common base configuration is shown in the fig. 20.5. The emitter to base voltage (V_{eb}) can be varied with the help of the rheostat Rh_1 whereas collector to base voltage (V_{cb}) can be changed by adjusting the rheostat Rh_2.

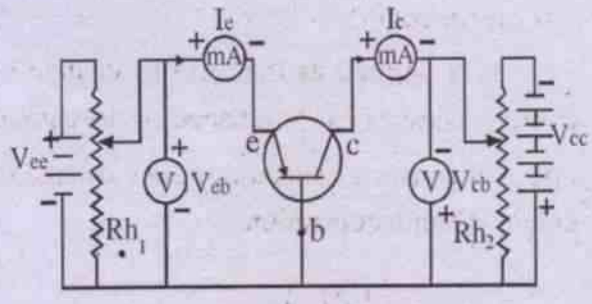


Fig. 20.5

a) Input characteristics :

The left side of the transistor of the circuit of Fig. 20.5 is the input side while right side is the output side. Obviously the input current is emitter current (I_e) and input voltage is emitter base voltage (V_{eb}) while output voltage is collector base voltage (V_{cb}). The plot of I_e against V_{eb} with V_{cb} as a parameter is the input characteristics for CB mode and the corresponding functional equation is

$$I_e = f(V_{eb}, V_{cb})$$

A set of typical static input characteristics is shown in the fig. 20.6 (a). The following points worth noting.

i) The curve for a given value of V_{cb} is just like the diode characteristic in forward bias.

ii) The emitter current I_e increases rapidly with small increaments in the V_{eb} indicating that input resistance is very low which is due to forward bias of input junction.

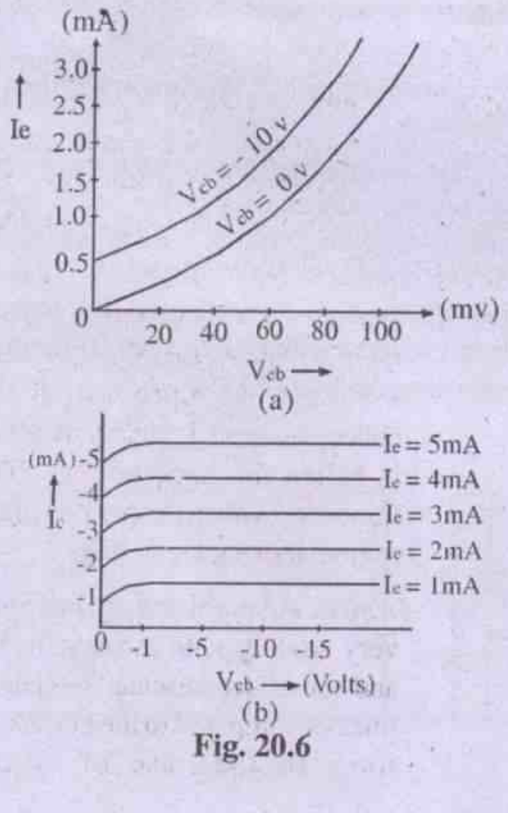


Fig. 20.6

iii) The emitter current I_e is almost independent of collector-base voltage V_{cb}. This leads to the conclusion that the emitter current and hence collector current is almost independent of collector voltage.

b) Output characteristics :

Here output quantities are collector current I_C, collector base voltage V_{cb} while input current is emitter current I_e.

The plots of I_c as a function of V_{cb} with I_e as a parameter are known as output characteristics of common base transistor. The fig. (20.6 b) shows its typical output characteristics. The following points may be noted from the output characteristics.

i) The collector current I_c varies with V_{cb} only at its very low values $(|V_{cb}| < 1V)$. The transistor is never operated in this region.

ii) When $(|V_{cb}| > 1V)$, the collector current becomes constant and independent of collector base voltage V_{cb} but depends only on I_e. The curves are extremely flat i.e. parallel to V_{cb} axis. This region is known as active region and the transistor is always operated in this region.

iii) The output resistance

$$r_o = \left(\frac{\Delta V_{cb}}{\Delta I_c}\right)_{I_e = \text{constant}}$$ is very high

since ΔI_c is very small as the collector junction is reverse biased.

iv) The transistor parameter

$$\alpha = \left(\frac{\Delta I_c}{\Delta I_e}\right)_{V_{cb}}$$ can be determined

from the output characteristics.

20.10: Common emitter (CE) characteristics of a transistor :

The circuit arrangement for studying CE mode characteristics of a pnp transistor is shown in the fig. 20.7. The base emitter voltage V_{be} can be varied with the help of the rheostat Rh_1 while collector emitter voltage V_{ce} by adjusting the rheostat Rh_2.

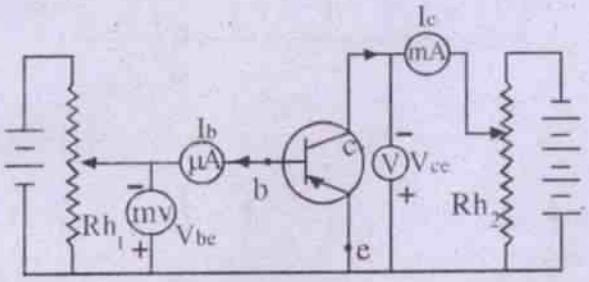


Fig. 20.7

a) Input characteristics :

Here input current is I_b, input voltage is V_{be} while output voltage is V_{ce}. The curves representing the variation of base current I_b with base emitter voltage V_{be} for a constant value of collector emitter voltage V_{ce} are called input characteristics or base characteristics of CE-mode transistor. The functional equation for input characteristics is

$$I_b = f(V_{be}, V_{ce})$$

A set of typical static input characteristics is shown in the fig. 20.8 (a). The following points may be noted from the characteristics.

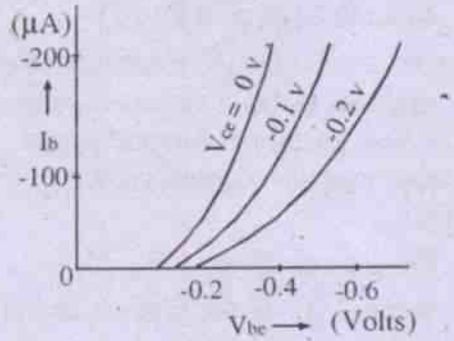


Fig. 20.8 (a)

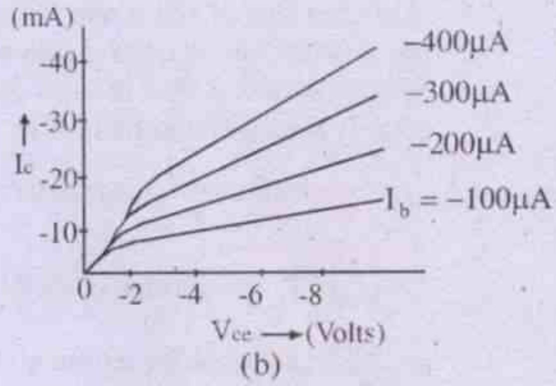


Fig. 20.8 (b)

i) The characteristics resemble that of a forward biased junction diode curve. This is expected since the base emitter section of the transistor is a diode and it is forward biased.

ii) As compared to CB arrangement, I_b increases less rapidly with V_{be}. Therefore input resistance of CE mode is higher than CB mode.

b) Output characteristics :

It is the curve between collector current I_c and collector-emitter voltage V_{ce} at constant base current I_b. A typical CE mode output characteristic curves of a pnp transistor are shown in fig. 20. 8(b). The following points may be noted from the characteristics.

i) The collector current I_c varies rapidly with V_{ce} for $|V_{ce}| < 1V$. The value of V_{ce} upto which the collector current I_c rapidly increases is called the knee voltage. The transistor is always operated in the region above knee voltage.

ii) Above knee voltage, I_c increases very slightly with increase in V_{ce} and hence is represented by slanted lines as compared to the horizontal lines in the case of output characteristics of CB mode. This region is known as the active region of the transistor and is used to amplify signals almost faithfully.

iii) The transistor parameter $\beta = \left(\frac{\Delta I_c}{\Delta I_b}\right)_{V_{ce}}$ at constant collector emitter voltage can be determined from output characteristics of CE mode.

20.11: Transistor as an amplifier :

The basic function of a transistor is to amplify. An amplifier is an electronic device which amplifies the magnitude of the applied weak input signal. We assume that the input signal is an a.c. signal. One important requirement during amplification is that only the magnitude of the signal should increase without any change in its shape (distortion). This process is known as faithful amplification. For faithful amplification, the transistor should be properly biased i.e. the input circuit should be forward biased while the output circuit should remain reverse biased throughout the presence of the signal.

As already discussed in the proceeding sections, there are three modes of using a transistor viz., i) common base, ii) common emitter and iii) common collector. Out of these three modes, the first two are used as amplifiers.

20.12 (a) : Common base amplifier :

Fig. (20.9 a) shows an amplifier circuit using an npn transistor in common base mode. The battery E_e supplies forward biasing voltage V_{eb} for the emitter base junction. The battery E_c maintains a potential difference V_{cb} between the collector and the base such that the collector base junction is in reverse bias. The input signal e_i fig.(20.9 b) to be amplified is connected in series with the biasing battery E_e in the emitter circuit. A high load resistance R_L is connected in the collector circuit and output voltage e_o is taken across R_L.

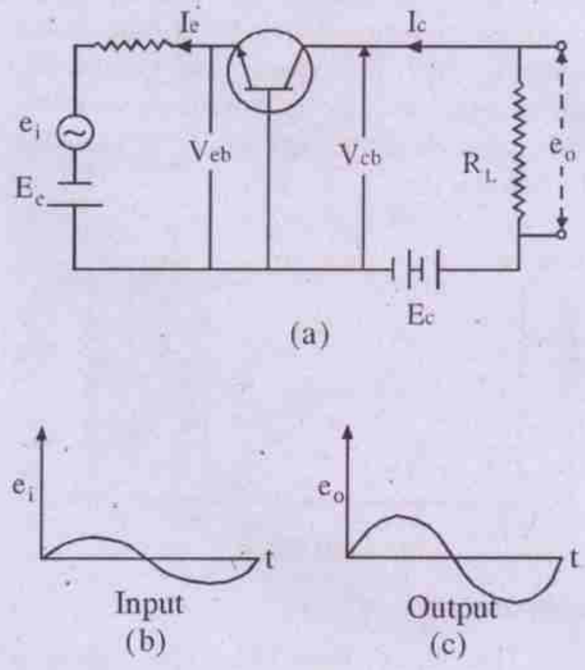


Fig. 20.9

Working :

When the input a.c. signal e_i is superimposed on the d.c. voltage E_e, the emitter base voltage varies with variation of the signal voltage and so does the emitter current and hence the collector current also varies. Since load resistance R_L is very high, variation in collector current produces a large variation in voltage across R_L fig. (20.9 c) thus amplifying the input signal.

Voltage gain A_V :

Input voltage, $e_i = R_i.\Delta I_e$

where R_i is the effective resistance of emitter base junction. It has a very low value as the emitter base junction is forward biased. ΔI_e is the change in emitter current, produced by signal voltage e_i.

Output voltage $e_o = R_L.\Delta I_c$

where ΔI_c is the change in collector current.

$$A_v = \frac{e_o}{e_i} = \frac{R_L.\Delta I_c}{R_i.\Delta I_e}$$

$$= \alpha.\frac{R_L}{R_i}$$

= current gain x resistance ratio

Since load resistance R_L is very high, input resistance R_i is very low, the resistance ratio $\frac{R_L}{R_i} >> 1$.

Power gain A_p :

$$A_P = \frac{\text{output power}}{\text{input power}}$$

$$= \frac{e_o.\Delta I_c}{e_i.\Delta I_e}$$

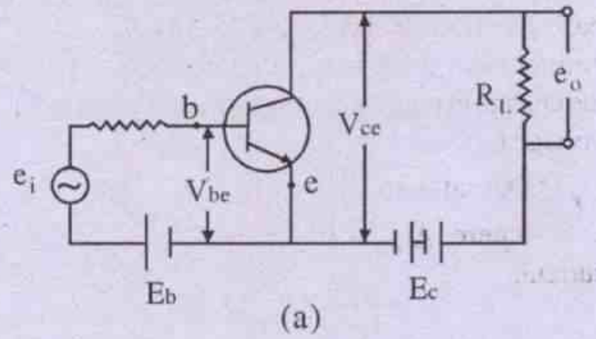
$$= \alpha.A_v$$

$$= \alpha^2.\frac{R_L}{R_i}$$

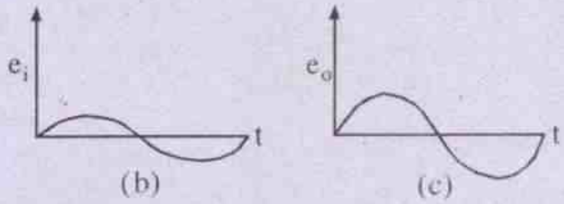
$A_v = \alpha^2$ x Resistance ratio

20.12 (b) : Common emitter amplifier :

Fig. (20.10 a) shows an amplifier circuit using an npn transistor in common emitter mode. The battery E_b supplies forward biasing voltage



(a)



(b) (c)

Fig. 20.10

V_{be} for the base emitter junction. The battery E_c maintains a potential difference V_{ce} between the collector and emitter such that collector emitter junction is in reverse bias. The input signal e_i Fig. (20.10 b) to be amplified is connected in series with the biasing battery E_b in the base circuit. A high load resistance R_L is connected in the collector circuit and the output voltage e_o is taken across R_L.

Working :

When the input signal e_i is superimposed on the d.c. voltage E_b, the base emitter voltage V_{be} varies with the variation of the signal voltage and so does the base current. This results in variation of the collector current. Since load resistance R_L is very high, variation in collector current in load resistance, produces a large variation in output voltage e_0 across R_L fig. (20.10 c), thus amplifying the input signal e_i.

Voltage gain A_v :

Input voltage, $e_i = R_i \cdot \Delta I_b$

where R_i is the effective resistance of base emitter junction. It has a low resistance as it is forward biased. ΔI_b is the change in base current, caused by signal voltage e_i.

Output voltage, $e_0 = R_L \cdot \Delta I_c$

where ΔI_c is the change in collector current.

$$A_v = \frac{\text{output voltage}}{\text{input voltage}}$$

$$= \frac{e_o}{e_i} = \frac{R_L.\Delta I_c}{R_i.\Delta I_b}$$

$$= \beta \frac{R_L}{R_i}$$

= current gain x Resistance ratio

Since $R_L >> R_i$, R_L / R_i known as resistance ratio is much greater than unity.

Power gain A_p :

$$A_P = \frac{\text{output power}}{\text{input power}}$$

$$= \frac{e_o.\Delta I_c}{e_i.\Delta I_b}$$

$$= \beta^2.\frac{R_L}{R_i}$$

$= \beta^2$ x Resistance ratio

20.13: Comparision between CB and CE amplifiers :

i) The current amplification factor for CB is α which is less than unity whereas current amplification factor for CE is β which is very much greater than unity.

ii) The resistance ratio R_L / R_i in CB amplifier is larger than CE amplifier.

iii) Voltage gain in CE amplifier is higher than that in CB amplifier.

iv) Power gain in CE amplifier is much higher than that in CB amplifier.

20.14: Advantages of a Transistor over a triode :

i) Transistors are robust, cheap and are very small in size.

ii) As there is no heating filament, there is no heating delay and no heating power is required.

iii) Transistor works at a very low operating voltage.

iv) Transistors consume less power and hence they are highly efficient.

v) They are capable of sustaining mechanical shocks as they are solid crystals.

vi) They have long life as there is no filament.

Disadvantages of a transistor over a triode :

Transistor can be damaged by over heating so that they are not suitable for large currents whereas triode can withstand large currents.

SUMMARY

1. Transistor :

The name "transistor" comes from the relation :

Transfer + resistor $\rightarrow$ transistor

It is a three terminal semiconductor triode. It is of two forms n-p-n and p-n-p.

2. Biasing a transistor :

Emitter base junction is forward biased while collector base junction is reverse biased.

3. Transistor parameters :

i) Current gain $\alpha = \left(\frac{\Delta I_c}{\Delta I_e}\right)_{V_{cb}}$

ii) Current gain $\beta = \left(\frac{\Delta I_c}{\Delta I_b}\right)_{V_{ce}}$

iii) Current gains $\alpha \& \beta$ are related as

$$\alpha = \frac{\beta}{1+\beta}$$

$$\beta = \frac{\alpha}{1-\alpha}$$

4. Voltage gain A_V :

It is the ratio of output voltage to input voltage.

i) Common base amplifier

$$A_v = \alpha \frac{R_L}{R_i}$$

ii) Common emitter amplifier

$$A_v = \beta \frac{R_L}{R_i}$$

5. Power gain A_P :

It is the ratio of output power to input power.

a) Common base amplifier

$$A_P = \alpha^2 \frac{R_L}{R_i}$$

b) Common emitter amplifier

$$A_P = \beta^2 \frac{R_L}{R_i}$$

SOLVED NUMERICAL PROBLEMS :

Ex.1: When the base current in a transistor is changed from $40\mu A$ to $80\mu A$, the collector current is changed from 1.5 mA to 4.5 mA. Find the current gain β.

Soln.

Change in base current, ΔI_b

$= 80 - 40 = 40\mu A$

Change in collector current, ΔI_c

$= 4.5 - 1.5 = 3$ mA

Current gain, $\beta = \frac{\Delta I_c}{\Delta I_b}$

$$= \frac{3 \times 10^{-3}}{40 \times 10^{-6}} = 75$$

Ex.2: In a transistor in common base configuration, change in emitter current is 1.0 mA and change in base current is 0.05 mA. Find current gain α.

Soln.

Change in emitter current, $\Delta I_e = 1.0$ mA

Change in base current, $\Delta I_b = 0.05$ mA

$\therefore$ Change in collector current,

$$\Delta I_c = \Delta I_e - \Delta I_b$$

$$= 1.0 - 0.05 = 0.95 \text{ mA}$$

Current gain, $\alpha = \frac{\Delta I_c}{\Delta I_e}$

$$= \frac{0.95}{1.0} = 0.95$$

Ex.3: A transistor is used in CE mode in an amplifier circuit. When a signal of 20 mV is added to the base emitter voltage, the base current charges by $20\mu A$ and the collector current changes by 2 mA. If the load resistance is $5\,K\Omega$, calculate (a) β (b) the input resistance R_{be} (c) the transconductance and (d) voltage gain.

Soln.

Change in base current, $\Delta I_b = 20\mu A$

Change in collector current, $\Delta I_c = 2mA$

Signal voltage $\Delta V_{be} = 20mV$

Load resistance, $R_L = 5\,K\Omega$

a) Current gain, $\beta = \frac{\Delta I_c}{\Delta I_b}$

$$= \frac{2 \times 10^{-3}}{20 \times 10^{-6}} = 100$$

b) Input resistance, $R_{be} = \frac{\Delta V_{be}}{\Delta I_b}$

$$= \frac{20 \times 10^{-3}}{20 \times 10^{-6}} = 1K\Omega$$

c) Transconductance, $g_m = \frac{\Delta I_c}{\Delta V_{be}}$

$$= \frac{2mA}{20mV} = 0.1 \text{ mho}$$

d) Output voltage, $e_o = R_L \times \Delta I_c$

$$= 5 \times 10^3 \times 2 \times 10^{-3}$$

$$= 10 \text{ V}$$

Input voltage e_i = signal voltage

$$= 20 \text{ mV}$$

Voltage gain, $A_v = \frac{e_o}{e_i}$

$$= \frac{10}{20 \times 10^{-3}} = 500$$

Ex.4: A transistor connected in CE mode, has input resistance 2K Ω and load resistance 5K Ω. If $\beta = 60$ and input signal 20 mV is applied, calculate (a) resistance ratio (b) voltage gain and (c) output signal.

Soln.

Load resistance $R_L = 5 \times 10^3 \Omega$

Input resistance $R_i = 2 \times 10^3 \Omega$

Current gain $\beta = 60$

Input signal $e_i = 20$ mV

a) Resistance ratio = $\frac{R_L}{R_i} = \frac{5 \times 10^3}{2 \times 10^3} = 2.5$

b) Voltage gain $A_v = \beta \times \frac{R_L}{R_i}$

$$= 60 \times 2.5 = 150$$

c) Output signal $e_o = A_v . e_i$

$$= 150 \times 20 \times 10^{-3}$$

$$= 3 \text{ volt}$$

Ex.5: A transistor is working in common emitter configuration with a load resistance of $10K\Omega$ and collector supply voltage V_{cc} is 10V. If the potential drop across load in collector circuit is 1.0 V and $\alpha = 0.95$ calculate

a) collector emitter voltage

b) base current

c) emitter current

Soln.

Load resistance $R_L = 10 \times 10^3 \Omega$

Collector supply voltage $V_{CC} = 10V$

Potential drops across the load $e_0 = 1V$

Current gain $\alpha = 0.95$

a) Collector emitter voltage

$$V_{CE} = V_{CC} - e_0 = 10 - 1 = 9 \text{ V}$$

b) P.d. across R_L, $e_0 = R_L \times I_C$

$$\therefore I_c = \frac{e_o}{R_L} = \frac{1}{10 \times 10^3} = 0.1mA$$

c) Current gain $\beta = \frac{\alpha}{1-\alpha}$

$$= \frac{0.95}{1-0.95} = 19$$

$$I_b = \frac{I_c}{\beta} = \frac{0.1}{19} = 0.0053mA$$

$$I_e = I_b + I_c = 0.0053 + 0.1 = 0.1053 \text{ mA}$$

MODEL QUESTIONS

A. Multiple Choice Questions :

1. In an npn transistor,
   a) collector current is away from the collector and emitter current is towards the emitter
   b) collector current is away from the collector and emitter current is also away from the emitter
   c) collector current is towards the collector and emitter current is away from the emitter
   d) collector current is towards the collector and emitter current is towards the emitter.

2. NPN transistor is formed by
   a) joining two pn junctions
   b) joininng three separate n, p, n types of material
   c) growing n, p, n type materials in a single crystal
   d) all of the above.

3. In a transistor
   a) the emitter has largest concentration of impurity
   b) the collector has largest concentration of impurity
   c) the base has largest concentration of impurity
   d) all the three regions have equal concentrations of impurity.

4. In a transistor ______ region is very thin.
   a) emitter
   b) base
   c) collector
   d) all the three regions are of equal thickness.

5. In a normal operation of a transistor
   a) both base-emitter and the base-collector junctions are forward biased
   b) both base-emitter and the base-collector junctions are reverse biased
   c) the base-emitter junction is reverse biased and base-collector junction is forward biased
   d) the base emitter junction is forward biased and base-collector junction is reverse biased.

6. The value of α
   a) is always less than 1
   b) is always greater than 1
   c) may be less or greater than 1
   d) none of the above.

7. The value of α

 a) increases with increase in thickness of the base region.

 b) increases with decrease in thickness of the base region.

 c) is independent of thickness of base region.

 d) none of the above.

8. In a transistor,

 a) emitter is lightly doped, base is heavily doped

 b) base and collector both are lightly doped

 c) base is lightly doped and emitter is heavily doped

 d) all regions are equally doped.

9. The value of β

 a) is always greater than 1

 b) is always less than 1

 c) may be less or greater than I

 d) none of the above.

10. In a transistor base current is

 a) more than collector current

 b) more than emitter current

 c) less than collector and emitter currents

 d) equal to collector and emitter currents.

11. In a common-emitter transistor amplifier circuit $\beta = 50$, input resistance $R_i = 500$ ohm and output resistance $R_L = 5\,K\Omega$. The voltage gain of the amplifier is

 a) 50 b) 500
 c) 5 d) 5000

12. For a transistor $\alpha = 0.95$, the value of γ is

 a) 19 b) 0.05
 c) 20 d) 190

13. In a common-base transistor amplifier circuit $\beta = 9$, input resistance is 100Ω and output resistance is $2\,K\Omega$. The voltage gain of the circuit is

 a) 180 b) 1800
 c) 18 d) 19

B. Very Short Answer Questions :

1. Define, α, β and γ of a transistor. [CHSE 01]

2. In a normal operation of a transistor base-emitter junction is forward biased or reverse biased.

3. Why base is thinner compared to emitter and collector in a transistor ? [CHSE 97 S]

4. In CE mode, the base current is 100 μ A. Calculate emitter current if $\beta = 100$. [CHSE 2000 Instant]

5. Why base is lightly doped in a transistor ?

6. What is the relation between α and β ?

7. Give the values of α, β and γ relative to unity.

8. Which impurities are used in different regions of an n-p-n transistor.

9. For larger current which one n-p-n or p-n-p transistor is preferred.

10. Transistor is made of which material : silicon or copper ?

11. Which mode has larger voltage gain : CB or CE ?

12. Which circuit has input signal to the base and out put from the collector : CB or CE ?

13. Does the base-emitter junction has forward or reverse bias ?

14. Collector current is controlled by base voltage or collector voltage ?

15. A Si transistor has 0.1 V forward bias. Will it conduct easily ? Why ?

16. What are the different types of junction transistors ?

C. **Short Answer Questions :**

1. How emitter and collector are biased with respect to base in a transistor ? [CHSE 95 S]

2. Draw the common base circuit diagram of an n-p-n transistor. [CHSE 94 A]

3. Define transistor constants α and β. Write the relation between them. [CHSE 2000]

4. Find the value of emitter current in a transister for which $\beta = 40$ and $I_b = 10\,\mu$ A. [CHSE 2001]

5. State the advantages of a transistor over vacuum triode. [CHSE 85 S]

6. What is the α value of a transistor whose β value is 100. [CHSE 92 A]

7. Draw the circuit diagram of a p-n-p transistor used as an amplifier. [CHSE 94 S]

8. The α-value of a transistor is 0.995. Calculate its β value.

9. Give the symbols for (i) n-p-n and (ii) p-n-p transistors.

10. Define static characteristics of a transistor. Give their types.

11. Why the junction triode is called a transistor ?

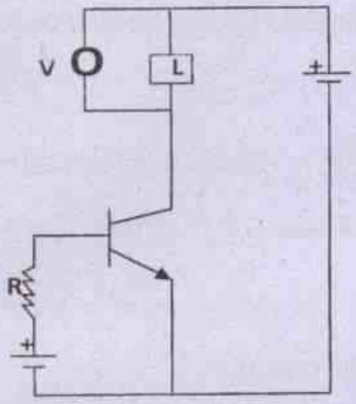
12. A transistor amplifier has collector supply voltage $V_{CC} = 50$ V and collector voltage $V_C = 10$V. If load resistance R_L is $200\,\Omega$, calculate collector current I_C.

13. If $V_b = 18.4$ V, $V_e = 17.8$ V, how much is V_{be} ?

14. What is the function of base region of a transistor ? Why this region is made thin and lightly doped ? [CBSE AI 2006]

15. In the given circuit Fig. (A) a voltmeter V is connected across the lamp L. What changes would occur at lamp L and voltmeter V if the resistance R is reduced in value ? Give reasons for your answer.

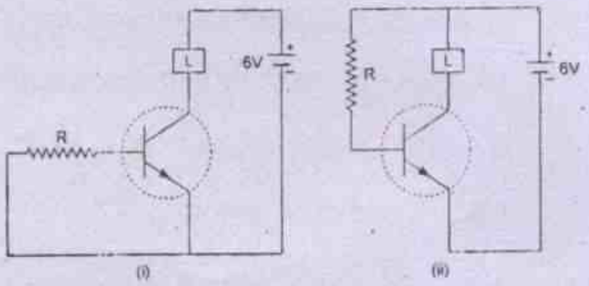
[CBSE 2002, 1999]



Fig(A)

16. In only one of the circuits given below the lamp L lights. Which circuit is it ? Give reasons for your answer.

[CBSE 2001]



D. Numerical Problems :

1. For a transistor in CE mode, $\beta = 50$ and voltage drop across $1K\Omega$ which is connected in the collector circuit is 2 volt. Find the base current.

2. A transistor is connected in CE mode in which collector supply is 9V and the voltage drop across load resistance R_L connected in the collector circuit is 1.0 V. If $R_L = 1K\Omega$ and $\alpha = 0.99$, determine (a) collector emitter voltage (b) base current.

3. In a transistor amplifier, the collector load resistance R_L is $2\ K\Omega$ and input resistance R_i is $1K\Omega$. If the current gain is 50, calculate voltage gain of the amplifier.

4. In a common base circuit, a resistance of $1K\Omega$ is connected in the collector base section. The voltage drop across it is 1.2 V. Find the base current if $\alpha = 0.96$.

5. A transistor with its emitter grounded has $\alpha = 0.95$. If the base current changes by 1mA, calculate change in collector current.

6. A p-n-p transistor in common base configuration offers an input resistance of $50\,\Omega$. If the current gain of the amplifier is 0.96, load resistance in the circuit is $5\ K\Omega$, find (a) voltage gain and (b) power gain of the transistor.

7. The input resistance of a common emitter amplifier is $2\ k\Omega$ and ac corrent gain is 20. If the load resistor used is $5\ k\Omega$,

calculate (i) the voltage gain of the amplifier (ii) the transconductance of the transistor used. [CBSE 1999]

8. In a common emitter mode of a transistor, the d.c.current gain is 20, the emitter current is 7 mA. Calculate (i) base current and (ii) collector current.

[CBSE Sample Paper]

9. A change of 0.2 mA in the base current causes a change of 5 mA in the collector current for a common emitter amplifier. (i) Find the a.c current gain of the transister. (ii) If the input resistance is 2 $k\Omega$, and is voltage gain is 75, calculate the load resirstor used in the circuit.

[CBSE AI 2005]

E. Long Answer Questions :

1. Obtain the relation between α and β of a transistor. With suitable circuit diagram describe the use of a p-n-p transistor as an amplifier. [CHSE 94 A]

2. With necessary circuit diagram describe use of an n-p-n transistor as an amplifier ?

[CHSE 95 A]

3. Explain with neat circuit diagram the working of an n-p-n transistor with brief description of its construction.

[CHSE 97 A]

4. What is a transistor ? Why it is so called ? Explain the working of an n-p-n or p-n-p transistor.

5. Define transistor parameters α, β and γ. Find a relation between them. How do you determine α and β from output characteristics of CE transistor ?

6. Draw the circuit diagram to study input and output characteristics of CB-npn transistor. How do you determine α and β from its characteristics ?

7. With a neat circuit diagram describe the use of n-p-n transistor as an amplifier in CE mode. [CHSE 2003]

F. True-False Type Questions :

1. In npn transistor emitter current is towards the emitter and collector current is away from the collector.

2. npn transistor is formed by growing n,p,n type meterials in a single crystal.

3. The value of β=90. Then the value of α is 0.99.

4. The value of α is always less than 1.

5. In a transistor emitter is heavily doped, base is lightly doped.

6. For a transistor $\alpha = 0.95$, then the value of γ is 0.05.

G. Fill-in-Blank Type Questions :

1. The emitter - base junction is always ____ biased.

2. The relation between β and γ in a transistor is ______.

3. On increasing the reverse bias to a large value in a pn junction, the diode current____.

4. The voltage gain in a common emitter amplifier when input resistance is $3\,\Omega$, and resistance is $24\,\Omega$ and $\beta = 60$ is ___.

5. In a transistor base current is ____ than collector and emitter current.

6. The value of β is always ____ than 1.

H. Correct the following sentences :

1. The emitter-base junction is always reverse biased.

2. The value of transistor constant β is always less than 1.

3. In a transistor base current is greater than collector and emitter current.

4. In a transistor emitter is heavily doped, base is moderately doped and collector is thinly doped.

5. The transistor constant α is always greater than 1.

6. The relation between transistor constants α and β is $\alpha = \beta/(1-\beta)$.

7. The relation between transistor constants β and γ is $\gamma = 1 - \beta$.

ANSWERS

A. Multiple Choice Questions :

1. (c) **2.** (c) **3.** (a) **4.** (b) **5.** (d) **6.** (a) **7.** (b) **8.** (c)
9. (a) **10.** (c) **11.** (b) **12.** (c) **13.** (a)

D. Numerical Problems :

1. 0.04 mA

2. (a) 8 V

(b) 0.0101 mA

3. 100

4. 0.05 mA

5. 19 mA

6. (a) 96

(b) 92.16

7. (i) 50 (ii) $10^{-2}\Omega^{-1}$,

8. (i) (1/3)mA (ii) (20/3) mA

9. (i) 25 (ii) 6 kΩ

F. **1.** False **2.** True **3.** True **4.** True **5.** True **6.** False

G. **1.** forward **2.** $\gamma = 1+\beta$ **3.** Suddenly increases **4.** 480 **5.** less **6.** greater.

21

Space Communication

21.1 Electromagnetic Wave

In an oscillating electric circuit the varying alternating current produces correspondingly varying magnetic and electric fields at different points in space. These may be looked upon as electrical and magnetic disturbances travelling in space. These fields are mutually perpendicular to each other and travel in space with speed of light ($\approx 3\times10^8 \, m/s$). This wave is usually called electromagnetic (e.m.) wave. Energy associated with this e.m. wave is called e.m. radiation (or energy).

James Clerk Maxwell unified the observations

(i) Electric field exists in the regions around electric charges obeying Gauss law in electrostatics. ($\vec{\nabla}.\vec{D} = \rho$)

(ii) Magnetic field is associated with moving charges obeying Ampire's circuital law (suitably modified)

$$\left(\oint \vec{B}.\vec{d\ell} = \mu_o(i+i_d) = \int_s (\vec{\nabla}\times\vec{B}).\vec{ds}\right)$$

(iii) Induced emf's are associated with changing magnetic fields obeying, Faraday's law of e.m. induction.

$$\left(\varepsilon = -\frac{d\phi}{dt} = \oint \vec{E}.d\ell\right)$$

(iv) Magnetic field associated with moving charges (or current) is solenoidal.

$$\left(\vec{\nabla}.\vec{B} = 0\right)$$

and established that e.m. waves are radiated by an oscillating electrical circuit.

In 1888 Heinrich Hertz, a German scientist carried out an experimental study on oscillating electric dipole and detected the existence of such waves.

21.2. Hertzian-dipole

The Hertzian apparatus consists of two metallic spheres Q_1 and Q_2 capable of sliding along two rods R_1 and R_2 having a spark gap G_1 in between (see fig. 21.1). The two rods are connected to the two terminals of the secondary of an induction coil. The receiving system consists of a circular wire having a spark gap G_2.

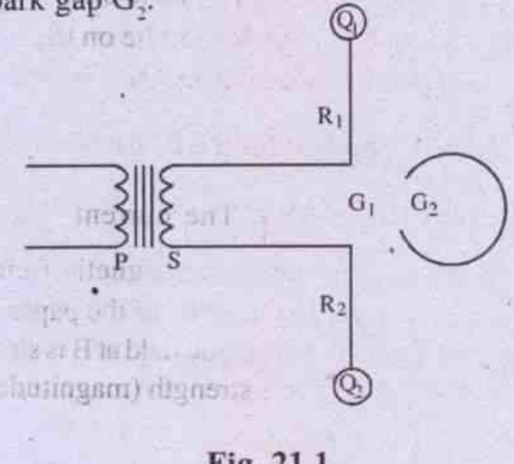


Fig. 21.1

The two spheres get charged by the induction coil. The charges oscillate and produce series of sparks across gap G_1. This results in radiation of energy. These radiations are received by the circuit G_2. After proper tuning (accomplished by sliding Q_1 and Q_2) sparks also appear at G_2. He was able to show that this radiation was e.m. wave as explained below qualitatively.

Let at any instant Q_2 be positively charged and Q_1 be negatively charged. At any

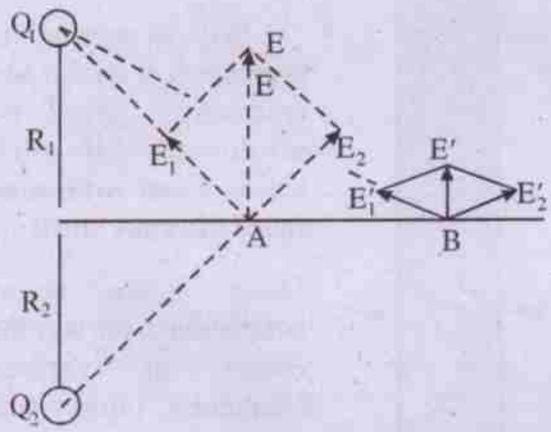


Fig. 21.2

point A the field due to charges on Q_2 shall be along $\vec{AE}_2$ and field due to charges on Q_1 shall be along $\vec{AE}_1$, with $|\vec{AE}_1| = |\vec{AE}_2|$ (see fig. 21.2). The resultant field shall be along $\vec{AE}$, which is euqally inclined to both of them and parallel to R_1 and R_2 and lie on the place of the paper. Considering another point B, we also find a resultant field $\vec{BE}'$ parallel to $\vec{AE}$; but with $|\vec{AE}| > |\vec{BE}'|$. The current flowing from R_2 to R_1 produces a magnetic field at A, perpendicular to the plane of the paper and directed into it. The magnetic field at B is similar to that at A, but of less strength (magnitude).

During oscillations when Q_1 carries +ve charges and Q_2 carries -ve charges the direction of electric field is reversed. Now the current flowing from R_1 to R_2 produces a magnetic field at A, perpendicular to the plane of paper but directed outwards. Similar fields are produced at B but of less strength.

As the magnitudes of charges on Q_1 and Q_2 change the strength of electric and magnetic fields change with time. This combination of varying electric and magnetic fields at right angles to each other gives rise to e.m. wave.

Maxwell theoretically explained this phenomenon and showed that $\bar{E}$ and $\bar{B}$ do not appear instantly at distance points but time is required for their propagation.

21.3 Radiation due to an oscillating dipole

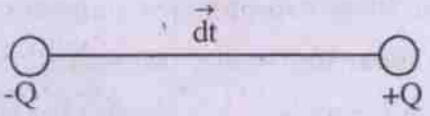


Fig. 21.3

Conside a dipole of moment $\vec{p} = Q\vec{d\ell}$. If the dipole is acted upon by a sinusoidally varying field of angular frequency ω, then instantaneous dipolemoment is given as

$$\bar{p} = \bar{p}_o e^{-i\omega t} \quad ...21.3.1$$

where $\bar{p}_o$ is the amplitude of the dipolemoment. Then it can be shown that the dipole radiates e.m. radiation and the total radiation is given by

$$P_t = \frac{p_o^2 \omega^4}{12\pi\varepsilon_o C^3} \quad ...21.3.2$$

Thus eqn. 21.3.2 shows that dipole-radiated power depends on the fourth power of angular frequency (hence frequency) and square of the amplitude of dipolemoment.

21.4 Radiation due to a small current :

An oscillating dipole may be treated as a small current element. It can be shown that the total radiated power by a current element is

$$P_t = 80\pi^2\left(\frac{d\ell}{\lambda}\right)^2 I_{rms}^2 = R_r I_{rms}^2 \quad ...21.4.1$$

where $R_r = 80\pi^2\left(\frac{d\ell}{\lambda}\right)^2$ ...21.4.2

is called **Radiation resistance.**

If $d\ell << \lambda$, then R_r is low compared to Ohmic resistance, hence most part of the energy is lost in the form of heat. For example for $d\ell = 0.01\lambda$, $R_r \simeq 0.08$ ohm and this is much smaller than the ohmic resistance. For efficient radiation of e.m. energy it is essential that the length of the antenna is comparable with wavelength. A half-wave antenna is commonly used to radiate e.m. energy into space.

21.5 Linear half-wave antenna :

A linear half-wave antenna is a straight conductor of length equal to half the space wavelength. If a linear conductor is set vertically on the ground, then only quarter free space wavelength ($\lambda/4$) of the conductor serves the purpose of half-wave antenna as additional $\frac{\lambda}{4}$ length is furnished by the ground reflection.

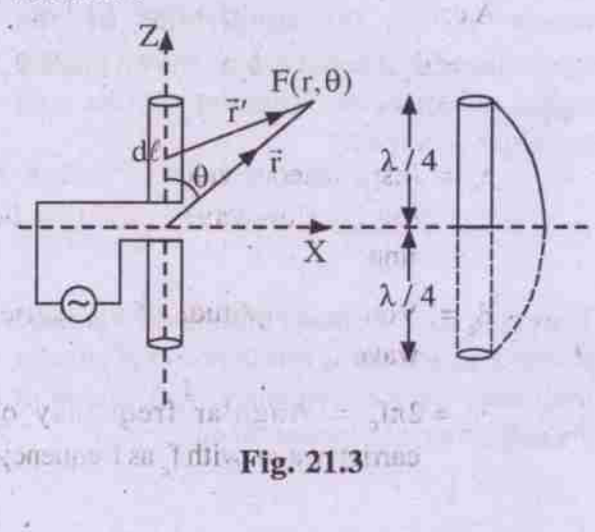


Fig. 21.3

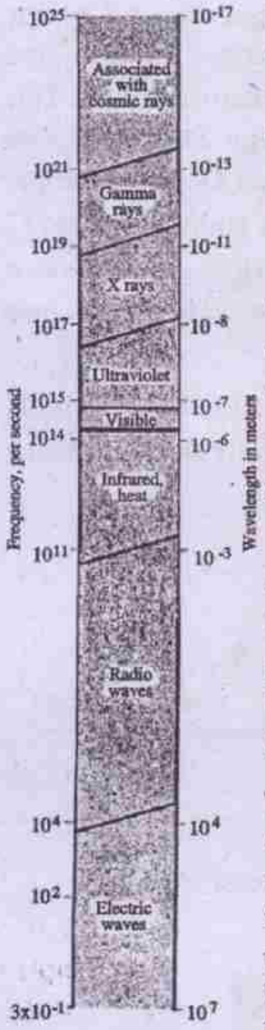


Fig. 21.4

Detailed calculations show that the total power radiated by such an antenna is given by

$$P_t = 73.2\, I_{rms}^2 \quad ...21.5.1$$

Thus such linear half-wave antenna emits e.m. radiation most efficiently.

21.6 Electromagnetic spectrum :

With an appropriate source e.m. waves of any frequency may be produced. All such waves travel with same speed in empty space.

There are major differences in the way the waves of various frequency ranges are produced and the methods by which they are studied. Fig. 21.4 indicates the e.m. spectrum (not all parts drawn upto seale). The various regions can be classified as follows:

1. Electric waves :

This is a broad region covering (a) long waves ($\lambda \sim 10^4$ m), (b) standard broadcasting waves ($\lambda \sim 0.3$ to 30×10^3 m),(c) microwaves ($\lambda \sim 0.3$ mm to 30cm)

2. Optical waves : ($\lambda \sim 10^{-3}$ m to 10^{-9} m)

This includes (a) infrared ($\lambda \sim 7.8\times10^{-7}$ m to 10^{-3} m), (b) visible ($\lambda \sim 3.9\times10^{-7}$ m to 7.8×10^{-7} m), (c) ultra violet ($\lambda \sim 3.9\times10^{-7}$ m to 5×10^{-10} m).

3. X-rays :

This includes the region $\lambda \sim 10^{-8}\,m$ to $10^{-12}\,m$

4. -rays :

This includes the region $\lambda < 10^{-12}\,m$.

21.7 Properties of e.m. waves :

E.m. waves exhibit the following properties.

i) They consist of mutually perpendicular electric and magnetic fields.

ii) Electric and magnetic fields are in phase with each other.

iii) They can travel in empty space.

iv) They travel with speed $\frac{1}{\sqrt{\mu_o \varepsilon_o}}$ through vaccum and with speed $\frac{1}{\sqrt{\mu\varepsilon}}$ in material medium.

v) E.m. waves obey the principle of superposition.

vi) The ratio of the electric and magnetic field strength of e.m. wave is equal to the speed of the wave.

21.8 Need for modulation

In radio communication the audio signal such as music or speech has to be transmitted over long distance. As these distances are very large, communicating wires are not used. For this reason radio communication is sometimes called "Wireless". The frequency range of audiosignal is from 20 Hz to 20 kHz. Since thê energy associated with a wave is directly proportional to the powers of frequency so energy associated with audio signal is small. Even if the audiowave is converted to electrical signals its range of transmission is small. In order to send the e.m. signal into space we use antenna. The theory of antenna as discussed in sec. 21.5 shows that for efficient transmission the antenna should have length $\ell = \lambda/4$. Therefore if the frequency of e.m. signal be 1 kHz, the length of the antenna should be $\ell = \frac{\lambda}{4} = \frac{C}{4\upsilon} = \frac{3\times10^{8}\,m}{4\times10^{3}\,Hz}$. A vertical antenna of this length is a practical impossibility. It is therefore not possible to transmit audio signal directly. Even if it were possible it might have posed another additional problem-signals from different stations would arrive at the receiving end over the same frequency range -, leading to utter confusion.

It is therefore necessary to broadcast at high frequency and make the high frequency wave carry the audiosignal. The high frequency wave is called "Carrier Wave". **The process of combining carrier wave and audio signal is called modulation.**

21.9. Modulation

The process of combining (R.F.) carrier-wave and the signal (audio or visual) so that some characteristics (e.g. amplitude, frequency or phase) of the carrier wave changes in accordance with the signal is known as modulation. Thus

(R.F) carrier-wave + signal (modulating-wave) = modulated-wave

21.9 (a) Types of modulation :

A carrier wave can be represented as

$$e_c = E_c \cos(\omega_c t + \phi_c) \qquad ...21.9.1$$

where,

e_c = Instantaneous value of voltage of the carrier-wave (taken to be sinusoidal)

E_c = Voltage amplitude of the carrier wave

$\omega_c = 2\pi f_c$ = Angular frequency of carrier-wave, with f_c as frequency

ϕ_c = The phase angle of the carrier wave

By suitably combining the signal any of the parameters E_c, ω_c and ϕ_c can be varied. Accordingly we have three possible types of modulation.

(i) **Amplitude modulation,** involving variation of amplitude E_c when ω_c and ϕ_c are maintained constant.

(ii) **Frequency modulation,** involving variations of ω_c (or f_c) when E_c and ϕ_c are kept constant.

(iii) **Phase modulation,** involving variation of ϕ_c when E_c and ω_c are kept constant.

21.10 Amplitude modulation (AM) :

Amplitude modulation is obtained by varying the amplitude of the carrier by the modulating signal. The change in amplitude from the unmodulated value is proportional to the instantaneous value of the modulating signal.

If the modulating signal be given as

$$e_m = E_m \cos\omega_m t \quad ...21.10.1$$

and carrier-wave be given as

$$e_c = E_c \cos(\omega_c t + \phi_c) \quad ...21.10.2$$

then the modulated-wave is given as

$$(e_c)_{AM} = (E_c)_{AM} \cos(\omega_c t + \phi_c) \quad ...21.10.3$$

where

$$(E_c)_{AM} = E_c + K_a E_m \cos\omega_m t$$

$$= E_c\left(1 + \frac{K_a E_m}{E_c}\cos\omega_m t\right)$$

$$\Rightarrow \quad (E_c)_{AM} = E_c(1 + m_a \cos\omega_m t) \quad ...21.10.4$$

where $m_a = K_a E_m / E_c$ is known as the **modulation index , modulation factor or depth of modulation.** Thus, modulation factor is defined as the ratio of the maximum deviation of the amplitude of modulated carrier wave to the ampitude of the unmodulated carrier amplitude.

i.e.

$$m_a = \frac{K_a E_m}{E_c} = \frac{\text{Max. change of the unmodulated carrier - wave}}{\text{Amplitude of unmodulated carrier - wave}} \quad ...21.10.5$$

The percentage modulation, designated as M_a is given as $M_a = m_a \times 100$. The effect of amplitude modulation is as shown in fig. 21.5.

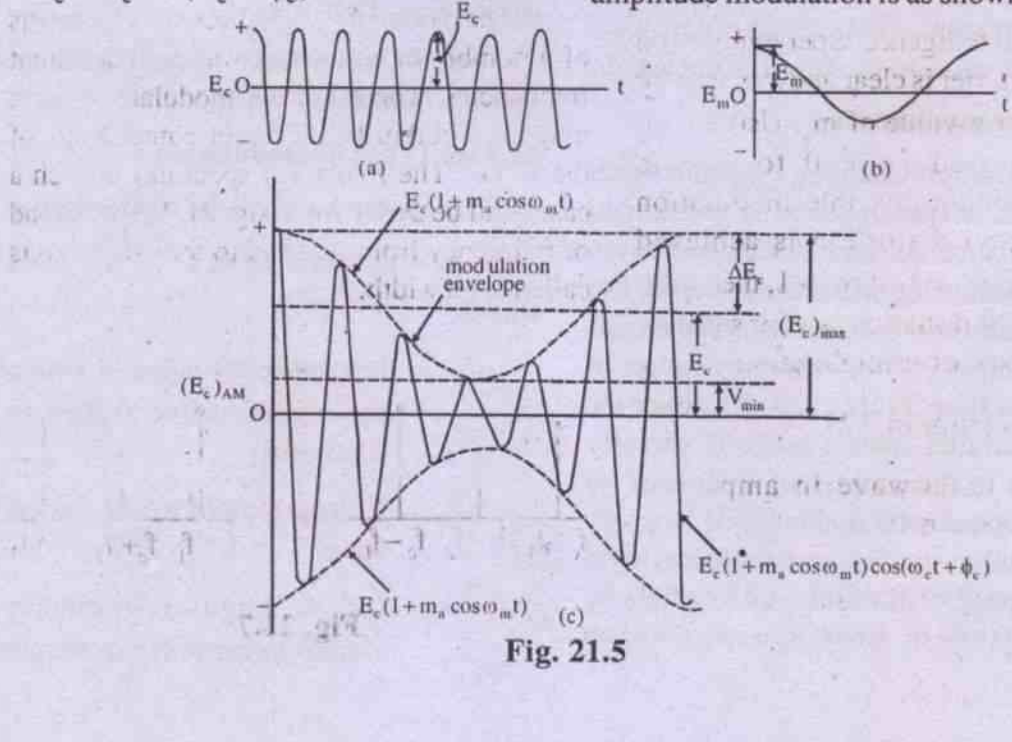


Fig. 21.5

The change in amplitude of carrier-wave is

$$\Delta E_c = (E_c)_{AM} - E_c = E_c m_a \cos\omega_m t \quad ...21.10.6$$

$$\Rightarrow \quad (\Delta E_c)_{max} = E_c m_a \quad ...21.10.7$$

and

$$[(E_c)_{AM}]_{max} = E_c(1+m_a) \quad ...21.10.8$$

$$[(E_c)_{AM}]_{min} = E_c(1-m_a) \quad ...21.10.9$$

This gives

$$m_a = \frac{[(E_c)_{AM}]_{Max} - [(E_c)_{AM}]_{Min}}{2E_c} \quad ...21.10.10$$

The quantities $[(E_c)_{AM}]_{Max}$ and $[(E_c)_{AM}]_{Min}$ can be measured from the wave forms of the modulated carrier wave [shown in fig. 21.5 (c)].

The quantity 'm_a' (or M_a) has the following significance :

(i) It gives the degree to which the signal modulates the carrier

(ii) The intelligence super imposed on the carrier is clear and stronger for a larger value of m_a. Usually m_a is varied from 0 to 1. The maximum possible modulation without distortion is achieved when $m_a = 1$. If $m_a > 1$, then there shall be distortion and this situation is called **over modulation.**

Frequency Spectrum in AM

The modulated-wave in amplitude modulation is represented as

$$(e_c)_{AM} = (E_c)_{AM}\cos(\omega_c t + \phi_c)$$

$$= E_c(1 + m_a\cos\omega_m t)\cos(\omega_c t + \phi_c)$$

$$= E_c\cos(\omega_c t + \phi_c) + E_c m_a \cos\omega_m t . \cos(\omega_c t + \phi_c)$$

$$= E_c\cos(\omega_c t + \phi_c) + \frac{1}{2}E_c m_a \cos [(\omega_c + \omega_m)t + \phi_c] + \frac{1}{2}E_c m_a \cos[(\omega_c - \omega_m)t + \phi_c] \quad ...21.10.11$$

Equation (21.10.11) shows that the amplitude modulated carrier-wave contains in addition to carrier frequency two other frequencies $(f_c + f_m)$ and $(f_c - f_m)$, respectively known as upper side-band frequency and lower side-band frequency. (see fig. 21.6)

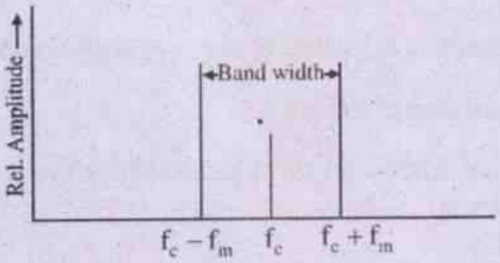


Fig. 21.6

The modulating signal very often consists of a number of sinusoidal waves of different frequencies. Therefore the modulation index may be different for different components of the signal. The frequency spectrum in such a case shall be as shown in fig. 21.7. The spread of frequency from one end to the other end is called band width.

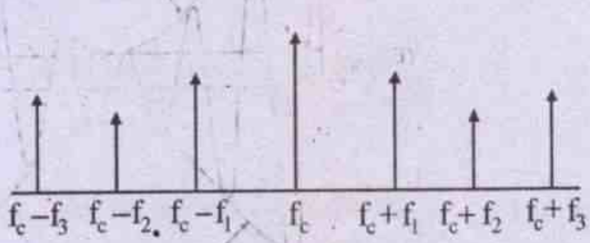


Fig. 21.7

Power in amplitude modulated wave :

Average power carried by the modulated wave is

$$\langle P_{CT}\rangle_{AM} = K\left[(E_c/\sqrt{2})^2 + (m_a E_c/2\sqrt{2})^2 + (m_a E_c/2\sqrt{2})^2\right]$$

$$= K.\frac{1}{2}E_c^2\left(1+\frac{m_a^2}{2}\right) \quad ...21.10.12$$

Where K is a constant. The power varried by the side-bands is

$$\langle P_{CS}\rangle_{AM} = K.\frac{m_a^2}{4}E_c^2 \quad ...21.10.13$$

(* This $K = \frac{X}{Z^2}$, where $\tilde{Z} = X + iY$, is the complex impedance of the cct.)

Hence fractional power carried by the side-bands is

$$\frac{\langle P_{CS}\rangle_{AM}}{\langle P_{CT}\rangle_{AM}} = \frac{m_a^2/2}{1+m_a^2/2} = \frac{m_a^2}{2+m_a^2} \quad ...21.10.14$$

The side-bands carry the signal. Hence strength of the signal depends on the fractional power of side-bands. For example we find if

$$m_a = 0, \quad \frac{\langle P_{CS}\rangle_{AM}}{\langle P_{CT}\rangle_{AM}} = 0$$

$$m_a = \frac{1}{2}, \quad \frac{\langle P_{CS}\rangle_{AM}}{\langle P_{CT}\rangle_{AM}} = \frac{1/4}{2+1/4} = \frac{1}{9} \equiv 11.1\%$$

$$m_a = 1, \quad \frac{\langle P_{CS}\rangle_{AM}}{\langle P_{CT}\rangle_{AM}} = \frac{1}{2+1} = \frac{1}{3} \equiv 33.3\%$$

Thus the maximum power associated with side-band shall be 33.3% of the total power associated with the modulated wave.

Limitations of AM

Amplitude modulation has the following limitations :

i) Low efficiency

ii) Small operating range

iii) Noisy transmission

iv) Poor audio quality

AM-Wave generation :

Fig. 21.8, shows the basic principle of obtaining an amplitude modulated carrier. The carrier-wave and the modulating signal are fed to an amplifier, called modulating amplifier.

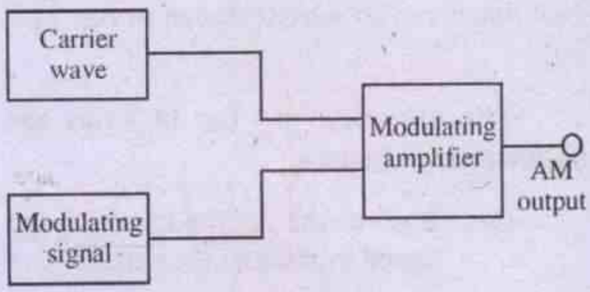


Fig. 21.8

AM circuits

The AM can be classified into two general categories.

i) Linear modulation method :

In this method the linear region of current-voltage characteristic of transistor is utilised. These are further classified as (a) collector modulation method (b) base modulation method (c) emitter modulation method, depending upon where the modulating signal is enjected into the modulating amplifier, as illustrated in figures 21.9, 21.10 and 21.11.

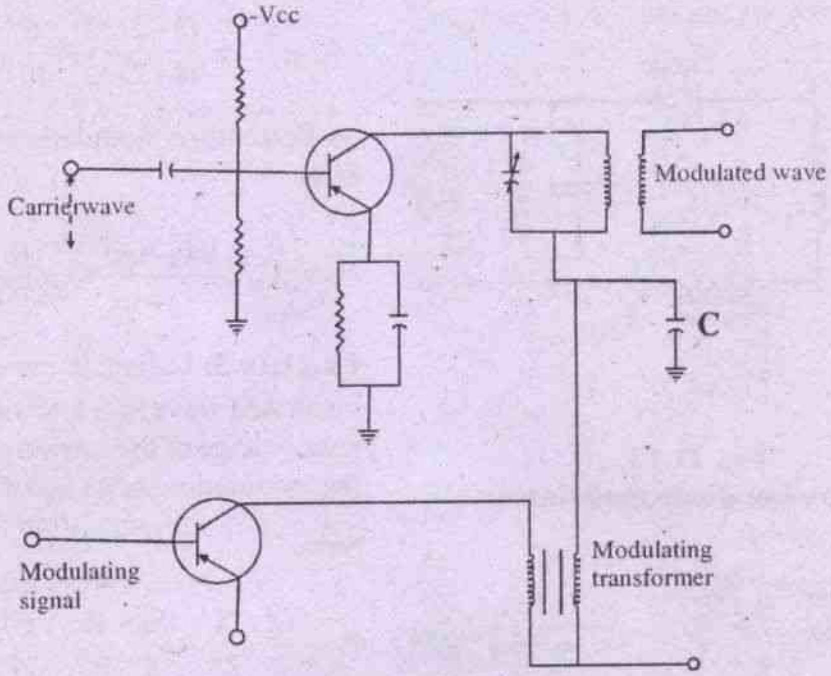


Fig. 21.9
(Collector modulation)

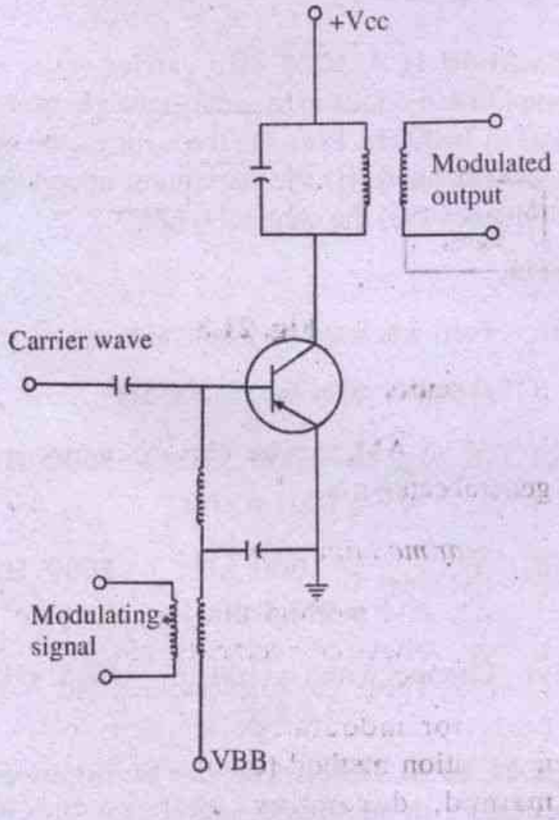


Fig. 21.10
(Base modulation)

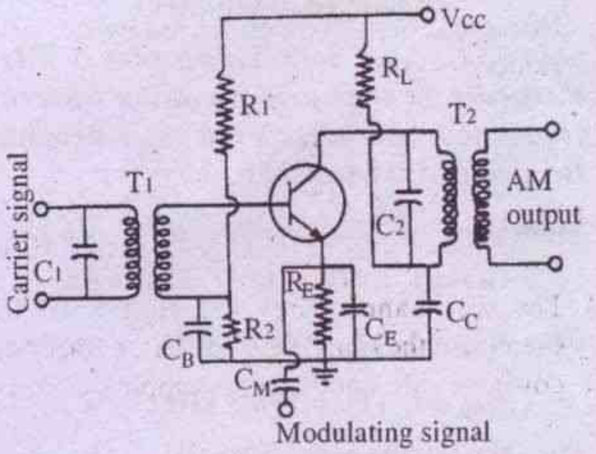


Fig. 21.11
(Emitter modulation)

ii) Square law modulation method :

This method makes use of the non-linear region of the current-voltage characteristics of transistor. This is further classified as (a) Square law diode modulator (b) Balanced modulator as shown in fig. 21.12 and 21.13.

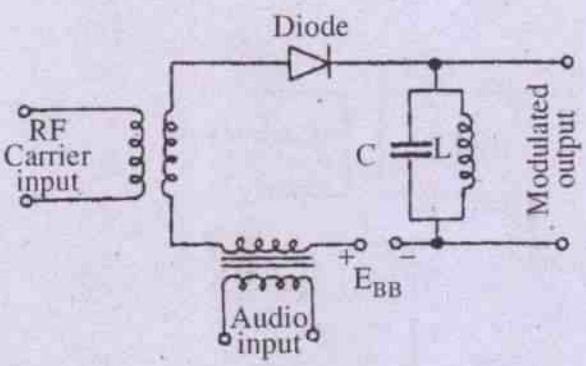


Fig. 21.12
(Square law diode modulator)

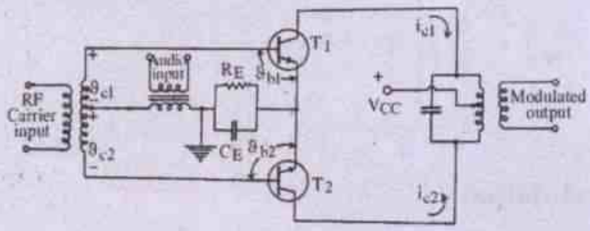


Fig. 21.13
(Balanced modulator)

Ex.21.10.1: An audiosignal of 1.5 kHz modulates the amplitude of a carrier-wave of frequency 600 kHz. Find (i) sideband frequencies (ii) bandwidth.

Soln.

Given $f_c = 600$ kHz, $f_m = 1.5$ kHz

(i) $f_{USB} = f_c + f_m = 601.5$ kHz

$f_{LSB} = f_c - f_m = 598.5$ kHz

(ii) Bandwidth $= 2f_m = 3$ kHz

Ex.21.10.2: The peak-to-peak value of AM voltage has a maximum value of 8V and minimum value of 2V. What is the percentage modulation and amplitude of the unmodulated carrier ?

Soln.

$$m_a = \frac{((E_c)_{AM})_{max} - ((E_c)_{AM})_{min}}{((E_c)_{AM})_{max} + ((E_c)_{AM})_{min}}$$

$$= \frac{(8-2)/2}{(8+2)/2} = \frac{6}{10} = 0.6$$

$\Rightarrow$ Percentage modulation $M_a = m_a \times 100 = 60\%$

$$\text{Now } E_c = \frac{((E_c)_{AM})_{max}}{1+m_a} = \frac{8/2}{1.6} = 2.5\text{V}$$

Ex.21.10.3: Calculate the power developed by an AM wave in a load of 100Ω when the peak voltage of the carrier-wave is 100V; and the modulation index is 0.4.

Soln.

$$P_c = \frac{E_c^2}{2}.\frac{X}{Z^2} = \frac{E_c^2}{2}.\frac{R}{R^2} = \frac{E_c^2}{2R} = \frac{(100)^2}{2 \times 100} = 50\text{W}$$

$$P_T = P_c\left(1+\frac{m_a^2}{2}\right) = 50\left(1+\frac{(0.4)^2}{2}\right) = 54\text{W}$$

Ex.21.10.4: A 1000 kHz carrier-wave is amplitude modulated by audiosignals between 400 to 1600 Hz. Find (1) frequency span on each sideband (ii) the maximum upperside frequency (iii) the channel width.

Soln.

(i) Frequency span of each side =

(1600 – 400) Hz = 1200 Hz

(ii) $(f_{uSB})_{max}$ = 1000 kHz + 1600 Hz = 1001.6 kHz

(iii) $(f_{LSB})_{min}$ = 1000 kHz – 1600 Hz = 998.4kHz

(iv) Channel width = (1001.6 – 998.4) kHz = 3.2 kHz

Ex.21.10.5: Determine the audiopower necessary to amplitude modulate a 20 KW carrier to a depth of 0.5 if the efficiency of the modulator system is 70 percent.

Soln.

$$P_S = \frac{1}{2}m_a^2 P_C = \frac{1}{2} \times (0.5)^2 \times 20\text{kw} = 2.5\text{kw}$$

Efficiency of modulator system $= \frac{P_s}{P_m} = 0.7$

$$\Rightarrow \quad \frac{2.5\text{kW}}{0.7} = P_m = 3.57\text{kW}$$

21.11 Frequency modulation (FM)

In frequency modulation, the carrier-wave frequency is modulated in accordance with the modulating signal; keeping the amplitude of carrier-wave intact. The instantaneous departure of carrier frequency from the umodulated value is proportional to the instantaneous value of the modulating signal.

If the modulating signal be given as

$$e_m = E_m \cos \omega_m t \quad ...21.11.1$$

and carrier-wave be given as

$$e_c = E_c \cos\theta(t) = E_c \cos(\omega_c t + \phi_c) \quad ...21.11.2$$

then the angular frequency of modulated wave is given as

$$\omega(t) = \omega_c + K_f E_m \cos \omega_m t \quad ...21.11.3$$

This gives frequency of modulated wave as

$$f(t) = \frac{\omega_c}{2\pi} + \frac{K_f E_m}{2\pi}\cos\omega_m t = f_c + \frac{K_f E_m}{2\pi}\cos\omega_m t \quad ...21.11.4$$

and the modulated wave as

$$(e_c)_{FM} = E_c \cos\left[\omega_c t + \frac{K_f E_m}{\omega_m}\sin\omega_m t + \phi_c\right] \quad ...21.11.5$$

The change in angular frequency and frequency of the carrier-wave is obtained as

$$\Delta\omega = \omega(t) - \omega_c = K_f E_m \cos\omega_m t \quad ...21.11.6$$

$$\Delta f = f(t) - f_c = \frac{K_f E_m}{2\pi}\cos\omega_m t \quad ...21.11.7$$

Defining $m_f = \frac{K_f E_m}{\omega_m}$, we find

$$\frac{(\Delta\omega)_{max}}{\omega_m} = \frac{(\Delta f)_{max}}{f_m} = m_f \quad ...21.11.8$$

and

$$\omega(t) = \omega_c + m_f \omega_m \cos\omega_m t \quad ...21.11.9$$

$$f(t) = f_c + m_f f_m \cos\omega_m t \quad ...21.11.10$$

Thus the quantity 'm_f' is a measure of the deviation in frequency and so called "frequency modulation index". The effect of frequency modulation can be inferred from fig. 21.14.

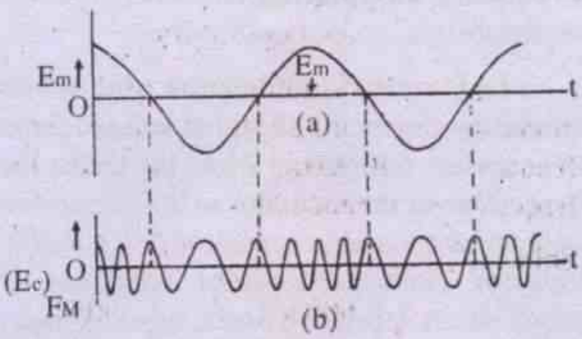


Fig. 21.14

Detailed analysis reveals that FM-wave consists of carrier component together with side frqeuencies such as $f_c \pm f_m$, $f_c \pm 2f_m$, $f_c \pm 3f_m$ ... and so on (see fig. 21.15). The amplitudes of these side frequencies are function of frequency modulation index 'm_f'. Like AM-wave the intelligence or information is contained in side frequencies. So if f_m be the modulating frequency, the bandwidth needed for transmission and reception is $2nf_m$, where 'n' isthe number of significant side frequencies.

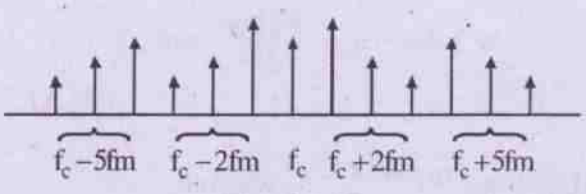


Fig. 21.15

Average power in FM-wave :

Detailed analysis show that the total average power in FM-wave remains constant for all conditions of modulation. As the modulation index increases more and more power is extracted from the carrier and redistributed among the side frequency components. As the average power does not vary with m_f, the rms value of voltage and current will also remain unattered at their respective unmodulated values.

Frequency modulating circuits *(Varactor modulator)*

In frequency modulation, the carrier frequency changes with signal voltage. So it is necessary to connect some device to the tank circuit of the oscillator so that the device will act as a variable capacitor or a variable inductor. The capacitance or inductance of this device is dependent on the signal voltage.

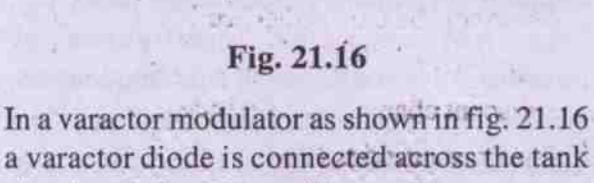


Fig. 21.16

In a varactor modulator as shown in fig. 21.16 a varactor diode is connected across the tank circuit of the RF-oscillator. The varactor diode when reverse biased its capacitance is given as

$$C_d = \frac{K}{\sqrt{E_B}}$$

where $E_B = E_o + E_m \cos\omega_m t$, is the voltage across the diode. So the instantaneous frequency of oscillation is

$$\omega = \frac{1}{\sqrt{L(C+C_d)}} = \frac{1}{\sqrt{L(C+KE_B^{1/2})}}$$

where L and C are respectively the inductance and capacitance of the tank circuit. This gives

$$f = f_c = \frac{1}{2\pi\sqrt{L(C+KE_B^{1/2})}} \text{ when } E_B = E_0$$

$$f_{max} = f_c + \Delta f = \frac{1}{2\pi\sqrt{L\left[C+K(E_o+E_m)^{-1/2}\right]}}$$

when $E_B = E_0 + E_m$

$$f_{min} = f_c - \Delta f = \frac{1}{2\pi\sqrt{L\left[C+K(E_o-E_m)^{-1/2}\right]}},$$

when $E_B = E_0 - E_m$

The capacitor C_0 in fig. 21.16 has a large value compared to the capacitance of the diode. But, however, it should have a reactance large in comparison with R even at the highest modulating frequency; so that the modulating signal is applied to the varactor diode but donot affect the tank circuit. Also the value of R is large so that the osciallator tuned circuit is not effectively loaded by the modulating source.

FM-wave transmission

Fig. 21.17 is a schematic diagram showing a simple and direct transmission of FM-wave.

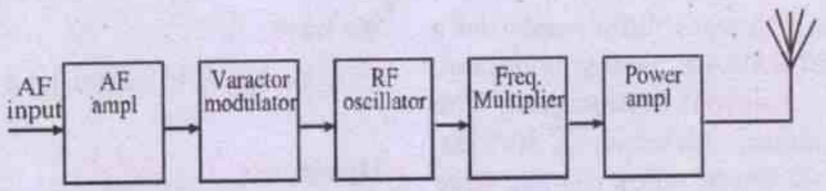


Fig. 21.17

But for frequency stability feed back principle is applied and the method as shown schematically in fig. 21.18 is adopted.

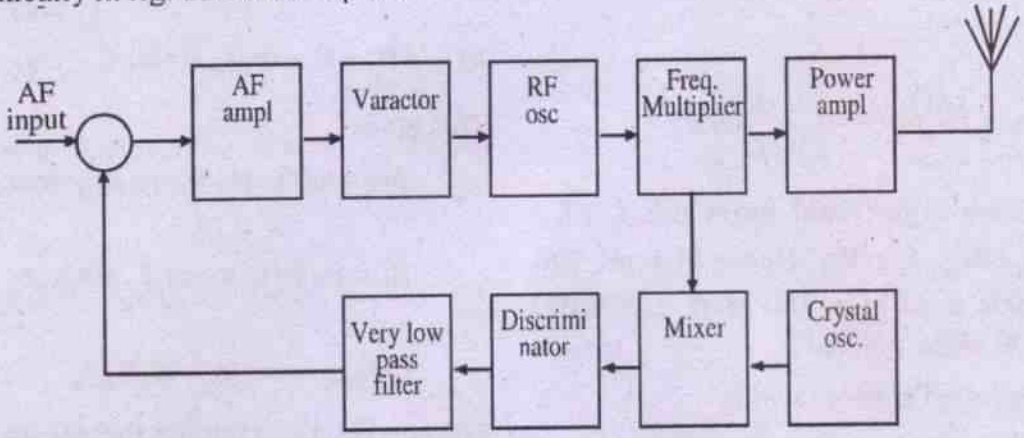


Fig. 21.18

Applications of FM :

i) In T.V. broad cast audio transmission is done by frequency modulation.

ii) In mobile and emergency services frequency modulation is used.

iii) Commercial programme in frequency range 88 to 108 MHz make use of frequency modulation.

Disadvantages of FM :

FM has few disadvantages :

i) Bandwidth of FM is about 8 times that of AM

ii) FM-reception is limited to the line of sight. Hence area of reception of FM is much less than AM.

iii) Transmitting and receiving systems in case of FM are more complex compared to those for AM. Hence FM is expensive.

Comparison of FM and AM:

AM	FM
1. Most of the power lies with carrier, hence useless.	1. More power lies with side-band frequencies, which contain the intelligence, hence useful.
2. Signal to noise ratio is low.	2. Signal to noise ratio is high.
3. Adjacent channel interference is high.	3. Adjacent channel interference is very low.
4. Power required is high.	4. Power required is low.
5. Noise level is high.	5. Noise level is low.
6. Amplitude modulation index 'm_a' ≤ 1.	6. No restriction on the value of frequency modulation index 'm_f'.

Ex.21.11.1: An FM transmitter sends out a 100 MHz carrier wave frequency modulated by a 15 kHz sinusoidal audiosignal. The maximum frequency deviation is 30 kHz. Find (i) the modulation index (ii) the three significant pairs of side frequencies (iii) the channel width required for these three side frequency pair.

Soln.

(i) $m_f = \frac{(\Delta f)_{max}}{f_m} = \frac{30kHz}{15kHz} = 2$

(ii) Three significant pairs are $f_c \pm f_m$, $f_c \pm 2f_m$, $f_c \pm 3f_m$. Hence they are 100 MHz ± 15 kHz, 100 MHz ± 30 kHz, 100 MHz ± 45 kHz.

(iii) Required channel width

= 2 x (3 x 15 kHz) = 90 kHz.

21.12 Phase modulation (PM) :

In phase modulation the instantaneous phase-angle of the carrier is changed by the modulating signal, keeping the amplitude of the carrier intact. The instantaneous departure of the phase from the unmodulated value is proportional to the instantaneous amplitude of the modulating signal and at a rate that is proportional to the modulating frequency.

If $e_m = E_m \cos\omega_m t$ ...21.12.1

and $e_c = E_c \cos(\omega_c t + \phi_c) = E_c \cos\theta(t)$ ...21.12.2

then

$$\theta_{PM}(t) = \omega_c t + \phi_c + K_P E_m \cos\omega_m t \quad ...21.12.3$$

where K_p is a constant of proportionality that determines the maximum variation in phase for a given signal intensity. Defining phase modulation index as

$m_P = K_P E_m$ = maximum deviation in phase ...21.12.4

We have

$$(e_c)_{PM} = E_c \cos(\omega_c t + \phi_c + m_p \cos\omega_m t) \quad ...21.12.5$$

Therefore

$$\frac{d\theta(t)}{dt} = \omega_{PM}(t) = \omega_c - m_p\omega_m \sin\omega_m t \quad ...21.12.6(a)$$

$$\Rightarrow f_{PM}(t) = f_c - m_p f_m \sin\omega_m t \quad ...21.12.6(b)$$

This gives

$$\Delta\omega = \omega(t) - \omega_c = -m_p\omega_m \sin\omega_m t \quad ...21.12.7$$

$$\Delta f = f(t) - f_c = -m_p f_m \sin\omega_m t \quad ...21.12.8$$

$$(\Delta f)_{max} = m_p f_m = K_p E_m f_m \quad ...21.12.9$$

The eqn.(21.12.9) implies that the maximum deviation in frequency of a PM-wave is proportional to the amplitude and frequency of the modulating signal.

Since m_p is small, eqn. (21.12.5) boils down to

$$(e_c)_{PM} = E_c\left[\cos(\omega_c t + \phi_c).\cos(m_p \cos\omega_m t) - \sin(\omega_c t + \phi_c)\sin(m_p \cos\omega_m t)\right]$$

$$= E_c\left[\cos(\omega_c t + \phi_c)\{1\} - \sin(\omega_c t + \phi_c).\{m_p \cos\omega_m t\}\right]$$

$$= E_c\left[\cos(\omega_c t + \phi_c) - m_p \sin(\omega_c t + \phi_c).\cos\omega_m t\right]$$

$$= E_c\left[\cos(\omega_c t + \phi_c) - \frac{m_p}{2}\sin(\omega_c t + \phi_c + \omega_m t) - \frac{m_p}{2}\sin(\omega_c t + \phi_c - \omega_m t)\right] \quad ...21.12.10$$

Eqn. (21.12.10) shows that only a pair of side-bands are to be considered in PM, if m_p is small. But when m_p is **appreciable** a large number of side-bands are available, separated from carrier by f_m, $2f_m$, $3f_m$ and so on.

PM-Wave transmission

The block diagram for phase modulated wave transmission is as shown below (fig. 21.19)

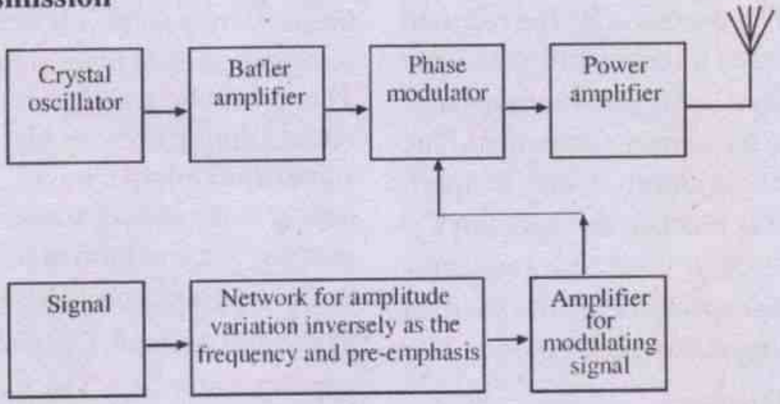


Fig. 21.19

21.13 Demodulation :

The modulated wave contains carrier wave as well as the side-band. The carrier and side-band frequencies are both in RF range. So if the modulated wave is fed directly to loud speaker no sound will be heard. Therefore it is necessary to separate out the signal from the modulated wave (side-bands of modulated wave).

The process of extracting the modulating signal from the modulated wave is called demodulation or detection.

As a token, we shall consider the detection of AM-wave. The detection of AM-wave involves two operations (i) rectification of modulated wave (ii) elimination of carrier component of the modulated wave. Naively we consider below "Diode detector" and Transistor detector.

Diode detector :

A diode detecter shown in fig. 21.20 is most widely used. The modulated wave sent by different transmitting stations are all received by the antenna A at the receiving station. By ajusting capacitor 'C' the tank circuit LC can be tuned to any desired station.

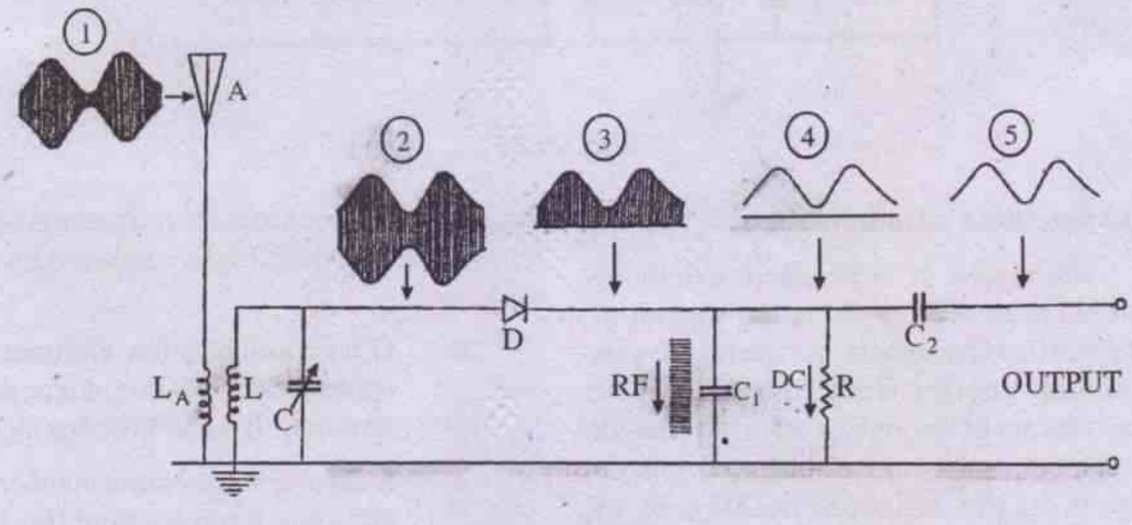


Fig. 21.20

Then the signals of the desired station are transferred to the tank circuit by the mutual induction between L_A and L. The modulated input is rectified by the diode D. The rectified signal is then passed on to the low pass fitter C_1 and R. The capacitor C_1 has low reactance and bypasses the RF carrier component. The d.c. component is shunted out through resistance R as it is blocked by capacitor C_2. The AF signal, which is of low frequency passes through the capacitor C_2. Thus the final output contains the audio-signal only.

Transistor as detector :

A transistor is essentially a square-law device. It can therefore be used as detector. As shown in fig. 21.21 the voltage divider R_1 - R_2 provides the necessary bias which keeps the transistor just at cut-off. As a result rectification takes place at base-emitter circuit. There will be small variations in the base current during positive half cycle of the input signal (modulated wave). These variations appear in the collector circuit in an amplified manner. These amplified signal appears across $R_C C_C$ combination. The RF-component is bypassed through C_C and AF component appears across R_C. The blocking capactitor C_B blocks the d.c. voltage but allows the rectified AF signal to pass on to the output.

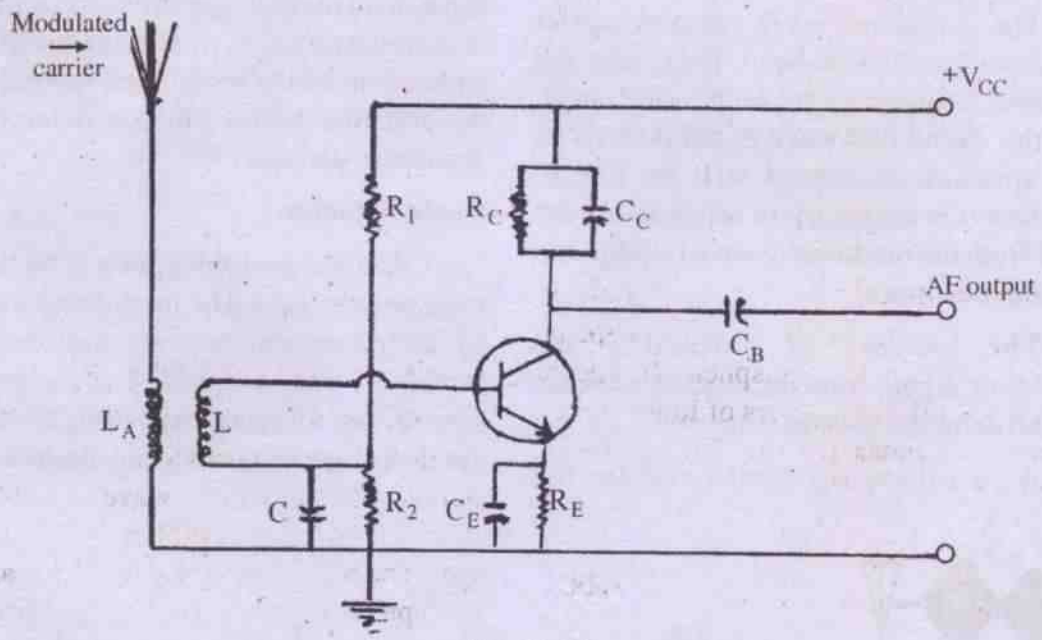


Fig. 21.21

21.14 Satellite Communication

The region of atmosphere extending from 50km to 500km above the surface of earth is called Ionosphere. As the region gets maximum energy from Sun so the air molecules are in the ionised state. As a result free electrons and positively charged ions are present in the Ionosphere. Degree of ionisation, however, is not uniform throughout the ionosphere. Accordingly the Ionosphere is divided into a number of layers or regions.

i) D-layer contains few electrons. It appears from 50-80 km, during day-time only. It is absent during night.

ii) E-layer contains more number of electrons. It spreads from 100-150 km.

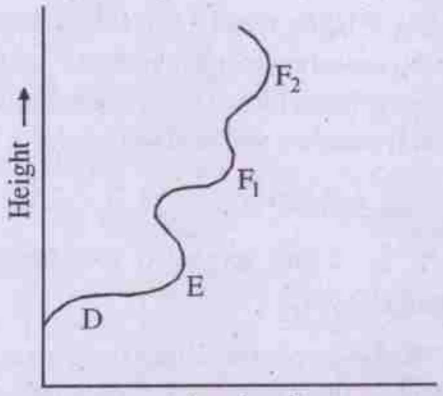


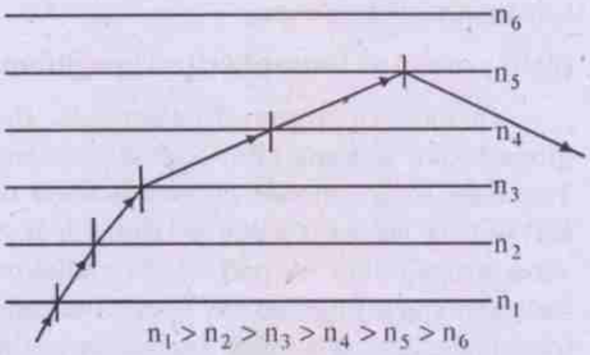
Fig. 21.22

iii) F-layer spreads between 200-400 km, with maximum concentration at 300km. But during day-time it splits into two separate layers called F_1 and F_2 around 220 km and 350 km respectively.

As Ionosphere contains charged particles, so it is a conducting medium. Hence the motion of radio-waves (e.m. waves) through it are affected by its characteristics. The speed of e.m. waves in Ionosphere is more than in ordinary atmosphere. Its speed also varies in different layers of Ionosphere. The refractive index (r.i.) of Ionosphere is given by

$$n = \sqrt{1 - \frac{81N}{f^2}} \qquad ...21.14.1$$

where N is electron density and f is frequency of e.m. wave. As we approach certain Ionospheric layer from below, the electron density gradually increases, hence r.i. decreases from lower sub-layer to upper sub-layer. Thus the motion of e.m. wave from lower sub-layer to upper sub-layer appears as going from denser to rarer medium. Therefore the radio-waves moves away from the normal (see fig. 21.13).



$n_1 > n_2 > n_3 > n_4 > n_5 > n_6$

Fig. 21.23

A stage comes when radio-waves suffer total internal reflection. This phenomenon is usually called reflection of radio waves by Ionosphere.

Modes of Propagation

There are three modes of propagation of radiowave transmitted by an antenna.

(i) Ground wave or surface wave propagation.

(ii) Sky wave or Ionospheric propagation.

(iii) Space wave propagation.

(1) Ground-wave Propagation :

Lower frequency radio-waves tend to travel along the surface of earth without attenuation. The ground wave is usable over a distance of 2000-3000 km during day and night. The transmitting and receiving antenna are kept close to the surface of earth. The ground-wave is vertically polarised, because horizontal electric field component gets short-circuited through earth and its intensity reduces quickly. Hence a vertical antenna is always necessary for ground-wave propagation. That is why antenna used by MW-broadcasting stations is always vertical.

The region around a transmitting station over which ground waves can be received satisfactorily is called primary service area. This depends on (i) power of transmitter (ii) location and type of antenna (iii) the region surrounding the antenna.

(2) Sky-wave or Ionospheric propagation:

In case of higher frequencies, the groundwave wakens after few kilometers. Therefore long distance communication of MF and HF waves is achieved through sky-wave propagation. A part of the radiation from antenna is directed sky ward. Since the Ionosphere acts as a refracting medium, the radio-waves are turned back after penetrating into the Ionosphere. The reflected ray reaches the surface of earth and is received by the receiver.

The D-layer of Ionosphere absorbs most of the radiation in the MF-range. Hence in day-time sky-wave communication of MW is not possible. But, however, during night the D-layer is absent and the E-layer reflects the MW. This is the reason why less number of MW stations are received during day time. On the otherhand HF (SW) waves penetrate through D and E layers and get reflected from F-layer. As F-layer is at a greater height, so SW cover a larger distance than the MW.

The sky-wave communication thus depends on the Ionosphere. Since condition of Ionosphere changes during day and night, so the sky-wave communication is different during day and night. Sky-wave communication is also affected by sunspot and solar flares.

In sky-wave communication total internal reflection by Ionosphere is the main cause. Since this depends on r.i. of medium and hence on frequency of incident wave (see eqn. 21.14.1). So there exists a highest frequency called **critical frequency** beyond which the radio-waves are not reflected by Ionosphere. Critical frequency for any Ionosphere layer is the highest frequency that is returned from particular layer on vertical incidence. This shall happen when r.i. n=0. This implies when $f = f_c = \sqrt{81N} = \sqrt{9N}$, n=0. For F-layer, $f_c = 10$ MHz. However since the radio waves are incident obliquely so highest frequency that is turned back is greater than the critical frequency and is given by

$$f_{max} = f_c \sec\phi \qquad ...21.14.2$$

where 'ϕ' is the angle of incidence at a particular layer.

Radiowaves travelling from antenna are incident on the Ionosphere at different angles. As the angle of incidence at the Ionosphere decreases, the distance from the transmitter at which the ray returns to the ground decreases. This behaviour continues upto a certain angle of incidence for which the distance is minimum. This minimum distance is called **skip distance**. With further decrease in angle of incidence, the distance to the point of return first increases and then near vertical incidence the wave will penetrate through the layer and not return to earth.

The **skip distance** represents the minimum distance from the transmitter at which a sky-wave of given frequency will be returned to earth by the Ionosphere. Hence a receiver placed within the skip-zone will not receive the signal by sky-wave.

On increasing the frequency the skip distance increases and on decreasing the frequency skip distance decreases. Hence corresponding to a particular skip distance there is a maximum frequency that can be used and this frequency is called **maximum usable frequency**.

As the skip distance varies in day and night, so in point-to-point HF communication like police-wireless, the transmitters change to a lower frequency after the sunset.

(3) Space wave propagation :

When frequency exceeds 30 MHz, ground-waves attenuate quickly and sky-waves are not returned back by Ionosphere. Hence communication is achieved by space-waves in the frequency range beyond 30 MHz ie VHF, UHF, microwaves etc. cellular phaone or mobile phones use the space-wave propagation mode.

For space-wave propagation the transmitting antenna is well removed from the earth so that ground does not affect it. This distance is actually measured in terms of the wave length of the radiation. For example for radiowave of frequency 300 MHz placed at a height of 5m may be physically closed to the ground but actually in terms of wavelength it is removed by 5λ from the ground. Hence electrically it is removed from the earth.

The radiowaves travel through the troposphere in practically straight path. Therefore obstacles like mountains, trees, high buildings etc. affect the propagation. It is essential that both the transmitting and receiving antennas **see** each other. Therefore the range of space wave communication depends on the height of both the transmitting and receiving antenna. The range 'd' is given by

$$d = \sqrt{2h_1} + \sqrt{2h_2} \qquad ...21.14.3$$

where h_1, h_2 are heights of transmitting and receiving antenna respectively above the sea level.

Since the height of antenna cannot be increased beyond certain limit, so range of space-wave communication is limited to a distance of 80-100 kms. These days radio-waves from earth are directed to geo-stationary satellites which in turn beams it over extensive areas over the earth.

21.15 Satellite Communication

Microwave radio systems on the ground (or in a satellite) propogate in a line of sight mode. Due to curvature of earth, the range of microwave transmission on earth is very small and limited (≈ 50 km). Therefore, such transmission requires repeater stations at about 50 kms interval. This becomes expensive as well as inconvenient. To overcome this now-a-days satellite communication is used.

A Communication satellite is a space craft in orbit around the earth which carries on board microwave receiving and transmitting equipment capable of relaying signals from one point on earth to other points.

Since all practical Geo-stationary satellite orbits are above the ionosphere, microwave frequencies must be used to penetrate the inonosphere.

Also, microwave frequencies are required to handle the wideband signal encountered in present-day communication net work.

A communication satellite receives the waves transmitted from earth station /stations and retransmits the same (at different frequencies) back to another earth station / stations.

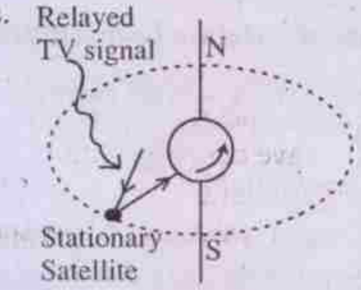
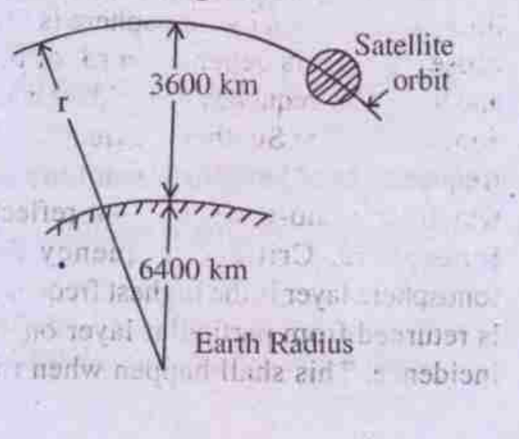


Fig. 21.24



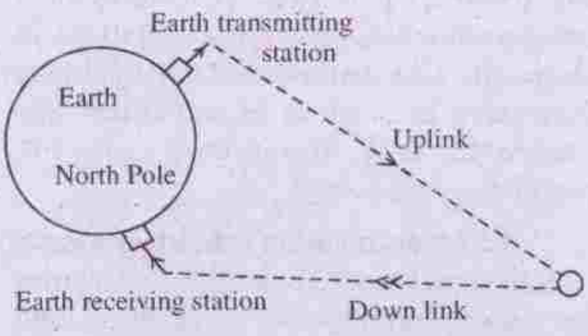


Fig. 21.26

In satellite communication system, a message signal is transmitted from an earth station via an uplink to satellite, amplified by electronic circuitry on board in the satellite and then transmitted from the satellite via a downlink to another earth station as shown in Fig. 21.26. The most popular frequency band for satellite communication is 6 GHz for the uplink and 4 GHz for the downlink.

In order to provide microwave communication over the entire surface of earth at least three geostationary satellites are required. Each geostationary satellite is 120^0 apart from the other.

Advantages of Satellite Communication :

i) A single relay station can cover large area

ii) It is very reliable

iii) The cost of communication is independent of distance of coverage.

iv) This can be utilised for mobile communication.

v) It is more accurate for search and navigation.

Disadvantages of Satellite Communication

i) The cost of placing a satellite is very high.

ii) The zone for geostationary satellite is crowded and at best 360 satellites can be accomodated.

iii) In telephone conversation, the response is not instant.

iv) It is not good from security point of view as it is open to all.

v) If any part of the system becomes faulty due to environmental conditions it is difficult to apply corrective measures.

Basic Idea about internet:

Internet has now become an essential part of our everyday life. It is a revolution in the field of exchange of ideas and informations. With the internet it is possible to access almost any information; Communicate with anyone else in the world or do much more. Let us discuss what actually internet consist of.

Internet is the global system of interconnected Computer networks that use the Internet Protocol suit (IP); to link billions of devices worldwide. It is actually a network of networks that connect millions of Computers globally so that any computer can communcate with anh other computer, so long as both of them are connected to the internet. For such facilities, there is a global network of physical cables which includes copper telephone wires, TV cables, fibre optic cables etc. Even wireless communications like wi-fi and 3G/4G rely on these cables to access internet.

A Brief History: The first stable link between multiple computers through the ARPANET occured in 1969. (ARPANET can be regarded as the predecessor to the Internet and was created by US defenses "Advanced Research Project Agency"(ARPA). It was a great success, but was limited only to certain academic and research organisation.

In 1983 ARPANET adopted the standard IP(Internet Protocol). Prior to this Various Computer networks did not have standard way to communicate with each other. Through IP, different computers on different networks are allowed to link with each other. This can be regarded as the begining of Internet. In March 1989 Tim Berners and Lee invented worldwide web(w.w.w).

Internet and world wide web are not synonymous. The w.w.w. is the primary application that we use in the internet. In other words it is a way of accessing information through internet. w.w.w. or simply called the web is a collection of different websites, which can be accessed through internet.

A website is composed of different texts, images and other resources. Once a computer is connected to the internet (i.e. computer is online) we can access or view the contents of the website, using web browser (such as Chrome, firefox, internet explorer etc.). Whenwe visit a website, the computer sends request to the server (where the website is stored). The server then retrieves the website and sends the required data back to the computer.

Uses:

The internet can be accessed almost everywhere by numerous means; including mobile internet devices, data cards, cellular routers etc. Some of the applicationsof internet are listed below.

i) Internet is used for sharing of Scientific or other knowledges of academic interest.

ii) E-mail is an important Communication service available in the internet, through which messages or files can be transferred electronally.

iii) We can use internet to access news to plan or book tour packages or persue our personal interests.

iv) Internet is used in banking services (net banking, e-banking) and online business for sell or purchase of goods.

v) There are also several social networking sites connecting people through the globe.

Mobile Telephony:

Now a days we are all familiar with mobile phones for which we are never out of touch with friends and relatives. Mobile phone works in a way completely different from land lines.

Mobile phone is a portable telephone that can make and receive calls through transmitting towers even when the user moves around rather than being fixed in one location. Radiowaves are used to transfer signals to and from the mobile phones. Most modern mobile telephone services use a cellular network architecture+ and for this reason mobile phones are called cell phones.

In addition to telephony, the cell phones we use now-a-days can support a variety of other services such as text messaging (SMS); MMS(multi-media message, service), e-mail, internet access, blue tooth, infrared, digital photography, computing etc. The phones with these added facilities are called smart phones.

How a cellphone works:

A cellphone is essentially a two way radio consisting of a radio transmitter and a radio receiver. When we speak into a cellphone, it converts the voice into electrical signal. This is done by a tiny microphone fitted into the handset. The signal is modulated into a radio frequency carrier wave and is transmitted to the nearest cellphone tower by means of a transmitter. The tower receives the signal and passes them on to its base station. From the base station, the calls are sent to their destination. The cellphone at the receiving end converts the signal into voice or anything originally sent.

+ For the operation of mobile phones, a given area (say a city) is divided into small cells. Each cell has a base station that consists of a tower and radio equipments.

GPS:

GPS is the abbreviation of Global positioning system. It is a space based nagivation system that provides location and time information in all weather conditions anywhere on or near the earth. The GPS operates independently of any telephonic or internet reception. GPS was created and realized by the VS department of defense and was originally run by 24 satellites. It became fully operational in 1995. A few other countries are working on new systems of their own, but right now US system is the only one used widely around the world.

The Current GPS consists of three major segments: a space segment, a control segment and a user segment. The space segment is composed of 24 to 32 satellites in medium earth orbit. These satellites orbit around the earth in approximately circular orbits at an altitude of 22,200km, with a time period equal to one half of a sideral day(11h, 58 min); So that the satellites pass over almost the same location everyday. The orbits are so arranged that at least six satellites are always within the line of sight from almost everywhere on the earths surface.

The control system is composed of a master control station; an alternate master control station and a host of dedicated and shared ground antenas and monitor stations.

The user segment is composed of hundreds of thousands of millitary and millions of Civil, commercial and scientific users of the standard positioning system. The GPS receivers are composed of an antenna (tuned to the frequencies transmitted by satellites), receiver-processors and a highly stable clock. There is also a display unit fitted to the receiver to provide location and speed information.

GPS satellites continually broadcast signals from space and the GPS receivers use these signals to calculate its three dimensional location (latitude, longitude and altitude) along with current time.

Applications: GPS provides critical positioning capabilities to military, civil, scientific and commercial astronomers. It is also used for disaster reliefs, emergency services, weather forecasting and surveying. Another common application of GPS is to get information about particular route to a place of internet.

MODEL QUESTIONS

A. Multiple choice type questions :

1. In e.m. wave angle between $\vec{E}$ and $\vec{B}$ is
   a) 120^0 b) 90^0
   c) 180^0 d) 45^0

2. The phase difference between $\vec{E}$ and $\vec{B}$ in e.m. wave is
   a) 120^0 b) 90^0
   c) 0^0 d) 45^0

3. The speed of e.m. wave in empty space is
   a) $\frac{1}{\sqrt{\mu_o \varepsilon_o}}$ b) $\sqrt{\mu_o \varepsilon_o}$
   c) $\frac{1}{\mu_o \varepsilon_o}$ d) $\sqrt{\frac{\mu_o}{\varepsilon_o}}$

4. If speed of e.m. wave be C then the ratio of the strengths of $\vec{E}$ and $\vec{B}$ in e.m. wave is
   a) C b) $\frac{1}{C}$
   c) $\frac{1}{\sqrt{C}}$ d) $\sqrt{C}$

5. If λ_1 and λ_2 be wavelengths of γ-rays and X-rays respectively then
   a) $\lambda_1 > \lambda_2$ b) $\lambda_1 < \lambda_2$
   c) $\lambda_1 = \lambda_2$ d) None of the above

6. Frequency modulation gives
   a) less noise b) more noise

7. In AM useful power is carried by
   a) side band b) carrier

8. In cell phones modulation used is
   a) AM b) FM

9. In FM useful power is carried by
   a) carrier b) sideband

10. Detection is done in
    a) receiver b) transmitter

B. Very short answer type questions :

1. What is the angle between $\vec{E}$ and $\vec{B}$ in e.m. wave ?
2. What is the phase difference between $\vec{E}$ and $\vec{B}$ in e.m. wave ?
3. Which has greater frequency-infrared or UV radiations ?
4. What is the order of magnitude of wave length of γ-rays ?
5. Write the wave length range of visible region.
6. Above what frequency radiation of electrical energy is possible ?
7. What is the percentage of power transmitted in AM ?
8. In cell phones what type of modulation is used ?

9. What is the band width in AM ?
10. Which modulation gives less noise ?
11. Which carries useful power in AM ?
12. What is sky wave propagation ? [CBSE 2004]
13. What is ground wave propagation ? [CBSE 2009]
14. What is space wave propagatio ? [CBSE 2010]
15. Give two examples of communication system which use space wave mode. [CBSE Sample Paper]
16. What is meant by band width of information signal ? [CBSE Sample Paper]
17. Name the types of modulation in analog communication. [CBSE Sample Paper]
18. Why is frequency modulation preferred over amplitude modulation ? [CBSE 2007]
19. What type of modulation is required for television broadcast ? [CBSE Sample Paper]
20. Why high frequency carrier waves are employed for transmission ? [CBSE Sample Paper]

C. Short Answer type questions :

1. Why is e.m. wave so named ?
2. What is the evidence in support of the theory that light is e.m. in nature ?
3. What should be the length of the antenna for efficient transmission ?
4. How does power radiated by a dipole depend on frequency ?
5. Mention two important properties of e.m. wave.
6. Define modulation.
7. Mention the types of modulation.
8. Define 'modulation index' in AM.
9. Define 'modulation index' in FM.
10. Write the expression for power carried by AM carrier wave.
11. By what percentage will the transmission range of a TV tower be affected when the height of the tower is increased by 21%. [CBSE 2009]
12. What is the range of frequencies used for TV transmission ? What is common between these waves and light waves ? [CBSE Delhi]

D. Unsolved Problems :

1. The peak-to-peak value of AM voltage has a maximum value of 5V and a minimum value of 1V. If the depth of modulation be 0.7, what is the amplitude of the unmodulated carrier wave ?

2. A 1KW carrier is amplitude modulated by an audio signal. If the modulation is 60%, determine the total power of the modulating wave.

3. In AM broadcast transmitter radiates a power of 50KW. If the modulation factor is 0.8, calculate the carrier power and power of the side frequencies.

4. A 80 MHz carrier is frequency modulated, the modulation index being A. The frequency of the information signal is 10 kHz. What is the maximum frequency deviation ?

5. A TV transmitter has a range of 50 km. What is the height of the TV transmission tower ? Radius of earth $R_e = 6.4 \times 10^6$ m. [CBSE 2004]

6. A TV tower has a hieght of 500 m at a given place. If radius of earth is 6400 km, what is its coverage range ?

[CBSE 2004]

E. Long answer type questions :

1. Describe complete e.m. spectrum from γ-rays to longwaves.

2. What is Hertzian Oscillator ? Describe how it produces e.m. wave.

3. Why modulation is required ? Name the types of modulation. Describe various aspects of AM.

4. Describe frequency modulation.

5. What do you mean by detection ? Describe how detection is achieved ?

F. True-False Type Questions :

1. In cell phones modulation used is AM.

2. In FM useful power is carried by side band.

3. Frequency modulation (FM) gives less noise.

4. Man made noises are amplitude variations

5. The distance upto which a TV signal can be transmitted depends upon radius of earth only.

G. Fill - in - Blank Type Questions :

1. In cell phones modulation used is ____.

2. Detection is done in ____.

3. In AM useful power is carried by ____.

4. In radio transmission the medium of transmission is ______.

5. In TV transmission picture signal is ____ modulated.

H. Correct the following sentences :

1. In e.m wave angle between E and B is 120°.

2. In e.m wave phase difference between E and B is 90°.

3. The speed of e.m wave is given as

4. In cell phones modulation used is AM.

5. In FM useful power is carried by carrier wave.

6. In TV transmission picture signal is frequency modulated.

7. Infrared radiation has greater frequency than UV radiation.

ANSWERS

A. Multiple choice type questions :

1. (b) **2.** (c) **3.** (a) **4.** (a) **5.** (b) **6.** (a) **7.** (a) **8.** (b) **9.** (b) **10.** (a)

B. Very short type questions :

1. 90^0

2. No phase difference

3. UV

4. 10^{-13} m

5. 4000 A^0 to 7000 A^0

6. 20 kHz

7. 33%

8. FM

9. $2f_m$

10. FM

11. sideband

C. Short Answer type questions :

1. It contains electric and magnetic fields

2. see text

3. $\lambda / 4$

4. see text $\left(P_t = p_o^2\omega^4 / 12\pi\varepsilon_o c^3\right)$

5. see text

6. see text

7. see text

8. see text

9. see text

10. see text

D. Unsolved Problems :

1. 1.43 V

2. 1.18 Kw

3. 37.88 KW, 12.12 KW

4. 40 kHz **5.** 195.3m **6.** 80 km

F. **1.** False **2.** True **3.** True **4.** True **5.** False

G. **1.** FM **2.** receiver **3.** side band **4.** space **5.** amplitude.

22

DIGITAL ELECTRONICS

22.1 Introduction:

ANALOG AND DIGITAL SIGNAL

22.1. Introduction:

Analog and digital signals are used to transmit information, usually electric signals. In both these techniques, the information, such as audio or video, is transformed into electric signals.

22.2 Analog Signal:

An **Analog Signal** is any continuous signal for which the time varying feature (variable) of the signal is a representation of some other time varying quantity i.e. analogous to another time varying signal (voltage/current).

because a signal varies over time, it is helpful to plot it on a graph where time is plotted on the horizontal X-axis and voltage/current on the vertical Y-axis. This graph of an Analog signal should be smooth and continuous as shown in fig. 22.1 below.

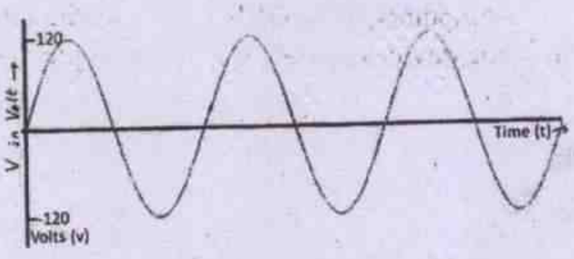


Fig. 22.1

While these signals may be limited to a range of maximum and minimum values, there are still an infinite number of possible values within that range. For example, the Analog voltage coming out of your wall socket might be clamped between -120V to +120V, but as you increase the resolution more and more, you discover infinite number of values that the signal can actually be (like 64.4V, 64.42V, 64.424V ... etc). Thus an Analog signal is transformed into electrical signals of varying amplitude.

Analog Circuits:

Most of the fundamental electronic components- *resistors, capacitors, inductors, diodes, transistors and operational amplifiers* - are all inherently analog. Circuit built with a combination of solely these components are usually analog.

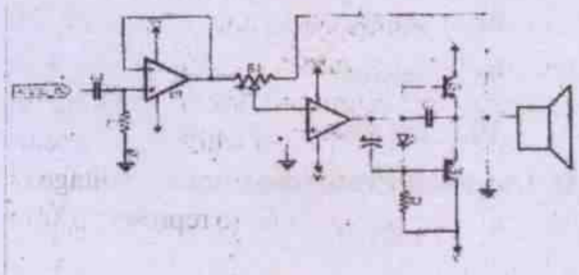


Fig. 22.2

An analog circuit can be very simple like two resistors combining to make a *voltage divider.*

Example:-

Video and audio transmissions are ofter transformed or recorded using analog signals. The composite video (see fig. 22.3) coming out of an old RCA jack for example is a coded analog signal ranging between 0 and 1.073 V.

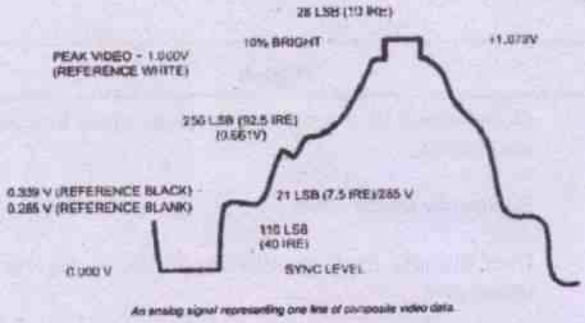


Fig. 22.3

Tiny changes in the signal have a huge effect on the colour or location of the video. Human voice,motion of wall clock, pure audio signals and alternating sinusoidal current are analog. The signal coming out of a microphone is full of analog frequencies and harmonics, which combine to make a beautiful music.

22.3 Digital Signal:

A **digital signal** uses discreate (discontinuous) values. Digital signals must have a finite set of possible values. Although digital representations are discrete, the information represented can be either discrete, such as numbers or letters, or continuous, such as sounds, and other measurements of continuous systems. Most commonly a digital signal uses binary values (0.1) to represent an information. '0' represents low (0) voltage or cut off voltage or off circuit while '1' represents high (say +5V) voltage or saturation voltage or on circuit. The voltage used to represent a '0' or

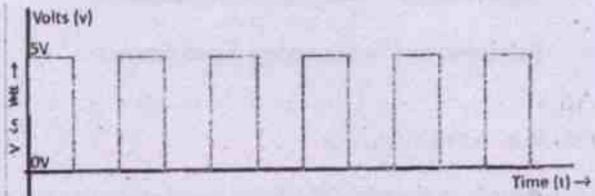


Fig.22.4

'1' are called logic levels. The timing graphs of these signals look like square waves (see fig.22.4).

Or a digital signal might be a discrete representation of an analog waveform, when viewed from a far distance. But when you look

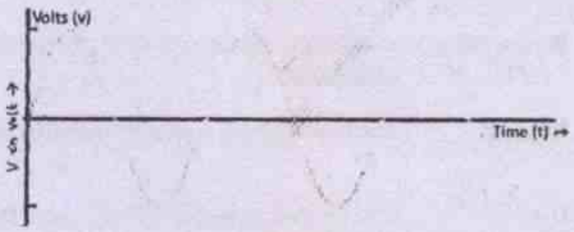


Fig. 22.5

from a close distance there are tiny discrete steps (see fig.22.5).

This is the big difference between analogand digital signals.

Digital Circuits:-

These circuits are usually made of a combination of transistors and logic gates (as discussed in sec. 22.10), and at higher levels, microcontrollers or other computing chips. These operate in digital realm.

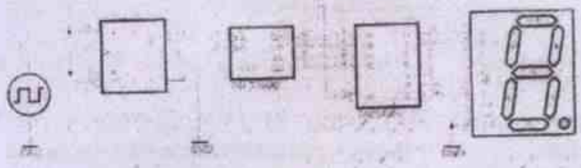


Fig.22.5

Example:

Computers, CDs, DVDs and other digital electronic devices operate on this principle.

Analog versus Digital Signal Comparison Chart:

	Analog	Digital:
Signal	Analog signal is a continuous signal which represents physical measurements.	Digital signals are discrete time signals generatee by digital modulation.
Waves	Denoted by sine waves	Denoted by square waves
Representation	Uses continuous range of values to represent information	Uses discrete to discontinuous values to represent information.
Example	Human voice in air, analog electronic devices.	Computers, CDs, DVDs and other digital electronic devices.
Uses	Can be used in analog devices only. Best suited for audio and video transmission.	Best suited for Computing and digital electronics.
Applications	Thermometer	PCs, PDAs
Bandwidth	Analog signal processing can be done in real time and consumes less bandwidth.	There is no guarantee that digital signal processing can be done in real time and consumes more bandwidth to carry out the same information.
Memory	Stored in the form of wave signal	Stored in the form of binary bit.
Power	Analog instrument draws large power	Digital instrument draws only negligible power
Cost	Low	Cost is high and not easily portable
Impedance	Low	High order of 100 megaohm
Errors	Analog instrument usually have a scale which is cramped at lower end and give considerable observational errors.	Dital instruments are free from observational like parallax and approximation errors.
Technology	Analog technology records waveforms as they are.	Samples analog waveforms into a limited set of numbers and records them.
Data transmissions	Subjected to deterioration by noise during transmission and write/read cycle.	Can be nose immune without deterioration during transmission and write/read cycle.
Response to Noise	More likely to get affected reducing accuracy	Less affected since noise response are analog in nature
Flexibility	Analog hardware is not flexible.	Digital hardware is flexible in implementation.
Users	Can be used in analog devices only. Best suited for audio and video transmission.	Best suited for Computing and digital electronics.
Applications	Thermometer	PCs, PDAs
Bandwidth	Analog signal processing can be done in real time and consumes less bandwidth.	There is no guarantee that digital signal processing can be done in real time and consumers more bandwidth to carry out the same information.
Uses	Can be used in analog signal processing can be done in real time and consumes less bandwith	Best suited for computing and digital electronics.

Memory	Stored in the form of wave signal	Stored in the form of binary bit
Power	Analog instrument draws large power	Digital instrument draws only negligible power
Cost	Low cost and portable	Cost of high and not easily portable.
Impedance	Low	High order of 100 megaohm
Errors	Analog instrument usually have a scale which is scamped at lower end and give considerable observational errors.	Digital instruments are free from observational errors like parallax and approximation errors.

22.4 Number system:

A digital circuit expresses the values in digits such as **'1'** (for **on state** or **high value**) and **'0'** (for **off state** or **low value**). Thus here only two numbers **'1'** and **'0'** are used; hence it is also called binary number system. So it is necessary to discuss about the number system.

There are four types of number systems in common use. They are (i) **Decimal** (ii) **Binary** (iii) **Octal** and (iv) **Hexadecimal** system.

A. Decimal number system:

In a decimal number system 10 symbols or coefficients: 0, 1, 2, 3, 4, 5, 6. 7. 8. 9 are used to write any number. A number in this system is built from blocks valued at 10^0, 10^1, 10^2, 10^3, 10^4, 10^5 ....etc. as one proceeds from left of the **decimal point** in a number. Similarly as one proceeds to the right of **decimal point** the weight for each position or the multiplying factors will be in order 10^{-1}, 10^{-2}, 10^{-3} .....etc.. Thus in this system the **base (radix)** about which the system is built is 10.

Table 22.4

Summary of Number Systems

Particulars	Decimal System	Binary System	Octal System	Hexadecimal System
Radix or base (N)	10	2	8	16
Symbols or Coefficients (0,1,....N)	0.1.2.3......8.9	0.1	0.1.2....6.7	0.1.2....8.9. A,B,C,D,E,F
Positional weight for the part to the left of the system point (integer part)	10^0. 10^1. 10^2....	2^0. 2^1. 2^2........	8^0. 8^1. 8^2........	16^0.16^1.16^2........
Positional weight for the part to the right of system point (the fractional point)	10^{-1}. 10^{-2}.10^{-3}....	2^{-1}. 2^{-2}.2^{-3}.....	8^{-1}.8^{-2}.8^{-3}.......	16^{-1}. 16^{-2}. 16^{-3}......

For illustration, consider the following examples

i) $5 = 5 \times 10^0$

ii) $25 = (2 \times 10^1) + (5 \times 10^0)$

iii) $925 = (9 \times 10^2) + (2 \times 10^1) + (5 \times 10^0)$

iv) $7835.45 = (7 \times 10^3) + (8 \times 10^2) + (3 \times 10^1) + (5 \times 10^0) + 4 \times 10^{-1}) + 5 \times 10^{-2})$

B. Binary number system:

The Binary number system uses only two numbers **'0'** and **'1'.** The **base (radix)** is 2. A number in this system is built from blocks valued at 2^0. 2^1, 2^2 ....etc. to the left of the **binary point** in a number. Similarly, as the digit is shifted to the right of the **binary point**, the positional weights are in order 2^{-1}, 2^{-2}, 2^{-3} ....etc.

For illustration consider the following examples.(the suffix indicates the system)

(i) $(1)_2 = 1 \times 2^0 = (1)_{10}$

(ii) $(10)_2 = 1 \times 2^1 + 0 \times 2^0 = (2)_{10}$

(iii) $(101)_2 = 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = (5)_{10}$

(iv) $(11.11)_2 = 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} = 2+1+0.5+0.25 = (3.75)_{10}$

C. Octal number system:

The Octal number system uses 8 symbols: 0, 1, 2, 3, 4, 5, 6, 7. The **base (radix)** is 8. The multiplying factor or positional weight are in the order 8^0, 8^1, 8^2, ...etc for the digits to the left of the **octal point**; and 8^{-1}, 8^{-2}, 8^{-3}...etc. for the positions to the right of the **octal point** in a number.

For illustration, consider the following examples:

(i) $(10)_8 = 1 \times 8^1 + 0 \times 8^0 = 8 + 0 = (8)_{10}$

(ii) $(32)_8 = 3 \times 8^1 + 2 \times 8^0 = 24 + 2 = (26)_{10}$

(iii) $(256)_8 = 2 \times 8^2 + 5 \times 8^1 + 6 \times 8^0 = 128 + 40 + 6 = (174)_{10}$

(iv) $(25.8)_8 = 2 \times 8^1 + 5 \times 8^0 + 8 \times 8^{-1} = 16 + 5 + 1 = (22)_{10}$

(v) $(26.4)_8 = 2 \times 8^1 + 6 \times 8^0 + 4 \times 8^{-1} = 16 + 6 + 0.5 = (22.5)_{10}$

D. Hexadecimal number system:

This system uses 16 symbols: 0, 1, 2, 3, ...9, A, B, C, D, E, and F. The alphabets A, B, C, D, E,& F are equivalent of decimal numbers 10, 11, 12, 13, 14, & 15, respectively. The **base (radix)** of the block is 16. The positional weights are in the order 16^0, 16^1, 16^2 ...etc for the positions to the left of the **hexadecimal point**, and 16^{-1}, 16^{-2}, 16^{-3} ...etc. for the positions to the right of the **hexadecimal point.** For illustration, consider the following examples:

(i) $(0001)_{16} = 0 \times 16^3 + 0 \times 16^2 + 0 \times 16^1 + 1 \times 16^0 = 0 + 0 + 0 + 1 = (1)_{10}$

(ii) $(000A)_{16} = 0 \times 16^3 + 0 \times 16^2 + 0 \times 16^1 + 10 \times 16^0 = 0 + 0 + 0 + 10 = (10)_{16}$

22.5 Conversion of a number from one system to another system:

We discuss here the conversion from decimal to binary and vice versa; as the others will follow in a similar fashion.

(i) **Decimal to Binary:**

The following examples will give sufficient idea about the conversion.

(a) Decimal number = 27

2	27		
2	13	1	↑ LSB
2	6	1	
2	3	0	
2	1	1	
	0	1	MSB

MBS- denotes maximum signigicant bit; LSB - denotes lowest significant bit

Thus $(27)_{10}=(11011)_2$

(a)Decimal number = 15.24

Integer part

2	15	
2	7 1	.... LSB ↑
2	3 1	
2	1 1	
	0 1	.... MSB

Decimal Part

$0.24 \times 2 = 0.48$	carry
$0.48 \times 2 = 0.96$ ...	0
$0.96 \times 2 = 0.92$ ...	0
$0.92 \times 2 = 0.84$ ...	1
$0.84 \times 2 = 0.68$ ...	1
$0.68 \times 2 = 0.36$ ...	1
$0.36 \times 2 = 0.72$ ...	1
0.....................0	↓ ....LSB

Binary equivalent = $(1111)_2$ Binary equivalent = $(0.0011110)_2$

So $(15.24)_{10} = (1111.0011110)_2$

(i) Binary to Decimal:

(a) $(1101)_2 = 1 \times 2^3 + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0 = 8 + 4 + 0 + 1$

$= (13)_{10}$

(b) $(11011.1101)_2 = 1 \times 2^4 + 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 + 1 \times 2^{-1} + 1 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4}$

$= 16 + 8 + 2 + 1 + 0.5 + 0.25 + 0.0625 = (26.8125)_{10}$

22.6 Binary Arithmetic Operation:

(A) Addition:

For addition of binary number the following rules need to be considered.

(a) Rule 1: $0 + 0 = 0$

(b) Rule 2: $0 + 1 = 1$

(c) Rule 3: $1 + 0 = 1$

(d) Rule 4: $1 + 1 = (10)_2$

Or $1 + 1 = 0$ with a carry of 1 to the immediate left column.

This procedure as in decimal system is continued from right to left.

Illustrations:

(i)
```
  1 0 1    ....  1st column 1+0 = 1
+ 1 1 0    ....  2nd column 0+1= 1
1 0 1 1    ....  3rd column 1+1= 10 (0,
                       with carry of 1)
```

(ii)
```
  1 1 .... 1st column 1+1 = 10 (0, with
                              carry of 1)
+ 1 1 1 .... 2nd column 1+1+1 = 1 0
1 0 1 0                        +   1
                 1 1 (i.e. 1 with carry of 1)
```

(iii)
```
    1 1 0 0 1 1
  + 1 0 1 1 0 1
  1 1 0 0 0 0 0
    1 1 1 1 ......... carried over digits
```

(iv)
```
    1 1 0 1 1
  + 1 0 1 1 1
  1 1 0 0 1 0
```

(A) Subtraction:

The following rules need to be considered during a binary subtraction.

(a) Rule 1: $0 - 0 = 0$

(b) Rule2: $1 - 0 = 1$

(c) Rule 3: $1 - 1 = 0$

(d) Rule 4: $0 - 1 = 10 - 1 = 01$ or 1

(This is done by borrowing 1 from the column to the left of the minuend)

Rule 4 seems strong but makes sense if we remember that $(10)_2 = (2)_{10}$ and we only deal with binary numbers '0' and '1'. Again using rule 4 it should be remembered that by borrowing '1', from the next left column the remaining of that minuend is reduced by 1. If the next column also happens to contain '0', it is changed to 1 and the succeeding is made '0's and the minuend are changed to '1' until a '1' is found, which is then changed to '0'.

Illustrations:

```
          0 10
(i)       1 1 0 1
        - 1 0 1 1
        ---------
          0 0 1 0

           1 10
(ii)      1 0 0 1
        - 0 1 1 1
        ---------
          1 0 1 0

         10  10 10
(iii)     1 0 1 1 0
        - 0 1 0 1 1
        -----------
          0 1 0 1 0
```

(C) Multiplication:

The procedure followed is identical to decimal multiplication, that is a series of shifts to the left and addition as per binary rules for addition. Following rules shall simplify the process.

(a) Rule 1: $0 \times 0 = 0$

(b) Rule 2: $0 \times 1 = 0$

(c) Rule 3: $1 \times 0 = 0$

(d) Rule 4: $1 \times 1 = 1$

The above is summarized in the Table No. 22.2 given below.

Table 22.2

x	0	1
0	0	0
1	0	1

Illustrations:

(i) $(101)_2 \times (1110)_2$ =

```
      1 1 1 0
    ×   1 0 1
    ---------
      1 1 1 0
    0 0 0 0
  1 1 1 0
  -------------
  (1 0 0 0 1 1 0)₂
```

(ii) $(111.01)_2 \times (1.10)_2$ =

```
      1 1 1.0 1
    ×     1.1 0
    -----------
        00000
      1 1 1 0 1
    1 1 1 0 1
  -----------------
  (1 0 1 0. 1 1 1 0)₂
```

(D) Binary Division:

The procedure is identical to decimal division. The divisor is subtracted from the dividend and a '1' is placed in the quotient. If the divisor cannot be subtracted, a zero appears in the quotient.

This continues until all numbers in the dividend have been brought down and the subtraction tallied in the quotient. The following rules are of use:

(a) Rule 1: $0 \div 1 = 0$

(b) Rule 2 : $1 \div 1 = 1$

(c) Rule 3: The divisor is subtracted from the dividend and a '1' is placed in the quotient. The subtraction is done according to binary rule.

Illustration:

```
(i)  1101 | 1011011 | 111
            1101
          -------
           10011
            1101
          -------
            1101
            1101
          -------
               0
```

Thus $(1011011)_2 \div (1101)_2 = (111)_2$

(ii)

```
101 | 11001 | 101
    | 101   |
    --------
      0010
       000
      -----
       101
       101
      -----
         0
```

Thus $(11001)_2 / (101)_2 = (101)_2$

22.7 Disadvantages of binary system:

The binary system has the following two disadvantages:

(a) It is difficult to deal with large binary numbers consisting of long string of binary '0' and '1' and to convert them into decimals.

(b) To express a large binary number in terms of decimal number one has to go through a lengthy conversion process using power of '2', since the direct substitution of decimal is not possible.

22.8 Logic:

Logic is a process of establishing a proposition by reasoning or arguments. Symbolic logic or **Boolean algebra** was developed by Google Boole in 1954 as a method of establishing the validity of thought or reason. His concept was that if simple proposition of basic logic could be represented by precise symbols, then such propositions could be expressed by algebraic equations.

Boolean algebra deals with variables that assume only two different values. In order to take mathematical or logical approach to a problem, we assign two possible values '0' and '1' to the variables and let the '0' and '1' represent two possibilities of the particular problem. Since a formal logic is based on the concept that a statement is either true or false, the ON or OFF state of a switch can be a convenient analogy. For example '0' might represent the OFF state/ Open switch/ False/No state while '1' represents the ON state/ Closed SWITCH/ True/Yes state.

22.8(i) Gate :

A switch that can be opened or closed is known as a **gate** and two positions of the switch (**ON** and **OFF**) can be regarded as **gate open** or **gate closed**. A vacuum tube or a semiconductor device can also be used as an **electronic switch** or **electronic gate**.

22.8(ii) Logic gate:

A number of electronic gates may be combined to form a switching circuit. Such a switching circuit can be so designed that it may have number of inputs but output will be obtained only when the inputs meet some specific conditions. Under all other conditions output remains zero.

Thus the switching circuit whose output is determined by the input satisfying certain conditions or logic is called a *logic gate* or *logic circuit*.

Hence a digital circuit with one or more input signals but only one output signal is called a logic gate. These logic gates are the basic building blocks from which most of the digital systems are built up.

22.8(iii) Types of Logic gates:

Different conditions for the input signals give different types of logic gates. There are **four** basic logic circuits (gates) used as building blocks in a complex digital system, such as digital computer, counting and scaling machines, communication systems and many other control systems used in industry and technology. They are:

(i) OR gate

(ii) AND gate

(iii) ` NOT gate

(iv) NOR gate

A. OR gate:

A gate that develops output when the signal is applied to any one or all inputs is called an **OR gate**. The electrical analogue of an OR gate is given below in fig. 22.1(a) and 22.1(b).:

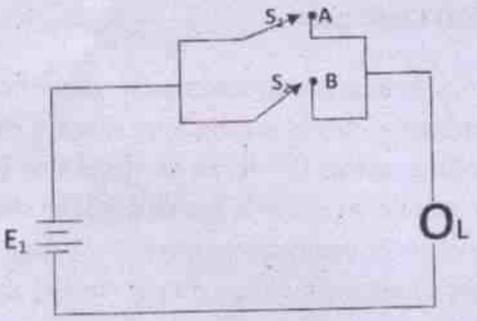


Fig. 22.1(a)

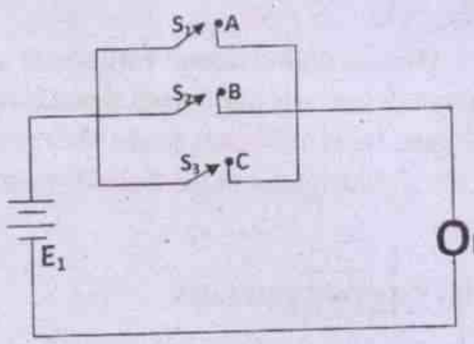


Fig. 22.1(b)

Here the switches S_1, S_2 and S_1, S_2 S_3 are in parallel. The lamp L will glow when any one or all the switches are closed. Stated logically, it says that **the output statement, 'Y' will be true (lamp will glow)** if either statement A, or statement B **(in Fig. 22.1(a))** is true (closed) or all the statements A and B are true (closed). Similarly if either statement A, or statement B or statement C (**in Fig. 22.1(b)**)is true (closed) or all the statements A,B and C are true (closed) the lamp will glow. In mathematical form OR function is written as

$$A + B = Y \qquad \textbf{22.7.1(a)}$$

$$A + B + C = Y \qquad \textbf{22.7.1(b)}$$

i.e. the laws for OR gate are the laws of addition, which are (i) A + 0 = A (ii) A + 1 = 1 (iii) A + A = A and (iv) A + A = 1(where A is the complement of A). The Truth Tables with 2- inputs and with 3-inputs are as given in Table Nos. 22.3 & 22.4 respectively.

Table No. 22.3

Input		Output
A	**B**	**Y**
0	0	0
1	0	1
0	1	1
1	1	1

Table No. 22.4

Inputs			Output
A	**B**	**C**	**Y**
0	0	0	0
0	0	1	1
0	1	0	1
1	0	0	1
1	1	0	1
0	1	1	1
1	0	1	1
1	1	1	1

The OR gate circuit symbol is given below in Fig. 22.2(a) & Fig. 22.2(b)

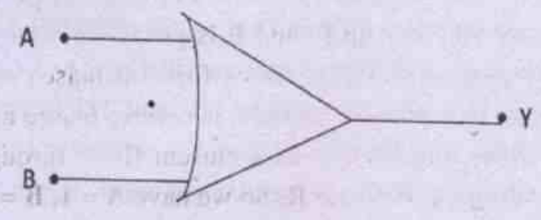


Fig. 22.2(a)

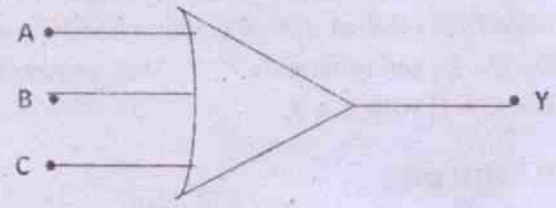


Fig. 22.2(b)

A.1 OR gate Electronic circuit:

A 1(a). Two-input OR gate:

A diode acts as a closed switch (ON) when conducting (Forward biased) and open switch (OFF) when non-conducting (Reversed biased). Hence it can be used to perform logical operation. The circuit is as shown in Fig. 22.3 below.

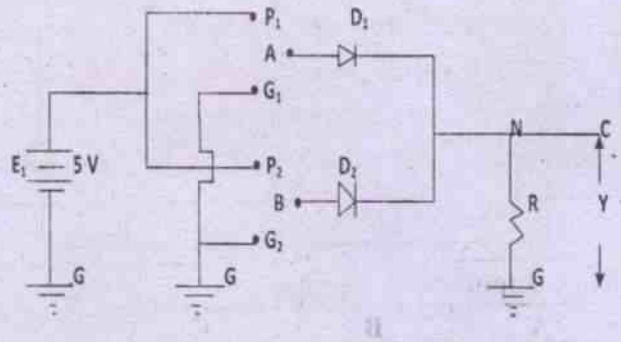


Fig. 22.3

The functioning of the circuit is as described below:

(i) A and B are grounded through G_1 and G_2 respectively, so that both the diodes are reverse biased and do not conduct. Hence the output voltage across the register R is zero. Thus we have **A =0, B = 0, with Y = 0.**

(ii) A is connected to the +ve terminal of the battery E_1 through P_1 and B is grounded through G_2. As a result diode D_1 is forward biased and conducts; whereas diode D_2 is reverse biased and does not conduct. Thus a current flows through diode D_1 and resistor R. So we have **A = 1, B = 0, with Y = 1.**

(iii) A is grounded though G_1 and B is connected to the +ve terminal of battery E_1 through P_2. As a result diode D_2 is forward biased and conducts; whereas diode D_1 is reverse biased and does not conduct. Thus a current flows through diode D_2 and resistor R. So we have **A = 0, B =1, with Y = 1.**

(iv) A and B are both connected to the +ve terminal of battery E_1 though P_1 and P_2 respectively. As a result both the diodes D_1 and D_2 are forward biased and conduct. Hence current flows through diodes D_1, D_2 and resistor R. So we have **A = 1, B = 1, with Y = 1.**

A.1 (b) Three-input OR gate:

We describe here a three-input OR gate in short as the steps shall be similar to above.

(i) A, B, and C all are grounded through G_1, G_2, G_3 respectively, so that diodes D_1, D_2, D_3 are all reverse biased and no current flows through D_1, D_2, D_3 and resistor R. Hence A = 0, B = 0, C = 0, with Y = 0.

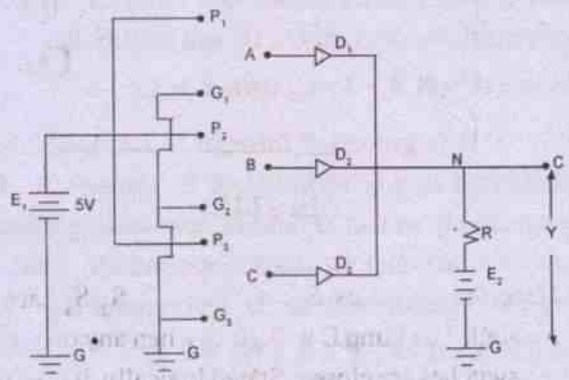


Fig. 22.4

(i) A and B are grounded through G_1, G_2 respectively, C is connected to +ve terminal of E_1 through P_3, so that D_3 is forward biased and conduct and D_1, D_2 do not conduct. Thus a current flows through D_3 and resistor R

Hence **A = 0 = B, C = 1, with Y = 1.**

(ii) A and C are grounded through G_1, G_3 respectively, B is connected to +ve terminal of E_1 through P_2, so that D_2 is forward biased and conduct and D_1, D_3 do not conduct. Thus a current flows through D_2 and resistor R.

Hence **A = 0 = C, B = 1, with Y = 1**.

(iii) B and C are grounded through G_2, G_3 respectively, A is connected to +ve terminal of E_1 through P_1, so that D_1 is forward biased and conduct and D_2, D_3 do not conduct. Thus a current flows through D_1 and resistor R.

Hence **A = 1, B = 0 =C, with Y = 1.**

(iv) C is grounded through G_3, A and B are connected to +ve terminal of E_1 through P_1, P_2; so that D_1 and D_2 are forward biased and conduct and D_3 does not conduct. Thus a current flows through D_1, D_2 and resistor R.

Hence **A = 1 = B, C = 0, with Y = 1**.

(v) A is grounded through G_1, B and C are connected to +ve terminal of E_1 through P_2 and P_3 respectively, so that D_2 and D_3 are forward biased and conduct and D_1 does not conduct. Thus a current flows through D_2, D_3 and resistor R.

Hence **A = 0, B = 1 =C, with Y = 1.**

(vi) B is grounded through G_2, A and C are connected to +ve terminal of E_1 through P_1, P_3 respectively so that D_1 and D_3 are forward biased and conduct and D_2 does not conduct. Thus a current flows through D_1, D_3 and resistor R.

Hence **A = 1 =C, B = 0, with Y = 1.**

(vii) A, B, and C all are connected to the +ve terminal of the battery through P_1, P_2, and P_3 respectively; so that D_1, D_2 and D_3 are all forward biased and conduct. Hence a current flows through D_1, D_2, D_3 and resistor R. Thus we have **A = B = C = 1, with Y = 1.**

B. AND gate:

A gate that develops output when signals are applied to all the inputs simultaneously is called an **AND** gate. Thus the AND gate is a logic gate that has two or more inputs but has only one output. Its output is low or '0' when any one of the input or all the inputs are low or '0' and the output is high or '1' when all the inputs are high or '1'. The electrical analogue of an AND gate is as given in Fig. 22. 5.

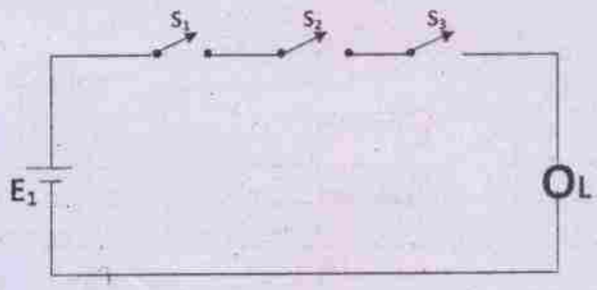


Fig. 22.5

Here a lamp L is connected in series with a source of emf and three switches S_1, S_2 and S_3. For our analysis the lamp L may be considered as the output (Y) and the three switches as the inputs. When all the inputs are applied (i.e. S_1, S_2 and S_3 are closed output is available (i.e. current in L)). So the AND function stated logically, "**statement Y will be true (i.e. lamp will glow) only if each of the statements A, B, C (i.e. switches S_1, S_2 and S_3 respectively) is true (i.e. closed)** ". Mathematically AND function is stated as:

$$A.\,B.\,C\ ....= Y \qquad 22.7.2$$

The dot placed between input symbols denotes the AND gate. More specifically **AND laws are laws of multiplication, which are (i) A.0 = 0 (ii) A.1 = A (iii) A.A = A and (iv) A. $\overline{A}$ = 0,(where $\overline{A}$ is the complement of A).** The truth table with two inputs and three inputs are given below in Table No. 22.5 & 22.6., with **'1' and '0' representing true and false statements respectively**

Table No. 22.5

Inputs		Output
A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

Table No. 22.6

Inputs			Output
A	B	C	Y
0	0	0	0
1	0	0	0
0	1	0	0
0	1	1	0
0	0	1	0
1	1	0	0
1	0	1	0
1	1	1	1

The AND gate circuit symbol is given below in Fig. No. 22.6 and 22.7 for 2-inputs and 3-inputs respectively.

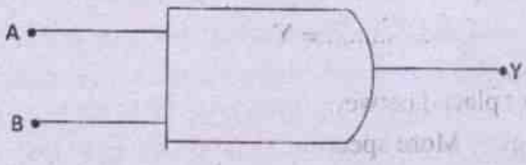


Fig. 22.6

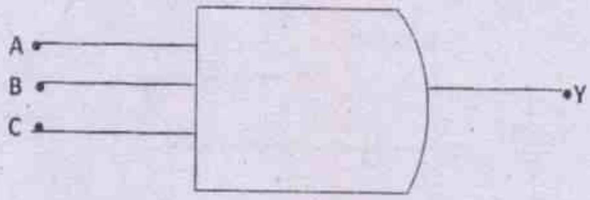


Fig.No.22.7

B.1 AND gate electronic circuit:

B.1 (i) Two-Input AND gate electronic circuit:

A diode acts as a closed switch (ON) when conducting (Forward biased) and open switch (OFF) when non-conducting (Reversed biased). Hence it can be used to perform logical operation. The circuit is shown below in Fig. 22.8. The input voltages are A and B, while the output voltage is Y. The negative terminals of the batteries are grounded and correspond to'0'state. The +ve terminal corresponds to '1' state (high). There are four input and output possibilities. The output Y is taken across the load R.

(a) A and B are grounded through G_1 and G_2 respectively, so that both the diodes are forward biased and conduct. As a result the supply voltage of +5V (E_2) drops across R. Consequently point N and C are driven to '0' volt. Therefore the output Y is zero. Thus we have **A = 0 =B, with Y = 0.**

(b) A is connected to the +ve terminal of battery E_1 through P_1 and B is grounded through G_2. As a result D_1 is reverse biased and does not conduct but D_2 is forward biased and conducts. Hence the supply voltage +5V (E_2) drops across R. Consequently points N and C are driven to '0' volt. Therefore output Y = 0. Thus we have **A = 1, B = 0, with Y = 0.**

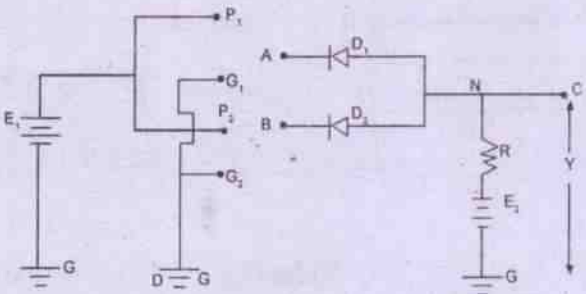


Fig. 22.8

(c) B is connected to the +ve terminal of battery E_1 through P_2 and A is grounded through G_1. As a result D_2 is reverse biased and does not conduct but D_1 is forward biased and conducts. Hence the supply voltage +5V (E_2) drops across R. . Consequently points N and C are driven to '0' volt. Therefore output Y = 0. Thus we have **A = 0, B = 1, with Y = 0.**

(d) A and B are connected to +ve terminal of battery E_1 through P_1 and P_2 respectively, so that both the diodes are reverse biased and do not conduct. As a result no current flows through D_1, D_2 and resistor R. Thus the output voltage at C is the +5V of battery E_2. Thus we have **A = 1, B = 1, with Y = 1.**

B.1(ii) Three-input AND gate:

The circuit is as shown in Fig.22.9. The input voltages are A, B and C while the output is Y.

(a) A, B, and C are grounded through G_1, G_2, and G_3 respectively, so that the diodes D_1, D_2 and D_3 are forward biased. Current flows through D_1, D_2, D_3 and resistor R. As a result supply voltage +5V (E_2) develops across R. So the point N and C are driven to '0'. Therefore Y = 0. Thus we have **A = 0 = B = C, with Y = 0.**

(b) B and C are grounded through G_2, and G_3 respectively and A is connected to +ve terminal of the battery E_1 through P_1, so that D_1 is reverse biased and does not conduct but , D_2 and D_3 are forward biased and conduct current. Consequently points N and C are driven to '0' volt. Therefore output Y = 0. Thus we have **A = 1, B = 0 = C, with Y = 0.**

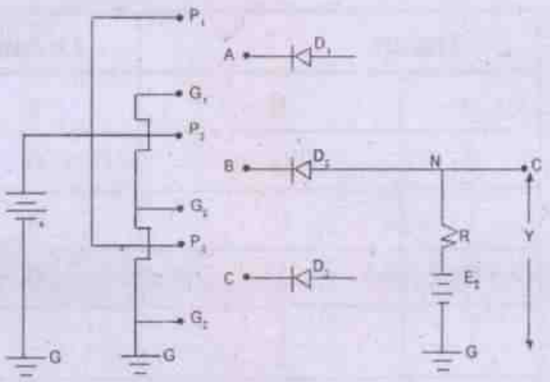


Fig. 22.9

(c) A and C are grounded through G_1, and G_3 respectively and B is connected to +ve terminal of the battery E_1 through P_2, so that D_2 is reverse biased and does not conduct but , D_1 and D_3 are forward biased and conduct current. Consequently points N and C are driven to '0' volt. Therefore output Y = 0. Thus we have **A = 0 = C, B = 1, with Y = 0.**

(d) A is grounded through G_1 and B and C are connected to +ve terminal of the battery E_1 through P_2 and P_3 respectively, so that D_2 and D_3 are reverse biased and do not conduct but , D_1 is forward biased and conducts current. Consequently points N and C are driven to '0' volt. Therefore output Y = 0. Thus we have **A = 0, B = 1 = C, with Y = 0.**

(e) A and B are grounded through G_1, and G_2 respectively and C is connected to +ve terminal of the battery E_1 through P_3, so that D_3 is reverse biased and does not conduct but , D_1 and D_2 are forward biased and conduct current. Consequently points N and C are driven to '0' volt. Therefore output Y = 0. Thus we have **A = 0 = B, C = 1, with Y = 0.**

(f) C is grounded through G_3, and A and B are connected to +ve terminal of the battery E_1 through P_1 and P_2 respectively, so that D_1 D_2 are reverse biased and do not conduct but , D_3 is forward biased and conduct current. Consequently points N and C are driven to '0' volt. Therefore output Y = 0. Thus we have **A = 1 = B, C = 0, with Y = 0.**

(g) B is grounded through G_2 and A and C are connected to +ve terminal of the battery E_1 through P_1 and P_3 respectively, so that D_1 and D_3 are reverse biased and do not conduct but , D_2 is forward biased and conducts current. Consequently points N and C are driven to '0' volt. Therefore output Y = 0. Thus we have **A =1 = C, B = 0, with Y = 0.**

(h) A, B, and C are all connected to +ve terminal of the battery E_1 through P_1, P_2, and P_3 respectively, so that the diodes D_1, D_2 and D_3 are reverse biased. No current flows through D_1, D_2, D_3 and resistor R. As a result output voltage at c is +5V of the battery (E_2). Therefore Y = 1. Thus we have **A =1 =B = C, with Y = 1.**

C. NOT gate:

The circuit that gives the output when input signal is not applied or the output is not present with the input signal applied, is called a NOT gate. Thus the NOT gate has only one input and one output.

The **NOT** gate inverts or complements the input as its output is just opposite of the input. Sometimes **NOT** gate is called **inverter.** Thus the **NOT** gate has output **low or '0'** when the input is **high or '1'**; and output is **high or '1'** when the input is **low or '0'**.

The NOT operation is represented by a bar i.e. $\overline{A}$ (complement of A) is output when A is the input. Mathematically we represent the statement with output (Y) and input (A) as:

$$Y = \overline{A} = \text{NOT } A \qquad \text{———} \qquad 22.7.3$$

This is called Boolean expression.

Thus if, $A = 0$, then $Y = \overline{0} = 1$

$A = 1$, then $Y = \overline{1} = 0$

Hence the **Truth table** for the **NOT gate** is given in table No.22.7

Table No. 22.7

Input A	Output Y
0	1
1	0

The circuit symbol for not gate is given in Fig. 22.10 below:

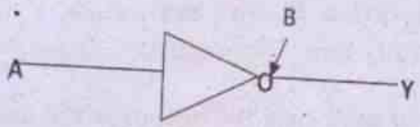


Fig.22.10

The bubble **B** denotes inversion, A and Y denote input signal and output respectively.

B.1 NOT gate electronic circuit:

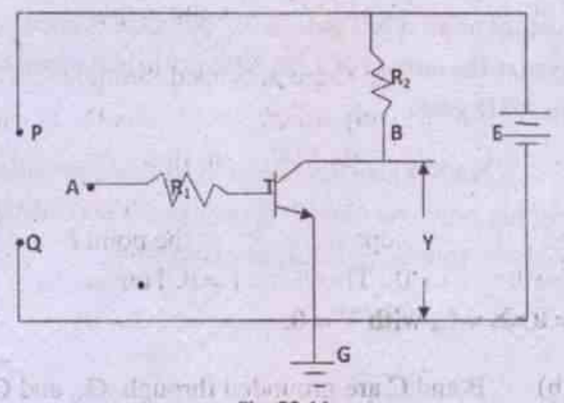


Fig. 22.11

Fig. 22.11 shows a typical NOT gate, where the input voltage is labeled as A and the output voltage as Y. The negative terminal of battery E is grounded and corresponds to '0' state (low). The positive terminal corresponds to '1' state (high). There are two input and output possibilities.

Here the transistor (T) is beyond cutoff.

(a) When no signal is applied at A (connecting A & Q) the base of transistor T becomes negative. The negative potential drives the transistor T to cutoff, so that the transistor does not conduct. As a result collector current is zero. Hence the collector supply voltage V_{cc} = +5V will appear across the output at Y. Thus we have A = 0 and Y = 1.

(b) If a sufficiently high voltage is applied at A (connecting A & P), the transistor is driven to saturation and conducts heavily. The collector current becomes so high that the supply voltage drops across R_2 and the output Y becomes practically zero. Thus we have A = 1 and Y = 0.

Thus in each case the output is the opposite of input.

D. NAND gate:

If the AND gate is followed by a NOT gate, then the combined gate is a NOT AND = NAND gate. Thus by connecting a NOT gate at the output of an AND gate, a NAND gate is created. Hence the output of a NAND gate is just opposite to AND gate.

A **NAND** gate has two or more than two inputs but has only one output. It operates in accordance with the following Boolean expression.

$$Y = \overline{A.B} \qquad 22.7.4(a)$$

$$Y = \overline{A.B.C} \qquad 22.7.4(b)$$

Where the dot between the symbols represents AND gate and the bar over the product gives the complement, which is NOT gate. It is also necessary to state the De Morgan's theorem, which we shall very often use.

De Morgan's Theorem:

Theorem 1: $\overline{A.B.C. .... N} = \bar{A} + \bar{B} + \bar{C} + ... + \bar{N}$

22.7.5(a)

Theorem 2: $\overline{A + B + C + ... N} = \bar{A}.\ \bar{B}.\ \bar{C} ....\bar{N}$

22.7.5(b)

Eqn. 22.7.4 leads to the following Truth tables (Table 22.8 & Table 22.9 respectively for 2-inputs and 3-inputs) for **NAND** gate.

Table 22.8

Inputs		AND output	NAND output
A	B	A.B	$Y=\overline{A.B}$
0	0	0	1
1	0	0	1
0	1	0	1
1	1	1	0

Table 22.9

Inputs			AND output	NAND output
A	B	C	A.B.C	$\overline{A.B.C}$
0	0	0	0	1
0	1	0	0	1
0	0	1	0	1
0	1	1	0	1
1	0	0	0	1
1	1	0	0	1
1	0	1	0	1
1	1	1	1	0

This shows that the output Y ='1' (high) whenever one or all the inputs are'0' (low) and the output Y = '0' (low) whenever all the inputs are '1' (high). The circuit symbol for the NAND gate is as given in Fig.22.12.

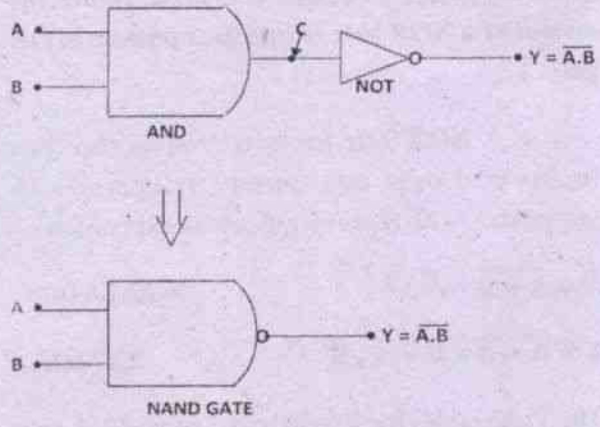


Fig. 22.12

D.1 NAND gate as Universal gate:

NAND gate is called a universal gate as its repeated use can produce other logic gates, as discussed below.

(i) NAND gate as NOT gate:

When two input NAND gates are joined together so that it has one input, the NAND gate becomes a NOT gate. This is discussed below.

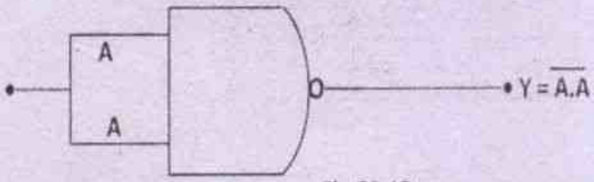


Fig.22.13

The Truth table corresponding to the circuit is as given in Table No. 22.10

Table 22.1o

Inputs		NAND output
A	B = A	$\overline{A.B}$
0	0	1
1	1	0

(ii) NAND gate as AND gate:

For this purpose two NAND gates are used as shown below in Fig. 22. 14

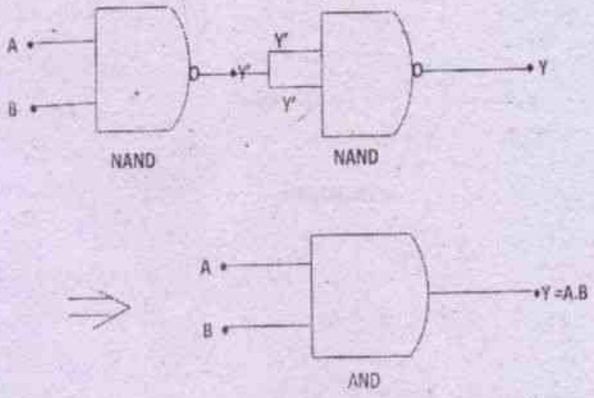


Fig. 22.14

The corresponding Truth table is as given below in Table No. 22.11

Table No. 22.11

Inputs		And output	NAND output	Final output
A	B	Y=A.B	Y'=$\overline{A.B}$	Y= $\overline{Y'.Y'}$ = $\overline{Y'}$
0	0	0	1	0
1	0	0	1	0
0	1	0	1	0
1	1	1	0	1

Truth combining two NAND gates obtain AND gate.

(i) NAND gate as Or gate:

For this purpose 3 NAND gates are used as shown in Fig. 22.15.

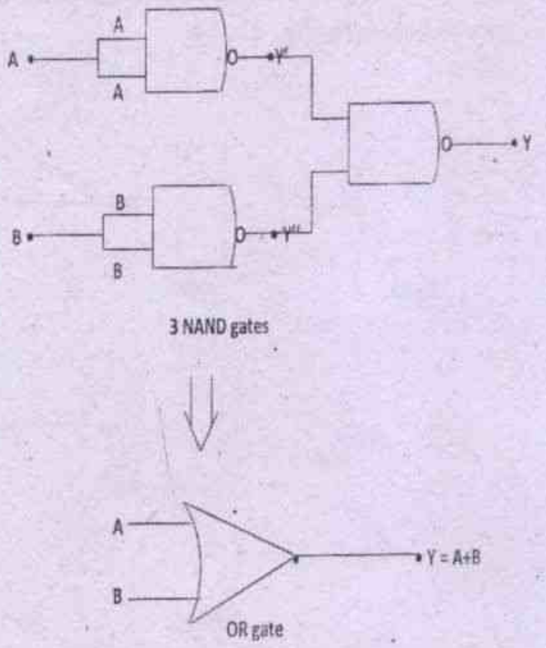


Fig. 22.15 : Conversion of 3 NAND gates to OR gate

Here $Y' = \overline{A.A} = \overline{A}$, $Y'' = \overline{B.B} = \overline{B}$

Then $Y = \overline{Y'.Y''} = \overline{A} \cdot \overline{B} = \overline{\overline{A+B}} = A + B$ **(Using De. Morgan theorem)**

Here the first two NAND gates are operated as NOT gates. The two outputs are fed to the 3rd NAND gate whose output is as of OR gate. The Truth table is given in Table No. 22.12.

Table No. 22.12

Inputs		$Y' = \overline{A}$	$Y'' = \overline{B}$	$Y = \overline{Y'.Y''}$ = A+B
A	B	Y'	Y''	Y
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

E. NOR gate:

If an OR gate is followed by a NOT gate, then combined gate will be a NOT $\rightarrow$ R = NOR gate. Thus by connecting a NOT gate at the output of an OR gate a NOR gate is created. **Hence the output of a NOR gate is just the opposite of OR gate.**

A **NOR** gate has two or more than two inputs and only one output. It operates in accordance with following Boolean expression.

$$Y = \overline{A+B} = \overline{A}.\overline{B} \qquad \textbf{22.7.6 (a)}$$

$$Y = \overline{A+B+C} = \overline{A}.\overline{B}.\overline{C} \qquad \textbf{22.7.6(b)}$$

The Truth tables for 2-input and 3-input NOR gate are given Table Nos. 22.13 & 22.14 respectively.

Table No. 22.13

A	B	A+B	$Y=\overline{A+B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

Table No. 22.14

A	B	C	A+B+C	$Y=\overline{A+B+C}$
0	0	0	0	1
0	1	0	1	0
0	0	1	1	0
0	1	1	1	0
1	0	0	1	0
1	0	1	1	0
1	1	0	1	0
1	1	1	1	0

The circuit symbol for NOR gate is as shown in Fig. 22.16.

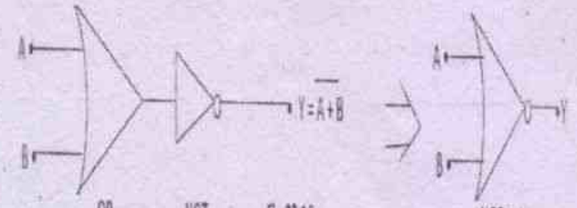


Fig.22.16

E 1. NOR gate as Universal gate:

(i) NOR gate as NOT gate:

When two inputs NOR gates are joined together so that it has one input then the NOR gate becomes NOT gate. The circuit symbol for this is given in Fig. 22. 17.

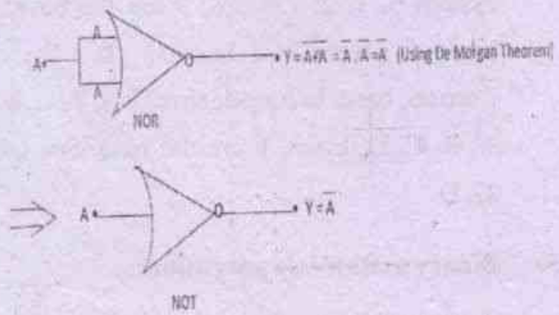


Fig. 22.17 (conversion of NOR gate to NOT gate)

The Truth table for the above is as given in Table No. 22.15.

Table No. 22.15

A	B=A	A+A	$Y=\overline{A+A}=\overline{A}$
0	0	0	1
1	1	1	0

(iii) NOR gate as OR gate:

For this purpose, two NOR gates are used as per the circuit symbol, shown in Fig. 22.18.

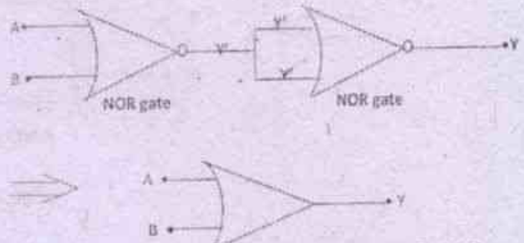


Fig. 22.18

Here $Y' = \overline{A+B}$**, and** $Y = \overline{Y'+Y'} = \overline{Y'} \cdot \overline{Y'}$

$= \overline{\overline{A+B}} = A + B$ **(using De Morgan Theorem)**

Here the output of first **NOR** gate is fed to second **NOR** gate whose inputs are joined so that it can act as a NOT gate. The resulting output is the output of an OR gate. The Truth Table for the above is given in Table No. 22.16.

Table No. 22.16

A	B	A + B	$Y'=\overline{A+B}$	$Y=\overline{Y'}=A+B$
0	0	0	1	0
0	1	1	0	1
1	0	1	0	1
1	1	1	0	1

The Truth Table above shows how combining two NOR gates one can obtain an OR gate.

(iv) NOR gate as AND gate:

For this purpose 3-NOR gates are used as shown in Fig. 22.19 (circuit symbol).

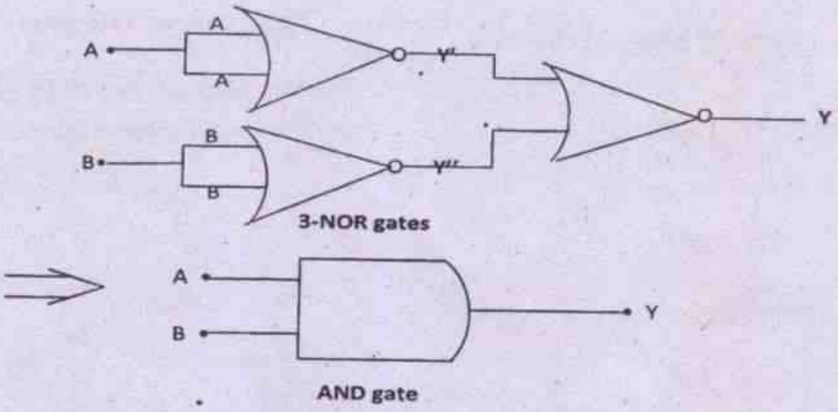


Fig. 22.19

Here $Y' = \overline{A+A} = \overline{A} \cdot \overline{A} = \overline{A}$ and Y'' $= \overline{B+B} = \overline{B} \cdot \overline{B} = \overline{B}$ (using De Morgan Theorem)

Then $Y = \overline{Y'+Y''} = \overline{A} + \overline{B} = \overline{A} \cdot \overline{B} = \overline{Y'} \cdot \overline{Y''}$ (using De Morgan theorem)

Here the first two NOR gates are operated as NOT gate. The outputs are fed to the third NOR gate whose output is as in AND gate. The Truth Table for the above is given in Table No. 22.17

A	B	$Y' = \overline{A}$	$Y'' = \overline{B}$	$Y = \overline{Y'+Y''} = A.B$
0	0	1	1	0
0	1	1	0	0
1	0	0	1	0
1	1	0	0	1

SUMMARY

1. **Digital circuit:** An electronic circuit that handles only two state (0, 1) operation of the digital signal is called a **digital circuit.**

2. **Digital electronics:** The branch of electronics that deals with digital circuit is known as **digital electronics.**

3. **Number system:** There are four types of number systems in common use. They are (i) Decimal (ii) Binary (iii) Octal (iv) Hexadecimal system

4. **Base or Radix:** It is defined as the number of different digits which can occur in each position in the number system. In decimal system 0, 1, 2, ...8, 9 are the base. In binary 0.1 is the base. In octal 0, 1, 2, ...6, 7 are the base. In hexadecimal 0,1, 2,.....8, 9, A, B, C, D, E and F are the base.(See table 22.1)

5. **Binary arithmetic operation:**

(a) **Binary Addition:** $0 + 0 = 0, 0 + 1 = 1, 1 + 0 = 1, 1 + 1 = (10)_2$

(b) **Binary Subtraction:** $0 - 0 = 0, 1 - 0 = 1, 1 - 1 = 0, 0 - 1 = 1\ 0 - 1 = 01$ **or 1**

(c) **Binary Multiplication:** $0 \times 0 = 0, 0 \times 1 = 0, 1 \times 0 = 0, 1 \times 1 = 1$

(d) **Binary Division:** $0 \div 1 = 0, 1 \div 1 = 1$

6. **Logic:** It is a process of establishing a proposition by reasoning or arguments.

7. **Boolean Algebra:** It deals with variables that assume only two possible values '0' and '1'.

8. **Gate:** A switch that can be opened or closed is known as a gate.

9. **Logic gate:** A digital circuit with one or more input signals but only one output signal is called a logic gate.

10. **OR gate:** OR gate is an addition gate. A gate that develops output (Y) when the signal is applied at any one or all inputs is called an OR gate. For OR gate $A + B + C + \ldots\ldots = Y$.

11. **AND gate:** And gate is a multiplication gate. A gate that develops output (Y) when signals are applied to all the inputs is called an AND gate. For AND gate $A . B . C \ldots\ldots\ldots\ldots = Y$.

12. **NOT gate:** NOT gate is an inversion gate. The circuit that gives the output (Y) when input signal is not applied or the output is not present with the input signal applied is called a NOT gate. For NOT gate $Y = \overline{A}$

13. **NAND gate:** NAND gate is complement of AND gate. If the AND gate is followed by a NOT gate, then the combined gate is a NAND gate. For NAND gate $Y = A . B$.

14. **NOR gate:** NOR gate is complement of OR gate. If an OR gate is followed by a NOT gate, then the combined gate is a NOR gate. For NOR gate $Y = \overline{A + B} = \overline{A} \cdot \overline{B}$ ·

15. **DeMorgan's theorems:**

(1) $\overline{A + B + C + \ldots\ldots\ldots\ldots} = \overline{A} \cdot \overline{B} \cdot \overline{C}$

(2) $\overline{A . B . C \ldots\ldots\ldots\ldots} = \overline{A} + \overline{B} + \overline{C}$

MODEL QUESTIONS

A. Multiple Choice Type Questions :

1. The equivalent decimal number of binary number $(11001.001)_2$ is

(a) 27.125 (b) 25.125

(c) 25.135 (d) 27.135

2. The equivalent binary number of the decimal number $(9.25)_{10}$ is

(a) $(1011.01)_2$ (b) $(1101.11)_2$

(c) $(1001.01)_2$ (d) $(111.01)_2$

3. The binary number $(10101)_2$ is equivalent to decimal number

(a) 12 (b) 19

(c) 27 (d) 21

4. The decimal number 49 is equivalent to binary number

(a) $(110001)_2$ (b) $(100011)_2$

(c) $(100001)_2$ (d) $(110111)_2$

5. When $(110011)_2$ is added to $(10110)_2$ one obtains

(a) $(1001001)_2$ (b) $(1110001)_2$
(c) $(1100000)_2$ (d) $(1100100)_2$

6. When $(01011)_2$ is subtracted from $(10110)_2$ one obtains

(a) $(01011)_2$ (b) $(11011)_2$

(c) $(01010)_2$ (d) $(01001)_2$

7. Adding $(1111)_2$, $(111)_2$ and $(1111)_2$ one obtains

(a) $(1100011)_2$ (b) $(100101)_2$

(c) $(1001001)_2$ (d) $(1100101)_2$

8. The digital system operates on

(a) Octal (b) decimal

(c) binary (d) hexadecimal

9. For positional value the binary system one uses power of

(a) 10 (b) 8

(c) 16 (d) 2

10. The binary addition 1 + 1 +1 gives

(a) 111 (b) 10

(c) 11 (d) 110

11. Multiplication of binary number $(10101)_2$ with $(101)_2$ gives

(a) $(1111001)_2$ (b) $(1101001)_2$

(c) $(1110011)_2$ (d) $(1100011)_2$

12. A logic gate is an electronic circuit which

(a) makes logic decisions
(b) works on binary algebra

(c) conducts only in one direction
(d) alternates between 0 and 1 values

13. A NOT gate

(a) implements logic addition
(b) is an any or all gate

(c) inverts the state
(d) implements the logic multiplication

14. An AND gate,

(a) is equivalent to parallel switching circuit.

(b) is equivalent to series switching circuit.

(c) implements logic addition.

(d) is an any or all gate.

15. The output of a 3-input OR gate is '0' only when its

(a) one of the input is '1'
(b) one of the output is '0'

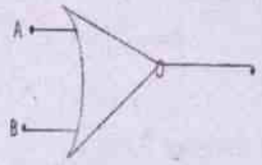
(c) all the inputs are '0'
(d) all the inputs are '1'

16. A NOR gate gives an output when all its inputs are

(a) ON (b) Positive

(c) High (d) Off

17. The circuit symbol below represents



(a) AND gate (b) NAND gate
(c) NOR gate (d) OR gate

18. The truth table below is of
(a) OR gate (b) AND gate

(c) NOR gate (d) NAND gate

Inputs		Output
A	B	Y
0	0	0
1	0	0
0	1	0
1	1	1

19. The gate which is building block of all gates is

(a) OR gate (b) AND gate

(c) NOT gate (d) NAND gate

20. The Boolean expression for NAND gate is

(a) A + B = Y (b) A . B = Y

(c) A = Y (d) A . B = Y

B. Very Short Answer Type:

1. An OR gate is a _________ switching circuit.

2. The output of 2-input OR gate is zero only when both the inputs are ___________.

3. An AND gate is a ___________ switching circuit.

4. The _________ gate is known as any or all gate.

5. _________ gate is called a universal gate.

6. The states '1' and '0' are required in _________ number system.

7. In the binary multiplication $(11)_2 \times (11)_2$ = _________

8. When all the inputs of AND gate are '1' its output will be ___________.

9. When all the inputs of NAND gate are '1' its output will be ___________

10. When all the inputs of NOR gate are '1' its output will be ___________

C. Short Answer Type:

1. Sketch the symbol and write the truth table of a 3-input NOR gate.

2. Sketch the symbol and write the truth table of a 3-input NAND gate.

3. Convert $(11.11)_2$ into its decimal equivalent.

4. Draw the circuit symbol of a 3-input OR gate.

5. Convert $(11)_{10}$ and $(20)_{10}$ into their binary equivalent and add them.

6. Convert $(0.85)_{10}$ into binary equivalent up to six bits.

7. Convert the given Boolean equation into logic diagram.

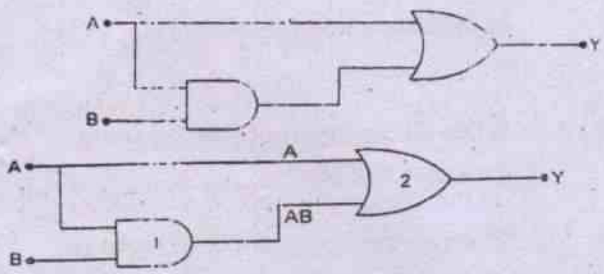
$$\overline{A.B.C} = \overline{A} + \overline{B} + \overline{C}$$

8. Why are NOR and NAND gates called Universal gates.

9. Give the truth table for 3-input OR gate.

10. State and explain AND law with one example.

11. Express by a truth table the output Y for all possible inputs A and B in the following circuit. [CBSE Sample Paper]



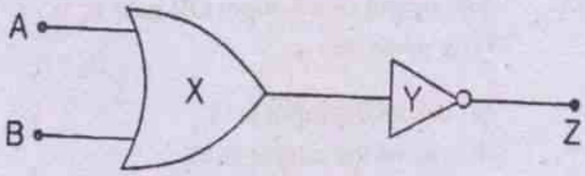
12. A logic gate has been otained by applying the negation (NOT) operation after OR gate. Name the so formed. Write the symbol and truth table of this gate.

[CBSE Sample Paper]

13. Draw a logic circuit digram showing how a NAND gate can be converted to NOT gate. [CBSE 1999]

14. Identify the logic gates marked X and Y in the fig. given below. Write down the output at Z when A = 1, B a and A= B =0.

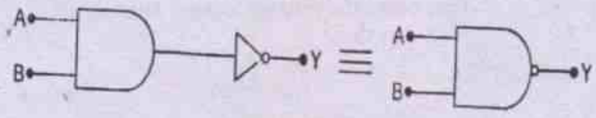
[CBSE 2001]



15. The output of a 2-input AND gate is fed to a NOT gate (as shown below). Write its truth table. [CBSE 2007]

D. **Numerical Problems:**

1. Convert the decimal number 23 to its binary equivalent.



2. Convert the decimal number 27 to its binary equivalent.

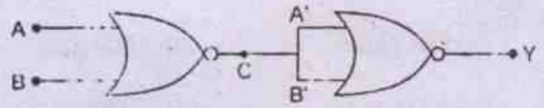
3. Convert the binary number $(10011001)_2$ to its decimal equivalent.

4. Find out the binary number of $(363)_{10}$

5. Find the binary (i) addition and (ii) subtraction of $(101010)_2$ and $(010101)_2$

6. Write the truth table for circuits shown in figure consisting NOR gate and identify the logic operation (OR, AND, NOT) which this circuit is performing.

[CBSE Sample Paper]



E. **Long Answer Type:**

1. What do you understand by logic gate? Discuss, how is an OR gate is realized from its electronic circuit. Give its circuit symbol and truth table.

2. What do you understand by logic gate? Discuss, how is an AND gate is realized from its electronic circuit. Give its circuit symbol and truth table.

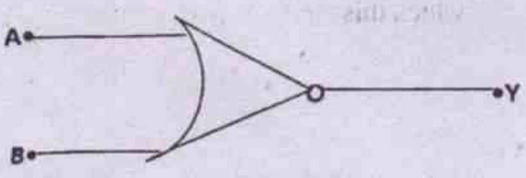
3. What do you understand by logic gate? Discuss, how is an NOT gate is realized from its electronic circuit. Give its circuit symbol and truth table.

4. What do you understand by logic gate? Discuss the formation and working of a NAND gate with circuit symbol and truth table.

5. What do you understand by logic gate? Discuss the formation and working of a NOR gate with circuit symbol and truth table.

F. Correct the following sentences:

1. An OR gate is a series switching circuit.
2. An AND gate is a parallel switching circuit.
3. The states '1' and '0' only are required in decimal number system.
4. The AND gate is called a universal gate.
5. $(23)_{10} = (10011)_2$
6. The circuit symbol below represents OR gate.



7. In binary system complement of 0 is 0.

G. True-False Type:

1. The binary equivalent of decimam number $(10.25)_{10}$ is $(1010.01)_2$.
2. A logic gate is an electronic circuit which makes logic decisions.
3. An AND gate is equivalent to parallel switching circuit.
4. An OR gate is equivalent to series switching circuit.
5. A NOT gate implements logic addition.

H. Fill-in-Blank Type

1. An OR gate is a _______ switching circuit.
2. The output of 2-input OR gate is zero only when both the inputs are _______.
3. An AND gate is a _______ switching circuit.
4. The _______ gate is known as any or all gate.
5. _______ gate is called a universal gate.
6. The states '1' and '0' are required in _______ number system.
7. In the bynary multiplication $(11)_2 \times (11)_2$ = _______.
8. When all the inputs of AND gate are '1' its output will be _______.
9. When all the inputs of NAND gate are '1' its output will be _______.
10. When all the inputs of NOR gate are '1' its output will be _______.
11. Boolean expression for NAND gate is _______.
12. A NOR gate gives output when all the inputs are _______.
13. The output of a 2-input Or gate is '0' when all the inputs are _______.
14. The bynary equivalent of $(11)_{10}$ is _______.
15. The decimal equivalent of $(10100)_2$ is _______.

ANSWERS

A. 1. (b) 2. (c) 3. (d) 4. (a) 5. (a) 6. (a) 7. (b) 8. (c) 9. (d) 10. (c) 11. (b) 12. (a) 13. (c) 14. (b) 15. (c) 16. (d) 17. (c) 18. (b) 19. (d) 20. (d)

B. 1. Parallel. 2. Zero. 3. Series. 4. OR. 5. NAND/NOR. 6. Binary. 7. $(1001)_2$ 8. One. 9. Zero. 10. Zero.

D. 1. $(10111)_2$ 2. $(11011)_2$ 3. 153

4. $(101101011)_2$ 5. (i) $(111111)_2$

(ii) $(010101)_2$

F. 1. True 2. True 3. False 4. false 5. False

G. Parallel 2. '0' 3. Series 4. OR 5. NAND/NOR 6. Binary 7. (1001)2 8. '1' 9. '0' 10. '0' 11. $\overline{A.B}$ =Y 12. Off 13. '0' 14. $(1011)_2$ 15.$(20)_{10}$

I
LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170						5	9	13	17	21	26	30	34	38
						0212	0253	0294	0334	0374	4	8	12	16	20	24	28	32	36
11	0414	0453	0492	0531	0569						4	8	12	16	20	23	27	31	35
						0607	0645	0682	0719	0755	4	7	11	15	18	22	26	29	33
12	0792	0828	0864	0899	0934						3	7	11	14	18	21	25	28	32
						0969	1004	1038	1072	1106	3	7	10	14	17	20	24	27	31
13	1139	1173	1206	1239	1271						3	6	10	13	16	19	23	26	29
						1303	1335	1367	1399	1430	3	7	10	13	16	19	22	25	29
14	1461	1492	1523	1553	1584						3	6	9	12	15	19	22	25	28
						1614	1644	1673	1703	1732	3	6	9	12	14	17	20	23	26
15	1761	1790	1818	1847	1875						3	6	9	11	14	17	20	23	26
						1903	1931	1959	1987	2014	3	6	8	11	14	17	19	22	25
16	2041	2068	2095	2122	2148						3	6	8	11	14	16	19	22	24
						2175	2201	2227	2253	2279	3	5	8	10	13	16	18	21	23
17	2304	2330	2355	2380	2405						3	5	8	10	13	15	18	20	23
						2430	2455	2480	2504	2529	3	5	8	10	12	15	17	20	22
18	2553	2577	2601	2625	2648						2	5	7	9	12	14	17	19	21
						2672	2695	2718	2742	2765	2	4	7	9	11	14	16	18	21
19	2788	2810	2833	2856	2878						2	4	7	9	11	13	16	18	20
						2900	2923	2945	2967	2989	2	4	6	8	11	13	15	17	19
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3224	3345	3365	3385	3402	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4320	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	6	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6002	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8

II
LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
50	6990	6993	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	8316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	5
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8476	8476	8482	8488	8494	8500	8501	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	3360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9561	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9998	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4

IV
ANTILOGARITHMS

	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	[illegible]	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	[illegible]	5117	1	2	4	5	6	7	8	9	10
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	12
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	9	11	13	15
.85	7079	7096	[illegible]	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	8	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	16	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	15	17	19
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	10	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20

V
NATURAL SINES

Degrees	0′ 0°.0	6′ 0°.0	12′ 0°.0	18′ 0°.0	24′ 0°.0	30′ 0°.0	36′ 0°.0	42′ 0°.0	48′ 0°.0	54′ 0°.0	1	2	3	4	5
0	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	.0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	.0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	15
5	.0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	.1045	1063	1080	1097	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	.1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	.1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	.1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	.1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	12	14
11	.1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	.2079	2096	2113	2130	2147	2164	2181	2198	2215	2232	3	6	9	11	14
13	.2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	.2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	.2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	.2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	.2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	.3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	.3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	.3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	.3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	.3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	.3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	.4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	.4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	.4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	.4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	.4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	.4848	4863	4879	4894	4909	4924	4939	4955	4970	.4985	3	5	8	10	13
30	.5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	.5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	.5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	.5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	.5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	.5736	5750	5764	5779	5793	5807	5821	5835	5120	5864	2	5	7	10	12
36	.5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	.6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	.6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	.6293	6307	6320	6334	6347	6361	6474	6388	6401	6414	2	4	7	9	11
40	.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	.6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	.6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	.6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	.6947	6959	6972	6984	6697	7009	7022	7034	7046	7059	2	4	6	8	10

VI
NATURAL SINES

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences 1	2	3	4	5
45	.7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	.7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	.7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	.7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	.7547	7558	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	.7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	.7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	.7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	.7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	.8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	.8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	.8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	.8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	.8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	.8572	8582	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	.8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	.9746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	.8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	.8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	.8988	8996	9903	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	.9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	.9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	.9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	.9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	.9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	.9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	.9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	.9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	.9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	.9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	.9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	.9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	.9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	.9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	.9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	.9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	.9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	.9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	.9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	.9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	.9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	.9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	.9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	.9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	.9998	9999	9999	9999	9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0
90	1.000														

VII
NATURAL COSINES

(Numbers in difference columns to be subtracted, not added)

Degrees	0′ 0°.0	6′ 0°.0	12′ 0°.0	18′ 0°.0	24′ 0°.0	30′ 0°.0	36′ 0°.0	42′ 0°.0	48′ 0°.0	54′ 0°.0	Mean Difference 1	2	3	4	5
0	1.000	1.000	1.000	1.000	1.000	1.000	.9999	.9999	.9999	.9999	0	0	0	0	0
1	.9998	9998	9998	9997	9997	9997	9996	9996	9995	9995	0	0	0	0	0
2	.9994	9993	9993	9992	9991	9990	9990	9989	9988	9987	0	0	0	1	1
3	.9986	9985	9984	9983	9982	9981	9980	9979	9978	9977	0	0	1	1	1
4	.9976	9974	9973	9972	9971	9969	9968	9966	9965	9963	0	0	1	1	1
5	.9962	9960	9959	9957	9956	9954	9952	9951	9949	9947	0	1	1	1	2
6	.9945	9943	9942	9940	9938	9936	9934	9932	9930	9928	0	1	1	1	2
7	.9925	9923	9921	9919	9917	9914	9912	9910	9907	9905	0	1	1	2	2
8	.9903	9900	9898	9895	9893	9890	9888	9885	9882	9880	0	1	1	2	2
9	.9877	9874	9871	9869	9866	9863	9860	9857	9854	9851	0	1	1	2	2
10	.9848	9845	9842	9839	9836	9833	9829	9826	9823	9820	1	1	2	2	3
11	.9816	9813	9810	9806	9803	9799	9796	9792	9789	9785	1	1	2	2	3
12	.9781	9778	9774	9770	9767	9763	9759	9755	9751	9748	1	1	2	3	3
13	.9744	9740	9736	9732	9728	9724	9720	9715	9711	9707	1	1	2	3	3
14	.9703	9699	9694	9690	9686	9681	9677	9673	9668	9664	1	1	2	3	4
15	.9659	9655	9650	9646	9641	9636	9632	9627	9622	9617	1	2	2	3	4
16	.9613	9608	9603	9598	9593	9588	9583	9578	9573	9568	1	2	2	3	4
17	.9563	9558	9553	9548	9542	9537	9532	9527	9521	9516	1	2	3	3	4
18	.9511	9505	9500	9494	9489	9483	9478	9472	9466	9461	1	2	3	4	5
19	.9455	9449	9444	9438	9432	9426	9421	9415	9409	9403	1	2	3	4	5
20	.9397	9391	9385	9379	9373	9367	9361	9354	9348	9342	1	2	3	4	5
21	.9336	9330	9323	9317	9311	9304	9298	9291	9285	9278	1	2	3	4	5
22	.9272	9265	9259	9252	9245	9239	9232	9225	9219	9212	1	2	3	4	6
23	.9205	9198	9191	9184	9178	9171	9164	9157	9130	9143	1	2	3	5	6
24	.9135	9128	9121	9114	9107	9100	9092	9085	9078	9070	1	2	4	5	6
25	.9063	9036	9048	9041	9033	9026	9018	9011	9003	8996	1	3	4	5	6
26	.8988	8980	8973	8965	8957	8949	8942	8934	8926	8918	1	3	4	5	6
27	.8910	8902	8894	8886	8878	8870	8862	8854	8846	8838	1	3	4	5	7
28	.8829	8821	8813	8805	8796	8788	8780	8771	8763	8755	1	3	4	6	7
29	.8746	8738	8729	8721	8712	8704	8695	8686	8678	8669	1	3	4	6	7
30	.8660	8652	8643	8634	8625	8616	8607	8199	8500	8581	1	3	4	6	7
31	.8572	8563	8554	8545	8536	8526	8517	8508	8499	8490	2	3	5	6	8
32	.8480	8471	8462	8453	8443	8434	8425	8415	8406	8396	2	3	5	6	8
33	.8387	8377	8368	8358	8348	8339	8329	8320	8310	8300	2	3	5	6	8
34	.8290	8291	8271	8261	8251	8241	8231	8221	8211	8202	2	3	5	7	8
35	.8192	8181	8171	8161	8151	8141	8131	8121	8111	8100	2	3	5	7	8
36	.8090	8080	8070	8059	8049	8039	8028	8118	8007	7907	2	3	5	7	9
37	.7986	7976	7965	7955	7944	7934	7923	7912	7902	7891	2	4	5	7	9
38	.7880	7869	7859	7848	7837	7826	7815	7804	7793	7782	2	4	5	7	9
39	.7771	7760	7749	7738	7727	7716	7705	7694	7683	7672	2	4	6	7	9
40	.7660	7649	7638	7627	7615	7604	7593	7581	7570	7559	2	4	6	8	9
41	.7547	7536	7524	7513	7501	7490	7478	7466	7455	7448	2	4	6	8	10
42	.7431	7420	7408	7396	7385	7373	7361	7349	7337	7325	2	4	6	8	10
43	.7314	7302	7290	7278	7266	7254	7242	7230	7218	7206	2	4	6	8	10
44	.7193	7181	7169	7157	7145	7133	7120	7108	7096	7083	2	4	6	8	10

VIII
NATURAL COSINES

(Numbers in difference columns to be subtracted, not added)

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences 1	2	3	4	5
45	.7071	7059	7046	7034	7022	7009	6997	6984	6972	6959	2	4	6	8	10
46	.6947	6934	6921	6909	6896	6884	6871	6858	6845	6833	2	4	6	8	11
47	.6820	6807	6794	6782	6769	6756	6743	6730	6717	6704	2	4	6	9	11
48	.6691	6678	6665	6652	6639	6626	6613	6600	6587	6574	2	4	7	9	11
49	.6561	6547	6534	6521	6508	6494	6481	6468	6455	6441	2	4	7	9	11
50	.6428	6414	6401	6388	6374	6361	6347	6334	6320	6307	2	4	7	9	11
51	.6293	6280	6266	6252	6239	6225	6211	6198	6184	6170	2	5	7	9	11
52	.6157	6143	6129	6115	6101	6088	6074	6060	6046	6032	2	5	7	9	12
53	.6018	6004	5990	5976	5962	5948	5934	5920	5906	5892	2	5	7	9	12
54	.5878	5864	5850	5835	5821	5807	5793	5779	5764	5750	2	5	7	9	12
55	.5736	5721	5707	5693	5678	5664	5650	5635	5621	5606	2	5	7	10	12
56	.5592	5577	5563	5548	5534	5519	5505	5490	5476	5461	2	5	7	10	12
57	.5446	5432	5417	5402	5388	5373	5358	5344	5329	5314	2	5	7	10	12
58	.3299	5284	5270	5255	5240	5225	5210	5195	5180	5165	2	5	7	10	12
59	.5150	5135	5120	5105	5090	5075	5060	5045	5030	5015	3	5	8	10	13
60	.5000	4985	4970	4955	4939	4924	4909	4894	4879	4863	3	5	8	10	13
61	.4848	4833	4818	4802	4787	4772	4756	4741	4726	4710	3	5	8	10	13
62	.4695	4679	4664	4648	4633	4617	4602	4586	4571	4555	3	5	8	10	13
63	.4540	4524	4509	4493	4478	4462	4446	4431	4415	4399	3	5	8	10	13
64	.4384	4368	4352	4337	4321	4305	4289	4274	4258	4242	3	5	8	11	13
65	.4226	4210	4195	4179	4163	4147	4131	4115	4099	4083	3	5	8	11	13
66	.4067	4051	4035	4019	4003	3987	3971	3955	3939	3923	3	5	8	11	14
67	.3907	3891	3875	3859	3843	3827	3811	3795	3778	3762	3	5	8	11	14
68	.3746	3730	3714	3697	3681	3665	3649	3633	3616	3600	3	5	8	11	14
69	.3584	3567	3551	3535	3518	3502	3486	3469	3453	3437	3	5	8	11	14
70	.3420	3404	3387	3371	3355	3338	3322	3305	3289	3272	3	5	8	11	14
71	.3256	3239	3223	3206	3190	3173	3156	3140	3123	3107	3	6	8	11	14
72	.3090	3074	3057	3040	3024	3007	2990	2974	2957	2940	3	6	8	11	14
73	.2924	2967	2890	2874	2857	2840	2823	2807	2790	2773	3	6	8	11	14
74	.2756	2780	2723	2706	2689	2672	2656	2639	2622	2605	3	6	8	11	14
75	.2588	2571	2554	2538	2521	2504	2487	2470	2453	2436	3	6	8	11	14
76	.2419	2402	2385	2368	2351	2334	2317	2300	2284	2267	3	6	8	11	14
77	.2250	2233	2215	2198	2181	2164	2147	2130	2113	2096	3	6	9	11	14
78	.2079	2062	2045	2028	2011	1994	1977	1959	1942	1925	3	6	9	11	14
79	.1908	1891	1874	1857	1840	1822	1805	1788	1771	1754	3	6	9	11	14
80	.1736	1719	1702	1685	1668	1650	1633	1616	1599	1582	3	6	9	12	14
81	.1564	1547	1530	1513	1495	1478	1461	1444	1426	1409	3	6	9	12	14
82	.1392	1374	1357	1340	1323	1305	1288	1271	1251	1236	3	6	9	12	14
83	.1219	1201	1184	1167	1149	1132	1115	1097	1080	1063	3	6	9	12	14
84	.1045	1028	1011	0993	0976	0958	0941	0924	0906	0889	3	6	9	12	14
85	.0872	0854	0837	0819	0802	0785	0767	0750	0732	0715	3	6	9	12	15
86	.0698	0680	0663	0645	0628	0610	0593	0576	0558	0541	3	6	9	12	15
87	.0523	0506	0448	0471	0454	0436	0419	0401	0384	0366	3	6	9	12	15
88	.0349	0332	0314	0297	0279	0262	0244	0227	0209	0192	3	6	9	12	15
89	.0175	0157	0140	0122	0105	0087	0070	0052	0035	0017	3	6	9	12	15
90	.0000														

IX
NATURAL TANGENTS

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences 1	2	3	4	5
0	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	.0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	.0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	.1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	.1228	1246	1263	1281	1299	1217	1334	1352	1370	1388	3	6	9	12	15
8	.1405	1423	1441	1459	1477	1493	1512	1530	1548	1566	3	6	9	12	15
9	.1384	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	.1944	1962	1980	1988	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	.2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	.2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	.2493	2512	2530	2549	2568	2586	2605	2623	2642	2661	3	6	9	12	16
15	.2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	.2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	.3057	3076	3096	3115	3134	3153	3132	3191	3211	3230	3	6	10	13	16
18	.3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	.3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	.3640	3659	3679	3699	3219	3739	3759	3779	3799	3819	3	7	10	13	17
21	.3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	.4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	.4245	4265	4286	4307	4327	4348	4369	4390	4411	4431	3	7	10	14	17
24	.4452	4473	4494	4515	4536	4457	4578	4599	4621	4642	4	7	11	14	18
25	.4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	.4877	4899	4921	4942	4964	4686	5008	5029	5051	5073	4	7	11	15	18
27	.5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	.5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	.5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	.6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	.6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	.6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	.6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	.7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	.7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	.7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	.8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	.8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	.9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	.9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	.9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

X
NATURAL TANGENTS

Degrees	0′ 0°.0	6′ 0°.1	12′ 0°.2	18′ 0°.3	24′ 0°.4	30′ 0°.5	36′ 0°.6	42′ 0°.7	48′ 0°.8	54′ 0°.9	Mean Differences 1	2	3	4	5
45	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1.0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1.0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1.1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1.1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1.1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1.2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1.2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1.3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1.3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1.4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1.4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1.5399	5458	5517	5577	5687	5697	5757	5818	5880	5941	10	20	30	40	50
58	1.6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1.6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1.7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1.8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1.8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1.9626	9711	9797	9883	9970	2.0057	2.0145	2.0233	2.0323	2.0413	15	29	44	58	73
64	2.0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2.1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2.2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2.3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	53	40	60	79	99
68	2.4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2.6031	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	109
70	2.7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2.9042	9208	9375	9544	9714	9887	3.0061	3.0237	3.0415	3.0595	53	58	87	116	145
72	3.0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3.2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3.4874	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3.7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4.0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4.3315	3662	4015	4374	4737	5107	5483	5864	6252	6646	Mean differences cease to be sufficiently accurate				
78	4.7046	7453	7867	8288	8716	9152	9594	5.0045	5.0504	5.0970					
79	5.1446	1929	2422	2924	3435	3955	4486	5026	5578	6140					
80	5.6713	7297	7894	8502	9124	9758	6.0405	6.1066	6.1742	6.2432					
81	6.3138	3859	4596	5350	6122	6912	7720	8548	9395	7.0264					
82	7.1154	2066	3002	3962	4947	5958	6996	8062	9158	8.0285					
83	8.1443	2636	3863	5126	6427	7769	9152	9.0579	9.2052	9.3572					
84	9.5144	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20					
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95					
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46					
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27					
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08					
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	57.30					
90	00														